CDS as Insurance: Leaky Lifeboats in Stormy Seas

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Abstract

In this paper we update the traditional insurance economics framework to incorporate key features of the credit default swap (CDS) market. First, we allow for insurer insolvency, with asymmetric information as to its probability. We find that stable insurers become less stable because they are forced to compete on price. When insurer type is known, increased competition among insurers can create instability for the same reason. Second, we allow the insured party to have heterogeneous motivations for purchasing CDS. For example, some may own the underlying asset and purchase CDS for risk management, while others buy these contracts purely for speculation. We show that speculators will choose to contract with less stable insurers, resulting in higher counterparty risk in this market relative to that of traditional insurance; however, a regulatory policy that disallows speculative trading can, perversely, cause market counterparty risk to increase. Third, we relax the standard assumption of contract exclusivity, which does not apply to the CDS market, by allowing the insured to purchase contracts from many insurers. In contrast to the traditional insurance model, we show that separation of risk type among insured parties can be achieved through insurer choice. We use our model to shed light on the debate over Central Counterparties (CCP). We show that requiring CDS contracts to be negotiated through CCPs can push stable insurers out of the market, mitigating the benefit of risk pooling.

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1 Introduction

Credit Default Swaps (CDS) have received considerable media attention since the beginning of the credit crisis in 2007.[1] There was public outrage over the use of U.S. tax payer money to pay (in full) the CDS claims that sellers, such as AIG, had sold to many major banks. In response to this and other episodes, policy-makers have been under pressure to implement regulatory reforms. For example, in 2008 the State of New York tabled legislation to have CDS sellers classified and regulated as insurers.[2] However, it is not clear whether sellers are an insurance provider, or simply a party to a derivative contract as in any other options market. While these issues are of widespread interest, the discourse on CDS is lacking in theoretical perspective. Our analysis provides a simple framework which elucidates key differences between these contracts and traditional insurance, and the consequences of these differences on market outcomes.

This paper updates the traditional insurance economics framework to account for features unique to the CDS market. We capture the pervasiveness of insurer instability by allowing for the possibility that sellers of CDS contracts become insolvent. We then incorporate three features into a standard model of insurance: First, we introduce privately observed heterogeneity of insurer quality to capture the opacity of the CDS market. With large sellers such as Ambac, MBIA and AIG suddenly and repeatedly downgraded by rating agencies, it would seem prudent to allow for this feature. Second, we consider insured parties who can differ on their motivation to insure. In particular, unlike traditional insurance markets, the CDS market is characterized by buyers who may or may not own the underlying risk. Furthermore, the number of buyers that use CDS purely for speculative purposes is roughly equal to those who use them for risk management (Fitch 2009, 2010). Finally, we allow an insured party to purchase protection on the same risk from multiple insurers. Similar to the market for life insurance, where an insured party can purchase policies from multiple insurers, there are few if any actions that a CDS seller can take to prevent the buyer from purchasing more protection elsewhere. We then apply the model to analyze the consequences of creating a central counterparty. This is a particularly relevant issue given the Dodd-Frank bill in the U.S., which requires a large portion of CDS trades to go through clearinghouses.[3]

We find that unstable insurers (i.e., those who are more likely to fail) can exist in equilibrium: either they are able to offer a sufficiently discounted price for the protection the provide, or they are able to camouflage themselves in an opaque market. We show that insurer specific and market counterparty risk (which is defined as the expected probability of insurer default) can endogenously

[1] In a credit default swap, an insurer agrees to cover the losses of the insured if pre-defined credit events (e.g., default) happen to some debt instrument. In exchange, the insured agrees to pay an ongoing premium at fixed intervals for the life of the contract. A CDS written on the debt of a single company is typically bought and sold through a dealer. When the underlying debt is more complicated (and so requires a non-standard contract), the CDS is completed directly between the two parties. For example, the CDS contracts that destabilized AIG were mainly direct contracts with major banks, written on complex mortgage related securities. The estimated notional size of the CDS market in 1998 was 180 billion dollars, by 2004 this number had grown to 6 trillion, and by the end of 2008 it was 41 trillion dollars (Stulz 2009). Note that this is a notional amount and no doubt overestimates the absolute economic value of all contracts, but the relative growth has been rapid.

[2] http://ins.state.ny.us/circltr/2008/cl08_19.htm

[3] Europe appears to be moving in a similar direction with the European Market Infrastructure Regulation (EMIR).

increase as competition increases among insurers, or when insurer quality is unknown. In addition, we show that when the proportion of buyers that use CDS for trading purposes increases, relative to those using them for risk management, more contracts will be written with unstable insurers. However, removing speculators from the market can decrease, or perversely increase market counterparty risk, depending on the nature of competition between insurers. When insured parties can differ according to their privately known risk type, we show that they may choose to separate on the choice of insurer, even when they can divide their contract among as many insurers as they wish. This is in contrast to a standard result in traditional insurance economics, which proposes that when insurers cannot preclude insured parties from purchasing insurance elsewhere, separation through a market mechanism cannot be achieved.[4] This separation however, does not occur when contracts must flow through a central counterparty (CCP). More importantly, we show that a CCP can force stable insurers out of the market.

The intuition behind our results is as follows. The insured party can choose to contract with a stable ('good') insurer, or with an unstable ('bad') insurer. The choice of insurer boils down to a tradeoff between the price (premium) and the degree of exposure to counterparty risk (probability of insurer insolvency). In our model, the bad insurer makes an investment that earns high returns but is illiquid and so cannot be used to help pay claims. The good insurer makes an investment that earns a lower return, but is a liquid asset, and so can improve its chances of being solvent when a claim is made. We assume that the return on the bad insurer's investment is sufficiently high that they are able to charge lower premia than the good insurer. In Section 3, we show that the resulting equilibrium can have good or bad insurers dominate the market. When the insured party is sufficiently averse to counterparty risk, the bad insurer will not be able to cut its premium enough to attract the insured party. In this case, only the good insurer exists, and it can extract positive profits. When the insured party has little aversion to counterparty risk, the bad insurer will control the market as insured parties become premium driven, rather than counterparty risk driven. In Section 3.2, we show that as insurer competition increases, the good insurer may be forced to compete on premium against new entrants, driving profits down and counterparty risk up. This result is similar in spirit to the banking literature that shows that when competition among banks increases, stability of the system can decrease. One mechanism that drives this result is a bank taking on a riskier portfolio as competition increases (see Boyd and De Nicoló (2005) or Vives (2010) for a summary of this literature). In contrast, our insurer's investment selection may remain the same, however competition gives them less resources to invest. Returning to the case of two insurers, we show in Section 3.3 how the equilibrium changes when the insured party does not know

[4]One exception is Rothschild (2009), who uses insurance on multiple contingencies to achieve separation. It is possible that a CDS contract specifies more than one contingency (e.g., default or restructuring); however, Rothschild (2009) requires that the insured parties have relative risks on contingencies that reverse (e.g., type 1 is more likely to default, whereas type 2 is more likely to restructure). Further, when markets are voluntary (as we consider), the results of that paper depend on the insured having a CRRA utility function. Since contingencies contained in a CDS contract are typically related to poor performance of the underlying, we cannot assume that relative risks of such triggers reverse depending on the insured party. Instead, we make the usual assumption that there is just one risk insured, and we do not require a CRRA utility function.

the insurer's quality. In particular, the good insurer can no longer drive out the bad insurer. This is because the bad insurer can simply charge the same premium as the good insurer and the insured cannot distinguish between the two. Competition between insurers then drives the equilibrium premium down to where the good insurer earns zero profit, thereby increasing counterparty risk.

In Section 4, we let there be insured parties who differ only on their aversion to counterparty risk, which we associate with having different motivations for using CDS. Recall that the two key factors in the choice of insurer are counterparty risk and premium. Those insured parties who use CDS purely for trading purposes, and perhaps do not even own the underlying asset (i.e., have no insurable interest), are more likely premium driven. On the other hand, buyers who use CDS for risk management would internalize the counterparty risk more, and would be willing to pay relatively more for stable protection. Traditional insurance markets are usually viewed as having risk averse insured parties. The analogue to CDS would be a market composed entirely of buyers using the contracts for risk management purposes. As more participants use CDS purely for speculation, we find that the market will be serviced more by unstable insurers. This is because the speculators prefer the lower premium that unstable insurers can offer. Simply removing speculators from the market may not solve the problem. Although such a policy can reduce the number of unstable insurers in the market, it can also have the perverse effect of making the otherwise stable insurers riskier. This is because removing buyers from the market creates more competition among the sellers. As in the competition result described above, this can drive down premia and increase counterparty risk.

In Section 5, we extend the model to allow for heterogeneity in the risk of insured parties (i.e., the risk of the underlying asset), as is standard in the insurance economics literature. We characterize an equilibrium in which the choice of insurer can separate the insured party types. Specifically, a relatively safe insured party contracts with a bad insurer to economize on premiums paid, whereas a riskier insured party contracts with a good insurer, at a higher premium, for fear of counterparty risk. In Appendix B, we show that this result is robust to a relaxation of the assumption of exclusivity, i.e., it holds even when insured parties can contract with more than one insurer. We permit a large number of insurers, split between good and bad types. Further, we assume that both insurer types are subject to idiosyncratic default risk; however, bad insurers are also subject to aggregate risk, such as an extreme market downturn that affects all these insurers at once. We show that there exists an equilibrium in which the safe insured party divides its contract over all of the bad insurers, whereas the risky insured party divides over the good insurers. The intuition for this result is the same for the two insurer case described above. The nature of the aggregate risk allows us to achieve separation without the assumption of exclusivity, because the safety of good insurers cannot be replicated by bad ones.

Finally, in Section 6 we consider the consequences of a central counterparty. A CCP acts as the buyer to every seller and the seller to every buyer. Participants in this market contribute to a fund designed to shelter each other from counterparty risk. Pirrong (2009) reports that few CCPs penalize sellers based on their counterparty risk. Given that the counterparty risk to which an insured party is exposed is now that of the entire pool of insurers (through co-insurance), differential premia based on insurer quality are not feasible. We consider the case in which there are a large number of insured parties and insurers. Given a CCP arrangement, counterparty risk is effectively pooled so that non-failing participants can absorb the losses of the failed ones. Therefore, insuring with a good insurer has little effect on the exposure of the insured to counterparty risk. Consequently, the insured party will contract with the bad insurer to obtain a better premium. Thus, the separating equilibrium described above ceases to exist. More importantly, insured parties will choose to insure solely with bad insurers so that in equilibrium, good insurers are pushed out of the market. This occurs because each individual insured party does not internalize the amount that their contract adds to the pooled counterparty risk, yielding an outcome similar in spirit to the problem of the commons. It is interesting to note that in this case, central organization is the cause and not the cure for this outcome. This result can also be interpreted as an example of the Lucas Critique, in that policy makers implementing a CCP should consider the reaction of market participants.

Literature Review

This paper contributes to the literature on counterparty risk, credit default swaps and insurance. Thompson (2010) considers a case with endogenous counterparty risk in financial insurance. It is shown that an insurer has a moral hazard problem and may not invest in the best interest of the insured party. Furthermore, it is shown that truthful revelation of insured type can be attained because revelation affects the investment decision of the insurer, and consequently, the counterparty risk to which the insured is exposed. In contrast to this paper, we explicitly model multiple insurers and so can analyze the composition of insurers in the market. In another related paper, Acharya and Bisin (2010) show that due to the opacity of over-the-counter markets (where many CDS trade), counterparty risk can occur because insurers may take positions which increase their likelihood of default. In contrast, we model a situation in which insurers have varying degrees of stability and show that, regardless of whether CDS markets are opaque, unstable insurance can be a feature of the equilibrium. Neither Thompson (2010) nor Acharya and Bisin (2010) analyze the affects of competition among insurers, the motivation to purchase CDS, the affects of mutual exclusion of contracts, or the commons problem that arises with a CCP, as is done in this paper.

