Managing Credit Booms and Busts: A Pigouvian Taxation Approach

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Abstract

We study a dynamic model in which the interaction between debt accumulation and asset prices magnifies credit booms and busts. We show that these feedback effects create an externality since borrowers do not internalize their contribution to aggregate volatility and therefore take on excessive leverage. As a result the economy suffers from excessive volatility, i.e. large booms and busts in both credit flows and asset prices. We propose a Pigouvian tax on borrowing that induces agents to internalize their externalities. In a sample calibration, the optimal magnitude of this tax is 2.41%. Our paper also develops a new numerical method of solving models with occasionally binding endogenous constraints.

1 Introduction

The interaction between debt accumulation and asset prices contributes to magnify the impact of booms and busts. Increases in borrowing and in collateral prices feed each other during booms. In busts, the feedback turns negative, with credit constraints leading to fire sales of assets and further tightening of credit. It has been suggested that prudential policies could be used to mitigate the build-up in systemic vulnerability during the boom. However, there are few formal welfare analyses of the optimal policies to deal with booms and busts in credit and asset prices.

This paper makes a step toward filling this gap with a dynamic optimizing model of collective and collateralized borrowing. We consider a group of individuals (the insiders) who enjoy a comparative advantage in holding an asset and who can use this asset as collateral on their borrowing from outsiders. The borrowing capacity of insiders therefore depends on asset prices.

Asset prices in turn are driven by the insiders' demand for loans, which is a function of their borrowing capacity. This introduces a mutual feedback loop between asset prices and credit flows: small financial shocks to insiders can simultaneously lead to large booms and busts in asset prices and booms and busts in credit flows.

The model attempts to capture, in a stylized way, a number of economic settings in which the systemic interaction between credit and asset prices may be important. The insiders could be interpreted as a group of entrepreneurs who have more expertise than outsiders to operate a productive asset, households putting a premium on owning their own homes, or a group of investors who enjoy an informational advantage in dealing with a certain class of financial assets. Alternatively, the group of insiders could also be interpreted, in an open economy context, as the residents of a country borrowing from foreign lenders. One advantage of studying these situations with a common framework is to bring out the commonality of the problems and of the required policy responses (although, in the real world, those policies pertain to different areas such as financial regulation, individual and corporate taxation, or capital controls).

Our main result is that such feedback loops in financial markets entail externalities that lead insiders to undervalue the benefits of conserving liquidity as a precaution against busts: an insider who has one more dollar of liquid net worth when the economy experiences a bust not only relaxes his private borrowing constraint, but also mitigates the feedback effects that worsen the bust, which relaxes the borrowing constraints of all other insiders. Not internalizing this spillover effect, the insider takes on too much debt during good times. In a benchmark calibration of our model, we find that it would be optimal to impose a 2.41 percent tax on borrowing by leveraged insiders to prevent them from taking on socially excessive debt.

Our model is related to the positive study of financial accelerator effects in closed and open economy macroeconomics. In DSGE models in the closed economy, Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) show that financial frictions amplify the response of an economy to fundamental shocks. However, models in this literature are traditionally solved by linearization, making them more appropriate to analyze regular business cycle fluctuations than systemic crises. In the open economy literature, Mendoza (2005) and Mendoza and Smith (2006), among others, have studied the nonlinear dynamics arising from financial accelerator effects during sudden stops in emerging market economies. All these papers provide a positive analysis of how financial frictions can amplify shocks to the economy, but do not characterize welfare-maximizing policies. The central focus of our paper is to fill this gap.

This paper is also related to analyses of the ongoing world-wide credit crisis that emphasize the amplifying mechanisms involving asset price deflation and deleveraging in the financial sector (e.g., Adrian and Shin, 2009; Brunnermeier, 2009). Some earlier contributions have clarified the externalities involved in credit booms and busts and drawn some implications for policy in the context of very stylized two- or three-period models (Korinek, 2008, 2009). By contrast, this paper gives a more realistic and quantitative flavor to the analysis, by considering an infinite-horizon model. This is particularly relevant for determining the optimal magnitude of regulatory measures in practice.

Benigno et al (2009) and Bianchi (2009) also characterize welfare-maximizing policies in dynamic optimization models with collateralized debt for policy analysis. Their papers focus on the role of exchange rate depreciations in emerging market crises. Our paper attempts to capture the essence of the problem in a more generic setting involving asset price deflation.

Finally, our paper makes an important technical contribution to the literature on DSGE models with occasionally binding endogenous constraints. We develop a new numerical solution method that allows us to solve such models in a very efficient novel way. We term this procedure the "endogenous gridpoint bifurcation method," which extends the endogenous gridpoints method of Carroll (2006) to an environment with endogenous constraints. This technical innovation enables us to analyze more complex models with more state variables than what has been computationally feasible in the existing DSGE literature with endogenous constraints, ultimately basing our policy guidance on richer and more realistic models of the economy.

2 The model

We consider a group of identical and atomistic individuals in infinite discrete time $t = 0, 1, 2, ...$ The utility of the representative individual at time t is given by,

$$U_t = E_t \left(\sum_{s=t}^{+\infty} \beta^{s-t} u(c_s) \right), \tag{1}$$

where $u(\cdot)$ is strictly concave and satisfies the Inada conditions. We will generally assume that utility has constant relative risk aversion

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

These individuals (the insiders) receive two kinds of income, the payoff of an asset that can serve as collateral, and an endowment income. The representative insider maximizes his utility under the budget constraint

$$c_t + d_t + \theta_{t+1} p_t = e_t + \theta_t (p_t + y_t) + \frac{d_{t+1}}{R}, \tag{2}$$

where c_t is his consumption, d_t his "outside debt" which we assume default-free, R is the riskless interest factor at which outsiders lend, p_t is the price of the asset, θ_t is the insider's holdings of the asset, e_t is the non-pledgeable endowment income, and y_t is the payoff of an asset that can be pledged as collateral in period t. We assume that y_t follows a stationary Markov process that will be specified later.[1] Furthermore, the endowment $e_t = e$ is constant (more generally, we could assume a stochastic process for e_t that covaries with y_t without affecting our qualitative findings).

