Strategic Complexity

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Motivation: Complexity in Finance

Financial products. E.g. securitized products.

"[Structured products] are detailed in lengthy prospectuses, many of them hundreds of pages long, describing the collateral, the allocation of cash flows from the pool of loans to the securities in various states of nature, the ratings of the securities, and other structural features." Shent, Torons, Valkanor (2017)

Regulation. E.g. tax code, banking rules.

"The Basel documents are not only thousands of pages long, they are also a hard read. An average sentence in the Basel documents consists of 25.7 words, often embedded into complex grammatical structures. In comparison, an average sentence of the British National Corpus, which is a collection of texts covering a broad range of modern British English, only consists of 21 words." Kolly and Muller (2018)

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- Evidence that complexity has been associated with lower quality.
  - Financial products: Célérier and Vallée (2015), Ghent at al (2017), Hoechle, Ruenzi, Schaub and Schmid (2017).
  - Regulation: Behn, Haselmann, Vig (2014), Colliard and Georg (2018).

- Questions that remain unanswered
  - Are increasingly complex products bad for finance?
  - When is complexity more likely to emerge?
  - What can rationalize the existence/persistence of complex products.

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Introduction

- We study the drivers of product complexity.
- Generally, a product is seen as more complex when
  - It has multiple provisions and contingencies.
  - It is difficult to understand (e.g. long, confusing, time-consuming).
- Our focus: complexity as a device to make products harder to understand/evaluate.
- Our Interest: to understand the strategic incentives to deviate from a product's *natural level of complexity.*

What we do: principal-agent model

- The principal wants a product that has to be designed by an agent.
- The agent chooses product quality and complexity, both hidden-actions.
- The agent proposes the product to the principal.
- The principal observes a signal about product quality and decides whether to accept the product or take her outside option.
- Strategic complexity: the principal is more likely to receive noisier information about a more complex product.

Applications

1. Financial products:
   - Banks design financial products that they offer to retail investors.
   - Product: e.g. savings account.
   - Investors choose whether to accept a product or not.

2. Regulation:
   - Regulators design rules that they propose to congress and the public.
   - Product: e.g. tax reform.
   - Congress and/or the public approve the rule or not.

3. Peer-reviewed publication process ...

In these examples, product design establishes:

- actions and transfers for different contingencies ($\sim$ quality).
- style and wording of descriptions and explanations ($\sim$ complexity).

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Preview of Results

- Complexity is not necessarily an indication of worse quality products: it is strategically used by all agents.
  - Producers of good quality products sometimes complexify them.
  - Producers of bad quality products sometimes simplify them.
  - This is true even though everyone is rational!
- As product demand increases (or competition decreases), products become more complex and their quality falls.
- As the *quality* of agents improves, however, both complexity *and* quality increase.
- This has important (empirical) implications: the relation between complexity and quality depends on the underlying drivers of heterogeneity.

The Model: Setup

Players: a principal (P) and a θ-type agent (A_θ) where $\theta \in \{H, L\}$.

- The agent is H-type with probability $p \in (0, 1)$.

Actions:

- A_θ chooses product output, $y_\theta \in \{G, B\}$, and complexity, $\kappa_\theta \in \{\underline{\kappa}, \bar{\kappa}\}$.
- P accepts, $a = 1$, or rejects, $a = 0$, the proposed product.

Payoffs:

- P gets $w(y)$ if she accepts a y-output product and w_0 o.w., where

$$w(B) < w_0 < w(G).$$

- A_θ gets $v_\theta(y)$ when a y-output product is accepted and zero o.w., where

$$v_H(G) = v_L(B) = \bar{v} > \underline{v} = v_H(B) = v_L(G).$$

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The Model: Information

- θ is the agent's private information, and so are his actions, $\{y_\theta, \kappa_\theta\}$.
- P observes signal $s \in \{G, B\}$ about product output, y, where

$$\mathbb{P}(s = g | y = B) = \mathbb{P}(s = b | y = G) = z \in \left[0, \frac{1}{2}\right].$$

  where $z \sim F(\cdot|\kappa)$ with pdf $f > 0$ and satisfies the MLRP property.

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Equilibrium Concept

We consider Perfect Bayesian Equilibria (PBE) of our game:

- Agent Optimality. A_θ's strategy must be optimal given P's strategy.
- Principal Optimality. P's strategy must be optimal given her beliefs.
- Belief Consistency. P's beliefs must be consistent with A_θ's strategies and updated using Bayes' rule.

We solve the model by backward induction ...

