The Fallacy of New Business Creation as a Disciplining Device for Managers

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DISCUSSION PAPER 398

February 2002

FINANCIAL MARKETS GROUP
AN ESRC RESEARCH CENTRE

LONDON SCHOOL OF ECONOMICS

ISSN 0956-8549-393

Non-Technical Summary:

The perspective of establishing their own firms in the future doubtlessly impacts managers' current behaviour. This paper investigates a potential negative externality of new business creation. We consider that in order to be able to create his own business tomorrow, which ensures an extra revenue, a manager must be perceived as good enough: then, he is able to pay back the initial outlay. We analyse a situation in which information about managers' talent is symmetric but incomplete and is updated at the end of each period. Thus, managers behave today so as to convince investors they are talented.

On the one hand, we show that a priori talented managers may indulge in undertaking risky projects today because such a choice renders more difficult the updating of beliefs process regarding their actual types. Unfortunately, this in turn leads them to perform less effort today, which comes at the expense of economic efficiency. Hence, the career concerns we examine do not discipline good managers. However, we show that in such a case, initial employers can reduce managerial slack by resorting to financial market monitoring.

On the other hand, the analysis regarding a priori untalented manager is reversed. A priori poorly talented managers want investors to change their beliefs. Thus, they prefer to choose less risky projects so as to facilitate the updating of beliefs process. Therefore, in that case, initial employers do not have to resort to costly financial market monitoring to discipline them.

Our analysis applies to different kinds of monitoring as well as to different kinds of informational incompleteness.

The Fallacy of New Business Creation as a Disciplining Device for Managers*

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Juin 2001

Abstract

This paper investigates a negative externality of new business creation. When being perceived as a good manager is a necessary condition to establish a firm in the future, we show that a priori talented managers may indulge in undertaking risky projects now. Indeed, such a choice renders more difficult the updating of believes process regarding their actual types. Unfortunately, this in turn leads them to perform less effort, which comes at the expense of economic efficiency. Hence, the career concerns we examine do not discipline good managers. However, we show that employers can reduce managerial slack by resorting to financial markets monitoring.

JEL Classification: G39, G32, D89
KEYWORDS: Career Concern, Business Creation, Market Monitoring, Choice of Risk

*This paper is part of Loss' PhD dissertation. We are grateful to Bruno Jullien for his comments on this work. We also want to thank Bruno Biais, Carole Haritcha! balet, Loana Loft, Estelle and Laetitia Malavolti, Javier Ortega, Pierre Picard, Jean Tirole, Raoul Turtle and Wilfried Zantman for insightful discussions.

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1 Introduction

Overall, the creation of new businesses is essential to economic development. The perspective of establishing his own enterprise in the future doubtlessly impacts a manager's current behavior. The present chapter focuses on the negative externalities this perspective has on his performance. More specifically, we argue that a priori good managers may indulge in choosing risky projects that induce them to perform suboptimal levels of effort, which decreases the total value of the firms they work in. Hence, career concerns do not always discipline managers. We emphasize this dark side of firm creation and investigate the adequate reaction of employers.

Creating and running one's own company is often recognized as being a personal goal that many managers share. On top of the satisfaction that making most of the important decisions gives, becoming an entrepreneur also increases one's ego as it enhances social status. Moreover, it allows formerly employees to appropriate a more important fraction of the surplus they contribute to create. Finally, embracing an entrepreneur's career also comes with private benefits such as pet projects or perks. In this chapter, we consider that becoming an entrepreneur allows to enjoy an extra revenue as compared to what the manager would earn within the company he is currently serving as an employee.

However, the founders of companies that have just graduated are relatively seldom. They often lack the reputation, the experience and/ or the cash resources that are necessary to establish their own businesses. To capture this in the simplest way, we consider a two-periods model where every manager works within a company during the first period while only managers who have a good reputation and enough funds can undertake their own project during the second period.

Reputation on the labor market is principally based on the manager's past activities. Thus, there exists a high level of uncertainty regarding these abilities when managers begin their professional life as neither themselves nor their employers know whether or not they are fit for the positions they hold. Hence, we assume that at the beginning of the first period (today) information is symmetric but incomplete about the managers' skills: the market (and the managers) forms a priori believes regarding their talents, taking into account their diplomas for example. To further keep the model as simple as possible, we make the assumption that there exists two types of managers only: good ones and bad ones.

However, as managers go on with their careers, both the market and themselves come to learn information regarding their competencies. Thus, a priori believes are updated with respect to available information. Accounting profits and stock prices represent two sources of hard information. Naturally, managers will take actions in an attempt to influence the market's believes. To phrase it differently, managers have career concerns. Indeed, according to DeMarzo and Duffie (1995), "Career concerns arise whenever the (internal or external) labor market uses a worker's current output to update the believes about the worker's ability and then bases future wages on these updated believes". As in traditional models of career concerns, the labor market anticipates theses actions in equilibrium and so draws the correct inference about ability from the observed output.

Here, the accuracy of the information that arises depends on two private choices and can be manipulated. On the one hand, managers have the opportunity to choose the informational content of the accounting data, using hedging techniques, among others. In the model we consider, managers decide to hedge or not against the idiosyncratic risk of the project. In other words, they can opt for a more or less risky version of the project they undertake. We suppose this choice to be observable but not verifiable. This reflects that accounting standards (see DeMarzo Duffie (1995) for more details) do not impose on those who run firms (i.e. managers) to disclose their hedging decisions. However, inside owners have privileged information regarding these hedging decisions: they observe the choices managers make but they are unable to write contracts contingent on this soft information.

On the other hand, inside owners of companies can use stock prices to elicit information. Indeed, rendering the stock more liquid enables a speculator that receives private information to disguise his orders more easily so as to make money against uninformed liquidity traders[1]. Thus, it enhances its incentives to gather private information on the firm, which allows to update more efficiently believes regarding the managers' abilities. In our framework, inside owners of companies choose whether to stay private or to go public. A more accurate information regarding manager's characteristics is useful since it leads to a level of effort that is nearer the first-best level.

To sum up, the present chapter analyses how the perspective of creating a new business in the future influences the current willingness of managers to let the market (and themselves) learn information regarding their characteristics as well as the employers' willingness to gather this information.

It seems reasonable that a condition for managers to have the opportunity to create their own firms is that the updated believes regarding their types are good enough, that is, they need to be perceived as good managers at the end of the first period. In this context, we show that opting for the risky version of the project prevents outside financiers from updating believes efficiently. Thus, we identify two opposite behaviors depending on whether a manager is a priori good or bad. On the one hand, a priori bad managers want the market to change its believes regarding their types. Hence, we prove that, provided that the extra revenue is attractive enough, they choose the less risky version of the project to facilitate the updating of believes process. On the other hand, a priori good managers want the market to keep its a priori about their talents. Therefore, we show that they are likely to opt for the risky version of the project so as to limit the updating process. Consequently, were the accounting data the only source of information, a priori good managers would perform a lower level of effort than a priori bad managers. This would decrease the total value of the firm. However, in equilibrium, inside owners of companies anticipate such behaviors. Thus, stock prices help improve the accuracy of information regarding actual managerial talent so as to incentivize a priori good managers to exert a higher level of effort than they would otherwise perform. We show that inside owners of companies resort to an IPO when managers are a priori of the good type whereas they remain private when managers are a priori of the bad type. To sum up briefly these results, employers substitute one source of information (stock prices) for the other (accounting profits).

More generally, the framework we adopt here allows for other sources of information. For example, direct supervision can replace monitoring by the market as far as engineers working in R&D departments are concerned. Then, the supervisor's variable choice could be the number of engineers he has under his control (assuming that more engineers render more difficult the assessment of their individual inputs).

[1]Of course, for the liquidity traders accepting to buy shares in the first place, the price they pay must take into account the loss they make when they sell their shares to the speculator.

Of course, this work is by no mean the first to point out adverse effects of business creation. In particular, a number of research has emphasized that managers often expropriate their current employer in the sense that they leave their companies with ideas or projects they developed therein (Aghion and Tirole (1994), Rajan and Zingales (1997)). However, they consider other mechanisms (promotions, trailer clauses) than the one we envision here. The present chapter deeply builds on the career concerns literature. The starting point of this literature is that managers are disciplined directly through the labor market: superior performances will generate high wage offers whereas poor performances will generate low wage offers. In such a context, explicit incentives may not be necessary (Fama (1980)). Holmström[2] (1982, 1999) investigates in details Fama's idea that career concerns induce efficient managerial behavior. He derives that under some narrow assumptions: neutrality with respect to risk and no discounting rate, Fama's suggestion is correct. Nevertheless, if managers have time preferences, Holmström proves that Fama's conclusion does not hold. Hence, there exists a complementarity between explicit and implicit incentives. Gibbons and Murphy (1992) show that career concerns still create important incentives, even in the presence of explicit incentive contracts. Thus, the optimal compensation contract optimizes total incentives, that is, the combination of the implicit incentives from career concerns and the explicit incentives from compensation contracts. This optimal combination varies with respect to several criteria. For example, explicit incentives should be stronger for workers close to retirement, because they are less sensitive to implicit incentives (end of their careers). The opposite applies for young managers, the current pay of whom should be separated from current performance. In this chapter, we have chosen not to tackle the explicit incentives issue. We do not mean to suggest that such incentives are irrelevant: employers actually use them in formal compensation contracts (Murphy (1998), Gibbons and Murphy (1992)). However, some constraints limit their utilization (regulated industries, Government agencies, difficulty to verify the input of each employee, and so on). Hence, implicit incentives play a critical role and we focus on this role here. The major- and crucial -point of departure of the present chapter vis à vis the above literature is that we endogenize the information that allows to update believes regarding the managers' talent. Indeed, this information depends both on the choice of managers and of their employers. Thus, the career concerns we consider (i.e. the perspective of creating a new firm) which usually serve as a disciplining device can trigger adverse managerial behaviors, here. Finally, our research is closely connected to the papers of DeMarzo and Duffie (1995), and Breeden and Viswanathan (1998). These authors examine financial hedging decisions by managers motivated by career concerns. Our work is closely related to theirs in the sense that hedging improves the information contained by corporate profits regarding management ability since it eliminates extraneous noise. However, and contrary to us, DeMarzo and Duffie consider risk averse managers and leave aside the information that is included in stock prices. Furthermore, they do not allow for different types of managers, which enables us to derive different behaviors depending on whether these managers are good or bad. Breeden and Viswanathan do consider two different types of agents, with unobservable hedging policy, but do not take into account the possibility to create a firm.

