Fund managers and defaultable debt*

Very preliminary, please do not circulate

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Abstract

We propose a model of defaultable debt where investors hire fund managers to invest their capital in a risky bond or in a riskless asset. The risky bonds are issued by a large number of borrowers who run risky projects and can decide to default ex-post. There is only a small fraction of informed fund managers who have privat information about the outcome of the risky project. Investors' search for informed managers generates career concerns that distort the investment decision of the uninformed fund managers. When the probability of default is sufficiently high, uninformed managers require a premium on risky bonds as this investment increases their probability of being fired. This is what we define "reputational premium". As the economic and financial conditions change, the reputational premium can switch sign. This generates an overreaction of the market leading to excess volatility of spreads, capital flows and economic activity.

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1 Introduction

In the last few years, before the subprime turmoil in August 2007, market observers seemed to be concerned about a growing "overenthusiasm" for risky investments, including high-yield corporate bonds, mortgage-backed assets and, in particular, emerging market bonds. As one observer puts it as early as 2005:

> "Bonds issued by Ecuador, which is politically very unstable, are among the riskiest bets in the emerging markets. It is hard to predict what will happen there next month, let alone in 10 years time. Yet buyers appear to be ready and willing to line up for a sale by the government of up to Dollars 750m in 10-years bonds, the first international bond offer since the country defaulted in 1999. The issue, [...] is the latest example that the prolonged love affair with emerging market debt is far from over." (December 9, 2005, Financial Times).

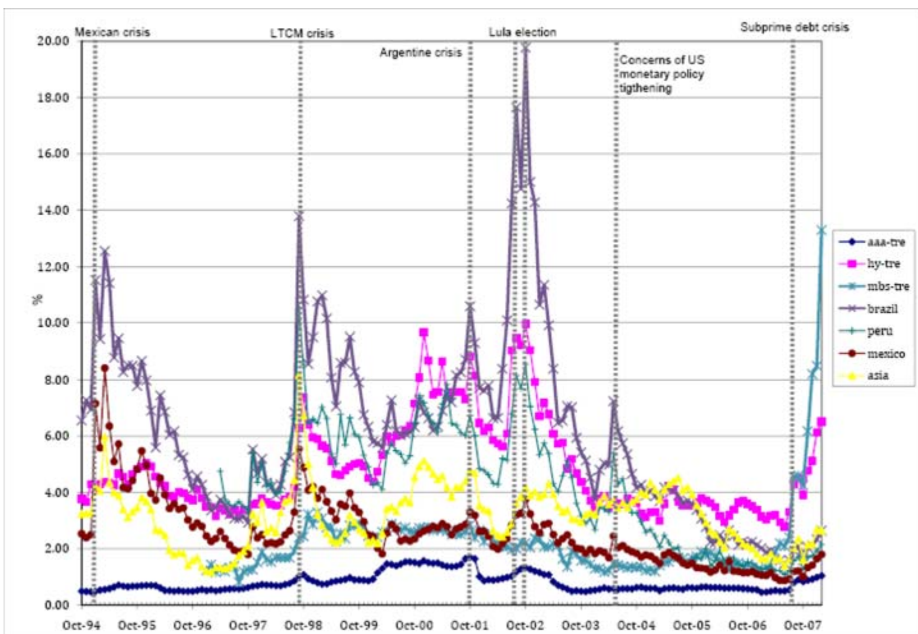

Figure 1: The JPMorgan EMBI+ spread for Asia, Brazil, Mexico, Peru, the yield spread of AAA corporate bonds and B-graded corporate bonds and the yield spread of BBB graded comercial morgate backed assets between October 1994 and February 2008. Source: Datastream, St. Louis Fed.

Figure 1 shows the patterns of the spreads of some emerging market bonds, comercial mortgage backed assets and high-yield corporate bonds between October 1994 and February 2008. They all peaked in 2002, and, after that, they started to decline and kept on declining even further after 2005. By April 2007, they shrank to historically very low levels. In particular, the spreads of all the emerging countries represented in Figure 1 were close to the level of the investment grade corporate spread.[1] Many argue that in 1996-97 there was a similar overenthusiasm for East-Asian and Russian bonds, right before the emergence of crises in these areas (e.g. Kamin and von Kleist, 1999, IMF, 1999b, Duffie et al., 2003). These episodes are in sharp contrast to crises episodes, when virtually all high-risk bond spreads jump up and capital tends to flow out from these markets; a phenomenon frequently dubbed as flight-to-liquidity or flight-to-quality.

We propose a stylized dynamic general equilibrium model where investors rationally allocate their capital to fund managers, who can invest in riskless bonds or finance defaultable risky projects. There is only a small fraction of informed fund managers who know the fundamentals of the risky project. We argue that fund managers' career concerns lead to rational "overinvestment" in good times and "underinvestment" in bad times, generating "excess volatility" of prices, capital flows and economic activity.

Our economy is populated by three types of agents: investors, fund managers, and borrowers. Investors delegate their portfolio decision to risk-neutral fund managers. Fund managers can invest either in riskless assets or in risky bonds issued by a large number of borrowers. Borrowers invest in risky projects and can default on them after observing the realized project's productivity. As shown in Figure 2, the model is structured on two sets of interactions: investors/managers and managers/borrowers.

On the one hand, the interaction between investors and managers shapes the managers career concerns. There is a small portion of informed fund managers who have private information about the productivity of the risky project. Using this information, they can formulate a more precise estimate of the default probability of the risky bond than the uninformed managers. At the end of each period, based on the manager's performance, each investor updates his belief and decides whether to keep his manager or to fire him and hire a new one. The

[1] As a columnist of the Wall Street Journal observes, the 5-year credit default swap spreads for Brazil, Peru, Columbia were at the record-tight levels of 0.70, 0.65 and 0.80 percentage point at the time when, for example, the Boston Scientific Corp, an investment grade company traded at 0.78 percentage point. (April 24, 2007, Tight spreads are emerging, WSJ).

firing decision of the investors distorts the investment decision of uninformed managers who would like to be perceived as informed managers.

On the other hand, the interaction between managers and borrowers determines the price of the risky bond, the probability of default and the level of economic activity in the economy. The investment choice of the fund managers determines the required rate of return on the bond for a given probability of default. The representative borrower issues bonds to cover her consumption and the fixed cost of the risky project. At the end of the period, she observes the productivity of the project and decides whether to pay back the outstanding debt or to default and suffer a cost. For a given price, her default rule determines the ex-ante probability of default on the bond. Hence, the equilibrium bond price and default probability are jointly determined by the conditions of both the financial market and the fundamentals of the risky projects. Even though borrowers are homogenous once they start the risky project, they are ex-ante heterogenous in their outside option. The measure of borrowers who choose to start the project for a given bond price determines the level of aggregate economic activity.

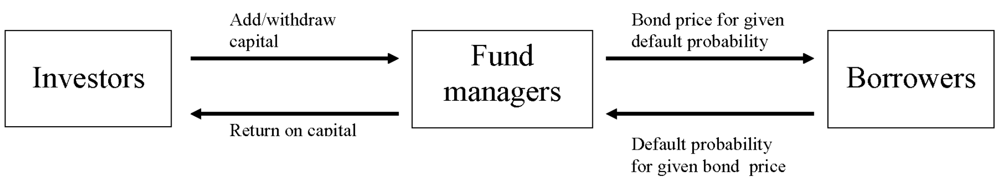

Figure 2: The structure of the model

The focus of our paper is to study the effect of the agency problem between investors and managers, that is, the outcome of the first interaction, on the equilibrium bond price, default frequency and economic activity, that is, the outcome of the second interaction.

Our main result is that managers' career concerns amplify the effect of fundamental shocks on the bond price, the probability of default and the level of economic activity. This amplification effect arises in general equilibrium as outcome of two reenforcing mechanisms. First, on the real side, when borrowing is more expensive, borrowers smooth its effect on their consumption over their lifetime by both decreasing the dollar amount of their borrowing and increasing the face value of their debt. Because of the latter, they default with larger probability. Second, on the financial side, career concerns impose a reputational premium on the spread of risky bonds that depends on the default probability. Uninformed fund managers try to time the market in order to behave as if they were informed and knew in advance if there would be default or not. Default will hurt the reputation of uninformed managers who invest in the risky bond, and no default will hurt the reputation of uninformed managers who invest in the riskless bond. Thus, when the probability of default is high, the reputational premium is positive to compensate for the foregone reputation. Vice-versa, when the default probability is low the risky bond will trade with a negative reputational premium, due to the reputational gain. The real side of the model implies that a larger return on bond leads to a larger probability of default. The financial side of the model implies that a larger probability of default leads to a larger return on bond, because of a larger reputational premium. These two mechanisms reinforce each other in equilibrium and generate excess volatility in bond prices: bond spreads are particularly low in good times and high in bad times. As in our model economic activity is lower when borrowing is more expensive, excess volatility in prices generates excess volatility in output.

We also explore an extension of the model where we introduce an alternative risky bond issued by a different group of borrowers. We show that career concerns introduce a common component in the required premium of the two bonds even if the underlying fundamentals are independent across the groups of borrowers. This result is in line with the large comovement of bond spreads shown in Figure 1. The channel of contagion in our model is different from the portfolio channel explored by Calvo (1999), and more recently, by Pavlova and Rigobon (2007), given that we assume that fund managers do not hold both types of risky bonds.

A natural application of our model would be to think of the borrowers as firms in an emerging economy. In this context, our results are in line with the empirical evidence that business cycle fluctuations in emerging countries are much more volatile than those in developed countries, and that such an excess volatility is partly driven by the volatility in bond spreads (Neumeyer and Perri, 2005, Uribe and Yue 2006). However, our result more generally applies to any type of credit market characterized by substantial fluctuations in the fundamentals of the underlying risk and by a crucial role of delegated portfolio management.

On the empirical side, our results are also broadly consistent with the puzzle that a large proportion of the variation in prices of both corporate and emerging market bonds cannot be explained by the variation of fundamentals and that a large part of this unexplained component is common across bonds (see Collin-Dufresne at al., 2000, Gruber et al., 2001, Westphalen, 2001). Furthermore, the recent papers of Singleton and Pan (2007) and Longstaff et al (2007) show that US financial market conditions have a large role in explaining the variation of emerging market spreads compared to emerging market fundamentals. Our model argues that fund managers' career concerns generate an important channel through which financial markets affect the pricing of debt.

Literature review. To our knowledge, this is the first paper to address the interaction between financial intermediation and endogenous default decision and its effect on the price of defaultable debt. Our work is related to several relatively unconnected areas of economics and finance. First, there is a growing literature which analyzes the effect of delegated portfolio management on traders' decisions and asset prices in general.[2] This literature is silent about the real effect of the agency frictions in financial markets.

Second, our paper is also related to the reputational herding models.[3] Just as our paper, these papers argue that decision makers with career concerns might choose inefficient decisions to convince their clients that they are informed. There are two main points of departure. On one hand, this literature traditionaly concentrates on partial equilibrium models while our focus is the price effect of the interaction of career concerns and endogenous default . On the other hand, these papers present mechanisms in which decision makers herd on others' decision because going against the average action is a bad signal about the ability of the decision makers. In our model, at the equilibrium prices, fund managers choose the inefficient action regardless of other managers' decision. That is, there are no strategic complementarities. In this literature, the closest paper to ours is Rajan (1994) as he shows that reputational herding might motivate bank executives to overextend credit in good times creating credit cycles by amplifying real shocks. Apart from the differences mentioned above, in contrast to our model Rajan (1994) predicts that in bad times banks provide the right amount of credit while we argue that in bad times managers invest too little in the risky bonds.

To our knowledge, the only work connecting the two groups above is Dasgupta and Prat (2006, 2008) who analyze the dynamics of equilibrium prices in a reputational herding model. As in our paper, in Dasgupta and Prat (2008) managers can choose the strategy with smaller monetary payoff to increase their future reputation and these reputational concerns affect prices. However, the way reputational concerns affect prices is very different in the two mod-

[2] See Dow and Gorton (1997), Shleifer and Vishny (1997), Allen and Gorton (1993), Cuoco and Kaniel (2007), Vayanos (2003), Gümbel (2005), Dasgupta and Prat (2006, 2008), Kondor (2007b), He and Krishnamurthy (2007).

[3] See Scharfstein and Stein (1990), Rajan (1994), Zweibel (1995) and Ottaviani and Sorensen (2006).

els. In their paper, managers trade over many periods before the true value of the asset is realized. An informed manager might ignore his signal and herd on the past action of other managers, because acting as everyone else reduces her reputation loss if his signal is wrong. If managers herd, their actions do not reveal their private information, so the price of the asset will not incorporate this information. In contrast, in our model, the private information of informed managers is never incorporated into prices although they always follow their signals. Reputational concerns affect prices, because the action with an ex-ante larger probability of success has a reputational advantage for the uninformed managers and this generates a reputational premium that is incorporated in the bond price.

Third, there is a related large literature focusing on the propagation and amplification of fundamental shocks due to the interaction between asset values, the value of firms' collateral and collateralized lending. This literature emerged from the papers of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) on the macro-side, and Gromb and Vayanos (2002) on the finance side[4]. The main difference of the implied amplification of these papers and our presented mechanism is that this literature predicts asymmetricity: as collateral constraints bind only in bad times, these models predict amplification of bad shocks only. In contrast, our mechanism also predicts amplification of good shocks through overprovision of credit when the probability of default is low.

Finally, our application on emerging markets is also related to the vast literature on sovereign debt, reversal of capital flows and financial crisis in emerging economies.[5] However, this literature abstracts away from the effects of intermediation in financial markets.

In the next section, we present the model. In Section 3, we define and characterize an equilibrium in the special case where borrowers are subject to an i.i.d. productivity process. In Section 4, we present the general case with a persistent productivity process and we perform a numerical exercise. In Section 5, we show the extension of the model with two risky bonds. In Section 6, we discuss the robustness of our model. Finally, Section 7 concludes. The appendix includes all the proofs.

[4]See also Aghion, Banerjee and Piketty (1999), Rampini (2003), Krishnamurthy (2003), Gai, Kondor and Vause (2005), Guerrieri and Lorenzoni (2007) on the macro side and Danielsson, Shin and Zigrand (2004), Morris and Shin (2004), Bernardo and Welch (2004) and Kondor (2007a) on the finance side.