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Equilibrium Characterization: Principal's Strategy

- After A proposes a product, P observes $\{s, z\}$ and forms beliefs

$$\mu(s, z) \equiv \mathbb{P}(y = G | s, z).$$

- Given her beliefs, she chooses her acceptance strategy to maximize her expected payoff

$$\max_{a \in \{0,1\}} a \cdot [\mu(s, z) w(G) + (1 - \mu(s, z)) w(B)] + (1 - a) \cdot w_0.$$

- Thus, P approves iff

$$\mu(s, z) \geq \omega \equiv \frac{w_0 - w(B)}{w(G) - w(B)}.$$

I will refer to ω as the (relative) outside option.

Principal's Strategy

Thus, for a given noise level z, there are three possibilities:

1. **Contingent decision, accept iff** $s = G$: $\mu(B; z) < \omega \leq \mu(G; z)$.
2. **Always accept**: $\omega \leq \mu(s; z)$, $\forall s$.
   - We say P is relatively optimistic when $\mu(s, 0.5) \geq \omega$, $\forall s$.
   - In the absence of information, P accepts the product.
3. **Always reject**: $\mu(s; z) < \omega$, $\forall s$.
   - We say P is relatively pessimistic when $\mu(s, 0.5) \leq \omega$, $\forall s$.
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Principal's Strategy

Probability of Approval

Proposition

If P is **optimistic***, the probability of having a y-output product accepted, given noise z, is:*

$$\mathbb{P}(a=1|y,z) = \begin{cases} \mathbb{P}(s=G|y,z) & \text{if } z \leq \bar{z} \\ 1 & \text{if } z > \bar{z} \end{cases}$$

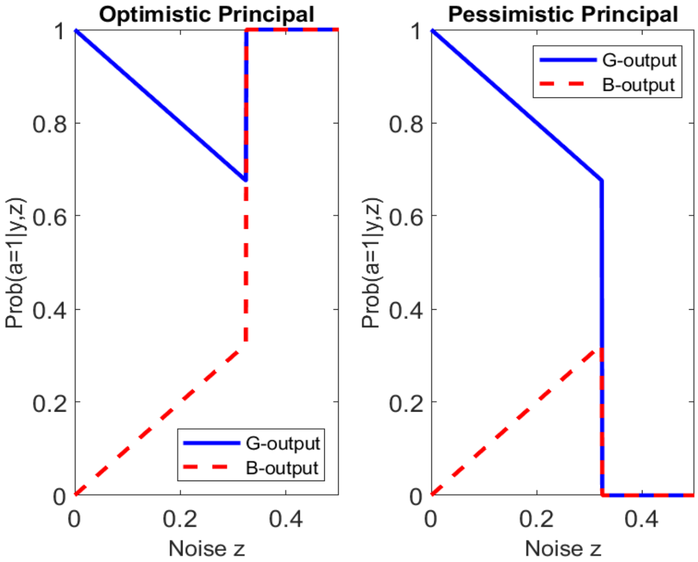
where $\mu(B;\bar{z}) = \omega$. Instead, if P is **pessimistic***, it is:*

$$\mathbb{P}(a=1|y,z) = \begin{cases} P(s=G|y,z) & \text{if } z \leq \bar{z} \\ 0 & \text{if } z > \bar{z} \end{cases}$$

where $\mu(G;\bar{z}) = \omega$.

Principal's Strategy in a Picture

Probability of Approval: $P(a = 1|y, z)$



Equilibrium Characterization: Agent's Strategy

- A_θ agent chooses $\{y, \kappa\}$ to maximize $\mathbb{E}\left[\mathbb{P}(a=1|y,z)|\kappa\right] \cdot v_\theta(y)$.
- Step 1: Find the optimal choice of complexity for a given output choice

$$\kappa_y \in \arg\max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\frac{1}{2}} \mathbb{P}(a=1|y,z) \cdot f(z|\kappa) \cdot dz$$

Note that κ_y is independent of θ.

- Step 2: Given $\{\kappa_G, \kappa_B\}$, find the optimal output choice

$$y_\theta \in \arg\max_{y \in \{B,G\}} \left(\int_0^{\frac{1}{2}} \mathbb{P}(a=1|y,z) \cdot f(z|\kappa_y) \cdot dz \right) \cdot v_\theta(y)$$

- A_θ's strategy: $\{m_\theta \equiv \mathbb{P}(y_\theta = B), \sigma_y \equiv \mathbb{P}(\kappa_y = \bar{\kappa})\}$.

