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by

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Abstract

We study the transmission of liquidity shocks from one sector of the economy to other sectors in a general equilibrium model with multiple trading venues connected by profit-seeking arbitrageurs. Arbitrageurs effectively provide liquidity to investors by intermediating trades between venues. The welfare impact on venue k of a liquidity shock on venue ℓ can go in either direction, depending on whether intermediated trades on k behave as complements or substitutes for such trades on ℓ. In addition to this direct effect through the arbitrage network, there is a feedback effect of an adverse shock reducing liquidity and arbitrageur profits, which leads to a lower level of intermediation, further reducing liquidity. We illustrate this contagion with examples of high-frequency trading in equity markets, shocks to one tranche of a collateralized debt obligation impacting investors in the other tranches, carry trade crashes, shocks to cross-country bank lending following the global financial crisis, and the bursting of the Japanese bubble in the early 1990s.

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1 Introduction

Modern financial markets are highly fragmented. The same assets are traded on multiple venues, such as exchanges, multilateral trading facilities, dark pools, and electronic communication networks. Closely related securities such as derivatives and exchange-traded funds (ETFs) on these assets in turn trade on multiple platforms.[1]

In this paper we study a model with segmented markets that are tied together by strategic arbitrageurs who effectively provide intermediation or liquidity services to investors on each market segment. As competition between arbitrageurs intensifies, the allocation and prices in this economy are close to those of a Walrasian economy. However, due to the underlying fragmented market structure, shocks can be propagated and amplified in ways they cannot be in a centralized, integrated economy. A local shock sets in motion a chain of events as arbitrageurs curtail or expand their activities in different markets, or exit the intermediation business altogether.

We argue that such adjustments are pervasive in the economy. The reason market prices usually appear, in the eyes of most users and observers, to be set on a centralized market is that intermediaries quote and trade across different venues, thereby bringing prices into line. However, when there is a market disruption and these intermediaries withdraw from trading and market making, as happens in flash crashes or when venues suffer from technical faults, liquidity evaporates and markets display large violations of the law of one price. Similarly, prior to the global financial crisis, the financing of small and medium-sized enterprises (SMEs) in the Euro area was cross-border intermediated by large international banks, resulting in some degree of market integration. But the shock of the financial crisis, and the regulatory costs that followed it, led to the withdrawal of many banks to their home regions and a splintering of SME financing into much more segmented national markets. We can also think of carry traders as liquidity intermediaries, carrying funding from countries with excess funds to countries with a high demand for credit and high interest rates, profiting from the spreads until such time as some shock puts their capital at risk and intermediation unravels.

We employ a general equilibrium model with multiple assets traded in multiple markets or venues that are linked by profit-seeking arbitrageurs. Each venue is populated by investors who can trade only on that venue. Arbitrageurs, on the other hand, possess the technology which allows them to trade across venues, or in other words, which allows them to act as intermediaries if they so wish. In order to focus on cross-market arbitraging we assume that there is no within-venue heterogeneity. Thus all trade is intermediated by arbitrageurs. We model these intermediaries as

[1] As an example, consider the SPY ETF. SPY enters into a no-arbitrage relationship with the portfolio of equities underlying the S&P500 index. In addition, there are over 2000 options on SPY. Each such option needs to satisfy no-arbitrage relationships not only with SPY, but also with all sorts of combinations of other options on SPY. Furthermore, SPY options are traded on six options exchanges simultaneously, adding another layer of law-of-one price relationships. Finally, options on SPY are closely related to options on the S&P500 itself as well as to options on the S&P500 futures contract.

imperfectly competitive, with entry into the intermediation business unrestricted but entailing a fixed cost (say in terms of human capital, software, or co-location of servers at the various market centers). In equilibrium, arbitrageurs serve to integrate markets, bringing marginal valuations on different venues into closer alignment.[2] The liquidity that they provide to investors can be measured by the utility gains realized by investors from the intermediated trades.

This framework allows us to study how a liquidity shock originating in one part of the economy is transmitted through the entire arbitrage network. More precisely, we investigate how a fall in the investor population of one venue affects arbitrageur trades, and hence the liquidity (which is synonymous with welfare in our model) of all venues. The overall effect can be decomposed into a direct effect for a given number of arbitrageurs, and a secondary effect stemming from the exit of arbitrageurs from the intermediation business as profits fall.

Consider a population shock on venue ℓ. A contraction of intermediated trades and hence of liquidity on ℓ is accompanied by a corresponding contraction on venues that are on the other side of these trades, and thus behave as *complements* of ℓ. Venues that compete for trades on ℓ, and hence act as *substitutes*, stand to gain, however. Over and above this direct transmission across the network, there is a feedback effect through which a detrimental liquidity shock lowers the number of intermediaries, which in turn lowers liquidity and so on.

We illustrate this type of contagion through a natural experiment that occurred on the London Stock Exchange when a server outage resulted in a suspension of trading, with knock-on effects on alternative trading venues. Other examples of contagion that we discuss are sudden stops linked to carry trades, the dramatic reduction in cross-country bank lending in the wake of the global financial crisis, and the bursting of the Japanese bubble in the 1990s. Finally, we work out an extended example of contagion from one tranche of a collateralized debt obligation (CDO) to the other tranches. The boom in CDOs was made possible not only by the low interest rate environment, but also by the arbitrage profits reaped by CDO structurers due to the difference between the price paid for debt, and the monies raised by selling tranches of that debt tailored to the needs of individual clienteles. Our framework offers a rationale for the CDO mechanism. Quite naturally, it also illustrates the dangers inherent in such a mechanism: should the demand for one of the tranches fall, this local liquidity shock ripples through all the tranches.

[2]While the "venue" metaphor is a helpful one and fits some situations exactly, such as latency arbitrage in which the same or similar securities are traded simultaneously on multiple trading venues, it is equally natural to think of the segmentation as being functional rather than geographical, e.g. in terms of investors restricted to certain asset classes (stock indices versus the underlying stocks, equities versus derivatives on those equities, on-the-run versus off-the-run bonds, investment grade versus junk bonds etc.). A trading venue can also be interpreted as an over-the-counter (OTC) market in which an intermediary trades with a clientele; the intermediary then tries to offload the exposure from this OTC trade either with offsetting OTC counterparties or in the organized markets.

Related Literature:

Our analysis is based on the segmented markets framework proposed by Rahi and Zigrand (2009), who employ it to study security design by arbitrageurs. In the present paper, we analyze the transmission of shocks between market segments. The results that we need from the earlier paper are summarized in Propositions 2.1 and 2.2.

The theoretical literature on market fragmentation focuses mainly on welfare comparisons of segmented and consolidated markets. Recent contributions include Malamud and Rostek (2017) and Chen and Duffie (2020), who consider multiple exchanges on which strategic traders compete in supply schedules. These papers do not study contagion. Gromb and Vayanos (2018) analyze the provision of liquidity by competitive arbitrageurs in a segmented markets setting, with arbitrage opportunities involving pairs of assets traded on different market segments. Arbitrageurs face a separate margin constraint for each asset, and this limits their positions. Arbitrageur diversification across multiple arbitrage opportunities induces contagion. In contrast, arbitrage in our model is limited by a cost of entering the arbitraging business, and by imperfect competition among arbitrageurs. We do not restrict the asset structure on any venue, nor do we restrict the arbitrage network that connects these venues. The pattern of contagion in our setup depends on the characteristics of the investors and of the tradeable assets on each venue.

There is an extensive empirical literature that supports our view of financial markets as multiple segments inhabited by distinct investor clienteles; see Rahi and Zigrand (2009) for a discussion of this literature. A growing body of research documents the impact of fragmentation on market quality as measured by bid-ask spreads, depth, transaction costs or informational efficiency; see Gomber et al. (2017) for a survey. Much less work has been done on the effects of cross-venue trades. Karolyi et al. (2012) argue that during periods of market stress, commonalities appear that are due to crisis-induced trades by cross-market arbitrageurs. Ben-David et al. (2018) and Agarwal et al. (2018) find empirical support for arbitrageur trading as the conduit for the transmission of liquidity shocks from ETFs to the underlying assets. We discuss additional evidence of market fragmentation and contagion in Section 6, in the context of specific examples and applications of our results.

The rest of the paper is organized as follows. In Section 2, we describe an economy with multiple trading venues connected by arbitrageurs, and characterize the (unique) equilibrium. In Section 3, we discuss the relationship between intermediation and welfare. Then we study the impact of a local shock on one of the venues. We describe the contagion-like welfare effects of this shock in Section 4, and the impact on asset prices in Section 5. Section 6 is devoted to applications. Section 7 concludes.

2 The Economy

We consider a two-period economy in which assets are traded at date 0 and pay off at date 1. Uncertainty, which is resolved at date 1, is described by S states of the world. Assets are traded on K "venues", with the set of venues also denoted by K.

Venue k is populated by a continuum of identical investors of mass I^k who can trade only on that venue. Each of these investors has date 0 endowment ω_0^k, date 1 (random) endowment ω^k, and quadratic preferences,

$$U^k(x_0^k, x^k) = x_0^k + E\left[x^k - \frac{\beta^k}{2}(x^k)^2\right],$$

where x_0^k is date 0 consumption, x^k is the random consumption at date 1, and β^k is a positive parameter. We assume that $1 - \beta^k\omega^k \geq 0$, which says that investors on k have nonnegative marginal utility of date 1 consumption in the absence of trade (or "autarky"). We will sometimes refer to investors on venue k as clientele k.

In addition to investors, there are N arbitrageurs (with the set of arbitrageurs also denoted by N) who can trade across venues. Unlike investors, who take prices as given, arbitrageurs are imperfectly competitive. They have no endowments, and care only about date 0 consumption.

There are J^k (non-redundant) assets available to agents on venue k, with the random payoff of a typical asset j denoted by d_j^k. Asset payoffs on venue k can then be summarized by the random payoff vector $d^k := (d_1^k, \ldots, d_{J^k}^k)$. Assets are in zero net supply.

