

Pension Plan Funding, Risk Sharing and Technology Choice

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Abstract

The paper presents an analysis of the impact of pension plan funding on workers' saving and portfolio behaviour. It shows that the impact of pension plan funding and asset allocation on the economy's technology choices depends upon the constraints facing workers in the capital market. The failure of equivalence propositions between defined benefit and defined contribution pension plans derives from the existence of borrowing and short-sales constraints. The two types of plan force workers against the constraints differently yielding an asymmetric impact on risk taking and technological choice in the economy and thereby on the equity premium. The outcome of the market economy is a risk sharing arrangement between the workers and rentiers. This leaves open the question of how best to share risk between generations. The argument that defined contribution pensions and individual savings are an effective market solution to risk sharing may conflict with the institutional arrangements needed to manage effective intergenerational risk smoothing.

1 Introduction

In many western economies the burden of retirement provision has been shifting from the state to individuals. In the United States and the United Kingdom, moreover, the balance of provision by the private sector has been shifting from defined benefit (DB) occupational plans to defined contribution (DC) occupational and personal money purchase plans. This shift almost certainly has important implications for the real behaviour of the economy.¹ The current paper provides some theoretical observations on this important issue.

Consider an economy that has no social security system so that savings for retirement are undertaken either on personal account (which could include a personal pension) or through an occupational pension plan.² If retirement is financed through a pension, then there is the choice between DB and DC. Both are assumed to be paid for through payroll contributions. In a deterministic world, in the absence of taxes and transaction costs, these two types of pension are equivalent. In a stochastic world, absent corporate default, a DB plan is equivalent to a package of riskless bonds. A DC plan is a savings account, which if it buys riskless bonds replicates a DB plan. A DB plan is insured by the company. The company's shareholders bear both company risk and pension plan risk. With a DC pension plan, the portfolio risk of the plan is borne by the plan members and shareholders bear only company risk.

Occupational pension plans are viewed as part of efficient compensation packages that are designed particularly to bind skilled workers to the sponsoring firm.³ Moreover, they can act as a commitment mechanism for bargaining with workers over current levels of wages. The firm can promise to put more money into the pension plan in return for workers foregoing current wages but it may not be able to commit to giving improved levels of compensation in future wages (see Ippolito, 1985).

The arguments in favour of DB over DC are to do with long-term contracting. A DB plan is deferred compensation the value of which is tied to the economic success of the plan sponsor. Workers are then committed to supplying high effort levels over the long run and do not gain from quitting or collectively shirking. If this is true, then firms have an incentive to let the company pension plan run a deficit and lever the firm's capital structure so as to subject workers to the risk of default on their pensions should the firm perform poorly. However, an efficient trade-off between risk sharing and long-run effort incentives can be

¹The reasons for the contraction in public provision are to do with pressure on government budgets and unfavourable demographics due to increased life expectancy and falling birth rates. The switch from DB to DC results from declining markets, plan deficits and problems of insolvency. Recent accounting changes such as FRS 17 in the UK and IAS 19 internationally mean that DB plan deficits have to be recognised on the profit and loss account which introduces volatility.

²We point out at this juncture that the analysis has nothing to say about risk sharing and retirement provision between workers. Of course there will be important distributional considerations such as, how to ensure that those workers with limited labour force participation provide for retirement. Women, disabled and those suffering extensive unemployment may suffer exclusion from sufficient coverage.

³Throughout the paper we abstract from tax considerations that may impact these issues.

achieved with a combination of a DC pension plan and a long-term labour contract with efficiency wages (see Lazear (1985)).

This paper examines how the risk bearing of the pension plan interacts with the economy's risk sharing, technology and financing choices. We also show how the outcome is affected by the way the pension plan is financed. The basic framework is based on Diamond and Geanakoplos (2004) and in particular models technologies in more or less the same way as they do. Unlike them, however, we do not include long-dated safe and risky land. The key difference is that we model occupational pensions that are funded by non-working rentiers and by workers themselves as part of their benefit arrangements. In addition workers can save on personal account and solve a portfolio allocation problem. But in solving this problem they may suffer from some restrictions on their financial market transactions.

The model allows us to capture the general interaction of pension plan funding with the savings and asset allocation decisions of workers, both when workers are constrained and unconstrained. This leads to a different and somewhat broader set of predictions than are obtained in Diamond and Geanakoplos. The framework is also used to investigate the implications of allowing firms to make simple capital structure choices. We also examine the implications of the type of pension plan, DB versus DC, for savings, asset market equilibrium and technology choices.

The model though simple allows us to derive results about the equity premium similar to those in the three-period overlapping generation model in Constantinides et al (2002). Our model predicts a large equity premium if the pension plan is either DB or DC with a high proportion invested in riskless assets. The pension plan holding of riskless assets forces workers to a corner in which they hold only risky assets but because of borrowing constraints are not able to offset the impact of the pension plan holding of riskless assets.

The final question we address is that of how best to share risk between generations? Allen and Gale (1997) provide a series of insights into the institutional arrangements that promote efficient intergenerational risk sharing. The present paper argues that defined contribution pensions and individual savings are an effective market solution to risk sharing. However, we note briefly that this may conflict with the institutional arrangements needed to manage effective intergenerational risk smoothing.

2 Savings, Portfolio Choice and Investment

2.1 Basic Model: Linear Technology

The following is a somewhat modified version of the model in Diamond and Geanakoplos (2004), which is a linear version of the type of model in Diamond (1965) with heterogeneous agents. At each date there are two groups of new agents, workers and rentiers, each represented by a single risk-averse member. Agents are assumed to be rational and able to make

optimal financial decisions. Workers supply labour, rentiers only own capital and employ labour. The labour contract has two components, wages and (occupational) pensions. The pension plan is fully funded, partly by workers and partly by rentiers as part of the workers' compensation package. The contribution and asset allocation decisions of the plan are taken as exogenous, although we will address some aspects of this assumption later in the paper.⁴

The economy has two technologies: a safe technology and a risky technology. Both technologies are linear, non-durable and exhibit constant returns to scale. Let p_t be the cost in terms of current consumption of producing a unit of riskless consumption at date t ; and q_t the price of risky investment in terms of current consumption goods. As marginal returns are fixed we only have a single intertemporal relative price, so if the risky technology is active we set $q_t = 1$. An investment at date t of $p_t k_{0t+1}$ in the safe technology yields $R_{t+1} p_t k_{0t+1}$ at date $t + 1$. $R_{t+1} = 1 + r_{t+1}$, with r_{t+1} being the riskless rate of interest. An investment of $q_t k_{1t+1}$ in the risky technology yields $R_{t+1}^* q_t k_{1t+1}$. R_{t+1}^* is assumed to be i.i.d. with mean \bar{R}^* and variance σ^{*2} . The return realisation of the risky technology is a function of the economy's technological realisation, θ_{t+1} , which is i.i.d. with finite mean and variance.⁵