[5]Atkenson (1991), Cole and Kehoe (2000), Aguiar and Gopinath (2006), Caballero and Krishnamurthy (2003), Calvo and Mendoza (2000), Benczur and Ilut (2005), Arellano (2006), Uribe and Yue (2006), Kovrijnykh and Szentes (2007).

2 The Model

The model is structured in three parts. First, the economy is populated by a large number of borrowers who need financing to undertake a risky project. They choose how much to borrow and under what circumstances to default, taking as given the cost of borrowing. Second, the international investors hire fund managers who decide whether to finance the risky project, or to invest their money in a risk-free bond. In any period, each investor decides whether to keep his manager, or to fire him and hire a new one, conditional on the realized returns. Third, the fund managers make their investment decisions, taking as given the probability of default of the risky project and the firing rule of the investors. We start by analyzing these three decision problems separately, taking as given the rest of the economy and, then, we merge them together to define the equilibrium concept.

2.1 The borrowers

The economy is populated by a large number of borrowers running the same risky project. They can borrow from financial markets by issuing one-period discount bonds and can ex-post decide to default. We can think of borrowers as firms in an emerging economy, or, more generally, firms with the same risk characteristics, or even property owners whose loans are behind the same mortgage backed asset.

Time is discrete and there are overlapping generations of borrowers who live for two periods. In each period a new generation is born, which is represented by a continuum of measure 1 of agents, indexed by i, with logarithmic utility. Consider agent i of the generation born at time t. When she is young, she has the choice to invest in a risky project with return a_{t+1}, distributed according to the cumulative distribution function $G(a_{t+1})$, or to enjoy an outside option $\bar{u}_t^i$. The distribution, $G(a_{t+1})$, has a finite first moment and its support is the positive real line or its subset. We assume that agents, within a generation, differ in their outside option $\bar{u}_t^i$, which is distributed according to the cumulative distribution function $H(\cdot)$ with real support and $i.i.d$ across time. However, they all have access to the same risky project, so that all the agents who become active borrowers face the same problem.

Let us present the behavior of an agent who has decided to become an active borrower at time t. To simplify notation we drop the superscript i whenever this does not cause any confusion. At time t, the agent chooses how much to borrow and how much to consume, taking as given the price of borrowing p_t. As she does not have any income when she is young, she has to cover both the fixed cost of the investment, F, and her consumption by borrowing, that is, her budget constraint when young is

$$p_t b_{t+1} \geq c_t + F, \tag{1}$$

where b_{t+1} represents the one-period discount bonds issued at time t, p_t represents the price of bonds issued at time t, and c_t represents consumption at time t.

When the agent is old, she collects the project pay-off a_t and has the option to default on her debt b_{t+1} at a cost $D(b_{t+1})$ in terms of utility.[6] The function $D(\cdot)$ satisfies mild conditions: *(i)* it is twice differentiable with $D'(b) > 0$, $D'(b)$ bounded away from zero and $\lim_{b \to 0} D'(b)$ finite; *(ii)* $D(b) > \log(1 + bD'(b))$ for all positive b.[7]

If the borrower chooses not to default she consumes her income after she repays her debt. If, instead, she decides to default, she can consume her entire income. Her budget constraint when old is

$$a_{t+1} - \left(1 - \chi_{t+1}(a_{t+1})\right) b_{t+1} \geq c_{t+1}, \tag{2}$$

where $\chi_{t+1} : \mathbb{R}_+ \mapsto \{0, 1\}$ denotes the default decision that the agent is making at time $t+1$, after observing the realization of a_{t+1}. However, if she decides to default she has a utility loss $D(b_t)$, so her objective function is

$$\log c_t + \beta \mathbb{E}\left[\log c_{t+1} - \chi_{t+1}(a_{t+1}) D(b_{t+1})\right]. \tag{3}$$

The problem for the representative active borrower is to maximize (3) subject to (1) and

[6] We do not take a stand on the exact source of the cost of default. This is a particularly debated issue in the case of sovereign debt of an emerging country. Since the seminal paper of Eaton and Greskovitz (1981), it is recognized that there must be some cost of default on sovereign debt to enforce repayment. The theoretical literature on sovereign default has explored alternative possible punishments, such as partial or full exclusion from financial markets, or other economic or political sanctions (Eaton and Greskovitz, 1981, Bulow and Rogoff, 1989), loss of reputation (Grossman et al., 1988, Atkeson, 1991, Cole and Kehoe, 1996), or worse future terms of borrowing (Chang and Sundaresan, 2001, Kovrijnykh and Szentes, 2007). In this paper we abstract from the specific form of punishment and simply assume that default is costly enough to support an equilibrium where it is not always optimal to default.

[7] To argue that our assumptions throughout the paper are not too strong, we build up two parametric examples in Appendix A. The first one assumes that the productivity shock is uniformly distributed and supposes a cost function which proves to be very tractable with distributions with finite support. The second one assumes that the productivity shock is lognormally distributed and the cost function is quadratic. As the second example represents a more standard environement, this is the basis for our numerical examples in the second part of the paper.

(2), taking p_t as given. The problem can be rewritten as

$$\begin{aligned}\max_{b_{t+1},\chi_{t+1},c_t} \log c_t + \beta \int_0^\infty & \log\left[a_{t+1} - \left(1-\chi_{t+1}\left(a_{t+1}\right)\right) b_{t+1}\right] dG(a_{t+1}) - \\ -\beta \int_0^\infty & \chi_{t+1}\left(a_{t+1}\right) D\left(b_{t+1}\right) dG(a_{t+1}) \\ s.t.\ p_t b_{t+1} = c_t + F. &\end{aligned} \tag{4}$$

Let V_t represents the value of investing in the risky project, given that the bond price is p_t. Recall that the agents of generation t differ for their outside option $\bar{u}_t^i$ and, hence, for their choice of becoming or not an active borrower. A young agent decides to become an active borrower if and only if the value of investing in the risky project is bigger than her outside option, that is, $V_t \geq \bar{u}_t^i$. Define B_{t+1} the aggregate supply of bonds, given that the bond price is p_t. It follows that

$$B_{t+1} = H\left(V_t\right) b_{t+1}.$$

With a slight abuse of notation, define $B\left(p_t\right)$ the aggregate supply of bonds conditional on the price p_t.

2.2 Investors and fund managers

The financial market is populated by a mass Γ of risk-neutral investors, indexed by j, who can invest one unit of capital at each time t. They can invest their capital only through fund managers. At the beginning of each period there is a mass 2Γ of potential, risk-neutral fund managers. They do not have any capital, and become active fund managers only when they are hired by some investor. An investor can hire only one fund manager and a fund manager can be hired only by a single investor, so that in each period there is a mass Γ of active managers. For simplicity, we fix the contract between investors and fund managers: fund managers keep a share γ of the revenues and leave the rest to the investors. Both investors and managers fully consume their net revenues in each period.

There are two types of fund managers: informed and uninformed. Only a small fraction $\bar{\varepsilon}$ of all potential managers are informed. At the end of any period, each fund manager has a probability $(1-\delta)$ to die, and $(1-\delta)\,2\Gamma$ newly born managers, $\bar{\varepsilon}$ of which are informed, join the pool of unemployed, keeping the mass of managers constant. The parameter δ can also be interpreted as a measure of the persistence of the information. Moreover, we denote by ε_t the probability that an unemployed manager is informed at the end of period $t-1$. The variable ε_t represents an aggregate state variable.

A fund manager can allocate his capital between a risk-less bond with gross return R and a defaultable risky bond with price $p(\varepsilon_t)$ and aggregate supply $B(\varepsilon_t)$. The return on the bond will be 0 if the borrowers default, or $1/p(\varepsilon_t)$ if they do not. If a manager is hired at time t by investor j, he gets a signal $s_t^j = \{n, d, 0\}$ about the productivity of the risky project, a_{t+1}, before making his investment decision. If the manager hired by investor j is informed, then he gets a signal that perfectly reveals whether borrowers will default, $s_t^j = d$, or will not default, $s_t^j = n$ otherwise. If he is uninformed, then he gets an uninformative signal, that is, with abuse of notation, $s_t^j = 0$. In this case, he will take as given the probability of default $q(\varepsilon_t) \equiv E\left(\chi_{t+1}(a_{t+1})\right)$. From now on, manager j stands for "the manager hired by investor j".

Manager j at time t, after receiving signal s_t^j, reports his investment strategy $\mu(s_t^j, \varepsilon_t)$ to an auctioneer. He can choose (1) a pure strategy of investing $\mu(s_t^j, \varepsilon_t)$ proportion of his capital in the risky bond with $\mu(s_t^j, \varepsilon_t) \in [0,1]$ and $1-\mu(s_t^j, \varepsilon_t)$ proportion of his capital in the riskless asset (2) to report that he is indifferent between investing all his capital into the risky bond or the riskless asset at the given prices, that is, $\mu(s_t^j, \varepsilon_t) = I$. The auctioneer sets the price and the mass of indifferent investors who will invest in the risky bond to ensure that the risky bond market clears. Let $\tilde{\mu}(\mu, \varepsilon_t, a_{t+1})$ be the realized investment strategy of a manager who chooses strategy μ at time t. Then $\tilde{\mu}(\mu, \varepsilon_t, a_{t+1}) = \mu$ if $\mu \in [0,1]$ and

$$\tilde{\mu}(I, \varepsilon_t, a_{t+1}) = \begin{cases} 1 \text{ with pr. } x(\varepsilon_t, a_{t+1}) \\ 0 \qquad\qquad \text{otherwise} \end{cases}, \tag{5}$$

where $x(\varepsilon_t, a_{t+1})$ is the equilibrium allocation rule, that is, the probability of an indifferent manager receiving the risky bond. With a slight abuse of notation, define $\tilde{\mu}_t^j$ the realization of $\tilde{\mu}(\mu(s_t^j, \varepsilon_t), \varepsilon_t, a_{t+1})$.

At the beginning of time t, each investor j has a manager working for him that he believes is informed with probability η_t^j. Let us define $U(\eta_t^j, \varepsilon_t)$ his expected utility at that stage. At the end of time t, he observes the realized profit of his manager, and hence his effective investment $\tilde{\mu}_t^j$, and the state a_{t+1}. Then, he can update his belief using the Bayes Rule, that is, $\eta_{t+1}^j = \zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1})$. Next, he chooses the firing rule $\phi(\eta_{t+1}^j, \varepsilon_{t+1})$ in order to maximize his expected utility from $t+1$ on, taking as given the risky bond price $p(\varepsilon_t)$, the allocation rule $x(\varepsilon_t, a_{t+1})$, the strategy of the fund managers $\mu(s_t^j, \varepsilon_t)$, and the default probability $q(\varepsilon_t)$.

The problem they solve is

$$U(\eta_t^j, \varepsilon_t) = (1-\gamma) E\left[\tilde{\mu}_t^j \frac{\chi_{t+1}(a_{t+1})}{p(\varepsilon_t)} + (1-\tilde{\mu}_t^j) R | \eta_t^j, \varepsilon_t\right] + \tag{6}$$
$$+\delta E\left[\max_{\phi}\left\{(1-\phi) U(\eta_{t+1}^j, \varepsilon_{t+1}) + \phi U(\varepsilon_{t+1}, \varepsilon_{t+1})\right\} | \eta_t^j, \varepsilon_t\right].$$

The probability ε_{t+1} of an unemployed manager to be informed at the end of period t is persistent and follows the low of motion $\varepsilon_{t+1} = \Psi(\varepsilon_t, a_{t+1})$ that is public information.[8] The investors' expected utility depends on ε_t because the investors use it to determine the probability that a new hire is informed and make their firing decision.

A fund manager with signal s_t^j chooses his investment strategy $\mu(s_t^j, \varepsilon_t)$ to maximize his expected utility, taking as given the risky bond price $p(\varepsilon_t)$, the firing rule adopted by the investors, $\phi(\zeta(\eta_t^j, \tilde{\mu}(\mu, \varepsilon_t, a_{t+1}), a_{t+1}), \varepsilon_{t+1})$, and the default rule followed by the borrowers. The problem for a fund manager with signal $s_t^j = (d, n, 0)$ is

$$W\left(s_t^j, \varepsilon_t\right) = \max_{\mu} \gamma E\left[\tilde{\mu}(\mu, \varepsilon_t, a_{t+1}) \frac{\chi_{t+1}(a_{t+1})}{p(\varepsilon_t)} + (1-\tilde{\mu}(\mu, \varepsilon_t, a_{t+1})) R | s_t^j, \varepsilon_t\right] + \tag{7}$$
$$+\delta E\left[1-\phi(\zeta(\eta_t^j, \tilde{\mu}(\mu, \varepsilon_t, a_{t+1}), a_{t+1}), \varepsilon_{t+1})] W(s_{t+1}, \varepsilon_{t+1}) | s_t^j, \varepsilon_t\right]$$

The key feature of this problem is that the fund managers know that their investment decision affects the investors' firing decision by changing the belief's update. This generates career concerns affecting investment decision that are at the core of our model.

2.3 Definition of equilibrium

Let us summarize the timing of the model. At the beginning of period t the productivity shock a_t is realized, and, hence, old borrowers make their default decision. The return of managers is also realized for each manager and it is shared between investors and managers. Then, some manager die and others are born and join the unemployed pool. Based on the return distribution of hired managers, investors with an alive manager decide whether to keep him or to fire him and hire a new one. Investors with a dead manager necessarily hire a new one. Next, hired managers receive the signal s_t and decide how to invest the investors' capital. At the same time, young agents decide whether to become active borrowers, how much to borrow and under what circumstances they will repay their loans. The bond market clears.

[8] See the appendix for the explicit derivation of the low of motion $\Psi(\varepsilon_t, a_{t+1})$.

Our equilibrium concept[9] is defined as follows.

