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Lerby M. Ergun
Andreas Uthemann

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Lerby Murat Ergun, Bank of Canada and Systemic Risk Centre, London School of Economics

Andreas Uthemann, Bank of Canada and Systemic Risk Centre, London School of Economics

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Higher-Order Uncertainty in Financial Markets: Evidence from a Consensus Pricing Service*

Lerby M. Ergun[1,2] and Andreas Uthemann[1,2]

[1]*Bank of Canada*
[2]*Systemic Risk Centre, LSE*

February 2020

Abstract

We assess the ability of an information aggregation mechanism that operates in the over-the-counter market for financial derivatives to reduce valuation uncertainty among market participants. The analysis is based on a unique dataset of price estimates for S&P 500 index options that major financial institutions provide to a consensus pricing service. We consider two dimensions of uncertainty: uncertainty about fundamental asset values and strategic uncertainty about competitors' valuations. Through structural estimation, we obtain empirical measures of fundamental and strategic uncertainty that are based on market participants posterior beliefs. We show that the main contribution of the consensus pricing service is to reduce its subscribers' uncertainty about competitors' valuations.

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*Uthemann (corresponding author): Financial Markets Department, Bank of Canada, 234 Wellington St W, Ottawa, ON K1A 0G9 (authemann@bank-banque-canada.ca). Ergun: Financial Markets Department, Bank of Canada (mergun@bank-banque-canada.ca). We are grateful for comments from Jason Allen, Enrique Salvador Aragó (discussant), Emanuel Moench (discussant), Jon Danielsson, Darrell Duffie, Jean-Sébastien Fontaine, Yifan Gong (discussant), Antonio Guarino, Myra Mohnen, Christine Parlour, Dongho Song, Haoxiang Zhu, Jean-Pierre Zigrand, IHS Markit's Totem team and seminar participants at the 2018 Paris December Finance meeting, 2018 NBER-NSF SBIES Conference, 2018 North American Summer Meeting of the Econometric Society, 2018 CEPR Spring Meeting in Financial Economics, 2017 European Summer Meeting of the Econometric Society, the Bank of Canada, Bank of England, Federal Reserve Board, Tinbergen Institute, Cass Business School, University of Sussex, Catolica Lisbon, Nova Lisbon, University of Bristol, King's College, London, Ecole Polytechnique, Paris, and the University of Copenhagen. This work has been supported by the Economic and Social Research Council (ESRC) through funding provided to the Systemic Risk Centre [grant number ES/K002309/1]. The views expressed in this paper are solely those of the authors and do not necessarily represent those of the Bank of Canada.

1 Introduction

In financial markets, prices serve a dual purpose. They aggregate dispersed information about asset values. At the same time, they help to coordinate actions among market participants. Empirical work on the informational content of prices typically focuses on their ability to reduce valuation uncertainty. However, their ability to reduce uncertainty about other market participants' actions and beliefs, that is strategic uncertainty, can be of equal importance in markets with strong coordination motives (Angeletos and Pavan (2007); Morris and Shin (2002)). Episodes of financial markets stress provide ample evidence for this fact. Lowenstein (2000), for example, gives a vivid account of the bond market at the height of the LTCM crisis on August 31, 1998:

> "It was as if a bomb had hit; traders looked at their screens, and the screens stared blankly back. [...] So few issues traded, you had to guess where they were." (p.159)

Prices provide a public signal about actions and beliefs. At an institutional level price data are used to value trading books, manage risk exposures and provide reference prices for trading. As the above quote illustrates, the lack of price data threatens the common understanding of market conditions among market participants. This can lead to costly coordination failures (Morris and Shin (2012)). Trading, however, is not the only available mechanism to generate prices. In many markets, alternative information aggregation mechanisms exist to address a temporary or permanent shortage of price information. A popular type of mechanism in financial markets is consensus pricing. A consensus pricing service collects price estimates from market participants and aggregates these estimates into a so-called consensus price. Consensus prices are available to market participants irrespective of the level of trading activity in the underlying market.

In this paper we develop a structural framework to quantify the price informativeness and informational efficiency of a consensus pricing mechanism. We evaluate these criteria for both valuation uncertainty and strategic uncertainty. The analysis is based on a novel dataset of individual price estimates that major derivatives dealers provide to a consensus pricing service.[1] We model dealers as Bayesian agents that learn about a time-varying fundamental asset value. Each dealer receives two noisy signals about this asset value. The first is a noisy private signal about the current fundamental value of the asset. The second signal is a consensus

[1] Derivatives dealers, typically large banks, stand ready to satisfy client demand for derivatives and manage the risk from these positions by either hedging it with other instruments or entering offsetting trades.

price; it is the average of all submitters' estimates of the asset value. The consensus price is modelled as an endogenous signal. It derives from the submitters' learning process and, consequently, its informational content is an equilibrium outcome.

Our measures of valuation and strategic uncertainty are based on submitters' beliefs. In particular, we use the variances of a submitter's posterior beliefs about the asset value to measure valuation uncertainty. We use the posterior variance of their beliefs concerning other submitters' valuations to measure strategic uncertainty. To evaluate the informational properties of the consensus price feedback, we perform two counterfactual experiments for the market's information structure. First, to gauge the informativeness of the consensus price, we counterfactually eliminate the consensus price mechanism and study how posterior beliefs are affected by this change in submitters' information sets. Second, we evaluate the informational efficiency of the consensus price, that is how well it aggregates dispersed information. To do so, we compare the actual information structure of the market to a counterfactual setting in which submitters pool their private information.

To estimate our model we use a proprietary panel dataset of consensus prices and the corresponding individual submissions for S&P500 index options. These data are collected by IHS Markit's Totem service, a major consensus pricing service in the derivatives market. Being able to track individual submitters' price estimates as well as the consensus feedback they receive allows us to estimate our structural model. An additional, unique feature of these data, is that broker-dealers submit to option contracts for a fixed underlying asset but varying times-to-expiration and moneyness, that is varying extremeness of the insured asset price movements. As times-to-expiration and moneyness of the options become more extreme, the dominant market structure changes from centralized options exchanges to over-the-counter (OTC) trading. Trade in the OTC market is bilateral without the intermediation of an exchange. In such markets, transaction prices tend to be proprietary information of the trading partners. The variation in dominate market structure allows us to estimate the importance of a consensus pricing service in information poor and rich environments.

The estimation of the structural model, contract by contract, reveals important variation in the informational properties of the consensus price for valuation and strategic uncertainty. For contracts that are mainly traded on exchanges, we find that a precise private signal renders the consensus price essentially redundant. Dealers put little weight on the consensus price when updating their beliefs. For contracts that are typically traded in the OTC segment of the market, dealers assign a higher weight to the consensus price. Surprisingly, we find that the consensus price is not an important source of information for contract valuation. The reduction in valuation uncertainty by gaining access to the consensus price is at most 4.6%. However, dealers find the consensus price to be highly informative about the strategic aspect of the market. For options with extreme contract terms the reduction in strategic uncertainty ranges from 5% to 37.8%. We find consensus prices in market segments that overlap with exchange-based trading to be fully efficient. For the extreme contracts that are traded OTC, a fully efficient price could contribute an additional 33.5% reduction in valuation uncertainty and an additional 66.1% in strategic uncertainty. This demonstrates the importance of publicly observable valuation data, such as benchmarks, to establish a shared understanding of market conditions in OTC markets.

The structural estimation framework developed in this paper makes a methodological contribution to the empirical measurement of the informational efficiency and informativeness of prices. The modern conceptual framework for these questions dates back to the early 1980s with seminal contributions by Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981).[2] However, as pointed out by Townsend (1983), determining the informational content of the price process in a dynamic equilibrium context poses significant technical challenges. Typically, the applied literature side-steps these problems. Most structural empirical analyses of price formation in stock markets, for example, follow the information structure in Easley, Kiefer, O'Hara, and Paperman (1996) and assume that asset values do not change intraday and become common knowledge at the end of a trading day. In this paper, we are particularly interested in the ability of the price process to track an uncertain, and importantly, changing asset value. This evolving source of uncertainty paired with privately informed market participants is a key source of belief heterogeneity and, hence, strategic uncertainty in our model. We adopt an algorithm developed in Nimark (2017) to solve the dynamic signal extraction problem and structurally estimate the model. It is well known that the mix between public and private information is an important determinant for the speed of information aggregation (Vives (1997), Amador and Weill (2012)). As we model the consensus price as an endogenous public signal, we can conduct counterfactual experiments on the market's information structure to evaluate the strength of informational externalities caused by public information.

A novel aspect of our empirical approach is the focus on strategic uncertainty.

[2]The modern literature on information aggregation is too large to do justice to here. Important contributions have focused on auctions (Pesendorfer and Swinkels (1997); Kremer (2002)), decentralized trading (Gale (1986), Golosov, Lorenzoni, and Tsyvinski (2014)), asset design (Ostrovsky (2012)) or the trade-off between market size and information heterogeneity (Rostek and Weretka (2012)).

Here, the structural approach is particularly useful as data on market participants' higher-order beliefs are typically not available. We can use model-implied higher-order beliefs to assess how price information impacts strategic uncertainty. Empirical work on auctions has taken a similar structural measurement approach to evaluate the strategic value of information. Hortaçsu and Kastl (2012) and Hortaçsu, Kastl, and Zhang (2018) work on Treasury auctions, for example, uses model-implied beliefs derived from a structural estimation to gauge the strategic value dealers derive from being able to observe client demand. Similarly, Boyarchenko, Lucca, and Veldkamp (2019) use a calibrated model to perform counterfactual informational experiments in the US Treasury market to evaluate the welfare implications of different information-sharing arrangements among dealers and clients. However, the source of strategic information in these models is order flow information rather than price data. More generally, we see the counterfactual experiments we perform on the market's information structure as an illustration of the usefulness of our structural approach for empirical work on information design (Bergemann and Morris (2019)).

This paper is, to our knowledge, the first to provide an empirical evaluation of the informational properties of a consensus pricing mechanism. The consensus pricing mechanism itself is widely used in financial markets. Many important financial benchmarks are consensus prices. It is also employed by information providers, such as Bloomberg, to calculate generic prices for a wide range of financial products. The manipulation of major interest rate benchmarks has led to a regulatory push to base benchmarks on transaction prices or firm quotes rather than expert judgment (IOSCO (2013), Financial Stability Board (2014)). However, in illiquid markets and during crisis times this might not always be possible. It is thus crucially important to understand whether, and how, a consensus pricing mechanism works in practice. Duffie, Dworczak, and Zhu (2017) show how benchmarks can reduce informational asymmetries in search markets and thereby increase the participation of less-informed agents. Here, we focus on the ability of consensus prices to aggregate dispersed information among symmetrically informed derivatives dealers. This also contributes more widely to understanding of the informational value of non-transaction based price information. Previous work in this area has focused on information aggregation mechanisms that operate in stock markets, in particular pre-opening prices (Biais, Hillion, and Spatt (1999), Cao, Ghysels, and Hatheway (2000)) and opening auctions (Madhavan and Panchapagesan (2000)).

The plan of the paper is as follows. Section 2 explains the Totem consensus pricing service and provides summary statistics for our data. Section 3 develops a theoretical model of consensus pricing. Sections 4 and 5 explain the estimation of the model and presents the results. Section 6 concludes.

2 Data

2.1 Consensus Pricing

The empirical analysis of the paper is based on data from a consensus pricing service for financial derivative contracts, IHS Markit's Totem service. Consensus pricing services allow market participants to anonymously share valuation information. The service, in turn, aggregates individual contributions into so-called consensus prices and feeds those back to the contributors. Consensus pricing services mostly operate in OTC markets where they address market participants' demand for reliable price data. These are needed to value books, manage risk exposures and provide reference prices for trading. Totem is such a service for the OTC derivatives market. The service started in February 1997 with 6 major OTC derivatives dealers. Since then, Totem has become the leading platform for OTC consensus price data with around 120 participants and a coverage of all major asset classes and types of derivatives contracts.

The pricing services typically operate at a monthly frequency. At the end of the month, contributors are issued spread sheets into which they enter the relevant price data. In addition to their estimate of the contract price itself, it includes other data used in the pricing of the contract such as discount factors, dividend yields, the price of the underlying asset, etc.[3] Price submissions that are deemed problematic do not enter the consensus price calculation and the submitting institution does not receive the consensus price for the submission period. The ability to deprive a submitting institutions of the consensus price information serves as an incentive mechanism to induce participants to submit high quality price data.[4] Accepted price submission are then used to calculate consensus prices, one for each derivatives contract. The consensus price for a given derivatives contract is

[3] All submitters are asked to provide their best estimate of the mid-quotes (at a pre-specified time on the so-called valuation day) for the set of derivatives contracts they participate in. For a detailed description of the submission process and the quantities submitted see Appendix 7.7.

[4] Manipulation incentives for consensus prices of OTC derivatives are generally weaker than for benchmark interest rates, such as Libor, that are compiled using a similar method. Unlike in the benchmark interest rate case, no financial products in the OTC derivatives markets are indexed to consensus prices. Hence, changes in consensus prices do not immediately impact an institution's profits and losses. Additionally, the Totem consensus pricing service has significantly more submitters than interest rate benchmarks, on average 30 per contract, which makes strategic manipulation of the consensus price more difficult.

the arithmetic mean of the accepted price estimates.[5] Totem provides contributors with the new consensus prices within 5 hours of their initial price submissions.

We have access to the full history of Totem contributors' price submissions for European put and call options on the S&P 500 index for the period December 2002 to February 2015. The individual institutions are anonymised, but we can track each institution's submissions over time and across contracts. Our baseline sample consists of option contracts with moneyness, expressed as the ratio of the option's strike price to the current index level, between 60 and 200 and times-to-expiration between six months and seven years. All options are either out-of-the-money or at-the-money. This implies that all options with moneyness below 100 are put options and those with moneyness above 100 are call options. In total we consider 78 distinct option contracts with, on average, 30 submitters per contract.[6]

2.2 Market structure

Put and call options on the S&P 500 index are arguably the central derivatives contract for the equity market. Their prices contain rich information on market participants' beliefs about future US stock market movements and risk premia. Furthermore, they are a key input for the pricing of more exotic derivatives products.

The dominant market structure for options trading depends on the terms of the contract. Option contracts with times-to-expiration of less than 6 months and strike prices close to the current index value are typically traded on options exchanges, such as the Chicago Board Options Exchange (CBOE), via limit order books. Price quotes, transaction prices and volumes are fully transparent and available to all market participants. For options with more extreme contract terms the dominant market structure is OTC. The OTC market is centred around a network of dealers, typically large banks, that act as market-makers and trade with each other or with clients such as insurers, asset managers, and pension funds. Trades are negotiated bilaterally, often over the phone, email or instant messaging. For OTC trades, both transaction price and volume remain proprietary information of the two parties involved in the trade.[7] Figure 1 displays the average on-exchange

[5]Typically, the highest and lowest price among the accepted prices are dropped from the sample before the mean is calculated. In the case of fewer than 7 accepted price submissions, the highest and lowest price are included in the mean.

[6]Tables 7 and 8 in the Appendix show the range of available option contracts with corresponding sample period and average number of submitters.

[7]Some dealers run proprietary electronic trading platforms on which they post price quotes.

trading activity for put and call options on the S&P 500 index for the period 1998 to 2015. On-exchange trading activity is tailing off in the time-to-expiration and the extremeness of strike price. In the "outer regions" of the time-to-expiration/strike price space, trading is exclusively OTC.[8]

2.3 Valuation differences among dealers

To provide a sense of the cross-sectional dispersion in Totem submitters' option valuations, the left panel of Figure 2 depicts the cross-sectional standard deviation of price submissions, averaged over the sample period. Throughout the paper, we express option prices in terms of Black-Scholes implied volatilities (IVs). This is the market convention for quoting option prices. It facilitates the comparison of option prices across times-to-expiration and strike prices. There is considerable variation in the dispersion in submitters' prices across the contract space. It is highest for short-dated options with extreme strike prices. For a given time-to-expiration, the dispersion is lowest for strike prices close to the current index level, that is a moneyness of 100. The price dispersion across submitters tends to decrease with time-to-expiration. The cross-sectional differences in Totem submissions are economically meaningful; they are of similar magnitude to bid-ask spreads observed on option exchanges in regions where OTC and on-exchange trading overlaps, but they display a low level of correlation with these bid-ask spreads over time, as seen in Figure 6 in the Appendix.

The right panel of Figure 2 shows how persistent individual submitters' deviations from the consensus price are. For each contract, we estimate the following AR(1) regressions

$$p^c_{i,t} - p^c_t = \beta^c \left(p^c_{i,t-1} - p^c_{t-1}\right) + \epsilon^c_{i,t},$$

where $p^c_{i,t}$ is institution i's price submission for contract c in period t and p^c_t is the corresponding consensus price. The right panel of Figure 2 reports the estimated

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In 2010, various electronic trading platforms introduced request-for-quote systems to further increase pre-trade price transparency. The regulatory reforms after the financial crisis have also introduced mandatory post-trade reporting to trade repositories for all OTC derivatives transactions. These are regulatory data and not available to market participants. Another source for price and volume information in OTC markets are central counterparties (CCPs). However, unlike for interest rate and credit derivatives, OTC equity derivatives trades are currently not subject to a central clearing mandate. The current proportion of OTC equity derivatives trades that is centrally cleared is negligible (see Financial Stability Board (2018)).

[8]For the overlapping region traders might choose an OTC over an on-exchange trade due to cost or market impact considerations. If a master agreement is already in place between two parties, direct trading can be cost saving, especially in case of large trades. Furthermore, large customized client originated trades might be difficult to hedge for the dealer if the original trade is publicized. Here, the pre-trade transparency of on-exchange trading is undesirable.