In the insurance economics literature, Ligon and Thistle (2009) provide a model in which mutual and stock insurers can co-exist. They show that mutual insurers may exist as a means for low-risk individuals to separate themselves. This parallels one of our results, wherein unstable insurers may exist to separate the market. Separation exists in their model because low-risk individuals can form a sufficiently small mutual wherein lower expected coverage relative to a stock insurer keeps the high-risk individuals out. In contrast, we consider two stock insurers with different levels of counterparty risk. Cummins and Mahul (2003) determine the optimal indemnity (contract size) in the case where the insurer and insured party have different beliefs about the probability that the insurer will fail. In contrast, we assume that the insurer has better information about its portfolio and so an asymmetric information problem arises. Further, the analysis of optimal indemnity is not relevant in our context due to the inability of insured parties in the CDS market to separate on ex-ante contract size (due to the non-exclusivity of contracts).

The paper proceeds as follows: Section 2 outlines the model. Section 3 considers the case when insurers are known, when there is increased competition among them, and when they are unknown. Section 4 allows insured parties to differ based on their motivation to purchase CDS. Section 5 analyzes the case in which there are multiple insurers, and in which the risk that is being insured is unknown to those insurers. Section 6 explores the consequences of a central counterparty and Section 7 concludes. Robustness Section 8 provides a discussion of a number of our assumptions and nontrivial proofs can be found in Appendix A.

2 Model: CDS as insurance

This section describes the market for insurance and its participants. The purchaser, whom we refer to as a bank, owns a risky asset which it wishes to insure. We refer to this asset as a loan. We will not model anything unique to a bank, however as banks are the largest purchasers of these types of contracts, we use this terminology for ease of exposition. The providers of insurance are simply referred to as insurers.

2.1 Banks

The fundamental characteristic of a bank is the desire to reduce risk. As in Thompson (2010), if the bank incurs a loss and has not insured this risk, it suffers the cost $Z \geq 0$.[5] If the bank has a loss for which it is insured, but the insurer cannot pay, it also suffers the cost Z. This cost could represent a regulatory penalty for exceeding some risk level, or an endogenous reaction to a shock to the bank's portfolio; however, we will not model this here. It is this cost that makes the bank averse to holding risk.

The bank's loan yields return R_B with probability p, otherwise it defaults with probability $1-p$ and returns nothing. The size of the loan is normalized to 1 and we assume that the bank must insure the full loan.[6] Therefore, in the event of a claim that is fulfilled, the bank will receive 1. In the event that a claim cannot be fulfilled, the bank is penalized Z. Denoting the premium (price)

[5] See robustness Section 8.2 for a more formal discussion about Z.

[6] This is done for simplicity. One could imagine that the bank chooses different amounts of insurance depending on the insurer type (note that different insurer types will be discussed in the next subsection). An endogenous contract size will not affect the qualitative results to be presented. Note also that we are ignoring a potential moral hazard, wherein the bank may lose the incentive to monitor its loan when it completely insures. We assume full insurance for simplicity, but all the results of the paper would go through if we assumed the bank insured only a fraction of its loan. That fraction could then be set such that moral hazard is eliminated. Even in the presence of a moral hazard problem, the only difference in our model would be that either p would decrease, R_B would decrease, or both. The insurer would simply alter its beliefs about the expected cost of a claim and the results of the model would follow through. For a more formal treatment of this moral hazard problem, see Bolton and Oehmke (2010), Parlour and Winton (2009) or Thompson (2007).

as P, and the probability that the insurer is solvent as q, the bank's expected payoff is

$$pR_B + (1-p)q - (1-p)(1-q)Z - P. \tag{1}$$

Note that in the event of a claim, the insurer fails with probability $1-q$ and for simplicity, pays nothing to the bank.[7] In other words, the bank is never fully insured against the loss provided $q < 1$.

2.2 Insurers

We allow for the possibility of insurer insolvency (with probability $1-q$), which we refer to as counterparty risk. Importantly, we allow the probability of insurer insolvency to be heterogeneous across insurers. This represents our first departure from the literature. We model this heterogeneity by considering two insurance providers, one relatively stable and the other unstable, referred to simply as "(G)ood" and "(B)ad" insurers.

Both insurers have an exogenous portfolio of assets represented by θ, that pays off at $t=1$. The portfolio consists of a draw from the distribution function $F(\theta)$, in which $\theta \in [\underline{\theta}, \overline{\theta}]$ and $\underline{\theta} < 0 < \overline{\theta}$. If an insurer does not sell an insurance contract, it is assumed to fail when its portfolio draw is between $[\underline{\theta}, 0]$. When it sells an insurance contract, the good insurer invests the premium it receives in a risk-free asset with return normalized to one, which is available at $t=1$. The bad insurer invests in a more profitable, but illiquid asset which has a rate of return $r > 1$, and is received at $t=2$. In other words, the bad insurer makes an investment that has no pledgable value at $t=1$.[8] Insurer $j \in \{G, B\}$, forms a belief b_j corresponding to the probability that a claim will not be made (when the underlying loan does not default). In our discussion thus far, we have assumed that there is only one type of bank, therefore the beliefs are $b_G = b_B = p$. In Section 5, we allow for heterogeneity across bank types, and so beliefs will be less trivial. Denoting the premium charged by insurer j by P_j, we write the payoff function for each insurer when they insure a loan of size 1. Note that insurer default is assumed to result in zero profit (i.e., limited liability).

$$\begin{aligned}
\pi_G &= p\left[\int_{-P_G}^{\overline{\theta}} (\theta + P_G)\, dF(\theta)\right] + (1-p)\left[\int_{(1-P_G)}^{\overline{\theta}} (\theta - 1 + P_G)\, dF(\theta)\right] && (2)\\
\pi_B &= p\left[\int_{0}^{\overline{\theta}} (\theta + rP_B)\, dF(\theta)\right] + (1-p)\left[\int_{1}^{\overline{\theta}} (\theta - 1 + rP_B)\, dF(\theta)\right] && (3)
\end{aligned}$$

Examining the limits of integration in the second terms of expressions (2) and (3), we can characterize the counterparty risk for each insurer. The probability that the good or bad insurer is solvent when a claim is made, is given by $q_G = 1 - F(1-P_G)$ and $q_B = 1 - F(1)$ respectively. Importantly, note that the bad insurer's probability of default is independent of the premium. This is not the

[7] A zero recovery value is assumed for simplicity and could be relaxed without changing the results.

[8] Note that for simplicity, we are implicitly assuming that insurers invest in different assets for reasons outside the model. In Robustness Section 8.1, we detail how the model could be modified to allow the investment decision to be endogenous.

case with the good insurer, since the probability of defaulting on a claim depends on the premium P_G. It is straightforward to see that if the premium increases, counterparty risk decreases for the good insurer, but remains constant for the bad insurer. This occurs because the good insurer uses the premium to improve the chances of solvency at $t = 1$, whereas the premium has no effect on the bad insurer's chances of being solvent at $t = 1$. It follows that $q_G > q_B$ whenever the premium is positive.

In the following lemma, we characterize a property of the premia which we use throughout the paper. First, define the zero profit premia P_G^0 and P_B^0, as the premium for which the good and bad insurer earn zero profit from selling a contract respectively.

Lemma 1 *There exists a return r^*, such that for all $r > r^*$, $P_G^0 > P_B^0$.*

Proof. *See Appendix A.*

The bad insurer invests the premium it receives from the insurance contract in an illiquid asset and earns a return r. It follows that if r increases, it would require a lower premium to attain the same expected payoff. For the remainder of the paper we assume that $r > r^*$ so that $P_G^0 > P_B^0$. In other words, the bad insurer is able to offer lower premia due to its favorable investment opportunity. If this were not true, the good insurer could offer the bank a lower premium *and* lower counterparty risk; trivially excluding the bad insurer from the market.

2.3 Timing

There are three time periods, in which we assume there is no discounting. At $t = 0$, an insurance contract is written by an insurer on a risky loan owned by the bank. At $t = 1$, the uncertainty about the insured loan and the insurer's portfolio is resolved. In this period, a claim is made if the loan defaults. The insurer either fulfils the claim if it is solvent, otherwise it fails and returns nothing to the bank. At $t = 2$, the payoff to the two period insurer investment is received. Figure 1 summarizes.

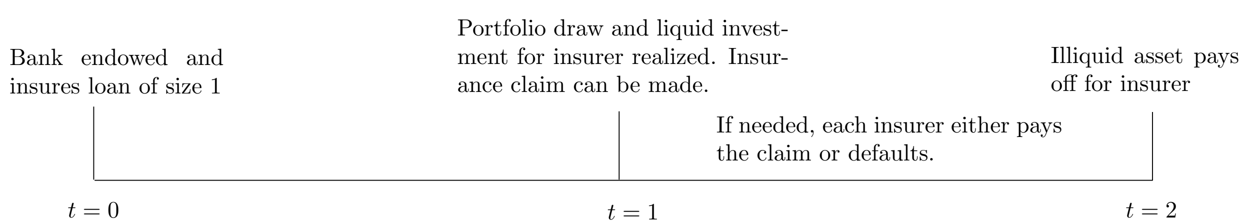

Figure 1: Timing of the Model

3 Equilibrium

3.1 Insurer type known

We begin by assuming that the bank can identify the insurer type. There are two outcomes in this simple insurance market. Either the good or the bad insurer dominates the market, and provides insurance for the bank.[9] Modifying expression (1), we give the bank's payoff function when insuring with insurer type j.

$$\Pi(j) = pR_B + (1-p)q_j - (1-p)(1-q_j)Z - P_j \tag{4}$$

We assume without loss of generality that a bank which is indifferent between contracting with a good and bad insurer opts for the former. Therefore, the good insurer will dominate the market when $\Pi(G) \geq \Pi(B)$. Similarly, the bad insurer will dominate the market when $\Pi(B) > \Pi(G)$.[10] The following lemma summarizes these equilibria.

Lemma 2 *There exist two equilibria in the market for insurance described above, which are characterized as follows:*

i. The good insurer provides insurance when

$$(1-p)(1+Z)(q_G - q_B) \geq P_G^0 - P_B^0, \tag{5}$$

where the equilibrium premium is $P_G^* = (1-p)(1+Z)(q_G - q_B) + P_B^0 \geq P_G^0$.

ii. The bad insurer provides insurance when

$$(1-p)(1+Z)(q_G - q_B) < P_G^0 - P_B^0, \tag{6}$$

where the equilibrium premium is $P_B^* = P_G^0 - (1-p)(1+Z)(q_G - q_B) - \epsilon \geq P_B^0$, *for* ϵ *small.*

Proof. *See Appendix A.*

The good insurer will dominate the market when the benefit of reduced counterparty risk, the left hand side of expression (5), more than compensates for the additional premia that the banks must pay, the right hand side of expression (5). It is straightforward to see that this will be true for large values of Z. Conversely, the bad insurer dominates the market when the premium discount it can offer exceeds the cost of the additional counterparty risk it poses to the banks. Regardless of

[9] Note that we are implicitly assuming that the bank does not split its contract over the two insurers. Allowing this would only complicate the analysis and would not change our qualitative results. One can imagine a transaction cost which induces this behavior. Alternatively, a straightforward restriction on the parameter space will also accomplish this. We explore the possibility of diversification by splitting the contract over many insurers in Section 5.

[10] Note that we are implicitly assuming that the bank chooses to purchase insurance (the bank's participation constraint is slack). The participation constraint discussed in Robustness Section 8.2.

which insurer dominates, premia are set just low enough to force the competitor out of the market and yield weakly positive profits to the remaining insurer.

In the remainder of this section, we describe forces which change the features of the equilibrium described above, with a focus on stability (measured by the probability of insurer default). Lemma 2 outlines two possible benchmarks from which this analysis could proceed. Presumably, the most interesting cases are those in which the counterparty risk posed by insurers increases. As we wish to highlight this phenomena, we present the results of this section using the case where the good insurer dominates (as per Lemma 2) as a benchmark; however, we will discuss the results in the alternative case where the bad insurer initially dominates the market.