Definition 1 *A stationary equilibrium is a sequence* $\{\mu(s_t^j, \varepsilon_t), \phi(\eta_{t+1}^j, \varepsilon_{t+1}), B(\varepsilon_t), q(\varepsilon_t)\}$*, a belief function* $\eta_{t+1}^j = \zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1})$*, an allocation function* $x(\varepsilon_t, a_{t+1})$*, a price* $p(\varepsilon_t)$*, and a low of motion* $\varepsilon_{t+1} = \Psi(\varepsilon_t, a_{t+1})$*, such that*

1. *investors maximize their expected utility, taking as given the price* $p(\varepsilon_t)$*, the allocation function* $x(\varepsilon_t, a_{t+1})$*, and the strategies of fund managers and borrowers;*

2. *fund managers maximize their expected utility, taking as given the price* $p(\varepsilon_t)$*, the allocation function* $x(\varepsilon_t, a_{t+1})$*, and the strategies of international investors and borrowers;*

3. *borrowers maximize their expected utility, taking as given the price* $p(\varepsilon_t)$*;*

4. *the bond market clears, that is,*

$$\int_0^\Gamma \tilde{\mu}_t^j(\mu, \varepsilon_t, a_{t+1}) dj = p(\varepsilon_t) B(\varepsilon_t),$$

   for any a_{t+1}*, where* $\tilde{\mu}_t^j(\mu, \varepsilon_t, a_{t+1}) = \mu$ *if* $\mu \in [0,1]$ *and* $\tilde{\mu}_t^j(I, \varepsilon_t, a_{t+1})$ *is given by (5),*

5. *investors' beliefs are consistent with the Bayes rule.*

We focus on the limiting case where $\bar{\varepsilon} \to 0$. In this limit, we look for a stationary limit equilibrium where the investment strategies, the firing rule, the bond supply, the default's probability and the price do not depend on the state ε_t. Thus, the only equilibrium objects that do vary with the state ε_t are the investors' beliefs. However, all the equilibrium objects do not depend on the distribution of beliefs, given that the level of η_t^j does not matter for the equilibrium, as long as $\eta_t^j \in (0,1)$, as we show in the appendix. This allows us not to keep track of the distribution of the beliefs in the population and to simplify the analysis. We will emphasize that even if the proportion of informed managers diminishes as $\bar{\varepsilon} \to 0$, our equilibrium does not converge to the natural equilibrium of the frictionless case with no asymmetric information of $\bar{\varepsilon} = 0$.

[9] The defined equilibrium can be implemented as a Perfect Bayesian Equilibrium of an augmented game where managers submit demand curves for the risky bond conditional on the state dependent allocation rule and a Walrasian auctioneer sets the price and a state dependent allocation rule which clears the bond market in each state.

Definition 2 *A stationary limit equilibrium consists of* $\{\mu(s_t^j), \phi(\eta_{t+1}^j, \varepsilon_{t+1}), B, q\}$, *a belief function* $\eta_{t+1}^j = \zeta(\eta_t^j, \mu_t^j, a_{t+1})$, *an allocation function* $x(a_{t+1})$, *a price* p, *and a low of motion* $\varepsilon_{t+1} = \Psi(\varepsilon_t, a_{t+1})$ *where each object is the limiting function of the sequence defined by the corresponding elements of a stationary equilibrium as* $\bar{\varepsilon} \to 0$.

3 Stationary Equilibrium

In this section we characterize the equilibrium, focusing on the limit case where $\bar{\varepsilon} \to 0$. In the limit, the amount of bond holdings b, the probability of default q, and the price p are constant over time. We proceed in three steps. First, we characterize the financial market *pricing rule,* that is, the equilibrium price for a given default probability. Then we characterize the borrowers' *repayment rule*, that is the optimal default rule conditional on the bond price. Finally, we show that these rules define a fixed point problem that determines the equilibrium bond price.

3.1 The pricing rule

Let us first characterize the financial market. Both investors and fund managers take as given the bond price p and the borrowers' strategy $\{b, \chi(a_{t+1})\}$, and, hence, q. We make the following assumption on Γ to ensure that the bond market clears as long as the bond price does not exceed $1/R$.

Assumption 1 *Assume that* Γ *is big enough such that*

$$\frac{1}{R} B\left(\frac{1}{R}\right) < \Gamma.$$

First consider a benchmark model with $\bar{\varepsilon} = 0$. In this case, all managers are uninformed, so investors will be indifferent between keeping the manager they started with and hiring a new one. In the natural equilibrium, investors do not systematically fire or hire managers based on their performance, so managers will maximize their period by period profit. Thus, the bond price is determined by the standard no-arbitrage condition

$$(1-q)\frac{1}{p} = R, \tag{8}$$

that is, the expected return on the bond must be equal to the return on the riskless asset, R.

Next, let us analyze our model with career concerns, where $\bar{\varepsilon} \to 0$, but $\bar{\varepsilon} > 0$. The next proposition characterizes the equilibrium strategies of investors and managers and the equilibrium bond price for a given probability of default and bond holding strategy.

Proposition 1 *Suppose that $\bar{\varepsilon} \to 0$, the probability of default q_t is a constant $q > (1 + 2\delta - \sqrt{1+4\delta})/2\delta$, and that there is a fixed, positive supply of bonds $\bar{B} \leq B\left(\frac{1}{R}\right)$. Let the bond price be*

$$p = \frac{(1-\delta q)(1-q)}{R[1-\delta(1-q)]}. \tag{9}$$

Then, the following strategies of investors and managers are optimal taking as given the strategies of the other players, under market clearing and a set of beliefs which are consistent with Bayes" rule:

1. *investors' firing rule*

$$\phi(\tilde{\mu}_t^j, a_{t+1}) = \begin{cases} 0 & if\ \tilde{\mu}_t^j = \chi(a_{t+1}) \\ 1 & otherwise \end{cases}; \tag{10}$$

2. *managers' strategies*

$$\mu(d) = 0,\ \mu(n) = 1,\ \mu(0) = I \tag{11}$$

   where the proportion of uninformed managers investing in the risky bond, $x(a_{t+1})$, is given by

$$\int_0^\Gamma \tilde{\mu}_t^j(s_t^j)dj = pB \tag{12}$$

   and (5).

This proposition shows that the optimal firing rule for the investors is to keep only the managers that invest in riskless bonds when there is default, and those who invest in the risky ones when there is no default. Then, the informed managers will follow their signal to avoid to be fired, and hence $\mu(d) = 0$ and $\mu(n) = 1$. Assumption 1 ensures that the market can clear if and only if a positive measure of uninformed managers invest in each of the two types of bonds. The proportion of uninformed managers who end up investing in the risky bond, $x(a_{t+1})$, has to be such that the market clears for the equilibrium price. From the managers' problem (7), the uninformed managers are indifferent if and only if

$$(1-q)\left(\gamma\frac{1}{p} + \delta W(0)\right) = \gamma R + q\delta W(0), \tag{13}$$

where

$$W(0) = \frac{\gamma R}{1 - \delta q}. \tag{14}$$

The left-hand side of equation (13) represents the expected payoff of a manager who invests in the risky bond. With probability $(1-q)$ borrowers do not default. In this case, the manager succeeded to pool with the informed managers, he is not fired, and gets continuation utility $W(0)$. If instead the manager invests in the risky bond and borrowers default, with probability q, he gets zero return. Moreover, the investor learns that the manager was not informed and fires him, so that he gets no continuation utility. Similarly, the right-hand side of equation (13) represents the expected payoff of a manager who invests in the risk-free bond. He gets a return R with certainty. However, he is not fired, and gets continuation utility $W(0)$, only if borrowers do default. Otherwise, the investor learns that he was not informed and fires him. Equation (14) gives the continuation value of being an uninformed manager who keeps the job. This condition is obtained by noticing that if a manager is indifferent in each point in time between investing in the risk-free asset or in the risky bond, his value function must be given by the value of always investing in the risk-free asset as long as with that strategy he is not fired. From combining equations (13) and (14) we immediately obtain the pricing condition (9). The lower bound on q implies that the return on the risky bond in the event of no default, $1/p$, is larger than R, i.e., the realized spread is non-negative.

Let us define the reputational premium Π, that is, the difference between the expected repayment and the risk free rate R

$$\Pi \equiv \frac{1-q}{p} - R. \tag{15}$$

This premium characterizes the price distortion generated by the career concerns of the uninformed fund managers. In the benchmark model with no career concerns, equation (8) immediately implies that this premium is equal to zero. In the case with a positive measure of informed managers, the reputational premium can be negative or positive. Typically, it is positive when q is sufficiently large and negative when q is sufficiently small. Betting on large probability events is especially attractive for an uninformed fund manager with career concerns, because it increases the chance that he will not make an unsuccessful decision and will not be fired. In contrast, even if the return compensates for the risk of default, holding a bond which pays off with small probability is especially unattractive for the uninformed fund manager, as it increases the chance of being fired. In equilibrium, this preference for large probability events is priced. Fund managers are willing to give up a part of their expected return for a large probability of not being fired.

3.2 The repayment rule

Borrowers choose their default rule and how much to borrow and to consume in order to solve problem (4), taking p as given. Let us first consider the default decision of an old borrower. For any given pair, a_t and a_{t+1}, she will default if and only if

$$\log a_{t+1} - D(b) - \log(a_{t+1} - b) > 0. \tag{16}$$

Note that the left hand side of the condition above is decreasing in a_{t+1}, thus there will be a threshold $\hat{a}$ such that the agent will repay if the shock $a_{t+1} \geq \hat{a}$, and will not repay otherwise. Hence, $q = G(\hat{a})$. This result is summarized in the following lemma.

Lemma 1 *For a given cost function, $D(\cdot)$, there exists a threshold $\hat{a}$ such that $\chi(a_{t+1}) = 1$ if $a_{t+1} \leq \hat{a}_{t+1}$ and $\chi(a_{t+1}) = 0$, otherwise, with*

$$\hat{a} = \frac{\exp\{D(b)\}}{\exp\{D(b)\} - 1} b. \tag{17}$$

The threshold $\hat{a}$ is increasing in b.

If we substitute back the budget constraint and the default decision $\chi(a_{t+1})$ into problem (4), it becomes a maximization problem over the borrowing decision only. Hence, the optimal policy b must satisfy the first order condition

$$\frac{p}{(pb - F)} - \beta \int_{\hat{a}}^{\infty} \frac{1}{(a_{t+1} - b)} dG(a_{t+1}) - \beta G(\hat{a}) \frac{dD(b)}{db} = 0, \tag{18}$$

where $\hat{a}_{t+1}$ solves equation (17). Then, using equation (18) together with (17), we can solve for the optimal amount of bonds supplied b, for a given price p. Next, we can plug b back into equation (17) and solve for the equilibrium default probability, $G(\hat{a})$, for a given price p. We will refer to this condition as the borrowers' optimal repayment rule.

Notice that for a given productivity distribution, not all the cost functions $D(\cdot)$ support an equilibrium with a non-trivial default decision. Intuitively, if the marginal cost of default is not large enough compared to the advantage of additional borrowing, the agent would always like to borrow more and default more often. In this case, the solution for problem (4) would not be a finite b. To be more precise, let $m(b)$ represent the total marginal cost of borrowing for a given level of debt b, that is,

$$m(b) \equiv \beta \int_{\hat{a}}^{\infty} \frac{1}{(a_{t+1} - b)} dG(a_{t+1}) + \beta G(\hat{a}) \frac{dD(b)}{db}, \tag{19}$$

where $\hat{a}$ is defined by (17). The first term is the cost of an additional unit of borrowing in terms of the foregone consumption of an old agent who pays back the debt, while the second term is the expected marginal increase of the cost of default. We make the following final assumption on $D(\cdot)$.[10]

Assumption 2 *The cost function $D(\cdot)$ is such that*

$$m'(b) > -m(b)^2 \text{ for any } b \in \mathbb{R}^+.$$

Under Assumption 2, the cost function $D(\cdot)$ is such that the second order condition of the representative agent problem is negative, and the problem has a unique solution, as stated in the following lemma.

Proposition 2 *For given p problem (4) has a unique solution.*

Finally, next lemma establishes three important properties of borrowers' optimal choices.

Lemma 2 *As the price of the bond, p, increases, (i) the face value of debt, b decreases, (ii) the value of capital borrowed pb increases, and (iii) the probability of default $G(\hat{a})$ decreases.*

The intuition behind these results is straightforward. If p increases, borrowing is cheaper so the budget constraint of the borrower is less stringent. As the borrower has decreasing marginal utility from consumption, she wants to enjoy the consumption benefit of cheaper borrowing in both periods. Thus, she decreases b, the face value of debt to increase consumption in the second period, but only to the extent that the borrowed amount, pb, also increases with p which allows for larger consumption in the first period. The decrease of b decreases $\hat{a}$ and the probability of default as the threshold for default, $\hat{a}$ is a monotonic function of b. This last result implies that the repayment rule, the probability of default as a function of the bond price, $G(\hat{a}(b(p)))$, is downward sloping in the (p, q) space.

[10] In Appendix A, we show in the context of our parametric examples that Assumption 2 is not necessarily strong.

Next, let us consider what happens to the aggregate value of capital inflows pB and to aggregate output. Define average aggregate output as $Y \equiv H(V)\,E(a_{t+1})$. The following proposition shows that both aggregate output and aggregate capital inflows are increasing with p.

Lemma 3 *Suppose that (2) holds. Then, as the price of borrowing p decreases, (i) the aggregate output Y decreases, and (ii) the aggregate value of capital borrowed pB decreases.*

3.3 Characterization of the equilibrium

In this section, we characterize the equilibrium of our model, which is jointly determined by the conditions of the financial market and the fundamentals of the borrowers.

In section 3.2, we have derived, for given price p, the endogenous probability of default of a representative borrower $q = G(\hat{a})$. In section 3.1, we have derived, for a given default probability q, the price p determined by the financial market. The next Proposition defines a fixed point problem combining the repayment rule and the pricing rule, and shows that the equilibrium is characterized by a stationary default rule and bond price $\{\hat{a}^*, p^*\}$.

Proposition 3 *Suppose that $\bar{\varepsilon} \to 0$. An equilibrium is characterized by a default rule and price $\{\hat{a}^*, p^*\}$ that solve the fixed point defined as follows:*

1. *given $\hat{a}$, p^* solves the pricing rule, that is,*

$$p^* = \frac{(1-\delta G(\hat{a}))\,(1-G(\hat{a}))}{R\left[1-\delta\left(1-G(\hat{a})\right)\right]}; \tag{20}$$

2. *given p, $\hat{a}^*$ solves the repayment rule, that is,*

$$\hat{a}^* = \frac{\exp\{D(b^*)\}}{\exp\{D(b^*)\}-1}b^*, \tag{21}$$

where b^* satisfies

$$\frac{p}{pb^*-F} - \beta\int_{\hat{a}^*}^{\infty}\frac{1}{(a-b^*)}dG(a) - \beta G(\hat{a}^*)\,D'(b^*) = 0. \tag{22}$$

Next, as a point of comparison, we describe the equilibrium in the benchmark case, when $\bar{\varepsilon} = 0$. This is also a fixed point $\left\{\hat{a}^b, p^b\right\}$, but it has to satisfy the no-arbitrage condition (8) and the repayment rule, $G(\hat{a})$.

