Should Derivatives be Senior?[*]

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Abstract

Derivative contracts, swaps, and repos enjoy "super-senior" status in bankruptcy in that they are exempt from the automatic stay on debt and collateral collection that applies to virtually all other claims. We propose a simple limited commitment corporate finance model to assess the effect of this exemption on firms' cost of borrowing and incentives to engage in swaps and derivatives transactions. Our model shows that while derivatives are value-enhancing risk management tools, super-seniority for derivatives is generally inefficient: collateralization and effective seniority of derivatives shifts credit risk to the firm's creditors, even though it is more efficient if this risk is borne by derivative counterparties. In addition, because super-senior derivatives dilute existing creditors, they may lead firms to take on derivative positions that are too large from a social perspective. Hence, derivatives markets may be inefficiently large in equilibrium.

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Derivatives enjoy special status in bankruptcy. Derivative counterparties are exempted from the automatic stay, and through netting, closeout, and collateralization provisions, they are generally able to immediately close out their positions and collect payment from a defaulted counterparty. Taken together, these provisions effectively make derivative counterparties senior to almost all other claimants in bankruptcy. Consequently, this special treatment in bankruptcy is often referred to as the 'super-seniority' of derivatives (see, e.g., Roe, 2010).

In this paper, we formally analyze the economic consequences of the super-seniority provisions for derivatives in a standard corporate finance model with limited commitment. We argue that, while derivatives are generally value-enhancing through their role as risk management tools, the super-senior status of derivatives is inefficient. The main reason is that collateralization and (effective) seniority of derivative contracts *shifts credit risk* from a firm's derivative counterparties to the firm's creditors. This leads to a loss of surplus: under very general conditions, it is more efficient if this credit risk were borne by derivative counterparties than the firm's creditors. In addition, we show that the super-senior status of derivatives may lead firms to take on derivative exposures that are excessively large from a social perspective (i.e., strictly larger than what is needed to hedge cash flow risk). In particular, in a form of debt dilution, firms acting in the best interests of their shareholders may use derivatives to gamble at the expense of creditors. The result is that in equilibrium derivatives markets are excessively large.

In our model a firm is financing a positive NPV investment with debt. Due to operational cash flow risk, the firm may not have sufficient funds to make required debt payments at an intermediate date. As the firm is not able to pledge future cash flows, it is then forced into a default and liquidation even though continuation would be efficient. We begin our analysis by showing that in this setting derivatives are valuable hedging tools: by transferring resources from high cash-flow states to low cash-flow states, derivatives can reduce, or even eliminate, costly default. Hence, the introduction of derivative markets can raise surplus relative to

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the benchmark case in which no derivatives are available. This result is in line with the existing literature on corporate risk management, which makes the general observation that when firms face external financing constraints and may be forced into inefficient liquidation they generally benefit from hedging cash flow risk (see, e.g., Smith and Stulz, 1985; Froot, Scharfstein, and Stein, 1993). The only novel contribution of our analysis here is to introduce hedging into a model of limited commitment.

The main novelty of our analysis is to consider how the super-seniority of derivatives affects the benefits of hedging. Although some legal scholars have informally argued that there may be costs associated with the exemption of the automatic stay for derivatives (e.g. Edwards and Morrison, 2005; Bliss and Kaufman, 2006; Roe, 2010), we offer the first formal analysis of this issue.[1] The conventional wisdom is that collateralization, super-seniority, and netting lower a firm's cost of hedging and should thus be beneficial overall. We show that this argument is flawed. Simply put, the reason is that, while reducing counterparty risk in derivatives markets, super-seniority also increases credit risk for the firm's creditors. We show that (as long as derivatives markets are not completely frictionless) this shift in risk from derivatives markets to debt markets is inefficient and results in a loss of overall surplus. The intuition for this result is simple and surprisingly robust. By increasing the firm's cost of debt and promised debt repayments, super-seniority of derivatives has the indirect effect of raising the firm's leverage and therefore also the firm's hedging demand. When derivatives markets involve frictions, this increased hedging demand results in a greater deadweight cost, so that credit risk is more efficiently borne in the derivative market than in the credit market. We first illustrate this result by comparing the two polar cases of *senior* and *junior* derivatives, and then show that the same intuition also holds in a more general setup that

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[1]Edwards and Morrison (2005) argue that one potential adverse consequence of the exemption of the automatic stay is that a firm in financial distress may fall victim to a run for collateral by derivatives counterparties. Roe (2010) argues that fully protected derivative counterparties have no incentive to engage in costly monitoring of the firm. In addition, commentators have pointed out that under the current rules firms may have an incentive to inefficiently masquerade their debt as derivatives, for example by structuring debt as total return swaps. In this article, we intentionally abstract away from runs and inefficient substitution away from debt. Our focus is on whether at the heart of the problem (i.e., before introducing to runs or the ability to masquerade debt as derivatives) there is a reason why derivatives should be senior to debt.

allows for endogenous partial collateralization of derivative positions.

We also show that under the status quo of senior derivatives, firms may have an incentive to take on derivative positions that are excessively large from a social perspective. This is the case whenever the payoff from the derivative contract is not perfectly correlated with the operational risk of the firm (in other words, when there is 'basis risk'). The reason is that, in the presence of basis risk, an increase in the firm's derivative position dilutes existing debtholders. The benefits from a unit increase in derivatives exposure fully accrue to the firm, while some of the cost of the derivative position is borne by existing creditors: in the event of default, derivative counterparties get paid before ordinary creditors, so that an increase in the firm's derivative position can leave existing creditors worse off. Effectively, the senior status of derivatives gives firms an incentive to speculate in the derivatives market over and above what is warranted for hedging purposes.

Our model thus predicts that under the status quo equilibrium derivative markets will be inefficiently large: the positions taken in derivatives, swaps and repo markets will be larger than is socially efficient. We show that this incentive to speculate disappears if the special treatment for derivatives in bankruptcy is removed. These results are consistent with the view that the special treatment of derivatives in bankruptcy may be one of the driving forces behind the tremendous growth of derivatives, swaps and repo markets in recent years. In particular, it may explain the further increase in the size of derivatives markets post 2005 bankruptcy reform, which strengthened the special treatment of derivatives in bankruptcy.

To the extent that the favorable bankruptcy treatment of derivatives may lead to inefficiencies, an important question is whether firms can 'undo the law', for example by committing not to collateralize derivative contracts, thus stripping them of their effective seniority. In this context, our model suggests that the super-seniority provisions for derivatives might have particular bite for financial institutions. While it may be possible to shield physical collateral from derivative counterparties (for example by granting collateral protection over plant and equipment to secured creditors), it is harder to shield unassigned cash from col-

lateral calls by derivative counterparties when a financial institution approaches financial distress. By the very nature of their business, it is too costly for financial institutions to assign cash as collateral to all depositors and creditors. (This would, almost by definition, eliminate their value added as a financial intermediary.) To the extent that firms are unable to contractually undo the effective super-seniority of derivatives, a change in the bankruptcy code that eliminates the special treatment of derivatives may be welfare-enhancing.

In our model, excessive derivatives positions can result because the firm can dilute existing creditors through activity in the derivative markets. This means that, in addition to the law literature on the bankruptcy exemption for derivatives and the literature on hedging (see the papers mentioned above), our model is also related to the literature on debt dilution, either through risk shifting as in Jensen and Meckling (1976), or through the issue of additional senior debt, as discussed, for example, in Fama and Miller (1972). Moreover, the fine line between hedging and speculation that we highlight in our paper is echoed in a recent paper by Biais, Heider, and Hoerova (2010), who show that when derivatives positions move way out of the money for one of the parties involved, this may adversely affect the counterparty's incentive to manage risk, resulting in counterparty risk.

The remainder paper is organized as follows. Section 1 briefly summarizes the special status of derivative securities in bankruptcy. Section 2 introduces the model. Section 3 analyzes a benchmark case without derivatives. Section 4 analyzes the equilibrium in the presence of derivatives and highlights the role of seniority and collateralization. Section 5 develops an extension of the baseline model that allows for tax benefits of debt. Section 6 concludes.

1　The Special Status of Derivatives

In this section we briefly summarize the special status of derivatives in bankruptcy and explain why derivatives are often referred so as 'super-senior.'[2] Strictly speaking, derivatives are not senior in the formal legal sense.[3] However, derivatives, swaps and repo counterparties enjoy certain rights that regular creditors do not enjoy. While not formally senior, these rights make derivatives effectively senior to regular creditors, at least to the extent that they are collateralized.

The most important advantages a derivative, repo or swap counterparty has relative to a regular creditor pertain to closeout, collateralization, netting, and the treatment of eve of bankruptcy payments, eve of bankruptcy collateral calls, and fraudulent conveyances. First, upon default, derivative counterparties have the right to terminate their position with the firm and collect payment by seizing and selling collateral posted to them. This differs from regular creditors who cannot collect payments when the firm defaults, because, unlike derivative counterparties, their claims are subject to the automatic stay. In fact, even if they are collateralized, regular creditors are not allowed to seize and sell collateral upon default, since their collateral, in contrast to the collateral posted to derivative counterparties, is subject to the automatic stay. Hence, to the extent that a derivative counterparty is collateralized at the time of default, collateralization and closeout provisions imply that the derivative counterparty is *de facto* senior to all other claimants.

Second, when closing out their positions with the bankrupt firm, derivative counterparties have stronger netting privileges than regular creditors. Because they can net out offsetting positions, derivative counterparties may be able to prevent making payments to a bankrupt firm that a regular debtor would have to make.[4]

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[2] The discussion in this section is kept intentionally brief and draws mainly on Roe (2010). For more detail on the legal treatment of derivatives, see also Edwards and Morrison (2005) and Bliss and Kaufman (2006).

[3] As pointed out by Roe (2010, p.5), "The Code sets forth priorities in §§ 507 and 726, and those basic priorities are unaffected by derivative status."

