Rollover Risk and Credit Risk[*]

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January 2010

Abstract

This paper models a firm's rollover risk generated by conflict of interest between debt and equity holders. When the firm faces losses in rolling over its maturing debt, its equity holders are willing to absorb the losses only if the option value of keeping the firm alive justifies the cost of paying off the maturing debt. Our model shows that both deteriorating market liquidity and shorter debt maturity can exacerbate this externality and cause costly firm bankruptcy at higher fundamental thresholds. Our model provides implications on liquidity-spillover effects, the flight-to-quality phenomenon, and optimal debt maturity structures.

Keywords: Short-term Debt Crisis, Endogenous Default, Flight to Quality, Liquidity Spillover, Debt Maturity Structure

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1 Introduction

The credit crisis of 2007/2008 highlights the severe rollover risk faced by financial institutions. Since financial institutions rely heavily on short-term debt to finance their illiquid long-term investment positions, they face the risk that they might not be able to roll over maturing debt when market conditions deteriorate, either for fundamental or liquidity reasons. In fact, failure to roll over maturing debt is the direct cause of the collapses of both Bear stearns and Lehman Brothers.[1] There is now an emerging literature analyzing rollover risk. Morris and Shin (2004, 2009) study rollover risk through a coordination problem between short-term creditors. He and Xiong (2009) show that fear of a firm's future rollover risk can lead creditors to preemptively run ahead of others. Acharya, Gale, and Yorulmazer (2009) show that when current asset owners and future buyers are all short of capital, high rollover frequency can lead to diminishing debt capacity of risky assets. These models all rely on an implicit assumption that firms are constrained from raising more equity during financial distress. However, this assumption is unrealistically strong as many firms were able to raise equity despite the recent credit crisis.

In this paper, we analyze the effect of rollover risk on firms' credit risk in a setting where firms can freely raise equity at market prices to absorb losses from rolling over maturing debt. Our model shows that despite this additional source of financing, firms are still exposed to rollover risk because of the intrinsic conflict of interest between equity and debt holders. When a firm faces a large loss from rolling over its maturing debt, any equity injection represents a bailout of maturing debt holders at the expense of equity holders. Equity holders are willing to do so to the extent that the equity value is positive, i.e., the option value of keeping the firm alive justifies the cost of absorbing the rollover losses. Our emphasis of this conflict is consistent with the observation that during the recent crisis many financial firms paid a substantial amount of dividends despite their financial distress and angry creditors; see, e.g., Scharfstein and Stein (2008). It also echoes Flannery (2005) and Duffie (2009), who highlight debt overhang as a crucial obstacle to recapitalizing banks and financial institutions in the aftermath of various historic financial crises, as well as the recent credit crisis.

We build on the structural credit risk model of Leland (1994, 1998) and Leland and Toft (1996). Ideal for our research question, this framework endogenously determines a firm's credit risk jointly with its equity valuation. Specifically, we allow a firm in Leland and Toft (1996) to use a debt structure mixed with fixed fractions of long-term and short-term debt. For each class of debt, the maturities of individual bonds are uniformly spread out over time.

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[1] See Brunnermeier (2009), Duffie (2009), Gorton (2009) and Krishnamurthy (2009) for comprehensive descriptions of the recent financial crisis.

When a bond matures, the firm issues a new bond with the same face value and maturity to replace it at market prices. As the bond price fluctuates over time, the firm faces rollover gains/losses, which are absorbed by its equity holders. In other words, rollover gains are paid out to the equity holders, while losses are paid off by issuing more equity at market prices. The equity price is determined by the firm's fundamental and expected future rollover gains/losses. The firm defaults endogenously when the equity value drops to zero.

Additionally, we incorporate an illiquid bond market into our model by assuming that investors need to pay a proportional cost when selling a bond, and that the cost for selling a long-term bond is higher than that for selling a short-term bond.[2] To capture the idea that investors can internalize the effect of market illiquidity by self-selecting the most appropriate bonds for themselves based on their liquidity needs, we follow Amihud and Mendelson (1986) by allowing two types of bond investors. Type-H investors are more likely to liquidate their bond positions by exogenous liquidity shocks, while type-L investors are less likely. In equilibrium, type-H investors self-select to hold the more liquid short-term bonds, while type-L investors are indifferent between holding the short-term and long-term bonds. The equilibrium liquidity premia of these bonds are not only determined by their marginal investors' expected future trading costs, but also by additional premia required to compensate them for forgoing the returns that they could earn from holding other bonds.

Our model allows us to analyze two key factors—market liquidity and debt maturity—in driving the credit risk of many financial institutions in the recent credit crisis. We show that, even in the absence of any constraint on the firm's ability to raise more equity, the deterioration of bond market liquidity can cause the firm to default at a higher fundamental threshold due to the surge in the firm's rollover losses. In other words, the deterioration of market liquidity increases not only the liquidity premium of the firm's bonds, but also its default probability. This mechanism originates from the intrinsic conflict of interest between debt and equity holders, and is similar in spirit to the debt overhang problem coined by Myers (1977). Equity holders have to bear all rollover losses to avoid bankruptcy, while maturing debt holders get paid in full. This unequal sharing of losses makes the equity value sensitive to the drop in bond prices and ultimately leads to costly bankruptcy at a high fundamental threshold. The premature bankruptcy reflects an important form of externality within the firm: equity holders only consider their own value when choosing whether to default, even though the bankruptcy is costly to both equity and debt holders.[3]

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[2] These assumptions are consistent with the recent empirical studies of corporate bond market liquidity, e.g., Bessembinder, Maxwell, and Venkataraman (2006), Edwards, Harris, and Piwowar (2007), Mahanti et al (2008), and Bao, Pan, and Wang (2009).

[3] This externality complements other types of externalities in debt crises. For example, when some

Shorter debt maturity can further exacerbate this externality. As we observed in the recent credit crisis, financial firms increasingly rely on overnight commercial paper and repo transactions, two extreme forms of short-term debt financing, to fund their investment positions. At this rapid rollover frequency, the equity holders are forced to immediately absorb any loss incurred to the firm's debt financing. This inflexibility reduces their option value of keeping the firm alive and thus causes the firm to default at a higher fundamental level. Our model thus highlights firms' debt maturity structure as another important determinant of credit risk, in addition to the widely recognized leverage effect.

The standard credit risk models, following Merton (1974), focus on firms' insolvency risk, i.e., the risk that firms' fundamental values could fall below their liability levels. However, a growing number of empirical studies, e.g., Collin-Dufresne, Goldstein, and Martin (2001), Longstaff, Mithal, and Neis (2005), Ericsson and Renault (2006), and Chen, Lesmond, and Wei (2007), find that liquidity is an important factor in firms' credit spreads. While these studies typically interpret this finding as a liquidity-premium effect,[4] our model highlights that through the rollover-risk channel, an increase in liquidity premium also feeds back to firms' endogenous default thresholds and causes higher default probabilities. This implication cautions against the common practice of decomposing credit spreads as simple sums of default premia and liquidity premia, and justifies the large liquidity-premium effect documented in the empirical studies. Moreover, it suggests time-varying market liquidity as an explanation for the common latent factors discovered by Duffie et al (2009) in firms' default probabilities.

The profound impact of market liquidity on firms' default risk illustrated by our model justifies the increasingly prevalent view among industry practitioners that credit ratings are an inadequate measure of credit risk. Rating agencies assign credit ratings largely based on firms' insolvency risk. In contrast, the 2008 Credit Assessment Guidelines, jointly issued by European Fund and Asset Management Association, European Securitization Forum, and Investment Management Association, caution against over-reliance of asset managers on credit ratings because of their failure to account for other important factors, such as liquidity risk:

> "Credit ratings are incomplete descriptions of riskiness. Credit ratings may be
> assessments of creditworthiness, but they are not assessments of the level of liq-

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creditors of a firm choose to run on the firm, their decision imposes a cost on other creditors by making the firm more likely to fail, e.g., He and Xiong (2009). Asset liquidations of some market participants also impose a cost on other participants holding the same assets, because the liquidations suppress the asset prices and thus weaken the financing capacities of the assets for other participants, e.g., Brunnermeier and Pedersen (2009) and Lorenzoni (2008).

[4]The only exception is Ericsson and Renault (2006), who develop a model to show that market illiquidity could also affect the renegotiation between debt and equity holders upon bankruptcy.

uidity, market or rating volatility risk nor should ratings be exclusively relied on for valuation purposes."

To make credit ratings a more efficient measure of credit risk, our model also suggests that they need to incorporate the so-called "maturity risk", i.e., an effect generated by firms' debt maturity structure on their credit risk, which we will discuss in Section 5.

Our model also features separate clienteles for the long-term and short-term bonds, endogenously determined by investors' heterogeneous liquidity needs and bonds' heterogeneous trading costs. This is consistent with a common observation that the markets for bonds with different maturities are highly segmented with different investor clienteles. Furthermore, our model highlights a rich set of channels for liquidity shocks to spill across these different debt market segments. For example, when type-H investors (who hold short-term bonds in equilibrium) become exposed to a higher probability of liquidity shocks, the liquidity premium of the short-term bonds rises. This, in turn, causes the liquidity premium of the long-term bonds to rise because the type-L investors' opportunity cost also rises with the liquidity premium of the short-term bonds. The increased liquidity premia of both short-term and long-term bonds also push up the firm's endogenous bankruptcy threshold, through which the credit spreads of the long-term bonds rise further.

Our model provides a new explanation for the widely observed flight-to-quality phenomenon: after major market liquidity disruptions, the prices of low quality bonds drop much more than those of high quality bonds. It is intuitive that a liquidity breakdown, by pushing down bond prices and raising firms' endogenous default thresholds, has a greater impact on the credit spreads and default probabilities of firms with weaker fundamentals, because they are closer to the default boundary. In contrast with the existing explanations of this phenomenon based on investors' investment constraints and preferences, e.g., Vayanos (2004), He and Krishnamurthy (2008), and Caballero and Krishnamurthy (2008), our model predicts that a surge in investor demand for market liquidity leads to not only higher liquidity premia in bond prices, but also higher bond default probabilities.

In our model, there are two opposing forces in determining a firm's optimal maturity structure. On one hand, more short-term debt leads to a higher expected bankruptcy cost because of increased rollover risk and higher future bankruptcy probability. On the other hand, short-term debt has a lower financing cost, which is consistent both with the aforementioned empirical finding that short-term bonds tend to be more liquid and with the theoretical argument that short-term debt is less information sensitive and thus suffers less from adverse-selection problems, e.g., Gorton and Pennacchi (1990). Our model suggests that firms with lower asset volatility, higher bankruptcy recovery rates, and higher secondary-market debt

liquidity tend to use a greater fraction of short-term debt. Our focus on market liquidity and future financial stability is different from the existing theories of optimal debt maturity based on the disciplinary role of short-term debt in preventing managers' asset substitution, e.g., Flannery (1994) and Leland (1998), and the theories based on private information of borrowers about their future credit ratings, e.g., Flannery (1986) and Diamond (1991). A recent paper by Diamond and He (2010) emphasizes the trade-off between short-term and long-term debt overhang, where investment plays a crucial role.

Our analysis shows that debt maturity structure should be used as part of a firm's liquidity management strategy. Despite its higher cost, long-term debt gives the firm the flexibility to delay realizing financial losses in adverse states, in which either market liquidity or the firm's fundamental deteriorates. This benefit is analogous to the role of cash reserves, the standard tool for risk management, e.g., Holmstrom and Tirole (2001) and Bolton, Chen, and Wang (2009). This implication of our model also echoes Brunnermeier and Yogo (2009), who argue that firms should shift to long-term debt as their fundamentals deteriorate.

The paper is organized as follows. Section 2 presents the model setting. We derive the debt and equity valuation and the firm's endogenous bankruptcy boundary in Section 3. Section 4 discusses the effects of market liquidity on the firm's credit risk, while Section 5 focuses on the effects of debt maturity. We discuss optimal maturity structure in Section 6. Section 7 concludes the paper. Appendix A provides the technical proofs, and Appendix B provides an extended model to analyze a temporary liquidity crisis.

2   The Model

We build on the structural credit risk model of Leland and Toft (1996) with two additional features. First, the firm uses a mix of short-term and long-term bonds, in addition to equity, to finance its operations. Second, and more importantly, bond markets are illiquid. The short-term and long-term bonds have different trading costs, and there are two types of investors facing different liquidity shock intensities. Our setting is generic and applies to both financial and non-financial firms, although the effects illustrated by our model are stronger for financial firms because they tend to have higher leverage and shorter debt maturity.

2.1   Firm Asset

The unlevered firm asset value $\{V_t : 0 \leq t < \infty\}$ follows a geometric Brownian motion in the risk-neutral probability measure:

$$\frac{dV_t}{V_t} = (r - \phi)\, dt + \sigma\, dZ_t, \tag{1}$$

where r is the constant risk-free rate in this economy, ϕ is the firm's constant cash payout rate, σ is the constant asset volatility, and $\{Z_t : 0 \le t < \infty\}$ is a standard Brownian motion. Throughout the paper, we refer to V_t as the firm fundamental.[5]

When the firm goes bankrupt, we assume that creditors can only recover a fraction α of the firm's asset value from liquidation. The liquidation loss $1 - \alpha$ can be interpreted in different ways, such as the loss from selling the firm's real asset to second-best users, loss of customers because of the bankruptcy, asset fire-sale losses, legal fees, etc. An important issue to keep in mind is that the liquidation loss represents a deadweight loss to both debt and equity holders ex ante, but is ex post borne only by debt holders.

2.2 Stationary Debt Structure

The firm maintains two classes of debt with maturities m_1 and m_2, respectively. Without loss of generality, we let class-1 debt to have a shorter maturity, i.e., $m_1 < m_2$. Each class of debt is identical to the one studied in Leland and Toft (1996). At each moment in time, class-i debt has a constant principal P_i outstanding and a constant annual coupon payment of C_i, where $i = 1$ or 2. The expiration of each class of debt is uniformly spread out across time. That is, during a time interval $(t, t + dt)$, a fraction $\frac{1}{m_i} dt$ of class-i debt matures and needs to be rolled over. Given the shorter maturity of class-1 debt, it has to be rolled over at a higher frequency $1/m_1$.

To focus on the firm's debt maturity structure and liquidity effects, we take the firm's total debt principal $P = \sum_{i=1}^{2} P_i$ and total coupon payment $C = \sum_{i=1}^{2} C_i$ as given. By taking the leverage level as given, we ignore many interesting issues related to the tradeoff between tax benefits and bankruptcy cost, which are analyzed by Leland (1994) and other following work such as Goldstein, Ju, and Leland (2001), Strebulaev (2007), and He (2009). For simplicity, we also assume that the principals and coupon payments of the two debt classes are in proportion:

$$C_i = \lambda_i C, \ P_i = \lambda_i P, \tag{2}$$

where λ_i represents the fraction of class-i debt, with $\lambda_1 + \lambda_2 = 1$.

Furthermore, following the Leland framework, we assume that the firm can commit to a stationary debt structure denoted by $(C, P, m_1, m_2, \lambda_1, \lambda_2)$. That is, the firm always main-

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[5]As in Leland (1994), we treat the unlevered firm value process $\{V_t : 0 \le t < \infty\}$ as the exogenously given state variable to focus on the effects of market liquidity and debt maturity. In our context, this approach is equivalent to directly modeling the firm's exogenous cash flow process $\{\phi V_t : 0 \le t < \infty\}$ as the state variable (i.e., the so-called EBIT model advocated by Goldstein, Ju, and Leland (2001)). For instance, Hackbarth, Miao, and Morellec (2006) use this EBIT model framework to analyze the effects of macroeconomic conditions on firms' credit risk.

tains the initially specified debt level represented by C and P, and the maturity structure specified by $(m_1, m_2, \lambda_1, \lambda_2)$. Thus, when a bond matures, the firm replaces it by issuing a new bond with identical maturity, principal value, coupon rate and seniority.

At each instant of time, there exists a continuum of class-i bonds in the firm with remaining time-to-maturity τ ranging from 0 to m_i. We measure these bonds by m_i units. Then each unit has a principal value of (recall (2))

$$p_i = \frac{P_i}{m_i} = \frac{\lambda_i}{m_i} P, \tag{3}$$

and an annual coupon payment of

$$c_i = \frac{C_i}{m_i} = \frac{\lambda_i}{m_i} C. \tag{4}$$

These bonds only differ in the time-to-maturity $\tau \in [0, m_i]$. Denote by $d_i(V_t, \tau)$ the value of one unit of class-i bond as a function of the firm fundamental V_t and its time-to-maturity τ.

The two classes of debt have the same priority in dividing the firm's assets during bankruptcy, i.e., the firm's liquidation value is divided among all debt holders on a pro rata basis. This assumption simplifies complication that long-term bonds are often secured by firm assets in reality. We believe this simplification is innocuous to our main results.

2.3 Debt Rollover and Endogenous Bankruptcy

When the firm pays off maturing bonds by issuing new bonds, the fluctuation in bond prices generates a rollover gain/loss, which needs to be absorbed by equity holders. Specifically, over a short time interval $(t, t + dt)$, the net cash flow to equity holders (omitting dt) is

$$NC_t = \phi V_t - (1 - \pi) C + \sum_{i=1}^{2} \left[d_i(V_t, m_i) - p_i \right]. \tag{5}$$

The first term is the firm's cash payout. The second term is the after-tax coupon payment, where π denotes the marginal corporate tax rate. The third term captures the rollover gain/loss when the firm pays off maturing bonds by issuing new bonds at market prices. In this transaction, dt units of both class-1 and class-2 bonds mature. The maturing class-i bonds require a principal payment of $p_i dt$. The market value of the newly issued bonds with identical principal value and maturity m_i is $d_i(V_t, m_i) dt$, which depends on the firm fundamental V_t and bond maturity m_i. When the bond value $d_i(V_t, m_i) dt$ drops, the equity holders have to absorb the rollover loss $\sum_{i=1}^{2} \left[d_i(V_t, m_i) - p_i \right] dt$ to prevent bankruptcy.