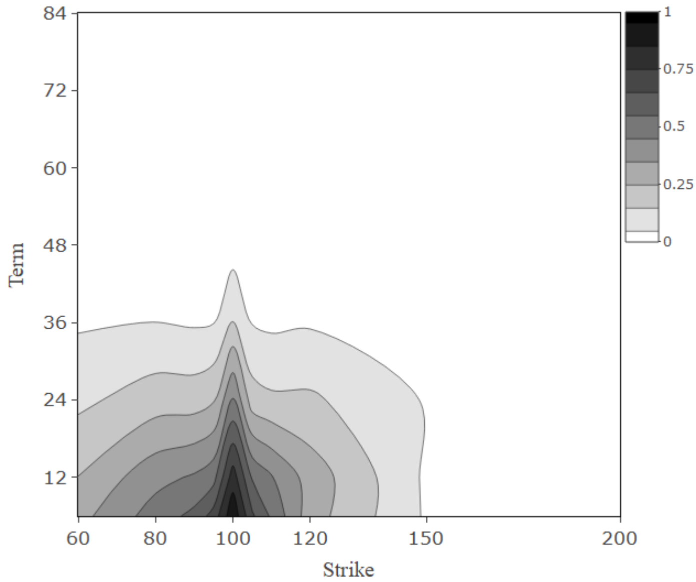
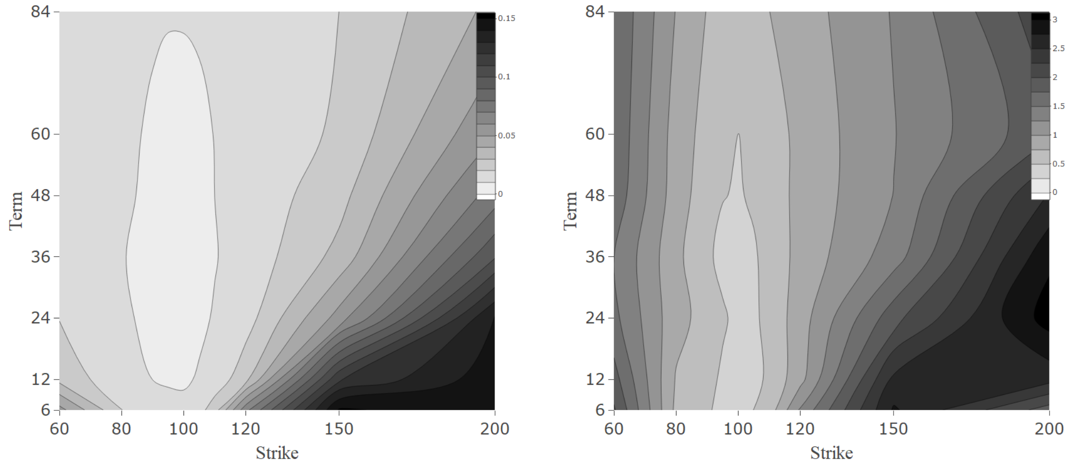


Figure 1: IHS Markit Totem surface vs on-exchange option trade

This figure presents the average percentage of days in a month options on the S&P500 index are traded between 1998 and 2015. For a particular submission date we sum the days of trade from the day after the previous submission date till the current submission date. Only days where the total volume is 10 contracts or more are included as trading days. The data includes all the put and call options traded at the specified moneyness. Due to the coarse grid of the options reported to the Totem service, exchange traded contracts in the proximity of a Totem contract are aggregated to one point. Proximity is defined here as less than half the distance to the next totem contract by moneyness and time-to-expiration. The data is provided by OptionMetrics.

β coefficients expressed as half-lives. Submitters' deviations from consensus are persistent for all contracts. The persistence pattern partially mirrors the cross-sectional dispersion in the left panel of Figure 2. The degree of persistence is U-shaped in the moneyness dimension. Deviations from consensus are least persistent for short-dated options with strike prices close to the current index level.

From the data, we draw some preliminary observations that guide our structural modelling:
1. All submitters are asked to provide their best estimate for the mid-quote of a given contract, i.e. a market-wide price. If all dealers had access to the same information and assumed the same structure for the market, they should all provide the same price estimate. In this paper we abstract from model disagreement or model uncertainty and assume that submitters form expectations via Bayesian updating. Under this interpretation of the data, the cross-sectional dispersion



(a) CS dispersion

(b) Half-lifes

Figure 2: The **left figure** displays the time-series average of the cross-sectional standard deviation of submitters implied volatility estimates to a particular contract. The **right figure** present half-lives estimates of the individual deviations from the contemporaneous consensus price. The half-lives are transformations from an AR(1) regression. The estimates are from a pooled ordinary least squares regression. The y-axis of each figures gives the time-to-expiration and the x-axis the moneyness of the options contract under consideration. The sample period of the data is December 2002 to February 2015 for the option contracts on the S&P500 index.

reveals informational frictions in the OTC market. These frictions vary across market segments.

2. The informational frictions revealed by the cross-sectional dispersion have to derive from submitting institutions' private information. Imperfect information that is observed by all submitters does not induce cross-sectional dispersion. However, the cross-sectional dispersion alone cannot identify the precision of private valuation information as both very precise and very imprecise private information imply low cross-sectional dispersion.

3. If the consensus price perfectly aggregated dispersed information, then all institutions should have the same expectation about the mid-quote after observing the current consensus price. Any deviation from consensus has to be driven by new private information. In this case, deviations from consensus cannot be persistent; an institution's past relative position to the consensus price has no predictive power for its future relative position. This is clearly rejected by the data. The positive persistence points to imperfect information aggregation and, consequently, long-lived private information at the level of individual submitters.

3 A Model of Consensus Pricing

We develop a model of consensus pricing that captures the most important features of the consensus pricing process and is able to generate key features of the data. The consensus price is the equilibrium outcome of a social learning process. Its informational content is an equilibrium outcome of the model. We obtain measures of valuation and strategic uncertainty based on submitters' beliefs.

3.1 The model

A large number of dealers, modelled as a continuum indexed by $i \in (0, 1)$, participate in a consensus pricing service. At discrete submission dates, indexed by $t = \{..., -1, 0, 1, ...\}$, each dealer i submits its best estimate for the current value of a latent stochastic process (θ_t), the fundamental value of a contract, to the service. The stochastic process itself evolves according to

$$\theta_t = \rho\, \theta_{t-1} + \sigma_u\, u_t \text{ with } u_t \sim N(0, 1), \tag{1}$$

and $-1 < \rho < 1$.[9]

At each submission date t, submitters observe two signals. Each institution receives a noisy private signal $s_{i,t}$ about θ_t,

$$s_{i,t} = \theta_t + \sigma_\eta\, \eta_{i,t} \text{ with } \eta_{i,t} \sim N(0, 1),$$

where $1/\sigma_\eta^2$ measures the precision of the private signal. All dealers receive signals are of equal quality.

Additionally, each institution observes last period's consensus price p_{t-1}. This timing assumption is a key difference to standard rational expectations equilibrium (REE) models. The available consensus price is a signal of last period's state.[10] The consensus price p_t is a noisy average of submitters' best estimates of θ_t. Submitter i's information set at the time of period t's consensus price submission consists of the (infinite) history of previous consensus prices and the private

[9] We do not explicitly model the economic forces responsible for the variation in fundamental values. A possible interpretation is based on demand-based option pricing models (see, for example, Gârleanu, Pedersen, and Poteshman (2009)). Changes in fundamental values derive from time-varying client demand that is satisfied by risk-averse broker-dealers. Under this interpretation, u_t is an aggregate demand shock for options with a specific strike price and maturity combination.

[10] This feature of the consensus pricing mechanism allows us to avoid certain technical difficulties that arise in the REE literature. Given that it is a signal of the past state of the market, it can never fully reveal the current state.

signals that i has observed up to period t, that is

$$\Omega_{i,t} = \{s_{i,t}, p_{t-1}, \Omega_{i,t-1}\}.$$

All dealers submit their best estimate of θ_t. For each dealer, we take this to mean its conditional expectation of θ_t given $\Omega_{i,t}$.[11] We denote this conditional expectation by

$$\theta_{i,t} = \mathbb{E}\left(\theta_t | \Omega_{i,t}\right),$$

and the corresponding cross-sectional average across submitters by

$$\bar{\theta}_t = \int_0^1 \theta_{i,t}\, di.$$

The consensus price is a noisy signal of this average expectation,

$$p_t = \bar{\theta}_t + \sigma_\varepsilon\, \varepsilon_t \text{ with } \varepsilon_t \sim N\left(0, 1\right). \tag{2}$$

Modelling the consensus price as a noisy public signal of average expectations is motivated by two considerations. First, Totem eliminates some submitted prices from the consensus calculations and, hence, the consensus price itself does not exactly correspond to the average submission. The parameter σ_ε can capture this divergence between consensus price and the cross-sectional average of submissions. Second, we want to allow for the possibility that the consensus price does not fully reveal the average expectation. Given a continuum of submitters, an assumption we need for technical tractability, a consensus price that fully reveals last period's average expectation allows submitters to perfectly learn last period's fundamental value. But this rules out long-lived private information which is needed to capture the persistence of the deviations of individual price submissions from the consensus price, a feature we observe in our data.

3.2 Learning from consensus prices

In order to characterise dealer i's submission to the consensus pricing service, we need to calculate the dealer's conditional expectation $\mathbb{E}\left(\theta_t | \Omega_{i,t}\right)$. Its information

[11] We do not specify submitters' preferences, which would determine why they value the consensus price information. Certain preference specifications could create an incentive to strategically manipulate the consensus price, for example in order to experiment or to gain a competitive advantage (see Brancaccio, Li, and Schürhoff (2017) for experimentation motives in OTC markets). However, given the assumption of a continuum of submitters (and mild technical restrictions on admissible submissions), no single submitter can influence the consensus price. Hence, asking submitters to submit their best estimate of θ_t is compatible with their incentives. Raith (1996) gives a theoretical analysis of the incentives for competitive firms to participate in (truthful) information sharing arrangements. Appendix 7.8 provides an example of how to approach welfare questions within the context of our model.

set $\Omega_{i,t}$, however, depends on all other dealers' submissions via the consensus price process (p_t). The information set is endogenous, as (p_t) is both an input and an output of the joint learning process of the consensus pricing participants. We adopt an iterative algorithm developed in Nimark (2017) to solve filtering problems with endogenous signals.[12]

The algorithm works as follows:

1. Start with any covariance-stationary process (p_t^0) that lies in the space spanned by linear combinations of current and past aggregate shocks (u_t) and (ε_t).

2. This consensus price process (p_t^0) yields information sets for all i and t defined recursively by $\Omega_{i,t}^0 = \{s_{i,t}, p_{t-1}^0, \Omega_{i,t-1}^0\}$.

3. Given information set $\Omega_{i,t}^0$, each dealer i can compute the conditional expectation $\mathbb{E}(\theta_t|\Omega_{i,t}^0)$ for each period t under the assumed stochastic process for (p_t^0).

4. Averaging the expectations across submitters yields a new consensus price process
$$p_t^1 = \int_0^1 \mathbb{E}(\theta_t|\Omega_{i,t}^0)\, di + \sigma_\varepsilon\, \varepsilon_t \text{ for all } t.$$

5. If the distance (in m.s.e.) between (p_t^0) and (p_t^1) is smaller than some pre-specified stopping criterion, stop. Otherwise, go to step 2 with (p_t^1) as the new consensus price process and so on.

Nimark (2017) shows that for any initial choice of (p_t^0) the sequence of price processes $\{(p_t^n)\}_n$ converges (in m.s.e.) to a unique limit process (p_t) that is the solution of the original filtering problem. The proof relies on the fact that the integral in step 4 is a contraction on the space of covariance-stationary price processes. This allows the calculation of bounds for the approximation error when stopping the algorithm after a finite number of steps. Setting the initial process (p_t^0) such that $p_t^0 = \theta_t + \sigma_\varepsilon\, \varepsilon_t$ allows the problem to be solved by a sequential application of the Kalman filter. Appendix 7.4 provides a detailed description of

[12] As first pointed out by Townsend (1983), signal extraction problems in which signals are equilibrium variables, such as prices, typically lead to an infinite state space representations. One direction of attack is to show that the original problem can be approximated arbitrary well by a finite state space. This is the approach taken here. Similar approaches have been developed in Sargent (1991)), Huo and Takayama (2015), and Huo and Pedroni (2017). For special cases, frequency domain techniques have been employed to obtain finite state space representations, e.g. Kasa (2000).

the solution algorithm applied to the above consensus pricing model.

If the algorithm stops after n steps, the equilibrium learning dynamics are well approximated by a linear state-space system with an $n+1$ dimensional state vector x_t having the fundamental value θ_t and the cross-sectional average expectation $\bar{\theta}_t$ as its first and second element respectively.[13] The state evolves according to

$$x_t = Mx_{t-1} + Nv_t \ \text{ with } \ v_t = (u_t, \varepsilon_{t-1})^\intercal, \ \ v_t \sim N(0, I_2).$$

The matrices M and N are known functions of the model parameters, namely $\{\rho, \sigma_u, \sigma_\varepsilon, \sigma_\eta\}$. A dealer's signals in period t can be expressed as noisy observations of the state,

$$s_{i,t} = e_1^\intercal \, x_t + \sigma_\eta \, \eta_{i,t} = \theta_t + \sigma_\eta \, \eta_{i,t},$$
$$p_{t-1} = e_2^\intercal \, x_{t-1} + \sigma_\varepsilon \, \varepsilon_{t-1} = \bar{\theta}_{t-1} + \sigma_\varepsilon \, \varepsilon_{t-1}.$$

Here, $e_n^\intercal$ is a vector with 1 in the n^{th} position, 0 otherwise. The two signals can be written in vector form as

$$y_{i,t} = D_1 \, x_t + D_2 \, x_{t-1} + B \, \epsilon_{i,t},$$

with $y_{i,t} = (s_{i,t}, \, p_{t-1})^\intercal$ and $\epsilon_{i,t} = (v_t^\intercal, \, \eta_{i,t})^\intercal$.

We can now use the Kalman filter to obtain dealer i's beliefs about θ_t and $\bar{\theta}_t$, the first two elements of x_t, given the information in $\Omega_{i,t}$. Given the linear-normal structure of the above state-space system, dealer i's beliefs are normally distributed,

$$x_t \mid \Omega_{i,t} \sim N\left(x_{i,t}, \Sigma\right),$$

where the conditional expectations about the state evolve according to

$$x_{i,t} = Mx_{i,t-1} + K\left(y_{i,t} - \left(D_1 M + D_2\right) x_{i,t-1}\right), \tag{3}$$

where K is a $(n+1) \times 2$ dimensional matrix of Kalman gains. Here K and the covariance matrix of dealers' beliefs Σ are known functions of the model parameters.[14]

[13] The kth element of x_t is the cross-sectional average of submitters' k^{th}-order expectation of θ_t given their information in period t. Appendix 7.4 provides a definition of these higher-order expectations.

[14] Given the infinite history of past signals, the covariance matrix Σ and the matrix of Kalman gains K are not time dependent. Also, Σ and K are not dealer-specific as dealers are symmetrically informed. They all receive signals of the same quality.

3.3 Valuation and strategic uncertainty

We use the covariance matrix of beliefs Σ to measure uncertainty. In particular, we focus on a dealer's uncertainty about the current fundamental value of a contract, θ_t, and its uncertainty about the cross-sectional average expectation, $\bar{\theta}_t$ of this value. We have already derived dealer i's beliefs about the state x_t. Restricted to θ_t and $\bar{\theta}_t$ these can be written as

$$\begin{pmatrix} \theta_t \\ \bar{\theta}_t \end{pmatrix} \mid \Omega_{i,t} \sim N\left(\begin{pmatrix} \theta_{i,t} \\ \bar{\theta}_{i,t} \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \right), \tag{4}$$

where dealer i's conditional expectations about θ_t and $\bar{\theta}_t$ are updated according to

$$\theta_{i,t} = \rho\, \theta_{i,t-1} + k_s\, (s_{i,t} - \rho\, \theta_{i,t-1}) + k_p \left(p_{t-1} - \bar{\theta}_{i,t-1}\right), \tag{5}$$

$$\bar{\theta}_{i,t} = m_2 \cdot x_{i,t-1} + \bar{k}_s\, (s_{i,t} - \rho\, \theta_{i,t-1}) + \bar{k}_p \left(p_{t-1} - \bar{\theta}_{i,t-1}\right). \tag{6}$$

The above covariance matrix of beliefs corresponds to the top left 2×2 sub-matrix of Σ. k_s and k_p are the Kalman gains for private signal and the consensus price, respectively. This is the weight a dealer puts on "news" in these signals when updating expectations about the fundamental value θ_t. They correspond to the first row of K in (3). Similarly, $\bar{k}_s$ and $\bar{k}_p$ are the Kalman gains for private signal and the consensus price, respectively, for the average expectation $\bar{\theta}_t$. They correspond to the second row of K.

Our measures of valuation and strategic uncertainty are based on the posterior variance of beliefs about θ_t and $\bar{\theta}_t$ given by σ_{11} and σ_{22}, respectively. They correspond to the variance of a dealer's forecast errors, $\theta_{i,t} - \theta_t$ and $\bar{\theta}_{i,t} - \bar{\theta}_t$, at the time of its consensus price submission. Of independent interest is the correlation between these two forecast errors, $\rho_{12} = \sigma_{12}/\sqrt{\sigma_{11}\sigma_{22}}$. It is a natural measure for the commonality of information in the sense that consensus price submitters interpret new valuation information in a similar way.[15]

To develop an intuition for the relationship between valuation uncertainty, strategic and informational commonality, it is best to split up the expectation updating for θ_t into two steps: (i) First, the dealer updates its expectations about θ_{t-1} after observing the consensus price p_{t-1}. Call this updated expectation $\theta^+_{i,t-1}$. Its posterior expectation for θ_t is now $\rho\, \theta^+_{i,t-1}$. (ii) Next, after having observed private information $s_{i,t}$ in period t, the dealer updates expectation about θ_t. This is the

[15] Section 5.1. in Angeletos and Pavan (2007) provides a more detailed discussion of the notion of informational commonality.

expectation it then submits to the consensus pricing service. We have

$$\theta_{i,t} = (1-k_s)\rho \underbrace{\left[\theta_{i,t-1} + \frac{k_p}{\rho(1-k_s)}(p_{t-1} - \bar{\theta}_{i,t-1})\right]}_{\theta^{+}_{i,t-1}} + k_s\, s_{i,t}\ ,$$

where the term in the square brackets gives the first updating step. The average expectation can then be expressed as

$$\bar{\theta}_t = (1-k_s)\rho\, \bar{\theta}^{+}_{t-1} + k_s\, \theta_t.$$

This allows us to link the forecast errors for θ_t and $\bar{\theta}_t$ as follows

$$\bar{\theta}_{i,t} - \bar{\theta}_t = (1-k_s)\left[\mathbb{E}(\rho\, \bar{\theta}^{+}_{t-1}|\Omega_{i,t}) - \rho\, \bar{\theta}^{+}_{t-1}\right] + k_s\,(\theta_{i,t} - \theta_t)\,.$$

The forecast error for $\bar{\theta}_t$ is a weighted sum of the forecast error for θ_t and the forecast error for the average prior expectation about θ_t before observing the private signal in period t. Submitter i's forecast errors for $\bar{\theta}_t$ and θ_t are perfectly correlated if the submitter knows the average expectation $\bar{\theta}^{+}_{t-1}$. In our model, where the only exogenous source of information is the private signals, this can only happen if the consensus price perfectly aggregates all dispersed information. In that case, all submitters have a common posterior expectation $\bar{\theta}^{+}_{t-1} = p_{t-1}$ and the average expectation is given by $\bar{\theta}_t = (1-k_s)\rho\, p_{t-1} + k_s\, \theta_t$. As forecast errors are perfectly correlated, strategic uncertainty mirrors valuation uncertainty, namely $\sigma_{22} = k_s^2\, \sigma_{11}$. Strategic uncertainty is necessarily smaller than valuation uncertainty as $0 \leq k_s \leq 1$; when updating their expectations, submitters put some weight on the commonly known consensus price p_{t-1}. As a result, they are less uncertain about the location of the average expectation than about the current value of the fundamental.

