The effect of risk preferences on the valuation and incentives of compensation contracts

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The effect of risk preferences on the valuation and incentives of compensation contracts

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Abstract

We use a comparative approach to study the incentives provided by different types of compensation contracts, and their valuation by risk averse managers, in a fairly general setting. We show that concave contracts tend to provide more incentives to risk averse managers, while convex contracts tend to be more valued by prudent managers. Thus, prudence can contribute to explain the prevalence of stock-options in executive compensation. We also present a condition on the utility function which enables to compare the structure of optimal contracts associated with different risk preferences.

The compensation package of top executives is widely regarded as an important governance mechanism, especially in big companies with dispersed ownership. Accordingly, most top executives and CEOs receive stocks and stock-options to align their interests with those of shareholders. This being said, there are many ways to provide monetary incentives as a function of a measure of performance: for example, good performance can be rewarded, or poor performance can be punished. Despite the popularity of stock-options, which have a convex payoff profile which reward managers who achieve positive stock returns, it is unclear whether and why this is more efficient than punishing managers who achieve negative stock returns with a concave payoff profile. To shed light on this important issue, we use a comparative approach in the context of a standard principal-agent model of effort choice to study the incentive and risk-sharing properties of convex and concave contracts. Our results suggest that effort incentives tend to be more effectively provided by concave contracts if managers are risk averse, whereas convex contracts tend to provide a higher expected utility to managers who are prudent, i.e., whose marginal utility is convex. Given that top executives are risk averse and prudent, the model predicts that the *structure* of the optimal contract which provides a given level of incentives is the outcome of a tradeoff between effort incentives and optimal risk sharing.[1]

In a standard version of the principal-agent problem, the utility function u of the manager interacts with the form of the compensation contract to affect both his level of incentives and his expected utility – hence the difficulty of obtaining relatively "general" optimality results.[2] Indeed, the form of the optimal contract, usually derived with the conditions in Holmstrom (1979), strongly depends on the assumed utility function and the assumed probability distri-

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[1]An interesting parallel is that, in a standard principal-agent model of effort choice with a risk averse agent, the *level* of incentives is determined by a tradeoff between effort inducement and optimal risk sharing. With a risk neutral principal, the latter is achieved with no incentives, while the former requires the provision of incentives by making the manager's pay sensitive to his performance. In particular, eliciting more effort requires a larger deviation from the first-best risk sharing rule, which is costly if and only if the agent is risk averse.

[2]One notable exception is the mechanism with arbitrarily large and very unlikely punishments proposed by Mirrlees (1975), which nevertheless relies on a utility function unbounded from below for feasible payments, and an arbitrarily large likelihood ratio at the left tail of the distribution.

bution of performances (e.g., Holmstrom (1979), Hall and Murphy (2002), Hemmer, Kim and Verrecchia (2000), Dittmann and Maug (2007)). Thus, it remains somewhat unclear which forces drive the optimality or suboptimality of different forms of compensation contracts in a standard moral hazard problem. This paper contributes to answering this question by highlighting the effect of the manager's risk aversion ($u'' < 0$) and prudence ($u''' > 0$) on the level of incentives provided by different contracts, and the expected utility they are associated with in equilibrium. Unless stated otherwise, we do not impose any condition on the probability distribution of performances except from the standard monotone likelihood ratio property (which ensures roughly speaking that better performances are more likely to be obtained with high rather than low effort), and we do not specify a functional form for the utility function. Thus, most of our results are quite general and widely applicable.

To start with, we consider the effect of risk aversion. For a manager with quadratic utility, which is by construction the only utility function with mean-variance preferences, we find that any compensation profile convex (respectively concave) in performance provides less (resp. more) effort incentives than a linear contract. Moreover, for a performance measure additive in effort and a symmetrically distributed noise term, we find that any convex compensation profile is dominated by a concave compensation profile for an agent with mean-variance preferences – we show that a related result is also applicable to models with a lognormal distribution for the stock price and a managerial effort which has a multiplicative effect on the performance measure, such as in Dittmann and Maug (2007). This is because it is possible to construct a concave contract with the same expected payment as the convex contract, i.e. the same cost to the firm, which gives the same utility to the manager but provides more effort incentives. Consequently, it is also possible to construct a concave contract which provides just as much effort incentives for a lower cost. Thus, it seems difficult to rationalize the existence of stock-options in the context of a standard principal-agent model of effort choice in which the manager is only risk averse.

This suggests that we should consider individual preferences with respect to higher-order

moments of the distribution of payments, including prudence, to explain the existence of compensation contracts such as stock-options in this type of model. Interestingly, we find that any given convex (respectively concave) compensation profile gives a higher (resp. lower) expected utility to the manager than a linear contract with the same expected payment and the same variance of payment if and only if the manager is prudent. More generally, we also show that increasing the convexity of any compensation profile while leaving the first and second moment of the distribution of payments unchanged increases the expected utility of a prudent agent, whereas the opposite holds if the contract becomes more concave instead.

The economic intuition for these results is the following. On the one hand, concave contracts tend to provide more incentives to risk averse managers because the covariance between marginal utility and the slope of payments is positive with a concave contract, so that concave contracts tend to concentrate incentives where the manager is most sensitive to changes in his pay. This effect had already been highlighted in Jenter (2002) and Dittmann and Maug (2007), but we present a more formal result and we also emphasize the existence of a countervailing effect. Indeed, on the other hand, prudent managers tend to place a higher value on convex contracts because they are averse to downside risk: the concavity of their utility function is decreasing in wealth. Roughly speaking, a prudent manager does not discount much an upward deviation from a given payment, but he is highly sensitive to a downward deviation – in utility terms. It is noteworthy that aversion to downside risk is not the same as loss aversion, and that aversion to downside risk is necessary but not sufficient for decreasing absolute risk aversion (for example, a utility function with Constant Absolute Risk Aversion (CARA) is prudent).

We emphasize that the implications of changes in prudence for the form of the optimal compensation profile are a priori ambiguous. This is because prudence is inextricably linked with risk aversion, so that a change in prudence typically does not leave risk aversion unchanged. For example, in the case of preferences with Constant Relative Risk Aversion (CRRA) and a normally distributed performance measure, we highlight that an increase in

prudence, however measured, is not accompanied by an increase in the convexity of the compensation profile, contrary to what the aforementioned results might suggest. This is because, in the case of CRRA utility, an increase in prudence also implies an increase in risk aversion. Nevertheless, when the first-order approach applies, we identify which transformations of the utility function unequivocally lead to an increase in the convexity (or concavity) of the optimal compensation profile. We also present a condition on risk preferences which enables to compare the relative curvatures of the optimal compensation profiles associated with two different utility functions.

Our approach is related to Ross (2004), who does not derive the optimal contract.[3] Instead, he compares a given contract to a linear contract, and establishes conditions under which this given contract provides more risk-taking incentives than a linear contract. We use a similar methodology: we compare any given convex or concave contract to a linear contract on two dimensions: risk sharing, and incentives. Thus, our comparative approach does not yield optimality results, but it enables us to study the properties of different contracts as they relate to the risk preferences embedded in the utility function.