[4] The advantages from netting are best illustrated through a simple example. Suppose that a firm has two counterparties, A and B. The firm owes $10 to A. The firm owes $10 to B, and, in another transaction,

Finally, derivative counterparties have stronger rights regarding eve of bankruptcy payments or fraudulent conveyances. While regular creditors often have to return payments made or collateral posted within 90 days before bankruptcy, derivative counterparties are not subject to those rules. Any collateral posted to a derivative counterparty at the time of a bankruptcy filing is for the derivative counterparty to keep.

Taken together, this special treatment of derivative counterparties puts them in a much stronger position than regular creditors. While they do not have priority in the strict legal sense, their special rights relative to other creditors make derivative counterparties effectively senior. While for most of the remainder of the paper we will loosely refer to derivatives as being senior to debt, this should be interpreted in the light of the special rights end effective priority of derivative counterparties discussed in this section.

2   Model Setup

We consider a firm (or bank) that can undertake a two-period investment project (or loan). The project requires an initial investment F at date 0 and generates cash flows at dates 1 and 2. At date 1 the project generates high cash flow C_1^H with probability θ, and low cash flow $C_1^L < C_1^H$ with probability $1-\theta$. At date 2 the project generates cash flow C_2. Following the realization of the first-period cash flow, the project can be liquidated for a liquidation value L. We assume that $0 \leq L < C_2$, implying that early liquidation is inefficient. For simplicity we normalize the liquidation value at date 2 to zero.

The firm has no initial wealth and finances the project by issuing debt.[5] A *debt con-*

B owes \$5 to the firm. Suppose that when the firm declares bankruptcy there are \$10 of assets in the firm. When creditor B cannot net its claims, he has to pay \$5 into the firm. The bankruptcy mass is thus \$15. A and B have remaining claims of \$10 each, such that they equally divide the bankruptcy mass and each receive \$7.5. The net payoff to creditor B is \$7.5-\$5 = \$2.5. When creditor B can net his claim, he does not need to make a payment to the firm at the time of default. Rather he now has a net claim of \$5 on the bankrupt firm. As before, A has a claim on \$10 on the firm. There are now \$10 to distribute, such that A receives 2/3*\$10 = \$6.66 and creditor B receives 1/3*\$10 = \$3.33. Hence, with netting B receives a net payoff of \$3.33, while without netting he only receives \$2.5.

[5]In the case of a bank, this means that beyond the minimum equity capital requirement, which we normalize to zero, the *banker* must raise the entire amount needed for the loan in the form of deposits. In

tract specifies a contractual repayment R at date 1.[6] If the firm makes this contractual payment, it has the right to continue the project and collect the date 2 cash flows. If the firm fails to make the contractual date 1 payment, the creditor has the right to discontinue the project and liquidate the firm. Liquidation can be interpreted as outright liquidation, as in a Chapter 7 cash auction, or as forcing the firm into Chapter 11 reorganization. In the latter interpretation L denotes the expected payment the creditor receives in Chapter 11. Both the firm and the creditor are risk neutral, and the riskless interest rate is zero. Unless we explicitly state otherwise, for most of our analysis we also normalize the firm's date 1 liquidation value to $L = 0$.

The main assumption of our model is that the firm faces a limited commitment problem when raising financing for the project, similar to Hart and Moore (1994, 1998) and Bolton and Scharfstein (1990, 1996). More specifically, we assume that only the minimum date 1 cash flow C_1^L is verifiable, and that all other cash flows can be diverted by the borrower. In particular, this means that the borrower can divert the amount $C_1^H - C_1^L$ at date 1 if the project yields the high return C_1^H. This means that after the date 1 cash flow is realized the firm can always claim to have received a low cash flow, default and pay out C_1^L instead of R. We also assume that at date 0 none of the date 2 cash flows can be contracted upon. One interpretation of this assumption is that, seen from date 0, the timing of date 2 cash flows is too uncertain and too complicated to describe to be able to contract on when exactly payment is due. Finally, to make financing choices non-trivial, we assume that $C_1^L < F$, such that the project cannot be financed with risk-free debt.

Next, we introduce derivative contracts into the analysis, which as with debt contracts, we do in the simplest possible way. Formally, a derivative contract specifies a payoff that is contingent on the realization of a *verifiable* random variable $Z \in \{Z^H, Z^L\}$. This random variable could be an asset price, a financial index, or a similar variable that is observable to both contracting parties and verifiable by a court. Verifiability is the crucial defining

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what follows, when we interpret the firm as a bank we also take it that the *creditor* is then a bank *depositor*.

[6] In the case of a bank R denotes the gross interest payment on deposits of size F.

characteristic of a derivative contract in our model: the ability to verify the derivative payoff means that in contrast to cash flows generated through the firms real operations, cash flows from derivatives positions can be contracted on without any commitment or enforceability problems.

A *derivative contract* of a notional amount X is a promise by the *derivative counterparty* to pay X to the firm if $Z = Z^L$, against a premium x that is payable from the firm to the derivative counterparty when $Z = Z^H$. For simplicity, we assume that Z^L is realized with the same probability as C_1^L, i.e., $\Pr\left(Z = Z^L\right) = 1 - \theta$. Hence, a long position in the derivative contract pays off with the same probability as receiving the low cash flow C_1^L. The derivative's usefulness for hedging the low cash flow outcome is then determined by the correlation of the derivative payoff with the low cash flow state. We parametrize this correlation through γ. Specifically, we assume that Z^L is realized conditional on $C_1 = C_1^L$ with probability γ:

$$\Pr\left(Z = Z^L | C_1 = C_1^L\right) = \gamma. \tag{1}$$

This means that if $\gamma = 1$, the derivative is a perfect hedge for the low cash flow state, since it pays out in exactly the same states in which the firm receives the low cash flow. When $\gamma < 1$, on the other hand, a long position in the derivative only imperfectly hedges the low cash flow state; with probability $(1 - \theta)(1 - \gamma)$ the derivative does not pay out X even though $C_1 = C_1^{L}.$[7]

When the firm enters a derivative position, the other side of the contract is taken by what we will loosely refer to as the *derivative counterparty*. This derivative counterparty could be a financial institution, an insurance company, or a hedge fund that is providing hedging services to the firm. Typically, providing this type of insurance is not free of costs for the derivative counterparty. For example, faced with a notional exposure of X, the

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[7]Note that we have chosen the unconditional payoff probability of the derivative to coincide with the probability that the low cash flow obtains (both are equal to $1 - \theta$). This is not necessary for the analysis. We could more generally assume that the derivative pays off with probability $1 - p$. Our setup has the convenient feature that when $\gamma = 1$, the derivative is a perfect hedge: it pays if and only if the firm's cash flow is low.

counterparty may face costs as it has to post collateral or set aside capital in order to fulfill capital requirements. In addition, the counterparty may want to hedge some of the exposure created by the derivative. However, if not all of this exposure is fully diversifiable (or if it is only diversifiable at a cost) the derivative counterparty incurs a deadweight cost for each unit of notional protection that it writes to the firm. We capture these costs in the simplest possible way, by assuming that when entering a derivative contract with a notional amount of X, the derivative writer incurs a deadweight *hedging cost* of $\rho(X)$,[8] where $\rho(X)$ is increasing ($\rho'(\cdot) > 0$) and weakly convex ($\rho''(\cdot) \geq 0$).[9] We will explicitly illustrate most of our findings for a linear hedging cost function $\rho(X) = \delta X$. However, qualitatively none of our main findings will depend on this particular functional form, in fact our main results continue to hold as long as $\rho(\cdot)$ is increasing.[10]

We assume that the firm enters the derivative contract only after it has signed the debt contract with the creditor. Moreover, when writing the debt contract the firm and the creditor cannot condition the repayment R on the particular realizations of Z. In other words, the debt contract cannot be made contingent on the realization of Z, and therefore the firm cannot generally commit ex-ante to taking on a particular derivative position.

This assumption reflects the idea that at the ex ante contracting stage it may not be known which business risks the firm needs to or can hedge in the future, and with what derivatives contracts. Contracting on a derivatives position at the ex-ante stage would essentially require the firm be able to fully specify a state-contingent long-term contract, which we rule out. More formally, assume that there is a continuum of Z-variables that may potentially be used to hedge the firm's business risk, but that at the ex-ante contracting stage it is not yet known which of these potential Z-variables will be the relevant one from a risk

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[8] In addition to the direct costs of hedging to the derivative writer, $\rho(X)$ may also contain the cost of potential systemic risk created by the derivative writer.

[9] While we take this cost of hedging as exogenous, the hedging cost could be derived from first principles. For example, in the model of demand-based option pricing of Gârleanu, Pedersen, and Poteshman (2009), the hedging cost arises endogenously because not all of the risk in the derivatives position can be hedged.

[10] The implications of our model are robust to introducing a similar deadweight cost also in debt markets. Please see the discussion on robustness following Proposition 4.

management perspective. However, once the firm is in operation and learns more about its business environment it can determine the relevant variable Z. This lack of knowledge on the relevant random variable Z ex ante, would effectively prevent the firm from contracting on a particular derivative position, or from making the debt contract contingent on the relevant Z-variable. It is then more plausible that the firm will choose its derivative position only after signing the initial debt contract. Note that this assumption also broadly reflects current market practice. Firms usually choose their derivative exposure for a given amount of debt only ex post. Moreover, in practice few (if any) bonds or loans include restrictions on future derivatives positions taken by the debtor.

Derivatives have economic value in our setting, since the correlation between the derivative payoff and the firm's operational risk can be used to reduce the firm's default risk. In particular, because income from a derivative position is verifiable, the derivative can be used to decrease the variability of the firm's cash flow at date 1. This effectively raises the verifiable cash flow the firm has available at date 1. From a welfare perspective this is beneficial, because by raising the low date 1 cash flow, the derivative may allow the firm to reduce the probability of default at date 1. When the derivative is a perfect hedge, it may even allow the firm to finance the project using risk-free debt, completely eliminating default. This reduction in (or elimination of) the probability of default is socially beneficial, because it reduces the probability that the firm is terminated inefficiently at date 1. In the presence of derivatives, the date 2 cash flow C_2 is thus lost less often, leading to a potential increase in surplus. Derivatives increase surplus whenever the gains from more efficient continuation at date 1 outweigh the cost of using derivatives, which is captured by the deadweight hedging cost $\rho\left(\cdot\right)$.

