Optimal Monetary Policy Rules

in a Rational Expectations Model

of the Phillips Curve

By

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Non-technical Summary

This paper departs from the recent theoretical literature on optimal monetary policy in two significant ways. First, it is based on a Phillips curve that embodies persistence in inflation, whereas those recent contributions dealing with persistence have been restricted to output. Second, unlike most of the work on time inconsistency and inflation bias, this paper assumes that the central bank targets the natural rate of output and consequently there is no inflation bias.

In our model inflation is generated by a rational expectations-augmented Phillips Curve, rather than a Lucas supply function; aggregate demand is determined by the real interest rate; both supply and demand are subject to stochastic shocks; the monetary instrument controlled by the authorities is the nominal interest rate which, in conjunction with the expected inflation rate determines the real interest rate; expectations in the model are rational, i.e., given by the expected value determined by the model. The authorities in our model, concerned about both deviations of output and inflation from their respective targets, do not control inflation directly, but can only influence it indirectly through the effect of their instrument on aggregate demand. Our model provides both a richer and more realistic characterization of the policy problem.

Using a dynamic programming approach, we derive optimal monetary policy rules both in the case where the cental bank follows a commitment strategy and where it pursues a discretionary procedure. These rules are state-contingent and shock-dependent in both cases.

Our results shed new light on the debate over commitment versus discretion. Numerical solutions show that in the state-contingent part there always exists a tradeoff between these two optimal rules in that the commitment rule involves smaller expected deviations of inflation from its target but larger

expected deviations of output from its target; in the shock-dependent part there can be situations in which the discretionary rule is more effective in reducing the impact of the random shock on inflation and less effective in reducing the random shock on output. Only in the latter case it is possible that one rule is superior; otherwise it is generally the case that a tradeoff exists between these two rules.

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Abstract

In this paper we construct a rational expectations model based on a Phillips curve that embodies persistence in inflation. As we assume that the central bank targets the natural rate of output, there is no inflation bias. We derive optimal monetary policy rules that are state-contingent and shock-dependent both in the case where the cental bank follows a commitment strategy and where it pursues a discretionary procedure. Numerical solutions show that in the state-contingent part there always exists a tradeoff between these two optimal rules in that the commitment rule involves smaller expected deviations of inflation from its target but larger expected deviations of output from its target; in the shock-dependent part there can be situations in which the discretionary rule is more effective in reducing the impact of the random shock on inflation and less effective in reducing the random shock on output. Only in the latter case it is possible that one rule is superior; otherwise it is generally the case that a tradeoff exists between these two rules.

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I. Introduction

The generally poor inflation performance of industrial countries in the postwar period has led a number of central banks to adopt explicit inflation targets in an attempt to improve this performance.[1] How best to achieve control over inflation has been the subject of ongoing debate among economists, with considerable attention devoted to the choices of policy instruments and rules. On the choice of instrument there are pros and cons of using the interest rate or the money stock as an instrument to achieve such control; on the choice of rules there are pros and cons of using commitment rules or discretionary procedures to guide such control.[2] While this debate will no doubt continue indefinitely, the instability in money demand equations associated with financial market innovations appears in fact to have led central banks to give more explicit emphasis to adjusting the interest rates which they influence directly as the primary channel for affecting inflation. The control of inflation therefore proceeds by affecting aggregate demand through the adjustment of interest rates.[3] One key element in this process is the link between aggregate demand and inflation, which is embodied in the Phillips curve.

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[1] The literature on this topic has grown enormously in recent years. For discussions of recent experience and relevant issues, see Ammer and Freeman (1995), Haldane (1995) and Leiderman and Svensson (1995).

[2] Friedman (1960), Kydland and Prescott (1977) and Barro and Gordon (1983) are among the key contributions to the enormous literature on the debate over rule versus discretion in monetary economics.

[3] The standard interest rate transmission channel though which the monetary authorities affect aggregate demand has recently been questioned by economists who emphasize the credit channel. See, for example, the papers and discussion in Thornton and Wheelock (1995).