Our results rely on the opposing effects of risk aversion and prudence on the form of the optimal contract. Prudence is less standard than risk aversion, but it is a reasonable assumption which is consistent with the empirical evidence on individual preferences. First, prudence implies a preference for distributions with a positive skewness, which has been documented for example in Kraus and Litzenberger (1976). Second, prudent agents have a precautionary saving motive, which has been validated empirically (Browning and Lusardi (1996), Gourinchas and Parker (2001)). Third, Scott and Horvath (1980) show that prudence is necessary for marginal utility to be positive for all wealth levels. Fourth, prudence is implied by decreasing absolute risk aversion, which is widely viewed as a reasonable assumption.

Many other factors besides risk preferences are obviously relevant in most moral hazard problems. In particular, since risk preferences are essentially preferences about moments of

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[3]In his words, the effort to characterize optimality – often in highly specific and parametric models – has crowded the comparative study of compensation contracts.

the probability distribution of higher order than the mean, our results are intrinsically related to the form of the probability distribution of performances. Accordingly, we also briefly analyze the effect of changes in the probability distribution of performances, as captured by the likelihood ratio, on the form of the optimal contract. We argue that this analysis can shed further light on the discrepancies between the different forms of optimal contracts identified in the literature.

1   The model

We use a standard principal-agent model in the spirit of Holmstrom (1979). At time 0, the manager exerts some effort e. Effort affects the probability distribution of a contractible measure of performance π, which is realized at time 1. We denote by $\psi(\pi|e)$ the probability density function of π conditional on e. We assume that the monotone likelihood ratio property holds (this condition guarantees that the likelihood that a given performance is the outcome of high rather than low effort is increasing in performance) but we do not impose any other condition on the probability distribution of performances.

At time -1, a risk-neutral principal offers a compensation contract to the manager.[4] This contract specifies the manager's payment W as a function of the measure of performance π.

As is standard in the literature, we assume that the objective function of the manager is additively separable in the utility of wealth and the cost of effort:

$$u(W(\pi)) - C(e) \tag{1}$$

The utility of wealth u is characterized by $u'(W) > 0$ and $u''(W) \leq 0$, and the cost of effort function is characterized by $C'(e) > 0$ and $C''(e) > 0$. For a given contract W considered, we assume as in Dittmann, Maug and Spalt (2010) that the cost function C is sufficiently convex

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[4]This hypothesis of risk-neutrality is common in the literature. It could be microfounded by assuming that the principal represents well-diversified shareholders.

for the first-order approach to be valid. This makes the problem tractable, as an infinity of incentive constraints can be replaced by the first-order condition to the manager's problem, which is satisfied at the equilibrium level of effort. We denote the reservation level of utility of the manager by $\bar{U}$.

2   Preliminary analysis and related literature

It is standard in the literature on the optimal structure of compensation to focus on the first step of optimal contracting in Grossman and Hart (1983), which consists in minimizing the (agency) cost of implementing a given level of effort, which we denote by $e^\star$. Any optimal contract is a solution to the first step problem, for a given $e^\star$. Therefore, if the optimal contract is convex in π for any $e^\star$ in a certain setting, say, then the model predicts that the optimal contract is convex.

With this approach, the problem is to minimize the expected cost of compensation subject to the participation constraint and the incentive constraint, which guarantee that the manager accepts the contract at $t = -1$, and exerts effort $e^\star$ at $t = 0$, respectively:

$$\min_{W(\pi)} E[W(\pi)|e^\star] \qquad \text{subject to} \tag{2}$$

$$E[u(W(\pi))|e^\star] - C(e^\star) \geq \bar{U} \tag{3}$$

$$E[W'(\pi)u'(W(\pi))|e^\star] = C'(e^\star) \tag{4}$$

Given this optimization problem, the optimal contract is given by the following condition, due to Holmstrom (1979):

$$\frac{1}{u'(W(\pi))} = \lambda + \mu \frac{\psi_e(\pi|e)}{\psi(\pi|e)} \tag{5}$$

where λ and μ are the Lagrange multipliers associated with the participation constraint and the incentive constraint, respectively. This condition makes clear that the form or curvature

of the optimal contract depends on the manager's preferences and the probability distribution of performances.

This condition may be applied to different settings. As the following results indicate, the form of the optimal contract strongly depends on the assumed probability distribution of performances and on the postulated functional form and parameters of the utility function:

CLAIM 1 (DITTMANN AND MAUG (2007)): With a lognormally distributed performance measure and a manager with CRRA utility with coefficient of relative risk aversion γ, the optimal contract takes the form:

$$W(\pi) = (\alpha_0 + \alpha_1 \ln(\pi))^{\frac{1}{\gamma}} \tag{6}$$

where α_0 and α_1 are two constants which are determined to satisfy the participation constraint and the incentive constraint.

Dittmann and Maug (2007) show that the optimal contract is concave for a range of plausible levels of relative risk aversion, and that its structure differs markedly from that of commonly observed CEO compensation contracts.

CLAIM 2 (CHAIGNEAU (2011)): With a lognormally distributed performance measure and a manager with HARA utility of the form $u(W) = \left(a + \frac{W}{b}\right)^{1-b}$, the optimal contract takes the form:

$$W(\pi) = \frac{1}{b}\Big(\frac{1-b}{b}(\alpha_0 + \alpha_1 \ln(\pi))^{\frac{1}{b}} - a\Big) \tag{7}$$

where α_0 and α_1 are two constants which are determined to satisfy the participation constraint and the incentive constraint.

Chaigneau (2011) shows that plausible preferences of the HARA class with decreasing

absolute risk aversion and decreasing relative risk aversion can generate an optimal contract which closely matches a typical CEO compensation contract.

CLAIM 3 (HEMMER, KIM, AND VERRECCHIA (2000)): Under some conditions to guarantee the validity of the first-order approach, if the likelihood ratio is linear in performance, and if the manager has HARA utility with decreasing absolute risk aversion and nondecreasing relative risk aversion and a relative risk aversion in-between one-half and one, then the optimal contract is convex in performance.

In particular, the likelihood ratio is linear in performance with the normal and the gamma distribution.

Comparing the result of Dittmann and Maug (2007) to that of Chaigneau (2011) shows that the form of the optimal contract is very sensitive to the postulated preferences of the manager. Likewise, comparing the result of Dittmann and Maug (2007) to that of Hemmer, Kim, and Verrecchia (2000) shows that, even with the same preferences (CRRA), the form of the optimal contract is very sensitive to the postulated probability distribution of performances. In the remainder of this paper, we will contribute to explain how and why risk preferences and the probability distribution of performances affect the form of the optimal contract.

3 The effect of risk aversion

To start with, we consider an economic agent who is risk averse ($u'' < 0$) but not prudent ($u''' = 0$). In this case, we will show that a convex compensation profile delivers less effort incentives than a linear compensation profile characterized by the same average payment and the same average pay-performance sensitivity.

For any given contract W, let e_W be the equilibrium level of effort which solves

$$\int_{-\infty}^{\infty} W'(\pi)u'(W(\pi))\psi(\pi|e_W)d\pi = C'(e_W) \tag{8}$$

For any given contract W, we construct the linear contract such that the compensation of the manager at any given π is equal to $a + b\pi$. The parameters a and b are set so that

$$b = E[W'(\tilde{\pi})|e_W] \qquad \text{and} \qquad a = E[W(\tilde{\pi})|e_W] - bE[\tilde{\pi}|e_W] \tag{9}$$

That is, for any given contract W, the associated linear contract is characterized by the same expected payment and the same average pay-performance sensitivity when evaluated at the equilibrium effort e_W (the pay-performance sensitivity of the linear contract is independent of effort). Let e_L be the equilibrium level of effort which solves

$$\int_{-\infty}^{\infty} bu'(a + b\pi)\psi(\pi|e_L)d\pi = C'(e_L) \tag{10}$$

We find that a concave (resp. convex) contract implements a higher (resp. lower) level of effort than the corresponding linear contract if the manager is risk averse and is not prudent:

PROPOSITION 1: If $u'' = 0$ then $e_W = e_L$. If $u'' < 0$ and $u''' = 0$, then $e_W > e_L$ if W is concave, and $e_W < e_L$ if W is convex.