To highlight the role of debt market illiquidity which is introduced in the next section, we assume that the firm can freely raise additional equity to pay for the rollover loss and coupon

payments without any friction, as long as the equity value remains positive.[6,7] In other words, equity holders have the option to keep servicing the debt (coupons and principals) in order to maintain the right to collect the future cash flows generated by the firm. Bankruptcy occurs endogenously when the firm fundamental drops to a critical threshold V_B at which equity value becomes zero. At this point, equity holders are no longer willing to absorb the rollover shortfalls by injecting more capital to meet coupon and principal payments, and the firm goes bankrupt. Then, the short-term and long-term bond holders divide the firm liquidation value αV_B on a pro rata basis.

Under the stationary debt structure specified earlier, the firm's bankruptcy boundary V_B is constant, which we will derive in the next section. As in any trade-off theory, bankruptcy involves a dead-weight loss. Endogenous bankruptcy is a reflection of the conflict of interest between debt and equity holders: when the firm fundamental is weak and bond prices are low, equity holders are not willing to bear the rollover losses to avoid the deadweight loss of bankruptcy. This situation resembles the debt-overhang problem suggested by Myers (1977), as equity holders voluntarily discontinue the firm by refusing to subsidize the maturing debt holders.

2.4 Secondary Bond Markets

We adopt a bond market structure similar to that in Amihud and Mendelson (1986). There are two types of bond investors, type-H and type-L. The two types differ in their demands for market liquidity. Each type-H investor is exposed to an idiosyncratic liquidity shock, which arrives according to a Poisson occurrence with intensity ξ_H. Upon the arrival of the liquidity shock, the bond investor has to exit by selling his bond holding in the secondary market. Each type-L investor is also exposed to an idiosyncratic liquidity shock, which arrives according to a Poisson occurrence with intensity ξ_L. The liquidity shocks are independent across investors. We let $\xi_H > \xi_L$, i.e., the type-H investors have greater liquidity needs than type-L investors. Furthermore, we assume that type-H investors arrive at the market with Poisson intensity λ_H and type-L investors arrive with intensity λ_L. We can intuitively interpret type-H investors as financial institutions facing higher redemption shocks from their investors, such as open-end mutual funds, and type-L investors as institutions with more secured funding, such as

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[6]This assumption is just for illustration and is by no means realistic. In fact, it is arguable that due to informational reasons, debt financing suffers from fewer frictions than equity financing. Besides, we broadly interpret equity issuance to include private placement, which is less vulnerable to informational problems.

[7]The new financing does not need to be equity, and just needs to be junior to existing debt. More specifically, as in the Leland (1994) framework, we can assume that strict bond covenants prevent the firm from increasing the amount of newly issued bonds with equal seniority to existing bonds (as an effort to reduce the rollover losses), because such an increase necessarily hurts the existing debt holders.

hedge funds and closed-end mutual funds.

When an investor suffers a liquidity shock, he sells his bond to the market maker at a fractional cost. The cost for a class-i bond is a fraction β_i of the bond value. In other words, the investor only recovers a fraction $1 - \beta_i$ of the bond value. We shall broadly interpret this cost either as a market impact of trade, e.g., Kyle (1985), or as a bid-ask spread, e.g., Amihud and Mendelson (1986).

As documented by a series of empirical papers, e.g., Bessembinder, Maxwell, and Venkataraman (2006), Edwards, Harris, and Piwowar (2007), Mahanti et al (2008), and Bao, Pan, and Wang (2009), the secondary markets for corporate bonds are highly illiquid. The illiquidity is reflected by a large bid-ask spread that bond investors have to pay in trading with dealers, as well as a potential price impact of the trade. Edwards, Harris, and Piwowar (2007) show that the average effective bid-ask spread on corporate bonds ranges from 8 basis points for large trades to 150 basis points for small trades. Bao, Pan, and Wang (2009) estimate that the average effective trading cost, which incorporates bid-ask spread, price impact and other factors, ranges from 74 to 221 basis points depending on the trade size. There is also large variation across different bonds with the same trade size. In particular, Mahanti et al (2008) and Bao, Pan, and Wang (2009) document an increasing pattern of the net trading cost with respect to bond maturity (the sum of bond age and time-to-maturity). This finding suggests that short-term debt is more liquid than long-term debt.[8] Thus, we impose that $\beta_1 < \beta_2$.

When an investor, either type H or L, arrives at the market, he buys one unit of bonds from the market maker. The investor can specify the class of the bond, i.e., class 1 or 2, but not the time-to-maturity of the bond. The bond is randomly chosen from the market maker's inventory and could have any time-to-maturity from 0 to m_i (for class i). Given the arrival and exit rates of each type of investors, standard results imply that the expected number of type-H investors is λ_H/ξ_H and that of type-L investors is λ_L/ξ_L. Because type-L investors have lower liquidity needs, it is more efficient for type-L investors to hold the less liquid long-term bonds. To make the analysis interesting, we assume that the number of type-L investors is sufficient for holding the long-term bonds but insufficient for absorbing all of the long-term and short-term bonds. We also assume that there is a large set of type-H investors, who are able to absorb all of the short-term bonds in a competitive fashion. Furthermore, both investors can invest in a risk-free bond, which offers a constant return of r without trading cost. The risk-free rate r provides the basis for investors to evaluate returns

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[8]Intuitively, the default probabilities of long-term bonds are usually higher than those of short-term bonds. As a result, there is more uncertainty in valuing long-term bonds. This in turn makes long-term bonds more exposed to asymmetric information problems between market participants, e.g., Gorton and Pennacchi (1990), which tend to make long-term bonds less liquid.

of the long-term and short-term bonds issued by the firm. While we focus on analyzing the bonds issued by one firm, it should be clear that there are many similar bonds issued by different firms in the market. As a result, the default of one firm does not affect the bond market equilibrium.

The bond issuance cost in the primary markets tends to be much lower than the trading cost in the secondary markets. Issuing commercial paper through dealers usually costs about 5 basis points.[9] While the average cost of raising capital through long-term debt is about 220 basis points, e.g., Lee, Lochhead, and Ritter (1996), the effective cost when spread out across the debt maturity, which is typically $5-10$ years, is still low relative to the secondary market trading cost. Thus, we ignore issuance costs in the model.[10]

In summary, our model captures the bond-market liquidity with four parameters: liquidity shock arrival intensities ξ_H and ξ_L, which represent the liquidity demands of the two types of bond investors; and bid-ask spreads β_1 and β_2, which represent the illiquidity of the short-term and long-term bonds. We derive an equilibrium in which type-L investors are the marginal investor of the less liquid long-term bonds, while the type-H investors are the marginal investors in the more liquid short-term bonds. In our later analysis, we use an unexpected rise in the value of ξ_H to analyze the effects of a surge in liquidity premium after a market disruption on the credit spreads of the long-term and short-term bonds. We focus on ξ_H because institutions with less secured financing, such as open-end mutual funds, are more exposed to market disruptions.

3   Valuation and Bankruptcy Boundary

In this section, we derive the debt and equity valuation and the firm's endogenous bankruptcy boundary.

3.1   Debt Market Equilibrium

We first derive the debt market equilibrium, in which the type-L and type-H investors are the marginal investors of the short-term and long-term bonds, respectively. In this subsection, we take the firm's bankruptcy boundary V_B as given.

Recall that $d_i(V_t, \tau; V_B)$ is the value of one unit of class-i bonds with time-to-maturity

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[9]See the Wikipedia website at http://en.wikipedia.org/wiki/Commercial_paper for more background information on commercial paper.

[10]It is easy to incorporate issuance costs in our model. Of course, the presence of issuance costs favors long-term debt over short-term debt, as short-term debt requires more frequent rollovers, e.g., Dangl and Zechner (2007).

$\tau < m_i$. We have the following standard partial differential equation for the bond value:

$$r_i d_i = -\frac{\partial d_i}{\partial \tau} + (r - \phi) V_t \frac{\partial d_i}{\partial V} + \frac{1}{2} \sigma^2 V_t^2 \frac{\partial^2 d_i}{\partial V^2} + c_i. \tag{6}$$

The left-hand side $r_i d_i$ is the required (dollar) return from the bond. As we will show in Proposition 1, in equilibrium the required rate of return r_i is constant for both types of bonds and higher than the risk-free interest rate r. The difference $r_i - r$ represents a liquidity premium, which, as we will show shortly, consists of two components. One component compensates the bond's marginal investor for his expected future trading cost; the other for his opportunity cost of capital.

There are four terms on the right-hand side, capturing expected changes in the bond value caused by the change in the time-to-maturity (the first term), the fluctuation in the firm's asset value V_t (the second and third terms), and the coupon payment (the fourth term).

Required Bond Returns We now give a proposition for the expected returns of the short-term and long-term bonds.

Proposition 1 *Suppose that the number of type-L investors is sufficient for holding the long-term bonds but insufficient for absorbing all of the long-term and short-term bonds, and that the number of type-H investors is sufficient for absorbing the short-term bonds. Further suppose that $\xi_L > \xi_H \beta_1$, which guarantees that the type-L investors' value function is bounded in equilibrium. Then, type-H investors only hold the short-term bonds and are the marginal investors in these bonds with a required return of*

$$r_1 = r + \xi_H \beta_1. \tag{7}$$

Type-L investors are the marginal investors in the long-term bonds and are indifferent between holding the long-term and short-term bonds. Their required return from the long-term bonds is

$$r_2 = r + \xi_H \beta_1 + \frac{\beta_2 - \beta_1}{1 - \beta_1} \left(\xi_L - \xi_H \beta_1 \right). \tag{8}$$

This proposition is in the same spirit as the endogenous liquidity clienteles derived in Amihud and Mendelson (1986), although the derivation is more involved due to the complication in dealing with re-investment of coupon and principal payments.[11] Due to their

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[11]In the setting of Amihud and Mendelson (1986), assets pay constant dividends and, as a result, always trade at constant prices. Since investors in their model always consume the dividends, there is no re-investment problem.

heterogeneous liquidity needs, the lower (higher) liquidity needs of type-L (type-H) investors make them the more efficient holder of the less (more) liquid long-term (short-term) bonds. In equilibrium, the limited number of type-L investors implies that type-L (type-H) investors are the marginal investors in the long-term (short-term) bonds.

Since there is a large number of type-H investors, they only earn an expected return of r on their capital by holding the more liquid short-term bonds. In other words, the type-H investors' outside option is investing in the risk-free bonds with return r. This implies that the required return of the short-term bonds r_1 is simply the risk-free rate r plus a premium $\xi_H \beta_1$, which compensates the type-H investors for their expected future trading cost.

The reasoning for type-L investors is more interesting. It is important to observe that type-L investors, who are the marginal investors in long-term bonds, have a better outside option than that offered by the risk-free bond; they can invest in short-term bonds. Since they have lower liquidity needs than type-H investors, type-L investors can earn a higher expected return than r by investing in short-term bonds. As a result, the required return of the long-term bonds needs to compensate the type-L investors' forgone opportunity from investing in the short-term bonds, in addition to the risk-free rate r and the expected future trading cost of long-term bonds $\xi_L \beta_2$. According to equation (8), this additional premium is given by

$$r_2 - (r + \xi_L \beta_2) = \frac{1 - \beta_2}{1 - \beta_1} \beta_1 \left(\xi_H - \xi_L \right) > 0.$$

Individual Bond Value   We have two boundary conditions to pin down the bond prices based on equation (6). At the bankruptcy boundary V_B, the bond holders share the firm's liquidation value proportionally. Thus, each unit of class-i bond gets

$$d_i \left(V_B, \tau; V_B \right) = \frac{P_i/m_i}{P} \alpha V_B = \frac{1}{m_i} \lambda_i \alpha V_B, \text{ for all } \tau \in [0, m_i] . \tag{9}$$

When $\tau = 0$, the bond matures and the bond holder gets the principal value p_i:

$$d_i \left(V_t, 0; V_B \right) = p_i, \text{ for all } V_t > V_B. \tag{10}$$

Similar to Leland and Toft (1996), we can solve for the individual bond value $d_i \left(V_t, \tau; V_B \right)$ based on equation (6) and boundary conditions (9) and (10). Let

$$v_t \equiv \ln \left(\frac{V_t}{V_B} \right). \tag{11}$$

We have

$$d_i \left(V_t, \tau; V_B \right) = \frac{c_i}{r_i} + e^{-r_i \tau} \left[p_i - \frac{c_i}{r_i} \right] \left(1 - F \left(\tau \right) \right) + \left[\frac{1}{m_i} \lambda_i \alpha V_B - \frac{c_i}{r_i} \right] G_i \left(\tau \right), \tag{12}$$

13

where

$$F(\tau) = N(h_1(\tau)) + \left(\frac{V_t}{V_B}\right)^{-2a} N(h_2(\tau)); \tag{13}$$

$$G_i(\tau) = \left(\frac{V_t}{V_B}\right)^{-a+z_i} N(q_{1i}(\tau)) + \left(\frac{V_t}{V_B}\right)^{-a-z_i} N(q_{2i}(\tau));$$

$$h_1(\tau) = \frac{(-v_t - a\sigma^2\tau)}{\sigma\sqrt{\tau}}; h_2(\tau) = \frac{(-v_t + a\sigma^2\tau)}{\sigma\sqrt{\tau}};$$

$$q_{1i}(\tau) = \frac{(-v_t - z_i\sigma^2\tau)}{\sigma\sqrt{\tau}}; q_{2i}(\tau) = \frac{(-v_t + z_i\sigma^2\tau)}{\sigma\sqrt{\tau}};$$

$$a = \frac{r - \phi - \sigma^2/2}{\sigma^2}; z_i = \frac{[a^2\sigma^4 + 2r_i\sigma^2]^{1/2}}{\sigma^2};$$

and $N(x) \equiv \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$ is the cumulative standard normal distribution.

Bond Yield The bond yield is typically computed as the equivalent return on a bond conditional on it being held to maturity without default or trading. Given the bond price derived in equation (12), the bond yield y_i is determined by solving the following equation:

$$d_i(V_t, m_i) = \frac{c_i}{y_i} \left(1 - e^{-y_i m_i}\right) + p_i e^{-y_i m_i} \tag{14}$$

where the right-hand side is the price of a bond with a constant cash flow c_i over time and a principal payment p_i at the bond maturity, conditional on no default or trading before maturity. The spread between y_i and the risk-free rate r is often called the credit spread of the bond. Since the bond price in equation (12) contains the effects of trading cost and bankruptcy cost, the credit yield contains a liquidity premium and a default premium. The focus of our analysis is to uncover the intricate interaction between the liquidity and default premia.

3.2 Equity Value and Endogenous Bankruptcy Boundary

Leland (1994, 1998) and Leland and Toft (1996) indirectly derive the equity value as the difference between the total firm value and the debt value. The total firm value is the unlevered firm value V_t, plus the total tax-benefit, minus the bankruptcy cost. This approach does not apply to our model because part of the total firm value is consumed by future trading costs. Thus, we directly compute the equity value $E(V_t)$ through the following ordinary differential equation:

$$rE = (r - \phi)V_t E_V + \frac{1}{2}\sigma^2 V_t^2 E_{VV} + \phi V_t - (1 - \pi)C + \sum_i [d_i(V_t, m_i) - p_i]. \tag{15}$$

14

The left-hand side is the required return for the equity holders. This term should be equal to the expected increment in the equity value, which is the sum of the terms on the right-hand side.

- The first two terms $(r - \phi) V_t E_V + \frac{1}{2}\sigma^2 V_t^2 E_{VV}$ capture the expected change in the equity value caused by the fluctuation in the firm's asset value V_t.

- The third term ϕV_t is the cash flow generated by the firm per unit of time.

- The fourth term $(1 - \pi) C$ is the after-tax coupon payment per unit of time.

- The fifth term $\sum_i [d_i (V_t, m_i) - p_i]$ is the equity holders' rollover gain/loss from paying off the maturing bonds by issuing new bonds at their market values.

Limited liability of equity holders provides the following boundary condition at V_B:

$$E(V_B) = 0. \tag{16}$$

Solving the differential equation in (15) is challenging because it contains the complicated bond valuation function $d_i (V_t, m_i)$ given in (12). We manage to solve this equation using the Laplace transformation technique detailed in Appendix A.2. Based on the equity value, we then derive the equity holders' endogenous bankruptcy boundary V_B based on the smooth-pasting condition that

$$E'(V_B) = 0.$$

Chen and Kou (2009) provide a rigorous proof of the optimality of the smooth-pasting condition in an endogenous-default model under a set of general conditions, which include finite debt maturity and a jump-and-diffusion process for the firm's unlevered asset value.

The results are summarized in the next proposition.