The representative rentier has an additive, strictly concave utility index, U . The rentier has a risky endowment of e_{yt}^s , drawn from a distribution with mean \bar{e} and variance σ_e^2 when young and makes a contribution to the workers pension plan of f_{yt}^s . This amount and how it is invested are taken as given in the rentier's optimisation problem. The residual is used to finance young consumption, c_{yt}^s , and savings of a_{yt}^s . The savings are used to purchase claims on the safe and risky technologies. The income stream from these investments is used to finance consumption when old, c_{ot}^s . Note that the subscript t refers to the individuals birth date, whilst y and o refer to youth and old age. The representative rentier thus solves the following optimisation problem:

$$\max\{U(c_{yt}^s) + E_t U(c_{ot}^s)\} \quad (1)$$

subject to

$$e_{yt}^s - f_{yt}^s = c_{yt}^s + a_{yt}^s \quad (2)$$

$$a_{yt}^s = p_t k_{0t+1}^s + q_t k_{1t+1}^s \quad (3)$$

$$c_{ot}^s = R_{t+1} p_t k_{0t+1}^s + R_{t+1}^* q_t k_{1t+1}^s \quad (4)$$

The representative worker also has an additive, strictly concave utility index, V . He

⁴A whole series of interesting and important issues arise if the contribution and asset allocation decisions of the plan are endogenous. We abstract from issues of over versus underfunding of the plan and plan deficits and surpluses. We also abstract from the conflicts that may arise, resulting from whether it is the plan sponsor or the beneficiaries that determine the plan's investment policy.

⁵The assumptions of i.i.d. shocks and non-durability of capital together with the regular structure of the model ensure the existence of a stationary equilibrium for the economy. Constantinides and Duffie (1996) and Storesletten et al (2001) have studied similar economies with durable capital in which the equilibria are stationary Markov.

supplies a unit of labour inelastically and divides his labour between the two technologies, being indifferent to where he works.⁶ He receives wages of w_{yt}^ω and makes pension contributions of f_{yt}^ω . The package of wages and pension benefits is determined to satisfy an exogenous participation condition. The worker's saving on personal account is denoted by a_{yt}^ω , which is divided between safe assets, k_{0t+1}^ω , and risky assets, k_{1t+1}^ω . Net resources finance consumption when young, c_{yt}^ω . Old consumption is determined by the return on personal saving and the pension plan. The worker's optimisation problem involves choosing a_{yt}^ω , k_{0t+1}^ω , k_{1t+1}^ω , with the pension contribution, f_{yt}^ω , and the plan asset allocation between safe assets, k_{0t+1}^f , and risky assets, k_{1t+1}^f , taken as given. The problem is stated as:

$$\max\{V(c_{yt}^\omega) + E_t[V(c_{ot}^\omega)]\} \quad (5)$$

subject to

$$w_{yt}^\omega - a_{yt}^\omega - f_{yt}^\omega = c_{yt}^\omega \quad (6)$$

$$a_{yt}^\omega = p_t k_{0t+1}^\omega + q_t k_{1t+1}^\omega \quad (7)$$

$$f_{yt}^s + f_{yt}^\omega = p_t k_{0t+1}^f + q_t k_{1t+1}^f \quad (8)$$

$$c_{ot}^\omega = R_{t+1} p_t [k_{0t+1}^f + k_{0t+1}^\omega] + R_{t+1}^* q_t [k_{1t+1}^f + k_{1t+1}^\omega] \quad (9)$$

Condition (8) combines the worker's and the rentier's contributions to the pension plan on the left-hand-side, with the asset allocation decision on the right-hand-side, which is taken as exogenous.

Firms choose capital intensity after the realisation of the current technology shock and simply maximise current profits. Taking the prices of safe and risky income streams as given, we have the following price equals marginal rate of transformation conditions:

$$p_t = \frac{1}{R_{t+1}} \text{ if } k_{0t+1} > 0 \text{ and } p_t \leq \frac{1}{R_{t+1}} \text{ if } k_{0t+1} = 0; \quad (10)$$

$$q_t = 1, \text{ if } k_{1t+1} > 0 \text{ and } q_t \leq 1 \text{ if } k_{1t+1} = 0 \quad (11)$$

The economy has three markets at date t , these are for current consumption, safe and risky assets. The demands for current consumption and safe and risky assets are normal and they are gross substitutes. We assume that rentiers' and workers' asset demand function can be aggregated, which is based on the assumption that they have parallel linear Engel curves. The asset market equilibrium conditions take the same general form as in Tobin (1969) and Foley and Sidrauski (1971)⁷: $H(p_t, q_t, e_t^s, w_{yt}^\omega)$ for the safe asset and $J(p_t, q_t, e_t^s, w_{yt}^\omega)$ for the risky asset; with $H_1 < 0$, $H_2 > 0$, $H_3 > 0$, $H_4 > 0$ and $J_1 > 0$, $J_2 < 0$, $J_3 > 0$, $J_4 > 0$.⁸

⁶The assumption that labour income is default free, allows us to allocate labour in this simple way.

⁷Fischer (1972) has a general discussion of the properties of asset demand functions and portfolio theory.

⁸Heaton and Lucas (2000) examine portfolio choice with both business income and wage income. They find that exposure to business income and wage income risk reduces stock holding relative to entrepreneurs with similar wealth. However, our model with a single "composite" risky asset precludes a discussion of

Consumption per capita is given by

$$c_t = c_t^y + c_{t-1}^o \quad (12)$$

where

$$c_t^y = c_{yt}^s + c_{yt}^w \text{ and } c_{t-1}^o = c_{ot-1}^s + c_{ot-1}^w \quad (13)$$

Equilibrium requires that the consumption and asset markets clear and that firms are making optimal investment decisions. Because of Walras' Law we can drop the current consumption goods market and consider only the markets for safe and risky assets. Hence the equilibrium values $\{p_t, q_t, k_{0t+1}, k_{1t+1}\}$ satisfy (10) and (11) and

$$H(p_t, q_t, e_t^s, w_{yt}^\omega) = k_{0t+1} \quad (14)$$

and

$$J(p_t, q_t, e_t^s, w_{yt}^\omega) = k_{1t+1} \quad (15)$$

It will be seen immediately that the model above yields a positive equity premium. The equity (risk) premium is defined as

$$E_t(R_{t+1}^*) - R_{t+1} \quad (16)$$

If $k_{1t+1} > 0$, then from the rentier's optimisation problem

$$U'(c_{yt}^s) = E_t[U'(c_{ot}^s)R_{t+1}] = E_t[U'(c_{ot}^s)R_{t+1}^*] \quad (17)$$

Since c_{ot}^s and R_{t+1}^* are perfectly correlated, $U'(c_{ot}^s)$ and R_{t+1}^* are negatively correlated. Hence $E_t(R_{t+1}^*) > R_{t+1}$.