Intuitively, the extent of incentives generated by a given sensitivity of pay to performance depends on the marginal utility at this point. With risk aversion, marginal utility is higher for low payments; and with a concave contract, high pay-performance sensitivities are associated with low payments.

If we add more structure on the probability distribution of performances, we can prove that a convex contract cannot be optimal if the manager has mean-variance preferences. More

precisely, we say that a manager has mean-variance preferences if his objective function over wealth with a contract $W(\pi)$ can be written as $f(E[W(\tilde{\pi})|e^\star], var[W(\tilde{\pi})|e^\star])$, with

$$\frac{\partial f(E[W(\tilde{\pi})|e^\star], var[W(\tilde{\pi})|e^\star])}{\partial E[W(\tilde{\pi})|e^\star]} > 0 \qquad \text{and} \qquad \frac{\partial f(E[W(\tilde{\pi})|e^\star], var[W(\tilde{\pi})|e^\star])}{\partial var[W(\tilde{\pi})|e^\star]} < 0 \quad (11)$$

This mean-variance criterion encompasses but is not limited to quadratic utility and an objective function linear in the mean and variance of payments.[5] Then we have the following result:

PROPOSITION 2: If the manager has mean-variance preferences as described in (11), and if the measure of performance is additive in effort and a symmetrically distributed noise term, then any compensation profile convex in performance is dominated by a compensation profile concave in performance.

When effort results in a rightward shift of the symmetric distribution of the performance measure, a convex contract cannot be optimal if the manager has mean variance preferences. This dominance result underlines the importance of considering preferences with respect to moments of the probability distribution of performances of higher order than the variance if the model is to generate contracts which are convex as a function of a symmetrically distributed performance measure.

It is noteworthy that Proposition 2 enables to rule out all contracts which are more convex than the logarithm function in a model with a lognormally distributed stock price where managerial effort has a multiplicative effect on firm value (e.g. Dittmann and Maug (2007)):

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[5]Strictly speaking, it encompasses quadratic utility for the range of payments such that marginal utility is positive, which is a standard restriction. On the one hand, for a quadratic utility function of the form $u(w) = \alpha x - \beta w^2$ for positive α and β, we have $u'(w) = \alpha - 2\beta w$, which is positive if and only if $w < \frac{\alpha}{2\beta}$. On the other hand, $E[u(w)] = \alpha E[x] - \beta(var(x) + (E[x])^2)$, so that $\frac{\partial E[u(w)]}{\partial var(x)} = -\beta$, which is indeed negative, and $\frac{\partial E[u(w)]}{\partial E[w]} = \alpha - 2\beta E[w]$, which is positive if and only if $E[w] < \frac{\alpha}{2\beta}$. It follows that on the interval of payments where marginal utility is positive ($w < \frac{\alpha}{2\beta}$), we necessarily have $E[w] < \frac{\alpha}{2\beta}$, so that the criterion described in (11) also captures quadratic utility.

COROLLARY 1: If the manager has mean-variance preferences, and if $\tilde{\pi}$ is multiplicative in effort and a lognormally distributed noise term, then any compensation profile such that $W''(\ln(\pi)) > 0$ is dominated.

In words, any compensation profile which is a convex transformation of the logarithm function is dominated. This obviously includes all compensation profiles which are linear or convex with respect to the performance π, but it also includes some compensation profiles which are concave with respect to π.

4   The effect of prudence

In this section, we show that prudent agents derive a higher expected utility from a convex contract than from a less convex contract (including a linear contract) characterized by the same expected payment and the same variance of payment.

Given any compensation profile W, we construct the linear compensation profile $a + b\pi$. The parameters a and b are determined so that, given that $e = e^\star$, both contracts have the same expected payment and the same variance of payment – we momentarily ignore effort to focus exclusively on the risk sharing aspect of the compensation profile.

Proposition 1 below shows that the manager's relative valuation of these two contracts depends on the convexity of his marginal utility:

PROPOSITION 3: Suppose that $e = e^\star$. If $u''' = 0$, then $E[u(W(\tilde{\pi}))] = E[u(a+b\tilde{\pi})]$ for any W. If $u''' > 0$, then $E[u(W(\tilde{\pi}))] < E[u(a+b\tilde{\pi})]$ if W is concave, and $E[u(W(\tilde{\pi}))] > E[u(a+b\tilde{\pi})]$ if W is convex.

A prudent manager $(u''' > 0)$ derives a higher expected utility from a convex contract than from the corresponding linear contract. This result generalizes to mean and variance

preserving convex or concave increasing transformations of any given contract W:

PROPOSITION 4: Suppose that $e = e^\star$. If $u''' = 0$, then $E[u(g(W(\tilde{\pi})))] = E[u(g(W(\tilde{\pi}))]$ for any increasing g. If $u''' > 0$, then $E[u(g(W(\tilde{\pi})))] < E[u(W(\tilde{\pi}))]$ if g is increasing and concave and preserves the mean and variance of W, and $E[u(g(W(\tilde{\pi})))] > E[u(W(\tilde{\pi}))]$ if g is increasing and convex and preserves the mean and variance of W.

That is, a more convex compensation profile with the same average payment and the same variance of payments is also associated with a higher expected utility if and only if the manager is prudent. This is because, for any given probability distribution of performances, a mean and variance preserving concave transformation of a compensation profile is equivalent to an increase in the downside risk of the payment distribution. But prudent agents are averse to downside risk, i.e., they apply a heavier discount to downward variations than to upward variations from a given payment.

5 A condition on risk preferences

When economic agents are both risk averse and prudent, the preceding results suggest that the structure or curvature of the optimal contract will be the outcome of a tradeoff between risk preferences.[6] Protecting a prudent manager against downside risk by offering a convex contract must be traded off against the property of a concave contract to concentrate steep changes in pay on regions where a risk averse manager (whose marginal utility is decreasing) is more sensitive to changes in his pay. In general, we cannot determine which effect will dominate, which can contribute to explain why the literature is inconclusive about the form of the optimal contract.

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[6]With a risk neutral agent ($u'' = 0$), any contract and the associated linear contract implement the same level of effort and provide the agent with the same expected utility. Thus, only the nonlinearity of the utility function may explain that two symmetrical contracts with opposite curvatures deliver different incentives, and are associated with a different expected utility.

Propositions 1 to 4 suggest that increasing the degree of prudence relative to the degree of risk aversion will tend to increase the convexity of the compensation profile. Of course, it is impossible to uniformly increase prudence while leaving risk aversion unchanged, because of the very definition of prudence ($u''' > 0$). We therefore cannot obtain simple comparative static results on this dimension.

