Realization Utility

Nicholas Barberis
Yale University

Wei Xiong
Princeton University

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Overview

- we propose that investors derive utility from *realizing gains and losses* on specific assets that they own
  - we label this "realization utility"
- e.g. if you buy a stock at \$40 and sell it at \$60
  - we assume that you receive a jolt of (positive) utility at the moment of sale, based on the size of the realized gain
- the underlying source of utility may be monetary or non-monetary
  - you are excited about the gain in wealth
  - you can boast about the transaction to friends and family
- why is the utility experienced at the moment of sale?
  - the transaction is "complete"
  - a gain can be fully savored, a loss must finally be acknowledged

Overview

In this paper:

- we propose a tractable model of realization utility, and derive its implications
  - both in partial equilibrium, but also in a full equilibrium
- we then link it to a wide range of applications
- and suggest some testable predictions

The model

- in our model, the investor derives utility from realized gains and losses
  - contrasts between previous work which draws no distinction between paper gains and realized gains
- we define the "gain" or "loss" as sale price minus purchase price
- and assume that realization utility is defined at the level of an individual asset
- what is the right functional form for realization utility?
  - we work primarily with a *linear* functional form, but also consider other specifications

The model

- realization utility is unlikely to be important for all investors in all circumstances
  - probably more important for individual investors
  - probably more important when the purchase price is more salient
- our benchmark application is the trading of individual stocks by retail investors

The model

Assets

- a risk-free asset, with net return of zero
- a large number of risky assets (stocks)
  - all stocks have the same price process

$$\frac{dS_{i,t}}{S_{i,t}} = \mu dt + \sigma dZ_{i,t}$$

The Investor

- at each time t, he either allocates all of his wealth to the risk-free asset, or all of his wealth to one of the stocks
  - time t wealth is W_t
- if he is holding stock at time t, let B_t be the cost basis of the position
- if he sells stock at time t, he pays a transaction cost kW_t

The model

- if the investor sells stock at time t, he receives realization utility equal to

$$u((1-k)W_t - B_t)$$

  - but only if he does not immediately buy back the same stock

- the investor also faces random liquidity shocks
  - the shocks arrive according to a Poisson process with parameter ρ
  - when a shock hits, the investor sells his holdings and exits the stock market

- the investor's task is to maximize the discounted sum of expected future utility flows
  - δ is the time discount rate
  - for now, we take $u(x) = x$

The model

- the value function of an investor holding stock at time t is $V(W_t, B_t)$
  - we assume $V(W, W) > 0$

- if τ' is the random future time at which a liquidity shock hits, the investor solves:

$$
\begin{aligned}
& V\left(W_t, B_t\right) \\
& = \max_{\tau \geq t} \quad E_t\{e^{-\delta(\tau-t)}[u((1-k)W_\tau - B_\tau) \\
& \qquad\qquad + V\left((1-k)W_\tau, (1-k)W_\tau\right)] I_{\{\tau<\tau'\}} \\
& \qquad + e^{-\delta\left(\tau'-t\right)} u((1-k)W_{\tau'} - B_{\tau'}) I_{\{\tau \geq \tau'\}}\}
\end{aligned}
$$

subject to

$$
\frac{dW_s}{W_s} = \mu ds + \sigma dZ_s, \qquad t \leq s < \tau'
$$

The model

Solution

- define $g_t = \frac{W_t}{B_t}$
- the investor sells a stock once it reaches a liquidation point $g_t = g_* > 1$, which satisfies:

$$(\gamma_1 - 1)\left(1 - \frac{\rho k(\rho + \delta)}{\delta(\rho + \delta - \mu)}\right) g_*^{\gamma_1} - \frac{\gamma_1}{1-k} g_*^{\gamma_1 - 1} + 1 = 0$$