Our formal description of derivatives contracts implicitly assumes that the firm faces no counterparty risk with respect to the payment by the derivative writer, X. We will stick to this simplifying assumption throughout our analysis, as our focus is primarily on counterparty risk emanating from the firm to the derivative writer, i.e., with respect to

the firm's payment of the premium x.[11] This risk is more or less severe depending on the extent to which the derivative contract is collateralized, and how this collateral is treated in bankruptcy.

As discussed in Section 1, under current U.S. bankruptcy law, any cash (or securities) that has been *assigned* by the firm as collateral to the derivatives writer in a *margin account* may be collected by the derivative writer if the firm defaults on its debt (or seeks bankruptcy protection). Typically, swaps and derivatives contracts will contain *termination clauses*, which bring forward the *settlement* of the contract to the time when the firm defaults. In practice, settlement then simply takes the form of the derivatives writer taking possession of the cash collateral in the margin account. Importantly, under current U.S. bankruptcy law, derivatives are exempt from the *automatic stay* that prevents collection of collateral for secured debtholders. This exemption provides a key seniority protection to derivatives that is not available to debtholders. However, any cash the firm holds that has not been assigned as collateral to a derivatives counterparty when the firm files for bankruptcy is stayed under chapter 11.[12] In addition, any cash that has been assigned as collateral to a creditor is also stayed.

The automatic stay exemption in bankruptcy has particular bite for financial firms (banks), for which it is more difficult to shield cash from derivatives counterparties. By the very nature of their business, it is too costly for banks to assign cash as collateral to their depositors and other creditors, and thereby contractually guarantee that creditors are always senior to derivatives counterparties. Assigning cash collateral in this way would simply negate their value added as financial intermediaries. What is more, once a bank is drained of its cash reserves it ceases to operate. The difficulty for banks is then that any cash that

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[11]Note, however, that the basis risk on the derivatives contract could also be interpreted as counterparty risk. For models that explicitly model counterparty risk emanating from the protection seller, see Thompson (2010) and Biais, Heider, and Hoerova (2010).

[12]Similarly, under the current FDIC resolution process there essentially no stay on derivative contracts. If not transferred to a new counterparty by 5pm EST on the business day after after the FDIC has been appointed receiver, derivative, swap, and repo counterparties can close out their positions and take possession of collateral. See, for example, Summe (2010, p.66).

is left unassigned ex ante may be assigned as collateral to derivative counterparties ex post, either as *initial margins* or through *margin calls (variation margin)* by derivatives counterparties. Therefore, the exemption from the automatic stay for derivatives offers derivatives counterparties a form of statutory seniority protection in financial firms that is difficult for these firms to undo contractually.

In what follows, we model the seniority of derivatives by first considering two extreme cases; first the case where derivatives are senior to debt and then the alternative extreme case in which derivatives are junior. The former situation is one where the premium x is fully collateralized, and where cash collateral in the amount of x can be seized by the derivative counterparty in the event of a default on debt payments. The cash the firm assigns as collateral to the derivatives margin account is obtained either from *retained earnings* or from the initial investment by the creditor. Retained earnings can be modeled by assuming that after the firm sinks the set-up cost F at date 0, the project first yields a sure return C_1^L at date 1^-. At that point it is still unknown whether the full period 1 return will be C_1^H or C_1^L; that is, the firm only knows that it will receive an incremental cash flow at date 1 of $\Delta C_1 = C_1^H - C_1^L$ with probability θ, and 0 with probability $(1 - \theta)$. To hedge the risk with respect to this incremental cash flow, the firm can then take a derivative position by pledging cash collateral $x \leq C_1^L$. Alternatively, the cash collateral x can be obtained from the creditor at date 0 by raising a total amount $F + x$ from the creditor. Either way of modeling cash collateral works in our setup.

In the other extreme case when derivatives are junior to debt, the premium x is simply not collateralized. In other words, no cash collateral is assigned to the derivative. Moreover, in this case the debt contract then specifies that it is senior to the derivative claim in bankruptcy. The key question in this polar case is whether the firm can commit not to collateralize its derivative position. Under current U.S. bankruptcy law it is difficult to make such a commitment, for any amount of cash the firm assigns to a derivative counterparty can simply be seized by the derivative writer when the firm files for bankruptcy. It is then

extremely difficult to recover any cash collateral that has been improperly assigned to the derivatives counterparty, so that the derivative is *de facto* senior. However, under different bankruptcy rules, for example if there was a general stay on all attempts to collect collateral, such a commitment may be contractually feasible.

Following the analysis of these two polar cases, we then also consider the more general, intermediate case in which derivatives can be *partially collateralized* by only assigning a limited cash collateral $\overline{x} \leq x$ to the derivatives counterparty. In this case, only the amount $\overline{x}$ can be seized by the derivatives writer in the event of default. The remaining amount the firm owes to the derivatives counterparty, $x - \overline{x}$, is then treated as a regular debt claim in bankruptcy. For simplicity we will assume that this remainder is junior to the claims of the debtholder. In practice, such a claim could be classified in the same priority class as debt. We do not explicitly consider this case, since the *pro-rata* allocation of assets to derivative counterparties and debtholders that arises in this case considerably complicates the formal analysis, without yielding any substantive additional economic insights.

3   Benchmark: No Derivatives

We first describe the equilibrium in the absence of a derivative market. The results from this section will provide a useful benchmark case against which we can evaluate the effects of introducing derivative markets in Section 4.

In the absence of derivatives, the firm always defaults if the low cash flow C_1^L realizes at date 1. We will refer to this outcome as a *liquidity default*. As $C_1^L < F$, the low cash flow is not sufficient to repay the face value of debt. Moreover, the date 2 cash flow C_2 is not pledgeable, and since the firm has no other cash it can offer to renegotiate with the creditor, the firm has no other option than to default when C_1^L is realized at date 1. The lender then seizes the cash flow C_1^L and shuts down the firm, collecting the liquidation value of the asset L. Early termination of the project leads to a social loss of $C_2 - L$, the additional cash flow

that would have been generated had the firm been allowed to continue its operations.

If the high cash flow C_1^H realizes at date 1, the firm has enough cash to service its debt. However, the firm may still choose not to repay its debt. We refer to this choice as a *strategic default*. A strategic default occurs when the firm is better off defaulting on its debt at date 1 than repaying the debt and continuing operations until date 2. In particular, the firm will make the contractual repayment R only if the following incentive constraint is satisfied:

$$C_1^H - R + C_2 \geq C_1^H - C_1^L + S, \tag{2}$$

where S denotes the surplus that the firm can extract in renegotiation after defaulting strategically at date 1. The constraint (2) says that, when deciding whether to repay R, the firm compares the payoff from making the contractual payment and collecting the entire date 2 cash flow C_2 to the payoff from defaulting strategically, pocketing $C_1^H - C_1^L$ and any potential surplus S from renegotiating with the creditor. Repayment of the face value R in the high cash flow state is thus incentive compatible only as long as the face value is not too high:

$$R \leq C_1^L + C_2 - S. \tag{3}$$

The surplus S that the firm can extract in renegotiation with the creditor after a strategic default depends on the specific assumptions made about the possibility of renegotiation and the relative bargaining powers when renegotiation takes place. To keep things simple, we will assume that the creditor can commit not to renegotiate with the debtor and will always liquidate the firm after a strategic default. In this case, $S = 0$.[13]

When the incentive constraint (2) is satisfied, the lender's breakeven constraint (under

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[13]This assumption is not crucial for our analysis. We could alternatively assume that renegotiation is possible after a strategic default. For example, one could imagine a scenario in which the firm has full bargaining power in renegotiation. In this case, after a strategic default, the firm would offer $C_1^L + L$ to the creditor, making him just indifferent between liquidating the firm and letting the firm continue. The surplus from renegotiation to the firm would then be given by $S = C_2 - L$ and the project can be financed whenever $F < C_1^L + L$. With slight adjustments, our results on the priority ranking of derivatives relative to debt (Section 4) also carry through in this alternative specification.

our simplifying assumption $L = 0$) is given by

$$\theta R + (1 - \theta) C_1^L = F, \tag{4}$$

which, given competitive debt markets, leads to an equilibrium face value of debt of

$$R = \frac{F - (1 - \theta) C_1^L}{\theta}.$$

Inserting this value of R into (3) we find that the project can be financed as long as

$$F \leq \overline{F} \equiv C_1^L + \theta C_2. \tag{5}$$

The *social surplus* generated in the absence of derivatives is equal to the firm's expected cash flows, minus the setup cost F:

$$\theta \left(C_1^H + C_2 \right) + (1 - \theta) C_1^L - F. \tag{6}$$

We summarize the credit market outcome in the absence of derivatives in the following Proposition.

Proposition 1 *In the absence of derivative markets, the firm can finance the project as long as $F \leq \overline{F} \equiv C_1^L + \theta C_2$. When the project can attract financing, the face value of debt is given by $R = \left[F - (1 - \theta) C_1^L \right] / \theta$, and social surplus is equal to $\theta \left(C_1^H + C_2 \right) + (1 - \theta) C_1^L - F$.*

Most importantly for the remainder of the paper, Proposition 1 establishes that, in the absence of derivatives, the firm is always shut down after a low cash flow realization at date 1. This early termination results in loss of the date 2 cash flow C_2, which means that the equilibrium is inefficient relative to the first-best (full commitment) outcome. As we will show in the following section, derivatives can reduce this inefficiency by reducing the risk of default at date 1.

4   Financing with Derivatives

We now turn to optimal financing in the presence of derivatives. Before analyzing the effect of adding derivatives to the model, we first establish a preliminary lemma about collateralization of derivatives positions. In particular, Lemma 1 states that once the face value of debt has been set, it is always optimal ex post to maximally collateralize the derivative contract. The reason is that once R is fixed, collateralization of the derivative contract makes hedging cheaper for the firm.

