On Physics and Finance

By

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ON PHYSICS AND FINANCE

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Abstract

This paper gives a short introduction of the academic field of financial asset pricing and relates some recent as well as historical developments in finance and in physics.

1   Introduction

This short essay is meant to fulfill a number of aims, none of which can be fully achieved in the short space available. First, it tries to give a first impression of what mathematical finance is all about. Second, it tries to relate some of the methods used in finance to some of the tools used in physics (or at least to some of the tools used in physics as perceived by a financial economist who is not also a physicist). And lastly it gives a broad overview of finance-related work done by academic physicists, as well as a biased opinion of such work. This short article then is, by necessity, partial, fragmentary and incomplete.

2   Some historical overlaps of physics and finance

One of the more common mathematical formulations for the stochastic time-series and cross-sectional behaviour of asset prices is to assume that asset prices are semi-martingales, and more restrictively that they are driven by Brownian Motions. This is chronologically one of the first concepts shared by mathematical finance and by physics.

Indeed, it seems that the first mathematical use of what is now called Brownian Motion or the Wiener process appears in Bachelier's 1900 thesis[4]. Bachelier was a mathematics student of Poincaré. But he used this (then still unnamed) process to model stock prices and to price all sorts of options in continuous time. And legend has it that Poincaré would never forgive him. The paper most commonly remembered as the first mathematical formulation of Brownian Motion is Einstein's[8]. Bachelier's contribution was not entirely forgotten by mathematicians, as Feller calls Brownian motion, in his book on probability theory, the "Wiener-Bachelier" process.

Bachelier used what amounts to $dS_t = \sigma dW_t$ as the model of stock price increments. This is not an appealing model for stocks, since it implies, among others, that prices have no drift and that the increments are independently

and identically distributed, both empirically rejected at decent levels of confidence.

The Brownian model then fell into oblivion until Paul Samuelson (1970 Nobel prize in economics) dug it out again in the sixties and assumed that stock prices solve the stochastic differential equation $dS_t = \mu S_t dt + \sigma S_t dW_t$. The obvious improvement here is that prices may now have a drift, and that the stock price increments are no longer identically distributed: asset price changes are more volatile the higher their levels. We shall come back to the maintained assumption of independence when we discuss some contributions by physicists to this debate in a later section.

Mathematical finance really took off after the celebrated Black-Scholes options pricing formula was published in 1973[5]. The contribution is actually more the logic of replication than the formula per se. They derived a second order linear parabolic partial differential equation (PDE) that the (unknown) pricing function would have to satisfy in order not to allow for any arbitrage. In other words, suppose we price a new redundant asset. Then the price we quote must be such that no-one can combine this new asset at that price with all the remaining assets at their respective prices into a portfolio that would generate money without needing an initial investment. In their simple model, it turns out that there is just one such price, and it solves the above-mentioned PDE with the corresponding boundary condition. This PDE can be transformed, modulo some changes of variables, into the heat equation. The resulting Cauchy problem can then be solved by Feynman-Kac probabilistic methods or by numerical methods such as finite-difference algorithms. Again, finance borrows heavily from physics.

Most of derivative pricing has been based on the no-arbitrage PDE until it was reformulated (fully done around 1979 by Harrison and Kreps[12]) in terms of equivalent martingale measures, i.e. probabilistic methods, in particular the Girsanov-Cameron-Martin Theorem. This scientific development seems to parallel the reformulation in quantum mechanics from a Hamiltonian approach involving operators in Hilbert space to a Lagrangian formulation involving Feynman path integrals. It is noteworthy, however, that

due to the influx of probabilists into the finance profession, the language of mathematical finance is now entirely the language of probability theory. A recent paper by two physicists, Rosa-Clot and Taddei[19], though, argues that these standard probabilistic methods are in fact equivalent to path integral methods.

3   Derivative Asset Pricing

So what does mathematical finance actually try to accomplish? One of its main aims is to price assets. Suppose you have got an asset that is characterized by its (unique) random payoff at some time T in the future, X_T. For instance, this could be the outcome of a coin flip: you get \$1 for heads and \$0 for tails. How much is that asset worth? Alternatively, it may be the payoff of a call option, $X_T = \max\{0, S_T - K\}$ where S_T is the stock price at T and where K is a prespecified deterministic strike price.