where $0 < \gamma_1(\mu, \sigma, \rho, \delta) < 1$

- the value function is $V(W_t, B_t) = B_t U(g_t)$, where

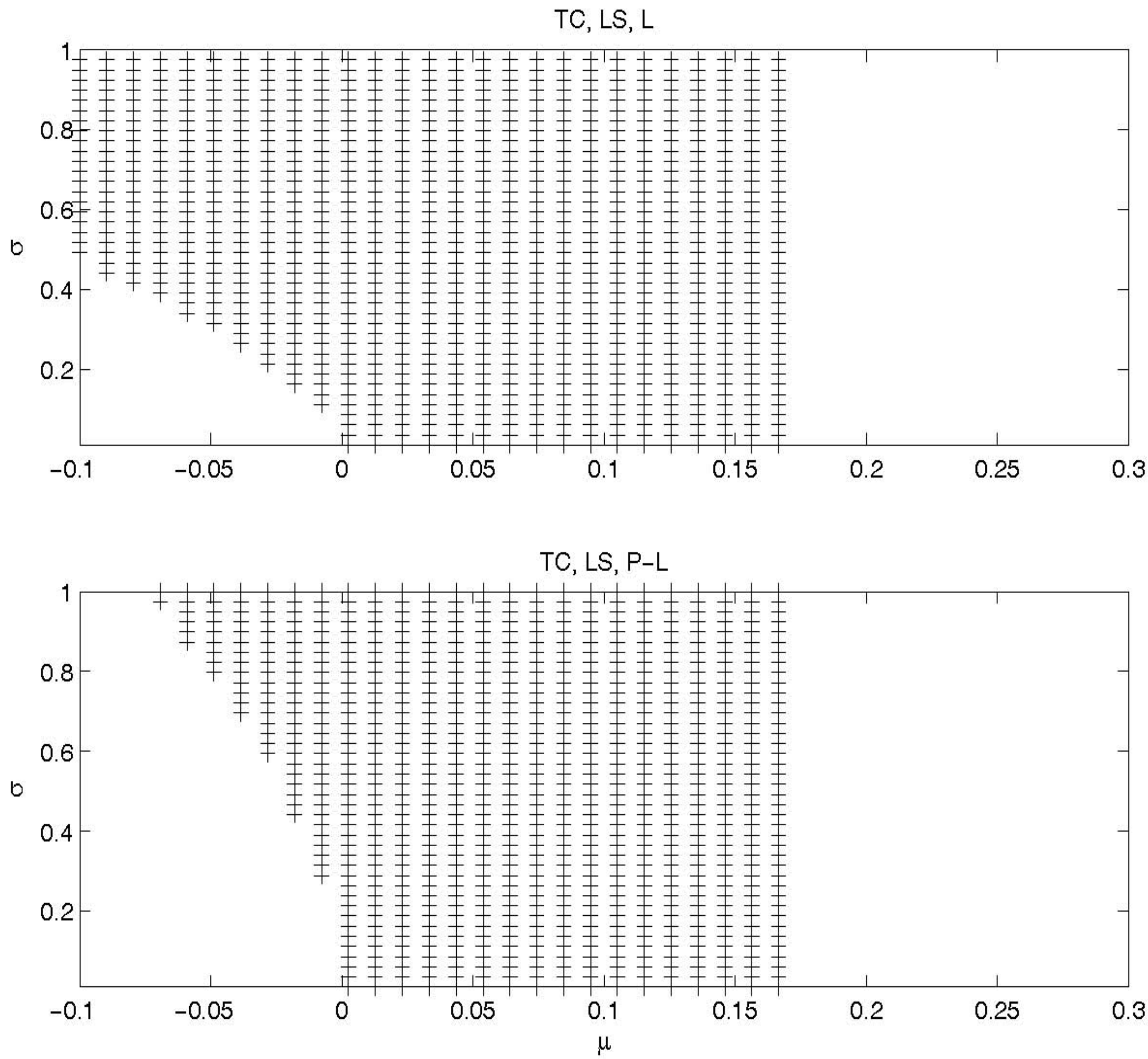
$$U(g_t) = \begin{cases} a g_t^{\gamma_1} + \frac{\rho(1-k)}{\rho+\delta-\mu} g_t - \frac{\rho}{\rho+\delta} & \text{if } g_t < g_* \\ (1-k) g_t (1 + U(1)) - 1 & \text{if } g_t \geq g_* \end{cases}$$

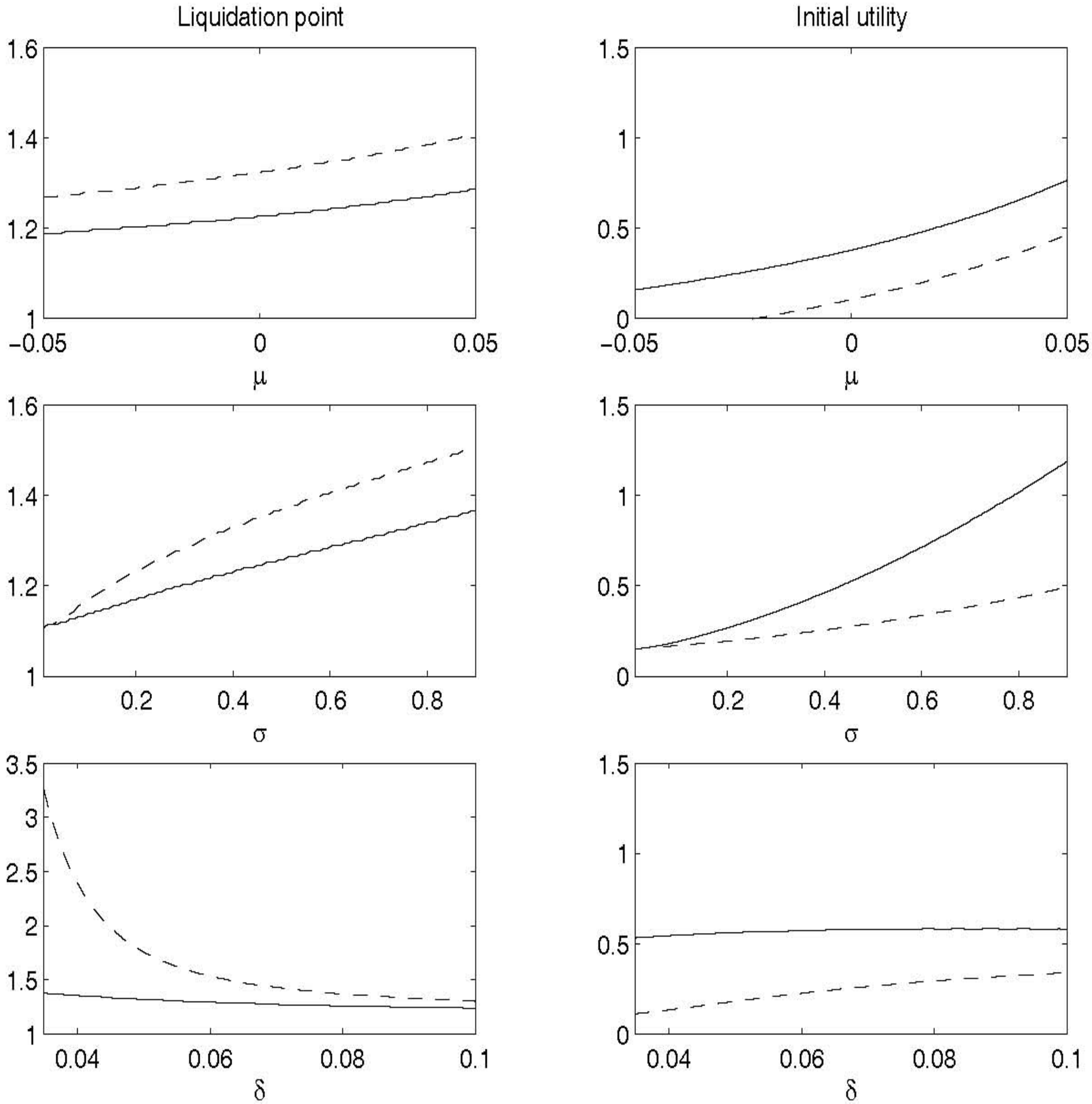
- under narrow framing, the solution to the one-stock problem also determines optimal behavior when the investor holds multiple stocks concurrently