Proposition 2 *Recall $v_t = \ln (V_t/V_B)$ defined in (11). The equity value $E(V_t)$ is*

$$
\begin{aligned}
E(V_t) &= V_t - \frac{\phi V_B}{z\sigma^2} \frac{e^{-\gamma v_t}}{\gamma + 1} - \frac{(1-\pi)C + \sum_{i=1}^{2}(1 - e^{-r_i m_i})\left[\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right]}{z\sigma^2}\left[\frac{1}{\eta} + \frac{(1 - e^{-\gamma v_t})}{\gamma}\right] \\
&+ \frac{1}{z\sigma^2}\sum_{i=1}^{2}\left\{ \begin{array}{l} e^{-r_i m_i}\left(\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right)[A_i(-a) + A_i(a)] \\ -\left(\frac{\lambda_i a V_B}{m_i} - \frac{C_i}{r_i m_i}\right)[A_i(-z_i) + A_i(z_i)] \end{array}\right\},
\end{aligned} \tag{17}
$$

where $a \equiv \frac{r - \phi - \sigma^2/2}{\sigma^2}$, $z \equiv \frac{(a^2\sigma^4 + 2r\sigma^2)^{1/2}}{\sigma^2}$, $\gamma \equiv a + z > 0$, $\eta \equiv z - a > 1$, and $A_i(\cdot)$ is defined as

$$A_i(x) \equiv \frac{1}{z + x}(K_i(v_t; a, -x, \gamma) + k_i(v_t; a, x, -\eta)),$$

15

where

$$K_i\left(v_t; a, x, \gamma\right) \equiv \left\{N\left(x\sigma\sqrt{m_i}\right) - e^{\frac{1}{2}\left(z^2 - x^2\right)\sigma^2 m_i} N\left(z\sigma\sqrt{m_i}\right)\right\} e^{-\gamma v_t}$$

$$+ e^{\frac{1}{2}\left(z^2 - x^2\right)\sigma^2 m_i} e^{-\gamma v_t} N\left(\frac{-v_t + z\sigma^2 m_i}{\sigma\sqrt{m_i}}\right) - e^{-(a+x)v_t} N\left(\frac{-v_t + x\sigma^2 m_i}{\sigma\sqrt{m_i}}\right),$$

and

$$k_i\left(v_t; a, x, -\eta\right) \equiv e^{\frac{1}{2}\left(z^2 - x^2\right)\sigma^2 m_i} e^{\eta v_t} N\left(\frac{-v_t - z\sigma^2 m_i}{\sigma\sqrt{m_i}}\right) - e^{-(a+x)v_t} N\left(\frac{-v_t + x\sigma^2 m_i}{\sigma\sqrt{m_i}}\right).$$

The endogenous bankruptcy boundary V_B is given by

$$V_B = \frac{\frac{(1-\pi)C + \sum_{i=1}^{2}\left(1 - e^{-r_i m_i}\right)\left(\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right)}{\eta} + \sum_{i=1}^{2}\left\{\begin{array}{c}\left(\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right)\left[b_i\left(-a\right) + b_i\left(a\right)\right] \\ + \frac{C_i}{r_i m_i}\left[B_i\left(-z_i\right) + B_i\left(z_i\right)\right]\end{array}\right\}}{\frac{\phi}{\eta-1} + \sum_{i=1}^{2}\frac{1}{m_i}\lambda_i \alpha\left[B_i\left(-z_i\right) + B_i\left(z_i\right)\right]}, \tag{18}$$

where

$$b_i\left(x\right) = \frac{1}{z+x}e^{-r_i m_i}\left[N\left(x\sigma\sqrt{m_i}\right) - e^{rm_i}N\left(-z\sigma\sqrt{m_i}\right)\right],$$

$$B_i\left(x\right) = \frac{1}{z+x}\left[N\left(x\sigma\sqrt{m_i}\right) - e^{\frac{1}{2}\left[z^2 - x^2\right]\sigma^2 m_i}N\left(-z\sigma\sqrt{m_i}\right)\right].$$

4 Market Liquidity and Credit Risk

In this section, we analyze the effects of deteriorating bond market liquidity on firms' credit spreads based on the model derived in the previous section. We first show that deteriorating market liquidity can exacerbate the conflict of interest between debt and equity holders and lead to higher default risk. We then analyze the spillover of liquidity shocks across different debt market segments. Finally, we discuss the flight-to-quality phenomenon caused by the varying impact of a market liquidity shock on different firms.

To facilitate our discussion, we use a set of baseline parameters:

$$r = 10\%, \ \phi = 3\%, \ \alpha = 0.5, \ \pi = 35\%, \ \sigma = 7\%, \ V_0 = 100, \ P = 90,$$

$$C = 9, \ m_1 = 0.25, \ m_2 = 5, \ \beta_1 = 0.2\%, \ \beta_2 = 2\%, \ \xi_H = 1, \ \xi_L = 0.8. \tag{19}$$

We choose the risk-free rate r to be 10%, the cash payout rate ϕ to be 3%, the firm's liquidation recovery rate α to be 50%, the tax rate π to be 35%, and the asset volatility σ to be 7%. These values are fairly standard, except that σ is on the low end relative to the value used by Leland. We choose a small volatility because we intend to analyze a financial firm

which uses high leverage to finance a relatively safe investment position. We let the initial value of the firm asset to be 100, the total principal value of the firm's debts P to be 90, and the total annual coupon payment C to be 9. These values imply that the firm's initial leverage is 73%. We choose the short-term debt maturity m_1 to be 3 months (commercial paper) and the long-term debt maturity m_2 to be 5 years (long-term corporate bond). We set their trading costs β_1 and β_2 to be 0.2% and 2%. These values are consistent with the empirical estimates of Bao, Pan, and Wang (2009). Finally, we let the arrival rate of liquidity shocks to the type-H and type-L bond holders ξ_H and ξ_L to be 1 and 0.8, which are broadly consistent with the average turnover rate of the corporate bonds in the data sample of Bao, Pan, and Wang (2009). For the analysis in this section with unexpected liquidity shocks, we treat λ_1 as exogenously given and fix it at a value of 42.8%, which is the optimal value we derive in Section 6. This value is close to the fraction of short-term debt of a typical financial firm in the Compustat data base.[12] Under this set of parameters, the firm's optimal default boundary V_B is 87.1.

4.1   Market Liquidity and Endogenous Defaults

Deteriorating bond market liquidity can exacerbate the conflict of interest between debt and equity holders when the firm is in financial distress. As we see in equation (5), when deteriorating liquidity pushes the market prices of the firm's newly issued bonds below their principal values, equity holders have to absorb the rollover loss in paying off the maturing debt:

$$\sum_{i=1}^{2} \left(d_i \left(V_t, m_i; \xi_H, \xi_L \right) - p_i \right),$$

where we write d_i as a function of ξ_H and ξ_L to emphasize the dependence of the rollover loss on the investors' demands for bond market liquidity. As a result, when the rollover loss becomes sufficiently large, equity holders will choose to stop servicing the debt even if the falling bond prices are caused by liquidity reasons.

To illustrate the effects of a liquidity shock, we conduct the following thought experiment. Suppose that the arrival rate of liquidity shocks to the type-H bond holders ξ_H has an unexpected increase from its baseline value 1. One can broadly interpret the unexpected increase in ξ_H as a surge in the demand for liquidity by institutions with less secured financing (e.g., open-end mutual funds) after a major market disruption, such as the recent failure of Lehman Brothers or the crisis of LTCM. After the shock, investors will demand a higher liquidity premium and drive down the bond prices. To analyze the effects of the shock,

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[12]For financial firms in the Markit CDS database, the average fraction of short-term debt was 44% right before the failure of Bear Stearns in March 2009. We thank Haitao Li for providing this number.
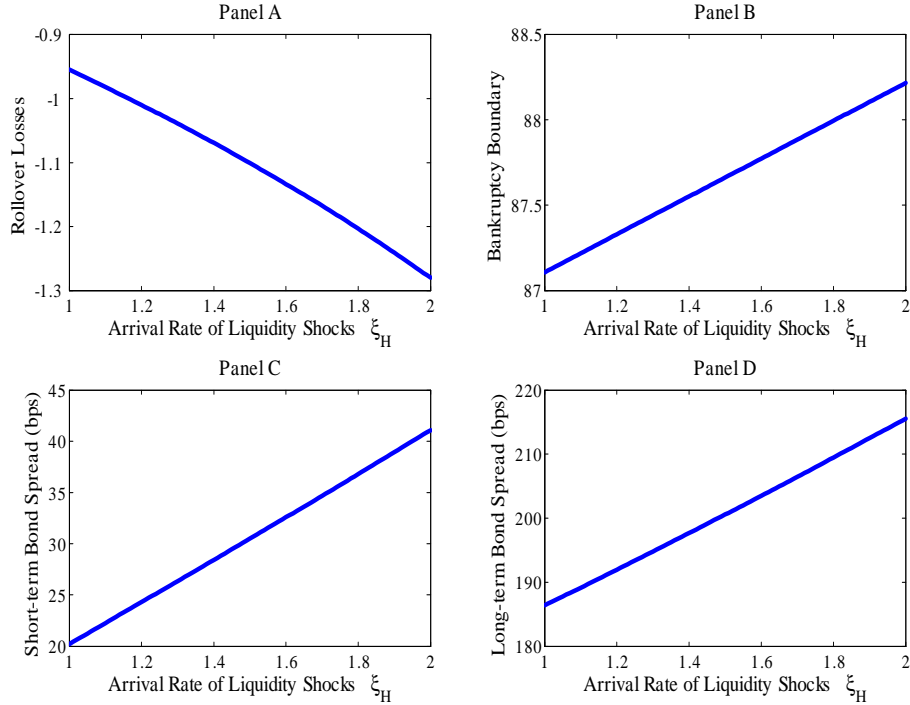
Figure 1: This figure shows the effects of a change in the arrival rate of liquidity shocks to type-H investors ξ_H on the firm's rollover loss and endogenous bankruptcy boundary, based on the baseline parameters given in (19), $\lambda_1 = 42.8\%$. Panel A plots the firm's rollover loss against ξ_H when $V_t = 97$. Panel B plots the firm's endogenous bankruptcy boundary V_B against ξ_H. Panels C and D plot the bond spreads of newly issued short-term and long-term bonds when $V_0 = 100$.

we hold the fraction of the firm's short-term debt at its initial value. For example, bond covenants and other operational restrictions prevent firms in real life from swiftly modifying their debt structures in response to sudden market fluctuations. For simplicity, we also treat the increase in ξ_H as permanent in the following discussion.[13]

Figure 1 illustrates the effects of a change in ξ_H on the equity holders' rollover loss and bankruptcy boundary. Panel A plots the rollover loss against ξ_H when $V = 97$. The line shows that the (absolute value of) rollover loss increases with ξ_H. That is, as the arrival rate of bond holders' liquidity shocks increases, the increased liquidity premium in bond prices makes it more costly for the equity holders to roll over the maturing bonds. Consequently,

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[13]In Appendix B, we extend our model to incorporate a temporary liquidity crisis, in which an increase in ξ_H mean-reverts back to its normal level according to a Poisson occurrence. This extension becomes more technically involved and requires substantial numerical analysis. The numerical results nevertheless show that, even in the case of a liquidity crisis with an expected length of 8 months, rolling over short-term debt with a maturity of three months makes the 8-month liquidity crisis relatively long, and, as a result, treating an increase in ξ_H as permanent or temporary only leads to modest differences in the impact on the long-term and short-term bond spreads.

Panel B shows that the firm's endogenous bankruptcy boundary V_B also increases with ξ_H. In other words, after a liquidity shock, equity holders will choose to default at a higher fundamental threshold. We formally prove these results in the following proposition. Panels C and D show that the bond spreads of newly issued short-term and long-term bonds increase with ξ_H.

Proposition 3 *All else equal, the bond prices d_i's decrease with the arrival rate of type-H bond holders' liquidity shocks ξ_H. Consequently, equity holders' endogenous default boundary V_B increases with ξ_H.*

The firm's endogenous bankruptcy decision is rooted in the conflict of interest between the debt and equity holders. When the firm's bond values fall (even for liquidity reasons as we illustrated here), the equity holders have to bear all the rollover losses to avoid bankruptcy, while the maturing debt holders get paid in full. This unequal sharing of losses causes the equity value to drop down to zero at V_B, at which point the equity holders stop servicing the debt. Could the debt and equity holders share the firm's losses, they would have avoided the social loss induced by the firm bankruptcy.

The implication of Proposition 3 is consistent with several empirical studies of market-liquidity effects on corporate bond spreads, e.g., Collin-Dufresne, Goldstein, and Martin (2001), Longstaff, Mithal, and Neis (2005), Ericsson and Renault (2006) and Chen, Lesmond, and Wei (2007). In particular, Chen, Lesmond, and Wei (2007) find that more illiquid bonds earn higher yield spreads, and that an improvement in liquidity causes a significant reduction in yield spreads. Consistent with their findings, our model incorporates liquidity premium as an important factor in bond spreads. Collin-Dufresne, Goldstein, and Martin (2001) find that proxies for both changes in the probability of future default based on standard fundamental-driven credit risk models and for changes in the recovery rate can only explain about 25% of the observed credit spread changes. On the other hand, they find that the residuals from these regressions are highly cross-correlated, and that over 75% of the variation in the residuals is due to the first principal component. While they cannot explain this systematic component, they attribute it to liquidity shocks. Our model explains their findings by suggesting that shocks to the aggregate demand for bond market liquidity can act as a common factor in individual bonds' credit spreads. Furthermore, our model shows that this liquidity factor affects not only the liquidity premium, but also future default probabilities. This amplification mechanism through endogenous firm default helps explain the large impact of the liquidity factor observed in the data. In the next subsection, we further illustrate how this mechanism can cause a spillover of liquidity shocks across different bond maturity segments.

4.2 Clientele and Liquidity Spillover Effects

As is well known, bond markets are highly segmented. For example, the market for commercial paper (short-term debt with maturities less than 9 months) operates on different quote conventions than the market for long-term corporate bonds. These markets also have separate institutional investor clienteles. Greenwood and Vayanos (2008, 2009) provide some recent evidence on market segmentation in the government bond markets. They interpret the existence of market segmentation as driven by investors' preferred habitats, i.e., individual investors/institutions have preferred debt maturities due to their different investment objectives. Our model also features separate clienteles for the short-term and long-term bonds, which are endogenously determined by their heterogeneous bond trading costs and heterogeneous investor liquidity needs. Type-H investors prefer to hold the more liquid short-term bonds because of their higher liquidity needs, while the lower liquidity needs of type-L investors make them the more efficient holders of the less liquid long-term bonds.

Despite the specific clienteles for the long-term and short-term bonds, liquidity shocks to one market segment can affect the credit spreads in the other market segment. Such spillovers occur through several channels. Following our discussion in the previous subsection, after an unexpected jump in the liquidity demand of type-H investors ξ_H, the credit spreads of *short-term* bonds (which are held by type-H investors) rise. More interestingly, the increase in ξ_H also drives up the credit spreads of the *long-term* bonds through three channels, even though the long-term bonds are held by type-L investors whose liquidity demand remains at the initial level ξ_L.

1. The first channel works through the equilibrium liquidity premium. Even though the increase in ξ_H does not directly affect type-L investors, the increase in the liquidity premium of the short-term bonds also increases the opportunity cost of type-L investors and thus causes their required return from the long-term bonds r_2 to go up, e.g., equation (8) of Proposition 1. With a slight abuse of notation, we write the required return as $r_2(\xi_H)$.

2. The second channel works through the increase in the firm's endogenous bankruptcy boundary V_B as a result of the increase in the type-H investors' required return from the short-term bonds $r_1(\xi_H)$.