Some recent work on the inflation process has emphasized the need to adjust interest rates quickly in order to avoid the buildup of inflationary pressure which is more difficult to reduce once it emerges. For example, Laxton, Meredith and Rose (1995) and Clark, Laxton and Rose (1996) have stressed the importance of nonlinearity in the relationship between inflation and excess demand, namely, where an increase in inflation caused by aggregate demand being above potential output is greater than the reduction in inflation when demand is below potential by the same amount. Such nonlinearity or asymmetry in the Phillips curve provides an incentive for policymakers to raise interest rates with alacrity to avoid periods of excess demand, as these require longer and/or more severe recessions to undo the inflation generated when output is above potential. The policy analysis in those papers is, however, based on rather arbitrary myopic and forward-looking monetary policy reaction functions. This paper, using a simplified version of the same model of the inflation process but with rational expectations, extends that work in a theoretical direction by deriving explicitly optimal feedback rules both in the case where the cental bank follows a commitment strategy and where it pursues a discretionary procedure, and provides an unbiased comparison of these two approaches to conduct monetary policy.

Our model is built on three equations: an objective function of the monetary authority concerned with both inflation and output stability; a Phillips curve that determines the inflation rate; and an equation for real aggregate demand which is determined by the real interest rate. The monetary instrument controlled by the authorities is the nominal interest rate, which in conjunction with the expected inflation rate determines the real interest rate. All expectations in the model are rational, i.e., given by the expected value determined by the model.

A feature of this model which distinguishes it from the related literature on time inconsistency (see Barro and Gordon (1983), Goodhart and Huang (1995), and Svensson (1995)) is that the authorities do not control inflation directly, but can only influence it indirectly through the effect of the interest rate on real aggregate demand. Thus there is one instrument to achieve two competing objectives in the short run, as both inflation and aggregate demand are subject to stochastic shocks.[4] In long-run equilibrium without shocks, the inflation and output targets can be achieved simultaneously. As discussed below, the inflation target of the monetary authority provides the nominal anchor that pins down the inflation rate.

More importantly, in our model we use a Phillips curve to characterize inflation rather than a Lucas supply function. This feature reflects an important difference from many other models analyzing the issue of optimal monetary policy. It is generally assumed that these two specifications are mirror images of each other. However, we would argue that this is in general not the case. The reason is that the Phillips curve is based on the assumption that there is persistence in the behavior of wages and prices. In the case of wages, this can arise - as is well known - from overlapping wage contracts. The persistence in prices or inflation is in contrast to the standard Lucas supply function, where prices adjust each period to equate supply and demand. Such market clearing does not underlie the Phillips curve, where the change in inflation is driven by excess demand or supply. Moreover, in the standard Lucas supply function there is a short-run tradeoff between output and inflation only in the current period when unanticipated shocks cause output to depart from its natural level. In such a model the equilibrating forces are so powerful that there is no significant policy problem.

We would argue that price stickiness and nominal wage rigidity are sufficiently pervasive that they need to be dealt with explicitly in analyzing the optimal monetary policy response to shocks to the economy. Thus a key element of our Phillips curve is a parameter that measures the degree of persistence in the inflation rate. Our model therefore encompasses the assumption of no persistence as an extreme case, but is more general in that a short-run tradeoff between output

[4] As described below, when the central bank can commit to a state-contingent rule for the interest rate, it has in effect two instruments: the *ex ante* expected interest rate and the *ex post* actual interest rate. However, these are not two fully independent instruments, which would be the case, for example, if fiscal policy were considered along with monetary policy.

and inflation lasts for more than one period. Nonetheless, it incorporates the natural rate hypothesis, so that there is no long-run tradeoff. A number of the findings in the paper depend importantly on the relationship between the degree of inflation persistence, which is parameterized in the model, and the slope of the Phillips curve. In particular, the size of the optimal adjustment in the nominal interest rate is quite sensitive to the relative magnitudes of these two parameters.

