The Maturity Rat Race*

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Abstract

We develop a model of endogenous maturity structure for financial institutions that borrow from multiple creditors. We show that a maturity rat race can occur: an individual creditor can have an incentive to shorten the maturity of its own loan to the institution, allowing him to adjust his financing terms or pull out before other creditors can. This, in turn, causes all other lenders to shorten their maturity as well, leading to excessively short-term financing. This force is most pronounced during volatile, crisis times. Overall, firms are exposed to unnecessary rollover risk.

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One of the central lessons of the financial crisis of 2007-09 is the importance of maturity structure for financial stability. In particular, the crisis vividly exposed the vulnerability of institutions with strong maturity mismatch—those who finance themselves short-term and invest long-term—to disruptions in their funding liquidity. This raises the question why financial institutions use so much short-term financing, and whether there may be excessive maturity mismatch in the financial system.

In this paper we analyze the maturity structure choice of a financial institution that can borrow at differing maturities from multiple creditors to finance long-term investments. We show that the equilibrium maturity structure will in general be inefficiently short-term—leading to excessive maturity mismatch, rollover risk, and inefficient creditor runs. The reliance on excessive short-term financing is caused by an externality that short-term creditors impose on the value of longer-term debt contracts. This externality arises for any maturity structure and, under certain conditions, successively unravels any form of long-term financing to the very short end: a *maturity rat race*. Moreover, since this externality is particularly strong during times of elevated volatility, the model predicts a particularly pronounced shortening of the maturity structure during financial crises—as seen in the fall of 2008.

The maturity rat race is inefficient. It leads to excessive rollover risk and causes inefficient liquidation of the long-term investment project after negative interim information. Moreover, because creditors anticipate the costly liquidations that occur when rolling over short-term debt is not possible, some positive NPV projects do not get started in the first place. This inefficiency stands in contrast to some of the leading existing theories of maturity mismatch. Diamond and Dybvig (1983) highlight the role of maturity mismatch in facilitating long-term investment projects while serving investors' liquidity needs that are individually random, but deterministic in aggregate. Calomiris and Kahn (1991) and Diamond and Rajan (2001) demonstrate the role of short-term financing and the resulting maturity mismatch as a disciplining device for bank managers. In these theories, maturity mismatch is an integral and desirable part of financial intermediation. This means that our model generates implications regarding the regulatory response to maturity mismatch that are very different from the theories mentioned above. In particular, to the extent that maturity mismatch results from our 'rat race mechanism,' a regulator may want to impose restrictions on short-term financing to preserve financial stability and reduce rollover risk. Our paper thus complements Diamond (1991) and Stein (2005) in arguing that financing may be excessively short-term. However, while the driving force in their models is asymmetric information about the borrower's type, we highlight a contractual externality among creditors of different maturities.[1]

Our model is based on a friction that applies particularly to financial institutions, rather than corporates in general: When offering debt contracts to its creditors, it is almost impossible for a financial institution to commit to an aggregate maturity structure. While corporates that tap capital markets only occasionally may be able to do this through covenants or the like, financial institutions, with their frequent sizable funding needs and opaque balance sheets, are more or less constantly active in the commercial paper market. This makes committing to a particular maturity structure difficult. This inability to commit means, for example,that when offering a long-term debt contract to one creditor, the financial institution cannot commit to simultaneously offer long-term debt contracts to all other creditors. The financial institution can, however, commit to the aggregate amount of financing raised, which means that no sequential banking problem emerges, as in Bizer and DeMarzo (1992) and Parlour and Rajan (2001).

The externality between creditors of differing maturities arises because creditors with shorter maturities can adjust their financing terms in response to interim information. When at the rollover date there is positive news about the profitability of the long-term investment, rolling over short-term debt is cheap. Since conditional on positive news it is more likely that the financial institution will be the residual claimant when the investment payoffs realize,

[1] In Flannery (1986) short-term financing also results from asymmetric information, but does not result in inefficiencies since there is no early liquidation.

the benefits of cheaply rolling over an additional unit of short-term debt are reaped disproportionately by the financial institution. When, on the other hand, negative news comes out, rolling over short-term debt is costly or even impossible, because short-term debtholders raise their face values or withdraw their funding. Conditional on bad news, however, the financial institution is less likely to be the residual claimant, such that the costs of rolling over an additional unit of short-term debt are disproportionately borne by the remaining long-term debtholders, whose financing terms remain unchanged. Due to the higher fraction of short-term creditors they will receive less, either because short-term creditors withdraw their funding early, or because short-term creditors have a larger claim on cash flows in default.

The same logic extends to settings in which short-term credit is rolled over multiple times before an investment pays off. In fact, in a model with multiple rollover periods the contractual externality leads to a successive unraveling of the maturity structure: If everyone's debt matures at time T, the financial institution has an incentive to start shortening an individual creditor's maturity, until everyone's maturity is of length $T-1$. Yet once everyone's maturity is of length $T-1$, there would be an incentive to give some creditors a maturity of $T-2$. In the extreme case, this process repeats until all financing is extremely short-term and rolled over every period.

The fundamental incentive to shorten the maturity structure is present whenever a financial institution borrows from multiple creditors. However, since the externality that causes maturity shortening stems from the short-term creditors' ability to update their financing terms in response to interim information, the incentive to shorten the maturity structure is particularly strong during times of high volatility, i.e. at times when investors expect a lot of information to be released before the long-term project matures. This means that the rat race mechanism is consistent both with the large amount of short-term financing that existed during the run-up of the crisis, and with the drastic further shortening of financial institutions' maturity structure during the fall of 2008. Specifically, in a refinement of the model in which writing a rollover contract has a small cost, the financial institution only has an incentive to shorten the maturity structure as long as sufficient information is expected to be learned by the rollover date. This leads to a moderate amount of maturity shortening during normal times, and drastic shortening during times of high volatility.

Of course, the incentive to shorten the maturity structure also depends on priority rules, covenants and the degree of collateralization. While in our baseline model we assume unsecured debt and equal priority among creditors, we then discuss the impact of other priority rules and the introduction of covenants. Seniority restrictions or covenants can weaken the maturity rat race, but generally do not eliminate it. Moreover, even when they can, financial institutions may be less willing to counteract the maturity rat race through covenants because they attach a high value to financing flexibility. Finally, even when by virtue of senioriy restrictions or covenants an equilibrium with long-term financing exists, the financial institution can still get caught in a 'short-term financing trap,' an inefficient short-term financing equilibrium that continues to exist even when long-term financing is also an equilibrium. The reason is that, given that all other lenders are only providing short-term financing, it is not individually rational for the financial institution to move an individual creditor to a longer maturity. In fear of getting stuck while others withdraw their funding, that creditor would require a very large face value, such that the financial institution is better off under all short-term financing, even though that maximizes the institution's rollover risk.

This paper relates to a number of recent papers on rollover risk. Brunnermeier and Yogo (2009) provide a model of liquidity risk management in the presence of rollover risk. Their analysis shows that liquidity risk management does not necessarily coincide managing duration risk. Acharya, Gale, and Yorulmazer (2009) show how rollover risk can reduce a security's collateral value. In contrast to our paper, they take short-term financing of assets as given, while we focus on why short-term financing emerges in the first place. In He and Xiong (2009) coordination problems among creditors with debt contracts of random maturity can lead to the liquidation of financially sound firms. Given a fixed expected rollover frequency, they show that each creditor has an incentive to raise his individual rollover threshold, inducing others to raise theirs as well. Unlike their dynamic global games setting, in which interest rates and maturity structure are exogenous, we focus on the choice of maturity with endogenous interest rates. Finally, Farhi and Tirole (2009) show how excessive maturity mismatch can emerge through collective moral hazard through the reaction of ex post monetary policiy in systemic crises.

The remainder of the paper is structured as follows. We describe our model setup in Section 1. In Section 2 we derive the equilibrium maturity structure and show how the inability to commit to an aggregate maturity structure leads to excessive short-term financing and a maturity rat race can lead to a complete unraveling of the maturity structure. In this section we also discuss the impact of seniority, covenants and how to what extend the rat race depends on macroeconomic conditions. Section 3 concludes.

1 Model Setup

Consider a risk-neutral financial institution that can invest in a long-term project. The investment opportunity arises at $t = 0$, is of fixed scale, and we normalize the required $t = 0$ investment cost to 1. At time T, the investment pays off a random amount θ, distributed according to a distribution function $F(\cdot)$ on the interval $\left[0, \bar{\theta}\right]$. Seen from $t = 0$, the unconditional expected payoff from investing in the long-term project is thus $E(\theta) = \int_0^{\bar{\theta}} \theta dF(\theta)$, and its net present value is positive when $E(\theta) > 1$. There is no time discounting.