Proposition 4 *An equilibrium of the benchmark economy, where* $\bar{\varepsilon} = 0$, $\{\hat{a}^b, p^b\}$ *solves the fixed point defined as follows:*

1. *given* $\hat{a}$, p^b *solves the pricing rule, that is,* $p^b = \left(1 - G\left(\hat{a}^b\right)\right)/R$;

2. *given* p, $\hat{a}^b$ *solves the repayment rule, that is, (21), where* b^* *satisfies (22).*

Figure 3 represents graphically the equilibrium both of the economy with career concerns (E) and of the benchmark economy (B). The equilibrium prices p^* and p^b correspond to the intersections of the repayment rule and the corresponding pricing rule, graphed in the space (p, q).

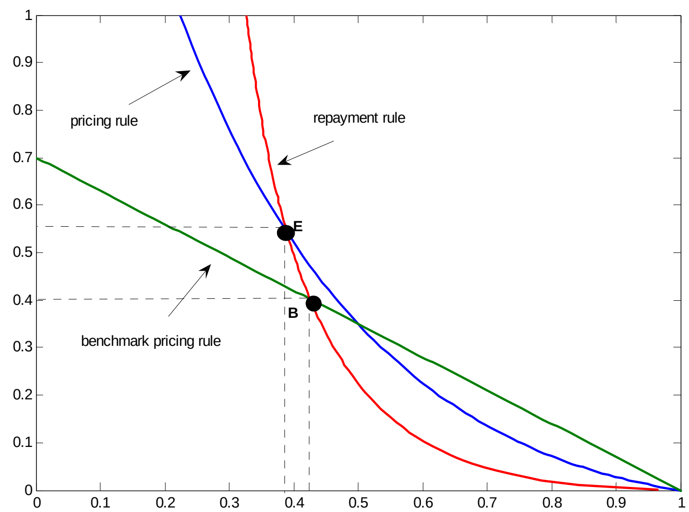

Figure 3: The solid line represents the repayment rule and the dashed curve and the dotted curve represent the pricing rule in the economy with career concerns and in the benchmark economy, respectively. Points E and B denote the equilibrium in the economy with career concerns and in the benchmark economy, respectively. Productivity is distributed according to a lognormal distribution with parameters 1.5 and 3.

In the baseline numerical exercise we have assumed that a is distributed as a lognormal random variable. In Figure 3, the parameters of the model are such that $p^* > p^b$, that is, such that the reputational premium is positive. By reducing the mean of a we can easily obtain the analogous figure where $p^* < p^b$ and the reputational premium is negative.

Proposition 2 shows that for any p, there is a unique $(\hat{a}, b)$ defined by (21) and (22). The next proposition proves the existence under some parameter restrictions. In this equilibrium, the repayment rule crosses the pricing rule from above in the (q,p) space, just as in Figure 3, i.e.,

$$\frac{\partial b^{-1}\left(\hat{a}^{-1}\left(G^{-1}\left(q\right)\right)\right)}{\partial q} > \frac{\partial p}{\partial q} = \frac{\partial\left(\frac{(1-\delta q)(1-q)}{R[1-\delta(1-q)]}\right)}{\partial q}|_{b=b^{*},p=p^{*}}.$$

For reasons clarified in the next part, we call an equilibrium with this property an *amplifying equilibrium*. We focus on amplifying equilibria throughout the paper.

Proposition 5 *Let us define b^{ex} as*

$$\frac{1}{b^{ex}} \equiv m\left(b^{ex}\right) \tag{23}$$

Suppose that $G(\hat{a}(b^{ex})) > (1+2\delta-\sqrt{1+4\delta})/2\delta$. Then there exist the threshold $F^{ex}>0$ that for any $F \leq F^{ex}$, an amplifying equilibrium $p^{*} \in \left(0, \frac{1}{R}\right), b^{*}$ exists.

3.4 Comparative statics

We now explore the properties of the equilibrium and analyze some comparative statics. In particular, we are interested in the reaction of the equilibrium both to shocks to financial markets and to shocks to the fundamentals of the borrowers. The first type of shocks affect the pricing rule and we refer to them as demand-side shocks; the second type affect the repayment rule and we label them supply-side shocks.

Notice that, depending on the parameters, two regimes are possible: the reputational premium might be negative or positive. The regime is determined jointly by the fundamentals of the risky project and the state of the financial market. For given fundamentals, the financial market can be such that the equilibrium is in any of the regimes. Similarly, for a given state of the financial market, the fundamentals can be such that the equilibrium is in any of the regimes. The following proposition formalizes the conditions that determine the equilibrium regime.

Proposition 6 *In equilibrium, one of the two following regimes arise: (i) if $G(\hat{a}(1/2R)) < 1/2$, the reputational premium is negative; (ii) if $G(\hat{a}(1/2R)) > 1/2$, the reputational premium is positive.*

Both demand-side and supply-side shocks can move the economy from one regime to the other. A typical demand-side shock can be represented by a change in the return of alternative investment opportunities, in our case a change in the risk-free rate, R. From equations (8) and

(20), it is easy to see that both pricing rules get flatter as R increases. Suppose the economy starts in a regime with negative reputational premium. Then, an increase in R can make the economy shift to a regime with positive premium. This shift amplifies the reduction of bond prices, capital flows, and production. We summarize this result in the following Proposition.

Proposition 7 *Suppose an amplifying equilibrium exists and $G(\hat{a}(1/2R')) < 1/2$. Then there is an $\tilde{R}$ that for any $R'', R'' > \tilde{R}$, $G(\hat{a}(1/2R'')) > 1/2$, i.e., then the reputational premium switches sign. Moreover, the price p, the value of capital flows pB, and the aggregate level of production Y, change more than in the benchmark model.*

Alternatively, the economy can move from one regime to another because of a supply-side shock. When the fundamentals of the borrowers deteriorate, the default probability may increase for any given price, and the repayment rule may shift to the right. For example, think of a shock to the productivity of borrowers. The next proposition shows that if the productivity of the borrowers is hit by a sufficiently large unfavorable shock , the default probability increases for a given bond price implying amplified price, capital flow and output response compared to the benchmark model.

Proposition 8 *Let us parametrize $G\left(a_{t+1}|\bar{a}\right)$ in a way that $\frac{\partial G(a_{t+1}|\bar{a})}{\partial \bar{a}} < 0$, that is, larger $\bar{a}$ values correspond to favorable second-order stochastic dominant shifts in the distribution of the productivity shock, and $\lim_{\bar{a}\to-\infty} G\left(a_{t+1}|\bar{a}\right) = 1$ for any positive a_{t+1}. Let us choose an $\bar{a}'$ that $(1+2\delta-\sqrt{1+4\delta})/2\delta < G\left(\hat{a}\left(b^{ex}\right)|\bar{a}'\right) < \frac{1}{2}$. Then there must be an $\tilde{a}$, $\tilde{a} < \bar{a}'$ and $\tilde{F}$ that for any $\bar{a}'' \leq \tilde{a}$ and $F < \tilde{F}$ an amplifying equilibrium exists for both $\bar{a}'$ and $\bar{a}''$ and*

$$\begin{aligned} G\left(\hat{a}\left(\frac{1}{2R}\right)|\bar{a}'\right) &< \frac{1}{2} \\ G\left(\hat{a}\left(\frac{1}{2R}\right)|\bar{a}''\right) &> \frac{1}{2}. \end{aligned}$$

Moreover, the price p, the value of capital flows pB, and the aggregate level of production Y, change more than in the benchmark model.

Propositions 7 and 8 show that as the financial environment or the borrowers' fundamentals change, the economy will switch between regimes with very low bond spreads, large capital inflows and large output of borrowers and regimes with high bond spreads and large capital outflows and reduced output of borrowers. The first type of regimes are frequently described in the financial press with terms like "abundant liquidity" or "traders reaching for yield". When the economy moves to the second type of regimes observers use terms like "flight-to-quality","flight-to-liquidity" or "drop in risk appetite" or "disappeared liquidity". In our model, these two types of regimes can arise even if fund managers are risk-neutral, managers' aggregate funds is constant, and even if the borrowers' fundamentals do not change. We argue that abrupt changes in risk-premium can be caused by managers' career concerns. In good times, when the default probability of credit instruments is low, it is very attractive for uninformed fund managers to invest in these instruments because if they stay away, they are likely to produce smaller returns than others and lose funds. In bad times, investing in the risk-free assets has the larger probability to increase their reputation, so they need extra premium for investing in the risky bond.

Our results also imply that fundamental shocks are amplified by the affect of fluctuation sign and level of the reputational premium leading to excess volatility of the bond price, the default probability, capital flows and output. This is consistent with the empirical evidence that shows that emerging market bond prices fluctuate more than what is accounted for by changes in probability of default. On the one hand, Broner, Lorenzoni and Schmukler (2007) argue that the premium over the expected repayment on emerging market bonds is especially high during crises times. On the other hand, Duffie et al. (2003) document that the implied short spread of Russian bonds was very low during the first 10 months of 1997. Moreover, their estimation shows that in one short interval in 1997, bond prices were so high that the implied default adjusted short spread was negative. Although this observation is model specific, it is still interesting to point out that this is inconsistent with most risk-aversion based explanation, but consistent with our model. Note also that the result that demand-side shocks can be important determinants of bond prices is broadly consistent with the empirical evidence that a large proportion of the variation in prices of both corporate bonds and emerging market bonds cannot be explained by the variation of fundamentals. Moreover, a large part of this unexplained component is common across bonds (see Collin-Dufresne at al., 2000, Gruber et al., 2001, Westphalen, 2001) and any demand-side shock will affect all bond prices simultaneously.

4 Persistent Productivity Shock

In this section, we introduce persistency in the productivity process. In particular, assume that a_{t+1} is distributed according to a first-order Markov process with cumulative density function $G(a_{t+1}|a_t)$. The environment is a natural generalization of the one with *i.i.d.* shock, where a_t represents an additional state variable. We look for Markovian equilibria.

4.1 Equilibrium characterization

The optimization problems for borrowers, investors and managers are natural generalizations of problems (4), (6), and (7), where a_t is added as a new state variable.

Hence, an equilibrium is an investment function $\mu(s_t^j, \varepsilon_t, a_t)$, a firing rule $\phi(\eta_{t+1}^j, \varepsilon_{t+1}, a_t)$, a supply of bonds $B(\varepsilon_t, a_t)$, a default probability $q(\varepsilon_t, a_t)$, a price $p(\varepsilon_t, a_t)$ and a belief updating rule $\zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1}, a_t)$ such that investors, fund managers, and borrowers maximize their expected utility taking prices and others' strategies as given, believes are consistent with the Bayes" rule and markets clear. We still focus on the case where $\bar{\varepsilon} \to 0$. We propose a Markovian equilibrium with very similar properties to the *i.i.d.* case as it is described in the next proposition.

Proposition 9 *Suppose that $\bar{\varepsilon} \to 0$ and there are default and pricing functions $\{\hat{a}^*(\cdot), p^*(\cdot)\}$ which solve the fixed point defined as follows:*

1. *given $\hat{a}(\cdot)$, $p^*(\cdot)$ solves the pricing rule, that is,*

$$p(a_t) = \frac{\gamma\left[1 - G(\hat{a}(a_t)|a_t)\right]}{W(0, a_t) - \delta \int_{\hat{a}(a_t)}^{\infty} W(0, a_{t+1})\, dG(a_{t+1}|a_t)}, \tag{24}$$

where $W(0, \cdot)$ satisfies

$$W(0, a_t) = \gamma R + \delta \int_0^{\hat{a}(a_t)} W(0, a_{t+1})\, dG(a_{t+1}|a_t). \tag{25}$$

2. *given $p(\cdot)$, $\hat{a}^*(\cdot)$ solves the repayment rule, that is,*

$$\hat{a}(a_t) = \frac{\exp\{D(b(a_t))\}}{\exp\{D(b(a_t))\} - 1} b(a_t), \tag{26}$$

where $b(\cdot)$ satisfies

$$\frac{p(a_t)}{p(a_t)\, b(a_t) - F} - \beta \int_{\hat{a}(a_t)}^{\infty} \frac{1}{(a_{t+1} - b(a_t))} dG(a_{t+1}|a_t) - \beta G(\hat{a}(a_t))\, D'(b(a_t)) = 0. \tag{27}$$

Then $\{\hat{a}^*(\cdot), p^*(\cdot)\}$ is a Markov-equilibrium with investors' firing rule and managers' strategies analogous to (10)and (11).

In our equilibrium investors and managers follow the same strategies as in the *i.i.d* case independently from the past realizations of the productivity shock, a_t. Borrowers repayment rule, $\hat{a}(a_{t+1})$, and the amount of the bonds issued, $b(a_t)$, are implicitly defined by equations (26) and (27) which are analogous to equations (21) and (22) with the only difference that all decision variables must be conditional on the past realization of a_t as the distribution of the productivity shock is persistent. The pricing rule (24) and the recursive formula for the value function (25) are also analogous to equations (13) and (14) implied by the observation that uninformed fund managers have to be indifferent whether to invest in the riskless asset or the risky bond.

Next, as a point of comparison, we propose the equilibrium for the benchmark economy with no career concerns.[11]

Proposition 10 *An equilibrium of the benchmark economy $\left\{\hat{a}^b(\cdot), p^b(\cdot)\right\}$ solves the fixed point defined as follows:*

1. *given $\hat{a}(\cdot)$, $p^b(\cdot)$ solves the pricing rule, that is, $p^b = [1 - G\left(\hat{a}^b|a_t\right)]/R$;*
2. *given $p(\cdot)$, $\hat{a}^b(\cdot)$ solves the repayment rule, that is, (21), where $b^d(\cdot)$ satisfies (22).*

4.2 Numerical example: Emerging market debt

In this section, we present a numerical example to illustrate the dynamic properties of our equilibrium when shocks are persistent. We discuss the results in the context of emerging market debt.

There are at least four major puzzles emphasized in the literature on the business cycle characteristics of emerging markets (see Neumeyer and Perri (2005), Uribe and Yue (2006), Arellano (2006), Aguiar and Gopinath (2006), Longstaff et al (2007)).