The implications of sluggish price adjustments, i.e., the absence of period-by-period market clearing, in a rational expectations model with a Lucas surprise supply function have been explored by McCallum (1978). He concludes that it remains the case that the distribution of the level of output is unaffected by the monetary policy feedback rule. We find that with price persistence, the optimal feedback rule does affect the behavior of output even though expectations are assumed to be formed rationally in our model, i.e., by the mathematical expectations of the relevant variables at the end of period t-1. The difference in results appears to reflect the fact that with inflation persistence, lagged inflation enters as a determinant of the current level of output. As shown below, this is a property of our model both without an optimum monetary policy feedback rule as well as with such a rule. Indeed, McCallum considers one alternative specification of his model which also has this property and notes (p.428) that "it is the mixing of a nominal price adjustment relation into the real aggregate supply process that is responsible for … [this non-neutrality]."[5]

It should be noted that optimal monetary policy with persistence has been analyzed in some of the contributions to the discussion of time inconsistency and optimal inflation contracts for central bankers, e.g., Lockwood and Philippopoulos (1994), Lockwood, Miller and Zhang (1994), Goodhart and Huang (1995) and Svensson (1995). However, in these models the persistence

[5] Fischer (1977) makes the general point that monetary policy will be effective in influencing output with rational expectations as long as output is determined as a function of more than one-period forecast error of the price level. Levine and Currie (1987) also make the point of policy effectiveness in the context of rather general linear rational expectations.

directly relates to output or employment rather than to prices or inflation. Such persistence is introduced by adding the lagged value of the deviation from potential output into the standard Lucas supply function, so that market clearing the monetary authority cannot achieve its inflation target every period even though it can commit. We find that optimal control necessarily reduces the impact of random shocks on inflation but that whether the effect of these shocks on output are damped depends on the sensitivity of output to the real interest rate and on the slope of the Phillips curve.

As the closed-form solution for the discretionary case is extremely complex, numerical methods have been employed to compare the interest rate decision rules and the behavior of inflation and output under the two policy regimes. The equations describing the behavior of these two variables have two components, a systematic state-contingent part and a random shock-dependent part. Using a wide range of plausible values for the parameters of the model to compare the coefficients of adjustment of inflation and output, we find that in the state-contingent part the expected deviation of inflation from its target level in the case of commitment is always smaller, but the expected deviation of output from its target level is always larger than in the case of discretion. This provides a clear demonstration of the tradeoff, in terms of expectations, between the two optimal monetary policy rules. In other words, our analysis implies that in terms of expected values, one policy rule is not unambiguously superior to the other.

We also compare the adjustment coefficients to the random shocks and find that both optimal rules reduce the impact of the inflation shock on the inflation and output. For each optimal rule, there is always a tradeoff between the impact of the inflation shock on inflation and output; that is, a larger reduction in the impact of the inflation shock on inflation involves a smaller offset of the random shock on output, and vice versa. The comparison shows that it is generally the case that the commitment rule leads to smaller departures of inflation from its target than discretion, the same result noted above regarding expectations of the two variables. However, there can be situations in which the discretionary rule generates a larger offset to the inflation shock on current-period inflation and a smaller offset of the shock on output. In this case the aggregation of these two effects that combines the adjustment coefficients in the systematic and random shock parts could lead to the result that one rule is superior to the other. In all other cases, there always exists a tradeoff between these two optimal rules.

The rest of this paper is organized as follows. The rational expectations model of the Phillips curve is presented in Section II. The optimal monetary response function under commitment is described in Section III. This is followed by the derivation of the optimal monetary rule under discretion in Section IV, which includes a comparison of the two policy rules. Section V provides some concluding comments.

