

# Feedback Effects of Commodity Futures Prices\*

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## Abstract

A widely held view posits that when speculators drive up the futures price of a commodity, real demand must fall. This paper develops a model to contrast this view through an informational feedback effect. Our model builds on two practical observations: 1) Futures prices of key industrial commodities such as copper and oil became barometers of global demand in the recent decade as a result of the rapid economic expansions of emerging economies; and 2) complementarity exists in industrial producers' production decisions as a result of their need to trade produced goods. In the presence of information frictions and production complementarity, an increase in commodity futures prices, even if driven by non-fundamental factors, signals strong global economic strength, and may thus induce increased commodity demand.

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# 1 Introduction

The dramatic boom-and-bust cycle of commodity prices in recent years has stimulated intensive debates in academic and policy circles regarding whether speculation in commodity futures markets might have affected commodity prices. Many economists (e.g., Krugman (2008a, 2008b) and Hamilton (2009)) hold the following argument on how speculation affects commodity futures markets: if some optimistic speculators drive commodity prices too high, consumers would reduce demand and commodity inventory would spike. This argument motivates the popular use of joint increases of commodity futures prices and inventory after controlling other fundamental factors to detect speculative effects in commodity markets (e.g., Kilian and Murphy (2010), Lombardi and van Robays (2011), Juvenal and Petrella (2012)). However, this argument ignores potentially important feedback effects of commodity futures prices. This paper develops a model to illustrate such feedback effects.

While the academic literature has long recognized that exogenous oil supply shocks can have a significant, adverse effect on the macroeconomy (e.g., Hamilton (1983)), there is an increasing recognition that demand factors also play important roles in driving commodity prices. In particular, Kilian (2009) provides evidence that oil prices move closely with global economic activity measured by an index of global shipping costs. In recent years, the rapid growth of many emerging economies such as China and India spurred growing demand for many commodities such as copper, oil, and soybeans. This new development on the demand side is also consistent with the significantly increased, positive correlations between commodity prices and stock prices in the US and emerging economies (e.g., Tang and Xiong (2010)). It is now common to see economic reports from the IMF, BIS and other institutions to use commodity prices to gauge the strength of the global economy—especially the strength of the emerging economies. Interestingly, in explaining the ECB’s decision to raise its key interest rate in March 2008 on the eve of the worst economic recession since the Great Depression, ECB policy reports cite high prices of oil and other commodities as a key factor, which reflects the fact that central banks across the world use commodity prices as key indicators in their policy analysis.

Our model highlights that when industrial producers across the world are uncertain about

the strength of the global economy and use commodity prices as barometers, speculation in commodity futures markets may affect commodity demand through an informational feedback channel. Our model features a set of markets related to a certain commodity. The core of the model is a futures market. In this market, industrial producers across the world take long positions to acquire the commodity as an input to their production. A group of financial traders, motivated by the increasing presence of commodity index traders in the commodity futures markets in recent years, also take long positions for investment purposes and unwind their positions before delivery. On the other side of the futures market, a group of intermediaries take short positions and purchase the commodity from decentralized spot markets to make delivery. Like the standard models of asset trading with asymmetric information (e.g., Grossman and Stiglitz (1980) and Hellwig (1980)), the futures market allows the industrial producers to aggregate dispersed private information about the strength of the global economy. Noise in financial traders' trading can affect the equilibrium futures price because other participants cannot separate their trading from producers' demand.

A key deviation of our model from the standard models of financial market equilibrium with asymmetric information is the presence of complementarity in producers' commodity demand. This is a result of people's need to trade produced goods for consumption purposes. As is common in the international macro literature (e.g., Obstfeld and Rogoff (1996)), it is more desirable for one producer to produce more and thus demand more of the commodity when producers of other goods produce more. To capture such complementarity, we adopt a modified setting of Angeletos and La'O (2012) with producers on different islands producing different goods and trading with each other.

Despite the non-linearity in the industrial producers' production decisions, we derive a unique log-linear, noisy rational expectations equilibrium in closed form. In the equilibrium, each producer's commodity demand is a log-linear function of his private signal and the futures price, while the futures price is a log-linear function of the global productivity and noise that originates from financial traders' aggregate position. This tractability originates from a key feature that the aggregate position of a continuum of producers remains log-linear as a result of the law of large numbers.

In our model, the futures price signals to each producer not only about the global pro-

ductivity but also other producers' production decisions. As a consequence, a price increase motivates each producer to increase his commodity demand. This information effect counters the reduced demand driven by an increased cost. The net of the information effect and the cost effect determines the elasticity of the producers' aggregate commodity demand to the futures price. When the complementarity in producers' commodity demand is small, the cost effect always dominates the information effect and the producers' demand elasticity is negative. However, when the complementarity is sufficiently strong, the information effect can dominate and cause the producers' demand elasticity to become positive. That is, the producers increase their commodity demand in response to a rising futures price.

Our model highlights noise in the financial traders' positions as a factor in the equilibrium futures price. As a result of information frictions, unexpected heavy buying by financial traders can lead to a higher futures price, which, under certain conditions, can in turn drive up producers' commodity demand and thus the spot price. This outcome contrasts the aforementioned argument that commodity price distortions caused by futures market speculation have to be accompanied by reduced commodity demand. Instead, our model points out that futures market speculation could distort commodity demand and prices through a new and more subtle channel. Under the taxonomy of Kilian and Murphy (2012) for structural models of commodity markets, this distortion arises through the flow demand of commodities rather than speculative demand (i.e., demand for inventory as opposed to consumption). Our model thus cautions against an over-emphasis on the use of joint increases of commodity prices and inventory as the sole measure of speculative effects in commodity markets.

This paper contributes to the emerging literature that analyzes whether the large inflow of financial investment to commodity futures markets might have affected commodity prices, e.g., Tang and Xiong (2010), Singleton (2011), Cheng, Kirilenko, and Xiong (2012), Hamilton and Wu (2012), Kilian and Murphy (2012), and Henderson, Pearson, and Wang (2012). See Singleton (2011) and Fattouh, Kilian, and Mahadeva (2012) for reviews of these and other related studies. This paper proposes a theoretical framework to highlight an information channel through which speculative trading in futures markets affects commodity demand and spot prices. This framework builds on the premise that financial traders' invest-

ment positions in commodity futures markets are not directly observable to other market participants and thus reinforces information frictions emphasized by Singleton (2011). Our analysis helps empirical researchers design sharper tests of the impacts of financial traders in commodity markets and confirms that policies designed to improve market transparency can help mitigate distortions induced by the information friction.

Our model adopts the setting of Angeletos and La'O (2012) for the goods market equilibrium to derive endogenous complementarity in producers' production decisions. As they focus on endogenous economic fluctuations as a result of the lack of centralized communication channels between different households, their model does not feature any centralized market trading. The centralized futures market and the feedback effects of futures prices thus differentiate our model from theirs.

The economics and finance literature has long recognized that trading in financial markets aggregates information and the resulting prices of financial securities can feed back to real world activities (e.g., Bray (1981) and Subrahmanyam and Titman (2001)). More recently, the literature emphasizes that such feedback effects can be particularly strong in the presence of strategic complementarity in agents' actions. Morris and Shin (2002) show that in such a setting, noise in public information has an amplified effect on agents' actions and thus on equilibrium outcomes. In our model, the commodity futures price serves such a role of public information and feeds back the effects of noise originated from financial traders to producers' production decisions. Similar feedback effects are also modeled in several other contexts, such as from stock prices to firm capital investment decisions and from exchange rates to policy choices of central banks (e.g., Angeletos, Lorenzoni and Pavan (2010) and Goldstein, Ozdenoren, and Yuan (2011, 2012)).

The paper is organized as follows. Section 2 describes our model setting, while Section 3 analyzes the equilibrium. We analyze the feedback effect in Section 4 and conclude the paper in Section 5. We provide the technical proofs to all of the propositions in the appendix.