Lemma 1 *Once financing has been secured and the face value of debt R has been set, it is optimal to fully collateralize the derivative position ex post. This is because, the cost of the derivative $x(\overline{x})$ is decreasing in the level of collateralization:*

$$\frac{\partial x(\overline{x})}{\partial \overline{x}} < 0. \tag{7}$$

Lemma 1 illustrates the conventional wisdom supporting the collateralization and effective seniority of derivatives: collateralization and seniority for derivatives makes hedging cheaper, which benefits the firm. By this rationale, it is often also argued that full collateralization and the concomitant seniority of derivative contracts is optimal, and that reducing collateralization or making derivative contracts junior to debt is undesirable, as it raises the cost of the derivative to the firm and makes hedging more expensive.

However, as we will argue below, changing the level of collateralization of derivatives, while holding the face value of outstanding debt constant is not the correct thought experiment. After all, in the event of default, debtholders and derivative counterparties hold claims on the same pool of assets. Varying the collateralization of derivatives must in equilibrium also have an impact on the pricing of the firm's debt. In fact, we will show below that once we allow the firm's terms in the debt market to adjust in response to the level of collateralization in derivatives markets, the argument for full collateralization and effective seniority for derivatives is reversed.

We show this by first considering the two extreme cases: senior derivatives and junior derivatives. These extreme cases contain most of the intuition for why it is optimal to make derivatives junior once we take into account the adjustment of the firm's borrowing costs in response to the treatment of derivatives in bankruptcy. We then show that this result generalizes to the intermediate case in which derivatives can be partially collateralized.

For the remainder of this section we will assume that the firm can commit to taking the optimal (i.e., surplus-maximizing) derivative position in any given priority structure. This abstracts away from the firm's potential incentive to take on an excessively large derivative position if the derivative dilutes existing debtholders. We will come back to the issue of dilution through excessively large derivative positions in Section 4.5, where we show that seniority for derivatives can lead firms to take on excessively large derivative positions.

4.1 Senior Derivatives

Senior derivatives (full collateralization of derivatives) is the natural starting point for our analysis because it most accurately reflects the current special bankruptcy status of derivatives discussed in Section 1. The required premium x for a derivative position of a notional size of X, is determined by the counterparty's breakeven constraint. When derivatives are senior, the derivative counterparty is always paid in full as long as $x \leq C_1^L$. The derivative counterparty then receives a payment of x whenever $Z = Z^H$, which happens with probability θ. When $x > C_1^L$, on the other hand, the counterparty cannot be fully repaid when the firm defaults, and then, as the senior claimant, receives the entire cash flow C_1^L. In the interest of brevity, we will focus on the first case, $x \leq C_1^L$, in the main text. The second case is covered in the appendix.

For the counterparty to break even, the expected payment received must equal the expected payments made, $X(1-\theta)$ plus the deadweight cost of hedging $\rho(X)$. The breakeven constraint is thus given by

$$x\theta = X(1-\theta) + \rho(X), \tag{8}$$

which yields a cost of the derivative of

$$x = \frac{(1 - \theta) X + \rho (X)}{\theta}.$$ (9)

The face value of debt, R, is determined by the creditor's breakeven condition. When derivatives are senior to the creditor and $x \leq C_1^L$, this breakeven condition is given by

$$[\theta + (1 - \theta) \gamma] R + (1 - \theta) (1 - \gamma) (C_1^L - x) = F.$$ (10)

This condition states that the expected payments received by the creditor must equal the initial outlay F. Note that the seniority of the derivative contract becomes relevant in the state when $C_1 = C_1^L$ and $Z = Z^H$, which occurs with probability $(1 - \theta) (1 - \gamma)$. In that case, the derivative counterparty is paid its contractual obligation x before the creditor can receive any payment. This leads to a face value of debt of

$$R = \frac{F - (1 - \theta) (1 - \gamma) (C_1^L - x)}{[\theta + (1 - \theta) \gamma]}.$$ (11)

The derivative can be a valuable hedging tool for the firm. In particular, when $\gamma = 1$ the derivative is a *perfect hedge* against the cash flow risk at date 1, such that the firm can completely eliminate default by taking a suitable position in the derivative market. When $\gamma < 1$, the derivative is only a partial hedge, as it sometimes does not pay X when $C_1 = C_1^L$ and sometimes pays X when $C_1 = C_1^H$. Nevertheless, hedging can still be valuable for the firm. While the derivative cannot eliminate default, it can still reduce the probability of default at date 1. When $\gamma < 1$, debt remains risky even under hedging. Moreover, since default occurs with positive probability when $\gamma < 1$, the seniority of derivatives relative to debt contracts is then relevant: in states in which the firm defaults and owes payments to both the creditor and protection seller, the protection seller will get paid first.

When hedging in the derivative market, under full commitment the *optimal derivative*

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position for the firm is the one that just eliminates default when the date 1 cash flow is low and the derivative pays X. This is achieved by setting

$$X = R - C_1^L. \tag{12}$$

Setting $X = R - C_1^L$, the derivative contract just eliminates default in states when $C_1 = C_1^L$ and $Z = Z^L$ (with probability $(1 - \theta)\gamma$). Increasing the derivative position beyond this level does not generate any additional surplus; it only increases the deadweight hedging cost ρ and is thus inefficient. As the derivative is an imperfect hedge, the firm still defaults when $C_1 = C_1^L$ and $Z = Z^H$ (with probability $(1 - \theta)(1 - \gamma)$). Using (9), (11), and (12) we can characterize the equilibrium under senior derivatives as follows.

Proposition 2 *Senior derivatives. Assume that derivatives are senior and that $x \leq C_1^L$. Under full commitment, the optimal derivative position is given by*

$$X = R - C_1^L. \tag{13}$$

This leads to a an equilibrium face value of

$$R = \frac{\theta F - (1 + \delta)(1 - \gamma)(1 - \theta) C_1^L}{\theta - (1 + \delta)(1 - \gamma)(1 - \theta)}, \tag{14}$$

and cost of the derivative of

$$x = \frac{(1 - \theta + \delta)\left[F - C_1^L\right]}{\theta - (1 + \delta)(1 - \gamma)(1 - \theta)}. \tag{15}$$

To gain intuition on the above results it is useful to consider the special case in which derivatives provide a perfect hedge against the cash flow risk at date 1 ($\gamma = 1$). In this case, debt becomes risk-free ($R = F$), so that the optimal derivative position is given by $X = F - C_1^L$. When the derivative is not a perfect hedge ($\gamma < 1$), on the other hand,

debt remains risky even in the presence of derivatives $(R > F)$ and the required derivative position increases to $R - C_1^L > F - C_1^L$.

The social surplus generated in the presence of derivatives depends on how effective derivatives are at hedging the firm's cash flow risks. In particular, when the derivative has more *basis risk* (lower γ), this reduces the effectiveness of the derivative as a hedging tool and thus the probability of continuation of the firm at date 1, $\theta + (1 - \theta)\gamma$. In addition, basis risk increases the costs of eliminating default, since the required derivative position, $R - C_1^L$, is strictly larger than the derivative position required in the absence of basis risk.

Corollary 1 *Social surplus.* *The social surplus when the firm chooses a derivative position of $X = R - C_1^L$ is given by*

$$\theta C^H + (1 - \theta) C_1^L + [\theta + (1 - \theta)\gamma] C_2 - F - \rho \left(R - C_1^L \right). \tag{16}$$

Derivatives raise social surplus relative to the outcome without derivatives when the gain from a greater likelihood of continuation of $(1 - \theta)\gamma$ outweighs the hedging cost:

$$(1 - \theta)\gamma C_2 - \rho \left(R - C_1^L \right) > 0, \tag{17}$$

where R is given by (14). When hedging costs are linear, this is satisfied whenever the hedging cost is not too large:

$$\delta < \delta^* = \frac{(1 - \theta)\gamma[\theta - (1 - \theta)(1 - \gamma)]C_2}{(1 - \gamma)\gamma(1 - \theta)^2 C_2 + \theta(F - C_1^L)}. \tag{18}$$

Assume for now that (17) is satisfied, so that derivatives can indeed add value. When (17) is satisfied, the socially optimal derivative position is given by $X = R - C_1^L$. When (17) is violated, on the other hand, it is optimal for the firm not to use derivatives at all. Corollary 1 shows that derivatives add value as long as the hedging cost δ is sufficiently low, or equivalently, as long as the setup cost F is not too large. The respective critical values

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for δ or F depend on the derivative's basis risk. In particular, when basis risk increases (γ decreases), this lowers the benefit from derivatives, $(1 - \theta) \gamma C_2$, while raising their cost, $\rho (R - C_1^L)$. While the reduction in benefits from derivatives is immediate from (17), the increase in the cost of derivatives arises from the higher required face value R for lower γ. This, in turn, implies that a larger derivative position is necessary in order to eliminate default in the states in which $C_1 = C_1^L$ and $Z = Z^L$, thus raising the cost of managing risk through derivatives. Hence, an increase in basis risk implies that derivatives add value for a strictly smaller set of combinations of hedging and setup costs.