The chapter is organized as follows. Section 4.2 introduces the model and discusses the most important assumptions. In Section 4.3, we determine both the equilibrium stock price and the

[2]For a general discussion on career concerns models, we shall refer the reader to Dewatripont, Jewitt and Tirole (1999 part I), who develop a general model of career concerns with multiple tasks and multiple signals.

conditions under which a manager can establish his own firm during the second period. Section 4.4 derives the optimal behaviors of both kinds of managers for both periods regarding their choices of level of effort and of risk. Concluding remarks follow. Proofs are relegated to the Appendix.

2 The model

We consider a two-period model. In the second period good managers have the opportunity to create their own businesses. In the first period all managers work within firms because they do not possess enough cash to become entrepreneurs.

2.1 First period

There exists an infinity of firms and an infinity of employees so that the latter are paid their marginal productivities. A firm's accounting profit π_1 (gross of the manager's wage W) is such that

$$\pi_1(e, \theta, \eta, \varepsilon_i) = e + \theta + \eta + \varepsilon_i,$$

where e represents the effort of the manager and θ represents his talent. Effort is unobservable and costs $\psi(e)$ to the manager, with $\psi(e)$ increasing and convex in e. Managerial talent is unknown from both managers and their employers so that information is incomplete but symmetric. θ is drawn from the distribution $\theta \sim N(\overline{\theta}; \sigma_\theta^2)$. Either $\overline{\theta} = \overline{\theta}^{\mathsf{sup}}$ and managers are assumed to be of the "good "type or $\overline{\theta} = \overline{\theta}_{\mathsf{inf}}$ and managers are of the "bad" type ($\overline{\theta}^{\mathsf{sup}} > \overline{\theta}_{\mathsf{inf}}$). We assume η and ε_i to be two different noises. The first one (with $\eta \sim N(0; \sigma_\eta^2)$) represents the aggregate risk of the market and is not under the control of the manager. Conversely, an adequate hedging policy allows the manager to reduce the idiosyncratic risk ε_i (with $i = 1, 2$) of the project. To be more specific, managers can choose between two versions of the project, Version 1 and Version 2. Version 1 eliminates the idiosyncratic risk, i.e. $\varepsilon_1 = 0$ with probability 1. However, hedging costs $c(\varepsilon_1) = c$. On the contrary, Version 2 does not provide hedging in the sense that $\varepsilon_2 \sim N(0; \sigma_\varepsilon^2)$. Thus, $c(\varepsilon_2) = 0$. We assume the level of risk to be observable but not verifiable so that no contract can be made contingent on it. This reflects that accounting standards do not impose on those who run firms (i.e. managers) to disclose their hedging decisions. However, inside owners have privileged information regarding these hedging decisions: they observe the choices managers make but they are unable to write contracts contingent on this soft information[3]. Finally, θ and η are independently distributed.

At the beginning of the first period, inside owners of companies can release a fraction τ of the shares they own via an IPO. We adopt a framework à la Kyle (1985). When companies are publicly traded, two categories of outside investors hold stocks. Liquidity traders as a whole buy equity (at the initial price P_0) for investment purposes but will have to sell y shares (at price P_1)[4] when unobservable liquidity shocks occur. The number of shares y is normally distributed

[3] Biais and Casamatta (1999) also study the case of managers exerting effort and choosing the risk of their ventures. In their paper, both choices are unobservable, which differs from our assumption that the choice of risk is observable.

[4] Lemma 2 clarifies the computation of P_1.

with mean 0 and variance σ_y^2. A speculator can collect information about the future value of the firm and earns money by trading on that information. The signal he privately observes is

$$s(e, \theta, \eta, \varpi_j) = e + \theta + \eta + \varpi_j,$$

where ϖ_j (with $j = 1, 2$) represents the observation error. This error can be of two types. Either the speculator chooses to receive a non-noisy signal, which requires an investigation effort that costs him $c(\varpi_1) = c$[5]. Then, $\varpi_1 = 0$ with probability 1. Or, he chooses to receive a less accurate signal $\varpi_2 \sim N(0; \sigma_{\varpi_2}^2)$, which costs him zero. The market cannot observe the speculator's choice (nor the signal).

If insiders owners of companies decide to resort to an IPO, they strategically choose the fraction of shares τ they release to outsiders. As will become apparent below, τ will affect the liquidity of the market. Hence, it will influence the speculator's incentives to collect the accurate signal. This will in turn impact the informational content of prices. However, inside owners can opt to remain private.

Profits and prices are observable by everyone but, here, we assume that employers cannot use them in a formal compensation contract. This is a shortcut to capture the idea that there exists constraints which limit the use of explicit incentives (regulated industries, Government agencies, difficulty to verify the input of each employee, and so on). Hence, implicit incentives are at the heart of our analysis. We assume that managers are paid a fixed wage $W\left(\overline{\theta}\right)$ at the end of the first period as is standard in career concerns models. Since the labor market is competitive, $W\left(\overline{\theta}\right)$ corresponds to the first-period marginal productivity of the manager[6]. Hence, managers exert effort and choose a level of risk solely to influence their revenues tomorrow.

2.2 Second period

In the second period managers have the opportunity to create their own businesses in a new industry provided that the updated believes regarding their types are sufficiently good[7]. Let $\mathbb{E}\left(\theta \mid \pi_1, P_1, \varepsilon_i\right)$ represent these believes, updated by taking into account the information that accrue at the end of the first period, i.e. the price of the stock and the profit. Establishing a new firm requires a high up-front involvement from the potential entrepreneur (e.g. meeting banks, lawyers, writing a business plan) before a project is actually undertaken. For simplicity, we assume that the new venture can either succeed or fail as in Holmström-Tirole (1997). When it succeeds, profits are equal to

$$\pi_2(e, \Delta, \theta, \eta, \varepsilon_i) = e + \Delta + \theta + \eta + \varepsilon_i,$$

[5] That $c(\varpi_1) = c(\varepsilon_1) = c$ simplyfies the computations without altering the results.

[6] Here, $W\left(\overline{\theta}\right) = \overline{\theta} + e^* - C$, where $\overline{\theta}$ corresponds to the managerial expected talent, e^* represents the level of managerial effort at the equilibrium, and C represents the generic cost born by the firm. The latter corresponds to the hedging cost as well as to the IPO cost, if any. See below for more details.

[7] At the end of the first period, inside owners of firms are aware of shocks realizations that have affected their results. Thus, they can infer the hedging policies of their competitors. Hence, second-period wages reflect the updating of the believes taken into account that the level of risk is observable. We implicitly assume managers can show to lenders their wages at the beginning of the second period so that lenders also update their believes. As a short cut, we only say that the choice of risk is observable.

where Δ represents the additional productivity of the new industry. When the venture fails, cash-flows are equal to zero. The neo-entrepreneur influences the probability of success of the new firm: if his involvement is high the venture is crowned with success with certainty while if his involvement is weak, the probability of success decreases to p (with $p < 1$). However, the entrepreneur receives a non-monetary and non-transferable private benefit B in the latter case. Undertaking a new venture also requires a financial investment I. We make the assumption-which parallels Holmström and Tirole's standard hypothesis (1997) -that talent let aside, a project needs managerial involvement to be profitable. In other words,

$$e^* - \psi(e^*) + \Delta - I > 0 > p\left[\widehat{e}^* + \Delta\right] - \psi(\widehat{e}^*) + B - I,$$

where e^* (respectively $\widehat{e}^*$) is the equilibrium level of effort when the new entrepreneur is involved (respectively not involved). For simplicity, investors are assumed to be competitive. All parties are risk-neutral and protected by limited liability[8].

So as to have the problem interesting, good (respectively bad) managers must be able (respectively unable) to create their own firms if the market keeps the same believes about their abilities tomorrow. Let $\widetilde{\theta}(.)$ denote the- endogenous -threshold above which a manager can establish his own venture. Thus, we assume[9]

$$\overline{\theta}^{\mathsf{sup}} \geq \widetilde{\theta}\left(W\left(\theta^{\mathsf{sup}}\right)\right) \text{ and } \overline{\theta}_{\mathsf{inf}} < \widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right).$$

The timing of events can be summarized as follows:

First period

1. At the beginning of the first period, all managers are hired by existing companies. They agree on the fixed wages which are paid at the end of the first period. Simultaneously, within each company, the controlling block of shareholders decides whether to undertake an IPO or not. In the former case, it chooses the fraction τ of the capital they release to liquidity traders at price P_0.

2. Each manager chooses the level of risk ε_i of the project he undertakes. This choice is observable but not contractible.

3. Then, each manager chooses his level of effort e, which is not observable.

4. By incurring a cost c, the speculator can increase the precision ϖ_j of the private signal s he receives regarding the profitability of the venture.

5. Then, liquidity traders face a liquidity shock and must sell y of their shares to the speculator. Market observes net order flows. The stock price P_1 is equal to the expected income conditionally on the net order flow.

[8]Since entrepreneurs are protected by limited liability, "involving" them requires the design of an incentive mechanism. Indeed, even if the project fails, which perfectly reveals that the entrepreneur was not involved enough, the latter cannot be hanged or sent to jail.

[9]We will later come back to this assumption and examine the importance of the distance between the a priori believes and the thresholds $\left(\left|\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right) - \overline{\theta}^{\text{sup}}\right| \text{ and } \left|\widetilde{\theta}\left(W\left(\overline{\theta}_{\text{inf}}\right)\right) - \overline{\theta}_{\text{inf}}\right|\right)$.

6. Wages are paid. First-period profits are realized.

7. Based on realized profits and market-clearing price P_1, believes are updated.

Second period

1. Either updated believes regarding a manager's type are good enough and the manager creates his own business or updated believes are not high enough and the manager remains in his first-period firm. In the first case, the new entrepreneur chooses his level of involvement after the financial investment has been sunk.

2. Then, whatever the case, managers choose their efforts and hedging policies.

3 Extra revenue and determination of equilibrium share price

Before investigating the optimal behaviors of both managers and inside owners of companies, we examine under which conditions a manager can establish his own business and determine the extra revenue he receives in such a case. Then, we compute the equilibrium share price that results from an IPO.