The interaction between price-taking investors and strategic arbitrageurs involves a Nash equilibrium concept with a Walrasian fringe. Let $y^{k,n}$ be the supply of the J^k assets on venue k by arbitrageur n, and $y^k := \sum_{n\in N} y^{k,n}$ the aggregate arbitrageur supply of these assets on venue k. For given y^k, $q^k(y^k)$ is the market-clearing asset price vector on venue k, with the asset demand of an investor on k denoted by $\theta^k(q^k)$. For vectors v and w in $\mathbb{R}^m$, $v \cdot w$ denotes the standard inner product, given by $\sum_{i=1}^m v_i w_i$.

Definition *A Cournot-Walras equilibrium (CWE) is an array of asset price functions, asset demand functions, and arbitrageur supplies, $\{q^k : \mathbb{R}^{J^k} \to \mathbb{R}^{J^k},\ \theta^k : \mathbb{R}^{J^k} \to \mathbb{R}^{J^k},\ y^{k,n} \in \mathbb{R}^{J^k}\}_{k\in K,\, n\in N}$, such that*

i. Investor optimization: For given q^k, $\theta^k(q^k)$ solves

$$\max_{\theta^k\in\mathbb{R}^{J^k}} x_0^k + E\left[x^k - \frac{\beta^k}{2}(x^k)^2\right],$$

subject to the budget constraints

$$x_0^k = \omega_0^k - q^k \cdot \theta^k,$$
$$x^k = \omega^k + d^k \cdot \theta^k.$$

ii. Arbitrageur optimization: For given $\{q^k(y^k), \{y^{k,n'}\}_{n'\neq n}\}_{k\in K}$, $y^{k,n}$ *solves*

$$\max_{y^{k,n}\in\mathbb{R}^{J^k}} \sum_{k\in K} y^{k,n}\cdot q^k\Big(y^{k,n}+\sum_{n'\neq n} y^{k,n'}\Big),$$

subject to the no-default constraint

$$\sum_{k\in K} d^k\cdot y^{k,n}\leq 0.$$

iii. Market clearing: $\big\{q^k(y^k)\big\}_{k\in K}$ *solves*

$$I^k\theta^k(q^k(y^k)) = y^k, \qquad \forall k\in K.$$

A complete characterization of the CWE can be found in Rahi and Zigrand (2009). We provide a brief synopsis of the relevant results in Propositions 2.1 and 2.2 below.

It is convenient to describe prices, trades and payoffs in terms of state-price deflators. Given a collection of J assets with random payoffs $d := (d_1,\ldots,d_J)$ and prices $q := (q_1,\ldots,q_J)$, a random variable p is called a state-price deflator if $q_j = E[d_jp]$ for every asset j, or more compactly, $q = E[dp]$. Consider the set of marketable payoffs, given by $M := \{z : z = d\cdot\theta,\ \text{for some portfolio } \theta\in\mathbb{R}^J\}$. For an arbitrary random variable z, let z_M denote the least-squares projection of z on M. If markets are incomplete, there are many state-price deflators p that price the payoffs in M identically, i.e. for which $E[zp]$ is the same for any given z in M. However, there is a unique state-price deflator that lies in M. This *traded* state-price deflator is p_M, the least-squares projection on M of any of the deflators p. We denote the set of marketable payoffs on venue k by M^k.

Proposition 2.1 (Cournot-Walras equilibrium: Rahi and Zigrand (2009)) *There is a unique CWE.*

i. Market-clearing asset prices, as a function of arbitrageur supplies, are given by

$$q^k(y^k) = E\left[d^k\left(p^k - \frac{\beta^k}{I^k}(d^k\cdot y^k)\right)\right], \qquad k\in K, \tag{1}$$

where $p^k := 1-\beta^k\omega^k$.

ii. Equilibrium arbitrageur supplies of state-contingent consumption are given by

$$d^k\cdot y^{k,n} = \frac{I^k}{(1+N)\beta^k}\left(p^k_{M^k} - p^A_{M^k}\right), \qquad k\in K, \tag{2}$$

where $p^A\geq 0$ *is a state-price deflator for the arbitrageurs.*

iii. Equilibrium asset prices on venue k are given by $\hat{q}^k = E[d^k \hat{p}^k]$, where

$$\hat{p}^k := \frac{1}{1+N}\, p^k + \frac{N}{1+N}\, p^A. \tag{3}$$

Thus $\hat{p}^k$ is an equilibrium state-price deflator for venue k.

iv. The equilibrium profits of an arbitrageur are given by

$$\Phi := \sum_{k \in K} \hat{q}^k \cdot y^{k,n} = \frac{1}{(1+N)^2} \sum_{k \in K} \frac{I^k}{\beta^k}\, E\big[(p^k_{M^k} - p^A_{M^k})^2\big]. \tag{4}$$

v. The equilibrium utilities of investors are given by

$$\mathcal{U}^k = U^k(\omega^k_0, \omega^k) + \left(\frac{N}{1+N}\right)^2 \frac{1}{2\beta^k}\, E\big[(p^k_{M^k} - p^A_{M^k})^2\big], \qquad k \in K. \tag{5}$$

From (1), $p^k - (\beta^k/I^k)(d^k \cdot y^k)$ is a state-price deflator for venue k when the supply of state-contingent consumption by arbitrageurs to venue k is $d^k \cdot y^k$. If arbitrageurs supply an additional unit of consumption in state s, the state-price deflator falls by β^k/I^k in that state. Accordingly, I^k/β^k is the depth of venue k. Setting $y^k = 0$, we see that p^k is an autarky state-price deflator for venue k.

The CWE is symmetric with all arbitrageurs supplying the same amount of state-contingent consumption to any given venue (equation (2)). The random variable p^A is a state-price deflator for the arbitrageurs in the sense that p^A_s is the arbitrageurs' marginal shadow value of consumption in state s.[3] Assuming for the moment that markets are complete on all venues, an arbitrageur supplies state s consumption to those venues which value it more than he does ($p^k_s - p^A_s > 0$). How much he supplies to venue k depends on the size of the mispricing $|p^k_s - p^A_s|$, on the depth I^k/β^k, with more consumption supplied the deeper the venue, and finally on the degree of competition N. If markets are incomplete, however, the difference between state prices may not be marketable. The arbitrageur would then supply state-contingent consumption as close to $p^k - p^A$ as permissible by the available assets d^k. The closest such choice is the projection $(p^k - p^A)_{M^k} = p^k_{M^k} - p^A_{M^k}$. The greater the number of arbitrageurs competing for the given opportunities, the smaller is each arbitrageur's residual demand, and so the less each one supplies. In the limit, as N approaches infinity, the equilibrium valuation on each venue converges to the arbitrageur valuation p^A, as we can see from (3).

[3] To be precise, p^A_s is the Lagrange multiplier attached to the arbitrageurs' no-default constraint in state s. More concretely, the algorithms used by latency arbitrageurs are known to revolve around the concept of a "micro price" that corresponds to the arbitrageur's marginal valuation, prompting the algorithm to buy if the actual price on a venue is below this value and to sell if it is above, as in equation (2).

Under a suitable restriction on the asset structure, p^A takes a very simple form. Let p^* denote the complete-markets Walrasian state-price deflator of the entire integrated economy in which investors do not face any participation constraints and there are no arbitrageurs. It can be shown that

$$p^* = \sum_{k \in K} \lambda^k p^k,$$

where

$$\lambda^k := \frac{\frac{I^k}{\beta^k}}{\sum_{j=1}^K \frac{I^j}{\beta^j}}, \qquad k \in K.$$

The state-price deflator p^* reflects the autarky valuation of each venue in proportion to its depth. Now consider the following spanning condition:

(S) Either (a) $M^k = M$, $k \in K$, or (b) $p^k - p^* \in M^k$, $k \in K$.

Under $\mathbf{S}$(a) we have a standard incomplete-markets economy in which all investors trade the same payoffs, though on different venues. Rahi and Zigrand (2009) show that an asset structure satisfying $\mathbf{S}$(b) is an optimal asset structure for arbitrageurs, and also an equilibrium security design in a game in which arbitrageurs choose the securities traded on each venue. Under $\mathbf{S}$, arbitrageur valuations are Walrasian:

Proposition 2.2 (Arbitrageur valuations: Rahi and Zigrand (2009))
Suppose the spanning condition $\mathbf{S}$ *holds. Then, arbitrageur valuations in the CWE coincide with valuations in the complete-markets Walrasian equilibrium, i.e. we can choose* $p^A = p^* = \sum_k \lambda^k p^k$. *Consequently,* $\lim_{N\to\infty} \hat{q}^k = E[d^k p^*]$.

This is the sense in which arbitrageurs serve to integrate markets. As the number of arbitrageurs goes to infinity, asset prices converge to those that would arise in the complete-markets Walrasian equilibrium.[4]

3 Intermediation and Welfare

In order to interpret the expressions for arbitrageur profits and investor welfare in the previous section, it is useful to define

$$\mathcal{L}^k := \left(\frac{N}{1+N}\right)^2 \frac{I^k}{2\beta^k} E\left[(p^k_{M^k} - p^A_{M^k})^2\right], \tag{6}$$
$$\mathcal{L} := \sum_{k \in K} \mathcal{L}^k.$$

[4] Convergence to the complete-markets Walrasian allocation holds under $\mathbf{S}$(b), but not necessarily under $\mathbf{S}$(a). Rahi and Zigrand (2014) show that, under $\mathbf{S}$(a), the allocation converges to the restricted-participation Walrasian equilibrium allocation. They also characterize p^A for a general asset structure (without imposing $\mathbf{S}$).

