

Optimal Interventions in Markets with Adverse Selection*

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Abstract

We study interventions to restore efficient lending and investment when financial markets fail because of adverse selection. We solve a design problem where the decision to participate in a program offered by the government can be a signal for private information. We characterize optimal mechanisms and analyze specific programs often used during banking crises. We show that programs attracting all banks dominate those attracting only troubled banks, and that simple guarantees for new debt issuances implement the optimal mechanism, while equity injections and asset buyback do not. We also discuss the consequences of moral hazard.

JEL: D02, D62, D82, D86, E44, E58, G01, G2

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An important insight of economic theory from the last forty years is that asymmetric information can lead to market collapse. George Akerlof demonstrated this phenomenon in a classic paper (Akerlof 1970). In many cases, economic and legal institutions arise endogenously to alleviate adverse selection (used car dealers provide expertise and guarantees, we have accounting standards and auditors, etc.) Although costly, these institutions allow the markets to function and often make government interventions unnecessary. If a collapse does occur, however – presumably because of a failure of the institutions designed to prevent it in the first place – a government might be tempted to intervene. This paper asks what form these interventions should take if the goal of policy is to improve the efficiency of resource allocation.

We characterize cost-minimizing interventions in an economy with endogenous borrowing and investment under asymmetric information. We derive the optimal mechanism and show that it can be implemented using standard financial contracts. In doing so we also make a methodological contribution by solving a class of mechanism design problems where the planner must deal with the presence of a competitive fringe and where the strategic decision to participate in a government’s program reveals information about private types.

In our benchmark model, firms receive profitable investment opportunities but they have private information about the value of their assets, as in the classic model of Myers and Majluf (1984). Because an important application of our work concerns the bailout of financial firms, we refer to our firms as banks and we interpret the investments as new loans. Inefficiencies occur when banks with good assets – who expect to repay more often than banks with poor assets – are unwilling to borrow at the prevailing market rate. Good banks then drop out of the market and lenders rationally demand a high interest rate. We first highlight the key ingredients for adverse selection to occur in equilibrium: risky investment opportunities and asymmetric information about the downside risk of legacy assets. Inefficiencies arise only when both are present.¹ We then derive the government intervention that restores efficient financing at the minimum expense of taxpayers’ money.

Our setup differs from the usual mechanism design framework because we allow our banks to borrow in the competitive private market where investors learn about the banks’ private information by observing their participation decisions. This implies that we have to solve a mechanism design problem with an endogenous competitive fringe. An appealing feature of our analysis is that we obtain a simple implementation of the optimal program using loan guarantees despite the complexity of the problem.

¹In normal times the quality of legacy assets is good enough to prevent the market from breaking down. In the Fall of 2008, however, it appeared that some large financial institutions might have worthless legacy assets.

The information structure at the time when firms decide whether to participate in a government program plays a crucial role in the analysis. We consider two cases: symmetric information at the participation stage (when firms must opt in or out of the government program before they learn the value of their legacy assets), and asymmetric information at the participation stage.² In both cases we characterize the optimal interventions and we also compare three specific programs: equity injection, asset-buyback and debt guarantee.

With symmetric information at the participation stage, the optimal program is actually profitable for the government because banks are willing to pay the expected net efficiency gain created by the program. Moreover, we show that equity injections, asset buybacks and debt guarantees can all be designed to yield the optimal outcome. In that sense, the nature of the intervention is irrelevant under symmetric information.

With asymmetric information at the participation stage, the government cares about which banks it attracts, and what impact this has on the private interest rates. For instance, in an equilibrium where the decision to opt in or out of the program reveals the type of the bank, the government provides a signaling device and all banks end up facing fair interest rates. In other equilibria, there is pooling in participation, the types are not revealed, and the private interest rates reflect the lack of information.³ The difficulty is that these interest rates determine the outside option of the participating banks, and therefore the ultimate cost of the program. Participation decisions affect outside options through signalling, and outside options affect the cost of the program through the participation constraints. Finally, the government must also design the financial contracts it offers to participating banks. This security design problem sits on top of the design of participation.

Our key insight for analyzing this complex problem is to study a particular set of relaxed optimization problems where only the participation decisions and their signaling properties are taken into account. We first show that there cannot exist a program that attracts only good banks, because any program that is attractive for good banks is also attractive for bad banks. Consequently, the government can only choose between programs that attract all banks and programs that attract only bad banks. Both of these programs are costly but we show that the participation cost is higher for programs that attract only bad banks. The intuition is that, in such programs, opting out is tempting because it signals a good type. This, in turn, implies that the program has to be more attractive and therefore more expensive.

²Mitchell (2001) reviews the evidence from past financial crises and explains why both cases are relevant in practice.

³The paper of Cramton and Palfrey (1995) formulates a refinement to impose restrictions on out-of-equilibrium beliefs. We choose not to select one particular refinement and provide a characterization valid for a range of out-of-equilibrium reactions.

Having derived a lower bound from the relaxed problem, we show that this minimum cost is achieved by a simple program that provides government guarantees on new debt issuances. We are thus able to show that debt guarantees attracting all banks are optimal among all conceivable mechanisms. Equity injections and asset buybacks on the other hand are not optimal. Asymmetric information therefore breaks the formal equivalence of the various interventions.

Finally, we provide three extensions to our benchmark model. We first show that our results still obtain when there is asymmetric information about new opportunities and not just about assets in place. We then consider the design of menus and the consequences of moral hazard. We show that menus of asset-buybacks and equity injections can also reach the lower bound on costs. Of course, they are significantly more complicated to set up and administer than the simple debt guarantee program. In our third extension we allow banks to choose the riskiness of their investments. This introduces moral hazard with respect to debt guarantees provided by the government. Interestingly, however, we find that this problem is mitigated by the endogenous response of private interest rates.

Several features of the financial market collapse in the Fall of 2008 suggest a role for asymmetric information. Not only did spreads widen (as they would in any case given the increase in counterparty risk), but transactions stopped in many markets. In the interbank market, only overnight loans remained. Banks refrained from lending to each other in part because they were afraid of not being repaid, as the assets that the borrowing bank would put as collateral could be in fact worth nothing (toxic). In the OTC market, the range of acceptable forms of collateral was dramatically reduced “leaving over 80% of collateral in the form of cash during 2008”, while the “repo financing of many forms of collateralized debt obligations and speculative-rate bonds became essentially impossible.” (Duffie 2009). Investors and banks were unable to agree on the price for legacy assets or for bank equity.

Governments stepped-in to try to alleviate the problem. In the US, the initial TARP program called for 700 billion to purchase illiquid assets from the banks. Subsequently other proposals were introduced and implemented with varying degrees of success. The main others were equity injection and debt guarantees. As of August 2009, there was 307 billion of outstanding debt issued by financial companies and guaranteed by the FDIC.⁴ The original TARP called for 700 billion to purchase illiquid assets from the banks. It was transformed into a Capital Purchase Program (CPP) to invest \$250 billion in U.S. banks. Treasury also insured 306 billions of Citibank’s assets, and 118 billion of Bank of America’s.⁵

⁴<http://www.fdic.gov/regulations/resources/tlgp/index.html>. Citigroup sold another 5 billion of guaranteed debt in September 2009. The program was set to expire at the end of October 2009.

⁵There is no consensus about which program is better. For instance, Soros (2009) and Stiglitz (2008) argue for equity injections, Bernanke (2009) is in favor of assets buybacks and debt guarantee, Diamond, Kaplan, Kashyap,

Our benchmark model is closely related to Heider, Hoerova, and Holthausen (2008) who analyze asymmetric information in the banking model of Diamond and Dybvig (1983) and Bhattacharya and Gale (1987). Heider, Hoerova, and Holthausen (2008) focus on the behavior of the interbank lending market while we focus on the design of government interventions. They also provide evidence consistent with adverse selection in the interbank market. Evidence about the presence of asymmetric information is also present in Gorton (2009) who explains how the complexity of securitized assets created asymmetric information about the size and the location of risk. Earlier banking crisis are analyzed by Corbett and Mitchell (2000) and Mitchell (2001). The recent market-breakdown is analyzed by Duffie (2009) with a particular focus on OTC and repo markets.

More generally, this paper is related to the works that study how government interventions can improve market outcomes in various contexts: In the context of borrower-lender relationships Bond and Krishnamurthy (2004) study optimal enforcement in credit markets in which the only threat facing a defaulting borrower is restricted access to financial market. They solve for the optimal level of exclusion, and link it to observed institutional arrangements. Golosov and Tsyvinski (2007) study the crowding out effect of government interventions in private insurance markets. There is also an extensive literature on how government interventions can improve risk sharing see, for example, Acemoglu, Golosov, and Tsyvinski (2008). For excellent surveys see Kocherlakota (2006), Kocherlakota (2009) and Golosov, Tsyvinski, and Werning (2006). Efficient bailouts are studied by Philippon and Schnabl (2009) in the context of debt overhang, and by Farhi and Tirole (2009) in the context of collective moral hazard.

We present our model in Section 1. We start by deriving some simple necessary conditions for the appearance of inefficiencies due to asymmetric information: investment opportunities must be risky, even conditional on private types, and there must be asymmetric information regarding the downside risk of legacy assets. Based on these simple initial results, we introduce our benchmark at the end of Section 1, and we characterize its decentralized equilibria in Section 2. We formally describe the mechanism design problem in Section 3. In Section 4 we characterize lower bounds on the costs of government interventions. Those bounds can actually be achieved by simple common interventions. This is shown in Section 5. Extensions and robustness of our findings are discussed in Section 6. We close the paper with some final remarks in Section 7.

Rajan, and Thaler (2008) view assets buybacks and equity injection as best alternatives, whereas, Ausubel and Cramton (2009) argue for a careful way to ‘price the assets, either implicitly or explicitly.’

1 The Model

1.1 Timing and technology

The model has a continuum of financial institutions indexed from 0 to 1. Financial institutions are financial companies such as commercial banks, investment banks, insurance companies, or finance companies. For simplicity, we refer to all of them as banks.

There are three dates $t = 0, 1, 2$. Banks start at time 0 with some exogenously given assets, which we refer to as legacy assets. At time 1 banks learn the value of the legacy assets on their balance sheets, and they receive the opportunity to make new loans. In order to exploit these opportunities they may need borrow from each other and from outside investors. To avoid confusion, we use the word “investments” to refer to the new loans that banks make at time 1, and we use “borrowing and lending” to refer to banks borrowing from each other and from outside investors. All returns are realized at time 2, and profits are paid out to investors. We assume that investors are risk-neutral and we normalize the risk-free rate to 0.

Initial assets and cash balance

Banks own two types of assets: cash and legacy assets. Cash is liquid and can be used for investments or for lending at date 1. Let c_t be cash holdings at the beginning of time t . All banks start at time 0 with c_0 in cash. Cash holdings cannot be negative: $c_t \geq 0$ for all t .

Long-term legacy assets deliver a random payoff $a \in [A_{\min}, A]$ at time 2, where $A_{\min} \geq 0$. The upper bound A represents the book value of the assets, but some of these assets may be impaired, and their true value can be less than A . We do not address the issue of outstanding long term debt and we refer the reader to Philippon and Schnabl (2009) for a model where debt overhang is the main friction.⁶

Information and new investments at time 1

At time 1 banks learn their type $\theta \in \Theta$ and they receive investment opportunities. Investments cost the fixed amount x at time 1 and deliver a random payoff $v \in [0, \infty]$ at time 2. At time 2 total bank income y depends on the realization of the two random variables, a and v :

$$y(i) = a + c_2(i) + v \cdot i, \tag{1}$$

where $i \in \{0, 1\}$ is a dummy for the decision to invest at time 1. The conditional distribution of (a, v) depends on the type θ and is denoted by $F(a, v|\theta)$. The type θ is privately revealed to the bank at time 1.