2.2 Workers do not Save on Personal Account

We begin by assuming workers do not save on personal account, $a_{yt}^\omega = 0$. Suppose that the pension plan were to invest only in safe assets, then workers bear no risk and rentiers bear all the risk. The pension plan holds only riskless claims against the safe technology or the riskless part of the risky technology's returns (a decomposition which we ignore for the time being). To support this allocation as an equilibrium p_t must be sufficiently high (r_{t+1} sufficiently low) so the equity premium has to be correspondingly high.⁹ However, at the proposed equilibrium allocation, there are gains to pension plan diversification. Diversification broadens risk bearing in the economy, so the economy can absorb more risky assets. If the pension plan diversifies its holdings into risky assets p_t declines and R_{t+1} rises.

these issues.

⁹Notice that here the equity premium is high when p_t is high. This means that a high equity premium corresponds to a low value of the riskless rate of interest.

This results in a lower value of the equity premium. Let k_{1t+1}^f increase at the expense of k_{0t+1}^f , then holding $q_t = 1$, rentiers sell the risky asset and buy the safe asset. However, given the normal goods assumption, aggregate risky investment goes up and aggregate safe investment goes down. As aggregate safe investment goes down, p_t goes down and r_{t+1} increases.

Applying a result of Arrow (1971), Diamond and Geanakopolos show the following, which is useful in our later discussion of DB versus DC pension plans.

Proposition 1. The diversification of the pension plan raises the welfare of both rentiers and workers.

Proof. Rentiers benefit from the fall in the price of the riskless asset, whilst the value of their risky portfolio remains constant. Workers who are old at the time of the substitution, whose pension plan allocation is already determined, benefit from the substitution only through the interest rate effect. Young workers gain by holding some small amount of the risky asset. Let $dk_{1t+1}^f = -p_t dk_{0t+1}^f$, then

$$\frac{dV}{dk_{1t+1}^f} = E_t[V'(c_{ot}^\omega)(E(R_{t+1}^*) - R_{t+1}) + V'(c_{ot}^\omega)\frac{dR_{t+1}}{dk_{1t+1}^f}p_t k_{0t+1}^f] > 0 \quad (18)$$

The second term is positive but in the neighbourhood of $k_{1t+1}^f = 0$,

$$E_t[V'(c_{ot}^\omega)(E_t(R_{t+1}^*) - R_{t+1})] = V'(c_{ot}^\omega)(E_t(R_{t+1}^*) - R_{t+1}) > 0 \quad (19)$$

and so the workers benefit from the equity premium. QED.

As Diamond and Geanakopolos note, further increases in k_{1t}^f will raise welfare until the workers reach their optimal risk exposure (portfolio equilibrium), unless rentiers reach a zero holding of the riskless asset first.

2.3 Workers do Save on Personal Account

Now suppose that workers can save on personal account, $a_{yt}^\omega > 0$. The pension plan is assumed to be diversified. Then if f_{yt}^ω is reduced, a_{yt}^ω will increase, $-df_{yt}^\omega = da_{yt}^\omega$, and we have neutrality because pension contributions and personal savings are perfect substitutes. This of course is not the case if f_{yt}^s is also cut. However, in this case, other things equal, the decline in f_{yt}^s will be met with an increase in w_{yt}^ω , $dw_{yt}^\omega = -df_{yt}^s$ and $da_{yt}^\omega = -d(f_{yt}^\omega + f_{yt}^s)$, thereby restoring the original equilibrium. If, for example, in the above a_{yt}^ω does not increase to offset the decline in pension contributions, the demand for riskless and risky assets declines, which given $q_t = 1$, leads to a decline in p_t . In turn aggregate safe and aggregate risky investment decline.¹⁰

¹⁰Hemming and Harvey (1983) examined the extent to which (defined benefit) pension plan and non-pension retirement saving are either substitutes or indeed complements. They found some evidence of complementarity, which of course runs entirely counter to the perfect substitutes requirement needed for neutrality. They do not, however, consider how this relationship is affected by the extent to which the

The next step is to consider how the institutional asset holdings of the pension plan interact with the worker's personal asset allocation, in particular when there are restrictions on borrowing and or short-sales.¹¹ Assume that the representative worker is at a portfolio optimum relative to a given equilibrium price system. If the pension plan substitutes risky for safe assets, this will be offset by an equivalent reduction in the holding of risky assets on worker's personal account. But, if the plan is a large part of the worker's retirement provision and if it invests a high proportion in the risky asset, workers may be forced to a corner. With short- sales restrictions they will be unable to undertake the necessary offsetting transactions. Thus if workers are forced to the corner with $k_{1t+1}^\omega = 0$ and cannot short-sell the risky asset, then there is effectively a decline in the demand for safe assets, so to restore equilibrium p_t must decline and r_{t+1} rises. This will lead to a reduction in the production of the safe asset and an increase in the production of the risky asset. There will also be a substitution of saving for consumption, since if workers are forced to bear more income risk they will increase precautionary saving.¹² This all has the general implication that the equity premium will be correspondingly lower than in the absence of the constraint. The opposite occurs if the pension plan invests heavily in the safe asset. Workers are then forced to the corner with $k_{0t+1}^\omega = 0$ and p_t and safe production must rise to restore equilibrium. In this case, the equity premium will be correspondingly higher.

The last result has similarities with that in Constantanides et al (2002). They develop a three-period overlapping generations model, in which the first (junior) generation is constrained not to be able to borrow to buy equity. This means that equity prices are lower and bond prices higher than otherwise, which raises the equity premium. In our model the same result arises if workers cannot undo the excess holding of the safe asset by the pension plan because they cannot borrow to buy shares in the risky technology.¹³

2.4 The Role of Government Debt

It is illustrative to briefly consider the impact of government bonds in the model presented above. Let the stock of government bonds be denoted by B_{gt} . Government bonds unlike corporate bonds imply an equivalent stream of taxes to pay interest and principal. We will not model the government's budget constraint here, nor will we give a detailed discussion of

occupational plan is funded through sponsor or worker contributions.

Borsch-Supan and Reil-Held (1997) review what is known about the relationship between the level risk and substitution among components of retirement income.

¹¹Mankiw (1986) was one of the first papers to investigate the implications of borrowing constraints on the equity premium. Lucas (1994) provides an detailed analysis the impact of borrowing and short sales constraints on the equity premium, when labour income is subject uninsurable shocks.