4.2 Junior Derivatives

We now consider the opposite extreme case, junior derivatives. As before, default by the firm occurs in the low cash flow state at date 1 when the derivative bet does not pay off. This happens again with probability $(1 - \gamma)(1 - \theta)$. Under seniority for derivatives, the protection seller was fully repaid in this state. This changes when derivatives are junior. Now the lender receives the entire cash flow C_1^L in default, whereas the protection seller receives nothing. This changes the protection seller's breakeven constraint, since now the protection seller only receives the premium x with probability $[\theta - (1 - \theta)(1 - \gamma)]$ rather than with probability θ. The protection seller's breakeven constraint is now given by

$$x^S [\theta - (1 - \theta)(1 - \gamma)] = (1 - \theta) X^S + \rho (X^S), \qquad (19)$$

(where the superscript S refers to the fact that debt is senior), which yields

$$x^S = \frac{(1 - \theta) X^S + \rho (X^S)}{\theta - (1 - \theta)(1 - \gamma)}. \qquad (20)$$

Debt is still risky, but since the creditor is now senior to the derivative counterparty, he receives the entire cash flow in the default state, so that the creditor's breakeven constraint

becomes

$$[\theta + (1 - \theta)\,\gamma]\,R^S + (1 - \theta)\,(1 - \gamma)\,C_1^L = F. \tag{21}$$

As a result, the face value of debt for the senior lender is lower than in the case where derivatives are senior:

$$R^S = \frac{F - (1 - \theta)\,(1 - \gamma)\,C_1^L}{\theta + (1 - \theta)\,\gamma}. \tag{22}$$

By the same argument as before, default can be eliminated in the state where $C_1 = C_1^L$ and $Z = Z^L$ by choosing the size of the derivative contract such that

$$X^S = R^S - C_1^L. \tag{23}$$

As before, default still occurs when $C_1 = C_1^L$ and $Z = Z^H$ when the derivative is an imperfect hedge. We can now use (20), (22) and (23) to characterize the equilibrium under junior derivatives.

Proposition 3 *Junior derivatives. Assume that derivatives are junior. Under full commitment, the optimal derivative position is given by*

$$X^S = R^S - C_1^L. \tag{24}$$

This leads to an equilibrium face value of

$$R^S = \frac{F - (1 - \theta)\,(1 - \gamma)\,C_1^L}{\theta + \gamma\,(1 - \theta)}, \tag{25}$$

and cost of the derivative of

$$x^S = \frac{(1 - \theta + \delta)\,[F - C_1^L]}{[\theta - (1 - \gamma)\,(1 - \theta)]\,[\theta + \gamma\,(1 - \theta)]}. \tag{26}$$

Analogously to before, we can use the results from Proposition 3 to calculate the surplus

when derivatives are junior.

Corollary 2 *With junior derivatives, social surplus is given by*

$$\theta C^H + (1 - \theta) C_1^L + [\theta + (1 - \theta)\gamma] C_2 - F - \rho \left(R^S - C_1^L \right). \tag{27}$$

When derivatives are junior, the introduction of derivatives raises social surplus relative to the outcome without derivatives whenever

$$(1 - \theta)\gamma C_2 - \rho \left(R^S - C_1^L \right) > 0. \tag{28}$$

where R^S is given by (25). When hedging costs are linear, this is satisfied whenever the hedging cost is not too large:

$$\delta < \delta^{**} = \frac{(1 - \theta)\gamma [\theta + \gamma (1 - \theta)] C_2}{F - C_1^L}. \tag{29}$$

Proposition 3 and Corollary 2 contain the key economic insight of our analysis. *When debt is senior, the required face value on the debt R^S is lower* than when derivatives are senior. In other words, when debt is senior to derivatives, the firm's cost of debt is lower despite the fact that the firm's hedging costs are higher. This is a striking result, which is robust to many changes in the model, and which is not entirely obvious a priori. The fact that the cost of debt is lower even though hedging costs are higher is critical, in particular, because according to (24) it implies that the size of the optimal derivative position is lower than when derivatives are senior. Indeed, from Corollaries 1 and 2 it is easy to observe that the optimal derivative position under senior debt, $R^S - C_1^L$, is smaller than the optimal derivative position under senior derivatives. As the deadweight cost of hedging is directly proportional to the size of the hedging position, it follows that the increase in the cost of debt that results when derivatives are senior reduces surplus. This is summarized in the following proposition.

Proposition 4 *Comparing surplus under junior and senior derivatives.* *Relative to the case without derivatives, junior derivatives are more likely to raise surplus than senior derivatives. When deadweight costs of hedging costs are linear, in particular, hedging with junior derivatives increases surplus for all $\delta \leq \delta^{**}$ while hedging with senior derivatives increases surplus for all $\delta \leq \delta^*$ where:*

$$\delta^{**} > \delta^*.$$

In addition, when hedging adds value both with senior and junior derivatives, surplus under junior derivatives is unambiguously higher than with senior derivatives. With linear hedging costs the difference in surplus is given by

$$\delta \left(R - R^S \right) = \delta \frac{(1 - \gamma)(1 - \theta)(1 - \theta + \delta)}{[\theta + \gamma(1 - \theta)][\theta - (1 + \delta)(1 - \gamma)(1 - \theta)]} \geq 0 \qquad (30)$$

Thus, the received wisdom that full collateralization and seniority of derivatives is desirable (Lemma 1) reverses once one takes into account the effects of collateralization of derivatives on the cost of debt. Proposition 4 shows that derivatives are more likely to add value when they are junior as opposed to when they are senior. Moreover, surplus is always higher under junior derivatives than under senior derivatives, except in two special cases. First, when the derivative is a perfect hedge ($\gamma = 1$), the firm never defaults, so that seniority of the derivative contract is irrelevant. Second, when there is no deadweight hedging cost ($\delta = 0$) seniority is irrelevant because of the Modigliani-Miller theorem: in frictionless markets, capital structure does not matter.

Robustness. The superiority of senior derivatives that is established in Proposition 4 is robust to a number of variations in the assumptions of our model. Most importantly, we want to stress that our result is not driven by the fact that there is a deadweight cost of hedging in the derivative markets, but no deadweight cost in the debt markets. The easiest way to see this is to consider the reverse case of what we have assumed up to now: if there was a

cost of risk only in the debt market, but not in the derivative market, it would obviously also be optimal to make debt senior, in order to minimize the risk borne by the debtholder. By the same logic, in a model in which there is a cost of risk both in debt and derivatives markets, the cost of risk in the debt market creates an additional reason why debt should be senior: when debt is senior, this minimizes the cost of hedging in the debt market (and thus the required face value of debt), which in turn minimizes the required derivative position, and thus the hedging cost in the derivative market.

To see this more specifically, consider the following example, in which we treat risk in the debt and derivative markets completely symmetrically and still obtain our main result. Assume that parties in each market (the creditor and the counterparty) incur a cost that is proportional to the potential loss they face when their contracts move against them. As before, for the counterparty this cost is proportional to X. For the creditor, this cost is proportional to the loss in case of default, which is given by $F - C_1^L - x$ when derivatives are senior and $F - C_1^L$ when derivatives are junior. It is straightforward to see that making derivatives junior reduces the hedging cost in both markets.

Alternatively, one could also impose a symmetric deadweight cost that is proportional to the volatility of the derivative and the debt contract payoff, respectively. While this is somewhat less tractable than our baseline model, also under this specification the ranking of junior and senior derivatives presented in Proposition 4 is preserved.

4.3   Partial Collateralization

Having compared the polar cases of senior and junior derivatives, we now characterize the equilibrium for the more general case of partial collateralization of the derivative contract. Under partial collateralization, the firm pledges a maximum amount $\overline{x} \leq x$ of collateral to the derivative counterparty.

Since the steps required to calculate the equilibrium are analogous to the discussion in the two polar cases, we illustrate them in the appendix. Intuitively, partial collateralization

makes the derivative contract senior up to the maximum amount $\overline{x} \leq x$. For the remaining amount $x - \overline{x}$, derivative counterparties are not collateralized and hold a regular debt claim. For simplicity we assume that this remaining claim is junior to the debtholder. As in the two polar cases discussed above, an increase in collateralization reduces the cost of the derivative, but increases the firm's cost of debt. We characterize the equilibrium for a general collateralization amount $\overline{x}$ in Proposition 5 below.

Proposition 5 *Partial collateralization.* *When the derivative is partially collateralized up to an amount $\overline{x} \leq x$, the optimal derivative position is given by*

$$X\left(\overline{x}\right) = R\left(\overline{x}\right) - C_1^L. \tag{31}$$

This leads to an equilibrium face value of

$$R\left(\overline{x}\right) = \frac{F - (1-\theta)(1-\gamma)\left(C_1^L - \overline{x}\right)}{\theta + \gamma(1-\theta)}, \tag{32}$$

and a cost of the derivative of

$$x\left(\overline{x}\right) = \frac{(1-\theta+\delta)\left[F - C_1^L\right] - (1-\gamma)(1-\theta)\left[\theta - (1-\gamma)(1-\theta) - \delta\right]\overline{x}}{\left[\theta - (1-\gamma)(1-\theta)\right]\left[\theta + \gamma(1-\theta)\right]}. \tag{33}$$

Proposition 5 shows that the case of partial collateralization lies between the two extreme cases above. We see that as collateralization increases, the cost of the derivative, $x\left(\overline{x}\right)$, decreases. At the same time, however, the required face value of debt increases, as an increase in collateralization of the derivative makes the debt contract riskier. This also means that the required derivative position, $R\left(\overline{x}\right) - C_1^L$, is monotonically increasing in the level of collateralization of the derivative.

This proposition shows that the surplus results from the extreme cases of senior and junior derivatives extend to a general setup with partial collateralization. In particular, as the equilibrium face value of debt rises when derivatives are more collateralized, the required

derivative position is larger. This reduces total surplus because the firm has to incur a larger hedging cost to eliminate default.

Corollary 3 *Surplus with partial collateralization.* *The surplus generated by the introduction of derivatives is decreasing in the level of collateralization of the derivative contract.*

4.4 Default due do Derivative Losses

Up to now we have (implicitly) assumed that the required debt and derivative payment are such that the firm meets its payment obligations when the firm receives the high cash flow C_1^H, but the derivative moves against the firm. While this helped simplify our analysis, this assumption is not quite innocuous. The reason is that the required payment $R\left(\overline{x}\right) + x\left(\overline{x}\right)$ may in fact exceed the available cash, such that the firm cannot meet its payment obligation, or alternatively, $R\left(\overline{x}\right) + x\left(\overline{x}\right)$ may be such that the firm has an incentive not to make its contractual payments and default. We now show that default due to derivative losses is more likely, the higher is the level of collateralization.