Once the project has been undertaken, more information is learned about its profitability. At any interim date $t = 1, ..., T-1$, a signal s_t realizes. We assume that for all signals up to time t, $\{s_1, .., s_t\}$, there is a sufficient statistic S_t, conditional on which the distribution of the project's payoff is given by $F(\theta|S_t)$, and its expected value accordingly by $E(\theta|S_t)$. For the remainder of the text, we will loosely refer to S_t, the sufficient statistic for the signal history, as the signal at time t. We assume that $F(\cdot)$ satisfies the strict monotone likelihood ratio property with respect to the signal S_t. This implies that when $S_t^A > S_t^B$, the updated distribution function $F\left(\theta|S_t^A\right)$ dominates $F\left(\theta|S_t^B\right)$ in the strict monotone likelihood ratio sense. The signal S_t is distributed according to the distribution function $G(\cdot)$. We refer to the highest possible signal at time t as S_t^H, and the lowest possible signal as S_t^L.

The long-term project can be liquidated prematurely at time $t < T$ with a continuous liquidation technology that allows to liquidate all or only part of the project. However, early liquidation yields only a fraction of the conditional expectation of the project's payoff, $\lambda E(\theta|S_t)$, where $\lambda < 1$. This implies that early liquidation is always inefficient—no matter how bad the signal realization S_t turns out to be, in expectation it always pays more to continue the project rather than to liquidate it at $t < T$. These liquidation costs reflect the deadweight costs generated by shutting down the project early, or the lower valuation that a second-best owner, who may purchase the project at an interim date, attaches to it.

Of course, in practice early liquidation must not always be inefficient—there may be projects for which liquidation at an interim date may yield more than continuation. If in addition the financial institution prefers continuation because of private benefits or empire building motives, some amount of short-term financing may be desirable, because if may help force liquidation in states where it is efficient. We intentionally rule out this possibility for the remainder of the paper. Our motivation for ruling out this case is to restrict the analysis to situations in which short-term debt has no inherent advantage and then show that short-term financing will nevertheless emerge as an equilibrium outcome. This allows us to isolate the effect of the contractual externality that leads to short-term financing in our model.

The financial institution has no equity capital and hence needs to raise the financing for the long-term project from a competitive capital market populated by a mass-one continuum of risk-neutral lenders, each with limited capital. We make two important assumptions on the financing choices the financial institution has at its disposal:

First, we assume that financing takes the form of debt contracts. We take debt contracts as given in this paper and do not study the optimality of these contracts from a security design perspective. Debt contracts with differing maturities are available, such that the financial institution has to make a choice about its maturity structure when financing the long-term project. A debt contract specifies a face value and a maturity date at which that face value is due. We refer to a debt contract with maturity T as a long-term debt contract. This long-term debt contract matches the maturity of the assets and liabilities of the financial institution, and when written at $t = 0$ it specifies a face value $D_{0,T}$ to be paid back at time T. By definition, long-term debt contracts do not have to be rolled over before maturity, which means that long-term debtholders cannot adjust their financing terms in response to the signals observed at the interim dates $t < T$.

In addition to long-term debt, the financial institution can also issue debt with shorter maturity, which has to be rolled over at some time $t < T$. A short-term debt contract written at $t = 0$ specifies a face value $D_{0,t}$ that comes due at date t, at which point this face value has to be repaid or rolled over. When short-term debt is rolled over at t, the outstanding face value is adjusted to reflect the new information contained in the signal S_t. In terms of notation, if debt is rolled over from time t to time $t + \tau$, we denote the rollover face value due at $t + \tau$ by $D_{t,t+\tau}(S_t)$. Short-term debtholders are atomistic and make uncoordinated rollover decisions at the rollover date. If short-term debtholders refuse to roll over their obligations at date t, some or even all of the long-term investment project may have to be liquidated early to meet the repayment obligations to the short-term debtholders. In the case of default at time $t \leq T$, we assume that there is equal priority between among all bonds maturing at time t. Consistent with U.S. bankruptcy procedures, we do not make a distinction between principal and accrued interest in the case of default. Equal priority thus implies that in the case of default at time $t \leq T$, the liquidation proceeds are split among the creditors proportionally to the face value of maturing debt. We assume that non-matured debt is not accelerated, or has lower priority than matured debt. This last assumption is for simplicity, and can be relaxed. For example, we could assume that long-term debt is accelerated and that liquidation proceeds at time t are shared between maturing debt contracts and accelerated longer term debt contracts proportional to the outstanding principal.

Our second assumption is that the financial institution has to finance itself using multiple creditors. This assumption reflects the dispersed creditor structures of financial institutions (and many other firms) observed in practice. The financial institution simultaneously offers debt contracts to a mass one continuum of potential lenders, and each creditor can only observe the contracts offered to him, but not the financing terms offered to other creditors. Importantly, when dealing with an individual creditor, the financial institution cannot credibly commit to the financing terms offered to other creditors. This implies that the financial institution cannot commit to a particular maturity structure (for example issuing only time T 'long-term' debt contracts). This inability to commit to an aggregate maturity structure arises naturally when a financial institution deals with many dispersed creditors. In our model, it is the key friction that generates equilibrium maturity structures that are excessively short-term.

2 The Equilibrium Maturity Structure

Given our setup, two conditions must be met for a maturity structure to constitute an equilibrium. First, in any equilibrium maturity structure all creditors must break even in expectation. This means that at $t = 0$ the financial institution has to issue a combination of debt contracts of different maturities that have an aggregate expected payoff equal to initial cost of undertaking the investment.

However, while creditor break-even is a necessary condition, it is not sufficient. A second condition arises because the financial institution deals bilaterally with multiple creditors and cannot commit to an aggregate maturity structure when entering debt contracts with individual creditors. In particular, for a maturity structure to be an equilibrium the financial institution must have no incentive to deviate from a conjectured equilibrium maturity structure by forming a coalition with an individual creditor and changing this creditors' maturity, while holding everybody else's financing terms fixed.

To illustrate this second requirement, consider for example a conjectured equilibrium in which all financing is in the form of long-term debt that matures at T. The 'no deviation' requirement asks whether the financial institution has an incentive to move one of the long-term creditors to a shorter maturity contract, given that all creditors anticipate financing to be purely long-term and set their financing terms such that they would just break even under all long-term financing. If this deviation is profitable, the all long-term financing outcome cannot be an equilibrium maturity structure.

We now examine the break-even and no deviation conditions in turn. For simplicity, in what follows we will initially focus on the financial institution's maturity structure choice when there is only one potential rollover date t. We will then show that the analysis can be extended to accommodate multiple rollover dates before the payoff date T. As we will show, when multiple rollover dates are possible, the maturity structure successively unravels to the very short end, strengthening the impact of the contractual externality we highlight.

2.1 Creditor Break-Even Conditions

Consider first the rollover decision of creditors' whose debt matures at $t < T$, and denote the fraction of creditors that has entered short-term debt contracts with maturity t by α. In order to roll over the maturing debt at t the financial institution has to issue new debt of face value $D_{t,T}(S_t)$, which conditional on the signal S_t must have the same value as the face value of the maturing debt, $D_{0,t}$. This means that the new short-term face value must be set such that

$$\int_0^{\bar{D}_T(S_t)} \frac{D_{t,T}(S_t)}{\bar{D}_T(S_t)} \theta dF(\theta|S_t) + D_{t,T}(S_t) \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t) = D_{0,t}, \tag{1}$$

where $\bar{D}_T(S_t) = \alpha D_{t,T}(S_t) + (1-\alpha) D_{0,T}$ denotes the aggregate face value due at time T. If default occurs at time T, the creditors rolling over at t receive a proportional share of the projects cash flows, $\frac{D_{t,T}(S_t)}{\bar{D}_T(S_t)}\theta$. When the financial institution does not default, the entire face value $D_{t,T}(S_t)$ is repaid. Equation (1) thus says that for rollover to occur, $D_{t,T}(S_t)$ must be set that in expectation the creditors receive their outstanding face value $D_{0,t}$. Both $\bar{D}_T(S_t)$ and $D_{t,T}(S_t)$ are also functions of α, the fraction of creditors with debt contracts that need to be rolled over at time t, but for simplicity of notation we will generally suppress that dependence.