Then, from (4), we can write the profits of an individual arbitrageur as

$$\Phi = \frac{2\mathcal{L}}{N^2}. \tag{7}$$

Since agents on any given venue are identical, they all have the same equilibrium utility, given by (5). Accordingly, the welfare of venue k, $\mathcal{W}^k := I^k\mathcal{U}^k$, is

$$\mathcal{W}^k = I^k U^k(\omega_0^k, \omega^k) + \mathcal{L}^k. \tag{8}$$

Thus $\mathcal{L}^k$ captures the welfare gains realized by investors on k through intermediation. In this sense, it is a welfare-based measure of the liquidity provided by intermediaries to venue k; accordingly, we will refer to it as venue k's liquidity. The term $E[(p^k_{M^k} - p^A_{M^k})^2]$ is the mean-square distance between venue k's autarky valuation p^k and the economy-wide valuation p^A, projected onto the set of marketable payoffs M^k. Liquidity on venue k is increasing in this distance, in the depth of the venue, I^k/β^k, and in the number of arbitrageurs N; note that aggregate arbitrageur supply to k is increasing in I^k/β^k and in N. Arbitrageur profits originating from venue k are proportional to $\mathcal{L}^k$. As N increases without bound, all the potential welfare gains from trade accrue to investors, while aggregate arbitrageur profits $N\Phi$ go to zero. We denote the limiting values of $\mathcal{L}^k$ and $\mathcal{L}$, as N goes to infinity, by $\bar{\mathcal{L}}^k$ and $\bar{\mathcal{L}}$, respectively:

$$\begin{aligned} \bar{\mathcal{L}}^k &:= \frac{I^k}{2\beta^k} E\left[(p^k_{M^k} - p^A_{M^k})^2\right], \\ \bar{\mathcal{L}} &:= \sum_{k\in K} \bar{\mathcal{L}}^k. \end{aligned} \tag{9}$$

$\bar{\mathcal{L}}^k$ is the maximal liquidity (or maximal welfare gains from intermediation) on venue k, given the available assets d^k. In practitioner language we can think of $\bar{\mathcal{L}}$ as the available untapped "global pool of liquidity", distributed across local liquidity pools waiting to be connected. The fraction of the available liquidity pool that is in fact exploited in equilibrium is $\mathcal{L}/\bar{\mathcal{L}} = [N/(1+N)]^2$.

The equilibrium level of intermediation is closely tied to the global pool of liquidity $\bar{\mathcal{L}}$. Suppose each arbitrageur must bear a fixed cost c in order to set up shop and intermediate across all markets. Then, from (7), ignoring integer constraints on N, N solves $c = 2\mathcal{L}/N^2$. Since $\mathcal{L}/\bar{\mathcal{L}} = [N/(1+N)]^2$, the following result is immediate:

Proposition 3.1 (Equilibrium level of intermediation) *Suppose $c \le \bar{\mathcal{L}}/2$. Then the equilibrium number of arbitrageurs N is given by*

$$N = \sqrt{2\bar{\mathcal{L}}c^{-1}} - 1.$$

The upper bound on c is needed to ensure that at least one arbitrageur finds it profitable to intermediate trades. This will be a standing assumption for the rest of the paper. The equilibrium N is decreasing in c, and grows without bound as c goes to zero. While it is convenient to ignore integer constraints on N for our analytical results in the next two sections, it is easy to calculate the natural number N that satisfies the free entry condition. It is given by

$$N = \mathrm{rd}\left(\sqrt{2\bar{\mathcal{L}}c^{-1}} - 1\right), \tag{10}$$

where the operator "rd" rounds the real number in parenthesis down to the next natural number. For this value of N, arbitrageurs make profits in equilibrium, but not enough to attract one further arbitrageur.

4 Transmission of Liquidity Shocks

In this section we study how liquidity shocks are transmitted across the economy. Starting from an initial equilibrium, we perturb fundamentals on one of the venues and analyze the economy-wide repercussions of this local shock. In order to simplify the analysis, we assume that the spanning condition **S** holds in a neighborhood of the equilibrium, i.e. either the security design is optimal, or the same set of payoffs are tradable on all venues. Then we can choose $p^A = p^* = \sum_k \lambda^k p^k$ by Proposition 2.2.[5]

We consider a shock to the investor population (or participation) I^ℓ on venue ℓ. A withdrawal of participants on venue ℓ lowers its depth I^ℓ/β^ℓ while keeping its autarky state-price deflator, $p^\ell = 1 - \beta^\ell\omega^\ell$, constant. Consequently p^ℓ plays a less prominent role in p^*.

Let

$$\vartheta^{k\ell} := \frac{E\left[(p^k_{M^k} - p^*_{M^k})(p^\ell_{M^k} - p^*_{M^k})\right]}{E\left[(p^k_{M^k} - p^*_{M^k})^2\right]}. \tag{11}$$

Thus $\vartheta^{k\ell}$ is the regression coefficient of the (projected) mispricing on venue ℓ, $p^\ell_{M^k} - p^*_{M^k}$, on the mispricing on venue k, $p^k_{M^k} - p^*_{M^k}$. This measure of covariation is a noncentral "beta" in the language of the CAPM. Ignoring integer constraints on N, we have the following result:

Proposition 4.1 (Contagion) *Consider a CWE and suppose the spanning condition* **S** *holds in a neighborhood of this equilibrium. Then the effect on venue k's liquidity $\mathcal{L}^k$ of a population shock on venue ℓ is given by*

$$\frac{d\log\mathcal{L}^k}{d\log I^\ell} = \underbrace{\mathbb{1}_{k=\ell} - 2\lambda^\ell\vartheta^{k\ell}}_{\left.\frac{d\log\mathcal{L}^k}{d\log I^\ell}\right|_N} + \frac{\bar{\mathcal{L}}^\ell}{N\bar{\mathcal{L}}}, \tag{12}$$

[5] The assumption that **S** holds in a neighborhood of the equilibrium allows us to set p^A equal to p^* both before and after the shock. This is clearly not an issue if the same payoffs are traded on all venues (condition **S**(a)). However, if we invoke **S**(b), the result should be interpreted as the long-run effect of a population shock, allowing for optimal adjustment of the security design. While it is difficult to obtain an analytical result if we fix the (initially optimal) security design, numerical examples can be worked out, as we do in Section 6.6.

and the effect on aggregate liquidity $\mathcal{L}$ is given by

$$\frac{d\log\mathcal{L}}{d\log I^\ell} = \frac{1+N}{N}\frac{\bar{\mathcal{L}}^\ell}{\bar{\mathcal{L}}}. \tag{13}$$

The proof is in the Appendix. The indicator function $\mathbb{1}_{k=\ell}$ takes the value 1 if $k=\ell$, and is zero otherwise.

There is a straightforward connection between liquidity shocks and welfare. From the expression for the welfare of venue k, $\mathcal{W}^k$, given by (8),

$$\frac{d\mathcal{W}^k}{dI^\ell} = U^\ell(\omega_0^\ell,\omega^\ell)\mathbb{1}_{k=\ell} + \frac{d\mathcal{L}^k}{dI^\ell}.$$

Thus, for venue $k\neq\ell$, a change in liquidity is synonymous with a change in welfare. For venue ℓ there is an additional term that captures the welfare loss corresponding to agents on ℓ who are "removed" from ℓ as a result of the population shock. Any loss of liquidity on ℓ is therefore equal to the welfare loss of agents who are still active on ℓ. As far as arbitrageurs are concerned, a liquidity shock has no effect on their welfare, since our assumption of free entry ensures that they make zero net profits in any equilibrium.

The overall effect on venue k's liquidity can be decomposed into two components: a direct effect for a given N, captured by the term $\mathbb{1}_{k=\ell} - 2\lambda^\ell\vartheta^{k\ell}$, and an indirect effect via entry or exit which is represented by the term $\bar{\mathcal{L}}^\ell/(N\bar{\mathcal{L}})$. Notice that the direct effect does not depend on the initial level of N, while the indirect effect is decreasing in N.

Consider first a venue $k\neq\ell$, and suppose N is fixed. The effect on venue k's liquidity is $-2\lambda^\ell\vartheta^{k\ell}$. If the parameter $\vartheta^{k\ell}$ is negative, venues k and ℓ are *complements* in the sense that arbitrageurs tend to buy on one when they are selling to the other, i.e. there is intermediated trade between the two venues. If venue ℓ experiences a reduction in its investor base, and a consequent deterioration of its depth, these intermediated trades become less valuable and less plentiful in equilibrium, thus reducing liquidity on k.

With endogenous N, this effect is exacerbated: fewer investors and lower depth on ℓ lead to less trade and to lower liquidity, which in turn leads to lower profits and thereby to fewer intermediaries, which in turn affects liquidity adversely and so forth. The net effect of this feedback loop is $\bar{\mathcal{L}}^\ell/(N\bar{\mathcal{L}})$. The effect is more pronounced the larger the role of venue ℓ in generating trades, as measured by its relative size $\bar{\mathcal{L}}^\ell/\bar{\mathcal{L}}$ (which is equal to $\mathcal{L}^\ell/\mathcal{L}$), and the smaller the initial N. A smaller initial N means that the feedback loop of liquidity on N and again of N on liquidity etc. is stronger as each arbitrageur is more powerful and holds a larger portfolio.

If, on the other hand, $\vartheta^{k\ell}>0$, valuations on venues k and ℓ are similar in the sense of being on average on the same side as the economy-wide valuation p^*. The two venues therefore compete for trades, and can be said to be *substitutes*. In this case, a shallower ℓ induces intermediaries to migrate to k, thereby increasing liquidity on k, for given N. The contagion effect operating through a lower N is however the same as in the case of complementary venues.