⁶This is without loss of generality if efficient renegotiation among creditors is possible. Impediments to renegotiation can create debt overhang. This issue is analyzed in Philippon and Schnabl (2009).

Banks can borrow at time 1 in a perfectly competitive market.⁷ After learning its type θ , a bank offers a contract (l, y^l) to the competitive investors, where l is the amount raised from investors at time 1, and y^l is the schedule of repayments to investors at time 2. The schedule y^l can be contingent on the income y realized at time 2. Formally, our model involves contracts designed by informed parties and offered to competitive investors.⁸ We specify later the exact nature of the optimal contracts offered. The bank's cash at time 2 as a function of its investment decision at time 1 is

$$c_2(i) = c_1 + l - x \cdot i. \quad (2)$$

Because the credit market is competitive and investors are risk neutral, in any candidate equilibrium, the participation constraint of investors implies:

$$E[y^l | i = 1] \geq l. \quad (3)$$

1.2 Symmetric information

We first consider the case where banks' types are observed by all market participants. Banks raise money at time 1 to finance their investments. Their goal is to maximize total value as of time 1: $E[a|\theta] + c_2(i) + E[v|\theta] \cdot i - E[y^l|\theta] \cdot i$, subject to the constraints (2) and (3). The bank will go ahead with the investment if

$$E[a|\theta] + c_2(1) + E[v|\theta] - E[y^l|\theta] \geq E[a|\theta] + c_2(0). \quad (4)$$

Because the private credit market is perfectly competitive the zero profit condition implies that (3) binds. Therefore $E[y^l|\theta] = x - c_1$, and equation (4) simply becomes $E[v|\theta] \geq x$. As expected, the requirement is simply that the net present value be positive. Note that $E[a|\theta]$ is irrelevant. Under symmetric information, investment decisions are independent of the quality of legacy assets on the banks' balance sheet.

1.3 When does adverse selection occur?

In this section we assume that the market does not observe θ . This is a necessary condition for market failure, but not a sufficient one. We therefore need to identify the conditions under which asymmetric of information does not matter before studying the case where it does. We use these results to construct our benchmark model.

⁷This borrowing and lending could take place between banks with investment projects and banks without investment projects (interbank lending), or between banks and outside investors.

⁸See Appendixes to Section 6 in Tirole (2006).

We assume for now that banks offer debt contracts to new lenders at time 1.⁹ Let r be the interest rate on the loans. The payoffs to investors are:

$$y^l(y, rl) \equiv \min(y, rl). \quad (5)$$

We now turn to the role of asymmetric information. The following proposition presents conditions under which asymmetric information does not matter.

Proposition 1 *The symmetric information allocation is an equilibrium under asymmetric information when banks are certain about the future payoff of the new project, or when the bank can issue risk free debt.*

Proof. See Appendix. ■

This proposition shows that two conditions must be satisfied for asymmetric information to matter. First, there must be uncertainty in v conditional on θ . The intuition here is one of risk shifting. The low quality borrower is tempted to finance a risky project on favorable terms by pretending to be a safe borrower. If there is no risk in the project conditional on θ , then this temptation disappears, and asymmetric information is inconsequential.

Second, there must be asymmetric information with respect to the downside risk of legacy assets. As long as the balance sheet can be pledged to new lenders even under pessimistic expectations, new projects can always be financed at a low rate, and asymmetric information is irrelevant. We can think of the case $A_{\min} \geq x - c_0$ as corresponding to the normal state of interbank flows. The scale of the new investment is small relative to the pledgeable part of the balance sheet, and all positive new projects can easily be financed irrespective of how risky they are.

1.4 Benchmark model

We now present our benchmark model, which is a special case of the general model presented above. Proposition 1 above has established two properties that are necessary for adverse selection to occur in the credit market. First, there must be risk in the new project, even conditional on private information. Second, there must be private information with respect to the legacy assets' ability to cover losses from new investments. These two insights allow us to construct the simplest model where borrowing and lending is sensitive to information.

⁹We will show in Lemma 1 that, under standard assumptions, it is optimal for the borrower to offer a debt contract to its creditors. Here we simply take as given the nature of the contract. This is without loss of generality since we only want to characterize conditions under which asymmetric information does not matter (enlarging the contract space would only reinforce our result).

We therefore construct our benchmark model with two possible types learnt at time 1: $\Theta = \{B, G\}$. The type determines the distribution of payoffs of legacy assets at time 2: $f_a(a|\theta)$. We define the ex-ante (time 0) distribution of types by

$$\pi \equiv \Pr(\theta = G).$$

In addition, we assume that all new projects are ex-ante identical: they deliver random payoffs v distributed on $[0, \infty)$ according to the density function $f_v(v)$. Note that the payoffs of the projects are independent from the value of the legacy assets. Let \bar{v} the expected value of v . To make the problem interesting, we assume that new projects have positive NPV and that banks need to borrow in order to invest:

Assumption A1: $\bar{v} > x > c_0$.

In our benchmark model, the income of the bank at time 2 conditional on investment at time 1 is $y = a + v$. The distribution of y is denoted by f and it is the convolution $f_a \cdot f_v$ of the distributions a and v . Since f_a depends on θ , the distribution f also depends on θ . We assume that the random payoff $y = a + v$ satisfies the monotone likelihood ratio property (MLRP) with respect to the bank type θ .¹⁰

Assumption A2 (MLRP): If $\theta > \theta'$ then $f(y|\theta)/f(y|\theta')$ is increasing in y .

Let us briefly discuss the special features of our model. The main simplifying assumption is that the investment opportunity is the same for all the banks, as in the standard model of Myers and Majluf (1984). This means that banks with bad assets have potentially the same lending opportunities than banks with good legacy assets. It also means that there is no asymmetric information with respect to the new opportunities.

We make this assumption for two reasons. The first reason is that, based on our reading of the 2008-2009 crisis, as well as on various interactions with bankers and investors, it seems that there is more asymmetric information with respect to legacy assets than with respect to new investment opportunities.¹¹

The second reason is tractability. We want to keep the benchmark model as simple and as close as possible to the workhorse model of Myers and Majluf (1984). As will become clear, it is quite

¹⁰This assumption covers many interesting special cases. For instance, $\theta = a$ and v uniform.

¹¹For instance, it appears possible for banks to provide good documentation on particular new loans they could make and securitize, but the sheer size and complexity of their balance sheets, as well as the ambiguity of their off-balance sheet exposures, means that banks necessarily know more than outside investors about the value of their legacy assets and liabilities. To be clear, we do not want to argue that asymmetric information about new investment cannot happen – it obviously can – but rather that it is not critical for our analysis, and that asymmetric information with respect to existing assets is even more likely.

complicated to analyze government interventions in our economy and we use this benchmark model to establish our main results. We relax this assumption in Section 6 and we show that most of our results generalize to the case where there is also asymmetric information with respect to new investments.¹²

2 Decentralized Equilibria

We now proceed to characterize the equilibria of our benchmark model under asymmetric information and without government interventions. Before doing so, we need to identify the nature of private contracts used at time 1.

2.1 Private contracts

Following Innes (1990) and most of the financial contracting literature (Tirole 2006), we impose the standard requirement that repayments y^l be weakly increasing in y .¹³ We also impose MLRP (Assumption A2) which is also standard in this literature.

Assumption A3 (Innes, 1990): The repayment y^l is increasing in y .

Under assumptions A2 and A3, the usual results obtain that debt contracts are optimal.¹⁴

Lemma 1 *Under Assumptions A2 and A3, it is optimal for banks to offer debt contracts to investors at time 1.*

Proof. See *Technical Appendix*. ■

The intuition behind Lemma 1 is straightforward. Debt contracts dominate equity contracts for the reasons emphasized in Myers and Majluf (1984). If we allow for any contingent repayment scheme $y^l(y)$ without the monotonicity constraint, the optimal contract is a live-or-die contract: $y^l = y$ up to a threshold beyond which $y^l = 0$. The monotonicity constraint introduced by Innes (1990) irons out this discontinuity and leads to a standard debt contract (see also Section 6.6 in Tirole (2006)).

¹²The main downside of the assumption of symmetric information with respect to new investments is that efficiency can be restored if spin-offs are feasible, i.e., if the cash flows of new investment can be entirely and credibly separated from the cash flows of legacy assets. In practice, such a separation is costly because it involves setting up an entire new bank. Moreover, it is clear from Proposition 1 that in normal times, pledging the legacy assets is a way to achieve the efficient allocation.

¹³The justification is that if repayments were to decrease with y , the borrower could secretly add cash to the bank's balance sheet by borrowing from a third party, obtain the lower repayment, repay immediately the third party, and obtain strictly higher returns. See also Section 3.6 in Tirole (2006).

¹⁴The proof is standard and be found in a separate document titled *Technical Appendix for "Optimal Interventions in Markets with Adverse Selection."*

2.2 Equilibria

Let us now characterize the equilibria of the benchmark model. Given that it is optimal for banks to offer debt contracts (Lemma 1), the repayment schedule is as in equation (5) and we can define the expected repayment as

$$\rho(\theta, rl) \equiv E[y^l | \theta] = \int_0^\infty y^l(y, rl) f(y|\theta) dy.$$

The function $\rho(\theta, rl)$ is clearly increasing in rl . The Monotone Likelihood Ratio Property (A2) implies First Order Stochastic Dominance. Since the function $y^l(y, rl)$ is increasing in y , this implies that $\rho(\theta, rl)$ is increasing in θ .

The fair interest rate $r_\theta(l)$ on a loan of value l for a bank with known type $\theta \in \{B, G\}$ is implicitly given by $l = \rho(\theta, r_\theta(l)l)$. To simplify our notations we will simply write r_θ , for $\theta \in \{B, G\}$ instead of $r_\theta(l)$ with the understanding that r_θ always depends on l . It is clear that for any given l , the fair rate is decreasing in θ . This fair interest rate would prevail in the symmetric information equilibrium or at a separating equilibrium. With asymmetric information in general, however, the interest rate cannot depend explicitly on θ .

We call an equilibrium *pooling* when all banks invest and face the same interest rate. In such an equilibrium, the interest rate r_π is pinned down by the zero profit condition $E[y^l] = l$:

$$l = \pi \rho(G, r_\pi l) + (1 - \pi) \rho(B, r_\pi l). \quad (6)$$

Similarly, we call an equilibrium *separating* when the types are revealed and the interest rates adjust accordingly. We will show that there is no separating equilibrium where the good types invest. The interest rate that bad banks face in a separating equilibrium where the good banks do not invest is simply r_B . It is clear that $r_B > r_\pi > r_G$.