3.1 Extra revenue

We first derive under which conditions a manager can create his own firm during the second period. Note that moral hazard does not prevent very talented managers (i.e. $\overline{\theta} \to \infty$) to establish a company. Indeed, profit is in expectation greater than the level of investment I. However, when managers are fairly talented, moral hazard imposes that the founder of a new business must be deeply involved so as to have the project exhibit enough cash-flows. Let D denote the payment to the investor. For incentive purposes, the entrepreneur must earn more when he is involved than when he is not involved into the new project. This is equivalent to

$$[e + \Delta + \mathbb{E}\left(\theta \mid \pi_1, P_1, \varepsilon_i\right) - \psi(e) - D] \geq p\left[\widehat{e} + \Delta + \mathbb{E}\left(\theta \mid \pi_1, P_1, \varepsilon_i\right) - D\right] - \psi(\widehat{e}) + B,$$

where e is the level of effort if the neo-entrepreneur is involved and $\widehat{e}$ is the level of effort if he is not involved. As the neo-entrepreneur's financial input $W(\overline{\theta})$ corresponds to the wage he received during the fi

Lemma 1 *A manager can establish his own firm if and only if the updated believes regarding his type are good enough, i.e. if and only if*

$$\mathbb{E}\left(\theta \mid \pi_1, P_1, \varepsilon_i\right) \geq \widetilde{\theta}(W(\overline{\theta})),$$

where

$$\widetilde{\theta}(W(\overline{\theta})) \equiv I - W(\overline{\theta}) - \frac{e_{SP}^* - p\widehat{e}_{SP}^* + \psi(\widehat{e}_{SP}^*) - \psi(e_{SP}^*)}{1-p} - \Delta + \frac{B}{1-p}.$$

Then, his extra revenue amounts to

$$\beta \equiv e_{SP}^* + \Delta - \psi\left(e_{SP}^*\right) - I.$$

Note that $\widetilde{\theta}(.)$ depends on the manager's first-period wage, hence on his type. We will come back to this point more precisely below, when the wages of the two types of employees are determined. If the manager creates his own firm, his expected gains are equal to the NPV of the project since investors are competitive. This amounts to

$$e_{SP}^* + \mathbb{E}\left(\theta \mid \pi_1, P_1, \varepsilon_i\right) + \Delta - \psi\left(e_{SP}^*\right) - I.$$

However managers, whatever their types can still work as employees within their first-period firms. In that case, their wages are equal to their expected marginal productivities $\mathbb{E}\left(\theta \mid \pi_1, P_1, \varepsilon_i\right)$ [10], which allows to determine the extra revenue they capture when they become entrepreneurs.

3.2 Equilibrium share price

In addition to the (gross) profit π_1, the price of the stock P_1 at the end of the first period helps updating believes. Let us be more specific about the determination of this price. We adopt Kyle's (1985) model of a market with informed traders. In such a framework, market participants first submit their demands. Then, prices are fixed so as to have arbitrageurs make zero expected trading profits (conditional on aggregate demand). As usual liquidity traders enable the informed speculator to disguise his trades; else prices would fully reveal his information and collecting information would be unprofitable ("no trade" theorem, Milgrom and Stokey (1982)).

When a manager chooses the less risky version of the project, the signal the speculator receives does not confer on him a private information. Hence, there exists no reason to trade. Besides, when the speculator does not monitor the firm, we assume the cost of an IPO to be greater than the marginal effort it triggers on the manager's side so that the founder of the company does not resort to an IPO in the first place. Thus, we compute the price of the stock in the only relevant case, that is, when the speculator chooses to receive a non-noisy signal (ϖ_1) while the manager opts for the risky version of the project (ε_2). In this work, we restrict our attention to the speculator's demand strategies that take the linear form

$$x(s) = \alpha + \lambda s.$$

[10] When managers stay within their first-period firms, they exert no effort during the second period as career concerns are absent. Hence, their expected marginal productivities are equal to $\mathbb{E}\left(\theta \mid \pi_1, P_1, \varepsilon_i\right)$.

Total demand amounts to $q = y + x(s)$. To prevent arbitrageurs from earning strictly positive profits, the price must satisfy the condition

$$P_1 = \mathbb{E}\left(\pi_1 - W(\overline{\theta}) \mid y + x(s) = q\right),$$

where the expectation is taken with respect to y and s, conditional on q and the assumption that the speculator's demand is such as described above. The speculator submits his order after he has received the signal s but unaware of total demand q. Hence, his optimal demand is given by

$$x^*(s) = \underset{x}{\operatorname{argmax}}\ x \times [\mathbb{E}\left(\pi_1 - W(\overline{\theta}) \mid s\right) - \mathbb{E}\left(P_1 \mid x\right)].$$

Choosing x the speculator takes into account that P_1 is a function of x and y. Combining the above conditions leads to market equilibrium. The speculator's linear demand is then characterized by the coefficients

$$\alpha = -\lambda(e^* + \overline{\theta}) \text{ and}$$
$$\lambda = \frac{\sigma_y}{(\sigma_\theta^2 + \sigma_\eta^2)^{1/2}},$$

which parallels Homlström and Tirole (1993)'s result[11]. As becomes apparent, the more volatile the market (i.e. the higher σ_y^2), the more the speculator can trade aggressively since it is easier to disguise his orders. The equilibrium price is characterized in the following lemma[12].

Lemma 2 *At the equilibrium, the stock price is given by*

$$P_1 = e^* + \overline{\theta} - W(\overline{\theta}) + \frac{\theta - \overline{\theta} + \eta}{2} + \frac{(\sigma_\theta^2 + \sigma_\eta^2)^{1/2}}{2}\frac{y}{\sigma_y}.$$

The speculator's expected (ex ante) revenue corresponds to[13]

$$ER \equiv \mathbb{E}\left(x^* \times [\mathbb{E}\left(\pi_1 - W(\overline{\theta}) \mid s\right) - \mathbb{E}\left(P_1 \mid x^*\right)]\right) = \frac{1}{2}\sigma_y(\sigma_\theta^2 + \sigma_\eta^2)^{1/2}.$$

Finally, let us define the equivalent normalized performance measure Z, which will render the analysis conducted in the next section more tractable. We define Z as

$$Z \equiv 2\left(P_1 + W(\overline{\theta}) - \frac{(e^* + \overline{\theta})}{2}\right),$$

where P_1 is such as computed in Lemma 2. Hence, Z reduces to[14]

$$Z = e + \theta + \eta + (\sigma_\theta^2 + \sigma_\eta^2)^{1/2}\frac{y}{\sigma_y}.$$

[11] Note that as the liquidity traders' offer is in expectation equal to zero, the speculator's demand must also be zero in expectation.

[12] The resolution of this problem is standard and can be found in Holmström-Tirole (1990) or in Biais, Foucault and Million (1997).

[13] As liquidity traders receive no private information, the price P_0 they pay for the shares they buy must allow them to make no loss (in expectation) when they face a liquidity shock that forces them to sell y stocks at price P_1.

[14] Note that Z is built both from public information variables and from hypothesized equilibrium values. To compute Z, we take $P_1(e, e^*) = e^* + \overline{\theta} - W + \frac{\theta - \overline{\theta} + \eta + e - e^*}{2} + \frac{(\sigma_\theta^2 + \sigma_\eta^2)^{1/2}}{2}\frac{y}{\sigma_y}$.

4 First period optimal behaviors

In this section, we first determine each kind of managers' levels of effort. Then, we derive the optimal level of risk they opt for. Finally, we investigate the decision of inside owners to resort or not to an IPO, which amounts to deciding the level of market monitoring by the speculator.

4.1 Managers' choices of effort

Since effort is costly, unobservable and does not increase first-period wages (which are already fixed at the beginning of the period), a manager exerts e solely to influence favorably the updating process, and thus his future expected gains. These gains correspond to the manager's expected ability over all possible values for π_1 and Z

$$\mathbb{E}_{\pi_1,Z}\left[\mathbb{E}_\theta(\theta \mid \pi_1, Z, e^*, \varepsilon_i)\right],$$

minus his cost of effort

$$\psi(e),$$

plus the probability to create a business times the extra revenue β which is obtained in such a case

$$\Pr\left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) \geq \widetilde{\theta}\right) \times \beta).$$

"$\Pr\left(\mathbb{E}(\theta \mid .) \geq \widetilde{\theta}\right)$" corresponds to the probability that the random variable $E(\theta \mid .)$ is higher than the threshold $\widetilde{\theta}$. To sum up, each manager chooses his effort level e^* so that

$$e^* = \underset{e}{\operatorname{argmax}} \mathbb{E}_{\pi_1,Z}\left[\mathbb{E}_\theta(\theta \mid \pi_1, Z, e^*, \varepsilon_i)\right] + \Pr\left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) \geq \widetilde{\theta}\right) \times \beta - \psi(e).$$

Assuming interior solution, the first-order condition for an equilibrium satisfies (detailed in the Appendix, proof of Proposition 1)

$$\underbrace{cov\left(\theta, \frac{\widehat{f}_e(\pi_1, Z \mid e^*, \varepsilon_i)}{\widehat{f}(\pi_1, Z \mid e^*, \varepsilon_i)}\right) + \frac{\partial}{\partial e}\left\{\Pr\left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) \geq \widetilde{\theta}\right) \times \beta\right\}}_{term\ A} = \psi'(e^*), \tag{1}$$

where

$$\widehat{f}(\pi_1, Z \mid .) = \int f(\pi_1, Z, \theta \mid .)\, d\theta$$

and $f(\pi_1, Z, \theta \mid .)$ respectively denote the marginal density of the observables and the joint density of the talent and of the observables, given the effort level e^* and the choice of version of the project ε_i. Besides, $\widehat{f}_e$ denotes the derivative with respect to effort of the marginal distribution. Overall, *term A* describes the manager's marginal incentives.

Depending on whether the firm goes public or stays private, the equilibrium choices of effort of the managers vary.

In the IPO case, equation (1) reduces to

$$\underbrace{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2}}_{term\ 1}+\underbrace{\frac{\sigma_\theta^2\sigma_{\varepsilon_i}^2}{\left(\sigma_\theta^2+\sigma_\eta^2\right)\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_i}^2\right)}}_{term\ 2}+\underbrace{v\left(\left(\widetilde{\theta}-\overline{\theta}\right)^2\right)}_{term\ 3}\times\beta=\psi'\left(e^*\right), \tag{2}$$

where the function $v(.)$[15] is decreasing in $\left(\widetilde{\theta}-\overline{\theta}\right)^2$. We derive the first two terms of equation (2) from the computation of the covariance in equation (1). Term 1 (respectively Term 2) represents the marginal gain of effort due to the incentives related to the accounting data π_1 (respectively the price P_1[16]) through the updating process. Term 3 indicates the marginal gain of effort due to the expected extra revenue β the manager earns when he becomes an entrepreneur during the second period. On the one hand, the higher the extra revenue, the higher these incentives: the attractiveness of creating a firm increases. On the other hand, the farther the manager's talent from the threshold that allows him to establish his firm (i.e. the higher $\left(\widetilde{\theta}-\overline{\theta}\right)^2$), the lower these incentives. Indeed, the higher $\left(\widetilde{\theta}-\overline{\theta}\right)^2$, the less effort impacts the probability to be above the threshold $\widetilde{\theta}$ which allows to create a business and obtain the extra revenue.

