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Abstract

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Introduction

Racetracks and lotteries are interesting financial markets. These investments have negative expected value for most investors. But they have wide appeal for entertainment, the possibility of small and large gains, the intellectual aspects of their study and for insights valuable in other financial markets. There are economic and behavorial effects and an extensive literature concerning biases, strategies, regulations and successes. Thaler and Ziemba (1988) in a well cited paper analyzed these markets including their market efficiency and rationality. They argued that

> "wagering markets have a better chance of being efficient because the conditions (quick, repeated feedback) are those which usually facilitate learning"

*Thanks to Scott Brown, Steve Moffitt and Leighton Vaughan Williams for helpful data and discussions.

This paper focuses on racetracks and lotteries, but other sports betting markets have been shown to be efficient. These include NFL football as shown by Pankoff (1968), Sauer, Brajer, Ferris and Marr (1988), Sauer (1998) and Ziemba and MacLean (2018). These papers show that football betting markets like the Las Vegas NFL market are efficient and and the spreads quoted reflect the true probability of winning. Like the stock market, there are winning strategies based on mean reversion, risk arbitrage.

Financial economists have long been interested in the efficiency of financial markets. Kendall [1953] examined the behavior of industrial share prices and spot prices for cotton and wheat. By analyzing serial correlations, prices appeared to follow a random walk. In the 1960s the focus was on defining efficiency and performing tests for a range of efficiency notions. Roberts [1967] defined weak, semi-strong, and strong form efficiencies as holding when stock market prices reflect all price information, all publicly available information, and all information, respectively. Most financial markets have been shown to be efficient in the weak form. The evidence for semi-strong is mixed; see Johnson and Sung (2008) for a survey of the racing literature. The strong form is largely inefficient. See Fama (1970) for a survey of this work in financial markets. The exceptions, termed anomalies, include seasonal patterns such as the small firm January effect, turn-of-the-month and year effects, holiday effect, day of week, time of day, and cross-sectional regularities that apply to stocks with low price to earnings ratios or with earnings surprises, etc. See the surveys by Hawawini and Keim [1995, 2000], Ziemba [1994, 2012, 2020] and Keim and Ziemba (2000) for more details.

Fama [1991] updated his earlier survey. Tests for return predictability focus on forecasting returns using variables such as interest rates and dividend yields. Event studies formalize the semi-strong form idea by testing whether or not there are adjustments of prices to specific public announcements. Finally, the strong form concept is studied through tests for private information. The evidence is that future returns are predictable from past returns, dividend yields and term structure variables. On the face of it, this is a violation of weak form efficiency. But, as suggested by Roll [1977], since every test of efficiency must be a joint one with a maintained equilibrium hypothesis of price formation (e.g., the capital asset or arbitrage pricing models), this violation is confounded by the joint hypothesis problem of whether there is a rational variation over time in expected returns or systematic deviations from fundamental value.

One issue that makes stock market efficiency and rationality hard to test is the fact that the markets do not have a fixed end point where the investments are settled. See Fama (1970, 1991) and Roll (1977) on this. There are, of course, futures markets in equities to hedge and essentially cash out cash equity positions. Also day traders have fixed end point strategies. While arguments can be made that increased returns may be occurring because of increased risk, which is difficult or impossible to measure accurately, there is evidence that most or all of the gains in securities markets have occurred during the seasonally anomalous periods. Ritter and Chopra [1989] and Cadsby and Ratner [1992] show, for example, that the only periods where risk as measured by the capital asset pricing model is rewarded with equity returns is precisely at the anomalous periods such as the day before holidays, the turn of the month, in the first two weeks of January for small stocks, etc. Ariel [1987], Lakonishok and Smidt [1988], Hensel, Sick and Ziemba [2000] and Ziemba [2012, 2020] showed that essentially all the stock market's gains during the 20th century in the U.S. occurred in the first half of the month. Event studies are more straightforward and less controversial since they are able to provide more clear cut evidence of the effect of new information. Regarding strong form tests there is considerable evidence that corporate insiders have private information that is not fully reflected in current prices.

MacLean and Ziemba (2020b) show that in sports having several good players is better than one superstar for success. The same is true for thoroughbred stud fees, see Cameron (2010), where having many good offspring is superior to one superstar. Also in executive stock buying where purchases by several executives is a better signal of future prospects than a single larger transaction, see Brown et al (2013).

Sports and betting markets are well suited for testing market efficiency and bettor rationality. This is because vast amounts of data are available, in the form of prices (for devising technical systems) and other information (for devising fundamental systems), and each bet has a specified termination point when its final asset value is determined. For rationality tests, markets with this latter property offer an advantage over markets, like securities markets, where current value depends upon future events and current expectations of future values. Also, some wagering markets have characteristics that reduce the problematic nature of the aforementioned joint hypothesis test. For instance, Dana and Knetter [1994] note that pointspread bets on National Football League games all have identical risk and return characteristics, as well as similar horizons. This allows a test of efficiency without specifying the bettors utility functions.

The special properties of sports betting and lotteries might lead one to speculate that they are even more efficient than financial markets. However, there is another aspect to these markets that confounds the notion of rationality: for them to be offered, the average bettor must lose. Indeed, given the transactions costs involved in these markets (e.g., about 13-30% for horseracing and about 50% for lotteries), the average losses are large. This has not stopped the search for profitable wagering systems, though, and there are some notable successes. For example, Thorp (1962) demonstrated that card-counters can win playing blackjack. This survey of research on horseracing, sports betting on football and basketball, and lotteries reports numerous studies of efficiency in these markets. Several profitable systems are also described, though[1]. The continued success of these winning systems tends to be related to some complicating factor in its development or execution. For instance, the

[1] Beyond the academic work surveyed here, evidence abounds of individuals who have successfully beaten the odds. See, for example, Akst [1989], Benter (1994, 2008) and Ziemba [2017, 2019a].

system may involve short odds and complex probability estimation (e.g. place and show wagering at the racetrack), it may rely on syndicates of bettors (e.g., cross-track horserace betting), it could require extremely long time horizons (e.g., lotteries), or extensive data collection and statistical work (e.g., fundamental handicapping systems for horseracing). The winning systems described are, of course, just a subset of the winning systems used in practice. The incentives to disclose details of a winning system may not be sufficient in some cases given that such an action typically reduces the systems profitability as others employ it. Finally, I also discuss optimal betting strategies for exploiting inefficiencies when they are present.

Sports betting and lotteries involve substantial transactions costs. Because it directly affects prices, the take - what the gambling establishment keeps for its operation - is properly accounted for in all of the analyses discussed in this survey. Another cost in these markets is for information (e.g., tip sheets at the racetrack). Costly information requires a redefinition of efficiency, one where prices are said to reflect information to the point where the cost of additional information just equals the benefit of acting on that information. But because they are difficult to measure, this survey ignores information costs (and other transactions costs beyond the take). Thus, the general findings of efficiency gain further support with the introduction of these other costs. In racing, the advent of rebaters has driven the take from 13-30% for various bets to about 11-13% for large bettors. Just like shopping at a wholesale market for food, large bettors get a quantity discount. The rebaters and the large bettors share a discount given by the track for the signals (outcomes).

The net effect of this is that these discounts bets are blended with the rest of the bets. Rebates at a lower level are available to small bettors. For example, a win bet might have a track-take of 15%. The rebate shop might take 5%, the bettor gets 5% off and the track gets only 5% for the *signal*, namely the results. So the rebate bettor pays effectively 10% and the track gets its 15% by charging the non-rebate wagerers more than 15%. For exotic wagers such as exactas (getting the first two finishers in exact order) and other wagers, the take is in the 20-25% area so the rebate is larger. The biggest race track betting market is Hong Kong. There the standard rebate gives 10% back on losing bets of $10,000 plus Hong Kong dollars. This update discusses changes and new information in these markets.

Racetrack betting markets

At a typical racetrack the market accepts wagers for about 20-60 minutes. These wagers come from those at the track and from simulcasts that feed money into the betting pools from all over the US and the world. In the past, there were separate pools for a given race and type of wager with different prices that could be arbitraged as discussed in Hausch and Ziemba (1990a, b). Now all the money goes into the same pools from the host track and many other betting venues.[2]

Racetrack betting is simply an application of portfolio theory. The racetrack offers many bets that involve the results of one to about twenty horses. Each race is a special financial market with betting over a short horizon then a race that takes one or a few minutes. Unlike the financial markets, one cannot stop the race when one is ahead or having the market going almost 24/7. There is a well-defined end point. Like standard portfolio theory, the key issues are to get the means right. In this case, it is the probabilities of various *ijkl* finishes for a superfecta or *ijk* for place and show bets, and to bet well. For the latter, the Kelly capital growth criterion is widely used and that maximizes the expected logarithm of final wealth almost surely. Transaction and price pressure odds changes fit well into the stochastic programming models discussed below; see Ziemba (2019a) for additional formulations and numerical results.

Professional syndicates or teams have been successful as hedge funds with gains approaching US$1 billion over several years for the most successful. This is not easy as the markets are quite efficient, see e.g. Ali (1977, 1979), Hausch and Ziemba (2008), and Hausch, Lo and Ziemba (1994, 2008) for a review of the literature. Besides the advantages of rebates, over half the betting is not recorded in the pools until the race is being run. This is because monies are bet near the start of the race and come from many off track sites which are combined with the on-track bets into the track pool. All this takes time. So estimates of future prices are crucial. Betting exchanges such as Betfair in London allow short as well as standard long bets. This allows for more arbitrage and the ability to take advantage of known biases.

Some of the bets involve multiple horses in a given race while others involve multiple races. The wagers are basically of two types: high probability of winning low payoff bets and low probability high payoff bets. The latter can return a million dollars or more. Table 1 describes a number of the bets.

[2] The arbitrages called *locks* in the gaming industry occur because the racetrack minimum payoff is 5% or 10% profit after the parimutuel payout. So with a super horse that is highly bet, you can construct a large bet on the super horse plus small bets on the other horses so a profit is guaranteed without risk for place or show wagers. Hausch and Ziemba (1990b) developed this and derived the conditions for the arbitrage to exist and showed how to calculate the bet sizes. If the bets on the non super horse are assumed to be equal in the 5% area, then a lock exists when: $K > 1 - \frac{Q((n-1)}{21(n-3)}$ where $K =$ the fraction of the show bet on the super horse, the show bet on the favorite is KS, S is the total show pool and the bets on the other horses are $\frac{(1-K)S}{(n-1)}$. A linear program can calculate the wagers for unequal bets. In the US, such a lock occurs about 10 times per year with a gain of about 2%. Ziemba and Hausch (1987) pointed out a flaw in the UK and Ireland betting rules. They have a minimum payout like the US which is to get your money back. However, the pool was split differently so that the payoffs for the losing bets do not pay for the winning bets because the net pool is shared equally after the take. This can lead to a minus pool where the house has a loss and an arbitrage exists for the players. In a followup paper, Jackson and Waldron (2003) fully analyze this and they successfully exploited it in 1998 in the UK and Ireland until the tracks eliminated the flaw in 1999 in the UK and 2000 in Ireland.

Table 1: Common US racetrack wagers

High probability low payoff	win, place, show
Low probability high payoff	pick 3-6
One horse is involved	win
Two horses are involved	place, exacta
Three horses are involved	show, triactor
Four horses are involved	superfecta
Two races are involved	double
Three races are involved	pick 3
Four races are involved	pick 4
Five races are involved	pick 5
Six races are involved	pick 6 and Rainbow pick 6
N races are involved	place pick all

Investing in traditional financial markets has many parallels with racetrack and lottery betting and much of the analysis is similar. Behavioral anomalies such as the favorite-longshot bias are pervasive and also exist and are exploitable in the S&P500 and FTSE100 futures and equity puts and calls options markets. Biases there favor buying high probability favorites and selling low probability longshots just like the high probability low payoff racing wagers. See Hodges, Tompkins and Ziemba (2004), Tompkins, Ziemba and Hodges (2008) and Ziegler and Ziemba (2015) for data and calculations. But in complex low probability high payoff exotic wagers such as the Pick 6, the bias reverses to overbet the favorite so one must include other value wagers in the betting program.

Fundamental information such as breeding is important and is especially useful for the Kentucky Derby and Belmont Stakes where horses have never run that far before. The idea is that more stamina is needed to win these races from the sires in the horses lineage. Since the horses have never run this distance before, a forecast of how they might do from their breeding is helpful. The key idea is that some stallions called chef-de-race impart consistent speed-stamina in their offspring. These stallions are identified and classified and it is they who essentially form the racetrack breed. The speed versus stamina is measured by a *dosage index* using the five categories: brilliant, intermediate, classic, solid and professional with more stamina and less speed in the latter categories. See Hausch, Bain and Ziemba (2006) and Gramm and Ziemba (2008) who study this by merging the odds (prices) with expert opinion (breeding measured by dosage) and the comprehensive book Roman (2016).

Figure 1 shows the average dosage index or speed over stamina for the average winner of the $1\frac{1}{4}$ mile Kentucky Derby and $1\frac{1}{2}$ mile Belmont stakes as well as a large number of high quality races at different distances. It was compiled from race data of Steve Roman. The conclusion that the winners of longer races have lower dosages which means more stamina

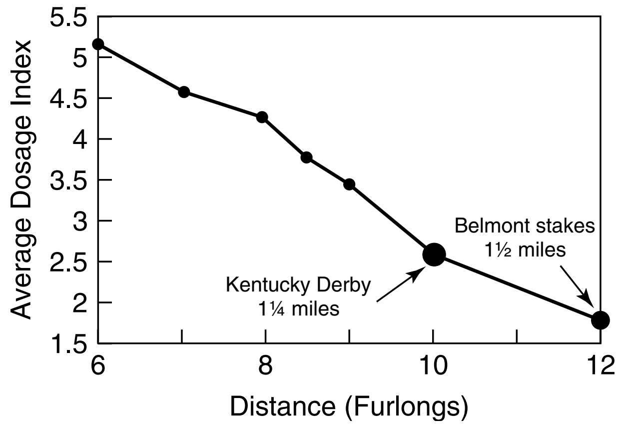


Figure 1: Average dosage of winners of $25,000 plus pure race by distance. Source: Roman (2016)

and less speed.

The calculations are simple and very useful. For example, a horse named *Stay Thirsty* has a dosage profile of 4-6-16-0-0. That is 4 brilliant points (pure speed); 6 intermediate points, 16 classic points, zero solid and zero professional (extreme stamina) points. These are categories on the speed-stamina space. You can think of this as a discrete probability distribution. Only the chef-de-race stallions (that is those that breed consistent characteristics in their offspring) in the horse's pedigree count: 16 for first generation sires, 8 each for the second generation, 4 each for the four third generation and finally 2 for the eight fourth generation sires. So each generation is equally important. If a chef is in two categories, the points are split. Some generations may have no chefs. Lists of these chefs-de-race stallions by category and decade that form the thoroughbred breed are in Roman (2016).

The dosage index is

$$\text{DI} = \frac{\text{Brilliant} + \text{Intermediate} + 1/2\ \text{Classic}}{\text{Solid} + \text{Professional} + 1/2\ \text{Classic}}$$

Despite its simplicity and crude tail weighting, the index does seem to work. An example of the pedigree and dosage of the 2005 Belmont Stakes winner, Afleet Alex, is in Tables 2 and 3.

The evidence is that horses with dosage indices above 4.00 do not win the Kentucky Derby. From 1979-2010 none did, though here have been a few exceptions since.