If submitters are uncertain about the average expectation $\bar{\theta}^{+}_{t-1}$ at the time of their consensus price submission in period t, then forecast errors for the fundamental value θ_t and the average posterior expectation $\bar{\theta}_t$ are no longer perfectly correlated; strategic uncertainty is no longer simply a scaled-down version of valuation uncertainty. This causes private information to be long-lived in the sense that submitters do not return to a common posterior after observing the consensus price and before observing new private signals. This drives a wedge between forecast errors. The lack of a common perspective on past market conditions, as measured by uncertainty about $\bar{\theta}^{+}_{t-1}$, partially feeds into current uncertainty about $\bar{\theta}_t$ through the weight put on priors when updating expectations.[16] For a submitter whose

[16]See Sethi and Yildiz (2016) for a related discussion of the difference between well-informed and well-understood information sources and the implications for information segregation in markets.

prior expectations for fundamental and average valuation are equal, this weight is given by $1-k$, where

$$k = k_s + \frac{k_p}{\rho}, \tag{7}$$

as can be seen from rewriting equation (5),

$$\theta_{i,t} = (1-k)\rho\,\theta_{i,t-1} + k_s\,s_{i,t} + k_p\,p_{t-1} + k_p\,\left(\theta_{i,t-1} - \bar{\theta}_{i,t-1}\right).$$

As long as the consensus price is a noisy signal of the average expectation, that is as long as $\sigma_\varepsilon > 0$, it cannot perfectly aggregate dispersed private information. Submitter i's posterior expectations after observing the consensus price, $\theta^+_{i,t-1}$ partially depend on $\theta_{i,t-1}$ and hence submitters do not return to a common market perception after observing the consensus price. Individual perceptions are partially dependent on the individual history of private signals, unless private signals are very precise (σ_η is low) or the fundamental value process is not persistent (ρ is close to 0). In that case the history of past private signals does not matter.

The fact that we model the consensus price as an endogenous signal has important consequences for the interpretation of the data and the implications of alternative information structures. First, a noisier signal, that is a higher σ_ε, does not necessarily correspond to a less informative signal about θ_t. Submitters react by shifting weight to the private signal, but this in turn increases the informational content of the consensus price. In equilibrium, the Kalman gain on the consensus price k_p can increase in σ_ε. This informational externality is similar to the mechanism discussed in Amador and Weill (2012). Secondly, shutting down the consensus price process does not destroy exogenous information, it simply prevents its aggregation. This fact allows us to conduct counterfactual experiments on the information structure. Modelling the consensus price as an exogenous public signal of θ_t would be problematic for this exercise.

4 Estimation

We estimate the parameters of the model presented in Section 3, namely $\phi = \{\rho, \sigma_u, \sigma_\varepsilon, \sigma_\eta\}$, separately for each options contract. For a given contract our data consists of the panel of Totem price submissions of individual dealers and the corresponding consensus price. We denote by $\iota_t \subset \{1, 2, .., S\}$ the set of dealers active in t where S is the total number of distinct dealers that have submitted to Totem over the course of our sample period. The time series of submissions is given by $(\mathbf{m}_t)_{t=1}^T$, where $\mathbf{m}_t = (m_{j,t})_{j\in\iota_t}$ is a $|\iota_t|$-dimensional vector consisting of the individual period t consensus price submissions. Our data set for a given contract, $(\mathbf{y})_{t=1}^{T}$, then consists of the time-series of dealers' price submissions for this contract and the corresponding consensus price, i.e. $\mathbf{y}_t = (p_t\,, \mathbf{m}_t)^{\intercal}$.[17]

4.1 Likelihood function and estimation

To estimate the model for a given contract, we now show how it can be cast into state-space form. The panel of individual price submissions and the time series of consensus prices constitute the available observations of the system.

Based on Section 3, the latent state space has the following dynamics,

$$x_t = M(\phi)\, x_{t-1} + N(\phi)\, v_t \quad , \quad v_t \sim N(\mathbf{0}, I_2),$$

where $v_t = (u_t\ \varepsilon_t)^{\intercal}$. $M(\phi)$ and $N(\phi)$ are obtained employing the previously explained solution algorithm for a given parameter vector ϕ.

We assume that dealer i's price submission for period t is its conditional expectation of θ_t,

$$m_{i,t} = \theta_{i,t}.$$

Based on Section 3, dealer i's conditional expectation of the current state x_t is a partial latent variable with dynamics

$$x_{i,t} = M(\phi)\, x_{i,t-1} + K(\phi) \left[\begin{pmatrix} s_{i,t} \\ p_{t-1} \end{pmatrix} - (D_1\, M(\phi) + D_2)\, x_{i,t-1} \right].$$

Dealer i's private signal $s_{i,t}$ is also modelled as a latent variable, that is not observed by the econometrician,

$$s_{i,t} = \theta_t + \sigma_\eta\, \eta_{i,t} \;\text{ with } \eta_{i,t} \sim N(0,1).$$

All shocks $\eta_{i,t}$ are uncorrelated across submitting dealers and time.

Furthermore, we assume that the observed consensus price of period t is equal to the average first-order belief of period t plus aggregate noise, that is

$$p_t = \bar{\theta}_t + \sigma_\varepsilon\, \varepsilon_t \;\text{ with } \varepsilon_t \sim N(0,1).$$

Given the linearity of the above system and the joint normality of all shocks, the likelihood function for the observed data $(\mathbf{y})_{t=1}^{T}$ can be derived using the Kalman

[17] To be precise, $m_{j,t}$ is the (demeaned) natural logarithm of the Black-Scholes implied volatility of submitter j's time t price submission, and p_t is the (demeaned) natural logarithm of the consensus Black-Scholes implied volatility calculated by Totem for the corresponding contract.

filter. We obtain maximum-likelihood estimates for the parameter vector ϕ using MCMC methods with diffuse priors. Appendix 7.5 provides a detailed derivation of the filter for the above model and discusses the estimation technique.[18] Appendix 7.1 reports parameter estimates and standard errors for $\rho, \sigma_u, \sigma_\varepsilon$, and σ_η for all contracts.

4.2 Identification

Appendix 7.6 provides a formal proof of identification for the model. Here, we give a short summary of which moments of the data help us to identify the structural parameters of the model. The time-series variance of the differences between p_t and cross-sectional average of submission identifies σ_ε. The speed at which individual deviations from the average submission mean-revert determines the weight submitters put on their prior expectations (as opposed to weight put on news in the current signal and consensus price). Knowing this weight allows us to isolate changes in price submissions that are due to new information a submitter received in a given period. As these news are linked to the current fundamental, the autocorrelation of these expectation updates that have been "cleaned" of prior expectations allow us to identify ρ, the persistence in the fundamental value process. The weight submitters put on their prior depends on how persistent the fundamental is and how high the quality of their new information is, i.e. the signal-to-noise ratio of their signals. Having identified ρ, we can now identify this signal-to-noise ratio from the weight submitters put on their prior expectations. The signal-to-noise ratio depends on the variance of the fundamental shocks, σ_u^2, and the precision of private signals and the consensus price as determined by σ_η and σ_ε. We have already identified σ_ε. The relative weight submitters put on the consensus price as opposed to the private signal depends on the relative precision of these two signals. This allows us to identify σ_η and, finally, σ_u from the signal-to-noise ratio.

4.3 Model fit and robustness checks

To judge how well the model fits the data, we compare the model implied cross-sectional dispersion of price submissions and the time-series volatility of the consensus price to their empirical analogues. The upper panel in Table 1 in the Appendix displays the ratio of the model implied cross-sectional standard deviation and the empirical cross-sectional standard deviation. The ratios for the different contracts

[18]For the estimation of the parameters we assume diffuse priors. We constrain $\sigma_u, \sigma_\varepsilon$, and σ_η to be positive and $0 < \rho < 1$. For each contract we run chains of length 100.000 with the Metropolis-Hastings algorithm and disregard the first 10.000 draws. We subsequently pick every 10^{th} draw to construct the posterior distribution of the parameters.

are between 0.909 and 1.125, which implies that the model is able to reproduce the size of the cross-sectional dispersion for the different contracts.

The model implied volatility of the consensus price is the unconditional variance of $p_t = \bar{\theta}_t + \sigma_\varepsilon \varepsilon_t$. The unconditional variance of $\bar{\theta}_t$ is the solution to a Lyapunov equation that defines the unconditional variance of the state x_t.[19] In the lower panel of Table 1 we see that the model is able to match the volatility of the consensus price for contracts with moneyness 80 to 120. For the contracts with a short time-to-expiration and the most extreme moneyness the model implied time-series volatility is two to three times higher than in the data. However, the wide posterior distributions of these estimated volatilities implies that we cannot reject that the model implied and data given consensus price volatility are different from one another at conventional confidence levels.

The sample period covers two peculiar time periods. The low volatility period from 2002 to 2006 and the Great Recession from 2007 to 2010. The estimated values may be driven solely by the dynamics in one of these periods. We find that our results do not change if we consider these two sample periods separately. Another potential split is that of submitters that participate for a limited time frame and routine submitters. We therefore exclude submitters who have submitted less than 40% of the total sample period. The parameter estimates are comparable to the whole sample period. Including only the 'routine' submitters makes the contrast between the at-the-money (ATM) and the out-of-the-money (OTM) options slightly larger.

5 Results

The estimated precision of the different types of information determines the relative weights dealers put on the their signals and their prior expectations. We show what this implies for the level of valuation and strategic uncertainty and how these uncertainties vary across market segments. Furthermore, we quantify the informativeness and efficiency of the consensus pricing mechanism via counterfactual experiments on the market's information structure.

5.1 Price versus private information

The only source of information in our framework is the private signals. The consensus price mechanism allows submitters to partially infer their competitors' private information. The informational value of private signals and the consensus price

[19] This Lyapunov equation is given by $V = M V M^\mathsf{T} + N N^\mathsf{T}$, where V is the unconditional variance of the state x_t.

for dealers is neatly summarized by their Kalman gains, k_s and k_p as given in (5) for the fundamental value, $\bar{k}_s$ and $\bar{k}_p$ as given in (6) for the average market valuation. Given our estimates of the parameter vector ϕ, we can compare the implied Kalman gains across contracts. This allows us to compare the relative importance of signals across the options market. Figure 3 shows these Kalman gains for contracts with fixed times-to-expiration of 1 and 5 years.

A key structural parameter for understanding the variation in the Kalman gains across market segments is $1/\sigma_\eta$, the precision of the private signal. The estimates for σ_η, given in Table 3 in the Appendix, show that dealers receive very precise private signals for contracts that overlap with active exchange-based trading activity.[20] Consequently, the implied Kalman gains in Figure 3 show that submitters put essentially full weight on their private signal and largely ignore the information contained in the consensus price when updating expectations about θ_t. For OTC options in market segments with low exchange-based trading activity, the private signals are estimated to be noisier. Therefore, increasingly more weight is given to the consensus price. When updating expectations about $\bar{\theta}_t$, the consensus price receives relatively higher weight for all contracts. This shows the importance of the consensus price mechanism for forming expectations about average market valuations.

When signals are imprecise, dealers put relatively more weight on their priors. This weight is given by $1-k$ where equation (7) expresses k in terms of the Kalman gains k_s and k_p. Table 4 provides estimates for k across market segments. We see that only for the most extreme contracts that are exclusively traded in the OTC market dealers do not put full weight on new information. For those contracts k is smaller than one. We also find that in these market segments, dealers' forecast errors are not perfectly correlated, that is ρ_{12} is smaller than 1. As described in Section 3.3, this leads to long-lived private information and, consequently, dispersed priors among submitters. Table 4 reports these correlations for all contracts.

[20]For contracts with time-to-expiration of 6 and 12 months we exclude the contracts with a moneyness of 200. For these contracts, prices are close to zero and crucially depend on the numerical precision used by Totem submitters when reporting prices. Additionally, the inversion of the prices to Black-Scholes implied volatilities can become numerically unstable.

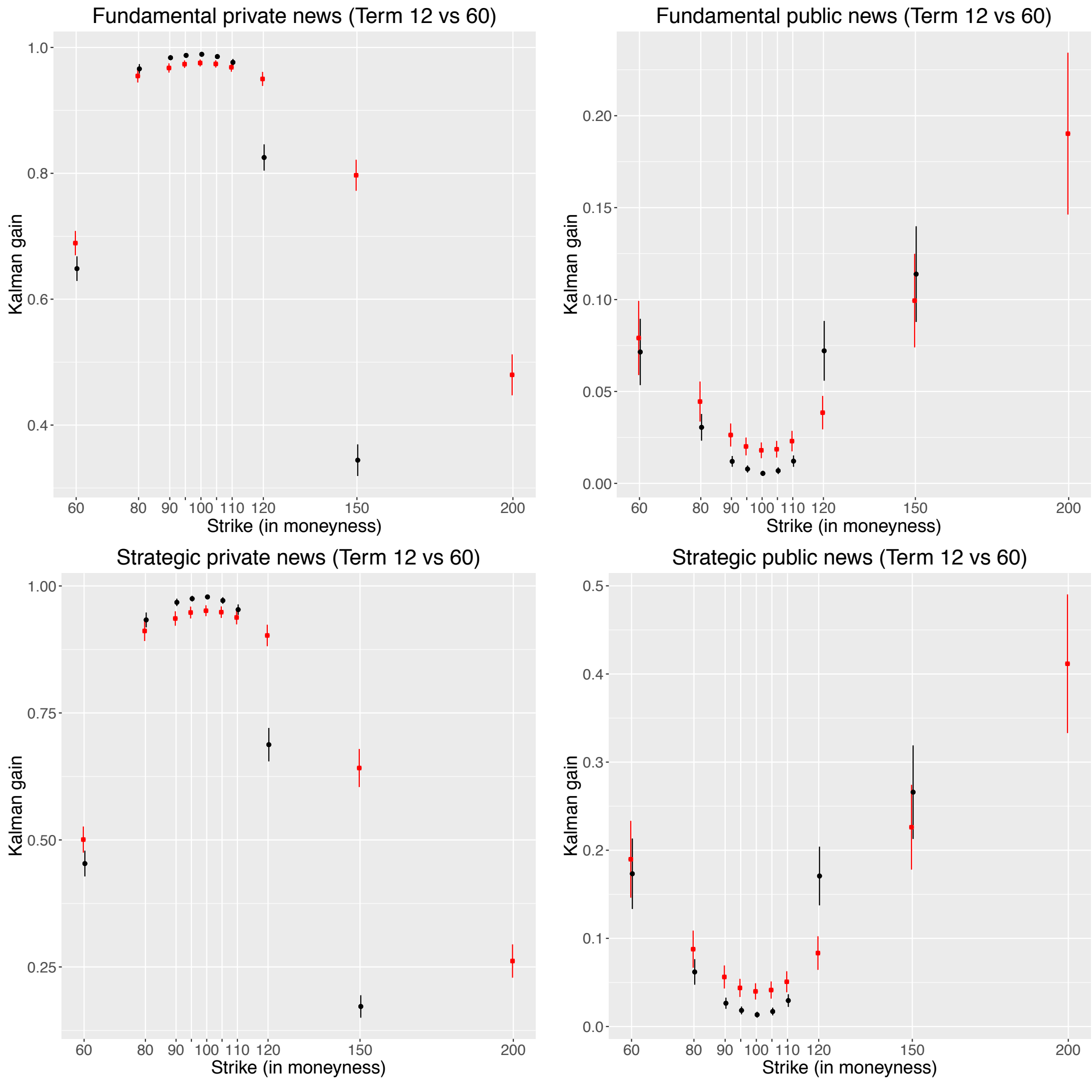


Figure 3: These figures present the Kalman gains with respect to their private signals and consensus feedback for their first- and second-order posterior believes. The horizontal axis denotes the moneyness of the option contracts. The black dots in the figures represent the Kalman gain extracted from the K matrix in Equation (4). The *top* figures depict the k_s and k_p elements. From left to right, these are the weights put on the private signal and public signal in updating the posterior believe about the fundamental. The *bottom* figures depict the $\bar{k}_s$ and $\bar{k}_p$ elements. From left to right, these are the weights put on the private signal and public signal in updating the posterior believe about the average believe. The two standard deviation of the posterior distribution of Kalman gain estimates are given by the bars surrounding the dots. The Kalman gains for the option contracts with a **time-to-expiration** of **12 months** are given by **black dots** and the estimates for the **60 month** contracts by **red squares**. The sample period is December 2002 to February 2015 for the option contracts on the S&P500 index. Data provided by IHS Markit's Totem service.