The model

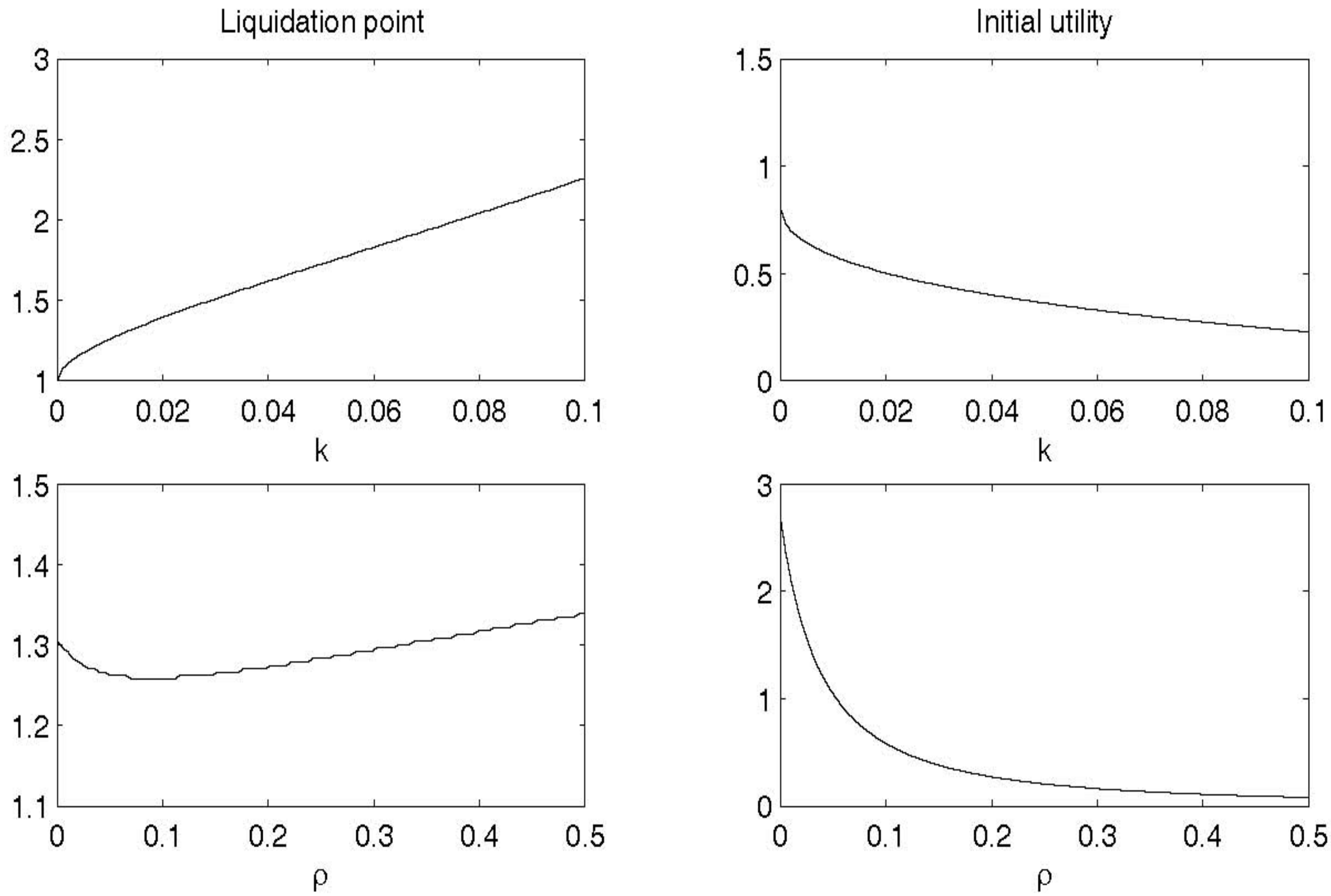
Results

- first, look at the range of μ and σ for which the investor is willing to buy a stock at time 0 *and* to sell it at a finite liquidation point
  - fix the other parameters at the benchmark values:
  $$(\delta, k, \rho) = (0.08, 0.01, 0.1)$$
  - find that the investor is willing to buy a stock with a *negative* expected return, so long as its volatility is sufficiently high
- also look at how the liquidation point and initial utility depend on μ, σ, δ, k, and ρ
  - when varying one parameter, keep the others fixed at the benchmark values:
  $$(\mu, \sigma, \delta, k, \rho) \;=\; (0.03, 0.5, 0.08, 0.01, 0.1)$$
  - most interestingly, initial utility is an *increasing* function of volatility σ