Short-term debt can be rolled over as long as the project's future cash flows are high enough such that the financial institution can find a face value $D_{t,T}(S_t)$ for which (1) holds. Given the equal priority assumption, the maximum the financial institution can pledge to the short-term creditors at time t is the entire expected future cash flow from the project. This is done by setting $D_{t,T}(S_t)$ to infinity, such that short-term debt holders effectively become equity holders and long-term debtholders are wiped out. This implies that rolling over short-term debt becomes impossible when the expected future cash flows conditional on the signal S_t are smaller than the maturing face value $\alpha D_{0,t}$ owed to the short-term creditors at t. This is the case when

$$\alpha D_{0,t} > \int_0^{\bar{\theta}} \theta dF(\theta|S_t). \tag{2}$$

The amount of pledgeable cash flow the financial institution has at his disposal to roll over debt at time t is increasing in the signal realization S_t. Denote by $\tilde{S}_t(\alpha)$ the signal for which (2) holds with equality:

$$\alpha D_{0,t} = \int_0^{\bar{\theta}} \theta dF\left(\theta|\tilde{S}_t(\alpha)\right) \tag{3}$$

When the signal realization S_t is below the critical value $\tilde{S}_t(\alpha)$, the financial institution cannot roll over his short-term obligations. This is because the dispersed lenders make their rollover decision in an uncoordinated fashion, such that they will find it individually rational to pull out their funding in a 'fundamental bank run' when $S_t < \tilde{S}_t(\alpha)$. The critical signal realization below which the project is liquidated is a function of the fraction of overall debt that has been financed short-term, α, and of the initial face value of short-term debt $D_{0,t}$.

This argumemt implicitly assumes that short-term debt cannot be restructured, such that uncoordinated rollover decisions lead to inefficient liquidation at the rollover date. One justification for this is that when creditors also respond to restructuring offers in an uncoordinated fashion, restructuring would be difficult or even impossible because of the holdout problems. This is analyzed in more detail in Gertner and Scharfstein (1991). Essentially, since the Trust Indenture Act of 1939 prohibits changing the timing or the payment amounts of public debt, debt restructuring must take the form of exchange offers, which usually require consent of a specified fraction of debtholders to go through. If each debtholder is small (as we assume in our model), he will not take into account the effect of his individual tender decision on the outcome of the exchange offer. This means that assuming that a sufficient number of other creditors accept the restructuring, an individual creditor prefers not to accept in order to be paid out in full. Since all creditors will follow this rationale, the exchange offer will not be successful.

Anticipating potential early liquidation, the $t = 0$ face value of short-term debt $D_{0,t}$ is determined by the short-term creditors' break-even constraint seen from $t = 0$, which is given by

$$\frac{1}{\alpha}\int_{S_t^L}^{\tilde{S}(\alpha)} \lambda E[\theta|S_t]\, dG(S_t) + \left[1 - G\left(\tilde{S}_t(\alpha)\right)\right] D_{0,t} = 1. \quad (4)$$

The interpretation of (4) is as follows. When $S_t < \tilde{S}(\alpha)$, the short-term creditors withdraw their funding and receive $\lambda E[\theta|S_t] = \lambda \int_0^{\bar{\theta}} \theta dF(\theta|S_t)$. The overall proceeds are then split among the α short-term creditors, which is why the term is preceded by $\frac{1}{\alpha}$. When $S_t \geq \tilde{S}(\alpha)$, short-term creditors roll over, in which case they are promised a new face value $D_{t,T}(S_t)$, which in expectation has to be worth $D_{0,t}$. Together, these two terms must be equal to one for short-term creditors to break even.

Now turn to the break-even condition for the long-term creditors. Since long-term creditors enter their debt contracts at $t = 0$ and cannot change their financing terms after that, they must break even taking an expectation across all signal realizations for which the project is not liquidated early, i.e. for all $S_t \geq \tilde{S}_t(\alpha)$. This leads to the long-term break-even condition

$$\int_{\tilde{S}_t(\alpha)}^{\infty} \left[\int_0^{\bar{D}(S_t)} \frac{D_{0,T}}{\bar{D}_T(S_t)} \theta dF(\theta|S_t) + D_{0,T} \int_{\bar{D}(S_t)}^{\bar{\theta}} dF(\theta|S_t) \right] dG(S_t) = 1. \tag{5}$$

The interpretation of equation (5) is as follows. Creditors with maturity T are only paid if the project is not liquidated at t, i.e. when $S_t \geq \tilde{S}_t(\alpha)$. When the project is not liquidated, they receive either their proportional share of the cash flow $\frac{D_{0,T}}{\bar{D}_T(S_t)}\theta$ if the financial institution defaults at time T, or they are paid back their entire face value $D_{0,T}$. Since the time-T face value of creditors that roll over at t depends on the signal S_t, long-term creditors must take an expectation across signal realizations when determining their break-even face values.

2.2 Profit to the Financial Institution and No Deviation Condition

We can now write down the expected profit for the financial institution. The financial institution will receive a positive cash flow at time T if two conditions are met. First, the project must not be liquidated at t, which means that $S_t \geq \tilde{S}_t(\alpha)$. Second, conditional on survival at t, the realized cash flow θ must exceed the aggregate face value owed to the creditors of different maturities, $\bar{D}_T(S_t)$. This means that we can write the expected profit the to the financial institution as

$$\Pi = \int_{\tilde{S}_t(\alpha)} \int_{\bar{D}_T(S_t)}^{\bar{\theta}} \left[\theta - \bar{D}_T(S_t)\right] dF(\theta|S_t) \, dG(S_t). \tag{6}$$

The inner integral of this expression is the expected cash flow to the institution given a particular signal realization S_t. The outer integral takes the expectation of this expression over signal realizations for which the project is not liquidated at time t.

What is the payoff to the financial institution of moving one additional creditor from a long-term debt contract to a short-term debt contract? To calculate this, take the derivative of (6) with respect to α, keeping in mind that $\bar{D}_T(S_t) = \alpha D_{t,T}(S_t) + (1-\alpha) D_{0,t}$. This yields.

$$\frac{\partial \Pi}{\partial \alpha} = \int_{\tilde{S}_t(\alpha)} \int_{\bar{D}_T(S_t)}^{\bar{\theta}} \left[D_{0,T} - D_{t,T}(S_t) - \alpha \frac{d}{d\alpha} D_{t,T}(S_t) \right] dF(\theta | S_t) \, dG(S_t) . \tag{7}$$

The intuitive interpretation for this expression is as follows. On the margin, the gain from moving one long-term creditor to short-term is given by the differences of the marginal cost of long-term and short-term credit. Because there is one less long-term creditor, the financial institution saves $D_{0,T}$ in states where it is the residual claimant, i.e. when $S_t \geq \tilde{S}_t(\alpha)$ and $\theta > \bar{D}_T(S_t)$. This gain has to be weighed against the marginal cost of short-term credit, which is given by $D_{t,T}(S_t) + \alpha \frac{d}{d\alpha} D_{t,T}(S_t)$. The first term of this sum is the cost of adding one more short-term creditor keeping the cost of the remaining units of short-term credit constant, while the second part of the sum captures the effect of adding one more short-term creditor on the cost of rolling over existing, infra-marginal units of short-term credit. When all financing is long-term, i.e. when $\alpha = 0$, there are no infra-marginal units of short-term credit, such that the term $\alpha \frac{d}{d\alpha} D_{t,T}(S_t)$ drops out. Note that in deriving this expression we made use of the fact that the derivatives with respect to the lower integral boundaries drop out, since both times the term inside the integral when evaluated at the boundary equals zero.

The no deviation condition implies that as long as at a conjectured equilibrium, in which creditors break even, we have

$$\frac{\partial \Pi}{\partial \alpha} > 0, \tag{8}$$

the financial institution has an incentive to move an additional creditor from long-term financing to a shorter maturity, keeping everybody else's financing terms fixed. An equilibrium maturity structure will thus either be characterized by $\frac{\partial \Pi}{\partial \alpha} = 0$ (with the appropriate second order condition holding), or it will be extreme maturity structures, either all long-term debt with $\alpha = 0$ and $\frac{\partial \Pi}{\partial \alpha} \leq 0$, or all short-term maturity with $\alpha = 1$ and $\frac{\partial \Pi}{\partial \alpha} \geq 0$.

Since short-term financing exposes the financial institution to rollover risk stemming from the short-term creditors' uncoordinated rollover decisions which can lead to inefficient liquidation, the efficient way to finance the long-term investment would be to choose a maturity structure that matches the maturity of the project with the maturity of the debt issued—in other words, financing the entire project with long-term debt. But can all long-term financing be an equilibrium? It turns out that this is not the case. We first illustrate this in a simple example, and then state a formal version of the result.