5.2 The uncertainty "smile"

The precision of signals as well as the relative weight submitters put on them are key determinants for their uncertainty about θ_t and $\bar{\theta}_t$. We measure these uncertainties by the posterior variance of dealers' beliefs, σ_{11} and σ_{22}. As the model-implied posterior beliefs are given by a normal distribution, a convenient way to display valuation and strategic uncertainty is as 95% posterior intervals centered around the consensus price of a contract as shown in Figure 4.

Both top figures show the well-known "smile" of the implied volatility curve. OTM put options (moneyness below 100) tend to be relatively more expensive than ATM put options or OTM call options reflecting market participants' demand for insurance against drops in the S&P500 index. The width of the posterior intervals shows that for options with more extreme strike prices (further away from moneyness 100), valuation and strategic uncertainty are also higher. These areas correspond to market segments in which trading is predominately or exclusively OTC as was previously shown in Figure 1. For options with moneyness 150 and time-to-expiration of 12 months, for example, the posterior intervals are on the order of 8 volatility points. This is substantial given that the average consensus price and time-series standard deviation for this contract are 13 and 3.8 volatility points, respectively. It reflects the low precision of the private signal for this contract and, consequently, a higher weight put on prior expectations which, in turn, is the source of sizable strategic uncertainty. This contrasts with posterior intervals well below one volatility point for ATM options. Here, private signals are estimated to be precise and dealers' submissions put most weight on the private signal. This implies low levels of σ_{11}. As all submitters are symmetrically informed and receive private signals from the same distribution, strategic uncertainty is small as well. These results illustrate that for the exclusively OTC traded areas of the option market, dealers are not only relatively uncertain about the correctness of their own option valuations, but also face substantial uncertainty about the relative position of their valuation to the average market valuation.

5.3 The informational properties of consensus prices

We use counterfactual experiments on the information structure to evaluate two informational properties of the consensus price: price informativeness and informational efficiency of the price. Furthermore, given the lagged nature of the consensus price, even a fully efficient price does not eliminate all uncertainty about asset values. Therefore, we also quantify the potential for further uncertainty reduction outside the scope of this consensus price mechanism.

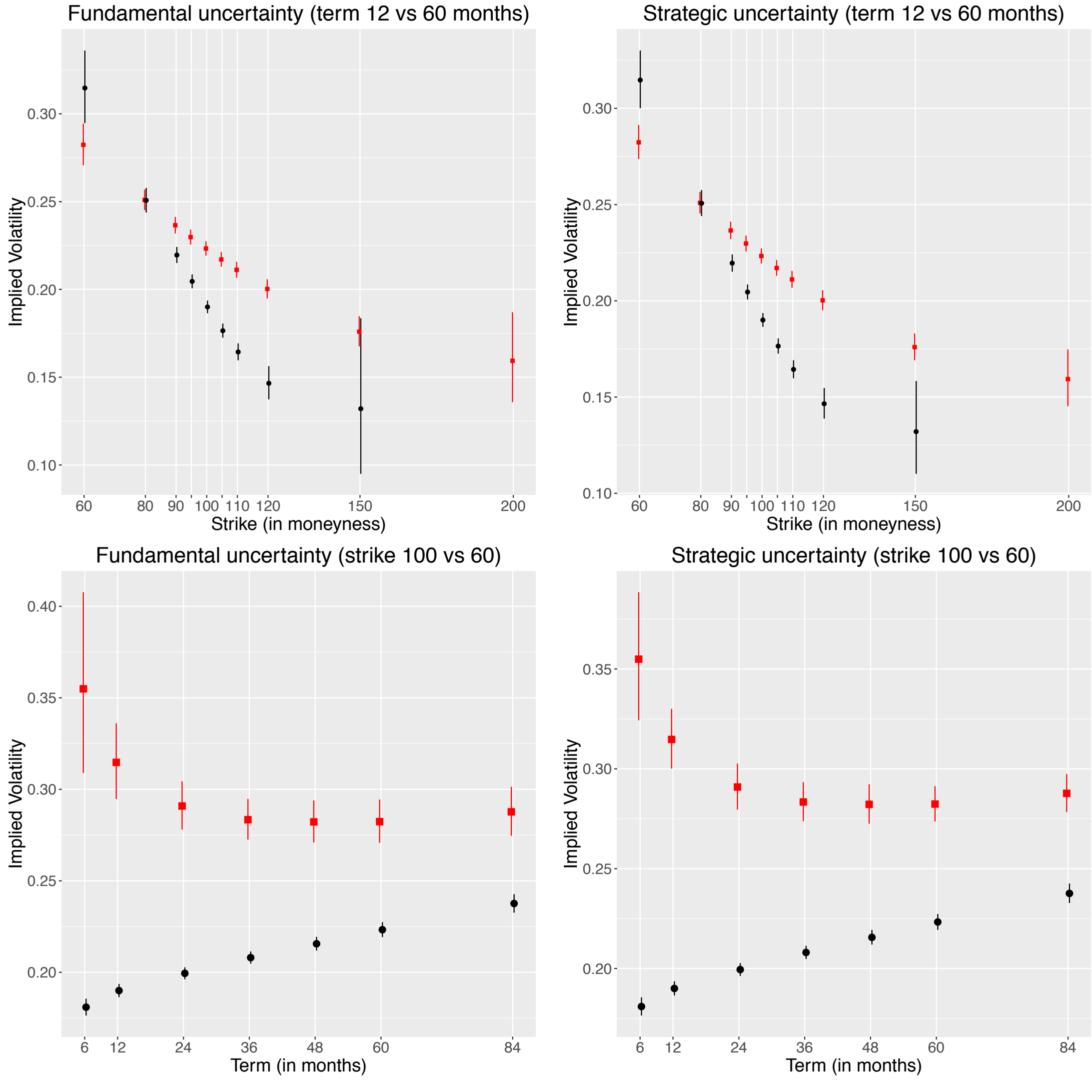


Figure 4: These figures present the variance of submitters' posterior beliefs expressed in terms of posterior intervals centred around the unconditional mean of implied volatilities, θ^m. The *left* figures depict the 95% posterior intervals for first-order beliefs, $[\theta^m \pm 1.96 \cdot \sigma_{11}]$, as given in (4). The figures on the *right* display the posterior intervals for second-order beliefs, $[\theta^m \pm 1.96 \cdot \sigma_{22}]$, as in (4). The two *top* panels depict the variances along the different levels of moneyness for the option contracts with a **time-to-expiration** of **12 months** and **60 months**. The two *bottom* panels show the term structure of the uncertainties for ATM options with moneyness **100** and OTM option with moneyness of **60**. The sample period is December 2002 to February 2015 for the option contracts on the S&P500 index. Data provided by IHS Markit's Totem service.

Price informativeness

To quantify how informative the consensus price is, we ask the following question: "By how much would a (non-participating) dealer's posterior uncertainty change if it gained access to the consensus price, keeping all else equal?" More precisely, we calculate the percentage change in the posterior intervals for valuation and strategic uncertainty, as specified in (4). To do so, we calculate the counterfactual posterior beliefs of a dealer who only has access to the private signal. We take this as the baseline informational setting against which to measure changes in uncertainty. Denote by $\hat{\Sigma}$ the covariance matrix of dealer i's posterior beliefs under this counterfactual information set, namely $\hat{\Omega}_{i,t} = \{s_{i,t}, \hat{\Omega}_{i,t-1}\}$. This covariance matrix can be obtained by solving a standard single-agent learning problem using parameter estimates for $\{\rho, \sigma_u, \sigma_\eta\}$.[21] The percentage reduction in a dealer's valuation uncertainty when gaining access to the consensus price is

$$\widehat{\Delta}_1 = \frac{\widehat{\sigma}_{11} - \sigma_{11}}{\widehat{\sigma}_{11}}. \tag{8}$$

Similarly, we denote the percentage reduction in strategic uncertainty by

$$\widehat{\Delta}_2 = \frac{\widehat{\sigma}_{22} - \sigma_{22}}{\widehat{\sigma}_{22}}. \tag{9}$$

Price efficiency

To understand how well the consensus price mechanism aggregates dispersed information, we compare the current consensus price to a fully efficient price that perfectly reveals last period's fundamental value, i.e. θ_{t-1}. As the price reveals θ_{t-1}, it provides submitters with a common prior before receiving new private signals. In addition to providing a benchmark for efficiency, this counterfactual also helps us understand how big an impediment the lack of a common prior is for creating a common understanding of market conditions. Denote by $\tilde{\Sigma}$ the counterfactual covariance matrix of posterior beliefs for a dealer who receives a fully efficient consensus price in the above sense. Again, we focus on the percentage reduction in valuation and strategic uncertainty as measured by posterior intervals,

$$\widetilde{\Delta}_1 = \frac{\sigma_{11} - \widetilde{\sigma}_{11}}{\widehat{\sigma}_{11}}, \tag{10}$$

and

$$\widetilde{\Delta}_2 = \frac{\sigma_{22} - \widetilde{\sigma}_{22}}{\widehat{\sigma}_{22}}. \tag{11}$$

[21] Appendix 7.9 derives the stationary posterior covariance matrices for first- and second-order beliefs for all counterfactional informational scenarios.

Residual informational frictions

Given an informationally efficient price, the residual uncertainty $\breve{\Delta} = 1 - \widehat{\Delta} - \widetilde{\Delta}$ is attributable to informational frictions that are outside of the scope of this consensus pricing mechanism. The scope for a further reduction in valuation and strategic uncertainty is given by

$$\breve{\Delta}_1 = \frac{\widetilde{\sigma}_{11}}{\widehat{\sigma}_{11}}. \tag{12}$$

and

$$\breve{\Delta}_2 = \frac{\widetilde{\sigma}_{22}}{\widehat{\sigma}_{22}}. \tag{13}$$

The influence of information structure on uncertainty

Figure 5 displays the percentage reductions in uncertainty under the different informational settings for contracts with a fixed time-to-expiration of 12 months. The dark gray region, i.e. $\widehat{\Delta}_j$, displays the informativeness of the price for the different contracts. The lack of uncertainty reduction in the moneyness range from 80 to 110 is to be expected as submitters solely rely on their precise private signal as previously revealed by the estimates of the Kalman gain k_s. For the more extreme contracts the consensus price is relatively more informative about θ_t; its Kalman gain k_p is higher. Table 5 in the Appendix shows similar patterns for other times-to-expirations. The reduction in valuation uncertainty is between 0% for the ATM contracts to 4.6% for the more extreme contracts. The bottom plot in Figure 5 shows that the consensus price signal is much more informative about $\bar{\theta}_t$. The reduction in strategic uncertainty ranges from 0.02% to 37.75% as can be seen in Table 5. The relative larger decrease in strategic uncertainty in comparison to valuation uncertainty points to the importance of the consensus price for learning about the average market valuation $\bar{\theta}_t$. This is also echoed by the difference between k_p and $\bar{k}_p$. Given the scarcity of shared trade data in market segments that are dominated by OTC trading, the ability of the consensus price to significantly reduce strategic uncertainty is both intuitive and important.

The light gray area in Figure 5 corresponds to $\widetilde{\Delta}_j$, the additional reduction in uncertainty due to a price that perfectly reveals θ_{t-1}. Knowing the past state eliminates two sources of uncertainty. The uncertainty that originates from the noise in the consensus price and the uncertainty that emanates from the dispersion in dealers' prior expectations. In the top and bottom panel of Figure 5, the lack of uncertainty reduction in the moneyness range from 80 to 110 is mainly due to the precision of private information. The signal-to-noise ratio σ_u/σ_η puts an upper bound on the weight the consensus price can receive when updating expectations.

The consensus price can at most reveal the past state while the private signal is a signal about the current state. This limits the potential impact of a fully efficient price on valuation uncertainty. For contracts with intermediate moneyness, little weight is put on prior expectations thus limiting the potential of a fully efficient price to reduce uncertainty by providing a common prior. For contracts with extreme moneyness, the relative imprecision of the private signal shifts weight towards the consensus price and the prior. This explains the up to 33.46% drop in valuation uncertainty and 62.10% drop in strategic uncertainty for the deep OTM call options as seen in Table 6 in the Appendix. The potential consequences of dispersed priors is highlighted in Sethi and Yildiz (2016). They illustrate that dispersion in priors can lead market participants to search out other participants with similar priors, leading to informational segmentation of markets. A focal point, such as a consensus price, helps to reduce dispersion in priors, reduces strategic uncertainty and creates a common understanding of market conditions.

The white area in the figures marks $\breve{\Delta}_j$, the potential uncertainty reduction outside of the scope of this consensus pricing mechanism. The reduced size of this area for the more extreme contracts illustrates the important role a consensus pricing mechanism can play in these market segments, especially in providing information about average market valuations. For contracts with moneyness between 80 and 110, informational frictions that could not be remedied by a perfectly efficient consensus pricing mechanism dominate. Reducing the remaining uncertainty would require changing the design parameters of the consensus pricing service. Increasing the frequency of the consensus service, for example, can be thought of as a lowering of σ_u. However, given the labor intensive nature of the consensus pricing process, running a more frequent services is costly. In appears that the marginal cost of increasing the frequency of the service exceeds the dealers' willing to pay for a marginal reduction in uncertainty.

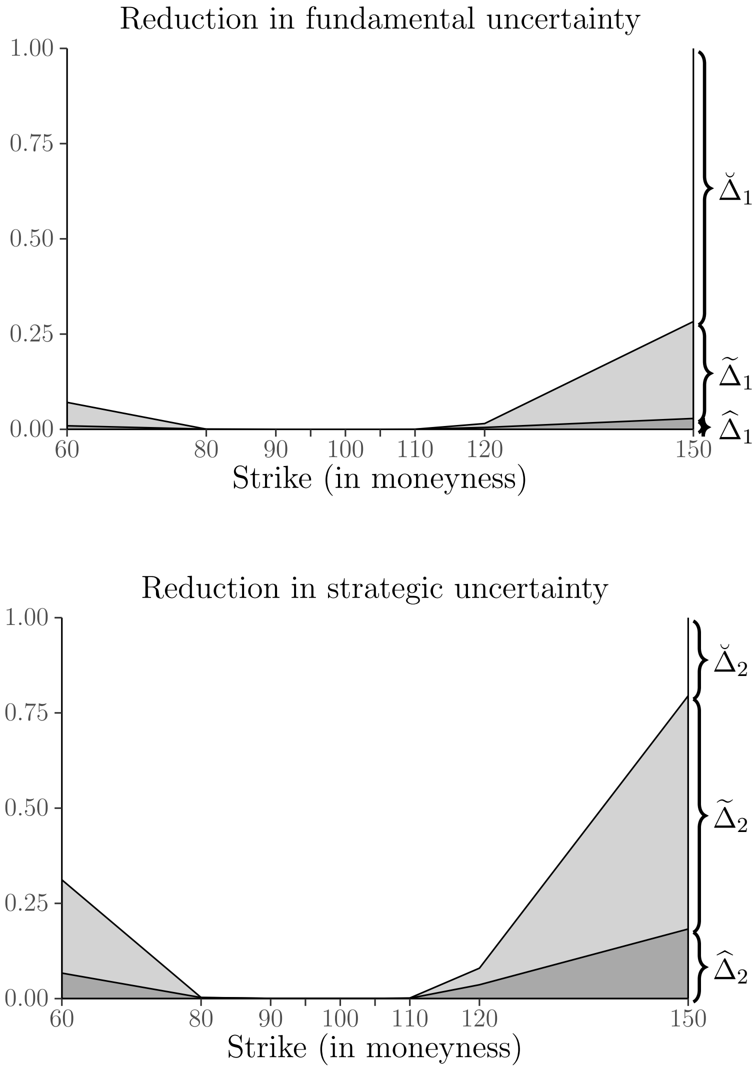


Figure 5: These two figures present the percentage reductions in valuation and strategic uncertainty under different informational settings. The upper figure presents the results for the percentage reduction in **valuation uncertainty** and the bottom figure presents the reductions in **strategic** uncertainty. The figures depict the uncertainty reductions along the different levels of moneyness for the option contracts with a **time-to-expiration** of **12 months**. In the base case setting, submitters only observes their private signal. This is indicated by the horizontal axis. The dark gray shade area is the percentage reduction in uncertainty due to observing the consensus price, i.e $\widetilde{\Delta}_i$ from in (8) and (9). The light gray area indicates the further reduction in uncertainty due to observing the past state, i.e. $\widehat{\Delta}_i$ in (10) and (11). The white area is the further reduction in uncertainty from learning outside of the market, i.e. $\breve{\Delta}$ in (12) and (13). The sample period is from December 2002 to February 2015.

6 Conclusion

In this paper we provide empirical evidence on the ability of consensus prices to reduce valuation uncertainty among major broker-dealers in the over-the-counter market for S&P500 index options. This evidence is based on a structural model of learning from prices. The estimation is based on a proprietary panel of price estimates that large broker-dealers have provided to a consensus pricing service for OTC derivatives. The structural model allows us to address three questions. First, how large is the valuation uncertainty of broker-dealers participating in the OTC market for S&P500 index options? Here, we consider two dimensions of uncertainty: a dealer's uncertainty about fundamental values and uncertainty about its valuations in relation to other market participants' valuations? Second, does the consensus price feedback help to reduce market participants' valuation uncertainty? Lastly, how well does the consensus pricing mechanism aggregate dispersed information.

Both fundamental and strategic valuation uncertainty vary substantially across the different market segments. We find higher uncertainty for option contracts with strike prices that correspond to more extreme index moves; these contracts are typically traded in the OTC segment of the market. Broker-dealers do not appear to rely heavily on the consensus price feedback to reduce fundamental uncertainty. The consensus price feedback is found to be most important for reducing strategic uncertainty, and particularly so for extreme option contracts. This result is consistent with the scarcity of shared valuation information for such extreme contracts. It stresses the importance of publicly observable valuation data, such as benchmarks, to establish a shared understanding of market conditions in OTC markets. Such a shared understanding can be particularly valuable during episodes of market stress where high levels of strategic uncertainty might cause derivatives markets to freeze up.