A dual qualifier is a horse whose dosage index is 4.00 or lower, which is the limit suggested for maximum speed in the pedigree for a Kentucky Derby winner, and within 10 pounds of

Table 2: Pedigree for the 2005 Belmont Stakes Winner Afleet Alex

<table>
<tr><td rowspan="8">Northern Afleet</td><td rowspan="4">Afleet</td><td rowspan="2">Mr. Prospector (B/C)</td><td>Raise a Native (B)</td></tr>
<tr><td>Gold Digger</td></tr>
<tr><td rowspan="2">Polite Lady</td><td>Venetian Jester</td></tr>
<tr><td>Friendly Ways</td></tr>
<tr><td rowspan="4">Nuryette</td><td rowspan="2">Nureyev (C)</td><td>Northern Dancer (B/C)</td></tr>
<tr><td>Special</td></tr>
<tr><td rowspan="2">Stellarette</td><td>Tentam</td></tr>
<tr><td>Square Angel</td></tr>
<tr><td rowspan="8">Maggy Hawk</td><td rowspan="4">Hawkster</td><td rowspan="2">Silver Hawk</td><td>Roberto (C)</td></tr>
<tr><td>Gris Vitesse</td></tr>
<tr><td rowspan="2">Strait Lane</td><td>Chieftain</td></tr>
<tr><td>Level Sands</td></tr>
<tr><td rowspan="4">Qualique</td><td rowspan="2">Hawaii</td><td>Utrillo II</td></tr>
<tr><td>Ethane</td></tr>
<tr><td rowspan="2">Dorothy Gaylord</td><td>Sensitivo</td></tr>
<tr><td>Gaylord's Touch</td></tr>
</table>

the top 2 year old horse on the experimental free handicapping ratings. Gramm and Ziemba (2008), Hausch, Bain and Ziemba (2006), Roman (2016) and Ziemba (2019a) show that such horses have superior performance in the Kentucky Derby and Belmont Stakes.

The breed is moving more towards speed and a number of horses with dosage indices above the historical 4.0 cutoff have been winners. In the long $1\frac{1}{2}$ mile Belmont almost all the winners have had low dosage below 3.00, meaning they have stamina in their pedigree. Since 2000 there have been some Kentucky Derby winners with dosage above 4.00 but barely any in the Belmont. Gramm and Ziemba (2008), Roman (2016) and Ziemba (2019a) study this and provide results.

In racing another major change is those markets are many more types of bets including ones that are essentially lotteries and betting exchanges in London and elsewhere that allow short as well as long wagers to hedge, eliminate or increase the investment positions. Since these are in continuous time, this allows for mean reversion risk arbitrage during the race as the horses are running well or poorly.

One wager that historically has had a large betting pool is the Pick6 and similar wagers as P3, P4, P5 in the US , Hong Kong and other venues. Payoffs are frequently large and even

Table 3: Dosage Index Calculation for 2005 Belmont Stakes Winner Afleet Alex

Generation	Sire	Brilliant	Intermediate	Classic	Solid	Professional
1	Northern Afleet					
2	Afleet					
	Hawkster					
3	Mr. Prospector	2		2		
	Nureyev			4		
	Silver Hawk					
	Hawaii					
4	Raise a Native	2				
	Venetian Jester					
	Northern Dancer	1		1		
	Tentam					
	Roberto			2		
	Chieftain					
	Utrillo II					
	Sensitivo					
	Total	**5**	**0**	**9**	**0**	**0**

NOTE: Dosage Index = $(5 + 0 + 9/2)/(0 + 0 + 9/2) = 2.11$.

larger with carryover but the probability of winning is low because it is difficult to pick all six winners or even five of six for a smaller consolation prize. At $2 per combination the wager must be high to have a reasonable chance of winning. Small bankrolled bettors often bet the top horses in the six races and they have a small chance of winning and if they win the payoff is likely to be small as there are likely to be many winners sharing the parimutuel pool. Thus giving large syndicates who can have more combinations an advantage.

The Rainbow Pick 6

The Rainbow Pick6 (called the Empire Six in New York state and other names at other racetracks) has become the substitute to the ordinary Pick6 which has lost favor as it it too expensive for bettors to have a reasonable chance of winning.

The idea is to create large payoffs from a small wager which is a lottery type idea mixed with fundamental handicapping skill. The Rainbow Pick6 tickets are ten or twenty cents rather than $2.

The bet has two prizes. The jackpot is awarded to the holder of the unique Pick 6 ticket. If there is no unique P6 winner but multiple P6 winners, then, after a 20% take, the net pool is given 60% to be divided by the multiple P6 winners to share and 40% goes to the carryover. If no one wins the P6 then the entire net pool goes to the carryover. So the expected value of a bet has three parts, namely:

EX= (Prob you win the unique P6) times (Value of unique P6 which is the carryover plus today's entire net pool) + (Prob you win a non-unique P6) times (Value of each share of the P6) + Rebate.

There is also no 5/6 consolation prize but a shared P6 second prize is awarded if there are two or more P6 winners. So with the the cheap 10 cent tickets, $300 will give the bettor $6000 worth of $2 P6 action which frequently wins the P6. So almost for sure the P6 will have multiple winners on most days. There have been very large payoffs as in lotteries. In February 2013, the carryover reached $3,107,159 by Thursday the 21st. The jackpot was not won on that day so the carryover reached $3,249,259.28 going into Friday.

It is clear that to have a chance to be the unique P6 winner you usually need 3, 4 or even 5 *bombs* (longshots). That is it is very unlikely to have so many longshots winning. Two bombs will not usually be enough because one can take all the horses in two races for say 10x10=100 combinations but that's only $10 times the bets made in other races. So the strategy to win the jackpot must be to take some potential bombs in 3, 4, 5 or even 6 races along with some more favored horses in the other races. Observe that using all is not worth much — you can win a shared second prize but it is too expensive so you might win the P6 but lose money on the bet. And taking all the horses in 3,4, and 5 races becomes very expensive fast. This usually focuses on 3-4 potential bombs in 4, 5, or 6 races but will likely lose as more favored horses win some of the time. Suppose we take the four longest odds horses in four races and we take two horses in the other two races. Then the cost = $\$0.10 \cdot 2^2 \cdot 4^4 = \$102.40$ less rebate.

But there are $\binom{6}{2}$ ways of doing this, that is 4 bombs and 2 other horses $= \frac{6!}{2!4!} = 15$. So we are up to $1536 with the question which of the 2 horses to pick in the other two races. So we might need more money. However we might need only 2 or 3 bomb possibilities in some of the other 4 races. So to do this, the cost is about $1500 to $3000 less the rather low rebate of about 3.5% per play on these wagers. You might need 10 or more such plays to win the jackpot and you may never win. But with a carryover at over $3 million, such a strategy has positive expectation and might work as the probability of winning can get close to one. In fact it did win on Friday February 22, 2013. The Rainbow P6 was Races 5-10. The payoffs of the winners for a $2 ticket were:

R5=$114; R6=$11.40; R7=$36.80; R8=$17.20; R9=$23.60 and R10=$23.00.

Playing the regular P6, the second prize requires different types of tickets and basically is not a good bet because of the 40% carryover and the large track take because going for the second prize has essentially zero chance of getting the jackpot namely the unique P6 win. Rebates bring back about 8% but still the effective payback = 60(-80)+.08=0.56 or a huge 44% track take.

But the jackpot has a positive expectation. However, that's hard to estimate as the probability of winning the first prize and the size of the shared second prize are both very hard to determine. While this approach gives a decent chance of eventually winning the first prize, it likely would give some second prizes but not enough to turn a profit unless the jackpot is won.

The parlay is
$0.1(\text{odds}_A+1)(\text{odds}_B+1)\ldots(\text{odds}_F+1)$ = \$0.1(57)(5.7)(18.4)(8.6)(11.8)(11.5)= \$697,663.23 But the parlay has six win track takes versus one P6 take but just on the new money bet on Friday. The carryover is not reduced by any additional track take.

The new money =

$$\frac{\$3,591,245.44 - \$3,240,259.28}{0.8} = \$438,732.70$$

where we adjust by the 20% win track take on that day's bets.

So if we adjust by the win track take of 17%, we have for the true parlay value

$$0.1\frac{(57)(5.7)(18.4)(8.6)(11.8)(11.5)}{(1-0.17)^6} = \$1,608,904.69$$

compared to the actual Rainbow P6 payout of \$3,591,245.44 which was the record payout for a single winner on the Rainbow 6 on February 22, 2013. The record total P6 pool was on July 2, 2007 at Hollywood Park where the pool reached \$10,870,852.60 with 13 winners of \$576,064.40 each.

If the pool is not won, there are periodic mandatory payout days usually at the end of meets. There was a mandatory payout on April 23, 2011 with a \$1.4 million carryover. The total pool reached \$5.1 million. There were 1,311 winning tickets each worth \$3,279.26. There was another mandatory payout on March 31, 2013, the last day of the meet. The pool had a carryover over \$2 million. So for this, it was optimal to go for second prize, namely, the regular P6 as there was no sense going for the unique P6. The P6 was won that day and paid \$3,932.32 for the \$0.10 ticket with win payoffs of Race 6 \$8.20, Race 7 %16.20, Race 8 \$13.80, Race 9 \$4.20, Race 10 \$22.00 and Race 11 \$9.80. The true parlay was below this payoff, namely 0.1(4.1)(8.2)(6.90)(2.10)(11.00)(14.90) = \$7,984.45.

In 2014 the Rainbow Pick 6 tickets were raised to 20 cents. Going into the May 26 mandatory payout, the carryover pool was \$6,303,426.30. On Sunday the pool was \$6,397.293/35 and Monday's, which was won, was \$6,678,939.12.

On March 31, 2014, one investor playing the \$2,721.60 ticket
1,2,4,5,7,8,9,10,12/1,2,4,5,7,8/1,2,3,6,8,9/1,2,3,4,5,7,8/1,3,4,5,6,7/6 won \$301,933.
Other payoffs were \$791,364, \$414,166.52 and \$327,110.71.

A single bettor, Danny Borislow, won the whole P6 pool on Sunday May 25, 2014, a day before the mandatory payout. The payoffs of Races 3-8 with winning numbers 1-8-6-1-6-5 were \$35.80, \$22.60, \$12.80, \$10.40, \$9.60, and \$12.80, respectively. Danny had two tickets each for \$7,603.20 with the winning ticket being all-all-all-1,4-all-all as #1 won the 4th leg. The second ticket had the same five "alls" plus two other horses in leg 4.

These races have two bombs and four 4-1 or 5-1 horses. Normally two bombs in the \$20-30 range and four \$10 horses would not yield just one unique P6 winner. The reason this bettor won was the low volume in the pool on Sunday as most bettors were preparing for Monday's mandatory payout. Also the short fields 6-6-6-10-7-13 allowed Danny to bet all the horses in five of the races and four deep in the fourth leg for his \$7,603.20 times 2 or a \$15,206.40 investment.

The net increase in the pool on Sunday was \$281,755.77 (total pool of 6,678,939.12 — carryover of 6,397,283.35), dividing by 0.80 gives the total bet of \$352,194.71. This was more than Saturday's increase in the carryover of \$93,857.05 or the \$117,321.31 bet.

The parlay with six win takes of 17% was $0.2(17.9)(11.3)(6.4)(5.2)(4.8)(6.4) = 41{,}358.62$. Then with no track take, the parlay was $4,135,861.62/(1-0.17)^6 = 126,502.02$ which compares to the payoff of 6,678,939.12 which had 20% taken off the total bet.

So bravo to Danny Borislow who seized a good opportunity to win the whole pool. Danny was the owner of the Magic Jack telephone device business. It is extremely unlikely that someone could win the whole pool on mandatory payout day as most likely there would be multiple P6 winners all going after a share of \$10+ million pool.

A sad postscript: on Monday July 21, 2014, Danny collapsed and died at age 52 after playing in an indoor soccer game in West Palm Beach, Florida.

The ordinary Pick6

The Rainbow Pick6 is designed to boost sales with large jackpots and cheap tickets. Las Vegas and Reno casinos also do this in various ways. For example, cheap keno tickets with the games deep into the casino draw customers into other games where the casinos profit edge is larger.

The strategy to play the ordinary \$2 Pick6 wisely depends on the handicapping of the horses' chances of winning. A Pick6 program can use individual probabilities to generate an overall probability of success in winning the P6 and collecting some 5/6 consolation prizes. In the 2009 Breeders Cup, there were three races with standouts but not certain winners and three that were wide open.

So you could play the Pick 6 in the following way: I thought about doing this but did not — it was a $2 million mistake. You have a single ticket with about 10 horses in the three wide open races and single the three standouts. That would cost about 10*10*10*1*1*1*$2=$2000, not a large Pick 6 ticket. You only win if all three standouts win and they did.

The payoffs for $2 win tickets were as follows

Race 4	Dancing in Silks	52.60	
Race 5	Value of York	63.20	
Race 6	Goldikova	4.80	the first standout
Race 7	Furthest Land	44.60	
Race 8	Conduit	3.80	the second standout
Race 9	Zenyatta	7.60	the third standout

The Pick 6 paid $1,838,305.20 for one winning 6/6 ticket and the 3*9=27 Pick 5/6 tickets (of the 10 losers in the three wide open races) paid 27*$4822.40 for a total of $130,204.80 plus the 6% rebate on the $2000 of $120 for a grand total of $1,968,630.00. It is not quite $2 million but as Johnnie Hooker played by Robert Redford in the Sting said: "it's not enough but it's close". Of course, taxes would take 25% at the track and be sorted out later when filing and my winning would depress the Pick 6 and Pick 5/6 prizes.

Summary: all three of the wide open races had winners that were competitive horses. So they would be on our ticket. But even if we bet on all horses, these three races, the ticket only costs 9*13*10*1*1*1=$2340. This ticket made a lot of sense so I should have played it.[3] It would have had 8+11+9=28 Pick 5/6s. Oh well, there is always next year.

Pool guarantee insurance

Another new feature of racetracks is pool guarantee sports insurance companies including SCA of Dallas, Texas run by bridge champion Robert Hammond. These companies guarantee the $1 million and other jackpots for the Pick6 and other exotic wagers. Ziemba (2019a) discusses such bets he and colleague Cary Fotias made including the Pick6 at the 2001 Breeders Cup at Belmont Racetrack. The insurance was the $2 to $3 million part, to guarantee a pool of at least $3 million. For example, if only $2.15 million was bet,

[3] Another way to play this is to have three sets of tickets in which you assume that at least two of the three standouts will win. So you have $N_1^i, N_2^i, N_3^i, N_4^i, 1, 1$ combinations, $i = 1, 2, 3$ all at $2 each. So depending on the N_j^i you likely have a larger ticket than the three singles approach. You might win more than one Pick 6 and more Pick 5/6s, but you might miss the Pick 6 as well unless the tickets are well spread.

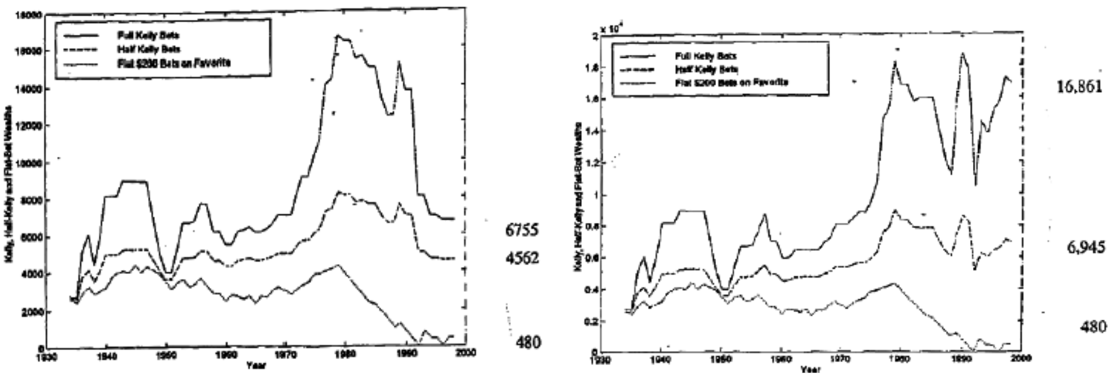
they would be liable for \$850,000. We studied and proposed to bet a random amount if needed.