The collateral asset can be exchanged between insiders in a perfectly competitive market. However, it cannot be sold to outsiders: θ_t must be equal to 1 in equilibrium. We do not allow insiders to sell the asset to outsiders and rent it back – insiders derive important benefits from the control rights that ownership provides. This assumption is of course extreme, and will be relaxed in later work (we will allow for $1 \geq \theta_t \geq \underline{\theta}$ for some $\underline{\theta} > 0$). But it is important to have positive asset holdings for insiders to issue collateralized debt and to be prone to debt deflation.

Furthermore, we assume that the only financial instrument which can be traded between insiders and outsiders is uncontingent debt. This assumption can be justified e.g. on the basis that shocks to the insider sector might not be perfectly observable to outsiders and therefore cannot be used to condition payments. This feature also corresponds to common practice across a wide range of financial relationships. More generally, the findings of Korinek (2009) suggest that our results on excessive exposure to binding constraints would continue to hold when insiders have access to state-contingent financial contracts.

The value of the collateral asset determines how much of insiders' short-term debt lenders are willing to roll over. Outside lenders do not roll over more than a fraction ϕ of the value of the collateral asset observed in the period that borrowing takes place

$$\frac{d_{t+1}}{R} \leq \phi \theta_t p_t. \tag{3}$$

A micro-justification – along the lines of Kiyotaki and Moore (1997) and others – would be that an indebted insider can walk away from the debt he just issued, in which case lenders can seize a fraction ϕ of his asset and sell it in the market. Equation (3) is then the incentive compatibility condition ensuring that debtors do not walk away from their debt.[2]

3 Laissez-faire equilibrium

3.1 Equilibrium conditions

The first-order conditions (derived in the appendix) are

$$u'(c_t) = \lambda_t + \beta R E_t \left[u'(c_{t+1}) \right], \tag{4}$$

[1]We could introduce growth at a rate G into the model. If the detrended payoff is Markov stationary and utility is CRRA, the model with growth, once detrended, is isomorphic to the model presented here with an interest rate of $RG^{1-\gamma}$.

[2]The constraint could involve the end-of-period holding of asset, θ_{t+1}. Appendix A.3 derives the equilibrium conditions in this case. More generally, the right-hand side could involve a constant term or be a nonlinear increasing function of p_t. The only important assumption, to obtain the debt deflation and asset price deflation mechanism at the core of the model, is that it depend on the price p_t.

$$p_t = \frac{\beta E_t\left[u'(c_{t+1})(y_{t+1}+p_{t+1})+\phi\lambda_{t+1}p_{t+1}\right]}{u'(c_t)}, \tag{5}$$

where λ_t is the costate variable for the borrowing constraint. The asset-pricing equation has a λ_{t+1} term in the numerator, which reflects the asset's extra utility as collateral in the next period. Using (2) and $\theta = 1$ and defining beginning-of-period liquid net wealth $m_t = e + y_t - d_t$ the collateral constraint (3) can also be written,

$$c_t \leq m_t + \phi p_t. \tag{6}$$

The state of the economy at the beginning of period t is summarized by (m_t, y_t). The equilibrium is characterized by three non-negative functions, $c(m_t, y_t)$, $p(m_t, y_t)$ and $\lambda(m_t, y_t)$ such that

$$u'(c(m,y)) = \lambda(m,y) + \beta R E\left[u'(c(m',y'))\right], \tag{7}$$

$$p(m,y) = \beta\frac{E\left[u'(c(m',y'))(y'+p(m',y'))+\phi\lambda(m',y')p(m',y')\right]}{u'(c(m,y))}, \tag{8}$$

$$\lambda(m,y) > 0 \Rightarrow c(m,y) = m + \phi p(m,y), \tag{9}$$

where next-period values are denoted with primes, and the transition equation for liquid net wealth is

$$m' = e + y' + R\left(m - c(m,y)\right). \tag{10}$$

Naturally, if y is i.i.d. the policy functions depend solely on m.

3.2 Deterministic case with $\beta R = 1$

We look at the equilibrium where e and y are deterministic and constant. This special case brings out, in a relatively simple context, some issues in the determination of the equilibrium that will also matter in more general and less tractable cases. We first assume $\beta R = 1$ since if there is no demand for precautionary savings one does not need to consider an impatient consumer in order to obtain a well-defined long-run steady state. We will look at the case $\beta R < 1$ in a second step.

Unconstrained equilibrium In an unconstrained steady state we have

$$p^{\text{unc}} = \frac{\beta}{1-\beta}y.$$

Starting from the initial condition d_1 (the debt due in period 1) the economy immediately settles in a steady state if

$$\frac{d_1}{R} \leq \phi p^{\text{unc}},$$

that is, if debt is lower than the maximum steady state debt level $\bar{d}$ of the economy, which is a threshold that is increasing in y and β,

$$d_1 \leq \bar{d} \equiv \frac{\phi y}{1-\beta}. \tag{11}$$

Since y is constant we can write the policy function in terms of liquid net wealth m only. For debt lower than this threshold, the consumption policy function takes a simple linear form

$$c(m) = \beta(e+y) + (1-\beta)m \text{ for } m \geq \bar{m} = e + y - \bar{d} \tag{12}$$

Constrained equilibrium If wealth is below the threshold $m_1 < \bar{m}$ or, equivalently, if the debt due in period 1 is above the threshold $d_1 > \bar{d}$, the borrowing constraint on d_2 will be binding. If the economy is constrained in period 1, the debt level satisfies $d_2 \leq \bar{d}$. This implies that from period 2 onwards, the economy continues in an unconstrained steady state as described by equation (12), and that the economy is constrained only for a single period, i.e., period 1.

Whether or not the economy is constrained in period 1, we thus have $d_2 \leq \bar{d}$. Conversely, to any $d_2 \leq \bar{d}$ we can associate an unconstrained period 1 equilibrium with

$$S^{\text{unc}} \begin{cases} c_1^{\text{unc}} = c_2 = e + y - (1-\beta)\, d_2, \\ m_1^{\text{unc}} = c_1^{\text{unc}} - d_2/R = e + y - d_2. \end{cases}$$

Note that both c_1^{unc} and m_1^{unc} are strictly decreasing in d_2. Furthermore to any $d_2 \in \left[0, \bar{d}\right]$, i.e. any debt level that is both *smaller than the threshold* $\bar{d}$ and *positive*, we can associate a constrained period-1 equilibrium through the following equations

$$\begin{aligned} \frac{d_2}{R} &= \phi p_1^{\text{con}} = \phi\beta \frac{u'(c_2)}{u'(c_1^{\text{con}})} (y + p^{\text{unc}}) \\ m_1^{\text{con}} &= c_1^{\text{con}} - d_2/R, \end{aligned}$$

where $c_2 = e + y - (1-\beta)d_2$.