1. Emerging market bond spreads are very volatile. Also output and capital flows are more volatile in small emerging market economies than developed economies of comparable

[11] The proof of this proposition is virtually identical to the proof of Proposition 4, with the only modification that the distribution of productivity shock, $G(\cdot)$, is replaced by its conditional counterpart. Thus, the proof is omitted.

size. Especially the magnitude of volatility of interest rates is hard to reconcile in models where bond prices are given by the no-arbitrage condition (8).

2. There is a complex interaction between the international financial environment, emerging market bond spreads and economic fundamentals of the country. The international financial environment affects spreads and economic fundamentals, economic fundamentals affect spreads and spreads affect economic fundamentals.

3. Unlike developed economies, emerging markets borrow more in good times and borrow less in bad times.

4. Emerging market bond spreads are countercyclical.

5. Occasionally, emerging markets default on their debt.

Our main contribution to this literature is that we present a mechanism which help to address the first and second puzzles. We show that the pricing rule taking into account managers' career concerns leads to much more volatile spreads, output and capital flows than the standard, no-arbitrage condition. Our equilibrium model also helps to disentangle the complex interaction between conditions in international financial markets, the economic fundamentals of the country and emerging market bond spreads.

We also show in this part that our model is consistent with the rest of the puzzles. However, given our non-standard assumptions on borrowers, we see this as a consistency check, rather than a major contribution.

As our model is stylized in many aspects, we do not match quantitatively the moments of the data. Instead, we focus on the qualitative properties of the model with special attention on the comparison between the cases with and without career concerns. Thus, we choose a simple parametrization. In our example

$$\ln a_{t+1} \tilde{} N\left(0.3, 0.3^2\right), \ \bar{u}_t^i \tilde{} N\left(E\left(V_t\right), Var\left(V_t\right)\right)$$

$$D\left(b_t\right) = 0.3b_t^2 + 1$$

$$Corr\left(\ln a_{t+1}, \ln a_t\right) = 0.1$$

$$\delta = 0.5, \ \beta = 0.96, \ R = 1/\beta, \ F = 0.01.$$

First, we show how the default probability, the bond price, and the reputational premium vary with the realization of the productivity shock. Let us start with the equilibrium behavior

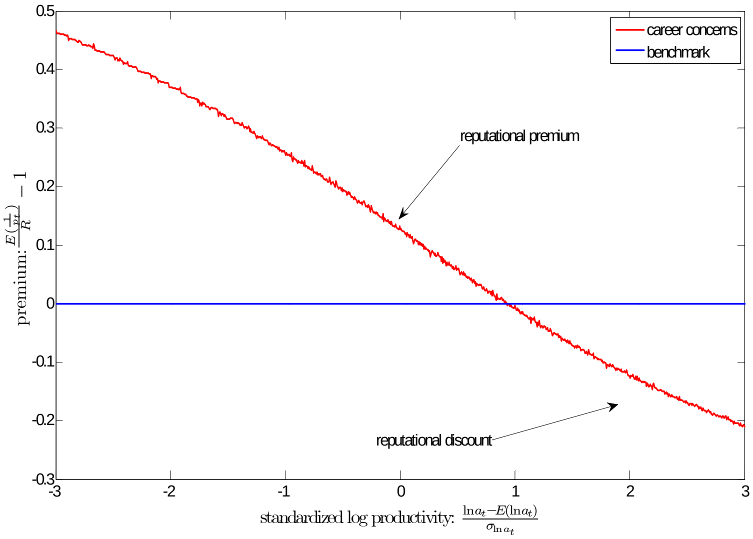

Figure 4: The figure shows the reputational premium as a function of the realization of the productivity shock. The dashed line is the premium with career concerns and the solid line shows the premium in the benchmark case.

in the benchmark economy. As a bad shock hits, the financial market will realize that, even for a given default rule, the probability of default will be higher and will require a lower bond price. As borrowing becomes more expensive, borrowers will then reduce their default cut-off, magnifying the reduction in the bond price. Hence, for low realizations of productivity, the default cut-off will be higher and the bond price lower. Now, consider the economy with career concerns. Suppose the default probability is high enough that the reputational premium is positive. In this case, the financial market will require a bond price even lower than the benchmark economy because of the reputational premium. Given that productivity is persistent, a bad realization of the shock will further increase the probability of default, increasing the fear of the fund managers of being fired and pushing the bond price further down. This implies that the reputational premium itself is higher after bad shocks. Figures 4 and 5 show how the reputational premium, the bond price, and the default probability vary in equilibrium with the different realizations of the productivity shocks.

Now, consider an economy that at time zero is hit by a shock. Figure 6 shows how the equilibrium prices react in expected terms to a bad and to a good shock, both with and without career concerns. Notice that the economy with career concerns reacts much more to

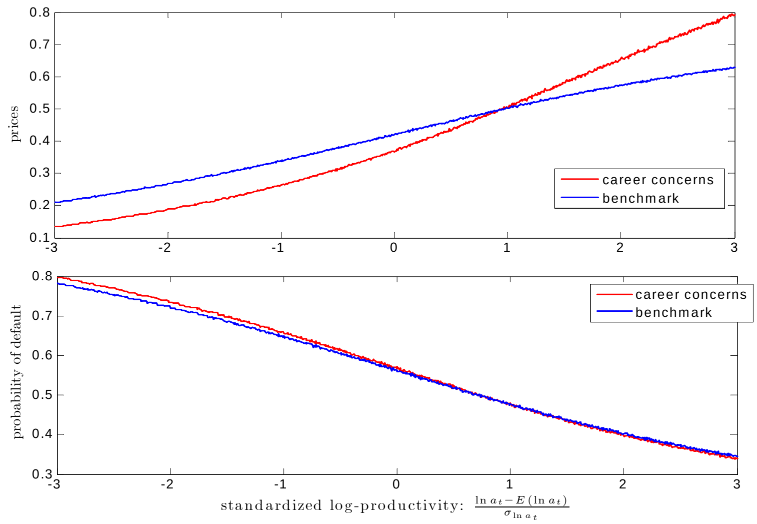

Figure 5: The upper panel shows the equilibrium price as a function of the productivity shock, while the lower panel shows the equilibrium probability of default as a function of the productivity shock. In both panels, the solid line represents the benchmark case and the dashed line represents the case with career concerns.

the shocks than the benchmark economy. Moreover, notice that in the economy considered, the reputational premium would be positive in expected terms and a good shock can actually make the economy shift regime.

Finally, we present some of the moments generated by our example. It is apparent that the implied interest rates are significantly more volatile with career concerns than in the benchmark case. The output and capital flows are also more volatile with career concerns. As a consistency check, we show that our model generates a negative covariance between interest rates and output and a positive covariance between output and capital flows consistently with empirical observations. The coefficients are also larger under career concerns in absolute terms.

	$\sigma^2(\cdot)$		$cov(\cdot, Y)$	
	without	with	without	with
	career concerns		career concerns	
$\frac{1}{p_t}$	0.25	0.93	-0.26	-0.55
$p_t B_t(p)$	0.016	0.03	0.05	0.11
$Y_t(p_t)$	0.33	0.43	-	-

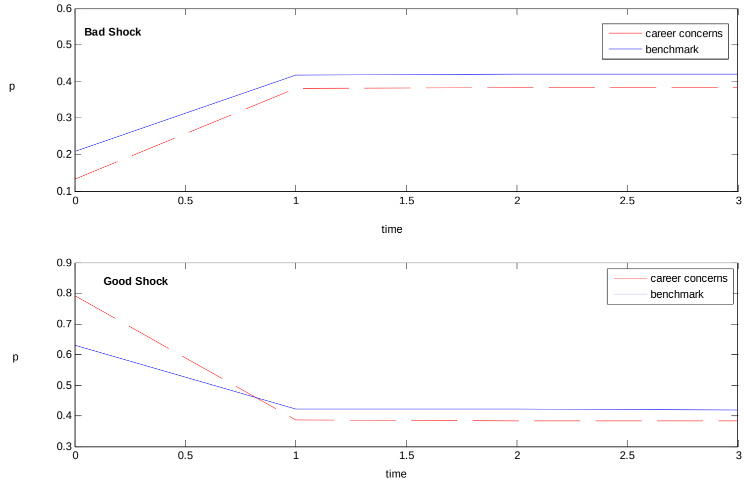

Figure 6: The two panels show the reaction of the equilibrium prices to a bad and a good shock, respectively. The solid line represents prices in the benchmark economy, and the dashed line prices in the economy with career concerns. At time zero productivity drops to the lowest possible realization in the first case and rises to the highest possible one in the second case.

5 Two groups of borrowers

In this section, we allow fund managers to lend to two different groups of borrowers. We show that even if the fundamentals of the two groups are independent, prices and default probabilities will be correlated if fund managers have career concerns.

5.1 Equilibrium characterization

Let us suppose that there are two groups of borrowers in the economy. The two groups are identical, except that group $r = A, B$ faces the productivity shock a_t^r. The stochastic Markov processes a_t^A and a_t^B are independent and described by the conditional cumulative density function, $G\left(a_{t+1}^r | a_t^r\right)$. We assume that an informed manager gets one of three possible informative signals, s_t^j, $s_t^j = n_A, n_B, d$. Signal n_r implies that group $r, r = A\,B$, will not default and signal d implies that both groups will default. If none of the groups default, informed managers get signal n_A or signal n_B with equal probability. We also change Assumption 1 as follows.

Assumption 3 *Assume that* Γ *is such that*

$$\sup_{a_t^r} \frac{1}{R} B\left(\frac{1}{R}, a_t^r\right) < \frac{1}{2}\Gamma.$$

In the following proposition, we propose a Markovian equilibrium very similar to the baseline equilibrium described in Proposition 9.

Proposition 11 *Suppose that* $\bar{\varepsilon} \to 0$ *and* $\{\hat{a}^{r*}(\cdot), p^{r*}(\cdot)\}$ $r = A, B$ *solves the fixed point defined as follows:*

1. *given* $\hat{a}^A(\cdot)$ $\hat{a}^B(\cdot)$, $p^{r*}(\cdot)$ *solves the pricing rule, that is,*

$$p^{r*}(a_t) = \frac{\gamma\left(1 - G\left(\hat{a}^r\left(a_t^r\right)|a_t^r\right)\right)}{W\left(a_t^A, a_t^B\right) - \delta\int_{\hat{a}(a_t^r)}^{\infty}\int_0^{\infty} W\left(a_{t+1}^A, a_{t+1}^B\right) dG(a_{t+1}^B|a_t^B)dG(a_{t+1}^A|a_t^A)}, \tag{28}$$

where $W(\cdot)$ *satisfies*

$$W\left(a_t^A, a_t^B\right) = \gamma R + .\delta \int_0^{\hat{a}^A\left(a_t^A\right)} \int_0^{\hat{a}^B\left(a_t^B\right)} W\left(a_{t+1}^A, a_{t+1}^B\right) dG\left(a_{t+1}^B|a_t^B\right) dG\left(a_{t+1}^A|a_t^A\right). \tag{29}$$

2. *given* $p^r(\cdot)$, $\hat{a}^{r*}(\cdot)$ *solves the repayment rule, that is,*

$$\hat{a}\left(a_t^r\right) = \frac{\exp\left\{D\left(b\left(a_t^r\right)\right)\right\}}{\exp\left\{D\left(b\left(a_t^r\right)\right)\right\} - 1} b^r\left(a_t^r\right), \tag{30}$$

where $b^r(\cdot)$ *satisfies*

$$\frac{p\left(a_t^r\right)}{\left(p\left(a_t^r\right) b\left(a_t^r\right) - 1\right)} - \beta \int_{\hat{a}(a_t^r)}^{\infty} \frac{1}{\left(a_{t+1}^r - b^r\left(a_t^r\right)\right)} dG\left(a_{t+1}^r|a_t^r\right) - \beta G\left(\hat{a}\left(a_t^r\right)|a_t^r\right) D'\left(b\left(a_t^r\right)\right) = 0. \tag{31}$$

Then $\{\hat{a}^*(a_t^r), p^*(a_t^r)\}$ *for* $r = A, B$ *is a Markov-equilibrium with*

3. *investors' firing rule*

$$\begin{aligned} \phi(\tilde{\mu}_t^{j,A}, a_{t+1}^A, a_{t+1}^B) &= \begin{cases} 0 & if\ \tilde{\mu}_t^{j,A} = \chi\left(a_{t+1}^A\right) \\ 1 & otherwise \end{cases}; \\ \phi(\tilde{\mu}_t^{j,B}, a_{t+1}^A, a_{t+1}^B) &= \begin{cases} 0 & if\ \tilde{\mu}_t^{j,B} = \chi\left(a_{t+1}^A\right) \\ 1 & otherwise \end{cases} \end{aligned}$$

4. *and managers' strategies*

$$\mu^r(d) = \mu^A(n_B) = \mu^B(n_A) = 0,\ \mu^r(n_r) = 1,\ \mu^r(0) = I$$

for $r = A, B$, *where the proportion of uninformed managers investing in the risky bond,* $x^s\left(a_{t+1}^A, a_{t+1}^B\right)$, *is given by the respective market clearing conditions analogous to 12.*

Observe that the prices of the two risky bonds are related through the common terms in the denominator in (28), even though the two group-specific shocks are independent. Thus, the price of each risky bond will depend on both productivity shocks.

To shed more light on the intuition let us focus again on the special case of i.i.d. distribution of productivity shocks. It is easy to see that the repayment rule, $G(\hat{a}^r)$, is the same in the two risky bond case and in the one risky bond case as (27) is identical to (31). Just as in the case with 1 group of borrowers, the equilibrium collapses to a a stationary equilibrium where bond prices are determined by the condition that uninformed managers have to be indifferent between investing in each asset. That is,

$$(1-q^r)\left(\frac{1}{p^r}+\delta W\right) = R + q^r q^{r'} \delta W \tag{32}$$

where q^r is the probability of default of the bond r for $r = A, B$, $r' = B, A$. Analogously to equation (13), the left hand side of the expression shows the expected pay-off of the uninformed manager who invests in asset r, and the right hand side is his expected pay-off if he invests in the risk-free asset. The new element is that with two bonds a manager investing in the risk-free asset is fired unless both risky bonds default. This is so, because if any of the bonds do not default, informed managers will choose to invest in a non-defaulting risk bond. The indifference condition implies the new value function

$$W = \frac{R}{1-\delta q^A q^B}.$$

Thus, the pricing rule changes to

$$p^r = \frac{(1-q^r)}{R}\frac{\left(1-\delta q^r q^{r'}\right)}{1-\delta(1-q^r)}$$

and

$$\frac{\partial p^r}{\partial q^{r'}} = -\delta q^r \frac{1-q^r}{R(1-\delta(1-q^r))} < 0.$$

That is, if the probability of default of one risky bond increases, the price of both risky bond will decrease. The intuition is immediate from the indifference conditions (32). The reputational cost of investing in the riskless asset depends on the probability that both bonds defaults. If both of the bonds do not default, the manager who invests in the riskless bond will be perceived to be uninformed and will lose his job. Thus, if the probability of default of any of the risky bonds decreases, the riskless asset will be less attractive, so the return on both bonds has to decrease to keep managers indifferent between different strategies.