Now consider the effect of a population shock on venue ℓ on its own liquidity. For fixed N, this effect is given by $(1 - 2\lambda^\ell)$. If λ^ℓ is small, this has the straightforward interpretation of the direct loss of liquidity due to the flight of investors. This is compounded by the consequent flight of intermediaries in the same way as for the rest of the economy. If λ^ℓ is non-negligible, however, there is a countervailing effect. Indeed, if $\lambda^\ell > 1/2$, $\mathcal{L}^\ell$ actually increases when the population on ℓ falls, for given N. This might at first appear odd, but the effect stems from the endogenous nature of Walrasian prices. Fewer investors on venue ℓ lower the depth of venue ℓ, and everything else constant, liquidity is lower. But the smaller size of this clientele also means that it will now play a less prominent role in the determination of the economy-wide valuation p^*. The valuation p^* will become more dissimilar from p^ℓ, thereby increasing the potential gains from trade between ℓ and the rest of the economy, stimulating intermediated trades and increasing liquidity on ℓ. If $\lambda^\ell > 1/2$, this effect is strong enough to compensate for the loss of depth, before accounting for the knock-on effect on the number of intermediaries.

Evidently, in an economy with many venues, loss of liquidity is more likely to go hand in hand with a decline in the number of active investors. But there might be situations where a dominant venue optimally limits or rations participants. It may be that the arrival of more (identical) investors can hurt local liquidity. The converse implication is that liquidity can suffer on a venue that experiences a rise in its investor population.

For $k \neq \ell$, assuming that $\lambda^\ell < 1/2$, it is easy to verify that

$$\frac{d \log \mathcal{L}^k}{d \log \mathcal{L}^\ell} > 1 \qquad \text{iff} \qquad 2\lambda^\ell(1 - \vartheta^{k\ell}) > 1 \tag{14}$$

for the population-type shocks considered above. Thus, if ℓ is large (but not too large) in terms of relative depth, and k is sufficiently complementary with respect to ℓ, a liquidity shock on ℓ has an even bigger impact on k than on ℓ itself. This is an illustration of the dictum that "when Russia sneezes, Brazil catches a cold".

If we measure the degree to which markets are integrated by N, we see that contagion (in the sense of an adverse spillover) is more pronounced the more fragmented markets are. More precisely, the expression in (12) is strictly decreasing in N, and is minimized as N goes to infinity and perfect integration is achieved. If k and ℓ are substitutes, this minimized value is negative; in this case the spillover of a negative population shock is actually benign.

Finally, consider the effect on aggregate liquidity given by (13). As one would expect, a negative shock always reduces aggregate liquidity, and hence arbitrageur profits and the level of intermediation N. As with contagion, this effect is greater if markets are more fragmented prior to the shock.

To summarize, the welfare impact of an adverse liquidity shock on venue ℓ (as measured by a lower I^ℓ) may be positive or negative on venue k, but the effect on overall welfare is always negative. Moreover, the greater the degree of fragmentation prior to the shock, the more pronounced is the impact on the overall economy and on any adversely affected venues; if there are any venues which benefit from the shock, this benefit is lower in a more fragmented economy.

5 Contagion and Asset Prices

We now turn to the effect of a liquidity shock on asset prices. It is instructive to consider the case where the same assets trade on all venues so that price comparisons are straightforward. Accordingly, we assume that $d^k = d$, all k. Then $q^* := E[dp^*]$ is the asset price vector implied by the hypothetical complete-markets state-price deflator for the entire integrated economy. Autarky asset prices on venue k are given by $\mathring{q}^k := q^k(0) = E[dp^k]$. These are prices at which investors on k choose not to trade. From Proposition 2.1 (iii), equilibrium asset prices on venue k are are given by

$$\hat{q}^k = E[d\hat{p}^k] = \frac{1}{1+N}\,\mathring{q}^k + \frac{N}{1+N}\,q^*.$$

Proposition 5.1 (Asset prices) *Suppose $d^k = d$, for all $k \in K$. Then the effect on equilibrium asset prices on venue k of a population shock on venue ℓ is given by*

$$\frac{d\hat{q}^k}{d\log I^\ell} = \underbrace{\frac{N}{1+N}\,\lambda^\ell(\mathring{q}^\ell - q^*)}_{\left.\frac{d\hat{q}^k}{d\log I^\ell}\right|_N} + \frac{\bar{\mathcal{L}}^\ell}{2(1+N)\bar{\mathcal{L}}}\,(q^* - \mathring{q}^k).$$

The proof is in the Appendix. As in our analysis of the effect of a population shock on the liquidity of venue k, the effect on asset prices can be decomposed into a direct effect, for given N, and the indirect effect of a change in N.

Consider first the direct effect. If venue ℓ in isolation values asset j more highly than the economy as a whole ($\mathring{q}^\ell_j > q^*_j$), an adverse participation shock on ℓ depresses the price of asset j on all venues. This is because the tendency of venue ℓ to pull up the price of this asset, via intermediated trades, is reduced when its weight in the economy is lower. Quite naturally, the effect is more pronounced the greater the degree of intermediation.

The indirect affect of a negative shock works through a lower level of intermediation which pulls the price of asset j on venue k closer to its autarky level $\mathring{q}^k_j$ and further away from the economy-wide valuation q^*_j. This effect is more pronounced the greater the initial degree of fragmentation.

The overall effect is unambiguous in sign if the economy-wide valuation q^*_j lies between the autarky valuations of k and ℓ. For example, if $\mathring{q}^k_j < q^*_j < \mathring{q}^\ell_j$, then $\mathring{q}^k_j < \hat{q}^k_j < q^*_j < \mathring{q}^\ell_j$, and a negative population shock on venue ℓ lowers $\hat{q}^k_j$.

6 Examples and Applications

In this section we show how our framework can be used to understand the diffusion of liquidity shocks in some recent market events.

6.1 High-Frequency Traders

One of the most disruptive recent changes in the financial industry has been the widespread proliferation of trading venues following the Regulation National Market System (Reg NMS) in the US and the Markets in Financial Instruments Directive (MiFID) in Europe. These created markets that are segmented very much like we assume in our setup, with the same stocks traded not only on several exchanges but also on alternative trading systems such as multilateral trading facilities (MTFs), electronic communication networks (ECNs) and various dark pools. The regulations, which were designed to enhance competition between trading venues, have in turn spawned a new breed of intermediary in the form of high-frequency traders (HFTs) or latency arbitrageurs[6] who trade simultaneously across multiple trading venues in order to exploit, and thus reduce or eliminate, price discrepancies. A very large percentage of trading volume has been attributed to such traders.[7] There is growing concern that competition in security markets in the US and Europe has led to trading liquidity becoming fragmented across too many venues. At the same time, the HFTs, who provide liquidity and help to align prices across venues, have been viewed with suspicion by the press, the traditional real-money investors and even by the regulators who to some extent created the need for this intermediation.

The Flash Crash of May 6th 2010, in which the Dow Jones index fell nearly 10% only to recover a few minutes later, has accelerated that discussion and has brought the topic of modern market making to the forefront. Attention has focused on the interconnectedness of trading venues and the implications for liquidity and welfare. For instance, a report by the CFTC and SEC (CFTC and SEC (2010)) points out that during the Flash Crash, "hot-potato volumes" spiked up as HFTs passed securities around in a musical chair-like fashion within and across trading venues, and shocks were transmitted across markets for stocks, options and futures in a complex fashion. When latency arbitrageurs withdrew from the markets and prices of identical securities diverged across trading venues, panic set in as market participants no longer trusted the price discovery mechanism.[8] This suggests that

[6]Besides pure HFT shops such as KCG, Virtu and Optiver, examples of HFTs include proprietary quantitative hedge funds and market makers at firms such as Citadel Group, D.E. Shaw Group and Renaissance Technologies, as well as trading desks in some of the major investment banks.

[7]Various sources estimate that the fraction of equity trades involving HFT algorithms is 60–70% in the US, 30% in the UK, 40% in Europe, and 30% in Japan (see, for instance, Beddington et al. (2013)). The TABB group estimates that annual aggregate profits from latency arbitrage currently exceed \$21bn, Donefer (2008) provides a range of \$15-25bn, and Strasbourg (2011) estimates that HFT profits in the US were around \$7.2bn in 2009.

[8]Consider for example the E-Mini index futures contract traded on CME Globex and the SPY

conventional measures of intermediation and liquidity provision may not adequately reflect market conditions when trading and liquidity are fragmented.

6.2 Trading Halt on the LSE

The UK FTSE stock market basically consists of the London Stock Exchange (LSE) as the main venue with around 60% of trading volume for FTSE-100 stocks, with BATS, Chi-X and Turquoise as the main MTFs.[9] Since these venues trade a large common set of securities, we can reasonably view them as being competing venues, or substitutes. On Thursday 26th of November 2009, the LSE halted trading at 10:33 due to a server error, placing all order books into auction mode until trading resumed at 14:00. If these venues were strong substitutes, our model would predict that a negative liquidity shock on the LSE would lead to higher liquidity on the MTFs. But the opposite happened. Liquidity dried up immediately on all the MTFs and recovered only on the dot at 14:00 (see Intelligent Financial Systems (2009)).

Our model suggests that these markets should instead be understood as complements, with arbitrageurs typically buying on one and selling on the other. By Proposition 4.1, an adverse shock to I^{LSE} has a negative impact on the liquidity of an MTF if and only if

$$2\lambda^{LSE}\vartheta^{LSE,MTF} < \frac{\bar{\mathcal{L}}^{LSE}}{N\bar{\mathcal{L}}}.$$

So all trading venues that are either weak enough substitutes or complements of the LSE would have their liquidity negatively affected by a liquidity shock to the LSE. In fact, (14) tells us that the impact on an MTF would be more pronounced than on the LSE itself if

$$2\lambda^{LSE}(1-\vartheta^{LSE,MTF}) > 1$$

(we can safely assume that $\lambda^{LSE} < 1/2$). This condition is more likely to be satisfied the larger the relative weight of the LSE in pricing the true value of stocks, and the greater the degree of complementarity. It would be an interesting empirical exercise to estimate these numbers.