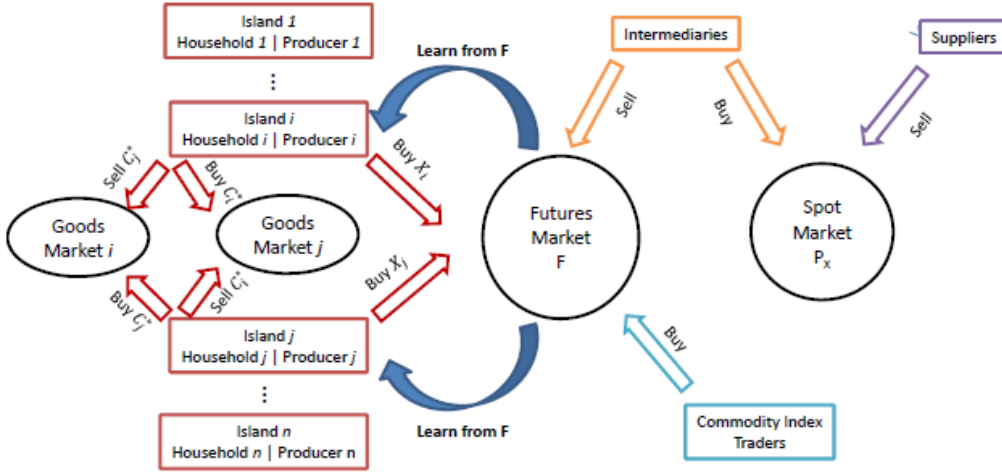


Figure 1: Structure of the Model

## 2 The Model

We develop a model with three dates  $t = 0, 1, 2$  to analyze the market equilibrium related to a commodity. One can think of this commodity as oil or copper, which is used across the world as a key production input. We adopt a modified setup of Angeletos and La’O (2012) to model a continuum of islands of total mass 1. Each island produces a single good, which can either be consumed at “home” or traded for another good produced “away” by another island. Their trading of the produced goods determines the goods’ prices.

In reality, spot markets of commodities are often decentralized, while centralized futures markets provide key platforms for different market participants to aggregate demands and supplies for commodities, and, more importantly, to aggregate private information about global productivity.<sup>1</sup> Our model features a centralized market for a futures contract written on the commodity, in addition to decentralized spot markets for the commodity and for the produced goods. The futures contract is traded at  $t = 0$  for delivery at  $t = 1$ . One can interpret this futures contract as a front-month or second-month futures contract traded in reality, which tends to attract much larger trading volume than more distant futures contracts.

<sup>1</sup>See Garbade and Silber (1983) for evidence that futures markets play a more important role in information discovery than cash markets for a set of commodities.

Figure 1 illustrates the structure of the model, while Table 1 summarizes the timeline. There are five types of agents: households on the islands, goods producers on the islands, a group of commodity suppliers, a group of intermediaries, and a group of commodity index traders. The goods producers trade only in the futures market at  $t = 0$ , and take delivery of the commodity for their production at  $t = 1$ . Their produced goods are distributed to the households on their respective islands at  $t = 2$ . The households then trade their goods with each other and consume. The commodity suppliers do not participate in the futures market and sell commodities in the spot markets to intermediaries, who intermediate between goods producers and commodity suppliers by shorting the futures contract and purchasing the commodity in the spot markets. The commodity index traders represent financial traders who trade the futures contract at  $t = 0$  for investment purposes and who always unwind their futures positions before the delivery at  $t = 1$ .

Table 1. Timeline of the Model

Agent Group	t=0	t=1	t=2
Households			trade and consume goods
Goods Producers	long futures	take futures delivery and produce	
Commodity Suppliers		sell commodity to intermediaries	
Intermediaries	short futures	buy commodity and make delivery	
CITs	long futures	unwind futures position	

## 2.1 Island households

Each island has a representative household. Following Angeletos and La’O (2012), we assume a particular structure for goods trading between households on different islands. Each island is randomly paired with another island at  $t = 2$ . The households on the two islands trade their goods with each other and consume both goods produced by the islands. For a pair of matched islands, we assume that the preference of the households on these islands over the consumption bundle  $(C_i, C_i^*)$ , where  $C_i$  represents consumption of the “home” good while  $C_i^*$  consumption of the “away” good, is determined by a utility function  $U(C_i, C_i^*)$ . The utility function is increasing in both  $C_i$  and  $C_i^*$ . This utility function specifies all “away”

goods as perfect substitutes, so that the utility of the household on each island is well-defined regardless of the matched trading partner. The households on the two islands thus trade their goods to maximize each's utility. We assume that the utility function of the island households takes the Cobb-Douglas form

$$U(C_i, C_i^*) = \left(\frac{C_i}{1-\eta}\right)^{1-\eta} \left(\frac{C_i^*}{\eta}\right)^\eta \quad (1)$$

where  $\eta \in [0, 1]$  measures the utility weight of the away good. A greater  $\eta$  means that each island values more of the away good and thus relies more on trading goods with other islands. Thus,  $\eta$  eventually determines the degree of complementarity in islands' goods production.

## 2.2 Goods producers

Each island has a locally-owned representative firm to organize its goods production. We refer to each firm as a producer. The production requires the use of the commodity as an input. To focus on the commodity market equilibrium, we exclude other inputs such as labor from production. Each island has the following constant-returns-to-scale production function<sup>2</sup>:

$$Y_i = AX_i, \quad (2)$$

where  $Y_i$  is the output produced by island  $i$ ,  $X_i$  is the commodity input, and  $A$  is the common productivity shared by all islands. For simplicity, we ignore the idiosyncratic component of each island's productivity. This simplification is innocuous for our qualitative analysis of how information frictions can affect commodity demand.

The productivity  $A$  is a random variable, which becomes only observable after the producers complete their productions at  $t = 1$ . We assume that  $A$  has a lognormal distribution:

$$\log A \sim \mathcal{N}(\bar{a}, \tau_A^{-1})$$

where  $\bar{a}$  is the mean of  $\log A$  and  $\tau_A^{-1}$  is its variance. At  $t = 0$ , the goods producer on each island observes a private signal about  $\log A$  :

$$s_i = \log A + \varepsilon_i$$

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<sup>2</sup>One can also specify a Cobb-Douglas production function with both commodity and labor as inputs. The model remains tractable but the formulas become more complex and harder to interpret.



where  $\varepsilon_i \sim \mathcal{N}(0, \tau_s^{-1})$  is random noise independent of  $\log A$ .  $\tau_s$  is the precision of the signal. The signal allows the producer on the island to form his expectation of the global productivity, determine his production decision, and trade in the futures market to purchase the commodity input. The futures market serves to aggregate all of the private signals dispersed among the producers regarding  $\log A$ . As each producer's private signal is noisy, the information friction also motivates him to use the publicly observed futures price to form his expectation.

At  $t = 0$ , the producer on island  $i$  maximizes his expected profit by choosing his commodity input  $X_i$ :

$$\max_{X_i} E_i [P_i Y_i] - F X_i \quad (3)$$

where  $E_i$  is shorthand for  $E[\cdot | \mathcal{I}_i]$ , and  $P_i$  is the price of the good produced by the island. The producer's information set  $\mathcal{I}_i = \{s_i, F\}$  includes his private signal  $s_i$  and the futures price  $F$ . The goods price  $P_i$ , which one can interpret as the terms of trade, is determined at  $t = 2$  based on the matched trade with another island.

### 2.3 Commodity suppliers

There is a group of suppliers of the commodity. The suppliers do not participate in the futures market. Instead, they directly sell to intermediaries in decentralized spot markets at  $t = 1$ . We assume the suppliers face a convex cost

$$\frac{k}{1+k} e^{-\xi/k} (X^S)^{\frac{1+k}{k}}$$

in supplying the commodity, where  $X^S$  is the quantity sold,  $k \in (0, 1)$  is a constant parameter, and  $\xi$  represents random noise. We assume that  $\xi$  has Gaussian distribution  $\mathcal{N}(\bar{\xi}, \tau_\xi^{-1})$  with  $\bar{\xi}$  as its mean and  $\tau_\xi^{-1}$  as its variance. Thus, given a spot price  $P_X$ , the suppliers face the following optimization problem:

$$\max_{X^S} P_X X^S - \frac{k}{1+k} e^{-\xi/k} (X^S)^{\frac{1+k}{k}}. \quad (4)$$

It is easy to determine the suppliers' optimal supply curve:

$$X^S = e^{\xi} P_X^k, \quad (5)$$

which shows  $\xi$  as uncertainty in the supply and  $k$  as the price elasticity of the commodity supply.<sup>3</sup>

## 2.4 Intermediaries

There is a group of identical intermediaries. Each intermediary is a price taker. Intermediaries take short positions in the futures market at  $t = 0$ . They then purchase commodities from commodity suppliers in the spot markets at  $t = 1$  to make delivery of their futures positions.

We assume that the intermediaries are risk-neutral and have a required return of  $\nu$  from their positions. That is, they are willing to take infinitely large positions if  $E [\log (F/P_X)|\mathcal{I}^F]$ , the expected return from shorting futures and buying the commodity in the spot markets, is higher than  $\nu$ . The required return  $\nu$  is constant and, in practice, depends on factors that determine the financial status and cost of capital of the intermediaries.