This being said, we can identify which type transformation of the utility function will lead to a more convex (or more concave) optimal compensation profile when the first-order approach is valid. Indeed, we can rewrite the Holmstrom (1979) condition in (5) as

$$W(\pi) = u'^{-1}\left(\left(\lambda + \mu \frac{\psi_e(\pi|e)}{\psi(\pi|e)}\right)^{-1}\right) \tag{12}$$

For any given increasing function h, we have

$$h(W(\pi)) = hu'^{-1}\left(\left(\lambda + \mu \frac{\psi_e(\pi|e)}{\psi(\pi|e)}\right)^{-1}\right) \tag{13}$$

Therefore, a transformation h convexifies (respectively concavifies) the optimal compensation profile $W(\pi)$ if and only if it convexifies (resp. concavifies) the inverse of marginal utility, u'^{-1}. That is, any convex transformation ($h'' > 0$) of u'^{-1} increases the convexity of the optimal compensation profile.

This leads us to an important condition on the utility function:

PROPOSITION 5: Consider two utility functions, u and v.

$$\text{If} \qquad -\frac{u'^{-1''}(x)}{u'^{-1'}(x)} < -\frac{v'^{-1''}(x)}{v'^{-1'}(x)} \qquad \text{for all } x \text{ in the relevant domain} \tag{14}$$

then the optimal contract for a manager with utility u is a convex transformation of the optimal contract for a manager with utility v.

The condition in (14) is generally easy to check. It enables to determine which changes in risk preferences make the compensation profile more or less convex.

The only other condition which relates the curvature of the optimal contract to risk preferences in a relatively general setting that we are aware of in the literature is Proposition 2 in Hemmer, Kim and Verrecchia (2000). However, they restrict attention to a subset of HARA utility functions with nondecreasing relative risk aversion, to convex compensation profiles, and to probability distributions with a likelihood ratio linear in performance.

To apply Proposition 5 to a simple example, consider the case of two CRRA utility functions with relative risk aversion of γ_u and γ_v, respectively. In this case, $u'^{-1}(x) = x^{-\frac{1}{\gamma}}$, so that the condition in (14) rewrites as

$$-\frac{\frac{1}{\gamma_u}(\frac{1}{\gamma_u}+1)x^{-\frac{1}{\gamma_u}-2}}{\frac{1}{\gamma_u}x^{-\frac{1}{\gamma_u}-1}} < -\frac{\frac{1}{\gamma_v}(\frac{1}{\gamma_v}+1)x^{-\frac{1}{\gamma_v}-2}}{\frac{1}{\gamma_v}x^{-\frac{1}{\gamma_v}-1}} \qquad \text{for all } x \text{ in the relevant domain} \qquad (15)$$

Or

$$-\frac{(\frac{1}{\gamma_u}+1)}{x} < -\frac{(\frac{1}{\gamma_v}+1)}{x} \qquad \text{for all } x \text{ in the relevant domain} \qquad (16)$$

which is satisfied if and only if $\gamma_u < \gamma_v$. That is, the convexity of the optimal compensation profile with CRRA utility is decreasing in the coefficient of relative risk aversion. Notice that this is in line with the result in (6). Finally, even though prudence itself ($u'''(x)$), the coefficient of absolute prudence $-\frac{u'''(x)}{u''(x)}$ (Kimball (1990)) or the measure of downside risk $\frac{u'''(x)}{u'(x)}$ (Keenan and Snow (2010)) are all increasing in the coefficient of relative risk aversion in the case of CRRA utility, we find that an increase in this coefficient leads to a more concave compensation profile. This is because both risk aversion and prudence increase following an increase in relative risk aversion, so that it is a priori not clear which effect will dominate.

These results emphasize that the calibration of preference parameters in a utility function is crucial. This is because it does not only determine the degree of risk aversion of the manager, but also the relative importance of risk aversion and prudence in the utility function, which in turn matters for the curvature of the optimal contract. Proposition 5 offers an easy way to

determine the effect of a change in risk preferences on the curvature of the optimal contract. A notable advantage of this condition is that it enables to compare the relative curvature of two compensation profiles associated with different preferences even when different forms of utility functions are used (for example CRRA and CARA), so that the degree of curvature cannot be directly compared by looking at the expressions which describe the optimal compensation profiles.

6   Valuation and incentives of calls and puts-based contracts

In this section, we assess quantitatively the magnitude of the incentive effect due to risk aversion and of the valuation effect due to prudence in a standard model of executive compensation. More specifically, we compare option-based compensation contracts to linear contracts on two dimensions: their valuation by a risk averse manager, and the incentives they provide.

[TO BE COMPLETED]

7   The distribution of performances and the curvature of the compensation profile

In this section, we study the effect of some changes in the probability distribution of performances on the curvature of the optimal contract, with an aim to explaining differences in the form of the optimal contract under different settings considered in the literature.

With a likelihood ratio $\frac{\psi_e}{\psi}$ strictly increasing in performance, the Holmstrom (1979) condition in (5) that describes the optimal contract when the first-order approach is valid may

be rewritten as

$$W(\pi) = g(\lambda_\pi + \mu_\pi l(\pi|e)) \tag{17}$$

where $l(\pi|e) \equiv \frac{\psi_e(\pi|e)}{\psi(\pi|e)}$, and $g(y) = u'^{-1}(\frac{1}{y})$. Note that g only depends on the utility function u, so that it is not affected by a change in the measure of performance.

We now define a new performance measure, x, which is such that the likelihood ratio l_x at any given x may be written as $l_x(x|e) = l(\phi(x)|e)$, where ϕ is increasing in x to guarantee that the monotone likelihood ratio property is preserved. Below, we will show that this transformation enables us to study several cases of interest. With this new performance measure, the optimal contract takes the form

$$W(x) = g(\lambda_x + \mu_x l(\phi(x)|e)) \tag{18}$$

An interesting question is whether the optimal contract is more convex under π or under x. It can be answered by comparing the measure of convexity $\frac{W''}{W'}$, which has already been used in Hemmer, Kim, and Verrecchia (2000), under both measures of performance. Some calculations yield

$$\frac{W''(\pi)}{W'(\pi)} = \frac{l''(\pi|e)}{l'(\pi|e)} + \mu_\pi l'(\pi|e)\frac{g''(\lambda_\pi + \mu_\pi l(\pi|e))}{g'(\lambda_\pi + \mu_\pi l(\pi|e))} \tag{19}$$

$$\frac{W''(x)}{W'(x)} = \frac{l''(\phi(x)|e)}{l'(\phi(x)|e)} + \mu_x l'(\phi(x)|e)\frac{g''(\lambda_x + \mu_x l(\phi(x)|e))}{g'(\lambda_x + \mu_x(\phi(x)|e))} + \frac{\phi''(x)}{\phi'(x)} \tag{20}$$

The first term in both (19) and (20) is zero if the likelihood ratio under the distribution of π is linear. This is the case for a wide range of distributions, including the normal distribution and the gamma distribution. The second term in both (19) and (20) is zero if g is linear in x, which is notably the case with log utility. The third term in (20) is zero if the transformation ϕ is linear. If all terms are zero, the transformation ϕ does not alter the curvature of the optimal contract. If not, then the curvature of the optimal contract typically changes.

This is due to the combination of three effects. The first two, as captured by the first two terms in either (19) or (20), are translation effects. Equation (17) (or (18)) implies that the

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curvature of the optimal contract at any given performance is a function of the convexity of both g and of the likelihood ratio at this performance. A transformation of the measure of performance will have two effects. First, it will shift the argument of the likelihood ratio l to a different point in the domain of l. To the extent that the likelihood ratio is nonlinear, this will have an impact on the convexity of the likelihood ratio (as a function of performance) at this point, and therefore on the convexity of the optimal contract. This effect is represented in the first term in either (19) or (20). Second, it will shift the argument of g to a different point in the domain of g. To the extent that g is nonlinear, this will have an impact on the convexity of g at this point, and therefore on the convexity of the optimal contract. This effect is represented in the second term in either (19) or (20).