2.3 One-step deviation: A Simple Example

Assume that the cash flow θ can only take two values at time T, θ^H and θ^L. The probability of the high cash flow is p, which seen from $t = 0$ is a random variable. p is realized at time t from a uniform distribution on $[0, 1]$. We assume that the high cash flow realization is sufficiently high to repay debt at time T, while the low cash flow realization leads to default, i.e. $\theta^L < 1$.

If all financing is long-term, the break-even condition of the long-term creditors (5) can be rewritten as

$$\frac{1}{2}\theta^L + \frac{1}{2}D_{0,T} = 1, \tag{9}$$

which leads to a face value of $D_{0,T} = 2 - \theta^L$. Calculating the financial institution's expected profit shows that it will earn the entire expected NPV of the project, $E[\theta] - 1$.

But is financing will all long-term debt an equilibrium maturity structure? To see this, let us check the no deviation condition derived above. First, we need to calculate the marginal cost of short-term financing starting from a conjectured equilibrium with all long-term financing. Since $\theta^L > 0$, the first short-term creditor can always be rolled over at time

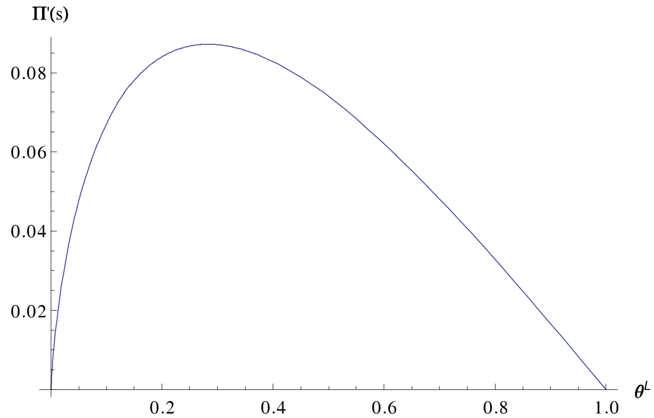

Figure 1: The payoff from deviating from all long-term financing ($\alpha = 0$). The payoff is strictly positive for all θ^L on the interval $(0, 1)$. This means that for those values, all long-term financing cannot be an equilibrium.

t, which implies that $D_{0,t} = 1$. The time t rollover condition (1) then reduces to

$$(1-p)\frac{D_{t,T}}{2-\theta^L}\theta^L + pD_{t,T} = 1, \tag{10}$$

which yields $D_{t,T} = \frac{2-\theta^L}{2p\left(1-\theta^L\right)+\theta^L}$. Using equation (7) we see that the deviation payoff from all long-term financing is given by

$$\frac{\partial \Pi}{\partial \alpha} = \int_0^1 p\left[D_{0,T} - D_{t,T}(p)\right] dp \tag{11}$$

Given the face values calculated above, this integral can be calculated analytically.[2]

The deviation payoff is illustrated in Figure 1. The figure shows that whenever the project is risky $\left(\theta^L < 1\right)$ and when there is some positive cash flow to be distributed in the case of default $\left(\theta^L > 0\right)$, the deviation is profitable. This means that except in the two extreme polar cases, all long-term financing cannot be an equilibrium.

The same argument holds for any maturity structure that involves some amount of long-term financing with maturity T. In other words, for any maturity structure with $\alpha <$

[2] Substituting in the expressions for the face values, we get $\frac{\partial \Pi}{\partial \alpha} = \frac{\left(2-\theta^L\right)\theta^L\left(\theta^L-1+\tanh^{-1}\left[1-\theta^L\right]\right)}{2\left(1-\theta^L\right)^2}$.

1, there is a profitable deviation for the financial institution. While we cannot solve for face values and the deviation payoff analytically for cases when $\alpha > 0$ (the break-even conditions become a complicated system of non-linear equations), the problem can be solved numerically, and it can be verified that for all $\alpha < 1$ the deviation payoff remains positive, even after taking into account the effect of the deviation on the infra-marginal units of short-term debt, i.e. $\alpha \frac{d}{d\alpha} D_{t,T}(S_t)$, and the fact that the project will get liquidated in states when $p\theta^H + (1-p)\theta^L < \alpha D_{0,t}$. This means that the only maturity structure that can be an equilibrium is $\alpha = 1$, i.e. all creditors are receive short-term contracts and roll over at time t. There is no deviation from all short-term financing. Starting from all short-term financing, if the financial institution would switch one short-term creditor to a long-term contract, that creditor would charge a higher face value than the short-term creditors, since he would anticipate that the short-term creditors liquidate the project for bad realizations of p, leaving the short-term creditor with no payoff at all. This means that all short-term financing is the only equilibrium.

2.4 One-step deviation: A More General Statement

The result we illustrated in the simple example above holds much more generally. In fact, under the condition that the product $\left(D_{t,T}(S_t) + \alpha \frac{d}{d\alpha} D_{t,T}(S_t)\right) \int_{\bar{D}(S_t)}^{\bar{\theta}} dF(\theta|S_t)$ has no inflection points when drawn as a function of the signal S_t, we can establish the following proposition. We conjecture that this condition is always satisfied when $F(\cdot|S_t)$ satisfies the monotone likelihood ratio property.

Proposition 1 ***One-step Deviation.*** *Under a regularity condition on $F(\cdot)$, in any conjectured equilibrium maturity structure with some amount of long-term financing, $\alpha \in [0,1)$, the financial institution has an incentive to increase the amount of short-term financing by switching one additional creditor from maturity T to the shorter maturity $t < T$, since $\frac{\partial \Pi}{\partial \alpha} > 0$. As a result, the maturity structure of the financial institution shortens to all time-t financing.*

What is the reason that the financial institution is unable to sustain a maturity structure in which it enters long-term contracts with all creditors? To see this, consider what happens when the institution moves one creditor from a long-term contract to a shorter maturity while keeping the remaining long-term creditors' financing terms fixed. The difference between long-term and short-term debt is that the face value of the short-term contract reacts to the signal observed at time t. When the signal is positive, rolling over the maturing short-term debt contract at time t is cheap. When, on the other hand, the signal is negative, rolling over the maturing short-term debt at t is costly or even impossible. The reason why the deviation to short-term financing is profitable for the financial institution is that rolling over short-term financing is cheap exactly in those states in which the financial institution is likely to be the residual claimant. This means that benefits from an additional unit of short-term financing accrue disproportionately to the financial institution. On the other hand, the signal realizations for which rolling over short-term debt is costly or even impossible are the states in which the financial institution is less likely to be the residual claimant. The costs that arise from an additional unit of short-term financing are thus disproportionately borne by the existing long-term debtholders.

Naturally, the remaining long-term creditors are hurt by the financial institution's deviation. However, note that when the financial institution moves an additional creditor to a short-term contract, the remaining long-term creditors do not lose on a state by state basis. This is because depending on the signal realization at rollover they can lose or gain. When the signal is bad, short-term creditors raise their face value, which means that in the case of default at time T long-term creditors will receive less under equal priority in default. When the signal realization is good, on the other hand, long-term creditors can gain. In that case short-term creditors lower their face value, such that in case of default, long-term creditors receive a larger proportional share of the liquidation mass. The reason that long-term creditors lose on average is because short-term creditors raise their face value when default is likely, while they lower their face value only in states when default is less likely. Thus in expectation the existing long-term creditors are worse off when the financial institution moves an additional creditor to short-term contract.

This rationale is not limited just to the initial deviation from a conjectured equilibrium in which all financing is through long-term debt. Rather, by the same argument *any* maturity structure that involves some amount of long-term debt cannot be an equilibrium. As long as the short-term debtholders can exploit some remaining long-term debtholders by imposing a contractual externality on the value of long-term debt, the financial institution gains from moving an additional creditor from a long-term to a short-term debt contract.

2.5 The Maturity Rat Race

Up to now we have focused on a one-step deviation with just one rollover date t. In this section we show how in a setup with multiple rollover dates the one-step deviation illustrated above can occur repeatedly, successively unraveling the maturity structure to the very short end. We refer to this powerful successive unraveling of the maturity structure as the *maturity rat race.*

The maturity rat race is illustrated in Figure 2. Consider starting in a conjectured equilibrium in which all debt is long-term, i.e. matures at time T. By our one-step deviation principle, if everyone's debt matures at time T, the financial institution has an incentive to start shortening some creditor's maturity until everyone's maturity is only of length $T-1$. As we showed above, moving an additional creditor from a contract with maturity T to a contract with maturity $T-1$ is profitable until all creditors have debt contracts with maturity $T-1$. But now consider the same deviation again, but from a conjectured equilibrium in which everyone's maturity is $T-1$. The one-step deviation principle applies again, meaning that now there is an incentive for the financial institution to give some creditors a maturity of $T-2$. The financial institution would do this until all creditors have an initial maturity of $T-2$, after which the whole process would repeat again, in an analogous manner. This implies that in equilibrium all financing is extremely short-term—the financial institution writes debt contracts of the shortest possible maturity with all creditors and rolls them over period by period.