II. The Model

We start with a standard utility or loss function for the monetary authority,

$$U_t = -(y_t - y_n)^2 - k(\pi_t - \pi^*)^2, \qquad (1)$$

where y_t is the level of output and π_t is the inflation rate. We assume that the monetary authority is the sole relevant government decision-making unit, so that we abstract from issues arising from different preferences over output and inflation between the government and the central bank.[7] The monetary authority has an exogenous inflation target, π^*, but it is also assumed that its policy actions take into account deviations of output from the natural rate, y_n, which is exogenous. The parameter k, which is between 0 and infinity,[8] is the weight given to inflation stabilization relative to output stabilization. Finally, as described above, we do not consider the possibility that the authorities may wish to achieve a target level of output that differs from the natural rate. The implications of this

[7] For a recent discussion of these issues, see Huang and Padilla (1995).

[8] In the case where k equals infinity, the monetary authority has only one objective, inflation stability. See Goodhart (1996) for more detailed discussion of this case.

assumption have been extensively analyzed by Goodhart and Huang (1995) and Svensson (1995), among others.

Unlike that of the monetary authorities, the behavior of private agents is not derived from the solution of a maximization process. This approach follows from our desire to explore the implications for optimal monetary feedback rules of a very specific kind of private sector behavior, namely, a Phillips curve that embodies persistence in inflation given by equation (2) below.[9]

$$\pi_t = \lambda\pi_{t-1} + (1 - \lambda)E\pi_t + \theta(y_t - y_n) + u_t. \qquad (2)$$

In this equation λ and θ are positive constant coefficients, the operator E denotes the rational expectation taken at the end of period t-1, and $u_t \in N(0, \sigma_u^2)$ is a random shock. For simplicity, inflation in the current period is a linear function of only the contemporaneous gap between the level of aggregate demand and the natural or capacity level of output. The parameter θ is a positive constant which measures the sensitivity of inflation to excess demand. As pointed out in Laxton, Meredith and Rose (1995), Clark, Laxton and Rose (1996), and Clark and Laxton (1996), there are good reasons to believe that θ is not constant but in fact depends in a nonlinear fashion on the cyclical position of the economy, namely, the positive effect on inflation generated when output is above capacity is greater than the negative effect arising when output is below capacity by the same amount. In particular, as noted by Lipsey (1960), the fact that the unemployment rate is bounded by zero implies that the rise in inflation when unemployment is below the NAIRU is greater than the fall in inflation for the same degree of slack in the labor market. In principle, such nonlinearity should be taken into account in our analysis of monetary policy. However, as we wish to obtain a closed form solution for the optimal feedback rule in the commitment case, we retain the linearity assumption in this paper. In our analysis below of the effect of changes in

[9] As the parameters of the equations describing private sector behavior are invariant to changes in the monetary policy rule, our results are still subject to the Lucas critique even though that rule is part of the information set of the private sector.

θ on this feedback rule, we discuss the results obtained in Bean (1996), who explores the implications of nonlinearity for optimal monetary policy in a simpler model than that presented here which does not embody rational expectations.

The novel feature of this specification of the Phillips curve is the explicit introduction of persistence in inflation, in the form of lagged inflation, together with inflation expectations. This type of specification is sometimes referred to as the "backward and forward-looking components" model -- see Buiter and Miller (1985). The backward-looking component here reflects inertia in inflation that can be derived, for example, from overlapping wage contracts based on Fischer (1977) and Taylor (1980), as is done very elegantly in Ireland and Wren-Lewis (1995) and by Fuhrer and Moore (1995).[10] The forward-looking component is represented by the rational expectation of current rate of inflation, i.e., all the determinants of inflation as embodied in the model and known to private agents up to the beginning of the period. Consistency is achieved by imposing the constraint that the sum of the coefficients on the two components sum to unity, so that in long-run equilibrium, $\pi_t = \pi_{t-1} = E\pi_t$. This constraint implies that the index of persistence, λ, must lie between zero (no persistence) and unity (complete persistence). Obviously, the standard Lucas surprise supply function is where λ is zero.

