Incentive constrained risk-sharing, segmentation and asset pricing

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Risk sharing and collateral

Financial markets: agents invest in/hold risky assets + share risk

Relatively risk tolerant agents (eg hedge fund, investment bank) insure risk averse (eg pension fund)

If agent i sells CDS or put against state ω, must pay if ω occurs

If agent i has no resource in ω: counterparty default

To avoid this, i must hold assets generating resources in state ω

Resources back promise made by i $\longrightarrow$ collateral

Imperfect collateral pledgeability $\longrightarrow$ risk sharing $\longrightarrow$ asset pricing

Imperfectly pledgeable collateral

Collateral = assets under agent's management/custody

For collateral to be valuable for creditor:

- Agent must manage assets optimally, instead of shirking - diverting - gambling
- Agent must not threaten to strategically default to obtain debt write-down

Pledgeable = what can be promised s.t. incentive compatibility constraint (IC) that agent does not misbehave

Endogenous incompleteness

Promise lots of insurance in state $\omega \implies$ IC does not hold (//debt overhang Myers 1977)

$\rightarrow$ to avoid misbehaviour (IC): promise only limited insurance

In spite of full set of AD securities,

$$\text{IC} \implies \text{endogenous incompleteness}$$

Endogenous segmentation

To share risk when insurance limited by IC, tilt asset allocation:

More risk averse hold safer assets
$\longrightarrow$ lower need to buy insurance from risk tolerant
$\longrightarrow$ by market clearing, more risk tolerant hold riskier assets

Different agents hold different portfolios of risky assets:
$\longrightarrow$ segmentation

Basis

Price of underlying asset $<$ Price of derivative
(Derivative $=$ replicating portfolio of AD securities)

Deviation from Law of One Price, cannot be arbitraged:

To arbitrage, sell expensive AD securities $\rightarrow$ precluded by IC

Basis $=$ shadow price of IC

Yet, derivative and underlying equally imperfectly pledgeable (and can equally be sold short)

Two premia

Expected return on asset held by agent i reflects two premia

$\longrightarrow$ Premium for covariance with consumption of i (not aggregate consumption, because endogenous incompleteness)

$\longrightarrow$ Premium for covariance with shadow price of IC_i

SML flat at top, steep at bottom

IC $\implies$ limited insurance
$\rightarrow$ high demand for low risk assets from more risk averse agents
$\rightarrow$ relatively high price (low return) for low β assets

Symmetrically relatively high price for high β assets

$\rightarrow$ Expected returns concave in β

Supply effects

Holding aggregate risk (total output in each state) constant

If many very low β and very high β assets

$\longrightarrow$ can allocate risk rather efficiently (risk averse buy low β, risk tolerant buy high β) without much need to trade derivatives

$\longrightarrow$ low shadow cost of IC

$\longrightarrow$ low basis

In contrast, low cross sectional dispersion of βs $\longrightarrow$ large basis

Literature

Kehoe Levine 1993, Alvarez Jermann 2000, Rampini Vish 2017: limited pledgeability of labor income $\implies$ limited insurance
$\neq$ we have imperfectly pledgeable but tradeable assets
$\to$ we study pricing of these assets (deviation from law of one price, concave SML, supply effects)

Financial constraints $\implies$ deviation from law of one price: Hindy Huang (1995), Gromb Vayanos (2002), Garleanu Pedersen (2011)
$\neq$ we have full set of AD securities (pricing results don't reflect exogenous market incompleteness, only IC constraint)

Garleanu Pedersen: different exogenous margin constraints for underlying and derivative $\to$ basis
$\neq$ we have same constraint for underlying and derivative, yet basis

Assets, markets and agents

Two dates: 0 and 1. State ω realized at date 1, with proba $\pi(\omega)$

Assets (trees): $j \in [0,1]$ with payoff (fruits): $d_j(\omega)$

tree supply $\bar{N}_j$ positive measure on $j \in [0,1]$
can be discrete, continuous or both

I types, each in measure 1, endowed with fraction of market $\bar{N}_{ij}$

Concave utility over date-1 consumption $U_i = \sum_\omega \pi(\omega) u_i(c_i(\omega))$

At date 0, can trade trees and complete set of state–ω contingent Arrow Debreu securities $\longrightarrow$ potential for risk–sharing

Investor i's program

Choose tree holdings: N_{ij} positive measure over $[0,1]$, Arrow securities: $a_i(\omega)$, to max U_i s.t.