The idea is to get to \$3 million and return the insurance company's money by winning Pick 6s and Pick 5/6s. It was risky as September 11 had just occurred and all the Arab owners such as Sheikh Mohammed of Dubai were not in attendance. Their horses and trainers were though. It turned out to be a glorious day so the crowd sent the Pick 6 pool well over \$4 million. Our client said you two can just play about \$25-30,000 of the tickets. So we had a \$2000 ticket twice and what we call a gorilla ticket for \$28,000. We had some 5/6s and got most of the money back. The Pick 6 paid about \$250,000. The race we lost was the sprint. Squirtle Squirt, which my handicapping colleague did not like at 9-1 beat the front running filly, Xtra Heat at 14-1, who we had and had led all the way until the finish. So if she had won we would have had three about \$450,000 Pick 6s plus more 5/6s. Squirtle Squirt had run at Belmont and had the top jockey Jerry Bailey and was trained by the recently deceased legendary trainer Broadway Bobby Frankel. Too bad. But the next week we won a similar case at Santa Anita, while guaranteeing a \$1 million Pick 6, collecting \$240,000 for the client and a nice bonus for us.

Betting strategies

Kelly and fractional Kelly betting is used extensively in racetrack betting. Full Kelly is the maximization of the expected logarithm of final wealth subject to constraints. That's an expected utility approach with $u(w) = logw$. Log with very low Arrow-Pratt risk aversion $-u''(w)/u'(w) = 1/w \cong 0$ is very risky short term despite wonderful long term growth properties. MacLean, Thorp and Ziemba (2010, 2011) provide an extensive treatment of the key ideas and major papers. Ziemba (2011) has a response to Professor Paul Samuelson's critiques. MacLean, Thorp, Zhao and Ziemba (2010) provide simulations of typical behavior.

Fractional Kelly is simply the idea to blend cash with the Kelly strategy similar to blends of cash and the market index in portfolio theory. This under the lognormal asset assumption amounts to a less risky negative power utility function rather than log which is the most risky utility function one would ever want to use. Fractional Kelly leads usually to less growth and more security and a less violent wealth path. Half Kelly is a frequently used strategy. It has 75% of the full Kelly growth but the security, measured by the probability of breaking even rising from 87% with full Kelly to 95.4% with half Kelly. For lognormal assets this is the negative power utility function $u(w) = -1/w$ and this is approximate for other return distributions. This is shown visually in Figure 2(a) for the Kentucky Derby from 1934 to 2005 and Figure 2(b) with the dosage filter to eliminate horses that cannot run $1\frac{1}{4}$ miles on the first Saturday in May of their three year old career. These use the place and show system originally devised in Hausch, Ziemba and Rubinstein (1981). A system that only bets on the favorite turns \$2500 into \$480 so is a loser; while the full and half Kelly systems have gains.



(a) Kelly, half Kelly and betting on the favorite (b) same as (a) with dosage filter

Figure 2: Wealth history of some Kentucky Derby bets, 1934-2005

In all cases the strategy to win is the same as in the financial markets:

1. get the mean right: thus one must have accurate probabilities of various outcomes. This is discussed below.

2. use the actual odds and a betting model such as the Kelly criterion to optimize the bet sizes, that is the allocations.

For situations with not many wagers, the Kelly capital growth maximize expected logarithm or, its safer version, fractional Kelly, is useful as a decision tool especially with many repeated bets. Then one has a stochastic program to maximize the expected utility using a logarithmic utility function of final wealth subject to various constraints. The Kelly strategy bets more on the attractive situations. In wagers where one makes hundreds of bets, it is often better to use a tree approach where many of the bets are of equal value. Besides being more convenient to make these multiple bets, this gets around integer problems as the wagers will be integers that can easily be bet. Whereas the Kelly optimization needs modifications to produce integer wagers. Also this approach can be computerized to print out the tickets and the higher probability wagers can be bet more to approximate a Kelly strategy.

The importance of accurate mean estimates

Table 4 and Figure 3 show that getting the mean right is a most important aspect of any portfolio decision problem. Chopra and Ziemba (1993) discuss that and look at the effect of errors in means, variances and covariances using the cash equivalent of the approximate versus exact optimal solutions. Basically it is in the ratio 20:2:1 for errors in means, variances and covariances in terms of error impact on certainty equivalent value. We measure risk aversion by the Arrow-Pratt risk aversion index $R_A(w) = -u''(w)/u'(w)$, where primes denote differentiation of the utility of wealth function u.

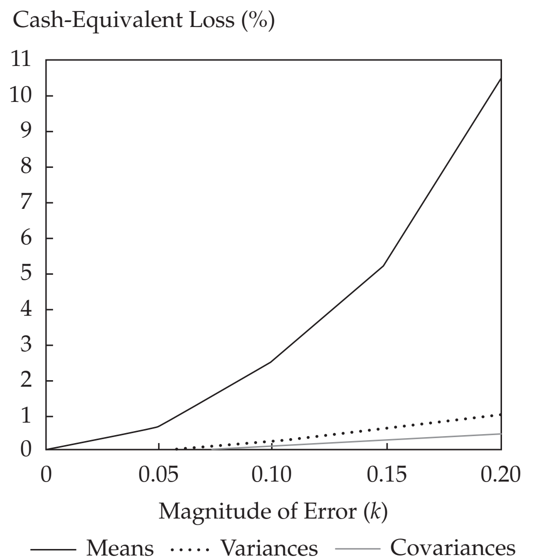


Figure 3: Typical relative importance of errors in means, variances and covariances in terms of certainty equivalent. Source: Chopra and Ziemba (1993)

Table 4: Average Ratio of CEL for Errors in Means, Variances and Covariances. Source: Chopra and Ziemba (1993)

t Risk Tolerance	Errors in Means vs Covariances	Errors in Means vs Variances	Errors in Variances vs Covariances
25	5.38	3.22	1.67
50	22.50	10.98	2.05
75	56.84	21.42	2.68
	↓	↓	↓
	Error Mean	Error Var	Error Covar
	20	2	1

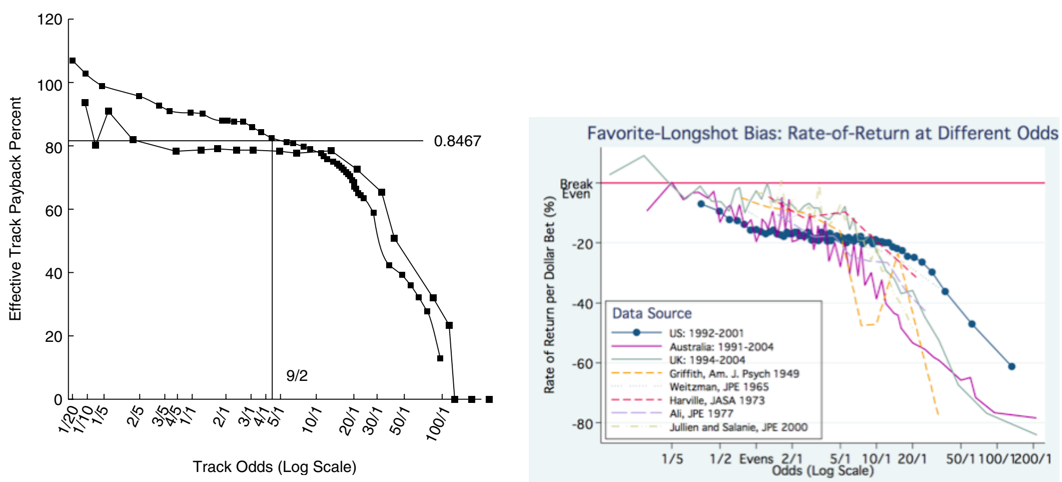
Low risk aversion utility functions such as log with $R_A = 1/w \cong 0$, the effect of the errors is more like 100:3:1 so getting the mean right is even more important. In horse racing, that is the probabilities for horses coming first, second, third, etc. Sports betting is similar.

The favorite-longshot bias

The favorite-longshot bias is the tendency in horseracing, sports betting, and financial options for the most likely outcome to be underbet and the less likely outcomes overbet. So people on average prefer poor choices and dislike the best possibilities. This bias has been well known to Irish and other bookmakers who actually create the bias with the bets they offer for the last 100+ years. Griffith (1949), McGlothin (1956) and Fabricant (1965) are early references.

Figure 4a shows the basic effect over time, the top curve as of 1986 and the more recent curve on the bottom.

Figure 4a shows the effective track payback less breakage (rounding down) for various odds levels in California. The curve slightly changes in different locals with different track takes. This data reflects more than 300,000 races over various years and tracks, in 1986 and in recent data. Figure 4b has an independent study and some earlier studies. Historically in the 1986 data, there was a small profit, about 3%, in betting horses to win at US odds of 3-10 (UK odds of 1.30 or less) and that at odds of 100-1, the fair odds were about 700-1 so that such bets were worth only about 13.7 cents per dollar bet.



(a) Source: Ziemba and Hausch (1986) (b) Source: Snowberg and Wolfers (2008, 2010)

Figure 4: Effective track payback less breakage

The bias curve is often different for different types of races. Higher quality races like the Kentucky Derby have flatter biases. See Ziemba and Hausch (1987) for the 1903-1986 graph. See also Tompkins, Ziemba and Hodges (2004, 2008) who demonstrate similar biases in the S&P500 and FTSE100 index futures options which is consistent with option pricing theory with positive risk premiums.

In the more recent data in Figure 4ab, the favorites are no longer underbet enough to turn a profit betting them. Also the curve is flat across longer odds horses. However, investors can still short longshots on betting exchanges like Betfair and make a profit.

What are the reasons for the favorite-longshot bias?

Thaler and Ziemba (1988) included the idea that there are more bragging rights from picking longshots than from favorites: *50-1 Wow, was I smart*, while 2-5 is an easy pick. Transaction costs are another factor: betting \$50 to win \$10 is hardly worth the effort. The bias is consistent with the Kahneman-Tversky (1979) prospect theory of actual human behavior and behavioral finance, see Barberis and Thaler (2003). There low probability gains are risk seeking and are overestimated and high probability gains are viewed as risk averse, so are underestimated. Mental accounting (Kahneman and Tversky, 1984) may be involved where the bettor is risk seeking in one area and risk averse in another.

Many studies, data, theories and references regarding the favorite-longshot bias are in Hausch, Lo and Ziemba (1994, 2008) and Vaughan Williams (2003). Historical graphs are in Ziemba (2019a). Additional literature on this bias was provided by Snyder (1978), Quandt (1986), Benter (1994, 2008), Hurley and McDonough (1995), Jullien and Salanie (2000), Shin (1992), Ottaviani (2008), Golec and Tamarkin (1998), Vaughan Williams and Paton (1997), Green, Lee and Rothschild (2019) and others. Snowberg and Wolfers (2008, 2010) discuss risk love versus misperceptions as explanations. Woodland and Woodland (1994) discuss baseball biases. Ziemba (1989) discusses hockey biases. Vaughan Williams et al (2016) discusses poker where misperception rather than risk-love better explains the bias. Forrest and McHale (2005) show the bias in tennis betting and Forrest and Simmons)(2005) discuss soccer. The bias is stronger late in race days when losing bettors try to catch up by betting more on long odds horses, see Metzger (1985), Gramm and McKinney (2009) and Ziemba (2019a). Asch and Quandt (1986) devise a place and show betting system based on late money.

Ziemba and Hausch (1987) show that UK and Irish bookies create the bias to balance their betting books. Busche and Hall (1986) and Busche (1994, 2008) and Benter (1994, 2008) show the bias is not in the Hong Kong and Japan data.

The bias is part of much human behavior. In the lottery context, management wants games that sell well that exploit the risk seeking low probability high payoff bias. With the help of UBC colleague Shelby Brumelle and Sandra Schwartz, I designed a bingo game with a \$100 million first prize, a \$10 million second prize, down to a \$10 lowest prize. The edge for the house was over 50% after selling the risk to insurance providers for the two top prizes. The game is based on patterns of chosen and not chosen bingo numbers.

Place and show and exotic optimization with transactions costs

The Dr Z system, co-developed with Donald Hausch with some early help from Mark Rubinstein, presents a winning method for betting on underpriced wagers by pricing the bets. The idea of the system is simple: use the data from a simple market, in this case the win probabilities to fairly price bets in the more complex markets, such as place and show. For example, with ten horses, there are 720 possible finishes for show. Then one searches for mispriced place and show opportunities. This is a weak form violation of the efficient market hypothesis based solely on prices. How much to bet depends on how much the wager is out of whack and it is a good application of the Kelly betting system. The formulation below shows such an optimization. There is a lot of data here on all the horses and not much time at the track. So a simplified approach is suggested. Don and I solved thousands of such models with real data and estimated approximation regression equations that only involve four numbers, namely, the amounts bet to win in the total pool and the horse under consideration for a bet. Plus the total place or show pool and the place or show bet on the horse under consideration.

These equations appear below. In the books Ziemba and Hausch (1984, 1986, 1987) and papers Hausch, Ziemba and Rubinstein (1981) and Hausch and Ziemba (1985), we study this in various ways, including different track takes, multiple bets for place and show on the same horse and how many can play the system before the edge is gone. This system revolutionized the way racetrack betting was perceived viewing it as a financial market not just a race to handicap. This led to pricing of wagers and the explosion of successful betting by syndicates in the US,Hong Kong and elsewhere using some of these ideas as discussed in Hausch, Lo, Ziemba (1994, 2008), Hausch and Ziemba (2008) and Ziemba (2017, 2019).

The effect of transactions costs which is called slippage in commodity trading is illustrated with the following place/show horseracing optimization formulation; see Hausch, Ziemba and Rubinstein (1981). Here q_i is the probability that i wins, and the Harville probability of an ij finish is $\frac{q_i q_j}{1-q_i}$, etc. That is $q_j/1 - q_j$ is the probability that j wins a race that does not contain i, that is, comes second to i. Q, the track payback, is about 0.82 (but is about 0.88 with professional rebates). The players' bets are to place p_j and show s_k for each of the about ten horses in the race out of the players' wealth w_0. The bets by the crowd are P_i with $\sum_{i=1}^{n} P_i = P$ and S_k with $\sum_{k=1}^{n} S_k = S$. The payoffs are computed so that for place, the first two finishers, say i and j, in either order share the net pool profits once each P_i and p_i bets cost is returned. The show payoffs are computed similarly. The maximum expected utility model is

$$\max_{p_i,s_i} \sum_{i=1}^{n} \sum_{\substack{j=i \\ j\neq i}}^{n} \sum_{\substack{k=i \\ k\neq i,j}}^{n} \frac{q_i q_j q_k}{(1-q_i)(1-q_i-q_j)} \log \left[\begin{array}{c} \frac{Q\left(P+\sum_{l=1}^{n} p_l\right)-(p_i+p_j+P_{ij})}{2} \\ \times \left[\frac{p_i}{p_i+P_i} + \frac{p_j}{p_j+P_j} \right] \\ + \frac{Q\left(S+\sum_{l=1}^{n} s_l\right)-(s_i+s_j+s_k+S_{ijk})}{3} \\ \times \left[\frac{s_i}{s_i+S_i} + \frac{s_j}{s_j+S_j} + \frac{s_k}{s_k+S_k} \right] \\ +w_0 - \sum_{\substack{l=i \\ l\neq i,j,k}}^{n} s_l - \sum_{\substack{l=i \\ l\neq i,j}}^{n} p_l \end{array} \right]$$

$$\text{s.t.} \sum_{l=1}^{n} (p_l + s_l) \leqslant w_0, \quad p_l \geqslant 0, \quad s_l \geqslant 0, \quad l = 1, \ldots, n,$$

While the Harville (1973) formulas make sense, the data indicate that they are biased. Savage (1957), Henery (1981), and Dansie (1983) discuss this Bayesian type formula more.