¹²With additive utility, decreasing absolute risk aversion is sufficient to guarantee that if workers are bearing more income risk in the future they will increase their saving. See Sandmo (1969) and Leland (1968).

¹³Our model is a two-period overlapping generations in which worker's benefit from a pension plan that enjoys contributions from rentiers. The asset allocation decisions of the plan are made independently of the worker's own optimisation problem. This model generates many of the properties on the demand for assets achieved in three-period over-lapping generations model in Constantanides et al.

the structure of current and future tax liabilities. We restrict ourselves to one fairly general observation that mirrors the argument in Diamond and Geanakoplos (2004).¹⁴

It is well known that government bonds are only net worth to the personal sector to the extent that the implied tax liabilities are not fully capitalised (see Barro (1974)). Begin by assuming that government bonds are held by the pension plan, so that

$$f_t = f_{yt}^s + f_{yt}^\omega = p_t k_{0t+1}^f + q_t k_{1t+1}^f + B_{gt}^f \quad (20)$$

Holding f_{yt}^s and f_{yt}^ω fixed, let k_{1t+1}^f increase, then there will be no effect on r_{t+1} as the substitution of risky for riskless assets in the plan involves the plan selling bonds that are purchased by rentiers, who reduce their investment in the safe technology and not risky production. The pension plan in turn holds claims on the risky technology at the expense of bonds. But if there are government bonds in the economy and $k_{0t+1} = 0$, then r_{t+1} increases and this raises taxes and to the extent that workers pay interest service taxes redistributes wealth from workers to rentiers.

3 Company Financial Policy

Firms in this model are technologies, the activity levels of which are chosen endogenously. We have seen how the investment policy of the pension plan can affect the technological choices of the economy, we now examine the role of the financial policy of the risky technology. To examine this, suppose that the risky technology has a balance sheet made up of debt and equity. Returns to the risky technology are given by $R_{t+1}^* q_t k_{1t+1}$, which are packaged as riskless debt which pays D_{1t+1} and equity which pays $R_{t+1}^* q_t k_{1t+1} - D_{1t+1}$. Then

$$D_{1t+1} = (1 + r_{t+1})B_{1t} \quad (21)$$

where B_{1t} is the value of corporate debt. The risky technology is a portfolio of two claims, levered equity, S_{1t} , and riskless debt:

$$q_t k_{1t+1} = B_{1t} + S_{1t} \quad (22)$$

First let $\{\alpha_{1t}^f, \beta_{1t}^f\}$ be the shares of equity and debt issued by the risky technology held by the pension plan

$$f_t = f_{yt}^s + f_{yt}^\omega = p_t k_{0t+1}^f + \alpha_{1t}^f S_{1t} + \beta_{1t}^f B_{1t} \quad (23)$$

The return on the pension plan is

$$R_{t+1} p_t k_{0t+1}^f + \alpha_{1t}^f (R_{t+1}^* q_t k_{1t+1}^f - D_{1t+1}) + \beta_{1t}^f D_{1t+1} \quad (24)$$

¹⁴For an earlier discussion of some of the relevant issue relating to the interaction of government debt and savings behaviour in a heterogeneous agent model, with firm technology choice, see Webb(1982).

We allow young rentiers and workers to borrow and lend with each other. Let $L_t^\omega \geq 0$ and $L_t^s \geq 0$ represent the holdings of personal loans (borrowing is negative) by workers and rentiers respectively. There are no direct loan market transactions with the pension plan. We assume that the market borrowing rate equals the lending rate, both denoted by ρ_{t+1} , and that $L_t^\omega + L_t^s = 0$. Let α_{1t}^s and β_{1t}^s represent the shares of the risky technology's equity and debt held in the portfolio of the representative rentier. The rentier's optimisation problem is written as

$$\max\{U(c_{yt}^s) + E_t U(c_{ot}^s)\} \quad (25)$$

subject to

$$e_{yt}^s - f_{yt}^s = c_{yt}^s + a_{yt}^s + L_t^s \quad (26)$$

$$a_{yt}^s = p_t k_{0t+1}^s + \alpha_{1t}^s S_{1t} + \beta_{1t}^s B_{1t} \quad (27)$$

$$c_{ot}^s = R_{t+1} p_t k_{0t+1}^s + \alpha_{1t}^s (R_{t+1}^* q_t k_{1t+1}^s - D_{1t+1}) + \beta_{1t}^s D_{1t+1} + (1 + \rho_{t+1}) L_t^s \quad (28)$$

Let $\{\alpha_{1t}^\omega, \beta_{1t}^\omega\}$ be the shares of equity and debt issued by the risky technology that are held in the worker's saving account. The worker's problem is

$$\max\{V(c_{yt}^\omega) + E_t[V(c_{ot}^\omega)]\} \quad (29)$$

subject to

$$w_{yt}^\omega - a_{yt}^\omega - f_{yt}^\omega = c_{yt}^\omega + a_{yt}^\omega + L_t^\omega \quad (30)$$

$$a_{yt}^\omega = p_t k_{0t+1}^\omega + \alpha_{1t}^\omega S_{1t} + \beta_{1t}^\omega B_{1t} \quad (31)$$

$$f_{yt}^s + f_{yt}^\omega = p_t k_{0t+1}^f + \alpha_{1t}^f S_{1t} + \beta_{1t}^f B_{1t} \quad (32)$$

$$\begin{aligned} c_{ot}^\omega = & R_{t+1} p_t [k_{0t+1}^f + k_{0t+1}^\omega] + (\alpha_{1t}^f + \alpha_{1t}^\omega) [R_{t+1}^* q_t (k_{1t+1}^f + k_{1t+1}^\omega) - D_{1t+1}] \\ & + (\beta_{1t}^f + \beta_{1t}^\omega) D_{1t+1} + (1 + \rho_{t+1}) L_t^\omega \end{aligned} \quad (33)$$

The worker's total direct and indirect holding of riskless income streams is then given by

$$p_t [k_{0t+1}^f + k_{0t+1}^\omega] - [(\alpha_{1t}^f + \alpha_{1t}^\omega) - (\beta_{1t}^f + \beta_{1t}^\omega)] B_{1ct} + L_t^\omega \quad (34)$$

Given the intensity with which the risky technology is operated, changes in the composition of liabilities financing this technology do not alter individual investor's budget sets if they can borrow and lend on personal account. Hence no new consumption opportunities are introduced. In other words, transactions on personal account are perfect substitutes for borrowing on corporate account. Given the financing of the pension plan and taking the returns on the safe and risky technologies as given as well as the prices p_t and q_t , workers and rentiers solve the above optimisation problems. Let $k_{0t+1}^f, k_{0t+1}^\omega, k_{0t+1}^s, k_{1t+1}^f, k_{1t+1}^\omega,$

k_{1t+1}^s , B_{1t} , S_{1t} , $\{\alpha_{1t}^f, \beta_{1t}^f\}$, $\{\alpha_{1t}^\omega, \beta_{1t}^\omega\}$, $\{\alpha_{1t}^s, \beta_{1t}^s\}$ and r_{t+1} denote the equilibrium values of the relevant variables.