The third effect may be called a likelihood ratio effect, for lack of a better term. The economic intuition is as follows. If the likelihood ratio becomes more concave as a function of performance, say – as is the case following a switch from the normal to the lognormal distribution, for instance – then high performances become relatively less informative about high effort, whereas low performances become relatively more informative about low effort. In such a case, effort is optimally elicited by rewarding good performances less, and by punishing poor performances more, which is achieved by concavifying the compensation profile.

When the likelihood ratio is linear in performance and the agent has log utility, only the likelihood ratio effect operates. Besides, in this case, if we define the function z by $l(\phi(x)|e) \equiv z(l(x|e))$, then, since the likelihood ratio is linear, differentiating twice both sides of this equation yields

$$z''(l(x|e))(l(\phi(x)|e))^2 = \phi''(x)l'(\phi(x)|e) \tag{21}$$

Since the likelihood ratio is increasing by assumption, this implies that z'' is convex (respectively concave) if and only if $\phi''(x)$ is convex (resp. concave). It follows that increasing the convexity (respectively concavity) of the likelihood ratio of performances is equivalent to increasing the convexity (resp. concavity) of the optimal compensation profile. Notice however that this is only true in the special case of log utility and a likelihood ratio linear in

performance.

This analysis can be applied to explain some discrepancies between the different forms of optimal contracts found in the literature. First, it can explain why the optimal contract in a CRRA-lognormal framework (Dittmann and Maug (2007)) is typically concave[7], while it is linear in a CRRA-normal framework (Hemmer, Kim, and Verrecchia (2000)). Indeed, suppose that π is normally distributed. If the agent has log utility, then g is linear (Hemmer, Kim, and Verrecchia (2000)). Since the likelihood ratio of the normal distribution is also linear, it follows that the optimal contract is linear in performance (π). In addition, for a lognormally distributed measure of performance ($x = \exp(\pi)$), we can write $l_x(x|e) = l(\ln(x)|e)$ (Dittmann and Maug (2007)). Now consider (19) and (20). In both cases, the first and second term are zero, which notably indicates that the optimal contract as a function of π is (indeed) linear. But the third term in (20) is negative because $\phi(\cdot) = \ln(\cdot)$ is concave, which indicates that the optimal contract as a function of x is (indeed) concave. Second, the result in Hemmer, Kim, and Verrecchia (2000) that an increase in skewness typically leads to a more convex contract, ceteris paribus, is explained by the second translation effect described above. Indeed, the likelihood ratio of the gamma distribution is linear, and a change in the skewness of this distribution implies a transformation ϕ that is linear, so that neither the first translation effect that we identified nor the likelihood ratio effect operate in this case.[8]

8 Conclusion

Given the centrality of incentives in modern economics, the determinants of the form of the optimal contract is an important question on which the literature has provided surprisingly little guidance so far. In a standard principal-agent model of effort choice, our analysis

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[7]More precisely, it is a linear transformation of the logarithm of the (lognormally distributed) performance measure.

[8]In line with this interpretation, it is noteworthy that this result in Hemmer, Kim, and Verrecchia (2000) (Proposition 3 of their paper) does not hold in the special case of log utility. Indeed, in this case, the function g defined above is linear, which implies that the second translation effect does not operate either, and the optimal contract remains linear for any level of skewness of the stock price distribution.

suggests that the form of the optimal contract is the outcome of a tradeoff between risk aversion and prudence. Since prudent managers tend to discount gains less and to be more averse to losses, we have shown that they place a higher value on contracts with downside risk protection and upside participation, which are precisely features embedded in typical executive compensation contracts. Thus, the effect of prudence can contribute to explain the convexity of typical executive compensation contracts.

The analysis also highlights the importance of the specification and calibration of a utility function in the context of the principal-agent model. In particular, simplifying assumptions such as CRRA preferences might misrepresent the relative importance of risk aversion and prudence in managers' preferences, which could explain some failures of the principal-agent model (e.g., Dittmann and Maug (2007)).

Our results have implications not just for executive compensation, but more generally for economic situations where moral hazard can be addressed by the provision of incentives. Indeed, a convex (respectively concave) contract is essentially a contract such that the pay-performance sensitivity is increasing (resp. decreasing) in performance. Thus, roughly speaking, a convex contract can be implemented with rewards or carrots, whereas a concave contract can be implemented with punishments or sticks.

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9   Appendix

Proof of Proposition 1:

For any contract W, the left-hand side of (8) is equal to

$$E[W'(\tilde{\pi})u'(W(\tilde{\pi}))] = cov(W'(\tilde{\pi}), u'(W(\tilde{\pi}))) + E[W'(\tilde{\pi})]E[u'(W(\tilde{\pi}))] \qquad (22)$$

For any linear contract $a + b\pi$, the left-hand side of (10) is equal to

$$E[bu'(a + b\tilde{\pi})] = cov(b, u'(a + b\tilde{\pi})) + bE[u'(a + b\tilde{\pi})] = bE[u'(a + b\tilde{\pi})] \qquad (23)$$

In the first part of the proof, we consider the case of $u'' < 0$ and $u''' = 0$. We will show that

$$cov(W'(\tilde{\pi}), u'(W(\tilde{\pi}))) > 0 \qquad (24)$$

if $u'' < 0$ and W is concave, and

$$E[W'(\tilde{\pi})]E[u'(W(\tilde{\pi}))] \geq bE[u'(a + b\tilde{\pi})] \qquad (25)$$

if $u'''' \leq 0$ and if W is concave.

First, if $u'' < 0$ and if W is increasing and concave, then

$$\frac{\partial}{\partial \pi}W'(\pi) = W''(\pi) < 0 \qquad \text{and} \qquad \frac{\partial}{\partial \pi}u'(W(\pi)) = W'(\pi)u''(W(\pi)) < 0 \qquad (26)$$

This implies that $cov(W'(\tilde{\pi}), u'(W(\tilde{\pi}))) > 0$.

Second, if $u''' = 0$, then the utility function may be written as $u(w) = \alpha x - \beta w^2$ for positive α and β. Then

$$E[u'(W(\tilde{\pi}))] = \alpha - 2\beta E[W(\tilde{\pi})] \qquad (27)$$

$$E[u'(a + b\tilde{\pi})] = \alpha - 2\beta E[a + b\tilde{\pi}] \qquad (28)$$

Since $E[a + b\tilde{\pi}] = E[W(\tilde{\pi})]$ by construction, we have

$$E[u'(a + b\tilde{\pi})] = E[u'(W(\tilde{\pi}))] \qquad (29)$$

Third, by construction of the linear contract associated with the given contract W (see

(9)), we have

$$E[W'(\tilde{\pi})] = b \qquad (30)$$

for any W.

We conclude that, if $u'' < 0$ and $u''' = 0$ and if W is concave, then

$$E[W'(\tilde{\pi})u'(W(\tilde{\pi}))] > E[bu'(a + b\tilde{\pi})] \qquad (31)$$

for any given e. Finally, we take into account the fact that e_W solves (8) and e_L solves (10). Since the right-hand-side of either (8) or (10) involves the same increasing and convex function C of e, and that we have shown that the left-hand-side of (8) is larger than the left-hand-side of (10) if W is concave, $u'' < 0$, and $u''' = 0$, we conclude that $e_W > e_L$ in this case.