In the no-IPO case, the sole measure of performance comes from the accounting profit π_1. Then, the first-order condition given by (2) reduces to

$$\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2}+t\left(\left(\widetilde{\theta}-\overline{\theta}\right)^2\right)\times\beta=\psi'\left(e^*\right), \tag{3}$$

where the function $t(.)$[17] is decreasing in $\left(\widetilde{\theta}-\overline{\theta}\right)^2$. Except for the incentives implied by the stock price, the same intuitions apply for equations (2) and (3).

Working backward, we now determine the choice of risk a manager makes regarding the project he has under his control during the first period.

4.2 Managers' choices of risk

Each manager has to choose between the two versions of the project, that is two levels of risk: ε_1 or ε_2.

4.2.1 The no-IPO case

Suppose the firm stays private. Since the first-period wage $W(\overline{\theta})$ is already determined, managers opt for the version of the project that maximizes their second-period revenues. Depending on this choice, the manager's expected revenue amounts to

$$W(\overline{\theta})+\mathbb{E}_{\pi_1}\left[\mathbb{E}_\theta(\theta\mid\pi_1,e^*,\varepsilon_i)\right]+\Pr\left(\mathbb{E}(\theta\mid\pi_1,e^*,\varepsilon_i)\geq\widetilde{\theta}\right)\times\beta-\psi\left(e^*\left(\varepsilon_i\right)\right)-c\left(\varepsilon_i\right).$$

[15] The function $v(.)$ is defined in the Appendix, proof of Proposition 1.
[16] To be more specific, in our model, we use Z rather than P_1.
[17] The function $t(.)$ is defined in the Appendix, proof of Proposition 1.

At the equilibrium, the market perfectly anticipates e^* and observes the choice of project version. Thus, $\mathbb{E}_{\pi_1}\left[\mathbb{E}_\theta(\theta \mid \pi_1, e^*, \varepsilon_2)\right]$ is equal to $\mathbb{E}_{\pi_1}\left[\mathbb{E}_\theta(\theta \mid \pi_1(e^*), e^*, \varepsilon_2)\right]$. This means that since the market anticipates e^*, we can apply the law of iterated expectations. Finally, the expectation of the conditional expectation is equal to the non-conditional expectation $\overline{\theta}$[18]. In other words, the market draws the correct inference about the manager's ability from the realized first-period output. Therefore, a manager only considers the impact his choice has on the probability of creating his firm- which drives the extra revenue β -and on the cost resulting from his effort. Using statistic rules for computing the conditional expectation in the case of normal laws[19], we find that

$$\mathbb{E}(\theta \mid \pi_1, e^*, \varepsilon_i) \sim N(\overline{\theta}; \frac{\sigma_\theta^4}{\sigma_\theta^2 + \sigma_\eta^2 + \sigma_{\varepsilon_i}^2}).$$

In other words, $E(\theta \mid \pi_1, e^*, \varepsilon_i)$ is centered on the non-conditional expectation $\overline{\theta}$ and its variance is decreasing with respect to $\sigma_{\varepsilon_i}^2$.

First consider the case of a priori good managers. Equation (3) shows that effort increases when performance becomes more informative in the sense that the variance of the measurement error $\eta + \varepsilon$ decreases[20]. Hence, the effort the manager performs when he opts for Version 1 $(\sigma_{\varepsilon_1}^2 = 0)$ of the project has a greater impact on the updated believes than the one he performs if he chooses Version 2 $(\sigma_{\varepsilon_2}^2 > 0)$. Therefore, choosing the less risky version of the project implies a higher equilibrium effort $(e^*(\varepsilon_1) > e^*(\varepsilon_2))$, which results in a higher cost for the manager. In addition, the manager incurs the hedging cost c. These are the "costs" effects. We have assumed that a priori good managers have the opportunity to establish their own firms provided that the updated believes $\mathbb{E}_\theta(\theta \mid \pi_1, e^*, \varepsilon_i)$ and the *ex ante* believes $\overline{\theta}^{\text{sup}}$ about their talents are equal. Thus, an a priori good manager prefers the believes regarding his type not to be modified. Hence, he wants to minimize the variance of $\mathbb{E}(\theta \mid \pi_1, e^*, \varepsilon_i)$. Since $var\left(\mathbb{E}(\theta \mid \pi_1, e^*, \varepsilon_i)\right)$ is decreasing in $\sigma_{\varepsilon_i}^2$, it leads him to favor the riskier version of the project. This is the "probability" effect. Hence, both the "costs" and the "probability" effects go into the same direction: opting for ε_2 today both decreases the cost resulting from the effort incurred by the manager at the equilibrium and maximizes the probability to have the opportunity to become an entrepreneur tomorrow (see Figure 1).

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[18]Note that, even if the choice of the project version were not observable, at the equilibrium, the market would also perfectly anticipate this choice so that $\mathbb{E}_{\pi_1}\left[\mathbb{E}_\theta(\theta \mid \pi_1, e^*, \varepsilon^*)\right] = \mathbb{E}_{\pi_1}\left[\mathbb{E}_\theta(\theta \mid \pi_1(e^*, \varepsilon^*), e^*, \varepsilon^*)\right] = \overline{\theta}$ would obtain.

[19]Applying statistic rules for computing conditional expectation in the case of normal law gives: $\mathbb{E}(\theta \mid \pi_1, e^*, \varepsilon_i) = \overline{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\eta^2 + \sigma_{\varepsilon_i}^2}\left[\theta + \eta + \varepsilon_i - \overline{\theta}\right]$

Donc, $\mathbb{E}(\theta \mid \pi_1, e^*, \varepsilon_i) \sim N(\overline{\theta}; \frac{\sigma_\theta^4}{\sigma_\theta^2 + \sigma_\eta^2 + \sigma_{\varepsilon_i}^2})$.

[20]By looking at Appendix, proof of Proposition 1, choice of effort by managers, it is straightforward to see that $t\left(\left(\widetilde{\theta} - \overline{\theta}\right)^2\right)$ is decreasing in $\sigma_{\varepsilon_i}^2$.

whith

$$\beta\left(\overline{\theta}_{\mathsf{inf}}\right) \equiv \frac{\psi\left(e^{*}\left(\varepsilon_{1}\right)\right)-\psi\left(e^{*}\left(\varepsilon_{2}\right)\right)+c}{\Phi\left[\frac{\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+\sigma_{\varepsilon_{2}}^{2}\right)^{\frac{1}{2}}}{\sigma_{\theta}^{2}}\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right)-\overline{\theta}_{\mathsf{inf}}\right)\right]-\Phi\left[\frac{\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)^{\frac{1}{2}}}{\sigma_{\theta}^{2}}\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right)-\overline{\theta}_{\mathsf{inf}}\right)\right]},$$

the extra revenue more than offsets the higher cost incurred by the manager due to his higher effort. This leads a priori bad managers to choose the less risky version of the project.

4.2.2 The IPO case

Consider the case where inside owners of companies resort to an IPO. What are the corresponding choices of the two types of managers?

As in the no-IPO case, both the "costs" effect and the "probability" effect induce a priori good managers to opt for the riskier version of the project (Version 2).

Regarding a priori bad managers, the analysis merely differs. Let us first consider the "probability" effect. When Version 2 of the project involves a high level of risk, i.e. when $\sigma_{\varepsilon_2}^2$ is high enough[22], the updating process is poor and resorting or not to an IPO- which is supposed to provide more or less information on the managers' type -only slightly impacts the probability to create a firm tomorrow[23]. If $\overline{\theta}_{\mathsf{inf}}$ is sufficiently smaller than $\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right)$, then this probability is low. Thus, opting for Version 1 of the project rather than for Version 2 implies a quite higher probability to become an entrepreneur regardless the decision to go public. Let us turn to the "costs" effect. The additional cost of effort that results from the choice of Version 1 rather than of Version 2 of the project is lower when the company goes public than when it remains private. This again originates in the relative easiness of the updating process in the former case. Gathering the two effects, and under the assumptions made above, it turns out that if a bad manager opts for the less risky version of the project when there is no IPO, he also makes the same decision if the firm goes public (see the Appendix, proof of Proposition 1, for a rigorous proof).

The above results are summarized in the following proposition.

Proposition 1 *Let the extra revenue related to the creation of a firm be attractive enough (i.e. $\beta \geq \beta\left(\overline{\theta}_{\mathsf{inf}}\right)$) and Version 2 of the project be risky enough. Whether or not the inside owners of companies resort to an IPO, a priori good managers opt for the riskier version of the project. Conversely, a priori bad managers choose the less risky version of the project.*

We can now analyze the inside owners' initial choice to resort or not to an IPO.

4.3 Staying private or going public

First-period wages are determined in a competitive labor market, simultaneously with the decision whether or not to undertake an IPO. This implies that at the equilibrium each wage is equal to the (expected) marginal productivity of each agent. Besides, inside owners choose the

[22] See the Appendix, proof of Proposition 1, for more details on the level of $\sigma_{\varepsilon_2}^2$.

[23] Note that $\sigma_{\varepsilon_2}^2$ high enough does not mean that Z is a sufficient statistic when estimating θ, for (π_1, Z).

initial dilution which maximizes the total expected revenue of their managers[24] so as to be able to attract them.

When a firm hires an a priori bad manager, there is no rationale to release part of the stocks since a priori bad managers opt for the less risky version of the project and because (costly) monitoring by the market cannot procure more information: $\pi_1(\varepsilon_1)$ is a sufficient statistic for $(\pi_1(\varepsilon_1), P_1)$ when estimating $\overline{\theta}$.

When a firm hires an a priori good manager, the IPO decision is more complex. The manager's utility reduces to

$$W\left(\overline{\theta}\right) - \psi\left(e^*\right) + \overline{\theta} + \Pr\left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) \geq \widetilde{\theta}\right) \times \beta. \tag{4}$$

$$\textit{where } W\left(\overline{\theta}\right) = \begin{cases} \overline{\theta} + e^* - ER & \textit{if IPO} \\ \overline{\theta} + e^* & \textit{if no IPO} \end{cases} \tag{5}$$

The first term in equation (4) corresponds to the first-period wage. According to (5), $W\left(\overline{\theta}\right)$ depends on whether or not there was an IPO. The second term in (4) corresponds to the first-period cost of effort incurred by the manager. The third term is the inference of the managers' talent: as before, $\mathbb{E}_{\pi_1,Z}\left[\mathbb{E}_\theta(\theta \mid \pi_1, Z, e^*, \varepsilon_i)\right] = \overline{\theta}$ and does not depend on the accuracy of the signal included in Z.

the threshold $\widetilde{\theta}\left(W\left(\overline{\theta}^{\mathsf{sup}}\right)\right)$ above which the manager is able to create his own business, which increases the probability to become an entrepreneur. On the other hand, dilution has a negative impact on the probability to become an entrepreneur. Indeed, monitoring by the market implies an easier revision of the believes, which is not in the interest of an a priori good manager.