TC, LS, L
TC, LS, P−L
σ
μ
1
0.8
0.6
0.4
0.2
−0.1
−0.05
0
0.05
0.1
0.15
0.2
0.25
0.3



Liquidation point
Initial utility
μ
σ
δ



Liquidation point
Initial utility
3
2.5
2
1.5
1
0
0.02
0.04
0.06
0.08
0.1
k
1.5
1
0.5
0
0
0.02
0.04
0.06
0.08
0.1
k
1.5
1.4
1.3
1.2
1.1
0
0.1
0.2
0.3
0.4
0.5
ρ
3
2
1
0
0
0.1
0.2
0.3
0.4
0.5
ρ

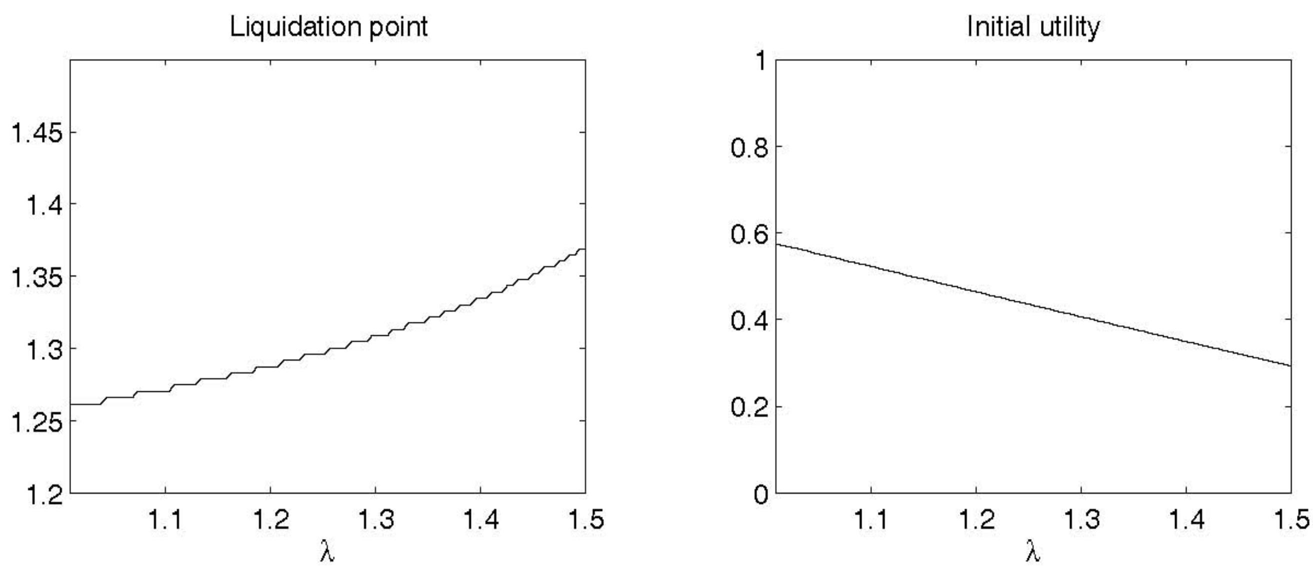
Alternative preference specifications

Piecewise-linear utility

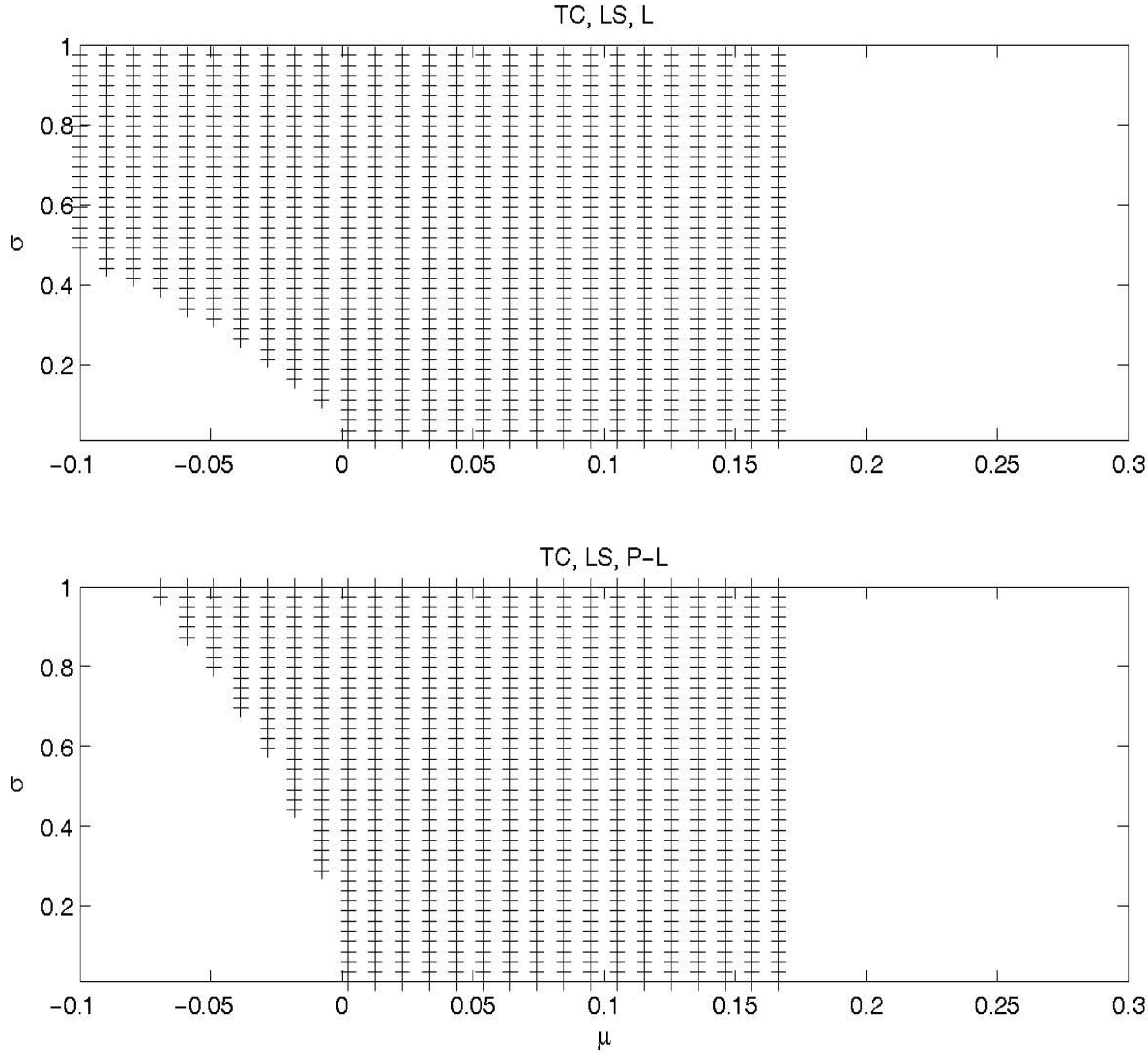
- an increase in loss sensitivity λ increases the liquidation point
- the investor is still willing to buy a negative expected return stock, so long as its volatility is sufficiently high
- initial utility is still increasing in volatility, so long as ρ and λ are not too high

