ISSN 0956-8549

Time-varying price discovery

By

Gustavo Fruet Dias
Marcelo Fernandes
Cristina Scherrer

FINANCIAL MARKETS GROUP DISCUSSION PAPER NO. 948

March 2026

Time-varying price discovery

Gustavo Fruet Dias*

School of Economics, University of East Anglia and CREATES

Marcelo Fernandes

Sao Paulo School of Economics, FGV

Cristina Mabel Scherrer

Norwich Business School, University of East Anglia and CREATES

This version: July 11, 2022

Abstract: We formulate a price discovery model in which the price discovery measures vary either locally, say, for instance, at intervals of 30 minutes or at a daily frequency. Given the empirical and theoretical evidence that price discovery measures relate to highly persistent fundamentals, we adopt a kernel-based approach that allows parameters to vary smoothly over time. The resulting kernel-based price discovery measures are consistent and asymptotically normal. Empirically, we compute daily estimates of price discovery for 30 stocks in the U.S. over a long time span, from 2007 to 2013. We show that there is significant daily variation in the component share measures with relative market informativeness alternating between the listing and competing exchanges.

JEL classification numbers: G10, G14, C10, C14, C32, C36, C58

Keywords: time-varying coefficient models, kernel estimation, price discovery, high-frequency data

* Corresponding author: 2.27, Registry, University of East Anglia, Norwich Research Park, Norwich, Norfolk, NR4 7TJ, UK. Tel. : +44 (0) 1603 592941. E-mail address: g.fruet-dias@uea.ac.uk.

1 Introduction

Price discovery has attracted a large empirical and methodological attention due to its importance for financial markets. The knowledge of whether and how price discovery changes over time is of high importance for market quality and efficient decision-making purposes. More specifically, the speed and efficiency in which markets incorporate information into prices reflect better capital allocation decisions (Subrahmanyam and Titman, 1999). This makes price discovery one of the most important functions of financial markets from the asset pricing point of view (O'Hara, 1997, 2003; Edmans and Goldstein (2012), 2012).

The price discovery analysis has gained further importance following the strong fragmentation of order flow in the late 2000s. This phenomenon followed regulatory policy changes aimed to increase competition, which allowed new exchanges, trading platforms, and internalizers to compete for the order flow. See, among others, Menkveld (2014, 2016) and O'Hara (2015).[1] In response to the greater competition between the listing exchanges and new entrants, important determinants of the price discovery process such as market share, for instance, are now changing over time. Therefore, the ability of these trading venues to impound information to the efficient price should also vary over time. This paper addresses the empirical and theoretical evidence supporting time variation in the price discovery process by proposing a time-varying price discovery model and showing how to make inference on daily (local) price discovery measures.

The importance of allowing for some sort of time variation in the price discovery process is not novel and it has been highlighted in the literature. The standard approach to capture this time-varying nature of price discovery is to estimate daily vector error correction (VEC) models (Hasbrouck, 2003; Chakravarty, Gulen and Mayhew, 2004; Hansen and Lunde, 2006; Mizrach and Neely, 2008). Based on these daily estimates of the VEC parameters, one can compute different price discovery measures. For example, the two most used price discovery measures are any variant of Hasbrouck's (1995) information share (IS) and the Gonzalo and Granger's (1995)-based component share (CS). Estimating individual daily VECMs essentially boils down to treating the VEC parameters as if they were independent across days. This contradicts the empirical evidence that the price discovery measures are related to highly persistent fundamentals, such as liquidity and volatility (see, for instance, Eun and Sabherwal, 2003; Figuerola-Ferretti and Gonzalo, 2010).

[1] Regulation ATS (alternative trading systems; RegATS) in 2000, and Regulation National Market System (Reg NMS) in 2007 in the U.S., and Markets in Financial Instruments Directive (MiFiDin) in 2007 in Europe promoted the existence of multiple trading venues.

Additionally, the factors understood to promote price discovery (e.g., market design, trading clienteles, and prevalence of high frequency trading) are not likely to change abruptly and hence should contribute to generate highly persistent price discovery measures.

In this paper, we formally address the persistence associated with the price discovery mechanism and formulate a time-varying price discovery model in which the price discovery measures vary either locally, say, for instance, at intervals of 30 minutes or at a daily frequency. From a time-varying Granger representation theorem decomposition, we derive daily (local) CS and IS measures. Next, we acknowledge that the VEC parameters are persistent over time (Hasbrouck, 2003; Eun and Sabherwal, 2003; Mizrach and Neely, 2008; and Frijns, Gilbert and Tourani-Rad, 2015) and then exploit the inter-daily information to obtain better finite-sample performance in the estimation of daily price discovery measures. In particular, we estimate the daily speed-of-adjustment parameters in the VEC using Giraitis, Kapetanios and Yates's (2013) kernel least squares (KLS) estimator. Given its non parametric nature, the KLS estimator allows for either deterministic or stochastic variation of unknown form in the VEC parameters, as opposed to the parametric nature of Ozturk, van der Welv and van Dijk's (2017) state-space approach for price discovery analysis, for instance. We establish the consistency and asymptotic distribution of the kernel least squares estimator and investigate the asymptotic properties of the price discovery measures. We show through an extensive Monte Carlo exercise that it compares favourably to price discovery measures based on daily VECMs. The results indicate that our estimation strategy is able to alleviate most of the noise in the daily VEC estimation and hence offer a more precise picture of the relative informativeness of each market. More specifically, we find that the KLS estimator improves its relative performance to the daily LS estimator as either or both the contemporaneous correlation among markets and the sampling interval increase. For example, in terms of root mean squared error, the KLS estimator improves on daily VEC approach by 56% on average.

Our empirical application examines the daily price informativeness of Arca, Nasdaq and New York Stock Exchange (NYSE) for both Nasdaq- and NYSE-listed stocks from 2007 to 2013. By entertaining such a long time span, we deviate from the current practice of considering at most one year of data in price discovery analyses at the high-frequency level (see, for instance, de Jong and Schotman, 2010; Benos and Sagade, 2016; Hasbrouck, 2018). Using high-frequency midquotes and transaction prices of 30 stocks that differ in terms of listing venues and trading activity, we find statistical evidence that there is indeed significant daily variation in the component share measures with relative market informativeness alternating between the listing venue and the competing exchange. In particular, Elliott and Müller's (2006) test strongly rejects the null hypothesis of time-invariant component share measures against the alternative of persistent time variation for virtually every stock.

We show that our daily estimates of the component share measures are more precise than the least squares estimates from daily VECMs in that the standard error of the former estimator is about three times smaller. When jointly assessing the informational content of midquotes and transaction prices, we find that the former strongly dominates the price discovery process, reinforcing our main set of results. Finally, to better understand the daily variation in the price discovery mechanism, we study the long-run relationship between price discovery and liquidity. The results reveal that the listing venue contribution to the price discovery increases with its relative volume. This is consistent with recent evidence from Brogaard, Brugler and Rösch (2021), who relate lower trading volume together with weaker price discovery to increase in trading costs. More specifically, we find that for virtually every stock, trading volume seems to respond more significantly to deviations from the long-run equilibrium than the price discovery measure.

The remainder of this paper is organized as follows. Section 2 presents the standard price discovery setting and introduces the time-varying price discovery model. Section 3 shows how to estimate price discovery measures accounting for daily (local) stochastic changes in the speed-of-adjustment parameter. Section 4 investigates the finite sample performance of the proposed kernel least squares estimator vis-à-vis the daily VEC model estimation. Section 5 investigates how the price informativeness of NYSE- and Nasdaq-listed stocks changes over time and the long-run relationship between price discovery and volume. Section 6 offers the concluding remarks. Finally, the Appendix collects the proofs.

2 Measuring price discovery

In this section, we present the reduced form price discovery cointegration model and review the derivation of the CS and IS measures. Next, we formally introduce time variation in the speed-of-adjustment parameters and formally derive the daily (local) price discovery measures.

2.1 The setting for price discovery

We consider a setting where a single asset trades in multiple trading venues. Let $P_t = (p_{1,t}, \ldots, p_{M,t})'$ be a $M \times 1$ vector of log-prices with M denoting the number of trading venues. Prices at different venues should not drift apart much, oscillating around the (latent) efficient price, as they refer to the same asset. In econometric terms, P_t is integrated of order one, $I(1)$, and price changes, $\Delta P_t = P_t - P_{t-1}$, are integrated of order zero, $I(0)$. Furthermore, prices at the different trading venues are expected to cointegrate. As these prices refer to the same asset, there are $r = M - 1$ cointegrating relationships, with log-prices sharing the asset's efficient price as the single common stochastic trend. The dynamics of the first differences of P_t can be represented by the VEC model:

$$\Delta P_t = \alpha\beta' P_{t-1} + \sum_{j=1}^{\ell} \Gamma_j \Delta P_{t-j} + \varepsilon_t, \quad t = 1, \ldots, T, \tag{1}$$

where α is a $M \times r$ error correction matrix; β is a $M \times r$ cointegrating matrix; r is the number of cointegrating vectors; Γ_ℓ with $\ell = 1, 2, ..., p$ are the $M \times M$ autoregressive matrix coefficients; ε_t is a zero-mean white noise process with a $M \times M$ non-diagonal covariance matrix Ω, and T is the number of observations. We assume without loss of generality that β is known and takes the form of $\beta = (I_r, -\iota_r)'$, where ι_r denotes a $r \times 1$ unit vector. In turn, α determines how quickly each market reacts to deviations from the long-run equilibrium $\beta' P_t$. In that, α also contains information about the price discovery mechanism. The matrix α reflects the adjustment that each market implements such that their prices do not deviate from the efficient latent price. This means that the closer the α of a given market is to zero, the less it adjusts to the efficient price. In the limit, if all elements in the mth row of α are equal to zero, the price at market m coincides with the efficient price, therefore leading the price discovery (see, for instance, excellent discussion in Johansen, 1995, p. 41). More naturally, however, is to decompose market prices into an $I(1)$ common stochastic trend (efficient price) and a covariance-stationary component to entertain a price discovery measure that relates exclusively with the efficient price (see, for instance, Hasbrouck, 1995; de Jong, 2002).

The Granger representation theorem (GRT) helps to illustrate the usefulness of the parameter α and, ultimately, provides a theoretical justification for introducing the CS and IS measures. The GRT decomposes the price vector into a permanent $I(1)$ component and a transitory covariance-stationary $I(0)$ component. To appreciate that, let $\alpha_\perp$ be the $M \times 1$ orthogonal complement of α, such that $\alpha'\alpha_\perp = 0$. Likewise α and β, $\alpha_\perp$ is not unique, meaning that we can adopt Hansen and Lunde's (2006) intuitive normalization in which the elements of $\alpha_\perp$ sum one, i.e., $\iota_M'\alpha_\perp = 1$, where ι_M is a $M \times 1$ unit vector. The GRT then reads:

$$P_t = \Xi \sum_{h=1}^{t} \varepsilon_h + \sum_{h=0}^{\infty} \Upsilon_h \varepsilon_{t-h} + \Xi \left(P_0 - \Gamma_1 P_{-1}, \ldots, \Gamma_\ell P_{-\ell} \right), \tag{2}$$

where $\Xi = \beta_\perp \left(\alpha_\perp' \Gamma \beta_\perp \right)^{-1} \alpha_\perp'$ with $\Gamma = I_M - \sum_{j=1}^{\ell} \Gamma_j$, Υ_h with $h = 1, \ldots$ is computed recursively through $\Delta\Upsilon_h = \alpha\beta'\Upsilon_{h-1} + \sum_{j=1}^{\ell} \Delta\Upsilon_{h-j}$ such that $\sum_{h=0}^{\infty} \Upsilon_h \varepsilon_{t_{i-h}}$ is a stationary process with $\lim_{h\to\infty} \Upsilon_h = 0$, and $(P_0 - \Gamma_1 P_{-1}, \ldots, \Gamma_\ell P_{-\ell})$ are initial values (Hansen, 2005). The stochastic common trend given by the first term on the right-hand side of (2) reflects the efficient price of the asset. It is also reassuring to observe that the stochastic trend, $\left(\alpha_\perp' \Gamma \beta_\perp \right)^{-1} \alpha_\perp' \sum_{h=1}^{t} \varepsilon_h$, is a martingale and so consistent with non-arbitrage requirements (see discussion in Hansen and Lunde, 2006).[2] In view that $\beta_\perp = \iota_M$, not only does Ξ have common rows, but also the efficient price relates to a weighted-average of the innovations at the different trading venues. To appreciate that, note that $\left(\alpha_\perp' \Gamma \beta_\perp \right)^{-1}$ is a scalar that simply rotates $\alpha_\perp$, such that the weights used to recover the efficient price are in fact given by $\alpha_\perp$. In that, the CS measure is defined in terms of the orthogonal complement of the speed-of-adjustment parameter and emerges naturally as price discovery metric, as it shows how the efficient price is related to the innovations in the different markets:

$$CS_m = \alpha_{m,\perp}, \quad m = 1, \ldots, M. \tag{3}$$

It then follows that the higher the CS_m, the more important market m is to the price discovery process. Furthermore, the CS measure also carries the desirable property of being invariant to the sampling frequency (Dias, Fernandes and Scherrer, 2020).[3] This result has two important implications. First and foremost, one can use prices sampled at lower frequencies (less prone to be affected by market microstructure noise) to make inference about the price discovery mechanism at the tick-by-tick frequency and ultimately in continuous time. In that, such property suits particularly well markets that operate at extremely fast time frames, e.g., microseconds and nanoseconds. Second, it is possible to compare the CS measures across assets and markets which possess distinct

[2] Alternatively, the CS measures also emerge as a valid price discovery measure under Gonzalo and Granger's (1995) permanent and transitory decomposition (PTD). As a drawback, the efficient price (common stochastic trend) that originate from this decomposition is not necessarily a martingale, (see, for instance, discussion in de Jong, 2002)

[3] Nevertheless, standard errors from the CS estimates generally increase with the sampling interval, reflecting that the noise-to-signal ratio increases with the sampling interval.

characteristics, e.g., actively traded stocks versus less-liquid assets.