It is tempting to say that the value of the coin toss should be \$50, since by repeating the experiment independently infinitely often, the strong law of large numbers applies. However, this ignores two facts. First, by introspection no-one would invest all of his or her savings in such an asset: one would presumably have to be somehow compensated to take on the risk. And second, it doesn't consider the market prices of related assets. If related securities are already traded in the marketplace, the new option we would like to price may have a value that can be deduced (maybe even unambiguously) from the prices of the remaining assets. Or, at the very least, we may be able to gauge the investor's risk attitudes and the resulting compensations they require to take on this particular market risk from the prices of all the other assets in the economy.

Mathematically, the fundamental pricing method used these days, from which the PDE method can be easily derived if asset prices and payoffs are Markovian, is the following. Assume that N risky assets are traded in the economy at prices $(S_t^1, \ldots S_t^N)$, and that there is a riskless money market account B_t (a sort of savings account that grows at the rate of interest) as well.

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The price S_t^0 of the new asset is determined as follows. There is no (approximate) arbitrage for asset prices $(S_t^0, S_t^1, \ldots S_t^N, B_t)$ and the corresponding cumulative dividend processes $(D_t^1, \ldots, D_t^N)$ if and only if there is a measure $\mathbb{Q}$, equivalent to the real world objective measure $\mathbb{P}$, under which all gains processes after deflation, $S_t^i/B_t + \int_0^t dD_s^i/B_s$, are martingales. Intuitively, this is an internal consistency condition. If the new asset is a redundant derivative with only the final payoff $X_T \equiv g(S_T^1, \ldots, S_T^N)$, then we can deduce $\mathbb{Q}$, the manner by which risks are priced, from the existing N assets, and then price the new asset unambiguously as $S_t^0/B_t = E_t^{\mathbb{Q}}\left[g(S_T^1, \ldots, S_T^N)/B_T\right]$.

If both the payoff of the new asset and S are Markovian, then the pricing function F, with $S_t^0 = F(t, S_t)$, satisfies the PDE $\mathcal{L}F(x,t) = r_t F(x,t)$ with boundary condition $F(x,T) = g(T,x)$, $(\forall x, \forall t \in (0,T))$, where the operator $\mathcal{L}F(x,t) \equiv F_t(x,t) + r_t F(x,t)' F_S(x,t) + \frac{1}{2}\mathrm{tr}\left[\sigma_t \sigma_t' F_{SS}(x,t)\right]$. The PDE can be easily computed numerically. Notice also the parallel between the martingale pricing method and the probabilistic Feynman-Kac solution to the PDE.

4　A Pricing Example: the Perpetual American Put

We make the convenient *Black-Scholes assumptions*, specifically that $dS_t = \mu S_t dt + \sigma S_t dW_t$, W_t a one-dimensional Brownian Motion, a constant interest rate r and continuous and frictionless trading.

As opposed to European options, American options can be exercised at any moment. Given an underlying stock price process S, an American Put pays off $\max\{K - S_\tau, 0\}$ whenever the holder wants to exercise the option, and the option then dies. K is called the strike price: it is the right to choose a time $\tau \leq T$ (T is the maturity) and to sell the stock worth S_τ for K, where K and T are contractually specified at the outset.

We assume there are no dividends. Still, an American put may get exercised before maturity. The reason is that the payoff of a put (as opposed to a call) is bounded by the strike price K (by limited liability). So if we're at $t < T$ and if S is low enough, say $S \approx 0$, we get K if we exercise at t.

Waiting will not be optimal, since the maximum we could get in the future will also be K, but discounted back to today this will be worth less than K. By continuity, it is optimal to exercise for S low enough. The task will be to find how low low enough is.

Intuitively, denote the *optimal exercise boundary* by $S^*(t)$. That S^* is an increasing function of time is evident. Given that we have not exercised in the past, the larger $T - t$ the higher the likelihood that S will drop deeply. Also, $S_T^* = K$. What is the put price process $Y_t = P(S, t)$? The payoff of the put now depends on time, $g(t, S) = (K - S_t)^+$.