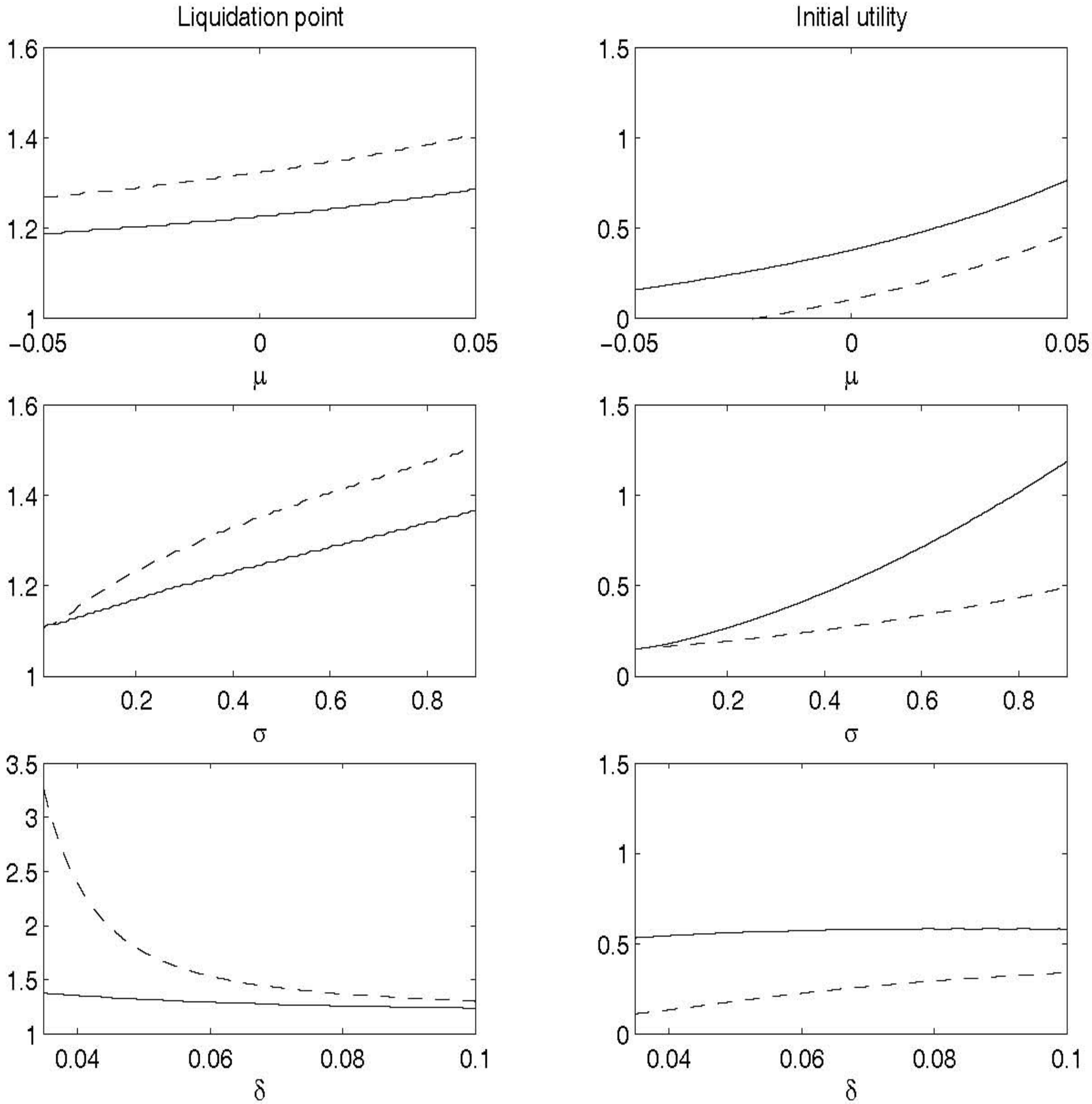
Hyperbolic discounting

- compared to exponential discounting, hyperbolic discounting puts more weight on the present, relative to the future
  - we therefore expect it to *lower* the liquidation point
- we follow Harris and Laibson (2004) and Grenadier and Wang's (2007) way of incorporating hyperbolic discounting into a continuous-time framework
- we confirm that hyperbolic discounting indeed leads to a lower liquidation point