Given a market rate r , a type θ with cash on hand c wants to invest if and only if $E[a|\theta] + \bar{v} - \rho(\theta, rl) \geq E[a|\theta] + c$. Since the loan is $l = x - c$, the investment condition under asymmetric information becomes:

$$\bar{v} - x > \rho(\theta, r(x - c)) - (x - c). \quad (7)$$

The term $\rho(\theta, r(x - c)) - (x - c)$ measures the informational rents paid by the bank. Clearly, the rents are zero if the interest rate correctly reflects the risks of the borrower, since $\rho(\theta, r(\theta)l) = l$. The information cost is positive when $r > r(\theta)$ and negative when $r < r(\theta)$. When good banks expect to pay large informational rents, they choose not to invest. We can therefore define a minimum cash level $\bar{c}(r)$ below which good types are not willing to invest:

$$\bar{c}(r) \equiv \bar{v} - \rho(G, r(x - \bar{c}(r))).$$

When we apply this formula to the pooling and separating rates, we obtain $c_\pi \equiv \bar{c}(r_\pi)$ and $c_B \equiv \bar{c}(r_B)$. These cutoffs allow us to describe the decentralized equilibria in our model:

Proposition 2 *There is no separating equilibrium where good types invest and bad types do not. If $c_0 < c_\pi$, the unique equilibrium is a separating one where only the bad types invest. If $c_0 > c_B$, the unique equilibrium is pooling where all types invest. If $c_0 \in [c_\pi, c_B]$, there are multiple equilibria.*

Proof. The first observation is to note that there is no separating equilibrium where the good types invest alone. In such an equilibrium, the interest rate would be $r_G < r_B$, and the bad types would always want to invest because $\rho(B, r_G(x - c_0)) < x - c_0$. In a pooling equilibrium, the interest rate must be r_π . It is clearly optimal for the bad types to invest when $r = r_\pi$ since again $\rho(B, r_\pi(x - c_0)) < x - c_0$. On the other hand, the good types chose to invest if and only if $\bar{v} - x > \rho(G, r_\pi(x - c_\pi)) - (x - c_\pi)$. Therefore there exists an equilibrium where all types of banks invest if and only if $c_0 \geq c_\pi$. In a separating equilibrium where only the bad banks invest, the interest rate must be r_B . It is clearly optimal for the bad types to invest since $\bar{v} > x$. On the other hand, the good types chose not to invest if and only if $c_0 < c_B$. Hence, there exists a separating equilibrium with only bad types investing if and only if $c_0 \leq c_B$. Finally, since $r_B > r_\pi$, we have $c_B > c_\pi$. ■

The intuition for Proposition 2 is simple. A bank of high quality knows it is likely to repay its lenders, while a bank of low quality knows that it is less likely to repay its lenders. The potential for adverse selection with respect to θ exists because the investment condition is more likely to hold for lower values of θ .

Note that the pooling equilibrium is efficient, and the separating equilibrium is inefficient.¹⁵ This observation together with the characterization in Proposition 2 implies that higher cash levels improve economic efficiency. Governments might therefore seek to establish the pooling equilibrium if and when it fails to happen. In the remaining of the paper, we assume that the decentralized equilibrium is inefficient, so there is a role for the government:

Assumption A4: $c_0 < c_\pi$

We are now going to study optimal interventions.

¹⁵If the scale of investment was a choice variable, the separating equilibrium would involve good banks scaling down to signal their types. With our technological assumption, they scale down to zero. The only important point is that in both cases the separating equilibrium is inefficient.

3 Mechanism Design with a Competitive Fringe

In this section we present the objective of the government and we formally define the mechanism design problem. The government's objective is simple. Let Ψ be the expected cost of the government program. We assume that there is a deadweight loss χ per dollar spend from raising taxes. Then, the efficiency cost of an intervention is $\chi\Psi$. The cost is 0 if the government decides not to intervene. Since bad banks always invest, the only choice of the government is to do nothing or to implement the efficient outcome where all banks invest. In this case, the design problem is to attain the efficient outcome at the smallest cost $\chi\Psi$. Conditional on intervening, the program of the government is therefore simply to minimize Ψ subject to the constraints that all types of banks invest and that participation be voluntary.

While the objective is simple, the mechanism design problem is non-standard. We have to take into account not only the usual adverse selection problem, but also the fact that the decision to participate in the government program may itself signal private information. This matters because we assume that the government does not close down the private lending market. Realistic interventions aim at unfreezing private credit markets, not at replacing them. We therefore always allow our banks to borrow in the competitive market and indeed we will see in Section 5 that optimal programs only provide partial funding to the banks. This means that we have to solve a mechanism design problem with a competitive fringe.

3.1 Strategy of the government

The government makes a take-it-or-leave-it offer of a menu of programs $\mathcal{M} = \{\mathcal{P}_1, \dots, \mathcal{P}_k\}$.¹⁶ We can describe any particular program in terms of the cash m injected at time 1, and the payments y^g received by the government at time 2, so we write $\mathcal{P} = \{m, y^g\}$. We allow the payments y^g to depend on the payoffs from legacy assets a , the payoffs from the new project v , and on the repayments to lenders y^l . To be consistent with our assumption on private contracts, we restrict $y^g(a, y, rl)$ to be increasing in a and $y = a + v$.

Assumption A5: $y^g(a, y, rl)$ is increasing in a and in y .

Assumption 5 is satisfied by asset buyback programs, any program based on equity payoffs (common stock, preferred stock, warrants, etc.) as well as all types of debt guarantee programs. Note that we do not allow the government to make its payment depend explicitly on the decision to invest.

¹⁶The banks are all ex-ante identical, so the government offers the same menu to all. All our results immediately apply when there is a heterogenous population of banks. The government programs would be conditioned on observable characteristics, such as size, or leverage.

We therefore rule out directed lending or outright nationalization, where the government would essentially tell the banks when to lend and to whom. With some abuse of notation we denote by \mathcal{M} the set of possible menus. The government's strategy σ^g is then to choose an element of \mathcal{M} .

3.2 Strategy of the banks

The strategy of the banks σ^b is made of two decisions. The first is a participation decision:

$$\mathcal{I}(\Theta) \rightarrow \{\mathcal{O} \cup \mathcal{M}\}.$$

The bank can opt out of the government programs by choosing \mathcal{O} , or it can participate in one of the programs offered by the government. The information set $\mathcal{I}(\Theta)$ depends on the timing that we consider. When participation is decided at time 0, the information set is the same for all types. When participation is decided at time 1, the information set contains the type $\theta \in \Theta$. The second decision is whether to invest or not. This decision is always made at time 1, and it also depends on the participation decision:

$$i : \Theta \times \{\mathcal{O} \cup \mathcal{M}\} \rightarrow \{0, 1\}.$$

If a bank participates in a program $\mathcal{P} = \{m, y^g\}$, its cash at time 1 becomes $c_1 = c_0 + m$, and the payments to the shareholders at time 2 become $y_2 - y^l - y^g$. The value for type θ of participating in the government program is therefore:

$$V(\theta, \mathcal{P}_\theta, i) = E \left[y(i) - y^l - y^g | \theta \right], \quad (8)$$

where $i \in \{0, 1\}$ indexes a bank's decision to invest or not. If the bank does not know its type at the participation stage it simply takes expectation of equation (8).

If the bank opts-out of the government program, it has the option to borrow on the private market at an interest rate consistent with the equilibrium strategies, as explained in the next subsection. For any market rate r , the outside option of a good bank is:

$$V(G, \mathcal{O}(r)) = E[a|G] + \max \{ \bar{v} - \rho(G, rl_0), c_0 \}. \quad (9)$$

If the rate is too high, the good bank's outside option is not to invest. Bad banks always invest since the rate cannot be worse than r_B . The outside option of a bad bank is therefore

$$V(B, \mathcal{O}(r)) = E[a|B] + \bar{v} - \rho(B, rl_0). \quad (10)$$

3.3 Competitive fringe

Whether a bank opts in or out, the interest rate at which it borrows on the private market must satisfy the break-even condition of competitive lenders. Since the market learns about a bank's assets through its participation decision, there will typically be a different rate for participating banks

than for non-participating ones. In both cases, for any loan l , the expected value of repayments $E[y^l|\sigma, l]$ equals the loan:

$$E[\min(y, rl) | \sigma, l] = l.$$

This expectation depends on the strategy profile $\sigma = (\sigma^g, \sigma^b)$ since it affects which banks opt in or out. In other words, the option not to participate \mathcal{O} depends on the menu that the government offers \mathcal{M} and on the strategies of the banks. This interrelationship does not exist in standard mechanism design, where outside options are fixed and independent from the designer's and the agent's actions. It exists, though in common agency problems where the outside option of an agent depends on the contract offered by the other principal. However, here we do not have such multi-principal competition. Rather, the principal's (the government's) mechanism induces a competitive market's response. Hence, we are dealing with a situation that we can call mechanism design with a “*competitive fringe*.”

3.4 Feasible Mechanisms

Despite the unusual features of the design problem, it is straightforward to see that the revelation principle applies and it is without loss of generality to consider mechanisms \mathcal{M} that consist of two menus: one for the bad banks and one for the good banks. We can treat an equilibrium where only type θ does not participate as $\mathcal{P}_\theta = \mathcal{O}$. With this simple convention, a feasible mechanism can be defined as follows:

Definition 1 *With symmetric information at the participation stage, a mechanism \mathcal{P} is **feasible** if it satisfies voluntary participation*

$$E[V(\theta, \mathcal{P}, 1)] \geq E[V(\theta, \mathcal{O})], \tag{11}$$

and the investment constraint

$$V(\theta, \mathcal{P}, 1) \geq V(\theta, \mathcal{P}, 0) \text{ for } \theta \in \{B, G\}. \tag{12}$$

*With asymmetric information at the participation stage, a mechanism $\mathcal{M} = \{\mathcal{P}_B, \mathcal{P}_G\}$ is **feasible** if it satisfies the participation constraints*

$$V(\theta, \mathcal{P}_\theta, 1) \geq V(\theta, \mathcal{O}) \text{ for } \theta \in \{B, G\}, \tag{13}$$

the incentive constraints

$$V(\theta, \mathcal{P}_\theta, 1) \geq V(\theta, \mathcal{P}_{\theta'}, i) \text{ for } \theta, \theta' \in \{B, G\} \text{ and } i = \{0, 1\}, \tag{14}$$

and the investment constraints

$$V(\theta, \mathcal{P}_\theta, 1) \geq V(\theta, \mathcal{P}_\theta, 0) \text{ for } \theta \in \{B, G\}. \quad (15)$$

Summing up, the problem we are dealing with is complex because the government can screen through two channels: the first one is the standard one through the menu offer, and the second one is through the participation decision. Additionally, the participation decision is influenced by the non-participation payoffs that depend on the market reaction, which is, in turn, endogenous to the mechanism. In order to find the optimal mechanism the government must choose whether to make everyone participate or not, and then decide the type and the size of the program.

Despite these complications, it turns out that we can identify the optimal mechanism and propose a simple and realistic implementation. Our strategy is to cut through unnecessary complications by using a simple accounting identity involving the participation constraints. It turns out that we can learn a lot simply by looking at the participation constraints. They not only allow us to derive lower bounds on the cost of interventions, they also tell us a lot about the feasibility of various types of interventions. This is what we do in Section 4. Then, in Section 5, we show that these bounds are actually achieved by some well-designed interventions.

4 Obtaining Lower Bounds of Government Program Costs

In this section we derive lower bounds on the costs of feasible government programs. We first need to characterize the minimum cost of a feasible program before finding ways to implement it. The following proposition characterizes the cost of any intervention as a function of the inside values:

Lemma 2 *Let $W \equiv E[a] + c_0 + \bar{v} - x$ be the total value when all banks invest. The cost to the government of any feasible program is:*

$$\Psi = E[V(\theta, \mathcal{P}_\theta)] - W.$$

Proof. In any program where $i(\theta) = 1$ for $\theta = G, B$, we must have $E[y_2|\theta] = E[a|\theta] + \bar{v} + c_2(1)$. From (2), we get $c_2(1) = c_0 + l - x + m$. Taking unconditional expectations of (8), we get that $E[V(\theta, \mathcal{P}_\theta)] = E[a] + \bar{v} + E[c_0 + l - x + m - y^l - y^g]$. Now, the break-even constraint of investors is $E[l - y^l] = 0$ and the expected cost of the government is by definition $\Psi = E[m - y^g]$. Therefore $E[V(\theta, \mathcal{P}_\theta)] = E[a] + c_0 + \bar{v} - x + \Psi$. ■

The intuition for Lemma 2 is as follows. The sum of expected net payoffs of banks, plus the expected income of lenders plus the government's profit (cost if negative), must equal the total value

of the economy (total value of legacy assets, plus the initial cash holdings of banks and the net present value of investment). Since we allow banks to opt out of the programs, their participation payoffs must be at least as large as their non-participation payoffs. Then the minimum cost of the government is equal to the total non-participation payoffs of the banks minus the total pie of the economy.

This simple insight allows us to derive lower bounds on the cost of various interventions. In our analysis it is crucial to distinguish between the cases where the participation decision is made under symmetric information (participation decision at time 0) and the one where this decision is made under asymmetric information (participation decision at time 1). We explore them in turn.