Another prominent price discovery measure extensively adopted in the literature is the Hasbrouck's (1995) IS. The IS measure takes into account the contemporaneous covariance (correlation) between markets and it is defined as the relative contribution of each market to the total variance of the innovation in the efficient price (see, for instance, Grammig, Melvin and Schlag, 2005; Lien and Shrestha, 2009; Fernandes and Scherrer, 2018). The IS for a given market m reads

$$IS_m = \frac{[\xi C]_m^2}{\xi \Omega \xi'}, \quad m = 1, \ldots, M, \tag{4}$$

where $[\ .\ ]_m$ denotes the mth element of the $M \times 1$ vector, C is the Cholesky decomposition of Ω such that $CC' = \Omega$, and $\xi = \left(\alpha'_\perp \Gamma \beta_\perp\right)^{-1} \alpha'_\perp$ is the common row of Ξ. The denominator of the IS measure consists of the unconditional variance of the efficient price returns that is computed from the first difference of the $I(1)$ component of (2), whereas the numerator accounts for the portion of information impounded by market m into the variance of the efficient price returns. Choosing C as the Cholesky decomposition carries the usual drawbacks associated with order variability.[4] The standard solution to this problem is to average the maximum and minimum IS measures computed over all possible orderings. The IS measure, however, carries an important caveat: it is not invariant to the sampling frequency. This follows because the correlation between markets increases with the sampling interval. This feature is particularly relevant for highly-liquid assets that operate at fast and integrated markets. In such cases, Dias et al. (2020) show that the IS measures computed for thickly traded NYSE- and Nasdaq-listed stocks are unable to identify the leading market as their estimates are equal to the limiting value of $1/M$. These findings hold even if one uses the continuous-time analogue of (4), where Ω is replaced by an estimate of the integrated covariance matrix using prices sampled at the frequency of one observation per minute.

2.2 The time-varying price discovery model

In this section, we extend the baseline reduced form price discovery model discussed in Section 2.1 to formally address time variation in the price discovery measures. The usual practise in the literature is to accommodate such time variation by running independent VEC models for each day in the sample (see, among others, Hasbrouck, 2003; Chakravarty et al., 2004; Hansen and

[4] Alternatively, one could use the spectral decomposition as in Fernandes and Scherrer (2018) or a the factor structure on the correlation matrix as proposed in Lien and Shrestha (2009) to obtain order invariant IS measures.

Lunde, 2006; Mizrach and Neely, 2008; Hasbrouck, 2018). Based on the parameter estimates of these daily VEC models, one can then compute daily estimates of the CS and IS measures.

However, it follows that treating the days in the sample as separate time series processes carries two important caveats. First, any persistence (time dependence across days) that the price discovery measures may exhibit is formally ignored. In turn, the implicit assumption that emerges from this strategy is that price discovery measures should not relate to highly persistent fundamentals, such as volume and volatility, for instance. This assumption goes against the empirical evidence. The ability of a trading venue to impound new information mostly depends on market features (e.g., cost structure, market design, technological infrastructure, and relative presence of high-frequency traders) and market characteristics (e.g., trading intensity, trading volume and volatility). It turns out that both change over time, but neither in a continuous nor in a brusk fashion. This is well in line with the empirical findings in Eun and Sabherwal (2003) and Frijns, Gilbert and Tourani-Rad (2015), who show that some of the main price discovery drivers are highly persistent over time (e.g., volume and volatility). The second caveat is associated with finite sample performance and consistency of the price discovery measures. Through an econometric perspective, throwing data away should never be an optimal strategy, as using only a single day yields smaller sample sizes and hence induces wider confidence intervals. In addition, estimating daily VEC models at high resolutions would imply relying on infill asymptotic theory for both consistency and inference on price discovery measures. However, if the true data generation process is defined in continuous time, the prices are sampled inside a fixed interval (say a trading day), and the sampling interval tends to zero, then it is not possible to identify the drift of a diffusion process (see, for instance, Prakasa Rao, 1983). In that, the usual LS estimator of the speed of adjustment parameters is inconsistent. Differently, identification requires both long span and infill asymptotic to hold (Phillips and Yu, 2009; Tang and Chen, 2009). In what follows, we extend the baseline model in (1) to accommodate the empirical evidence that price discovery measures are persistent and all available observations are used for estimation purposes.

The price discovery measures are essentially a function of the speed-of-adjustment parameters. In that, it suffices to allow α to be a time varying parameter matrix in order to generate time-varying CS and IS measures. Keeping that in mind, we start by refining our notation. Assume prices within a trading day are observed equidistantly, such that there is n intraday observations per trading day. For instance, the usual trading day in the U.S. market lasts for 6.5 hours (23,400 s), and thus, sampling one observation per minute yields $n = 390$ intraday observations. Next, denote D as the number of trading days in our sample, such that $T = nD$ with T denoting the total number of observations in the sample. Next, we allow for the parameters in (1) to change over time at the daily frequency, while being constant intradaily. More specifically, the speed-of-adjustment parameters follow a bounded local stable stochastic process at the daily frequency in order to cope with the required level of persistence found empirically. This flexible parametrization accommodates the highly persistent time variation in the price discovery mechanism identified in previous studies (see, for instance, Hasbrouck, 1995, 2003; Mizrach and Neely, 2008). We denote the daily speed-of-adjustment parameter as $\alpha^{(d)}$. Finally, we also allow the corresponding autoregressive matrices $\Gamma_1^{(d)}, \ldots, \Gamma_\ell^{(d)}$ to vary over time, just as $\alpha^{(d)}$, so the reduced for price discovery model fully reflects the daily variation on the information processing. It then follows that the dynamics of the first differences of P_t on a given day, d, can be represented by the time-varying VEC model:

$$\Delta P_t = \alpha^{(d)} \beta' P_{t-1} + \sum_{j=1}^{\ell} \Gamma_j^{(d)} \Delta P_{t-j} + \varepsilon_t, \qquad d = 1, \ldots, D, \quad t = 1, \ldots, nD = T, \tag{5}$$

where $\alpha^{(d)}$ is a $M \times r$ error correction matrix; $\beta = (I_r, -\iota_r)'$ is a $M \times r$ cointegrating matrix that remains constant across days; $r = M - 1$ is the number of cointegrating vectors; $\Gamma_\ell^{(d)}$ with $\ell = 1, 2, ..., p$ are the $M \times M$ autoregressive matrix coefficients; ε_t is a zero-mean white noise process with a $M \times M$ non-diagonal covariance matrix Ω, and n and D are the number of intraday observations and days, respectively. The VEC model in (5) is a generalization of Hansen's (2003) VEC model with structural changes. Finally, it is worth pointing out that (5) remains a valid representation of cointegrated prices as long as the roots of $\mathcal{C}(z) = \left|(1-z)I_M - \alpha^{(d)}\beta' z - \sum_{j=1}^{\ell} \Gamma_j^{(d)} (1-z) z^j\right| = 0$ are outside the unit circle or equal to one for all $d = 1, 2, \ldots, D$ (Hansen, 2003).

Another possible approach is to require that (5) applies only locally. The local approximation framework has attracted substantial attention in the realized measures and asset pricing tests literature using high-frequency data (see, for instance, Andersen, Archakov, Cebiroglu and Hautsch, 2021a; Andersen, Thyrsgaard and Todorov, 2021b) and fits well the price discovery analysis. In the context of extremely fast markets, it is possible to re-parametrize (5) to apply only locally, say, for instance, for sampling intervals of 30 minutes. In such cases, d will then denote the local interval in which the speed-of-adjustment parameters are held constant and n will be the number of observations contained within this time interval. This is in line with recent empirical evidence that new information is impounded to the efficient price within a few minutes, (see, among others Dufour and Engle, 2000; Hansen and Lunde, 2006; Santosh, 2016). This flexible parameterization gives rise to intraday price discovery analysis.

To formally derive the corresponding time-varying price discovery measures, it is necessary to decompose the process in (5) into $I(1)$ and $I(0)$ components. Very interestingly, auxiliary result in the proof of Hansen's (2003) Lemma 5 extends the GRT to our time-varying setting. More specifically, denote $\Xi^{(d)} = \beta_\perp \left(\alpha_\perp^{(d)\prime}\Gamma^{(d)}\beta_\perp\right)^{-1}\alpha_\perp^{(d)\prime}$ with $\Gamma^{(d)} = I_M - \sum_{j=1}^{\ell}\Gamma_j^{(d)}$ and let $\Upsilon_h^{(d)}$ with $h = 1, \ldots,$ be computed recursively through $\Delta\Upsilon_h^{(d)} = \alpha^{(d)}\beta'\Upsilon_{h-1}^{(d)} + \sum_{j=1}^{\ell}\Delta\Upsilon_{h-j}^{(d)}$. Next, note that the tth observation within any day $d \in [1, D]$ is given by $(d-1)n \leq t \leq dn$. It then follows that the representation of P_t obtained from the GRT reads

$$P_t = \left\{\Xi^{(d)} \sum_{h=(d-1)n+1}^{t} \varepsilon_h\right\} + \left\{\sum_{h=0}^{t-1}\Upsilon_h^{(d)}\varepsilon_{t-h} + \Upsilon_t^{(d)}P_{(d-1)n}\right\} + \left\{\Xi^{(d)}\left(P_{(d-1)n} - \sum_{j=1}^{\ell}\Gamma_j P_{(d-1)n-j}\right)\right\}, \quad t = (d-1)n+1, \ldots, dn. \tag{6}$$

The first term in (6) corresponds to the $I(1)$ stochastic trend. Again, it is reassuring to note that it remains a martingale and hence reflects the latent efficient price. The second term is not the usual $I(0)$ process, as it depends on the price levels, $P_{(d-1)n}$. However, recall that as h increases, $\Upsilon_h^{(d)}$ converges to zero exponentially fast. This is particularity useful in our price discovery setting, as $(d-1)n \leq t \leq dn$ and the number of intraday observations is usually large, say $n = 390$ at the 1-minute sampling interval. This means that the deviation from stationarity given by $\Upsilon_t^{(d)}P_{(d-1)n}$ is quantitatively negligible and plays no role in the price discovery analysis. Furthermore, $\Upsilon_t^{(d)} \approx 0$ implies that the information process happens within the same trading day, which is consistent with recent empirical evidence (see, for instance, Scherrer, 2021). If (5) and (6) were to apply only locally, the term that accounts for the deviation from stationarity would have to converge to zero within this time interval, so that it remains negligible. Recent empirical evidence for thickly traded stocks in corroborates the parameterization of (5) and (6) over local intervals (see, among others Santosh, 2016; Scherrer, 2021). Finally, the third term consists of initial values sampled from the previous trading day, $d-1$. The GRT in (6) provides the necessary ingredients to formulate daily (local) price discovery measures. More specifically, we construct the daily (local) price discovery measures using the stochastic trend: $\left(\alpha_\perp^{(d)\prime}\Gamma^{(d)}\beta_\perp\right)^{-1}\alpha_\perp^{(d)\prime}\sum_{h=(d-1)n+1}^{t}\varepsilon_h$.

We start by defining the daily CS measures in terms of the orthogonal complement of the daily speed-of-adjustment parameters. This follows because $\left(\alpha_{\perp}^{(d)\prime}\Gamma^{(d)}\beta_{\perp}\right)^{-1}$ essentially rotates the non-unique orthogonal complement of $\alpha^{(d)}$, and hence carries no information to the price discovery process. It then follows that the daily CS reads

$$CS_m^{(d)} = \alpha_{m,\perp}^{(d)}, \quad m = 1, \ldots, M. \tag{7}$$

The $CS_m^{(d)}$ shows how the efficient price is related to the innovations in the different markets on a given day d. It carries the same intuition as the standard CS measure, meaning that the higher $CS_m^{(d)}$ the more important market m is to the price discovery process. The $CS_m^{(d)}$ remains invariant to the sampling frequency, meaning that one could make inference about the price discovery in markets that operate at extremely fast time frames using prices sampled at lower frequencies, such as one observation per minute.[5]

Next, we focus on the daily IS measures. Similarly to the daily CS measures, it suffices to use the first term in (6) to come up with the relative contribution of each market to the total variance of the efficient price return sin any day d. It then follows that the daily IS measure reads:

$$IS_m^{(d)} = \frac{\left[\xi^{(d)}C\right]_m^2}{\xi^{(d)}\Omega\xi^{(d)\prime}}, \quad m = 1, \ldots, M, \tag{8}$$

where $[\ .\ ]_m$ denotes the mth element of the $M \times 1$ vector, C is the Cholesky decomposition of Ω such that $CC' = \Omega$, and $\xi^{(d)} = \left(\alpha_{\perp}^{(d)\prime}\Gamma^{(d)}\beta_{\perp}\right)^{-1}\alpha_{\perp}^{(d)\prime}$ is the common row of $\Xi^{(d)}$. It is important to note that the daily IS measures are not invariant to the sampling frequency, meaning that it also converges to $1/M$ as the sampling interval increases. This remains an important caveat in the context of markets that operate at time-scales of seconds or microseconds. Finally, in the case (5) and (6) apply locally, the resulting CS and IS measures could be automatically interpreted in terms of intraday price discovery measures, as they both relate to the latent efficient price within the local interval d.

[5] Note that there still exists a trade off between frequency and noise-to-signal ratio. We specifically address this trade off in out Monte Carlo study.

3 Estimation of the time-varying price discovery

This section presents a consistent and asymptotically normal estimator of the daily speed-of-adjustment parameters and discusses the asymptotic properties of the price discovery measures.

The standard practice in the literature is to capture the daily variation in price discovery by estimating a daily VEC model (see, for instance, Hasbrouck, 2003; Chakravarty et al., 2004; Mizrach and Neely, 2008). However, this parametrization is not consistent with empirical evidence suggesting that price discovery measures are related to highly persistent fundamentals, such as trading volume, for instance, and that news (permanent innovations to the efficient price) are incorporated within the five minutes, (see, for instance, Eun and Sabherwal, 2003; Scherrer, 2021). As opposed to this approach, we propose an estimation method for price discovery that is not independent across days. We employ Giraitis et al.' (2013) kernel-based estimator to retrieve daily estimates of the VEC parameters. The kernel least squares (KLS) estimator exploits the assumption that the daily variations in the alpha and Gamma matrices are persistent processes (either deterministic or stochastic) in order to obtain more efficient estimators.

We start by rewriting the baseline time-varying VEC model in (5) in a more compact notation

$$\Delta P_t = B^{(d)\prime} X_t + \varepsilon_t, \tag{9}$$

where $X_t = (P_{t-1}'\beta, \Delta P_{t-1}', \ldots, \Delta P_{t-\ell}')'$ is a $(M\ell + r) \times 1$ vector of covariance stationary regressors and $B^{(d)}$ is a $(M\ell + r) \times M$ random coefficient matrix that collects the free parameters in (1), namely, $B^{(d)} = (\alpha^{(d)}, \Gamma_1^{(d)}, \ldots, \Gamma_\ell^{(d)})'$. Regardless of whether the variation in $B^{(d)}$ is stochastic or deterministic, the matrix of regressors X_t is the same for all equations of (9), and thus, it is possible to estimate the free parameters in each of the M equations separately.