5.2 A numerical example

We consider two symmetric groups of borrowers with the same fundamentals as in our numerical example of the baseline case. The productivity processes of the two countries follow the same Markov-process. Assume that there is no fundamental link between the two bonds, that is, the two productivity processes are independent.

We conduct an experiment very similar to the one which leaded to Figure 6. At time 0, the economy of group A is hit by a large negative or positive shock. We check how the prices of both bonds react to these shocks with and without career concerns. The results are shown in Figure 7. Dashed lines show the price responses with career concerns. Solid lines show the price responses in the benchmark case. Starred lines show the price responses of the bond of the group with unaffected productivity process. Naturally, with no career concerns the price of the bond with unaffected fundamentals remains constant. However, with career concerns both prices respond to the shock. There is a reputational link which leads to comovement in bond prices, even if the underlying fundamentals are independent.

6 Robustness

The main result of our paper is that managers' career concerns amplify real shocks leading to large capital inflows and very low required bond returns in good times and large capital outflows and very high required returns in bad times. The intuition behind this result is based only on three critical points.

First, there must be some "informed" managers who are capable of persistently producing superior returns. This is sufficient to induce investors to periodically reallocate their capital from managers with low realized returns to managers with high realized returns. This introduces a reputational concern of "uninformed" managers who, other parameters equal, will prefer bets with large probability of success.

Second, the capital share of uninformed managers has to be sufficiently high that their preferences are getting priced. If this assumption holds, assets with high probability of low return realization will trade with a reputational premium while assets with low probability of a low return realization will trade with a reputational discount.

Third, the sensitivity of the investment of borrowers has to be sufficiently insensitive to the changes in the cost of capital. In our model, the size of necessary investment is fixed and

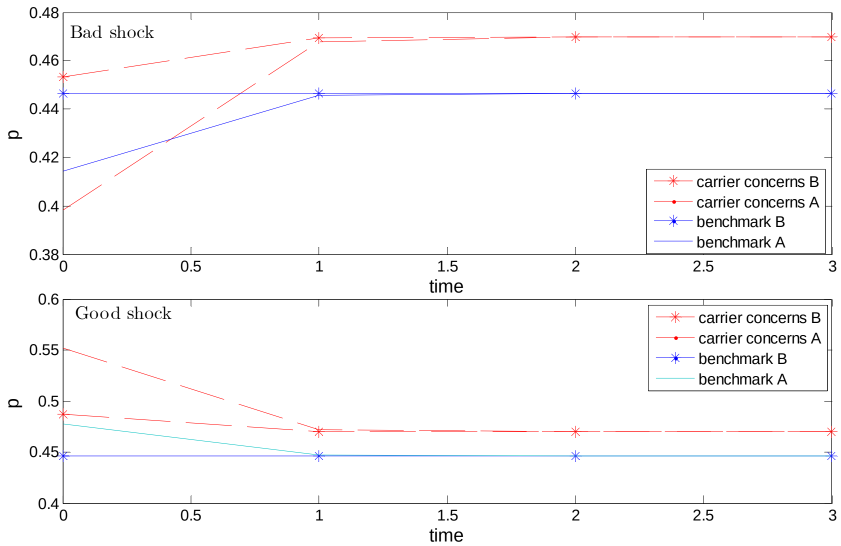

Figure 7: The two panels show the reaction of the equilibrium prices to a bad and a good shock, respectively. Dashed lines show the price response with career concerns. Solid lines show the price response in the benchmark case. Starred lines show the price response of the bond of the group with unaffected productivity process. At time zero, productivity of one group drops to the lowest possible realization in the first case and rises to the highest possible one in the second case. The productivity of the other group is unaffected.

perfectly insensitive to price changes. This implies that as the cost of borrowing increases, borrowers have to smooth its effect over their lifetime consumption by decreasing their dollar amount of borrowing when young, but increasing the face value of their debt which they have to pay back when they are old. The latter implies that larger required return on the issued assets leads to higher probability of default. This provides the feedback which reinforces the reputational premium or discount leading to amplification of real shocks.

Although any sets of assumptions which satisfy these three points should be sufficient to generate our mechanism, our particular modelling choices are necessary to keep our model analytically tractable. In the rest of this section, we discuss the role of some of our strongest assumptions.

Diminishingly small proportion of informed managers. It is possible to analytically characterize the equilibrium for the case when $\bar{\varepsilon}$ is bounded away from zero and the productivity shock is i.i.d. This equilibrium would be similar to the presented stationary limit equilibrium except that all variables would fluctuate together with the stochastic fluctuation of the proportion of informed managers within the hired pool, ε_t . The main reason why we focus on the limiting equilibrium is that we consider this fluctuation an artifact of our structure (in particular, of the assumption that the proportion of informed managers in the economy, $\bar{\varepsilon}$, is fixed and common knowledge) and not a phenomenon based on economic intuition.

Perfect signals. Talented fund managers know whether the borrowers will default or not for sure. The main implication of this assumption is that we do not have to keep track each manager's past performance. The idea is that because of the perfect signal, if a manager makes a mistake, it will reveal that he must be uninformed and he will be fired. Thus, all hired fund managers at the beginning of a given period must have chosen the right strategy in all previous periods when they were hired. This property keeps the model tractable. However, it is clear that this assumption is not critical for our mechanism. We need only that informed fund managers have a sufficient informational advantage to induce investors to chase performance.

Fixed, linear contract. Fund managers get a fixed share of the return of their portfolio in each period. This assumption corresponds to the typical, real-world, mutual fund contract which is a flat proportion of the managed assets. In contrast, the typical contract for a hedge fund is piecewise linear, with 2% on managed assets and 20% on the upside relative to a pre-specified benchmark. Allowing for similar performance fees would not change our qualitative results. For example, let us suppose that the benchmark is between the riskless return, R, and the return on the risky bond when the borrowers do not default, $\frac{1}{p_t}$. Let γ^H denote the performance fee. Then the indifference condition, (13), modifies to

$$(1-q)\left(\gamma^H \frac{1}{p} + \delta W(0)\right) = \gamma R + q\delta W(0),$$

where

$$W(0) = \frac{\gamma R}{1-\delta q}.$$

Consequently, the pricing curve is modified to

$$\frac{\gamma^H}{\gamma} \frac{(1-q)(1-q\delta)}{R(1-\delta+q\delta)} = p.$$

Thus, this modification would only increase the price proportionally for any probability of default, leaving our qualitative results unchanged.

However, we do not explore the problem whether there is an optimal contract, and if there is, whether an optimal contract would change our results. Although, it is an interesting problem, this question is out of the scope of our paper.

Risk neutrality. Fund managers in our model are risk neutral. This ensures that the demand of managers for the risky bond is perfectly inelastic which considerably simplifies the solution of the model. This assumption also helps to emphasize that our mechanism is orthogonal to arguments on changing risk aversion. Admittedly, this assumption also makes it harder to contrast our results to empirical facts. In reality, prices of risky bonds do contain a premium for risk. The presented reputational premium or discount should be an additional element in prices. Thus, it is very possible that even when a bond is traded with a reputational discount, in reality this would not result in a negative expected excess return on bond, only it would decreases the positive risk premium to a very small positive level. It is also because of risk neutral managers that our model, unlike standard real business cycle models, automatically generates a negative covariance between interest rates and output. In standard real business cycle models, the covariance is positive because the higher demand for bonds in good times pushes up interest rates which overweights the effect that better fundamentals should push interest rates down. Because risk neutrality implies flat bond demand, the first effect is not present in our model.

7 Conclusion

In this paper we have introduced an equilibrium model of delegated portfolio management with endogenous default. In our model, investors hire fund managers to invest their capital either in a defaultable bond or in a riskless one. Looking at the past performance, investors update their beliefs on the information of their fund managers. This leads to career concerns that affect the funds' investment decisions, generating a "reputational premium". When the probability of default is sufficiently high, fund managers prefer to invest in safe bonds even at a lower expected return to reduce the probability of being fired. The reputational premium can switch sign in response to shocks, both to the financial market and to the fundamentals of the borrowers (for example, to the economic conditions of an emerging economy). This can generate an overreaction of the market leading to excess volatility of spreads, capital flows, and output. In an extension, we also show that the presented reputational mechanism can lead to contagion between assets with no fundamental links.

We consider our work as a first step to understand how career concerns of fund managers affect the real economy. Our model concentrates on one particular feature of managers incentives: namely that they prefer strategies with high ex ante probability of success even if the expected pay-off of this strategy is low. An interesting next step is to consider the effect of other features, for example that managers with outstanding performance might attract a disproportionate share of funds. Moreover, we leave for future research a more detailed calibration to shed light on the quantitative importance of our mechanism.

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Appendix A: Parametric examples

Example 1 (finite support) *Let us suppose that a_{t+1} is uniformly distributed between $a^{\max}$ and $a^{\max}-\varphi$ and the cost function is*

$$D\left(b_t\right)=\ln\left(1+\frac{b_t}{\lambda\left(b^{\max}-b_t\right)}\right)$$

where λ and $b^{\max}$ are fixed, positive parameters with $0<\lambda<1$ and $b^{\max}\leq a^{\max}$.

Under this paramethrization

$$m\left(b\right)\equiv\beta\frac{1}{\varphi}\left(\ln\frac{a^{\max}-b}{\left(b^{\max}-b\right)\lambda}\right)+\beta\frac{b^{\max}\left(\left(b\left(1-\lambda\right)+b^{\max}\lambda\right)-\left(a^{\max}-\varphi\right)\right)}{\varphi\left(b^{\max}-b\right)\left(b\left(1-\lambda\right)+b^{\max}\lambda\right)}.$$

It is easy to check that $\frac{\partial m(b)}{\partial b}>0$, thus (2) holds for any λ and $b^{\max}$.

Example 2 (infinite support) *Let us suppose that a_{t+1} is lognormally distributed and $D\left(b_t\right)=\alpha_1+\alpha_2 b_t^2$ where α_1 is sufficiently high that the cost function satisfies $D\left(b\right)>\log\left(1+bD'\left(b\right)\right)$.*

Under this parametrization, although $\lim_{b\to\infty}m\left(b\right)=\infty$, $m\left(b\right)$ might be non-monotonic. However, (2) still holds for a wide range of parameters. In particular, we choose $\ln a_{t+1}\tilde{}N\left(0.3,0.3^2\right)$, $D\left(b_t\right)=0.3b_t^2+1$ in our numerical exercises.

B Appendix B: Proofs

B.1 Proof of Proposition 1

Let us fix $\bar{\varepsilon}$ at an arbitrary positive level and define ρ_t as the proportion of informed managers who are unemployed at the beginning of period t. After a mass $\left(1-\delta\right)2\Gamma$ of unemployed managers die and a new mass $\left(1-\delta\right)2\Gamma$ is born, there are going to be $\left[\delta\rho_t+\left(1-\delta\right)2\bar{\varepsilon}\right]\Gamma$ of informed unemployed and a total mass of $\left(2-\delta\right)\Gamma$ unemployed mangers. Hence, the probability that an unemployed manager is informed at the moment of hiring is $\varepsilon_t\equiv\left[\delta\rho_t+\left(1-\delta\right)2\bar{\varepsilon}\right]/\left(2-\delta\right)$. The informed unemployed managers at time $t+1$ are going to be the ones that were informed at time t who survived, plus the newly born ones, minus the proportion of informed of the newly hired ones. Define μ_t^c the proportion of uninformed managers who make the same investment decisions of the informed guys and hence are not fired, that is,

$$\mu_t^c\left(\varepsilon_t,a_{t+1}\right)\equiv\left\{\begin{array}{ccc}x\left(\varepsilon_t,a_{t+1}\right) & if & a_{t+1}>\hat{a}_{t+1}\\ 1-x\left(\varepsilon_t,a_{t+1}\right) & if & a_{t+1}<\hat{a}_{t+1}\end{array}\right\}.$$

It follows that $\rho_{t+1} = \tilde{\Psi}(\rho_t, a_{t+1})$ where

$$\tilde{\Psi}(\rho_t, a_{t+1}) = [\delta\rho_t + (1-\delta)\, 2\bar{\varepsilon}] \left[\frac{1 - \delta\,[1 - (2\bar{\varepsilon} - \rho_t) - (1 - (2\bar{\varepsilon} - \rho_t))\, \mu_t^c]}{(2-\delta)}\right]. \tag{33}$$

It is then immediate to derive the law of motion for ε_t, that is, $\varepsilon_{t+1} = \Psi(\varepsilon_t, a_{t+1})$, where

$$\Psi(\varepsilon_t, a_{t+1}) = \frac{\delta\tilde{\Psi}\left(\frac{(2-\delta)\varepsilon_t - (1-\delta)2\bar{\varepsilon}}{\delta}, a_{t+1}\right) + (1-\delta)\, 2\bar{\varepsilon}}{2-\delta}.$$