Proposition 2 ***The Maturity Rat Race.*** *When many rollover dates are possible, successive application of the one-step deviation principle results in a complete unraveling of the maturity structure to the minimum rollover interval.*

Intuitively, the maturity rat race is a simple extension of the one-step deviation principle stated in proposition 1. Starting from any conjectured time τ at which all creditors roll over for the first time, it is a profitable deviation for the financial institution to move a creditor to a shorter maturity contract, keeping all other creditors' financing terms fixed. While in the original one-step deviation this increases the financial institutions' expected payoff at time T, in this case the deviation increases the financial institution's expected continuation value at time τ. Save for this, the proof of the maturity rat race is very similar to the proof of the one-step deviation.

Conceptually, Proposition 2 demonstrates the power of the simple contractual externality that arises when a financial institution cannot commit to an aggregate maturity structure. Not only does it result in an shortening of the maturity structure, it results in a successive shortening to the very short end of the maturity structure. This successive unraveling maximizes rollover risk and the possibility of inefficient liquidation.

2.6 Discussion: Excessive Rollover Risk and Underinvestment

The maturity mismatch that arises in our model is inefficient. Maturity mismatch does not arise to be able to serve the investors' interim liquidity need while still being able to invest in the long-term project, as in Diamond and Dybvig (1983). Neither does maturity mismatch serve a beneficial role by disciplining bank managers, as in Calomiris and Kahn (1991) or Diamond and Rajan (2001). In fact, recall that in setting up our model we have intentionally 'switched off' these channels, since in our model, there are no liquidity shocks,

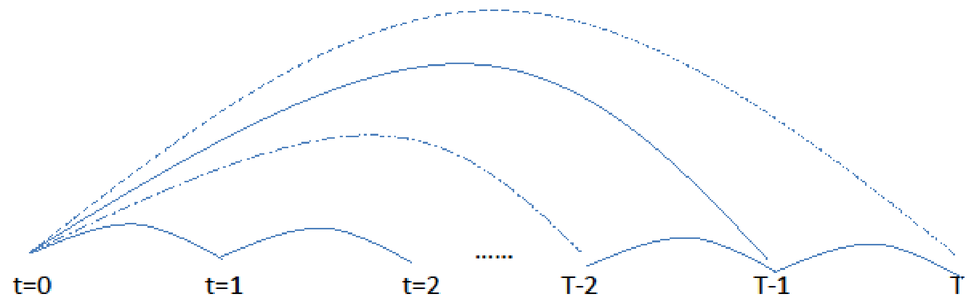

Figure 2: **Illustration of the Maturity Rat Race.** Start in a conjectured equilibrium in which all financing has maturity T (dashed line). In that case there is a profitable deviation to moving some creditors to and initial maturity of $T-1$ and then roll over from $T-1$ to T. However, once all creditors' initial maturity is $T-1$, there is an incentive to move some creditors to an initial maturity of $T-2$. The process repeats until all financing has the shortest possible maturity and is rolled over from period to period.

and no disciplining role for short-term debt. Maturity mismatch arises in our model as the result of the contractual externality between short-term and long-term debtholders described above.

Since our model is set up such that matching maturities by financing the long-term project via long-term debt is always efficient, the maturity structure that emerges in our model is excessively short-term and clearly inefficient. In particular, the excessive reliance on short-term financing leads to excessive rollover risk and underinvestment, since it can prevent some positive NPV projects from being financed. This is stated more formally in the following two corollaries. For simplicity, we state the two corollaries for the case with only one rollover date.

Corollary 3 ***Excessive rollover risk.*** *The equilibrium maturity structure ($a = 1$) exhibits excessive rollover risk when conditional on the worst interim signal the expected cash flow of the project is less than the initial investment 1, i.e.* $\int_0^{\bar{\theta}} \theta dF\left(\theta | S_t^L\right) < 1$.

Corollary 4 ***Some positive NPV projects will not get financed.*** *As a result of the maturity rat race, some positive NPV projects will not get financed. Only projects whose*

NPV exceeds $(1-\lambda)\int_{S_t^L}^{\tilde{S}_t(1)}\int_0^{\bar{\theta}}\theta dF(\theta|S_t)\,dG(S_t)$ *will be financed in equilibrium.*

Corollary 3 states that the maturity rat race leads to a positive amount of rollover risk when, conditional on the worst signal, rolling over short-term debt fails at date t. Corollary 4 states that this rollover risk in turn can make projects that have positive NPV in absence of early liquidation unprofitable. Consider a positive NPV project with expected cash flow $\int_0^{\bar{\theta}}\theta dF(\theta) > 1$.When the project is finance entirely through short-term debt, the project will be liquidated at date t for any signal realization $S_t < \tilde{S}_t(1)$, since the uncoordinated rollover decision of the short-term creditors makes continuation of the project infeasible. Given this positive probability of liquidation at time t, the pledgeable worth of the project is given by the expected cash flows minus expected liquidation costs,

$$\underbrace{\int_0^{\bar{\theta}}\theta dF(\theta)}_{\text{Expected cash flows}} - \underbrace{(1-\lambda)\int_{S_t^L}^{\tilde{S}_t(1)}\int_0^{\bar{\theta}}\theta dF(\theta|S_t)\,dG(S_t)}_{\text{Value destruction from early liquidation}}.$$

This means that as a result of the maturity rat race and the resulting rollover risk, some positive NPV projects will not be financed in equilibrium, since in order to receive financing, from creditors that anticipate equilibrium liquidation costs, the expected cash flows of the project must exceed its setup cost plus the expected liquidation costs.

Note that both Corollary 3 and 4 become more powerful if we allow for many rollover dates. This is because allowing for more rollover dates leads to a successive unraveling of the maturity structure to the minimum contract length, which increases the equilibrium amount of rollover risk and the probability of inefficient liquidation.

2.7 Discussion: Maturity Shortening Before and During the Crisis

In our model, the incentive to shorten the maturity structure depends on how much information is released at the potential interim rollover dates. This is easiest to see by considering the extreme case in which no signals are observed before maturity of the project. In that case, the ability to adjust promised face values or to pull out financing at the rollover dates has no value to creditors with shorter maturities. There is thus no incentive to shorten the maturity structure. On the other hand, the incentive to shorten the maturity structure is particularly strong during volatile times, when a lot of information may be released before investments mature. Financial crises are one important example of periods with heightened volatility fo this kind. This feature of the model can explain the additional shortening of the maturity structure that occured since the beginning of the current crisis.

To make this more explicit, we add a small refinement to the model: We assume that writing a rollover contract has a small proportional cost. This has the effect that the deviation to increase the amount of short-term financing is only profitable as long as the deviation payoff exceeds the amount of the rollover cost. This in turn depends on how much information is expected to be released by the rollover date in question. During normal times there may be point at which there is no further incentive to shorten the maturity structure. Thus, while there is a a general incentive to shorten the maturity structure to some extent, during normal times we would not necessarily expect to see complete unraveling to the very short end. However, when volatility spikes up, such that more information is expected to be released at interim dates, the incentive to shorten the maturity structure becomes stronger, leading to further shortening of the maturity structure. In the extreme case of unusually high volatility the maturity structure unravels completely to the shortest possible maturity. This means that our model is consistent both with the relatively large reliance on short-term financing that existed already during the run-up to the financial crisis and with the massive further shortening of the maturity structure that occured in the fall of 2008, as documented (in the commercial paper market) by Krishnamurthy (2009). This is summarized in Corollary 5.