6.3 Currency Carry Trades

The basic currency carry trade involves borrowing in one country and lending to another in order to exploit interest rate differentials. Evidence shows that most

ETF traded on NYSE, both of which track the S&P500. During the Flash Crash, trading in the E-Mini was paused for 5 seconds while trading in SPY continued. Uncertainties about pricing accuracy, exacerbated by the uncoordinated introduction of circuit breakers, led many arbitrageurs to cease operating their cross-market strategies. For four minutes, very profitable arbitrage mispricings occurred (see CFTC and SEC (2010) and Nanex (2010)). For a detailed analysis of financial stability in computer-based trading environments, the reader is referred to Chapter 4 of Beddington et al. (2013).

[9] BATS acquired Chi-X in 2011. They were separate entities at the time of the trading halt on the LSE.

of the carry trade is done by specialized, highly levered carry traders, akin to our arbitrageurs,[10] suggesting that a segmented markets framework is the right one to think of the carry trade (see Lee et al. (2020) for an insightful analysis of carry). The profits of carry traders are typically viewed as a compensation for offering liquidity to agents in the recipient country, the high interest rate in that country reflecting a large unmet demand for credit.

While our two-period real setup is too pared-down to explain many of the fascinating dynamics of the carry trade, it does highlight the intermediation role that carry traders play, and the general equilibrium effects of their trades. In order to illustrate the carry trade most simply, we assume that only the riskfree asset is traded in each country, which corresponds to a venue in our model. Let $\mathring{r}^i$ be the riskfree interest rate in country i in autarky, and r^* the riskfree rate in the hypothetical complete-markets integrated world economy with no arbitrageurs. Specializing our model to this asset structure we obtain the following result (see the Appendix):

Proposition 6.1 *Suppose $d^k = 1$, for all $k \in K$. Then arbitrageur supply on venue k is*

$$y^{k,n} = \frac{I^k}{(1+N)\beta^k} \frac{r^* - \mathring{r}^k}{(1+r^*)(1+\mathring{r}^k)},$$

liquidity on venue k is

$$\mathcal{L}^k = \left(\frac{N}{1+N}\right)^2 \frac{I^k}{2\beta^k} \left(\frac{r^* - \mathring{r}^k}{(1+r^*)(1+\mathring{r}^k)}\right)^2,$$

and

$$\vartheta^{k\ell} = \frac{(r^* - \mathring{r}^\ell)/(1+\mathring{r}^\ell)}{(r^* - \mathring{r}^k)/(1+\mathring{r}^k)}.$$

It is convenient to partition the set of countries into "funding countries" where the autarky interest rate is lower than the Walrasian interest rate r^*, and "receiving countries" where the autarky interest rate is higher than r^*. Quite naturally, arbitrageurs borrow from funding countries and lend to receiving countries. All countries benefit from the carry trades; those that benefit the most are the ones with the highest depth I^k/β^k, and the greatest discrepancy between their local interest rate and the economy-wide rate r^*. Carry trades channel excess savings to locations where the funds are needed the most.

We see from the sign of $\vartheta^{k\ell}$ that a funding and a receiving country are complements, while countries that belong to the same group are substitutes. Hence a liquidity shock to a receiving country negatively affects all funding countries and, for given N, positively affects all other receiving countries. The effects of a shock to a funding country are analogous. Furthermore, from (14), the impact of a shock to a

[10] Arbitrageurs in our model are infinitely levered, having no initial capital. Their no-default constraints as well as their strategic behavior limit the positions that they take. Anecdotally, the main FX carry traders are hedge funds, private-equity firms and non-financial corporations.

country ℓ in one group on a country k in the other group is magnified, in the sense that $d\log\mathcal{L}^k > d\log\mathcal{L}^\ell$, if $2\lambda^\ell(1-\vartheta^{k\ell}) > 1$ (assuming that $\lambda^\ell < 1/2$).

It is well-known that carry trades can lead to so-called "sudden stops" or "carry crashes". In our model, a participation-like credit shock originating in a receiving country ℓ can have a big impact on a funding country k by the time all direct and indirect effects are accounted for. The size of this impact is increasing in λ^ℓ (provided $\lambda^\ell < 1/2$), and in $|\vartheta^{k\ell}|$. In other words, the effect is larger the greater the discrepancy between $\mathring{r}^\ell$ and r^* and the more average k is (the closer $\mathring{r}^k$ is to r^*).[11] The same analysis goes through for a negative population-liquidity shock to a funding country. Our general equilibrium analysis also explicitly describes knock-on effects on countries that are not linked directly. For instance, although no funds flow between two funding countries k_1 and k_2, a negative shock to k_1 will affect credit liquidity on k_2 positively through the substitution effect, and negatively through a lower number of carry traders.

6.4 European Credit Fragmentation

The financing of small and medium-sized enterprises (SMEs) in Europe has always been a segmented market. International banks play the role of cross-border direct as well as wholesale and interbank liquidity providers, the same role that is played by arbitrageurs in our model. Several studies show that credit markets have become more fragmented since the global financial crisis (Cetorelli and Goldberg (2011), Milesi-Ferretti and Tille (2011), Bremus and Fratzscher (2015), Bruno and Shin (2015), Lane (2015)), and that the decline in cross-border direct and wholesale bank lending is particularly pronounced and persistent in the Euro area.

Bremus and Neugebauer (2018) show that in the Euro area new bank credit to small firms declined by nearly 40% between 2008 and the beginning of 2014, with SMEs reporting deteriorating credit availability in many Euro area countries. Access to finance has been particularly problematic in the periphery countries, with spreads between loan rates for small and large loans rising significantly. In contrast, only about 6% of German firms listed access to finance as their most pressing problem at the end of 2013. The authors note that these differences across countries point to strong fragmentation of credit markets in the Euro area.

In modeling terms, we can stylize this episode through a shock to $I^{\text{periphery}}$. Reduced overall demand for loans in the periphery countries due to the downturn leads in our model to a decrease in cross-border intermediation N and lower liquidity in both receiving and funding countries, in turn raising funding spreads between SMEs in the periphery and those in the center. It is also likely that the series of very strict bank regulations following the crisis reduced the profitability of such cross-border loans. This can be captured in our model by an increase in the intermediation cost c, leading to a further reduction in lending volumes and liquidity, and an increase in

[11] This is because we measure the effect on liquidity $\mathcal{L}^k$ relative to its initial level, which goes to zero as $\mathring{r}^k \uparrow r^*$.

spreads. These are also the findings of De Haan et al. (2017) based on micro-data for a set of Euro area banks.

6.5 Japan-US in the Early 1990s

As a further illustration of contagion, this time of the macro type, consider the liquidity shock emanating from Japan at the end of the 1980s and beginning of the 1990s, as documented for example by Peek and Rosengren (1997). We can interpret this shock as a drop in the Japanese local investor base. While Japan was a major financial power, it is safe to assume that it did not account for more than half of the world's financial depth. Given that the flow of capital was from Japan to the US, Japan and the US were complements, and on average asset prices were higher in Japan than in the rest of the world. The adverse shock to Japanese liquidity depressed stock prices in Japan. The authors found that the result of this liquidity shock was a sharp decline in Japanese investment in the US, which in turn adversely affected liquidity in the US, an instance of contagion along the lines suggested by our model.

6.6 CDO Boom and Bust

In this section we provide an extended example of the mechanism underlying CDOs in order to study how a liquidity shock to one tranche affects investors in other tranches. Proposition 4.1 does not apply since the asset structure does not satisfy the spanning condition **S** in a neighborhood of the initial equilibrium (though it does satisfy **S** *at* the equilibrium). The example allows us to go beyond the local shocks considered in our previous results, and investigate the effect of shocks of arbitrary size.

The profit to intermediaries from structuring and marketing CDOs ultimately stems from the fact that the tranched cash flows can be sold for more than the procurement cost of the cash flows from credit, such as loans and mortgages. In this example, we assume that there are four clienteles. Venue 4 represents the clientele from which the credit originates, modeled as a single security with payoff d^4. Suppose there are three states of the world, and the promised cash flows from credit are 3. Due to default, however, the effective cash flows are $d^4 = (3, 2, 1)$, where we write the random variable d^4 as a vector of state-contingent payoffs. In other words, in state 1 all loans are repaid, in state 2 two-thirds are repaid, and in state 3 only one-third are repaid. For simplicity, we assume that the three states are equally probable.

Intermediaries slice the cash flows from venue 4 into three tranches. The supersenior tranche is sold off to the highest bidders, here represented by investors on venue 1. We assume that the supersenior tranche always pays off,[12] with $d^1 = (1, 1, 1)$. The mezzanine tranche, paying off $d^2 = (1, 1, 0)$, is sold to the highest bidding clientele, on venue 2. Notice that the mezzanine tranche suffers a loss in state 3. Finally, the

[12] This is irrelevant for our results. With more states, superseniors can default as well.

highest bidders for the junior tranche are investors on venue 3. The junior tranche only pays off in state 1 as it is the first to absorb any losses: $d^3 = (1,0,0)$. To summarize, the asset structure is:

$$d^1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad d^2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad d^3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad d^4 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}. \tag{15}$$

We construct an economy in which the equilibrium strategies of the arbitrageurs consist of buying the debt on venue 4, tranching it, and selling each tranche off to the clientele that values it most. We are interested in the transmission of liquidity shocks across this economy. In particular, based on current accounts of the subprime crisis, the relevant question is what the repercussions on overall liquidity are of a diminished clientele for the supersenior tranche.