The parameter matrix $B^{(d)}$ could be easily estimated by a rolling window OLS estimation method. To this end, define the $(M\ell + r) \times 1$ vector $B_m^{(d)}$ as the mth column of $B^{(d)}$, containing all the parameters of the mth equation of (9). The rolling window OLS estimator then reads

$$\widetilde{B}_m^{(d)} = \left(\sum_{t=(d-1)n+1-\widetilde{d}n}^{dn+\widetilde{d}n} X_t X_t' \right)^{-1} \sum_{t=(d-1)n+1-\widetilde{d}n}^{dn+\widetilde{d}n} X_t \Delta P_{m,t}, \quad m = 1, ..., M, \tag{10}$$

where $\widetilde{d} \geq 0$ is an integer denoting the size of the rolling window in the same unit of measurement as d, and $\Delta P_{m,t}$ is the mth row of ΔP_t. However, this strategy carries an important drawback: it gives equal weights to all observations within an exogenously defined rolling window. This feature fails to acknowledge the fact that the observations that are nearer to the trading day d are meant to be more informative for the estimation of $B^{(d)}$ than those further away. In this context, the size of the rolling window matters. The usual practise in the price discovery literature is to set the size of the rolling window equal to one trading day, i.e., $\widetilde{d} = 0$, and compute individual daily VEC models for each trading day. As a result, estimates of $B^{(d)}$ are computed with only n intraday observations, e.g., $n = 390$ for prices sampled at 1-minute sampling interval. This strategy is not optimal, as it discards relevant information contained in the neighbourhood of trading day d.

We explicitly address this issue by adopting the KLS estimation method to carry out estimation and inference on $B^{(d)}$ and consequently on the daily CS and IS measures. The KLS estimator can be seen as an weighted least squares estimator, where the weights are given by a kernel function. More specifically, the KLS estimator is a kernel based generalisation of a rolling window OLS estimation method that gives higher (lower) weights to the observations that are in the neighbourhood (further away) of trading day d. To appreciate that, note that the KLS estimator collapses to the standard rolling window OLS estimator if one adopts the uniform kernel function.[6] The KLS estimator then reads

$$\widehat{B}_m^{(d)} = \left(\sum_{t=1}^{T} K\left(\frac{nd-t}{H}\right) X_t X_t' \right)^{-1} \sum_{t=1}^{T} K\left(\frac{nd-t}{H}\right) X_t \Delta P_{m,t}, \quad m = 1, ..., M, \tag{11}$$

where $K(\cdot)$ and H respectively denote a kernel function and the corresponding bandwidth. To conduct inference regarding the daily CS and IS measures and hence establish its consistency and asymptotic normality, it is necessary state specific functional form assumptions about $B^{(d)}$, the kernel function, and regularity conditions for the disturbances. In what follows, we discuss these assumptions in details.

First, we incorporate the desired level of persistence on the time-varying parameter matrix $B^{(d)}$, so that it reflects the empirical evidence that price discovery measures relate to fundamentals that change in a smooth fashion. Furthermore, we parametrize $B^{(d)}$ such that the roots of $\mathcal{C}(z) = \left|(1-z)I_M - \alpha^{(d)}\beta' z - \sum_{j=1}^{\ell} \Gamma_j^{(d)} (1-z) z^j \right| = 0$ are outside the unit circle or equal to one for all $d = 1, 2, \ldots, D$. In what follows, we assume that $B^{(d)}$ is a bounded stochastic process that meets

[6] The uniform kernel, also known as flat kernel, is defined as $K(u) = 1/2$ for $|u| \leq 1$.

this condition.

Assumption RM *$B^{(d)}$ forms a sequence of random matrices satisfying $\mathcal{C}(z) = 0$ for $|z| \geq 1$, $\sup_{d \leq D} \left\| B^{(d)} \right\| = O_p(1)$, and $\sup_{\imath:|\imath-d|\leq h} \left\| B^{(d)} - B^{(\imath)} \right\|_{sp}^2 = O_p(h/d)$ for $h = o(d) \to \infty$ as $d \to \infty$.*

The local stability conditions in Assumption RM are very mild, holding for the bounded random walk process we consider in our Monte Carlo study, for instance. Note that we implicitly assume that the elements of $B^{(d)}$ vary over time in a stochastic fashion as in Giraitis, Kapetanios and Yates (2018). Alternatively, in the case of deterministic variation in $B^{(d)}$, asymptotic normality of the KLS estimator would require the parameters to satisfy a Lipschitz condition (Robinson, 1989).

The next assumption regulates the kernel properties and the rate at which the bandwidth grows.

Assumption K *The kernel function $K(v)$ is nonnegative for any $v \in \mathbb{R}$. It is continuous and bounded, with a bounded first derivative, such that $\int K(v)\,\mathrm{d}v = 1$ and $\int [K(v)]^2\,\mathrm{d}v = c_K < \infty$. There is also a constant $c > 0$ such that $K(v) = O\left(e^{-cv^2}\right)$. As for the bandwidth, for fixed n, $H = o(nD) \to \infty$ as $D \to \infty$.*

Most standard kernels in the literature (e.g., the flat, Epanechnikov and Gaussian kernels) meet the conditions in Assumption K. It now remains to regulate the higher-order moments of errors and regressors, so as to ensure that the limiting distribution of the KLS estimator is well defined.

Assumption COV *The error ε_t forms a covariance stationary martingale difference sequence with finite fourth moment uniformly over t, and orthogonal to the regressors X_t. The latter are covariance stationary such that $\mathbb{E}(X_t X_t') = Q_X < \infty$ uniformly over t and T and that $\frac{1}{\kappa} \sum_{\kappa=0}^{\infty} \sup_{1 \leq i,\imath,j,\jmath \leq 2\ell+1} \sup_1 |Cov(X_{i,t} X_{\imath,t}, X_{j,t+\kappa} X_{\jmath,t+\kappa})| < \infty$. Finally, ε_t and X_t are such that $\zeta_t = (X_t \varepsilon_{1,t}, X_t \varepsilon_{2,t})'$ has a finite covariance matrix $Q_{\zeta,t} = \mathbb{E}[\zeta_t \zeta_t']$ uniformly over t, with $\frac{1}{T} \sum_{t=1}^{T} Q_{\zeta,t} \xrightarrow{p} Q_\zeta$, $\frac{1}{H} \sum_{|t-nd|<H} \left[K\left(\frac{nd-t}{H}\right)\right]^2 \zeta_t \zeta_t' \xrightarrow{p} c_K\, Q_\zeta$, and $\mathbb{E}(\zeta_{i,t} \zeta_{\imath,t} \zeta_{j,t} \zeta_{\jmath,t}) < \infty$ for all $i, \imath, j, \jmath = 1, \ldots, 4\ell + 2$.*

Under Assumptions RM, K, and COV, we firstly establish the asymptotic normality of the KLS estimator for the long span (i.e., $T \to \infty$ with n being fixed and $D \to \infty$) asymptotics. More specifically, Assumptions COV, K and RM coincide with Giraitis, Kapetanios and Marcellino's (In Press) Assumptions 1 to 3 in the context of exogenous regressors, and hence asymptotic normality of the KLS estimator readily follows as $D \to \infty$ from their Theorem 3. We then employ the delta method to back out the asymptotic behavior of the time-varying component share measures. To this end, recall that as the orthogonal complement of $\alpha_\perp^{(d)}$ is not unique, we can impose the normalization $\iota_M'\alpha_\perp^{(d)} = 1$. This is achieved by computing $\alpha_\perp^{(d)}$ as follows:

$$\alpha_\perp^{(d)} = \frac{1}{\vartheta}\left(\iota_M - \alpha^{(d)\prime}\left(\alpha^{(d)\prime}\alpha^{(d)}\right)^{-1}\alpha^{(d)\prime}\iota_M\right), \tag{12}$$

where $\vartheta = \iota_M'\iota_M - \iota_M'\alpha^{(d)}\left(\alpha^{(d)\prime}\alpha^{(d)}\right)^{-1}\alpha^{(d)\prime}\iota_M$. Next, the delta method requires the $M \times Mr$ matrix $\Lambda_\perp^{(d)}$ whose mth row is the vector of derivatives of the mth function with respect to $\alpha_\delta^{(d)\prime}$. It is straight forward to compute $\Lambda_\perp^{(d)} \equiv \frac{\partial\alpha_\perp^{(d)}}{\partial\alpha_\delta^{(d)\prime}}$ numerically based on (12) for any $M \geq 2$. For illustrative purposes, it is also possible to obtain closed form expressions for both $\alpha_\perp^{(d)}$ and $\Lambda_\perp^{(d)}$ in the case $M = 2$. The expressions for $\alpha_\perp^{(d)}$ and $\Lambda_\perp^{(d)}$ then simplify to

$$\alpha_\perp = \left(\frac{\alpha_2}{\alpha_2 - \alpha_1}, -\frac{\alpha_1}{\alpha_2 - \alpha_1}\right)', \tag{13}$$

and

$$\Lambda_\perp^{(d)} \equiv \frac{\partial\alpha_\perp^{(d)}}{\partial\alpha^{(d)\prime}} = \begin{pmatrix} \frac{\alpha_2^{(d)}}{\left(-\alpha_1^{(d)}+\alpha_2^{(d)}\right)^2} & -\frac{\alpha_1^{(d)}}{\left(-\alpha_1^{(d)}+\alpha_2^{(d)}\right)^2} \\ -\frac{\alpha_2^{(d)}}{\left(-\alpha_1^{(d)}+\alpha_2^{(d)}\right)^2} & \frac{\alpha_1^{(d)}}{\left(-\alpha_1^{(d)}+\alpha_2^{(d)}\right)^2} \end{pmatrix}, \tag{14}$$

respectively. Finally, consistency of the IS measure follows directly from consistency of the estimates of $\alpha_\perp^{(d)}$ and Ω and application Slutsky theorem. The next result documents the asymptotic normality of the KLS estimators of the time-varying VEC model parameters and of the daily component shares, providing analytical expressions for their asymptotic variances. In addition, it states the consistency of the IS measure estimates.

Theorem *Let Assumptions COV, K and RM hold, and let* $b^{(d)} = vec\left(B^{(d)}\right)$ *and* $\widehat{b}^{(d)} = vec\left(\widehat{B}^{(d)}\right)$. *If* $H = o(\sqrt{T})$,

$$\sqrt{H}(\widehat{b}^{(d)} - b^{(d)}) \xrightarrow{d} N\left(0, c_K\left[I_M \otimes Q_X^{-1}\right]Q_\zeta\left[I_M \otimes Q_X^{-1}\right]\right), \tag{15}$$

where $Q_X = \mathbb{E}[X_t X_t']$ *and* $\otimes$ *denotes the Kronecker product. In turn,*

$$\sqrt{H}(\widehat{\alpha}_\perp^{(d)} - \alpha_\perp^{(d)}) \xrightarrow{d} N\left(0, c_K\, \Lambda_\perp^{(d)} \mathbb{K}_{rM} R_\alpha \left[I_M \otimes Q_X^{-1}\right] Q_\zeta \left[I_M \otimes Q_X^{-1}\right] R_\alpha' \mathbb{K}_{rM}' \Lambda_\perp^{(d)'}\right), \tag{16}$$

R_α *as the deterministic matrix that selects the elements of* $\alpha^{(d)}$ *from* $\widehat{b}^{(d)}$*, such that* $R_\alpha \widehat{b}^{(d)} = vec\left(\alpha^{(d)'}\right)$*, and* $\mathbb{K}_{rM}$ *is the* $rM \times rM$ *commutation matrix, such that* $vec\left(\alpha^{(d)}\right) = \mathbb{K}_{rM}\, vec\left(\alpha^{(d)'}\right)$*. Finally,* $\widehat{IS}_m^{(d)} \xrightarrow{p} IS_m^{(d)}$*.*

To consistently estimate the asymptotic covariance matrix, it suffices to substitute $\widehat{Q}_X = \frac{1}{H}\sum_{|t-nd|<H} K\left(\frac{nd-t}{H}\right) X_t X_t'$ and $\widehat{Q}_\zeta = \frac{1}{H}\sum_{|t-nd|<H} \left[K\left(\frac{nd-t}{H}\right)\right]^2 \widehat{\zeta}_t \widehat{\zeta}_t'$ into (15) and (26), with $\widehat{\zeta}_\tau = (X_t \widehat{\varepsilon}_{1,t}, X_t \widehat{\varepsilon}_{2,t})'$, given that $\widehat{Q}_\zeta \xrightarrow{p} c_K\, Q_\zeta$ due to the consistency of $\widehat{\varepsilon}_t$ and to Assumptions K and COV.