Note that the Phillips curve described by (2) is consistent with rational expectations. This can be seen clearly by the following two-step operation: first, taking the rational expectation of (2), which gives $E\pi_t = \pi_{t-1} + (\theta/\lambda)(Ey_t - y_n)$, and second, substituting it into (2), which results in $\pi_t = \pi_{t-1} + (\theta/\lambda)(Ey_t - y_n) + \theta(y_t - Ey_t) + u_t$. It follows that $\pi_t - E\pi_t = \theta(y_t - Ey_t) + u_t$, so that the difference between

[10] There is considerable empirical evidence of persistence in inflation. See, for example, Chadha, Masson and Meredith (1992), Clark, Laxton and Rose (1995), and Fuhrer and Moore (1995). There is, of course, a great deal of autocorrelation in the time series of inflation itself which may be a reflection of past inflation being a proxy for expected future inflation. The papers cited above attempt to distinguish between expected future inflation and the pure persistence or inertia in inflation.

the realized actual value and the rational expectation of inflation reflects only random disturbances.

It should be clear that Phillips curve in equation (2) ties down only the change in the rate of inflation and not the level of inflation itself. As a consequence, if there were no monetary control, the level of inflation would follow a random walk and would vary without bounds over time. The equilibrium level of the inflation rate obviously must be tied down by a nominal anchor outside the dynamics of the inflation process. As noted above, this is accomplished in the usual fashion through the loss function for the monetary authorities that includes π^*, the target rate of inflation. As shown below in Sections III and IV, in long-run equilibrium the control exercised by the central bank ensures that the actual inflation rate is equal to the target level.

Consequently the inflation rate is tied down but the price level is not determined in the model. This could be done by adding a money demand equation. However, this would not change the properties of the model as the price level plays no role. In effect, the monetary authority adjusts the supply of money for a given money demand to achieve the desired nominal interest rate. In this sense, the money market is always in equilibrium and therefore is redundant. Of course, in a more complete model in which economic behavior is affected by nominal magnitudes deflated by the price level, a money demand equation would be essential. As the focus of our analysis here is solely on the rate of inflation, this additional complexity is not needed.

Aggregate demand is given by equation (3) as a function of the real interest rate.

$$y_t - y_n = -\phi(i_t - E\pi_t - \alpha) + v_t. \qquad (3)$$

In the above equation, ϕ is a positive constant coefficient, $\alpha > 0$ is the long-run equilibrium real interest rate, $v_t \in N(0, \sigma_v^2)$ is a random shock which is assumed to be independent of u_t, and i_t denotes the nominal interest rate, the instrument under the monetary authority's control.[11] As we wish to explore the effect of persistence that arises directly in the inflation process *per se*, we ignore lags in the effect of the real interest rate on demand. The implications of persistence in aggregate demand have been examined in a theoretical context by Goodhart and Huang (1995) and empirically by Clark, Laxton and Rose (1996) and Fuhrer and Moore (1995). The real interest rate is equal to the nominal interest rate minus the expected inflation rate in the current period.[12] The monetary authority is assumed to vary the nominal interest rate directly to achieve the level of excess demand needed to affect the inflation rate. Thus inflation control is achieved only indirectly through variations in aggregate demand via changes in the nominal interest rate relative to inflation expectations.

Taking the rational expectation of equations (2) and (3) and after some algebraic manipulations, we have the following reduced-form equations for inflation and output, respectively:

$$\pi_t = \pi_{t-1} - \phi\theta[(i_t - Ei_t) + (Ei_t - \pi_{t-1} - \alpha)/(\lambda - \phi\theta)] + u_t + \theta v_t. \quad (4)$$

$$y_t = y_n - \phi[(i_t - Ei_t) + \lambda(Ei_t - \pi_{t-1} - \alpha)/(\lambda - \phi\theta)] + v_t. \quad (5)$$

These two equations are useful because they show the relationship between π_t and y_t and both the state variable, π_{t-1}, and the control variable(s). In the case of discretion, the sole control variable is i_t. With commitment, by contrast, the monetary authority minimizes the loss function (1) not only with respect to i_t but also Ei_t as well. In this case expectations regarding the policy stance of the

[11] Note that it is necessary to distinguish between π_t and i_t. In our model π_t is the inflation outcome. Although it can be a target, as treated in Svensson (1995), π_t cannot also be a monetary policy instrument as i_t is the only instrument in this economy.