For place and show, the win favorite-longshot bias and the second and third finish bias tend to cancel so the corrected Harville formulas are not needed here. For other bets to correct for this, professional bettors adjust the Harville formulas, using, for example, discounted Harville formulas,[4] to lower the place and show probabilities for favorites and raise them for the longshots; see papers such as Benter (1994, 2008) in Hausch, Lo and Ziemba (1994, 2008) and papers by Henery, Stern, Lo, and Lo and Bacon-Shone and others in Hausch and Ziemba (2008).

Rebate is added to final wealth inside the large brackets by adding the rebate rate times all the bets,winners and losers.

This is a non-concave program but it seems to converge when nonlinear programming algorithms are used to solve such problems. But a simpler way is via expected value regression approximation equations using 1000s of sample calculations of the NLP model. These are

$$\text{Ex Place}_i = 0.319 + 0.559 \left(\frac{w_i/w}{p_i/p} \right)$$

[4] The discounted probabilities come from

$$q_i^* = \frac{q_i^\alpha}{\sum_i^n q_i^\alpha}$$

for α about 0.81 then one uses the q_i^* in the second place position. For third one uses α^2 about 0.64 and for fourth place α^3. These empirical numbers vary over time and by track. This is more important for exacta, trifecta and superfecta pricing than place and show because for the latter the win bias from the favorite-longshot and the second and third biases tends to cancel.

$$\text{Ex Show}_i = 0.543 + 0.369 \left(\frac{w_i/w}{s_i/s} \right).$$

The expected value (and optimal wager) are functions of only four numbers - the totals to win and place for the horse in question and the totals bet. These equations approximate the full optimized optimal growth model. See Hausch and Ziemba (1985) for more on this plus additional features. This is used in calculators. See the discussion in Ziemba and Hausch (1986) and Ziemba (2019) for a discussion of typical use at the first Breeders' Cup in 1994.

An example is the 1983 Kentucky Derby.

	Totals	#8 Sunny's Halo	Expected Value Per Dollar Bet	Optimal Bet (W_0=1000)
Odds		5-2		
Win	3,143,669	745,524		
Show	1,099,990	179,758	1.14	52

Sunny's Halo won the race

Win	Place	Show
7.00	4.80	4.00

Π = \$52

15 second bet!

Watch board in lineup
while everyone is at the TV

Here, Sunny's Halo has about 1/6 of the show pool versus 1/4 of the win pool so the expected value is 1.14 and the optimal Kelly bet is 5.2% of one's wealth.

You might ask: does the system still work in 2020 and what is changed?

Syndicates exist that break even on their wagers yet make millions on the rebate. Indeed the tracks generally require this of the betting exchanges to provide the signal which is the data. So the effect of the rebate and betting exchange cost is paid by the other bettors who actually pay more than the stated overall take.

Regarding the original 1980s Dr Z system, I am still using it with John Swetye and we wager with rebate searching for bets at 80 racetracks. Basically the system still works but the task is not easy as there is a lot of syndicate competition. One successful six month period, with a $5000 bankroll, the system lost 7%, but received a 9% rebate. The total wagers were $1.5 million recycling the bankroll, giving a 2% or $30,000 profit.

Pricing exotic wagers

The basic idea from Ziemba, Hausch and Rubinstein (1981) was to price the bets and then make the good ones. Some of these are discussed here. More types of wagers and actual bets are described in Ziemba (2019a). All the wagers in the US, Hong Kong and elsewhere can be modelled this way and some of this is done by the syndicates that are successful.

A less commonly used and known bet called the *place pick all* is available at Santa Anita, for example. The idea is to create a ticket with I horses over I races where you have either the winner or the second place horse in each race i, $i = 1, \ldots, I$. The number of races I varies from about 7 to 12.

The probability that a ticket with j the chosen horse in race i *wins* that race is the probability that j is first plus the probability that j is second, namely

$$p_{ij} + \sum_{k=1,\ldots,K_i} p_{ik} \frac{q_{ij}}{1 - q_{ik}} \text{ for } k \neq j$$

where the discounted Harville probability is

$$q_{ik} = \frac{p_{ik}^{\alpha}}{\sum p_{ik}^{\alpha}} \text{ and } \alpha \cong 0.81 \text{ and is track dependent.}$$

These are discounted Harville formulas, see papers in Hausch, Lo and Ziemba (1994, 2008) for more on this.

Then the chance that a given ticket with $i = 1, \ldots, I$ is a winner is

$$\hat{p}_{ij} = \prod_{i=1,\ldots,I} \left\{ p_{ij} + \sum_{k=1,\ldots,K_i} p_{ik} \frac{q_{ij}}{1 - q_{ik}} \right\}$$

Some stochastic programming optimization formulations

There are basically three strategies for the optimization: Kelly expected log optimization, probability weighting betting on all positive expectation wagers above a cutoff, and the tree tickets approach that approximates the Kelly strategy.

The expected log problems are typically solved using a non-linear programming code such as CONOPT which has produced good results even though the problems are non-concave. The bets must be computed very fast as the odds are changing. While this may not be general but because of an epsilon optimality convergence criteria, the Minos-Stanford code may converge to a non-optimal strategy. Hence, CONOPT is safer.

In general, the Kelly Elog optimization given modern computing can be used for essentially all the bets, even possibly Hong Kong's triple trio, namely, getting the 1-2-3 finish in any order in three races with 14 horses in each race of which many horses are 200-1 but they can still finish 3rd. This has 48 million combinations. I focus here on the US bets and Elog Kelly optimization for high probability low payoff bets and the tickets approach for the low probability high payoff events which can closely approximate the Kelly strategy and yields easily implemented tickets that are integers.

The simplest bet is the exacta. To win you must get the winner plus the second place finisher. This uses elements of the place pick all formulation except it is just for one race and it is not first **or** second but first **and** second. First is easy, it is just p_i the probability that i wins. Second uses the discounted Harville formula so it is $\frac{q_i}{1-q_i}$ where $q_i = \frac{p_i^\alpha}{\sum p_i^\alpha}$, where $\alpha \cong 0.81$. The probability of an ij finish is $\frac{p_i q_j}{1-q_i}$. Let s_{ij} be our bet on an ij finish. The Kelly optimization problem is

$$\begin{aligned}\max_{x\in K} E_w \log W \quad = \quad & \sum_{i=1}^{I}\sum_{j\neq i}^{J} p_i \frac{q_j}{1-q_i} \log \Bigg\{ W_0 + r\sum_{i=1}^{I}\sum_{j\neq i}^{J} x_{ij} \\ & + Q\frac{\left(E + \sum_{i=1}^{I}\sum_{j\neq i}^{J} x_{ij}\right)}{E_{ij}+x_{ij}} \left(\frac{x_{ij}}{x_{ij}+E_{ij}}\right) \\ & - \sum_{i=1}^{I}\sum_{j\neq i}^{J} x_{ij} \Bigg\}\end{aligned}$$

where r is the rebate percent payable on all bets, losers and winners, E is the total exacta bet by the crowd, with the E_{ij} their bets on ij and Q is the track payback. The constraints can include a maximin bet on any combination ij as well as on each i and on the total bet. The wealth is final wealth plus rebate plus profits minus the bets and E_w is the expected value. Other high probability low payoff bets are similar.

Lets now consider a tickets model as well as an expected log model. I supervised an unpublished MSc thesis on this at the Oxford Math Department, see Assamoi (2010). Some of the theory is there but no calculations. Before we consider this, let us do it for the place pick all. I am not aware of any published research or calculations, real or simulated, on this bet.

Let x_{ij} be the bet that either the chosen horse j will win or come second in race i. The probability of winning is $\hat{p}_{ij}$ as given above.

The Kelly formulation is

$$\begin{aligned}\max_{x\in K} E_w \log W &= \sum_{i=1}^{I}\sum_{j\neq i}^{J} \hat{p}_{ij}\log\left\{W_0 + r\sum_{i=1}^{I}\sum_{j\neq i}^{J} x_{ij}\right. \\ &\quad + Q\frac{\left(PL+\sum_{i=1}^{I}\sum_{j\neq i}^{J} x_{ij}\right)}{E_{ij}+PL_{ij}}\left(\frac{x_{ij}}{x_{ij}+PL_{ij}}\right) \\ &\quad \left. -\sum_{i=1}^{I}\sum_{j\neq i}^{J} x_{ij}\right\}\end{aligned}$$

which is very similar to the exacta formulation where PL is the place pick all pool with PL_{ij} bet on ij, r is the rebate percent and Q is the track payback percent.

The probability weighting approach is frequently used instead of Kelly optimization which has very few large bets and can have a violent wealth path. There one finds the bets with expected value edges and weights them. In Hong Kong this is popular in wagers with low probability high payoffs to diversify the overall wager better. Examples of violent wealth paths in financial markets are shown in Ziemba (2005) and Gergard and Ziemba (2012) for Warren Buffett and George Soros who are full Kelly investors.

The ticket formulation breaks the picks, the js, into categories I, II and III for each race j. I's are high value and high probability of winning horses. II's are major contenders and III's are longer odds horses who could upset the favorites. Suppose there are eight races so I=8, where n_{ij} is the number of horses in category ij.

	1	2	3	4	5	6	7	8
I	N_{1I}	N_{2I}						
II	N_{1II}	N_{2II}						
' II	N_{1III}	N_{2III}						

Given the probabilities and other factors, the horses are put into I, II or III or not considered in each race i. The score 8 tickets have all I's and they have the most money on them. There are $\prod_{i=1}^{8} N_{iI}$ of these. The score 9 have 7 ones and one II. There are 8 such tickets with lower bets. The score 10 tickets have 6 I's and 2 twos or 7 I's and one III with even lower bets. There are $\binom{8}{2}$ and eight of these with the lowest bets. One might go to score

11 and have bet sizes to approximate a Kelly strategy. The number of tickets gets very large here as is the cost. A computer program can be used to generate these tickets. A sample printout is below.

Example: the 9/11 Pick3: don't trust odds from newspaper stories

On September 11, 2011, the tenth anniversary of the attacks on New York and Washington, the first three races at Belmont had winners 9-1-1. The Pick3 paid \$18.60. The parlay on the three track takes versus one for the Pick3 paid $4.20 * 4.20/2 * 6.20/2 = \$30.5$, which is more than the Pick3 payoff without even factoring in the two extra tack takes. So the 9-1-1 Pick3 was over bet just like popular numbers in lotteries are. The track take for the Pick3 is 26% and for win 16%.

The fair value of the Pick3 with zero track take was \$18.60/0.74=\$25.14 and for the parley

$$30.5\left\{\frac{1}{0.84}\right\}^3 = \$51.34,$$

so the numbers 9-1-1 were over bet. A newspaper article said the odds of such an outcome were a million to one. Actually by looking at the final odds on the charts for these three races, we can estimate that. Adjusting for the favorite-longshot bias, probability of winning is

$$\frac{Q + \Delta Q(odds)}{odds + 1}.$$

Referring to a table in Ziemba (2019a) for the ΔQs and the payoffs below, the odds of 9-1-1 occurring are about one divided by the probability of this outcome namely

$$\left\{\frac{0.84 - .02315}{2.1}\right\}\left\{\frac{0.84 - .02315}{2.1}\right\}\left\{\frac{0.84 - .0345}{3.45}\right\} = \\ 0.3890 * 0.3890 * 0.2335 = 0.0353$$

since the payoffs for win were \$4.20, \$4.20 and \$6.90 for each \$2 bet. The chance of the 9-1-1 outcome then is equal to about 1/.0353= 28.31. This means that there was a 1 in 28 chance of the 9-1-1 payoff, not 1 million to 1.

An example of the multiple ticket to approach to the Pick 6 is Santa Anita, March 6, 2002 with \$202,790 carryover from Sunday's wagers. This is included here because it had a behavioral finance element that was crucial to its success.

Race	3	4	5	6	7	8
Win Payoff	3.60	4.60	12.00	9.20	18.00	7.20

1-5 shot out of the money

Advantage $\equiv \prod_{i=1}^{6}(1+\text{edge}_i)$

Pick 6 \$86,347.60 – one large transaction cost

Parlay \$15,452.40 - six races, six smaller transactions costs

Bet = 1922	1 Pick 6		Score 6	\$4
Rebate = 154	11 Pick 5/6	692.00	7	\$58
Net bet = 1768	Gross =	93,961.80	8	\$376
			9	\$1484
				1922

Score 9: you win if the score is 9 or less, here 3 I's, 2 II's, total 9 so we won.

Race	3	4	5	6	7	8
I	$\boxed{1}$, 3	$\boxed{3}$	7	3	2	$\boxed{2}$
II	7	1,5	1,2,$\boxed{5}$,6	4,7,8,$\boxed{10}$	1,$\boxed{4}$	5.9
III	5,6	2,4,8		1,2,5		1,6,7,8

8 was scratched

This \$1922 ticket was actually 66 separate tickets.
The winning ticket will have one 6/6 winner and multiple 5/6 winners

			Morning Line
Race 7	1	Madame Pietra	3-1
	2	Love at Noon	3-5
	3	Harvest Girl	8-1
	4	Filigree	4-1
	5	Farah Love	12-1

- This was the behavioral key.
- Filigree, the third choice in the morning line went off at 8-1.
- The 3rd and 5th races back, he ran faster than the favorite Love at Noon ran in his last two races. So he had a chance to win and he did.
- Love at Noon went off at 1-5 and had most of the P5 money

In the following Equiform numbers, the top number is final speed number, other numbers are pace within the race.