Liquidation point
1.45
1.4
1.35
1.3
1.25
1.2
1.1
1.2
1.3
1.4
1.5
λ
Initial utility
1
0.8
0.6
0.4
0.2
0
1.1
1.2
1.3
1.4
1.5
λ


TC, LS, L
1
0.8
0.6
0.4
0.2
σ
−0.1
−0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
TC, LS, P−L
1
0.8
0.6
0.4
0.2
σ
−0.1
−0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
μ



Liquidation point
Initial utility
1.6
1.4
1.2
1
1.5
1
0.5
0
−0.05
0
0.05
μ
0.2
0.4
0.6
0.8
σ
3.5
3
2.5
2
1.5
0.04
0.06
0.08
0.1
δ

Asset pricing

Assets

- a risk-free asset, in perfectly elastic supply, and with a net return of zero
- N risky assets, each in limited supply, which can differ in their expected return and standard deviation
  - price process for stock i is:

$$\frac{dS_i}{S_i} = \mu_i dt + \sigma_i dZ_{i,t}$$

Investors

- a continuum of realization utility investors
- allow for transaction costs, liquidity shocks, and piecewise-linear utility
- investors are *homogeneous:* δ, ρ, and λ are the same for all investors
- transaction costs can differ across stocks: k_i for stock i

Asset pricing

- the condition for equilibrium is:

$$V\left(W,W\right)=0$$

- if τ' is the random future time at which a liquidity shock arrives, the decision problem for an investor holding stock i at time t is:

$$\begin{aligned} & V\left(W_t,B_t\right) \\ & =\max_{\tau\geq t}\quad E_t\{e^{-\delta(\tau-t)}u((1-k_i)W_\tau-B_\tau)I_{\{\tau<\tau'\}} \\ & \qquad +e^{-\delta\left(\tau'-t\right)}u((1-k_i)W_{\tau'}-B_{\tau'})I_{\{\tau\geq\tau'\}}\} \end{aligned}$$

- given δ, ρ, λ, σ_i, and k_i, the condition

$$V(W,W)=0$$

allows us to solve for the expected return μ_i

Applications

The disposition effect

- individual investors have a greater propensity to sell stocks trading at a *gain* relative to purchase price, rather than stocks trading at a loss
  - standard hypotheses fail to fully explain this
- our model shows that realization utility, coupled with a positive discount factor, predicts a strong disposition effect
- realization utility *alone* does not predict a disposition effect; an additional ingredient is needed
  - e.g. a positive time discount factor
  - or a prospect theory utility function (Shefrin and Statman, 1985; Barberis and Xiong, 2006)
- realization utility may also be a useful way of thinking about the disposition-type effects uncovered in other settings
  - e.g. in the housing market (Genesove and Mayer, 2001)

Applications

- Weber and Camerer (1995) provide useful experimental support for the realization utility view of the disposition effect
- in a laboratory setting, they ask subjects to trade six stocks over a number of periods
  - each stock has some probability of going up in each period, ranging from 0.35 to 0.65
  - subjects are not told which stock is associated with which up-move probability
- subjects exhibit a disposition effect
- more interestingly, in one condition, the experimenter liquidates subjects' holdings and then allows them to reallocate however they like
  - subjects do *not* re-establish their positions in prior losers

Applications

Excessive trading

- individual investors trade a lot in their brokerage accounts, but destroy value in the process
  - gross returns are on a par with benchmarks, but *net* returns underperform
  - Barber and Odean (2000)
- our model suggests an explanation for this trading / performance puzzle
  - people sell in order to enjoy the experience of realizing a gain

Applications

Excessive trading, ctd.