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7 Appendix

7.1 Tables

Table 1: Matching **cross-sectional dispersion** and **consensus price volatility**

	60	80	90	95	100	105	110	120	150	200
6	1.021	1.043	1.045	1.050	1.052	1.058	1.115	1.125	0.961	·
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.012)	(0.014)	·
12	1.026	1.050	1.039	1.047	1.056	1.049	1.058	1.103	0.988	·
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.013)	·
24	1.079	1.052	1.047	1.053	1.064	1.063	1.164	1.082	0.999	0.909
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.011)	(0.012)	(0.016)
36	1.058	1.040	1.043	1.048	1.052	1.059	1.060	1.061	1.013	0.928
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.014)
48	1.036	1.033	1.038	1.038	1.033	1.034	1.035	1.041	1.019	0.960
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.014)
60	0.992	1.028	1.041	1.039	1.038	1.037	1.040	1.043	1.014	1.003
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.015)
84	0.986	1.026	1.021	1.019	1.015	1.012	1.015	1.023	1.005	0.913
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)	(0.021)	(0.012)	(0.012)	(0.012)	(0.014)

(a) Matching cross-sectional dispersion

	60	80	90	95	100	105	110	120	150	200
6	1.437	1.058	1.138	1.053	1.045	1.401	1.155	1.242	2.595	·
	(0.271)	(0.243)	(0.294)	(0.224)	(0.277)	(0.780)	(0.315)	(0.213)	(0.629)	·
12	1.110	1.028	1.117	1.320	1.104	0.990	1.120	1.014	2.573	·
	(0.264)	(0.201)	(0.313)	(0.553)	(0.252)	(0.219)	(0.263)	(0.228)	(0.755)	·
24	1.086	1.030	1.050	1.413	1.213	1.033	1.095	1.183	1.483	3.635
	(0.286)	(0.203)	(0.214)	(0.733)	(0.484)	(0.290)	(0.324)	(0.394)	(0.476)	(2.366)
36	1.019	1.158	1.049	2.062	1.199	0.931	1.383	1.017	1.061	1.761
	(0.214)	(0.593)	(0.336)	(1.594)	(0.365)	(0.158)	(0.644)	(0.254)	(0.347)	(0.379)
48	0.987	1.203	1.090	0.985	1.086	1.027	1.346	1.011	0.968	1.490
	(0.203)	(0.561)	(0.327)	(0.203)	(0.339)	(0.229)	(0.838)	(0.244)	(0.246)	(0.420)
60	1.296	1.373	1.157	1.009	1.106	1.043	1.070	1.300	0.980	1.324
	(0.418)	(0.642)	(0.409)	(0.247)	(0.338)	(0.299)	(0.306)	(0.579)	(0.254)	(0.433)
84	0.942	1.149	1.142	1.098	0.968	1.029	0.989	1.054	1.045	1.055
	(0.179)	(0.526)	(0.379)	(0.341)	(0.181)	(0.222)	(0.209)	(0.265)	(0.265)	(0.234)

(b) Matching volatility consensus price

These two tables present the mean and standard deviation of the ratio of the raw moments of the data versus the model implied moments for each contract. The upper table displays the ratio of the model implied cross-sectional dispersion versus the average cross-sectional standard deviation in the data. The lower table displays the ratio of the model implied volatility of the consensus price to the volatility of the consensus price observed in the data. The model implied volatility is given by the unconditional volatility of the first-order believe plus σ_ϵ. The first row and first column of each table denotes the moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviation of the posterior distribution of the ratios is given in parenthesis below its mean (0.000 signifies standard deviations below 0.0005). The sample period of the data is January 2002 to December 2015 for the option contracts on the S&P500 index.

Table 2: Estimates ρ and σ_u

	60	80	90	95	100	105	110	120	150	200
6	0.950	0.911	0.930	0.923	0.920	0.945	0.949	0.956	0.950	·
	(0.019)	(0.031)	(0.028)	(0.026)	(0.028)	(0.030)	(0.022)	(0.015)	(0.021)	·
12	0.967	0.930	0.939	0.949	0.941	0.930	0.949	0.967	0.969	·
	(0.015)	(0.024)	(0.027)	(0.028)	(0.022)	(0.025)	(0.022)	(0.013)	(0.017)	·
24	0.935	0.940	0.943	0.956	0.947	0.938	0.945	0.962	0.970	0.971
	(0.025)	(0.021)	(0.020)	(0.026)	(0.028)	(0.024)	(0.023)	(0.020)	(0.015)	(0.017)
36	0.939	0.943	0.941	0.969	0.952	0.932	0.958	0.948	0.963	0.946
	(0.022)	(0.026)	(0.023)	(0.026)	(0.023)	(0.021)	(0.025)	(0.021)	(0.017)	(0.021)
48	0.935	0.949	0.947	0.938	0.944	0.942	0.953	0.945	0.959	0.943
	(0.022)	(0.025)	(0.022)	(0.021)	(0.024)	(0.022)	(0.026)	(0.020)	(0.017)	(0.022)
60	0.982	0.956	0.951	0.941	0.948	0.941	0.945	0.956	0.963	0.937
	(0.012)	(0.028)	(0.022)	(0.021)	(0.021)	(0.025)	(0.024)	(0.027)	(0.016)	(0.026)
84	0.968	0.947	0.949	0.947	0.939	0.945	0.940	0.945	0.941	0.954
	(0.011)	(0.025)	(0.026)	(0.023)	(0.019)	(0.019)	(0.022)	(0.024)	(0.023)	(0.017)

(a) Mean and standard deviation ρ

	60	80	90	95	100	105	110	120	150	200
6	0.079	0.092	0.099	0.106	0.116	0.120	0.115	0.111	0.166	·
	(0.001)	(0.005)	(0.005)	(0.006)	(0.006)	(0.007)	(0.006)	(0.002)	(0.005)	·
12	0.047	0.076	0.082	0.086	0.091	0.095	0.095	0.073	0.135	·
	(0.001)	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.002)	(0.003)	·
24	0.064	0.065	0.070	0.072	0.075	0.078	0.080	0.073	0.078	0.135
	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.005)
36	0.056	0.059	0.063	0.065	0.067	0.069	0.070	0.068	0.061	0.110
	(0.002)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.003)
48	0.054	0.056	0.059	0.061	0.062	0.064	0.065	0.065	0.057	0.097
	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.003)	(0.002)	(0.002)
60	0.033	0.055	0.055	0.057	0.058	0.059	0.060	0.061	0.051	0.089
	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)
84	0.033	0.050	0.051	0.052	0.053	0.054	0.055	0.056	0.062	0.058
	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.001)

(b) Mean and standard deviation σ_u

These two tables present the mean and standard deviation of the estimate of the persistence of the process for the fundamental, ρ, and the variance of the shock to the fundamental, σ_u. The structural model is estimated with Bayesian analysis through MCMC methods. The first row and first column of each table denotes the moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviation of the posterior distribution of the parameter is given in parenthesis below its mean (0.000 signifies standard deviations below 0.0005). The sample period of the data is December 2002 to February 2015 for the option contracts on the S&P500 index.

Table 3: Estimates σ_ε and σ_η

	60	80	90	95	100	105	110	120	150	200
6	0.121	0.004	0.007	0.009	0.011	0.016	0.021	0.151	0.328	·
	(0.008)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.002)	(0.010)	(0.013)	·
12	0.055	0.004	0.006	0.007	0.009	0.011	0.014	0.036	0.262	·
	(0.004)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.003)	(0.016)	·
24	0.002	0.004	0.006	0.007	0.008	0.009	0.010	0.015	0.114	0.270
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.008)	(0.016)
36	0.003	0.004	0.005	0.006	0.007	0.008	0.008	0.011	0.054	0.183
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.004)	(0.012)
48	0.003	0.004	0.005	0.006	0.006	0.007	0.007	0.009	0.034	0.121
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)	(0.009)
60	0.032	0.002	0.004	0.005	0.005	0.006	0.006	0.007	0.024	0.083
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.006)
84	0.034	0.003	0.003	0.004	0.004	0.004	0.004	0.005	0.005	0.069
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.005)

(a) Mean and standard deviation σ_ε

	60	80	90	95	100	105	110	120	150	200
6	0.096	0.022	0.015	0.013	0.013	0.018	0.030	0.131	0.380	·
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.020)	·
12	0.041	0.014	0.010	0.010	0.010	0.011	0.015	0.036	0.281	·
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.011)	·
24	0.024	0.012	0.009	0.008	0.008	0.009	0.013	0.018	0.093	0.395
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.023)
36	0.021	0.011	0.009	0.008	0.008	0.009	0.010	0.015	0.049	0.231
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.011)
48	0.022	0.012	0.010	0.009	0.009	0.009	0.011	0.014	0.035	0.155
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.006)
60	0.025	0.012	0.010	0.009	0.009	0.010	0.011	0.014	0.027	0.116
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.004)
84	0.029	0.014	0.011	0.011	0.011	0.011	0.012	0.014	0.023	0.069
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)

(b) Mean and standard deviation σ_η

These tables presents the mean and standard deviation of the estimate of the noise of the public signal, σ_ε, and the noise of the private signal submitter i receives, σ_η. The structural model is estimated with Bayesian analysis through MCMC methods. The first row and first column of each table denotes the moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviation of the posterior distribution of the parameter is given in parenthesis below its mean (0.000 signifies standard deviations below 0.0005). The sample period of the data is December 2002 to February 2015 for the option contracts on the S&P500 index.

Table 4: Weight on new information

	60	80	90	95	100	105	110	120	150	200
6	0.625	0.998	0.996	0.995	0.995	0.990	0.978	0.649	0.465	·
	(0.012)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.013)	(0.012)	·
12	0.733	0.998	0.996	0.996	0.995	0.993	0.989	0.901	0.490	·
	(0.011)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.010)	(0.014)	·
24	0.999	0.997	0.995	0.995	0.995	0.993	0.990	0.976	0.639	0.440
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.013)	(0.017)
36	0.997	0.995	0.995	0.995	0.994	0.993	0.991	0.984	0.781	0.513
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.012)	(0.017)
48	0.996	0.995	0.994	0.994	0.993	0.993	0.991	0.986	0.860	0.613
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.013)	(0.019)
60	0.778	0.999	0.994	0.994	0.994	0.993	0.992	0.989	0.900	0.703
	(0.011)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.015)	(0.018)
84	0.747	0.997	0.996	0.996	0.995	0.995	0.994	0.993	0.994	0.671
	(0.012)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.016)

(a) k

	60	80	90	95	100	105	110	120	150	200
6	0.954	1.000	1.000	1.000	1.000	1.000	1.000	0.958	0.915	·
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.003)	·
12	0.974	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.922	·
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)	·
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.956	0.909
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.004)
36	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.982	0.925
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.004)
48	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.993	0.947
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)
60	0.981	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.964
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)
84	0.976	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.962
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)

(b) ρ_{12}

This table presents the mean and standard errors of the Kalman gain, k. The Kalman gain gives the weight submitters put on new information and $1 - k$ shows how much weight is put on the prior. ρ_{12} is the correlation between the forecast error for asset value and average valuations. The first row and first column of each table denotes the moneyness and time-to-expiration, respectively, of the options under consideration. The sample period of the data is December 2002 to February 2015 for the option contracts on the S&P500 index.

Table 5: Counterfactual - **No consensus price**

	60	80	90	95	100	105	110	120	150	200
6	1.40	0.12	0.02	0.01	0.00*	0.01	0.11	1.61	2.99	·
	(0.17)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.19)	(0.31)	·
12	0.94	0.05	0.01	0.00*	0.00*	0.00*	0.01	0.50	2.93	·
	(0.12)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.07)	(0.34)	·
24	0.63	0.04	0.01	0.01	0.00*	0.00*	0.02	0.07	1.54	4.56
	(0.12)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.19)	(0.54)
36	0.59	0.04	0.01	0.01	0.01	0.01	0.01	0.06	1.15	3.31
	(0.12)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.14)	(0.41)
48	0.76	0.07	0.02	0.01	0.01	0.01	0.02	0.06	0.93	3.15
	(0.15)	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.12)	(0.39)
60	0.93	0.09	0.04	0.02	0.02	0.02	0.03	0.09	0.76	3.16
	(0.12)	(0.02)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.02)	(0.10)	(0.37)
84	1.22	0.20	0.09	0.07	0.06	0.07	0.08	0.14	0.54	1.94
	(0.16)	(0.04)	(0.02)	(0.02)	(0.01)	(0.10)	(0.02)	(0.03)	(0.11)	(0.25)

(a) *Decrease in valuation uncertainty*: $\hat{\Delta}_1$

	60	80	90	95	100	105	110	120	150	200
6	10.70	0.71	0.12	0.05	0.03	0.09	0.78	12.14	23.75	·
	(1.26)	(0.16)	(0.03)	(0.01)	(0.01)	(0.02)	(0.16)	(1.37)	(2.16)	·
12	7.18	0.28	0.06	0.03	0.02	0.04	0.10	3.77	22.40	·
	(0.86)	(0.06)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.46)	(2.50)	·
24	3.61	0.26	0.06	0.04	0.03	0.03	0.12	0.54	11.58	37.75
	(0.71)	(0.06)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.11)	(1.37)	(4.44)
36	3.45	0.28	0.08	0.05	0.04	0.05	0.09	0.40	8.68	26.16
	(0.68)	(0.06)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.09)	(0.99)	(3.08)
48	4.44	0.44	0.15	0.10	0.08	0.10	0.15	0.44	6.96	24.05
	(0.87)	(0.10)	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.10)	(0.84)	(2.84)
60	7.11	0.53	0.25	0.16	0.14	0.15	0.22	0.56	5.59	23.64
	(0.87)	(0.12)	(0.06)	(0.04)	(0.03)	(0.03)	(0.05)	(0.12)	(0.68)	(2.61)
84	9.24	1.19	0.53	0.43	0.39	0.44	0.49	0.88	3.18	14.53
	(1.12)	(0.26)	(0.12)	(0.10)	(0.09)	(0.58)	(0.11)	(0.19)	(0.64)	(1.78)

(b) *Decrease in strategic uncertainty*: $\hat{\Delta}_2$

These two tables present the counterfactual results of the percentage decrease in valuation uncertainty when moving from an setting without consensus price to a setting with consensus price feedback. The upper table presents the results for the percentage decrease in **valuation uncertainty**, $\hat{\Delta}_1$ in (8). The lower table presents the results for the percentage increase in **strategic uncertainty**, $\hat{\Delta}_2$ in (9). The first row and first column of each table denotes the moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviation of the posterior distribution of the parameter is given in parenthesis below its mean (0.00 signifies standard deviations below 0.005). The sample period is from December 2002 to February 2015.

Table 6: Counterfactual - **Perfect Consensus price**

	60	80	90	95	100	105	110	120	150	200
6	12.02	0.01	0.00*	0.00*	0.00*	0.01	0.07	11.35	27.01	·
	(0.52)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.52)	(1.03)	·
12	6.15	0.00*	0.00*	0.00*	0.00*	0.00*	0.01	1.02	25.45	·
	(0.35)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.16)	(0.84)	·
24	0.01	0.01	0.00*	0.00*	0.00*	0.00*	0.01	0.07	11.99	33.46
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.52)	(1.21)
36	0.02	0.01	0.00*	0.00*	0.00*	0.01	0.01	0.03	4.71	24.14
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.34)	(1.00)
48	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.03	2.19	16.80
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.27)	(0.82)
60	4.60	0.00*	0.01	0.01	0.01	0.01	0.01	0.03	1.24	11.55
	(0.33)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.24)	(0.74)
84	6.13	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.04	10.88
	(0.39)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.63)

(a) *Reduction in valuation Uncertainty*: $\tilde{\Delta}_1$

	60	80	90	95	100	105	110	120	150	200
6	41.31	0.02	0.01	0.01	0.01	0.04	0.27	38.80	62.69	·
	(1.43)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.07)	(1.49)	(1.09)	·
12	24.49	0.01	0.01	0.01	0.01	0.02	0.05	4.35	61.23	·
	(1.28)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.74)	(1.52)	·
24	0.03	0.02	0.01	0.01	0.01	0.02	0.05	0.27	40.68	62.10
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.07)	(1.48)	(1.95)
36	0.07	0.03	0.02	0.02	0.02	0.02	0.04	0.13	18.68	57.87
	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.04)	(1.31)	(1.75)
48	0.09	0.03	0.03	0.02	0.02	0.03	0.04	0.12	8.94	46.67
	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(1.16)	(1.89)
60	18.72	0.01	0.03	0.02	0.02	0.03	0.04	0.10	5.06	35.14
	(1.29)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(1.05)	(1.94)
84	23.65	0.04	0.03	0.03	0.03	0.03	0.04	0.07	0.13	36.56
	(1.41)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.04)	(1.82)

(b) *Reduction in Strategic Uncertainty*: $\tilde{\Delta}_2$

These two tables present the counterfactual results of the percentage reduction in valuation uncertainty when moving from the current information structure to an information structure where the consensus price perfectly reveals last period's state (0.00* means below 0.005). The upper table presents the results for the percentage reduction in **valuation uncertainty**, $\tilde{\Delta}_1$ in (10). The lower table presents the results for the percentage reduction in **strategic uncertainty**, $\tilde{\Delta}_2$ in (11). The first row and first column of each table denotes the moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviation of the posterior distribution of the parameter is given in parenthesis below its mean (0.00 signifies standard deviations below 0.005). The sample period is from December 2002 to February 2015.