4 Monte Carlo study

We assess the performance of the KLS estimator relative to the standard approach of estimating time-varying measures of price discovery using independent daily VECMs. The main goal is to investigate the role played by the choices of sampling frequency and bandwidth parameters in the performance on both estimators. Notably, the choice of sampling frequency affects the estimation of $\alpha^{(d)}$ in the two following ways: First, the finite sample performance of both estimators deteriorates as the the sampling interval increases. This follows because the rate of convergence of both estimators depend on n, and n decreases at these lower resolutions. Second, the noise-to-signal ratio used to identify the speed-of-adjustment parameters increases with the sampling interval, meaning that a higher portion of the adjustment across markets takes place contemporaneously, which ultimately contributes reduces the signal used to estimate $\alpha^{(d)}$. We contemplate one asset traded at $M = 2$ trading venues. We focus on the CS measures, given that these measures are invariant to the sampling frequency.[7]

In order to fully appreciate the effect of the sampling frequency on the estimation of the speed-of-adjustment parameters and hence on the CS measures, it is necessary to work out the discrete-time price processes from a continuous time setting. In this way, the properties from the data generation process are preserved across the different sampling frequencies. The reason is twofold. First, the speed of adjustment parameter at the different sampling frequencies is a deterministic

[7] Results for the IS are available upon request.

function of the sampling interval. This means that one cannot simply define ad-hoc alphas on each sampling interval while keeping a coherent noise-to-signal ratio across these different sampling intervals. Second, the correlation between markets is also a function of the alphas and the sampling intervals, meaning that it also cannot be set exogenously across the different sampling frequencies. To address these issues, we follow Dias et al. (2020) and adopt the reduced-rank multivariate Ornstein-Uhlenbeck (RR-OU) process as the data generation process for each day $d = 1, \ldots, D$. The daily RR-OU process then reads:

$$\mathrm{d}P_t = \boldsymbol{\alpha}^{(d)}\beta' P_t \,\mathrm{d}t + C^{(d)}\,\mathrm{d}W_t, \tag{17}$$

where $P_t = (P_{1,t}, \ldots, P_{M,t})'$ is an $M \times 1$ vector collecting the log-prices of each of the M trading venues, $\boldsymbol{\alpha}^{(d)}$ and β are $M \times R$ full-rank matrices with $\boldsymbol{\alpha}^{(d)}$ denoting the speed-of-adjustment parameter matrix in continuous time, W is an $M \times 1$ vector of Brownian motions, and $C^{(d)}$ is an $M \times M$ matrix such that the covariance matrix $\Sigma^{(d)} = C^{(d)}C^{(d)'}$ is positive definite that also varies on a daily basis. The exact discretization of the RR-OU process collapses to a homoskedastic Gaussian VEC(0) model. Notably, without loss of generality, this is equivalent to set $\Gamma_1 = \ldots = \Gamma_\ell = 0$ and ε_t as an iid Gaussian process in (5). To obtain the exact discrete-time counterpart of (17), we assume prices in a given day are observed regularly and equidistantly over the unit interval $[0, 1]$ that characterizes, say, one trading day (calendar-time sampling, as discussed in Hansen and Lunde, 2006). The length of each interval in $[0, 1]$ is $\delta = 1/n$ with n denoting the total number of intervals (intraday observations), and the starting values for each day d is defined as the last intraday observation from the previous day. In this set-up, price changes at day d and at interval length δ reads

$$\Delta P_t = \alpha_\delta^{(d)}\beta' P_{t-1} + \varepsilon_t, \quad \text{with } t = (d-1)n + 1, \ldots, dn, \tag{18}$$

where $\alpha_\delta^{(d)} = \boldsymbol{\alpha}^{(d)}(\beta'\boldsymbol{\alpha}^{(d)})^{-1}\left[\exp(\delta\beta'\boldsymbol{\alpha}^{(d)}) - I_r\right]$, with I_r denoting a r-dimensional identity matrix. The innovation ε_t is iid Gaussian with zero mean and covariance matrix given by $\Omega_\delta^{(d)} = \int_0^\delta \exp\left(u\boldsymbol{\alpha}^{(d)}\beta'\right)\Sigma^{(d)}\exp\left(u\beta\boldsymbol{\alpha}^{(d)'}\right)\,\mathrm{d}u$.

The data-generation process we entertain requires that the elements of $\alpha_\delta^{(d)}$ follow independent bounded random walk processes at the daily frequency. In order to guarantee that $\alpha_\delta^{(d)}$ remains identifiable in all sampling intervals, we restrict the speed-of-adjustment parameters in their lowest frequency. The bounds are such that $\alpha^{(d)}_{1,1/78} \in [-0.49, -0.01]$ and $\alpha^{(d)}_{2,1/78} \in [0.01, 0.49]$ at the 5-minute interval ($\delta = 1/78$). These bounds ensure that the eigenvalues of $\alpha^{(d)}_{\delta}\beta'$ are strictly smaller than one in magnitude for any $0 < \delta \leq 1/78$. Next, we define the level of persistence associated with the speed-of-adjustment parameters. Similarly to Giraitis et al. (2013), we assume that

$$\alpha^{(d)}_{1,1/78} = \bar{\alpha}\left(\frac{a^{(d)}_1}{\max_{0\leq d^*\leq d}\left|a^{(d^*)}_1\right|} - 1\right) - 0.01 \qquad \alpha^{(d)}_{2,1/78} = \bar{\alpha}\left(\frac{a^{(d)}_2}{\max_{0\leq d^*\leq d}\left|a^{(d^*)}_2\right|} + 1\right) + 0.01,$$

with $\bar{\alpha} = 0.24$, so as to satisfy the upper and lower bounds, and $(a^{(d)}_1, a^{(d)}_2)'$ following independent driftless random walk processes driven by white noise innovations. We then back out the continuous-time $\boldsymbol{\alpha}^{(d)}$ by imposing $\beta = (1, -1)'$ and inverting the matrix exponential operator:

$$\boldsymbol{\alpha}^{(d)}\beta' = \left(\frac{1}{78}\right)^{-1} \log\left(\alpha^{(d)}_{\delta=1/78}\beta' + I_M\right), \tag{19}$$

where $Z = \log(A)$ if Z is such that $\exp(Z) = A$ for any square matrix A. Finally, we can map the the discrete-time alphas from $\boldsymbol{\alpha}^{(d)}$ for all sampling intervals $\delta \in (1/780, 1/390, 1/195, 1/130, 1/78)$ and hence compute the true CS measures at those sampling frequencies.

As for the daily continuous-time covariance matrix $\Sigma^{(d)}$, we assume that

$$\ln\sigma^2_{m,d} = \phi_0 + \phi_1 \ln\sigma^2_{m,d-1} + \varsigma\, \upsilon_{m,d}, \qquad \text{for} \quad m = 1, 2 \quad \text{and} \quad d = 1, 2, \ldots, D \tag{20}$$

with $\sigma^2_{1,d}$ and $\sigma^2_{2,d}$ denoting the diagonal elements of $\Sigma^{(d)}$. The volatility innovations $\upsilon_{1,d}$ and $\upsilon_{2,d}$ are Gaussian white noises with a constant correlation of 0.95 and unit variances. As in Jacquier, Polson and Rossi (1994), we fix the autoregressive parameter to 0.98 and calibrate ϕ_0 and ς in (20) such that the expected annual volatility is 20% and the coefficient of variation given by $\mathbb{V}(\sigma^2_{m,d})/\mathbb{E}(\sigma^2_{m,d}) = \exp\left(\varsigma/(1-\phi^2_1)\right) - 1$ is equal to 1/2. Finally, we entertain different levels of contemporaneous correlation between markets given by $\rho \in \{0, 0.3, 0.5, 0.7, 0.9\}$.

Using (18) and the discrete-time speed-of-adjustment parameter implied by (19), we simulate prices within a given day from (18) at the 1-second interval (i.e., $\delta = 1/23,400$ for a trading day of 6.5 hours) for $D = 500$ trading days (about 2 years). We then sample prices at fixed intervals of 1/2, 1, 2, 3, and 5 minutes, corresponding to δ values of 1/780, 1/390, 1/195, 1/130, and 1/78, respectively. As a result, sample size ranges from 39,000 to 390,000 intraday observations.

We estimate $\alpha_{\delta}^{(d)}$ by LS as in the daily VEC approach and by KLS as in (11) using the Epanechnikov kernel, with bandwidth $H \in \{n^{8/10}\sqrt{D}, n^{9/10}\sqrt{D}, n\sqrt{D}\}$. Table 1 documents that the root mean squared errors of the KLS estimates of $\alpha_{\delta,\perp,1}^{(d)}$ relative to their LS counterparts over 1,000 replications as well as their bias. It should be noted that given the normalization of the orthogonal complements, the bias magnitude and the relative root mean squared errors (RRMSE) of the $\alpha_{\delta,\perp,1}^{(d)}$ and $\alpha_{\delta,\perp,2}^{(d)}$ estimates are identical by construction.

Both estimators are clearly unbiased in that sample biases are very close to zero in magnitude. There is a clear decreasing pattern with the sampling interval and bandwidth value, whereas the magnitude of the bias tend to increase with the amount of correlation across markets. As for the latter, higher contemporaneous correlations among markets contribute to increase the noise-to-signal ratio and hence deteriorates the finite sample performance of both estimators. As for the RRMSE figures, they show that the KLS-based component shares are much more precise than the LS estimates almost regardless of the amount of correlation between markets, sampling frequency, and bandwidth choice. The difference in performance increases with the contemporaneous correlation but declines with the sampling frequency. This happens because estimating independent VEC models for each day becomes more difficult not only at lower frequencies due to the smaller sample sizes but also as correlation increases due to the lesser amount of information. Also, because the noise-to-signal ratio increases with the sampling interval, using observations in the neighbourhood of d strongly benefits the KLS estimator. This follows because the inter-daily persistence associated with the alphas helps attenuating the noise-to-signal ratio. For example, in terms of root mean squared error, the KLS estimator improves on daily VEC approach by 56% and 76% on average for the 1- and 5-minute intervals, respectively.

All in all, the Monte Carlo results strongly favor the KLS estimator over the LS estimator for any contemporaneous correlation, bandwidth choice, and covariance matrix specification.[8]

5 Price informativeness

In this section, we estimate the time-varying CS measures for a broad set of NYSE- or Nasdaq-listed stocks using high-frequency data from January 2007 to December 2013. Such a long time span is unusual for price discovery analyses in that most studies typically consider much shorter periods

[8] The KLS estimator also outclasses the LS estimator when we allow for a stochastic covariance matrix, that is, $C^{(d)}$ in (17) evolving intradaily. Results available upon request.

of up to 12 months (see, for instance, de Jong and Schotman, 2010; Riordan and Storkenmaier, 2012; Benos and Sagade, 2016; Ozturk et al., 2017; Hasbrouck, 2018). This large dataset provides us a richer analysis of the time-varying characteristic of the price discovery measure. The CS measures is better suited than the IS measures for two reasons. First, the CS is invariant to the sampling frequency, meaning that we can use prices sampled at lower frequencies such as one observation per minute and still learn about the price discovery process in frequencies as high as the one associated with the continuous-time process. Second, the IS measure converges to the limiting value of $1/M$ as the sampling interval increases. This drawback is particularly evident in our analysis, as we compute price discovery for highly liquid assets that trade at trading platforms populated with high-frequency traders. The faster the markets operate, the faster the correlation between markets converge to the limiting value of one. This is indeed what we observe in our sample. For example, the contemporaneous correlation between the markets residuals for BAC is 0.9929 at the 1-minute sampling interval, which yields IS measures of approximately 0.50 in the two-market setting analysis. In what follows, we first carry out Elliott and Müller's (2006) test to assess the time-varying nature of the CS measures in a formal manner. We then estimate the time-varying CS measures by both LS and KLS and investigate how they relate to liquidity.

5.1 Data

Our data set consists of 30 stocks that markedly differ in terms of industry, listing venue, and trading activity. We group them into three subsamples: 10 Nasdaq-listed stocks, 10 actively-traded NYSE-listed stocks and 10 less-actively-traded NYSE-listed stocks. The first group consists of a random sample from the Nasdaq-100 stock market index constituents that have been trading since January 2007: Adobe Systems (ADBE), Align Technology (ALGN), Amazon.com (AMZN), CA Technologies (CA), Expedia (EXPE), Alphabet (GOOG), Micron Technology (MU), Starbucks Corporation (SBUX), Vodafone Group (VOD), and Wendy's (WEN).

As for the actively-traded NYSE-listed stocks, we select at random from the S&P 500 index constituents: Bank of America (BAC), General Electric (GE), Hewlett-Packard (HPQ), International Business Machines (IBM), J.C. Penney Company (JCP), JP Morgan Chase (JPM), Coca-Cola Company (KO), Altria Group (MO), Verizon Communications (VZ), and ExxonMobil (XOM). Finally, we randomly select 10 less-liquid NYSE-listed stocks from the Russell 1000 index constituents: Canon (CAJ), Cooper Companies (COO), Dolby Laboratories (DLB), Kirby Corporation (KEX), Lazard (LAZ), Corporate Office Properties Trust (OFC), Everest Re Group (RE), Regal Beloit Corp (RBC), RPC Inc (RES), Rollins (ROL) and Thor Industries (THO). These stocks exhibit, on average, 70% less trading intensity, as measured by the number of trades, than the actively-traded stocks in the previous group. See also Figure 4 for the trading volume of each stock.

We extract quotes data from TAQ and implement the same cleaning filters as in Barndorff-Nielsen, Hansen, Lunde and Shephard (2009), discarding any observation with a zero quote, negative bid-ask spread, or outside the main trading hours (9:30 to 16:00). We also discard any data point either with a bid-ask spread higher than 50 times the median spread on that day or with a midquote deviating by more than 10 mean absolute deviations from a rolling centered median of 50 observations. Finally, we take the median bid and ask quotes at each second in the presence of multiple ticks. We then synchronize the NYSE and Nasdaq midquotes by sampling at regularly spaced intervals of 1 minute. This not only alleviates concerns with market microstructure noise (see, among others, Hupperets and Menkveld, 2002; Grammig, Melvin and Schlag, 2005) but also helps with the convergence rate of the KLS estimator given that the $n = 390$ intraday observations at this frequency allows for a reasonably large bandwidth H. Table 2 details the cleaning process and provides the final number of time-series observations for each stock.

Although using midquotes is standard in the price discovery literature (see, among others, Hasbrouck, 1995; Baillie, Booth, Tse and Zabotina, 2002; Menkveld, Koopman and Lucas, 2007), we redo our analysis using transaction prices. Accordingly, we also collect transactions data from TAQ, including prices, number of trades, and trading volume. We handle transactions data using the same cleaning filters as in Barndorff-Nielsen et al. (2009). In particular, we discard any observation with a zero trade, outside the main trading hours, or with a flag for trade correction or abnormal sale condition. In addition, we take the median over prices with the same time stamp and delete any price above/below the ask/bid quote plus/minus the bid-ask spread.

5.2 Time variation in the component shares

There is seemingly a consensus in the literature that the price discovery processes may change over time, with many studies running daily VEC specifications to address this issue. Additionally, empirical evidence suggests that the price discovery changes with some highly persistent market indicators such as trading volume and volatility. For instance, Figuerola-Ferretti and Gonzalo (2010) posit an equilibrium model of commodity spot and future prices in which the speed-of-adjustment parameters of a discrete-time VEC depend on the relative number of market participants. As a result, they establish a direct link between component shares and market activity indicators, such as relative volume or trade intensity.

We start with a formal test of whether component shares change over time. In particular, we employ Elliott and Müller's (2006) test for the null hypothesis of constant speed-of-adjustment parameters against the alternative hypothesis that they display persistent variation in time. In the context of one asset trading at two markets, time-varying speed-of-adjustment parameters automatically imply that the CS measures also change over time. The Elliott-Müller test is convenient because it accommodates well enough the sort of variation we describe in Section 2.2. In addition, the test entails good size and power properties even under conditional heteroskedasticity. The asymptotic distribution of the test statistic is nonstandard, with small values indicating rejection of the null hypothesis of parameter stability. The critical values for a bivariate VEC model are respectively -12.80, -14.32, and -17.57 at the 10%, 5%, and 1% significance levels, regardless of the lag structure.