The possibility of default due to derivative losses also implies that derivatives can serve as hedging tools only if the ex ante setup cost lies below a cutoff value $F\left(\overline{x}\right)$. This cutoff value is decreasing in the level of collateralization, which means that derivatives can serve as hedging tools for a larger set of ex-ante projects when there is less collateralization.

The reason behind this result is again that a higher level of collateralization of the derivative contract leads to a *larger* overall required payment $R\left(\overline{x}\right) + x\left(\overline{x}\right)$ in states where the derivative moves against the firm. While more collateralization generally decreases the cost of the derivative $x\left(\overline{x}\right)$, this is more than outweighed by the concomitant increase in the face value of debt $R\left(\overline{x}\right)$. This makes default more likely because it increases the chance of fundamental or strategic default in the state when the firm receives the high cash flow, but the derivative moves against the firm.

Proposition 6 *Default due to losses on the derivative position. The firm meets its payment obligations when it receives the high cash flow but the derivative moves against the firm as long as:*

$$R\left(\overline{x}\right) + x\left(\overline{x}\right) \leq \min\left[C_1^H, C_1^L + C_2\right].\tag{34}$$

The higher the level of collateralization for derivatives, the less likely it is that this condition holds:

$$\frac{\partial R\left(\overline{x}\right)}{\partial \overline{x}} + \frac{\partial x\left(\overline{x}\right)}{\partial \overline{x}} = \frac{\delta\left(1-\gamma\right)\left(1-\theta\right)}{\left[\theta - \left(1-\gamma\right)\left(1-\theta\right)\right]\left[\theta + \gamma\left(1-\theta\right)\right]} > 0\tag{35}$$

Proposition 6 shows that both fundamental and strategic default are more likely when the derivative is more highly collateralized. This also implies that

Corollary 4 *Derivatives can be used to hedge the low cash flow state without causing default in the high cash flow state as long as*

$$F \leq F\left(\overline{x}\right) = \Gamma_0 C_1^L + \Gamma_1 \min\left[C_1^H, C_1^L + C_2\right] - \Gamma_2\overline{x}.\tag{36}$$

where Γ_0 and Γ_1 are positive constants and

$$\Gamma_2 = \frac{\left(1-\gamma\right)\left(1-\theta\right)\delta}{\theta + \gamma\left(1-\theta\right) + \delta}\tag{37}$$

Since $\Gamma_2 \geq 0$, $F\left(\overline{x}\right)$ is decreasing in the level of collateralization.

4.5 Hedging or Speculation?

Up to now we have assumed that, irrespective of the treatment of derivatives in bankruptcy, the firm can commit to taking the optimal derivative position. In this section we highlight another major inefficiency that results from the current preferential treatment of derivatives in bankruptcy: in the absence of commitment, the firm may take excessively large derivative positions, essentially speculating in the derivative markets at the expense of its existing creditors.

We illustrate this motive for speculation for the case in which derivatives are senior (full collateralization) and the model parameters are such that derivatives can add value. As we will show, under some conditions the derivative position chosen by the firm will strictly exceed the position chosen by a social planner. By a similar argument, one can also show that under senior derivatives the firm may choose to hold derivatives even in cases when it is socially optimal to hold no derivatives at all.

Assume that (17) is satisfied, so that a social planner would choose a derivative position that just eliminates default: $X = R - C_1^L$, where R is the face value at which the creditor breaks even given the derivative position. As seen before, this face value is given by (14). Now consider the firm's private incentives. Clearly, the firm would never want to take a derivative position that is smaller than X. It may, however, have an incentive to take a derivative position that exceeds X. To see this, consider the firm's objective function below. The firm's privately optimal derivative position X^B maximizes the firm's private payoff, subject to the constraint that $X^B \geq R - C_1^L$:

$$
\max_{X^B \geq R - C_1^L} \theta \left[C_1^H - R + \frac{1-\theta}{\theta} (1-\gamma) X^B - \left[1 - \frac{1-\theta}{\theta} (1-\gamma) \right] x \left(X^B \right) \right]
$$
$$
+ (1-\theta) \gamma \left[C_1^L + X^B - R \right] + \left[\theta + (1-\theta) \gamma \right] C_2. \tag{38}
$$

where the premium $x \left(X^B \right)$ the firm pays for the derivative is determined by the protection seller's breakeven constraint (8).

When choosing a derivative position, the firm maximizes (38), taking the face value of debt R, which has already been determined, as given. To see why the firm may over-speculate in derivatives markets, it is instructive to look at the firm's marginal payoff from increasing its derivative position beyond X, the optimal derivative position:

$$
\underbrace{1-\theta}_{\text{marginal derivative payoff}} - \underbrace{\left[1 - \frac{1-\theta}{\theta} (1-\gamma) \right]}_{\leq 1} \underbrace{\left[1 - \theta + \rho'(X) \right]}_{\text{marginal cost of derivative}} \gtrless 0 \tag{39}
$$

The first term is the extra derivative payoff to the firm from increasing its derivative position by one unit beyond X. It is equal to $(1-\theta)$ because an increase in the derivative's notional value generates an additional dollar for the firm with probability $(1-\theta)$. The second term is the share of the marginal cost of an additional unit of the derivative that is borne by the firm. The full marginal cost of an additional unit in notional derivative exposure is given by its actuarially fair marginal cost $(1-\theta)$ plus the increase in the hedging cost $\rho'(X)$. However, this cost is only borne by the firm in states in which it is the residual claimant. In the default state, the marginal cost of the derivative is paid by the creditor. Hence, since the firm does not internalize the full cost of increasing its derivative position beyond X, it may have an incentive to over-speculate. This is the case when the dilution effect on existing creditors outweighs the share of the hedging cost borne by the firm. The social cost of overinsurance is given by the increase in deadweight hedging costs, $\rho\left(X^B\right) - \rho\left(X\right)$.

To illustrate this explicitly, consider the firm's privately optimal derivative position under linear hedging costs. Using (39), we find that the firm's privately optimal derivative position coincides with the optimal derivative position when the derivative has relatively little basis risk $\gamma \geq \overline{\gamma}$. When the derivative has significant basis risk, $\gamma < \overline{\gamma}$, on the other hand, the firm will enter a derivative position that is too large from a social perspective. This implies that the firm will choose to over-speculate in derivatives markets whenever the derivative's basis risk is sufficiently large. Given a linear hedging cost, when the firm chooses to over-speculate it will choose a derivative position that completely expropriates the creditor in the default state (it will choose a position X^B such that $x\left(X^B\right) = C_1^L$). This is summarized in the following Proposition.

Proposition 7 *Senior derivatives lead to excessively large derivatives positions. When the firm cannot commit to a derivative position ex ante, the firm's privately optimal derivative position coincides with the optimal derivative position only if $\gamma \geq \overline{\gamma}$. When $\gamma < \overline{\gamma}$,*

the firm enters a derivative position that is too large from a social perspective, where

$$\overline{\gamma} = 1 - \frac{\delta\theta}{(1-\theta)(1-\theta+\delta)}. \tag{40}$$

Given the linear hedging cost, when the firm chooses to over-speculate it chooses a derivative position that completely expropriates the creditor in the default state: it chooses a position X^B such that $x\left(X^B\right) = C_1^L$. This yields

$$X_{\gamma<\overline{\gamma}}^B = \frac{\theta}{1-\theta+\delta}C_1^L. \tag{41}$$

The social loss from excessively large derivative positions in the linear cost case is then given by

$$\delta(X^B - X) = \delta\left[\frac{\theta}{1-\theta+\delta}C_1^L - \frac{\theta}{\theta-(1-\theta)(1-\gamma)(1+\delta)}(F - C_1^L)\right]. \tag{42}$$

The incentive to over-speculate in derivatives markets, however, disappears when derivatives are junior. To see this, note that the borrower's surplus is unchanged relative to (38), except that the premium for the derivative $x(X^B)$ is now determined by (19):

$$x\left(X^B\right) = \frac{(1-\theta)X^B + \rho\left(X^B\right)}{\theta-(1-\theta)(1-\gamma)}. \tag{43}$$

Taking the first-order condition with respect to X^B shows that with junior derivatives the firm has no incentive to take an excessively large derivative position. The marginal payoff from increasing the derivative position beyond $X^S = R^S - C_1^L$ is given by $-\rho'\left(R^S - C_1^L\right) < 0$. This is what one should expect: with junior derivatives the firm bears the full marginal cost of an additional unit of derivative exposure. Since the derivative is priced at actuarially fair terms net of the deadweight hedging cost, on net the firm cannot gain from increasing its derivative exposure.

Proposition 8 *Junior derivatives lead to efficient derivative positions.* *When derivatives are junior, the private incentives of the firm are aligned with surplus maximization and the firm has no incentive to over-speculate in derivative markets. The firm chooses the efficient derivative position.*

Our model thus predicts that under the current exemption of derivatives from the automatic stay in bankruptcy, derivatives markets are larger than they would be absent the special treatment. In fact, firms may have an incentive to take derivatives exposures that are excessively large from a social perspective. This result occurs even though derivatives are fundamentally value-enhancing in our model as risk management tools. As an immediate consequence, aggregate derivative markets are larger when derivatives are senior to other claims on the firm. This may explain why, over the years, industry organizations that may have an interest in increasing the open interest in derivatives markets, such as the International Swaps and Derivatives Association (ISDA), have lobbied to preserve (or even extend) the special status of derivatives in bankruptcy (see Morgan (2008)). However, as our model shows, this increase in the size of the derivative market may in fact be undesirable from a welfare perspective.

Proposition 7 shows that the incentives to take on excessively large derivative positions are tightly linked to the basis risk of the derivative contract available to the firm. As shown above, when the derivative has no basis risk, or when basis risk is sufficiently small, the firm has no incentives to take excessively large positions. When, on the other hand, there is a sufficient amount of basis risk, the firm may have an incentive to take on excessive derivative positions, thereby diluting existing creditors. Rather than being a hedging tool, the derivative becomes a vehicle for speculation.