3. The third channel works through the increase in the firm's endogenous bankruptcy boundary V_B as a result of the increase in the type-L investors' required return from the long-term bonds $r_2(\xi_H)$.
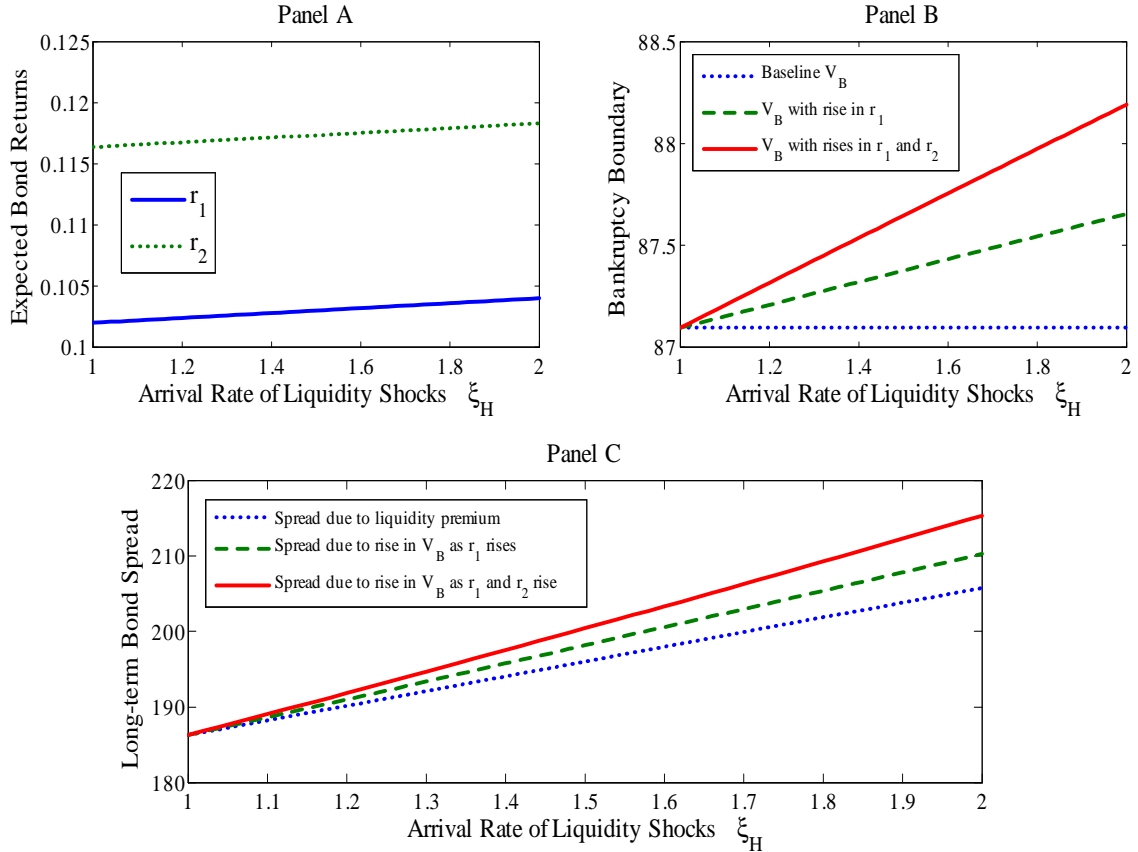
Figure 2: Decomposing the spillover effects of an increase in ξ_H on the credit spread of long-term bond, based on the baseline parameters given in (19) and by fixing the firm's short-term debt fraction at $\lambda_1 = 42.8\%$ and the current firm fundamental at $V_0 = 100$. Panel A plots the required returns of both short-term and long-term bonds (r_1 and r_2) against ξ_H. Panel B plots the firm's endogenous bankruptcy boundary V_B against ξ_H, with the dotted line representing the baseline level, the dashed line representing the change in V_B in response to rise in r_1 only, and the solid line representing the net change in V_B in response to rises in both r_1 and r_2. Panel C plots the credit spread of a newly issued long-term bond against ξ_H, with the dotted line representing the change in the credit spread with V_B fixed at the baseline level, the dashed line representing the change in the credit spread with V_B changing in response to a rise in r_1, and the solid line representing the net change in the credit spread in response to the change in both r_1 and r_2.

Figure 2 illustrates these channels. Panel A shows that as ξ_H rises, both r_1 and r_2 (the required returns from the short-term and long-term bonds) also rise. Panel B shows that as ξ_H increases, the firm's endogenous bankruptcy boundary V_B also rises and the increase of V_B can be attributed to the increases in both $r_1(\xi_H)$ and $r_2(\xi_H)$. Panel C shows that the credit spread of a newly issued long-term bond also increases with ξ_H, and each of the three channels described above makes a significant contribution to this increase. In this plot, the dotted line represents the change in the long-term bond credit spread due to the liquidity premium, i.e., fixing V_B at the baseline level; the dashed line represents the change in the long-term bond credit spread when V_B responds only to the change in $r_1(\xi_H)$; finally, the solid line represents the total change in the long-term credit spread when V_B responds to the changes in both $r_1(\xi_H)$ and $r_2(\xi_H)$.

The effect of an increase in investor demand for market liquidity on the firm's default probability, combined with the spillover effects illustrated in this section, demonstrates the profound impact of market liquidity on firms' credit risk. These results also suggest that time-varying liquidity could contribute a common factor to firms' default probabilities. A recent empirical study by Duffie et al (2009) indeed finds that the probability of extreme default losses on portfolios of U.S. corporate debt is much greater than would be estimated under the standard assumption that default correlation arises from exposure to observable fundamental factors. Instead, their result points to the presence of common latent factors, which are a potential reflection of liquidity effects illustrated by our model.

4.3 Flight to Quality

It is common to observe the so-called flight-to-quality phenomenon after major liquidity disruptions in financial markets—prices (credit spreads) of low quality bonds drop (rise) much more than those of high quality bonds. Recent episodes include the stock market crash of 1987, the events surrounding the Russian default and the crisis of LTCM in 1998, the events after the attacks of 9/11 in 2001, and the credit crisis of 2007/2008. The BIS (1999) report documents that during the 1998 LTCM crisis, which is widely regarded as a market-wide liquidity shock, the yields of speculative-grade corporate bonds and emerging market bonds rose much more than those of investment-grade bonds. A recent BIS report by Fender, Ho, and Hordahl (2009) shows that in a two-month period around the bankruptcy of Lehman Brothers in September 2008, the US five-year CDX high yield index spread shot up from around 700 basis points to over 1500, while the corresponding investment grade index spread widened from 150 basis points to a little above 250. Acharya, Amihud, and Bharath (2009) provide systematic evidence that the exposures (or betas) of speculative-
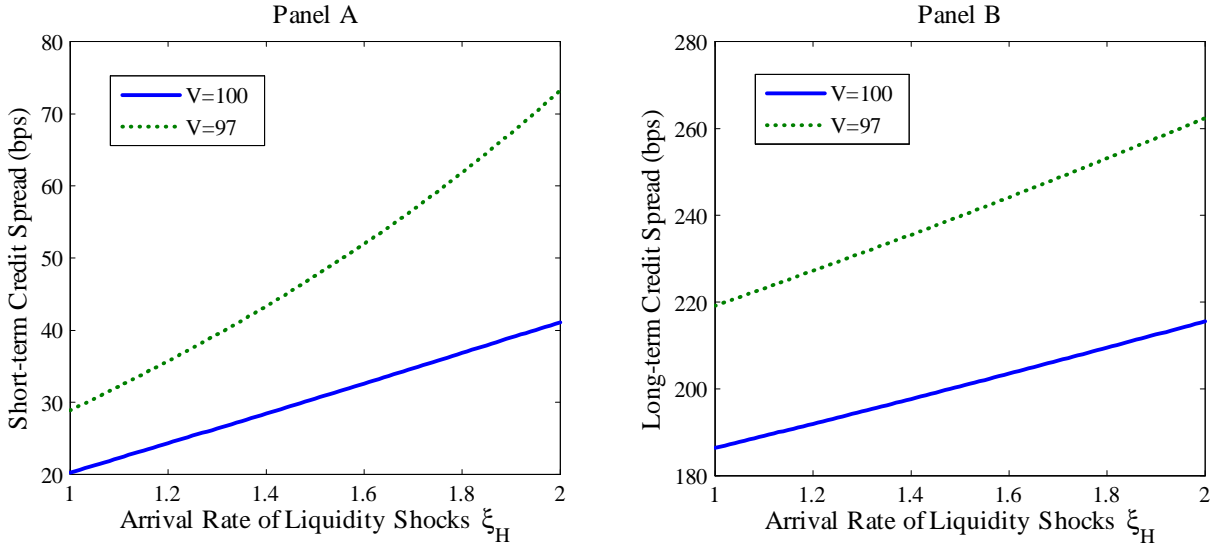
Figure 3: This figure plots the credit spreads of the newly issued short-term and long-term bonds of two firms with different fundamentals against the arrival rate of type-H investors' liquidity shocks ξ_H, based on the baseline parameters given in (19) and by fixing the firms' short-term debt fraction at $\lambda_1 = 42.8\%$. One of the firms has a fundamental of $V = 100$, while the other firm has $V = 97$.

grade corporate bonds to market liquidity shocks rise substantially during times of high macroeconomic stress, illiquidity and volatility.

Can our model explain the flight-to-quality phenomenon? To address this question, we examine two firms that are identical, except that one firm has a higher fundamental ($V = 100$) than the other ($V = 97$). We compare the changes in the credit spreads of these two firms' newly issued short-term and long-term bonds as the intensity of the type-H bond holders' liquidity shocks ξ_H jumps from 1 to 2. Figure 3 shows that the credit spreads of the weaker firm are more sensitive to the increase in ξ_H than those of the stronger firm. The intuition is simple. The increase in the bond holders' liquidity needs pushes up the firms' endogenous bankruptcy boundary. Since the weaker firm is closer to the bankruptcy boundary to begin with, the same increase of endogenous bankruptcy boundary has a greater effect on the weaker firm. As a result, its credit spreads shoot up more than those of the stronger firm.

Our explanation of the flight-to-quality phenomenon is different from the existing ones. The BIS report (1999) attributes flight-to-quality to suddenly increased risk aversion of market participants. Vayanos (2004) provides an explanation based on professional fund managers' career concerns, He and Krishnamurthy (2008) develop a model to show that falling intermediary capital can cause households to fly from risky assets to risk-free bonds, and Caballero and Krishnamurthy (2008) argue for Knightian uncertainty. These expla-

nations are all based on considerations from the investor side. Our model focuses on the financing issues from the firm side and shows that deterioration of market liquidity increases rollover risk and makes the weaker firms more likely to eventually fail.

Corroborating our theory, Fender, Ho, and Hordahl (2009) show that soon after the market liquidity breakdown caused by the failure of Lehman Brothers in September 2008, the default rates of speculative-grade bonds increased significantly from the very low levels (around 1%) observed in early 2008 to nearly 5% in March 2009, and were expected to rise further. The recent bankruptcy of General Growth Properties in April 2009, one of the largest mall operators in the US, nicely illustrates how the deteriorating credit market liquidity put pressure on lower-quality firms:

> "*Despite bargaining for months with its creditors, General Growth faced dwindling options for handling its more than $25 billion in debt, largely in the form of short-term mortgages that will come due by next year. The company has been severely wounded by the trouble in the financial markets, which has wreaked havoc on its ability to refinance that debt.*" The New York Times (April 16, 2009)

5 Amplification by Short-term Debt

The ongoing financial crisis reveals that many financial firms rely heavily on short-term debt such as commercial paper and overnight repos to finance their investment positions. Commercial paper typically has a maturity of less than 270 days, while overnight repos have an extremely short maturity of one day. What is the effect of short-term debt on the firm's exposure to the liquidity shocks?

To examine this question, we compare two otherwise identical firms, one with a short-term debt fraction λ_1 of 42.8% and the other with 30%. Figure 4 plots both firms' rollover loss and endogenous bankruptcy boundary against the arrival rate of the type-H bond holders' liquidity shocks ξ_H. Panel A shows that the rollover loss of the firm with higher λ_1 rises much faster with the increase in ξ_H. This is because short-term debt needs to be rolled over at a higher frequency. As a result, when bond prices drop below their principal values, equity holders have to pay off the losses incurred from the debt at a faster speed. The heavier financial burden increases the risk that equity value will fall to zero and causes equity holders to quit servicing the debt at a higher fundamental threshold. Indeed, Panel B shows that the firm with the higher short-term debt fraction has a higher bankruptcy boundary V_B. Taken together, this figure illustrates that short-term debt can further exacerbate the conflict of interest between the debt and equity holders in financial distresses, and thus increase the
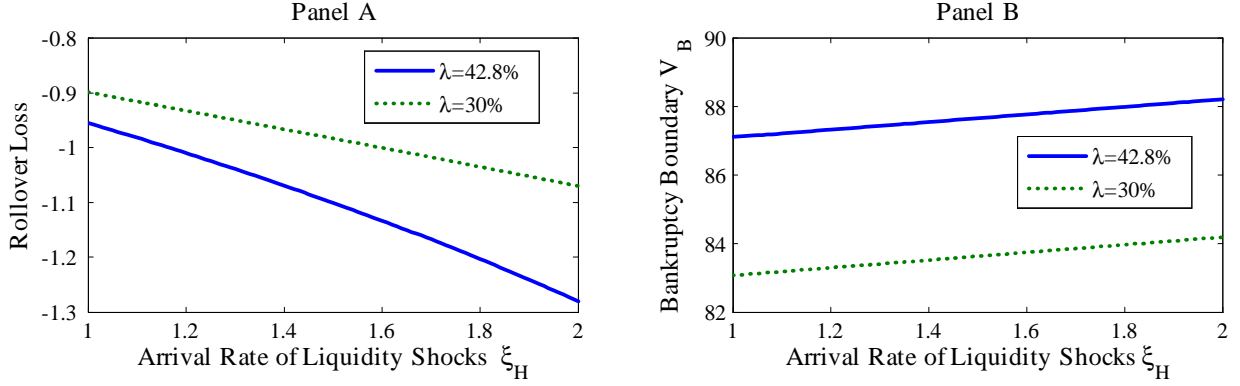
Figure 4: This figure plots the rollover loss and bankruptcy boundary against the arrival rate of type-H investors' liquidity shocks ξ_H for two otherwise identical firms, except one has a short-term debt fraction λ_1 of 42.8% and the other 30%. This figure is based on the baseline parameters given in (19). The rollover loss is measured when the firm fundamental is at $V = 97$.

firm's default risk.

To further illustrate the intuition, we introduce $\bar{d}_i$ to normalize the market value of the newly issued short-term and long-term bonds, so that

$$\bar{d}_i (V_t, m_i) = \frac{m_i}{\lambda_i} d_i (V_t, m_i) \tag{20}$$

corresponds to the value of a class-i bond with a coupon rate C and a principal P (recall equations (3) and (4)). These two normalized bonds differ only in their maturities, which allows us to rewrite the firm's net rollover gain/loss in $(t, t + dt)$ as (ignoring the net coupon payment C which is independent of maturity structure)

$$\left[\sum_{i=1}^{2} \lambda_i \frac{\bar{d}_i (V_t, m_i) - P}{m_i} \right] dt. \tag{21}$$

For each class of debt, the rollover loss is proportional to the normalized loss $\bar{d}_i (V_t, m_i) - P$ and the rollover frequency $\frac{1}{m_i}$.

To understand the role of maturity in rollover losses in equation (21), let us first discuss the normalized loss $\bar{d}_i (V_t, m_i) - P$. Note that short-term debt is more liquid than long-term debt $(\beta_1 < \beta_2)$. As a result, if default is not a concern, i.e., the firm's fundamental is strong, then we have $\bar{d}_1 > \bar{d}_2$. In words, short-term debt has a *smaller* rollover loss for each unit of normalized bond. This then makes the dramatic effect of short-term debt on the firm's bankruptcy boundary more surprising.

The key to this effect lies in the rollover frequency $\frac{1}{m}$, i.e., a shorter maturity m_i means a higher rollover frequency, which amplifies the rollover loss. This effect lies at the heart
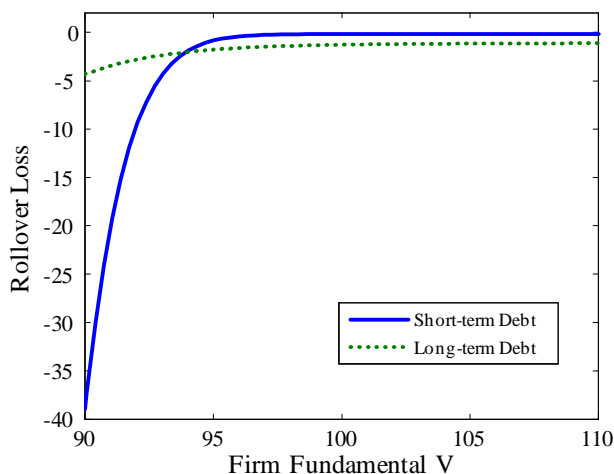
Figure 5: This figure plots the firm's rollover loss at different firm fundamentals for each unit of short-term and short-term debt (with same face value $P = 90$ and coupon $C = 9$). This figure is based on the parameters given in (19) and $\lambda_1 = 42.8\%$.

of how short-term debt causes the mounting financial burden on the firm exactly when its fundamental is weak. For illustration, Figure 5 plots the firm's loss from rolling over its short-term and long-term debt with respect to the firm fundamental. The figure shows that when the firm's fundamental is strong, short-term debt does provide a smaller rollover loss than long-term debt.[14] However, when the firm's fundamental is weak and thus close to bankruptcy, short-term debt generates a much larger rollover loss than long-term debt. In other words, the rollover gain/loss from short-term debt is highly skewed on the downside, while that from long-term debt is relatively flat.

From a contracting point of view, these different rollover gain/loss profiles are due to the fact that short-term debt is a "harder" claim relative to long-term debt. Essentially, short-term bond holders do not share gains/losses with equity holders to the same extent as long-term debt holders do, and, as a result, short-term debt leads to greater rollover losses borne by equity holders in bad times than long-term debt. This is similar in spirit to the debt-overhang problem initially suggested by Myers (1977). See Diamond and He (2010) for a recent study, which further analyzes the effects of short-term debt overhang on firms' default/investment decisions during bad times.

This problem has a direct impact on the value of the equity holders' embedded real option of keeping the firm alive. Even if the current fundamental is weak, equity holders

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[14]Because of the low coupon rate specified in this illustration (i.e., the bonds are not issued at par by setting $C = P/r$), the firm always has a rollover loss, which serves in this situation as part of the interest payment for its debt. This amounts to a level shift in rollover losses in Figure 5 and will not affect the relative comparison between long-term and short-term debt.

could choose to absorb the rollover losses out of hope for a future fundamental comeback. The negatively skewed payoff from rolling over short-term debt, which is designed to avoid sharing the losses with equity holders, makes keeping the firm alive costly and the option less valuable. In contrast, the flat payoff from rolling over long-term debt makes the option more valuable as long-term debt holders are sharing some of the losses with equity holders in bad times.

We can formally prove a set of results related to the discussion above. First, we can show that between two firms, one purely financed by short-term debt and the other purely financed by long-term debt, the short-term financed firm has a higher default boundary under some sufficient conditions.