The second column in Table 3 displays the Elliott-Müller test results, which are overwhelmingly in favour of time-varying speed-of-adjustment parameters. Specifically, we cannot reject at the usual significance levels the stability of the speed-of-adjustment parameters over time only for MU, JPM and XOM. We reject the null at the 1% significance level for every less-liquid NYSE-listed stock as well as for most Nasdaq-listed and actively-traded NYSE-listed stocks. This provides strong evidence that the price discovery indeed changes over time, corroborating previous findings in the empirical and theoretical literature on price discovery.

The second and third panels of Table 3 report the median and standard deviation of the LS and KLS estimates of the speed-of-adjustment parameters and of the CS measures over the sample period for each stock. The subscripts 1 and 2 in the parameter estimates denote Nasdaq and NYSE/Arca markets, respectively.[9] The LS and KLS median estimates of the component share are similar enough to lead to the same conclusion about overall market leadership. However, the standard deviations of the daily VEC least-squares estimates are much larger than the corresponding figures for the KLS estimates (threefold for the CS measures, for instance). This is not surprising given the better finite-sample performance of the KLS estimator (see Section 4), and accordingly, we restrict attention in the next section to the KLS-based price discovery analysis.

[9] For the Nasdaq-listed stocks, we use Arca as the competing trading venue, as NYSE does not trade equities listed on other exchanges.

5.3 Daily evolution of the time-varying CS measures

Before discussing the CS estimates, it is important to provide a couple of details about model selection and estimation. First, we set the bandwidth to $H = n\sqrt{D}$ as in Giraitis et al. (2013) given that the Monte Carlo results confirm that this bandwidth choice performs very well. Second, we determine the lag structure by minimizing the Bayesian information criterion (BIC), though selecting the most parsimonious specification in which we cannot reject the absence of residual autocorrelation at the 5% significance level as in Hansen and Lunde (2006) does not change the qualitative results. As expected, we find only one cointegrating vector for every pair of stock prices using Johansen's maximum eigenvalue and trace tests at the 1% significance level.[10]

Figures 1 to 3 plot the KLS estimates of the daily component shares and their respective 95% confidence intervals for each group of stocks we consider. Figure 1 focuses on the Nasdaq-listed stocks trading on Nasdaq and Arca. The price discovery changes markedly over time, with relative market informativeness alternating between trading venues. Even though results vary substantially across stocks, Nasdaq seems more important than Arca in general. This is not very surprising given that Nasdaq not only is the listing venue for these stocks but also offers more liquidity. See the trading volume dynamics in Figure 4. Finally, the standard errors are generally small enough to distinguish between the price discovery measures in the two markets for most of the days in our sample. This is mainly due to the use of 1-minute data, which alleviates the slow convergence rates of the KLS estimator while retaining the signal that identifies the CS measures from the data.

Figure 2 unveils the price discovery for the actively-traded NYSE-listed stocks trading on Nasdaq and NYSE. As before, market leadership alternates between the two exchanges, with mixed results especially in the beginning of the sample period. The Nasdaq contribution to the price discovery picks up for the majority of the stocks as of 2008, likely due to a severe decline in the NYSE market share (see Figure 4). This coincides with the period of increase in market fragmentation in the U.S. market and significant gain in market share of the new entrant markets (see, for instance, Menkveld, 2014, 2016; O'Hara, 2015). The NYSE gradually recovers its relevance in the price discovery process for most of the stocks as from mid 2010 and, by mid 2011, it leads the price discovery for 7 of the 10 stocks in this group. The price discovery contribution typically moves in tandem with relative liquidity and NYSE experiences much more volume than Nasdaq on average.

Figure 3 exhibits the daily component share estimates for the less-actively-traded NYSE-listed

[10] The results for the most parsimonious congruent specification and of the cointegration analyses are available upon request.

stocks. In contrast to the previous results, NYSE clearly dominates Nasdaq in terms of market leadership for every single stock in this group. This corroborates the importance of market activity and trade intensity to the price discovery in that the trading volume at NYSE is much larger than at the Nasdaq for the less liquid stocks. Furthermore, it is also in line with Frijns and Schotman's (2009) evidence that market makers lead the price discovery process for less liquid stocks.

In the next section, we check whether our results are sensitive to the use of midquotes rather than transaction prices. We not only redo the empirical analysis using transaction prices for a subset of five actively-traded NYSE-listed stocks, but also consider a price system with both midquotes and transaction prices to assess whether the latter brings about any additional information on the price discovery. Finally, we also exploit transactions data to link the daily variation in the price discovery measures to relative volume across markets.

5.3.1 Taking advantage of transactions data

Most price discovery analyses employ midquotes not only because they usually have a higher information content than transactions data, but also because they do not suffer from the bid-ask bounce that plagues transaction prices (Menkveld et al., 2007). Transaction prices indeed exhibit different statistical properties, e.g., negative autocorrelation (see Hasbrouck, 2007 for a comprehensive analysis of the Roll model of trade prices). We thus run two robustness checks for five actively-traded NYSE-listed stocks.[11] First, we estimate the time-varying CS measures using their transactions data. As before, we choose the VEC lag structure that minimizes the BIC criterion, which turns out to entail no residual autocorrelation up to the 10th lag at the 5% significance level. The plots in the first column of Figure 5 indicate a very similar CS pattern than we observe using midquotes in that time variation in the CS measures is evident with NYSE and Nasdaq alternating over time as the most informative market.

Second, we consider a price system with both midquotes and transaction prices to understand which contributes most to the price discovery. The plots in the second column of Figure 5 reveal a very clear dominance of midquotes, indicating that they are more informative than transaction prices. As before, there is strong evidence that the CS measures change over time. In short, these results support our choice of using midquotes instead of transactions data as standard in the price discovery literature.

[11] The results for the Nasdaq-listed stocks are qualitatively very similar, and hence we do not report them. They are of course available upon request.

5.3.2 Long-run relationship between CS measures and trading volume

Finally, to better understand the daily variation in the price discovery, we investigate the relationship between price discovery measures and relative volume as a proxy for liquidity. Both time series are very persistent, indicating the presence of unit roots. This is not surprising given that we allow for the component shares to follow a bounded random walk. Spurious inference arising from standard regressions with highly persistent (or even nonstationary) variables is well documented in the literature, (see, for instance Andersen and Varneskov, 2021). We address this problem and estimate a VEC model for the daily CS estimates and the logarithm of the relative volume at the listing venue.[12] Standard cointegration tests strongly suggest a long-run positive relationship between component shares and relative volume. This is in line with Figuerola-Ferretti and Gonzalo's (2010) model as well as with the empirical findings of Eun and Sabherwal (2003) and Frijns et al. (2015).[13]

Table 4 shows that every cointegrating vector is such that the contribution to the price discovery of a given trading venue increases with liquidity. This is consistent with the recent evidence in Brogaard et al. (2021). They find that both liquidity and price discovery decrease in response to an increase in trading fees. The relative volume coefficient estimates in the cointegrating vector are on average about -1/2 for the Nasdaq-listed stocks and twice that for the NYSE-listed stocks (on average, -1.1437 for the very liquid stocks and -0.9427 for the less liquid stocks). The listing venue also matters for the response of the CS estimates to their long-run relationship with the relative volume. In particular, the corresponding speed-of-adjustment parameters are always significant at the 5% level for the Nasdaq-listed stocks, whereas they are significant for less than half of the NYSE-listed stocks. Finally, the relative volume seems to react in a significant manner to the long-run relationship for every stock apart from ADBE.

The VEC specifications linking price discovery to liquidity have a good fit. This is especially so for the Nasdaq-listed stocks, whose average adjusted R^2 is 0.7675, with a median of 0.8330. Altogether, it seems that regardless of liquidity and listing venue issues, relative volume and price discovery adjust to maintain their long-run relationship.

[12] Depending on the stock, we need between 5 and 9 lags in the VAR specification to cope with persistence. To simplify matters, we carry out the cointegration analysis using a VAR(9) specification for every stock, which leads to VEC representations with 8 lags.

[13] As before, the unit root and cointegration test results are available from the authors upon request.

6 Conclusion

This paper carries three main contributions. First, we formally show how to identify daily measures of price discovery from a daily version of the Granger representation theorem. More specifically, we show that the usual CS measure can be used as price discovery measure as long as the speed-of-adjustment parameters are held constant intradaily or at least locally such that $\lim_{t\to\infty} \Upsilon_t^{(d)} = 0$.

Second, we acknowledge the fact that price discovery measures are highly persistent processes and make use of Giraitis et al.'s (2013) KLS method to estimate daily component and information share measures. By exploiting the inter-dependence across days, the KLS approach yields more efficient estimates than we would otherwise obtain by treating the daily variation in the VEC parameters as independent over time. We establish the asymptotic properties of the time-varying speed-of-adjustment parameters and price discovery measures. Monte Carlo simulations confirm under different scenarios that the KLS estimator easily outperforms the standard practice in this literature of estimating component shares for each day using individual VEC specifications.

Third, we assess the informativeness of Nasdaq and NYSE/Arca for a set of 30 stocks. Our empirical results are based on an unusually long time span, covering quotes and transaction prices from January 2007 to December 2013. We find statistical evidence that the component shares indeed change over time for virtually every stock in our sample. Our estimates indicate that market leadership alternates over time, depending on the relative liquidity of each trading venue. Finally, we assess the relationship between daily price discovery measures and trading volume. We find that these variables are cointegrated and the contribution to the price discovery of a given trading venue increases with liquidity.

Appendix: Proof

It follows from the orthogonality condition that $[\mathbb{E}(X_t X_t')]^{-1}\mathbb{E}(X_t \Delta P_{m,t})$ minimizes the LS objective function for the mth equation of (9). Giraitis et al. (2013, 2018) propose the use of sample kernel-based counterparts of the expectation terms, leading to the KLS estimator in (11). The only difference is that they allow for the parameters to change at each instant of time, whereas we restrict variation by assuming that parameters vary only at a lower (daily) frequency.

This modification yields

$$\widehat{B}_m^{(d)} = \left(\sum_{t=1}^{T} K\left(\frac{nd-t}{H}\right) X_t X_t'\right)^{-1} \sum_{t=1}^{T} K\left(\frac{nd-t}{H}\right) X_t \Delta P_{m,t} \tag{21}$$

$$= B_m^{(d)} + \left(\sum_{t=1}^{T} K\left(\frac{nd-t}{H}\right) X_t X_t'\right)^{-1} \sum_{t=1}^{T} K\left(\frac{nd-t}{H}\right) X_t \varepsilon_{m,t}, \qquad m = 1,\ldots,M \tag{22}$$

with $H = o(D) \to \infty$ as $D \to \infty$.[14] It then follows that

$$\sqrt{H}(\widehat{B}_m^{(d)} - B_m^{(d)}) = \left(\frac{1}{H}\sum_{t=1}^{T} K\left(\frac{nd-t}{H}\right) X_t X_t'\right)^{-1} \frac{1}{\sqrt{H}} \sum_{t=1}^{T} K\left(\frac{nd-t}{H}\right) X_t \varepsilon_{m,t}, \quad m = 1,\ldots,M.$$

It is straightforward to show along the same lines as in Giraitis et al.'s (In Press) Lemma 4 that

$$\frac{1}{H}\sum_{t=1}^{T} K\left(\frac{nd-t}{H}\right) X_t X_t' = \frac{1}{H}\sum_{|t-nd|<H} K\left(\frac{nd-t}{H}\right) X_t X_t' + o_p(1/H),$$

$$\frac{1}{\sqrt{H}}\sum_{t=1}^{T} K\left(\frac{nd-t}{H}\right) X_t \varepsilon_{m,t} = \frac{1}{\sqrt{H}}\sum_{|t-nd|<H} K\left(\frac{nd-t}{H}\right) X_t \varepsilon_{m,t} + o_p(H^{-1/2}).$$

This means that

$$\sqrt{H}(\widehat{B}_m^{(d)} - B_m^{(d)}) = \widehat{Q}_X^{-1} \frac{1}{\sqrt{H}} \sum_{|t-nd|<H} K\left(\frac{nd-t}{H}\right) X_t \varepsilon_{m,t} + o_p(1), \tag{23}$$

where $\widehat{Q}_X = \frac{1}{H}\sum_{|t-nd|<H} K\left(\frac{nd-t}{H}\right) X_t X_t'$ is common to both markets. We may then stack the VEC parameters and their estimates into

$$\sqrt{H}(\widehat{b}^{(d)} - b^{(d)}) = \left(I_M \otimes \widehat{Q}_X^{-1}\right) \frac{1}{\sqrt{H}} \sum_{|t-nd|<H} K\left(\frac{nd-t}{H}\right) \zeta_t + o_p(1). \tag{24}$$

Along similar lines to Giraitis et al.'s (2018) Lemma 5, it follows from Assumptions K and COV that $\lim_{H\to\infty} \mathbb{E}(\widehat{Q}_X) = Q_X$ for $\int K(v)\,\mathrm{d}v = 1$ and that $\lim_{H\to\infty} \mathbb{V}[\widehat{Q}_X(\imath,\jmath)] = \lim_{H\to\infty} \frac{c_K}{H}\,\mathbb{V}(X_{\imath,t}X_{\jmath,t}) = 0$ for all $\imath,\jmath = 1,2,...,2\ell+1$ with $\widehat{Q}_X(\imath,\jmath)$ denoting the $(\imath,\jmath)$th element of $\widehat{Q}_X$. Altogether, this implies that $\widehat{Q}_X \xrightarrow{p} Q_X$, so that $\widehat{Q}_X^{-1} \xrightarrow{p} Q_X^{-1}$. Assumption COV ensures that ζ_t is a vector of martingale difference sequences such that $\frac{1}{H}\sum_{|t-nd|<H} \left[K\left(\frac{nd-t}{H}\right)\right]^2 \zeta_t\zeta_t' \xrightarrow{p} c_K\, Q_\zeta$. It then follows

[14] Note that it suffices to assume that $n = 1$ and $D = T$ to recover the standard KLS setting in which the random matrix changes at every point in time.

from a standard central limit theorem for martingale arrays and Davidson's (1994) Theorem 19.2 that $\sqrt{H}(\widehat{b}^{(d)} - b^{(d)}) \xrightarrow{d} \mathcal{N}\left(0, c_K \left[I_M \otimes Q_X^{-1}\right] Q_\zeta \left[I_M \otimes Q_X^{-1}\right]\right)$. Asymptotic normality of the CS estimates readily ensues from a straightforward delta method application. To appreciate that, define $\Lambda_\perp^{(d)} \equiv \frac{\partial \alpha_\perp^{(d)}}{\partial \alpha_\delta^{(d)\prime}}$ and R_α as the deterministic matrix that selects the elements of $\alpha^{(d)}$ from $\widehat{b}^{(d)}$, such that $R_\alpha \widehat{b}^{(d)} = \text{vec}\left(\alpha^{(d)\prime}\right)$:

$$R_\alpha = \begin{pmatrix} I_r & 0_r & 0_{r\times rM+M^2\ell} \\ 0_{r\times(r+m\ell)} & I_r & 0_{r\times r(M-2)+M(M-\ell)} \\ 0_{r\times 2(r+m\ell)} & I_r & 0_{r\times r(M-3)+M(M-2\ell)} \\ \vdots & \vdots & \vdots \\ 0_{r\times(m-1)(r+m\ell)} & I_r & 0_{r\times +M(M-(M-1)\ell)} \end{pmatrix}. \tag{25}$$

Next, define $\mathbb{K}_{rM}$ as the commutation matrix with dimension $rM \times rM$, such that $\text{vec}\left(\alpha^{(d)}\right) = \mathbb{K}_{rM}\text{vec}\left(\alpha^{(d)\prime}\right)$ and $\text{vec}\left(\alpha^{(d)}\right) = \mathbb{K}_{rM} R_\alpha \widehat{b}^{(d)}$. By applying the delta method, the asymptotic distribution of the CS measures read

$$\sqrt{H}(\widehat{\alpha}_\perp^{(d)} - \alpha_\perp^{(d)}) \xrightarrow{d} N\left(0, c_K\, \Lambda_\perp^{(d)} \mathbb{K}_{rM} R_\alpha \left[I_M \otimes Q_X^{-1}\right] Q_\zeta \left[I_M \otimes Q_X^{-1}\right] R_\alpha' \mathbb{K}_{rM}' \Lambda_\perp^{(d)\prime}\right), \tag{26}$$

which completes the proof.