[12] Taking the expectation at the end of period t-1 is quite standard. See McCallum (1978). It seems plausible to assume that aggregate demand is a function of the currently observed interest rate, i_t , but only expected, not actual inflation, for at least three reasons. First, the interest rate is a market-clearing price that is observed on a daily basis, whereas the periodicity (monthly, quarterly, and annual) and reporting delays are much longer for inflation. Second, as discussed below, there may be an informational asymmetry between the central bank and the private sector. Third, because there is persistence in inflation, making an expenditure decision on the basis of expected inflation would involve a relatively low forecast error.

authorities are fully incorporated in deriving the feedback rule that determines that stance. By examining (4) and (5) it can be seen that an increase in i_t unambiguously reduces both output and inflation, whereas the effect on the expected interest rate depends on the value of $(\lambda - \phi\theta)$. However, as shown below, after optimal control has been taken into account, the behavior of π_t and y_t does not depend on this particular relationship among the parameters.

Finally, these two equations can be expressed more compactly by using $E\pi_t$ and Ey_t. This can be done by taking expectations of (4) and (5), which upon substitution yield:

$$\pi_t = E\pi_t - \phi\theta(i_t - Ei_t) + u_t + \theta v_t. \tag{6}$$

$$y_t = Ey_t - \phi(i_t - Ei_t) + v_t. \tag{7}$$

Equation (6) shows that in this model actual inflation is determined by expected inflation plus random disturbance terms, and similarly for output.

III. The Optimal Commitment Policy Rule

III.1 General Considerations

In the literature on optimal monetary policy with time inconsistency, a standard result is that a commitment strategy by the central bank is one way to overcome the inflation bias resulting from attempts to achieve a level of output higher than the natural rate. As shown by Svensson (1995), in this case the commitment solution leads to a better outcome than discretion because the latter results in too high an inflation rate. However, as a commitment strategy is generally viewed as infeasible, alterative ways of improving on the discretionary outcome have been suggested in the literature, e.g., delegation to a conservative central banker (Rogoff (1985)) and linear inflation contracts (Walsh (1995)).

In our view the difficulties involved in implementing a commitment strategy on the part of the monetary authorities have been exaggerated. We find rather more persuasive the argument of McCallum (1995) that while there are pressures from dynamic inconsistency, it is not necessary for the central bank to succumb to them. There is nothing to prevent a central bank from behaving in a committed fashion and abstain from attempting to exploit expectations that are predetermined period by period. Hence the absence of a "commitment technology" does not necessarily imply that the central bank will behave in an unconstrained discretionary manner.

Therefore in this paper we have chosen to examine the effects of alternative assumptions about the conduct of monetary policy where time inconsistency does not arise as a result of aiming at an output level greater than the natural rate. Consequently we do not immediately bias the results by formulating a contrived problem that can be easily alleviated by a commitment strategy. Instead, we only consider optimal policy rules where there is no inflation bias and compare commitment with discretionary solutions. This approach provides a fair test of the value added of the self-imposed constraint of commitment in comparison with a policy rule based on discretion.

When the central bank is committed to a state contingent rule in conducting monetary policy, this implies -- as noted by Svensson (1995) -- that the monetary authority internalizes the impact of its decision rule on the expectations of the private sector. With this approach to monetary policy there are in effect two decision variables or instruments: the actual *ex post* and the expected *ex ante* interest rate each period. The optimal response function is then derived by taking account of how the economic system responds to both control variables. By contrast, under discretion the central bank does not make a commitment to follow a state-contingent policy rule. It is no longer bound by this constraint, but as a result it loses one policy instrument, which is the expectation of the interest rate. In this case the optimal rule is derived by minimizing the loss function of the monetary authority only with respect to the actual *ex post* interest rate.