We assume that the option is redundant and we conjecture that, by no-arbitrage, Y/B is a $\mathbb{Q}$–martingale in the continuation region $\mathcal{C} \equiv \{(x, t) : P(x, t) > (K - x)^+\} = \{(x, t) : x > S^*(t)\}$. In terms of PDEs,

$$\mathcal{L}P = rP \qquad \text{on } \mathcal{C} \tag{1}$$

$$P(x, T) = (K - x)^+ \tag{2}$$

$$P(S^*(t), t) = (K - S^*(t))^+ \tag{3}$$

$$S^* \text{ maximizes the value of the option, or } \frac{\partial P}{\partial S}(S^*(t), t) = -1 \tag{4}$$

Equation (3) is the *value-matching* condition (follows by continuity from $P(x, t) = (K - x)^+$ in the stopping region, (which is closed)). Equation (4) is needed to determine the exercise boundary. The second condition of that last item is called the *smooth-pasting* boundary condition, or the *high-contact* boundary condition, and it can be justified by arbitrage arguments.

The technical difficulty is that we need to simultaneously solve for the put price and for the exercise boundary (which is a function). The PDE depends on the free-boundary, and the free-boundary (for instance via smooth-pasting) depends on the function $P(\cdot)$. Even making all the Black Scholes assumptions does not help. The following special case of Perpetual Puts (Merton[18]) makes this problem tractable.

If the put never matures, then $T - t$ is always infinite, independent of t. Hence S^* will simply be a nonnegative real number. But maximizing to find

a number is easier than the variational methods needed to find an optimal function. Also, P will not depend on t directly. The Pricing PDE becomes:

$$rP = rSP_S + \frac{1}{2}\sigma^2 S^2 P_{SS} \tag{5}$$

$$P(\infty) = 0 \tag{6}$$

$$P(S^*) = K - S^* \tag{7}$$

$$S^* \text{ maximizes the value of the option, or } \frac{\partial P}{\partial S}(S^*) = -1 \tag{8}$$

The PDE became an ODE with generic solution $P(S) = a_1 S + a_2 S^{-\gamma}$ with $\gamma \equiv 2r/\sigma^2$. The first boundary condition tells us that $a_1 = 0$, and the second one that $a_2 = (K - S^*)S^{*\gamma}$. We find that $P(S) = (K - S^*)\left(\frac{S}{S^*}\right)^{-\gamma}$. Lastly, we maximize over S^*, the FOC being $S^* = \frac{\gamma K}{1+\gamma}$ so that $P(S) = \frac{K}{1+\gamma}\left[(1+\gamma)\frac{S}{\gamma K}\right]^{-\gamma}$. We can verify our intuition that the reason we'd like to exercise early is the time-value of money. Setting $r = 0$ implies that $\gamma = 0$, and hence $S^* = 0$.

5 Behavioural Content and Economic Equilibrium

So far there was very little economic or behavioural content in our discussion, pricing was rather mechanical. This was because of two assumptions. First, we assumed that the underlying asset price processes were known and given to us. And second, the new asset or derivative asset we were set out to price was assumed to be redundant, i.e. the new asset's payoffs could be completely expressed and replicated in terms of the underlying assets (the so-called complete markets assumption), which was why pricing was unambiguous.

In the real world, it happens of course that markets are not complete, so that we cannot unambiguously determine $\mathbb{Q}$ from existing data. In that case, we have to model the dynamic behaviour of the stock market participants (investors, banks, hedge funds, speculators, investment funds, governments

etc.) whose continual interactions determine $\mathbb{Q}$ and therefore asset prices. In other words, of all the possible pricing measures, we have to pick one of them, hopefully the one that prices the new asset closest to the unknown value the market puts on it.

Asset prices are determined by what economists hundreds of years ago, and somewhat confusingly, called "equilibrium." The asset prices that we observe satify the condition that at those prices, demand equals supply dynamically at every moment in time. If demand, say, was greater than supply at a price and at a moment in time, then the seller could have sold his asset at a higher price, and some investors who demanded the asset were willing to pay the price, or even more, but they were artificially rationed. An efficient working of markets guarantees that at any moment in time demand equals supply: every participant buys or sells exactly the quantity they intended to at that price. Furthermore, investors' information is updated by the information revealed by prices, which in turn affects demands. At an equilibrium, these effects are incorporated into both prices and demands.

In the recent physics of finance literature, this simple point tends to be misunderstood because some physicists attach a different meaning to the word "equilibrium," a meaning of constancy and stability, while economists mean market-clearing and purposeful behaviour. Given that asset prices are quite volatile over time, this seems to be at odds with a steady "equilibrium."