Proposition 4 *Suppose that $\xi_i \beta_j = 0$ for $i \in \{H, L\}$ and $j \in \{1, 2\}$, and $P = \frac{C}{r}$. Then, $V_B(1) > V_B(0)$, i.e., the endogenous bankruptcy boundary of a 100% short-term financed firm is higher than that of a 100% long-term financed firm.*

Proposition 4 imposes two sufficient conditions. First, the bonds' liquidity premia are zero either because the investors' liquidity needs are zero or the bonds' trading costs are zero. Second, the bond's principal value is identical to the discounted value of the perpetual stream of its coupons, i.e., the firm faces no rollover loss when there is no default risk. These conditions are somewhat strong, as the complex expression of V_B in equation (18) prevents us from deriving the result of Proposition 4 under more relaxed conditions. However, by continuity arguments, the result must hold when the model parameters are close to the specified conditions (i.e., either ξ_i's or β_i's are small and the principal P is close to C/r). Furthermore, our numerical exercises also show that the result holds for a wide range of parameter values.

We can further prove that if the result of Proposition 4 holds, the firm's bankruptcy boundary is monotonically increasing in the fraction of its short-term debt.

Proposition 5 *Suppose that $V_B(1) > V_B(0)$, i.e., the endogenous bankruptcy boundary of a 100% short-term financed firm is higher than that of a 100% long-term financed firm. Then, $V_B(\lambda_1)$ is monotonically increasing with λ_1, i.e., the greater the fraction of the firm's short-term debt, the higher its endogenous bankruptcy boundary.*

This proposition provides a key factor in our analysis of the firm's optimal debt maturity structure in Section 6.

In assigning credit ratings, rating agencies tend to ignore the effects of firms' debt maturity structures. Gopalan, Song, and Yerramilli (2009) find that firms with a higher proportion
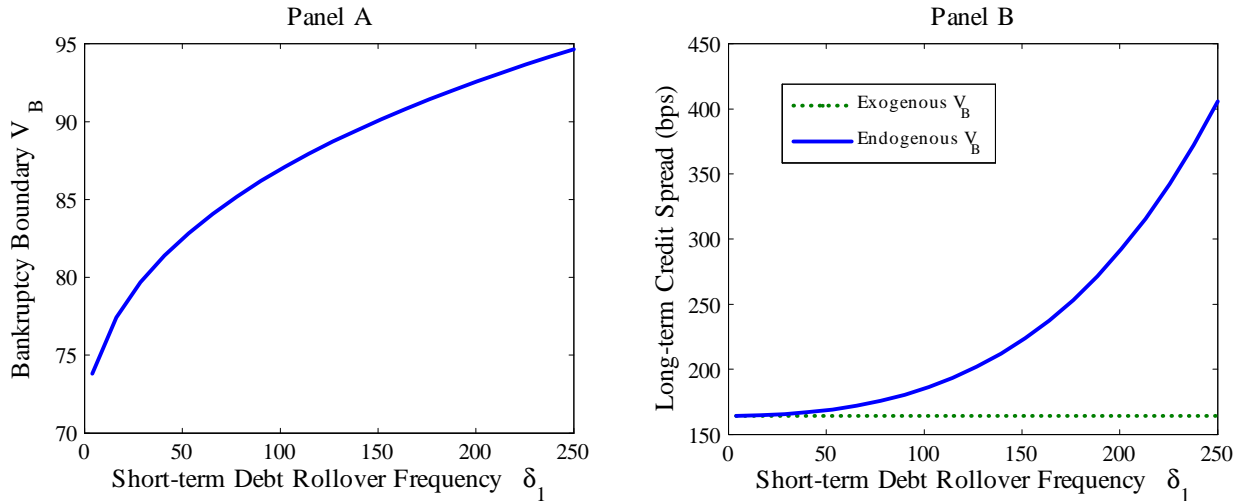
Figure 6: This figure shows the effect of shortening the maturity of the short-term debt, based on the model parameters given in (19) and fixing the short-term debt fraction λ_1 at 5%. Panel A plots the firm's bankruptcy boundary V_B against the short-term debt rollover frequency δ_1, and Panel B plots the credit spread of newly issued long-term bond against δ_1.

of short-term debt are more likely to experience multi-notch credit-rating downgrades. Their evidence suggests that credit ratings underestimate the so-called "maturity risk", an effect demonstrated by Proposition 5. Interestingly, rating agencies have recently started to incorporate this risk into credit ratings. For example, one of the major rating agencies, Standard & Poor's, has recently improved its approach to rating speculative-grade credits by adjusting for maturity risk:

> "Although we believe that our enhanced analytics will not have a material effect on the majority of our current ratings, individual ratings may be revised. For example, a company with heavy debt maturities over the near term (especially considering the current market conditions) would face more credit risk, notwithstanding benign long-term prospects." Ratings Direct (2008, page 6).

Repo Financing In the months leading up to its bankruptcy, Lehman Brothers was rolling over 25% of its debt every day through overnight repos. Repos are a type of collateralized lending agreement, in which a borrower finances the purchase of a financial security using the security as collateral. Overnight repos have an extremely short maturity of one day. What is the effect of overnight repos on the bankruptcy risk of a firm? To illustrate the impact of repo financing, we consider a hypothetical firm with the baseline parameters given in (19). We reduce the maturity of the short-term debt m_1 from 3 months to 1 day (overnight

repos). We denote by $\delta_1 = \frac{1}{m_1}$ the rollover frequency of the short-term debt, which is 4 for commercial paper with a 3-month maturity and 250 for overnight repos. We also fix the short-term debt fraction at 5% to focus on the effect of shortening the maturity.[15] Figure 6 shows that shortening maturity to 1 day has a large impact on default probability even with a small fraction of debt is short-term. Panel A shows that as the rollover frequency increases from 4 to 250, the endogenous bankruptcy boundary V_B increases from slightly above 74 to 95. As a result of the substantial increase in V_B, the credit spread of newly issued long-term bonds rises from 160 basis points to 405. This figure shows that simply shortening the maturity of a small fraction of the firm's debt from 3 months to 1 day can have a dramatic impact on the firm's financial stability.

Brunnermeier and Pedersen (2009), Geanakoplos (2009), and Shleifer and Vishny (2009) emphasize the destabilizing effect of the "haircut" (or margin) of the repos, i.e., creditors will increase the haircut when market liquidity deteriorates or when price volatility spikes. The increased margin requirement forces equity-constrained borrowers to liquidate their positions at firesale prices, resulting in a margin spiral. In sharp contrast, our model shows that even in the absence of any equity constraint on borrowers, the high rollover frequency of overnight repos can still lead to a debt crisis. Essentially, the repo rollover acts as a mark-to-market mechanism, forcing borrowers to absorb the losses accumulated in their positions every day through margin calls. The heavy financial burden on the borrowers can in turn motivate them to default at a higher fundamental threshold.[16]

Our focus on the rollover risk of short-term debt complements a recent study by Acharya, Gale, and Yorulmazer (2009). They study a setting in which asset owners have no capital and need to use the purchased risky asset as collateral to secure short-term debt funding. They show that the high rollover frequency associated with short-term debt can lead to diminishing debt capacity. Different from their model, our model demonstrates the severe consequences of short-term debt even in the absence of any constraint on equity issuance. This feature also differentiates our model from other papers related to rollover risk, e.g., Morris and Shin (2004, 2009) and He and Xiong (2009), which all assume implicit constraints on raising more equity when firms are in distress.

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[15] We reduce the short-term debt fraction from 42.8% in the previous illustrations to 5% here because a higher fraction, when combined with the high rollover frequency of overnight repos, would have caused the firm's bankruptcy boundary to be higher than the initial firm fundamental $V_0 = 100$.

[16] Since bankruptcy leads to a social loss, it is tempting to argue that debt restructuring, such as swapping debt into equity or lengthening the debt maturities, would be Pareto improving. However, such modifications of the debt agreements are already defined as a credit event, which would trigger many credit default swap contracts to pay out. As a result, even if such debt modifications avoid the social loss, they would hurt some parties and thus run into resistance in practice.
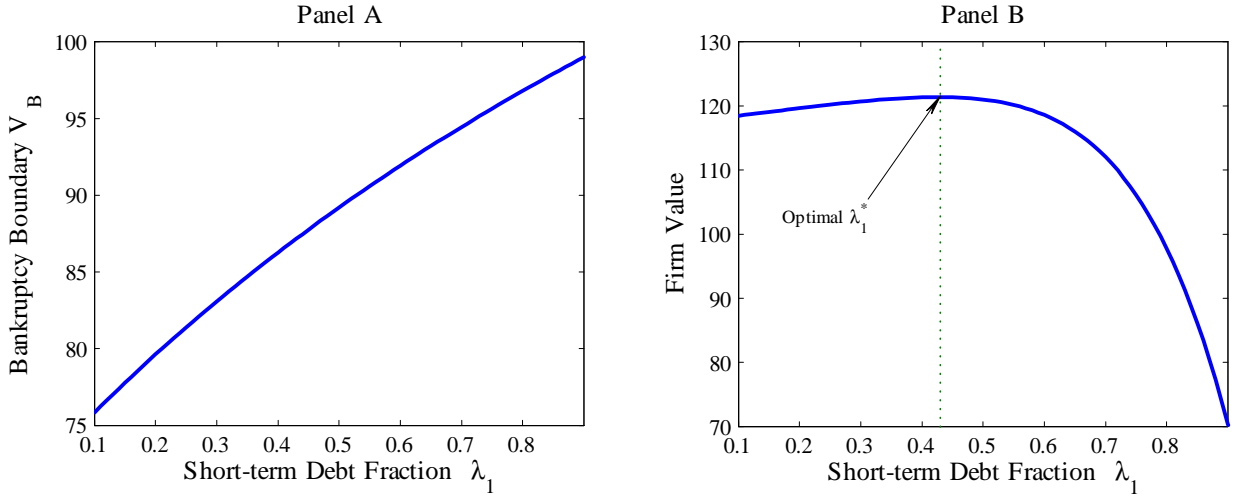
Figure 7: This figure plots the firm's endogenous bankruptcy boundary V_B and the total firm value against the firm's short-term debt fraction λ_1, based on the baseline parameters given in (19).

6   Optimal Debt Maturity Structure

In our model, there are two opposing forces working together to determine the firm's ex ante optimal debt maturity structure. On one hand, short-term debt is more liquid and therefore is cheaper for the firm.[17] On the other, short-term debt exacerbates the conflict of interest between debt and equity holders and therefore increases the firm's future default probability. In this section, we examine this tradeoff between short-term debt's cheaper financing cost and higher expected bankruptcy cost in determining the firm's optimal maturity structure.

Consider the firm's optimal maturity structure choice at time 0. The firm's objective is to maximize the total firm value, the sum of equity, short-term and long-term bonds:

$$\max_{\lambda_1 \in [0,1]} \ E\left(V\right) + D_1\left(V\right) + D_2\left(V\right).$$

This objective is also consistent with that of the equity holders at time 0 before the bonds are issued. Since the objective is a continuous function of λ_1 and λ_1 takes values in a closed set $[0,1]$, there must exist an optimum.

Figure 7 plots the firm's endogenous bankruptcy boundary V_B and the total firm value against the firm's short-term debt fraction λ_1. Panel A shows that V_B increases with λ_1, consistent with our discussion before. Panel B shows that the total firm value is maximized when $\lambda_1^* = 42.8\%$. This interior optimum reflects the tradeoff between the short-term debt's cheaper financing cost and higher expected bankruptcy cost.

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[17]The lower cost of short-term debt financing could also arise from other sources, such as an upward-sloping yield curve and investor preference for short-term debt, e.g., Greenwood, Hanson, and Stein (2009).
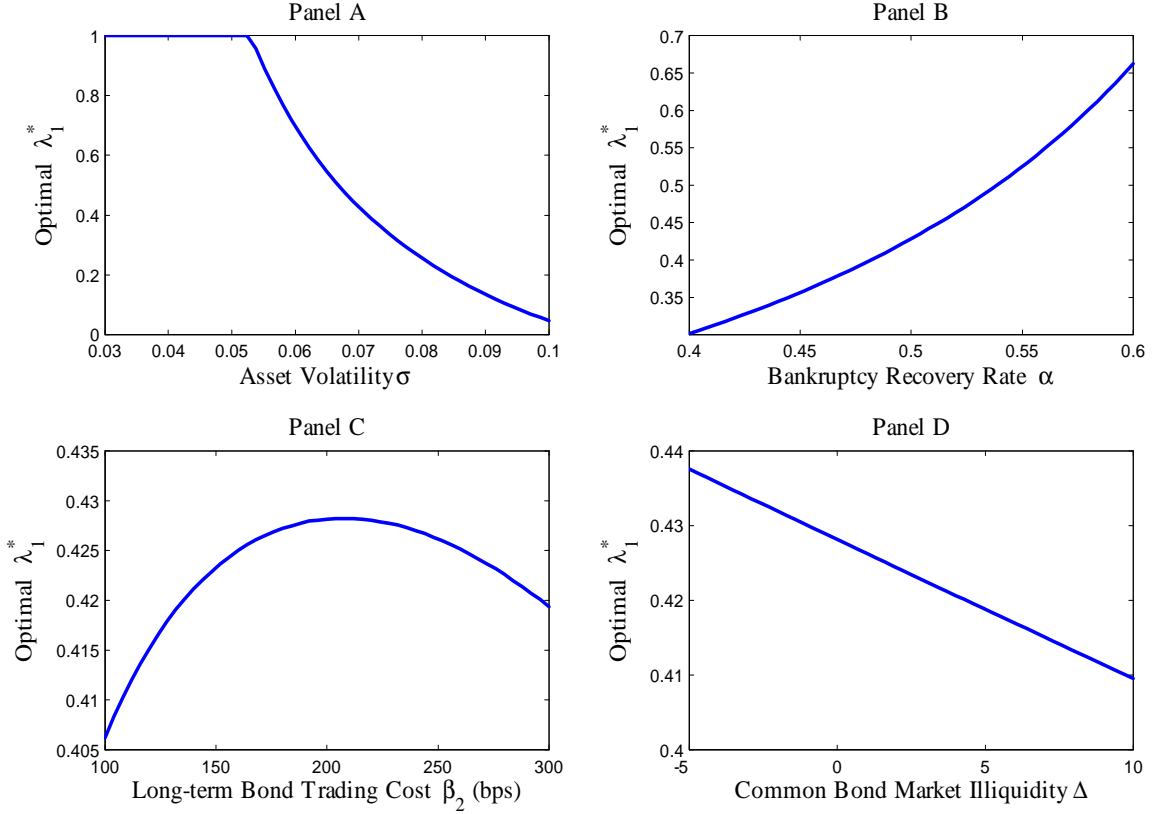
Figure 8: This figure shows how firm characteristics affect the firm's optimal short-term debt fraction λ_1^*, based on the baseline parameters given in (19). Panel A plots λ_1^* against the firm's asset volatility σ. Panel B plots λ_1^* against the bankruptcy recovery rate α. Panel C plots λ_1^* against the long-term debt trading cost β_2. Finally, Panel D plots λ_1^* against the common component Δ in the trading cost β_1 and β_2 of the firm's short-term and long-term bonds.

Figure 8 illustrates how firm characteristics affect its optimal short-term debt fraction λ_1^*, based on the baseline parameters given in (19). Panel A shows that λ_1^* decreases with the firm's asset volatility σ. As the volatility increases, it raises the firm's future default probability and therefore its expected bankruptcy cost. As a result, it is desirable for the firm to use less short-term debt to reduce the bankruptcy cost. The figure also shows that, in the region where the asset volatility is low (lower than 5.2%), the cheaper-cost effect dominates and induces the firm to use 100% short-term debt. Panel B shows that λ_1^* increases with the firm's bankruptcy recovery rate α. As α increases, the expected bankruptcy cost becomes smaller. As a result, the firm can take advantage of the cheaper financing cost of short-term debt more aggressively.

Panel C shows that there is a non-monotonic relationship between λ_1^* and the long-term debt trading cost β_2: λ_1^* first increases with β_2 when it is relatively low and decreases with

β_2 when it becomes high. This plot again reflects the tradeoff between the financing cost and expected bankruptcy cost. As β_2 increases, the direct effect is that the long-term debt becomes more expensive. This effect makes the short-term debt more attractive, and thus explains the increasing part of the plot. When β_2 increases, an indirect effect is that it induces the firm to set a higher bankruptcy threshold, resulting in a higher future default probability. This indirect effect makes the short-term debt less attractive on the margin, and explains the decreasing part of the plot.

If the trading costs of the short-term and long-term debt, β_1 and β_2, increase together, then the substitution effect between the two types of debt is void and only the second (indirect) effect through the firm's endogenous default is in operation. Panel D of Figure 8 shows that as the common component in β_1 and β_2 increases, the optimal short-term debt fraction λ_1^* decreases. This is because the increased market illiquidity makes the firm more likely to default in the future. As a result, the bankruptcy-cost effect becomes more important.

The existing theories of firms' debt maturity choice have mostly focused on the disciplinary role of short-term debt in preventing managers' asset substitution, e.g., Flannery (1994) and Leland (1998), and private information of borrowers about their future credit ratings, e.g., Flannery (1986) and Diamond (1991). These theories have had some success in explaining the data, as shown by Barclay and Smith (1995) and Guedes and Opler (1996). Our model provides a new hypothesis, which relates firms' debt maturity structure to market-liquidity considerations.