Corollary 5 ***The Rat Race is more powerful during volatile times, such as financial crises.*** *The incentive to shorten the maturity structure is stronger, the more information is released at potential interim rollover dates before the investment matures.*

2.8 The Effect of Seniority Restrictions and Covenants

There are a number of interesting avenues to extend the model, including the effect of covenants, seniority and collateralization. In particular, all three of these may counteract the maturity rat race. For example, seniority of the long-term debtholders would diminish the short-term debtholders' ability to exploit long-term debtholders by raising their face value in response to negative information that arrives at rollover dates. This is because if default occurs at time T and long-term debtholders are senior (in contrast to our equal priority assumption), short-term debtholders will not receive a larger share of the liquidation mass. However, seniority itself will not necessarily eliminate the incentive to shorten the maturity structure. Rather than increasing their face value at date t, in the presence of seniority the short-term creditors may decide to pull out their financing in response to negative news at t. This again would impose a negative externality on existing long-term creditors (as early liquidation is inefficient). In other words, even if long-term creditors have *de jure* seniority, they will still not always have *de facto* seniority if short-term creditors can pull out their funding early. Covenants may also restrict the ability of short-term creditors to impose externalities on long-term creditors. Consider for example a covenant that restricts raising the face value of short-term debt above a certain threshold. Again, covenants on the face value of short-term debt may not always prevent the financial institution's incentive to increase short-term financing at the expense of long-term creditors, unless there are also restrictions on short-term creditors withdrawing their funding at the interim dates t. But if both types of restrictions are in place, short-term debt essentially becomes long-term debt.

Finally, even in cases where seniority restrictions or covenants can restore an equilibrium in which all financing is long-term, a second, inefficient equilibrium in which all financing is short-term may still continue to exist. The reason is that, given that all other lenders are only providing short-term financing, it is not individually rational for the financial institution to move an individual creditor to long-term financing. This is because to induce an individual creditor to move from a short-term to a long-term contract when everybody else is extending only short-term credit, the financial institution has to promise a high interest rate in order to compensate the long-term creditor for the risk that the remaining short-term creditors may pull out their funding at time t, leaving the long-term creditor stranded. At that interest rate, however, the financial institution is better off under all short-term financing. Through this mechanism, a financial institution with dispersed creditors can get caught in a short-term financing trap, and only a coordinated, simultaneous move by a critical mass of creditors would allow the financial institution to regain access to long-term credit markets.

3 Conclusion

We provide a model of equilibrium maturity structure for financial institutions that deal with multiple creditors. Our analysis shows that a contractual externality between long-term and short-term debtholders can lead to an inefficient shortening of the maturity structure when the financial institution deals with creditors on a bilateral basis and cannot commit to an aggregate maturity structure. The incentive to shorten the maturity structure is particularly strong during periods of high volatility, such as financial crises. In our model the resulting maturity mismatch is inefficient, which stands in contrast to a number of other existing theories of maturity mismatch. To the extent that maturity mismatch is driven by the forces outlined in this paper, it may be desirable to include limits on maturity mismatch in future financial regulation.

4 Appendix

[Appendix is still incomplete.]

Proof of Proposition 1: In order to prove that there is a profitable deviation from any conjectured equilibrium in which a fraction $1-\alpha$ of the project is financed through long-term debt maturing at time T, we need to show that

$$\int_{\tilde{S}_t(\alpha)}^{S_t^H} \left[\int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t) - \int_{\bar{D}_T(S_t)}^{\bar{\theta}} \frac{D_{t,T}(S_t) + \alpha \frac{d}{d\alpha} D_{t,T}(S_t)}{D_{0,T}} dF(\theta|S_t) \right] dG(S_t) > 0, \quad (12)$$

or

$$\int_{\tilde{S}_t(\alpha)}^{S_t^H} \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t)\, dG(S_t) - \int_{\tilde{S}_t(\alpha)}^{S_t^H} w(S_t) \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t)\, dG(S_t) > 0, \quad (13)$$

where

$$w(S_t) = \frac{D_{t,T}(S_t) + \alpha \frac{d}{d\alpha} D_{t,T}(S_t)}{D_{0,T}}. \quad (14)$$

Clearly, without the weighting term $w(S_t)$, (13) is equal to zero. We thus have to prove that once we add the weighting term, the expression is positive.

Let us first illustrate the strategy of the proof by returning to our simple example from section 2.3. Consider starting from a conjectured equilibrium in which all financing is long-term debt, i.e. $\alpha = 0$. In that case, the first term in (13) is given by $\int_{\tilde{S}_t(\alpha)}^{H} \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t) = \frac{1}{2}$, the unconditional probability that the high cash flow realizes. This is depicted by the horizontal line in Figure 3. The second term in (13) becomes $\int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t) = p$, which is the upward sloping line in Figure 3. Since $E[p] = \frac{1}{2}$, we know that without the weighting term $w(S_t)$, area A must have the same size as area B, since the expectation of the two terms must be equal.

In order to show that the inequality in (13) holds, we must now consider what the weighting term $w(S_t)$ does to the upward sloping line. This is shown in Figure 4. The weighting term results in a concave transformation of the original upward sloping line. Moreover, by substituting into the rollover condition for the short-term creditor, we can show that when $p = \frac{1}{2}$ (a neutral signal realizes, in the sense that the conditional probability at time t is equal to the unconditional probability at time 0) $D_{t,T} = D_{0,T}$, which means that for $p = \frac{1}{2}$ the weighting term $w(S_t) = 1$. Together this implies after applying the weighting term, area

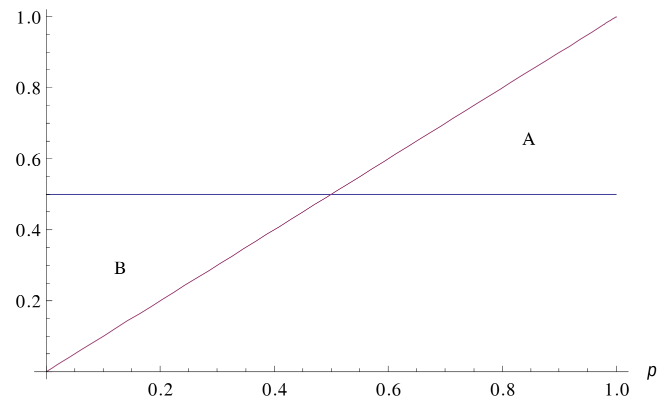

Figure 3: Before the weighting term $w(s)$ is applied, we know that area A has the same size as area B.

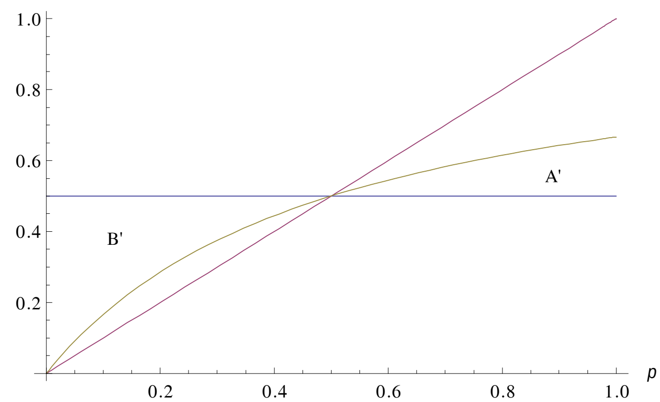

Figure 4: After applying the weighting term $w(s)$ area A' is smaller than area B', which establishes the inequality.

A' is smaller than area B', which gives the desired inequality.

We now want to make the same argument more generally. Throughout the proof we will use the following additional notation. We define as the 'distribution neutral signal', the signal for which $\int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t^*) = \int_{\tilde{S}_t(\alpha)}^{S_t^H} \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t)$. In words, S_t^* is the signal for which the conditional probability of repayment at time T seen from time t is equal to the unconditional probability of repayment as seen from time 0. Second, define as the 'equal face value signal' the signal S_t' for which $D_{t,T}(S_t') = D_{0,T}$, i.e. the signal for which the face value of long-term debt is equal to the face value of the maturing short-term debt that has

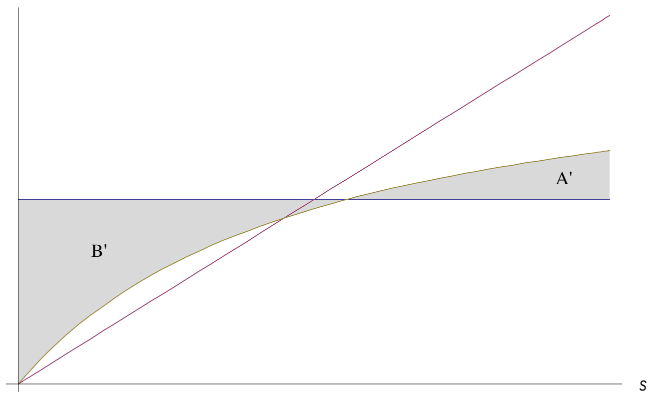

Figure 5: Illustration of proof that $A' < B'$ in the general case.

been rolled over at time t.