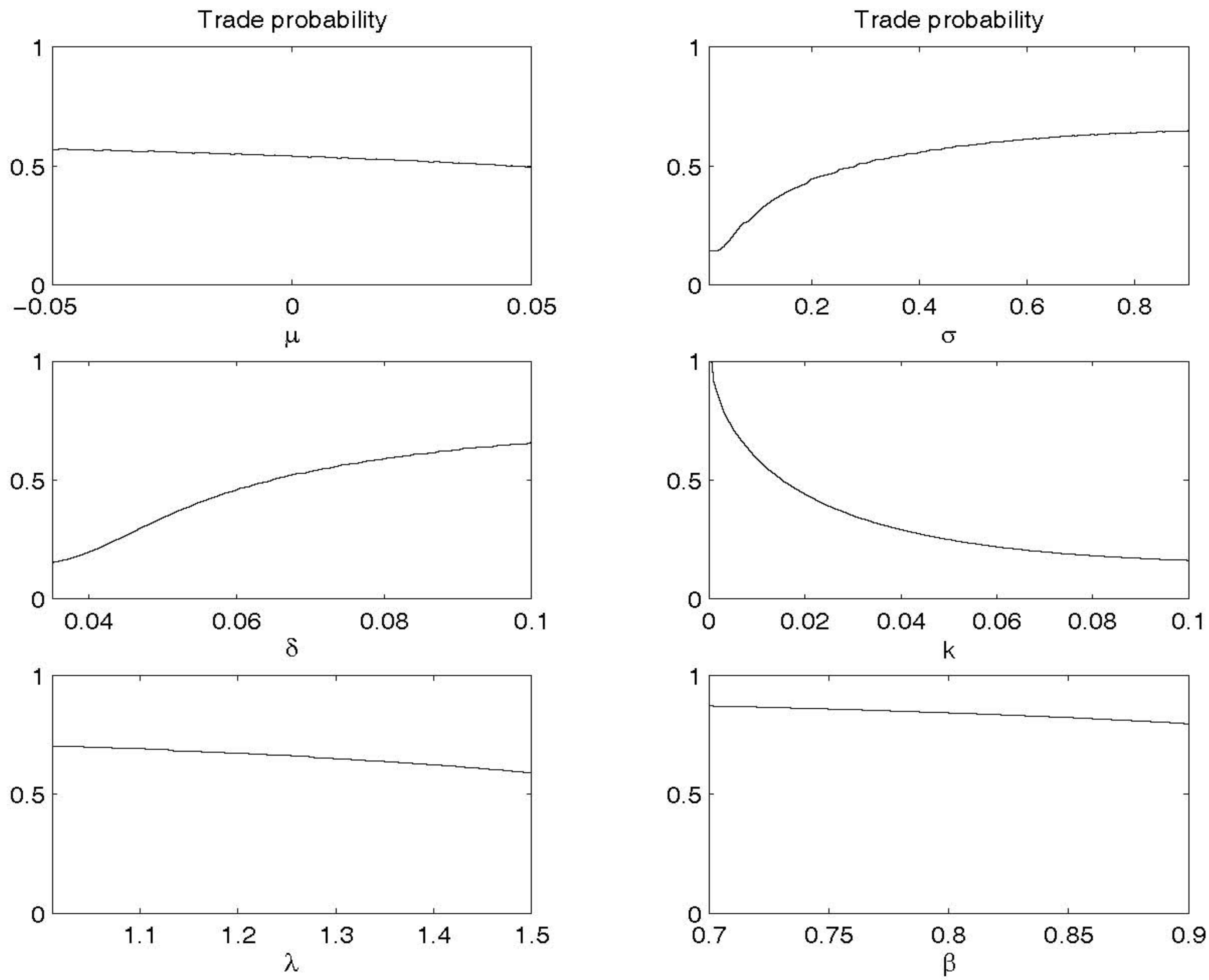
- we can compute the probability that, within a year of first buying a stock, the investor sells it:

$$
\begin{aligned}
&1 - e^{-\rho} \\
&+e^{-\rho}\left[N\left(\frac{-\ln g_* + \left(\mu - \frac{\sigma^2}{2}\right)}{\sigma}\right)\right. \\
&\qquad\left. + e^{\left(\frac{2\mu}{\sigma^2}-1\right)\ln g_*} N\left(\frac{-\ln g_* - \left(\mu - \frac{\sigma^2}{2}\right)}{\sigma}\right)\right]
\end{aligned}
$$

- look at how this probability depends on μ, σ, δ, k, and λ
  - keep the other parameters fixed at their benchmark values:

$$(\mu, \sigma, \delta, k, \rho, \lambda) = (0.03, 0.5, 0.08, 0.01, 0.1, 1.5)$$

- magnitudes are comparable to those observed for discount brokerage accounts



Trade probability
Trade probability
1
0.5
0
−0.05
0
0.05
μ
0.2
0.4
0.6
0.8
σ
0.04
0.06
0.08
0.1
δ
0
0.02
0.04
0.06
0.08
0.1
k
1.1
1.2
1.3
1.4
1.5
λ
0.7
0.75
0.8
0.85
0.9
β

Applications

Underperformance of individual investors

- in some studies, individual investors underperform benchmarks even before transaction costs
- may be related to our prediction that investors are willing to buy stocks with negative expected returns, so long as their volatility is sufficiently high

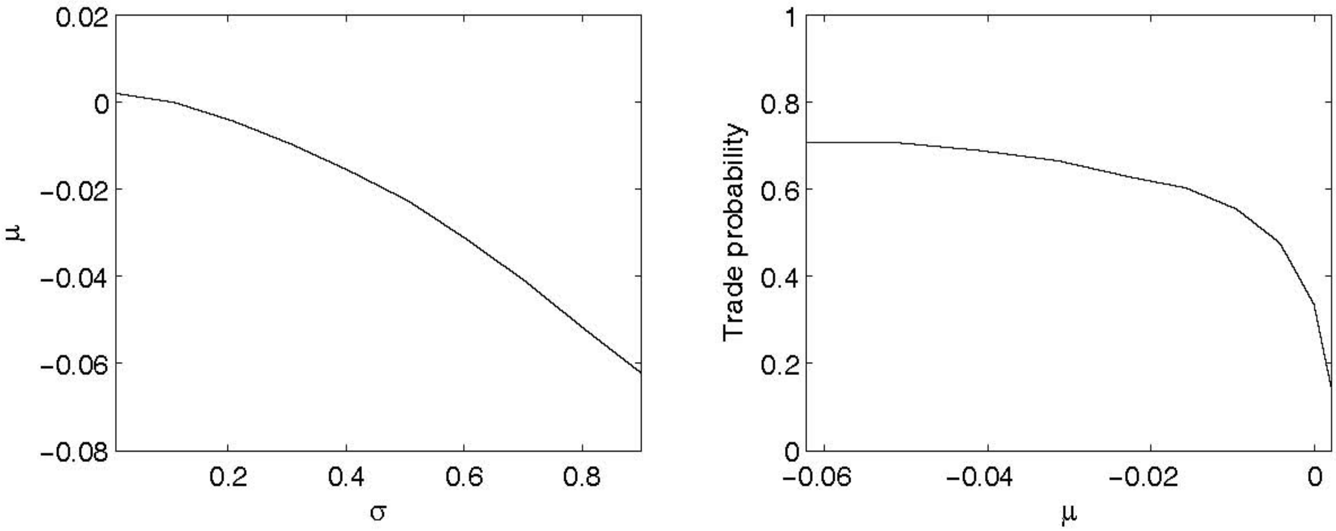
Turnover in bull and bear markets

- we observe more trade in rising, rather than in falling markets (Griffin, Nardari, and Stulz, 2007)
- our model predicts this
  - the investor is more willing to sell in a rising market
  - and therefore also more willing to buy