EQUIFORM ®

7TH SAX MARCH 6 - 6 FURLONGS DIRT

```
        MADAME PIETRA      (118)            40

  70    72¾   67¾   70½   64¾  w76½  w71¾   64½
       ///// /////
 80¾   71¾   73¾   79    77¾   78    77    67
SA6½ SA-6½   SA8  DMR6   SA6   SA6  DMR6  HOL6
 70¾   71    67¾
 82    70¾         81    83¾   81    63    68¾
JAN24 10/17 09/27 SEP02 03/16 JAN31 AUG05 05/28

        LOVE AT NOON      (118)            44

 72¾   73¾   74½   75½  w78¾ w^70½

 71¾   82½   75¾   74¾   85    70¾
SA8½   SA7 HOL8½  CD8½  CD6½   SA6
 69¾   74½   72    72¾   76¾
       84                85¾   77¾
01/20 DEC29 06/16 05/26 05/05 04/07

        HARVEST GIRL      (118)            19

w71½  71½  68½   --    70¾   71    66¾   66¾   68¾  w71¾   68    69¾  w68½  69½  w67½   61½
     /////       /////                              /////       /////             /////
 80½   77   80½   71½   83¾   81½   82½   79¾   81    78½   75¾   79½   77¾   82    79½   69¾
 SA6   SA8 HOL6½ BM8½ FPX6½  SR6   SOL6   GG6 SA-6½ FPX6½  BMP8  SOL6   GG6 HOL5½  BM5½ BMF5½
      68¾   69   65¾   71½                           69½   71¾   68
 87         83          98   81½   82¾   80½   84½   83¾         78¾   77¾   75½   86½   80
02/14 01/05 11/21 10/13 09/14 08/06 JUL14 11/25 10/28 09/19 08/12 07/22 05/31 MAY18 09/12 08/15

        FILIGREE      (118)            40

 71¾   67¾   74    71½  w74½  w71¾   71½   71½   70½   70    70    70½  w73¾  w70¾  w70    68¾

 77¾   72    82½   78½   75¾   74¾   75¾   73¾   70½   81½   61½   77    74    77¾   78    73¾
SA6½  SA5½  DMR6 DMR6½ HOL6½  HOL6   SA7  SA8½   SA6  SA5½   SA9 FPX6½ DMR6½ DMR6½ HOL6½   SA6
 71½               70¾   72½               67¾   70                61½   68½   68    70    70
 77½   75½   85¾   73¾   73½   74¾   75          67½   75½         74    68¾   77½   79    75¾
01/24 JAN02 09/05 07/23 06/27 05/28 04/16 03/03 01/31 JAN01 10/28 09/30 09/03 JUL31 05/26 04/16

        FARAH LOVE      (120)            26

 73   w70¾  w70½   67    66¾   65    63¾   67    62¾  w66    69½  w66
/////                               ///// /////
 78¾   74½   79    72    72¾   72    76¾   76¾   71½   76¾   74½   67
SA-6½  SA6  SA6½  HOL6   SA6   SA6 SA-6½ HOL5½ HOL8½ SA6½  SA6½  HOL6
 71¾         69¾                     62½         65¾   65¾   69
 81¾   77¾   82¾   72    68½   74¾   80    66½         84    70    70¾
02/07 01/17 01/05 DEC09 02/16 01/31 12/30 12/20 11/23 11/01 OCT11 07/23
```

Normally we approximate Kelly with more money on higher probability wagers, but in the following bets we made equal $2 bets.

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$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 3 /(7) 2 /(8) 2
$2.00 ;P6; (3) 7 /(4) 3 /(5) 7 /(6) 3 /(7) 2 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 1 ,5 /(5) 7 /(6) 3 /(7) 2 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 2 ,3 ,5 ,6 /(6) 3 /(7) 2 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 4 ,7 ,8 ,10 /(7) 2 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 3 /(7) 1 ,4 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 3 /(7) 2 /(8) 5 ,9
$2.00 ;P6; (3) 5 ,6 /(4) 3 /(5) 7 /(6) 3 /(7) 2 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 2 ,4 ,8 /(5) 7 /(6) 3 /(7) 2 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 1 ,2 ,5 /(7) 2 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 3 /(7) 2 /(8) 1 ,6 ,7 ,8
$2.00 ;P6; (3) 7 /(4) 1 ,5 /(5) 7 /(6) 3 /(7) 2 /(8) 2
$2.00 ;P6; (3) 7 /(4) 3 /(5) 2 ,3 ,5 ,6 /(6) 3 /(7) 2 /(8) 2
$2.00 ;P6; (3) 7 /(4) 3 /(5) 7 /(6) 4 ,7 ,8 ,10 /(7) 2 /(8) 2
$2.00 ;P6; (3) 7 /(4) 3 /(5) 7 /(6) 3 /(7) 1 ,4 /(8) 2
$2.00 ;P6; (3) 7 /(4) 3 /(5) 7 /(6) 3 /(7) 2 /(8) 5 ,9
$2.00 ;P6; (3) 1 ,3 /(4) 1 ,5 /(5) 2 ,3 ,5 ,6 /(6) 3 /(7) 2 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 1 ,5 /(5) 7 /(6) 4 ,7 ,8 ,10 /(7) 2 /(8) 2
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$2.00 ;P6; (3) 1 ,3 /(4) 1 ,5 /(5) 7 /(6) 1 ,2 ,5 /(7) 2 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 2 ,3 ,5 ,6 /(6) 1 ,2 ,5 /(7) 2 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 1 ,2 ,5 /(7) 1 ,4 /(8) 2
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 1 ,2 ,5 /(7) 2 /(8) 5 ,9
$2.00 ;P6; (3) 7 /(4) 3 /(5) 7 /(6) 3 /(7) 2 /(8) 1 ,6 ,7 ,8
$2.00 ;P6; (3) 1 ,3 /(4) 1 ,5 /(5) 7 /(6) 3 /(7) 2 /(8) 1 ,6 ,7 ,8
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 2 ,3 ,5 ,6 /(6) 3 /(7) 2 /(8) 1 ,6 ,7 ,8
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 4 ,7 ,8 ,10 /(7) 2 /(8) 1 ,6 ,7 ,8
$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 3 /(7) 1 ,4 /(8) 1 ,6 ,7 ,8
$2.00 ;P6; (3) 7 /(4) 1 ,5 /(5) 2 ,3 ,5 ,6 /(6) 3 /(7) 2 /(8) 2
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$2.00 ;P6; (3) 7 /(4) 1 ,5 /(5) 7 /(6) 3 /(7) 1 ,4 /(8) 2
$2.00 ;P6; (3) 7 /(4) 1 ,5 /(5) 7 /(6) 3 /(7) 2 /(8) 5 ,9
$2.00 ;P6; (3) 7 /(4) 3 /(5) 2 ,3 ,5 ,6 /(6) 4 ,7 ,8 ,10 /(7) 2 /(8) 2
$2.00 ;P6; (3) 7 /(4) 3 /(5) 2 ,3 ,5 ,6 /(6) 3 /(7) 1 ,4 /(8) 2
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$2.00 ;P6; (3) 1 ,3 /(4) 3 /(5) 7 /(6) 4 ,7 ,8 ,10 /(7) 1 ,4 /(8) 5 ,9
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	Amount bet	
Score 6	$2	$4
Score 7	$2	$58
Score 8	$2	$376
Score 9	$2	$1,484
Score 10	$2	$0
Score 11	$2	$0
Total 0.54%		$1,922

The key idea was: Consider two overlapping probability distributions. Suppose A is better but B overlaps. So if you choose a point from B it may be better from one chosen from A.

Professional racetrack betting syndicates

I had a hand with several of the major syndicate hedge fund teams through the *beat the racetrack* books Ziemba and Hausch (1984, 1986, 1987) and Hausch, Lo and Ziemba (1994, 2008) and other contacts. Hausch and I both talked to Bill Benter, the top racetrack syndicate organizer, early in his Hong Kong career. He had started betting but had not put together a successful syndicate yet. So he quizzed us on the Dr Z system and other ideas in phone calls. We did help him a bit but as he said "we were academics spreading knowledge and he was a businessman so could not pay us". He did have other paid consultants on factor models and he pioneered successfully using 80+ factor models of two types:

1. predict the fair odds probabilities of various horses outcomes and compare these to the public's odds, or
2. include the track odds as one of the variables to get even better probability estimates, see Sung and Johnson (2008).

Then bet with the Kelly criterion, probability weighting or the tree method.

I do not know if Benter picked up the Kelly from Ziemba and Vickson (1975) or from Ed Thorp's blackjack writings. Benter had been a blackjack player and Thorp introduced Kelly betting there as *Fortune's formula* so that may be where he learned it. A key early paper Thorp (1971) is in Ziemba and Vickson.

Benter pioneered the use of such models. I had a bit of a hand in there as the major paper on this was published while I was the *Management Science* departmental editor for finance and I processed and accepted it for publication. That's the Bolton and Chapman (1986) paper which along with the only paper Benter published are reprinted in Hausch, Lo and Ziemba. Chapman (1994, 2008) using Hong Kong data is in our book.

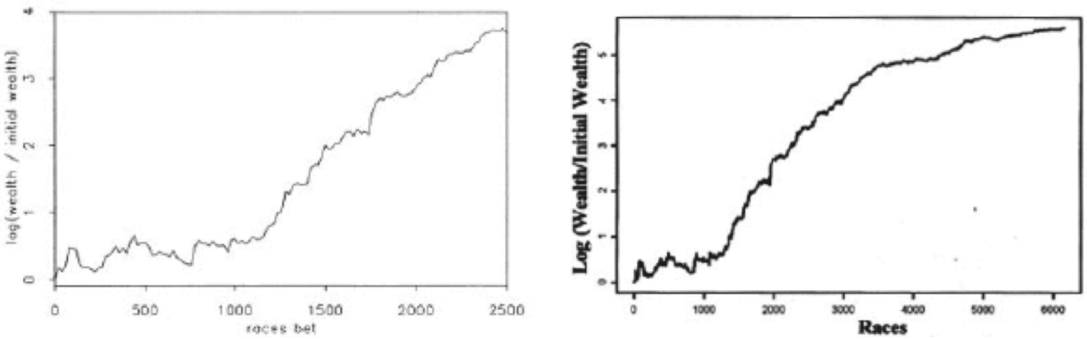
I met Benter in 1993 at the *Informs* meeting in Phoenix where I organized the finance sessions and helped on the racing sessions. I recall correcting Benter's (1994) paper in Hausch, Lo, and Ziemba which had one good new development. As discussed above, in the Dr Z method, the biases to win and being second and third tend to cancel so in the work I did with Donald Hausch, we did not need to make any changes except say that because of approximations, bets should not be made to place or show unless the expected value was significantly above break even. We suggested 1.10 for the best races at the best tracks and 1.14 and 1.18 for lesser races. This worked well for US place and show betting. Benter and others found that the Dr Z system did not really work well in Hong Kong as the biases there were different. Also, he discovered how to correct the second, third, etc biases through the discounted Harville formulations that are discussed above.

Victor Lo did his PhD thesis in Hong Kong, directed by statistician John Bacon-Shone on this problem and much of his research is in Hausch, Lo, Ziemba along with papers by others on this. Bacon-Shone has a joint paper in Hausch and Ziemba (2008) with the late Alan Woods who had his own small betting team in the Philippines after he left Benter.

Benter's real contribution is shown in Figure 5. Namely, he made it all work and in the process became a very rich man with total profits in the US$1 billion area. His paper in Hausch, Lo and Ziemba plus the other papers made our book a cult item with originals selling for $2000 up to $12,000 on EBay and Amazon. Originals are still trading at high prices, about $600. I sold one for $1400 to one of the copycat syndicates in Australia who I was consulting for. Another syndicate wanted to buy up all the Hausch, Lo and Ziemba books and burn them keeping one for their research. I decided to make a second edition which was published with a new preface in 2008 along with the sports and lottos handbook (Hausch and Ziemba, 2008).

The gains in Hong Kong by Benter's team and others were in a market without rebates and high commissions. But they utilized several advantages.

1. Hong Kong Chinese betters favor and dislike certain numbers from their culture which makes horses with these numbers differ from the true odds.



(a) to 1994 (b) to 2001

Figure 5: Benter's Hong Kong Racing Syndicate returns

2. Most of the horses are the same ones, mostly Australian geldings running in almost all the races on just two racetracks, so prediction was easier than in the US.
3. Data feeds were every 12 seconds and later every minute giving access to pool odds which could be successfully used.
4. The market was deep with huge betting so the price impact was low and lastly, they could bet electronically into the pools.

Since the mainland takeover of Hong Kong in 1997, there have been some changes. But the syndicates continue and trade in many markets today in 2020 such as Japan and Korea as well as in the US, Canada and Europe. My personal experience consulting extensively for one other syndicate is that the setup cost for the research and computer implementation is a major time and financial undertaking. Like most markets it was easily earlier and much more difficult now. Syndicates with many workers (up to 300 for the leading one in Australia) and good experience have an edge on new ones.

Conclusion

Racetrack betting remains a very active set of markets. The basic betting problems are various versions of portfolio management. The problems are stochastic programs usually one period but with non-concave objective functions because of the fractional functions inside the objective function that are needed so the effect of the syndicates on the odds is considered. But the problems are easily solved and for many situations there are simplified strategies. The objective is usually the Kelly expected log criterion but in cases of low probability high payoff bets there can be hundreds or thousands of separate tickets and the bets must be integers. So a tickets network tree (with equal bets on different combinations) or a probability weighting approach is useful and the Kelly strategy to bet more on the higher probability outcomes can be approximated.

The racetrack market is small compared to the financial markets such as bonds, currency and stock markets but there's enough there for about ten syndicates in the US, Australia and Hong Kong and elsewhere to make US$50-100 million or more per year. It is not an easy market to enter at a high level as the setup costs are high (about $1 million), the competition is fierce and prediction is difficult. Consider a grass race at a mile with seven horses: one has not run in a year but did well then; one has only run on dirt; one has never run past 6 furlongs (three quarters of a mile); one was racing in France long distances $1\frac{1}{2}$ miles plus on grass losing consistently and the others have run similar distances on grass but not on this racetrack. Add in jockey and trainer changes and you see why "it is a supreme intellectual challenge" as argued by Andy Beyer, a noted racetrack writer. The models try to bypass this with probabilities through prediction models and optimization. Much research on the economics of gambling is in Vaughan Williams and Siegel (2013)

Lotteries

Figure 6 provides a taxonomy for the types of games people can invest in. Games are classified by: 1) whether the chance of winning is purely luck or can be influenced with skill; and 2) whether the payoff upon winning is predetermined or can be improved with skill. Luck-luck games allow no possibility of discovering a profitable strategy, and so as markets are trivially efficient. On the other hand, there need not be a guarantee of efficiency for luck-skill games such as lotto (where we discuss a strategy of betting unpopular numbers, which does not affect the probability of winning but does affect the payoff upon winning) and skill-luck games (which are relatively uncommon). Blackjack, a skill-skill game, allows a profitable strategy. For horseracing, another skill-skill game, we review findings that certain forms of wagers are efficient while others are not.

		CHANCE OF WINNING	
		COMPLETE LUCK	**SKILL INVOLVED**
PAYOFF	**COMPLETE LUCK**	Scratch lottery games with fixed payment	Example: Pay $1 for a chance to pick all winners of hockey games on a particular day. From those who have all correct selections, one name is randomly drawn and awarded $100,000.
	SKILL INVOLVED	Lotto, such as 6/49, with some or all pari-mutuel payoffs	Sports pool games Horseracing Blackjack Sports betting

Figure 6: Taxonomy of games. Source: adapted from Ziemba, Brumelle, Gautier and Schwartz [1986].

Unlike most financial securities markets, the average lottery and sports betting participant must lose. We may, indeed, choose to differentiate gambling and investing by their expected returns, using the terms gambling when the expected profit is negative and investing when the expected profit, including all transactions costs and risk adjustments, is positive. Obviously a willingness to assume risk in the face of negative expected returns is inconsistent with the traditional assumptions that 1) individuals maximize the expected utility of wealth and 2) utility functions are concave, i.e., risk aversion. Instead of the second assumption, Friedman and Savage [1948] assumed a utility function that is convex in a neighborhood of the individuals present wealth but concave over higher and lower wealths. Given their different payoff distributions, simultaneously purchasing lottery tickets and insurance can be consistent with this form of a utility function. Markowitz [1952] offered a functional form that eliminates some behavior admitted by Friedman and Savages form that is not generally observed. He also pointed to the possibility of a utility function that recognizes the fun of gambling. Conlisk [1993] formalized this notion and found his model to be largely consistent with actual risk-taking behavior. Lane and Ziemba [2008] study hedging strategies for jai alai, a sport not considered in this survey. Haigh (2008) discusses the statistics of lotteriesmes. and Vaughan Williams (2012) discusses national lotteries and other gambling ga.