## 2.5 Commodity index traders

Since the mid-2000s, more and more institutional investors such as pension funds and insurance companies have started to treat commodities as a new asset class like stocks and bonds. These institutional investors regularly allocate a fraction of their portfolios to investing in futures and swap contracts written on commodities. They take only long positions, and typically close out their futures positions before maturity and roll into more distant futures contracts (i.e., replacing maturing contracts by more distant ones). As a result, their trading does not directly affect the physical supply and demand of commodities. The core of our model is to analyze whether their trading can feed back to commodity supply and demand through the futures prices.

Specifically, we introduce a group of commodity index investors (CITs). CITs take only long positions in the futures market at  $t = 0$  and unwind their positions at  $t = 1$  before

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<sup>3</sup>By letting the suppliers sell the commodity according to their marginal cost, our model ignores any potential feedback effect from the futures price to the supply side. In a more general setting with multiple rounds of spot market trading, suppliers and other agents may have incentives to store the commodity over time based on their expectations of future demands. Then, the futures price can feed back to these agents' storage decisions. We leave an analysis of such a feedback effect to the supply side to future research and instead focus on the feedback effect to the demand side.

delivery. For simplicity, we assume that on the long side of the futures market, the aggregate position of CITs and producers is given by the producers' aggregate position multiplied by a factor  $e^\theta$ :

$$e^\theta \int_{-\infty}^{\infty} X_i(s_i, F) d\Phi(\varepsilon_i),$$

where  $e^\theta$  represents the contribution of CITs. This multiplicative specification is useful for ensuring the tractable log-linear equilibrium of our model. From an economic perspective, this specification implies that CITs' trading tends to amplify the producers' demand in the futures market. This feature is broadly consistent with the expansion and contraction of CITs' commodity futures positions in the recent commodity price boom-and-bust cycle, as summarized by Cheng, Kirilenko, and Xiong (2012).

As a key source of information frictions in our model, we assume that  $\theta$  is random and unobservable by other market participants. This assumption is realistic since in practice market participants cannot observe others' positions and thus cannot directly measure the size of CITs' positions.<sup>4</sup> Specifically, we assume that from the perspective of other market participants,  $\theta$  has a normal distribution independent of other sources of uncertainty in the model:

$$\theta \sim \mathcal{N}(\bar{\theta}, \tau_\theta^{-1})$$

with a mean of  $\bar{\theta}$  and variance of  $\tau_\theta^{-1}$ . The mean of this distribution captures the part that is predictable to other market participants, while the variance represents uncertainty in the CITs' trading that is outside other market participants' expectations.

The presence of CITs introduces an additional source of uncertainty to the long side of the futures market, as both goods producers and intermediaries cannot observe  $\theta$  at  $t = 0$ . At  $t = 1$ , CITs unwind their positions with intermediaries, who then need to deliver only  $\int X(\varepsilon_i) d\Phi(\varepsilon_i)$  units of the commodity to goods producers.

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<sup>4</sup>Despite that large traders need to report their futures positions to the CFTC on weekly basis, ambiguity in trader classification and netting of positions taken by traders who are involved in different lines of business nevertheless makes the aggregate positions provided by the CFTC's public Commitment of Traders Report imprecise. See Cheng, Kirilenko, and Xiong (2012) for more detailed discussion of the trader classification and netting problems in the CFTC's Large Trader Reporting System and a summary of positions taken by commodity index traders.

## 2.6 Joint equilibrium of different markets

Our model features a noisy rational expectations equilibrium of a number of markets: the goods markets between each pair of matched islands, the spot markets for the commodity, and the centralized commodity futures market. The equilibrium requires clearing of each of these markets:

- At  $t = 2$ , for each pair of randomly matched islands  $\{i, j\}$ , the households of these islands trade their produced goods and clear the market of each good:

$$\begin{aligned} C_i + C_j^* &= AX_i, \\ C_i^* + C_j &= AX_j. \end{aligned}$$

- At  $t = 1$ , in the decentralized spot markets of the commodity, the intermediaries' demand equals the supply of the suppliers,  $e^{-\theta} X^F = X^S$ .
- At  $t = 0$ , in the futures market for the commodity, the aggregate long position taken by the goods producers equals the aggregate short position taken by the intermediaries:

$$e^\theta \int_{-\infty}^{\infty} X_i(s_i, F) d\Phi(\varepsilon_i) = X^F(F),$$

where each producer's optimal short position  $X_i(s_i, F)$  depends on his private signal  $s_i$  and the futures price  $F$ , and the intermediaries' optimal position  $X^F(\nu, F)$  depends on the futures price  $F$ . The producers' long positions are integrated over the noise  $\varepsilon_i$  in their private signals.

## 2.7 Comments on the setting

We choose to focus on information frictions in the commodity markets that arise from the inability of market participants to directly observe the global economic strength and CITs' investment positions. This modeling choice is motivated by the finding of Kilian and Murphy (2012) and others that the demand side is the relevant channel for explaining the buildup in commodity prices since mid-2000s. It is also worth mention that one can readily extend our setting to incorporate supply-side uncertainty by letting the supplier's supply curve and

the intermediaries' required return be uncertain. In fact, we have pursued such a more general setting in an earlier draft of the paper. We are able to obtain a similar log-linear equilibrium as in the currently simplified setting, but with a greater number of random factors in the equilibrium futures price. This generality comes at the expense of greater presentation complexity and the gain of more subtle interactions between different sources of uncertainty in the market participants' learning problem.

To focus on the informational effects, we also choose to ignore hedging-induced trading by suppliers and producers in the futures market. See Cheng, Kirilenko, and Xiong (2012) for a recent study of how suppliers' hedging needs interacted with financial traders' financing needs during the recent financial crisis to result in convective risk flows in commodity futures markets. Our model is static in nature and thus not suited for analyzing the dynamics of the futures curve and the interaction of the futures curve with commodity inventory. See Alquist and Kilian (2010) for a recent study on this important issue.

## 3 The Equilibrium

### 3.1 Goods market equilibrium

We begin our analysis of the equilibrium with the goods markets at  $t = 2$ . For a pair of randomly matched islands,  $i$  and  $j$ , the representative household of island  $i$  possesses  $Y_i$  units of the good produced by the island while the representative household of island  $j$  holds  $Y_j$  units of the other good.<sup>5</sup> They trade the two goods with each other to maximize each's utility function given in (1). The following proposition, which resembles a similar proposition in Angeletos and La'O (2012), describes the goods market equilibrium between these two islands.

**Proposition 1** *For a pair of randomly matched islands,  $i$  and  $j$ , their representative households' optimal consumption of the two goods is*

$$C_i = (1 - \eta) Y_i, C_i^* = \eta Y_j, C_j = (1 - \eta) Y_j, C_j^* = \eta Y_i.$$

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<sup>5</sup>Here we implicitly treat a representative household as representing different agents holding stakes in an island's good production, such as workers, managers, suppliers of inputs, intermediaries, etc. We agnostically group their preferences for the produced goods of their own island and other islands into the preferences of the representative household.

The price of the good produced by island  $i$  is

$$P_i = \left( \frac{Y_j}{Y_i} \right)^\eta. \quad (6)$$

Proposition 1 shows that each household divides its consumptions between the home and away good with fractions  $1 - \eta$  and  $\eta$ , respectively. When  $\eta = 1/2$ , the household consumes the two types of goods equally. The price of each good is determined by the relative output of the two matched islands. One island's good is more valuable when the other island produces more. This feature is standard in the international macroeconomics literature (e.g., Obstfeld and Rogoff (1996)) and implies that each goods producer needs to take into account the production decisions of producers of other goods.

### 3.2 Producer's production decision

By substituting the production function in (2) into (3), the expected profit of the goods producer on island  $i$ , we obtain the following objective:

$$\max_{X_i} E [AP_i X_i | s_i, F] - F X_i.$$

In a competitive goods market, the firm will produce to the level that the marginal revenue equals the marginal cost:

$$E [AP_i | s_i, F] = F.$$

By substituting in  $P_i$  from Proposition 1, we obtain

$$X_i = \left\{ \frac{E [AX_j^\eta | s_i, F]}{F} \right\}^{1/\eta} \quad (7)$$

which depends on the producer's expectation  $E [AX_j^\eta | s_i, F]$  of the global productivity  $A$  and the production decision  $X_j$  of its randomly matched trading partner, island  $j$ . This expression demonstrates the complementarity in producers' production decisions. A larger  $\eta$  makes the complementarity stronger as the producer relies more on selling its goods to the other island.

The futures price  $F$  is an important source of information for the producer to form his expectation of  $E [AX_j^\eta | s_i, F]$ , which serves as the main channel for the futures price to feed

back into the producer's commodity demand and thus the commodity's spot price. The presence of complementarity strengthens this feedback effect relative to the standard model of asset trading with asymmetric information.