Lemma 4 *The following series of inequalities hold for any $t \geq 0$ any $\bar{\varepsilon} > 0$:*

$$\bar{\varepsilon} > \rho_{t+1} > 0.$$

Proof. The proof proceeds by induction. We know that $\rho_0 = \bar{\varepsilon}$. Then

$$\rho_1(a_{t+1}, \bar{\varepsilon}) = \bar{\varepsilon}\,[1 - \delta(1-\bar{\varepsilon})(1-\mu_t^c)],$$

and $\bar{\varepsilon} > \rho_1 > 0$, so that the statement is true for ρ_1. Now let us suppose that it is true for ρ_t. Observe that

$$\frac{2\bar{\varepsilon}(1-\delta) + \delta\rho_t}{2-\delta} > 0,$$

so, from (33), ρ_{t+1} is increasing in μ_t^c. Hence, for a fixed ρ_t, the maximum ρ_{t+1} is achieved when $\mu_t^c = 1$ and the minimum one when $\mu_t^c = 0$. With slight abuse of notation, define $\rho_{t+1}(\mu_t^c, \rho_t) \equiv \tilde{\Psi}(\rho_t, a_{t+1})$. First, we show that for any $\rho_t \in (0, \bar{\varepsilon})$, $\rho_{t+1} < \bar{\varepsilon}$. If this is true for $\mu_t^c = 1$, then it is true for all μ_t^c. Then, notice that

$$\rho_{t+1}(1, \rho_t) = \frac{\delta\rho_t + (1-\delta)\, 2\bar{\varepsilon}}{2-\delta},$$

which is increasing in ρ_t, and that

$$\rho_{t+1}(1, \bar{\varepsilon}) = \bar{\varepsilon}.$$

It follows that

$$\rho_{t+1}(\mu_t^c, \rho_t) < \bar{\varepsilon}$$

for any $\mu_t^c \in (0,1)$ and $\rho_t < \bar{\varepsilon}$. Next, observe that

$$\rho_{t+1}(0, \rho_t) = -\frac{\delta^2}{2-\delta}\rho_t^2 + \delta\frac{1-\delta-2\bar{\varepsilon}+4\delta\bar{\varepsilon}}{2-\delta}\rho_t + 2\bar{\varepsilon}(1-\delta)\,\frac{1-\delta+2\delta\bar{\varepsilon}}{2-\delta},$$

is quadratic and concave and

$$\rho_{t+1}(0, 0) = 2\bar{\varepsilon}(1-\delta)\,\frac{1-\delta+2\delta\bar{\varepsilon}}{2-\delta} > 0$$

and

$$\rho_{t+1}(0,\bar{\varepsilon}) = \bar{\varepsilon}(1-(1-\bar{\varepsilon})\delta) > 0$$

This implies that

$$\rho_{t+1}(\mu_t^c,\rho_t) > \rho_{t+1}(0,\rho_t) > 0$$

for all $\rho_t \in (0,\bar{\varepsilon})$, completing the proof. ■

The proof Proposition 1 proceeds in 3 steps: first, we show that, for any given $\bar{\varepsilon} > 0$, given the equilibrium firing rule, the investment strategies are optimal, second we show that, for any given $\bar{\varepsilon}$ smaller than a positive constant, given the equilibrium investment strategies, the firing rule is optimal, third we show that given the optimal investment strategies of managers, when $\bar{\varepsilon} \to 0$, the price of the bond must be given by (9).

Step 1. Suppose that managers follow the investment strategies (11), that is, follow their signal when there are informed and randomize when they are not. Then, given that $1/p_t > R$, it follows that the managers will always prefer to hire informed managers who never make "mistakes". Suppose investor j has hired at the beginning of time t a manager. At the end of the period the investor observes the investment realization $\tilde{\mu}_t^j$ and the productivity realization a_{t+1}. Then if $a_{t+1} \leq \hat{a}_{t+1}$ and $\tilde{\mu}_t^j = 1$, or $a_{t+1} > \hat{a}_{t+1}$ and $\tilde{\mu}_t^j = 0$, he realizes that the his manager is not informed, that is, $\eta_{t+1}^j = 0$, and fires him, given that there are no cost of firing, while there is always a positive probability that a new manager is informed, that is, $\rho_{t+1} > 0$, from the previous lemma. On the other hand, if the manager does not make a mistake, that is, if $a_{t+1} \leq \hat{a}_{t+1}$ and $\tilde{\mu}_t^j = 0$, or $a_{t+1} > \hat{a}_{t+1}$ and $\tilde{\mu}_t^j = 1$, then he does not fire him if and only if the updated belief on the manager η_{t+1}^j is higher than the probability that a new hire is informed, that is, $\eta_{t+1}^j \geq \varepsilon_{t+1}$. In this case,

$$\begin{aligned}
\eta_{t+1}^j &= \zeta(\eta_t^j, \tilde{\mu}_t^j, a_{t+1}) = \frac{\Pr(\tilde{\mu}_t^j = \chi_{t+1}(a_{t+1})\,|s_t \neq 0)\eta_t^j}{\Pr(\tilde{\mu}_t^j = \chi_{t+1}(a_{t+1})\,|s_t \neq 0)\eta_t^j + \Pr(\tilde{\mu}_t^j = \chi_{t+1}(a_{t+1})\,|s_t = 0)(1-\eta_t^j)} \\
&= \frac{\eta_t^j}{\eta_t^j + \mu_t^c(1-\eta_t^j)}
\end{aligned}$$

Notice that if the manager is a new hire, then $\eta_t^j = 2\bar{\varepsilon} - \rho_{t+1}$. In this case

$$\eta_{t+1}^j = \frac{2\bar{\varepsilon} - \rho_{t+1}}{2\bar{\varepsilon} - \rho_{t+1} + \mu_t^c(a_{t+1})\left(1 - \left(2\bar{\varepsilon} - \rho_{t+1}\right)\right)}.$$

Given that $\mu_t^c \in [0,1]$, then

$$\eta_{t+1}^j \geq 2\bar{\varepsilon} - \rho_{t+1} \geq \frac{2\bar{\varepsilon}(1-\delta) + \delta\rho_{t+1}}{2-\delta},$$

given that from previous lemma $\rho_{t+1} < \bar{\varepsilon}$, and hence $\eta^j_{t+1} \geq \varepsilon_{t+1}$. Moreover, notice that the investors' believes about any other manager who is working but was hired before time t must be higher than the one that has been hired at time t, given that if he was not fired he never made any mistake. Hence, a fortiori, $\eta^j_{t+1} \geq \varepsilon_{t+1}$, completing the proof.

Step 2. Suppose now that the investors follow the strategy (10).

1. We show that informed managers must follow their signal, that is, $\mu(d) = 0$ and $\mu(n) = 1$. Given that $\frac{1}{p_t} > R$, and investors fire fund managers who invest in the risky bond when $s_t = d$, or invest in the riskless asset when $s_t = n$, informed fund managers maximize both their monetary and their continuation value if they follow the proposed strategy.

2. We show that some uninformed managers must invest in the risky bond and some must invest in the riskless asset, that is, $\mu(0) \in (0,1)$ and none of them will have a portfolio with positive weights of both assets. The market clearing condition for the risky bond market can be written as

$$[(2\bar{\varepsilon} - \rho_t)\, 1_{s_t = n} + (1 - (2\bar{\varepsilon} - \rho_t))\, \mu_t(0)]\, \Gamma = B.$$

   Notice that assumption (1) implies that there must always be some uninformed managers that invest in the risky bonds and always some that do not. If no uninformed manager was investing in risky bonds, then there would be excess supply, since

$$(2\bar{\varepsilon} - \rho_t)\, \Gamma < \bar{B}$$

   when $\bar{\varepsilon}$ is sufficiently small. If instead all the uninformed managers were investing in the risky bonds, even if no uninformed was doing the same, there would be excess demand, since

$$(1 - (2\bar{\varepsilon} - \rho_t))\, \Gamma > \frac{1}{R} B \left(\frac{1}{R}\right) > \bar{B}$$

   when $\bar{\varepsilon}$ is sufficiently small. This implies that uninformed managers must be indifferent between investing all their capital in the risky bond or in the riskless asset. These strategies must also dominate any portfolios with positive weights of both assets because with such portfolio the manager would be fired for sure regardless of the pay-off of the risky bond. This completes the proof of step 2.

Step 3. Given steps 1 and 2, the uninformed manager has to be indifferent between investing in the risk-free asset or the risky bond. From the market clearing condition, let us define

$$x\left(\varepsilon_t, a_{t+1}\right) = \left\{ \begin{array}{cc} x_t^d \equiv \frac{pB(p)}{\Gamma(1-(2\bar{\varepsilon}-\rho_t))} & if \quad a_{t+1} < \hat{a}_{t+1} \\ x_t^n \equiv \frac{pB(p)}{\Gamma(1-(2\bar{\varepsilon}-\rho_t))} - \frac{(2\bar{\varepsilon}-\rho_t)}{(1-(2\bar{\varepsilon}-\rho_t))} & if \quad a_{t+1} \geq \hat{a}_{t+1} \end{array} \right\}$$

the equilibrium proportion of uninformed managers investing in the riskless bond if there is default and if there is no default. Given that $x_t^d > x_t^n$, if the uninformed manager has to invest in the risky bond (riskless asset) to clear the market, he will update the default probability upward (downward) as

$$\begin{aligned} \Pr\left(d|\tilde{\mu}_t^j = 1\right) &= \frac{\Pr\left(\tilde{\mu}_t^j = 1|d\right)\Pr\left(d\right)}{\Pr\left(\tilde{\mu}_t^j = 1|d\right)\Pr\left(d\right) + \Pr\left(\tilde{\mu}_t^j = 1|n\right)\Pr\left(n\right)} = \frac{x_t^d q}{x_t^d q + x_t^n\left(1-q\right)} > q \\ \Pr\left(d|\tilde{\mu}_t^j = 0\right) &= \frac{\left(1-x_t^d\right)q}{\left(1-x_t^d\right)q + \left(1-x_t^n\right)\left(1-q\right)} < q \end{aligned}$$

However, as $\bar{\varepsilon} \to 0$, $x_t^d \to x_t^n$ and $\Pr\left(d|\tilde{\mu}_t^j = 1\right), \Pr\left(d|\tilde{\mu}_t^j = 0\right) \to q$, so the information content of the equilibrium action diminishes. Thus, in the limit, given the investors firing rule, the indifference condition is

$$\left(1-q\right)\left(\gamma\frac{1}{p} + \delta W\left(0\right)\right) = \gamma R + q\delta W\left(0\right),$$

where

$$W\left(0\right) = \frac{\gamma R}{1-\delta q}.$$

The expression for $W\left(0\right)$ is the consequence of the fact that if managers are indifferent in each point in time whether to invest in the risky bond or the riskless asset, their value function is the value of investing always in the riskless asset. This completes the proof.

B.2 Proof of Lemma 1

The monotonicity of the left hand side of (16) implies the threshold (17). The assumption that $D\left(b\right) > \log\left(1 + bD'\left(b\right)\right)$ ensures that $\hat{a}$ is increasing in b.

B.3 Proof of Proposition 2

Using expression (19), we can rewrite the first order condition of the problem (18) as

$$\frac{p_t}{p_t b_{t+1} - F} - m\left(b_{t+1}\right) = 0. \tag{34}$$

By deriving with respect to b_{t+1}, we obtain the second order condition

$$-\left(\frac{p_t}{p_t b_{t+1}-F}\right)^2-\frac{\partial m\left(b_{t+1}\right)}{\partial b_{t+1}}<0.$$

When $D(\cdot)$ satisfies (2), the second order condition is immediately satisfied, completing the proof.

B.4 Proof of Lemma 2

1. First, notice that for a given a_t

$$\frac{dG\left(\hat{a}_{t+1}\right)}{dp_t}=\frac{dG\left(\hat{a}_{t+1}\right)}{\hat{a}_{t+1}}\frac{d\hat{a}_{t+1}}{b_{t+1}}\frac{db_{t+1}}{dp_t}. \tag{35}$$

For given a_t, let us define the function

$$\Phi\left(p_t,b_{t+1}\right)\equiv\frac{p_t}{p_t b_{t+1}-F}-m\left(b_{t+1}\right).$$

The first order condition (34) implies $\Phi\left(p_t,b_{t+1}\right)=0$. Applying the implicit function theorem, we obtain

$$\frac{db_{t+1}}{dp_t}=-\frac{\frac{\partial\Phi\left(p_t,b_{t+1}\right)}{\partial p_t}}{\frac{\partial\Phi\left(p_t,b_{t+1}\right)}{\partial b_{t+1}}}, \tag{36}$$

where

$$\frac{\partial\Phi\left(p_t,b_{t+1}\right)}{\partial p_t}=-\frac{p_t b_{t+1}}{\left(p_t b_{t+1}-1\right)^2},$$

and $\partial\Phi\left(p_t,b_{t+1}\right)/\partial b_{t+1}<0$ because it coincides with the second order condition of problem 4, which is satisfied by lemma 2. It follows that $db_{t+1}/dp_t<0$. Moreover, by differentiating (17), we get

$$\frac{d\hat{a}_{t+1}}{db_{t+1}}=\frac{1}{1-\exp\left\{-D\left(b_{t+1}\right)\right\}}\left[1-\frac{D'\left(b_{t+1}\right)b_{t+1}}{\exp\left\{D\left(b_{t+1}\right)\right\}-1}\right]$$

and the assumption that $D\left(b_{t+1}\right)>\log\left(1+b_{t+1}D'\left(b_{t+1}\right)\right)$ implies immediately that $d\hat{a}_{t+1}/db_{t+1}>0$. Hence, from (35), it follows that $dG\left(\hat{a}_{t+1}\right)/dp_{t+1}<0$, competing the proof of the first claim of the proposition.

2. Notice that

$$\frac{dp_t b_{t+1}}{dp_t}=b_{t+1}+p_t\frac{db_{t+1}}{dp_t},$$

where from (36)

$$\frac{db_{t+1}}{dp_t}=\frac{p_t b_{t+1}}{\left(p_t b_{t+1}-F\right)^2}\left[m\left(b_{t+1}\right)^2+\frac{\partial m\left(b_{t+1}\right)}{\partial b_{t+1}}\right]^{-1}.$$

After some algebra, we obtain

$$\frac{dp_t b_{t+1}}{dp_t} = b_{t+1}\left[1 + m\left(b_{t+1}\right)^2\left[m\left(b_{t+1}\right)^2 + \frac{\partial m\left(b_{t+1}\right)}{\partial b_{t+1}}\right]^{-1}\right]$$

Using condition (2), we see that $dp_t b_{t+1}/dp_t > 0$, completing the proof.

B.5 Proof of Lemma 3

With a slight abuse of notation, let us define $V(p)$ the expected utility of an active borrower as a function of p. Agent j will decide to become an entrepreneur as long as his outside option is smaller than the value of being an active entrepreneur, that is, $\bar{u}^j \leq V(p)$. From the envelope theorem

$$V'(p) = \frac{b}{pb - F} > 0,$$

and hence,

$$\frac{\partial H\left(V(p)\right)}{\partial p} = h\left(V(p)\right)\frac{b}{pb - F} > 0.$$

This also implies that aggregate output Y increases with p. Together with the second part of Lemma 2, this also implies that pB also increases with p, completing the proof.