After simple manipulations of the first equation this can be re-written

$$S^{\text{con}} \begin{cases} c_1^{\text{con}} = [e + y - (1-\beta)\, d_2] \left(d_2/\bar{d}\right)^{1/\gamma}, \\ m_1^{\text{con}} = c_1^{\text{con}} - d_2/R. \end{cases}$$

We depict both systems as a function of d_2 in figure 1. The two downward-sloping linear curves represent $m_1(d_2)$ and $c_1(d_2)$ over the unconstrained region. The two upward-sloping concave lines represent the two functions over the constrained region. We indicate the maximum debt level d_2 by a vertical dotted line, and we truncate the m_1 and c_1 functions at that threshold.

Let us consider the constrained system. If d_2 is equal to zero, c_1^{con} and m_1^{con} are also both equal to zero. If d_2 converges toward $\bar{d}$ (dotted vertical line), c_1^{con}

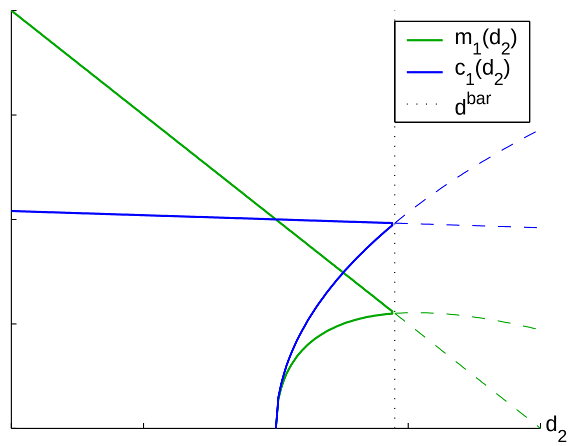

Figure 1: Systems S^{unc} and S^{con} as a function of d_2

and m_1^{con} respectively converge to $e+y\,(1-\phi)$ and $\bar{m}=e+y-\bar{d}$, and so do c_1^{unc} and m_1^{unc}, i.e. the constrained and the unconstrained functions intersect. Thus a necessary condition for $c(m_1)$ to be increasing along the constrained branch is $\bar{m}>0$, or

$$\bar{d} < e + y \tag{13}$$

If this condition was not satisfied, then the constrained branch of $c(m_1)$ would be downward-sloping over some range and there would be multiple equilibria: the same level of beginning-of-period wealth m_1 could be associated with two levels of consumption – one constrained and one unconstrained.

Conversely, assume that condition (13) is satisfied. Then, $c(m_1)$ is increasing with m_1 on the constrained branch if c_1^{con} and m_1^{con} are both increasing with d_2. We have

$$\frac{\partial c_1^{\text{con}}}{\partial d_2} = \left[\frac{1}{\gamma d_2} - \frac{1-\beta}{e+y-(1-\beta)d_2}\right] c_1^{\text{con}} = \left[\frac{e+y-(1-\beta)\,(1+\gamma)\,d_2}{\gamma d_2\,[e+y-(1-\beta)d_2]}\right] c_1^{\text{con}},$$

which is always positive in the relevant interval if this is true for $d_2=\bar{d}$, that is if

$$\bar{d} < \frac{e+y}{(1-\beta)(1+\gamma)}.$$

This condition is weaker than (13) for typical values of β and γ. Next, m_1^{con} is increasing with d_2 if

$$\begin{aligned} \frac{\partial m_1^{\text{con}}}{\partial d_2} &= \frac{\partial c_1^{\text{con}}}{\partial d_2} - 1/R \\ \text{or} \quad \frac{e+y-(1-\beta)\,(1+\gamma)\,d_2}{\gamma\,(\bar{d})^{1/\gamma}} \cdot (d_2)^{\frac{1-\gamma}{\gamma}} &> 1/R = \beta \end{aligned}$$

This condition cannot be satisfied for d_2 close to 0 if $\gamma < 1$ (because the l.h.s. is close to 0). Thus one must have $\gamma \geq 1$. Conditional on this the l.h.s. is decreasing with d_2 so one simply needs to check that the condition is satisfied for $d_2 = \bar{d}$, that is

$$\bar{d} \leq \frac{e+y}{1+\gamma-\beta}.$$

Since $\gamma \geq 1 > \beta$ this condition is stronger than (13). Using (11) to substitute out $\bar{d}$, this condition can be rewrittern in terms of y,

$$y \leq \frac{1-\beta}{\phi\gamma-(1-\phi)(1-\beta)} e.$$

Taken together, systems S^{unc} and S^{con} constitute a parametric representation of the equilibrium in period 1 (with d_2 as the parameter). This parameterization prefigures our “endogenous gridpoints bifurcation method” – when debt d_2 is negative there is a single solution for the period 1 equilibrium, when d_2 turns positive, the set of solution for the period 1 equilibria “bifurcates” into a constrained arm S^{con} and an unconstrained arm S^{unc}. The two systems S^{unc} and S^{con} characterize how c_1 varies with m_1, i.e., the equilibrium policy function $c(m_1)$. As d_2 varies in its possible interval of values, system S^{unc} gives the unconstrained branch of this function for all $m \geq \bar{m}$ – which we already derived in (12). The constrained branch of $c(m_1)$ for $m < \bar{m}$ is implicitly defined by S^{con}. Our results are summarized in the following proposition.

Proposition 1 *If y is constant and $\beta R = 1$, the credit constraint binds for one period at most. The consumption of insiders is a continuous and increasing function of their wealth $c(m)$ if and only if $\gamma \geq 1$ and*

$$y \leq \frac{1-\beta}{\phi\gamma-(1-\phi)(1-\beta)} e. \tag{14}$$

If condition (14) is not satisfied, there may be multiple equilibria. Although self-fulfilling credit and asset price busts are an interesting phenomenon, we prefer, in the current paper, to limit our attention to the case where the consumption function is ”well-behaved”. Thus in the numerical calibrations we will assume values that satisfy the conditions in the proposition above.