With a commitment solution, the monetary authority's maximization problem involves the additional constraint that the *ex ante* expected nominal interest rate, Ei_t, must be equal to its committed interest rate, $E_{t-1}[i_t]$.[13, 14] Hence, its problem is:

$$\max_{i_t,\, Ei_t} E \quad \beta^t U_t$$
subject to (4), (5) and $Ei_t = E_{t-1}[i_t]$

This is a dynamic programming problem with one state-variable, π_{t-1}, and two control variables, i_t and Ei_t, and where β is the discount factor. The solution can be obtained by solving the following problem:

$$V(\pi_{t-1}) = \max_{i_t,\, Ei_t} E\{-(y_t - y_n)^2 - k(\pi_t - \pi^*)^2 + \beta V(\pi_t)\}, \tag{8}$$
subject to (4), (5) and $Ei_t = E_{t-1}[i_t]$.

For the linear-quadratic problem such as ours, $V(\pi_t)$ must also be quadratic. Without loss of generality, we can write $V(\pi_t) = c_0 + 2c_1\pi_t + c_2\pi_t^2$, so that $V'(\pi_t) = 2(c_1 + c_2\pi_t)$. Using this condition together with (4), (5) and $Ei_t = E_{t-1}[i_t]$, we obtain two first-order conditions from (8) with respect to i_t and Ei_t, respectively:

$$2\phi(y_t-y_n) + 2k\phi\theta(\pi_t-\pi^*) - 2\beta\phi\theta(c_1 + c_2\pi_t) + \Lambda_{t-1} = 0, \tag{9}$$

$$-E\{2\phi[1-\lambda/(\lambda-\phi\theta)](y_t-y_n) + [2k\phi\theta(\pi_t-\pi^*)-2\beta\phi\theta(c_1+c_2\pi_t)][1-1/(\lambda-\phi\theta)] + \Lambda_{t-1}\} = 0, \tag{10}$$

where Λ_{t-1} is the Lagrange multiplier of $Ei_t = E_{t-1}[i_t]$.

Eliminating Λ_{t-1} by adding (9) and (10) gives us:

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13 Where Ei_t is a variable, which is denoted as i^e in Svensson (1995), while $E_{t-1}[i_t]$ is a function of i_t.

14 Since Ei_t is the private sector's rational expectations on i, one may wonder how can the central bank commit to these expectations. Notice, first, that such a rational expectation is endogenously derived from the central bank's dynamic programming problem by the private sector and the central bank. Committing to such a rule means that the monetary authority internalizes the impact of its decision rule on the expectations of the private sector. Moreover, the surveying of the financial markets' expectations of the inflation level by the Bank of England as a part of its conduct of monetary policy indicates that central banks indeed take into consideration the information on the private sector's expectations into its decision making. Committing to Ei_t has been widely used in the recent literature as a commitment strategy, e.g., Lockwood, Miller and Zhang (1994), Goodhart and Huang (1995), and Svensson (1995).

$$2\phi\{[\lambda(Ey_t\text{-}y_n)+k\theta(E\pi_t\text{-}\pi^*)\text{-}\beta\theta(c_1+c_2E\pi_t)]/(\lambda\text{-}\phi\theta)+$$
$$(y_t\text{-}Ey_t)+\theta(k\text{-}\beta c_2)(\pi_t\text{-}E\pi_t)\} \;=\; 0,$$

which upon substituting for y_t - Ey_t and π_t - $E\pi_t$ gives:

$$2\phi[\lambda(Ey_t\text{-}y_n) \;+\; k\theta(E\pi_t\text{-}\pi^*) - \beta\theta(c_1+c_2E\pi_t)]/(\lambda\text{-}\phi\theta)$$
$$-\phi(i_t\text{-}Ei_t)+v_t+\theta(k\text{-}\beta c_2)[-\phi\theta(i_t\text{-}Ei_t)+u_t+\theta v_t)]\}=0. \qquad (11)$$