For thousands of years choosing by lots has been used as one means of resolving disputes. The first lottery of a more traditional form, where one pays for a chance to win, dates at least to the Middle Ages in Italy (Ziemba, Brumelle, Gautier and Schwartz [1986], hereafter ZBGS). The first European lottery occurred in the reign of Augustus Caesar to raise money to rebuild Rome. The next known European lottery was in the 15th century in the Low Countries before probability theory was formulized.

Prior to the 20th century, lotteries were successfully used in the United States for local and state governments, and to fund numerous causes, such as universities. Corruption, fraud and moral opposition together with lottery restrictions imposed by Congress ended legalized lotteries by the end of the 19th century, with 35 states going so far as to explicitly prohibit them in their constitutions [Clotfelter and Cook, 1991, p. 38]. State lotteries continued to be nonexistent[5] until 1964 when New Hampshire introduced its lottery. Since then the United States has seen an explosive resurgence of lotteries. By 1991, the District of Columbia and 32 states offered lotteries. US lottery sales are large and expanding. In 2009 the state lotteries sold $58.3 billion, growing to over $91 billion in 2019.

Clotfelter and Cook [1990] mention that in the course of a year, 60% of the adults who live in lottery states play the lottery at least once [p. 105]. They also report that per capita sales in lottery states has increased from (in 1989 dollars) $22 in 1975 to $108 in 1989 [p. 105]. The present popularity of lotteries is more widespread than just the United States; in 1986, over 100 countries offered legalized lotteries [ZBGS, 1986, p. 2]. See ZBGS for

[5]Other lottery possibilities were available, though such as charity raffles, foreign lotteries like the Irish Sweepstakes, and illegal lotteries.

more on the history and on the practice of lotteries.

Despite their popularity, with expected returns typically of 40-60%, lotteries are usually a poor investment.[6] This range is even lower (10-20%) if prizes are not tax-free or if they are paid in installments over say twenty years, as they typically are in the U.S. Canadian and U.K. prizes are paid in cash and are tax free. (See ZBGS for calculations on the effects of tax and payment in installments.)

Lotteries take several forms. A simple version has players buy pre-numbered tickets followed by a random drawing. Instant scratch-off games allow one to determine immediately if a prize has been won. Another form is the numbers game that requires players to match a randomly generated three- or four-digit number. Lastly, players of lotto games attempt to match five to seven numbers (with six most common) drawn from 50 or so numbers (with 49 most common), with the actual choice of the parameters varying state by state. A feature distinguishing the numbers and lotto games from the other two forms is the players act of choosing his or her numbers. For reasons not easily explained by traditional economics, the feature of choice is of tremendous importance. This was illustrated by Langer [1975] who conducted two lotteries where tickets cost $1 and all the money collected was awarded to the winner, i.e., the payback was 100%. Players in the first lottery were assigned their tickets while those in the second lottery chose theirs. As the winner was randomly drawn, subjects in both lotteries had the same chance of winning. However, Langer found that ticket holders in the two lotteries viewed their situations differently. When individually approached to sell their tickets before the drawing, those in the first lottery demanded a mean payment with of $1.96, while in the second lottery the mean was $8.67. Langer referred to this phenomenon as the illusion of control, that choosing ones ticket improves in some way the likelihood one will win, see also Kahneman (2011). States seem to appreciate this phenomenon and lotteries involving choices are very common.

The pre-numbered and instant scratch-off games allow a state to establish winning payoffs that exactly conform to any payback percentage. For instance, if the instant scratch-off game has $1 tickets and a $100 prize, then a 40% payback can be guaranteed by printing 0.4% winning tickets. The numbers game can also involve fixed payoffs. For instance, if the game is to pick the three-digit number of that is randomly drawn from the 1000 possible three-digit numbers, then a prize of $400 is a 40% payback. The difference here is that the state averages a 60% return, but it is not guaranteed. If the winning number has disproportionately many bettors then the states return will be less than 60% and the possibility exists that it could even be negative. Despite this difference to the state, the

[6] An exception was the inaugural offering of a new lottery in British Columbia. To create a keen interest in its game, participants received six tickets for the price of one, for an expected return of $0.385 times 6 or $2.31, a 131% edge. Ziemba (1995) discusses actually doing this and the procedure to buy tickets when it was optimal to buy as many as possible with the high constant mean and the variance going lower with more purchases.

advice to bettors remains: no profitable betting scheme exists for lottery games of this sort of and each bets expected return equals the states payback percentage.

For the numbers game, Clotfelter and Cook [1993] document a tendency for the public to choose numbers relatively less often immediately after they have been drawn. They describe this pattern as a form of the gamblers fallacy, the belief that if an event just occurred, then the likelihood that it will occur again falls.[7] I saw this in the Canadian Lotto 6/49. They test the picking machine on TV before the actual official draw and those numbers affect the numbers bought for the official draw.

Inefficiencies with unpopular numbers

Fixed payoffs for lotteries are not the only possibility. Parimutuel payoffs are used by all states for lotto games and by Massachusetts for its numbers game. The parimutuel method allows a state to guarantee its percentage take by having the payoff to winners decreasing in the number of winners. Given that all numbers are equally likely[8], no system can be developed that will improve the likelihood of winning any of the lotteries that have been described. But, if a numbers or lotto game employs parimutuel payoffs, then by choosing unpopular numbers, upon winning one is likely to share the given prize with fewer other winners. If some numbers are sufficiently unpopular, bets with positive expected return may exist despite the lotterys low payout rate. Chernoff's [1980, 1981] study of the Massachusetts number game, where players pick a number from 0000 to 9999, found that numbers with 0s, 9s and to a lesser extent 8s tended to be unpopular. He showed that by concentrating on the unpopular numbers, bets with a positive expected return were possible. Clotfelter and Cook [1991] provided some evidence of this, too, with three days of 1986 data from Maryland's three-digit numbers game. The most popular three-digit choice was 333 which was 9.93 times more common than the average. The seven most popular choices were all triples - 333, 777, 555, 444, 888, 666, 999 - and all were at least five times more popular than the average number. The least popular was 092, picked 0.23 times as often as the average number, and was followed in unpopularity by 086, 887, 884, and 968, all 0.25 times as popular as the average.

Lotto with its possibility of prizes of tens of millions of dollars is one of the most popular games and it has received the most media attention. It involves matching six numbers drawn without replacement from fifty or so total possible numbers. If T is the total possible numbers and D is the number drawn, then the probability of matching is one in

[7] Metzger [1985] considered the gamblers fallacy at the racetrack, and found support for the hypothesis that betting on the favorite should be more attractive after a series of longshots have won than after a series of wins by favorites.

[8] Johnson and Klotz [1993], on the basis of 200 Lotto America winning combinations, suggest that each number may not be equally likely in some lotto games. They found that, roughly, small numbers are drawn more frequently than large numbers. They suggest that it may be a consequence of the mechanical mixing process, that small-numbered balls are dropped into the urn first. ZBGS for the Canadian 6/49 shows statistically equal choice chances for all 49 numbers.

T!/(D!(T - D)!). So, for example, the probability of winning when six numbers are drawn from 49 is one in 13,983,816. Most games have prizes for matching fewer than all the drawn numbers, too, but it is common for about half the prize money to go to the grand prize. The long odds mean that none of the perhaps millions of bettors might win in a given week (the usual period over which lotto is played). In this event, the grand prize jackpot is carried over to the next week. ZBGS studied whether unpopular numbers and the carryover can allow a profit. Using several methods, they determined that there were unpopular numbers, they were virtually the same ones year to year, and they tended to be high numbers (non-birthdays, etc.) and those ending in 0s, 9s and 8s. For instance, a regression method based on actual payoffs generated the following as the twelve most unpopular numbers: 32, 29, 10, 30, 40, 39, 48, 12, 42, 41, 38 and 18. They were 1530% less popular than average. The most popular number, 7, was selected nearly 50% more often than the average number. Using a maximum entropy distribution approach, Stern and Cover [1989] identified 20, 30, 38, 39, 40, 41, 42, 46, 48 and 49 as the ten most unpopular numbers while 3, 7, 9, 11, 25, and 27 were the six most popular.[9]

ZBGS showed that expected returns of \$1.50 without carryover and up to \$2.25 with carryover per dollar bet are possible.[10] Does this imply that lotto games can be profitable, though? To see that it may not, consider a hypothetical game where you pay \$1, choose a number between 1 and one million, and if your number matches the one that is randomly selected then you win \$2 million. In spite of your edge, you are likely to go bankrupt before winning the jackpot. A reduced wager will increase the likelihood that you will eventually hit the correspondingly-reduced jackpot before you go bankrupt, but your expected wealth will suffer. MacLean, Ziemba and Blazenko [1992] analyzed this problem using a model balancing growth versus security of wealth and found that lotteries are an impractical way for modestly endowed investors to enhance their long-term wealth. For instance, by wagering an optimally small amount each round, ones initial stake can be increased tenfold before losing half the stake with a probability close to one. However, millions of years of wagering are required on average to have high confidence of winning. For example, consider the hypothesized data in Table 5 and the results in Figure 7. With a more attractive set of prizes, the probability is arbitrarily close to one for sufficiently small wagers, see MZB (1992).

Rather than make optimally small wagers in the face of small probability gambles, growth may be improved by increasing the probability of success. For lotteries, this can be accomplished by buying more than one combination of numbers. It may even be possible in the

[9]See also Joe [1987]. Clotfelter and Cook [1991] provided another example of popular numbers from Maryland's lotto, which had 40 total possible numbers. On the particular day they analyzed, players picked the 1-2-3-4-5-6 combination over 2000 times more frequently than the average pick. Had this been the winning combination (at a chance of one in 3,838,380), winners would have collected only \$193.50!

[10]This uses the model expected return = $0.45F_1 \ldots F_6$ where 0.45 is the payback and the F_is are the ratios of equal versus unpopular number probabilities. For carryovers, the take is higher.

Table 5: Lotto 6/49 Data. Source: MacLean, et. al. (1992).

Prizes	Prob.	Value	Contribution
Jackpot	1/13983816	$6M	42.9
Bonus	1/2330636	$0.8M	34.3
5/6	1/55492	M	9.0
4/6	1/1032	$5,000	14.5
3/6	1/57	$150	17.6
Edge			18.1%
Kelly bet			0.00000011
Number of Tickets with 10M bankroll			11

face of a substantial carryover to profitably purchase most, or perhaps all, of the combinations. There have been times when this would have been profitable. See the discussion below on buying the pot. In practice, though, the transactions costs are enormous because tickets must be purchased one at a time. Furthermore, there is the worry that others might also be covering all the numbers, to your joint detriment.[11]

Lotto typically involves drawing six numbers. Different states have different total possible numbers, though, resulting in very different probabilities of winning. In 1990 the extremes were one chance in 974,000 (36 numbers and 2 picks per ticket) in Delaware and one chance in 22,957,480 (53 total numbers and 1 pick per ticket) in California [Cook and Clotfelter, 1993, p. 635]. Cook and Clotfelter [1993] explain this as a tradeoff states must make between the size of the jackpot and a players estimate of the likelihood that he or she will win. The former is easily learned through advertisements and the media. The latter, according to Cook and Clotfelter, is generally not well understood but tends to be based on the frequency with which someone wins [p. 634]. Thus, Delaware could increase its total possible numbers to 53 like California but, given its population, on average there would be many weeks between winners. This would lower the public's view of the likelihood of winning and the attractiveness of purchasing a ticket. On the other hand, given California's population, even with 53 total possible numbers there will usually be a winner each week. This nonrational means of probability assessment causes a scale effect whereby per capita expenditure increases with the population base of the lottery. Smaller states cannot exploit this scale effect themselves but can through forming consortia with other states, as happens with the Tri-State lottery (involving Maine, New Hampshire and Vermont) and the States constituting Lotto America.

Sports lotteries

[11] A related opportunity arises with horseracing pick-sixes (pick the winners of six consecutive races) if there are substantial carryovers. Covering all pick-six possibilities is easily accomplished at the track and may be profitable if few others behave likewise.

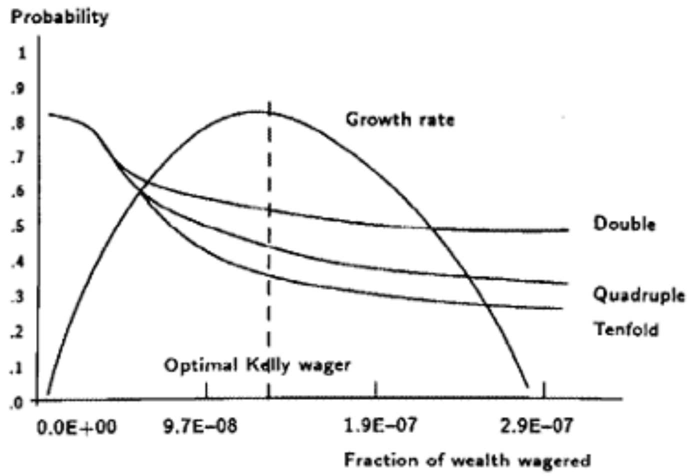


Figure 7: Lotto 6/49 - Probability of multiplying before losing half of ones fortune vs bet size. Source: Maclean et. al. (1992)

Besides Las Vegas and other legalized gambling locales, there are extensive sports lottery games, mostly run by the states. My own experience is largely as a consultant to the BC Lottery Commission, Singapore Pools, Mansion in Gibraltar and particularly the Canadian National Sports Pool.

The idea behind sports lotteries is to design games that players like and bet on and feel that with skill they can make profits. So the goal of management is to have the game look winnable with skill but in reality it is designed to be close to random with a negative expected value after the house take. A typical lottery has about 15 games and the players must pick correctly a home win or loss or tie in each game. The first prize is 15/15, second 14/15 and third 13/15.

Most sports have a home advantage. For example, in hockey, baseball and other sports, the home team is designed for the stadium. The simple model for a favorable betting system is to combine the home and away win-loss record with the home bias. One can use this to establish market efficient odds for the management. For the purposes of the lottery, a tie much be defined. In baseball a tie is a one run game. In NFL and basketball a tie is ± 3 points. So just like in regular lotto games with unpopular numbers where you can rearrange the ticket numbers so the popular numbers are in unpopular positions, in sports lottos you can design the tickets so the chances of win, loss, tie are 1/3 each or such that the game is unwinnable in reality but looks winnable with skill by the bettors. However, sophisticated computer models similar to professional racetrack models, can be used to beat sports pools. I did this in a court case for the Federal government of Canada in a dispute with Quebec who had a game in violation of the Canadian criminal code. The model was able to beat the Quebec game, see Ziemba (2017).

Buying the pot in lotto games

The carryover feature of lotto games, when the top prize is not won, builds up the jackpot pool on the next draw. Similar to the racing Rainbow Pick 6, with some skill, lotto games can have a way to create positive expected value bets. It is these large jackpot pools that generate sales for the hope of players to become rich.

You can guarantee to win the jackpot and many other lesser prizes by buying all the numbers. So the question is when is this a good idea and how do you do it? In Ziemba et al (1986) they show that two conditions must be met: a large carryover so the expected value of a random ticket is positive and there are not many tickets sold. Situations exist where these conditions are met. In Lotto BC a 5 of 40 game in British Columbia and Rhode Island there was a design feature that led to this. There was a 5/5 jackpot paying 91% of the net pool and little in the 4/5 and other prizes. So only the jackpot was paid. Racing is similar with its Pick 3, 4 and 5 wagers that only pay for the jackpot. As the jackpot carryover grew in Lotto BC, the ticket sales instead of rising with a larger jackpot actually fell because the public thought that the game was not winable. With only 658,008 combinations, the buying the pot strategy was very doable.