Maturity Structure as Liquidity Management   In fact, our analysis shows that debt maturity structure should be used as part of a firm's liquidity management strategy. As discussed in Section 5, despite its higher cost, long-term debt gives the firm more flexibility to delay realizing financial losses in adverse states, either when the firm's fundamental or bond market liquidity deteriorates. This benefit is analogous to the role of cash reserves, the standard tool for risk management, e.g., Holmstrom and Tirole (2001) and Bolton, Chen, and Wang (2009). Keeping cash is costly for a firm, but it allows the firm to avoid future financial constraints and to take advantage of future investment opportunities.

Brunnermeier and Yogo (2009) argue that firms should shift to long-term debt as their fundamentals deteriorate. This argument is consistent with the basic result of our model that the bankruptcy cost (or the loss of flexibility) from using short-term debt becomes higher as the firm's fundamental or bond market liquidity deteriorates. This effect motivates the firm to use less short-term debt in these states. This argument is appealing, but it is also

countered by other conflicts between debt and equity holders. As pointed out by Leland (1994), adjusting debt policy in his model by retiring or issuing debt ex post is infeasible to the extent that it will hurt either equity or long-term debt holders. This argument also applies to our setting in adjusting the debt maturity structure ex post.[18] A systematic analysis of these arguments is important, but is a challenge beyond our current framework. We leave it for future research.

7   Conclusion

This paper provides a model to analyze effects of a firm's rollover risk on its credit risk, through the conflict of interest between debt and equity holders. When the firm faces losses in rolling over its maturing debt, its equity holders are willing to absorb the losses only if the option value of keeping the firm alive justifies the cost of paying off the maturing debt holders. As liquidity shocks push down bond prices, they amplify the conflict of interest between the debt and equity holders because, to avoid bankruptcy, the equity holders have to absorb all of the losses from rolling over maturing bonds at the reduced market prices. As a result, equity holders choose to default at a higher fundamental threshold even if firms can freely raise more equity. A greater fraction of short-term debt further exacerbates the debt crisis by forcing the equity holders to realize the rollover loss at a higher frequency. Our model thus provides a new perspective to understand the financial instability created by overnight repos, an extreme form of short-term financing, to many financial firms.

Our model also features separate clienteles for long-term and short-term bonds, endogenously determined by heterogeneous liquidity needs and heterogeneous bond trading costs. We highlight different channels through which liquidity shocks to one market segment affect the credit spreads in the other segment, and provide a new explanation for the widely ob-

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[18]Regarding issuing more debt, see the related discussion in footnote 7. It is clear that increasing λ_1 leads to a higher bankruptcy boundary V_B, and therefore hurts long-term debt holders. Now suppose that the firm adjusts λ_1 downward. One possible policy is to issue more long-term debt to replace the maturing short-term debt, until a certain desirable maturity structure is achieved. In the interim period, the replaced short-term debt has coupon (principal) $\frac{\lambda_1}{m_1}C$ $(\frac{\lambda_1}{m_1}P)$. Therefore, to maintain the firm's net coupon and debt principal, the firm needs to issue $\frac{\lambda_1}{m_1}\frac{m_2}{\lambda_2}$ units of long-term debt. The net financing effect, relative to the base case without the maturity adjustment, is

$$-\frac{\lambda_1}{m_1}\overline{d}_1\left(V_t, m_1\right) + \frac{\lambda_1}{m_1}\frac{m_2}{\lambda_2}\frac{\lambda_2}{m_2}\overline{d}_2\left(V_t, m_2\right) \propto -\overline{d}_1\left(V_t, m_1\right) + \overline{d}_2\left(V_t, m_2\right).$$

Since the short-term debt is safer than the long-term debt, typically $\overline{d}_1 > \overline{d}_2$ and the above term is negative. This heuristic argument implies that the financial burden on the equity holders actually increases during the adjustment process, and therefore it is not in the interest of the equity holders to reduce the short-term debt fraction. For a related analysis of equity holders' ex post incentive to reduce leverage in response to poor firm performance, see Dangl and Zechner (2007).

served flight-to-quality phenomenon. We also examine a tradeoff between short-term debt's cheaper financing cost and higher future bankruptcy cost in determining firms' optimal debt maturity structure and liquidity management strategy.

Appendix A   Proofs for Propositions

A.1   Proof for Proposition 1

Because of the large number of type-H investors in the market, competition implies that each type-H investor earns only an expected return of r on his capital. It is natural to hypothesize that type-H investors are the marginal investors in the more liquid short-term bond. Then, the short-term bond price satisfies the following equation

$$rd_1 = -\frac{\partial d_1}{\partial \tau} + (r - \phi) V_t \frac{\partial d_1}{\partial V} + \frac{1}{2}\sigma^2 V_t^2 \frac{\partial^2 d_1}{\partial V^2} + c_1 - \xi_H \beta_1 d_1,$$

which is equivalent to

$$r_1 d_1 = -\frac{\partial d_1}{\partial \tau} + (r - \phi) V_t \frac{\partial d_1}{\partial V} + \frac{1}{2}\sigma^2 V_t^2 \frac{\partial^2 d_1}{\partial V^2} + c_1, \tag{22}$$

where

$$r_1 = r + \xi_H \beta_1$$

is the expected return of the bond. The expected return is higher than r because of the additional liquidity premium $\xi_H \beta_1$.

We now derive the required return for type-L investors, who are the marginal investors in the less liquid long-term bonds. We first solve for the type-L investors' value function, and then verify that they have no strict incentive to switch from investing in the less liquid long-term bonds to investing in the more liquid short-term bonds. Finally, we verify that type-H investors have no incentive to switch from investing in the more liquid short-term bonds to investing in the less liquid long-term bonds.

Because of the limited number of type-L investors, they earn an extra rent on their capital in addition to the riskfree rate r. We denote the value function of a type-L investor as $u(w)$, which is a function of his capital w. We conjecture that $u(\cdot)$ has a linear form (and verify shortly):

$$u(w) = Qw,$$

where the constant Q (to be determined shortly) measures the marginal utility of each unit of capital for type-L investors. It is intuitive that Q is related to the return offered by the long-term bond. More specifically, the HJB equation for a type-L investor from investing in the long-term bond is

$$rQw = Qw \frac{1}{d_2}\left[-\frac{\partial d_2}{\partial \tau} + (r - \phi) V_t \frac{\partial d_2}{\partial V} + \frac{1}{2}\sigma^2 V_t^2 \frac{\partial^2 d_2}{\partial V^2} + c_2 \right] + \xi_L \left[(1 - \beta_2) w - Qw\right].$$

Here, the left-hand-side represents the type-L investor's required return. On the right-hand-side, the first term represents the expected bond return, and the second term represents the expected effect from a liquidity shock. Rearranging terms, we can write

$$r_2 d_2 = -\frac{\partial d_2}{\partial \tau} + (r - \phi) V_t \frac{\partial d_2}{\partial V} + \frac{1}{2}\sigma^2 V_t^2 \frac{\partial^2 d_2}{\partial V^2} + c_2,$$

where

$$r_2 = r + \xi_L \left(1 - \frac{1 - \beta_2}{Q}\right) \tag{23}$$

is the expected return from holding the long-term bond.

Note that type-L investors can also invest in the short-term bond. Thus, the expected return from investing in the short-term bond determines type-L investors' opportunity cost of capital:

$$rQw = Qw\frac{1}{d_1}\left[-\frac{\partial d_1}{\partial \tau} + (r - \phi) V_t \frac{\partial d_1}{\partial V} + \frac{1}{2}\sigma^2 V_t^2 \frac{\partial^2 d_1}{\partial V^2} + c_1\right] + \xi_L\left[(1 - \beta_1) w - Qw\right],$$

which is equivalent to

$$\left[r + \xi_L\left(1 - \frac{1 - \beta_1}{Q}\right)\right] d_1 = -\frac{\partial d_1}{\partial \tau} + (r - \phi) V_t \frac{\partial d_1}{\partial V} + \frac{1}{2}\sigma^2 V_t^2 \frac{\partial^2 d_1}{\partial V^2} + c_1.$$

This equation guarantees that type-L investors are indifferent between holding less liquid long-term bonds and more liquid short-term bonds. Since d_1 satisfies equation (22), we must have

$$r + \xi_L\left(1 - \frac{1 - \beta_1}{Q}\right) = r_1 = r + \xi_H \beta_1,$$

which implies that

$$Q = \frac{1 - \beta_1}{1 - \frac{\xi_H}{\xi_L}\beta_1}.$$

This result also implies that the parameter restriction $\xi_L > \xi_H \beta_1$ ensures that Q is well-defined; in fact, if $\xi_L < \xi_H \beta_1$ then type-L investors' value is unbounded. Combining this result with equation (23), we have

$$r_2 = r + \xi_H \beta_1 + \frac{(\beta_2 - \beta_1)}{1 - \beta_1}\left(\xi_L - \xi_H \beta_1\right).$$

Next, we consider the incentive of a type-H investor to hold a long-term bond. The expected instantaneous return of a type-H investor from holding the long-term bond is

$$\frac{1}{d_2}\left[-\frac{\partial d_2}{\partial \tau} + (r - \phi) V_t \frac{\partial d_2}{\partial V} + \frac{1}{2}\sigma^2 V_t^2 \frac{\partial^2 d_2}{\partial V^2} + c_2 - \xi_H \beta_2 d_2\right],$$

which, by using equation (23), is

$$r_2 - \xi_H \beta_2 = r - (\beta_2 - \beta_1)\left(\xi_H - \xi_L\right) < r.$$

Thus, the type-H investor strictly prefers a short-term bond to a long-term bond.

A.2 Proof for Proposition 2

We omit the time subscript in V_t and v_t in the following derivation. The equity value satisfies the following ODE:

$$rE = (r - \phi)VE_V + \frac{1}{2}\sigma^2 V^2 E_{VV} + d_1(V, m_1) + d_2(V, m_2) + \phi V - \left[(1 - \pi)C + \frac{P_1}{m_1} + \frac{P_2}{m_2}\right].$$

Since $v = \ln\left(\frac{V}{V_B}\right)$, we have

$$rE = \left(r - \phi - \frac{1}{2}\sigma^2\right)E_v + \frac{1}{2}\sigma^2 E_{vv} + d_1(v, m_1) + d_2(v, m_2) + \phi V_B e^v - \left[(1 - \pi)C + \frac{P_1}{m_1} + \frac{P_2}{m_2}\right],$$
$$(24)$$

with the boundary conditions

$$E(0) = 0 \text{ and } E_v(0) = l,$$

where the free parameter l is to be determined by the boundary condition when $v \to \infty$ (where the equity value is linear in V).

Define the Laplace transformation of $E(v)$ as

$$F(s) = L[E(v)] \equiv \int_0^\infty e^{-sv} E(v)\, dv.$$

Then, applying the Laplace transformation to both sides of the ODE (24), we have:

$$rF(s) = \left(r - \phi - \frac{1}{2}\sigma^2\right)L[E_v] + \frac{1}{2}\sigma^2 L[E_{vv}] + L[d_1(v, m_1) + d_2(v, m_2)]$$
$$+ \frac{\phi V_B}{s - 1} - \frac{(1 - \pi)C + \frac{P_1}{m_1} + \frac{P_2}{m_2}}{s}.$$

Note that

$$L[E_v] = sF(s) - E(0) = sF(s)$$

and

$$L[E_{vv}] = s^2 F(s) - sE(0) - E_v(0) = s^2 F(s) - l;$$

thus we have

$$\left[r - \left(r - \phi - \frac{1}{2}\sigma^2\right)s - \frac{1}{2}\sigma^2 s^2\right]F(s) = L[d_1(v, m_1)] + L[d_2(v, m_2)] - \frac{1}{2}\sigma^2 l$$
$$+ \frac{\phi V_B}{s - 1} - \frac{(1 - \pi)C + \frac{P_1}{m_1} + \frac{P_2}{m_2}}{s}.$$

Define η and γ to be the solutions to following quadratic equation, i.e.,

$$r - \left(r - \phi - \frac{1}{2}\sigma^2\right)s - \frac{1}{2}\sigma^2 s^2 = -\frac{1}{2}\sigma^2(s - \eta)(s + \gamma);$$

one can easily show that $\eta > 1$ and $\gamma > 0$. Then,

$$\frac{1}{2}\sigma^2 F(s)$$

$$= -\frac{1}{(s-\eta)(s+\gamma)}\left\{L\left[d_1(v,m_1)\right] + L\left[d_2(v,m_2)\right] + \frac{\phi V_B}{s-1} - \frac{(1-\pi)C + \delta_1 P_1 + \delta_2 P_2}{s} - \frac{1}{2}\sigma^2 l\right\}$$

$$= -\frac{1}{\eta+\gamma}\left(\frac{1}{s-\eta} - \frac{1}{s+\gamma}\right)\left\{L\left[d_1(v,m_1)\right] + L\left[d_2(v,m_2)\right] + \frac{\phi V_B}{s-1}\right.$$

$$\left. -\frac{(1-\pi)C + \frac{P_1}{m_1} + \frac{P_2}{m_2}}{s} - \frac{1}{2}\sigma^2 l\right\}. \tag{25}$$

Recall

$$d_i(v,m_i) = \frac{C_i}{r_i m_i} + e^{-r_i m_i}\left(\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right)(1 - F(m_i)) + \left(\frac{1}{m_i}\lambda_i \alpha V_B - \frac{C_i}{r_i m_i}\right)G_i(m_i),$$

where $F(m_i)$ and $G_i(m_i)$ are given in equation (13), i.e.,

$$F(m_i) = N\left(\frac{(-v - a\sigma^2 m_i)}{\sigma\sqrt{m_i}}\right) + e^{-2av}N\left(\frac{(-v + a\sigma^2 m_i)}{\sigma\sqrt{m_i}}\right);$$

$$G_i(m_i) = e^{(-a+z_i)v}N\left(\frac{(-v - z_i\sigma^2 m_i)}{\sigma\sqrt{m_i}}\right) + e^{-(a+z_i)v}N\left(\frac{(-v + z_i\sigma^2 m_i)}{\sigma\sqrt{m_i}}\right);$$

where

$$a = \frac{r - \phi - \sigma^2/2}{\sigma^2}; \quad z_i = \frac{[a^2\sigma^4 + 2r_i\sigma^2]^{1/2}}{\sigma^2}; \quad z = \frac{[a^2\sigma^4 + 2r\sigma^2]^{1/2}}{\sigma^2}.$$

Plugging $d_i(v,m_i)$ in (25), we have

$$\frac{1}{2}\sigma^2 F(s) = -\frac{\frac{1}{s-\eta} - \frac{1}{s+\gamma}}{\eta+\gamma}\left\{\frac{\phi V_B}{s-1} - \frac{(1-\pi)C + \sum_i(1 - e^{-r_i m_i})\left(\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right)}{s} - \frac{1}{2}\sigma^2 l\right\} \tag{26}$$

$$-\frac{\frac{1}{s-\eta} - \frac{1}{s+\gamma}}{\eta+\gamma}\sum_i\left\{-e^{-r_i m_i}\left(\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right)L\left[F(m_i)\right] + \left(\frac{1}{m_i}\lambda_i \alpha V_B - \frac{C_i}{r_i m_i}\right)L\left[G_i(m_i)\right]\right\}.$$

Call the first line in (26) $\widehat{F}(s)$, and it is easy to work out its Laplace inverse as

$$\widehat{E}(v) = -\frac{\phi V_B}{\eta+\gamma}\left[\frac{1}{\eta-1}(e^{\eta v} - e^v) + \frac{1}{\gamma+1}(e^{-\gamma v} - e^v)\right]$$

$$+\frac{(1-\pi)C + \sum(1 - e^{-r_i m_i})\left[\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right]}{\eta+\gamma}\left[\frac{1}{\eta}(e^{\eta v} - 1) - \frac{1}{\gamma}(1 - e^{-\gamma v})\right]$$

$$+\frac{1}{2}\sigma^2 l\frac{1}{\eta+\gamma}(e^{\eta v} - e^{-\gamma v})$$

$$= V - \frac{\phi V_B}{\eta+\gamma}\left[\frac{1}{\eta-1}e^{\eta v} + \frac{1}{\gamma+1}e^{-\gamma v}\right]$$

$$+\frac{(1-\pi)C + \sum(1 - e^{-r_i m_i})\left[\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right]}{\eta+\gamma}\left[\frac{1}{\eta}(e^{\eta v} - 1) - \frac{1}{\gamma}(1 - e^{-\gamma v})\right]$$

$$+\frac{1}{2}\sigma^2 l\frac{1}{\eta+\gamma}(e^{\eta v} - e^{-\gamma v}).$$

Call the second line in (26) $\sum_i \overline{F}(s)$. One can show that

$$
\begin{aligned}
(\eta+\gamma)\,\overline{F}_i &= e^{-r_i m_i}\left(\frac{P_i}{m_i}-\frac{C_i}{r_i m_i}\right)\frac{1}{\eta}\left(\frac{1}{s-\eta}-\frac{1}{s}\right)\left[N\left(-a\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left((s+a)^2-a^2\right)\sigma^2 m_i}\right]\\
&\quad -e^{-r_i m_i}\left(\frac{P_i}{m_i}-\frac{C_i}{r_i m_i}\right)\frac{1}{\gamma}\left(\frac{1}{s}-\frac{1}{s+\gamma}\right)\left[N\left(-a\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left((s+a)^2-a^2\right)\sigma^2 m_i}\right]\\
&\quad +e^{-r_i m_i}\left(\frac{P_i}{m_i}-\frac{C_i}{r_i m_i}\right)\frac{1}{2a+\eta}\left(\frac{1}{s-\eta}-\frac{1}{s+2a}\right)\left[N\left(a\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left((s+a)^2-a^2\right)\sigma^2 m_i}\right]\\
&\quad -e^{-r_i m_i}\left(\frac{P_i}{m_i}-\frac{C_i}{r_i m_i}\right)\frac{1}{\gamma-2a}\left(\frac{1}{s+2a}-\frac{1}{s+k_2}\right)\left[N\left(a\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left((s+a)^2-a^2\right)\sigma^2 m_i}\right]\\
&\quad -\left(\frac{1}{m_i}\lambda_i\alpha V_B-\frac{C_i}{r_i m_i}\right)\frac{1}{a-z_i+\eta}\left(\frac{1}{s-\eta}-\frac{1}{s+a-z_i}\right)\left[N\left(-z_i\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left((s+a)^2-z_i^2\right)\sigma^2 m_i}\right]\\
&\quad +\left(\frac{1}{m_i}\lambda_i\alpha V_B-\frac{C_i}{r_i m_i}\right)\frac{1}{k_2-a+z_i}\left(\frac{1}{s+a-z_i}-\frac{1}{s+\gamma}\right)\left[N\left(-z_i\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left((s+a)^2-z_i^2\right)\sigma^2 m_i}\right]\\
&\quad -\left(\frac{1}{m_i}\lambda_i\alpha V_B-\frac{C_i}{r_i m_i}\right)\frac{1}{a+z_i+\eta}\left(\frac{1}{s-\eta}-\frac{1}{s+a+z_i}\right)\left[N\left(z_i\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left((s+a)^2-z_i^2\right)\sigma^2 m_i}\right]\\
&\quad \left(\frac{1}{m_i}\lambda_i\alpha V_B-\frac{C_i}{r_i m_i}\right)\frac{1}{\gamma-a-z_i}\left(\frac{1}{s+a+z_i}-\frac{1}{s+\gamma}\right)\left[N\left(z_i\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left((s+a)^2-z_i^2\right)\sigma^2 m_i}\right].
\end{aligned}
$$