In fact, quite the opposite is true. It is easy to show[13] that asset prices need to be of unbounded variation in order to preclude arbitrage. In some sense, if asset prices were not so irregular, they could be somehow predicted locally, and investors would be able to construct money-machines. But if that was the case, every investor would be on the same side of the market, say buy, and demand could not equal supply. I.e. the auctioneer, or the specialist, or the market maker as the case may be, could not clear markets and asset markets would collapse. So at every moment in time there are demand and a supply functions that aggregate investors' beliefs, random pieces of information and needs, causing prices to randomly change over time to accomodate demand and supply and to induce market clearing at

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that price.

Economic equilibrium also does not mean that all limit orders are matched, as some authors seem to believe. Of course there are at any moment agents out there who would like to buy an asset if only its price dropped sharply, or sell an asset if only the price rose dramatically, which is reflected by unmatched limit orders. Market clearing implies that there is no rationing of any sort. It is the "invisible hand" that sets the right incentives and harmonizes the many traders' diverse needs and opinions, even considering that participants learn from prices.

So in order to model the stochastic behaviour of asset prices, one needs to model demand, supply and the mechanism of market clearing, and this is where economics and behaviour enters the picture.

6   A simple general equilibrium model.

To illustrate how asset price behaviour can be deduced from traders' purposeful interactions, we briefly outline an excessively simplified toy model.

Time flows in $[0, T]$. There are $A + 1$ assets, with dividend processes $(0, D^1, \ldots, D^A)$. We want to find their price processes $(S^0, \ldots, S^A)$.

Investor h tries to maximize his increasing and concave (reflecting risk-aversion) "utility function" $U(c) = E\left[\int_0^T u^h(c_t, t)\right]$ by choosing a self-financing trading strategy θ^h such that

$$\theta_t^h \cdot S_t = \int_0^t \theta_s^h (dS_s + dD_s) - \int_0^t p_s (c_s^h - e_s^h) ds \qquad (9)$$

$$\theta_T^h \cdot X_T = 0 \qquad (10)$$

where e^h is investor h's non-financial (labour) income stream and where p_t is the consumption price process. The investor solves this problem anew in each period, having updated his or her information from the information revealed by prices and income.

Formally, an *equilibrium* is then defined as a collection

$$\{S; \; p; \; (c^h, \theta^h)_{h=1}^H\} \qquad (11)$$

such that

1) given S and p all investors maximize their utility by choosing optimal (c^h, θ^h), and

2) markets clear,

$$\sum_{h=1}^{H} \theta^h = 0; \qquad \sum_{h=1}^{H} [c^h - e^h] = 0 \tag{12}$$

Implicitly, as a consistency requirement, agents refine their information via observing prices, which in turn gives rise to demand functions which, when crossed, generate the observed prices. For simplicity, assume that markets are complete. It can then be shown that equilibrium asset prices satisfy

$$S_t = \frac{1}{u_c^\lambda(e_t, t)} E_t \left[\int_t^T u_c^\lambda(e_s, s) dD_s \right]$$

for a particular vector λ, and where

$$u^\lambda(e, t) = \sup_{x \in \mathbb{R}^H} \sum_{h=1}^{H} \lambda^h u^h(x^h, t) \quad \text{s.t.} \quad \sum_h x^h = e \tag{13}$$

Recall that by the Fundamental Theorem of Asset Pricing there is no arbitrage iff there is a measure $\mathbb{Q}$ equivalent to $\mathbb{P}$ s.t.

$$S_t = E_t^{\mathbb{Q}} \left[\int_t^T \exp\left(-\int_t^s r_u du \right) dD_s \right] \tag{14}$$

while we just showed that at equilibrium

$$S_t = \frac{1}{u_c^\lambda(e_t, t)} E_t \left[\int_t^T u_c^\lambda(e_s, s) dD_s \right] \tag{15}$$

It follows that we can think of $\exp\left(-\int_t^s r_u du \right) \frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{u_c^\lambda(e_s, s)}{u_c^\lambda(e_t, t)}$. In other words, we pinned down $\mathbb{Q}$ from more fundamental processes, namely the labour incomes. In a more complete model, these labour incomes themselves are endogenous, of course, because they depend on the supply and demand of labour.

Also in equilibrium $S_t = \frac{1}{u_c^\lambda(e_t,t)} E_t \left[\int_t^T u_c^\lambda(e_s,s) dD_s \right]$. The naive strong law of large numbers on the other hand would have predicted something like $S_t = E_t \left[\exp\left(-\int_t^s r_u du\right) dD_s \right]$. The strong law of large numbers, contrary to common sense and empirical evidence, values a dollar of payoff per unit of probability in each state of the world equally, whether you're rich or poor, whether you're risk averse or not, whether you're retired or saving, whether prices of consumption commodities are high or low, whether GDP is high or low, whether you're in a bull or a bear market.