A natural question to ask is what would happen if the firm had a choice of derivative instruments? Would it choose to hedge, or would it choose to speculate at its creditors' expense? To answer this question, imagine that after signing the debt contract and after identifying the relevant risks to hedge (i.e. after observing the relevant Z-variable), the

firm can choose among a number of derivative contracts that differ in their basis risk: $\gamma \in$ $[\gamma_{\min}, \gamma_{\max}]$.

To gauge the firm's incentives, reconsider the objective function (38). In particular, note that this objective function is linear in γ. This implies that when the firm has a choice of γ it will choose one of two extreme strategies. Either it is optimal for the firm to maximize the hedging benefits of the derivative by choosing $\gamma = \gamma_{\max}$. Alternatively, however, it may be optimal for the firm to maximize speculation by choosing $\gamma = \gamma_{\min}$. In this case the firm minimizes the hedging benefit of the derivative and maximizes the dilution effect on existing creditors. Differentiating (38) with respect to γ, we see that the firm has an incentive to hedge whenever

$$C_1^L + C_2 - R - x\left(X^B\right) > 0. \tag{44}$$

This implies that the firm has an incentive to hedge whenever, the continuation value C_2 is sufficiently large. When C_2 is small, on the other hand, it is (ex-post) optimal for the firm to choose a derivative instrument with maximum basis risk, in order to dilute existing creditors through speculation in the derivative market.

Proposition 9 *Choice of basis risk.* *Under no commitment, when the firm can choose the basis risk of the derivative, the firm speculates (chooses maximum basis risk) when $C_1^L + C_2 - R - x\left(X^B\right) < 0$. The firm hedges (chooses minimum basis risk) when the reverse condition holds.*

5   Hedging and Taxes

So far we have abstracted from corporate tax considerations that may affect the optimal choice of priority ordering of derivatives and debt. If the firm can benefit from a *tax shield* in the form of interest deductions from its taxable earnings, as is the case in practice, a natural question to ask is whether the presence of such a debt tax shield may affect our conclusion that it is optimal for debt to be senior to derivatives. To be able to address

this question we introduce a corporate tax rate $\tau > 0$ into our model. Specifically, we now assume that the firm must pay corporate taxes on its earnings exceeding debt repayments. Moreover, we allow the firm to make a leverage decision by raising more than the required setup cost F at date 0. For simplicity we assume that any excess financing raised through the promised face value R is spent (consumed) by the firm at date 0.

No derivatives: Consider first the firm's optimal choice of leverage in the absence of derivatives. As the firm's low earnings C_1^L at date 1 are too low to be able to meet its debt repayments R, it does not pay any corporate taxes when C_1^L is realized. But when the firm's high earnings C_1^H are realized it now also faces a tax liability of $\tau(C_1^H - R)$. In addition, the date 2 earnings C_2 are also taxable and result in a date 2 tax bill of τC_2.

As before, the firm can still choose to default strategically when its date 1 realized earnings are C_1^H, in which case the firm diverts the difference in earnings $C_1^H - C_1^L$ and is liquidated.[14] As a result of corporate taxation, the firm has more to gain from strategic default, as the after-tax value of continuation under truthful disclosure of its earnings is lower: The firm's incentive constraint is now given by

$$(1 - \tau)(C_1^H - R) + (1 - \tau)C_2 \geq C_1^H - C_1^L. \tag{45}$$

As can be readily seen from this constraint, the higher is the corporate tax rate τ, the lower is the maximum repayment R that the firm can credibly promise to make at date 1 conditional on earning C_1^H.[15]

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[14]The firm can divert $C_1^H - C_1^L$ by reporting that its realized earnings were only C_1^L to its creditors and the tax authorities. Given that the firm faces no tax liabilities when it reports C_1^L it is able to divert the full amount $C_1^H - C_1^L$.

[15]Note that our modeling of corporate taxation in the context of a model of corporate financing with limited commitment implicitly assumes that the tax authorities have a better collection technology available for collecting tax liabilities than creditors. In fact, we assume that in the face of evidence of positive earnings (such as the firm's ability to meet its debt obligations) the tax authorities are able to fully collect the firm's tax liability. While this is clearly an extreme assumption, which we make for simplicity, it is in line with the existing literature. See Desai, Dyck, and Zingales (2007) for an analysis of corporate taxation and corporate governance and for evidence consistent with the view that strategic default is worsened with higher corporate taxes in environments with weak governance (i.e., worse commitment problems for firms). Nonetheless, we could relax this assumption by allowing for imperfect tax collection and still obtain our main results on the optimality of senior debt.

Given this corporate tax regime, what is the optimal capital structure for the firm? In other words, what is the optimal level of the promised debt repayment R? Except for the limited commitment problem, our model is a standard binomial example of a static tradeoff theory of leverage (see, e.g., Brennan and Schwartz (1978)). In a pure tradeoff analysis of expected bankruptcy costs versus tax shield benefits in the context of our model there can only be two candidate values for optimal debt repayments at date 1, $R = C_1^L$ or $R = C_1^H$. The lower level maximizes the debt tax shield subject to avoiding default and costly liquidation, while the higher level provides a complete tax shield at the risk of liquidating the firm should date 1 realized cash flow be C_1^L.

As can be immediately seen, in a model with limited commitment the classical tradeoff analysis no longer applies: the firm would not be able to raise sufficient financing to cover the setup cost F by promising only $R = C_1^L$, and a promise of $R = C_1^H$ may not be credible as it may violate the firm's incentive constraint (45). The only feasible levels of R with limited commitment are such that

$$R \geq \underline{R} \equiv \frac{F - (1 - \theta)\, C_1^L}{\theta}, \tag{46}$$

so that the firm is able to raise sufficient financing to cover the setup cost F, and

$$R \leq \overline{R} \equiv C_1^H + C_2 - \frac{C_1^H - C_1^L}{1 - \tau}, \tag{47}$$

so that the firm's incentive constraint (45) holds.

Without much loss of generality we henceforth assume that τ is sufficiently high, such that $C_2 \leq \left(C_1^H - C_1^L \right) / (1 - \tau)$, and therefore that $\overline{R} \leq C_1^H$. Under this assumption, it is easy to see that the optimal level of promised repayment R^* is such that the firm's incentive constraint (45) binds, $R^* = \overline{R}$. Suppose, by contradiction that $R < \overline{R}$, then the firm's expected payoff is given by

$$\theta(1-\tau)(C_1^H - R + C_2) + [\theta R + (1-\theta)C_1^L - F]. \tag{48}$$

The first term represents the firm's expected earnings after debt repayments and taxes, and the second term represents the firm's financial slack (the firm's cash holdings net of the capital expenditure F).[16] Consider now an incremental promise of $dR > 0$ above R. This increases the firm's expected payoff by $\theta\tau dR > 0$, so that any promise $R < \overline{R}$ cannot be optimal. In sum, the firm's optimal choice of leverage in the absence of derivatives is given by the highest incentive compatible promised repayment $\overline{R}$. This is the repayment that maximizes the tax shield benefits of debt.

Derivatives in the presence of taxes: We now let the firm also take a derivative position X with basis risk, i.e., $\gamma < 1$. That is, as before the firm agrees to pay a premium x in the event that the random variable $Z \in \{Z^L, Z^H\}$ takes the value Z^H against the payment X by the insurance seller in the event that $Z = Z^L$.[17]

Consider first the situation where the *derivative is senior*, such that in default the counterparty is paid off first and receives x, while creditors only receive $C_1^L - x$. As before, given the hedging cost, the optimal derivative position just hedges the firm's operational risk, i.e., $X = R - C_1^L$. Given this derivative position, the firm's incentive constraint (45), in the presence of corporate taxes, becomes:[18]

$$(1-\tau)(C_1^H - x - R) + (1-\tau)C_2 \geq C_1^H - C_1^L \tag{49}$$

Note that the premium x is a cost that reduces taxable earnings in this state of nature. Should the firm choose to strategically default by hiding its true realized gross earnings C_1^H

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[16] Against a promised repayment of R, creditors are willing to lend a maximum amount, at zero market interest rates, of $\theta R + (1-\theta)C_1^L$.

[17] As before the random variable Z is positively correlated with the firm's date 1 cash flow, with $\Pr(C_1^H \wedge Z_H) = \theta - (1-\theta)(1-\gamma)$, $\Pr(C_1^H \wedge Z_L) = (1-\theta)(1-\gamma)$, $\Pr(C_1^L \wedge Z_H) = (1-\theta)(1-\gamma)$, and $\Pr(C_1^H \wedge Z_L) = (1-\theta)\gamma$.

[18] Note that there is also an incentive constraint that governs strategic default in the state $C_1^H \wedge Z^L$. However, since $X = R - C_1^L$, the firm's taxable income in this state is given by $C_1^H + X - R = C_1^H - C_1^L$, such that the firm's payoff and incentives to default strategically in this state are independent of R.

and reporting only C_1^L it would still be able to divert the amount $C_1^H - C_1^L$, which explains the form of the modified constraint (49).

As long as the cost of hedging δ is not too high, the same reasoning as in the case without derivatives leads to the conclusion that it cannot be optimal to set

$$R < \overline{R}(x) \equiv C_1^H - x + C_2 - \frac{C_1^H - C_1^L}{1 - \tau}. \tag{50}$$

However, this now no longer implies that $R = \overline{R}(x)$, as the firm could choose to strategically default in equilibrium in state $C_1^H \wedge Z^H$ and only repay the promised amount R in the states $C_1^H \wedge Z^L$ and $C_1^L \wedge Z^L$ when it gets the insurance payment $X = R - C_1^L$. We will show, however, that it is optimal for the firm to have debt be senior to derivatives whether it chooses an optimal promised repayment $R = \overline{R}(x)$ (such that the incentive constraint is satisfied) or $R > \overline{R}(x)$ (such that the incentive constraint is violated).