By inverting the equations for m_1 in the systems S^{unc} and S^{con}, we arrive at a function $d'(m_1)$ expressing the next-period debt level, given initial net worth m_1. Given consumption levels $c_1 = c(m_1)$ and $c_2 = e + y - (1-\beta)d'(m_1)$, we obtain the period 1 asset price function $p(m_1)$ as

$$p(m_1) = \left(\frac{c(m_1)}{e+y-(1-\beta)d'(m_1)}\right)^{\gamma} \cdot p^{\text{unc}} \tag{15}$$

We have depicted a sample calibration of the policy functions $c(m_1)$ and $d'(m_1)$ as well as the price function $p(m_1)$ in figure 2. The parameter values used are listed in the following table:

β	R	γ	e	y	ϕ
.96	$1/\beta$	2	.8	.2	.09

Table 1: Parameter values for deterministic case with $\beta R = 1$

Note the different behavior of the functions in the constrained ($m_1 < \bar{m}$) and unconstrained regions ($m_1 \geq \bar{m}$), which are separated by a dotted vertical line. For low levels of initial liquid net worth $0 \leq m_1 < \bar{m}$, debt is constrained by the low level of asset prices, $d_2(m_1)/R = \phi p_1(m_1)$ and there is "debt deflation." Over this region, consumption, debt, and asset prices respond strongly to small changes in m_1, as the economy is subject to amplification effects. Assume for instance a small drop in m_1: given the binding constraint, this forces the agent to reduce consumption, which leads to a feedback spiral of declining asset prices, falling borrowing limits and debt levels, and further decreases in consumption c_1. As the constraint tightens and the agent borrows less, first period consumption c_1 has to decline whereas consumption in period 2 and after rises because of the lower level of debt in the future. Both effects reduce the agent's MRS and push the asset price lower, as given by equation (15).

On the other hand, if $m_1 \geq \bar{m}$, the equilibrium is unconstrained and the economy behaves in the standard way: consumption is perfectly smooth $c_1 = c_2$ and the level of asset prices is independent of wealth $p(m_1) = p^{\text{unc}}$. For lower levels $0 \leq m_1 < \bar{m}$, the borrowing constraint is binding so that $c_1 < c_2$ and there is asset price deflation.

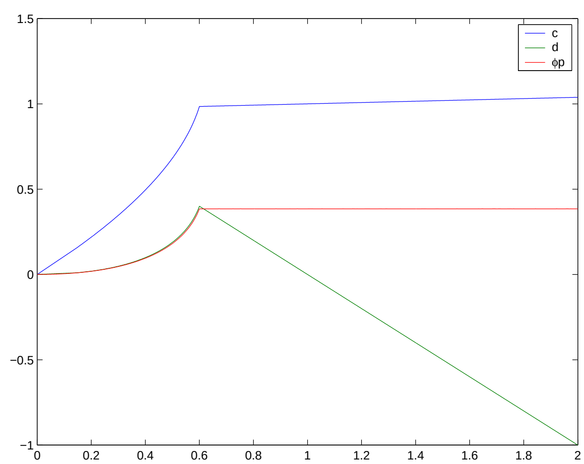

Figure 2: Policy functions for deterministic case with $\beta R = 1$

3.3 General case with $\beta R < 1$

Let us go back to the general case. In order to generate a persistent motive for borrowing, we need to assume that insiders are impatient relative to outsiders, i.e. $\beta R < 1$. (With trend growth at a growth factor G, we could allow for situations with $\beta R \geq 1$ as long as $\beta R G^{1-\gamma} < 1$.)

We may make conjectures about the form of the solution by analogy with the deterministic case studied in the previous section. The attention will be restricted to equilibria in which the consumption function $m \mapsto c(m, y)$ is a continuously increasing function of wealth for any y. Like in the deterministic case, this rules out multiple equilibria. Let us denote by $\underline{m}(y)$ the level of wealth for which consumption is equal to zero for a given y,

$$c(\underline{m}(y), y) = 0.$$

By analogy with the deterministic case, we would expect the insiders to be credit-constrained in a wealth interval $m \in [\underline{m}(y), \overline{m}(y)]$, and to be unconstrained for $m \geq \overline{m}(y)$. The thresholds $\underline{m}(y)$ and $\overline{m}(y)$ are key endogenous variables to determine in deriving the equilibrium.

It is not difficult to see that the lower threshold must be equal to zero,

$$\forall y, \ \underline{m}(y) = 0.$$

This results from the facts that $c(m, y) \leq m + \phi p$, and that p converges to zero as c goes to zero (by equation (5)). Since $m = e + y - d$ must always be positive and the level of debt is set before the realization of y, we must have

$$d \leq e + \min y'|y,$$

where $\min y'|y$ is the lowest possible realization of y' in the following period. The upper threshold, $\overline{m}(y)$, must be determined computationally.

The numerical resolution method that we develop here is an extension of the endogenous grid points method of Carroll (2006) to the case with endogenous credit constraints. The basic idea of Carroll (2006) is to perform backwards time iteration on the agent's optimality conditions, i.e. to define a grid $\mathbf{d}^g$ of next period debt levels d' and combine the next period policy functions with agent's optimality conditions to obtain current period policy functions until the resulting functions converge. As we illustrated in section 3.2, when the agent's next period debt level d' turns positive in a model of endogenous borrowing constraints, there is a bifurcation in the solution for the current period policy functions, i.e., there is a constrained and an unconstrained arm. We therefore term the following solution method the "endogenous gridpoints bifurcation method." Note that as the standard endogenous gridpoints method of Carroll (2006), our solution method is very efficient since it avoids computationally intensive numerical rootfinding operations.

The state of the economy in a given time period is fully captured by the pair (d', y). We therefore define a grid $\mathbf{d}^g$ of debt levels and a grid $\mathbf{y}^g$ containing the possible realizations of the output shock. Furthermore, we define $\mathbf{d}^g_+ = \mathbf{d}^g \cap \Re^+_0$ the set of all non-negative gridpoints in $\mathbf{d}^g$.