### 3.3 Intermediaries' short position

Intermediaries take short positions in the futures market and then purchase the commodity in the spot markets to make delivery. In equilibrium, they will trade so that the expected return from their positions exactly equals the required return  $\nu$

$$E \left[ \log \left( \frac{F}{P_X} \right) \middle| \mathcal{I}^F \right] = \nu. \quad (8)$$

The clearing of the spot market requires that  $e^{-\theta} X^F = X^S = e^\xi P_X^k$ , which implies that

$$\log P_X = \frac{1}{k} \log X^F - \frac{1}{k} \theta - \frac{1}{k} \xi. \quad (9)$$

By substituting this spot price into (8), we obtain the following equation:

$$E \left[ \log F - \frac{1}{k} \log X^F + \frac{1}{k} \theta + \frac{1}{k} \xi \middle| \mathcal{I}^F \right] = \nu.$$

This equation gives the logarithm of the intermediaries' aggregate short position  $X_F$  :

$$\log X^F = k \log F + E [\theta | \mathcal{I}^F] - k\nu + \bar{\xi}, \quad (10)$$

which linearly increases with  $\log F$ . As the intermediaries are risk-neutral, they are able to perfectly insulate the futures market and thus the producers against uncertainty regarding the commodity supply in the spot markets. Also note that they are willing to absorb  $E [\theta | \mathcal{I}^F]$ , i.e., their expectation of  $\theta$ , at no price impact. However, a larger realization of  $\theta$  than  $E [\theta | \mathcal{I}^F]$  will result in a price impact in  $\log F$  as the intermediaries cannot differentiate it from the real demand of the producers.

### 3.4 Futures market equilibrium

By clearing the aggregate long position taken by producers and CITs with the short position by intermediaries, we derive the futures market equilibrium. Despite the nonlinearity in the producer's production decision, we obtain a unique log-linear equilibrium in closed form.

The following proposition summarizes the futures price and positions taken by different participants in the equilibrium.

**Proposition 2** *At  $t = 0$ , the futures market has a unique log-linear equilibrium: 1) The futures price is a log-linear function of  $\log A$  and  $\theta$ :*

$$\log F = h_A \log A + h_\theta \theta + h_0, \quad (11)$$

with the coefficients  $h_A \in [0, 1]$ ,  $h_\theta > 0$ , and  $h_0$  given by

$$h_A = 1 - \left( \frac{1}{2} \eta^2 \tau_s^{-2} \tau_\theta^{-1} \tau_A^3 \right)^{1/3}. \quad (12)$$

$$\left( \sqrt[3]{1 + \sqrt{1 + \frac{4}{27} \eta^2 \tau_s^{-2} \tau_\theta^{-1} \tau_A^3}} + \sqrt[3]{1 - \sqrt{1 + \frac{4}{27} \eta^2 \tau_s^{-2} \tau_\theta^{-1} \tau_A^3}} \right),$$

$$h_\theta = \sqrt{\tau_A^{-1} \tau_\theta h_A (1 - h_A)}, \quad (13)$$

$$h_0 = (1 - h_A) \left[ \bar{a} + \frac{1}{2\tau_A} + \frac{\tau_s}{\tau_A^2} (1 - h_A) \right] - h_\theta \bar{\theta}. \quad (14)$$

2) The long position taken by the producer on island  $i$  is a log-linear function of his private signal  $s_i$  and  $\log F$ :

$$\log X_i = l_s s_i + l_F \log F + l_0, \quad (15)$$

with the coefficients  $l_s$ ,  $l_F$ , and  $l_0$  given by

$$l_s = \frac{1}{\eta} \frac{\tau_s}{\tau_A} (1 - h_A),$$

$$l_F = k - \left( \tau_\theta + \frac{h_\theta^2}{h_A^2} \tau_A \right)^{-1} \tau_\theta h_\theta^{-1}, \quad (16)$$

$$l_0 = \bar{\xi} + \left( \tau_\theta + \frac{h_\theta^2}{h_A^2} \tau_A \right)^{-1} \left( \tau_\theta \bar{\theta} - \frac{h_\theta}{h_A} \tau_A \bar{a} \right) + (k - l_F) h_0 - \frac{1}{2} l_s^2 \tau_s^{-1} - k\nu.$$

Proposition 2 shows that each producer's logarithmic position  $\log X_i$  is a linear function of his private signal and the futures price, while the logarithmic futures price aggregates the dispersed signals of the producers to partially reveal the global productivity  $A$ . The futures price does not depend on any producer's signal noise as a result of the aggregation across a large number of producers with independent noise. This feature is similar to Hellwig (1980).



The futures price also depends on the CITs' position  $\theta$ , which serves the same role as noise trading in the standard models of asset market trading with asymmetric information.

It is well known that asset market equilibrium with asymmetric information is often intractable due to the difficulty in aggregating different participants' positions. Most existing models adopt the setting of Grossman and Stiglitz (1980) and Hellwig (1980), which features CRRA utility for agents and normal distributions for asset fundamentals and noise trading. Under this setting, the equilibrium asset price is a linear function of the asset fundamental and noise trading, while each agent's asset position is a linear function of the price and his own information. This setting is unsuitable for analyzing learning and feedback effects from asset prices to asset fundamentals in a macroeconomic setting as feedback effects tend to make the asset fundamentals deviate from normal distributions.

The log-linear equilibrium derived in Proposition 2 resembles the standard linear equilibrium in Grossman and Stiglitz (1980) and Hellwig (1980) but nevertheless incorporates the feedback effect of the equilibrium futures price to producers' nonlinear commodity demands (e.g., equation (7)). In fact, each producer's demand has a log-normal distribution (e.g., equation (15)). As shown by equation (29) in the Appendix, the producers' aggregate demand remains log-normal as a result of the law of large numbers. This is the key feature that ensures the tractability of our model.<sup>6</sup>

### 3.5 A benchmark without information frictions

To facilitate our discussion on the effects of information frictions on commodity demand, it is useful to establish a benchmark without any information friction. Suppose that the global productivity  $A$  and CITs' position  $\theta$  are both observable by all market participants. Then, the producers can choose their optimal production decisions without any noise interference. The following proposition characterizes this benchmark.

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<sup>6</sup>It is also worth noting that our setting is different from the setting of Goldstein, Ozdenoren, and Yuan (2012). Their model features stock market trading with asymmetric information and a feedback effect from the equilibrium stock price to firm investment. While the equilibrium stock price is non-linear, they ensure tractability by assuming each trader in the asset market is risk-neutral and faces upper and lower position limits. Our model does not impose any position limit and instead derives each producer's futures position through his interior production choice.

**Proposition 3** *When both  $A$  and  $\theta$  are observed by all market participants, there is a unique, competitive equilibrium. In the equilibrium, 1) the producers have a symmetric commodity demand curve:*

$$\forall i \text{ and } j, X_i = X_j = \begin{cases} 0 & \text{if } F > A \\ [0, \infty) & \text{if } F = A \\ \infty & \text{if } F < A \end{cases} ;$$

2) *the intermediaries' supply is*

$$\log X^F = k \log F + k\theta - k\nu + \bar{\xi};$$

3) *the futures price is given by  $\log F = \log A$  and a producer's equilibrium demand is  $\log X_i = k \log A - k\nu + \bar{\xi}$ . This competitive equilibrium is Pareto efficient.*

Proposition 3 shows that when  $A$  and  $\theta$  are observable, there is a unique equilibrium despite the complementarity in the producers' production decisions. The uniqueness of the equilibrium comes from the competitiveness of the equilibrium. The perfect competition between producers leads to inelastic, downward demand for the commodity. That is, their demand is infinity if the futures price  $F$  is below  $A$ , zero if  $F$  is above  $A$ , and anywhere between zero and infinity if  $F$  equals  $A$ . This elastic demand curve dictates the equilibrium futures price to be  $A$ , but leaves the equilibrium commodity demand to be determined by the intermediaries' upward sloping supply curve. As a result, the complementarity between producers does not lead to multiple equilibria, in which producers coordinate on certain high or low demand levels. In the unique equilibrium, the producers' aggregate commodity demand increases with the global productivity  $\log A$  with a coefficient of  $k$  and decreases with the intermediaries' required return  $\nu$  with a coefficient  $-k$ .

This equilibrium is Pareto efficient. Thus, any deviation induced by informational frictions must at least hurt the welfare of some agents. This motivates us to use this equilibrium outcome as the benchmark to examine effects of information frictions on commodity demand.

## 4 Feedback Effects of Futures Prices

In the presence of information frictions, the futures price provides a channel for each producer to infer the global productivity in addition to his private signal. Through this channel, an

increase in the futures price, even if driven by random noise in the futures market, can affect the producers' commodity demand beyond the usual cost effect.