Moffitt and Ziemba (2019 ab) further analyze buying the pot and use one important fact: no matter how many ticket combinations are played by players picking their chosen or randomly selected numbers, there are bound to be some combinations that are not covered. Indeed, even in lotto games with millions of combinations, it is typical that there are 20-40% of the combinations not covered. They show that buying the pot has a positive expected return if

$$a + (t + c)(1 - x)E\left[\frac{1}{1 + X}\right] - t > 0$$

where t is the total number of tickets in the lotto game, $a \geqslant 0$ is the ticket equivalent of the carryover, $0 \leqslant x < 1$ is the lotto take of the betting pool, c is the number of tickets bought by the crowd and X is the random number of winning tickets held by the crowd.

The jackpot is shared equally $a+(t+c)(1-x)$ by the syndicate which has one winning ticket and any other players with the winning jackpot combination. Typical expected returns are in the 10-25% area for the syndicate. Moffitt did this successfully for jai alai carryovers on mandatory payout days consulting for Susquehanna. The optimal strategy for the crowd is $q = \frac{1}{t}$, that is buy all unique tickets with no doubles. But when the crowd bets not equally, as they do in all lotto games, with doubles and misses, then the expected syndicate edge is $E(q(\frac{1}{1+X})$. Moffitt and Ziemba (2019 a) discuss general lotto games and Moffitt and Ziemba (2019 b) discuss the 6/49 with many prize levels played in Canada and other countries. These papers describe the previous literature. Major syndicates are doing this in lotto games and Pick6 and other racing wagers making millions.

The football betting market Bettors on National Football League (NFL) games are offered a point spread. For example, suppose team A is a 10 points favorite over team B. Then a bet on A pays only if A wins by at least 11 points while a wager on B pays only if B either wins or loses by fewer than 10 points. If A wins by exactly 10 points then wagers are usually refunded. Typically bettors pay $11 for a $10 profit when they win. This provides the bookmaker a commission and means that a bettor has to beat the spread 52.4% of the time to break even. When the actual point spread equals the offered point spread, the bookmaker receives no return. Otherwise, by perfectly balancing the wagers, a bookmaker can guarantee a profit of 4.55% (since $21 is paid for each $22 wagered). The Las Vegas sports books, which dominate the market, offer opening point spreads on the coming weeks game. These spreads may change over the week, but bettors receive the spread offered at the time they placed their bet.[12]

The efficiency of NFL betting rests on the accuracy of the point spreads. An obvious and common approach to study their accuracy is to regress actual point spread on the offered point spread. If, say, bettors tend to wager on underdogs then, to balance the books, the bookmaker has to offer point spreads lower than unbiased expectations about actual point spreads. Bettor biases should be reflected in the point spread offered and, if they are sufficiently large, should allow profitable betting opportunities, which would reject efficiency.

Let A_1 be the actual point spread in game i and let P_i, be the point spread offered. The following equation can be estimated:

$$A_i = \beta_1 + \beta_2 P_i + \epsilon_i, \tag{1}$$

where ϵ_i is the error term. The efficiency test is the joint hypothesis that $\beta_1 = 0$ and $\beta_2 = 1$. Pankoff [1968], Zuber, Gandar and Bowers [1985] and Sauer, Brajer, Ferris and Marr [1988] all found significant support for the hypothesis. Gandar, Zuber, O'Brien and Russo's [1988] results are similar for both opening and closing point spreads (the point spread can change over the betting period as bookmakers attempt to balance their books). Their large t-statistic on β_2 and low R^2 (3.4% for closing data) suggest that while the point spread for any particular game is a poor predictor of the actual point spread, it is

[12]These dynamics are also present in horserace wagering against bookies. Wagering on jai alai is similar [see Lane and Ziemba, 2008], too, but its odds change during the contest as points are scored rather than before the contest as with sports betting. Parimutuel betting is different, though; its odds change over the course of the betting period as betting patterns change, but payoffs to all bettors are based only on the final odds.

a good predictor of the average actual point spread for a group of games with this point spread.

While these results support market efficiency, they are not directly useful in answering whether there might be technical rules that are economically profitable. Vergin and Scriabin [1978] used NFL data from 1969-1972 to consider various rules, such as betting on the underdog when the point spread exceeds some specified level, and identified several profitable strategies. Using 1975-1981 data, Tryfos, Casey, Cook, Leger and Pylypiak [1984] demonstrated that most of these strategies were unprofitable or, if profitable, not at the 5% significance level. Those that were significantly profitable all required a syndicate taking advantage of different point spreads in different cities. Gandar, Zuber, O'Brien and Russo [1988] found similar negative results for these strategies using their 1980-1985 data.

Golec and Tamarkin [1991] discussed how a model such as (7) can mask specific biases: 'consider that β_1 measures the average of the biases that do not change with the magnitude of the point spread. If half the observations in a data sample include a positive bias and the other half a negative bias of equal magnitude, then $\beta_1 = 0$ [p. 314]. The problem is that (7) can deal with only one bias. For instance, a bias in favor (or against) the home team can be considered by defining the data, P_i, relative to the home team. But if there is also a bias for (or against) the favored team, that can confound measuring the home team bias. To specifically test for possible biases for the favorite and the home teams, Golec and Tamarkin used the following model:

$$A_i = \beta_1 + \beta_2 P_i + \beta_3 H_i + \beta_4 F_i + \epsilon_i,$$

where H_i, is a dummy variable that is one for home teams and zero otherwise, and F_i, is another dummy variable that is one if the team is favored and zero otherwise. Here the test of efficiency is that $\beta_1 = \beta_3 + \beta_4 = 0$ and $\beta_2 = 1$. Their empirical results for NFL games from 1973-1987 indicated that bettors tend to underestimate the home field advantage and overestimate the distinction of being the favorite. Interestingly, they showed that the home field bias is disappearing over time while the underdog bias is actually growing. Despite demonstrating these biases, profits are shown to be slim at best in the face of the bookmakers commission. Neither bias is present in college football.

Sauer, Brajer, Ferris and Marr [1988] considered explanatory variables beyond the point spread, such as the number of wins prior to this game, fumbles, interceptions, penalties, yards passed, etc. Regressing these variables on the difference between the offered and the actual point spreads, they were unable to reject the hypothesis that their coefficients are all jointly zero. They concluded that these variables add essentially no information beyond that already in the point spread. Dana and Knetter [1994] allowed two modifications. Since fumbles, interceptions and penalties affect the game but are relatively uninformative about a teams ability, they accounted for these unsystematic sources of noise. Further, they used a nonlinear function of past point spreads. There is scant support for any of their models achieving the minimum 52.4% winners needed for profitable wagering.

What is the probability that a team favored to win a football game by p points does win the game? Stern [1991] showed that the margin of victory for the favorite is approximately normally distributed with mean equal to the point spread and standard deviation estimated at 13.86. The probability of winning a game is then:

$$Pr(F > U|P = p) = 1 - N\left(\frac{-p}{13.86}\right) = \left(\frac{p}{13.86}\right)$$

where F and U represent actual points scored by the favorite and the underdog, respectively, and $N(\cdot)$ is the standard normals cumulative distribution function. A linear approximation to the probability of winning is:

$$Pr(F > U|P = p) = 0.50 = 0.03p.$$

This formula is accurate to within 0.0175 for $p < 6$ and is based on data from the 1981, 1983, and 1984 NFL seasons. The normal approximation is accurate on current data, see Ziemba and MacLean (2018). This approximation is useful for a variety of applications, e.g., estimating the probability distribution of games won by a team, the probability a team makes the playoffs, and the probability distribution of season or playoff outcomes for particular teams. MacLean and Ziemba (2019, 2020a) show that the NFL odds are still efficient and the best teams have the best players. Ziemba and MacLean (2018) found that mean reversion risk arbitrage strategies are workable and profitable for ten years of actual Betfair long-short wagers.

The basketball betting market

Do athletes have performances that run in streaks? Gilovich, Vallone and Tversky [1985] using data from the 1980-81 season for the Philadelphia 76ers found that consecutive shots, if anything, were negatively autocorrelated. Hence there is no hot hand. They also let college players take shots while the players and other observers bet on the outcomes. Both players and observers made larger bets after players had just made shots, although bet size and actual performance were uncorrelated.[13] Camerer [1988, p. 1257] argued that . . . (b)elief in the hot hand is a mistake generated by persistent misunderstanding of randomness. People usually expect more alternations and fewer long streaks than actually occur in random series.

[13] Albright [1993] studied hitting streaks of baseball players and found no evidence of streaks beyond those expected by a statistical model of randomness.

If the hot hand is believed to exist within a game, then bettors might also believe in hot and cold streaks across games. And if point spreads reflect mistaken belief in hot hands then winning-streak teams should do worse than expected. For NBA regular season games from 1983-1986, Camerer [1988] found this effect to be very weak. The effect for losing streaks is slightly stronger, but in neither case is the bias sufficient to overcome the bookmakers transactions costs. Camerer's test is premised on the myth of the hot hand. Using more data and a test that can also detect the presence of the hot hand, Brown and Sauer [1993b] demonstrated that the market believes in the hot hand. Neither the hypothesis that the hot hand is real nor that it is a myth could be rejected, though.

Brown and Sauer [1993a] examined the error term in a point spread pricing model. While the models ex ante predictions explained 85% of the variation in point spreads, the error term has significant predictive power. Hence the error term contains unobserved fundamentals, not just noise.

Sauer [1991] showed that the Las Vegas Market point spreads offered at 5 p.m. Eastern time on the day of 5636 NBA games are an unbiased estimate of the actual difference in scores. In a subsample of 700 games that involved injuries to star players, the teams with the injured stars performed more than a point worse than the point spread. Obviously this is a nonrepresentative sample, though, because it consists of the games in which the injured star did not play, but not the games where the injured player decided after 5 p.m. to play. Accounting for the likelihood that a star with a nagging injury will play, the point spreads provided unbiased estimates of actual outcomes.

References

Akst, D. (1989). This is like stealing. *Forbes 13* (November), 142–144.

Albright, C. (1993). A statistical analysis of hitting streaks in baseball. *Journal of the American Statistical Association 88* (424), 1175–1183.

Ali, M. (1977). Probability and utility estimates for racetrack bettors. *Journal of Political Economy 85*, 803–815.

Ali, M. (1979). Some evidence of the efficiency of a speculative market. *Econometrica 47*, 387–392.

Ariel, R. A. (1987). A monthly effect in stock returns. *Journal of Financial Economics 18*, 161–174.

Asch, P. and R. Quandt (1986). *Racetrack Betting: The Professors' Guide to Strategies.* Auburn House, Dover, MA.

Asch, P. and R. Quandt (1987). Efficiency and profitability in exotic bets. *Economica 59*, 278–298.

Assamoi, K. V. (2010). Optimal investment strategies with Kelly capital growth criterion. Thesis: MSc in Mathematical Finance, Christ Church, University of Oxford.

Barbaris, N. and R. H. Thaler (2003). A survey of behavioural finance. In G. M. Constantinides, M. Harris, and R. M. Stulz (Eds.), *Handbook of the Economics of Finance*, Volume Volume 1, pp. 1053–1128. North Holland.

Benter, W. (1994). Computer based horse race handicapping. In D. B. Hausch, V. Lo, and W. T. Ziemba (Eds.), *Efficiency of Racetrack Betting Markets*, pp. 173–182. Academic Press.

Benter, W. (2008). Computer based horse race handicapping. In D. B. Hausch, V. Lo, and W. T. Ziemba (Eds.), *Efficiency of Racetrack Betting Markets* (2 ed.)., pp. 173–182. Academic Press.

Beyer, A. (2013). Gulfstream gimmick is irrestable at this point, but play it sensibly. *The Washington Post* (April 13).

Bolton, R. N. and R. G. Chapman (1986). Searching for positive returns at the track: a multinomial logit for handicapping horse races. *Management Science 32*, 1040–1–59.

Breiman, L. (1961). Optimal gambling systems for favorable games. In *Proceedings of the Fourth Berkeley Symposium, University of California Press, Berkeley, CA*, pp. 65–85.

Brown, S., J. J. Cao, and E. Powers (2013). To investment newsletters move markets? *Financial Management 42*, 315–338.

Brown, W. and R. Sauer (1993a). Does the baseball market believe in het hot hand? comment. *American Economic Review* (December), 1377–1386.

Brown, W. and R. Sauer (1993b). Fundamentals or noise? evidence from the point spread betting market. *Journal of Finance 84* (4), 1193–1209.

Busche, K. (1994). Efficient market results in an Asian setting. In D. B. Hausch and W. T. Ziemba (Eds.), *Efficiency of Racetrack Betting Markets*, pp. 615–616. Academic Press.

Busche, K. and C. Hall (1984). An exception to the risk preference anomaly,. *Journal of Business 61*, 337–346.

Cadsby, C. B. (1992). The CAPM and the calendar: Empirical anomalies and the risk-return relationship. *Management Science 38*(11), 1543–1439.

Cadsby, C. B. and M. Ratner (1992). Turn-of-the-month and pre-holiday effects on stock returns: some international evidence. *Journal of Banking and Finance 16*, 497–509.

Camerer, C. F. (1988). Can asset markets be manipulated? A field experiment with rcetrack betting. *Journal of Political Economy 106*, 457–482.

Cameron, R. (2010). The determinants of thoroughbred stud fees. Honors Thesis, Emory University.

Canfield, B., B. Fauman, and W. Ziemba (1987). Efficient market adjustment of odds prices to reflect track biases. *Management Science 33*, 1428–1439.

Chapman, R. G. (1994). Still searching for positive returns at the track: empirical results from 2000 Hong Kong races. In D. B. Hausch, V. Lo, and W. T. Ziemba (Eds.), *Efficiency of Racetrack Betting Markets*, pp. 173–181. Academic Press.

Chernoff, H. (1980). *An Analysis of the Massachusetts Numbers Game*. Technical Report (Massachusetts Institute of Technology. Department of Mathematics). Department of Mathematics, Massachusetts Institute of Technology.

Chernoff, H. (1981). How to beat the massachusetts numbers game. *The Mathematical Intelligencer 3*(4), 166–172.

Chopra, V. K. (1993). Improving optimization. *Journal of Investing*, 51–59.

Chopra, V. K. and W. T. Ziemba (1993). The effect of errors in mean, variance and co-variance estimates on optimal portfolio choice. *Journal of Portfolio Management 19*, 6–11.

Clotfelter, C. T. and P. J. Cook (1990). On the economics of state lotteries. *The Journal of Economic Perspectives 4*(4), 105–119.

Clotfelter, C. T. and P. J. Cook (1991). *Selling Hope*. Harvard University Press.

Clotfelter, C. T. and P. J. Cook (1993). The 'gambler's fallacy' in lottery play. *Management Science 39*(12), 1521–1525.

Conlisk, J. (1993). The utility of gambling. *Journal of Risk Uncertainty 6*, 255–275.

Cook, P. J. and C. T. Clotfelter (1993). The peculiar scale economies of lotto. *The American Economic Review 83*(3), 634–643.

Copeland, T. E. and D. Friedman (1992). The market value of information: Some experimental results. *The Journal of Business 65*(2), 241–66.

Dana, J. D. and M. Knetter (1994). Learning and efficiency in a gambling market. *Management Science 40*(10), 1317–1328.

Dansie, B. (198368). A note on permutation probabilities. *Journal of the American Statistical Association* (312-316).

Fabricand, B. (1965). *Horse Sense*. MaKay, NY.

Fama, E. F. (1970). Efficient capital markets: a review of theory and empirical work.

Journal of Finance 25, 383–417.

Fama, E. F. (1991). Efficient capital markets II. *Journal of Finance 46*, 1575–1617.