In iteration step k, we start with a triplet of functions $\tilde{c}_k(m, y)$, $\tilde{p}_k(m, y)$ and $\tilde{\lambda}_k(m, y)$ where the beginning-of-period liquid net worth $m = e + y - d$ and where $\tilde{c}_k(m, y)$ and $\tilde{p}_k(m, y)$ are weakly increasing in m and $\tilde{\lambda}_k(m, y)$ is weakly decreasing in m for a given y. For each $d' \in \mathbf{d}^g$ and $y \in \mathbf{y}^g$ we solve the system of optimality conditions from section 3.1 under the assumption that the borrowing constraint is loose, noting that $m' = e' + y' - d'$:

$$\begin{aligned}
c^{\text{unc}}(d', y) &= \beta R E\left\{\tilde{c}_k(m', y')^{-\gamma} | y\right\}^{-\frac{1}{\gamma}}, \\
p^{\text{unc}}(d', y) &= \frac{\beta E\left\{\tilde{c}_k(m', y')^{-\gamma} \cdot [y' + \tilde{p}_k(m', y')] + \phi \tilde{\lambda}_k(m', y') \tilde{p}_k(m', y') | y\right\}}{c^{\text{unc}}(d', y)^{-\gamma}}, \\
\lambda^{\text{unc}}(d', y) &= 0, \\
m^{\text{unc}}(d', y) &= c^{\text{unc}}(d', y) - \frac{d'}{R}.
\end{aligned}$$

Lemma 2 *For suitable parameter values (in particular low ϕ), it can be shown that the unconstrained functions $c^{unc}(d', y)$, $p^{unc}(d', y)$ and $m^{unc}(d', y)$ are decreasing in d' for a given y.*

Proof. [tk] ■

By the same token, we can solve for the constrained branch of the system for each non-negative $d' \in \mathbf{d}^g_+$ and $y \in \mathbf{y}^g$ under the assumption that the borrowing constraint is binding in the current period as

$$\begin{aligned}
p^{\text{con}}(d', y) &= \frac{d'}{\phi R}, \\
c^{\text{con}}(d', y) &= \left[\frac{\beta E\left\{\tilde{c}_k(m', y')^{-\gamma} \cdot [y' + \tilde{p}_k(m', y')] + \phi \tilde{\lambda}_k(m', y') \tilde{p}_k(m', y') | y\right\}}{p^{\text{con}}(d', y)}\right]^{-\frac{1}{\gamma}}, \\
\lambda^{\text{con}}(d', y) &= c^{\text{con}}(d', y)^{-\gamma} - \beta R E\left[\tilde{c}_k(m', y')^{-\gamma} | y\right], \\
m^{\text{con}}(d', y) &= c^{\text{con}}(d', y) - \frac{d'}{R}.
\end{aligned}$$

Lemma 3 *For suitable parameter values (in particular low ϕ), the functions $c^{con}(d', y)$, $p^{con}(d', y)$ and $m^{con}(d', y)$ are increasing in d' for a given y, and the function $\lambda^{con}(d', y)$ is decreasing in d'.*

Proof. [tk] ■

We determine for each level of $y \in \mathbf{y}^g$ the next period debt threshold $\bar{d}'(y)$ s.t. the borrowing constraint in the unconstrained system is just marginally binding, i.e., such that

$$\frac{d'}{R} = \phi p^{\text{unc}}(d', y)$$

This is the highest possible debt level d' that the economy can support for a given level of y. By construction of this threshold, the unconstrained and constrained arms of consumption coincide $c^{\text{unc}}\left(\bar{d}'(y), y\right) = c^{\text{con}}\left(\bar{d}'(y), y\right)$ and similarly for the other policy variables. This threshold debt level corresponds to a beginning-of-period liquid net wealth $\overline{m}(y) = m^{\text{unc}}\left(\bar{d}'(y), y\right) = m^{\text{con}}\left(\bar{d}'(y), y\right)$. The lowest possible level of m is $\underline{m}(y) = m^{\text{con}}(0, y)$. Given our monotonicity results above, we can construct for each y the step $k+1$ policy function $\tilde{c}_{k+1}(m, y)$ for the interval $\underline{m}(y) \leq m < \overline{m}(y)$ by interpolating on the pairs $\{(c^{\text{con}}(d', y), m^{\text{con}}(d', y))\}_{y \in \mathbf{y}^g, d' \in \mathbf{d}^g_+,, d' \leq \bar{d}'(y)}$ where $0 \leq d' < \bar{d}'(y)$, and then for the interval $m \geq \overline{m}(y)$ by interpolating on the pairs $\{(c^{\text{unc}}(d', y), m^{\text{unc}}(d', y))\}_{y \in \mathbf{y}^g, d' \in \mathbf{d}^g, d' \leq \bar{d}'(y)}$ for $d' < \bar{d}'(y)$. The resulting consumption function $\tilde{c}_{k+1}(m, y)$ is again monotonically increasing in m. We proceed in the same manner for the policy functions $\tilde{p}_{k+1}(m, y)$ and $\tilde{\lambda}_{k+1}(m, y)$, which are, respectively, monotonically increasing and decreasing in m for a given y. The iteration process is continued until the distance between two successive iterations $\tilde{c}_k(m, y)$ and $\tilde{c}_{k+1}(m, y)$ (or other policy functions) is sufficiently small.

3.3.1 Simulation of deterministic case

We first simulate the deterministic case where y is constant, maintaining the assumption that $\beta R < 1$. The parameter values for this case are given in the following table:

β	R	γ	e	y	ϕ
.94	1.04	2	.9	.1	.20

Table 2: Parameter values for deterministic case with $\beta R < 1$

Figure 3 depicts a graph of the resulting policy functions. (In this and future graphs, we denote insiders' debt level in terms of their liquid net worth $w = -d$ to correspond to the standard notation in the literature.)

As in the deterministic example with $\beta R = 1$, the policy functions of insiders are "kinked" at the threshold level of debt $\bar{d}'$ at which borrowing constraints become binding. For debt above this level, the economy experiences debt deflation; consumption and asset prices decline rapidly; and the constrained amount of debt that can be carried into the next period $d' = -w' = \phi R p$ shrinks. For debt below the threshold, the economy behaves similarly to a neoclassical economy.

In figure 4, we magnify part of the graph to focus on the dynamics in the sector. We assume that insiders start out in autarky with $d = 0$ and, thanks to financial liberalization, obtain access to borrowing, subject to the financial constraint. Since $\beta R < 1$, insiders have an incentive to take on debt. We illustrate the path of the economy by the zigzag line in the figure starting at zero.