7.2 Totem Submission Statistics

Table 7: Range of the times series for plain vanilla options

	moneyness																		
term	20	30	40	50	60	70	80	90	95	100	105	110	120	130	150	175	200	250	300
1	08/09	08/09	08/15	13/15	07/15	13/15	02/15	02/15	02/15	02/15	02/15	02/15	02/15	09/15	07/15	09/15	07/15	-	-
2	08/09	08/09	08/15	13/15	08/15	13/15	08/15	08/15	08/15	08/15	08/15	08/15	08/15	09/15	08/15	09/15	08/15	-	-
3	08/09	08/09	08/15	13/15	07/15	13/15	02/15	02/15	02/15	02/15	02/15	02/15	02/15	09/15	07/15	09/15	07/15	-	-
6	08/15	08/15	08/15	13/15	98/15	13/15	98/15	98/15	98/15	98/15	98/15	98/15	98/15	09/15	99/15	09/15	02/15	08/15	08/15
9	08/15	08/15	08/15	13/15	07/15	13/15	07/15	07/15	07/15	07/15	07/15	07/15	07/15	09/15	07/15	09/15	07/15	08/15	08/15
12	08/15	08/15	08/15	13/15	98/15	13/15	98/15	98/15	98/15	98/15	98/15	98/15	98/15	09/15	99/15	09/15	02/15	08/15	08/15
18	08/15	08/15	08/15	13/15	07/15	13/15	07/15	07/15	07/15	07/15	07/15	07/15	07/15	09/15	07/15	09/15	07/15	08/15	08/15
24	08/15	08/15	08/15	13/15	98/15	13/15	98/15	98/15	98/15	98/15	98/15	98/15	98/15	09/15	99/15	09/15	02/15	08/15	08/15
30	08/15	08/15	08/15	13/15	07/15	13/15	07/15	07/15	07/15	07/15	07/15	07/15	07/15	09/15	07/15	09/15	07/15	08/15	08/15
36	08/15	08/15	08/15	13/15	98/15	13/15	98/15	98/15	98/15	98/15	98/15	98/15	98/15	09/15	99/15	09/15	02/15	08/15	08/15
48	08/15	08/15	08/15	13/15	98/15	13/15	98/15	98/15	98/15	98/15	98/15	98/15	98/15	09/15	99/15	09/15	02/15	08/15	08/15
60	08/15	08/15	08/15	13/15	98/15	13/15	98/15	98/15	98/15	98/15	98/15	98/15	98/15	09/15	99/15	09/15	02/15	08/15	08/15
72	10/15	10/15	10/15	13/15	10/15	13/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15
84	08/15	08/15	08/15	13/15	99/15	13/15	99/15	99/15	99/15	99/15	99/15	99/15	99/15	09/15	99/15	09/15	02/15	08/15	08/15
96	10/15	10/15	10/15	13/15	10/15	13/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15
108	10/15	10/15	10/15	13/15	10/15	13/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15	10/15
120	08/15	08/15	08/15	13/15	99/15	13/15	99/15	99/15	99/15	99/15	99/15	99/15	99/15	09/15	99/15	09/15	03/15	08/15	08/15
144	08/15	08/15	08/15	13/15	05/15	13/15	05/15	05/15	05/15	05/15	05/15	05/15	05/15	09/15	05/15	09/15	05/15	08/15	08/15
180	08/15	08/15	08/15	13/15	05/15	13/15	05/15	05/15	05/15	05/15	05/15	05/15	05/15	09/15	05/15	09/15	05/15	08/15	08/15
240	11/15	11/15	11/15	13/15	11/15	13/15	11/15	11/15	11/15	11/15	11/15	11/15	11/15	11/15	11/15	11/15	11/15	11/15	11/15

This table gives the coverage of the data for the specific contracts on the S&P 500 Index. The table reports the start and end year that a contract covers.

Table 8: Average number of submitters

	moneyness										
term	60	80	90	95	100	105	110	120	130	150	200
6	27	31	31	31	31	31	31	31	29	27	·
9	26	29	29	29	29	29	29	29	29	26	·
12	27	30	30	30	30	30	30	30	28	27	19
24	27	30	30	30	30	30	30	30	28	26	19
36	26	29	29	29	29	29	29	29	27	26	18
48	26	29	29	29	29	29	29	29	26	25	18
60	25	28	28	28	28	28	28	28	26	25	18
84	24	25	25	25	25	25	25	25	24	23	17

This table provides the average number of submitters for the specific options on the S&P 500 Index. These are the accepted prices per contract for the dates that the contract is polled. In our analysis we ignore submissions with a price of 0. The data sample is from December 2002 till February 2015.

7.3 Additional figures

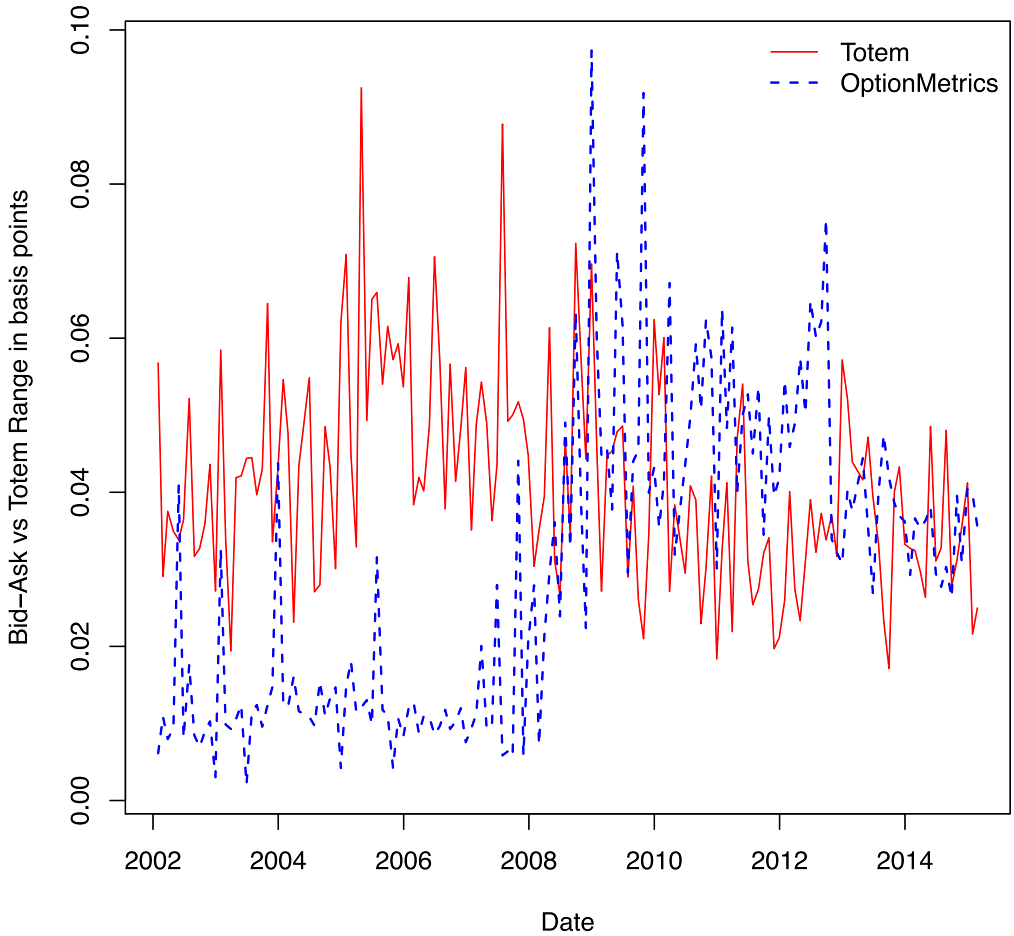

Figure 6: Bid-Ask spread vs Submission Range IV (S&P 500)

The figure above displays the range of the price submissions to the Totem service and bid-ask spread on traded options from OptionMetrics. This is for a contract with time-to-experation of 6 months and moneyness 100. The bid-ask spread is given by the best difference between the best closing bid price and best closing ask price across all US option exchanges. The options in the Totem service are matched to the traded options in the OptionMetrics database. On a given Totem valuation date we match OptionMetrics option contracts that are a close proxy for the totem option contracts. We search for contracts with a ± 10 days-to-maturity and a ±1 moneyness value. When multilple options match the criteria an average is taken of their bid-ask spread.

7.4 Solution algorithm

Here we show how to apply the algorithm developed in Nimark (2017) to solve the consensus pricing problem of section 3. We adopt the following standard notation for higher-order beliefs, defining recursively

$$\theta_t^{(0)} = \theta_t,$$
$$\theta_{i,t}^{(k+1)} = \mathbb{E}\left(\theta_t^{(k)} | \Omega_{i,t}\right) \text{ and } \theta_t^{(k+1)} = \int_0^1 \theta_{i,t}^{(k+1)} \, di \text{ for all } k \geq 0.$$

We denote institution i's hierarchy of beliefs up to order k by

$$\theta_{i,t}^{(1:k)} = \left(\theta_{i,t}^{(1)}, ..., \theta_{i,t}^{(k)}\right)^\intercal$$

and for the hierarchy of average beliefs up to order k, including the fundamental value $\theta_t^{(0)}$ as first element,

$$\theta_t^{(0:k)} = \left(\theta_t^{(0)}, \theta_t^{(1)}, ..., \theta_t^{(k)}\right)^\intercal.$$

The solution procedure proceeds recursively. It starts with a fixed order of beliefs $k \geq 0$ and postulates that the dynamics of average beliefs $\theta_t^{(0:k)}$ are given by the VAR(1)

$$\theta_t^{(0:k)} = M_k \, \theta_{t-1}^{(0:k)} + N_k \, w_t \tag{14}$$

with $w_t = (u_t, \varepsilon_t)^\intercal$ and $\theta_t^{(n)} = \theta_t^{(k)}$ for all $n \geq k$.

Institution i's signal can be expressed in terms of current and past average beliefs, $\theta_t^{(0:k)}$ and $\theta_{t-1}^{(0:k)}$, and the period t shocks w_t and $n_{i,t}$. The private signal can be written as

$$s_{i,t} = e_1^\intercal \, \theta_t^{(0:k)} + \sigma_\eta \, \eta_{i,t}$$

where e_j denotes a column vector of conformable length with a 1 in position j, all other elements being 0. Similarly, we can express the consensus price p_t as

$$p_t = \theta_{t-1}^{(1)} + \sigma_\varepsilon \, \varepsilon_t = e_2^\intercal \, \theta_{t-1}^{(0:k)} + \sigma_\varepsilon \, \varepsilon_t.$$

Denote the vector of signals by $y_{i,t} = (s_{i,t}, p_t)^\intercal$. We can now express the signals in terms of current average beliefs and shocks,

$$y_{i,t} = D_{k,1} \, \theta_t^{(0:k)} + D_{k,2} \, \theta_{t-1}^{(0:k)} + R_w \, w_t + R_\eta \, \eta_{i,t} \tag{15}$$

where

$$D_{k,1} = \begin{bmatrix} e_1^\intercal \\ 0_{k+1}^\intercal \end{bmatrix}, \quad D_{k,2} = \begin{bmatrix} 0_{k+1}^\intercal \\ e_2^\intercal \end{bmatrix}, \quad R_\eta = \begin{bmatrix} \sigma_\eta \\ 0 \end{bmatrix} \text{ and } R_w = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_\varepsilon \end{bmatrix}.$$

We thus obtain a state space representation of the system from the perspective of institution i. Equation (14) describes the dynamics of the latent state variable $\theta_t^{(0:k)}$, equation (15) is the observation equation that provides the link between the current state and i's signals. Using a

Kalman filter that allows for lagged state variables (see Nimark (2015)) allows us to express institution i's beliefs conditional on the information contained in $\Omega_{i,t}$ as

$$\theta_{i,t}^{(1:k+1)} = M_k\,\theta_{i,t-1}^{(1:k+1)} + K_k\left[y_{i,t} - D_{1,k}\,M_k\,\theta_{i,t-1}^{(1:k+1)} - D_{2,k}\,\theta_{i,t-1}^{(1:k+1)}\right], \tag{16}$$

where K_k is the (stationary) Kalman gain. Substituting out the signal vector in terms of current state and shocks, this can equivalently be written as

$$\begin{aligned}\theta_{i,t}^{(1:k+1)} = {} & [M_k - K_k(D_{1,k}M_k + D_{2,k})]\,\theta_{i,t-1}^{(1:k+1)} \\ & + K_k(D_{1,k}M_k + D_{2,k})\theta_{t-1}^{(0:k)} + K_k(D_{1,k}N_k + R_w)w_t + K_k R_\eta\,\eta_{i,t}.\end{aligned}$$

Averaging this expression across all submitters, assuming that by a law of large numbers $\int_0^1 \eta_{i,t}\,di = 0$, average beliefs are then given by

$$\begin{aligned}\theta_{t}^{(1:k+1)} = {} & [M_k - K_k(D_{1,k}M_k + D_{2,k})]\,\theta_{t-1}^{(1:k+1)} \\ & + K_k(D_{1,k}M_k + D_{2,k})\theta_{t-1}^{(0:k)} + K_k(D_{1,k}N_k + R_w)w_t.\end{aligned}$$

Combined with the fact that $\theta_t^{(0)} = \rho\,\theta_{t-1}^{(0)} + \sigma_u\,u_t$ we now obtain a new law of motion for the state

$$\theta_t^{(0:k+1)} = M_{k+1}\,\theta_{t-1}^{(0:k+1)} + N_{k+1}\,w_t$$

with

$$M_{k+1} = \begin{bmatrix} \rho\,e_1^\intercal & 0 \\ K_k(D_{1,k}M_k + D_{2,k}) & 0_{k\times 1}\end{bmatrix} + \begin{bmatrix} 0 & 0_{1\times k} \\ 0_{k\times 1} & M_k - K_k(D_{1,k}M_k + D_{2,k})\end{bmatrix} \tag{17}$$

and

$$N_{k+1} = \begin{bmatrix} \sigma_u\,e_1^\intercal \\ K_k(D_{1,k}N_k + R_w)\end{bmatrix}. \tag{18}$$

Note, however, that now the state space has increased by one dimension from $k+1$ to $k+2$. This is a consequence of the well-known infinite regress problem when filtering endogenous signals. When filtering average beliefs of order k, institutions have to form beliefs about average beliefs of order k. But this implies that equilibrium dynamics are influenced by average beliefs of order $k+1$, and so on for all orders $k \geq 0$.

In practice, the solution algorithm works as follows. We initialise the iteration at $k = 0$ with $M_0 = \rho$ and $N_0 = \sigma_u$, which implies that $\theta_t^{(1)} = \theta_t^{(0)}$ for all t. Consequently, the consensus price of the first iteration is given by[22]

$$p_t^{[1]} = \theta_{t-1}^{(0)} + \sigma_\varepsilon\,\varepsilon_t.$$

This yields a Kalman gain K_0 (here a two-dimensional vector) which can then be used to obtain M_1 and N_1 via equations (17) and (18) and so on until convergence of the process $p_t^{[n]}$ has been achieved according to a pre-specified convergence criteria after n steps. The highest order belief that is not trivially defined by lower order beliefs is then of order n.

[22] Superscripts in square brackets denote iterations of the algorithm.

7.5 Kalman Filter for Estimation

For a given contract, that is a given time-to-expiration, moneyness, and option type (put or call), our data consists of two time series. Let S be the total number of institutions that have submitted to Totem over the course of our sample and let $\iota_t \subset \{1, 2, .., S\}$ be the set of institutions active in t.[23] Our sample of submissions is then given by $(\mathbf{m}_t)_{t=1}^T$, where $\mathbf{m}_t = (m_{j,t})_{j \in \iota_t}$ is a $|\iota_t|$-dimensional vector consisting of the individual period t consensus price submissions. We assume that consensus price submissions are institution i's best estimate of θ_t plus uncorrelated measurement error[24]

$$m_{i,t} = \theta_{i,t}^{(1)} + \sigma_\psi \, \psi_{i,t} \;\text{ with }\; \psi_{i,t} \sim N(0, 1). \tag{19}$$

Following our model, we assume that the consensus price of period $t-1$, which we call p_t, equals the average first-order belief of period $t-1$ plus aggregate noise, that is

$$p_t = \theta_{t-1}^{(1)} + \sigma_\varepsilon \, \varepsilon_t.$$

Our data set for a given contract, $(\mathbf{y})_{t=1}^T$, then consists of the time-series of institutions' price submissions for this contract and the corresponding consensus price, i.e. $\mathbf{y}_t = (p_t \,, \mathbf{m}_t)^\intercal$.[25]

To estimate the model, we fix the maximum order of beliefs at $\bar{k} = 4$ and assume that the system has reached its stationary limit.[26] Average beliefs then evolve according to (14), namely

$$\theta_t^{(0:\bar{k})} = M_{\bar{k}} \, \theta_{t-1}^{(0:\bar{k})} + N_{\bar{k}} \, w_t,$$

where $M_{\bar{k}}$ and $N_{\bar{k}}$ are functions of the parameters ϕ defined recursively by equations (17) and (18) and $w_t = (u_t \,, \varepsilon_t)^\intercal \sim N(\mathbf{0}_2, I_2)$.[27] The dynamics of institutions i's conditional beliefs $\theta_{i,t}^{(1:\bar{k})}$ can be expressed in terms of deviations from average beliefs, $\hat{\theta}_{i,t}^{(1:\bar{k})} \equiv \theta_{i,t}^{(1:\bar{k})} - \theta_t^{(1:\bar{k})}$, as

$$\hat{\theta}_{i,t}^{(1:\bar{k})} = Q_{\bar{k}} \, \hat{\theta}_{i,t-1}^{(1:\bar{k})} + V_{\bar{k}} \, \eta_{i,t},$$

where

$$Q_{\bar{k}} = \left[M_{\bar{k}} - K_{\bar{k}} (D_{1,\bar{k}} M_{\bar{k}} + D_{2,\bar{k}}) \right] \;\text{ and }\; V_{\bar{k}} = K_{\bar{k}} \, R_\eta.$$

Given the linearity of the above system and the assumed normality of shocks the likelihood function for the observed data $(\mathbf{y})_{t=1}^T$ with $\mathbf{y}_t = (p_t \,, \mathbf{m}_t)^\intercal$ can be derived using the Kalman filter. We define $\alpha_t = (\theta_t^{(0:\bar{k})} \,, \hat{\theta}_{1,t}^{(1:\bar{k})} \,, ..., \hat{\theta}_{S,t}^{(1:\bar{k})} \,, \varepsilon_t)^\intercal$ to be the state of the system in t.

The *transition equation* of the system in state space form is then given by

$$\alpha_t = T \alpha_{t-1} + R \, \epsilon_t$$

[23] If an institution does not submit a price in t, we treat this as a missing value. However, it is assumed that this institution received both the consensus price and the private signal about the fundamental in that period.

[24] In our main specification we assume that there is not measurement error, i.e. $\sigma_\psi = 0$.

[25] To be precise, $m_{j,t}$ is the (demeaned) natural logarithm of the Black-Scholes implied volatility of submitter j's time t price submission, and p_t is the (demeaned) natural logarithm of the consensus Black-Scholes implied volatility calculated by Totem for the corresponding contract.