4.1 Participation under symmetric information

Let us now study interventions at time 0, i.e. before banks learn their types. At this point there is no asymmetric information between the government and the banks, so the government program must be designed in such a way, so as to attract banks voluntarily and to ensure that banks want to invest given the government intervention. Given that banks do not know their types, the participation constraint is simply equation (11) defined earlier.

Proposition 3 *If banks opt in the government program before they learn the quality of their legacy assets, then the program delivers at most a profit of $\pi(\bar{v} - x)$ to the government.*

Proof. Because banks decide to participate before they learn their type, their decision to opt in or out does not convey any information. A bank that opts out ends up in the decentralized equilibrium. Under assumption A4, only the bad types would invest. The outside value is therefore:

$$E[V(\theta, \mathcal{O})] = \pi(E[a|G] + c_0) + (1 - \pi)E[a + v + c_0 + l - x - y^l|B] = W - \pi(\bar{v} - x).$$

From the participation constraint and Lemma 2, we get $\Psi \geq E[V(\theta, \mathcal{O})] - W$. The government can always reduce its costs by uniformly increasing y^g so the participation constraint binds, and we get $\Psi_0^* = -\pi(\bar{v} - x)$, where Ψ_0^* stands for the lower bound cost under symmetric information.

■

The intuition behind Proposition 3 is simple. In the inefficient separating equilibrium, the good types do not invest. The government intervention enables all banks to invest. The net welfare gain is equal to $\pi(\bar{v} - x)$. Since the new lenders who come in at time 1 must break even on average, the welfare gains must accrue to the government and the initial shareholders, the banks. However, because the government makes a take-it-or-leave-it offer at time 0, it can extract all the surplus.

We will show in Section 5 that equity injections, asset buybacks and debt guarantees can all be designed to achieve this maximum profit.

4.2 Participation under asymmetric information

Let us now consider participation decisions at time 1, when banks have private information regarding their legacy assets. These interventions are more difficult to analyze because banks know how much their assets are worth but the government does not. Not only does this create adverse selection issues for the government, it also implies that the decision to participate in the government program may signal some information about the value of their assets, and therefore influence the market rates offered to participating and non participating banks. This is why we named this design problem as a “*mechanism design problem with a competitive fringe.*”

4.2.1 Programs that attract only one type of banks

The decision to participate in the government program can signal the type of the bank. Hence the mere ability of the government to design a program that attracts only a subset of types of banks, alleviates the asymmetric information problem for non-participating banks as well. In fact, in the two-type model we are considering solves it completely.

Interventions that attract good banks seem particularly appealing because good banks would be willing to pay to participate in the government program, since by doing so they can borrow at a low interest rate, since they are separated from the bad banks. Our first result demonstrates that such government interventions do not exist: if a program is designed to attract good banks it will necessarily attract bad banks as well.

Proposition 4 *There cannot exist a government program that attracts only good banks.*

Proof. See Appendix. ■

Proposition 4 is an important part of our paper, in terms of contribution and economic intuition. The contribution is clear: the Proposition shows that it is not possible for the government to design a program only for the good types. Any intervention is therefore bound to attract risky banks, and be politically costly. Critics will charge the government with too much generosity for bad banks, but the model says this generosity is unavoidable. Proposition 4 and its proof are also key to understand the rest of our paper. The general idea is that it is easier to attract bad types than to attract good types. This means that the participation constraint of the good types will bind, while the participation constraint of the bad type may not. The proof makes it clear that our

results generalize to any distribution of types (not simply good and bad). We could also allow the distribution of v to be type dependent provided that MLRP holds.

Consider now a program designed to attract only bad banks. Given such a program, which we index by \mathcal{B} , participation reveals bad type, whereas, non-participation reveals good type. Then, the non-participation value for a good bank is $V(G, \mathcal{O}(r_G)) = E[a|G] + c_0 + \bar{v} - x$ and the non-participation value for a bad bank is $V(B, \mathcal{O}(r_G)) = E[a|B] + \bar{v} - \rho(B, r_G l_0)$. Given these values, we can derive a lower bound for the cost of designing such a program:

Proposition 5 *The minimum cost of a program that attracts only bad banks is equal to the informational rents of the bad banks:*

$$\Psi_B^* = (1 - \pi)(l_0 - \rho(B, r_G l_0)).$$

Proof. Calculations similar to the ones done in the proof of Lemma 2 show that $\Psi_B^* = (1 - \pi)(V(B, \mathcal{P}_B) - (E[a|B] + c_0 + \bar{v} - x))$. The participation constraint of the bad type is $V(B, \mathcal{P}_B) \geq V(B, \mathcal{O}(r_G))$. Therefore $\Psi_B^* \geq (1 - \pi)(l_0 - \rho(B, r_G l_0))$. ■

The intuition behind Proposition 5 is straightforward. Separating the types simply requires paying informational rents to the bad types, so the cost of the government program is at least as big as these informational rents.

4.2.2 Programs that attract all banks

Let us now consider a program designed to attract all banks. Given such a program, deriving the banks' non-participation payoffs is delicate, because they depend on the out-of-equilibrium belief of investors regarding a bank that would unexpectedly opt out of the program. Let \tilde{r} be the interest rate a bank would face if it decided to opt out of the government program. In general, this rate \tilde{r} could be anywhere between r_G and r_B . We later show (in a similar fashion as we did in Proposition 4) that the participation constraint for good banks always implies the participation constraint for bad banks. Therefore, if we introduce a perturbation of the model where bank managers face an idiosyncratic disutility of participating in a government program (say because of political pressures or constraints on executive compensation), it is easy to see that the non-participating population would include a larger fraction of good banks than of bad banks. This would imply an interest rate $\tilde{r} < r_\pi$. In what follows we assume that the market perception about a bank dropping out from the

government program is favorable enough to induce an interest rate \tilde{r} that is low enough for good banks to invest:¹⁷

Assumption A6: \tilde{r} is such that $\bar{v} - x > \rho(G, \tilde{r}l_0) - l_0$.

Note that A6 makes it harder for the government to attract good types. We are going to show that programs designed to attract all banks are cheaper than programs that attract just troubled banks, *even* under this conservative assumption. As before, we can obtain a lower bound on the cost for a program designed to attract both kinds of banks:

Proposition 6 *The lowest possible cost for a program that attracts all banks is*

$$\Psi_{\Pi}^* = l_0 - \pi\rho(G, \tilde{r}l_0) - (1 - \pi)\rho(B, \tilde{r}l_0).$$

This minimum cost Ψ_{Π}^ is always positive and lower than the minimum cost Ψ_B^* for a program that attracts only bad banks.*

Proof. Since all banks participate in a pooling equilibrium, we know from Lemma 2 that $\Psi = E[V(\theta, \mathcal{P}_\theta)] - W$. Using the participation constraints and the outside options (9) and (10), we have $\Psi \geq E[V(\theta, \mathcal{O}(\tilde{r}))] - W$. We know that $W = E[a] + c_0 + \bar{v} - x$, and we can compute

$$\begin{aligned} E[V(\theta, \mathcal{O}(\tilde{r}))] &= \pi(E[a|G] + \bar{v} - \rho(G, \tilde{r}l_0)) + (1 - \pi)(E[a|B] + \bar{v} - \rho(B, \tilde{r}l_0)) \\ &= E[a] + \bar{v} - \pi\rho(G, \tilde{r}l_0) - (1 - \pi)\rho(B, \tilde{r}l_0). \end{aligned}$$

Hence the lowest bound for the cost $E[V(\theta, \mathcal{O}(\tilde{r}))] - W$ is equal to $\Psi_{\Pi}^* = l_0 - \pi\rho(G, \tilde{r}l_0) - (1 - \pi)\rho(B, \tilde{r}l_0)$. To finish the proof, notice that under A6 we know that $\tilde{r} < r_\pi$, and therefore $\Psi_{\Pi}^* > 0$. Moreover, since $\tilde{r} \geq r_G$, we have $\Psi_{\Pi}^* \leq l_0 - \pi\rho(G, r_G l_0) - (1 - \pi)\rho(B, r_G l_0)$. Since $\rho(G, r_G l_0) = l_0$, we see that $\Psi_{\Pi}^* \leq (1 - \pi)(l_0 - \rho(B, r_G l_0)) = \Psi_B^*$. ■

This Proposition suggests that programs that attract all banks have the potential to dominate programs that attract only bad banks. The reason is that programs that attract only bad banks have the perverse effect of creating the most attractive outside option for participating banks that consider opting out of the program. This forces the government to create a program that is generous and therefore costly.

¹⁷We have considered a number of refinements. Some have no bite, and the ones that do all imply that $\tilde{r} < r_\pi$. Some knife-edge cases even imply $\tilde{r} = r_G$. We do not need $\tilde{r} = r_G$, we only need the much weaker condition A6. To avoid using a particular and somewhat arbitrary refinement, we choose to state A6 as an assumption instead of a result. As explained in the text, however, it is clear that A6 would immediately follow from small noise in participation. Finally, we note that the empirical evidence supports our results since dropping out of a bailout program is typically received by an increase in market value (see the discussion in Acharya and Sundaram (2009) for instance).

Notice, however, that the lower bound for programs that attract all banks might be harder to achieve than that of programs that attract only bad banks. This is because reaching the lower bound requires that the government program be designed so as to make the participation constraints of *both* types of banks binding. It also presupposes that the constraints that matter for the design are the participation ones and not the incentive, nor the investment constraints.

So far we have derived bounds for what the government can expect to achieve at the best possible program that is designed to attract troubled banks and at the best one designed to attract all banks. We now proceed to ask whether, and under which circumstances, equity injections, asset buybacks and debt guarantees can reach these optimal bounds.

5 Implementation

In this section we study three government interventions that are often used during a financial crisis: recapitalization (equity injections), asset buybacks, and debt guarantees. Our first main result is that the form of the intervention is irrelevant when participation is decided under symmetric information. Our second main result is that debt guarantees are optimal under asymmetric information.

In this section, we restrict our analysis to the case where the government offers a unique program. We study menus of programs in Section 6.

5.1 Descriptions of programs

We start by describing each intervention.

- **Equity injection \mathcal{E} :** the government offers cash m_α against a share α of equity returns, $y^g = \alpha (y - y^l(y, rl))$
- **Asset buyback \mathcal{A} :** the government offers to buy an amount Z of legacy assets for cash m_z . If a bank opts in the program, the face value of its legacy assets decreases by Z . The payoffs to the government are $y^g = a \frac{Z}{A}$.
- **Debt guarantee \mathcal{D} :** the government offers to guarantee debt issuance up to an amount S for a fee ϕ paid up-front: $m = -\phi S$. Private lenders accept an interest rate of 1 on the guaranteed debt, so the date 1 budget constraint becomes

$$x = c_0 + (1 - \phi) S + l^u,$$

where l^u is the unsecured loan. The government will have to make payments in case of default, that is, whenever $a + v < S$.¹⁸

Note that these specific programs all belong to the general class of mechanisms we wrote down earlier. In particular, all these interventions satisfy the monotonicity condition A5 that y^g be increasing in $a + v$.

5.2 Symmetric information at participation stage

We start by considering the performance of optimal versions of these programs when banks make their participation decisions $t = 0$. At that point banks do not have any private information when they decide whether to participate in the program or not. Under symmetric information, we find that all programs can achieve the maximum profits for the government of $\pi(\bar{v} - x)$. We then show that this equivalence breaks down under asymmetric information, where debt guarantees are best.

Theorem 1 *Irrelevance of Intervention Choice under Symmetric Information.* *When banks make their participation decisions at time 0 equity injection, asset buybacks and debt guarantee can all be designed to achieve the lowest implementation cost derived in Proposition 3.*

Proof. See Appendix. ■

The critical point of the Theorem is that the interventions can actually make sure that (7) is satisfied and at the same time the participation constraint (11) holds with equality. Of course, if interventions under symmetric information were feasible, the banks could also raise private money from investors by issuing debt or equity before learning their types.