The choice to stay private or go public depends on which of the "costs" and the "probability "effects dominates the other. The inside owners of the company opt for an IPO if $\beta \leq \beta\left(\overline{\theta}^{\mathsf{sup}}\right)$, with

$$\beta\left(\overline{\theta}^{\mathsf{sup}}\right) \equiv \frac{\left[e^*\left(\pi_1, Z\right) - \psi\left(e^*\left(\pi_1, Z\right)\right) - c\right] - \left[e^*\left(\pi_1\left(\varepsilon_2\right)\right) - \psi\left(e^*\left(\pi_1\left(\varepsilon_2\right)\right)\right)\right]}{\Phi\left[\frac{\left(\widetilde{\theta}\left(W\left(\overline{\theta}^{\mathrm{sup}}\right)\right) - \overline{\theta}^{\mathrm{sup}}\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_2}^2\right)}{\left(\begin{array}{c}\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4 \\ +2\sigma_{\varepsilon_2}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\end{array}\right)^{\frac{1}{2}}}\right] - \Phi\left[\frac{\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)^{\frac{1}{2}}}{\sigma_\theta^2}\left(\widetilde{\theta}\left(W\left(\overline{\theta}^{\mathsf{sup}}\right)\right) - \overline{\theta}^{\mathsf{sup}}\right)\right]} \tag{6}$$

The intuition is that for low enough values of the extra revenue, the impact the negative effect of an IPO (i.e. the easier revision of believes) has on a priori good managers' wealth is low, and is more than offset by the positive effect (increased first-period net wage and decrease of $\widetilde{\theta}\left(W\left(\overline{\theta}^{\mathsf{sup}}\right)\right)$) of going public.

Note that the variance of the equivalent normalized performance measure Z remains equal to $VarZ = 2(\sigma_\theta^2 + \sigma_\eta^2)$ whatever the choice of dilution. Therefore, initial owners choose τ just to induce the speculator to learn the non-noisy signal, that is $\sigma_y = \overline{\sigma_y} \equiv \frac{2c}{(\sigma_\theta^2 + \sigma_\eta^2)^{1/2}}$. Raising τ such that $\sigma_y > \overline{\sigma_y}$ would increase the expected gross gain of the speculator (ER), without improving the precision of the signal $\left(\frac{1}{VarZ}\right)$. Inducing the speculator to collect information only requires to have his expected gross gain when he is active offset his cost of monitoring (i.e. gathering no information pays zero), that is $ER = c$.

The next proposition corresponds to a general rule for the decision whether or not to undertake an IPO, depending on the managers' a priori type.

Proposition 2 *Let the extra revenue related to the creation of a firm be in the interval* $\left[\beta\left(\overline{\theta}_{\mathsf{inf}}\right); \beta\left(\overline{\theta}^{\mathsf{sup}}\right)\right]$ *and Version 2 of the project be risky enough. Then, inside owners of companies resort to an IPO when managers are a priori of the good type, whereas they do not if managers are a priori of the bad type.*

As shown below, when inside owners decide to release part of their capital, they choose a level of dilution such that the gain of the speculator just compensates his cost, that is $ER = c$. Finally, we make the implicit assumption that c is small enough so that $e^*\left(\pi_1, Z\right) - c > e^*\left(\pi_1\right)$.

The Appendix, proof of proposition 2, details the technical conditions on Δ and $\overline{\theta}_{\text{inf}}$ under which we have effectively: $\beta\left(\overline{\theta}_{\text{inf}}\right) \leq \beta \leq \beta\left(\overline{\theta}^{\text{sup}}\right)$.

Consider now the (most) general case of a priori good managers. Suppose that β takes intermediate values, then the optimal policy depends on the distance between the manager's a priori talent and the threshold that would allow him to establish his firm, i.e. $\left|\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)-\overline{\theta}^{\text{sup}}\right|$. A closer look at equation (6) shows that $\beta\left(\overline{\theta}^{\text{sup}}\right)$ depends on this distance. Therefore, the optimal decision to go public depends on it. More precisely, the optimal decision wether or not to go public is the following.

If the distance $\left(\text{i.e. } \left|\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)-\overline{\theta}^{\text{sup}}\right|\right)$ is small enough, the probability to create a firm tomorrow when there is market monitoring is of the same order of magnitude as- though smaller than - the probability to create a firm in the absence of market monitoring (and approaches $\frac{1}{2}$). Hence, the threshold $\beta\left(\overline{\theta}^{\text{sup}}\right)$ above which it becomes optimal not to undertake an IPO becomes very high. In that case, monitoring by the market is optimal since it allows to reduce the inefficiency coming from the choice of project version by a priori good managers, without modifying the expected extra revenue[28]:

$$\Pr\left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) \geq \widetilde{\theta}\right) \times \beta \simeq \Pr\left(\mathbb{E}(\theta \mid \pi_1, e^*, \varepsilon_i) \geq \widetilde{\theta}\right) \times \beta.$$

For intermediate values of $\left|\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)-\overline{\theta}^{\text{sup}}\right|$, the difference between the probabilities to be able to create a firm depending on whether there is an IPO or not becomes significant. Going public decreases the probability to become an entrepreneur. Thus, $\beta\left(\overline{\theta}^{\text{sup}}\right)$ decreases so that it is not necessarily optimal to resort to market monitoring.

Finally, if $\left|\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)-\overline{\theta}^{\text{sup}}\right|$ is high, the difference in probabilities is very small. Whether there is an IPO or not, the probability to create a firm tomorrow is high

$$(\Pr\left(\mathbb{E}(\theta \mid \pi_1, e^*, \varepsilon_i) \geq \widetilde{\theta}\right) \simeq \Pr\left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) \geq \widetilde{\theta}\right) \longrightarrow 1).$$

Thus, $\beta\left(\overline{\theta}^{\text{sup}}\right)$ is very high and it turns out to be optimal to use market monitoring, since that allows to reduce the first-period inefficiency without modifying the expected extra revenue[29].

The following proposition summarizes the above results.

Proposition 3 *Suppose β takes intermediate values. Then, it is optimal to undertake an IPO when $\left|\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)-\overline{\theta}^{\text{sup}}\right|$ is small enough or high enough. When $\left|\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)-\overline{\theta}^{\text{sup}}\right|$ takes intermediate values, an IPO is not necessarily optimal.*

[28]In that case, $\beta\left(\overline{\theta}^{\text{sup}}\right)$ will be high. This case can correspond to a case where $\beta\left(\overline{\theta}^{\text{sup}}\right) \geq \beta\left(\overline{\theta}_{\text{inf}}\right)$ and Proposition 2 will apply. See appendix, proposition 2, for more details on that.

[29]Here again, $\beta\left(\overline{\theta}^{\text{sup}}\right)$ will be high. This case can also correspond to a case where $\beta\left(\overline{\theta}^{\text{sup}}\right) \geq \beta\left(\overline{\theta}_{\text{inf}}\right)$ and Proposition 2 will apply. See appendix, proposition 2, for more details on that.

Finally, first-period firms with a priori good managers undertake an IPO so as to induce them to exert a higher effort. However, even if the firm goes public, managers exert a lower effort than when Version 1 of the project is chosen[30].

Testable empirical implications are the following. We predict that firms undertaking an IPO should have a priori more talented managers than firms that do not. Firms going public should also face a higher turnover as they managers become entrepreneurs more frequently.

5 Concluding Remarks

In this chapter, we have focused on implicit incentives and left aside explicit devices. Obviously, within the setting of general risk-neutrality we adopt, explicit incentives would allow to obtain the first-best. Nevertheless, we know from Holmström (1979) that when managers are risk averse, the optimal explicit incentives contract trades off incentives and insurance. Then, it is impossible to achieve the first-best. In that context, Gibbons and Murphy (1992) showed that implicit incentives have a role to play, particularly for managers who are at the beginning of their career. Therefore, it would be worth developing the idea of fallacy of new business creation as a disciplining device in the context of risk averse managers by combining explicit and implicit incentives. Our results- a priori good managers' behaviors, reaction of inside owners -should be robust to such an extension.

More generally, the framework we develop here allows for other sources of information. As suggested above, direct supervision can replace monitoring by the market as far as engineers working in R&D departments are concerned. In such a case, the supervisor's variable choice could be the number of engineers he has under his control since this would alter the assessment of their individual inputs.

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6 Appendix

6.1 Proof of Proposition 1

First, let us determine the choice of effort on the managers' side.

6.1.1 Choice of effort by managers

Managers choose an optimal effort level e^* which maximizes the following objective function

$$e^* = \underset{e}{\operatorname{argmax}} \left\{ \mathbb{E}_{\pi_1, Z} \left[\mathbb{E}_\theta (\theta \mid \pi_1, Z, e^*, \varepsilon_i) \right] + \Pr \left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) \geq \widetilde{\theta} \right) \beta \right\} - \psi (e) .$$

Assuming interior solution, the unique solution satisfies the first order condition

$$\frac{\partial}{\partial e} \left[\int \int \left(\int \theta \frac{f(\theta, \pi_1, Z \mid e^*, \varepsilon_i)}{\widehat{f}(\pi_1, Z \mid e^*, \varepsilon_i)} d\theta \right) d\widehat{F}(\pi_1, Z \mid e, \varepsilon_i) + \Pr \left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) \geq \widetilde{\theta} \right) \beta \right] \Bigg|_{e=e^*} = \psi' (e^*)$$

or

$$\int \int \int \theta \frac{\widehat{f}_e (\pi_1, Z \mid e^*, \varepsilon_i)}{\widehat{f}(\pi_1, Z \mid e^*, \varepsilon_i)} f(\theta, \pi_1, Z \mid e^*, \varepsilon_i) \, d\pi_1 dZ d\theta + \frac{\partial \Pr \left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) \geq \widetilde{\theta} \right) \beta}{\partial e} = \psi' (e^*) . \tag{7}$$

where

$$\widehat{f}(\pi_1, Z \mid .) = \int f(\pi_1, Z, \theta \mid .) \, d\theta$$

and $f(\pi_1, Z, \theta \mid .)$ denote respectively the marginal density of the observables and the joint density of the talent and of the observables, given the effort level e^* and the choice of version of the project ε_i. $\widehat{f_e}$ denotes the derivative with respect to effort of the marginal distribution. Finally, "*cov*" and "$\Pr \left(\mathbb{E}(\theta \mid .) \geq \widetilde{\theta} \right)$" correspond respectively to the covariance of two random variables, and to the probability of the random variable $E(\theta \mid .)$ to be higher than the treshold $\widetilde{\theta}$.