Forrest, D. and I. McHale (2005). Longshot bias: insights from the betting market on mens' professional tennis. In L. Vaughan Williams (Ed.), *Information efficiency in financial and betting markets*, pp. 215–230. Cambridge University Press.

Forrest, D. and R. Simmons (2005). Efficiency of the odds on english professional football matches. In L. Vaughan Williams (Ed.), *Information efficiency in financial and betting markets*, pp. 330–338. Cambridge University Press.

Friedman, M. and I. Savage (1948). The utility analysis of choices involving risk. *Journal of Political Economy 56*, 279–304.

Gandar, J., R. Zuber, T. O'Brien, and B. Russo (1988). Testing rationality in the point spread betting market. *Journal of Finance 43*, 995–1008.

Gergaud, O. and W. T. Ziemba (2012). Great investors: their methods, results and evaluation. *Journal of Portfolio Management 28*(4), 128–147.

Gilovich, T., R. Vallone, and A. Tversky (1985). The hot hand in basketball: On the misperception of random sequences. *Cognitive Psychology 17*, 295–314.

Golec, J. and M. Tamarkin (1991). The degree of inefficiency in the football betting market. *Journal of Financial Economics 30*, 311–323.

Golec, J. and M. Tamarkin (1998). Bettors love skewness, not risk, at the horse track. *Journal of Political Economy 106*, 205–225.

Gramm, M. and C. N. McKinney (2009). The effect of late money on betting market efficiency. *Applied Economics Letters 16*, 369–372.

Gramm, M. and W. T. Ziemba (2008). The dosage breeding theory for horse racing predictions. In D. B. Hausch and W. T. Ziemba (Eds.), *Handbook of Sports and Lottery Markets*, pp. 307–340. North Holland.

Green, E. A., H. Lee, and D. Rothschild (2019). Paralax and tax. Technical Report October 11, University of Pennsylvania.

Grifith, R. (1949). Odds adjustments by American horse-race bettors. *American Journal of Psychology 62*.

Haigh, J. (2008). The statistics of lotteries. In D. B. Hausch and W. T. Ziemba (Eds.), *Chapter 23 - Handbook of Sports and Lottery Markets*, Handbooks in Finance, pp. 481 – 502. San Diego: Elsevier.

Hanoch, G. and H. Levy (1969). The efficiency analysis of choices involving risk. *Review of Economic Studies 36*, 335–346.

Harville, D. A. (1973). Assigning probabilities to the outcomes of multi-entry competitions. *Journal of the American Statistical Association 68*, 312–316.

Hausch, D. B., R. Bain, and W. T. Ziemba (2006). An application of expert information to win betting on the Kentucky Derby, 1981-2001. *European Journal of Finance 12*(4), 283–302.

Hausch, D. B., V. Lo, and W. T. Ziemba (Eds.) (1994). *Efficiency of Racetrack Betting Markets*. Academic Press.

Hausch, D. B., V. Lo, and W. T. Ziemba (Eds.) (2008). *Efficiency of Racetrack Betting Markets* (2 ed.). World Scientific, Singapore.

Hausch, D. B. and W. T. Ziemba (1985). Transactons costs, extent of inefficiencies, entries and multiple wagers in a racetrack betting model. *Management Science 31*, 381–394.

Hausch, D. B. and W. T. Ziemba (1990a). Arbitrage strategies for cross track betting on major horseraces. *Journal of Business LXIII*, 61–78.

Hausch, D. B. and W. T. Ziemba (1990b). Locks in racetrack minus pools. *Interfaces* (May-June), 41–48.

Hausch, D. B. and W. T. Ziemba (Eds.) (2008). *Handbook of Sports and Lottery Markets.* North Holland.

Hausch, D. B., W. T. Ziemba, and M. E. Rubinstein (1981). Efficiency of the market for racetrack betting. *Management Science XXVII*, 1435–1452.

Hawawini, G. and D. Keim (1995). On the predictability of common stock returns: Worldwide evidence. In *Finance, Handbooks in Operations Research and Management Science*, Volume 9, pp. 497–544. North Holland, Amsterdam.

Hawawini, G. and D. B. Keim (2000). The cross-section of common stock returns: a review of the evidence and some new findings. In D. B. Keim and W. T. Ziemba (Eds.), *Security Market Imperfections in World Wide Equity Markets*, pp. 3–43. Cambridge University Press.

Henery, R. (1981). Permutation probabilities as models for horse races. *Journal of the Royal Statistical Society 5Cr. B. 43*, 86–91.

Hensel, C. R., G. Sick, and W. T. Ziemba (2000). In D. B. Keim and W. T. Ziemba (Eds.), *Security Market Imperfections in World Wide Equity Markets*, pp. 179–202. Cambridge University Press.

Hodges, S., R. G. Tompkins, and W. T. Ziemba (2004). The long-shot biases in gambling and options markets. Working Paper, UBC.

Jackson, D. and P. Waldron (2003). Pari-mutuel place betting in great britain and ireland: An extraordinary opportunity. In L. Vaughan Williams (Ed.), *The Economics of Gambling*, pp. 18–29. Routledge.

Joe, H. (1987). An ordering of dependence for distribution of k-tuples, with applications to lotto games. *The Canadian Journal of Statistics / La Revue Canadienne de Statistique 15*(3), 227–238.

Johnson, R. and J. Klotz (1993). Estimating hot numbers and testing uniformity for the lottery. *Journal of the Royal Statistical Society 5Cr. B. 88*(422), 662–668.

Kahneman, D. (2011). *Thinking fast and slow.* Doubleday.

Kahneman, D. and A. Tversky (1979a). Choices, values and frames. *Econometrica 47*, 263–291.

Kahneman, D. and A. Tversky (1979b). Prospect theory: an analysis of decisions under risk. *Econometrica 47*(2), 263–92.

Keim, D. B. and W. T. Ziemba (Eds.) (2000). *Security Market Imperfections in World*

Wide Equity Markets. Cambridge University Press.

Kelly, J. (1956). A new interpretation of information rate. *Bell System Technology Journal 35*, 917–26.

Kendall, M. (1953). The analysis of economic time-series, Part I: Prices. *Journal of the Royal Statistical Society 96*(1), 11–25.

Lakonishok, J. and S. Smidt (1988). Are seasonal anomalies real? a ninety-year perspective. *Review of Financial Studies 1*, 403–425.

Lane, D. and W. T. Ziemba (2008). Arbitrage and risk arbitrage in team jai alai. In D. Hausch and W. T. Ziemba (Eds.), *Handbook of Sports and Lottery Markets*, pp. 253–271. North Holland.

Langer, E. (1975). The illusion of control. *Journal of Personal Social Psychology 32*(2), 311–328.

MacLean, L. C., E. O. Thorp, Y. Zhao, and W. T. Ziemba (2011). How does the Fortune's Formula-Kelly capital growth model perform? *Journal of Portfolio Management* (Summer, in press).

MacLean, L. C., E. O. Thorp, and W. T. Ziemba (2010). The good and bad properties of the Kelly and fractional Kelly capital growth criterion. *Quantitative Finance* (August-September), 681–687.

MacLean, L. C., E. O. Thorp, and W. T. Ziemba (2011a). *The Kelly Capital Growth Investment Criterion*. World Scientific.

MacLean, L. C., E. O. Thorp, and W. T. Ziemba (Eds.) (2011b). *The Kelly Investment Criterion: Theory and Practice*. World Scientific.

MacLean, L. C. and W. T. Ziemba (2019). The 2018-2019 NFL season, playoffs and super bowl. *Wilmott*.

MacLean, L. C. and W. T. Ziemba (2020a). NFL 2019-2020 update and review of the season and superbowl. *Wilmott*.

MacLean, L. C. and W. T. Ziemba (2020b). NFL team composition: are the best players on the best teams? *Wilmott*.

MacLean, L. C., W. T. Ziemba, and G. Blazenko (1992). Growth versus security in dynamic investment analysis. *Management Science 38*(11), 1562–1585.

Markowitz, H. (1952). The utility of wealth. *Journal of Political Economy 60*, 151–158.

McGlothin, W. (1956). Stability of choices among uncertain alternatives. *American Journal of Psychology 69*.

Metzger, M. A. (1985). Biases in betting: An application of laboratory findings. *Psychological Report 56*(3), 883–888.

Moffitt, S. D. and W. T. Ziemba (2019a). Does it pay to buy the pot in the Canadian 6/49 Lotto? Implications for lottery design. *Wilmott* (May), 42–52.

Moffitt, S. D. and W. T. Ziemba (2019b). A risk arbitrage strategy for lotteries. *Wilmott* (March), 52–62.

Pankoff, L. (1968). Market efficiency and football betting. *Journal of Business* (41), 203–214.

Quandt, R. (1986). Betting and equilibrium. *Quarterly Journal of Economics 101*, 201–207.

Ritter, J. and H. Chopra (1989). Portfolio rebalancing and the turn-of-the-year effect. *Journal of Finance 44*, 149–166.

Roberts, H. V. (1959). Stock market patterns and financial analysis: methodological suggestions. *Journal of Finance 14*(1), 1–10.

Roll, R. (1977). A critique of the asset pricing theory's test, Part 1: onn past and potential testibility of the theory. *Journal of Financial Economics 4*, 129–176.

Roman, S. A. (2016). Pedigree and performance in thoroughbred racing. report on website: saroman7.winsite.com/dosage. .

Sauer, R. (1991). An injury process model of forecast bias in nba point spreads. Technical report, Clemson University.

Sauer, R. (1998). The economics of wagering markets,. *Journal of Economic Literature 36*, 2021–64.

Sauer, R., V. Brajer, S. Ferris, and L. Marr (1988). Hold your bets: another look at the efficiency of the gambling market for national football league games. *Journal of Political Economy* (February), 206–213.

Savage, I. (1957). Contributions to the theory of rank order statistics - the trend case. *Annuals of Mathematical Statistics 28*, 968–977.

Shin, H. (1992). Prices of contingent claims with insider trades and the favorite-longshot bias. *Economic Journal 102*(411), 426–435.

Snowberg, E. and J. Wolfers (2008). Examining explanations of a market anomaly: preferences or perceptions? In D. B. Hausch and W. T. Ziemba (Eds.), *Handbook of Sports and Lottery Markets*, pp. 103–136. North Holland.

Snowberg, E. and J. Wolfers (2010). Explaining the favorite-longshot bias: Is it risk-love or misperceptions? *Journal of Political Economy 118*, 723–746.

Sobel, R. and T. Raines (2003). An examination of the empirical derivatives of the favorite-longshot bias in racetrack betting. *Applied Economics 35*, 371–385.

Stern, H. (1991). On the probability of winning a football game. *The American Statistican 45*(3).

Stern, H. and T. Cover (1989). Maximum entropy and the lottery. *Journal of the American Statistical Association 84*(408), 980–885.

Sung, M. C. and J. Johnson (2008)). Semi-strong form efficiency in the horse race betting market. In D. B. Hausch and W. T. Ziemba (Eds.), *Handbook of Sports and Lottery Markets*, pp. 275–306. North Holland.

Swidler, S. and R. Shaw (1995). Racetrack wagering and the "uninformed" bettor: A study of market efficiency. *The Quarterly Review of Economics and Finance 35*, 305–314.

Thaler, R. H. and W. T. Ziemba (1988). Anomalies: parimutuel betting markets: racetracks and lotteries. *Journal of Economic Perspectives 2*, 161–174.

Thorp, E. O. (1962). *Beat the Dealer*. Random House.

Tompkins, R., W. T. Ziemba, and S. Hodges (2008). The favorite-longshot bias in the S&P500 and FTSE100 index futures options: the return to bets and the cost of insurance. In D. B. Hausch and W. T. Ziemba (Eds.), *Handbook of Sports and Lottery Markets*, pp. 161–180. North Holland.

Tryfos, P., S. Casey, S. Cook, G. Leger, and B. Pylypiak (1984). The profitability of wagering on NFL games. *Management Science 30*(1), 123–132.

Vaughan Williams, L. (Ed.) (2003). *The Economics of Gambling*. Routledge.

Vaughan Williams, L. (Ed.) (2005). *Information efficiency in financial and betting markets*. Cambridge University Press.

Vaughan Williams, L. (Ed.) (2012). *The Economics of Gambling and National Lotteries*. Edward Elgar Publishers.

Vaughan Williams, L. and D. Paton (1997). Why is there a favorite-longshot bias in british racetrack betting markets. *The Economic Journal 107*, 150–158.

Vaughan Williams, L. and D. S. Siegel (Eds.) (2013). *The oxford handbook of the economics of gambling*. Oxford University Press.

Vaughan Williams, L., M. Sung, P. A. F. Fraser-Mackenzie, J. Pierson, and J. E. V. Johnson (2016). Towards an understanding of the origins of the favourite–longshot bias: Evidence from online poker markets, a realmoney natural laboratory. *Economica 85*(4).

Woodland, L. M. and B. M. Woodland (1994). Market efficiency and the favorite-longshot bias: the baseball betting market. *Journal of Finance 49*, 269–279.

Ziegler, A. J. and W. T. Ziemba (2015). Returns from investing in S&P500 futures options, 1985-2010. In A. Mallarias and W. T. Ziemba (Eds.), *Handbook of Futures Markets*, pp. 643–688. World Scientific.

Ziemba, W. T. (1984). The favorite-long shot bias in hockey: betting on the 1982 Stanley Cup Playoffs. Technical report, Facutly of Commerce, University of British Columbia, March.

Ziemba, W. T. (1994). Worldwide security market regularities. *European Journal of Operational Research 74*, 198–229.

Ziemba, W. T. (2005). The symmetric downside risk sharpe ratio and the evaluation of great investors and speculators. *Journal of Portfolio Management Fall*, 108–122.

Ziemba, W. T. (2008). Efficiency of racetrack, sports and lottery betting markets? In D. B. Hausch and W. T. Ziemba (Eds.), *Handbook of Sports and Lottery Markets*, pp. 183–222. North Holland.

Ziemba, W. T. (2011). Place and show. *Wilmott Magazine* (November), 32–42.

Ziemba, W. T. (2012). *Calendar anomalies and arbitrage*. World Scientific.

Ziemba, W. T. (2014). *Stochastic programming and optimization in horserace betting*.

Ziemba, W. T. (2015). Response to Paul A Samuelson letters and papers on the Kelly capital growth investment model. *Journal of Portfolio Management 42*(1), 153–167.

Ziemba, W. T. (2017). *Adventures of a modern renaissance academic in investing and gambling*. World Scientific.

Ziemba, W. T. (2019a). *Exotic Betting at the Racetrack*. World Scientific.

Ziemba, W. T. (2019b). The Pick6 and the Rainbow Pick6. *Wilmott* (November), 70–80.

Ziemba, W. T. (2020). US Stock Market Prediction. Working Paper.

Ziemba, W. T. and D. B. Hausch (1984). *Beat the Racetrack*. Harcourt, Brace and Jovanovich.

Ziemba, W. T. and D. B. Hausch (1986). *Betting at the Racetrack*. Dr Z Investments, San Luis Obispo, CA.

Ziemba, W. T. and D. B. Hausch (1987). *Dr Z's Beat the Racetrack*. William Morrow.

Ziemba, W. T., S. Lleo, and M. Zhitlukhin (2017). *Stock market crashes: predictable and unpredictable and what to do about them*. World Scientific.

Ziemba, W. T. and L. C. McLean (2018). *Dr Z's NFL Guidebook*. World Scientific.

Ziemba, W. T. and R. G. Vickson (Eds.) (1975). *Stochastic Optimization Models in Finance*. Academic Press, New York.

Ziemba, W. T. and R. G. Vickson (Eds.) (2006). *Stochastic Optimization Models in Finance* (2 ed.). World Scientific.

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