An alternative – and perhaps more relevant – interpretation of the symmetric information outcome is that the government can exert pressure and force participation in its program. Since the government has no reason to impose a loss on the industry as a whole, we can assume ‘forced’ participation subject to the average bank breaking-even (or subject to a diversified household owning shares and bonds of the banks breaking-even). This is then formally equivalent to our model with symmetric information at the participation stage. The important point is that this type of arm-twisting intervention cannot be privately enforced. This might explain why banks do not raise money by themselves on private markets and why government interventions might sometimes be more successful.

¹⁸This is also equivalent to the government providing a junior loan with face value S and an interest rate of $1/(1 - \phi)$.

The historical evidence clearly shows that both cases (arm-twisting and voluntary) are relevant. Corbett and Mitchell (2000) explain that “In Mexico, Japan and Thailand banks could voluntarily apply for programs while in Korea and Malaysia some coercion seems to have been applied.” Similarly, most observers have suggested that banks were ‘required’ to participate in the initial equity injection program of October 2008.¹⁹ On the other hand the FDIC debt guarantee programs and the various Fed lending programs are voluntary. Interestingly, the Treasury was explicitly concerned about the stigma effect of its equity program if it allowed some banks to drop out. We address this important issue in the next section.

5.3 Asymmetric information

We now examine the implementation of optimal programs when the banks make their participation decisions $t = 1$, once they know the value of their assets. We have shown in Proposition 4 that there does not exist a program that can attract only good banks. We therefore only need to consider programs that attract only bad banks, and programs that attract all banks. For interventions that attract only bad banks, we again obtain an irrelevance result.

Proposition 7 *All three interventions are optimal among the programs that only attract bad banks*

Proof. We need to check the participation and investment constraints. From the proof of Proposition 4 we see that if the participation constraint binds for type B then for type G , it must be the case that $V(G, \mathcal{O}(r_G)) \geq V(G, \mathcal{P}_G)$. Hence if a program is designed so that bad banks are indifferent between participating and not, good banks must at least weakly prefer to drop-out. In all programs, it is clear that one can make the participation binding for bad types by setting high enough values for αZ or ϕ .²⁰ For the investment constraints, the proof follows from the fact that the participation decision reveals the type, and therefore all banks face a fair interest rate. ■

The intuition for this result is based on the fact that the program reveals the type of the bank: The good banks do not participate and raise money on private markets at a low interest rate. The bad banks must be convinced to participate. Hence, the program must be generous enough. But once participation is ensured, investment follows automatically because all banks face fair interest rates. All three programs can be designed to give the bad banks their expected informational rents.

¹⁹“The Bush administration will announce a plan to rescue frozen credit markets that includes spending about half of a total of \$250 billion for preferred shares of nine major banks [...] None of banks getting government money was given a choice about it, said one of the people familiar with the plans.” (Bloomberg, Oct. 13, 2008).

²⁰Full details are available upon request.

Let us now consider programs that attract all banks. For such programs, our main result is that debt guarantees are optimal: debt guarantees achieve efficient investment at the minimum cost for the government.

Proposition 8 *Guaranteeing new debt is optimal among programs that attract all banks. It achieves the lower bound of Proposition 6.*

Proof. Notice first that no bank wants to issue guaranteed debt without investing. If the bank does not invest, it must carry the cash $c_0 + (1 - \phi)S$ and repay S , so shareholders receive $\max(a + c_0 - \phi S, 0)$ which is decreasing in S irrespective of a as long as $\phi \geq 0$. After joining the program, the choice is therefore to issue guaranteed debt and invest, or not to invest at all. The inside value of type θ is

$$V(\theta, \mathcal{P}_\theta) = \int_0^\infty (y - y^u(y, rl^u) - y^s(y, S)) f(y|\theta) dy$$

where $y^u(y, rl^u) = \min(y, rl^u)$, $y^s(y, S) = \min(y - y^u, S)$ and where l^u stands for the unsecured debt which is equal to $l^u = l_0 - (1 - \phi)S$. Under A6, all types invest if they opt out, therefore:

$$V(\theta, \mathcal{O}(\tilde{r})) = \int_0^\infty (y - y^l(y, \tilde{r}l_0)) f(y|\theta) dy.$$

First we show that the investment constraint is always weaker than the participation constraint: The value without investing is $V(\theta, 0) = \int_0^\infty (a + c_0) f(a|\theta) da$, while the outside value is in general (even when A6 fails) $V(\theta, \mathcal{O}(\tilde{r})) = \max(V(\theta, 0), \int_0^\infty (y - y^l(y, \tilde{r}l_0)) f(y|\theta) dy)$, implying that the outside option is always at least as good as participating without investing for all types. Therefore the participation constraint $V(\theta, \mathcal{P}_\theta) \geq V(\theta, \mathcal{O}(\tilde{r}))$ always implies the investment constraint $V(\theta, \mathcal{P}_\theta) \geq V(\theta, 0)$.

Now, the value of participation is:

$$V(\theta, \mathcal{P}_\theta) - V(\theta, \mathcal{O}(\tilde{r})) = \int_0^\infty \gamma(y) f(y|\theta) dy$$

where

$$\gamma(y) \equiv \min(y, \tilde{r}l_0) - \min(y, S + rl^u).$$

The key is to make the participation constraints binding for all types. This is achieved by setting $S + rl^u = \tilde{r}l_0$, which we can write as

$$(r(1 - \phi) - 1)S = (r - \tilde{r})l_0.$$

This implies that the function $\gamma(y)$ is identically equal to zero for both types. Therefore, $\Psi_{\Pi}^D = \Psi_{\Pi}^*$.

■

Notice that debt guarantees achieve the lower bound for the cost function. They are thus optimal among all conceivable programs that attract all banks, not just among the three programs for which we provide detailed analysis. The intuition behind Proposition 8 is contained in the proof. The first idea is that the investment constraints are implied by the participation constraints. The second idea is that the participation constraints of both types are equivalent. Hence a debt guarantee program achieves the bound derived in Proposition 6. To complete our analysis, we show that asset buybacks and equity injections do not reach the lower bound, and that a pure equity injection program dominates a pure asset buyback program.

Proposition 9 *Among programs that attract all banks under asymmetric information, equity injections do worse than debt guarantees, and asset buybacks do worse than equity injections.*

Proof. See Appendix. ■

The intuition behind Proposition 9 is the following. Either the investment constraint binds, or it does not. If the outside option of banks is very high or if investment is easy to achieve, then the investment constraint does not bind. It is then optimal to offer a pure cash transfer to the banks (i.e., a program where $Z = \alpha = 0$). The interventions are then (trivially) equivalent and achieve the lower bound. In the more interesting and more relevant case where the investment constraint binds, the programs must be generous enough to sustain investment. In this case, setting α or Z to zero is suboptimal since this would leave slack in the participation constraints. The government then sets Z and α so that the participation constraint of the good types binds. But in this case we can show that the participation constraint of the bad type is always slack. Unlike debt guarantees, equity injections and asset buybacks cannot be designed to make both participation constraints binding. This wedge means that the bad types earn rents relative to their outside option in excess of what is implied by voluntary participation. These unnecessary rents are strictly positive as soon as α or Z is strictly positive. Finally, we can show that the excess rents are higher for equity than for asset buybacks. The intuition for this last result is that asset buybacks maximize the adverse selection problem because they are most sensitive to private information. Equity injections suffer less because they incorporate the cash flows from the new project about which information is symmetric.

Proposition 6 shows that the lower bound for costs is lower for programs that attract all banks than for programs that only attract bad banks. Proposition 8 shows that debt guarantees achieve this minimum cost. We therefore obtain our main theorem:

Theorem 2 *Optimality of Debt Guarantee.* *The optimal government intervention to achieve the efficient level of investment is to set up a debt guarantee program in which all banks participate.*

Our Theorem establishes that the optimal way to restore efficient lending are debt-guarantees. This is the best not only among equity injection or assets buybacks, but among all conceivable mechanisms.

6 Extensions

In this section we provide three important extensions to our main results. The first extension is to consider asymmetric information with respect to new investment opportunities. The second extension is to analyze menus of contracts. The third extension is to consider the consequences of moral hazard in addition to adverse selection.

6.1 Asymmetric information about new loans

We have assumed so far that the distribution of v is independent of the bank's type. Let us now relax this assumption, while maintaining assumptions A2 and A3. We replace A1 by:

Assumption A1': $E[v|G] > E[v|B] > x > c_0.$

Note that A1' implies that both types still have positive NPV projects.²¹ Under A1', we can show that all our results apply except for the comparison of equity injections and asset buybacks in Proposition 9.

Let us start with the decentralized equilibrium. It is still optimal to offer debt contracts and the expected repayment is still given by $\rho(\theta, rl) \equiv \int_0^\infty y^l(y, rl) f(y|\theta) dy$. We still have the ranking for the "fair" rates $r_B > r_\pi > r_G$ which follows from A2. The investment condition, however, becomes $E[v|\theta] - x > \rho(\theta, r(x - c)) - (x - c)$ for type θ , so it is now possible for that condition to hold for good banks and not for bad banks. This is a significant difference, but it does not alter the results of Proposition 2 for the following simple reason: Under assumption A1', there still cannot exist an equilibrium where only the good types invest because in such an equilibrium the interest rate would be r_G which is less than r_B and would therefore make it optimal for bad banks to invest. Therefore Proposition 2 remains unchanged.

Nothing changes from Section 3 to 5, except that \bar{v} must be replaced by $E[v|G]$ in Proposition 3. The only significant change is in Proposition 9. It is still true that debt guarantees dominate

²¹We have also analyzed the case where the bad type has a negative NPV project. In this case, the main difference is that there can exist equilibria where no bank invests.

other interventions, but it is not necessarily true that equity dominates asset buybacks, since it is theoretically possible to have more asymmetric information with respect to v than with respect to a .

6.2 Menus

We have analyzed in Section 5 the design of simple interventions and we have shown that debt guarantees are optimal, while equity injections and asset buybacks are not optimal in general. We now allow the government to offer menus of contracts. It is important to emphasize that the results of Section 4 are valid with or without menus, since they only rely on the participation constraints and the competitive fringe. Hence we know we will not improve upon the simple debt guarantee program, but we can hope to obtain the same lower bound for the cost with equity or asset buybacks.

In what follows we appeal to the revelation principle and look at menus with two options: one for good banks, another for bad banks. For the case of equity the menu takes the form α_G, m_G and α_B, m_B . In order to be feasible each of these options must satisfy the participation constraints as before, and additionally incentive-compatibility constraints. However, observe that now, even though the government is designing a program to attract all banks, the interest rate that banks face is not a pooling rate r_π . Good banks choose the option α_G, m_G and face an interest rate of r_G , whereas bad banks choose option α_B, m_B and face an interest rate of r_B . Given that all banks face a fair interest rate when they choose to participate, they will always invest, hence there is no need to consider investment constraints.

The cost-minimizing equity program for the government solves:

$$\min_{\alpha_G, m_G, \alpha_B, m_B} \pi \left(m_G - \alpha_G E \left[y - y^l(y, r_G l_{m_G}) | G \right] \right) + (1 - \pi) \left(m_B - \alpha_B E \left[y - y^l(y, r_B l_{m_B}) | B \right] \right)$$

subject to incentive constraints (IC_B) and (IC_G), participation constraints (PC_G) and (PC_B), and non-negativity constraints on equity shares α_G and α_B . The constraints can be found in the Appendix. Note that the constraints that the α 's must be less than 1 are ignored because they are implied by the participation constraints.

Similarly, we can formulate the government program that the cost-minimizing menus of asset buybacks m_G, Z_G and m_B, Z_B and debt guarantees ϕ_G, S_G and ϕ_B, S_B .