Applications

Negative volatility premium

- Ang et al. (2005) show that stocks with high daily volatility over the past month have *low* subsequent returns
- realization utility investors like stocks with high volatility
  - if there are many such investors in the economy, they may bid up the prices of these stocks
- we can see this in the equilibrium model from before
  - suppose there are a large number of stocks, with standard deviations ranging from 0.01 to 0.9
  - set $(\delta, \rho, \lambda) = (0.08, 0.1, 1.5)$ and let $k = 0.01$ for all stocks
  - use the condition $V(W, W) = 0$ to compute the expected return for each stock
- the model indeed predicts a negative relationship between expected return and volatility in the cross-section



0.02
0
−0.02
−0.04
−0.06
−0.08
μ
0.2
0.4
0.6
0.8
σ
1
0.8
0.6
0.4
0.2
0
Trade probability
−0.06
−0.04
−0.02
0
μ

Applications

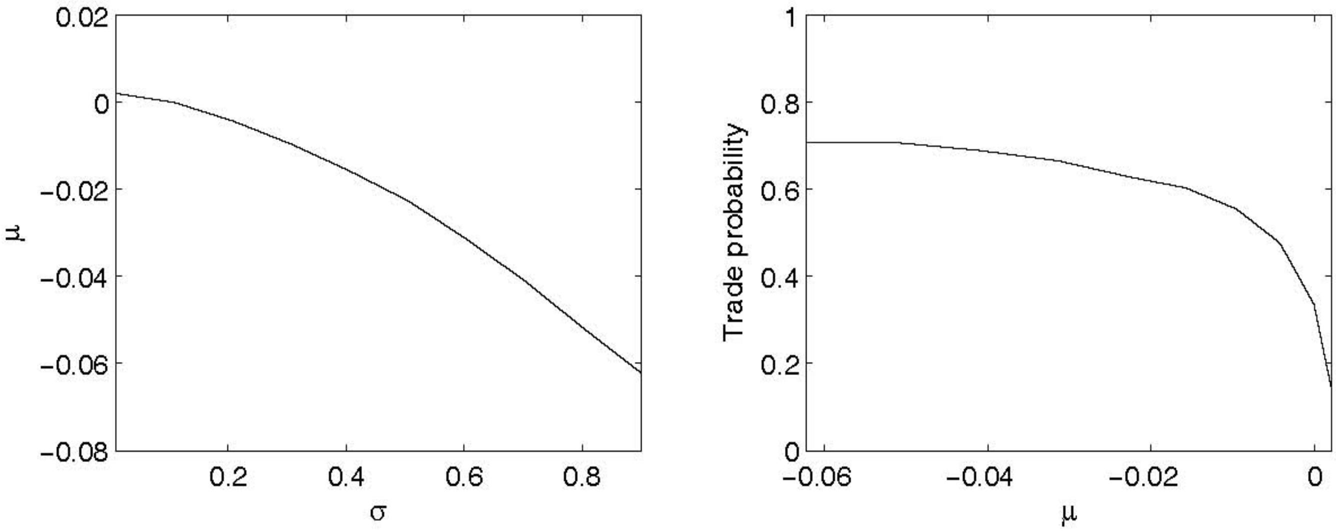
Heavy trading of highly valued assets

- assets which are highly valued, perhaps over-valued, are also heavily traded (Hong and Stein, 2007)
  - e.g. growth stocks vs. value stocks
  - e.g. technology stocks in the late 1990s
  - e.g. shares at the center of famous bubble episodes (South Sea bubble)
- our model predicts this coincidence, and also that it will occur when underlying asset uncertainty is particularly high
  - high $\sigma \Rightarrow$ asset price is pushed up
  - high $\sigma \Rightarrow$ asset is more heavily traded

Applications

Heavy trading of highly valued assets, ctd.

- e.g. consider the equilibrium model from before, with the same parameterization
  - for each stock, compute not only the expected return, but also the probability it is traded within a year of purchase
- the model indeed predicts a negative relationship between expected return and trade probability
- realization utility may offer an alternative to the differences of opinion / short-sale constraints approach to this set of facts


0.02
0
−0.02
−0.04
−0.06
−0.08
μ
0.2
0.4
0.6
0.8
σ
1
0.8
0.6
0.4
0.2
0
Trade probability
−0.06
−0.04
−0.02
0
μ

Testable predictions

- the most natural predictions to test are those related to turnover
- some are obvious:
  - when transaction costs are lower, the investor trades more
- some are hard to test:
  - the investor holds stocks with higher average returns for longer, before selling them
- but some are novel and testable:
  - the investor holds stocks with higher volatility for *shorter* periods, before selling them
    - Zuckerman (2006) confirms this
  - more impatient investors will trade more frequently

Conclusion

We propose that investors derive utility from *realizing* gains and losses

- we present a tractable model of realization utility and derive its implications
  - both in partial equilibrium and in a full equilibrium
- we then link it to a range of applications
- and suggest some testable predictions