Now consider an alternative equilibrium in which other things equal the risky technology has a reduced amount of debt. Let \widehat{k}_{0t+1}^f , $\widehat{k}_{0t+1}^\omega$, \widehat{k}_{0t+1}^s , \widehat{k}_{1t+1}^f , $\widehat{k}_{1t+1}^\omega$, \widehat{k}_{1t+1}^s , \widehat{B}_{1t} , \widehat{S}_{1t} , $\{\widehat{\alpha}_{1t}^f, \widehat{\beta}_{1t}^f\}$, $\{\widehat{\alpha}_{1t}^\omega, \widehat{\beta}_{1t}^\omega\}$, $\{\widehat{\alpha}_{1t}^s, \widehat{\beta}_{1t}^s\}$ and \widehat{r}_{t+1} denote the new equilibrium values of the relevant variables. For simplicity of illustration, let the new level of \widehat{B}_{1t} equal to zero. We assume that workers and rentiers can borrow and lend at the same rate as firms. Suppose to begin with that in the new equilibrium $r_{t+1} = \rho_{t+1} = \widehat{r}_{t+1}$. Keeping the level and composition of funding of the pension plan fixed, the plan simply replaces debt one for one with unlevered equity,

$$f_{yt}^s + f_{yt}^\omega = p_t \widehat{k}_{0t+1}^f + \widehat{\alpha}_{1t}^f \widehat{S}_{1t} \quad (35)$$

and this yields

$$R_{t+1} p_t \widehat{k}_{0t+1}^f + \widehat{\alpha}_{1t}^f R_{t+1}^* q_t \widehat{k}_{1t+1}^f \quad (36)$$

at date $t + 1$. Suppose that in the original situation, $-[(\alpha_{1t}^f + \alpha_{1t}^\omega) - (\beta_{1t}^f + \beta_{1t}^\omega)] < 0$, now that $\widehat{B}_{1t} = 0$ and given the new values of $\widehat{\alpha}_{1t}^f > 0$ and $\widehat{\beta}_{1t}^f = 0$ the representative worker should increase borrowing to

$$\widehat{L}_t^\omega = L_t^\omega - [(\alpha_{1t}^f + \alpha_{1t}^\omega) - (\beta_{1t}^f + \beta_{1t}^\omega)] B_{1t} < 0$$

With the increased borrowing on personal account the worker increases his direct equity holding in the risky technology by

$$(\widehat{\alpha}_{1t}^\omega - \alpha_{1t}^\omega) \widehat{S}_{1t} = [(\alpha_{1t}^f + \alpha_{1t}^\omega) - (\beta_{1t}^f + \beta_{1t}^\omega)] B_{1t} \quad (37)$$

which implies that

$$c_{ot}^\omega = R_{t+1} p_t [\widehat{k}_{0t+1}^f + \widehat{k}_{0t+1}^\omega] + (\widehat{\alpha}_{1t}^f + \widehat{\alpha}_{1t}^\omega) [R_{t+1}^* q_t (\widehat{k}_{1t+1}^f + \widehat{k}_{1t+1}^\omega)] + (1 + \rho_{t+1}) \widehat{L}_t^\omega \quad (38)$$

Now consider rentiers, in the original situation,

$$c_{ot}^s = R_{t+1} p_t k_{0t+1}^s + \alpha_{1t}^s (R_{t+1}^* q_t k_{1t+1}^s - D_{1t+1}) + \beta_{1t}^s D_{1t+1} + (1 + \rho_{t+1}) L_t^s \quad (39)$$

where $\alpha_{1t}^s - \beta_{1t}^s = [(\alpha_{1t}^f + \alpha_{1t}^\omega) - (\beta_{1t}^f + \beta_{1t}^\omega)] > 0$. In the new situation

$$c_{ot}^s = R_{t+1} p_t \widehat{k}_{0t+1}^s + \widehat{\alpha}_{1t}^s R_{t+1}^* q_t \widehat{k}_{1t+1}^s + (1 + \rho_{t+1}) \widehat{L}_t^s \quad (40)$$

To replicate the original situation the rentier should lend

$$\widehat{L}_t^s = L_t^s + [(\alpha_{1t}^f + \alpha_{1t}^\omega) - (\beta_{1t}^f + \beta_{1t}^\omega)] B_{1t} > 0 \quad (41)$$

and so increase lending by exactly the additional amount of borrowing undertaken by the workers and keep their share of equity in the risky technology constant. But if $\widehat{\alpha}_{1t}^s = \alpha_{1t}^s$, then $\widehat{\alpha}_{1t}^f + \widehat{\alpha}_{1t}^\omega = \alpha_{1t}^f + \alpha_{1t}^\omega$, and condition (38) is equivalent to (33). This means that the budget sets of workers and rentiers are unchanged, so that the same choices of risky and safe assets are obtained as in the original equilibrium. Moreover, the market equilibrium conditions remain the same. In particular, the increase in the demand for equity by the workers net of the demand from the pension plan exactly matches the increase in the supply net of this demand. Furthermore, the decline in the demand for riskless corporate bonds exactly matches the reduction in the supply, with borrowing and lending between workers and rentiers netting out. Hence our original assumption that $r_{t+1} = \rho_{t+1} = \widehat{r}_{t+1}$ is fixed is justified and so p_t remains unchanged.¹⁵

Now, as in subsection 2.3, allow for some possible corners arising from the impact of borrowing and short-sales constraints. The points we wish to make here are best seen if we again consider the pension plan to be large relative to worker's other saving. First suppose that the pension plan is heavily weighted towards riskless assets, so that workers' holding of the riskless income stream is less than the desired value from their optimisation problem. Then workers should reduce their holding of corporate bonds issued by the risky technology and their holding of the safe technology, k_{0t+1}^ω , and buy equity in the risky technology. Rentiers must buy $\beta_{1t}^\omega B_{1t}$ from workers and increase their direct holding of the riskless asset, k_{0t+1}^s . There will be downward pressure on p_t . The lower value of p_t will lead to a decrease in riskless production and a reduced equity premium. But suppose that k_{0t+1}^ω and $\beta_{1t}^\omega B_{1t}$ are both zero, then workers must borrow on personal account, using pension plan assets as collateral, to purchase risky assets, now given by levered equity. If this transaction cannot be undertaken, then the investment policy of the pension plan is constraining the behaviour of the economy, so that the workers are bearing too little risk and the equity premium will be higher than otherwise. In other words, adding corporate borrowing, in and of itself, is not enough to overcome the constraining effect that limited borrowing to finance security markets transactions has on the equity premium.¹⁶