Proposition 10 *The optimal equity menu is given by $\alpha_G^* = 0$, $\alpha_B^* = 0$, and m_G, m_B are such that $y^l(y, r_G l_{m_G}) = y^l(y, \tilde{r} l_0) = y^l(y, r_B l_{m_B})$ which ensures that IC_B, PC_B and PC_G hold with equality. The optimal asset-buyback menu satisfies $Z_G^* = Z_B^* = 0$ and m_G^* and m_B^* be such that*

$y^l(y, r_G l_{m_G}) = y^l(y, r_B l_{m_B}) = y^l(y, \tilde{r} l_0)$ which ensures that IC_B , IC_G , PC_B and PC_G hold with equality. Finally, the optimal debt-guarantee menu satisfies $y^u(y, r_B l^u) + y^s(y, S_B) = y^u(y, r_G l^u) + y^s(y, S_G) = y^l(y, \tilde{r} l_0)$, which again ensures that IC_B , IC_G , PC_B and PC_G hold with equality. All menus achieve the minimal cost, exactly as the simple debt-guarantee program, namely Ψ_{Π}^* .

Proof. See Appendix. ■

We conclude that menus can be designed to be as efficient as simple debt guarantee programs. In both the case of equity and the one of asset buybacks these are pure cash injection programs. The government offers two different cash levels one to the banks that report B , m_B and one to the banks that report G , m_G . Banks who choose m_i face a corresponding rate r_i for $i = B, G$. The cash levels are such that the repayments to lenders are the same irrespective of the option chosen. Moreover, they are equal to the repayments that banks would have to make without the government's assistance in the market, that is $y^l(y, r_G l_{m_G}) = y^l(y, r_B l_{m_B}) = y^l(y, \tilde{r} l_0)$. Then all incentive and participation constraints hold with equality. The feature that the government can offer different options that generate the same repayments that can be made equal to the repayments without participation, is also the key feature of the optimal debt guarantee program.

6.3 Moral hazard

In the benchmark model we have assumed that all new projects are identical and all have positive NPV. Under these assumptions, the government's objective is to obtain the maximum amount of lending at the minimum cost, and we have shown that a debt guarantee program designed to attract all banks is the most cost-effective intervention. In reality, however, banks can sometimes control the characteristics of their new lending opportunities. In particular, they can choose various degrees of riskiness in the new loans that they extend. It is therefore important to understand how endogenous risk-taking could affect our results on the optimality of debt guarantees.

We introduce a new project with random payoff v' , with $E[v'] < E[v]$. Project v' is riskier than project v in the sense of second order stochastic dominance (SOSD). The investment cost is the same x as in the project v .

We assume that the choice of the project is observable by private lenders, but cannot be enforced by the government:

Assumption A7: Moral hazard. The choice of project v' cannot be controlled by the government. It is however observable by private lenders.

It is important to understand the motivation behind assumption A7. Our point is to ask when

government interventions can create inefficiencies. If v' is not observable by private investors, risk shifting could occur in the decentralized equilibrium without intervention. While risk shifting would be a greater issue on average, it would be unrelated to the government's intervention, and would therefore not affect the optimality of debt guarantee programs. By contrast, Assumption A7 implies that there is no risk shifting without government intervention.

Proposition 11 *The decentralized equilibrium without intervention, and the equilibria with equity injection or asset buyback are unaffected by the availability of project v' .*

Proof. Consider first the equilibria without interventions. Because v' is observable, the lending rates from the private sector depend on whether v or v' is chosen. In a separating equilibrium, the banks obtain the fair interest rates. Therefore they always chose the projects with the higher NPV. In the pooling case, it is easy to see that good types dislike project v' because it generates a higher interest rate and because good types are more likely to repay their debts. Bad types cannot chose v' without revealing their types. Hence the project v' is never chosen in a decentralized equilibrium. Asset buybacks do not change this result because the net payments from the government do not depend on the project choice. Finally, in the case of equity banks maximize shareholder value and make the same project choices as in the decentralized equilibrium. ■

Under equity injection or asset buyback, risky projects do not interfere with the government program. We now turn to debt guarantees. The issue with debt guarantees is that the subsidies are higher when the projects are riskier. The government's debt guarantee program may therefore increase risk taking as banks and private investors take advantage of the implicit subsidy. The worst case scenario is clearly one where the government guarantees all new debt issuances. Let us consider first the case of a full guarantee program that attracts only bad banks.

Lemma 3 *A full guarantee program that attracts only bad banks induces risk shifting when $\rho(B, r_\phi l_0) - \rho'(B, r_\phi l_0) > E[v] - E[v']$, where $r_\phi = 1/(1 - \phi)$ and ρ' are expected repayments under the risky project.*

Proof. In the full guarantee case the participating bad banks do not issue uninsured debt. The program is equivalent to borrowing the full amount at an interest rate of $1/(1 - \phi)$. Bad banks would chose v' over v if and only if:

$$\rho(B, r_\phi l_0) - \rho'(B, r_\phi l_0) > E[v] - E[v'] .$$

■

The key in the previous Lemma is that we consider a separating equilibrium where all debt is insured. This provides the maximum incentives for risk shifting. Since we have shown that interventions that attract all banks dominate the ones that only attract bad banks, we are more interested in consequences of moral hazard in this case. Choosing the risky project in this case is costly for two reasons. First, the cost of uninsured borrowing goes up. Second, the choice might reveal the bad type, since risk shifting is always more tempting for the bad types than for the good types. Consider therefore a pooling equilibrium where all types participate and invest in v . If the bad types choose v , they pool and obtain the value function

$$V_B = E [y - \min (y, r_\pi l^u + S) | B].$$

If they choose v' they obtain

$$V'_B = E [y' - \min (y', r'_B l^u + S) | B].$$

They choose the risky project if and only if

$$E [\min (y, r_\pi l^u + S) - \min (y', r'_B l^u + S) | B] > E [v - v'] .$$

Risk shifting has one benefit and two costs. The benefit is the lower repayment on the insured debt S . The first cost is the NPV loss $E [v - v']$. The second cost is the higher rate $r'_B > r_\pi$. It is important to understand that r'_B exceeds r_π for two reasons. The first is the riskiness of the project which increases the rate from r_π to r'_π . The second is the revelation of information which increases the rate from r'_π to r'_B . In equilibrium, we have $E [\min (y', r'_B l^u) | B] = l^u$, since r'_B is the fair rate while $E [\min (y, r_\pi l^u) | B] < l^u$. This reputation cost applies to the part of borrowing that is not insured, $l^u = x - c_0 - (1 - \phi) S$.

Finally, we can also find equilibria where good types risk shift. In this case, the bad types obviously risk shift as well. The key constraint is for the good types. In this equilibrium, they could reveal their good type by not risk shifting. If the good types choose v' , they pool and obtain the value function

$$V'_G = E [y' - \min (y', r'_\pi l^u + S) | G],$$

while if they choose v they obtain

$$V_G = E [y - \min (y, r_G l^u + S) | G].$$

They choose the risky project if and only if

$$E [\min (y, r_G l^u + S) - \min (y', r'_\pi l^u + S) | G] > E [v - v'] .$$

The opportunity cost of risk shifting is therefore again equal to the loss of NPV plus the opportunity cost of reputation. Note that this opportunity cost is higher for good types since they suffer relatively more from high rate than the bad types.²² The equilibrium where all types risk shift is therefore less likely to happen than the equilibrium where only the good types risk shift.

We can summarize our discussion by the following proposition.

Proposition 12 *Risk shifting is most likely to occur when the debt guarantee is large enough to induce the bad types to select the risky project even though this choice reveals their type. Risk shifting is less likely to occur in the optimal program that attracts all banks than in a program attracting only the bad banks.*

The good news here is that the optimal debt guarantee program (attracting all banks with as little guarantee as possible) is the least likely to induce risk shifting. Risk shifting only occurs when the subsidy is high enough to dominate both the NPV loss and the negative signaling.

It is also worth pointing out that risk shifting can also make debt guarantees more attractive. It is easy to see that good types could be willing to select safer projects even if they had lower NPV because such anti-risk shifting would function as a costly signaling device for good banks to reveal their types. In this case the availability of a low-risk, low-return project would change the decentralized equilibrium. Good banks would become too conservative in their lending policies during a crisis in order to signal their types. A debt guarantee could then be even more attractive since it would lean against this conservatism bias.

We conclude that the risk-shifting problem might not be as damaging for government interventions as one would have predicted before our analysis. Either risk-shifting is observed by market participants and the endogenous response of private lending rates puts discipline on the banks, or risk-shifting is private information to the bank, but then the moral hazard problems occurs with or without the government. In addition, anti-risk-shifting by good types might actually be more of a concern during a crisis than risk shifting by bad types. This would only make debt guarantee programs more attractive. If, despite all these economic forces, the risk-shifting problem is so severe that even the NPV loss and the negative signaling cannot prevent it, then the government should implement the optimal outcome using a menu of equity injections or asset buybacks. We have seen that these menus are as cost-effective as the simple debt guarantee program (Proposition 10), and that equity injections and asset buybacks do not create incentives for risk shifting (Proposition 11).

²²It is easy to see that $E[\min(y, r_G l^u + S) - \min(y', r'_\pi l^u + S) | G] < E[\min(y, r_\pi l^u + S) - \min(y', r'_B l^u + S) | B]$

7 Conclusion

We provide a complete characterization of optimal interventions to restore efficient lending and investment in markets where inefficiencies arise because of asymmetric information. In doing so we make two contributions. On the technical side, we solve a non-standard mechanism design problem with two information sensitive decisions (investing or not, participating or not) and in the presence of a competitive fringe (the government does not shut down the private markets).

On the normative side, our main results are as follows. If participation decisions occur under symmetric information (or equivalently under forced participation), we obtain an irrelevance result: all common interventions (equity injections, asset buybacks and debt guarantees) can be designed to restore efficient lending at the minimum cost for the government.

In the more interesting case where participation is subject to adverse selection, the mere existence of a government program affects the borrowing costs of all banks, even the ones that do not participate in the program. These endogenous borrowing costs determine the outside option of participating banks and therefore the cost of implementing the program. We find that it is impossible to design a program that attracts only good types, and that programs that attract all types are less costly than programs that attract only bad types. Our most remarkable result is that the optimal intervention can be implemented by offering a simple program of debt guarantees. This simple program is optimal among all possible interventions aimed at increasing investment. If we consider moral hazard in addition to adverse selection, we find that the optimality of debt guarantees can be overturned by risk shifting but that this problem is significantly mitigated by the fact that good banks want to signal their types by choosing safe projects.

A Proof of Proposition 1

Let us first write the investment condition, conditional on r and l :

$$i(\theta) = 1 \iff E[\max\{a + v - rl, 0\} | \theta] > E[a|\theta] + c_1. \quad (16)$$

Suppose first that the type is $\theta = (\theta^a, v)$ where θ^a indexes the conditional distribution of a , $F(a|\theta^a)$. We have

$$E[\max(a + v - rl, 0) | \theta] = \int_{rl-v}^A (a + v - rl) dF(a|\theta^a).$$

If $rl > v$, then the investment condition (16) is clearly violated. So banks with $v < rl$ would never invest. But if only banks with $v > rl$ invest, then debt is risk free and $r = 1$, and, since $l = x - c_1$, the investment condition is simply the first best rule:

$$E[a|\theta^a] + v - l > E[a|\theta^a] + c_1 \iff v > x.$$

The second part of the proposition is simpler. Suppose now that the bank can issue risk free debt, i.e., that $A_{\min} \geq x - c_0$. Recall that $y^l = \min(y, rl)$. Now, we also have that $y = a + v \geq A_{\min} > x - c_0 = l$. So $r = 1$ satisfies the participation constraint of lenders. With $r = 1$, $\max(a + v - rl, 0) = a + v - rl$ and the investment condition becomes

$$E[v|\theta] > l + c_0 = x$$

which is the first best investment rule.