## 4.1 Price elasticity of commodity demand

We first examine the price elasticity of an individual producer's demand for the commodity. Equation (7) reveals two forces: first, a higher futures price reduces the demand through a direct cost effect in the denominator; second, a higher price may increase the demand through the producer's learning from the price about the global productivity and other producers' production decisions, as captured by the term  $E [AX_j^\eta | s_i, F]$  in the numerator. As discussed before, the strength of this informational feedback effect increases with the complementarity parameter  $\eta$ .

The net effect of the direct cost effect and the indirect feedback effect determines  $l_F$  the price elasticity of each producer's demand, which is given in equation (16) of Proposition 2. The following proposition proves that the price elasticity is negative when  $\eta$  is sufficiently small but is positive if  $\eta$  is sufficiently large.

**Proposition 4** *The following necessary and sufficient condition ensures that  $l_F > 0$ :*

$$\eta > k^{-1} (\tau_A + k^2 \tau_\theta)^{-1} \tau_s.$$

The result that  $l_F$  is positive only when the complementarity effect is sufficiently strong highlights the key difference of our model to the standard models of asset trading with asymmetric information, e.g., Grossman and Stiglitz (1980) and Hellwig (1980). In these models, the price elasticity of traders' demand for financial assets is negative despite that they also extract information from the traded assets' prices regarding the assets' fundamentals.

The result that producers' demand curves can be upward sloping is in sharp contrast to the benchmark case without information frictions, and thus highlights the important effect of information frictions in commodity markets. It also should be noted that despite that the demand curve can be upward sloping, its slope  $l_F$  is always lower than the slope of the intermediaries' supply curve  $k$ . Thus, the equilibrium is stable.

## 4.2 Real consequence of futures market speculation

As CITs trade only in the futures market and do not take or make any delivery, their trading is a form of pure speculation in the futures market. Proposition 2 shows that in the presence of uncertainty about  $\theta$ , a larger  $\theta$  leads to a higher futures price as  $h_\theta > 0$ . This is because information frictions prevent other market participants from separating CITs' trading from the producers' real demand.

The higher futures price induced by a larger  $\theta$  also feeds back to each producer's commodity demand and thus the spot price through the information effect and cost effect discussed earlier. By substituting equation (11) into (15), we have a producer's commodity demand:

$$\log X_i = l_s s_i + l_F h_A \log A + l_F h_\theta \theta + l_F h_0 + l_0.$$

Then, the aggregate demand of the producers is

$$\log \left[ \int_{-\infty}^{\infty} X_i(s_i, F) d\Phi(\varepsilon_i) \right] = l_F h_\theta \theta + (l_s + l_F h_A) \log A + l_0 + l_F h_0 + \frac{1}{2} l_s^2 \tau_s^{-1}. \quad (17)$$

By substituting equation (17) into (9), we obtain the spot price

$$\log P_X = \frac{1}{k} \left[ l_F h_\theta \theta + (l_s + l_F h_A) \log A + l_0 + l_F h_0 + \frac{1}{2} l_s^2 \tau_s^{-1} \right] - \frac{1}{k} \xi.$$

Taken together, a larger  $\theta$  not only leads to a higher futures price but also affects producers' demand and the spot price. The impact of  $\theta$  on the producers' aggregate demand and the spot price are both determined by the coefficient  $l_F h_\theta$ . As  $h_\theta > 0$ ,  $l_F < 0$  implies that  $\theta$  has a negative impact on the aggregate demand and the spot price, while  $l_F > 0$  ensures that  $\theta$  has a positive impact on the aggregate demand and the spot price. The necessary and sufficient condition listed in Proposition 4 clearly separates these two cases. In the first case, the cost effect dominates the information effect—when CITs' trading drives up the futures price, producers cut down their production in response to the increased production cost. In the latter case, the information effect dominates—producers increase their production in response to the higher futures price as a result of their higher expectations about the global economic strength and other producers' production.

The emergence of these two cases clarifies the widely held argument mentioned at the beginning of the paper that when speculators drive up the futures price of a commodity, real

demand must fall. While this argument holds in the first case, it fails in the latter. The key here is that this argument ignores the potential informational feedback effect from the increased futures price to commodity demand in the presence of demand complementarity.

### 4.3 Implications for market transparency

Information frictions are essential for CITs' trading to impact the futures price and producer demand. Equation (17) shows that the logarithmic aggregate demand is a linear function of  $\log A$  and  $\theta$ , with the impact of  $\theta$  captured by the term  $l_F h_\theta \theta$ . We can measure the economic impact of  $\theta$  by the the uncertainty induced by  $\theta$  to the aggregate demand:

$$V_\theta = l_F^2 h_\theta^2 \tau_\theta^{-1}.$$

The following proposition characterizes the dependence of  $V_\theta$  on uncertainty in  $\theta$ .

**Proposition 5** *If  $l_F > 0$ ,  $V_\theta$  decreases with  $\tau_\theta$ .*

Under the condition of Proposition 5, greater uncertainty about the CITs' futures position (i.e., smaller  $\tau_\theta$ ) leads to a greater impact of their trading on the aggregate commodity demand and thus the spot price. This is because greater uncertainty makes it more difficult for producers and intermediaries to separate the CITs' trading from the producers' real demand for the commodity. This result thus supports financial policies that enhance transparency in commodity futures markets so that market participants can more precisely measure trading as a result of actual commodity demand as opposed to speculative trading.

By highlighting the feedback effect originated from information frictions as the key channel for speculative trading in futures markets to affect commodity price and demand, our model also suggests that imposing position limits on speculators, a widely discussed policy proposal in the recent years, does not address the central information frictions that confront participants in commodity markets and thus may not be effective in reducing any potential distortion caused by speculative trading.

### 4.4 Implications for structural models

In the presence of both supply and demand shocks simultaneously affecting commodity markets, economists commonly use structural VAR models to separate and estimate the

effects of supply and demand shocks. For example, Kilian (2009) develops a widely used, structural VAR model for the dynamics of global oil production, global economic activity, and oil price. By decomposing the shocks in the economy to three orthogonal sources: an oil supply shock, an aggregate demand shock, and an oil specific demand shock based on certain identification restrictions, this study finds that the aggregate demand shock has a bigger impact on the oil market than previously thought.

More recently, a wave of studies employ structural VAR models to quantify speculative effects in the oil market, e.g., Kilian and Murphy (2010), Lombardi and van Robays (2011), Juvenal and Petrella (2012). While these studies differ in implementation, they all use identification restrictions motivated by the same economic argument that any price effect driven by speculation must be positively associated with an inventory increase. Specifically, Kilian and Murphy (2010) specify a fourth type of independent shock—a speculative demand shock—in addition to the three types employed in Kilian (2009). While analyzing this type of speculative demand shock is appealing, this identification strategy is incomplete and ignores speculative effects transmitted to commodity demand through the informational feedback channel. More precisely, as illustrated by our model, industrial producers cannot differentiate noise in futures prices from genuine information about the global economic strength and react to noise by increasing their commodity demand. This feedback effect thus occurs through flow demand driven by the aggregate demand shock, according to the taxonomy of Kilian and Murphy, as opposed to speculative demand.

Put differently, our model shows that commodity demand is not necessarily always driven by economic fundamentals. To the extent that noise in futures markets can affect futures prices, it can affect producers' expectations and thus their commodity demand. Therefore, it is important not to blindly attribute increases in commodity demand to improved economic fundamentals. Instead, one should be cautious in separating fundamental and noise-driven demand. This empirical identification problem is challenging but central in evaluating whether speculation in futures markets affects spot market dynamics.

Our model also provides another implication on the slope of commodity demand curve. In the presence of the informational feedback effect, the demand curves can be downward, flat, or even upward sloping, despite the presence of the standard cost effect that decreases

demand with price. Thus, it can be difficult to interpret or sign demand elasticities without taking into account the feedback effect.

Taken together, our model motivates structural models to account for the informational feedback effect in order to systematically evaluate the effects of speculative trading on commodity markets. Instead of treating expectations as part of structural shocks, it is important to explicitly model agents' expectations and analyze the interaction between their expectations and different sources of shocks (including non-fundamental shocks).