To simplify our calculations, we assume that all investors have the same preference parameter $\beta^k = 1/4$. Furthermore, we assume that venues 2, 3 and 4 have the same population, which we normalize to one (i.e. $I^2 = I^3 = I^4 = 1$). We denote the population on venue 1 by I (i.e. $I^1 = I$). We shall reduce I to reflect investor flight from the supersenior CDO tranche. Date 1 endowments are as follows:

$$\omega^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \omega^2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \omega^3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \omega^4 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}.$$

The corresponding autarky state-price deflators, given by $p^k = 1 - \beta^k \omega^k$, are:

$$p^1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad p^2 = \begin{bmatrix} 1 \\ 1 \\ \frac{3}{4} \end{bmatrix}, \quad p^3 = \begin{bmatrix} 1 \\ \frac{3}{4} \\ \frac{3}{4} \end{bmatrix}, \quad p^4 = \begin{bmatrix} 0 \\ \frac{1}{4} \\ \frac{1}{2} \end{bmatrix}.$$

Thus clientele 1 has the highest willingness to purchase the supersenior payoff d^1. Likewise, clienteles 2 and 3 are the highest bidders for the mezzanine and junior tranches, d^2 and d^3, respectively.

To understand the rationale for the CDO structure, consider first the benchmark case in which $I = 1$. Then the complete-markets Walrasian state-price deflator for the integrated economy, $p^* = \sum_k \lambda^k p^k$, is given by

$$p^* = \frac{1}{4}(p^1 + p^2 + p^3 + p^4) = \begin{bmatrix} \frac{3}{4} \\ \frac{3}{4} \\ \frac{3}{4} \end{bmatrix}.$$

It is easy to check that, given the asset structure (15), $p^k - p^* \in M^k$ for all k, i.e. condition $\mathbf{S}$(b) is satisfied. Thus this asset structure is an optimal security design for arbitrageurs. The arbitrageurs' valuation p^A is equal to p^*, by Proposition 2.2. Equilibrium arbitrageur supplies can be calculated from (2):

$$y^{1,n} = y^{2,n} = y^{3,n} = -y^{4,n} = \frac{1}{1+N}.$$

For every unit of d^4 that arbitrageurs buy, they sell one unit each of the tranches d^1, d^2 and d^3.

Compare this, for instance, to the case in which a pass-through security is sold to all investors. Then the asset structure is $(3, 2, 1)$ on all venues. Since condition $\mathbf{S}$(a) is satisfied, the arbitrageurs' valuation is still equal to p^*. Arbitrageur supplies are:

$$y^{1,n} = \frac{1}{1+N}\frac{6}{14}, \qquad y^{2,n} = \frac{1}{1+N}\frac{5}{14}, \qquad y^{3,n} = \frac{1}{1+N}\frac{3}{14}, \qquad y^{4,n} = -\frac{1}{1+N}.$$

Details of these and other calculations below are in the Appendix. Maximal liquidity $\bar{\mathcal{L}}^k$ is unchanged for venue 4 (since for this venue the pass-through security is the same one as in the optimal security design (15)), but is lower for the other venues. The equilibrium level of intermediation is therefore lower as well, leading to lower liquidity and welfare on all four venues.

While the CDO structure is optimal for $I = 1$, it is not so for other values of I. In particular, we are interested in what happens if appetite for the supersenior tranche diminishes, given this CDO structure. For $I \neq 1$, the spanning property $\mathbf{S}$ fails, which means that we cannot use the convenient condition $p^A = p^*$. We show in the Appendix that

$$p^A = \frac{3}{17I+3}\begin{bmatrix} 4I+1 \\ 4I+1 \\ 9I-4 \end{bmatrix} \tag{16}$$

is a Lagrange multiplier vector for the arbitrageurs' first-order condition, and therefore a valid state-price deflator, provided $I \geq 4/9$, which we will henceforth assume.[13] Arbitrageur supplies are:

$$y^{1,n} = y^{2,n} = y^{3,n} = -y^{4,n} = \frac{1}{1+N}\frac{20I}{17I+3}.$$

Thus the pattern of trade is the same as in the benchmark case of $I = 1$. These trades are simply scaled down as I falls. Notice that arbitrageur trades are exactly

[13]The results are less clear-cut when I falls below $4/9$. This is because there are not enough investors to absorb consumption in state 3, so it ends up in the hands of the arbitrageurs. Then our assumption that arbitrageurs only care about consumption at date 0, which is fairly innocuous as long as the asset structure does not deviate too far from one that satisfies $\mathbf{S}$, starts to matter.

offsetting, so that $\sum_k y^{k,n} d^k = 0$. Equilibrium asset prices are given by:

$$\begin{aligned}
\hat{q}^1 &= 1 - \frac{N}{1+N}\,\frac{5}{17I+3},\\
\hat{q}^2 &= \frac{2}{3} - \frac{N}{1+N}\,\frac{10I}{3(17I+3)},\\
\hat{q}^3 &= \frac{1}{3} - \frac{N}{1+N}\,\frac{5I}{3(17I+3)},\\
\hat{q}^4 &= \frac{1}{3} + \frac{N}{1+N}\,\frac{70I}{3(17I+3)}.
\end{aligned} \tag{17}$$

Maximal economy-wide liquidity is

$$\bar{\mathcal{L}} = \frac{100I}{3(17I+3)}. \tag{18}$$

As I falls, so does $\bar{\mathcal{L}}$. This means that, even for fixed N, overall liquidity $\mathcal{L}$, which is given by $(\frac{N}{1+N})^2\bar{\mathcal{L}}$, falls. In fact, the same is true for the liquidity of tranches 2 and 3, and the liquidity of the underlying debt. Moreover, as I falls, intermediaries start going out of business, with N given by (10). This exacerbates the drying up of liquidity.

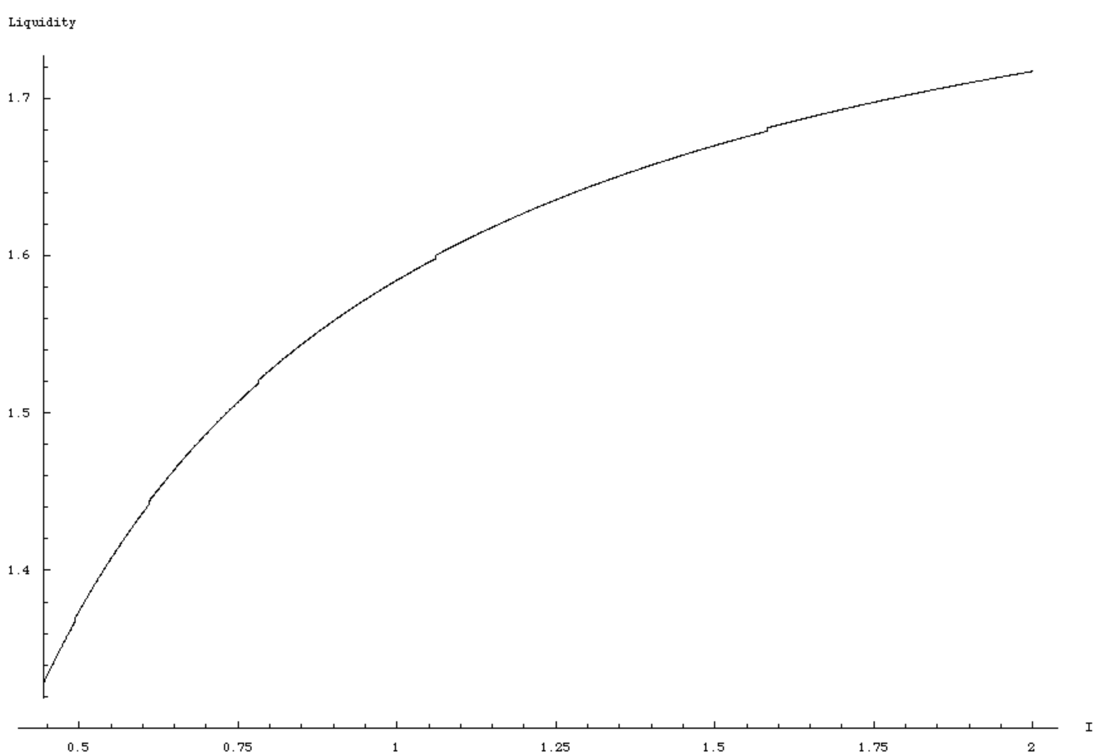

Figure 1: Overall liquidity, $\mathcal{L}$, as a function of I

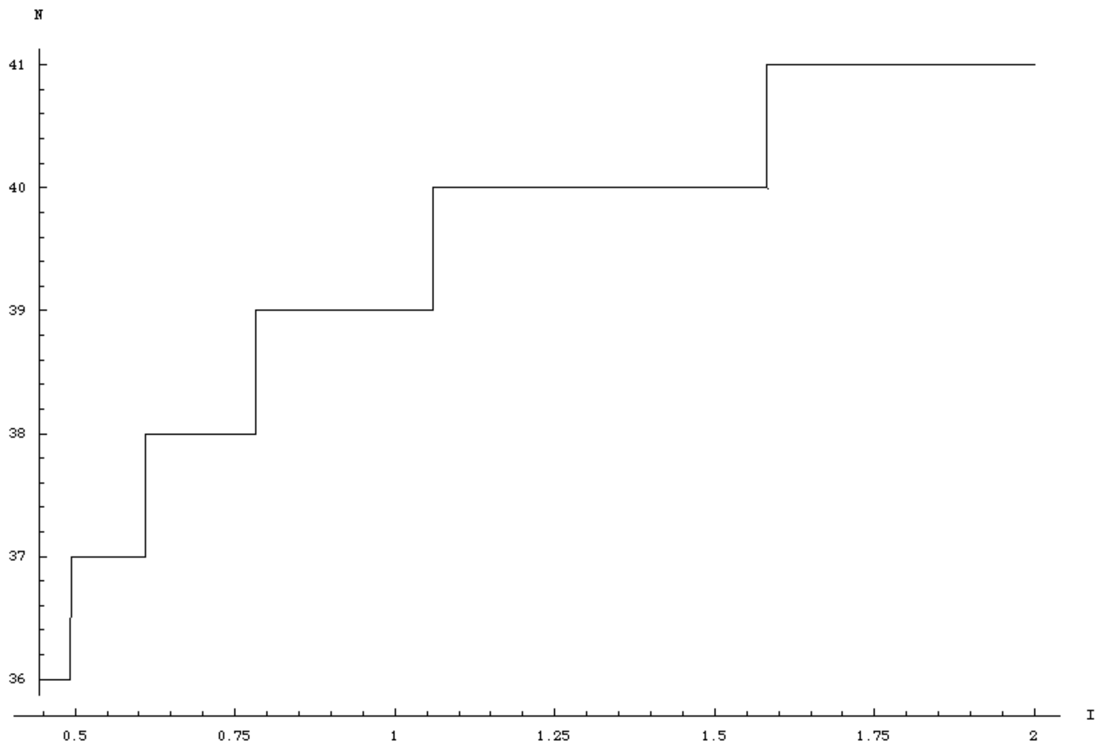

Figure 2: Equilibrium Number of Arbitrageurs, N, as a function of I

Figures 1 and 2 illustrate the effects on liquidity and intermediation of a change in I, both above and below 1, for $c = .001$. That there is contagion is evident: as the natural clientele for the supersenior tranche is eroded, the entire CDO market seizes up. A 50% decline in the size of this clientele (starting from $I = 1$) causes overall liquidity to decline by more than 13%. This effect aggregates the impact of a change in I on relative depths, on shadow prices p^A, as well as on N. The plots for the liquidity of tranches 2 and 3, and for the liquidity of the securitized debt, are similar to that for overall liquidity.