B Proof of Proposition 4

Consider a candidate equilibrium where the good types choose a particular program \mathcal{P} while the bad types stay out (choose \mathcal{O}). Consider first the case where $y^g(a, y)$ depends only on y . If y^g depend also on a , the program is less attractive for good types, all else equal, so this is without loss of generality. Given a program designed to attract only good banks, participation would reveal good type, non participation reveals bad type. The interest rate would be r_G for participating banks, and r_B for non-participating banks. Consider a bank of type θ . The inside value for this bank is

$$V(\theta, \mathcal{P}) = \int_0^\infty \left(y - y^l(y, r_G l) - y^g(y) \right) f(y|\theta) dy,$$

where $l = x - c_0 - m$ and $y^l(y, r_G l) = \min(y, r_G l)$. The outside value for type θ depends on whether it invests or not. Let us first consider the case where it invests if it opts out (we will later show that the option of not investing only reinforces our result). The outside value for type θ is then

$$V(\theta, \mathcal{O}) = \int_0^\infty \left(y - y^l(y, r_B l_0) \right) f(y|\theta) dy,$$

where $l_0 = x - c_0$ and $y^l(y, r_B l_0) = \min(y, r_B l_0)$. Therefore we have

$$V(\theta, \mathcal{P}) - V(\theta, \mathcal{O}) = \int_0^\infty \gamma(y) f(y|\theta) dy,$$

where

$$\gamma(y) \equiv \min(y, r_B l_0) - \min(y, r_G l) - y^g(y).$$

The next step of the proof focuses on the function $\gamma(y)$. The complication in the proof is that the differential payoff function γ is non monotonic, so there is no general result on how $E[\gamma(y) | \theta]$ changes with θ . In particular, FOSD is not useful here. The key is therefore MLRP and the ensuing single crossing property for $V(\theta, \mathcal{P}_\theta) - V(\theta, \mathcal{O})$. Notice first that we must have $r_G l < r_B l_0$,

otherwise good banks would invest with high interest rate r_B , which is impossible since we focus on the case where inefficiency arises because $c_0 < c_\pi$ (Assumption A4). For $y < r_G l$, we have $\gamma = 0$. For $y \in [r_G l, r_B l_0]$, we have $\gamma(y) \equiv y - r_G l - y^g(y) \geq 0$ since $y^g(y) \leq y - r_G l$. For $y > r_B l_0$, we have $\gamma(y) \equiv r_B l_0 - r_G l - y^g(y)$ decreasing. If γ is always positive, all types obviously participate. If γ is sometimes positive, the above discussion implies that there is a unique cutoff \hat{y} such that $\gamma(y) \geq 0$ on $[0, \hat{y}]$ and $\gamma(y) \leq 0$ on $[\hat{y}, \infty]$. Define the marginal type θ^* for whom

$$V(\theta^*, \mathcal{P}) - V(\theta^*, \mathcal{O}) \equiv 0. \quad (17)$$

For this type we must have

$$\int_0^{\hat{y}} \gamma(y) f(y|\theta^*) dy = \int_{\hat{y}}^{\infty} -\gamma(y) f(y|\theta^*) dy.$$

The following step uses Assumption A2. Consider a type $\theta < \theta^*$. From MLRP, we know that $\frac{f(y|\theta^*)}{f(y|\theta)}$ is increasing. Therefore, for all $y < \hat{y}$, we have $\frac{f(y|\theta^*)}{f(y|\theta)} < \frac{f(\hat{y}|\theta^*)}{f(\hat{y}|\theta)}$ and therefore since $\gamma(y) \geq 0$ on $[0, \hat{y}]$ we have

$$\int_0^{\hat{y}} \gamma(y) f(y|\theta^*) dy \leq \frac{f(\hat{y}|\theta^*)}{f(\hat{y}|\theta)} \int_0^{\hat{y}} \gamma(y) f(y|\theta) dy.$$

Similarly, for all $y > \hat{y}$, we have $\frac{f(y|\theta^*)}{f(y|\theta)} > \frac{f(\hat{y}|\theta^*)}{f(\hat{y}|\theta)}$ and therefore since $\gamma(y) \leq 0$ on $[\hat{y}, \infty]$ we have

$$\int_{\hat{y}}^{\infty} -\gamma(y) f(y|\theta^*) dy \geq \frac{f(\hat{y}|\theta^*)}{f(\hat{y}|\theta)} \int_{\hat{y}}^{\infty} -\gamma(y) f(y|\theta) dy.$$

Using (17), we get

$$\frac{f(\hat{y}|\theta^*)}{f(\hat{y}|\theta)} \int_0^{\hat{y}} \gamma(y) f(y|\theta) dy \geq \frac{f(\hat{y}|\theta^*)}{f(\hat{y}|\theta)} \int_{\hat{y}}^{\infty} -\gamma(y) f(y|\theta) dy,$$

and therefore $\int_0^{\infty} \gamma(y) f(y|\theta) dy \geq 0$ and type θ wants to opt in. This shows that if a particular type opts in, all the types below must also opt in. Hence there is no equilibrium where only the good types participate.

To complete the proof, we must return to our two simplifying assumptions, namely that $y^g(a, y)$ depends only on y and that the types invest if they opt out. Relaxing these assumptions only reinforces our results. If $y^g(a, y)$ increases in a , this lowers the inside value for high types more than for low types. Similarly, the option of not investing is relatively more attractive to high types, hence it increases the outside value for high types more than for low types. In both cases, the participation becomes relatively less attractive for high types than for low types compared to our benchmark calculations.

C Proof of Irrelevance Theorem 1

C.1 Equity injection

In a pure equity injection program denoted by \mathcal{E} the government injects cash m_α in return for a fraction α of equity. The government's cost function is

$$\Psi_0^\mathcal{E} = m_\alpha - \alpha E[a + c_2 + v \cdot i - y^l].$$

To make sure that all banks invest, the government must inject $m_\alpha \geq c_\pi - c_0$. Given such a government program, all firms invest, so $i = 1$ and $c_2 = 0$ for all types. Then, the cost function becomes

$$\Psi_0^\mathcal{E} = (1 - \alpha) m_\alpha - \alpha (E[a] + c_0 + \bar{v} - x).$$

By participating, a bank knows that it will be able to invest irrespective of its type, and that it will receive cash m_α . In return, it will give up a fraction α of its equity. It is immediate to see that at the optimum the participation constraint binds. It implies:

$$\alpha (E[a] + c_0 + \bar{v} - x) = (1 - \alpha) m_\alpha + \pi (\bar{v} - x).$$

By setting m_α equal to the minimum level necessary to achieve investment we get that

$$\alpha = \frac{c_\pi - c_0 + \pi (\bar{v} - x)}{(E[a] + c_0 + \bar{v} - x) + c_\pi - c_0} \leq 1.$$

This program is optimal because it is feasible and achieves the minimum cost (which is actually a profit here):

$$\Psi_0^{\mathcal{E}} = -\pi (\bar{v} - x) = \Psi_0^*.$$

C.2 Asset buyback

In an asset buyback program denoted by \mathcal{A} the government offers to buy an amount Z of legacy assets for cash m_z . If a bank opts in the program, the face value of its legacy assets decreases by Z . The payoffs to the government are $y^g = aZ/A$. Define the fraction of buyback by $z \equiv Z/A$. The government cost of an equity injection program is

$$\Psi_0^{\mathcal{A}} = m_z - E[az].$$

Given that good types do not invest alone, the participation constraint at time 0 is simply

$$(1 - z) E[a] + c_0 + \bar{v} - x + m_z \geq E[a] + c_0 + (1 - \pi) (\bar{v} - x),$$

which simplifies to

$$m_z - zE[a] \geq -\pi (\bar{v} - x).$$

Note, however, that the cash injection m must be high enough to make the good types willing to invest. The various investment constraint are tedious to check, but it turns out that the proof is much simpler. Consider a program where $Z = A$, i.e. where the government buys back all the assets. The government resolves the asymmetric information problem since banks have no assets left. Moreover, by setting $m_z = E[a] - \pi (\bar{v} - x)$, this program is optimal since it achieves the lower bound on costs,

$$\Psi_0^{\mathcal{A}} = -\pi (\bar{v} - x) = \Psi_0^*.$$

This is not the only way to implement the ex-ante optimum with asset buy backs – there may exist other programs where the government leaves some asymmetric information in the market – but it is clearly sufficient for our proof.

C.3 Debt guarantee

In a debt guarantee program denoted by \mathcal{D} the banks issue at time 0 debt S guaranteed by the government guarantees in exchange for a fee ϕS . The cash balance at time 1 is $c_0 + (1 - \phi) S$. At time 1, the banks borrow an unsecured amount l^u and invest. The budget constraint requires $l^u = x - (1 - \phi) S - c_0$. The payments at time 2 are $y^u = \min(y, r^u l^u)$, $y^s = \min(y - y^u, S)$. The expected cost for the government is

$$\Psi_0^{\mathcal{D}} = -\phi S + S - E[y^s(y, S)].$$

The first term is the fees received at time 1, while $S - E[y^s(y, S)]$ are the expected payments needed at time 2 to make the secured debt holders whole. In equilibrium, the rate r^u is such that $E[y^u(y, r^u l^u)] = l^u$. The ex-ante participation constraint for the banks is therefore

$$E[a] + \bar{v} + c_0 + (1 - \phi) S - x - E[y^s(y, S)] \geq E[a] + c_0 + (1 - \pi) (\bar{v} - x),$$

or

$$E[y^s(y, S)] - (1 - \phi)S \leq \pi(\bar{v} - x).$$

As in an asset buyback program, however, we must ensure investment by all banks. The value after investment is $V(\theta, 1) = E[\max(a + v - S - r^u l^u, 0) | \theta]$ while value without investment is $V(\theta, 0) = E[\max(a + c_0 - \phi S, 0) | \theta]$. We are going to show that a full guarantee satisfies the investment constraint. In a full guarantee we have $(1 - \phi)S = x - c_0$ and $l^u = 0$. In this case

$$V(\theta, 1) = E[\max(a + v - S, 0) | \theta] = E[E[\max(a + v - S, 0) | a, \theta] | \theta]$$

Since the max function is convex,

$$E[\max(a + v - S, 0) | a, \theta] \geq \max(a + E[v] - S, 0).$$

Now, since $(1 - \phi)S = x - c_0$ and $E[v] > x$ we have $E[v] - S > c_0 - \phi S$, therefore

$$\max(a + E[v] - S, 0) \geq \max(a + c_0 - \phi S, 0).$$

It follows that

$$V(\theta, 1) \geq E[\max(a + c_0 - \phi S, 0) | \theta] = V(\theta, 0).$$

Therefore a full guarantee program always satisfies the investment constraint. The full guarantee program only requires $(1 - \phi)S = x - c_0$, so it is always possible to adjust S and ϕ to make the ex-ante participation bind with equality while satisfying the full guarantee constraint. This achieves the minimum cost $\Psi_0^D = \Psi_0^*$.