Alternatively, suppose that the pension plan holding of the risky income stream is more than the desired value, then this should lead to a decline in $\alpha_{1t}^\omega S_{1t}$ and k_{1t+1}^ω until either the optimum portfolio is achieved or $k_{1t+1}^\omega = 0$. If this latter constraint is binding, then workers will want to short-sell risky equity, S_{1t} , and use the proceeds to invest in riskless assets. Then rentiers must borrow from workers and sell risky assets. In this case p_t will

¹⁵This result is nothing more than a version of the Modigliani-Miller Proposition for our model, much in the spirit of Stiglitz (1969). We note here that the result generalises to risky corporate debt provided individuals can undertake the necessary transactions on personal account. Hellwig (1981) provides a complete discussion of the necessary arrangements in a general environment. However, in the current context risky corporate debt raises no further basic consideration beyond those discussed below.

¹⁶This result means that institutional asset holding through the pension plan is constraining the economy's risk bearing. However, the evidence is that individuals who are not covered by pension plans have only limited participation in the equity market. This may reflect low levels of wealth or risk tolerance and not constraints on capital market transactions.

increase as will production of the safe asset. This will also result in a higher value of the equity premium. However, again if workers cannot short-sell equity, the investment policy of the pension plan will constrain the economy to bear too much risk and in this case the equity premium will be lower than otherwise.

4 Defined Benefit versus Defined Contribution Pension Plans

From a risk-sharing perspective, the usual argument given in favour of DB pension plans is that they are a mechanism for ensuring that workers bear little income risk when old and that this risk is better borne by the younger generation or rentiers. The former point assumes that workers bear relatively little risk when young, which is the case in the current model but may not be so in reality; or that they are more risk averse when they are older.

A DB pension plan specifies a benefit rate to be paid to old workers that is typically related to final earnings, which in the current abstract context is some fraction, γ , of wages, w_{yt}^ω . If wages are high because the productivity shock θ_t is high, this means that unless investment returns are expected to be correspondingly higher, contribution levels, f_{yt}^s and f_{yt}^ω must be higher. If f_{yt}^ω is fixed, then f_{yt}^s must be higher. But with fixed γ , workers' old age consumption is guaranteed relative to their wages when young, which means rentiers must bear more risk when old and pay larger pension contributions when young. The risk to rentiers when old rises if the pension plan invests in risky assets and returns are insufficient to meet the DB pension promise, in which case the rentier, as plan sponsor, must make up the deficit. Of course, if there is a surplus this will typically be captured by the sponsor.¹⁷ Then, to smooth consumption, rentiers will reduce wages for workers at date t and pass the risk onto the generation of workers at date $t + 1$, or reduce investment in the risky technology. In other words the economy's risk bearing will be reduced.¹⁸ This will manifest itself in terms of a rise in p_t and a rise in the equity premium.

If the DB plan pays a fixed fraction of wages, then it must hold sufficient riskless assets, or rentiers themselves hold sufficient riskless assets to make up any possible shortfall. We will assume the former. The pension plan asset is effectively held by workers, and it is they who may, particularly if the plan is relatively large, find themselves constrained to hold too much of the riskless assets. Hence, if the pension plan is DB, we can think of it as a riskless asset in the worker's portfolio with value $p_t k_{0t+1}^f + \alpha_{1t}^f B_{1t}$. Then applying Proposition 1, if $a_{yt}^\omega = 0$, workers will be under-diversified. On the other hand if $a_{yt}^\omega > 0$, and workers can hold risky assets, the higher the value of riskless assets held by the plan, $p_t k_{0t+1}^f + \alpha_{1t}^f B_{1t}$, the lower the value of $p_t k_{0t+1}^\omega + \alpha_{1t}^\omega B_{1t}$ chosen by workers. For sufficiently high pension plan investment in riskless assets, workers will choose $p_t k_{0t+1}^\omega + \alpha_{1t}^\omega B_{1t} = 0$, so that a_{yt}^ω is invested entirely in risky assets. Unless workers can short-sell the safe technology, they are forced

¹⁷In reality, plan surpluses may be split between the plan sponsor and plan member.

¹⁸This runs counter to the implications of the typical planner's problem which will, given welfare weights, smooth consumption as much as possible both within and between generations.

to a corner and the economy will under-invest in the risky technology. In other words, if the worker is unable to undo his lack of exposure to risk through the DB plan he would be better off with a DC scheme that invests a positive amount in the risky technology.¹⁹ Then as in the discussion of Proposition 1, part of the welfare gain to diversification will flow to rentiers who benefit from the economy's improved risk sharing as reflected in asset prices, in particular a lower value of p_t .

Although a DB plan can replicate a DC plan if contribution and benefit rates can be varied over time, that is not the nature of a DB plan. The model that we have understates the advantages of the DC over the DB plan. The argument is that the DC plan offers worker's greater risk sharing. In a model with many risky technologies that are not perfectly correlated the DC plan can diversify holdings across the full range of available technologies and obtain the well-understood gains to diversification. The DB plan can also invest in a diversified portfolio but as the gains and losses relative to the fixed pension plan promise accrue to the plan sponsor the additional risk is borne by rentiers and should be treated as part of their equity investment. However, there is a further consideration. The DB plan is linked to sponsor contributions and if the plan is in deficit it is subject to the risk that the sponsor, who is the residual claimant, may default. The DC scheme, however, leaves the residual risk with the plan members, who are the workers, so in this case the workers are subject to plan portfolio risk but not to the risk of default of the plan sponsor.

The arguments in favour of DB over DC are outside of the current framework. These arguments are to do with long-term contracting. One is that the DB plan is deferred compensation, the value of which is tied to the economic success of the plan sponsor, so workers are committed to supplying high effort levels over the long-run and do not gain from quitting. If this argument is true, then firms have an incentive to let the pension plan run a deficit and lever the firm's capital structure and so subject the workers to the risk of default on their pensions should the firm perform poorly. However, an efficient trade-off between risk sharing and long-run effort incentives can be achieved with a combination of a DC pension and a long-term labour contract with a temporally increasing efficiency wage schedule.