Figure 5 depicts the resulting path of consumption, debt and the asset price, where we have indicated the time of financial liberalization by the vertical dotted

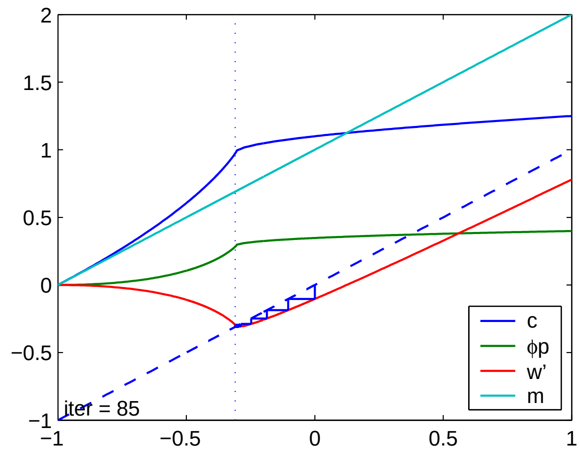

Figure 3: Policy functions for deterministic case with $\beta R < 1$

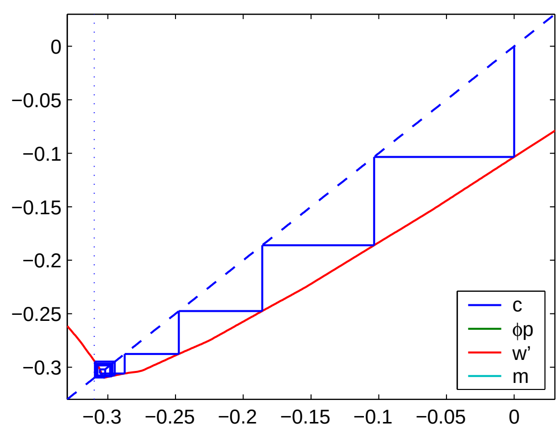

Figure 4: Endogenous cycle dynamics for deterministic case with $\beta R < 1$

line. Right after financial markets are opened, insiders' consumption (top line) spikes up – this raises their debt level over time (bottom line) until the borrowing constraint is hit.

When the constraint becomes binding, the asset price declines and insiders' maximum debt level falls, leading to a lower level of debt and higher asset prices next period. In the long run, the economy oscillates between the constrained state and the unconstrained state – an endogenous cycle emerges. Starting from a situation in which insiders have no debt ($d_1 = 0$) one converges toward a situation in which the consumption of insiders is both lower and more volatile. Along the adjustment path they benefit from a temporary consumption boom financed by debt.

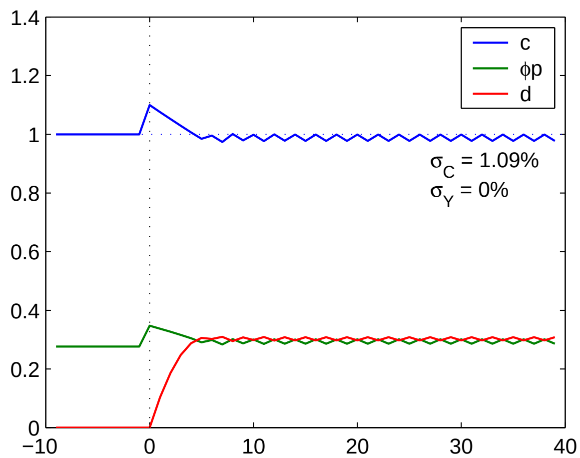

Figure 5: Path of consumption and debt after financial liberalization

The existence of an endogenous cycle depends on the slope of the w'-curve in the constrained region. Specifically, if the downward-sloping part of the w'-curve is steeper than -1, the system possesses a steady state with an endogenous cycle and will converge to this steady state with probability one. There is also an unstable deterministic equilibrium at the intersection of the w'-curve with the 45° line. If insiders starts out in that equilibrium, they will remain there indefinitely in the absence of shocks, but the probability that this equilibrium is reached for different starting values of w is zero. If the w'-curve is flatter than -1, the deterministic equilibrium is the unique equilibrium and the system converges to this equilibrium from any admissible initial level of wealth. The slope of the w'-curve depends chiefly on the parameter ϕ.

Proposition 4 *If $\beta R < 1$ and output y is constant, the economy possesses a deterministic steady state at $w' = w$. This is the only steady state for $\phi < \hat{\phi}$. For a looser borrowing constraint $\phi \geq \hat{\phi}$, the deterministic steady state is unstable and the economy converges with probability one towards a stochastic steady state in which constrained and unconstrained periods alternate.*

3.3.2 Simulation of stochastic case

Consider now the case where y is stochastic. Booms and busts can be modeled by assuming a simple two-state Markov process for y. Assume that the return on the collateral asset can be high, $y = y_H$, or low, $y = y_L$ and that booms are likely to persist for some time, i.e., there is a high probability of staying in the high state. We assume the following parameter values for our simulation, where we denote P the matrix of transition probabilities:

β	R	γ	e	y_L	y_H	P	ϕ
.94	1.04	2	.9	.08	.12	$\begin{pmatrix} .9 & .1 \\ .5 & .5 \end{pmatrix}$	.20

Table 3: Parameter values for stochastic simulation with $\beta R < 1$

The resulting debt and consumption dynamics are shown in figure.

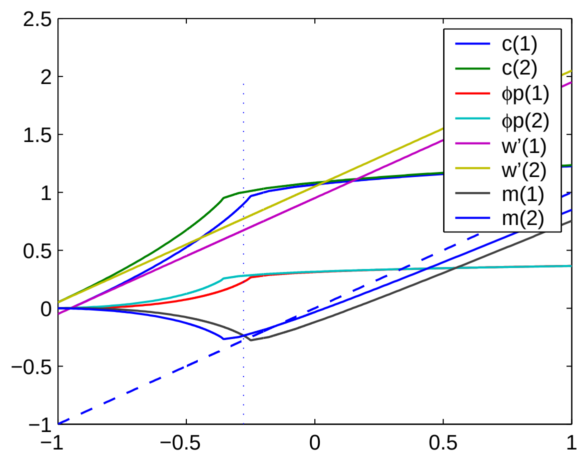

Figure 6: Policy functions for stochastic case with $\beta R < 1$.