7   On "econophysics" as a discipline

There is a growing litterature written by academic physicists on finance, sometimes called "econophysics", which is published in their own journals and evolves largely apart from academic finance. Many authors seem to ignore the vast finance literature that has already tackled many of those problems and to avoid contacts with academic financial economists generally. Economists are typically not aware of the "econophysics" literature, which would be a shame if wheels got reinvented on both sides. Some relevant web sites are `http://www.mailbase.ac.uk/lists/finance-and-physics`, and `http://www.unifr.ch/econophysics` or `http://www.econophysics.org`. The working papers on these sites vary from naive, uninformed, arrogant and amusing (to economists) conjectures to serious and scientific analysis, and it is hard to give an overall impression.

One has to welcome the (re)discovered interest by physicists to financial problems because finance is a fascinatingly complex field. The hope is that new tools may contribute via technical problem solving, via original ideas or via good empirical work. However, the somewhat uneasy relation between physicists and economists has already been described by Doyne Farmer (of the Santa Fe Institute, also co-heading his own financial trading company, called "Prediction Company") who wrote a nice piece on "Physicists attempt to scale the ivory towers of finance," in which he makes points similar to the ones I raise here, for instance that the problem solving approaches are quite

different, and that it is not obvious that methods that work in physics also work in finance. Another survey paper is "Econophysics - A new area for computational statistical physics ?" by Dietrich Stauffer (to be found on http://www.unifr.ch/econophysics/). In the end of the day, one cannot assume that economic decision makers as well as the resulting actions and prices behave like lifeless particles or feedback rules following automata. Banks, investors, government etc. behave purposefully, they interact and play games, they anticipate each other's actions, and they try to change the rules of the game if that may remove inefficiencies. Behaviour and prices are endogenous and each variable depends on each other variable via complicated relations like expectations about future actions of the Fed and how agents perceive them to affect inflation, GDP etc. That's what makes economics a social science.

8   On some recent work on finance by physicists

There seem to be 3 main fields of research where physicists have contributed to finance: empirical statistical regularities in prices, market crashes and derivatives pricing. I will address them in turn, and illustrate them with the help of some representative papers, both from physicists and from economists.

8.1   Empirical statistical regularities in prices

For a given unit of time, log returns should be normally distributed since any log-return $\ln p(t + \tau) - \ln p(t)$ can be decomposed into many log-returns over arbitrarily small subintervals, at least if these sub-returns are iid. However, it has been known for a long time (at least since the 50s, first known reference in 1915) that returns over short horizons are not normal, they have "fat tails." Refer for instance to Fama's[9][10] and Mandelbrot's[17] seminal articles and to the references given therein. It was thought then that returns have a stable Levy distribution. Based on daily prices, the characteristic exponent

α was estimated to be 1.7.

Some economists pointed out that variances seem to be defined, though, an observation that invalidates the assumption of Levy processes. They addressed these issues by using random clock changes or stochastic volatility (volatility clustering), methods to mix normals, thereby yielding fat tails, but keeping second moments finite. In independent work, it has recently been shown both in the econophysics literature and in the financial literature that indeed the tail index ranges from 3 to 5, incompatible with a Levy distribution.

Physicists also analyzed the exact power laws in the tails, and conclude that the price-formation process cannot be fully understood in terms of central limit theorems at all, even in generalized form, confirming Clark's results from 1973. They also characterize how large the central part of the distribution that does converge nicely is, and how slowly the tails converge. Since that central part and its cutoffs change with the scale chosen, they confirm that the process underlying prices must have nontrivial temporal structure. Physicists also pointed out that some of the proposed solutions by economists for this dependency (like stochastic volatility models, ARCH/GARCH models etc.) do address some issues, but not others, since they do not connect the behaviour on multiple timescales.