Symmetrically, if W is convex, it can be shown that

$$E[W'(\tilde{\pi})u'(W(\tilde{\pi}))] < E[bu'(a + b\tilde{\pi})] \qquad (32)$$

for any given e. Using the same reasoning as above, this implies $e_W < e_L$ if $u'' < 0$ and $u''' = 0$.

In the second part of the proof, we consider the case of $u'' = 0$. If $u'' = 0$, then, for any W,

$$\frac{\partial}{\partial \pi} u'(W(\pi)) = W'(\pi)u''(W(\pi)) = 0 \qquad (33)$$

This implies that $cov(W'(\tilde{\pi}), u'(W(\tilde{\pi}))) = 0$. If $u'' = 0$, then $u'(w) = \alpha > 0$ for any w, so that

$$E[u'(W(\tilde{\pi}))] = \alpha = E[u'(a + b\tilde{\pi})] \qquad (34)$$

We conclude that, for any contract W, if $u'' = 0$ then

$$E[W'(\tilde{\pi})u'(W(\tilde{\pi}))] = E[bu'(a + b\tilde{\pi})] \qquad (35)$$

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for any given e. Using the same reasoning as above, this implies $e_W = e_L$ if $u'' = 0$.

Q.E.D.

Proof of Proposition 2:

First, we show that any convex contract is such that $\frac{\partial}{\partial e} var[W(\tilde{\pi})|e^\star] < 0$. Note that, for any level of effort e, we can write $\pi = e + \epsilon$ where without loss of generality we set $E[\tilde{\epsilon}] = 0$. For any contract,

$$\frac{\partial}{\partial e} var[W(\tilde{\pi})|e] = E\Big[\frac{\partial}{\partial e}(W(e + \epsilon) - E[W(e + \tilde{\epsilon})])^2\Big]$$

$$= 2E\Big[f(\pi) - E[f(\tilde{\pi})])(f'(\pi) - E[f'(\tilde{\pi})]\Big] \tag{36}$$

If W is increasing and convex in π, then

$$\frac{\partial W(\pi)}{\partial \epsilon} = \frac{\partial W(\pi)}{\partial \pi}\frac{\partial \pi}{\partial \epsilon} = W'(\pi) > 0$$

$$\frac{\partial W'(\pi)}{\partial \epsilon} = \frac{\partial W'(\pi)}{\partial \pi}\frac{\partial \pi}{\partial \epsilon} = W''(\pi) > 0$$

Therefore,

$$cov(W(\tilde{\pi}), W'(\tilde{\pi})) > 0 \tag{37}$$

Moreover,

$$cov(W(\tilde{\pi}), W'(\tilde{\pi})) = E\Big[W(\pi) - E[W(\tilde{\pi})]\Big]\Big(W'(\pi) - E[W'(\tilde{\pi})]\Big]$$

This expression is positive because of (37). Applying these two results to (36) shows that if W is convex then $\frac{\partial}{\partial e} var[W(\tilde{\pi})|e^\star] < 0$.

Second, given any convex contract W, consider the contract which is symmetrical to W with respect to the point $\{e^\star, W(e^\star)\}$ (this contract can be built in two steps: first by considering the contract symmetrical to W with respect to the horizontal line going through

$\{e^\star, W(e^\star)\}$, then by considering the contract symmetrical to this new contract with respect to the vertical line going through $\{e^\star, W(e^\star)\}$. We denote the contract thus obtained by V. We now show that V has the following characteristics:

$$var[V(\tilde{\pi})|e^\star] = var[W(\tilde{\pi})|e^\star] \tag{38}$$

$$E[V'(\tilde{\pi})|e^\star] = E[W'(\tilde{\pi})|e^\star] \tag{39}$$

$$\frac{\partial}{\partial e}var[V(\tilde{\pi})|e^\star] < 0 \tag{40}$$

We construct the function Y as the function symmetrical to W with respect to the horizontal line going through the point $\{e^\star, W(e^\star)\}$, so $Y(\pi) = -W(\pi) + 2W(e^\star)$. Its variance writes as

$$var[Y(\tilde{\pi})] \equiv E\left[\left(Y(\tilde{\pi}) - E[Y(\tilde{\pi})]\right)^2\right] = E\left[\left(-W(\tilde{\pi}) + 2W(e^\star) + E[W(\tilde{\pi})] - 2W(e^\star)\right)^2\right]$$

$$= E\left[\left(W(\tilde{\pi}) - E[W(\tilde{\pi})]\right)^2\right] \equiv var[W(\tilde{\pi})] \tag{41}$$

Now we construct the function V as the function symmetrical to Y with respect to the vertical line going through the point $\{e^\star, W(e^\star)\}$:

$$V(\pi) = Y(\pi - 2(\pi - e^\star)) = Y(-\pi + 2e^\star)$$

Because the probability distribution of $\tilde{\pi}$ is centered around its mean $e^\star$, by assumption, we have $\psi(-\pi + 2e^\star) = \psi(\pi)$.

We first prove (38)

$$E[V(\tilde{\pi})] \equiv \int_{-\infty}^{\infty} V(\pi)\psi(\pi)d\pi = \int_{-\infty}^{\infty} Y(-\pi + 2e^\star)\psi(\pi)d\pi$$

$$= \int_{-\infty}^{\infty} Y(\pi)\psi(-\pi + 2e^\star)d\pi = \int_{-\infty}^{\infty} Y(\pi)\psi(\pi)d\pi \equiv E[Y(\tilde{\pi})] \tag{42}$$

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The second equality uses $Y(\pi) = -W(\pi) + 2W(e^\star)$, the third involves a change of variable, and the fourth uses the symmetry of the probability distribution of $\tilde{\pi}$. Likewise,

$$var[V(\tilde{\pi})] \equiv \int_{-\infty}^{\infty} \left(V(\pi) - E[V(\tilde{\pi})]\right)^2 \psi(\pi) d\pi = \int_{-\infty}^{\infty} \left(Y(-\pi + 2e^\star) - E[Y(\tilde{\pi})]\right)^2 \psi(\pi) d\pi$$

$$= \int_{-\infty}^{\infty} \left(Y(\pi) - E[Y(\tilde{\pi})]\right)^2 \psi(-\pi + 2e^\star) d\pi = \int_{-\infty}^{\infty} \left(Y(\pi) - E[Y(\tilde{\pi})]\right)^2 \psi(\pi) d\pi \equiv var[Y(\tilde{\pi})] \quad (43)$$

Combining (41) and (43),

$$var[V(\tilde{\pi})] = var[Y(\tilde{\pi})] = var[W(\tilde{\pi})] \quad (44)$$

We now prove (39). According to the definition of Y, $Y'(\pi) = -W'(\pi)$, so that

$$E[Y'(\tilde{\pi})] = -E[W'(\tilde{\pi})] \quad (45)$$

Furthermore, $V'(\pi) = -Y'(-\pi + 2e^\star)$. Using the same steps as above,

$$E[V'(\tilde{\pi})] \equiv \int_{-\infty}^{\infty} V'(\pi)\psi(\pi) d\pi = \int_{-\infty}^{\infty} -Y'(-\pi + 2e^\star)\psi(\pi) d\pi$$

$$= \int_{-\infty}^{\infty} -Y'(\pi)\psi(-\pi + 2e^\star) d\pi = \int_{-\infty}^{\infty} -g'(\pi)\psi(\pi) d\pi \equiv -E[Y'(\tilde{\pi})] \quad (46)$$