We now establish two properties about the weighting function $w(S_t)$. First, we can show that the weighting function is decreasing and equal to one at the equal face value signal, which is weakly smaller than the distribution neutral signal, i.e. $S_t' \leq S_t^*$. For a proof of this property see Lemma 1 below. This also implies that for the most positive signal, $w(S_t)\int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t) < \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t)$. Second, we can show that

$$\lim_{S_t \to \tilde{S}_t(\alpha)} w(S_t) \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t) = \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t) = 0. \tag{15}$$

This is shown in Lemma 2 below.

Together, these properties imply that, following a similar argument to the illustrative example above, as long as the product of the weighting function $w(S_t)$ and $\int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t)$ has no inflection points, the resulting area A' must be smaller than B'. This is shown in figure 5. On the far left, the plot starts at $\tilde{S}_t(\alpha)$. On the far right we have the highest signal S_t^H. The properties established above pin down three points on this plot. $w(S_t)\int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t)$ starts at 0, intersects $\int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t)$ when $w(S_t) = 1$ at S_t', and ends below $\int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF\left(\theta|S_t^H\right)$. If $w(S_t)\int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t)$ has no inflection points, it cannot be that $A' > B'$. This completes the proof.

Lemma 1: There is a signal realization $S_t' \leq S_t^*$ for which $w(S_t') = 1$. When $S_t > S_t'$ then $w(S_t) < 1$, and when $S_t < S_t'$ then $w(S_t) > 1$.

Proof: Because we consider a continuous signal space, $D_{t,T}(S_t)$ is continuous in S_t. From the break-even conditions and continuity there must be a value S_t' for which $D_{t,T}(S_t', \alpha) = D_{0,T}$. Moreover, when $D_{t,T}(S_t', \alpha) = D_{0,T}$, (22) implies that evaluated at $S_t = S_t'$, $\frac{d}{d\alpha} D_{t,T}(S_t) = 0$.

This implies that $w(S_t') = 1$. When $S_t > S_t'$, the break-even conditions (5) and (1) imply that $D_{t,T}(S_t) < D_{0,T}$. Moreover, equation (22) implies that $\frac{d}{d\alpha} D_{t,T}(S_t) < 0$. (for calculation see below). Together, this implies that $w(S_t) < 1$. Finally, when $S_t < S_t'$, $D_{t,T}(S_t) > D_{0,T}$ and $\frac{d}{d\alpha} D_{t,T}(S_t) > 0$, which implies that $w(S_t) > 1$.

Also note that the signal S_t' has to lie weakly below the neutral signal S_t^*. We will give an intuitive proof for this property. Start at $\alpha = 0$, which is depicted in the left panel of Figure 6. $D_{0,t}(0)$ is a constant, whereas $D_{t,T}(S_t, 0)$ is decreasing in the signal realization. From above we know that when $\alpha = 0$, then $S_t' = S_t^*$, i.e. the signal for which $w(S_t) = 1$ is the neutral signal S_t^*. Now consider what happens when α is increased. First, the curve for $D_{t,T}(S_t)$ pivots. This is because $\frac{d}{d\alpha} D_{t,T}(S_t) > 0$ when $D_{t,T}(S_t) > D_{0,T}$ and $\frac{d}{d\alpha} D_{t,T}(S_t) < 0$ when $D_{t,T}(S_t) < D_{0,T}$. Second, since long-term creditors would not be able to break even anymore after α has been increased, $D_{0,T}$ shifts upward. This is illustrated in the right panel of Figure 6. Clearly, the intersection of the two dashed lines must be to the left of S_t^*, which implies that $S_t' < S_t^*$ Now the same procedure can be repeated. Starting from the two dashed lines, a further increase in α would tilt the dashed curve around its intersection with the dashed line; and the dashed line would move upwards. As before, the point of intersection, S_t', would move to the left.

Lemma 2: $\lim_{S_t \to \tilde{S}_t(\alpha)} w(S_t) \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta | S_t) = 0$.

Proof: In order to prove this relation, we show separately that

$$\lim_{S_t \to \tilde{S}_t(\alpha)} D_{t,T}(S_t) \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta | S_t) = 0$$

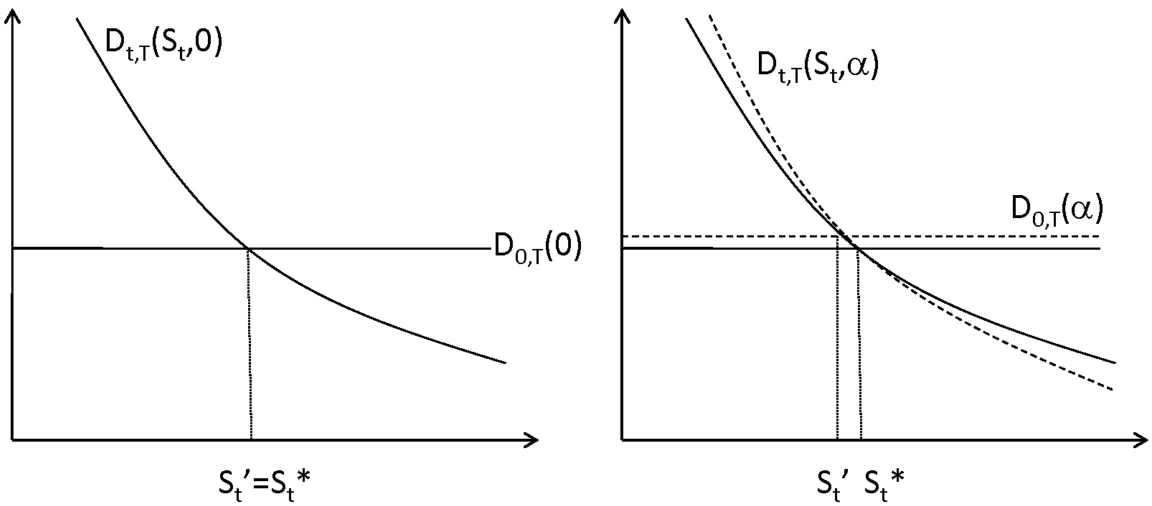

Figure 6: Illustration of the proof that $S_t' \leq S_t^*$.

and

$$\lim_{S_t \to \tilde{S}_t(\alpha)} \alpha \frac{d}{d\alpha} D_{t,T}(S_t) \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t) = 0.$$

For both parts, we use the fact that $\int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF(\theta|S_t) = 0$ is equal to zero when $\bar{D}_T(S_t) = \bar{\theta}$. Since for $\bar{D}_T(S_t) = \bar{\theta}$ both $D_{t,T}(S_t)$ and $\alpha \frac{d}{d\alpha} D_{t,T}(S_t)$ are finite, the expressions must be zero at $\tilde{S}_t(\alpha)$ (which is the signal realization for which the short-term face value tends to infinity).

Calculation of $\frac{d}{d\alpha} D_{t,T}(S_t)$: From the break-even condition for the short-term creditors (1) we know that

$$\int_0^{\bar{D}_T(S_t)} \frac{D_{t,T}(S_t)}{\bar{D}_T(S_t)} \theta dF(\theta|S_t) + \int_{\bar{D}_T(S_t)}^{\bar{\theta}} D_{t,T}(S_t) dF(\theta|S_t) = 1. \tag{16}$$

We can use this relation to find $\frac{d}{d\alpha} D_{t,T}(S_t)$using the implicit function theorem. Denote the LHS of (16) by $H(D_{t,T}(S_t), \alpha)$,

$$H(D_{t,T}(S_t), \alpha) = \int_0^{\bar{D}_T(S_t)} \frac{D_{t,T}(S_t)}{\bar{D}_T(S_t)} \theta dF(\theta|S_t) + \int_{\bar{D}_T(S_t)}^{\bar{\theta}} D_{t,T}(S_t) dF(\theta|S_t). \tag{17}$$

Then by the implicit function theorem

$$\frac{d}{d\alpha}D_{t,T}\left(S_t\right) = -\frac{\frac{\partial H}{\partial \alpha}}{\frac{\partial H}{\partial D_{t,T}}}, \tag{18}$$

where

$$\frac{\partial H}{\partial \alpha} = \int_0^{\bar{D}_T(S_t)} \frac{-D_{t,T}\left(S_t\right)\left[D_{t,T}\left(S_t\right) - D_{0,T}\right]}{\left[\bar{D}_T\left(S_t\right)\right]^2}\theta dF\left(\theta|S_t\right) \tag{19}$$

$$\frac{\partial H}{\partial D_{t,T}} = \int_0^{\bar{D}_T(S_t)} \frac{\bar{D}_T\left(S_t\right) - \alpha D_{t,T}\left(S_t\right)}{\left[\bar{D}_T\left(S_t\right)\right]^2}\theta dF\left(\theta|S_t\right) + \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF\left(\theta|S_t\right). \tag{20}$$