Since the likelihood ratio has zero mean, i.e. $\mathbb{E} \left(\frac{\widehat{f_e}}{\widehat{f}} \right) = 0$, the first part on the left-hand side of equation (7) is such that

$$\int \int \int \theta \frac{\widehat{f}_e (\pi_1, Z \mid e^*, \varepsilon_i)}{\widehat{f}(\pi_1, Z \mid e^*, \varepsilon_i)} f(\theta, \pi_1, Z \mid e^*, \varepsilon_i) \, d\pi_1 dZ d\theta = cov \left(\theta, \frac{\widehat{f_e}}{\widehat{f}} \right) . \tag{8}$$

In our case, the marginal density can be factored as $\widehat{f}(\pi_1, Z \mid e^*, \varepsilon_i) = g(\pi_1 \mid e^*, \varepsilon_i) \, h(Z \mid \pi_1, e^*, \varepsilon_i)$, with

$$
\begin{aligned}
g\left(\pi_1 \mid e^*, \varepsilon_i\right) &\propto \exp\left(-\frac{1}{2}\frac{\left(\pi_1-\left(e+\overline{\theta}\right)\right)^2}{\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2}\right), \\
\text{which implies that } \frac{g_e\left(.\right)}{g\left(.\right)} &= \frac{\left(\theta-\overline{\theta}\right)+\eta+\varepsilon_i}{\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2}, \text{ and} \qquad (9) \\
h\left(Z \mid \pi_1, e^*, \varepsilon_i\right) &\propto \exp -\frac{1}{2}\frac{\left(Z-\mathbb{E}\left(Z \mid \pi_1, e^*, \varepsilon_i\right)\right)^2}{Var\left(Z \mid \pi_1, e^*, \varepsilon_i\right)},
\end{aligned}
$$

By applying statistic rules for computing expectations and variance in case of normal law, we obtain:

$$
\begin{aligned}
\mathbb{E}\left(Z \mid \pi_1, e^*, \varepsilon_i\right) &= e+\overline{\theta}+\frac{\sigma_\theta^2+\sigma_\eta^2}{\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2}\left(\left(\theta-\overline{\theta}\right)+\eta+\varepsilon_i\right), \text{ and} \\
Var\left(Z \mid \pi_1, e^*, \varepsilon_i\right) &= \frac{\left(\sigma_\theta^2+\sigma_\eta^2\right)^2+2\left(\sigma_\theta^2+\sigma_\eta^2\right)\varepsilon_i}{\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2}
\end{aligned}
$$

$$
\text{which implies that } \frac{h_e\left(.\right)}{h\left(.\right)} = \frac{Z-\mathbb{E}\left(Z \mid \pi_1, e^*, \varepsilon_i\right)}{Var\left(Z \mid \pi_1, e^*, \varepsilon_i\right)}, \qquad (10)
$$

Combining (9) and (10) allows to rewrite (8) as

$$
cov\left(\theta, \frac{\widehat{f}_e}{\widehat{f}}\right) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2}+\frac{\sigma_\theta^2\sigma_{\varepsilon_i}^2}{\left(\sigma_\theta^2+\sigma_\eta^2\right)^2+2\left(\sigma_\theta^2+\sigma_\eta^2\right)\sigma_{\varepsilon_i}^2}. \qquad (11)
$$

Now turn to the second part on the left-hand side of equation (7). Applying statistic rules for computing conditional expectation in the case of normal laws gives

$$
\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) = \frac{\left(\begin{array}{c} \overline{\theta}\left(\left(\sigma_\theta^2+\sigma_\eta^2\right)^2+2\left(\sigma_\theta^2+\sigma_\eta^2\right)\sigma_{\varepsilon_i}^2-\sigma_\theta^2\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)\right)-e^*\left(\sigma_\theta^2\sigma_{\varepsilon_i}^2+\left(\sigma_\theta^2+\sigma_\eta^2\right)\sigma_\theta^2\right) \\ +\left(\sigma_\theta^2+\sigma_\eta^2\right)\sigma_\theta^2\left(\theta+\eta+e+\varepsilon_i\right)+\sigma_\theta^2\sigma_{\varepsilon_i}^2\left(\theta+\eta+e+\left(\sigma_\theta^2+\sigma_\eta^2\right)^{\frac{1}{2}}\frac{y}{\sigma_y}\right) \end{array}\right)}{\left(\sigma_\theta^2+\sigma_\eta^2\right)^2+2\left(\sigma_\theta^2+\sigma_\eta^2\right)\sigma_{\varepsilon_i}^2},
$$

which leads to

$$
\begin{gathered}
\Pr\left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i) \geq \widetilde{\theta}\right) = \\
1-\Phi\left(\frac{\left(\widetilde{\theta}-\overline{\theta}\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_i}^2\right)+\left(e^*-e\right)\sigma_\theta^2\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)}{\left[\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4+2\sigma_{\varepsilon_i}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\right]^{\frac{1}{2}}}\right).
\end{gathered}
$$

Thus, the second part on the left-hand side of equation (7) can be rewritten as

$$\left.\frac{\partial \Pr\left(\mathbb{E}(\theta \mid \pi_1, Z, e^*, \varepsilon_i^*) \geq \widetilde{\theta}\right)}{\partial e}\right|_{e=e^*} =$$

$$A \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\frac{\left(\widetilde{\theta}-\overline{\theta}\right)^2\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_i}^2\right)^2}{\left[\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_\theta^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4+2\sigma_{\varepsilon_i}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\right]}\right], \tag{12}$$

$$\text{with A} \equiv \frac{\sigma_\theta^2\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)}{\left(\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_\theta^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4+2\sigma_{\varepsilon_i}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\right)^{\frac{1}{2}}}.$$

According to (7), combining (11) and (12) and rearranging shows that if there is an IPO, the manager exerts an effort e^* that verifies

$$\begin{gathered}\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2}+\frac{\sigma_\theta^2\sigma_{\varepsilon_i}^2}{\left(\sigma_\theta^2+\sigma_\eta^2\right)\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_i}^2\right)}+ \\ \frac{\beta\sigma_\theta^2\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)}{\left[\begin{array}{c}\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4 \\ +2\sigma_{\varepsilon_i}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\end{array}\right]^{\frac{1}{2}}}\frac{1}{\sqrt{2\pi}}\exp-\frac{1}{2}\left[\frac{\left(\widetilde{\theta}-\overline{\theta}\right)^2\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_i}^2\right)^2}{\left[\begin{array}{c}\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4 \\ +2\sigma_{\varepsilon_i}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\end{array}\right]}\right] = \psi'\left(e^*\right).\end{gathered} \tag{13}$$

For reading convenience, we have denoted in the text

$$v\left(\left(\widetilde{\theta}-\overline{\theta}\right)^2\right) \equiv$$

$$\frac{\sigma_\theta^2\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)}{\left[\begin{array}{c}\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4 \\ +2\sigma_{\varepsilon_i}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\end{array}\right]^{\frac{1}{2}}}\frac{1}{\sqrt{2\pi}}\exp-\frac{1}{2}\left[\frac{\left(\widetilde{\theta}-\overline{\theta}\right)^2\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_i}^2\right)^2}{\left[\begin{array}{c}\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4 \\ +2\sigma_{\varepsilon_i}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\end{array}\right]}\right].$$

The proof regarding the no-IPO case follows the same lines. Thus, the level of effort performed by the manager satisfies

$$\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2}+\frac{\beta}{\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)^{\frac{1}{2}}}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\frac{\left(\widetilde{\theta}-\overline{\theta}\right)^2\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)}{\sigma_\theta^4}\right] = \psi'\left(e^*\right). \tag{14}$$

For reading convenience, we have denoted

$$t\left(\left(\widetilde{\theta}-\overline{\theta}\right)^2\right) \equiv \frac{1}{\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)^{\frac{1}{2}}}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\frac{\left(\widetilde{\theta}-\overline{\theta}\right)^2\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)}{\sigma_\theta^4}\right].$$

Finally, let us come back to footnote Nb025. In order to compare the actual level of effort of the manager to the first-best, we compute the latter which verifies

$$1+\frac{\beta}{\left(\sigma_\theta^2+\sigma_\eta^2\right)^{\frac{1}{2}}}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\widetilde{\theta}-\overline{\theta}\right)^2\frac{\left(\sigma_\theta^2+\sigma_\eta^2\right)}{\sigma_\theta^4}\right]=\psi\left(e^{FB}\right).$$

Now, we can examine the choice of version of the project by a priori bad managers in the no-IPO case.

6.1.2 Choice of risk of the first period project when the firm remains private and managers are of the bad type

An a priori bad manager chooses Version 1 of the project (i.e. ε_1) if his expected revenue when doing so is higher than the one he earns if he chooses Version 2 of the project (i.e. ε_2), which reduces to

$$\begin{bmatrix} \mathbb{E}_{\pi_1}\left[\mathbb{E}_\theta(\theta\mid\pi_1,e^*,\varepsilon_1)\right]-\psi\left(e^*\left(\varepsilon_1\right)\right)-c \\ +\Pr\left(\mathbb{E}(\theta\mid\pi_1,e^*,\varepsilon_1)\geq\widetilde{\theta}\right)\beta \end{bmatrix}\geq\begin{bmatrix} \mathbb{E}_{\pi_1}\left[\mathbb{E}_\theta(\theta\mid\pi_1,e^*,\varepsilon_2)\right]-\psi\left(e^*\left(\varepsilon_2\right)\right) \\ +\Pr\left(\mathbb{E}(\theta\mid\pi_1,e^*,\varepsilon_2)\geq\widetilde{\theta}\right)\beta \end{bmatrix}. \tag{15}$$

It is possible to rewrite (15) as a condition on β. The bad managers favor the riskless version if and only if $\beta\geq\beta\left(\overline{\theta}_{\mathsf{inf}}\right)$ with

$$\beta\left(\overline{\theta}_{\mathsf{inf}}\right)\equiv\frac{\psi\left(e^*\left(\varepsilon_1\right)\right)-\psi\left(e^*\left(\varepsilon_2\right)\right)+c}{\Phi\left[\frac{\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)^{\frac{1}{2}}}{\sigma_\theta^2}\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right)-\overline{\theta}_{\mathsf{inf}}\right)\right]-\Phi\left[\frac{\left(\sigma_\theta^2+\sigma_\eta^2\right)^{\frac{1}{2}}}{\sigma_\theta^2}\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right)-\overline{\theta}_{\mathsf{inf}}\right)\right]}. \tag{16}$$

Now, consider the choices managers make after the firm has gone public.