3.2 Competition

We now analyze how competition affects the equilibrium outlined previously. In addition to the good and bad insurers, let us consider an insurer type $j = M$ (middle), which has the same initial portfolio. Assume that this insurer invests half of its premia in the liquid asset, and half in the illiquid asset. The payoff of the new insurer is given as follows.

$$\begin{aligned}\pi_M &= p\left[\int_{-\frac{1}{2}P_M}^{\overline{\theta}}\left(\theta + \frac{1}{2}P_M(1+r)\right)dF(\theta)\right] \\ &\quad + (1-p)\left[\int_{(1-\frac{1}{2}P_M)}^{\overline{\theta}}\left(\theta - 1 + \frac{1}{2}P_M(1+r)\right)dF(\theta)\right] \qquad (7)\end{aligned}$$

It follows that in the event of a claim, the probability of solvency of insurer type M is given by $q_M = 1 - F(1 - \frac{1}{2}P_M)$, where $q_B < q_M < q_G$. A straightforward extension of Lemma 1 yields $P_G^0 > P_M^0 > P_B^0$, so that the zero profit premium of the middle type is between that of the good and bad insurers. Consider the case in which expression (5) holds so that the bank chooses to insure with the good over the bad insurer. We define market counterparty risk as the expected counterparty risk to which the bank is exposed. The following proposition shows that market counterparty risk weakly increases as competition increases.

Proposition 1 *When the good insurer dominates the market (as per Lemma 2), increased competition causes market counterparty risk to increase. This occurs regardless of which one of two possible equilibria arise. In the first case, the good insurer continues to dominate, which occurs when*

$$(1-p)(1+Z)(q_G - q_M) \geq P_G^0 - P_M^0, \qquad (8)$$

where the equilibrium premium is $P_G^* = (1-p)(1+Z)(q_G - q_M) + P_M^0 \geq P_G^0$*. Alternatively, the middle insurer may dominate, which occurs when*

$$(1-p)(1+Z)(q_G - q_M) < P_G^0 - P_M^0, \qquad (9)$$

where the equilibrium premium is $P_M^* = P_G^0 - (1-p)(1+Z)(q_G - q_M) - \epsilon \geq P_M^0$*, for* ϵ *small.*

These equilibria can be shown to exist in the same way as outlined in the proof of Lemma 2. To gain the intuition behind this result, we consider both situations described in the proposition. Initially, the good insurer provides coverage as described in Lemma 2. Thus the premium and associated counterparty risk are implied by the equality of (5), which we denote $\widehat{P_G}$ and $\widehat{q_G} = 1 - F(1 - \widehat{P_G})$. First, consider the situation in which the good insurer continues to dominate when another competitor is introduced. Define the equilibrium premium charged by the good insurer when the middle insurer is introduced by $\widetilde{P_G}$, which in turn defines $\widetilde{q_G} = 1 - F(1 - \widetilde{P_G})$. The premium and counterparty risk in this case are defined by the equality of (8). Contrasting this with the equilibrium with no middle insurer, it follows that $\widehat{P_G} \geq \widetilde{P_G}$. In other words, the new competitor forces the good insurer to lower its premium, which results in an increase in counterparty risk since $\widehat{q_G} \geq \widetilde{q_G}$. Alternatively, the new competitor may take the market, which occurs when (9) is satisfied. Since $q_M < q_G$, market counterparty risk is always higher in this case.[11]

We can also consider the case when the bad insurer dominates the good insurer initially (expression (6) is satisfied). When this occurs, there will be two possible outcomes when the middle insurer is added. First, the bad insurer dominates the middle insurer and counterparty risk remains unchanged. Although the middle insurer may force the bad insurer to cut its premium, recall that $1 - q_B = F(1)$, so that the risk of insolvency of the bad insurer is independent of the premium. In the second case, the middle insurer will dominate the bad insurer and market counterparty risk will decrease.

3.3 Unknown Insurer

We now return to the case of two insurers (good and bad) and consider the consequences of asymmetric information regarding the quality of the insurance provider. In Section 3.1, the good insurer can dominate with perfect information. With asymmetric information regarding insurer type, the bad insurer can simply offer a contract with the same premium as the good insurer and no information would be revealed. Conversely, the bad insurer will be revealed if it sets $P_B < P_G^0$, since the good insurer would never offer a contract that earns negative profit. The following proposition characterizes the impact of this informational asymmetry on market counterparty risk.

Proposition 2 *When the good insurer dominates under perfect information (as per Lemma 2), market counterparty risk as well as the individual counterparty risk of the good insurer will increase when insurer type is unknown.*

Proof. *See Appendix A.*

[11] Note that the form of competition is not vital to Proposition 1. Alternatively, we could assume that increased competition comes in the form of additional insurers of the same type (i.e., good and bad types). With two insurers of the same type, the premium is driven down to that which earns zero profit à la Bertrand competition. Since counterparty risk of the good insurer increases as the premium decreases, the result follows.

The intuition behind this result is as follows. When the insurer type is known and the good insurer dominates, it charges the highest premium such that the bank still prefers to insure with it rather than the bad insurer (who charges the lowest premium it can). When the insurer type is unknown, the good insurer is forced to cut its premium or else give up the entire market to the bad insurer who can undercut it and still not reveal itself. In equilibrium, the premium charged by both insurers is P_G^0. Since the good insurer (weakly) reduces its premium and $1 - q_G = F(1 - P_G)$, it follows that the good insurer becomes individually less stable. Furthermore, bad insurers now participate in the market, so that market counterparty risk unambiguously increases.

When the bad insurer dominates under perfect information, there are two equilibria that can arise when insurer type is unknown (a straightforward condition would determine which one prevails in equilibrium). First, the bad insurer may choose to reveal itself by setting $P_B^* < P_G^0$ and dominate the market. The bad insurer does this to obtain the insurance contract with certainty, rather than charging P_G^0 and allowing the good insurer to stay in the market, thereby reducing its chances it will obtain the contract. Since the counterparty risk of the bad insurer is independent of the premium ($1 - q_B = F(1)$), if it chooses to reveal itself and take the market, market counterparty risk remains unchanged. Conversely, when the bad insurer prefers the higher premium P_G^0 over obtaining the contract with certainty, the presence of the good insurer causes market counterparty risk to fall since the expected counterparty risk to which the bank is exposed decreases.

4 Incentives to Insure

A fundamental difference between a standard insurance market and that for CDS are the incentives for purchasing these types of contracts. For example, some buyers may own the underlying loan being insured, while others do not. More generally, some participants may use CDS entirely for trading purposes, while others may use them for risk management purposes. Fitch (2009, 2010) use surveys to gauge the motivation of global banks to use credit derivatives (of which CDS represents more than 90% (Fitch 2010)). They find that hedging/credit risk management and speculation are the two most common reasons to use credit derivatives (with similar prevalence).

To the best of our knowledge, there has not been a paper which analyzes the impact of buyer's incentives on the market for CDS. We can do this simply in our model, as the incentive to purchase insurance is captured by Z. One would expect that those who purchase CDS for risk management purposes will view counterparty risk differently than those who purchase it for speculation (i.e., speculators will internalize the cost of counterparty risk less). Further, although it does not directly follow that those who purchase CDS for risk management will own the underlying loan, it is reasonable to expect that those who do not own the underlying loan are more likely to purchase CDS for speculation than for risk management. This has interesting consequences for policies aimed at increasing stability, since ownership of the underlying loan can be easily observed. Consider Germany's recent ban on the practice of buying CDS without owning the underlying risk, and

China's intent on creating a CDS market with this same restriction.[12]

Modifying the base model from Sections 2 and 3, we create a market for CDS as simply as possible. In addition to the good and bad insurers, we let there be two types of banks who differ on Z, which we denote Z_L and Z_H (to be defined below). For simplicity, we assume that a bank insures with its own insurer, and that the size of each contract is one. At the end of the section, we discuss how our results obtain when the model is enriched. The following lemma finds the value of Z for which a bank is indifferent between insuring with a good or bad insurer.

Lemma 3 *Define $\hat{Z}$ as the level of Z for which the bank is indifferent between insuring with the good or bad insurer at the zero profit premium for each insurer. Thus, with P_G^0 and P_B^0, a bank with $Z < \hat{Z}$ will prefer to contract with the bad insurer and a bank for which $Z > \hat{Z}$ will prefer the good insurer. The expression for $\hat{Z}$ is given by $\hat{Z} = \frac{P_G^0 - P_B^0}{(1-p)(q_G - q_B)} - 1$.*

Proof. *See Appendix A.*

We interpret $\hat{Z}$ by considering the two relevant components of a contract from the perspective of a bank: counterparty risk and premium. A bank trades off a higher premium against increased counterparty risk in its choice of insurer. A bank for which $Z < \hat{Z}$ is less averse to counterparty risk and so insures with the bad insurer, as it is able to offer a lower premium than the good insurer. A bank for which $Z > \hat{Z}$ is sufficiently averse to counterparty risk to compensate for the increase in premium at the good insurer. For the remainder of the section, let $Z_L < \hat{Z}$ and $Z_H > \hat{Z}$.

Given the market for CDS we have introduced, we define market counterparty risk as the average counterparty risk to which banks are exposed. In light of Lemma 3, we can explore the difference between a market for CDS and that for traditional insurance in a relatively simple way. When modeling a market for insurance, it is customary to assume that the insured party has exposure to the underlying risk (i.e., has an insurable interest). An insured party is typically modeled as being risk averse and so willing to pay a risk premium when purchasing insurance. As discussed above, in the market for CDS, some buyers purchase protection purely for speculation. As the number of speculators (i.e., Z_L types) increases, so does the relative amount of insurance sold by bad insurers. It is reasonable to assume that the CDS market has more speculators than a traditional insurance market, so it follows that the market for CDS will tend to have lower quality sellers.

The existence of speculators and bad insurers implies that CDS markets are generally characterized by higher market counterparty risk. Although this is a "mechanically" trivial consequence of our framework, it adds a new element to the policy debate on the CDS market. Ideally, a policy maker whose mandate is to reduce counterparty risk could simply remove bad insurers; however, the quality of the counterparty is often not observable to the bank, so it is unlikely that it will be observable to a regulator.[13] A second-best alternative may be to remove the Z_L banks from the market, similar to the recent proposals described above which disallow CDS to be purchased by

[12] http://www.bloomberg.com/news/2010-09-13/china-plans-to-introduce-credit-default-swaps-by-year-end-official-says.html, http://dealbook.nytimes.com/2010/05/19/germany-bans-naked-shorts-and-c-d-s-s/

[13] For a discussion on this issue see Pirrong (2009).

those who do not own the underlying loan. Although it is possible that those who own the loan could purchase CDS on speculation, it is more likely that this policy will reduce the number of buyers for which $Z = Z_L$ more than it would for buyers with $Z = Z_H$. We look at the extreme case in which the Z_L bank can be eliminated. For a policy maker concerned about counterparty risk in CDS markets, we show that removing the bank which demands insurance from the bad insurer, may reduce counterparty risk, but can actually increase it depending on the structure of the market. We consider the following two polar cases of market competition.

Case 1: Bertrand competition within each insurer type.

Case 2: No Bertrand competition within each insurer type.

We can think of the first case as having multiple insurers of the same type who are able to compete in the market, while in the second case there is only one good and one bad insurer. The following proposition characterizes the impact of removing the Z_L bank from the market, where P_G^* and P_G^{**} are defined as the good insurer's equilibrium premium before and after this is done.

Proposition 3 *In case 1, a policy that removes Z_L banks will decrease market counterparty risk. In case 2, such a policy will make the good insurer riskier and consequently may increase or decrease market counterparty risk. When $2F(1 - P_G^{**}) > F(1 - P_G^*) + F(1)$, market counterparty risk increases.*

Proof. *See Appendix A.*

In the first case, the insurers are driven down to zero profit due to competition within types. When the Z_L banks are removed, the bad insurers cannot compete in the market and so drop out (as per Lemma 3). The good insurers still face Bertrand competition and so profits are zero and the risk of the good insurer remains unchanged. Thus, average counterparty risk in the market falls since bad insurers drop out.