During the boom phase, before doubts about the creditworthiness of CDOs and related products became prevalent, demand for tranches was in part fueled by the quest for yield in a low interest rate environment. In our model, the CDO mechanism leads to lower prices of the various tranches than would have obtained in its absence (i.e. $\hat{q}^k < \mathring{q}^k, k = 1, 2, 3$). In other words, the CDOs allow the credit and money markets to deliver higher yields. Likewise, the CDO mechanism allows debtors to borrow at a more attractive rate ($\hat{q}^4 > \mathring{q}^4$).

Everything else constant, higher demand for the supersenior tranche leads to higher supersenior prices (we show in the Appendix that this is true in spite of the countervailing effect of higher N), as well as higher prices for the underlying securitized debt. Concurrently, prices for the other tranches fall – and yields rise – since these investors find more counterparties for their trades. And if, on the contrary, demand for the supersenior tranche wanes, these effects are reversed: prices for tranches 2 and 3 rise and the corresponding yields fall as arbitrageurs are forced to reduce their shorts and buy back those tranches.

The crisis events unfolding in the credit markets from Summer 2007 onwards cannot be fully captured by this simple version of our model. Contrary to our assumptions here, banks in the real world did have their own capital and used it to keep the supersenior tranches when they found no buyers for them. They went on structuring CDOs and selling the remaining lower graded tranches off, pocketing the "arbitrage" profits (they were arbitrage trades for the structuring desks, who sold the supersenior tranches to the treasury department of the same organization, but not for the intermediary as a whole). This overextension into CDOs then became plain when an "unexpected" state was realized wherein the supersenior tranches were no longer perceived to pay back their face value. More elaborate versions of our model can be constructed to allow for arbitrageur capital and for default, but this is beyond the scope of this paper.

7 Conclusion

In this paper we study the transmission of liquidity shocks in a world in which trading is fragmented across multiple venues or platforms linked together by profit-maximizing intermediaries. The intermediation can be functional or geographic, and can encompass a variety of trading activities, from arbitrage between derivatives and the underlying markets to the huge industry of latency arbitrage across multiple lit and dark trading venues that has been in the regulatory limelight recently. The intermediaries form endogenous links across markets, and these links determine how local shocks spread through the system.

We show how the diffusion of liquidity shocks in a number of recent market events can be understood through the lens of interconnected markets described in the paper. For instance, the outage on the LSE in November 2009 brought liquidity across all related MTFs crashing down, even though the alternative venues were supposed to pick up the liquidity lost on the LSE. Our model suggests that most of the trades on the MTFs were arbitrage trades between these venues and the LSE.

The impact of a local shock on the size of the intermediation sector has a feedback multiplier effect on liquidity – for instance, a negative liquidity shock forces some intermediaries to exit, thus reducing liquidity, inducing more intermediaries to exit, and so forth. We illustrate this with an example of contagion in CDO markets, wherein a demand shock to one tranche reverberates through the entire system, impacting the liquidity of all the other tranches. By interpreting trading venues as countries, our setup can also shed light on cross-border investment flows following a shock in one country, as in the case of the bursting of the Japanese bubble and its effect on the US stock market.

Appendix

In the Appendix we adopt matrix notation in order to simplify the proofs. We represent d^k, the asset payoffs on venue k, by the $S \times J^k$ matrix R^k whose j'th column lists the state-by-state payoffs of the j'th asset. The set of traded payoffs M^k is then the column space of R^k.

Let Π be the diagonal matrix whose diagonal elements are the probabilities of the states, $\pi_1, \ldots, \pi_S$. A state-price deflator for (q, R) is a vector $p \in \mathbb{R}^S$ such that $q = R^\top \Pi p$.[14] In other words, state-price deflators can be viewed as vectors instead of random variables. Similarly, the expectation $E[xy]$ can be written as $x^\top \Pi y$, where the random variables x and y are viewed as vectors in $\mathbb{R}^S$. In our finite-dimensional setting, the inner product space L^2 is the space $\mathbb{R}^S$ endowed with the inner product $\langle x, y\rangle_2 := x^\top \Pi y$. Then $x_{M^k} = P^k x$, where P^k is the orthogonal projection operator in L^2 onto M^k, given by the idempotent matrix

$$P^k := R^k ({R^k}^\top \Pi R^k)^{-1} {R^k}^\top \Pi. \tag{19}$$

An explicit derivation of P^k can be found in Rahi and Zigrand (2009). P^k depends on R^k only through the span M^k. The L^2-norm of $x \in \mathbb{R}^S$ is $\|x\|_2 := (x^\top \Pi x)^{\frac{1}{2}}$. Note that ${P^k}^\top \Pi P^k = \Pi P^k$.

In this notation, equilibrium arbitrageur supply on venue k (equation (2)) is

$$R^k y^{k,n} = \frac{I^k}{(1+N)\beta^k} P^k (p^k - p^A), \tag{20}$$

the liquidity on venue k (equation (6)) is

$$\begin{aligned} \mathcal{L}^k &= \left(\frac{N}{1+N}\right)^2 \frac{I^k}{2\beta^k} \|P^k(p^k - p^A)\|_2^2 \\ &= \left(\frac{N}{1+N}\right)^2 \frac{I^k}{2\beta^k} (p^k - p^A)^\top \Pi P^k (p^k - p^A), \end{aligned} \tag{21}$$

the maximal liquidity on venue k for given R^k (equation (9)) is

$$\bar{\mathcal{L}}^k = \frac{I^k}{2\beta^k} (p^k - p^A)^\top \Pi P^k (p^k - p^A), \tag{22}$$

and $\vartheta^{k\ell}$ (equation (11)) can be written as

$$\vartheta^{k\ell} = \frac{(p^k - p^*)^\top \Pi P^k (p^\ell - p^*)}{(p^k - p^*)^\top \Pi P^k (p^k - p^*)}. \tag{23}$$

[14] The symbol $\top$ denotes "transpose". We adopt the convention of taking all vectors to be column vectors by default, unless transposed.

Proof of Proposition 4.1 Since **S** holds in a neighborhood of the equilibrium, we can set p^A equal to p^*. Then, from (21), we can write $\mathcal{L}^k$ as a function of I^ℓ, p^* and N:

$$\mathcal{L}^k(I^\ell, p^*(I^\ell), N(I^\ell)) = \frac{I^k}{2\beta^k}\left(\frac{N}{1+N}\right)^2 (p^k - p^*)^\top \Pi P^k (p^k - p^*).$$

We will need the following derivatives:

$$\begin{aligned}
\frac{\partial \mathcal{L}^k}{\partial I^\ell} &= \frac{\mathcal{L}^\ell}{I^\ell}\mathbb{1}_{k=\ell}\,, \\
\frac{\partial \mathcal{L}^k}{\partial p^*} &= -\frac{I^k}{\beta^k}\left(\frac{N}{1+N}\right)^2 \Pi P^k (p^k - p^*), \\
\frac{\partial \mathcal{L}^k}{\partial N} &= \frac{2\mathcal{L}^k}{N(1+N)},
\end{aligned} \tag{24}$$

where $\mathbb{1}_{k=\ell}$ denotes the indicator function that takes value 1 if $k = \ell$, and zero otherwise. Furthermore, since

$$p^*(I^\ell) = \frac{\frac{I^\ell}{\beta^\ell}p^\ell + \sum_{k\neq\ell}\frac{I^k}{\beta^k}p^k}{\frac{I^\ell}{\beta^\ell} + \sum_{k\neq\ell}\frac{I^k}{\beta^k}},$$

we have

$$\frac{\partial p^*}{\partial I^\ell} = \frac{\lambda^\ell}{I^\ell}(p^\ell - p^*) = \frac{\lambda^k \beta^k}{I^k \beta^\ell}(p^\ell - p^*). \tag{25}$$

The total derivative of $\mathcal{L}^k$ with respect to I^ℓ is

$$\frac{d\mathcal{L}^k}{dI^\ell} = \underbrace{\frac{\partial \mathcal{L}^k}{\partial I^\ell} + \left(\frac{\partial \mathcal{L}^k}{\partial p^*}\right)^\top \frac{\partial p^*}{\partial I^\ell}}_{\frac{d\mathcal{L}^k}{dI^\ell}\big|_N} + \frac{\partial \mathcal{L}^k}{\partial N}\frac{\partial N}{\partial I^\ell}. \tag{26}$$