When $y = y_H$, the price of collateral is high, allowing insiders to increase the level of their debt. There is a credit boom. During the boom, debt converges to a level that keeps insiders unconstrained (they keep a precautionary margin of safety if the risk of a bust is not too small). If the boom is long enough, debt exceeds the threshold that makes the economy vulnerable to a credit crunch if y falls to y_L. When y falls, there is a bust with a sharp contraction of credit and downward overshooting in the price of the asset. The economy then oscillates between the constrained and the unconstrained state (like in the deterministic case)—until y goes back up to y_H, allowing a new boom to take place.

Starting from a situation with no debt, there will be a period during which consumption is both higher on average and less volatile (honeymoon of financial liberalization), but in the long run consumption may be made more volatile (with a fat tail on the downside) by credit crunches and procyclical credit flows.

4 Social planner

4.1 The social planner solution

We assume that the social planner of the economy determines the amount of insiders' borrowing, but does not directly interfere in asset markets—that is, the social planner takes as given that insiders trade the collateralizable asset at a price that is determined by their private optimality condition (5). The social optimum differs from the laissez-faire equilibrium because the social planner internalizes that future asset prices and insiders' borrowing capacity depend on the aggregate level of debt accumulated by insiders. A possible motivation for this setup is that decentralized agents are better than the planner at observing the fundamental payoffs of financial assets, while only the social planner has the capacity of internalizing the costs of debt deflation dynamics that may arise from high levels of debt.

In period t, the social planner chooses the debt level of the representative insider, d_{t+1}, before the asset market opens at time t. The asset market remains perfectly competitive, i.e., individual market participants optimize on θ_{t+1} subject to (2), yielding the optimality condition (5). We look for time-consistent equilibria in which the social planner optimizes on d_{t+1} taking the future policy functions $c(m,y)$ and $p(m,y)$ as given. (Although we do not change the notation, those policy functions are not the same as in the laissez-faire equilibrium.)

We define a new function that says how the current price level depends on the beginning-of-period state, (m,y), and on the level of debt chosen by the social planner,

$$p\left(m,y,d'\right)=\beta\frac{E\left[u'(c\left(m',y'\right))(y'+p\left(m',y'\right))+\phi\lambda(m',y')p(m',y')|y\right]}{u'(m+d'/R)}, \quad (16)$$

where $m'=e+y'-d'$. This function has the same form as (8), reflecting the fact that the asset market remains perfectly competitive. The only difference is that the social planner internalizes that he can affect the price of the asset through his decision on the current level of aggregate debt. Increasing d' lowers the marginal utility of consumption (the denominator in (16)), which tends to increase the price of the asset.[3]

Since insiders can still not borrow more than a fraction ϕ of the value of their asset holdings, the social planner sets d' subject to the constraint

$$\frac{d'}{R}\leq\phi p\left(m,y,d'\right), \quad (17)$$

where m is the insiders' *aggregate* wealth. The right-hand side may be increasing in d', in which case the social planner relaxes the credit constraint by increasing aggregate debt. However, if ϕ is small enough, the right-hand side increases less with d' than the left-hand side, so that this inequality determines an *upper bound* on aggregate debt. We assume that this is the case (otherwise (17) would determine a lower bound on aggregate debt).

[3]The sign of the variation of the numerator with d' is however ambiguous.

The social planner's credit constraint can be rewritten in reduced form,

$$\frac{d'}{R} \leq \phi \overline{p}(m, y), \tag{18}$$

where $\overline{p}(m, y)$ is the level of $p(m, y, d')$ such that (17) is an equality. This function is increasing in m because $p(m, y, d')$ is increasing in m for any y and d' (as can be readily seen from (16)). Note that per the definition of the function $\overline{p}(\cdot, \cdot)$, we have $\overline{p}(m, y) = p(m, y)$ for all the states (m, y) in which the social planner's constraint is binding.

The social planner solves the same optimization problem as decentralized agents, except that he takes $\theta_t = 1$ as given in the aggregate budget constraint, and that his credit constraint is given by (18), which internalizes the feedback effects of changes in consumption on the level of asset prices. As shown in the appendix, the social planner's Euler equation is,

$$u'(c_t) = \lambda_t + \beta R E_t \left(u'(c_{t+1}) + \lambda_{t+1} \phi \frac{\partial p_{t+1}}{\partial m_{t+1}} \right). \tag{19}$$

The derivative of the next-period asset price with respect to aggregate wealth, $\partial p_{t+1}/\partial m_{t+1}$, is positive. Comparing (4) and (19), this implies that the social planner favors more saving (less consumption and less debt) than in the decentralized equilibrium. The saving wedge is proportional to the expected product of the shadow cost of the credit constraint times the derivative of the debt ceiling with respect to wealth. This reflects that the social planner internalizes the endogeneity of next period's asset price and credit constraint to this period's aggregate saving.

Decentralized agents are aware of the risk of credit crunch and maintain a certain amount of precautionary savings (they issue less debt than if this risk were absent), but they do not internalize the contribution of their precautionary savings to reducing *systemic* risk. With the social planner, precautionary savings is augmented by a systemic component (i.e., the social planner implements a policy of *systemic precautionary saving*).

4.1.1 Simulation of deterministic case

We simulate the deterministic case for the same parameter values as what is given in table 2. Figure 7 below shows the amount of next-period wealth w' chosen by decentralized agents and by the social planner as a function of current-period wealth. The social planner consumes less and borrows less for a given level of initial wealth. In fact, in the given example, the social planner will reduce insiders' exposure to binding borrowing constraints sufficiently so that the oscillating equilibrium disappears and the economy converges to a stable steady state.