## 4.5 Empirical implications and further discussion

The key implication of our model is that non-fundamental factors originating from the futures markets can feed back to commodity demand and spot prices through the informational channel of futures prices.<sup>7</sup> To test this implication, constructing a valid measure of non-fundamental factors that affect futures prices is a challenge. The large inflow of financial investment to commodity futures markets in the recent years is a potential candidate. Several considerations, however, complicate the direct use of the positions of hedge funds and commodity index traders (as released by the weekly reports of the Commodity Futures Trading Commission). First, these financial traders might possess private information about demand and supply of commodities. As a result, while their trading is treated as exogenous in our model, it may be correlated with economic fundamentals in reality. Thus, it is necessary to either control for the fundamental-related investment flow or ideally find an instrument to extract investment flow that is unrelated to economic fundamentals. Second, financial traders may trade to either fulfill their own investment purposes or to facilitate the trading needs of other market participants. In the former case, the changes of their futures positions are positively correlated with the changes of futures prices, while in the latter case, their trades provide liquidity and are negatively correlated with price changes.<sup>8</sup> This consideration also cautions against blindly using financial traders' positions as a measure of non-fundamental factors in commodity futures markets.

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<sup>7</sup>There is evidence that firm investment reacts to stock prices. Chen, Goldstein and Jiang (2007) and Bakke and Whited (2010) find that the sensitivity of investment to price (or Tobin's Q) is stronger when the firm's stock price incorporates more private information.

<sup>8</sup>See Cheng, Kirilenko and Xiong (2012) for an analysis of these two types of trades in the commodity futures markets during the financial crisis.

Our model is most relevant for industrial commodities whose demands are closely tied to global economic strength. Crude oil and copper are examples of such commodities. On the other hand, the demands for some other commodities, such as cocoa and coffee, are less correlated with economic growth. We expect the feedback effects of futures prices to be stronger for the commodities in the first group than in the latter. This contrast provides ground for potential cross-sectional tests of feedback effects.

We view prices of the front-month or second-month futures contracts as the most relevant futures prices for empirical tests of feedback effects. This is because in reality front-month and second-month contracts tend to attract most trading and thus fulfill most of the information aggregation role in commodity markets. It is useful to note that the prices of more distant futures contracts often deviate from the front contracts. The price spread between the front and distant contracts reflects confounding factors not considered in our model, such as the convenience yield from holding physical commodities and cost for arbitrageurs to arbitrage mispricing on the futures price curve.

The feedback effect illustrated by our model originates from information frictions about global economic fundamentals. While noise in commodity futures markets can distort people's expectations for a certain period of time, we expect them to eventually correct their expectations with updated information. Thus, the feedback effect is likely to be relevant only for short- and medium-terms. A systematic evaluation of the horizons of feedback effects requires a dynamic model beyond our current framework, which we leave for future research.

A dynamic framework can also help address another issue ignored by our model—commodity storage. In practice, commodity suppliers and other speculators can choose to stock up based on their expectations of commodity prices in the future. To the extent that futures prices may affect the expectations of these agents, another feedback effect may arise through their decisions to store commodities over time. This feedback effect complements the feedback effect through producers' commodity demand and can further exacerbate the impacts of non-fundamental factors in commodity markets.



## 5 Conclusion

This paper develops a model to examine an information channel for commodity futures prices to feed back to commodity demand and spot prices. When goods producers extract information about the global productivity and other goods producers' production decisions from the futures price of a commodity, an increase in the futures price does not simply represent a higher cost of production, but also signals a strong global economy and more trading opportunities for their produced goods. We characterize the conditions for the indirect information effect to dominate the direct cost effect and thus for producers to increase their commodity demand in response to the increased futures price. Through this information channel and under certain sufficient conditions, our model also shows that by driving up the futures price, a noise shock to the futures market can cause both commodity demand and the spot price to rise. This outcome contrasts a widely held argument that commodity price increases driven by futures market speculation must be accompanied by inventory spikes. Our model thus cautions against an over-emphasis on using this argument to identify speculative effects in commodity markets. Instead, our model provides a conceptual framework for designing sharper empirical tests of such effects. By highlighting the feedback effect originated from information frictions as a key channel for speculation in futures markets to affect commodity demand and prices, our model supports policies that enhance market transparency.

## Appendix Proofs of Propositions

### A.1 Proof of Proposition 1

Consider the maximization problem of the household on island  $i$ :

$$\max_{C_i, C_i^*} \left( \frac{C_i}{1-\eta} \right)^{1-\eta} \left( \frac{C_i^*}{\eta} \right)^\eta$$

subject to the budget constraint

$$P_i C_i + P_j C_i^* = P_i Y_i. \tag{18}$$

The two first order conditions with respect to  $C_i$  and  $C_i^*$  are

$$\left(\frac{C_i^*}{C_i}\right)^\eta \left(\frac{1-\eta}{\eta}\right)^\eta = \lambda_i P_i \quad (19)$$

$$\left(\frac{C_i}{C_i^*}\right)^{1-\eta} \left(\frac{\eta}{1-\eta}\right)^{1-\eta} = \lambda_i P_j \quad (20)$$

where  $\lambda_i$  is the Lagrange multiplier for his budget constraint. Dividing equations (19) and (20) leads to

$$\frac{\eta}{1-\eta} \frac{C_i}{C_i^*} = \frac{P_j}{P_i}$$

which is equivalent to  $P_j C_i^* = \frac{\eta}{1-\eta} P_i C_i$ . By substituting this equation back to the household's budget constraint in (18), we obtain  $C_i = (1-\eta) Y_i$ .

The market clearing of the island's produced goods requires  $C_i + C_j^* = Y_i$ , which implies that  $C_j^* = \eta Y_i$ .

The symmetric problem of the household of island  $j$  implies that  $C_j = (1-\eta) Y_j$ , and the market clearing of the goods produced by island  $j$  implies  $C_i^* = \eta Y_j$ .

The first order condition in equation (19) also gives the price of the goods produced by island  $i$ . Since the household's budget constraint in (18) is entirely in nominal terms, the price system is only identified up to  $\lambda_i$ , the Lagrange multiplier. Following Angeletos and La'O (2012), we normalize  $\lambda_i$  to 1. Then,

$$P_i = \left(\frac{C_i^*}{C_i}\right)^\eta \left(\frac{1-\eta}{\eta}\right)^\eta = \left(\frac{\eta Y_j}{(1-\eta) Y_i}\right)^\eta \left(\frac{1-\eta}{\eta}\right)^\eta = \left(\frac{Y_j}{Y_i}\right)^\eta.$$

## A.2 Proof of Proposition 2

We first conjecture that the futures price and each island producer's long position take the following log-linear forms:

$$\log F = h_0 + h_A \log A + h_\theta \theta \quad (21)$$

$$\log X_i = l_0 + l_s s_i + l_F \log F \quad (22)$$

where the coefficients  $h_0$ ,  $h_A$ ,  $h_\theta$ ,  $l_0$ ,  $l_s$ , and  $l_F$  are determined by the equilibrium conditions.

Define

$$z \equiv \frac{\log F - h_0 - h_\theta \bar{\theta}}{h_A} = \log A + \frac{h_\theta}{h_A} (\theta - \bar{\theta})$$

which is a sufficient statistic of information contained in  $F$ . Then, conditional on observing his private signal  $s_i$  and the futures price  $F$ , the island producer  $i$ 's expectation of  $\log A$  is

$$E[\log A \mid s_i, \log F] = E[\log A \mid s_i, z] = \frac{1}{\tau_A + \tau_s + \frac{h_A^2}{h_\theta^2} \tau_\theta} \left( \tau_A \bar{a} + \tau_s s_i + \frac{h_A^2}{h_\theta^2} \tau_\theta z \right),$$

and his conditional variance of  $\log A$  is

$$\text{Var}[\log A \mid s_i, \log F] = \left( \tau_A + \tau_s + \frac{h_A^2}{h_\theta^2} \tau_\theta \right)^{-1}.$$

According to equation (7),

$$\log X_i = \frac{1}{\eta} \{ \log (E_i [AX_j^\eta]) - \log F \}. \quad (23)$$

By substituting in equation (22) for  $\log X_j$  into the expression above, we obtain

$$\begin{aligned} E_i [AX_j^\eta] &= E_i \{ \exp [\log A + \eta (l_0 + l_s s_j + l_F \log F)] \} \\ &= \exp [\eta (l_0 + l_F \log F)] \cdot E_i \{ \exp [\log A + \eta l_s s_j] \}. \end{aligned}$$

We first derive the conditional expectation in the above equation

$$\begin{aligned} &E[\exp(\log A + \eta l_s s_j) \mid s_i, \log F] = E[\exp((1 + \eta l_s) \log A + \eta l_s \varepsilon_j) \mid s_i, \log F] \\ &= \exp \left\{ (1 + \eta l_s) E_i[\log A] + \frac{(1 + \eta l_s)^2}{2} \text{Var}_i[\log A] + \frac{\eta^2 l_s^2}{2} \text{Var}_i[\varepsilon_j] + (1 + \eta l_s) \eta l_s \text{Cov}_i[\varepsilon_j \log A] \right\}. \end{aligned}$$