Combining (45) and (46) yields

$$E[V'(\tilde{\pi})] = E[W'(\tilde{\pi})] \quad (47)$$

We now prove (40). On the one hand, using the definition of Y,

$$\frac{\partial}{\partial e} var[Y(\tilde{\pi})|e] = \frac{\partial}{\partial e} var[-W(\tilde{\pi}) + 2W(e^\star)|e] = \frac{\partial}{\partial e} var[-W(\tilde{\pi})|e] = -\frac{\partial}{\partial e} var[W(\tilde{\pi})|e] \quad (48)$$

On the other hand,

$$\frac{\partial}{\partial e}var[Y(\tilde{\pi})|e] = \frac{\partial}{\partial e}E[(Y(\pi) - E[Y(\tilde{\pi})])^2 = 2E\Big[\Big(\frac{\partial}{\partial \pi}Y(\pi) - \frac{\partial}{\partial \pi}E[Y(\tilde{\pi})]\Big)(Y(\pi) - E[Y(\tilde{\pi})])\Big]$$

Using the definition of V,

$$\frac{\partial}{\partial e}var[V(\tilde{\pi})] = \frac{\partial}{\partial e}E\Big[\Big(V(\pi) - E[V(\tilde{\pi})]\Big)^2\Big]$$

$$= \int_{-\infty}^{\infty} -2\Big[\frac{\partial}{\partial \pi}Y(-\pi + 2e^\star) - \frac{\partial}{\partial \pi}E[Y(-\tilde{\pi} + 2e^\star)]\Big]\Big(Y(-\pi + 2e^\star) - E[Y(-\tilde{\pi} + 2e^\star)]\Big)\varphi(\epsilon)d\epsilon$$

$$= \int_{-\infty}^{\infty} -2\Big[\frac{\partial}{\partial \pi}Y(\pi) - \frac{\partial}{\partial \pi}E[Y(\tilde{\pi})]\Big]\Big(Y(\pi) - E[Y(\tilde{\pi})]\Big)\varphi(-\epsilon)d\epsilon$$

$$= \int_{-\infty}^{\infty} -2\Big[\frac{\partial}{\partial \pi}Y(\pi) - \frac{\partial}{\partial \pi}E[Y(\tilde{\pi})]\Big]\Big(Y(\pi) - E[Y(\tilde{\pi})]\Big)\varphi(\epsilon)d\epsilon = -\frac{\partial}{\partial e}var[Y(\tilde{\pi})|e] \qquad (49)$$

Combining (48) with (49) yields

$$\frac{\partial}{\partial e}var[h(\tilde{\pi})|e] = -\frac{\partial}{\partial e}var[f(\tilde{\pi})|e] \qquad (50)$$

for any e.

Third, we show that a contract $\delta V(\pi) + w$ with $\delta = 1$ and $w = 2E[W(\tilde{\pi})] - 2W(e^\star) \equiv w^\star$ induces a higher effort than the contract $W(\pi)$, gives the same expected utility to the agent, and is as costly to the firm. The third claim follows directly from (42) and the definition of Y:

$$E[Y(\tilde{\pi})] = E[-W(\tilde{\pi}) + 2W(e^\star)] = 2W(e^\star) - E[W(\tilde{\pi})] \qquad (51)$$

So, with $w = w^\star$, we have

$$E[V(\tilde{\pi}) + w^\star|e^\star] = E[W(\tilde{\pi})|e^\star] \qquad (52)$$

Moreover, a contract $\delta V(\pi) + w$ satisfies the participation constraint as an equality if and

only if

$$f(E[\delta V(\tilde{\pi}) + w|e^\star], var[\delta V(\tilde{\pi})|e^\star]) = \bar{U} \tag{53}$$

In particular, because of (38) and

$$f(E[W(\tilde{\pi})|e^\star], var[W(\tilde{\pi})|e^\star]) = \bar{U} \tag{54}$$

then we know that a contract $\delta V(\pi) + w$ with $\delta = 1$ and $w = w^\star$ is such that the participation constraint in (53) is satisfied. Next, a contract $\delta V(\pi) + w$ induces the equilibrium level of effort $e^\star$ if and only if

$$\frac{\partial f(E[\delta V(\tilde{\pi})|e^\star] + w, var[\delta V(\tilde{\pi})|e^\star])}{\partial E[V(\tilde{\pi})|e^\star]} E[\delta V'(\tilde{\pi})|e^\star]$$

$$+ \frac{\partial f(\delta E[V(\tilde{\pi})|e^\star] + w, var[\delta V(\tilde{\pi})|e^\star])}{\partial var[V(\tilde{\pi})|e^\star]} \frac{d}{de} var[\delta V(\tilde{\pi})|e^\star] = C'(e^\star) \tag{55}$$

Because of (39), (40), and $\frac{\partial}{\partial e} var[W(\tilde{\pi})|e^\star] < 0$, and because of the assumption (11) on preferences, we know that the left-hand-side of (55) is higher with the contract $\delta V(\pi) + w$ with $\delta = 1$ and $w = w^\star$ than it would be with the contract $W(\pi)$. Moreover, because C is convex, (55) shows that the level of effort induced by a contract $\delta V(\pi) + w$, for any w, is strictly increasing in the left-hand-side of (55). Therefore, the contract $\delta V(\pi) + w$ with $\delta = 1$ and $w = w^\star$ induces a higher level of effort than the level of effort $e^\star$ induced by the contract $W(\pi)$.

Fourth, we show that there exists a contract $\delta V(\pi) + w$, for $\delta < 1$ and $w < w^\star$, which induces the same effort $e^\star$ as the contract $W(\pi)$, gives the same expected utility to the agent, but which is less costly to the firm.

The left-hand-side of the incentive constraint in (55) is decreasing in δ and independent of w. In addition, it is equal to zero for $\delta = 0$, in which case the level of effort induced is too low since $C' > 0$ by assumption, and it induces a level of effort higher than $e^\star$ for $\delta = 1$ (see above). Therefore, for any value of w, there exists a value of $\delta \in (0, 1)$ which is such that (55)

is satisfied as an equality, i.e., which induces the level of effort $e^\star$. Denote this value by $\delta^\star$ and set $\delta = \delta^\star$, where $\delta^\star \in (0,1)$. Then, for any value of w, the contract $\delta^\star V(\pi) + w$ induces the same effort $e^\star$ as the contract $W(\pi)$.

Since $\delta^\star \in (0,1)$, the contract $\delta^\star V(\pi) + w$ has a lower variance than the contract $V(\pi) + w^\star$:

$$var[\delta^\star V(\tilde{\pi}) + w | e^\star] > var[V(\tilde{\pi}) + w^\star | e^\star] \tag{56}$$

Moreover, since the contract $V(\pi) + w^\star$ satisfies the participation constraint in (53) as an equality, our assumption on mean-variance preferences in (11) imply that the contract $\delta^\star V(\pi) + w$ satisfies the participation constraint as an equality if and only if

$$E[\delta^\star V(\tilde{\pi}) + w | e^\star] < E[V(\tilde{\pi}) + w^\star | e^\star] = E[W(\tilde{\pi}) | e^\star] \tag{57}$$

where the equality follows from (52). This shows that the expected cost of compensation in equilibrium $E[\delta^\star V(\tilde{\pi}) + w | e^\star]$ with the concave contract $\delta^\star V(\tilde{\pi}) + w$, which induces the same effort $e^\star$ as the contract $W(\pi)$ and gives the agent the same expected utility $\bar{U}$, is lower than the expected cost of compensation in equilibrium $E[W(\tilde{\pi}) | e^\star]$ with the convex contract $W(\pi)$.