$t = 1$ BC: consumption = fruits of trees + payoff AD security

$$c_i(\omega) = \int_j d_j(\omega) dN_{ij} + a_i(\omega)$$

$t = 0$ BC: initial endowment $\geq$ portfolio held (trees + AD)

$$\int_j p_j d\bar{N}_{ij} \geq \int_j p_j dN_{ij} + \sum_{\omega} q(\omega) a_i(\omega)$$

Incentive compatibility constraint (IC): slack if $a_i(\omega) \geq 0$, otherwise

$$\begin{array}{ccc} -a_i(\omega) & \leq & (1-\delta) \int_j d_j(\omega) dN_{ij}. \\ \text{liability} & \leq & \text{pledgeable income} \end{array}$$

Portfolio margining, with state by state constraint

Equilibrium

Consumption plans $c_i(\omega)$ and tree holdings N_{ij}, prices for Arrow securities $q(\omega)$ and trees p_j, s.t.

Agents maximize expected utility of time 1 consumption given price and budget and IC constraint and markets clear

$$\sum_i a_i(\omega) = 0, \sum_i N_{ij} = \bar{N}_j$$

Equilibrium constrained Pareto optimal: complete markets + no price in constraint

Existence: because IC imposes only additional linear constraints

Uniqueness: with two CRRA types with $\gamma \leq 1$

First order condition w.r.t. consumption

$$\pi(\omega)u_i'(c_i(\omega)) + \mu_i(\omega) = \lambda_i q(\omega) \text{ if } c_i(\omega) > 0$$

Increasing $c_i(\omega)$ increases Eu_i and relaxes IC, but tightens BC

If IC slack, MRS equal across agents // pricing kernel $M = q/\pi$

$$\frac{u_i'(c_i(\omega_1))}{u_i'(c_i(\omega_2))} = \left(\frac{q(\omega_1)}{\pi(\omega 1)}\right) / \left(\frac{q(\omega_2)}{\pi(\omega_2)}\right) = \frac{M(\omega_1)}{M(\omega_2)}$$

If IC binds ($\mu_i(\omega) > 0$): wedge between agents MRS/imperfect risk-sharing $\rightarrow$ AD securities pricing kernel reflects agent's marginal utility $u_i'(c_i(\omega)$ *and* shadow cost of IC $A_i(\omega)$

$$M(\omega) = \frac{u_i'(c_i(\omega))}{\lambda_i} + \frac{\mu_i(\omega)}{\lambda_i \pi(\omega)} = \frac{u_i'(c_i(\omega)}{\lambda_i} + A_i(\omega)$$

First order condition w.r.t. tree holdings

$$p_j = E\left[M(\omega)d_j(\omega) - A_i(\omega)\delta d_j(\omega)\right] \text{ if } n_{ij} > 0$$

1^{st} term: asset's cash flows, valued at pricing kernel $M(\omega)$
2^{nd} term: shadow cost of IC for i when buying j

$$\text{basis}: \; p_j < E\left[M(\omega)d_j(\omega)\right]$$

price of underlying < price of replicating AD portfolio

Not arb opportunity:

Arb $\rightarrow$ buy "underpriced"/sell "overpriced"
$\rightarrow$ hit IC constraint

Basis without exogenously different constraints for different assets
($\neq$ Garleanu Pedersen)

Discount versus premium

Geanakoplos (2008), Geanakoplos Zame (2014): collateral *premium* $\neq$ here: basis, i.e., *discount*

No contradiction, different benchmarks

Collateral premium: Asset price $>$ value of cash flows for i

$$p_j > \frac{1}{\lambda_i} E[u_i^{'}(c_i(\omega))d_j(\omega)]$$

also true in our model

Basis: Asset price $<$ price of replicating derivatives

$$p_j < E\left[M(\omega)d_j(\omega)\right]$$

only in our model ($\delta > 0$)

Endogenous segmentation

FOC tree holdings

$$p_j = \max_i v_{ij}, \text{ where } v_{ij} = E\left[M(\omega)d_j(\omega) - A_i(\omega)\delta d_j(\omega)\right]$$

$E\left[M(\omega)d_j(\omega)\right] =$ "common value" same for all

$-E\left[A_i(\omega)\delta d_j(\omega)\right] =$ "endogenous private value" shadow cost IC_i

Trees held by agents who value them most, because they have the lowest shadow cost

Different trees held by different agents, priced by different kernels

$\neq$ exogenous segmentation: segmentation varies with environment (supply, initial endowment, risk aversion), shocks to different institutions affect different assets differently