In view that $\widehat{\varepsilon}_t$ is consistent, $\widehat{\Omega} \xrightarrow{p} \Omega$ ensues from standard HAC asymptotic theory (see, for instance, Corollary 6.11 in White, 2000), so that $\widehat{IS}_m^{(d)} \xrightarrow{p} IS^{(d)}$ follows from straightforward applications of the Slutsky's theorem.. ■

Acknowledgments

We are indebted to the participants at the 14th Annual SoFiE conference in Cambridge, UK. Fernandes thanks financial support from FAPESP FAPESP/Brazil (Grants 2013/22930-0, 2019/05798-7, and 2019/08243-6) and CNPq/Brazil (302278/2018-4). We thank for the opportunity to run the computational analysis on the High Performance Computing Cluster supported by the Research and Specialist Computing Support service at the University of East Anglia. The usual disclaimers apply.

References

Andersen, T., Archakov, I., Cebiroglu, G., Hautsch, N., 2021a, Local mispricing and market microstructure noise: A parametric perspective, forthcoming at the Journal of Econometrics.

Andersen, T., Thyrsgaard, M., Todorov, V., 2021b, Recalcitrant betas: Intraday variation in the cross-sectional dispersion of systematic risk, Quantitative Economics.

Andersen, T., Varneskov, R. T., 2021, Consistent inference for predictive regressions in persistent economic systems, Journal of Econometrics.

Baillie, R. T., Booth, G. G., Tse, Y., Zabotina, T., 2002, Price discovery and common factor models, Journal of Financial Markets 5, 309–321.

Barndorff-Nielsen, O. E., Hansen, P. R., Lunde, A., Shephard, N., 2009, Realized kernels in practice: Trades and quotes, Econometrics Journal 12, C1–C32.

Benos, E., Sagade, S., 2016, Price discovery and the cross-section of high-frequency trading, Journal of Financial Markets 30, 54–77.

Brogaard, J., Brugler, J., Rösch, D., 2021, Competition and Exchange Data Fees, Working Paper.

Chakravarty, S., Gulen, H., Mayhew, S., 2004, Informed trading in stock and option markets, Journal of Finance 59, 1235–1257.

Davidson, J., 1994, Stochastic Limit Theory, Oxford University Press.

de Jong, F., 2002, Measures of contributions to price discovery: A comparison, Journal of Financial Markets 5, 323–327.

de Jong, F., Schotman, P. C., 2010, Price discovery in fragmented markets, Journal of Financial Econometrics 8, 1–28.

Dias, G. F., Fernandes, M., Scherrer, C., 2020, Price Discovery in a Continuous-Time Setting, Journal of Financial Econometrics.

Dufour, A., Engle, R. F., 2000, Time and the price impact of a trade, Journal of Finance pp. 2467–2498.

Edmans, P. B. A., Goldstein, I., 2012, The Real Effects of Financial Markets, Annual Review of Financial Economics 4, 339–360.

Elliott, G., Müller, U. K., 2006, Efficient tests for general persistent time variation in regression coefficients, Review of Economic Studies 73, 907–940.

Eun, C. S., Sabherwal, S., 2003, Cross-border listings and price discovery: Evidence from U.S.-listed Canadian stocks, Journal of Finance 58, 549–575.

Fernandes, M., Scherrer, C. M., 2018, Price discovery in dual-class shares across multiple markets, Journal of Futures Markets 38, 129–155.

Figuerola-Ferretti, I., Gonzalo, J., 2010, Modelling and measuring price discovery in commodity markets, Journal of Econometrics 158, 95–107.

Frijns, B., Gilbert, A., Tourani-Rad, A., 2015, The determinants of price discovery: Evidence from US-Canadian cross-listed shares, Journal of Banking and Finance pp. 457–468.

Frijns, B., Schotman, P., 2009, Price discovery in tick time, Journal of Empirical Finance 16, 759–776.

Giraitis, L., Kapetanios, G., Marcellino, M., In Press, Time-varying instrumental variable estimation, Journal of Econometrics.

Giraitis, L., Kapetanios, G., Yates, T., 2013, Inference on stochastic time-varying coefficient models, Journal of Econometrics 179, 46–65.

Giraitis, L., Kapetanios, G., Yates, T., 2018, Inference on multivariate heteroscedastic stochastic time varying coefficient models, Journal of Time Series Analysis 39, 129–149.

Gonzalo, J., Granger, C., 1995, Estimation of common long-memory components in cointegrated systems, Journal of Business and Economic Statistics 13, 27–35.

Grammig, J., Melvin, M., Schlag, C., 2005, Internationally cross-listed stock prices during overlapping trading hours: Price discovery and exchange rate effects, Journal of Empirical Finance 12, 139–164.

Hansen, P. R., 2003, Structural changes in the cointegrated vector autoregressive model, Journal of Econometrics 114, 261–295.

Hansen, P. R., 2005, Granger's representation theorem: A closed-form expression for I(1) processes, Econometrics Journal 8, 23–38.

Hansen, P. R., Lunde, A., 2006, Realized variance and market microstructure noise, Journal of Business and Economic Statistics 24, 127–161.

Hasbrouck, J., 1995, One security, many markets: Determining the contributions to price discovery, Journal of Finance 50, 1175–1198.

Hasbrouck, J., 2003, Intraday price formation in U.S. equity index markets, Journal of Finance 58, 2375–2399.

Hasbrouck, J., 2007, Empirical Market Microstructure: The Institutions, Economics, and Econometrics of Securities Trading, Oxford University Press, Inc.

Hasbrouck, J., 2018, Price discovery in high resolution, Halbert White Jr. Memorial JFEC invited lecture, forthcoming at Journal of Financial Econometrics.

Hupperets, E. C., Menkveld, A. J., 2002, Intraday analysis of market integration: Dutch blue chips traded in Amsterdam and New York, Journal of Financial Markets 5, 57–82.

Jacquier, E., Polson, N. G., Rossi, P. E., 1994, Bayesian analysis of stochastic volatility models, Journal of Business and Economic Statistics 12, 371–389.

Johansen, S., 1995, Likelihood-Based Inference in Cointegrated Vector Autoregressive Models, Oxford University Press.

Lien, D., Shrestha, K., 2009, A new information share measure, Journal of Futures Markets 4, 377–395.

Menkveld, A. J., 2014, High-frequency traders and market structure, Financial Review 49, 333–344.

Menkveld, A. J., 2016, The economics of high-frequency trading: Taking stock, Annual Review of Financial Economics 8, 1–24.

Menkveld, A. J., Koopman, S. J., Lucas, A., 2007, Modeling around-the-clock price discovery for cross-listed stocks using state space methods, Journal of Business and Economic Statistics 25, 213–225.

Mizrach, B., Neely, C. J., 2008, Information shares in the US Treasury market, Journal of Banking and Finance 32, 1221–1233.

O'Hara, M., 1997, Market Microstructure Theory, Blackwell Publishers, Oxford.

O'Hara, M., 2003, Presidential Address: Liquidity and Price Discovery, Journal of Finance 58, 1335–1354.

O'Hara, M., 2015, High frequency market microstructure, Journal of Financial Economics 116, 257–270.

Ozturk, S., van der Welv, M., van Dijk, D., 2017, Intraday price discovery in fragmented markets, Journal of Financial Markets 32, 28–48.

Phillips, P. C. B., Yu, J., 2009, A two-stage realized volatility approach to estimation of diffusion processes with discrete data, Journal of Econometrics.

Prakasa Rao, B., 1983, Asymptotic theory for non-linear least squares estimator for diffusion processes, Statistics: A Journal of Theoretical and Applied Statistics.

Riordan, R., Storkenmaier, A., 2012, Latency, liquidity and price discovery, Journal of Financial Markets 15, 416–437.

Robinson, P. M., 1989, Nonparametric estimation of time-varying parameters, in: P. Hackl (ed.), Statistical Analysis and Forecasting of Economic Structural Change, Springer-Verlag, Berlin, pp. 253–264.

Santosh, S., 2016, The speed of price discovery: Trade time vs clock time, Working Paper.

Scherrer, C., 2021, Information processing on equity prices and exchange rate for cross-listed stocks, Journal of Financial Markets.

Subrahmanyam, A., Titman, S., 1999, The Going-Public Decision and the Development of Financial Markets, Journal of Finance.

Tang, C. Y., Chen, S. X., 2009, Parameter estimation and bias correction for diffusion processes, Journal of Econometrics.

White, H., 2000, Asymptotic Theory for Econometricians, Academic Press.

Table 1: Relative performance of the daily component share estimators

We document the bias of the KLS and LS estimators of $\alpha^{(d)}_{\delta,\perp,1}$ as well as their relative root mean squared error (RRMSE) for sample sizes of $D = 500$ days over 1,000 replications. RRMSE figures below unit imply better performance of the KLS estimator. The instantaneous correlation between markets ρ ranges from 0 to 0.90, whereas the sampling frequency ranges from one observation per 30 seconds to one observation per 5 minutes: $\delta \in \{1/780, 1/390, 1/195, 1/130, 1/78\}$. We compute the KLS estimator using a bandwidth $H = n^{b/10}\sqrt{D}$, with $b \in \{8, 9, 10\}$.

		$100 \times \text{bias}(\widehat{\alpha}^{(d)}_{\delta,\perp,1})$					$\text{RRMSE}(\widehat{\alpha}^{(d)}_{\delta,\perp,1})$				
	ρ	$\delta = 1/780$	$\delta = 1/390$	$\delta = 1/195$	$\delta = 1/130$	$\delta = 1/78$	$\delta = 1/780$	$\delta = 1/390$	$\delta = 1/195$	$\delta = 1/130$	$\delta = 1/78$
least squares	0.00	-0.01	-0.03	0.06	0.01	0.12					
	0.30	-0.02	-0.05	-0.02	0.06	0.05					
	0.50	-0.03	-0.04	0.04	0.02	0.06					
	0.70	-0.03	-0.08	0.06	-0.04	0.01					
	0.90	-0.05	-0.32	0.13	0.14	0.17					
$H = n^{8/10}\sqrt{D}$	0.00	-0.02	-0.01	0.00	0.00	0.03	0.68	0.56	0.42	0.37	0.29
	0.30	-0.02	-0.01	-0.01	0.04	0.05	0.40	0.45	0.36	0.33	0.27
	0.50	-0.03	-0.02	0.02	0.02	0.06	0.45	0.40	0.33	0.31	0.26
	0.70	-0.04	-0.03	0.01	-0.02	0.00	0.42	0.35	0.30	0.28	0.25
	0.90	-0.07	-0.03	0.08	0.09	0.16	0.34	0.27	0.26	0.26	0.23
$H = n^{9/10}\sqrt{D}$	0.00	-0.02	-0.01	0.00	0.00	0.03	0.82	0.63	0.44	0.37	0.27
	0.30	-0.03	-0.02	-0.02	0.02	0.05	0.48	0.49	0.36	0.31	0.25
	0.50	-0.03	-0.01	0.02	0.03	0.07	0.50	0.42	0.32	0.28	0.23
	0.70	-0.05	-0.03	0.00	-0.01	-0.03	0.43	0.34	0.28	0.25	0.21
	0.90	-0.07	-0.03	0.06	0.08	0.13	0.30	0.24	0.22	0.21	0.19
$H = n\sqrt{D}$	0.00	-0.03	-0.02	-0.03	0.00	0.01	1.02	0.74	0.49	0.39	0.27
	0.30	-0.03	-0.03	-0.02	-0.02	-0.02	0.57	0.59	0.40	0.32	0.23
	0.50	-0.04	-0.02	0.00	0.00	0.02	0.60	0.48	0.34	0.28	0.21
	0.70	-0.05	-0.04	-0.01	-0.02	-0.03	0.50	0.37	0.28	0.23	0.19
	0.90	-0.08	-0.04	0.05	0.05	0.08	0.31	0.23	0.19	0.18	0.16

Table 2: Data description

We report summary statistics for raw and cleaned data for Nasdaq and NYSE/Arca. For the Nasdaq-listed stocks in the top panel, we report summary statistics of Nasdaq and Arca, whereas we consider Nasdaq and NYSE for the actively- and less-actively-traded stocks listed at NYSE respectively in the middle and bottom panels. In particular, we display the number of quotes (in millions) for each stock on the two trading venues before any cleaning filter (raw data) as well as after the implementation of the cleaning procedure (clean data). We also report the daily average number of quotes (in thousands) for both trading venues, and the total number of days we have for each stock in the sample period (January 2007 to December 2013).