5 Pension Plan Asset Allocation

We have treated the pension plan asset allocation decision as exogenous. In a complete financial market, for any given contribution policy the asset allocation policy of the plan will be irrelevant. The earlier discussion in this paper shows that with incomplete financial markets an exogenously determined pension plan asset allocation policy can have real implications for asset holding in the economy and consequently the prices of assets and

¹⁹Exley, Mehta and Smith (1997) discuss the relationship of the asset allocation of DB plans relative to shareholders' personal portfolios. They argue that offsetting transactions needed to obtain neutrality are easy to implement. The present paper does not take this view.

the choice of technology. With incomplete markets there are two polar cases worth noting. First consider a DC plan. If, holding contribution levels fixed, a fully rational representative worker chooses the asset allocation policy of the pension plan, he will consolidate the plan with his private portfolio decisions and we will obtain neutrality. Now consider a DB pension plan. In the absence of sponsor default, all the risk of the plan investment policy is borne by the rentiers. Then it is they who will consolidate the plan's investment policy with their other portfolio decisions and again we have neutrality.

With a DC plan, if the rentiers choose the plan investment policy and the plan is relatively large, rentiers may benefit from forcing the workers to a corner in which they hold too much of the risky asset. This is because in the current model, at the margin they benefit from the decline in the value of the safe asset, whilst at the same time the value of their holding of the risky technology remains constant. This suggests that even at this level, there is scope for disagreement on the asset allocation policy of the pension plan. This policy will be decided by bargaining between the workers and rentiers over the general structure of the benefit package in the context of incomplete markets. The problem will be further complicated if we drop the representative agent assumption and if we extend the analysis to include some of the complex principal-agent problems that arise when discussing delegated portfolio management.

6 Concave Technology and Long Run Effects

We retain the assumption that the riskless technology is linear. However, we can now consider convex costs of transforming consumption goods into risky capital assets. Production is assumed to be homogeneous of degree one with labour augmenting technology shocks, $g(k_{1t}, \theta_{1t})$, with $g_1 > 0$, $g_{11} < 0$ and $g_2 > 0$. The technology variable θ_{1t} is labour augmenting and has a stochastic i.i.d. structure.

We retain the assumption that capital is non-durable. The cost of producing k_{1t+1} units of capital at date $t + 1$ is $k_{1t+1} + \phi(k_{1t+1})$.²⁰ The adjustment costs are given by $\phi(k_{1t+1})$ with $\phi(0) = 0$, $\phi' > 0$ and $\phi'' > 0$.

Total date t consumption per capita produced by the risky technology is given by

$$c(k_{1t}, k_{1t+1}) = g(k_{1t}, \theta_{1t}) - k_{1t+1} - \phi(k_{1t+1}) \quad (42)$$

Adjustment costs mean that the marginal rate of transformation between capital and consumption set equal to the consumption price of capital goods is given by:

$$q_t = \left[1 + \frac{\partial \phi}{\partial k_{1t+1}} \right] \quad (43)$$

²⁰If capital is durable then the adjustment cost function can be written as $\phi(k_{1t+1}, k_1)$, which is convex. If there are additional costs of transforming consumption goods into capital linked to the financial structure of the technology, these costs can also be allowed for in this function.

Firms are assumed to maximise profits each period after the realisation of the periods stochastic shock. The marginal products of labour and capital are

$$R_t^* = \left[\frac{\partial g}{\partial k_{1t}} \right] / q_{t-1} \quad (44)$$

$$w_{1t} = g(k_{1t}, \theta_{1t}) - \phi(k_{1t+1}) - \frac{\partial g}{\partial k_{1t}} k_{1t} \quad (45)$$

both of which are functions of the technology shock at date t .

As we assume that only the risky technology is concave, wages are determined by the intensity with which the risky technology is used. In particular, if other things equal the risky technology is used more intensely, wages will rise and the marginal product of risky capital will decline.

Firms maximise profits and

$$p_t = \frac{1}{R_{t+1}} \text{ if } k_{0t+1} > 0, p_t \leq \frac{1}{R_{t+1}} \text{ if } k_{0t+1} = 0 \quad (46)$$

$$q_t = \left[1 + \frac{\partial \phi}{\partial k_{1t+1}} \right], \text{ if } k_{1t+1} > 0 \text{ and } q_t \leq \left[1 + \frac{\partial \phi}{\partial k_{1t+1}} \right] \text{ if } k_{1t+1} = 0 \quad (47)$$

The rentier's optimisation problem is the same as before, with first order conditions:

$$U'(c_{yt}^s) = E_t[U'(c_{ot}^s)R_{t+1}] = E_t[U'(c_{ot}^s)R_{t+1}^*] \quad (48)$$

The worker's optimisation problem is also as before with first-order conditions:

$$V_1(c_{yt}^w) = E_t[V'(c_{ot}^w)R_{t+1}] = E_t[V'(c_{ot}^w)R_{t+1}^*] \quad (49)$$

Suppose that workers have logarithmic utility so they have a constant savings rate; labour supply is exogenous. Then saving and consumption are perfectly correlated with disposable income. As we have full depreciation of capital and i.i.d. technology shocks, in the absence of rentiers, the representative worker's old consumption and young consumption are equally risky.²¹ In a model with rentiers, the rentiers will bear a significant share of the economy's capital risk but the workers bear all the economy's labour income risk when young, so that their consumption when young will be riskier than their old consumption. Policies that attempt to reduce retirement risk inevitably shift the risk, at least in part, onto future workers. A similar issue arises in the discussion of DB versus DC pension plans to which we turn below.

This model with concave technology arising from adjustment costs can be used to examine the simple dynamics of a shift from DB to DC pension plans. A DB pension plan is effectively held by workers, and it is they who may, particularly if the plan is relatively

²¹If capital does not depreciate fully old consumption will be less risky than young consumption.

large find themselves constrained to hold too much of the riskless assets. Then if they are constrained not to be able to borrow against plan assets to buy risky assets, q_t and k_{1t+1} will be lower and p_t and k_{0t+1} higher than otherwise. The equity premium will be higher than otherwise, with a correspondingly lower value of the riskless rate. The shift to DC will relax this constraint if the plan then holds risky assets. This will lead to a corresponding decrease in the equity premium as the expected return on risky assets decreases (with the increase in k_{1t+1}) and the riskless rate rises (with the decrease in k_{0t+1}). Of course there is the potential for the DC plan to take too much risk and if workers are constrained by short-sales constraints q_t and k_{1t+1} will be constrained to be too high and p_t and k_{0t+1} too low in equilibrium. The equity premium will also be lower than otherwise.