In the second case, the good (bad) insurer contracts with the Z_H (Z_L) bank as in case one; however, there is no Bertrand competition before the Z_L banks are removed. Therefore, both insurers extract positive profits from the contracts. When the Z_L bank is removed, the bad insurer then competes with the good insurer for the remaining bank, thereby eroding profits for both insurers. In equilibrium, the good insurer cuts its premium sufficiently to attract the remaining bank and the bad insurer drops out of the market as in case one. Since its premium is forced down due to competition, the good insurer becomes riskier. Whether the net affect on market counterparty risk is negative or positive depends on how much the good insurer must cut its premium. Market counterparty risk will increase when $2F(1 - P_G^{**}) > F(1 - P_G^*) + F(1)$, where $1 > 1 - P_G^{**} \geq 1 - P_G^*$. Given the ordering of premia in equilibrium, it is clear that this can be satisfied by any number of distribution functions $F(\cdot)$. The proof of the proposition characterizes this outcome using the example of the uniform distribution. It follows that policy makers must be cognizant that eliminating some buyers from the market may drive premia down, working against the reduction of counterparty risk that arises when bad insurers drop out.

Although we model the market for insurance in a simple way, our results survive when we consider a more complex framework. If we allow for many banks and insurers, it will still follow that Z_L banks will prefer the bad insurers. As long as premia are not set at the zero profit level (so that insurers have some market power) and there is some competition between insurer types, the impact of removing Z_L type buyers from the market will have the same implications for market counterparty risk. Essentially, such a policy amounts to a decrease in demand, which results in increased competition for existing buyers and puts downward pressure on premia.

5 Separation and Mutual Exclusion

In their seminal papers, Rothschild and Stiglitz (1976) and Wilson (1977) develop a framework in which insurers can screen individuals through a menu of contracts. Screening provides the means to deal with an asymmetric information problem facing providers, as unobservable insured party risk types will reveal themselves through their choice of contract. Up to this point, we have not considered risk heterogeneity among the purchasers of insurance (banks), so that the traditional form of asymmetric information has not been analyzed. This section evaluates a similar outcome to the separating equilibrium in Rothschild and Stiglitz (1976). We do this by extending the analysis to allow for multiple bank risk types, which cannot be directly observed by the insurer.

Before considering the existence of separating equilibria, we make note of a fundamental assumption which is commonly made in models of insurance; that insured parties cannot purchase insurance from more than one provider. Without this assumption, which is referred to as exclusivity, separation of risk types through a market mechanism cannot generally be achieved.[14] The nature of CDS markets makes the exclusion assumption implausible, as it is not possible for a seller to restrict a buyer from purchasing insurance elsewhere. For presentation purposes, we present the results of this section under the assumption of exclusivity, as this provides the intuition with a simple extension of our previous model. We conclude with a discussion of how our results obtain without exclusivity. The interested reader can refer to Appendix B, where we formally analyze this case.

We enrich the environment outlined in Section 3.1 to allow two types of loans that a bank can insure; a safe type (S) and a risky type (R), both of size 1. We assume that the safe (risky) loan succeeds with probability p_S (p_R), and the bank is privately endowed with one or the other with equal probability. Further, we continue to assume that contracts are exclusive and that the bank insures both loans completely, i.e., the contract size is 1 regardless of loan type. The analysis is similar to that in Section 3.1, however we need to modify the beliefs that the insurers have about the probability of a claim. Recall that insurer $j \in \{G, B\}$, forms a belief b_j corresponding to the

[14] For a further discussion of why a separating equilibrium will not generally exist with non-exclusivity in the context of life insurance, see Hoy and Polborn (2000).

probability that a claim will not be made (when the underlying loan does not default). We rewrite the payoff for each insurer, noting that the premium is a function of beliefs.

$$\pi_G = b_G \left[\int_{-P_G}^{\overline{\theta}} (\theta + P_G)\, dF(\theta) \right] + (1 - b_G) \left[\int_{1-P_G}^{\overline{\theta}} (\theta - 1 + P_G)\, dF(\theta) \right] \tag{10}$$

$$\pi_B = b_B \left[\int_{0}^{\overline{\theta}} (\theta + rP_B)\, dF(\theta) \right] + (1 - b_B) \left[\int_{1}^{\overline{\theta}} (\theta - 1 + rP_B)\, dF(\theta) \right] \tag{11}$$

These payoffs are akin to those described in equations (2) and (3). The difference is that the probability of a claim, which we previously referred to as p, is replaced with the insurer's beliefs about the type of bank it has contracted with. Since bank types are unknown to insurers, we focus our attention on Bayesian Nash Equilibria.

Analogous to Section 3.1, there exists a pooling equilibrium in which the good insurer dominates the market and provides insurance for both safe and risky banks, and another in which the bad insurer dominates and provides insurance for either bank type. Unlike Section 3.1, there exists a separating equilibrium in which both insurers are active. In this case, the safe bank insures with the bad insurer and the risky bank with the good insurer.[15] Before characterizing such an equilibrium, we rewrite the payoff function for bank $i \in \{S, R\}$, when insuring with insurer type j as follows.

$$\Pi(i, j) = p_i R_B + (1 - p_i)q_j - (1 - p_i)(1 - q_j)Z - P_j \tag{12}$$

Consider the first case in which the good insurer dominates the market. This will occur when both bank types prefer the good insurer, $\Pi(S, G) \geq \Pi(S, B)$ and $\Pi(R, G) \geq \Pi(R, B)$, with the former being the binding condition. Similarly, we define the case under which the bad insurer will dominate the market. In this case, $\Pi(S, B) > \Pi(S, G)$ and $\Pi(R, B) > \Pi(R, G)$, where the latter is the binding condition. The final case in which the bank types separate occurs when $\Pi(S, B) > \Pi(S, G)$ and $\Pi(R, G) \geq \Pi(R, B)$. The following lemma characterizes the set of equilibria in this market.[16]

Lemma 4 *There are three equilibria in this market.*

i. When Z is high, there exists an equilibrium in which the good insurer contracts with both safe and risky banks at a single (pooling) premium.

ii. When Z is low, there exists an equilibrium in which the bad insurer contracts with both safe and risky banks at a single (pooling) premium.

[15] In the proof to Lemma 4, the separating equilibrium where the good insurer contracts with the safe bank and the bad insurer with the risky bank is ruled out.

[16] Lemma 4 characterizes two pooling equilibria which do not have an analogue in Rothschild and Stiglitz (RS) (1976). In their screening model, the insurer is free to choose a menu of contracts, namely any contract size and premium combination. In our model, counterparty risk plays the same role as contract size in RS (1976). To see this, note that the difference in counterparty risk between the good and bad insurer alters the expected coverage from an ex-ante perspective, whereas changing the contract size in RS (1976) alters the actual level of coverage. It is because insurers are flexible in their choice of contract size in RS (1976) that pooling is never optimal. In our framework, the level of counterparty risk is not a choice variable, so that pooling equilibria can be supported.

iii. For intermediate values of Z, there exists an equilibrium in which the banks separate wherein the bad (good) insurer contracts with the safe (risky) bank.

Proof. *See Appendix A.*

When Z is high, both the safe and risky bank are highly averse to counterparty risk, and are willing to pay more for the increased protection that the good insurer offers. When Z is low, both the safe and risky banks care little about counterparty risk, and so insure with the bad insurer at a lower premium. A separating equilibrium exists when the risky bank is willing to pay a higher premium and be revealed as risky, since it obtains lower counterparty risk at the good insurer. Similarly, the safe bank reveals itself and pays a lower premium, however it suffers higher counterparty risk at the bad insurer. This equilibrium will arise when the safe bank is concerned less about the risk of insurer default (due to its lower probability of making a claim), whereas the risky bank is driven more by counterparty risk (due to its higher probability of making a claim). In the proof of the lemma, we provide the formal conditions needed to establish existence, as well as characterize the equilibrium premia.

In our previous discussion, which culminated in Lemma 4, we assumed that banks buy insurance exclusively from one provider. As discussed above, this assumption is not appropriate in the CDS market, as insurers cannot preclude banks from purchasing more protection elsewhere. Proposition 5 in Appendix B shows that a separating equilibrium, in which unobserved bank type is revealed, can exist even when a bank can split its contract among many insurers. A formal argument requires a more complicated model, which can be found in Appendix B, but the general intuition is fairly simple. As long as the bank cannot replicate the insurance it receives through the good insurer, by simply purchasing more insurance from the bad insurer, a separating equilibrium can exist. In Appendix B, we consider a case with many good and bad insurers in which the latter are exposed to aggregate risk, so that all may fail in some event. In other words, bad insurers effectively cannot provide complete coverage. In this environment, either bank will divide insurance across providers to reduce idiosyncratic default risk. In the separating equilibrium, the safe bank will contract solely with bad insurers and bear the aggregate risk in exchange for a lower premium, while the risky bank will contract solely with good insurers and pay a higher premium in exchange for more complete protection.

6 Central Counterparties

In the wake of the credit crisis that began in 2007, law makers around the world have been tabling regulations to move CDS from over-the-counter markets to a formal central counterparty (CCP) arrangement.[17] In the absence of a central counterparty, contracts are bilateral and take one of two forms. First, and most commonly, contracts are negotiated through a dealer. In these types of transactions, a buyer purchases protection from a counterparty located by a dealer. Second,

[17]See Bliss and Steigerwald (2007) for an introduction to and discussion on CCPs.

trading may be done without a dealer, where a buyer approaches a seller directly. In a CCP arrangement, all transactions flow through a central counterparty which acts as the buyer to every seller and the seller to every buyer. In this arrangement, participants provide capital and post margins (collateral) that the CCP can use to cover default losses. Furthermore, the CCP can require participants to make additional payments if needed to cover losses. Therefore, a CCP pools default risk across participants (or members).[18]

In our model, bad insurers are forced to set a lower premium because they pose a greater risk of default (assuming insurer type is known). Arora et al. (2009) provide evidence that counterparty risk is priced in CDS contracts, so that premia can vary depending on the quality of the seller. Importantly, the CCP forces a single premium on the market because traders view counterparty risk as being only that of the CCP. As discussed above, a CCP requires capital (for a default fund) and collateral in case of contract non-performance. Typically, CCPs demand collateral according to the quality of the asset being insured, but less so based on the quality of the counterparties (Pirrong 2009). In what follows, we assume that insurers are either unwilling or cannot condition collateral or contributions to the default fund based on insurer quality.[19]

A comprehensive analysis of a CCP arrangement is beyond the scope of this paper, however our framework can be used to address an issue that has been largely ignored in the debate thus far. Assume there are many insurers of both types, that there is Bertrand competition within both insurer types and that each insurer contracts with its own bank. Let there be N banks, who each contract with one insurer. As in Section 4, banks may differ in their aversion to counterparty risk. We assume that there are N_G banks for which $Z = Z_H$ and N_B banks for which $Z = Z_L$, so that $N = N_G + N_B$. When there is no CCP, Lemma 3 implies that in equilibrium Z_H (Z_L) banks insure with good (bad) insurers, so that the number of good (bad) insurers in the market is N_G (N_B). Furthermore, since there is Bertrand competition within insurer type, all premia are determined by the zero profit conditions defined in Section 2.2.

We now analyze the imposition of a CCP on this market, which we conceptualize as a scheme to pool the risk of insurer default. Mutualization requires each insurer to contribute to a pool of funds that the CCP can use in the event of insurer failure. We denote the size of this pool by m. The CCP will pay out claims as long as it is solvent, but fails if the number of insurers which have defaulted on claims is too high. We assume collateral requirements are not insurer specific (and the risk of a claim is the same with every bank), so for simplicity we normalize collateral to zero. Since all contracts are of size 1, the CCP will default when m insurers who have been faced with a claim have defaulted. The default risk of the CCP can be characterized as follows (note that

[18]This can be contrasted with the analysis in Appendix B, which shows that a risk-averse ($Z > 0$) bank will divide a contract across many insurers to reduce idiosyncratic risk of counterparty default. A CCP however, forces the diversification of counterparty risk among all insurers, so that a bank that insures with one insurer receives the diversification benefit as if it were to contract with every insurer.