Over and above measuring the fatness, it seems primordial to try to understand what CAUSES tails to be fat. Some economists[7][1] argue that the central limit theorem (CLT) fails because sampling is random. The number of individual effects added together to give the return during a day is variable and in fact random, making the standard CLT inapplicable. Ané and Geman[1] for instance showed empirically that when one uses the cumulative number of transactions as "business time" then the resulting returns (1 minute, 5 minutes, 10 minutes and 15 minutes) under business time are normal. They show that the densities conditioned on the number of trades are normal. Olsen and Associates have similarly done extensive work on high-frequency foreign exchange data, see for instance [20]. Interestingly, their research staff consists mainly of physicists.

This line of research that aims to characterize and explain the statistical properties of returns is still booming, and joint research would certainly be useful.

8.2   Market crashes

The methods of investigation in this subfield of econophysics are inspired largely by the physics of critical phenomena, in particular by the idea of log-periodic oscillations.

The basic idea goes as follows (from a paper by Laloux, Potters, Cont, Aguilar and Bouchaud[14]): financial crashes are the analogue of critical points in statistical mechanics, where the response to a small external perturbation becomes infinite, because all the subparts of the system respond cooperatively. If one further assumes that 'log-periodic' corrections are present, then one can try to use the oscillations seen on the markets as precursors to predict the crash time, which is the time where those oscillations accumulate. The authors do point out that there is no theoretical evidence substantiating the claim that crashes are critical points - not even speaking of log-periodic oscillations. They then go on and show that some crashes were not predicted, but more importantly that most predictions of crashes did not happen. In other words, no statistical significance has been assessed, and it is not obvious that money could be consistently made. Of course, if the theory works and money can be made, then the profit opportunity vanishes quickly since many people will try to sell short just before the crash, causing the crash to happen already at that time, which by induction will cause a crash instantaneously, without anyone being able to benefit from the insight.

A paper by Bouchaud and Cont[6] proposes ad-hoc non-linear Langevin equations as a model of stock market fluctuations. The paper assumes an ad-hoc price-setting rule and feedback effects of prices on themselves. These rules drive their results. The techniques used are very useful, but again for my taste I would like to see why agents and market makers should behave that way. Also, prices cannot be thought of as exogenous processes that crash according to some law. This kind of assumes what was to be shown. Prices

are the outcomes of trades and expectations of zillions of agents with different motivations to trade, and the prices themselves influence the motivations.

I would like to contrast these approaches with the ones chosen by economists studying the same topics. Prices on many markets are set by informed market makers. Economists, for instance Madrigal and Scheinkman[16], use rational models. Traders possess private and heterogenous information, and the market maker acts strategically to maximize profits. The market maker must now consider that the prices he sets affect both the information he acquires (through the order flow) and the information he releases back to the market. They show that the equilibrium price as a function of the order flow displays a discontinuity, which can be interpreted as a market crash.

Economists Avery and Zemski[2] on one hand, and Lee[15] on the other hand, construct models of herd behaviour, informational cascades, bubbles and crashes entirely built upon rational optimizing agents. The main ingredients are event uncertainty and composition uncertainty (Avery and Zemski) or trading costs (Lee).

8.3   Derivative Pricing

Obviously many physicists work on Wall Street, their technical skills being highly valued. Monte-Carlo simulations, finite difference methods and neural networks have been fruitfully used in finance for a long time.

Academically, some papers introduced a path integral approach to derivatice security pricing, reformulating the models in terms of quantum mechanical formalism, see for instance Baaquie[3]. This paper also claims to have shown that the price of the stock option is the analogue of the Schrodinger wave-function of quantum mechanics and to have obtained the exact Hamiltonian and Lagrangean of the system. Papers along this line reformulate standard finance models in a different language, but it is too early to report extensions and new insights. The hope is to find accelerated numerical methods.

9 Conclusion

The field of finance recently received a lot of attention by physicists, and my hope is that the interaction between financial economists and physicists will lead to new insights into the way financial markets work. A great first step would be to submit research papers to the appropriate finance journals for refereeing and for dissemination among the finance profession. There is a lot to be gained from both sides.

I could not find a better way to conclude than citing Doyne Farmer[11],

> With some justification, many economists think that the entry of physicists into their world reflects merely audacity, hubris, and arrogance. Physicists are not known for their humility, and some physicists have presented their work in a manner that plays into that stereotype.
>
> ...
>
> Many of the physicists know very few empirical facts and are largely ignorant of the literature in economics and finance... Physicists like me should stop reinventing the wheel.

Maybe one could add that financial economists like me should stop presuming that many physicists simply look for a way to have fun and to recycle physics methods, and invite physicists to take part in financial workshops and conferences.

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