Consider first the case where there is no strategic default in equilibrium and $R = \overline{R}(x)$. In this case the premium paid by the firm on the senior derivative position is given by:

$$x = \frac{(1 - \theta + \delta)\left[\overline{R}(x) - C_1^L\right]}{\theta}, \tag{51}$$

and, from the incentive constraint (49) we know that

$$\overline{R}(x) + x = C_1^H + C_2 - \frac{C_1^H - C_1^L}{1 - \tau}. \tag{52}$$

In contrast, when *derivatives are junior* the optimal promised repayment under no equilibrium strategic default is given by:[19]

$$\overline{R}(x^S) + x^S = C_1^H + C_2 - \frac{C_1^H - C_1^L}{1 - \tau}, \tag{53}$$

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[19]Recall that the superscript S refers to the case in which debt is senior and derivatives junior.

where from the counterparty's breakeven condition we know that

$$x^S = \frac{(1 - \theta + \delta)\left[\overline{R}(x^S) - C_1^L\right]}{\theta - (1 - \gamma)(1 - \theta)}. \tag{54}$$

Substituting for the equilibrium values of x and x^S and solving for the equilibrium expressions for $\overline{R}(x^S)$ and $\overline{R}(x)$ it is straightforward to show (see appendix for details) that $\overline{R}(x^S) < \overline{R}(x)$. In other words, the face value of debt that maximizes the tax benefit is lower under junior derivatives than under senior derivatives. This, in turn, implies that the required derivative position and the concomitant deadweight cost of hedging are smaller when derivatives are junior, i.e., $\delta X^S < \delta X$.

Consider next the situation where the firm strategically defaults in equilibrium in state $C_1^H \wedge Z^H$. In this case the firm only pays taxes in state $C_1^H \wedge Z^L$, where its tax liability is given by

$$(C_1^H + X - R)\tau = (C_1^H - C_1^L)\tau, \tag{55}$$

as $X = R - C_1^L$. Since in this case the firm's tax liability is independent of the level of promised repayment R, it is optimal for the firm to minimize the face value R so as to minimize the deadweight cost of insurance $\delta X = \delta(R - C_1^L)$. As in the analysis without corporate taxes, the required face value of debt R is minimized when derivatives are junior: $R^S < R$.

We thus conclude that also in the presence of corporate taxation it is optimal for the firm to have debt be senior to derivatives. The argument for why debt should be senior to derivatives in the absence of corporate taxation essentially transposes to the case where the firm is subject to corporate taxes: it is less costly for the firm to maximize the tax benefits of deb when derivatives are junior to debt. This was not obvious *a priori* as the higher face value of debt when derivatives are senior would seem to imply a benefit in the form of a higher debt tax shield. However, as we have shown, this potential benefit of a higher debt tax shield is always outweighed by the higher deadweight cost of hedging.

6 Conclusion

This paper develops a simple model to analyze in a tractable and transparent way the implications of granting super-seniority protection to derivatives, swaps, and repos. These protections have been put in place with the main objective of providing stability to derivatives markets, without any systematic analysis of the likely consequences for firms' overall costs of borrowing and hedging incentives. The presumption of the ISDA and policy makers has basically been that the effects of super-protection of derivatives on firms' cost of debt are negligible and do not require any in-depth analysis. Our analysis suggests, however, that the strengthening of derivatives' treatment in bankruptcy may have been socially harmful. While seen in isolation the super-protection lowers the cost of hedging, this is more than offset by a greater cost of debt and a greater incentive to over-hedge. Based on our analysis, it appears that, at a minimum, further research is required into the consequences for firms' cost of borrowing before one can conclude that the super-priority status of derivatives is warranted.

7 Appendix

Proof of Lemma 1: The steps needed to calculate the cost of the derivative as a function of the level of collateralization $\overline{x}$ are given below in the section characterizing the equilibrium under partial collateralization. Holding R fixed and assuming that $\overline{x} \leq C_1^L$, we know that

$$x\left(\overline{x}\right) = \frac{(1-\theta)\left[R - C_1^L\right] + \rho\left(R - C_1^L\right) - (1-\theta)\left(1-\gamma\right)\overline{x}}{\theta - (1-\theta)\left(1-\gamma\right)}. \tag{56}$$

This implies that, when R is held fixed,

$$\frac{\partial x\left(\overline{x}\right)}{\partial x} = -\frac{(1-\theta)\left(1-\gamma\right)}{\theta - (1-\theta)\left(1-\gamma\right)} < 0.$$

This means that when we take face value of debt as given, the cost of the derivative

is decreasing in the level of collateralization of the derivative as long as $\bar{x} \leq C_1^L$. When $\bar{x} > C_1^L$, a further increase in collateralization does not change the payoff of the derivative counterparty, such that in this region the cost of the derivative is unchanged.

Senior Derivatives when $x > C_1^L$: In this section we describe the equilibrium under senior derivatives when $x > C_1^L$, which we left out in the main body of the text for space considerations. The main difference to the case discussed in the text is that the equations that the breakeven conditions for the derivative counterparty and the creditor change. When $x > C_1^L$, when the firm defaults, the derivative counterparty receives the entire cash flow, while the creditor receives nothing. Hence, the equilibrium is characterized by

$$X = R - C_1^L \tag{57}$$

$$R = \frac{F}{\theta + \gamma (1 - \theta)} \tag{58}$$

$$x = \frac{(1 - \theta) X + \rho (X) - (1 - \gamma) (1 - \theta) C_1^L}{\theta - (1 - \theta) (1 - \gamma)}. \tag{59}$$

Under linear hedging costs, we can solve for x in terms of the underlying parameters:

$$x = \frac{F (1 - \theta + \delta)}{[\theta - (1 - \gamma) (1 - \theta)] [\theta + \gamma (1 - \theta)]} - \frac{C_1^L [1 - \theta + \delta + (1 - \gamma) (1 - \theta)]}{[\theta - (1 - \gamma) (1 - \theta)]} \tag{60}$$

$$= \frac{(1 - \theta + \delta) [F - C_1^L] - (1 - \gamma) (1 - \theta) [\theta + \gamma (1 - \theta) - (1 - \theta + \delta)] C_1^L}{[\theta - (1 - \gamma) (1 - \theta)] [\theta + \gamma (1 - \theta)]} \tag{61}$$

Characterization of Equilibrium under Partial Collateralization: This section contains the breakeven conditions used to derive the equilibrium under partial collateralization (Proposition 5). Under partial collateralization, the required derivative position is given by

$$X (\bar{x}) = R (\bar{x}) - C_1^L. \tag{62}$$

The creditor's and derivative counterparty's breakeven conditions are given by

$$[\theta + \gamma (1 - \theta)] R + (1 - \theta)(1 - \gamma)(C_1^L - \overline{x}) = F \tag{63}$$

$$[\theta - (1 - \theta)(1 - \gamma)] x (\overline{x}) + (1 - \theta)(1 - \gamma) \overline{x} = (1 - \theta) [R (\overline{x}) - C_1^L] + \rho (R (\overline{x}) - C_1^L),$$

which implies that, under linear hedging costs,

$$R (\overline{x}) = \frac{F - (1 - \theta)(1 - \gamma)(C_1^L - \overline{x})}{\theta + \gamma (1 - \theta)} \tag{64}$$

$$x (\overline{x}) = \frac{(1 - \theta)[R (\overline{x}) - C_1^L] + \delta [R (\overline{x}) - C_1^L] - (1 - \theta)(1 - \gamma) \overline{x}}{\theta - (1 - \theta)(1 - \gamma)} \tag{65}$$

Substituting (64) into (65) yields the expression for $x (\overline{x})$ given in the Proposition.

Proof of Proposition 6: Assume that the firm receives the high cash flow C_1^H but has to make a payment $x (\overline{x})$ on its derivative position. The firm will meet its total payment obligation $R (\overline{x}) + x (\overline{x})$ under two conditions. First, the cash available to the firm must be sufficient, which is the case whenever

$$C_1^H - [R (\overline{x}) + x (\overline{x})] \geq 0. \tag{66}$$

Second, the firm must have no incentive to default strategically. This is the case whenever

$$C_1^H - [R (\overline{x}) + x (\overline{x})] + C_2 \geq C_1^H - C_1^L. \tag{67}$$

The left hand side is the payoff from making the contractual payment and continuing, whereas the right hand side is the payoff from declaring default, pocketing $C_1^H - C_1^L$ and letting the creditor and the derivative counterparty split C_1^H. Overall, the firm will thus meet its contractual obligations if

$$R (\overline{x}) + x (\overline{x}) \leq \min [C_1^H, C_1^L + C_2]. \tag{68}$$

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Equation (35) follows from taking the derivatives of equations (32) and (33) and simplifying.

Proof of Corollary 4: The result follows from substituting (32) and (33) into (34) and simplifying. The constants not given in the main text are

$$\Gamma_0 = \frac{(1-\theta)(1-\gamma)[\theta-(1-\gamma)(1-\theta)]+1-\theta+\delta}{\theta+\gamma(1-\theta)+\delta}, \tag{69}$$

$$\Gamma_1 = \frac{[\theta-(1-\gamma)(1-\theta)][\theta+\gamma(1-\theta)]}{\theta+\gamma(1-\theta)+\delta}. \tag{70}$$

Hedging and Taxes: Following the substitution of the equilibrium values of x and x^S we obtain the following two equations:

$$\overline{R}(x^S) + \left(\frac{(1-\theta+\delta)}{\theta-(1-\gamma)(1-\theta)}\right)(\overline{R}(x^S) - C_1^L) = C_1^H + C_2 - \frac{C_1^H - C_1^L}{1-\tau} \tag{71}$$

and

$$\overline{R}(x) + \left(\frac{(1-\theta+\delta)}{\theta}\right)(\overline{R}(x) - C_1^L) = C_1^H + C_2 - \frac{C_1^H - C_1^L}{1-\tau}. \tag{72}$$

Note that the right-hand sides of equations (71) and (72) are the same constants. Using $\overline{R}(x^S) > C_1^L$ and $\overline{R}(x) > C_1^L$ and the fact that

$$\frac{(1-\theta+\delta)}{\theta-(1-\gamma)(1-\theta)} > \frac{(1-\theta+\delta)}{\theta}, \tag{73}$$

it follows that $\overline{R}(x^S) < \overline{R}(x)$.

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