Agent's Strategy: Complexity

Proposition

In any equilibrium, if principal is **optimistic**, *the agent who designs:*

- *B-output product always complexifies.*
- *G-product complexifies if* $\bar{z} \leq \hat{z}$, *and simplifies otherwise (mix if* $\bar{z} = \hat{z}$*).*

In contrast, if principal is **pessimistic**, *the agent who designs:*

- *B-product simplifies if* $\bar{z} \leq \hat{z}$, *and complexifies otherwise (mix if* $\bar{z} = \hat{z}$*).*
- *G-product always simplifies.*

where $\hat{z}$ *is uniquely defined by* $\int_0^{\hat{z}} z \cdot f(z|\underline{\kappa})dz = \int_0^{\hat{z}} z \cdot f(z|\bar{\kappa})dz$.

Let's now study the choice of product output y ...

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Agent's Strategy: Output

Proposition

The agent choice of product output is as follows:

- A_H *agent chooses the* B*-product w.p.* $m_H = 0$.
- A_L *agent chooses the* B*-product w.p.* $m_L \in [0, 1]$, *where* $m_L = 1$ *if*

$$\underbrace{\mathbb{P}(a = 1 | G, \kappa_G) \cdot \underline{v}}_{L\text{-type's payoff if } y = G} - \underbrace{\mathbb{P}(a = 1 | B, \kappa_B) \cdot \bar{v}}_{L\text{-type's payoff if } y = B} < 0.$$

Equilibrium

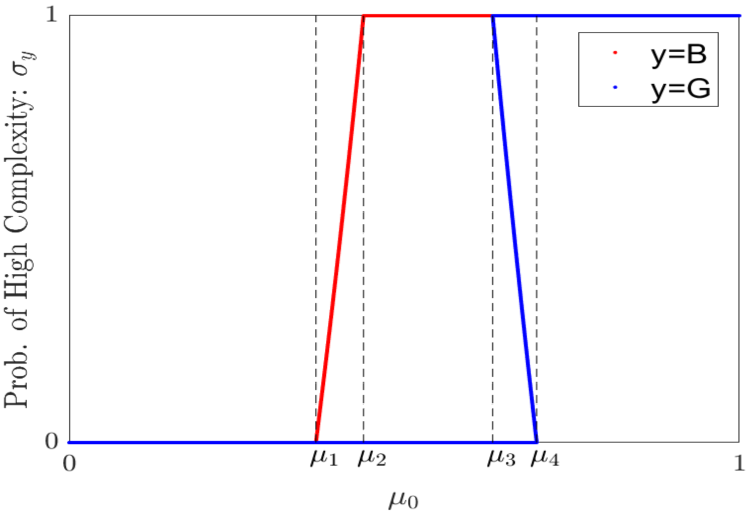
- We have characterized P's and A_θ's strategies for given principal prior belief, μ_0, and interim beliefs, $\hat{\mu} : z \mapsto [0,1]$.
- But these must be consistent with equilibrium outcomes:

$$\mu^* = p + (1 - m_L^*)(1 - p)$$
$$\hat{\mu}^*(z) = \frac{\mu^*}{\mu^* + (1 - \mu^*)\ell^*(z|\sigma_G^*, \sigma_B^*)}$$

- Partial Equilibrium: fix μ_0, and find the fixed point(s) on σ_y's.
- Full Equilibrium: also find fixed point(s) on m_θ's to obtain μ^*, σ_y^*'s.