By recognizing that  $\text{Cov}_i[\varepsilon_j \log A] = 0$  and substituting in the expressions of  $E_i[\log A]$ ,  $\text{Var}_i[\log A]$ , and  $\text{Var}_i[\varepsilon_j]$ , we obtain that

$$\begin{aligned} &\log E[\exp(\log A + \eta l_s s_j) \mid s_i, \log F] \\ &= (1 + \eta l_s) \left( \tau_A + \tau_s + \frac{h_A^2}{h_\theta^2} \tau_\theta \right)^{-1} \left( \tau_A \bar{a} + \tau_s s_i + \frac{h_A^2}{h_\theta^2} \tau_\theta z \right) \\ &\quad + \frac{(1 + \eta l_s)^2}{2} \left( \tau_A + \tau_s + \frac{h_A^2}{h_\theta^2} \tau_\theta \right)^{-1} + \frac{\eta^2 l_s^2}{2} \tau_s^{-1}. \end{aligned}$$

Consequently, equation (23) gives

$$\begin{aligned} \log X_i &= \frac{1}{\eta} \{ \log (E_i [AX_j^\eta]) - \log F \} \\ &= (l_0 + l_F \log F) + \frac{1}{\eta} \log E_i [\exp(\log A + \eta l_s s_j)] - \frac{1}{\eta} \log F \\ &= l_0 + \left( l_F - \frac{1}{\eta} \right) \log F + \left( \frac{1 + \eta l_s}{\eta} \right) \left( \tau_A + \tau_s + \frac{h_A^2}{h_\theta^2} \tau_\theta \right)^{-1} \\ &\quad \left( \tau_A \bar{a} + \tau_s s_i + \frac{h_A^2}{h_\theta^2} \tau_\theta \frac{\log F - h_0 - h_\theta \bar{\theta}}{h_A} \right) + \frac{(1 + \eta l_s)^2}{2\eta} \left( \tau_A + \tau_s + \frac{h_A^2}{h_\theta^2} \tau_\theta \right)^{-1} + \frac{\eta l_s^2}{2} \tau_s^{-1}. \end{aligned}$$

For the above equation to match the conjectured equilibrium position in equation (22), the constant term and the coefficients of  $s_i$  and  $\log F$  have to be identical. We thus obtain the following equations for determining the coefficients in (22):

$$l_0 = l_0 + \left( \frac{1 + \eta l_s}{\eta} \right) \left( \tau_A + \tau_s + \frac{h_A^2}{h_\theta^2} \tau_\theta \right)^{-1} \left( \tau_A \bar{a} - \frac{h_A}{h_\theta^2} \tau_\theta (h_0 + h_\theta \bar{\theta}) \right) \quad (24)$$

$$+ \frac{(1 + \eta l_s)^2}{2\eta} \left( \tau_A + \tau_s + \frac{h_A^2}{h_\theta^2} \tau_\theta \right)^{-1} + \frac{\eta l_s^2}{2} \tau_s^{-1},$$

$$l_s = \left( \frac{1 + \eta l_s}{\eta} \right) \left( \tau_A + \tau_s + \frac{h_A^2}{h_\theta^2} \tau_\theta \right)^{-1} \tau_s, \quad (25)$$

$$l_F = l_F - \frac{1}{\eta} + \left( \frac{1 + \eta l_s}{\eta} \right) \left( \tau_A + \tau_s + \frac{h_A^2}{h_\theta^2} \tau_\theta \right)^{-1} \frac{h_A}{h_\theta^2} \tau_\theta. \quad (26)$$

By substituting equation (26) into (25), we have

$$l_s = \frac{1}{\eta} \frac{\tau_s}{\tau_\theta} \frac{h_\theta^2}{h_A}. \quad (27)$$

Substituting this expression into equation (25), we also have that

$$h_\theta^2 = \tau_A^{-1} \tau_\theta h_A (1 - h_A). \quad (28)$$

For  $h_\theta$  to be real, it is necessary that  $h_A \in [0, 1]$ , a condition we will verify later.

From equation (24) and our expressions for  $l_s$  and  $h_\theta$ , we obtain  $h_0$  given in equation (14).

We now use the market clearing condition for the futures market to determine three other equations for the coefficients in the conjectured log-linear futures price and log-linear producer position. Aggregating equation (22) gives the aggregate long position of the producers:

$$\begin{aligned} \int_{-\infty}^{\infty} X_i(\varepsilon_i) d\Phi(\varepsilon_i) &= \int_{-\infty}^{\infty} \exp[l_0 + l_s s_i + l_F \log F] d\Phi(\varepsilon_i) \\ &= \int_{-\infty}^{\infty} \exp[l_0 + l_s (\log A + \varepsilon_i) + l_F (h_0 + h_A \log A + h_\theta \theta)] d\Phi(\varepsilon_i) \\ &= \exp[(l_s + l_F h_A) \log A + l_F h_\theta \theta + l_0 + l_F h_0] \int_{-\infty}^{\infty} \exp[l_s \varepsilon_i] d\Phi(\varepsilon_i) \\ &= \exp \left[ (l_s + l_F h_A) \log A + l_F h_\theta \theta + l_0 + l_F h_0 + \frac{1}{2} l_s^2 \tau_s^{-1} \right]. \end{aligned} \quad (29)$$

Equation (10) implies that  $\log X^F = k \log F + E[\theta | \mathcal{I}^F] - k\nu + \bar{\xi}$ . Define

$$z_\theta \equiv \frac{\log F - h_0}{h_\theta} = \frac{h_A}{h_\theta} \log A + \theta.$$

Then,

$$\begin{aligned} E[\theta|\mathcal{I}^F] &= E[\theta|z_\theta] = \left(\tau_\theta + \frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1} \left[\tau_\theta\bar{\theta} + \frac{h_\theta^2}{h_A^2}\tau_A\left(z_\theta - \frac{h_A}{h_\theta}\bar{a}\right)\right] \\ &= \left(\tau_\theta + \frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1} \left[\tau_\theta\bar{\theta} + \frac{h_\theta^2}{h_A^2}\tau_A\left(\frac{\log F - h_0}{h_\theta} - \frac{h_A}{h_\theta}\bar{a}\right)\right]. \end{aligned}$$

Thus,

$$\begin{aligned} \log X^F &= \left[k + \left(\tau_\theta + \frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1} \frac{h_\theta}{h_A^2}\tau_A\right] (h_A \log A + h_\theta\theta) \\ &\quad - k\nu + \bar{\xi} + kh_0 + \left(\tau_\theta + \frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1} \left(\tau_\theta\bar{\theta} - \frac{h_\theta}{h_A}\tau_A\bar{a}\right). \end{aligned}$$

Then, the market clearing condition  $\log\left[e^\theta \int_{-\infty}^{\infty} X_i(\varepsilon_i) d\Phi(\varepsilon_i)\right] = \log X^F$  requires that the coefficients of  $\log A$  and  $\theta$  and the constant term be identical on both sides:

$$l_s + l_F h_A = \left[k + \left(\tau_\theta + \frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1} \frac{h_\theta}{h_A^2}\tau_A\right] h_A, \quad (30)$$

$$1 + l_F h_\theta = \left[k + \left(\tau_\theta + \frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1} \frac{h_\theta}{h_A^2}\tau_A\right] h_\theta, \quad (31)$$

$$l_0 + l_F h_0 + \frac{1}{2}l_s^2\tau_s^{-1} = -k\nu + \bar{\xi} + kh_0 - \frac{1}{\tau_\theta + \frac{h_\theta^2}{h_A^2}\tau_A} \left(\tau_\theta\bar{\theta} - \frac{h_\theta}{h_A}\tau_A\bar{a}\right). \quad (32)$$

Equation (31) directly implies that

$$l_F = k - \left(\tau_\theta + \frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1} \tau_\theta h_\theta^{-1}. \quad (33)$$

Equations (30) and (31) together imply that

$$l_s = h_\theta^{-1} h_A.$$

Combining this equation with (27) leads to

$$h_A^2 = \frac{1}{\eta} \frac{\tau_s}{\tau_\theta} h_\theta^3, \quad (34)$$

which, together with (28), implies that

$$h_A = \frac{\tau_s^2 \tau_\theta}{\eta^2 \tau_A^3} (1 - h_A)^3. \quad (35)$$

This equation has a unique real root  $h_A \in [0, 1]$ . First note that the RHS of equation (35) is monotonically decreasing in  $h_A$  and the LHS is monotonically increasing in  $h_A$ . Thus,

if there is a root, the root must be unique. Next, note that at  $h_A = 0$  the RHS is above the LHS while at  $h_A = 1$  the RHS is below the LHS. Then, according to the intermediate value theorem, there must be a root inside  $[0, 1]$ .