In our model with i.i.d shocks and non-durable capital, if a DC plan replaces a DB plan, even in part, there is a one-period adjustment to a new steady state. However, if capital is durable, so there is less than one hundred per-cent depreciation, the adjustment process will be spread out over time.²²

7 Macroeconomic Factors

A social planner will not just be concerned with how the economy can share risk between workers and rentiers at a point in time but also with how the economy can best allocate risk intertemporally between generations. The planner will maximise the weighted sum of the welfare of the representative workers and rentiers living at all dates.²³ We will abstract from distributional issues at a point in time and also assume that the social insurance scheme weights different generations equally. Hence, the object of social security is to equalise the welfare of generations. In presenting the argument we abstract from some of the complex issues that arise if such schemes affect the incentives of agents.

The pension plan arrangement that we have investigated so far has payments from rentiers and workers into a fund that then invests in the economy's technologies to achieve either a DB; or if the contribution rate is fixed, a DC return that depends upon the plan investment policy. Under either policy, each generation of workers and rentiers will have to bear consumption risk arising from shocks to endowments and technology. In an overlapping generations structure there is the possibility of establishing institutional arrangements for risk sharing between the young and old at a point in time. At date t , young rentiers and workers can make transfers to old rentiers and workers. The young can gain from this if they are in turn beneficiaries when they are old. However, in a stochastic environment with non-diversifiable shocks at a point in time, smoothing can lead to the impoverishment of the young generation, particularly workers that are suffering negative shocks to their

²²Abel (2002) and Storesletten (2001) both examine dynamical models with durable capital and persistence in shocks. In a similar way the current model could be extended to give a complete treatment of the dynamics of q and k_1 .

²³Bohn (2003) gives a detailed and informative analysis of market outcomes and general planner allocations.

labour income. With commitments to the old generation, large negative shocks can only be accommodated at a point in time if either the young rentiers save more and workers work more; or if the economy has accumulated a stock of assets that can be drawn upon in adverse circumstances without damaging the economy's productive capabilities. The latter draws our attention to the possibility of smoothing risk across the complete sequence of generations, with shocks at any given date being absorbed through a buffer-stock of assets. In this context social insurance is a DB scheme, where payments to the retired generation are linked to the economy's long-run average income; similarly the consumption of the younger generation is linked to this average.

We follow Allen and Gale (1997) and consider an economy which, at each date has resources available as the result of the return on production from the economy's risky technology, plus unproductive storage. The technology's return is assumed to be i.i.d., with positive mean and finite variance. However, the result generalises to include a stochastic endowment, the important assumption is that the economy's income from all sources is i.i.d. The economy's resources are used to finance consumption at date t , $c_t^y + c_{t-1}^o$, plus next period's risky production and additions to unproductive storage.

In a stationary economy the set of choices is the same for all generations. In a market equilibrium with endowment and technology risk, even if the risky technology is not used, second-period consumption is risky. We want to ensure that each generation has the same resources (with probability one). To ensure that each generation achieves constant utility with probability one requires a sufficiently large stock of reserve assets, which if nothing is stored at the initial date must be accumulated. A long-lived intermediary with a sufficient stock of safe assets could provide a steady stream of consumption and smooth the effect of risk on any given generation.

Allen and Gale show that the long-run average utility of a sequence of generations can be maximised with a policy that, through the accumulation of a buffer-stock of riskless assets, yields constant utility over time. Each generation deposits their endowment and capital with a financial institution in return for the feasible dividend stream, which if there is a sufficient stock of reserve assets is a constant stream.²⁴ Let the long-run mean of the economy's income, z_t , be denoted by \bar{z}_t . At each date the new generation observes the state of the system. If the endowment plus the return on the risky technology, in our notation $e_t^y + R_t^* q_{t-1} k_{1t} > \bar{z}$, then under the Allen and Gale policy the generation is paid \bar{z} . But if $e_t^y + R_t^* q_{t-1} k_{1t} < \bar{z}$ and if the stock of reserve assets is sufficient, \bar{z} is also paid. An implication of the above line of reasoning is that, with i.i.d. endowment and technology returns, except for the initial accumulation phase the economy should be one hundred per-cent invested in the risky technology.

If the above scheme can be implemented, so that generations benefit from intergenera-

²⁴In an open economy if a generation puts more into the scheme than it consumes it runs a trade surplus, which in turn is its net accumulation of world safe assets. Then the premium paid by each generation would then measure the maximum trade surplus that the generation is willing to run ex ante

tional insurance, each generation of workers and rentiers should be willing, ex ante, to pay a maximum insurance premium equal to the consumption goods equivalent of the total gain in expected utility from the scheme. However, if R_t^* and or e_t^y are low and the stock of reserve assets, s_t , is also low such that $e_t^y + R_t^* q_{t-1} k_{1t} + s_t < \bar{z}$, then this is the maximum that can be paid to the two generations at this date. Having observed the state of the economy at date t , the young generation at this date sacrifices the upside benefits of risk bearing for no downside protection. In other words the insurance scheme is only attractive if it has sufficient reserves. Any generation that inherits low reserves would be better-off bearing the risk rather than paying insurance premia into a nearly empty pot. If this happens, then this generation of workers and rentiers will be collectively better-off deviating from the DB scheme operated by the long-lived intermediary to the private market outcome analysed previously with a mix of DC pensions and private savings.²⁵

8 Conclusion

The paper has presented a simple yet illustrative analysis of the impact of pension plan funding on workers' other saving and portfolio behaviour. The analysis shows that the impact of pension plan funding and asset allocation on the economy's technology choices depends upon the constraints facing workers in the capital market. If occupational pension plans make up a significant proportion of workers retirement income then DB plans are likely to distort the economy's portfolio composition and technology choices towards the safe technology. This will be undone to the extent that workers take offsetting transactions with the other elements of their portfolio, which theory predicts they will. However, this would mean that we observe members of DB schemes holding significant amounts of equity on personal account that may involve borrowing against pension plan assets to purchase them. However, there is little evidence of either. DC plans on the other hand will generally bear more risk and consequently the economy is likely to push the economy towards riskier investments. The failure of equivalence propositions between DB and DC schemes derives from the existence of borrowing and short-sales constraints. The different natures of these two types of scheme means that they force the workers against the constraints asymmetrically and this explains the asymmetric impact on risk taking in the economy and thereby on the equity premium.

We noted that the outcome of the market economy is a risk sharing arrangement between the workers and rentiers. This left open the question of how best to share risk between generations. These issues were briefly discussed at the end of the paper. However, the argument that DC pensions and individual savings are an effective market solution to risk sharing

²⁵From a technical point of view the intertemporal risk smoothing allocation violates the economy's disintermediation constraint at this date. That is the expected utility to a given generation from dealing privately on capital markets using the available set of risk sharing opportunities, including the pension plan arrangements exceeds the expected utility from depositing their endowment and capital with a financial institution in return for the feasible dividend stream.

conflicts with the institutional arrangements needed to manage effective intergenerational risk smoothing.

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