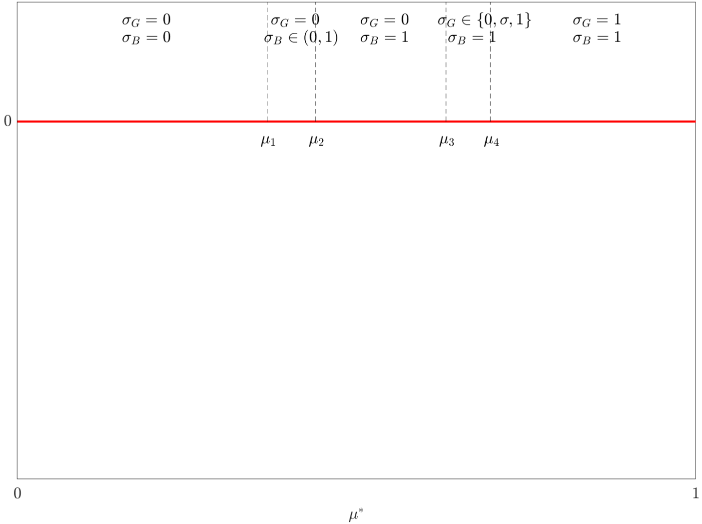
Partial Equilibrium in a Picture

Complexity Determination for given μ_0



Full Equilibrium in a Picture

Output and Complexity Determination



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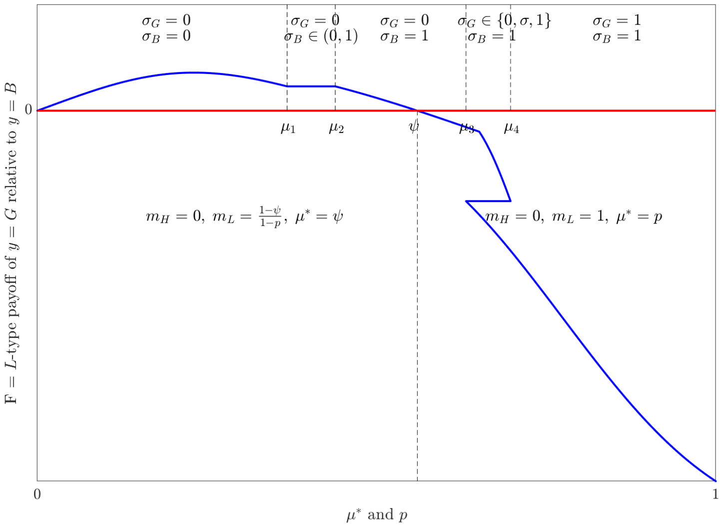
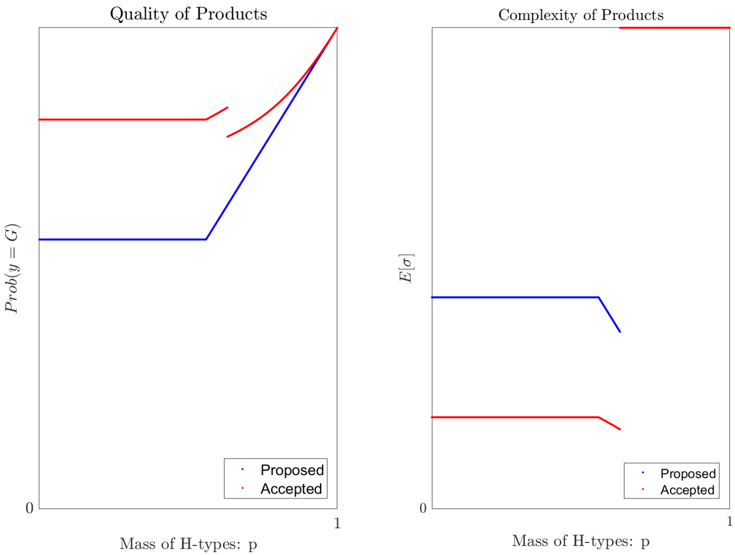


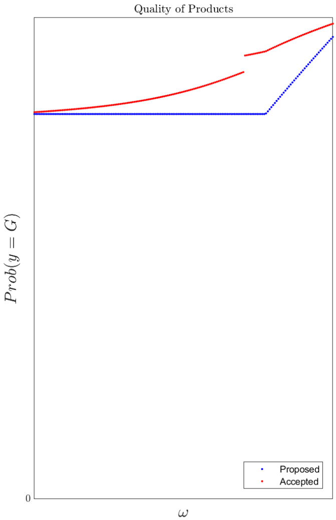
Comparative statics: changes in p

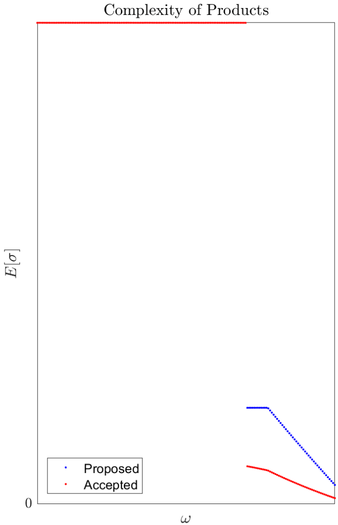
$\uparrow p$: the number of good advisors/regulators is high



Comparative statics: changes in ω

$\uparrow \omega$: demand is low or competition is high





What have we learned?

1. Are increasingly complex products bad for finance?
   - Not necessarily. All agents use complexity strategically, even when they design good quality products/rules.

2. When is complexity more likely to arise?
   - When demand for products is high ($\downarrow \omega$).
   - When competition among agents is low ($\downarrow \omega$).
   - When the trust in the agent is high ($\uparrow \mu$, e.g. $\uparrow p$).

3. What can rationalize the existence/persistence of complex products?
   - In our model, if the demand for financial products increases:
     - complexity increases while product quality falls.
     - negative correlation between quality and complexity.
   - Same is true for rules when urgency to pass regulation increases.

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