We can directly solve for the unique real root of equation (35). Define  $H_A = 1 - h_A$ . Then, the equation becomes

$$H_A^3 + \eta^2 \tau_s^{-2} \tau_\theta^{-1} \tau_A^3 H_A - \eta^2 \tau_s^{-2} \tau_\theta^{-1} \tau_A^3 = 0.$$

This is a depressed cubic polynomial of the form  $x^3 + px + q = 0$ , which has one real and two complex roots. Following Cardano's method, the one real root is given by

$$H_A = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}.$$

Since  $q = -p$ , the above simplifies to

$$H_A = p^{1/3} \left( \sqrt[3]{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{p}{27}}} + \sqrt[3]{\frac{1}{2} - \sqrt{\frac{1}{4} + \frac{p}{27}}} \right).$$

Substituting in  $p = \eta^2 \tau_s^{-2} \tau_\theta^{-1} \tau_A^3$ , we arrive at the expression for  $h_A$  in equation (12).

By substituting the expression for  $h_A$  back into equation (28), we obtain a quadratic equation for  $h_\theta$ . We take the positive root given in equation (13) to fulfill the necessary condition for  $h_A$  to be real. With  $h_A$  and  $h_\theta$  determined,  $l_s$  is then given by (27),  $l_F$  by (33),  $h_0$  by (24), and  $l_0$  by (32).

### A.3 Proof of Proposition 3

We keep the same setting outlined in the main model, except letting  $A$  and  $\theta$  be observable by all market participants. We first derive the equilibrium. In this setting, each producer's private signal  $s_i$  becomes useless as  $A$  is directly observable. We can still use equation (7) to derive producer  $i$ 's optimal commodity demand:

$$X_i = \frac{A^{1/\eta}}{F^{1/\eta}} \{E[X_j^\eta | A, F, s_i]\}^{1/\eta}.$$

As the producers now share the same information about  $A$ , they must have the same expectation about their future trading partners' production decisions. As a result,  $X_i = X_j$  for any  $i$  and  $j$ . Then, the above equation implies that in equilibrium  $F = A$ . Each producer

has an inelastic demand whenever  $F \leq A$ . That is,

$$X_i = \begin{cases} 0 & \text{if } F > A \\ [0, \infty) & \text{if } F = A \\ \infty & \text{if } F < A \end{cases} .$$

Market clearing of the futures market requires that the sum of the producers' aggregate commodity demand and CITs' long position equals the short position of intermediaries, i.e.,  $e^\theta X_i = X^F$ . Equation (10) thus gives the producers' aggregate commodity demand:  $k \log A - k\nu + \bar{\xi}$ . Note that as  $\theta$  is observable,  $E[\theta | \mathcal{I}^F] = \theta$ . It is clear that this equilibrium is unique.

We now use the First Fundamental Theorem of Welfare to show that the competitive equilibrium derived in this setting is Pareto efficient. We can rewrite the optimization problem of the representative household on island  $i$  as

$$\max_{C_i, C_i^*} \left( \frac{C_i}{1 - \eta} \right)^{1 - \eta} \left( \frac{C_i^*}{\eta} \right)^\eta$$

subject to the following budget constraint with transfers:

$$P_i C_i + P_j C_i^* = W_i^f + W_i^s .$$

where  $W_i^f = P_i Y_i - F X_i$  is the profit from the producer on the island and  $W_i^s = F X_i$  is the net profit returned from the intermediaries, commodity suppliers and CITs to the representative household on the island. By netting out the intermediaries, suppliers and CITs, we can view them together as one group of competitive firms in the economy that solves the following profit maximization problem to supply the commodity to island households by shorting the futures contract:

$$\max_{X^F} F e^{-\nu} X^F - \frac{k}{1 + k} e^{-\bar{\xi}/k} (X^F)^{\frac{1+k}{k}} .$$

The first order condition of this optimization leads to the following supply curve of commodity futures:

$$\log X^F = k \log F - k\nu + \bar{\xi} ,$$

which is identical to equation (10) derived earlier. Since  $F = A$ , we arrive at

$$\log X = k \log A - k\nu + \bar{\xi} .$$

We have thus constructed a competitive price equilibrium with transfers, where the transfers to each island household are  $W_i^f$  from the island's producer and  $W_i^s$  from the intermediaries, commodity suppliers and CITs. Competitive firms maximize profits, there is an efficient use of social resources, the derived consumption bundle  $\{C_i, C_i^*\}$  also maximizes each household's utility. Since the household's utility is locally non-satiated, we can apply the First Fundamental Theorem of Welfare to say that the allocation in the no information friction benchmark is Pareto efficient.

#### A.4 Proof of Proposition 4

By Equation (33), for  $l_F \geq 0$ , we must have

$$l_F h_\theta = k h_\theta - \frac{\tau_\theta h_A^2}{\tau_\theta h_A^2 + h_\theta^2 \tau_A} \geq 0. \quad (36)$$

As  $\eta \rightarrow 0$ ,  $h_\theta \rightarrow 0$  and this condition is impossible to satisfy. By using equations (28) and (34), one can rewrite  $h_\theta$  as

$$h_\theta = \eta \tau_s^{-1} \tau_A \frac{h_A}{1 - h_A}.$$

By substituting the above into the first term of (36) and by making use of (28) for the second, one can rewrite (36) as

$$\begin{aligned} l_F h_\theta &= k \eta \tau_s^{-1} \tau_A \frac{h_A}{1 - h_A} - \frac{\tau_\theta h_A^2}{\tau_\theta h_A^2 + \tau_\theta h_A (1 - h_A)} \\ &= k \eta \tau_s^{-1} \tau_A \frac{h_A}{1 - h_A} - h_A \geq 0. \end{aligned}$$

Since  $h_A \in (0, 1)$ , for this condition to be satisfied, a necessary and sufficient condition is

$$h_A \geq h^* = 1 - k \eta \tau_s^{-1} \tau_A.$$

As  $h_A$  is the unique root of equation (35), this condition is equivalent to the following condition on the two sides of equation (35):

$$LHS(h^*) < RHS(h^*).$$

By substituting  $h^*$  into the two sides, we obtain the following condition:

$$\eta > k^{-1} (\tau_A + k^2 \tau_\theta)^{-1} \tau_s.$$



## A.5 Proof of Proposition 5

By using equation (31) and (28), we rewrite

$$V_\theta = l_F^2 h_\theta^2 \tau_\theta^{-1} = \left( k \sqrt{\tau_A^{-1} \tau_\theta h_A (1 - h_A)} - h_A \right)^2 \tau_\theta^{-1}.$$

By substituting with (35), we have

$$V_\theta = l_F^2 h_\theta^2 \tau_\theta^{-1} = \left( k \eta \tau_s^{-1} \tau_A \frac{h_A}{1 - h_A} - h_A \right)^2 \tau_\theta^{-1}.$$

Then, direct calculation leads to

$$\frac{\partial V_\theta}{\partial \tau_\theta} = \tau_\theta^{-1} l_F h_\theta \left[ 2 \left( k \eta \tau_s^{-1} \tau_A \frac{1}{(1 - h_A)^2} - 1 \right) \frac{\partial h_A}{\partial \tau_\theta} - \tau_\theta^{-1} l_F h_\theta \right].$$

Recognize that

$$\begin{aligned} k \eta \tau_s^{-1} \tau_A \frac{1}{(1 - h_A)^2} - 1 &= \left( k \eta \tau_s^{-1} \tau_A \frac{h_A}{(1 - h_A)} - h_A (1 - h_A) \right) \frac{1}{h_A (1 - h_A)} \\ &= (l_F h_\theta + h_A^2) \frac{1}{h_A (1 - h_A)}. \end{aligned}$$

Note that  $h_A \in (0, 1)$  and that

$$\frac{\partial h_A}{\partial \tau_\theta} = -\frac{\tau_\theta^{-2} \eta^2 \tau_s^{-2} \tau_A^3 h_A}{3(1 - h_A)^2} < 0.$$

By substituting again with (35), one finds that

$$\frac{\partial V_\theta}{\partial \tau_\theta} = -\tau_\theta^{-2} l_F h_\theta \left[ \frac{2}{3h_A} (l_F h_\theta + h_A^2) + l_F h_\theta \right] = -\left( \frac{2}{3h_A} + 1 \right) \tau_\theta^{-2} (l_F h_\theta)^2 - \frac{2}{3} \tau_\theta^{-2} h_A l_F h_\theta.$$

Then,  $l_F h_\theta > 0$  is sufficient for  $\frac{\partial V_\theta}{\partial \tau_\theta}$  to be negative.

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