B.6 Proof of Propositions 3 and 4

Proposition (3) is a simple consequence of Proposition (1), Lemma (1) and Proposition (2). Proposition (4) is a simple consequence of the frictionless pricing rule (8), Lemma (1) and Proposition (2)

B.7 Proof of Proposition 5

First note that the following lemma holds.

Lemma 5 *If the fixed investment cost increases, the default probability increases for any given bond price*

Proof. The equilibrium condition is

$$k(b,F) = \frac{p}{(pb - F)} - \int_{\hat{a}}^{\infty}\frac{1}{(a_{t+1} - b)}dG(a_{t+1}) - G(\hat{a})\frac{dD(b)}{db} = 0.$$

Using the implicit function theorem it is straightforward to show that

$$\frac{db}{dF} = -\frac{\frac{\partial k(b,F)}{\partial F}}{\frac{\partial k(b,F)}{\partial b}} > 0$$

given that

$$\frac{dk}{dF} = \frac{p}{(pb - F)^2} > 0$$

and $\partial g(b, F)/\partial b < 0$ by Lemma 2. Hence, from equation (17) we get that $d\hat{a}/dF > 0$ and hence $dG(\hat{a})/dF > 0$, completing the proof. ■

Note also that because of Assumption 2, the function $\frac{1}{b} - m(b)$ is monotonic, and because of our assumptions on $D(b)$,

$$\begin{aligned} \lim_{b \to 0} \left(\frac{1}{b} - m(b) \right) &= \infty \\ \lim_{b \to \infty} \left(\frac{1}{b} - m(b) \right) &< 0, \end{aligned}$$

hence, there is a unique b^{ex} which solves (23). Note that (23) is the limit of the first order condition (18) as $F \to 0$ for any p. Thus, as $F \to 0$, the probability of default for any p converges to $G(\hat{a}(b^{ex}))$ and the curve of the repayment rule becomes vertical in the (q, p) space.

Now, consider $p = \frac{1}{R}$. From the pricing rule, one can show that this price is required by the financial market, when the default probability $F(\hat{a})$ satisfies

$$q = \frac{1}{2\delta} \left(1 + 2\delta - \sqrt{1 + 4\delta} \right).$$

Thus, condition

$$G(\hat{a}(b^{ex})) > (1 + 2\delta - \sqrt{1 + 4\delta})/2\delta$$

, together with the facts that both the pricing rule and the repayment rule are monotonically decreasing and as $F \to 0$ the slope of the repayment rule decreases without bound ensure that there exists an F^{ex} that for any $F < F^{ex}$ when the default probability $G\left(\hat{a}\left(\frac{1}{R}\right)\right)$, the price required by the financial market is smaller than $\frac{1}{R}$. Furthermore, there will be a sufficiently small $\bar{p}$ that when the default probability is $G(\hat{a}(\bar{p}))$, the price required by the financial market is larger than $\bar{p}$. Thus, for any $F < F^{ex}$, there exists a fixed point (p, q) for the problem defined in Proposition 3.

B.8 Proof of Proposition 6

The pricing rule with career concerns crosses the pricing rule of the benchmark case at the $q = \frac{1}{2}$, $p = \frac{1}{2R}$ point. The conditions of the Proposition check whether for $p = \frac{1}{2R}$ the repayment rule is to the left or to the right from this intersect on Figure 3

B.9 Proof of Proposition 7

The first part of the Proposition is a consequence of the facts that (i) the repayment rule implies $p = 0$ if $q = 1$ for any R (ii) the slope of the pricing rule,

$$\frac{\partial p}{\partial q} = -\frac{1}{R}\frac{\delta(1-q)(1-\delta(1-q)) + (1-\delta q)}{(1-\delta(1-q))^2}$$

is monotonically increasing in R with $\lim_{R\to 0}\frac{\partial p}{\partial q} = -\infty$. The second part of the lemma is a consequence the definition of reputational premium and of Lemmas 2 and 3.

B.10 Proof of Proposition 8

From Proposition (existence) and its proof it must be clear that if the conditions in this Proposition hold and there is an $\tilde{a}$ that for any $\bar{a}'' \leq \tilde{a}\ \frac{1}{2} < G(\hat{a}(b^{ex})|\bar{a}'')$, then the statement of the Proposition holds. Thus, we only prove that such $\tilde{a}$ exists.

With a slight abuse of notation let us define $b(q,\bar{a}) = \hat{a}^{-1}\left(G^{-1}(b|\bar{a})\right)$, the bond holding consistent with a probability of default q at $\bar{a}$ where we used $q = G(\hat{a}(b)|\bar{a})$. From this, let us also define $q^{ex}(\bar{a})$ as the solution of $\frac{1}{b(q^{ex},\bar{a})} = m(b(q^{ex},\bar{a}))$. Observe that if $\Phi(\bar{a}, b(q^{ex},\bar{a})) = \frac{1}{b(q^{ex},\bar{a})} - m(b(q^{ex},\bar{a})) \equiv 0$ then

$$\frac{dq^{ex}(\bar{a})}{d\bar{a}} = -\frac{\frac{\partial\Phi(\bar{a},b(q,\bar{a}))}{\partial b}\frac{\partial b(q,\bar{a})}{\partial \bar{a}} + \frac{\partial\Phi(\bar{a},b(q,\bar{a}))}{\partial \bar{a}}}{\frac{\partial\Phi(\bar{a},b(q,\bar{a}))}{\partial b}\frac{\partial b(q,\bar{a})}{\partial q}}|_{q=q^{ex}}.$$

Assumption 2 implies that $\frac{\partial\Phi(\bar{a},b(q,\bar{a}))}{\partial b} < 0$. Assumption result $\frac{\partial \hat{a}}{\partial b} > 0$ implies that $\frac{\partial b(q,\bar{a})}{\partial q} > 0$ and assumption $\frac{\partial G(a|\bar{a})}{\partial \bar{a}} < 0$ and $\frac{\partial \hat{a}}{\partial b} > 0$ implies $\frac{\partial b(q,\bar{a})}{\partial \bar{a}} > 0$.

Note that

$$\frac{\partial\Phi(\bar{a}, b(q,\bar{a}))}{\partial\bar{a}} = \int_{\hat{a}(b(q,\bar{a}))}^{\infty}\frac{-1}{(a_{t+1} - b(q,\bar{a}))}\frac{\partial g(a_{t+1}|\bar{a})}{\partial\bar{a}}da_{t+1}.$$

As $\lim_{a_{t+1}\to 0} g(a_{t+1}|\bar{a}) = 0$ there must be an $\bar{a}'''$ that for any $\bar{a}'' < \bar{a}'''$ $\frac{\partial g(a_{t+1}|\bar{a})}{\partial\bar{a}} \leq 0$ for all $a_{t+1} > \hat{a}$. Thus, if $\bar{a}'' < \bar{a}'''$,$\frac{\partial\Phi(\bar{a},b(q,\bar{a}))}{\partial\bar{a}}|_{\bar{a}=\bar{a}''} \leq 0$ and

$$\frac{dq^{ex}(\bar{a})}{d\bar{a}} = -\frac{\frac{\partial\Phi(\bar{a},b(q,\bar{a}))}{\partial b}\frac{\partial b(q,\bar{a})}{\partial \bar{a}} + \frac{\partial\Phi(\bar{a},b(q,\bar{a}))}{\partial \bar{a}}}{\frac{\partial\Phi(\bar{a},b(q,\bar{a}))}{\partial b}\frac{\partial b(q,\bar{a})}{\partial q}}|\bar{a} = \bar{a}'' < 0.$$

Given that $\lim_{\bar{a}\to-\infty} G(a_{t+1}|\bar{a}) = 1$ for any positive a_{t+1}, $\lim_{\bar{a}\to-\infty} q^{ex} = 1$ must hold also. Thus, an $\tilde{a}$ that for any $\bar{a}'' \leq \tilde{a}\ \frac{1}{2} < G(\hat{a}(b^{ex})|\bar{a}'')$ must exist. This concludes the first part of the proof.

The second part of the lemma is a consequence the definition of reputational premium and of Lemmas 2 and 3.

B.11 Proof of Propositions 9

The proof is virtually identical to the proof of Proposition 3 with the following differences.

1. In borrowers' first order condition, (18), $G(a_{t+1})$ must be replaced by $G(a_{t+1}|a_t)$.This implies expressions (27) and (26). Thus, the probability of default, $G(\hat{a}(a_t)|a_t)$, will depend on the realization of past shocks.

2. If the probability of default depends on past shocks, then the value of being an uninformed hired manager also depends on past shocks. Consequently, the indifference condition of uninformed managers, (13), is replaced by

$$(1-q_t)\left(\gamma\frac{1}{p(a_t)}+\delta\int_{\hat{a}(a_t)}^{\infty}W(0,a_{t+1})\,dG(a_{t+1}|a_t)\right)=\gamma R+q\delta\int_0^{\hat{a}(a_t)}W(0,a_{t+1})\,dG(a_{t+1}|a_t). \tag{37}$$

   Both sides of this expression show the uninformed manager's value of being hired conditional on a_t. The right hand side implies expression (25) which replaces (14). Substituting (25) and the new repayment rule into (37) gives (24).

Thus, if $(p^*(\cdot),\hat{a}^*(\cdot))$ is the fixed point of the system described in the Proposition, then uninformed managers are indifferent whether to invest in the risky bond or the riskless asset and each borrower optimize her value function for the given price. For the proposed firing rule and $(p^*(\cdot),\hat{a}^*(\cdot))$, informed managers prefer to follow their signals, as in each period this provides both a larger monetary gain and a larger probability of being rehired. Given the strategies of managers, investors' firing rule is optimal as it is shown in Proposition (1).

B.12 Proof of Proposition 11

The proof is analogous to the proof of Proposition 9 (which builds on the results of Proposition 1) with the following modifications. We use the same notation as in previous proofs with the superscript $r=A,B$ to distinguish variables referring to different groups of borrowers.

1. We have to show that if informed managers follow the proposed strategies then investors also follow the proposed strategies (analogously to Step 1 in the proof of Proposition 1). At the end of period t the investor j observes the investment realization $\tilde{\mu}_t^{j,A},\tilde{\mu}_t^{j,B}$ and (from the pay-off on bonds) can infer whether the productivity realization a_{t+1}^r is smaller or larger than the threshold $\hat{a}\left(a_{t+1}^r\right)$. Then if $a_{t+1}^r\leq\hat{a}\left(a_{t+1}^r\right)$ and $\tilde{\mu}_t^{j,r}=1$, or

$a^r_{t+1} > \hat{a}\left(a^r_{t+1}\right)$ and $\tilde{\mu}^j_t = 0$, he fires the manager as the manager cannot be informed, i.e., $\eta^j_{t+1} = 0$, and $\rho_{t+1} > 0$.Similarly, if $\tilde{\mu}^{j,r}_t = 1$ and $\tilde{\mu}^{j,r'}_t = 0$ and $a^r_{t+1} > \hat{a}\left(a^r_{t+1}\right)$ and $a^{r'}_{t+1} \leq \hat{a}\left(a^{r'}_{t+1}\right)$, then the manager is kept as $\eta^j_{t+1} \geq \varepsilon_{t+1}$ by the same argument as in Step 1 of the proof of Proposition 1. We still have to show that if $\tilde{\mu}^{j,r}_t = 1$ and $\tilde{\mu}^{j,r'}_t = 0$ and $a^r_{t+1} > \hat{a}\left(a^r_{t+1}\right)$ and $a^{r'}_{t+1} > \hat{a}\left(a^{r'}_{t+1}\right)$, that is, none of the risky bonds default and the manager invest in one of them, the manager is kept as $\eta^j_{t+1} \geq \varepsilon_{t+1}$. For this observe that if the manager is a new hire, then $\eta^j_t = 2\bar{\varepsilon} - \rho_{t+1}$ and in this case

$$
\begin{aligned}
\eta^j_{t+1} &= \frac{\frac{1}{2}\left(2\bar{\varepsilon} - \rho_{t+1}\right)}{\frac{1}{2}\left(2\bar{\varepsilon} - \rho_{t+1}\right) + \mu^c_t\left(a_{t+1}\right)\left(1 - \left(2\bar{\varepsilon} - \rho_{t+1}\right)\right)} = \frac{\frac{1}{2}\left(2\bar{\varepsilon} - \rho_{t+1}\right)}{\frac{p^r B(p^r)}{\Gamma}} > \\
&> \left(2\bar{\varepsilon} - \rho_{t+1}\right) > \frac{2\bar{\varepsilon}\left(1-\delta\right) + \delta\rho_{t+1}}{2-\delta} > \varepsilon_{t+1}.
\end{aligned}
$$

where we used that the proportion of uninformed manager investing in asset r must be $\frac{p^r B(p^r)}{\Gamma\left(1-\left(2\bar{\varepsilon}-\rho_{t+1}\right)\right)} - \frac{\frac{1}{2}\left(2\bar{\varepsilon}-\rho_{t+1}\right)}{\left(1-\left(2\bar{\varepsilon}-\rho_{t+1}\right)\right)}$ by market clearing and Assumption 3 and that $\rho_{t+1} < \bar{\varepsilon}$. By the same argument as before, $\eta^j_{t+1} \geq \varepsilon_{t+1}$ must hold for every manager who has been hired for many periods.

2. Under the equilibrium strategies, the indifference condition (37) is replaced by

$$
\begin{aligned}
&\left(1 - G\left(\hat{a}\left(a^A_{t+1}\right)\right)\right)\gamma\frac{1}{p\left(a^A_t\right)} + \delta\int_{\hat{a}\left(a^A_t\right)}^{\infty}\int_0^{\infty} W\left(0, a^A_{t+1}, a^B_{t+1}\right) dG(a^B_{t+1}|a^B_t)dG(a^A_{t+1}|a^A_t) = \\
&\qquad = \gamma R + \delta\int_0^{\hat{a}\left(a^A_t\right)}\int_0^{\hat{a}\left(a^B_t\right)} W\left(0, a^A_{t+1}, a^B_{t+1}\right) dG(a^B_{t+1}|a^B_t)dG(a^A_{t+1}|a^A_t).
\end{aligned}
$$

and the analogous condition for asset B. The right hand side of this expression implies (29) which replaces (25). Substituting (29) into (32) implies (28).