[19]In practice, CCPs can and sometimes do try to enforce higher capital charges (and higher collateral) to riskier counterparties. The relevance of this assumption is discussed below, but we note that the results of this section will survive provided that the CCP does not perfectly condition on counterparty quality.

$q_B = 1 - F(1)$ and $q_G = 1 - F(1 - P_G^0)$).

$$
\begin{aligned}
1 - q_{ccp} \quad = \quad & \sum_{i=m}^{N} \sum_{j=0}^{i} \binom{N_G}{i-j} \binom{N_B}{j} \left[(1-p)(1-q_G)\right]^{i-j} \left[(1-p)(1-q_B)\right]^{j} \\
& \times [p + (1-p)q_G]^{N_G-(i-j)} [p + (1-p)q_B]^{N_B - j}
\end{aligned}
\tag{13}
$$

This expression captures all the possible combinations for which m or more insurers default, weighted by the probability of each outcome. To determine the optimal size of the default pool m requires added structure which is beyond the scope of this paper. However, because our results are not dependent on the specific form of the CCP objective, we focus our attention on the simple case in which every insurer must fail before the central counterparty fails. For example, this would be true if the remaining solvent insurers could be forced to help cover all losses. Therefore, expression (13) collapses to

$$1 - q_{ccp} = (1 - q_G)^{N_G} (1 - q_B)^{N_B}. \tag{14}$$

The following proposition characterizes the equilibrium with a CCP and represents the main result of this section.

Proposition 4 *In the presence of a CCP, the bad insurers will dominate the market and push the good insurers out when*

$$(1-p)(1+Z)(q_G - q_B)(1-q_G)^{N_G-1}(1-q_B)^{N_B} < P_G^0 - P_B^0, \tag{15}$$

which is satisfied for large N.

Proof. *See Appendix A.*

This result is best understood by comparing (15) to the case in which a bank chooses to contract with a bad insurer in the absence of a CCP, given by (6), which we re-write here for convenience.

$$(1-p)(1+Z)(q_G - q_B) < P_G^0 - P_B^0 \tag{16}$$

Comparing (15) and (16), it is straightforward to see that the additional counterparty risk to which the bank is exposed from a bad insurer is smaller in the CCP case, since $(1-q_G)^{N_G-1}(1-q_B)^{N_B} < 1$. With a CCP, the change in individual counterparty risk from switching to a bad insurer (the left hand size of (15)) approaches zero as either $N_G \to \infty$, $N_B \to \infty$ or both. Thus, in a large market, the good insurer presents no benefit of reduced counterparty risk since the additional counterparty risk that a single bank adds to the CCP by choosing a bad over a good insurer is negligible. Instead, the insurers compete solely on premium. Given the assumption that $r > r^*$ (see Lemma 1), the bad insurer can offer a lower premium ($P_B^0 < P_G^0$). Accordingly, the bad insurer will simply undercut the good insurer and obtain the contract with the bank. No single bank would then wish to remain at the good insurer, so that in equilibrium, only bad insurers are left in the market. This result is similar in spirit to the classic problem of the commons. Although as a group, they may be worse off for only insuring with bad insurers (through increased default risk of the CCP or an increase in the capital charge/collateral requirement to offset the increased risk), a single bank does not internalize the counterparty risk since it is individually too small.

As a straightforward corollary of the above proposition, we note that the existence of a CCP with a large number of members eliminates the possibility of a separating equilibrium as discussed in Section 5. Obviously, banks will not separate on insurer type when the counterparty risk to which they are exposed is the same regardless of the insurer in which they contract.

This result can be interpreted as an example of the Lucas critique, in that policy-makers considering the imposition of a CCP must consider the reaction of market participants to the policy. The remedy to the problem is obvious: the CCP should penalize unstable insurers in this market. Pirrong (2009) reports that this may not be possible since it is not clear whether CCPs can deduce the quality of the insurer, especially if market participants cannot deduce the quality themselves. Nonetheless, to mitigate the problem posed in Proposition 4, our analysis suggests that CCPs should condition capital requirements and collateral on the quality of the counterparty. In this way, the bad insurer would have a higher cost of participation in the market and thus may not be able to undercut the premium of the good insurer.

A brief discussion about marking to market and counterparty risk is warranted. With daily mark to market, the CDS seller would have to post additional collateral if the quality of the underlying asset deteriorates. When the quality of the insurer falls at the same time as the underlying asset, the increase in collateral will help mitigate the counterparty risk to which the buyer is exposed. However, it could be that the underlying asset becomes safer at the same time as the insurer becomes riskier. In this case, the decrease in collateral exacerbates the counterparty risk. Therefore, it is clear that mark to market cannot eliminate the problem in Proposition 4.

The results from this section are meant to highlight a very specific point relevant to the debate over CCPs. There are many factors that must be considered in determining whether such an arrangement would be beneficial to the market. For example, there is certainly a diversification benefit that comes with co-insurance which may outweigh the endogenously lower quality individual insurance that we consider. Further, there are other possible benefits such as netting that CCPs can provide.[20] A formal welfare analysis is left for future research.

7 Conclusion

In this paper we update the traditional insurance economics model to account for features unique to the market for credit default swaps. We show that when the counterparty risk of the insurer is unknown, unstable insurers can exist in equilibrium and otherwise stable insurers can destabilize. Increased competition among insurers is also shown to potentially destabilize good

[20]See Bliss and Steigerwald (2007).

insurers. Further, we show that when some buyers of CDS use the instrument purely for speculation (and potentially have no insurable interest), the market will be characterized by more unstable insurers; however, removing these traders may cause market counterparty risk to increase. We also analyze the case when contracts can be split over multiple providers. Contrary to standard results in the insurance economics literature, we show that with counterparty risk (some of which must be aggregate risk), the insured parties can separate based on the type of insurers with whom they contract. Finally, we apply our analysis to the ongoing debate on central counterparties. We show that in such an arrangement, the stable insurers can be driven out of the market due to their inability to compete on premia.

8 Robustness

8.1 Insurer Investment Choice

We model heterogeneity between the two insurer types as simply as possible. Given that the two insurers are identical before contracts are issued, there is an obvious question of why they would invest in different assets. We recognize that investing in the liquid asset may not be credible for the good insurer, given that it could earn a higher profit investing in the illiquid asset. This could easily remedied by allowing heterogeneity along two dimensions. First, by making our insurers different before contracting and second, by allowing the investment choice to be an optimal decision variable, as in Thompson (2010). For example, instead of endowing both insurers with a portfolio draw from $F(\theta)$, as is done in our paper, let the good (bad) insurer receive a draw from a distribution $\mathcal{G}(\theta)$ ($\mathcal{B}(\theta)$). Next, let the proportion that the good (bad) insurer invests in the liquid asset be given by β_G (β_B), with the remainder invested in the illiquid asset. Each insurer can now solve for its optimal investment decision given its portfolio distribution. Using the usual notation for premia, counterparty risk is now defined in a similar way as in Section 2, $1 - q_G = \mathcal{G}(1 - \beta^*_G P_G)$ and $1 - q_B = \mathcal{B}(1 - \beta^*_B P_B)$, where the asterisk represents the optimal portfolio choice. We can then impose the appropriate restrictions on the distribution functions to ensure $q_G(\beta^*_G) > q_B(\beta^*_B)$, i.e., so that our bad insurer has higher counterparty risk.

What will drive the difference in investment choice between the good and bad insurer is how valuable the liquid asset is relative to the illiquid asset. As in expressions (2) and (3), the insurers invest in the liquid asset to reduce the probability of insolvency in the state of the world when a claim is made. Given the new distributions, the reduction in counterparty risk from investing in the liquid asset for the good (bad) insurer is: $\mathcal{G}(1) - \mathcal{G}(1 - \beta^*_G P_G)$ ($\mathcal{B}(1) - \mathcal{B}(1 - \beta^*_B P_B)$). Therefore, the more mass that the distribution function has in this region, the higher the benefit and the more that will be invested in the liquid asset. We imagine that the bad insurer is endowed with a riskier portfolio than the good insurer, in the sense that it has more mass on high and low outcomes (for example, portfolio draws near $\overline{\theta}$ and $\underline{\theta}$). Consequently, the bad insurer invests more in the illiquid asset (and reaps higher returns) because investments in the liquid asset are unlikely to be sufficient for it to remain solvent in the event of a bad draw. Conversely, we imagine that the good insurer has a portfolio with less risk so that bad portfolio draws are less extreme and can therefore be offset by investing in the liquid asset. All of the results of the paper would then follow with the new definition of counterparty risk.

8.2 Z and Risk Aversion

It is worthwhile to contrast the parameter Z in our model with standard utility assumptions made in most insurance papers. Typically, a non-linear utility function is used for the insured party that puts different weights/utility value on high and low outcomes. A standard risk averse utility function will put relatively more negative weight on the bad outcomes (e.g., an 'accident') versus the high outcome (e.g., no 'accident'). As such, insurance is purchased to protect the risk averse individual that may cost more than the expected loss from the accident. In our model, we use the simplest formulation possible that captures insurance without a non-linear function. In particular, we put a weight Z on the bad outcome (i.e., the loan fails). As such, the utility in the good state (i.e., the loan does not fail) is simply equal to the monetary payoff. In our model we let $Z \in [0, \infty)$. To understand this range, we consider the condition under which a bank (with probability of default 1-p) is indifferent between purchasing and not purchasing insurance, i.e., its participation constraint.

$$\begin{aligned} pR_B + (1-p)q - (1-p)(1-q)Z - P &= pR_B - (1-p)Z \\ \Rightarrow P &= (1-p)q(1+Z) \end{aligned} \tag{17}$$

Therefore, when $Z = 0$, $P = (1-p)q$, which is the actuarially fair premium, i.e., the bank pays the expected value of the coverage. This corresponds to the usual insurance result with a risk neutral agent. When $Z > 0$, the bank is willing to pay greater than the expected value in return for coverage. This represents the usual risk premium that an insurance provider can extract due to the risk aversion of the insured party.

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Appendix A

Proof of Lemma 1

Equation (3) characterizes the profit of the bad insurer. Setting this equal to $\int_0^{\bar{\theta}} \theta dF(\theta)$, provides an expression which implicitly defines the zero-profit premium P_B^0 as a function of r, the return on premia invested. It is straightforward to show that P_B^0 becomes arbitrarily small as r becomes arbitrarily large. Differentiating with respect to r yields

$$\left(P_B^0 + r\frac{dP_B^0}{dr}\right)\left[p\int_0^{\bar{\theta}} dF(\theta) + (1-p)\int_1^{\bar{\theta}} dF(\theta)\right] = 0 \tag{18}$$

$$\Rightarrow P_B^0 + r\frac{dP_B^0}{dr} = 0 \Rightarrow \frac{dP_B^0}{dr} = -\frac{P_B^0}{r} < 0. \tag{19}$$

Thus $P_B^0(r)$ is strictly decreasing in r, and $\lim_{r\to\infty} P_B^0 = 0$. Since the zero profit premium for the good insurer is a positive finite number, there exists a finite r^* such that $P_G^0 = P_B^0(r^*)$ and $P_G^0 > P_B^0(r^*)$ for all $r > r^*$.

■

Proof of Lemma 2

First, consider the case in which the good insurer dominates. The existence of such an equilibrium can be ensured when the bank's aversion to counterparty risk is high. As Z becomes arbitrarily large, $(1-p)(1+Z)(q_G - q_B) \geq P_G^0 - P_B^0$ must hold, so that the bank prefers the good insurer at the zero profit premium. The good insurer's optimal premium P_G^*, is that which satisfies (5) with equality, as described in the lemma.