Define

$$
\begin{aligned}
M_i\left(v;x,w,p,q\right) &\equiv L^{-1}\left\{\left(\frac{1}{s+p}-\frac{1}{s+q}\right)\left[N\left(y\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left((s+x)^2-w^2\right)\sigma^2 m_i}\right]\right\}\\
&= \left\{N\left(w\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left[(p-x)^2-w^2\right]\sigma^2 m_i}N\left((p-x)\sigma\sqrt{m_i}\right)\right\}e^{-pv}\\
&\quad +e^{\frac{1}{2}\left[(p-x)^2-w^2\right]\sigma^2 m_i}e^{-pv}N\left(\frac{-v+(p-x)\sigma^2 m_i}{\sigma\sqrt{m_i}}\right)\\
&\quad -\left\{N\left(w\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left[(q-x)^2-w^2\right]\sigma^2 m_i}N\left((q-x)\sigma\sqrt{m_i}\right)\right\}e^{-qv}\\
&\quad -e^{\frac{1}{2}\left[(q-x)^2-w^2\right]\sigma^2 m_i}e^{-qv}N\left(\frac{-v+(q-x)\sigma^2 m_i}{\sigma\sqrt{m_i}}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
K_i\left(x,w,p\right) &\equiv \left\{N\left(w\sigma\sqrt{m_i}\right)-e^{\frac{1}{2}\left[(p-x)^2-w^2\right]\sigma^2 m_i}N\left((p-x)\sigma\sqrt{m_i}\right)\right\}e^{-pv}\\
&\quad +e^{\frac{1}{2}\left[(p-x)^2-w^2\right]\sigma^2 m_i}e^{-pv}N\left(\frac{-v+(p-x)\sigma^2 m_i}{\sigma\sqrt{m_i}}\right)-e^{-(x+w)v}N\left(\frac{-v+w\sigma^2 m_i}{\sigma\sqrt{m_i}}\right).
\end{aligned}
$$

Then

$$
M_i\left(v;x,w,x+w,q\right)=-K_i\left(x,w,q\right),\ M_i\left(v;x,w,p,x+w\right)=K_i\left(x,w,p\right).
$$

Therefore (note that $\frac{2}{\sigma^2}\frac{1}{\eta+\gamma} = \frac{1}{z\sigma^2}$),

$$
\begin{aligned}
E(v) &= \frac{2}{\sigma^2}\left(\widehat{E}(v) + \sum \overline{E}_i\right) \tag{27}\\
&= V - \frac{\phi V_B}{z\sigma^2}\left[\frac{e^{\eta v}}{\eta-1} + \frac{e^{-\gamma v}}{\gamma+1}\right] + \frac{l}{2z}\left(e^{\eta v} - e^{-\gamma v}\right)\\
&\quad + \frac{(1-\pi)C + \sum\left(1 - e^{-r_i m_i}\right)\left[\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right]}{z\sigma^2}\left[\frac{1}{\eta}\left(e^{\eta v} - 1\right) - \frac{1}{\gamma}\left(1 - e^{-\gamma v}\right)\right]\\
&\quad + \sum_i \left[
\begin{array}{l}
\frac{e^{-r_i m_i}\left(\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right)}{z\sigma^2}\left[
\begin{array}{l}
\frac{1}{\eta}K_i\left(v;a,-a,-\eta\right) + \frac{1}{\gamma}K_i\left(v;a,-a,\gamma\right)\\
+\frac{1}{\gamma}K_i\left(v;a,a,-\eta\right) + \frac{1}{\gamma-2a}K_i\left(v;a,a,\gamma\right)
\end{array}\right]\\
+\frac{\left(\frac{1}{m_i}\lambda_i \alpha V_B - \frac{C_i}{r_i m_i}\right)}{z\sigma^2}\left[
\begin{array}{l}
-\frac{1}{z-z_i}K_i\left(v;a,-z_i,-\eta\right) - \frac{1}{z+z_i}K_i\left(v;a,-z_i,\gamma\right)\\
-\frac{1}{z+z_i}K_i\left(v;a,z_i,-\eta\right) - \frac{1}{z-z_i}K_i\left(v;a,z_i,\gamma\right)
\end{array}\right]
\end{array}\right].
\end{aligned}
$$

There is one free parameter $l = E'(0)$ to be pinned down by the boundary condition at $v \to \infty$. The equity value has to grow linearly when $V \to \infty$. Since $e^{\eta v} = \left(\frac{V}{V_B}\right)^\eta$ and $\eta > 1$, to avoid explosion we need the coefficient of $e^{\eta v}$ in $E(v)$ to collapse to 0. Collecting coefficients of $e^{\eta v}$, we require that (note that $-\eta - a = -z$, $\gamma = 2a + \eta$, $\frac{1}{2}\left[z^2 - a^2\right]\sigma^2 m_i = r m_i$):

$$
\begin{aligned}
0 &= -\frac{\phi V_B}{\eta-1} + \left[(1-\pi)C + \sum\left(1 - e^{-r_i m_i}\right)\left[\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right]\right]\frac{1}{\eta} + \frac{\sigma^2}{2}l\\
&\quad + \sum_i\left\{
\begin{array}{l}
e^{-r_i m_i}\left(\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right)\left[
\begin{array}{l}
\frac{\left\{N\left(-a\sigma\sqrt{m_i}\right) - e^{r m_i}N\left(-z\sigma\sqrt{m_i}\right)\right\}}{\eta}\\
+\frac{\left\{N\left(a\sigma\sqrt{m_i}\right) - e^{r m_i}N\left(-z\sigma\sqrt{m_i}\right)\right\}}{\gamma}
\end{array}\right]\\
+\left(\frac{1}{m_i}\lambda_i \alpha V_B - \frac{C_i}{r_i m_i}\right)\left[
\begin{array}{l}
-\frac{\left\{N\left(-z_i\sigma\sqrt{m_i}\right) - e^{\frac{1}{2}\left[z^2 - z_i^2\right]\sigma^2 m_i}N\left(-z\sigma\sqrt{m_i}\right)\right\}}{a - z_i + \eta}\\
-\frac{\left\{N\left(z_i\sigma\sqrt{m_i}\right) - e^{\frac{1}{2}\left[z^2 - z_i^2\right]\sigma^2 m_i}N\left(-z\sigma\sqrt{m_i}\right)\right\}}{a + z_i + \eta}
\end{array}\right]
\end{array}\right\}. \tag{28}
\end{aligned}
$$

This condition gives l as an expression of primitive parameters and the bankruptcy boundary V_B. More importantly, this implies that the constant coefficient of $e^{\eta v}$ should be zero. This simplifies the expression of K_i that involves $-\eta$ to

$$
\begin{aligned}
K_i\left(x,w,-\eta\right) &= e^{\frac{1}{2}\left[(p-x)^2 - w^2\right]\sigma^2 m_i}e^{\eta v}N\left(\frac{-v + (-\eta - x)\sigma^2 m_i}{\sigma\sqrt{m_i}}\right) - e^{-(x+w)v}N\left(\frac{-v + w\sigma^2 m_i}{\sigma\sqrt{m_i}}\right)\\
&\equiv k_i\left(x,w,-\eta\right).
\end{aligned}
$$

The smooth pasting condition implies that $E'(V_B) = 0$, or $E'_v(0) = l = 0$. Then we can use condition (28) to obtain V_B. With these results, we have the closed-form expressions for $E(v)$ and V_B stated in Proposition 2.

A.3 Proof of Proposition 3

We first fix the default boundary V_B. According to the Feynman-Kac formula, PDE (6) implies that at time 0, the price of a bond with time-to-maturity τ satisfies

$$
d_i(V_0, \tau; V_B) = \mathbb{E}_{V_0}\left[\int_0^{\tau \wedge \tau_B}e^{-r_i s}c_i ds + e^{-r_i\cdot(\tau \wedge \tau_B)}d_i\left(\tau \wedge \tau_B\right)\right], \tag{29}
$$

where $\tau_B = \inf\{t : V_t = V_B\}$ is the first hitting time of V_t to V_B. V_t follows (1), and $d_i(\tau \wedge \tau_B)$ is defined by the boundary conditions in (9) and (10):

$$d_i(\tau \wedge \tau_B) = \begin{cases} \frac{1}{m_i}\lambda_i\alpha V_B & \text{if } \tau \wedge \tau_B = \tau_B \\ p_i & \text{if } \tau \wedge \tau_B = \tau \end{cases}.$$

Proposition 1 shows that both r_1 and r_2 increase with ξ_H. Because in equation (29) ξ_H enters d_i in the discount rate r_i, and an increase in ξ_H raises the discount rate, a path-by-path argument implies that d_i decreases with ξ_H.

Similarly, the equity value can be written as

$$E(V_0, \tau; V_B) = \mathbb{E}_{V_0}\left\{\int_0^{\tau_B} e^{-rs}\left[\phi V_s - (1-\pi)C + \sum_{i=1}^2 (d_i(V_s, m_i - s; \xi_H) - p_i)\right] ds\right\},$$

where we write the dependence of d_i on ξ_H explicitly. Again, a path-by-path argument implies that once fixing V_B, E decreases with ξ_H.

We now consider two different values of ξ_H: $\xi_1 < \xi_2$. Denote the corresponding default boundaries as $V_{B,1}$ and $V_{B,2}$. We need to show that $V_{B,1} < V_{B,2}$. Suppose that the opposite is true, i.e., $V_{B,1} \geq V_{B,2}$. Since the equity value is zero on the default boundary, we have

$$E(V_{B,1}; V_{B,1}, \xi_1) = E(V_{B,2}; V_{B,2}, \xi_2) = 0,$$

where we expand the notation to let the equity value $E(V_t; V_B, \xi_H)$ explicitly depend on V_B (the default boundary) and ξ_H (the liquidity shock arrival of type-H investors). Also, the optimality of the default boundary implies that

$$0 = E(V_{B,1}; V_{B,1}, \xi_1) > E(V_{B,1}; V_{B,2}, \xi_1).$$

Since E decreases with ξ_H, $E(V_{B,1}; V_{B,2}, \xi_1) > E(V_{B,1}; V_{B,2}, \xi_2)$. Because $V_{B,1} \geq V_{B,2}$ according to our counter-factual hypothesis, $E(V_{B,1}; V_{B,2}, \xi_2) < 0$. This contradicts to limited liability which says that

$$E(V_t; V_{B,2}, \xi_2) \geq 0 \text{ for all } V_t \geq V_{B,2}.$$

Therefore $V_{B,1} < V_{B,2}$.

A.4   Proof of Proposition 4

When the firm is only financed by one class of debt, and $\xi_i\beta_j = 0$ for $i \in \{H, L\}$ and $j \in \{1, 2\}$, this setting is identical to Leland and Toft (1996). The endogenous default boundary in this case is

$$V_B = \frac{\frac{C}{r}\left(\frac{\widehat{A}}{rm} + \widehat{B}\right) - P\frac{\widehat{A}}{rm} - \frac{\tau C\gamma}{r}}{1 + (1-\alpha)\gamma + \alpha\widehat{B}},$$

where

$$\widehat{A} = 2ae^{-rm}N\left(a\sigma\sqrt{m}\right) - 2zN\left(z\sigma\sqrt{m}\right)$$
$$-\frac{2}{\sigma\sqrt{m}}n\left(z\sigma\sqrt{m}\right) + \frac{2e^{-rm}}{\sigma\sqrt{m}}n\left(a\sigma\sqrt{m}\right) + z - a;$$
$$\widehat{B} = \left(2z + \frac{2}{z\sigma^2 m}\right)N\left(z\sigma\sqrt{m}\right) + \frac{2}{\sigma\sqrt{m}}n\left(z\sigma\sqrt{m}\right) - z + a - \frac{1}{z\sigma^2 m}.$$

With a slight abuse of notation, we denote V_B as a function of maturity m. If $P = \frac{C}{r}$,

$$V_B = \frac{\frac{C}{r}\widehat{B} - \frac{\tau C\gamma}{r}}{1 + (1-\alpha)\gamma + \alpha\widehat{B}} = \frac{\frac{C}{r} - \frac{\tau C\gamma}{rB'}}{\frac{1+(1-\alpha)\gamma}{B'} + \alpha}.$$

Note that $\frac{\partial V_B}{\partial m}$ has the same sign as $\frac{\partial \widehat{B}}{\partial m}$. Then, simple algebra shows that (note $z\sigma\sqrt{m} > 0$)

$$\frac{\partial \widehat{B}}{\partial m} = \frac{1}{z\sigma^2 m^2}\left(1 - 2N\left(z\sigma\sqrt{m}\right)\right) < 0.$$

Therefore, $V_B(m_1) > V_B(m_2)$.

A.5   Proof of Proposition 5

We have

$$V_B(\lambda_1) = \frac{\frac{(1-\pi)C + \sum\left(1-e^{-r_i m_i}\right)\left[\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right]}{\eta} + \sum_i \left\{ \begin{array}{c} \left(\frac{P_i}{m_i} - \frac{C_i}{r_i m_i}\right)\left[\frac{1}{\eta}b(-a) + \frac{1}{\gamma}b(a)\right] \\ + \frac{C_i}{r_i m_i}\left[B_i(-z_i) + B_i(z_i)\right] \end{array} \right\}}{\frac{\phi}{\eta-1} + \sum_i \frac{1}{m_i}\lambda_i\alpha\left[B_i(-z_i) + B_i(z_i)\right]}$$

$$= V_B(1)\frac{\lambda_1\left[\frac{\phi}{\eta-1} + \frac{\alpha[B_1(-z_1)+B_1(z_1)]}{m_1}\right]}{\frac{\phi}{\eta-1} + \sum_i \frac{1}{m_i}\lambda_i\alpha\left[B_i(-z_i) + B_i(z_i)\right]}$$

$$+ V_B(0)\frac{\lambda_2\left[\frac{\phi}{\eta-1} + \frac{\alpha[B_2(-z_2)+B_2(z_2)]}{m_2}\right]}{\frac{\phi}{\eta-1} + \sum_i \frac{1}{m_i}\lambda_i\alpha\left[B_i(-z_i) + B_i(z_i)\right]}.$$

Define

$$w(\lambda_1) \equiv \frac{\lambda_1\left[\frac{\phi}{\eta-1} + \frac{\alpha[B_1(-z_1)+B_1(z_1)]}{m_1}\right]}{\frac{\phi}{\eta-1} + \sum_i \frac{1}{m_i}\lambda_i\alpha\left[B_i(-z_i) + B_i(z_i)\right]},$$

then we can write $V_B(\lambda_1) = V_B(1)w(\lambda_1) + V_B(0)(1 - w(\lambda_1))$. Because

$$\frac{1}{w(\lambda_1)} = \frac{\frac{\phi}{\eta-1} + \sum_i \frac{1}{m_i}\lambda_i\alpha\left[B_i(-z_i) + B_i(z_i)\right]}{\lambda_1\left[\frac{\phi}{\eta-1} + \frac{\alpha[B_1(-z_1)+B_1(z_1)]}{m_1}\right]}$$

$$= 1 + \frac{1-\lambda_1}{\lambda_1}\frac{\frac{\phi}{\eta-1} + \frac{\alpha[B_2(-z_2)+B_2(z_2)]}{m_2}}{\frac{\phi}{\eta-1} + \frac{\alpha[B_1(-z_1)+B_1(z_1)]}{m_1}}$$

is decreasing in λ_1, $w(\lambda_1)$ is increasing in λ_1. Our claim follows.