6.1.3 Choice of version of the project after an IPO

The level of effort $e^*\left(Z,\varepsilon_2\right)$ which corresponds to the IPO case and the choice of Version 2 of the project verifies equation (13) with $\varepsilon_i=\varepsilon_2$. Regarding the IPO case and the choice of Version 1 of the project, it is possible to rewrite (13) as

$$\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2}+\frac{\beta}{\left(\sigma_\theta^2+\sigma_\eta^2\right)^{\frac{1}{2}}}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\frac{\left(\widetilde{\theta}-\overline{\theta}\right)^2\left(\sigma_\theta^2+\sigma_\eta^2\right)}{\sigma_\theta^4}\right]=\psi'\left(e^*\right). \tag{17}$$

We now compare the level of efforts respectively implied by Version 1 and Version 2 of the project once an IPO has been undertaken (respectively given by equation (17) and by equation (13) and $\varepsilon_i=\varepsilon_2$). Note we have

1.

$$\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2} > \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2} + \frac{\sigma_\theta^2\sigma_{\varepsilon_2}^2}{\left(\sigma_\theta^2+\sigma_\eta^2\right)\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_2}^2\right)},$$

2.

$$\text{assuming that } \sigma_{\varepsilon_2}^2 > \sigma_\theta^2+\sigma_\eta^2 \qquad (\mathbf{A}_1), \tag{18}$$

$$\text{we have} \qquad \frac{1}{\left(\sigma_\theta^2+\sigma_\eta^2\right)^{\frac{1}{2}}} > \frac{\sigma_\theta^2\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)}{\left[\begin{array}{c}\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4\\ +2\sigma_{\varepsilon_2}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\end{array}\right]^{\frac{1}{2}}},$$

3. $$\exp -\tfrac{1}{2}\left[\frac{\left(\widetilde{\theta}-\overline{\theta}\right)^2\left(\sigma_\theta^2+\sigma_\eta^2\right)}{\sigma_\theta^4}\right] > \exp -\tfrac{1}{2}\left[\frac{\left(\widetilde{\theta}-\overline{\theta}\right)^2\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_i}^2\right)^2}{\left[\begin{array}{c}\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_i}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4\\ +2\sigma_{\varepsilon_i}^4\sigma_\theta^4\left(\sigma_{\theta\mathrm{wj}}^2\right.\end{array}\right.}\right.$$

$\overset{+}{\varepsilon}$

$$\widehat{\beta}\left(\overline{\theta}_{\mathsf{inf}}\right) \equiv \frac{\psi\left(e^{*}\left(\varepsilon_{1}, Z\right)\right)-\psi\left(e^{*}\left(\varepsilon_{2}, Z\right)\right)+c}{\Phi\left[\frac{\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathrm{inf}}\right)\right)-\overline{\theta}_{\mathrm{inf}}\right)\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+2\sigma_{\varepsilon_{2}}^{2}\right)}{\left(\begin{array}{c}\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+\sigma_{\varepsilon_{2}}^{2}\right)\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)^{2}\sigma_{\theta}^{4} \\ +2\sigma_{\varepsilon_{2}}^{4}\sigma_{\theta}^{4}\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)\end{array}\right)^{\frac{1}{2}}}\right]-\Phi\left[\frac{\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)^{\frac{1}{2}}}{\sigma_{\theta}^{2}}\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right)-\overline{\theta}_{\mathsf{inf}}\right)\right]}. \tag{19}$$

We need now to check that $\beta\left(\overline{\theta}_{\mathsf{inf}}\right) > \widehat{\beta}\left(\overline{\theta}_{\mathsf{inf}}\right)$, respectively given by (16) and (19). Let us first compare the numerators of these two thresholds. Since choosing Version 1 of the project implies that π_1 is a sufficient statistic for (π_1, Z) when estimating θ, then $\psi\left(e^{*}\left(\varepsilon_{1}, Z\right)\right) = \psi\left(e^{*}\left(\varepsilon_{1}\right)\right)$. Then, compare $e^{*}\left(\varepsilon_{2}, Z\right)$ and $e^{*}\left(\varepsilon_{2}\right)$, which verify respectively equation (13) and equation (3). We have

1. $\dfrac{\sigma_{\theta}^{2}}{\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+\sigma_{\varepsilon_{2}}^{2}}+\dfrac{\sigma_{\theta}^{2}\sigma_{\varepsilon_{2}}^{2}}{\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+2\sigma_{\varepsilon_{2}}^{2}\right)} > \dfrac{\sigma_{\theta}^{2}}{\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+\sigma_{\varepsilon_{2}}^{2}}$,
2. $\dfrac{\sigma_{\theta}^{2}\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+\sigma_{\varepsilon_{2}}^{2}\right)}{\left[\begin{array}{c}\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+\sigma_{\varepsilon_{2}}^{2}\right)\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)^{2}\sigma_{\theta}^{4} \\ +2\sigma_{\varepsilon_{2}}^{4}\sigma_{\theta}^{4}\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)\end{array}\right]^{\frac{1}{2}}} > \dfrac{1}{\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+\sigma_{\varepsilon_{2}}^{2}\right)^{\frac{1}{2}}}$,
3. assuming

$$\sigma_{\varepsilon_{2}}^{2} > \frac{\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)^{2}}{\sigma_{\varepsilon_{2}}^{2}}+\frac{\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)}{2} \qquad (\mathbf{A}_2), \tag{20}$$

we have
$$\frac{\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+\sigma_{\varepsilon_{2}}^{2}}{\sigma_{\theta}^{4}} > \frac{\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)^{2}\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+2\sigma_{\varepsilon_{2}}^{2}\right)^{2}}{\left[\begin{array}{c}\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+\sigma_{\varepsilon_{2}}^{2}\right)\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)^{2}\sigma_{\theta}^{4} \\ +2\sigma_{\varepsilon_{2}}^{4}\sigma_{\theta}^{4}\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}\right)\end{array}\right]}.$$

Consequently, under $(\mathbf{A}_2)$ we obtain that $e^{*}\left(\varepsilon_{2}, Z\right) > e^{*}\left(\varepsilon_{2}\right)$. To sum up, since $e^{*}\left(\varepsilon_{2}, Z\right) > e^{*}\left(\varepsilon_{2}\right)$, and $e^{*}\left(\varepsilon_{1}, Z\right) = e^{*}\left(\varepsilon_{1}\right)$, we have

$$\psi\left(e^{*}\left(\varepsilon_{1}, Z\right)\right)-\psi\left(e^{*}\left(\varepsilon_{2}, Z\right)\right)+c < \psi\left(e^{*}\left(\varepsilon_{1}\right)\right)-\psi\left(e^{*}\left(\varepsilon_{2}\right)\right)+c,$$

which means that the numerator of $\beta\left(\overline{\theta}_{\mathsf{inf}}\right)$ is greater than the numerator of $\widehat{\beta}\left(\overline{\theta}_{\mathsf{inf}}\right)$. Let us now compare the denominators of these two thresholds.

1. $\Phi\left[\dfrac{\left(\sigma_{\theta}^{2}+\sigma_{\eta}^{2}+\sigma_{\varepsilon_{2}}^{2}\right)^{\frac{1}{2}}}{\sigma_{\theta}^{2}}\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right)-\overline{\theta}_{\mathsf{inf}}\right)\right]$ is increasing in $\sigma_{\varepsilon_{2}}^{2}$,

2.

$$\text{assuming that} \qquad \sigma_{\varepsilon_2}^2 < \frac{3}{2}\left(\sigma_\theta^2 + \sigma_\eta^2\right) \qquad (\mathbf{A}_3), \tag{21}$$

$$\text{we have } J \equiv \frac{\left(\sigma_\theta^2+\sigma_\eta^2\right)\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_2}^2\right)}{\left(\begin{array}{c}\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4 \\ +2\sigma_{\varepsilon_2}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\end{array}\right)^{\frac{1}{2}}} \text{ increasing in } \sigma_{\varepsilon_2}^2.$$

Under assumption $(\mathbf{A}_3)$, $\Phi[J]$ is increasing in $\sigma_{\varepsilon_2}^2$. This means that for $\sigma_{\varepsilon_2}^2$ high enough,

$$\Phi\left[\frac{\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)^{\frac{1}{2}}}{\sigma_\theta^2}\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right)-\overline{\theta}_{\mathsf{inf}}\right)\right] \simeq \Phi\left[J\times\left(\widetilde{\theta}\left(W\left(\overline{\theta}^{\mathsf{sup}}\right)\right)-\overline{\theta}^{\mathsf{sup}}\right)\right].$$

Hence, for $\sigma_{\varepsilon_2}^2$ high enough the denominator of $\widehat{\beta}\left(\overline{\theta}_{\mathsf{inf}}\right)$ is close to the denominator of $\beta\left(\overline{\theta}_{\mathsf{inf}}\right)$, whereas the numerator is lower. Thus, for $\sigma_{\varepsilon_2}^2$ high enough, we have $\widehat{\beta}\left(\overline{\theta}_{\mathsf{inf}}\right) < \beta\left(\overline{\theta}_{\mathsf{inf}}\right)$. Note that $\sigma_{\varepsilon_2}^2$ high enough does not mean $\sigma_{\varepsilon_2}^2 \longrightarrow +\infty$. Thus, Z is not a sufficient statistic for (π_1, Z) when estimating θ.

Finally, we need to check that assumptions $(\mathbf{A}_1)$, $(\mathbf{A}_2)$ and $(\mathbf{A}_3)$, respectively given by (18), (20) and (21) are compatible.

- $(\mathbf{A}_1)$ and $(\mathbf{A}_2)$ are compatible.

  It suffices to choose $\sigma_{\varepsilon_2}^2 > \max\left(\sigma_\theta^2+\sigma_\eta^2, \dfrac{\left(\sigma_\theta^2+\sigma_\eta^2\right)^2}{\sigma_{\varepsilon_2}^2} + \dfrac{\left(\sigma_\theta^2+\sigma_\eta^2\right)}{2}\right)$.

- $(\mathbf{A}_1)$ and $(\mathbf{A}_3)$ are compatible. It suffices to choose $\sigma_\theta^2+\sigma_\eta^2 < \sigma_{\varepsilon_2}^2 < \frac{3}{2}\left(\sigma_\theta^2+\sigma_\eta^2\right)$.

- $(\mathbf{A}_2)$ and $(\mathbf{A}_3)$ are compatible. $\frac{3}{2}\left(\sigma_\theta^2+\sigma_\eta^2\right) > \dfrac{\left(\sigma_\theta^2+\sigma_\eta^2\right)^2}{\sigma_{\varepsilon_2}^2} + \dfrac{\left(\sigma_\theta^2+\sigma_\eta^2\right)}{2} \Leftrightarrow \sigma_{\varepsilon_2}^2 > \left(\sigma_\theta^2+\sigma_\eta^2\right)$ which corresponds to $(\mathbf{A}_1)$.

