Mitigating Fire Sales with Contracts: Theory and Evidence

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Introduction

- Two parts
  1. A model of fire-sales
  2. Evidence on a fascinating historical episode of a bail-out coordinated by a clearing house

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  2. Evidence on a fascinating historical episode of a bail-out coordinated by a clearing house
- I will comment on part 1 almost exclusively

Model, Comments

- $(1+\delta)$ measure of risk-neutral investors, each owning a unit of the risky asset
- unit measure of CARA market makers with risk aversion γ
- $t = 0, 1, 2$
- In period 2, risky asset $R \sim N(\mu, \sigma^2)$

- In period 1, two states: $\omega = (0,1)$ implying price $p_1(\omega)$:
  - w.p λ default state $(\omega = 1)$:
    - δ fraction of investors default sell all their holdings and get 0 utility
    - remaining 1 measure of investors have to sell everything iff $p_1(1) \leq \kappa$ (set $\bar{e} = 1$ for simplicity)
  - w.p. $(1-\lambda)$ no default ($\omega = 0$) , no one liquidates, $p_1(0) \geq \kappa$.

Investor's problem

$$\max_{s_0} s_0 p_0 + ((1-\lambda) + \lambda(1-\delta) 1_{p_1>\kappa})((1-s_0)\mu) +$$
$$+ \lambda(1-\delta) 1_{p_1<\kappa}(1-s_0) p_1 + \lambda\delta 0$$

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- **C1:** If $\mu - \delta\gamma\sigma^2 > \kappa > \mu - (1+\delta)\gamma\sigma^2$, there would be multiple equilibria. I would prefer that route. Instead it is assumed that $\kappa > \mu - \delta 2\gamma\sigma^2, \rightarrow$, unique equilibrium. (Language is mixed. "Coordination failure".)

- unique equilibrium where the constraint binds for $\omega = 1$:

$$\max_{s_0, s_1} s_0 p_0 + ((1-\lambda))((1-s_0)\mu) + \lambda(1-\delta)(1-s_0)p_1$$

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$$\max_{s_0, s_1} s_0 p_0 + ((1-\lambda))((1-s_0)\mu) + \lambda(1-\delta)(1-s_0)p_1$$

- to have an interior optimum, we need

$$p_0 = (1-\lambda)\mu + \lambda(1-\delta)p_1 \, (1)$$

- (if $>$ then $s_0 = 1$, if $<$, then $s_0 = 0$)

Market maker's problem

$$\max_{d_0,d_1} E_{R,\omega}\left(-\exp\left(-\gamma\left(d_0\left(R-p_0\right)+d_1\left(R-p_1\left(\omega\right)\right)\right)\right)\right)$$

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- **C2:** Instead, G solves for prices by assuming that MM makes zero utility from trading. Why? (Free entry? But then, each would end up with ε holdings and no risk premium)

- Period 1: market maker arrives with position d_0 and ω :

$$V_1(d_0,\omega) = \max_{d_1} E_R\left(-\exp\left(-\gamma\left(d_0\left(R-p_0\right)+d_1\left(R-p_1\left(\omega\right)\right)\right)\right)\right)$$

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- if $\omega = 1$, all liquidated: $D_0 + D_1 = 1 + \delta$, $p_1(1) = \mu - (1+\delta)\gamma\sigma^2$, $d_1 = 1 + \delta - d_0$
- if $\omega = 0$, no default, $d_1 = D_1 = 0$ implying $p_1(0) = \mu - d_0\gamma\sigma^2$. (we have to check back that $\mu - d_0\gamma\sigma^2 > \kappa$)

- Remarks:
  - changing D_0 has no effect on $p_1(1)$ as long as all has to liquidate everything
  - for interior equilibrium we need

$$p_0 = (1-\lambda)\mu + \lambda(1-\delta)\left(\mu - (1+\delta)\gamma\sigma^2\right)$$

- in Period 0 market maker solves

$$
\begin{aligned}
&\max_{d_0} E_\omega V_1(d_0,\omega) = \\
=&\max_{d_0} \lambda E_R\left[-\exp\left(-\gamma\left(d_0\left(R-p_0\right)+\left(1+\delta-d_0\right)\left(R-p_1(1)\right)\right)\right)\right]+ \\
&+\left(1-\lambda\right)E_R\left[-\exp\left(-\gamma\left(d_0\left(R-p_0\right)\right)\right)\right]
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- giving (after substitution of $p_1(1)$)

$$\frac{\mu - p_0 + \lambda(1+\delta)\gamma\sigma^2}{(1-\lambda)\gamma\sigma^2} = d_0$$

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- giving (after substitution of $p_1(1)$)

$$\frac{\mu-p_0+\lambda(1+\delta)\gamma\sigma^2}{(1-\lambda)\gamma\sigma^2}=d_0$$

- substituting in p_0, we have

$$\frac{\lambda}{1-\lambda}\left(\frac{\mu\delta}{\sigma^2\gamma}+(1+\delta)(2-\delta)\right)=D_0=d_0=S_0=(1+\delta)s_0$$

Claims and comments

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1. planner solution: no one sells anything in period 0 and 1 (except the defaulting agents)
    - **C3:** Why?
        - suppose no-fire sales: if I know that with some probability I will default and get 0 utility, why is it efficient not to sell and enjoy some consumption before default?
        - suppose $\kappa > \mu - \delta\gamma\sigma^2$, hence, fire sales: No-selling is not a feasible allocation for planner as violates the constraint. Right benchmark is the second best.

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   - **C4:** Given C3, unique equilibrium seems to be constrained efficient.
   - to check, ask the question: Would welfare increase if we change a choice in the decentralized solution? No: increasing s_0, d_0 would not even change $p_1\,(1)$. (Unless there another equilibrium with no binding constraint.)

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   - **C5:** this is an incomplete market problem. Investors are not allowed to save towards $\omega = 1$ only they are forced to save towards both aggregate states. The proposed contract/institution completes the market, i.e., it gets rid of a constraint. (subject to free-riding)

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   - **C6:** There are similar problems which do lead to inefficiencies: there is Pareto improvement without violating constraints. Why not going with that? (E.g. Moore (2013) variant of Lorenzoni (2008), or Davila-Korinek(2018))
     - hopefully argument goes through: why an institution which solves some fire-sale problems but not others would be attractive?

- **C6:** Now it should be clear why the multiple equilibria route might be simpler. Two equilibria, one Pareto dominates the other. CCP can push the economy to one.
  - It is like D-D and deposit insurance: a good point to make.

Sum up

- In the current form model harms the paper much more than it helps
- But there are easy fixes:
  - go with a standard one
  - or claim equilibrium selection instead of eliminating inefficiency.