The case in which the bad insurer dominates follows the same logic, but is somewhat more involved given the simplified framework outlined in Section 2. The bad insurer offers a lower premium, but higher counterparty risk. Intuitively, it will dominate the market if the added risk is small relative to the discount on premium it offers. Existence requires

$$(1-p)(1+Z)(q_G - q_B) < P_G^0 - P_B^0. \tag{20}$$

From the proof to Lemma 1, we can see that P_B^0 becomes arbitrarily small as r becomes large. Let this be the case, let $Z = 0$ and replace $q_B = 1 - F(1)$ and $q_G = 1 - F(1 - P_G^0)$ so that (20) becomes

$$(1-p)[F(1) - F(1 - P_G^0)] < P_G^0. \tag{21}$$

To prove existence, let $F(\cdot)$ be uniform over $[\underline{\theta}, \overline{\theta}]$. Expression (21) then becomes

$$P_G^0 > \frac{(1-p)}{\underline{\theta} - \overline{\theta}} P_G^0, \tag{22}$$

which holds since $(1-p)/(\underline{\theta} - \overline{\theta}) < 1$. Note that $\overline{\theta} > 1$, otherwise the bad insurer would always default when faced with a claim. The equilibrium premium is the maximum premium for which (6) is still satisfied. This is sufficient to prove the result, but we note that existence in the case where the bad insurer dominates can be shown for any $F(\cdot)$ through a simple generalization of Z. In Section 2, we assume that when the bank suffers a loss and is uninsured, or is insured but the insurer cannot pay, it suffers the cost Z. Implicit in this is that the insurer pays nothing in the event of a default. This simplification is made to avoid unnecessary complication, and is not vital to our results. In reality, it would be more common for there to be a recovery value when the underlying asset of a CDS defaults. This feature can be added to the model with a simple modification to Z. Let ϕ be the recovery value. We can then define $\widetilde{Z} = Z - \phi$ and all of the results in the paper follow through by replacing Z with $\widetilde{Z}$. In particular, if we rewrite (20) with the general form of Z described above, we obtain the following.

$$(1-p)(1+\widetilde{Z})(q_G - q_B) < P_G^0 - P_B^0 \tag{23}$$

Thus, to prove the existence of an equilibrium where the bad insurer dominates we can simply let $Z = 0$, and $\phi \to 1$, so that $\widetilde{Z} \to -1$ and (23) holds trivially. The intuition is that when there is almost full recovery, there is little difference in counterparty risk. Since the bad insurer offers a lower premium, it is preferred.

■

Proof of Proposition 2

With no information about insurer type, banks will insure with the provider who offers the lowest premium (unless the premium is below P_G^0). Given this, the insurers will compete on premia until it falls to P_G^0. Below this premium, the bad insurer would be revealed and the good insurer would drop out of the market. Thus, the equilibrium is characterized as one in which both insurers offer coverage at the premium P_G^0, and the bad insurer earns a positive profit.

With perfect information over insurer type and all insurance provided by the good insurer, the premium is $P_G^* \geq P_G^0$ such that (5) holds with equality. The market counterparty risk is then given by $1 - q_G = F(1 - P_G)$. It follows that,

$$\frac{d(1-q_G)}{dP_G} = -dF(1-P_G) \leq 0. \tag{24}$$

With asymmetric information over insurer type, $P_G^* = P_G^0$. Thus counterparty risk for the good insurer is (weakly) higher with asymmetric information. Coupled with the participation of the bad insurer, market counterparty risk unambiguously increases.

■

Proof of Lemma 3

The bank's payoff from insuring with the good and bad insurers are given as follows.

$$\Pi(G) = pR_B + (1-p)q_G - (1-p)(1-q_G)Z - P_G \tag{25}$$

$$\Pi(B) = pR_B + (1-p)q_B - (1-p)(1-q_B)Z - P_B \tag{26}$$

We now define $\hat{Z}$ as that which equates these expressions.

$$\hat{Z} = \frac{P_G - P_B}{(1-p)(q_G - q_B)} - 1 \tag{27}$$

Inserting the zero profit premia yields the expression characterized in Lemma 3.

■

Proof of Proposition 3

The result is straightforward in case 1. Given that there is Bertrand competition within each type of insurer, the premium is always that which earns zero profit. Initially, market counterparty risk is given by $(2 - q_B - q_G)/2$, where $q_B = 1 - F(1)$ and $q_G = 1 - F(1 - P_G^0)$. Once the Z_L bank is removed from the market, the bad insurer drops out (by Lemma 3), but the good insurer cannot alter its premium. Therefore, market counterparty risk is now $1 - q_G$. Since q_G is unchanged, and $q_G > q_B$, market counterparty risk decreases.

The second case is less obvious. We begin by defining the initial equilibrium and then characterize the change in market counterparty risk when the Z_L bank is removed. Initially, there are two banks and two insurers. The Z_H bank is most attractive to both insurers, as this type is willing to pay a higher premium for insurance, yet poses no additional risk. Thus, by Lemma 3, we restrict our attention to the case when the good insurer contracts with Z_H and the bad insurer with Z_L. Given this, a unique set of equilibrium premia are determined by the following set of participation and incentive constraints.

$$q_B(1-p)(1+Z_L) \geq P_B \tag{PCL}$$

$$q_G(1-p)(1+Z_H) \geq P_G \tag{PCH}$$

$$P_G - P_B \geq (1-p)(1+Z_L)(q_G - q_B) \tag{ICL}$$

$$P_G - P_B \leq (1-p)(1+Z_H)(q_G - q_B) \tag{ICH}$$

The Inequality PCL (PCH) ensures that the Z_L (Z_H) bank will purchase insurance from the bad (good) insurer, rather than go without. Inequality ICL (ICH) ensures that the Z_L (Z_H) bank contracts with the bad (good) insurer rather than the competitor. Recall that we assume that an insurer only insures one bank so that a deviating bank is still the only party with whom the new insurer contracts.[21] This could be relaxed to allow multiple contracts per insurer, but would add undue complication without changing the qualitative results. Expanding ICH, we have

$$P_G \le q_G(1-p)(1+Z_H) - [q_B(1-p)(1+Z_H) - P_B]. \tag{28}$$

The second term on the right hand side is negative since

$$q_B(1-p)(1+Z_H) - P_B > q_B(1-p)(1+Z_L) - P_B \ge 0, \tag{29}$$

where the second inequality follows from PCL. Thus, (28) shows that PCH is redundant and can be ignored. Furthermore, in equilibrium, ICH must be satisfied with equality, otherwise the good insurer could increase the premium and still attract the Z_H bank. This implies

$$P_G - P_B = (1-p)(1+Z_H)(q_G - q_B) > (1-p)(1+Z_L)(q_G - q_B), \tag{30}$$

so that ICL can also be ignored. Finally, in equilibrium the bad insurer will increase its premium until the Z_L bank is just indifferent to purchasing the contract or not so that PCL is satisfied with equality. To summarize, the equilibrium premia in this situation are $P_B^* = q_B(1-p)(1+Z_L)$ and $P_G^* = (1-p)(q_G - q_B)(1+Z_H) + P_B^*$.

We now consider the equilibrium when the Z_L banks are removed. We know from Lemma 3 that this will drive the bad insurer out of the market. However, this constrains the good insurer by changing ICH, which determines the premium in equilibrium. As the bad insurer no longer has a contract, it will offer the lowest premium possible in an attempt to lure the Z_H bank, namely P_B^0. Thus, the new premium offered by the good insurer is $P_G^{**} = (1-p)(q_G - q_B)(1+Z_H) + P_B^0$. Since $P_G^{**} \le P_G^*$, the good insurer will become less stable (weakly).

Market counterparty risk in the initial equilibrium is $(2-q_B-q_G)/2$, where q_B and q_G are implied by the premia P_B^* and P_G^* defined above. When the Z_L bank is removed, market counterparty risk is simply $1-q_G$, which is defined by the premium P_G^{**}. Since $P_G^{**} \le P_G^*$, and the default risk of the good insurer is decreasing in the premium, the affect on market counterparty risk is ambiguous. Using the definition of q and the relevant premia, we can derive the following condition under which market counterparty increases (as stated in the proposition).

$$2F(1-P_G^{**}) > F(1-P_G^*) + F(1) \tag{31}$$

Clearly, a distribution function can be chosen that will satisfy this condition. In the case where $F(\cdot)$ is uniform, condition (31) is simply $2P_G^{**} < P_G^*$. Using the expressions for P_G^* and P_G^{**} we

[21] We make this simplifying assumption for expositional purposes. Since both banks have the same probability of a claim, the insurer would be indifferent between the two for a given premium. Therefore, we assume they choose to contract with the deviating bank. Allowing each insurer to contract with both banks greatly complicates the payoff function of the insurers, but would not change our result.

write

$$2P_G^{**} - P_G^* = (1-p)(1+Z_H)(2q_G(P_G^{**}) - q_B - q_G(P_G^*)) + 2P_B^0 - q_B(1-p)(1+Z_L). \tag{32}$$

Using the uniform assumption for q_G and q_B yields:

$$2P_G^{**} - P_G^* = \frac{(1-p)(1+Z_H)}{\overline{\theta} - \underline{\theta}}(2P_G^{**} - P_G^*) + 2P_B^0 - q_B(1-p)(1+Z_L) \tag{33}$$

Therefore,

$$2P_G^{**} - P_G^* = \frac{(\overline{\theta} - \underline{\theta})[2P_B^0 - q_B(1-p)(1+Z_L)]}{\overline{\theta} - \underline{\theta} - (1-p)(1+Z_H)}. \tag{34}$$

Consider the case in which r becomes arbitrarily large, so that $P_B^0 \to 0$ as shown in Lemma 1. It follows that when $Z_H < \frac{\overline{\theta} - \underline{\theta} - (1-p)}{1-p}$, then $2P_G^{**} < P_G^*$. Note that $\overline{\theta} > 1$ and $\underline{\theta} < 0$ (if $\overline{\theta} < 1$, the bad insurer would always default when faced with a claim).

■

Proof of Lemma 4

Since there are effectively four participants in this market, a Bayesian Nash equilibrium is attained when all four have no incentive to change their behavior. The banks choose an insurer and the insurers optimize payoffs through their choice of premia. First, we characterize the payoff functions for both types of banks when insuring with both types of insurer.

$$\Pi(S,G) = p_S R + (1-p_S)q_G - (1-p_S)(1-q_G)Z - P_G \tag{35}$$
$$\Pi(S,B) = p_S R + (1-p_S)q_B - (1-p_S)(1-q_B)Z - P_B \tag{36}$$
$$\Pi(R,G) = p_R R + (1-p_R)q_G - (1-p_R)(1-q_G)Z - P_G \tag{37}$$
$$\Pi(R,B) = p_R R + (1-p_R)q_B - (1-p_R)(1-q_B)Z - P_B \tag{38}$$

We first set up conditions for each bank type which determine with whom they contract. We then consider the behavior of the insurers, which is characterized by the premia.