Equilibrium expected excess returns

FOC wrt holdings: $p_j = E\left[M(\omega)d_j(\omega) - A_i(\omega)\delta d_j(\omega)\right]$, if $n_{ij} > 0$

Define risky return: $R_j(\omega) = \dfrac{d_j(\omega)}{p_j}$, risk-free return: $R_f = \dfrac{1}{E[M(\omega)]}$

$$E\left[R_j(\omega)\right] - R_f = -R_f Cov(M(\omega), R_j(\omega)) + R_f E[A_i(\omega)\delta R_j(\omega)]$$

1^{st} premium > 0 if $R_j(\omega)$ large when $M(\omega)$ low ($\neq$ frictionless CCAPM, $M(\omega)$ does not mirror agg. consumption, not even individual consumption, bc IC prevents full risk-sharing)

2^{nd} premium > 0 if nonpledgeable income $\delta R_j(\omega)$ large when IC binds (for agents holding the asset): varies across assets because $\neq$ assets held by $\neq$ agents

2 states - 2 types

- aggregate output in bad state ω_1 < in good state ω_2
- type 1 more risk tolerant, type 2 more risk averse: CRRA $\gamma_1 < \gamma_2$

Continuum of trees indexed by $j \in [0, 1]$

If payoff in good state $d_j(\omega_2)$ large relative to payoff in bad state $d_j(\omega_1) \longrightarrow$ large consumption β

Simple specification: Large $j \longrightarrow$ large consumption β

$$d_j(\omega) = (1 - j)\mathbf{1}(\omega = \omega_1) + j\mathbf{1}(\omega = \omega_2)$$

Equilibrium segmentation

$\exists$ k, s.t., risk tolerant type 1 hold trees $j > k$ (high β), risk averse type 2 hold trees $j < k$ (low β)

1^{st} best $\longrightarrow$ large share of aggregate consumption for risk averse in bad state ω_1 $\longrightarrow$ implement by holding low β assets and purchasings state ω_1 Arrow securities from risk tolerant type

2^{nd} best: IC precludes large sale of bad state ω_1 Arrow securities by risk tolerant (otherwise tempted to default) $\longrightarrow$ engineer as much insurance as possible with trees $\longrightarrow$ risk averse holds asset with relatively high payoff in ω_1: low β

$\neq$ types hold $\neq$ portfolios: risk tolerant tilts towards high β

Equilibrium asset prices

Asset $j > k$, held by risk-tolerant agent 1$\longrightarrow$ basis reflects shadow price of agent 1's IC (binds in bad state ω_1)

$$p_j = E\left[M(\omega)d_j(\omega)\right] - A_1(\omega_1)\delta d_j(\omega_1)$$

Asset $j < k$, held by risk-averse agent 2$\longrightarrow$ basis reflects shadow price of agent 2's IC (binds in good state ω_2)

$$p_j = E\left[M(\omega)d_j(\omega)\right] - A_2(\omega_2)\delta d_j(\omega_2)$$

Beta and basis

Consumption β increases as $d_j(\omega_2)$ increases & $d_j(\omega_1)$ decreases

Among assets held / risk-tolerant agent 1 (which tend to have high β)

Larger β (lower dividend when IC1 binds, $d_j(\omega_1)$) $\longrightarrow$ lower basis $A_1(\omega_1)\delta d_j(\omega_1)$

Among assets held / agent 2 (which tend to have low β)

Lower β (low $d_j(\omega_2)$) $\longrightarrow$ lower basis $A_2(\omega_2)\delta d_j(\omega_2)$

$\longrightarrow$ basis inverse U-shaped with β: smallest for very low β and very high β, largest for intermediary β

In terms of expected returns

Low basis for very high and very low β assets

$\longrightarrow$ Low expected returns for very high and very low β assets

$\longrightarrow$ SML steep at bottom and flat at top

Black (1972), Frazzini Pedersen (2010), Hong and Sraer (2016)

Supply effects

Holding aggregate risk constant, i.e., holding aggregate output in each state constant

Large cross sectional dispersion of $\beta \longrightarrow$ some assets with very large or very low $\beta \longrightarrow$ low basis

Low cross sectional dispersion of $\beta \longrightarrow$ high basis on average

Conclusion

Simple one-period GE asset pricing model + standard corporate finance friction $\implies$

- Endogenous segmentation
- Basis: underlying $<$ derivative
- SML steep at bottom flat at top
- Lower dispersion of $\beta \rightarrow$ larger basis