Now, let us examine the decision to go public when firms hire a priori good managers.

6.2 Proof of Proposition 2 and 3

At the equilibrium, initial owners of companies choose to undertake an IPO if and only if doing so maximizes the total expected revenue of the manager they hire, that is iff

$$\left(\begin{array}{c}\overline{\theta}+e^*\left(\pi_1\left(\varepsilon_2\right),Z\right)-\psi\left(e^*\left(\pi_1\left(\varepsilon_2\right),Z\right)\right)-c \\ +\mathbb{E}_{\pi_1,Z}\left[\mathbb{E}_\theta(\theta\mid\pi_1,Z,e^*,\varepsilon_2)\right]+\Pr\left(\mathbb{E}(\theta\mid\pi_1,Z,e^*,\varepsilon_2)\geq\widetilde{\theta}\right)\beta\end{array}\right)$$
$$\geq\left(\begin{array}{c}\overline{\theta}+e^*\left(\pi_1\left(\varepsilon_2\right)\right)-\psi\left(e^*\left(\pi_1\left(\varepsilon_2\right)\right)\right) \\ +\mathbb{E}_{\pi_1}\left[\mathbb{E}_\theta(\theta\mid\pi_1,e^*,\varepsilon_2)\right]+\Pr\left(\mathbb{E}(\theta\mid\pi_1,e^*,\varepsilon_2)\geq\widetilde{\theta}\right)\beta\end{array}\right)$$

This reduces to $\beta \leq \beta\left(\overline{\theta}^{\mathsf{sup}}\right)$, with

$$\beta\left(\overline{\theta}^{\mathsf{sup}}\right) \equiv$$
$$\frac{\left[e^*\left(\pi_1\left(\varepsilon_2\right), Z\right) - \psi\left(e^*\left(\pi_1\left(\varepsilon_2\right), Z\right)\right) - c\right] - \left[e^*\left(\pi_1\left(\varepsilon_2\right)\right) - \psi\left(e^*\left(\pi_1\left(\varepsilon_2\right)\right)\right)\right]}{\Phi\left[\frac{\left(\widetilde{\theta}\left(W\left(\overline{\theta}^{\mathrm{sup}}\right)\right)-\overline{\theta}^{\mathrm{sup}}\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_2}^2\right)}{\left(\begin{array}{c}\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4 \\ +2\sigma_{\varepsilon_2}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\end{array}\right)^{\frac{1}{2}}}\right] - \Phi\left[\frac{\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)^{\frac{1}{2}}}{\sigma_\theta^2}\left(\widetilde{\theta}\left(W\left(\overline{\theta}^{\mathsf{sup}}\right)\right) - \overline{\theta}^{\mathsf{sup}}\right)\right]} \tag{22}$$

Now let us compare the thresholds $\beta\left(\overline{\theta}_{\mathsf{inf}}\right)$ and $\beta\left(\overline{\theta}^{\mathsf{sup}}\right)$. We first make some preliminary statements.

1) Regarding $\beta\left(\overline{\theta}_{\mathsf{inf}}\right)$ given by (16):

- Its numerator $\psi\left(e^*\left(\varepsilon_1\right)\right) - \psi\left(e^*\left(\varepsilon_2\right)\right) + c$ is positive, since $e^*\left(\varepsilon_1\right)$ is higher than $e^*\left(\varepsilon_2\right)$ and since $\psi\left(e\right)$ is an increasing function of e.
- Its denominator

$$\Phi\left[\frac{\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)^{\frac{1}{2}}}{\sigma_\theta^2}\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right) - \overline{\theta}_{\mathsf{inf}}\right)\right] - \Phi\left[\frac{\left(\sigma_\theta^2+\sigma_\eta^2\right)^{\frac{1}{2}}}{\sigma_\theta^2}\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right) - \overline{\theta}_{\mathsf{inf}}\right)\right]$$

  corresponds to the difference between two probabilities.

  If $\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right) = \overline{\theta}_{\mathsf{inf}}$, both probabilities are equal to $\frac{1}{2}$. The denominator is equal to 0 and $\beta\left(\overline{\theta}_{\mathsf{inf}}\right)$ tends to the infinity.

  If $\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right) \gtrsim \overline{\theta}_{\mathsf{inf}}$, the denominator is strictly positive. Hence, $\beta\left(\overline{\theta}_{\mathsf{inf}}\right)$ is positive and finite.

  If $\widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right) \gg \overline{\theta}_{\mathsf{inf}}$ and sufficiently high, these probabilities are equal to 1. Hence, $\beta\left(\overline{\theta}_{\mathsf{inf}}\right)$ tends to the infinity.

2) Regarding $\beta\left(\overline{\theta}^{\mathsf{sup}}\right)$ given by (22):

- Its numerator $\left[e^*\left(\pi_1, Z\right) - \psi\left(e^*\left(\pi_1, Z\right)\right) - c\right] - \left[e^*\left(\pi_1\left(\varepsilon_2\right)\right) - \psi\left(e^*\left(\pi_1\left(\varepsilon_2\right)\right)\right)\right]$ is positive since it corresponds to the difference of profit resulting from the choice of effort associated with the information set $\left(\pi_1\left(\varepsilon_2\right), Z\right)$ in the first case and with the information set $\pi_1\left(\varepsilon_2\right)$ in the second case. This effort is nearer the first-best level in the first case.

- Its denominator

$$\Phi\left[\frac{\left(\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)-\overline{\theta}^{\text{sup}}\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)\left(\sigma_\theta^2+\sigma_\eta^2+2\sigma_{\varepsilon_2}^2\right)}{\left(\begin{array}{c}\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)\left(\sigma_\theta^2+\sigma_\eta^2\right)^2\sigma_\theta^4\\+2\sigma_{\varepsilon_2}^4\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\eta^2\right)\end{array}\right)^{\frac{1}{2}}}\right]-\Phi\left[\frac{\left(\sigma_\theta^2+\sigma_\eta^2+\sigma_{\varepsilon_2}^2\right)^{\frac{1}{2}}}{\sigma_\theta^2}\left(\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)-\overline{\theta}^{\text{sup}}\right)\right]$$

corresponds to the difference between two probabilities.

- If $\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)=\overline{\theta}^{\text{sup}}$, both probabilities are equal to $\frac{1}{2}$. The denominator is then equal to 0 so that $\beta\left(\overline{\theta}^{\text{sup}}\right)$ tends to the infinity.

  If $\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)\gtrsim\overline{\theta}^{\text{sup}}$, the denominator is strictly positive. Hence, $\beta\left(\overline{\theta}^{\text{sup}}\right)$ is positive and finite.

  If $\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)\gg\overline{\theta}^{\text{sup}}$ and sufficiently high, these two probabilities are equal to 1. Hence, $\beta\left(\overline{\theta}^{\text{sup}}\right)$ tends to the infinity.

The proof of proposition 3 is direct from point 2). If β takes an intermediate value, then if $\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)=\overline{\theta}^{\text{sup}}$ or $\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)\gg\overline{\theta}^{\text{sup}}$, $\beta<\beta\left(\overline{\theta}^{\text{sup}}\right)$ and it is optimal to undertake an IPO. But, if $\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)\gtrsim\overline{\theta}^{\text{sup}}$ then we can have either $\beta<\beta\left(\overline{\theta}^{\text{sup}}\right)$ or $\beta\geq\beta\left(\overline{\theta}^{\text{sup}}\right)$. It is then not necessarily optimal to undertake an IPO.

3) Moreover,

$$\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)=\underbrace{I-\frac{e_{SP}^*-p\widehat{e}_{SP}^*+\psi(\widehat{e}_{SP}^*)-\psi(e_{SP}^*)}{1-p}-\Delta+\frac{B}{1-p}}_{term\ 1}-W\left(\overline{\theta}^{\text{sup}}\right),$$

$$\widetilde{\theta}\left(W\left(\overline{\theta}_{\text{inf}}\right)\right)=\underbrace{I-\frac{e_{SP}^*-p\widehat{e}_{SP}^*+\psi(\widehat{e}_{SP}^*)-\psi(e_{SP}^*)}{1-p}-\Delta+\frac{B}{1-p}}_{term\ 1}-W\left(\overline{\theta}_{\text{inf}}\right).$$

Term 1 is independent of the type of the agent since e_{SP}^*, $\widehat{e}_{SP}^*$, Δ, p and B do not depend on $\overline{\theta}$. $W\left(\overline{\theta}_{\text{inf}},\varepsilon_1\right)=\overline{\theta}_{\text{inf}}+e^*\left(\overline{\theta}_{\text{inf}},\varepsilon_1\right)-c$ is strictly increasing in $\overline{\theta}_{\text{inf}}$. Indeed, by using equation (14)

$$\frac{d\ e^*\left(\overline{\theta}_{\text{inf}},\varepsilon_1\right)}{d\ \overline{\theta}_{\text{inf}}}=$$

$$\frac{\overline{\theta}_{\text{inf}}\overbrace{\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\text{inf}}\right)\right)-\overline{\theta}_{\text{inf}}\right)}^{>0\text{ by definition}}\frac{\beta}{\sqrt{2\pi}}\frac{\left(\sigma_\theta^2+\sigma_\eta^2\right)^{\frac{1}{2}}}{\sigma_\theta^4}\exp-\frac{1}{2}\left(\widetilde{\theta}\left(W\left(\overline{\theta}_{\text{inf}}\right)\right)-\overline{\theta}_{\text{inf}}\right)^2\frac{\sigma_\theta^2+\sigma_\eta^2}{\sigma_\theta^4}}{\psi''(e^*)}>0.$$

Thus, for example if we consider the cases where either $\overline{\theta}^{\text{sup}}\gtrsim\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)$ or $\overline{\theta}^{\text{sup}}\gg\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)$, we adjust Term 1 in such a way that $\overline{\theta}^{\text{sup}}$ is just above $\widetilde{\theta}\left(W\left(\overline{\theta}^{\text{sup}}\right)\right)$ or much higher than $\widetilde{\theta}\left(W\left(\overline{\theta}^{\mathsf{sup}}\right)\right)$. Then, we choose $\overline{\theta}_{\mathsf{inf}}$ sufficiently low - such that $W\left(\overline{\theta}^{\mathsf{sup}}\right) > W\left(\overline{\theta}_{\mathsf{inf}}\right)$, $\overline{\theta}^{\mathsf{inf}} < \widetilde{\theta}\left(W\left(\overline{\theta}_{\mathsf{inf}}\right)\right)$ and such that $\beta\left(\overline{\theta}_{\mathsf{inf}}\right)$ is finite. In that case, we obtain $\beta\left(\overline{\theta}^{\mathsf{sup}}\right) > \beta\left(\overline{\theta}_{\mathsf{inf}}\right)$.