Pooling at the Good Insurer:

Consider first the case in which the good insurer dominates the market. This will occur when $\Pi(S,G) \geq \Pi(S,B)$ and $\Pi(R,G) \geq \Pi(R,B)$, which are characterized as follows.

$$(1-p_S)(1+Z)(q_G - q_B) \geq P_G - P_B \tag{39}$$
$$(1-p_R)(1+Z)(q_G - q_B) \geq P_G - P_B \tag{40}$$

Should the premia be such that (39) holds, then condition (40) is satisfied by default. Intuitively, these conditions are satisfied when both bank types care more about counterparty risk (Z) and less about the difference in premia between the two insurers. If (39) is satisfied, the (pooling) beliefs of the good insurer are defined by $b_G = (2 - p_R - p_S)/2$, while the beliefs of the bad insurer are defined off the equilibrium path. We use off the equilibrium path beliefs for the bad insurer that make this equilibrium least likely to exist, namely $b_B = 1 - p_S$. When the good insurer dominates, the bad insurer will cut its premium as low as it can and is still driven out of the market. Therefore, the premium the bad insurer charges is $P_B^0\boldsymbol{(1-p_S)}$, where we write the beliefs of the insurer as an argument of the premium function in bold to avoid confusion with the multiplication operator. We now rewrite expression (39) to include beliefs.

$$(1-p_S)(1+Z)(q_G - q_B) \geq P_G\,\boldsymbol{((2-p_R-p_S)/2)} - P_B^0\boldsymbol{(1-p_S)} \tag{41}$$

The good insurer will maximize profit by setting a premium which just satisfies (41) with equality, so that $P_G^*\,\boldsymbol{((2-p_R-p_S)/2)} \geq P_G^0\,\boldsymbol{((2-p_R-p_S)/2)}$. We simply wish to show existence, so we look at the limiting case where $Z \to \infty$. In this case, (41) is clearly satisfied for $P_G^*\,\boldsymbol{((2-p_R-p_S)/2)} = P_G^0\,\boldsymbol{((2-p_R-p_S)/2)}$. It remains to be determined whether either insurer has an incentive to change its premium. Given that the bad insurer sets the lowest possible premium, it only has the possibility of raising its premium. Since no bank contracts with it at $P_B^0\boldsymbol{(1-p_S)}$, then no bank will contract with it at a higher premium. Since the good insurer already appeals to both bank types, it has no incentive to lower its premium. It may however, want to increase its premium to as high as the risky bank can tolerate, losing the safe bank in the process. The premium will be set such that (40) holds with equality where $P_B = P_B^0\boldsymbol{(1-p_S)}$. However, the payoffs for the banks converge in the limiting case, so that

$$\lim_{Z\to\infty}(1-p_S)(1+Z)(q_G - q_B) = \lim_{Z\to\infty}(1-p_R)(1+Z)(q_G - q_B).$$

Effectively, both bank's payoffs are dominated by the aversion to counterparty risk and the insurer has no way of determining bank type. Thus, it cannot extract any extra premium from the risky bank.

Pooling at the Bad Insurer:

We now consider the case in which the bad insurer dominates the market. This will occur when $\Pi(S,G) < \Pi(S,B)$ and $\Pi(R,G) < \Pi(R,B)$, which are characterized as follows.

$$(1-p_S)(1+Z)(q_G - q_B) < P_G - P_B \tag{42}$$

$$(1-p_R)(1+Z)(q_G - q_B) < P_G - P_B \tag{43}$$

Should the premia be such that (43) holds, then condition (42) is satisfied by default. Intuitively, these conditions are satisfied when the banks put little weight on counterparty risk (Z) and are more driven by the premium. If (43) is satisfied, the (pooling) beliefs of the good insurer are defined by $b_B = (2 - p_R - p_S)/2$, while the beliefs of the bad insurer are defined off the equilibrium path. We take off equilibrium path beliefs for the good insurer that make this equilibrium least likely to exist, $b_G = 1 - p_S$. When the bad insurer dominates, the good insurer charges the premium $P_G^0\boldsymbol{(1 - p_S)}$ and still does not insure either type. We rewrite expression (43), explicitly specifying the beliefs.

$$(1 - p_R)(1 + Z)(q_G - q_B) < P_G^0\boldsymbol{(1 - p_S)} - P_B\,\boldsymbol{((2 - p_R - p_S)/2)} \tag{44}$$

The bad insurer will maximize profit by setting a premium which just satisfies (44), so that $P_B^*\,\boldsymbol{((2 - p_R - p_S)/2)} \geq P_B^0\,\boldsymbol{((2 - p_R - p_S)/2)}$. To show existence, we show that (44) holds when $P_B^*\,\boldsymbol{((2 - p_R - p_S)/2)} = P_B^0\,\boldsymbol{((2 - p_R - p_S)/2)}$. Similar to Lemma 2, this can be shown by setting $Z = 0$ and placing appropriate restrictions on the distribution function $F(\cdot)$. Alternatively, as discussed in the proof to Lemma 2, we could analyze the limiting case when the recovery value $\phi \to 1$, so that $Z \to -1$. In this case, (44) is trivially satisfied.

It remains to be determined whether either insurer has an incentive to change its premium. The bad insurer is the only one that may have the incentive to do so. It can raise its premium as high as possible, such that (42) still holds, where $P_G^* = P_G^0\boldsymbol{(1 - p_S)}$. In this case, it gives up the risky bank in exchange for a higher premium. As with the previous case of pooling at the good insurer, we can rule this type of deviation out by considering a limiting case in which the left hand side of inequalities (42) and (43) converge. A value of $Z = 0$ is not sufficient to attain this however, as the good insurer may provide a higher expected return for the risky bank even when $Z = 0$ (when the banks are risk neutral), as it poses less risk of default. Thus we let $Z = 0$ and $p_R \to p_S$ and see that in the limit, the bad insurer has no means by which to insure only the safe bank. Alternatively, we can consider the limiting case with a recovery value, in which Z gets arbitrarily close to -1. In this case, it is easy to see that the left hand side of (42) and (43) both approach 0, so the amount by which the bad insurer can raise its premium and obtain only the safe bank gets arbitrarily small.

Separating Equilibrium:

A separating equilibrium is most likely to exist when there is a significant difference between risky and safe banks. With this in mind, we prove existence by fixing Z and focusing on the limiting case in which the probability that the safe bank makes a claim is arbitrarily small, so that $p_S \to 1$. We will then show that a separating equilibrium cannot exist when Z takes extreme values as with the pooling equilibria discussed above.

In examining this case, it is necessary to be explicit about a bank participation constraint which imposes a ceiling on the premium. Recall that banks receive the penalty Z if they suffer a loss. It follows that if the premium is too high, they will simply choose not to insure. Using expression (1), we can determine the premium under which a bank i, potentially insuring with an insurer j, will be indifferent to purchasing the contract.

$$P_{ij}^{max} = q_j(1 - p_i)(1 + Z) \tag{45}$$

From expressions (35)-(38), the following inequalities characterize the conditions that must hold for a separating equilibrium in which a safe bank insures with the bad insurer, while a risky bank insures with the good insurer.

$$(1-p_S)(1+Z)(q_G-q_B) < P_G\boldsymbol{(1-p_R)} - P_B\boldsymbol{(1-p_S)} \tag{46}$$
$$(1-p_R)(1+Z)(q_G-q_B) \geq P_G\boldsymbol{(1-p_R)} - P_B\boldsymbol{(1-p_S)} \tag{47}$$

We propose a candidate equilibrium in which the risky (safe) bank contracts with the good (bad) insurer, with $P_B^* = P_{SB}^{max}$ and P_G^* that which satisfies (47) with equality. Note that when (47) holds with equality, then (46) must also hold, as $p_R < p_S$.

These premia satisfy (46) - (47), so a risky (safe) bank will indeed contract with a good (bad) insurer. What remains to be shown is that the insurers will not benefit from deviating and offering different premia. Consider the bad insurer first. The premium P_{SB}^{max}, is defined as in expression (45) so that the safe bank is just indifferent to purchasing the contract. If it increases the premium, it will no longer participate in the market. Alternatively, it may wish to reduce its premium and insure both bank types. However, in the limiting case where $p_S \to 1$ this will not be optimal because P_{SB}^{max} is lower than the zero profit pooling premium, $P_{SB}^{max} \leq P_B^0(\mathbf{2 - p_R - p_S/2})$. To see that this is true, note that as $p_S \to 1$, $P_{SB}^{max} \to 0$.

Now consider the good insurer. As the good insurer's premium is that which just satisfies (47) with equality, it cannot increase the premium or it will lose the risky bank and not participate in the market. Alternatively, it may wish to lower its premium and take the whole market (pooling over both types). To do this, P_G^* must fall until (46) is satisfied. In the limiting case, the left hand side of (46) approaches zero, thus, to satisfy this expression we must have $P_G^* \to P_{SB}^{max} \leq P_B^0\boldsymbol{(2-p_R-p_S/2)} < P_G^0\boldsymbol{(2-p_R-p_S/2)}$. Therefore, such a deviation is not profitable and our candidate is an equilibrium.

Now consider the case in which p_S fixed. If $Z \to \infty$, (46) and (47) cannot be simultaneously satisfied. The same is the case when $Z = 0$ (with suitable restrictions on the distribution function $F(\cdot)$), or alternatively for $Z \to -1$ when we allow for recovery values. Therefore, separation will only occur for intermediate values of Z as described in the lemma.

As a final note, we can see that the separating equilibrium in which the safe (risky) bank insures with the good (bad) insurer is easily ruled out. Such an equilibrium would require the following.

$$(1-p_S)(1+Z)(q_G-q_B) \geq P_G\boldsymbol{(1-p_S)} - P_B\boldsymbol{(1-p_R)}$$
$$(1-p_R)(1+Z)(q_G-q_B) < P_G\boldsymbol{(1-p_S)} - P_B\boldsymbol{(1-p_R)}$$

These expressions cannot be satisfied simultaneously since $1-p_S < 1-p_R$.

■

Proof of Proposition 4

Consider one Z_H bank switching from a good to a bad insurer. Default risk of the CCP, $1-q_{CCP}$, changes from $(1-q_G)^{N_G}(1-q_B)^{N_B}$ to $(1-q_G)^{N_G-1}(1-q_B)^{N_B+1}$. Given the probability of a claim, $(1-p)$, the cost in terms of increased counterparty risk follows by subtracting these two numbers. Under Bertrand competition, the benefit of insuring with the bad insurer is the difference in premium $P_G^0 - P_B^0$. Thus, if $P_G^0 - P_B^0 \geq (1-p)(1-q_G)^{N_G-1}(1-q_B)^{N_B}(q_G - q_B)$, every Z_H will unilaterally switch to the bad insurer, so that in equilibrium $N_G = 0$.

■

Appendix B: Formal Analysis of Section 5

We extend the analysis of Section 5 to allow banks to divide their contracts (each of size one) with as many insurers as they wish, and let there be a countably infinite number of good and bad insurers. As defined previously, each insurer receives an independent portfolio draw from the same distribution $F(\theta)$. We enrich the modeling of insurer default and allow aggregate risk that affects bad insurers, but does not affect good insurers. Specifically, with probability $1-q_A$, all of our bad insurers default at once.[22] Therefore, we redefine $1-q_B = 1-\widetilde{q_B}q_A$, where $1-\widetilde{q_B}$ represents the idiosyncratic risk of a bad insurer (which can be thought of as the usual counterparty risk of the bad insurer as in previous sections), and the default risk for a good insurer remains unchanged.

When banks contract with many insurers, it is possible that some default while others do not. Thus, we redefine our cost of default Z as $Z(x)$, where x represents the percentage of a bank's insurers that fail when a claim is made, and where $Z'(x) > 0$, $Z''(x) > 0$ and $Z(0) = 0$. This definition of Z implies the following.

Lemma 5 *When $Z(x)$ is strictly convex, the bank will insure with as many insurers as possible.*

Proof:

Consider the expected profit of a bank that insures a risky asset (that defaults with probability p), with k identical insurers. Since insurers are identical, we consider the size of each contract, $1/k$ and the premium, P as the same across insurers. Profits are characterized as follows.

$$pR + (1-p)\underbrace{\sum_{i=0}^{k}\binom{k}{i}q^{k-i}(1-q)^i\left[\frac{k-i}{k} - Z\left(\frac{i}{k}\right)\right]}_{S_k} - P. \tag{48}$$

Where q $(1-q)$ is the probability that an insurer is solvent (insolvent) in the event of a claim. We first show that increasing the number of insurers keeps the payoff constant if banks were risk

[22] This structure is assumed for simplicity. The good insurers could be exposed to aggregate risk, but to a lesser degree. Furthermore, we could have partially correlated default risk instead of perfectly correlated risk. Neither simplification affects the qualitative results.

neutral (i.e., $Z(x) = 0$ for every x). In step 2 of this proof, we will show that (48) is concave and so our banks will behave in a risk averse manner and so they will choose the number of insurers to minimize the variance of their payoff. In other words, we show that decreasing the number of insurers is a mean preserving spread, and so is not desirable to a risk averse bank; a standard result.