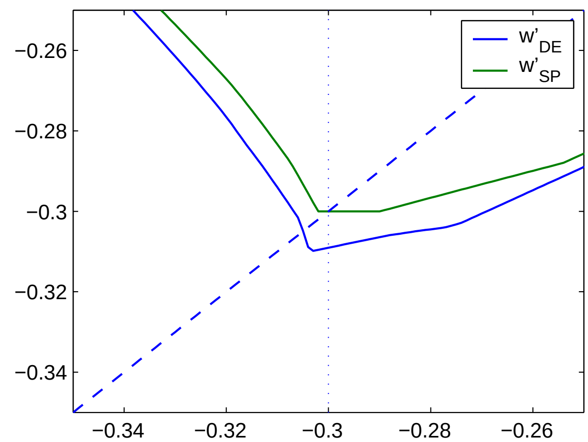

Figure 7: Optimal borrowing choices of decentralized insiders and social planner

4.1.2 Simulation of stochastic case

4.1.3 Pigouvian taxation

The social planner's Euler equation also provides guidance for how the socially optimal equilibrium can be implemented via taxes on external borrowing. Decentralized agents undervalue the social cost of debt by the term $\phi E_t\left[\lambda_{t+1}\frac{\partial p_{t+1}}{\partial m_{t+1}}\right]$ on the right-hand side of the social planner's Euler equation (19), which depends on the state of the economy (m_t, y_t). The planner's equilibrium can be implemented by a Pigouvian tax $\tau_t = \tau(m_t, y_t)$ on borrowing that introduces a wedge in insiders' Euler equation and that is rebated as a lump sum transfer $T_t = \tau_t w_{t+1}/R$:

$$c_t = e_t + y_t + w_t - \frac{w_{t+1}}{R}(1+\tau_t) + T_t$$

This modifies insiders' Euler equation to

$$u'(c_t) = \lambda_t + (1+\tau_t)\,\beta R E_t\left[u'\left(c_{t+1}\right)\right]$$

The tax replicates the constrained social optimum as chosen by the constrained planner if it is chosen such that

$$\begin{aligned}(1+\tau_t)\,E_t\left[u'(c_{t+1})\right] &= E_t\left[u'(c_{t+1}) + \lambda_{t+1}\phi\frac{\partial p_{t+1}}{\partial m_{t+1}}\right]\\ \text{or}\quad \tau(m_t, y_t) &= \frac{\lambda_{t+1}\phi\frac{\partial p_{t+1}}{\partial m_{t+1}}}{E_t\left[u'(c_{t+1})\right]}\end{aligned}$$

where all variables are evaluated at the social optimum.

The tax would be levied at time t when the borrowing decision w_{t+1} for next period is made; therefore such a measure avoids any commitment problems. A

Ramsey-equivalent approach would be to impose a tax whenever borrowing constraints are binding and the externality materializes. However, this would potentially face two important political economy constraints: First, it would require that higher taxes are imposed in the midst of large downturns – precisely when consumption among insiders falls sharply. Secondly, it would create a commitment problem for the planner – the measure is only effective if insiders in period t when borrowing choices are made believe that the tax will indeed be imposed in period $t + 1$.

In our simulations, we find that the optimal magnitude of this tax is on average 2.41%.

5 Model with Capital

6 Extensions

6.1 Debt moratorium and bailouts

6.2 FDI Liberalization

6.3 Debt maturity

6.4 Endogenous return

7 Conclusion

This paper has developed a simple model to study the optimal policy response to booms and busts in credit and asset prices. We found that decentralized agents do not internalize that their borrowing choices in boom times render the economy more vulnerable to credit and asset price busts involving debt deflation in bust times. Therefore their borrowing imposes an externality on the economy.

In our baseline calibration, a social planner would impose an ex-ante tax of 2.41% per dollar on insider borrowing so as to reduce the debt burden of insiders and mitigate the decline in consumption in case of crisis.

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A Solution of Benchmark Model

A.1 Laissez-faire

Decentralized agents solve the Lagrangian

$$\mathcal{L}_t = E_t \sum_{s=t}^{+\infty} \beta^{t-s} \left\{ u \left(e_s + \theta_s (y_s + p_s) + \frac{d_{s+1}}{R} - d_s - \theta_{s+1} p_s \right) + \lambda_s \left[\phi \theta_s p_s - \frac{d_{s+1}}{R} \right] \right\},$$

Given CRRA utility, this implies the first-order conditions

$$\begin{aligned} FOC\,(d_{s+1}) \quad &: \quad c_s^{-\gamma} = \beta R E_s \left[c_{s+1}^{-\gamma} \right] + \lambda_s, \\ FOC\,(\theta_{s+1}) \quad &: \quad p_s c_s^{-\gamma} = \beta E_s \left[c_{s+1}^{-\gamma} (y_{s+1} + p_{s+1}) + \phi \lambda_{s+1} p_{s+1} \right]. \end{aligned}$$

A.2 Social planner

The social planner maximizes the utility of the representative insider subject to the budget constraint (2) taking $\theta_t = 1$ as given, and to the credit constraint (18). The Lagrangian of the social planner is

$$\mathcal{L}_t^{SP} = E_t \sum_{s=t}^{+\infty} \beta^{t-s} \left\{ u \left(e + y_s + \frac{d_{s+1}}{R} - d_s \right) + \lambda_s \left[\phi \bar{p}(e + y_s - d_s, y_s) - \frac{d_{s+1}}{R} \right] \right\},$$

FOC(d_{t+1}):

$$u'(c_t) = \lambda_t + \beta R E_t \left[u'(c_{t+1}) + \phi \lambda_{t+1} \frac{\partial \bar{p}(m_{t+1}, y_{t+1})}{\partial m} \right].$$

Using the fact that $\bar{p}(\cdot,\cdot) = p(\cdot,\cdot)$ in the constrained states, we have

$$\lambda_{t+1} \frac{\partial \bar{p}(m_{t+1}, y_{t+1})}{\partial m} = \lambda_{t+1} \frac{\partial p(m_{t+1}, y_{t+1})}{\partial m},$$

so that the Euler condition can be written like (19).

A.3 Alternative Specification of Constraint

If the collateral constraint in subsection A.1 was written in terms of future asset holdings

$$\frac{d_{s+1}}{R} \leq \phi \theta_{s+1} p_s,$$

the second first-order condition would read as

$$p_s (c_s^{-\gamma} - \phi \lambda_s) = \beta E_s \left[c_{s+1}^{-\gamma} (y_{s+1} + p_{s+1}) \right].$$

In this case, we would have

$$p_s = \frac{E_s \left[c_{s+1}^{-\gamma} (y_{s+1} + p_{s+1}) \right]}{c_s^{-\gamma} - \phi \lambda_s},$$

As long as $\phi < 1$, there is again a positive feedback effect from c_s to p_s, giving rise to debt deflation dynamics that are equivalent to our benchmark specification.