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Appendix B Modeling a Temporary Liquidity Crisis

In this appendix, we model a temporary liquidity crisis. Suppose that at $t = 0$ the intensity ξ_H of Poisson liquidity shocks to type-H investors unexpectedly jumps to a higher level ξ_H^{cr} (where the superscript cr stands for crisis). One can interpret this event as a crisis that dramatically increases the rate of investor redemption faced by financial institutions. At a future time τ_κ the liquidity-shock intensity ξ_H^{cr} reverts back to its normal level ξ_H permanently. Suppose that τ_κ follows an exponential distribution with parameter κ (i.e., the crisis ends at a Poisson event with intensity of κ). For simplicity we assume that the liquidity-shock intensity of type-L investors does not rise in the crisis.

The breakout of the crisis causes investors' required returns from the short-term and long-term bonds to change. Following Proposition 1, we specify the required return for the short-term bonds to be

$$r_1^{cr} = r + \xi_H^{cr}\beta_1$$

and the required return for the long-term bonds to be

$$r_2^{cr} = r + \xi_H^{cr}\beta_1 + \frac{\beta_2 - \beta_1}{1 - \beta_1}\left(\xi_L - \xi_H^{cr}\beta_1\right).$$

Consequently, the HJB equation for the class-i bond in the crisis becomes

$$r_i^{cr}d_i^{cr} = -\frac{\partial d_i^{cr}}{\partial \tau} + (r - \phi)V\frac{\partial d_i^{cr}}{\partial V} + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 d_i^{cr}}{\partial V^2} + c_i + \kappa\left(d_i^{cr} - d_i\right).$$

Denote by V_B^{cr} the endogenous bankruptcy boundary in crisis, which is determined later. Because in the crisis the required bond returns rise, equity holders incur more rollover losses relative to normal times. This suggests that $V_B^{cr} > V_B$, i.e., equity holders default at a higher fundamental boundary in crisis. Moreover, it implies that when ξ_H^{cr} reverts back to ξ_H, i.e., when the economy returns to normalcy, the firm is strictly above the normal-period default boundary and we can evaluate the debt and equity values based on the results that we have derived in Section 3.

B.1 Bond Value

We need to calculate the value of a class-i bond $d_i^{cr}(V_0 = V, \tau)$ in the crisis period, where τ is the time-to-maturity. There are two cases to consider:

1. If $\tau_\kappa < \tau$, i.e., the crisis ends before the bond matures. At τ_κ, investors who hold class-i bonds get $d_i(V_{\tau_\kappa}, \tau - \tau_\kappa)$ which is the bond value in the normal period. Then

$$
\begin{aligned}
d_i^{cr}(V, \tau; \tau_\kappa < \tau) &= \int_0^{\tau_\kappa} e^{-r_i^{cr}s}c_i\left(1 - F^{cr}(s; V; V_B^{cr})\right)ds \\
&\quad + e^{-r_i^{cr}\tau_\kappa}\mathbb{E}_{V_0=V}\left[d_i(V_{\tau_\kappa}, \tau - \tau_\kappa)\bigg|\inf_{0<s<\tau_\kappa} V_s > V_B^{cr}\right] \\
&\quad + \int_0^{\tau_\kappa} e^{-r_i^{cr}s}\frac{\alpha\lambda_i V_B^{cr}}{m_i}f^{cr}(s; V; V_B^{cr})ds,
\end{aligned}
$$

where $F^{cr}(s; V; V_B^{cr})$ is the cumulative distribution function of the first hitting time for V at V_B^{cr}, and $f^{cr}(s; V; V_B^{cr})$ is its density. The first term is the value of expected coupon payments until τ_κ, the second term is the expected value at τ_κ without bankruptcy, and the third term is the value of expected bankruptcy recovery. By simplifying the first and the third terms, we have

$$\frac{c_i}{r_i^{cr}} - \frac{c_i}{r_i^{cr}}\left(1 - F^{cr}(\tau_\kappa; V; V_B^{cr})\right)e^{-r_i^{cr}\tau_\kappa} + \left(\frac{\alpha\lambda_i V_B^{cr}}{m_i} - \frac{c_i}{r_i^{cr}}\right)\int_0^{\tau_\kappa} e^{-r_i^{cr}s}f^{cr}(s; V; V_B^{cr})\,ds$$

$$= \frac{c_i}{r_i^{cr}} - e^{-r_i^{cr}\tau_\kappa}\frac{c_i}{r_i^{cr}}\left(1 - F^{cr}(\tau_\kappa; V; V_B^{cr})\right) + \left(\frac{\alpha\lambda_i V_B^{cr}}{m_i} - \frac{c_i}{r_i^{cr}}\right)G_i^{cr}(\tau_\kappa),$$

where

$$v = \ln\frac{V}{V_B^{cr}}; F^{cr}(\tau) = N(h_1(\tau)) + \left(\frac{V}{V_B^{cr}}\right)^{-2a} N(h_2(\tau));$$

$$G_i^{cr}(\tau) = \left(\frac{V}{V_B^{cr}}\right)^{-a+z_i^{cr}} N(q_{1i}^{cr}(\tau)) + \left(\frac{V}{V_B^{cr}}\right)^{-a-z_i^{cr}} N(q_{2i}^{cr}(\tau)); \qquad (30)$$

$$h_1(\tau) = \frac{(-v - a\sigma^2\tau)}{\sigma\sqrt{\tau}}; h_2(\tau) = \frac{(-v + a\sigma^2\tau)}{\sigma\sqrt{\tau}};$$

$$q_{1i}^{cr}(\tau) = \frac{(-v - z_i^{cr}\sigma^2\tau)}{\sigma\sqrt{\tau}}; q_{2i}^{cr}(\tau) = \frac{(-v + z_i^{cr}\sigma^2\tau)}{\sigma\sqrt{\tau}};$$

$$a = \frac{r - \phi - \sigma^2/2}{\sigma^2}; z_i^{cr} = \frac{[a^2\sigma^4 + 2r_i^{cr}\sigma^2]^{1/2}}{\sigma^2}.$$

To calculate the second term $\mathbb{E}_{V_0=V}\left[d_i(V_{\tau_\kappa}, \tau - \tau_\kappa)\Big|\inf_{0<s<\tau_\kappa} V_s > V_B^{cr}\right]$, note that (e.g., Black and Cox (1976), equation (7))

$$\Pr_{V_0=V}\left(\inf_{0<s<\tau_\kappa} V_s > V_B^{cr}\Big| V_{\tau_\kappa} \geq y\right)$$

$$= N\left(\frac{\ln V - \ln y + a\sigma^2\tau_\kappa}{\sigma\sqrt{\tau_\kappa}}\right) - \left(\frac{V}{V_B^{cr}}\right)^{-2a} N\left(\frac{2\ln V_B^{cr} - \ln V - \ln y + a\sigma^2\tau_\kappa}{\sigma\sqrt{\tau_\kappa}}\right),$$

which implies that

$$\Pr_{V_0=V}\left(\inf_{0<s<\tau_\kappa} V_s > V_B^{cr}\Big| V_{\tau_\kappa} \in dy\right) \qquad (31)$$

$$= \frac{1}{\sigma y\sqrt{\tau_\kappa}}\left[n\left(\frac{\ln V - \ln y + a\sigma^2\tau_\kappa}{\sigma\sqrt{\tau_\kappa}}\right) - \left(\frac{V}{V_B^{cr}}\right)^{-2a} n\left(\frac{2\ln V_B^{cr} - \ln V - \ln y + a\sigma^2\tau_\kappa}{\sigma\sqrt{\tau_\kappa}}\right)\right]dy.$$

Let us denote

$$M(\tau_\kappa, V; V_B^{cr}) \equiv e^{-r_i^{cr}\tau_\kappa}\mathbb{E}_{V_0=V}\left[d_i(V_{\tau_\kappa}, \tau - \tau_\kappa; V_B)\Big|\inf_{0<s<\tau_\kappa} V_s > V_B^{cr}\right]$$

$$= e^{-r_i^{cr}\tau_\kappa}\int_{V_B^{cr}}^{\infty} d_i(V_{\tau_\kappa}, \tau - \tau_\kappa; V_B)\Pr\left(\inf_{0<s<\tau_\kappa} V_s > V_B^{cr}\Big| V_{\tau_\kappa} \in dy\right),$$

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where $d_i\left(V_{\tau_\kappa},\tau-\tau_\kappa;V_B\right)$ is the normal-period bond value with time-to-maturity $\tau-\tau_\kappa$ given in (12):

$$d_i\left(V_{\tau_\kappa},\tau-\tau_\kappa;V_B\right)=\frac{c_i}{r_i}+e^{-r_i(\tau-\tau_\kappa)}\left[p_i-\frac{c_i}{r_i}\right]\left(1-F\left(\tau-\tau_\kappa\right)\right)+\left[\frac{\lambda_i\alpha V_B}{m_i}-\frac{c_i}{r_i^{cr}}\right]G_i\left(\tau-\tau_\kappa\right).$$
$$(32)$$

2. If $\tau_\kappa>\tau$, the bond matures before the crisis ends, and we have

$$\begin{aligned}d_i^{cr}\left(V,\tau;\tau_\kappa>\tau\right)&=\int_0^\tau e^{-r_i^{cr}s}c_i\left(1-F^{cr}\left(s;V;V_B^{cr}\right)\right)ds+e^{-r_i^{cr}\tau}p_i\left(1-F^{cr}\left(\tau_\kappa;V;V_B^{cr}\right)\right)\\&\quad+\int_0^\tau e^{-r_i^{cr}s}\frac{\alpha\lambda_iV_B^{cr}}{m_i}f^{cr}\left(s;V;V_B^{cr}\right)ds,\end{aligned}$$

which has the same closed-form expression for bond price in equation (12) but with $\xi=\xi^{cr}$:

$$d_i^{cr}\left(V,\tau;\tau_\kappa>\tau\right)=Q_i\left(V_t,\tau;V_B^{cr}\right)\equiv\frac{c_i}{r_i^{cr}}+e^{-r_i^{cr}\tau}\left[p_i-\frac{c_i}{r_i^{cr}}\right]\left(1-F^{cr}\left(\tau\right)\right)+\left[\frac{\lambda_i\alpha V_B^{cr}}{m_i}-\frac{c_i}{r_i^{cr}}\right]G_i^{cr}\left(\tau\right).$$

Taking these two cases together, we have

$$\begin{aligned}d_i^{cr}\left(V,\tau\right)&=\int_0^\infty\kappa e^{-\kappa\tau_\kappa}d_i^{cr}\left(V,\tau;\tau_\kappa\right)d\tau_\kappa\\&=\int_0^\tau\kappa e^{-\kappa\tau_\kappa}d_i^{cr}\left(V,\tau;\tau_\kappa<\tau\right)d\tau_\kappa+\int_\tau^\infty\kappa e^{-\kappa\tau_\kappa}d_i^{cr}\left(V,\tau;\tau_\kappa>\tau\right)d\tau_\kappa\\&=\int_0^\tau\kappa e^{-\kappa\tau_\kappa}d_i^{cr}\left(V,\tau;\tau_\kappa<\tau\right)d\tau_\kappa+Q_i\left(V_t,\tau;V_B^{cr}\right)e^{-\kappa\tau},\end{aligned}$$

where for $\tau_\kappa<\tau$ we have

$$\begin{aligned}&d_i^{cr}\left(V,\tau;\tau_\kappa<\tau\right)\\&=\frac{c_i}{r_i^{cr}}-e^{-r_i^{cr}\tau_\kappa}\frac{c_i}{r_i^{cr}}\left(1-F^{cr}\left(\tau_\kappa;V;V_B^{cr}\right)\right)+M\left(\tau_\kappa;V;V_B^{cr}\right)+\left(\frac{\alpha\lambda_iV_B^{cr}}{m_i}-\frac{c_i}{r_i^{cr}}\right)G_i^{cr}\left(\tau_\kappa\right).\end{aligned}$$

Piece by piece, we can show that

$$\begin{aligned}d_i^{cr}\left(V,\tau\right)&=Q_i\left(V_t,\tau;V_B\right)e^{-\kappa\tau}+\frac{c_i\left(1-e^{-\kappa\tau}\right)}{r_i^{cr}}\\&\quad-\frac{\kappa}{r_i^{cr,\kappa}}\frac{c_i}{r_i^{cr}}\left(1-e^{-r_i^{cr,\kappa}\tau}-G_i^{cr}\left(\tau;\kappa\right)+e^{-r_i^{cr,\kappa}\tau}F^{cr}\left(\tau;V;V_B^{cr}\right)\right)\\&\quad+\left(\frac{\alpha\lambda_iV_B^{cr}}{m_i}-\frac{c_i}{r_i^{cr}}\right)\left(-G_i^{cr}\left(\tau\right)e^{-\kappa\tau}+G_i^{cr}\left(\tau;\kappa\right)\right)+\int_0^\tau\kappa e^{-\kappa\tau_\kappa}M\left(\tau_\kappa;V;V_B^{cr}\right)d\tau_\kappa,\end{aligned}$$

where $r_i^{cr,\kappa}\equiv\kappa+r_i^{cr}$ and $G_i\left(\tau;\kappa\right)$ is $G_i\left(\tau\right)$ in (30) but with z_i^{cr} replaced by $z_i^{cr,\kappa}\equiv\frac{\left[a^2\sigma^4+2r_i^{cr,\kappa}\sigma^2\right]^{1/2}}{\sigma^2}$. We have to numerically evaluate

$$\int_0^\tau\kappa e^{-\kappa\tau_\kappa}M\left(\tau_\kappa;V;V_B^{cr}\right)d\tau_\kappa=\int_0^\tau\kappa e^{-r_i^{cr,\kappa}\tau_\kappa}\int_{V_B^{cr}}^\infty d_i\left(V_{\tau_\kappa},\tau-\tau_\kappa\right)\Pr\left(\inf_{0<s<\tau_\kappa}V_s>V_B^{cr}\bigg|V_{\tau_\kappa}\in dy\right)d\tau_\kappa.$$

Note that we have derived closed-form expressions for the integrands in (31) and (32). We use the *Matlab* function *dblquad* to numerically calculate this integral and obtain $d_i^{cr}\left(V,m_i;V_B^{cr}\right)$.

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B.2 Equity Value

To calculate the crisis-period equity value E^{cr}, we have

$$rE^{cr} = (r - \phi) V E_V^{cr} + \frac{1}{2}\sigma^2 V^2 E_{VV}^{cr} + \phi V - (1 - \pi) C + \sum_i [d_i^{cr}(V, m_i) - p_i] + \kappa (E - E^{cr}).$$

At the bankruptcy boundary V_B^{cr},

$$E^{cr}(V_B^{cr}) = 0 \text{ and } E^{cr\prime}(V_B^{cr}) = 0.$$

At $V = \infty$, there is no default, and one can show that

$$E^{cr}(V \to \infty) = V - (1 - \pi)\frac{C}{r} + \frac{1}{r}\frac{r \sum_i [d_i^{cr}(\infty, m_i) - p_i] + \kappa \sum_i \left(\frac{c_i}{r_i} - p_i\right)(1 - e^{-r_i m_i})}{r + \kappa},$$

where

$$d_i^{cr}(\infty, m_i) - p_i = \frac{c_i}{r_i}\frac{r_i + \kappa}{r_i^{cr,\kappa}}\left(1 - e^{-r_i^{cr,\kappa} m_i}\right) + \frac{\kappa\left(p_i - \frac{c_i}{r_i}\right)}{r_i^{cr,\kappa} - r_i}\left(e^{-r_i m_i} - e^{-r_i^{cr,\kappa} m_i}\right) + p_i\left(e^{-r_i^{cr,\kappa} m_i} - 1\right).$$

We use the standard shooting method (*Matlab* function *ode15s*) to compute E^{cr} and V_B^{cr}.

B.3 Numerical Results

We present some numerical results in Table B.1 to compare the difference between treating a liquidity crisis as permanent or temporary. The baseline case is $\xi_H^{cr} = \xi_H = 1$ (normal period), where we calculate the bankruptcy boundary, short-term bond spread and long-term bond spread. For the temporary-crisis case, we set $\kappa = 1.5$ so that the crisis will last on average for 8 months before ξ_H^{cr} reverts back to its normal level ξ_H. For $\xi_H^{cr} > \xi_H = 1$, we are particularly interested in the difference between the temporary-crisis case ($\kappa = 1.5$) and the permanent-crisis case ($\kappa = 0$). We present the results for $\xi_H^{cr} = 2$ and 3. As expected, there are some quantitative differences in the bonds spreads between these two cases—the bond spreads of both long-term (LT) and short-term (ST) bonds rise less in the temporary-crisis case than in the permanent-crisis case. But the differences are nevertheless modest relative to the increases in these bond spreads from their baseline levels.

Table B.1 Permanent Liquidity Crisis ($\kappa = 0$) vs. Temporary Liquidity Crisis ($\kappa = 1.5$)

	V_B^{cr}		ST Spread (bps)		LT Spread (bps)	
Baseline $\xi_H^{cr} = \xi_H = 1$	87.11		20.22		186.33	
$\xi_H^{cr} > \xi_H$	perm.	temp.	perm.	temp.	perm.	temp.
$\xi_H^{cr} = 2$	88.22	87.96	41.10	37.84	215.58	195.45
$\xi_H^{cr} = 3$	89.32	88.84	64.77	55.66	248.55	200.34

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