We now set up the payoff function for the bank when it insures with k and $k+1$ insurers (S_k and S_{k+1} respectively). Each term in the sum S_k consists of the claim payed out $(k-i)/k$, less the cost of counterparty risk $Z(i/k)$ when the fraction i/k insurers default (the result of which may be negative) weighted by the probability that i/k insurers default. First, note that regardless of the number of insurers (k), the bank will receive R with probability p and pay the insurance cost P, so we focus solely on the sum S_k. Expanding the sum (past $k = 3$) and separating the claims paid from the cost of default, we have the following.

$$\begin{aligned} S_k &= q^k + \binom{k}{1} q^{k-1}(1-q)\left(\frac{k-1}{k}\right) + \binom{k}{2} q^{k-2}(1-q)^2\left(\frac{k-2}{k}\right) + \ldots + \binom{k}{k-1} q(1-q)^{k-1}\left(\frac{1}{k}\right) \\ &\quad - \sum_{i=0}^{k} \binom{k}{i} q^{k-i}(1-q)^i Z\left(\frac{i}{k}\right) \end{aligned}$$

$$\begin{aligned} S_{k+1} &= q^{k+1} + \binom{k+1}{1} q^{k}(1-q)\left(\frac{k}{k+1}\right) + \binom{k+1}{2} q^{k-1}(1-q)^2\left(\frac{k-1}{k+1}\right) + \ldots + \binom{k+1}{k} q(1-q)^{k}\left(\frac{1}{k+1}\right) \\ &\quad - \sum_{i=0}^{k+1} \binom{k+1}{i} q^{k+1-i}(1-q)^i Z\left(\frac{i}{k+1}\right) \end{aligned}$$

In step 1 we show that when the bank payoff function is linear, i.e., $Z(\cdot) = 0$, then $S_k = S_{k+1}$.

Step 1

Let $Z(x) = 0 \ \forall x$. The insurance payout when there are k insurers is as follows.

$$\begin{aligned} \sum_{i=0}^{k} \binom{k}{i} q^{k-i}(1-q)^i \frac{k-i}{k} &= q \sum_{i=0}^{k-1} \binom{k-1}{i} q^{k-1-i}(1-q)^i \\ &= q(1-q+q)^{k-1} \\ &= q \end{aligned}$$

The first equality follows since the kth term in the sum is zero and $\binom{k}{i} = \binom{k-1}{i}\frac{k}{k-i}$. The second equality follows from binomial theorem $(x+y)^n = \sum_{i=0}^{n}\binom{n}{i}x^i y^{n-i}$, where $y = q$, $x = 1-q$ and $n = k-1$. Similarly, we examine the insurance payout with $k+1$ insurers

$$\sum_{i=0}^{k+1} \binom{k+1}{i} q^{k+1-i}(1-q)^i \frac{k+1-i}{k+1} = q \sum_{i=0}^{k} \binom{k}{i} q^{k-i}(1-q)^i \frac{k+1}{k+1-i}\,\frac{k+1-i}{k+1} = q.$$

Therefore, $S_k = S_{k+1}$ so that changing the number of insurers does not change the payoff for a bank with $Z(\cdot) = 0$. We now show that when $Z(\cdot)$ is convex, the bank will prefer more insurers to fewer.

Step 2

We wish to appeal to standard results on mean preserving spreads and risk aversion. First, it is simple to see that adding more insurers decreases the variance of the payoff. Intuitively, as more insurers are added, it makes extreme events (such as every insurer defaulting at the same time) less likely. To show that our objective function is concave in payoff (or wealth as it is traditionally viewed in standard portfolio theory), we let $y = \frac{k-i}{k}$ be the payout from insurers, and so the cost of default becomes $Z(1-y)$. Ignoring the constant terms, the payoff for a fixed number of insurer defaults (defined as $U(y)$) can be represented as follows.

$$U(y) = y - Z(1-y) \tag{49}$$

The derivative with respect to y is

$$1 + Z'(1-y) > 0. \tag{50}$$

Where the sign follows because $Z'(\cdot) > 0$. Taking and signing the second order condition yields the following.

$$-Z''(1-y) < 0 \tag{51}$$

Where the sign follows because $Z''(\cdot) > 0$. Thus, our objective function is concave. Therefore, since insurance with k insurers is a mean preserving spread of $k+1$ insurers, a bank will insure with as many insurers as possible.

■

Due to the convexity of $Z(\cdot)$, the bank is effectively risk averse. The benefit of insuring with more insurers is that it makes the banks return more predictable by making tail events (e.g., every insurer defaulting and the bank incurring a cost $Z(1)$) less likely. The result that a risk averse agent prefers more predictable returns is then a standard one. Given that there is a countably infinite number of insurers, Lemma 5 tells us that the banks will insure with a continuum of insurers. We now turn to the main result of the section. The following proposition shows that there exists a separating equilibrium wherein a risky bank insurers with only good insurers, and a safe bank insures with only bad insurers.

Proposition 5 *There exists a separating equilibrium when the insurance market is non-exclusive.*

Proof.

Let the beliefs of the insurers correspond to a separating equilibrium in which the risky bank insurers with only good insurers, and the safe bank insures with only bad insurers. Since insurers are identical, we assume that all good (bad) insurers charge P_G (P_B). For expositional purposes, we drop the belief argument on the premia. The payoff of the safe bank if it insures with a measure ψ of bad insurers and $1-\psi$ of good insurers is given as follows. Note that we highlight the argument of $Z(\cdot)$ in bold to emphasize that Z is now a function.

$$\begin{aligned}\Pi(S, \psi B, (1-\psi)G) &= p_S R + \psi(1-p_S)q_B + (1-\psi)(1-p_S)q_G \\ &\quad -(1-p_S)(1-q_A)Z\left(\boldsymbol{\psi}\left(\mathbf{1}-\widetilde{\boldsymbol{q_B}}\right)+(\mathbf{1}-\boldsymbol{\psi})\left(\mathbf{1}-\boldsymbol{q_G}\right)\right) \\ &\quad -(1-p_S)q_A Z\left(\boldsymbol{\psi}+(\mathbf{1}-\boldsymbol{\psi})\left(\mathbf{1}-\boldsymbol{q_G}\right)\right) - \psi P_B - (1-\psi)P_G\end{aligned}$$

We wish to find when the safe bank chooses $\psi = 1$, i.e., insures exclusively with bad insurers. This can be assured when $\Pi(S, \psi B, (1-\psi)G)$ is strictly increasing in ψ. Differentiating our profit function with respect to ψ yields the following.

$$\begin{aligned}\frac{P_G - P_B}{1-p_S} &> (q_G - q_B) + (1-q_A)(q_G - q_B)Z'\left(\boldsymbol{\psi}\left(\mathbf{1}-\widetilde{\boldsymbol{q_B}}\right)+(\mathbf{1}-\boldsymbol{\psi})\left(\mathbf{1}-\boldsymbol{q_G}\right)\right) \\ &\quad + q_A q_G Z'\left(\boldsymbol{\psi}+(\mathbf{1}-\boldsymbol{\psi})\left(\mathbf{1}-\boldsymbol{q_G}\right)\right)\end{aligned} \tag{52}$$

Since $Z''(\cdot) > 0$, the right hand side of (52) is increasing in ψ. To show existence, we use the value that makes condition (52) least likely to hold, $\psi = 1$.

$$\frac{P_G - P_B}{1-p_S} > (q_G - q_B) + (1-q_A)(q_G - q_B)Z'\left(\mathbf{1}-\widetilde{\boldsymbol{q_B}}\right) + q_A q_G Z'\left(\mathbf{1}\right) \tag{53}$$

Repeating a similar exercise for the risky bank, we wish to find when the payoff is decreasing in ψ, i.e., the risky bank insures exclusively with good insurers. The condition under which it would insure with only good insurers is given as follows.

$$\begin{aligned}\frac{P_G - P_B}{1-p_R} &< (q_G - q_B) + (1-q_A)(q_G - q_B)Z'\left(\boldsymbol{\psi}\left(\mathbf{1}-\widetilde{\boldsymbol{q_B}}\right)+(\mathbf{1}-\boldsymbol{\psi})\left(\mathbf{1}-\boldsymbol{q_G}\right)\right) \\ &\quad + q_A q_G Z'\left(\boldsymbol{\psi}+(\mathbf{1}-\boldsymbol{\psi})\left(\mathbf{1}-\boldsymbol{q_G}\right)\right)\end{aligned} \tag{54}$$

Since $Z''(\cdot) > 0$, the right hand side of (54) is increasing in ψ. We use the value that makes condition (54) least likely to hold, $\psi = 0$.

$$\frac{P_G - P_B}{1-p_R} < (q_G - q_B) + (1-q_A)(q_G - q_B)Z'\left(\mathbf{1}-\boldsymbol{q_G}\right) + q_A q_G Z'\left(\mathbf{1}-\boldsymbol{q_G}\right) \tag{55}$$

We now characterize the two equilibrium premia. Given competition within insurer types, an insurer must charge its zero profit premium, conditional on its beliefs about bank type. Therefore, in a separating equilibrium, the premia are: $P_G = P_G^0(\mathbf{1}-\boldsymbol{p_R})$ and $P_B = P_B^0(\mathbf{1}-\boldsymbol{p_S})$. To demonstrate that the separating equilibrium exists, we use the extreme case in which $p_S \to 1$. Since $P_G^0(\mathbf{1}-\boldsymbol{p_R}) > P_B^0(\mathbf{1}-\boldsymbol{p_S})$, it follows that (53) is satisfied and (55) is satisfied for some finite $Z'\left(\mathbf{1}-\boldsymbol{q_G}\right)$.

We are left with determining whether any insurers (or positive measure of insurers) would like to change its premium. It is straightforward to see that the premium cannot increase since that insurer would be removed from the market. Next, if a bad insurer cut its premium in an attempt to gain the risky type, it will make negative profit since it is currently charging its zero profit premium given the safe bank. If it was to insure the risky type as well, it would require a higher premium to break even (i.e., the break-even pooling premium). Next, consider a good insurer who cuts its premium to gain the safe bank in addition to the risky bank. As $p_S \to 1$, (53) implies that it must reduce its premium to $P_G \leq P_B^0\boldsymbol{(1-p_S)}$. In this case it would earn negative profit since Lemma 1 implies that $P_G^0\boldsymbol{(1-p_S)} \geq P_B^0\boldsymbol{(1-p_S)}$ and we know that $P_G^0\boldsymbol{(1-p_S)} < P_G^0\boldsymbol{(2-p_R-p_S/2)}$.

■

The intuition behind this result is as follows. Each bank type holds an infinitesimally small amount with each insurer. There exists a parameter range in which the safe bank insures with the full set of bad insurers, and no good insurers. Conversely, the risky bank insures with the full set of good insurers, and no bad insurers. Banks do not insurer with both insurer types in this case because uncertainty with bad insurers cannot be eliminated because of the aggregate risk. The risky bank prefers to insure with only good insurers to avoid this uncertainty, whereas the safe bank prefers the lower premium at the bad insurers.

In the separating equilibrium, our banks insure with one type of insurer, even though each was permitted to split its contract over insurer type. We can go even further than this in addressing contract non-exclusivity. Although we have not explicitly modeled it here, this equilibrium can still exist with a formal choice of contract size by the banks. Although the banks can attain certainty by insuring with an infinite number of good insurers, they cannot attain this with the bad insurers. If they were able to attain certainty by insuring with an infinite number of bad insurers, then they could replicate the protection that good insurers provide by simply increasing the contract size with bad insurers. However, in the state of the world in which all bad insurers fail due to aggregate risk, a larger contract size with bad insurers will not increase the payout. Therefore, banks cannot replicate good insurance from the bad insurers so that the risky bank prefers to pay the extra premium to insure with the good insurers.