[26] Allowing $\bar{k}$ greater than 4 does not change the estimates noticeably.

[27] We use $0_{n \times m}$ to denote a $n \times m$ matrix of zeros, 1_n is a (column) vector containing n ones, and I_n is an n-dimensional identity matrix.

where

$$T = \left(\begin{array}{c} M_{\bar{k}}\,,\, 0_{\bar{k}+1\times S\bar{k}+1} \\ 0_{S\,\bar{k}\times\bar{k}+1}\,,\, I_S \otimes Q_{\bar{k}}\,,\, 0_{S\,\bar{k}\times 1} \\ 0_{2\times\bar{k}+1+S\bar{k}+1} \end{array} \right)\,,\; R = \left(\begin{array}{c} N_{\bar{k}}\,,\, 0_{\bar{k}+1\times S} \\ 0_{S\bar{k}\times 2}\,,\, I_S \otimes \sigma_\eta\, V_{\bar{k}} \\ I_2\,,\, 0_{2\times S} \end{array} \right)$$

and $\epsilon_t = (u_t\,, \varepsilon_t\,, \eta_{1,t}\,, ..., \eta_{S,t})^\mathsf{T} \sim N\left(\mathbf{0}_{2+S}, I_{2+S}\right)$.

We now derive the *observation equation* for the system given by

$$\mathbf{y}_t = Z_{1,t}\,\alpha_t + Z_{2,t}\,\alpha_{t-1} + \phi_t.$$

First note that the consensus price p_t can be expressed in terms of the past state vector α_t as

$$p_t = e_2^\mathsf{T}\,\theta_{t-1}^{(1)} + \sigma_\varepsilon\,\varepsilon_t.$$

Next, note that we can write institution i's submission $m_{i,t}$ as

$$m_{i,t} = \theta_{i,t}^{(1)} + \sigma_\psi\,\psi_{i,t} = \theta_t^{(1)} + x_{i,t}^{(1)} + \sigma_\psi\,\psi_{i,t}.$$

The above derivations allow us to write c_t, $Z_{1,t}$ and $Z_{2,t}$ in terms of the parameters of the model. We start by defining an auxiliary matrix J_t that allows us to deal with missing submissions by some institutions in period t. Recall that $\iota_t \subset \{1, 2, .., S\}$ is the set of institutions submitting in t. Let $\iota_{k,t}$ designate the k-th element of the index ι_t. J_t is a $(|\iota_t| \times S)$ matrix whose k-th row has a 1 in position $\iota_{k,t}$ and zeros otherwise.

We thus have

$$\phi_t = \left(\begin{array}{c} 0 \\ \sigma_\psi\, J_t\,(\psi_{1,t}, ..., \psi_{N,t})^\mathsf{T} \end{array} \right) \;\text{ with }\; \Gamma_t = \mathbb{E}\left(\phi_t\,\phi_t^\mathsf{T}\right) = \left(\begin{array}{cc} 0 & \mathbf{0}_{|\iota_t|}^\mathsf{T} \\ \mathbf{0}_{|\iota_t|} & \sigma_\psi^2\, I_{|\iota_t|} \end{array} \right).$$

Furthermore, we have $Z_{1,t} = J_t\,Z_1$ and $Z_{2,t} = J_t\,Z_2$ where

$$Z_1 = \left(\begin{array}{c} 0_{1\times 1+\bar{k}+S\bar{k}}\,,\, \sigma_\varepsilon \\ 0\,,\,1\,,\, e_1^\mathsf{T} \\ 0\,,\,1\,,\, e_{\bar{k}+1}^\mathsf{T} \\ \vdots \\ 0\,,\,1\,,\, e_{(S-1)\bar{k}+1}^\mathsf{T} \end{array} \right),\;\text{ and }\; Z_2 = \left(\begin{array}{c} 0, 1, 0_{1\times(\bar{k}-2)+S\bar{k}+1} \\ 0_{1\times 1+\bar{k}+S\bar{k}+1} \\ \vdots \\ 0_{1\times 1+\bar{k}+S\bar{k}+1} \end{array} \right).$$

Given a prior for the state of the system at $t = 1$, $\alpha_1 \sim N(\mathbf{a}_1, P_1)$, we can now apply the usual Kalman filter recursion to derive the likelihood function for our data $(\mathbf{y}_t)_{t=1}^T$ given the parameter vector ϕ denoted $L\left((\mathbf{y}_t)_{t=1}^T \mid \phi\right)$.

7.6 Proof of identification

Strategy of proof The proof of identification proceeds in two steps. First, we establish identification for the model under the assumption that submitting institutions take the consensus price to be an exogenous signal of the past state, i.e. $p_t = \theta_{t-1} + \varepsilon_t$. This is the model of the first step in Nimark's (2017) solution algorithm. Second, once we have established identification of the first-step model, we proceed by induction. In particular, we argue that if the model is identified at step n of the algorithm, it is also identified at step $n+1$. This then establishes identificiation of the model at all steps of the algorithm.

A. Identification with exogenous consensus price signal

If submitters assume that the consensus price is an exogenous signal of the (past) state, then individual submitters' first-order beliefs are updated according to

$$\theta_{i,t} = \rho\,\theta_{i,t-1} + (k_{11}\;k_{12}) \begin{pmatrix} \theta_t + \eta_{i,t} - \rho\,\theta_{i,t-1} \\ \theta_{t-1} + \varepsilon_t - \theta_{i,t-1} \end{pmatrix}.$$

We can write this as

$$\theta_{i,t} = (1-k)\rho\,\theta_{i,t-1} + k\,\rho\,\theta_{t-1} + k_{11}\,u_t + k_{12}\,\varepsilon_t + k_{11}\,\eta_{i,t}, \tag{20}$$

where the Kalman gains k_{11} and k_{12} are given by

$$k_{11} = \frac{\zeta + \rho^2\,k}{\zeta + \rho^2 + \psi/(1-\psi)} \;\text{ and }\; k_{12} = \rho(k - k_{11}) \;\text{ with}$$

$$k = \frac{1}{2} + \frac{1}{2\rho^2}\left\{\left[(1-\rho)^2 + \xi\right]^{\frac{1}{2}}\left[(1+\rho)^2 + \xi\right]^{\frac{1}{2}} - (1+\xi)\right\},$$

$$\xi = \frac{\zeta}{\psi}\;,\;\; \psi = \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} \;\text{ and }\; \zeta = \frac{\sigma_u^2}{\sigma_\varepsilon^2}.$$

The average first-order belief is then

$$\bar{\theta}_t = (1-k)\rho\,\bar{\theta}_{t-1} + k\,\rho\,\theta_{t-1} + k_{11}\,u_t + k_{12}\,\varepsilon_t,$$

with corresponding (step 2) consensus price process

$$p_t = \bar{\theta}_{t-1} + \varepsilon_t.$$

This implies the following dynamics for the consensus price,

$$p_t = (1-k)\rho\,p_{t-1} + k\,\rho\,\theta_{t-2} + k_{11}\,u_{t-1} + (k_{12} - (1-k)\rho)\varepsilon_{t-1} + \varepsilon_t. \tag{21}$$

Observed data We assume that our observed data consists of a panel of individual first-order beliefs for N submitting institutions $\{\{\theta_{i,t}\}_{i=1}^N\}_{t=1}^T$ that evolve according to (20), and the corresponding time-series of consensus prices $\{p_t\}_{t=1}^T$ generated by the process specified in (21).

We now show how the distribution of the above data identifies the model parameters of interest, namely $\{\rho, \sigma_\varepsilon^2, \sigma_\eta^2, \sigma_u^2\}$.

1. Deviations of the consensus price from average expectations identify σ_ε^2.
We obtain estimates for the error ε_t from the difference between the current consensus price and the past mean submission as

$$\varepsilon_t = p_t - \bar{\theta}_{t-1}.$$

We can thus identify σ_ε^2 from the time-series variance of the estimated errors.

2. Individual deviations from average expectations identify $(1-k)\rho$.
Individual deviations from the consensus, $\hat{\theta}_{i,t} = \theta_{i,t} - \bar{\theta}_t$ are given by

$$\hat{\theta}_{i,t} = (1-k)\rho\,\hat{\theta}_{i,t-1} + k_{11}\,\eta_{i,t}.$$

Individual deviations follow an AR(1) process. Deviations from consensus mean-revert more quickly if submitters put less weight on past information (higher k), or if the fundamental value process is less persistent (low ρ). We can therefore identify $(1-k)\rho$ from the auto-covariances of individual deviations from the current mean submission.

3. Persistence in consensus price updates identify ρ *and hence* k *via* $(1-k)\rho$.
Having identified $(1-k)\rho$ we can obtain $\omega_t = p_t - (1-k)\rho\,p_{t-1}$ from our data, where

$$\omega_t = k_{11}\,u_{t-1} + k\,\rho\left(\frac{u_{t-2}}{1-\rho L}\right) + (k_{12} - (1-k)\rho)\varepsilon_{t-1} + \varepsilon_t.$$

ω_t is a noisy measure of the fundamental news submitters receive in period t. By subtracting $(1-k)\rho\,p_{t-1}$ from p_t it "cleans out" their prior beliefs. For sufficiently long lags, ω_t's auto-correlation exclusively comes from its dependence on the fundamental process and not the aggregate noise, ε_t. Its auto-covariances thus allow us to identify the persistence in the process of θ_t. In particular, we can see that the auto-covariances of ω_t have to satisfy

$$Cov(\omega_t, \omega_{t-3}) = \rho\,Cov(\omega_t, \omega_{t-2}).$$

The ratio of these auto-covariances thus identify ρ,

$$\rho = Cov(\omega_t, \omega_{t-3})/Cov(\omega_t, \omega_{t-2}),$$

which together with $(1-k)\rho$ then allow us to identify $1-k$, i.e. the persistence in individual expectations due to informational frictions.

4. The weight submitters put on the consensus price when updating expectations identifies σ_η^2 *and hence* σ_u^2 *via* k.
k determines how much weight submitters put on new information as opposed to their priors. It is given by

$$k = \frac{1}{2} + \frac{1}{2\rho^2}\left\{\left[(1-\rho)^2 + \xi\right]^{\frac{1}{2}}\left[(1+\rho)^2 + \xi\right]^{\frac{1}{2}} - (1+\xi)\right\},$$

$$\text{where } \xi = \frac{\zeta}{\psi} \text{ with } \psi = \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} \text{ and } \zeta = \frac{\sigma_u^2}{\sigma_\varepsilon^2}.$$

It is a function of ξ, which is a ratio of the variance of the shocks to the fundamental value to the variance of the signal noises and can thus be seen as a measure of the signal to noise ratio. k is monotonically increasing in ξ; a higher signal to noise ratio implies a higher weight on current signals. Hence, having already identified k, we can also identify ξ.

In turn, the weights submitters put on the private signal and the consensus price can be expressed in terms of k, ξ, and ψ, namely

$$k_{11} = \frac{\xi\,\psi + \rho^2\,k}{\xi\,\psi + \rho^2 + \psi/(1-\psi)} \quad \text{and} \quad k_{12} = \rho(k - k_{11}).$$

It can be shown that, for a given k, the weight on the private signal, k_{11}, is monotonically decreasing and the weight on the consensus price, k_{12}, monotonically increasing in ψ for $\psi \in (0,1)$; a relatively more noisy private signal will lead submitters to shift weight from the private signal to the consensus price (given k). As we have already identify k and ξ, knowing either k_{11} or k_{12} will allow us to identify ψ. Given ψ we can then back out σ_η^2 and ζ, which yields σ_u^2.

We now proceed to show identification of k_{12}, which by the previous argument establishes identification of the model. To do so, we return to the individual expectation updating equation,

$$\theta_{i,t} = (1-k)\rho\,\theta_{i,t-1} + k_{11}\,\rho\,\theta_{t-1} + k_{12}\,p_t + k_{11}\,\eta_{i,t} + k_{11}\,u_t.$$

We also have

$$\theta_{i,t-1} = (1-k)\rho\,\theta_{i,t-2} + k_{11}\,\theta_{t-1} + k_{12}\,p_{t-1} + k_{11}\,\eta_{i,t-1}.$$

Multiplying the latter expression by ρ and subtracting from the former eliminates the unobservable θ_{t-1}. We obtain an expression in terms of observables and shocks,

$$\theta_{i,t} - \rho\,\theta_{i,t-1} = (1-k)\rho(\theta_{i,t-1} - \rho\,\theta_{i,t-2}) + k_{12}\,(p_t - \rho\,p_{t-1}) + k_{11}(\eta_{i,t} - \rho\,\eta_{i,t-1}) + k_{11}\,u_t.$$

Note that we have already identified $(1-k)\rho$. Define

$$y_{i,t} = \theta_{i,t} - \rho\,\theta_{i,t-1} - (1-k)\rho(\theta_{i,t-1} - \rho\,\theta_{i,t-2}).$$

We can then identify the coefficient k_{12} from the covariance of $y_{i,t}$ and $p_t - \rho\,p_{t-1}$ noting that

$$y_{i,t} = k_{12}\,(p_t - \rho\,p_{t-1}) + k_{11}(\eta_{i,t} - \rho\,\eta_{i,t-1}) + k_{11}\,u_t.$$

This is possible as p_t is a signal based on information available in $t-1$ plus ε_t. It is not correlated with the shock u_t. Furthermore, the idiosyncratic noise terms $\eta_{i,t}$ and $\eta_{i,t-1}$ are uncorrelated with the consensus price process by construction.

B. Establishing identification by induction

Suppose we have established identification of the model parameters by our observed data for step n of the algorithm. That is, any two distinct sets of parameters ϕ_1 and ϕ_2 imply distinct distributions of the observable data. In particular, the step n consensus price process that submitters will assume in step $n+1$ differs across the two parameter sets. This necessarily implies that that the distribution of individual expectations will differ across the two parameter sets in step $n+1$. But this then establishes identification of the model at step $n+1$ of the algorithm. ■

7.7 Valuation submission process

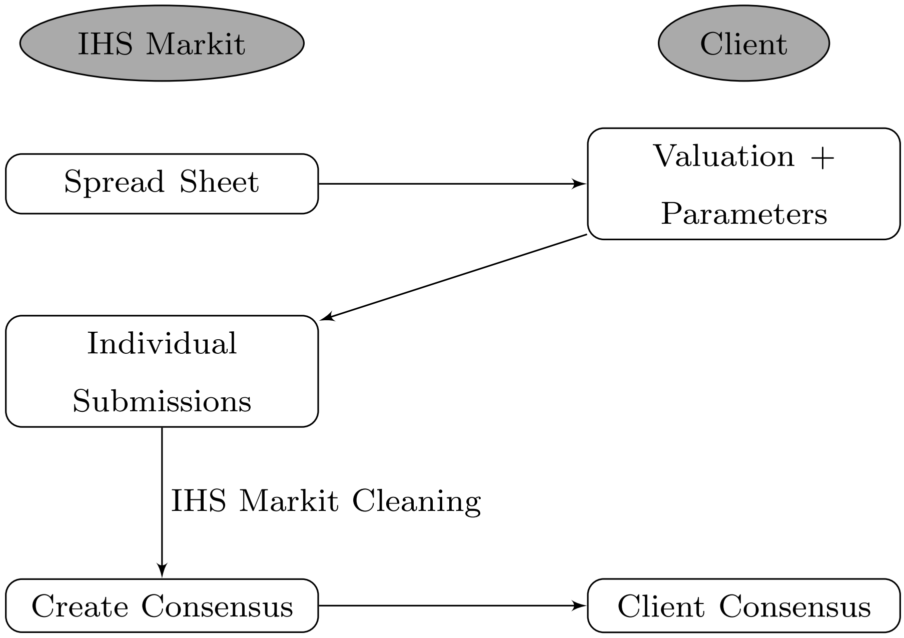

Figure 7: Diagram - Submission process

Figure 7 depicts a diagram of the submission process to IHS Markit's Totem service for plain vanilla index options.[28] Totem issues on the last trading day of each month a spread sheet to $N_{K,T}$ submitters. Here K is the moneyness of the contract defined as the strike price divided by the spot price multiplied by 100 and T is the time-to-expiration of the contract in months. Participating submitters are required to submit their mid-price estimate for a range of put options with a moneyness between 80 and 100 and a range of call option with a moneyness ranging from 100 to 120 with a time-to-expiration of 6 months. Submitters which want to submit to any other contracts with a different maturity or/and different moneyness are required to submit to all the available strike price and time-to-expiration combinations which lie in between the required contracts and the additionally demanded contracts.

Submitter i submits its mid-price estimate for different out of the money put and call options,

[28]Data provided by IHS Markit^{TM} - Nothing in this publication is sponsored, endorsed, sold or promoted by IHS Markit or its affiliates. Neither IHS Markit nor its affiliates make any representations or warranties, express or implied, to you or any other person regarding the advisability of investing in the financial products described in this report or as to the results obtained from the use of the IHS Markit Data. Neither IHS Markit nor any of its affiliates have any obligation or liability in connection with the operation, marketing, trading or sale of any financial product described in this report or use of the IHS Markit Data. IHS Markit and its affiliates shall not be liable (whether in negligence or otherwise) to any person for any error in the IHS Markit Data and shall not be under any obligation to advise any person of any error therein.

$P_t^i(p,K,T)$ and $P_t^i(c,K,T)$ respectively. The inputs which are required in addition to the mid-price estimates are:

- Their discount factor $\beta_t^i(T)$.
- Reference level R_t^i (This is the price of a futures contract with maturity date closest to the valuation date, i.e t.)
- Implied spot level $S_t^i(K,T)$ (Implied level of the underlying index of the futures contract)

There are strict instructions on the timing of the valuation of the contract and the reference level used. To address any issues which might still arise with respect to valuation timing and the effect it could have on the comparability of the prices, the various prices are aligned according to the following mechanism.