	raw ('000,000)		clean ('000,000)		obs per day ('000)		number of days
	Nasdaq	NYSE/Arca	Nasdaq	NYSE/Arca	Nasdaq	NYSE/Arca	
ADBE	37	19	9	7	5.20	3.83	1,734
ALGN	188	136	23	22	13.09	12.56	1,734
AMZN	202	80	21	17	12.37	9.57	1,734
CA	176	76	19	16	11.19	8.98	1,734
EXPE	604	212	32	27	18.74	15.81	1,734
GOOG	230	94	22	19	12.90	10.88	1,734
MU	218	140	20	19	11.46	10.93	1,734
SBUX	50	28	11	9	6.39	5.44	1,734
VOD	524	204	31	27	18.05	15.60	1,734
WEN	262	114	23	20	13.44	11.65	1,734
BAC	523	503	31	34	17.85	19.05	1,734
GE	363	427	29	31	16.53	17.78	1,734
HPQ	326	277	26	28	14.84	15.86	1,734
IBM	122	149	21	25	11.96	14.13	1,734
JCP	175	149	20	22	11.55	12.26	1,734
JPM	696	542	32	33	18.43	18.64	1,734
KO	244	205	23	25	13.27	14.44	1,734
MO	178	204	22	26	12.40	14.90	1,734
VZ	264	257	25	29	14.44	16.19	1,734
XOM	503	417	31	33	18.10	18.84	1,734
CAJ	27	30	8	9	4.58	4.99	1,734
COO	21	29	7	9	4.09	5.42	1,734
DLB	22	37	7	10	4.26	6.05	1,734
DNB	24	30	8	10	4.67	5.49	1,734
OFC	29	42	9	11	5.13	6.36	1,734
RBC	25	30	8	10	4.76	5.67	1,734
RE	21	26	7	8	4.13	4.88	1,734
RES	21	28	7	9	3.82	5.24	1,734
ROL	11	28	5	8	2.74	4.51	1,734
THO	21	29	7	9	4.22	5.19	1,734

Table 3: Elliott-Müller test for daily variation in the component share estimates

The column "EM test" reports the test statistics of the Elliott-Müller test for the null hypothesis of time-invariant VEC parameters. The asymptotic critical values for a bivariate VEC are respectively -17.57, -14.32, and -12.80 at the 1%, 5%, and 10% significance levels, with ***, **, and * denoting the corresponding rejections. The remaining columns report the median and standard deviation (within parentheses) over the entire sample period of the LS and KLS daily estimates of the speed-of-adjustment parameters and of the Nasdaq component share $\alpha^{(d)}_{\perp,1}$. The subscript 2 refers to Arca for the Nasdq-listed stocks in the top panel, and to NYSE for the actively- and less-actively-traded stocks listed at NYSE in the middle and bottom panels, respectively.

	EM test	least squares			kernel least squares		
		$\alpha^{(d)}_1$	$\alpha^{(d)}_2$	$\alpha^{(d)}_{\perp,1}$	$\alpha^{(d)}_1$	$\alpha^{(d)}_2$	$\alpha^{(d)}_{\perp,1}$
ADBE	-30.37***	-0.42 (0.71)	0.49 (0.72)	0.54 (0.83)	-0.13 (0.19)	0.29 (0.30)	0.67 (0.28)
ALGN	-203.75***	-0.21 (0.26)	0.52 (0.27)	0.71 (0.37)	-0.16 (0.10)	0.40 (0.17)	0.69 (0.12)
AMZN	-28.80***	-0.39 (0.64)	0.51 (0.64)	0.57 (0.68)	-0.18 (0.18)	0.24 (0.22)	0.61 (0.19)
CA	-57.48***	-0.41 (0.65)	0.47 (0.66)	0.53 (0.82)	-0.16 (0.16)	0.27 (0.23)	0.64 (0.18)
EXPE	-15.74**	-0.35 (0.65)	0.52 (0.66)	0.59 (0.71)	-0.22 (0.19)	0.39 (0.24)	0.63 (0.30)
GOOG	-21.21***	-0.39 (0.39)	0.49 (0.36)	0.56 (0.40)	-0.26 (0.21)	0.34 (0.18)	0.56 (0.25)
MU	-7.69	-0.54 (0.81)	0.34 (0.80)	0.39 (0.93)	-0.18 (0.23)	0.11 (0.19)	0.57 (0.25)
SBUX	-58.77***	-0.40 (0.72)	0.53 (0.72)	0.57 (0.76)	-0.14 (0.26)	0.22 (0.28)	0.62 (0.38)
VOD	-30.61***	-0.54 (0.54)	0.34 (0.54)	0.39 (1.09)	-0.08 (0.33)	0.08 (0.31)	0.44 (0.52)
WEN	-35.01***	-0.44 (0.56)	0.31 (0.56)	0.43 (0.70)	-0.28 (0.24)	0.20 (0.16)	0.45 (0.30)
BAC	-18.67***	-0.59 (0.74)	0.36 (0.74)	0.38 (0.76)	-0.36 (0.29)	0.19 (0.23)	0.47 (0.30)
GE	-28.21***	-0.61 (0.72)	0.33 (0.72)	0.35 (0.77)	-0.29 (0.25)	0.20 (0.16)	0.45 (0.24)
HPQ	-56.10***	-0.48 (0.80)	0.46 (0.80)	0.48 (0.89)	-0.19 (0.35)	0.24 (0.37)	0.49 (0.45)
IBM	-21.94***	-0.55 (0.44)	0.35 (0.44)	0.39 (0.50)	-0.36 (0.22)	0.23 (0.17)	0.39 (0.19)
JCP	-57.18***	-0.53 (0.66)	0.36 (0.65)	0.39 (0.72)	-0.35 (0.24)	0.19 (0.16)	0.37 (0.24)
JPM	-3.45	-0.42 (0.90)	0.52 (0.90)	0.55 (0.93)	-0.26 (0.30)	0.27 (0.33)	0.54 (0.31)
KO	-14.74**	-0.52 (0.70)	0.39 (0.70)	0.43 (1.12)	-0.26 (0.26)	0.12 (0.17)	0.39 (0.19)
MO	-42.00***	-0.55 (0.73)	0.38 (0.73)	0.40 (0.89)	-0.27 (0.45)	0.12 (0.35)	0.38 (0.40)
VZ	-18.63***	-0.52 (0.74)	0.41 (0.73)	0.44 (1.23)	-0.22 (0.27)	0.17 (0.22)	0.44 (0.28)
XOM	-3.51	-0.40 (0.73)	0.54 (0.74)	0.57 (0.99)	-0.20 (0.26)	0.27 (0.32)	0.56 (0.28)
CAJ	-287.48***	-0.43 (0.22)	0.21 (0.19)	0.32 (0.28)	-0.42 (0.14)	0.15 (0.11)	0.30 (0.13)
COO	-132.32***	-0.50 (0.24)	0.18 (0.20)	0.26 (0.86)	-0.39 (0.19)	0.08 (0.10)	0.19 (0.13)
DLB	-143.27***	-0.51 (0.23)	0.17 (0.22)	0.23 (0.64)	-0.46 (0.21)	0.09 (0.11)	0.24 (0.18)
DNB	-55.93***	-0.49 (0.23)	0.21 (0.20)	0.29 (0.44)	-0.43 (0.21)	0.10 (0.08)	0.24 (0.23)
OFC	-469.83***	-0.56 (0.23)	0.18 (0.21)	0.24 (1.15)	-0.45 (0.21)	0.12 (0.08)	0.21 (0.11)
RBC	-1,790.14***	-0.47 (0.20)	0.17 (0.20)	0.25 (0.34)	-0.43 (0.15)	0.11 (0.12)	0.23 (0.16)
RE	-37.61***	-0.49 (0.22)	0.20 (0.20)	0.29 (0.33)	-0.46 (0.22)	0.09 (0.13)	0.23 (0.14)
RES	-68.25***	-0.49 (0.21)	0.18 (0.20)	0.26 (0.35)	-0.49 (0.20)	0.11 (0.11)	0.25 (0.17)
ROL	-201.00***	-0.46 (0.20)	0.12 (0.16)	0.20 (0.85)	-0.45 (0.19)	0.07 (0.08)	0.16 (0.12)
THO	-134.02***	-0.52 (0.22)	0.19 (0.19)	0.26 (0.53)	-0.51 (0.21)	0.12 (0.08)	0.25 (0.10)

Table 4: Long-run relationship between price discovery and liquidity measures

We report the results of a VEC(8) for the daily price discovery measure and relative volume at the listing venue. We report the cointegrating vector estimates and their standard errors (note that we omit the one related to the price discovery measure as we force its loading to one) as well as the corresponding speeds of adjustment for the price discovery measure and for the relative volume. Finally, the last column displays the adjusted R^2 to gauge how much the VEC explains of the overall variation of the daily changes in the price discovery measure.

	cointegrating vector		speed of adjustment		R^2
	intercept	relative volume	component share	relative volume	
ADBE		−0.2107 (0.0253)	−0.0020 (0.0004)	0.0443 (0.0471)	0.8339
ALGN		−0.3196 (0.0269)	−0.0015 (0.0004)	0.1971 (0.0505)	0.8321
AMZN		−0.5301 (0.0488)	−0.0014 (0.0004)	0.0765 (0.0274)	0.8530
CA		−0.5557 (0.0352)	−0.0011 (0.0003)	0.1430 (0.0342)	0.8448
EXPE		−0.2456 (0.0263)	−0.0024 (0.0005)	0.1232 (0.0477)	0.6496
GOOG		−0.4878 (0.0354)	−0.0048 (0.0013)	0.1264 (0.0328)	0.4437
MU		−0.7771 (0.0713)	−0.0014 (0.0004)	0.0552 (0.0203)	0.8273
SBUX		−0.4962 (0.0274)	−0.0009 (0.0003)	0.1644 (0.0388)	0.8770
VOD		−0.5485 (0.1147)	−0.0008 (0.0003)	0.0797 (0.0257)	0.8638
WEN		−0.7331 (0.1962)	−0.0004 (0.0002)	0.0611 (0.0214)	0.6495
BAC	−0.0749 (0.1732)	−1.6909 (0.3827)	-7.33×10^{-5} (0.0002)	0.0417 (0.0097)	0.4772
GE		−1.0747 (0.0825)	−0.0011 (0.0006)	0.1242 (0.0228)	0.4387
HPQ		−1.9721 (0.2823)	-7.16×10^{-5} (0.0003)	0.0482 (0.0102)	0.2347
IBM		−0.5962 (0.0617)	−0.0049 (0.0014)	0.1211 (0.0282)	0.2520
JCP		−1.8361 (0.2824)	-5.33×10^{-5} (0.0005)	0.0403 (0.0091)	0.2648
JPM	−0.3339 (0.0926)	−1.1122 (0.2241)	−0.0010 (0.0004)	0.0678 (0.0140)	0.3263
KO		−0.6748 (0.0501)	−0.0054 (0.0013)	0.1499 (0.0323)	0.2405
MO		−0.4463 (0.0552)	−0.0040 (0.0014)	0.1324 (0.0362)	0.2416
VZ		−0.9991 (0.0864)	−0.0032 (0.0012)	0.1224 (0.0238)	0.1291
XOM		−1.0343 (0.0897)	−0.0011 (0.0007)	0.1150 (0.0214)	0.3262
CAJ	1.8425 (0.3354)	−1.9411 (0.2674)	−0.0013 (0.0010)	0.1048 (0.0163)	0.2618
COO		−0.8027 (0.0782)	−0.0029 (0.0015)	0.1240 (0.0272)	0.2571
DLB		−0.9201 (0.0507)	−0.0022 (0.0013)	0.1812 (0.0284)	0.4880
DNB		−0.9477 (0.0768)	−0.0032 (0.0015)	0.1193 (0.0229)	0.4829
OFC		−0.5100 (0.0221)	−0.0043 (0.0015)	0.2536 (0.0477)	0.2745
RBC	0.4429 (0.1501)	−1.1042 (0.1401)	−0.0012 (0.0008)	0.1637 (0.0256)	0.2699
RE	0.4619 (0.1594)	−1.1997 (0.1619)	−0.0012 (0.0012)	0.1379 (0.0223)	0.2810
RES		−0.8941 (0.0446)	−0.0009 (0.0008)	0.2108 (0.0320)	0.2652
ROL		−0.4790 (0.0343)	−0.0012 (0.0008)	0.1832 (0.0404)	0.2576
THO		−0.6285 (0.0357)	−0.0032 (0.0011)	0.2305 (0.0395)	0.2421

Figure 1: KLS estimates of the daily component shares for the Nasdaq-listed stocks

The plots depict the KLS estimates of $\alpha_{\perp}^{(d)}$ with their 95% confidence bands (in shades) for the 10 actively-traded Nasdaq listed stocks. We fix the bandwidth at $n\sqrt{D}$, where n is the number of intraday observations (average of 390 observations per day) and $D = 1,735$ is the number of trading days. We normalize the orthogonal complements such that the elementwise estimates sum up to one, i.e., $\widehat{\alpha}_{1,\perp}^{(d)} + \widehat{\alpha}_{2,\perp}^{(d)} = 1$, with subscripts 1 and 2 denoting Nasdaq and Arca, respectively.