D Proof of Proposition 9

D.1 Equity Injection

Conditional on investment, the private loan is $l_\alpha = x - c_0 - m_\alpha$ and the private rate is r_α consistent with all banks participating in equilibrium. The total inside value of equity for type θ conditional on investment is:

$$V(\theta, \mathcal{E}(\theta)) = (1 - \alpha) E[y - \min(y, r_\alpha l_\alpha) | \theta].$$

The interest rate r_α is pinned down by l_α via the zero profit condition

$$l_\alpha = E[\min(y, r_\alpha l_\alpha)]. \quad (18)$$

The initial shareholders retain a fraction $1 - \alpha$. The outside option for type θ is $V(\theta, \mathcal{O}(\tilde{r})) = E[y - \min(y, \tilde{r} l_0) | \theta]$ with $l_0 = x - c_0$, and the participation constraint is therefore:

$$V(\theta, \mathcal{E}(\theta)) \geq V(\theta, \mathcal{O}(\tilde{r})). \quad (19)$$

The investment constraint does not depend on α since all shareholders (the government and the old ones) are treated equally. The investment constraint for type θ is:

$$V(\theta, \mathcal{E}(\theta)) \geq (1 - \alpha) (E[a | \theta] + c_0 + m_\alpha). \quad (20)$$

The government chooses α and m_α to minimize

$$\Psi_{\text{II}}^\mathcal{E} = m_\alpha - \alpha E[y - \min(y, r_\alpha l_\alpha)], \quad (21)$$

subject to participation, investment constraints for all banks and to the interest rate equilibrium condition (18).

As in the proof of Proposition 4 we can show that the participation constraint for the good type implies the participation constraint for the bad type. Moreover, since $r_\alpha(m_\alpha) \leq r_B(m_\alpha)$, the investment constraint for bad banks is always satisfied. Then, the solution depends on whether (19) binds for $\theta = G$, or whether (20) for $\theta = G$ binds. Suppose first that the investment constraint

(20) is slack. Given any cash level m , we can make the participation constraint bind for the good type by choosing α such that

$$\alpha(m) = \frac{\int_0^\infty (\min(y, \tilde{r}l_0) - \min(y, r_\alpha l_\alpha)) f(y|G) dy}{E[y - \min(y, r_\alpha l_\alpha) | G]} \quad (22)$$

which must be non-negative. Moreover, we can make the participation constraints for both types of banks to bind by setting $y^l(y, \tilde{r}l_0) = y^l(y, rl)$ which is achieved by a cash level that satisfies $r_\alpha l_\alpha = \tilde{r}l_0$ or $r(x - c_0 - m_\alpha) = \tilde{r}(x - c_0)$, or

$$\hat{m} = \left(1 - \frac{\tilde{r}}{r}\right) (x - c_0). \quad (23)$$

Note that this is an implicit definition since r depends on m through (18). With this cash level we have $\alpha(\hat{m}) = 0$ and the participation constraints for both types of banks bind and the cash injection reaches the lower bound

$$\Psi_{\Pi}^{\mathcal{E}} = \Psi_{\Pi}^* = \hat{m}.$$

Note, however, that this only happens when $\alpha = 0$, so it can only be optimal for the government to transfer cash without asking for equity. Moreover, with this cash level we have that $r_\alpha l_\alpha = \tilde{r}l_0$. Assumption A6 only guarantees that $E[y - \min(y, \tilde{r}l_0) | G] \geq E[a|G] + c_0$, so it is clear that the investment constraint (20) may be violated at cash level \hat{m} . When the investment constraint (20) binds for the good type, it pins down m_α . Since this cash level is above the one that sets $\alpha = 0$, we obtain a non-negative α from (22). In this case the participation constraint for the bad type is slack and the cost of equity injection does not reach the lower bound. The cost of this program is then:

$$\Psi_{\Pi}^{\mathcal{E}} - \Psi_{\Pi}^* = (1 - \pi) (V(B, \mathcal{E}(B)) - V(B, \mathcal{O}(\tilde{r}))) > 0.$$

D.2 Asset buybacks

The government offers to buy an amount Z of legacy assets for cash m_z . If a bank opts in the program, the face value of its legacy assets decreases by Z . The payoffs to the government are $y^g = a \frac{Z}{A}$. Define the fraction of buyback by $z \equiv Z/A$. The total value conditional on participation and investment for type θ is

$$V(\theta, \mathcal{A}(\theta)) = E[y - za - \min(y - za, r_z l_z) | \theta]$$

where $l_z = x - c_0 - m_z$, and where r_z is pinned down by l_z via the zero profit condition

$$l_z = E[\min(y - za, r_z l_z)]. \quad (24)$$

The non-participation payoff for type θ is $V(\theta, \mathcal{O}(\tilde{r})) = E[y - \min(y, \tilde{r}l_0) | \theta]$ with $l_0 = x - c_0$. The participation constraint for type θ is

$$V(\theta, \mathcal{A}(\theta)) \geq V(\theta, \mathcal{O}(\tilde{r})). \quad (25)$$

The investment constraint is

$$V(\theta, \mathcal{A}(\theta)) \geq (1 - z) E[a|\theta] + c_0 + m_z. \quad (26)$$

The government chooses z and m_z to minimize

$$\Psi_{\Pi}^A = m_z - zE[a], \quad (27)$$

subject to participation, investment constraints for all banks and to the interest rate equilibrium condition (24). As in the proof of Proposition 4 we can show that the participation constraint for the good type, implies the participation constraint for the bad type. Moreover, since $r_z(m_z) \leq r_B(m_z)$, the investment constraint for bad banks is always satisfied. Then, the solution depends on whether

(25) binds for $\theta = G$, or whether (26) for $\theta = G$ binds. First we look at the case where the participation constraint for good banks binds, whereas the investment constraints are slack. In this case the government can set $Z = 0$ and $m_z = \hat{m}$ as for the equity injection analyzed above. The program achieves the lower bound on costs

$$\Psi_{\Pi}^A = \Psi_{\Pi}^* = \hat{m}.$$

However, this program is not feasible if the investment constraint is violated at \hat{m} . In that case, the government sets m_z so that the investment constraint is satisfied with equality. This cash level is higher than \hat{m} and the government chooses a strictly positive Z to make the participation constraint bind for the good type. As in the case of equity, the participation constraint for the bad type is slack and the cost of equity injection does not reach the lower bound:

$$\Psi_{\Pi}^A - \Psi_{\Pi}^* = (1 - \pi) (V(B, \mathcal{A}(B)) - V(B, \mathcal{O}(\tilde{r}))) > 0.$$

D.3 Comparisons

We want to compare the costs of equity injections and asset buybacks. When the outside option is very high the required cash injection is high and the investment constraint is slack. In this case, it is optimal to do a pure cash injection program, and the two interventions are (trivially) equivalent. The interesting case is when the investment constraint binds. In this case, the government always chooses a non zero value for α and z , and the comparison of the costs $\Psi_{\Pi}^{\mathcal{E}}$ and Ψ_{Π}^A is not trivial. From our previous propositions, we know that since the good type participation constraint always binds, the failure to reach the minimum cost is driven by the gap in the participation constraint of the bad type

$$\begin{aligned} \Psi_{\Pi}^{\mathcal{E}} - \Psi_{\Pi}^* &= (1 - \pi) (V(B, \mathcal{E}(B)) - V(B, \mathcal{O}(\tilde{r}))) \text{ and} \\ \Psi_{\Pi}^A - \Psi_{\Pi}^* &= (1 - \pi) (V(B, \mathcal{A}(B)) - V(B, \mathcal{O}(\tilde{r}))), \end{aligned}$$

Then

$$\begin{aligned} \Psi_{\Pi}^A > \Psi_{\Pi}^{\mathcal{E}} &\Leftrightarrow V(B, \mathcal{A}(B)) > V(B, \mathcal{E}(B)) \\ &\Leftrightarrow E[y - za - \min(y - za, r_z l_z) | B] > (1 - \alpha) E[y - \min(y, r_{\alpha} l_{\alpha}) | B] \\ &\Leftrightarrow \Xi^{\mathcal{E}} \geq \Xi^A \end{aligned}$$

where

$$\begin{aligned} \Xi^{\mathcal{E}} &\equiv (1 - \alpha) E[\min(y, r_{\alpha} l_{\alpha}) | B] + \alpha E[y | B] \\ \Xi^A &\equiv E[\min(y - za, r_z l_z) | B] + z E[a | B] \end{aligned}$$

Let $V(B, \mathcal{O}(\tilde{r})) = V_G^{out}$. When V_G^{out} is high enough we know that $\Psi_{\Pi}^A = \Psi_{\Pi}^{\mathcal{E}}$. In a separate appendix we show that $\Xi^{\mathcal{E}} \geq \Xi^A$ when V_G^{out} goes down, establishing for low values of V_G^{out} , we have that $\Psi_{\Pi}^A > \Psi_{\Pi}^{\mathcal{E}}$, that is equity is cheaper.²³

E Optimal Menus: Proof of Proposition 10

E.1 Equity Menus

The revelation principle implies that without loss we can assume that each program consists of an option for good banks and an option for bad banks:

$$\alpha_G, m_G \text{ and } \alpha_B, m_B.$$

²³All missing details can be found in a separate document titled *Technical Appendix for "Optimal Interventions in Markets with Adverse Selection."*

Then, the participation payoff for type θ bank is $V(\theta, \mathcal{P}_\theta) = (1 - \alpha_\theta) E [a + v - y^l(y, r_\theta l_{m_\theta}) | \theta]$, where $l_{m_\theta} = x - c_0 - m_\theta$, whereas the non-participation payoff for type θ is $V(\theta, \mathcal{O}(\tilde{r})) = E [a + v - y^l(y, \tilde{r}l_0) | \theta]$, where $l_0 = x - c_0$. Given that all types face a fair interest rate when they participate and choose the option corresponding to their type, the investment constraints is irrelevant here. The constraints are

$$\begin{aligned}
IC_B & : (1 - \alpha_B) E [a + v - y^l(y, r_B l_{m_B}) | B] \geq (1 - \alpha_G) E [a + v - y^l(y, r_G l_{m_G}) | B] \\
IC_G & : (1 - \alpha_G) E [a + v - y^l(y, r_G l_{m_G}) | G] \geq (1 - \alpha_B) \max \left\{ E [a | G] + c_0, E [a + v - y^l(y, r_B l_{m_B}) | G] \right\} \\
PC_G & : (1 - \alpha_G) E [a + v - y^l(y, r_G l_{m_G}) | G] \geq E [a + v - y^l(y, \tilde{r}l_0) | G] \\
PC_B & : (1 - \alpha_B) E [a + v - y^l(y, r_B l_{m_B}) | B] \geq E [a + v - y^l(y, \tilde{r}l_0) | B] \\
\alpha_G & : \alpha_G \geq 0 \\
\alpha_B & : \alpha_B \geq 0
\end{aligned}$$

Observe that the incentive constraint for good banks depends on whether m_B is high enough to make them willing to invest even though, when they deviate they are perceived as bad banks and face an interest rate of r_B . Hence, depending on the

$$\max \left\{ E [a | G] + c_0, E [a + v - y^l(y, r_B l_{m_B}) | G] \right\}$$

there are two cases to consider. Notice that m_B is one of the unknowns, so a good approach seems to be to solve the problem in each case and check which one is internally consistent. In what follows we show that the solution is the same in both cases.

Suppose that good banks invest when they choose the option for the bad banks, that is $E [a + v - y^l(y, r_B l_{m_B}) | G] = \max \{ E [a | G] + c_0, E [a + v - y^l(y, r_B l_{m_B}) | G] \}$. Consider the following menu: $\alpha_G^* = 0$, $\alpha_B^* = 0$, and m_G, m_B are such that $y^l(y, r_G l_{m_G}) = y^l(y, \tilde{r}l_0) = y^l(y, r_B l_{m_B})$. Notice that in this menu, since $r_B \neq r_G$, we have that $m_G \neq m_B$ which is necessary in order to achieve separation. With such a menu we have that the incentive and the participation constraints of both types hold with equality. Hence this menu is feasible and it achieves the minimal cost for the government. Notice also that in deriving this program we have assumed that good banks invest when they choose the option for the bad banks. This is indeed the case, which follows from the fact that $y^l(y, r_B l_{m_B}) = y^l(y, \tilde{r}l_0)$ and from Assumption A6.²⁴

The proofs for the case of asset buybacks and the case debt guarantees are analogous and can be found in a separate document titled *Technical Appendix for "Optimal Interventions in Markets with Adverse Selection."*

²⁴All missing details available upon request.

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