Hence the effect on $\mathcal{L}^k$ for given N is

$$\left.\frac{d\mathcal{L}^k}{dI^\ell}\right|_N = \frac{\mathcal{L}^\ell}{I^\ell}\mathbb{1}_{k=\ell} - \frac{\lambda^k}{\beta^\ell}\left(\frac{N}{1+N}\right)^2 \varphi^{k\ell}, \tag{27}$$

where

$$\varphi^{k\ell} := (p^k - p^*)^\top \Pi P^k (p^\ell - p^*).$$

We now solve for $\partial N/\partial I^\ell$. Since $cN^2 = 2\mathcal{L}$ (setting $\Phi = c$ in equation (7)), $N(I^\ell)$ satisfies the identity

$$c[N(I^\ell)]^2 \equiv 2\sum_k \mathcal{L}^k(I^\ell, p^*(I^\ell), N(I^\ell)).$$

Implicit differentiation gives us

$$\frac{\partial N}{\partial I^\ell} = \frac{1}{cN}\sum_k \left(\left.\frac{d\mathcal{L}^k}{dI^\ell}\right|_N + \frac{d\mathcal{L}^k}{dN}\frac{\partial N}{\partial I^\ell}\right)$$
$$= \frac{1}{cN}\left[\frac{\mathcal{L}^\ell}{I^\ell} - \frac{1}{\beta^\ell}\left(\frac{N}{1+N}\right)^2 \sum_k \lambda^k \varphi^{k\ell} + \frac{2\mathcal{L}}{N(1+N)}\frac{\partial N}{\partial I^\ell}\right].$$

Under the spanning condition **S**, either $P^k = P$ or $P^k(p^k - p^*) = p^k - p^*$. In both cases $\sum_k \lambda^k \varphi^{k\ell} = 0$. Using $cN^2 = 2\mathcal{L}$ once again, we obtain

$$\frac{\partial N}{\partial I^\ell} = \frac{(1+N)\mathcal{L}^\ell}{2I^\ell \mathcal{L}}. \tag{28}$$

Substituting (24), (27) and (28) into (26) gives us

$$\frac{d\mathcal{L}^k}{dI^\ell} = \frac{\mathcal{L}^\ell}{I^\ell}\mathbb{1}_{k=\ell} - \frac{\lambda^k}{\beta^\ell}\left(\frac{N}{1+N}\right)^2 \varphi^{k\ell} + \frac{\mathcal{L}^k\mathcal{L}^\ell}{NI^\ell\mathcal{L}}. \tag{29}$$

Multiplying through by $I^\ell/\mathcal{L}^k$, we get

$$\frac{d\log \mathcal{L}^k}{d\log I^\ell} = \mathbb{1}_{k=\ell} - 2\lambda^\ell \frac{\varphi^{k\ell}}{\varphi^{kk}} + \frac{\mathcal{L}^\ell}{N\mathcal{L}}.$$

Noting that $\varphi^{k\ell}/\varphi^{kk} = \vartheta^{k\ell}$, and $\mathcal{L}^\ell/\mathcal{L} = \bar{\mathcal{L}}^\ell/\bar{\mathcal{L}}$, gives us equation (12). Summing (29) over k, and using the fact that $\sum_k \lambda^k \varphi^{k\ell} = 0$, we get

$$\frac{d\mathcal{L}}{dI^\ell} = \frac{\mathcal{L}^\ell}{I^\ell} + \frac{\mathcal{L}^\ell}{NI^\ell}$$
$$= \frac{1+N}{N}\frac{\mathcal{L}^\ell}{I^\ell}.$$

Multiplying through by $I^\ell/\mathcal{L}$, and noting again that $\mathcal{L}^\ell/\mathcal{L} = \bar{\mathcal{L}}^\ell/\bar{\mathcal{L}}$, gives us equation (13). $\square$

Proof of Proposition 5.1 Let $R^k = R$, all k. Since condition **S**(a) is satisfied, we can set p^A equal to p^*. Using (3),

$$\hat{q}^k = R^\top \Pi \hat{p}^k$$
$$= R^\top \Pi \left(\frac{1}{1+N}p^k + \frac{N}{1+N}p^*\right).$$

We have

$$\frac{d\hat{q}^k}{dI^\ell} = \underbrace{\left(\frac{\partial \hat{q}^k}{\partial p^*}\right)^\top \frac{\partial p^*}{\partial I^\ell}}_{\left.\frac{d\hat{q}^k}{d\alpha}\right|_N} + \frac{\partial \hat{q}^k}{\partial N}\frac{\partial N}{\partial I^\ell}.$$

Using (25) and (28),

$$\left.\frac{d\hat{q}^k}{dI^\ell}\right|_N = \frac{N}{1+N}\,\frac{\lambda^\ell}{I^\ell}\,R^\top \Pi(p^\ell - p^*)$$
$$= \frac{1}{I^\ell}\,\frac{N}{1+N}\,\lambda^\ell(\mathring{q}^\ell - q^*),$$

and

$$\frac{\partial \hat{q}^k}{\partial N}\,\frac{\partial N}{\partial I^\ell} = \frac{\mathcal{L}^\ell}{2(1+N)I^\ell \mathcal{L}}\,R^\top \Pi(p^* - p^k)$$
$$= \frac{1}{I^\ell}\,\frac{\bar{\mathcal{L}}^\ell}{2(1+N)\bar{\mathcal{L}}}\,(q^* - \mathring{q}^k).$$

The result follows. $\square$

Proof of Proposition 6.1 Arbitrageur supply $y^{k,n}$ is given by (20), liquidity $\mathcal{L}^k$ by (21), and $\vartheta^{k\ell}$ by (23). We specialize these formulas to the case in which $R^k = \mathbf{1}$ for all k, where $\mathbf{1}$ is an S-vector of ones, i.e. $\mathbf{1}^\top := (1\ \ 1\ \ \ldots\ 1)$. Let π denote the vector of probabilities of the S states, i.e. $\pi^\top := (\pi_1\ \ \pi_2\ \ \ldots\ \ \pi_S)$. From (19), $P^k = \mathbf{1}\pi^\top$, and $\Pi P^k = \pi\pi^\top$. Using Proposition 2.2, we can choose $p^A = p^*$. Noting that

$$\pi^\top p^k = E(p^k) = \mathring{q}^k = \frac{1}{1+\mathring{r}^k},$$

for all k, and

$$\pi^\top p^* = E(p^*) = q^* = \frac{1}{1+r^*},$$

we obtain the desired formulas for $y^{k,n}, \mathcal{L}^k$ and $\vartheta^{k\ell}$. $\square$

Detailed calculations for the CDO example in Section 6.6:

The optimization problem of an arbitrageur is studied in Rahi and Zigrand (2009). In particular, it follows from equations (A11) and (A12) in that paper that p^A is a Lagrange multiplier vector for this optimization problem if $p^A \geq 0$ and solves

$$\sum_k \frac{I^k}{\beta^k} P^k(p^k - p^A) = 0. \tag{30}$$

In this example, $\beta^k = 1/4$ for all k, $I^1 = I$, and $I^2 = I^3 = I^4 = 1$.

Consider the asset structure (15). From (19), the projection matrices P^k are:

$$P^1 = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}, \quad P^2 = \frac{1}{2}\begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}, \quad P^3 = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \quad P^4 = \frac{1}{14}\begin{bmatrix}9 & 6 & 3\\ 6 & 4 & 2\\ 3 & 2 & 1\end{bmatrix}.$$

It is straightforward to verify that the vector p^A given by (16) solves (30), provided $I \geq 4/9$. Arbitrageur supplies can then be obtained from (20), and asset prices from Proposition 2.1 (iii). Maximal liquidity on each venue can be calculated from (22):

$$\bar{\mathcal{L}}^1 = \frac{100I}{(17I+3)^2}, \quad \bar{\mathcal{L}}^2 = \frac{200I^2}{3(17I+3)^2}, \quad \bar{\mathcal{L}}^3 = \frac{100I^2}{3(17I+3)^2}, \quad \bar{\mathcal{L}}^4 = \frac{1400I^2}{3(17I+3)^2}.$$

Summing these we obtain the overall maximal liquidity $\bar{\mathcal{L}}$, given by (18). Since $\bar{\mathcal{L}}$ is increasing in I, so is N. From the formulas for $\hat{q}^k$ in (17), we see that $\hat{q}^2$ and $\hat{q}^3$ are decreasing in I, while $\hat{q}^4$ is increasing in I.

In order to sign $\partial\hat{q}^1/\partial I$, we first calculate $\partial N/\partial I$, ignoring integer constraints on N. From Proposition 3.1, we have $(1+N)^2 = 2\bar{\mathcal{L}}/c$, so that

$$\begin{aligned}
\frac{\partial N}{\partial I} &= \frac{1}{c(1+N)} \frac{\partial \bar{\mathcal{L}}}{\partial I} \\
&= \frac{1+N}{2\bar{\mathcal{L}}} \frac{\partial \bar{\mathcal{L}}}{\partial I} \\
&= \frac{1+N}{2} \frac{\partial \log \bar{\mathcal{L}}}{\partial I} \\
&= \frac{3(1+N)}{2I(17I+3)}.
\end{aligned}$$

We have

$$\begin{aligned}
\frac{\partial \hat{q}^1}{\partial I} &= \frac{85N}{(1+N)(17I+3)^2} - \frac{5}{(1+N)^2(17I+3)} \frac{\partial N}{\partial I} \\
&= \frac{85N}{(1+N)(17I+3)^2} - \frac{15}{2I(1+N)(17I+3)^2} \\
&= \frac{5}{(1+N)(17I+3)^2} \left[17N - \frac{3}{2I} \right],
\end{aligned}$$

which is positive, given our assumption that $I \geq 4/9$, and that c is small enough to ensure that $N \geq 1$.

For the case of the pass-through security $(3,2,1)$, and $I = 1$, we have $p^A = p^* = (3/4, 3/4, 3/4)$. The projection matrix P^k is the same for all k and equal to P^4 above. Arbitrageur supplies can be calculated from (20). $\square$

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