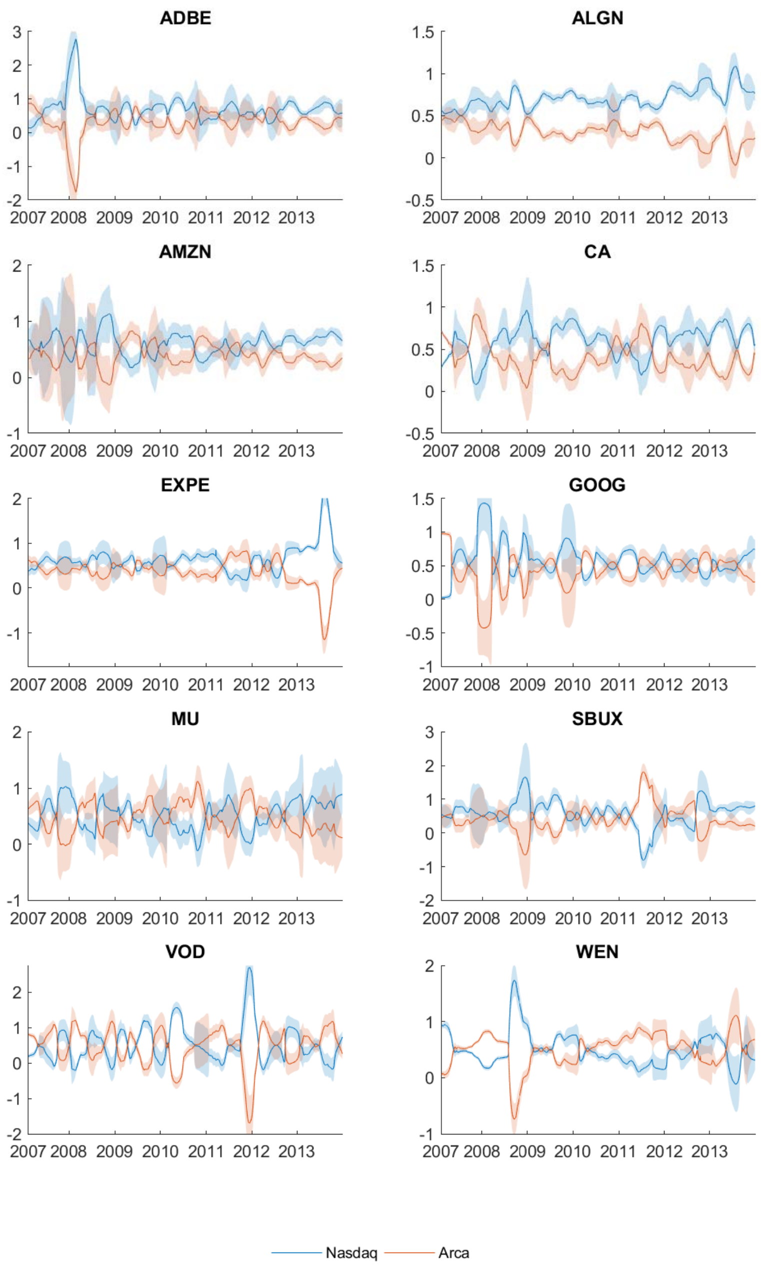

Figure 2: KLS estimates of the daily component shares for the actively-traded NYSE-listed stocks

The plots depict the KLS estimates of $\alpha_{\perp}^{(d)}$ with their 95% confidence bands (in shades) for the 10 actively-traded NYSE-listed stocks. We fix the bandwidth at $n\sqrt{D}$, where n is the number of intraday observations (average of 390 observations per day) and $D = 1,735$ is the number of trading days. We normalize the orthogonal complements such that the elementwise estimates sum up to one, i.e., $\widehat{\alpha}_{1,\perp}^{(d)} + \widehat{\alpha}_{2,\perp}^{(d)} = 1$, with subscripts 1 and 2 denoting Nasdaq and NYSE, respectively.

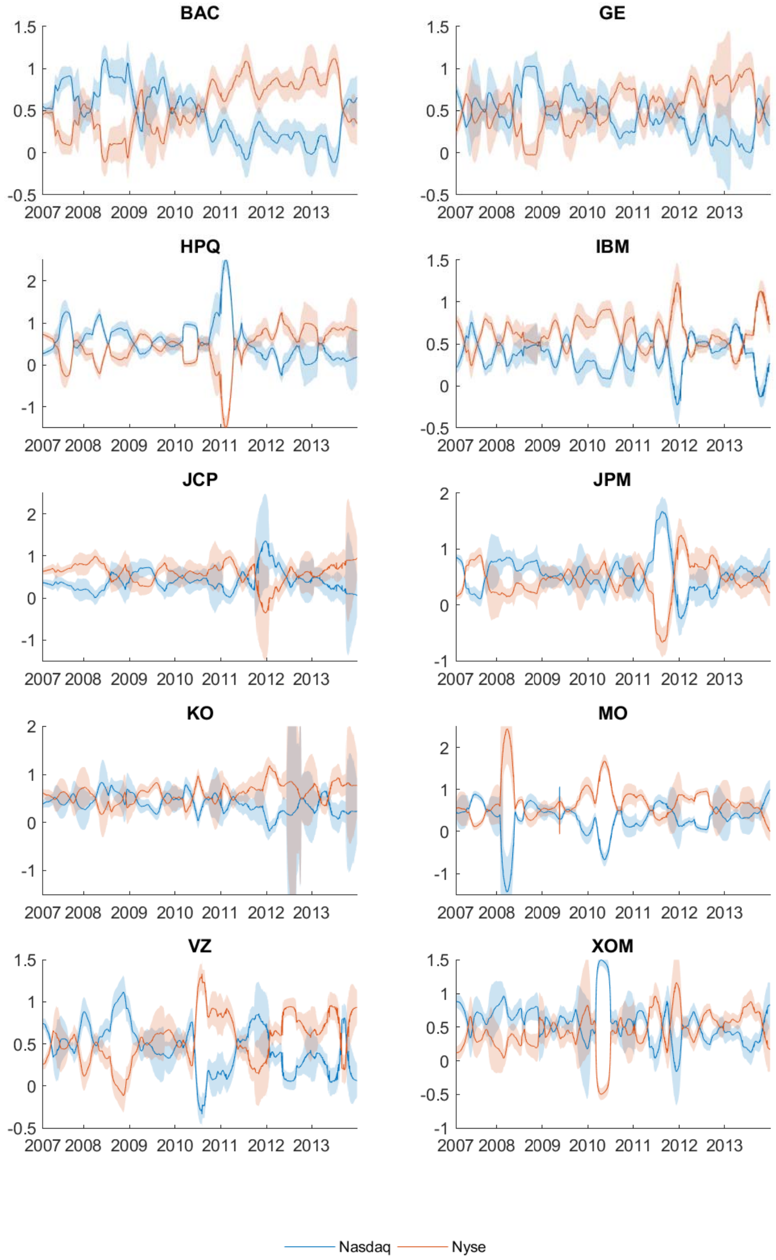

Figure 3: KLS estimates of the daily component shares for the less-liquid NYSE-listed stocks

The plots depict the KLS estimates of $\alpha_{\perp}^{(d)}$ with their 95% confidence bands (in shades) for the 10 less-actively-traded NYSE-listed stocks. We fix the bandwidth at $n\sqrt{D}$, where n is the number of intraday observations (average of 390 observations per day) and $D = 1,735$ is the number of trading days. We normalize the orthogonal complements such that the elementwise estimates sum up to one, i.e., $\widehat{\alpha}_{1,\perp}^{(d)} + \widehat{\alpha}_{2,\perp}^{(d)} = 1$, with subscripts 1 and 2 denoting Nasdaq and NYSE, respectively.

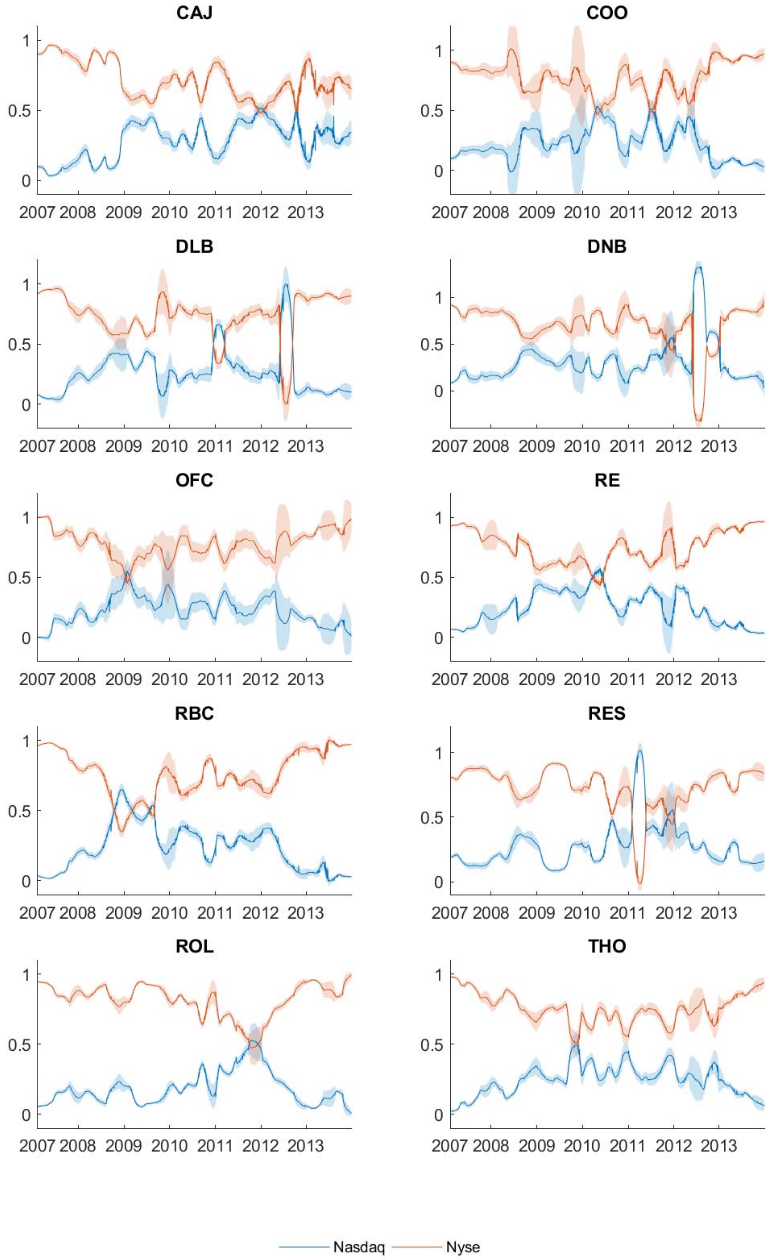

Figure 4: Relative trading volume

The plots portray the volume shares of Nasdaq and NYSE/Arca for each stock. For example, the volume share of Nasdaq for a NYSE-listed stock corresponds to the trading volume at Nasdaq over the sum of the trading volume at Nasdaq and NYSE. Recall that, for Nasdaq-listed stocks, we report volume shares of Nasdaq and Arca.

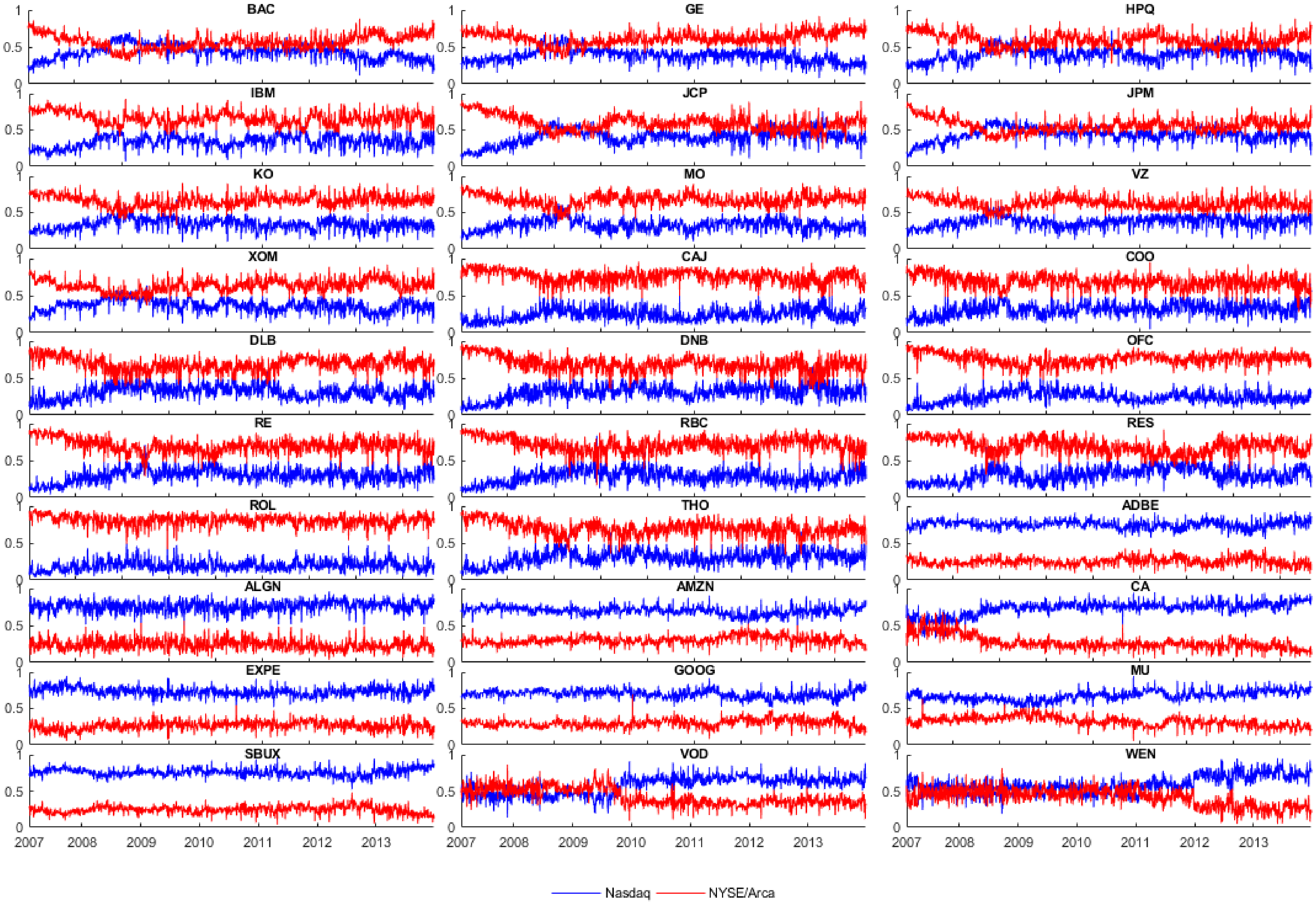

Figure 5: Trades and Quotes: KLS estimates of the daily component shares, with 95% confidence intervals

The plots portray the KLS daily estimates of $\alpha_{\perp}^{(d)}$ with their 95% confidence bands (in shades) for 5 actively-traded NYSE-listed stocks. We fix the bandwidth at $n\sqrt{D}$, where n is the number of intraday observations (average of 390 observations per day) and $D = 1,735$ is the number of trading days. We normalize the orthogonal complements such that the element wise estimates sum up to one, i.e., $\widehat{\alpha}_{1,\perp}^{(d)} + \widehat{\alpha}_{2,\perp}^{(d)} = 1$, with subscripts 1 and 2 denoting Nasdaq and NYSE, respectively. The first column presents the estimates using transactions prices, whereas the second column displays the estimates considering both quotes and trades at NYSE.