1. $\text{Basis}_i = {R^i}_t - S_t^i(K=100, T=6)$
2. $S_t^{i*}(K,T) = \underset{i}{\text{mode}}\left[{R^i}_t\right] - \text{Basis}_i$
3. Remove from $S_t^{i*}(K,T)$ the lowest, highest and erroneous adjusted spot levels.
4. $\bar{S}_t(K,T) = \underset{i}{\text{mode}}\left[{R^i}_t\right] - \frac{1}{N^*(K,T)}\sum_{i=1}^{N^*(K,T)} S_t^i(K,T)$

This consensus implied spot from the at-the-money 6 month option is used for all other combinations of K and T. The submitted prices are restated in terms of $\bar{S}_t(K,T)$, giving: $\widehat{p}_t^i(\{c,p\},K,T) = {}^{P_t^i(\{c,p\},K,T)}/_{\bar{S}_t(K,T)}$.

Given the submitted quantities a security analyst calculates various implied quantities to validate the individual submissions. The security analyst utilizes put-call parity on ATM options to retrieve the relative forward, i.e

$$f_t^i(K,T) = \frac{\widehat{p}_t^i(c,K,T) - \widehat{p}_t^i(p,K,T)}{\beta_t^i(T)} + 1$$

The above inputs are then used in the Black-Scholes model,

$$\widehat{p}_t^i(c,K,T) = \beta_t^i(T)\left[f_t^i(K,T)\,N(d_1) - KN(d_2)\right]$$

$$d_1 = \frac{\ln\left(\frac{f}{K}\right) + \left(\frac{\sigma^2}{2}\right)\Delta T_t}{\sigma\sqrt{\Delta T_t}} \quad \text{, where} \quad \Delta T_t = \frac{\text{days(T)}}{365.25}$$

$$d_2 = d_1 - \sigma\sqrt{\Delta T_t}$$

to back-out the implied volatility, $\sigma_i(K,T)$.

When reviewing submissions, security analysts compare the implied volatilities against other submitted prices and market conditions by taking the following points into consideration:

- The number of contributors
- Market activity & news
- Frequency of change of variables
- Market conventions

- In a one way market, is the concept of a mid-market price clearly understood?
- The distribution and spread of contributed data

In addition to these criteria security analyst also visually inspect the ATM implied volatility term structure and the implied volatility along the moneyness for a given term.[29] After the vetting process the security analyst proceeds to the aggregation of the individual submissions into the consensus data.

Given the Black-Scholes model they back out $\sigma_i(K,T)$ and aggregate it into the consensus implied volatility.

$$\bar{\sigma}\left(K,T\right) = \frac{1}{n_{(K,T)} - n^r} \sum_{i=1}^{n_{K,T}-n^r} \sigma_i\left(K,T\right)$$

Here n^r are the number of excluded prices. The exclusions consist of the lowest, highest and rejected prices. The highest and lowest acceptable $\sigma_i\left(K,T\right)$ are consistent and reasonable IV's, but are excluded to safeguard the stability of the consensus IV.[30] The same process takes place for the submitted prices.[31]

The submitters of which the pricing information is not rejected receive from the security analyst the consensus information.[32,33] The consensus data includes the average, standard deviation, skewness and kurtosis of the distribution of accepted prices and implied volatilities. They also include the number of submitters to the consensus data.

[29] Also referred to as the skew or smile.

[30] If the number of acceptable prices is 6 or below the highest and lowest submissions are included in the consensus price calculations.

[31] The feedback also includes the number of accepted prices, distance between the largest and smallest price, the second, third and fourth moment of the cross-sectional distribution. In our setup the contributers are only uncertain about the mean of the cross-sectional distribution.

[32] The security analysis of totem aim to return the consensus price to the eligiable contributers within 5 hours of the submission deadline.

[33] Submitters who consistently submit accepted prices, might receive a wild card for a rejected submission. Submitters who received a wild card will receive the consensus data even though their submission got rejected.

7.8 Welfare Analysis

Considering a dealer's uncertainty about $\bar{\theta}_t$ in addition to uncertainty about the fundamental value θ_t itself is motivated by strategic complementarities among dealers in the OTC options market. A key source of such complementarities is risk sharing among dealers. When an individual dealer enters into a trade with a client, an important consideration in its pricing decision is to what extent it can share this risk by entering into off-setting trades with other dealers.[34] This creates a coordination incentive among dealers. Lower uncertainty about the average valuation $\bar{\theta}_t$ corresponds to a better ability to forecast other dealers' actions which facilitates coordination. Here, we employ a "reduced-form" model of market behaviour to evaluate the welfare consequences of different information structures for the OTC derivatives market. We derive a dealer's ex-ante expected steady state utility and show how it depends on the covariance matrix of his beliefs.

We assume that in every period each dealer $i \in [0, 1]$ undertakes a payoff-relevant action $a_{i,t}$. Dealer i's expected per-period payoff given information set $\Omega_{i,t}$ is

$$u\left(a_{i,t}, \theta_t, a_t\right) = -\left(\theta_t - a_{i,t}\right)^2 - \lambda\left(a_t - a_{i,t}\right)^2$$

where $\lambda > -1/2$ and $a_t = \int a_{i,t}\, di$ is the average action in t. A first-order condition wrt $a_{i,t}$ yields the optimal action for i in period t, namely

$$a_{i,t} = (1-\beta)\,\mathbb{E}_{i,t}\left(\theta_t\right) + \beta\,\mathbb{E}_{i,t}\left(a_t\right),$$

where $\beta = \lambda/(1+\lambda)$. As $\lambda > -1/2$ we have $|\beta| < 1$ and we can solve the above expression "forward" iteratively substituting out a_t (see Morris and Shin (2002) or Woodford (2003)) to get

$$a_{i,t} = (1-\beta)\sum_{k=0}^{\infty}\beta^k\,\theta_{i,t}^{(k+1)}$$

and

$$a_t = (1-\beta)\sum_{k=0}^{\infty}\beta^k\,\theta_t^{(k+1)}.$$

We substitute these expressions for the optimal action back into the utility function to obtain

$$-\left[(1-\beta)\sum_{k=0}^{\infty}\beta^k\left(\theta_t - \theta_{i,t}^{(k+1)}\right)\right]^2 - \left(\frac{\beta}{1-\beta}\right)\left[(1-\beta)\sum_{k=0}^{\infty}\beta^k\left(\theta_t^{(k+1)} - \theta_{i,t}^{(k+1)}\right)\right]^2.$$

Now note that

$$\mathbb{E}_{i,t}\left[\sum_{k=0}^{\infty}\beta^k\left(\theta_t - \theta_{i,t}^{(k+1)}\right)\right]^2 = \left(\frac{1}{1-\beta}\right)^2\mathbb{E}_{i,t}\left(\theta_t - \theta_{i,t}^{(1)}\right)^2 + \sum_{k=1}^{\infty}\beta^{2k}\left(\theta_{i,t}^{(1)} - \theta_{i,t}^{(k+1)}\right)^2 + 2\sum_{k=1}^{\infty}\sum_{l=k+1}^{\infty}\beta^{k+l}\left(\theta_{i,t}^{(1)} - \theta_{i,t}^{(k+1)}\right)\left(\theta_{i,t}^{(1)} - \theta_{i,t}^{(l+1)}\right),$$

[34] The aggregate dealer sector typically holds a non-zero net position in S&P500 index options, that is the client demands for index options do not net to zero. For empirical evidence on the net dealer exposure in the index options market and its pricing implications, see Gârleanu, Pedersen, and Poteshman (2009).

and

$$\mathbb{E}_{i,t}\left[\sum_{k=0}^{\infty}\beta^k\left(\theta_t^{(k+1)}-\theta_{i,t}^{(k+1)}\right)\right]^2=\sum_{k=1}^{\infty}\beta^{2(k-1)}\,\mathbb{E}_{i,t}\left(\theta_t^{(k)}-\theta_{i,t}^{(k+1)}\right)^2+$$

$$2\sum_{k=1}^{\infty}\sum_{l=k+1}^{\infty}\beta^{k+l-2}\,\mathbb{E}_{i,t}\left(\theta_t^{(k)}-\theta_{i,t}^{(k+1)}\right)\left(\theta_t^{(l)}-\theta_{i,t}^{(l+1)}\right)+$$

$$\sum_{k=1}^{\infty}\beta^{2(k-1)}\left(\theta_{i,t}^{(k+1)}-\theta_{i,t}^{(k)}\right)^2+2\sum_{k=1}^{\infty}\sum_{l=k+1}^{\infty}\beta^{k+l-2}\left(\theta_{i,t}^{(k+1)}-\theta_{i,t}^{(k)}\right)\left(\theta_{i,t}^{(l+1)}-\theta_{i,t}^{(l)}\right).$$

Expected utility in period t can then be expressed as

$$U(\Omega_{i,t})\equiv\max_{a_{i,t}}\mathbb{E}\left[u(a_{i,t},\theta_t,a_t)|\Omega_{i,t}\right]=-Var(\theta_t|\Omega_{i,t})-\beta\left(1-\beta\right)\sum_{k=1}^{\infty}\beta^{2(k-1)}\,Var\left(\theta_t^{(k)}|\Omega_{i,t}\right)-$$

$$2\beta\left(1-\beta\right)\sum_{k=1}^{\infty}\sum_{l=k+1}^{\infty}\beta^{k+l-2}\,Cov\left(\theta_t^{(k)},\theta_t^{(l)}|\Omega_{i,t}\right)-G(\Omega_{i,t})$$

where

$$G(\Omega_{i,t})=(1-\beta)^2\sum_{k=1}^{\infty}\beta^{2k}\,\left[\mathbb{E}\left(\theta_t-\theta_t^{(k)}|\Omega_{i,t}\right)\right]^2+$$

$$+\,2(1-\beta)^2\sum_{k=1}^{\infty}\sum_{l=k+1}^{\infty}\beta^{k+l}\,\mathbb{E}\left(\theta_t-\theta_t^{(k)}|\Omega_{i,t}\right)\mathbb{E}\left(\theta_t-\theta_t^{(l)}|\Omega_{i,t}\right)+$$

$$\beta\left(1-\beta\right)\sum_{k=1}^{\infty}\beta^{2(k-1)}\,\left[\mathbb{E}\left(\theta_t^{(k)}-\theta_t^{(k-1)}|\Omega_{i,t}\right)\right]^2+$$

$$2\beta\left(1-\beta\right)\sum_{k=1}^{\infty}\sum_{l=k+1}^{\infty}\beta^{k+l-2}\,\mathbb{E}\left(\theta_t^{(k)}-\theta_t^{(k-1)}|\Omega_{i,t}\right)\mathbb{E}\left(\theta_t^{(l)}-\theta_t^{(l-1)}|\Omega_{i,t}\right).$$

We now calculate the ex-ante expectation of steady-state utility under a common prior. The steady state covariance matrix is constant. Let $\Sigma_{k+1,l+1}$ denote submitter i's steady-state covariance between $\theta_t^{(k)}$ and $\theta_t^{(l)}$ for all $k,l\geq 0$. Furthermore, under the common prior assumption we have $\mathbb{E}\left[\mathbb{E}(\theta_t^{(k)}-\theta_t^{(l)}|\Omega_{i,t})\right]=\mathbb{E}(\theta_t^{(k)}-\theta_t^{(l)})=0$. The ex-ante expectation of steady-state utility is then given by

$$U=-\Sigma_{1,1}-\beta\left(1-\beta\right)\sum_{k=1}^{\infty}\beta^{2(k-1)}\,\Sigma_{k+1,k+1}-2\beta\left(1-\beta\right)\sum_{k=1}^{\infty}\sum_{l=k+1}^{\infty}\beta^{k+l-2}\,\Sigma_{k+1,l+1}-G.$$

where

$$
\begin{aligned}
G = (1-\beta)^2 \sum_{k=1}^{\infty} \beta^{2k} \left(\Sigma_{1,1} - 2\Sigma_{1,k+1} + \Sigma_{k+1,k+1}\right) + \\
+ 2(1-\beta)^2 \sum_{k=1}^{\infty} \sum_{l=k+1}^{\infty} \beta^{k+l} (\Sigma_{1,1} - \Sigma_{1,k+1} - \Sigma_{1,l+1} + \Sigma_{k+1,l+1}) + \\
\beta\,(1-\beta) \sum_{k=1}^{\infty} \beta^{2(k-1)} (\Sigma_{k+1,k+1} - 2\Sigma_{k+1,k} + \Sigma_{k,k}) + \\
2\beta\,(1-\beta) \sum_{k=1}^{\infty} \sum_{l=k+1}^{\infty} \beta^{k+l-2} (\Sigma_{k+1,l+1} - \Sigma_{k+1,l} - \Sigma_{k,l+1} + \Sigma_{k,l}).
\end{aligned}
$$

Assuming $|\beta|$ is "small" and ignoring terms of order $O(\beta^2)$, ex-ante expected steady-state utility is approximately

$$U \approx -(1+\beta)\Sigma_{1,1} - 2\beta\,\Sigma_{2,2} + 2\beta\,\Sigma_{1,2}.$$

This illustrates how strategic uncertainty, as measured by the variance of second-order beliefs, translates affects dealers' welfare. Different information structures for the market imply different covariance matrices Σ. The above expected utility can then be used to perform welfare comparisons across information structures.

7.9 Covariance Matrices for Counterfactual Scenarios

7.9.1 Consensus price perfectly reveals past state

If the consensus price perfectly aggregates dispersed information we have

$$p_t = \theta_{t-1}.$$

In this case all submitters start period t with a common prior about θ_t, namely $\rho\,\theta_{t-1}$, and there is no higher-order uncertainty before receiving new signals. This is because every submitter knows that every submitter knows that ... the average expected value of θ_t before receiving period t signals is $\rho\,\theta_{t-1}$.

Submitter i's expectations about the fundamental given signal $s_{i,t} = \theta_t + \eta_{i,t}$ can be obtained by the standard updating formula as state θ_t and signal $s_{i,t}$ given θ_{t-1} are jointly normally distributed:

$$\mathbb{E}_{i,t}\left(\theta_t\right) = \theta_{i,t} = \rho\,\theta_{t-1} + k_1\left(s_{i,t} - \rho\,\theta_{t-1}\right) = \rho\,\theta_{t-1} + k_1(u_t + \eta_{i,t})\,,$$

where k_1 is the Kalman gain

$$k_1 = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\eta^2}.$$

It follows that the average expectation is

$$\bar{\theta}_t = \rho\,\theta_{t-1} + k_1\,u_t.$$

Now define the random vector

$$X_t = \left[\theta_t - \rho\,\theta_{t-1}\,,\,\bar{\theta}_t - \rho\,\theta_{t-1}\right] = \left[u_t\,,\,k_1\,u_t\right],$$

and

$$y_{i,t} = s_{i,t} - \rho\,\theta_{t-1} = u_t + \eta_{i,t}.$$

X_t and $y_{i,t}$ are jointly normally distributed. Thus, the covariance of X_t given $y_{i,t}$ is

$$Var\left(X_t|y_{i,t}\right) = \Sigma_{xx} - \Sigma_{xy}\left(\sigma_y^2\right)^{-1}\Sigma_{xy}^{\mathsf{T}},$$

where Σ_{xx} is the variance of X_t and Σ_{xy} is the covariance of X_t and $y_{i,t}$, namely

$$\Sigma_{xx} = \begin{bmatrix} \sigma_u^2 & k_1\sigma_u^2 \\ k_1\sigma_u^2 & k_1^2\,\sigma_u^2, \end{bmatrix}\ ,\ \Sigma_{xy} = \left[\sigma_u^2\ ,\ k_1\,\sigma_u^2\right]^{\mathsf{T}}.$$

As $\rho\,\theta_{t-1}$ is known in t, $Var((\theta_t, \bar{\theta}_t)^{\mathsf{T}}|\Omega_{i,t}) = Var((\theta_t, \bar{\theta}_t)^{\mathsf{T}}|\theta_{t-1}, y_{i,t}) = Var\left(X_t|y_{i,t}\right)$. It follows that

$$Var((\theta_t, \bar{\theta}_t)^{\mathsf{T}}|\Omega_{i,t}) = \begin{bmatrix} \frac{\sigma_u^2\,\sigma_\eta^2}{\sigma_u^2+\sigma_\eta^2} & \frac{\sigma_u^4\sigma_\eta^2}{(\sigma_u^2+\sigma_\eta^2)^2} \\ \frac{\sigma_u^4\sigma_\eta^2}{(\sigma_u^2+\sigma_\eta^2)^2} & \frac{\sigma_u^6\sigma_\eta^2}{(\sigma_u^2+\sigma_\eta^2)^3} \end{bmatrix}.$$

7.9.2 No consensus price feedback

Without consensus price feedback the stationary expectation dynamics of submitter i are given by

$$\theta_{i,t} = \rho\,\theta_{i,t-1} + k_1\,(s_{i,t} - \rho\,\theta_{i,t-1})\,,$$

where k_1 is the stationary Kalman gain. k_1 is the solution to the system of two equations in two unknowns, k_1 and σ^2,

$$k_1 = \frac{\sigma^2}{\sigma^2 + \sigma_\eta^2}\;,\; \sigma^2 = \rho^2(1-k_1)\sigma^2 + \sigma_u^2.$$

The average stationary expectation then evolves according to

$$\bar{\theta}_t = (1-k_1)\rho\,\bar{\theta}_{t-1} + k_1\rho\,\theta_{t-1} + k_1\,u_t.$$

We can now write the dynamics for $(\theta_t, \bar{\theta}_t)^\mathsf{T}$ in state space form, with transition equation

$$\begin{pmatrix} \theta_t \\ \bar{\theta}_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ k_1\rho & (1-k_1)\rho \end{bmatrix} \begin{pmatrix} \theta_{t-1} \\ \bar{\theta}_{t-1} \end{pmatrix} + \begin{bmatrix} 1 \\ k_1 \end{bmatrix} u_t$$

and measurement equation

$$y_{i,t} = (1\;,\;0)\begin{pmatrix} \theta_t \\ \bar{\theta}_t \end{pmatrix} + \eta_{i,t}.$$

The stationary covariance matrix for the state given the history of signals up to t, $Var((\theta_t, \bar{\theta}_t)^\mathsf{T} | \{s_{i,t-j}\}_{j=0}^{\infty})$ can now be derived with a standard Kalman filter.

THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

Economic and Social Research Council

Systemic Risk Centre

The London School of Economics
and Political Science
Houghton Street
London WC2A 2AE
United Kingdom

tel: +44 (0)20 7405 7686
systemicrisk.ac.uk
src@lse.ac.uk