Q.E.D.

Proof of Corollary 1:

In this setting, we can write

$$\tilde{\pi} = h(e) \times \alpha \times \exp(\tilde{\varepsilon}) \tag{58}$$

Where h is increasing in e, α is a constant and $\tilde{\varepsilon}$ is normally distributed. Therefore

$$\ln(\tilde{\pi}) = \ln(h(e)) + \ln(\alpha) + \tilde{\varepsilon} \tag{59}$$

So $\ln(\tilde{\pi})$ is additive in an increasing function of effort, a constant, and a normally distributed random variable. Since the normal distribution is symmetric around its mean, we can apply Proposition 2 to $\ln(\tilde{\pi})$. Therefore, any compensation profile W which is a convex transformation of $\ln(\pi)$ is dominated.

Q.E.D.

Proof of Proposition 3:

Denote by G the cumulative distribution function (c.d.f.) of the random variable $\tilde{\pi}$. For a given increasing function h defined over the real line, we denote by F the c.d.f. of the random variable $h(\tilde{\pi})$.

For a given W, setting $\tilde{y} \equiv W(\tilde{\pi})$ and denoting the c.d.f. of $\tilde{y}$ by F_W, this notably implies that

$$\int_{-\infty}^{\infty} W(\pi)dG(\pi) = \int_{-\infty}^{\infty} ydF_W(y) \tag{60}$$

and

$$\int_{-\infty}^{\infty} u(W(\pi))dG(\pi) = \int_{-\infty}^{\infty} u(y)dF_W(y) \tag{61}$$

for any function u.

For a given W, set a and b so that $E[a + b\tilde{\pi}] = E[W(\tilde{\pi})]$ and $var[a + b\tilde{\pi}] = var[W(\tilde{\pi})]$. As above, for the function $a + b\tilde{\pi}$ defined over the real line, we denote by F_L the c.d.f. of the random variable $\tilde{z} \equiv a + b\tilde{\pi}$.

Given G, W, and the associated F_W, we have

$$F_W(\alpha) = Pr[W(\pi) < \alpha] = Pr[\pi < W^{-1}(\alpha)] = Pr[a+b\pi < a+bW^{-1}(\alpha)] = F_L(a+bW^{-1}(\alpha)) \tag{62}$$

In addition, by definition of the inverse function,

$$F_W^{-1}(F_W(\alpha)) = \alpha \tag{63}$$

Substituting for the value of $F_W(\alpha)$ derived in (62),

$$F_W^{-1}(F_L(a + bW^{-1}(\alpha))) = \alpha \tag{64}$$

Lastly, setting $\alpha = W(\pi)$,

$$F_W^{-1}(F_L(a + b\pi)) = W(\pi) \tag{65}$$

For $b > 0$, $F_W^{-1}(F_L(\pi))$ is respectively globally convex, concave, linear, if and only if $F_W^{-1}(F_L(a + b\pi))$ is respectively globally convex, concave, linear. Then (65) implies that $F_W^{-1}(F_L(\pi))$ is convex (with respect to π) if W is convex (with respect to π), that $F_W^{-1}(F_L(\pi))$ is concave if W is concave, and that $F_W^{-1}(F_L(\pi))$ is linear if W is linear.

We now use the result that any c.d.f. F_W is more skewed to the right than the c.d.f. F_L if and only if $F_W^{-1}(F_L(\pi))$ is convex (e.g. Van Zwet (1964), Chiu (2010)). Therefore, given that $F_W^{-1}(F_L(\pi))$ is convex if W is convex (cf. the preceding paragraph), we conclude that F_W is more skewed to the right than F_L if W is convex.

Because of (61), comparing the expected utility of the contract $W(\pi)$ to the expected utility of the contract $a + b\pi$ reduces to comparing on the one hand

$$\int_{-\infty}^{\infty} u(y)dF_W(y) \tag{66}$$

and on the other hand

$$\int_{-\infty}^{\infty} u(z)dF_L(z) \tag{67}$$

By construction, F_L has the same mean and the same variance as F_W. Moreover, we have shown that F_W is more skewed to the right than F_L if W is convex, and less skewed to the right than F_L if W is concave. We can now apply Theorem 2 in Chiu (2010): if W is convex and $u''' > 0$, then

$$\int_{-\infty}^{\infty} u(y)dF_W(y) > \int_{-\infty}^{\infty} u(z)dF_L(z) \tag{68}$$

Equivalently,

$$\int_{-\infty}^{\infty} u(W(\pi))dG(\pi) > \int_{-\infty}^{\infty} u(a + b\pi)dG(\pi) \tag{69}$$

Likewise, if W is concave and $u''' > 0$,

$$\int_{-\infty}^{\infty} u(W(\pi))dG(\pi) < \int_{-\infty}^{\infty} u(a + b\pi)dG(\pi) \tag{70}$$

Q.E.D.

Proof of Proposition 4:

Denote by F_W the c.d.f. of $W(\pi)$, and by F_g the c.d.f. of $g(W(\pi))$. First,

$$F_W(\pi) = Pr(W(\pi) < \pi) = Pr(g(W(\pi)) < g(\pi)) = F_g(g(\pi)) \tag{71}$$

Second, by definition of the inverse function,

$$F_g^{-1}(F_g(g(\pi))) = g(\pi) \tag{72}$$

Third, substituting for $F_g(g(\pi))$ in (72),

$$F_g^{-1}(F_W(\pi)) = g(\pi) \tag{73}$$

If g is convex (resp. concave), then $F_g^{-1}(F_W(\pi))$ is convex (resp. concave), so that F_g is more (resp. less) skewed to the right than F_W.

If $g(W(\pi))$ is a mean and variance preserving transformation of $W(\pi)$, the same arguments used in Proposition 1 then lead to the conclusion that if $u''' > 0$ and if g is convex (resp. concave), then $E[u(g(W(\tilde{\pi})))] > E[u(W(\tilde{\pi}))]$ (resp. $E[u(g(W(\tilde{\pi})))] < E[u(W(\tilde{\pi}))]$), whereas if $u''' > 0$ then $E[u(g(W(\tilde{\pi})))] = E[u(W(\tilde{\pi}))]$.

Q.E.D.

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Proof of Proposition 5:

We must show that if

$$-\frac{u'^{-1''}(x)}{u'^{-1'}(x)} < -\frac{v'^{-1''}(x)}{v'^{-1'}(x)} \qquad \text{for all } x \text{ in the relevant domain} \tag{74}$$

then $u'^{-1}(x)$ is a convex transformation of $v'^{-1}(x)$. But this directly follows from a standard result in financial economics (e.g. Proposition 2 in Gollier (2001)) that a function f is a convex transformation of a function g if and only if

$$-\frac{f''(x)}{f'(x)} < -\frac{g''(x)}{g'(x)} \qquad \text{for all } x \text{ in the relevant domain} \tag{75}$$

Setting $f(x) = u'^{-1}(x)$ and $g(x) = v'^{-1}(x)$ completes the proof.

Q.E.D.

10   Bibliography

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