Substituting this into (18) yields

$$\frac{d}{d\alpha}D_{t,T}\left(S_t\right) = -\frac{\int_0^{\bar{D}_T(S_t)} \frac{-D_{t,T}(S_t)\left[D_{t,T}(S_t)-D_{0,T}\right]}{\left[\bar{D}_T(S_t)\right]^2}\theta dF\left(\theta|S_t\right)}{\int_0^{\bar{D}_T(S_t)} \frac{\bar{D}_T(S_t)-\alpha D_{t,T}(S_t)}{\left[\bar{D}_T(S_t)\right]^2}\theta dF\left(\theta|S_t\right) + \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF\left(\theta|S_t\right)}. \tag{21}$$

We can simplify this expression by taking constants outside the integral and by multiplying numerator and denominator by $\left[\bar{D}_T\left(S_t\right)\right]^2$. This yields

$$\frac{d}{d\alpha}D_{t,T}\left(S_t\right) = \frac{D_{t,T}\left(S_t\right)\left[D_{t,T}\left(S_t\right) - D_{0,T}\right]\int_0^{\bar{D}_T(S_t)} \theta dF\left(\theta|S_t\right)}{\left(1-\alpha\right)D_{0,T}\int_0^{\bar{D}_T(S_t)} \theta dF\left(\theta|S_t\right) + \left[\bar{D}_T\left(S_t\right)\right]^2 \int_{\bar{D}_T(S_t)}^{\bar{\theta}} dF\left(\theta|S_t\right)}. \tag{22}$$

Proof of Proposition 2: Assume that the first date at which all creditors roll over is date $t \leq T$. We want to consider a deviation from a conjectured equilibrium in which all creditors first roll over at time t, and then roll over every period after that until T. Of course, when $t = T$, the project is financed entirely through long-term debt and the proof of Proposition 1 implies that there is an incentive to shorten the maturity structure to $T - 1$. When $t < T$, on the other hand, we need to extend the proof of Proposition 1. Intuitively, rather than showing that the deviation raises the expected time T payoff of the financial institution, we now show that it raises the expected time t continuation value.

Let V_t be the time-t continuation value for the financial institution. This continuation value is a function of three state variables. The first is the face value of debt that has to be rolled over at time t. Consistent with our earlier notation, we denote the aggregate face value maturing at time t by $\bar{D}_t$. The aggregate face value that needs to be rolled over at time t is the sum of the face value issued at time 0 and at the potential earlier rollover date $t-1$, i.e. $\bar{D}_t = \alpha D_{t-1,t}\left(S_{t-1}\right) + (1-\alpha)D_{0,t}$. The second state variable is the time-t distribution of the final cash flow. A sufficient statistic for this distribution is the time t signal S_t. The third state variable is the remaining time to maturity, $T-t$ (which is also equal to the number of the remaining rollover dates). Together this implies that, conditional on all the information released up to time t, we can write the time t continuation value for the financial institution as

$$V_t\left(\bar{D}_t, S_t, T-t\right). \tag{23}$$

Seen from $t=0$, the expected continuation value for the entrepreneur at time t is then given by

$$\int_{\tilde{S}_{t-1}}^{\infty}\int_{\tilde{S}_t}^{\infty} V_t\left(\bar{D}_t, S_t, T-t\right) dG\left(S_t|S_{t-1}\right) dG\left(S_{t-1}\right), \tag{24}$$

where $\tilde{S}_{t-1}$and $\tilde{S}_t$ are the signals below which the project is liquidated at times t and $t-1$, respectively, because rollover fails. Note that because the face value of the debt that is rolled over at $t-1$ depends on the signal at $t-1$, we have to take an expectation over the S_{t-1} when calculating the expected continuation value at time t.

Now take the derivative of (24) with respect to α. This yields

$$\int_{\tilde{S}_{t-1}}^{\infty}\int_{\tilde{S}_t}^{\infty} \frac{\partial V_t}{\partial \bar{D}_t}\frac{d\bar{D}_t}{d\alpha} dG\left(S_t|S_{t-1}\right) dG\left(S_{t-1}\right). \tag{25}$$

To prove that there is a profitable deviation from a conjectured equilibrium in which all creditors roll over for the first time at time t, we need to show that this expression is positive. From the definition $\bar{D}_t = \alpha D_{0,t-1}\left(S_{t-1}\right) + (1-\alpha)D_{0,t}$ we know that $\frac{d\bar{D}_t}{d\alpha} =$

$D_{0,t-1}(S_{t-1}) + \frac{dD_{0,t-1}(S_{t-1})}{d\alpha} - D_{0,t}$. This means that we need to show that

$$\int_{\tilde{S}_{t-1}}^{\infty} \int_{\tilde{S}_t}^{\infty} \frac{\partial V_t}{\partial \bar{D}_t} \left[D_{0,t-1}(S_{t-1}) + \frac{dD_{0,t-1}(S_{t-1})}{d\alpha} - D_{0,t} \right] dG(S_t|S_{t-1})\, dG(S_{t-1}) > 0. \tag{26}$$

Since $D_{0,t}$ is constant, we can rewrite (26) as

$$\int_{\tilde{S}_{t-1}}^{\infty} \int_{\tilde{S}_t}^{\infty} \frac{\partial V_t}{\partial \bar{D}_t} \left[\frac{D_{0,t-1}(S_{t-1}) + \frac{dD_{0,t-1}(S_{t-1})}{d\alpha}}{D_{0,t}} - 1 \right] dG(S_t|S_{t-1})\, dG(S_{t-1}) > 0 \tag{27}$$

or alternatively

$$\int_{\tilde{S}_{t-1}}^{\infty} w(S_{t-1}) \int_{\tilde{S}_t}^{\infty} \frac{\partial V}{\partial \bar{D}_t} dG({}_t|S_{t-1})\, dG(S_{t-1}) - \int_{\tilde{S}_{t-1}}^{\infty} \int_{\tilde{S}_t}^{\infty} \frac{\partial V}{\partial \bar{D}_t} dG(S_t|S_{t-1})\, dG(S_{t-1}) > 0 \tag{28}$$

where we defined

$$w(S_{t-1}) = \frac{D_{0,t-1}(S_{t-1}) + \frac{dD_{0,t-1}(S_{t-1})}{d\alpha}}{D_{0,t}}. \tag{29}$$

Note that without the weighting term $w(S_{t-1})$ expression (28) is equal to zero. To sign the expression, we will now establish a number of properties. Whith these properties, the remainder of the proof goes through using the same steps as the proof of Proposition 1. First, $\frac{\partial V}{\partial \bar{D}_t}$ is negative for any signal that leads to continuation at time t, i.e. $S_t > \tilde{S}_t$. Second, $\frac{\partial V}{\partial \bar{D}_t}$ is more negative for higher S_t, i.e. $\frac{\partial^2 V}{\partial \bar{D}_t \partial S_t} < 0$. This is the case since the continuation value is essentially an option. The further the option is in the money, the more likely the entrepreneur will be the residual claimant. This means that the continuation value is more sensitive to changes in $\bar{D}_t$, the further the option is in the money, i.e. for higher S_t. Third, as before from the breakeven conditions for the $t-1$ creditors and the t-creditors we also know that $w(S_{t-1}) < 1$ for positive $t-1$ signal realizations $S_{t-1} > S^*_{t-1}$ and $w(S_{t-1}) > 1$ for negative $t-1$ signal realizations $S_{t-1} < S^*_{t-1}$. Finally, $w(S_{t-1}) = 1$, when $S_{t-1} = S'_{t-1}$, the 'equal face value' signal realization at time $t-1$. As before, when $\alpha = 0$, the distribution neutral signal is the same as the equal face value signal, i.e. $S^*_{t-1} = S'_{t-1}$. With these properties in hand, the proof works analogously to the proof of Proposition 1, with the exception that the signs in (28) are reversed compared to (13), since $\frac{\partial V}{\partial \bar{D}_t}$ is negative.

Proof of Corollary 3: Since early liquidation is always inefficient in this model, the socially optimal level of rollover risk is zero. Any positive probability of liquidation means that there is excessive rollover risk. The unraveling of the maturity structure to all short-term financing leads to positive rollover risk when conditional on the worst interim signal the expected cash flow is less than 1, i.e.

$$\int_0^{\bar{\theta}} \theta dF\left(\theta | S_t^L\right) < 1.$$

Proof of Corollary 4: Proof follows directly from the discussion in the main text.

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