Equation (11) is the optimal feedback rule under commitment expressed as a function of the parameters of the model and two coefficients, c_1 and c_2, which are derived below. Taking the expectation of equation (11) and dividing its both sides by $2\phi/(\lambda\text{-}\phi\theta)$, we have:

$$\lambda(Ey_t - y_n) \;+\; k\theta(E\pi_t - \pi^*) - \beta\theta(c_1 \;+\; c_2E\pi_t) \;=\; 0. \qquad (12)$$

One way to interpret equations (11) and (12) is through specifying the information structure in our model as follows: both the monetary authority and the private sector have no information about the shocks at the beginning of each period; then the monetary authority observes the shocks and conducts its policy; and finally, near the end of each period, the private sector observes the shocks and the outcome of monetary policy. Hence, the only information asymmetry occurs in the *interim* of each period, and both the monetary authority and the private sector have exactly the same information *ex ante* and *ex post* of each period.[15] Because the private sector only observes the shocks *ex post*, it is too late for it to take any action to offset the response of the monetary authority to shocks. We also describe below the solution for the case where there is no information asymmetry.

As the monetary authority conducts its policy after observing the shocks u_t and v_t, equation (11) determines the optimal feedback rule. It is important to note that equation (12) is the expected optimal policy rule at the beginning of period t not only from the monetary authority's perspective, but also from the private

[15] This may raise the question why the authorities do not simply inform the public of these observed shocks when they occur. There are several possible answers. One such answer is that the authorities may do so, but this will not affect the private sector behavior because wages and prices are assumed to be sticky in the current period. This is further explained in the next footnote.

sector's perspective because it cannot observe the shocks until the end of period t. However, as the private sector can observe shocks at the end of each period, it can in principle check the monetary authority's commitment at the end of each period.[16] If the private sector discovers any cheating by the monetary authority, it can punish the authority through a well-defined trigger mechanism, as in Barro and Gordon (1983). This provides one mechanism through which the commitment solution is sustainable. As a result, the expected monetary authority's optimal policy rule at the beginning of the period t is the correct conjecture on the part of the private sector because of the commitment of the monetary authority not to deviate from this rule. Thus the commitment to the rule clearly determines the expectations of the private sector.

As the solution procedure of the optimal feedback rule for the monetary authority, i.e., equation (11), involves both the state-contingent variable π_{t-1} and the two random shocks u_t and v_t, it is quite complicated. However, given that the optimal feedback rule for the monetary authority is a linear function of state-contingent and shock-dependent components, i.e., it has both a systematic state-contingent part and a random shock-dependent part, it is simpler to determine each part of the optimal feedback rule separately. The first step of the solution procedure therefore is to derive only the state-contingent part of the optimal feedback rule, and the second step is to derive only the shock-dependent part. Note that the first step of the procedure is equivalent to solving the expected optimal feedback rule determined by equation (12). As equation (12) always has to be satisfied, we can impose (12) on (11). Thus in the second step of the procedure the random shock-dependent part can be simply solved using the

[16] In practice, of course, the public finds it hard enough to observe what the shocks have been, even after the event, far less whether the authorities have abided by a complicated feedback rule like equation (10) above. What actually occurs is that the authorities make a subjective estimate, based on their expectations of the probability distribution of the shocks, u_t and v_t, of what bounds of $\pi_t - \pi^*$ and $y_t - y_n$ their feedback rule can deliver. Then they commit to keeping within such bounds. The public can more easily see whether the outcomes remain within the pre-commitment ranges.

condition that $-\phi(i_t - Ei_t) + v_t + \theta(k-\beta c_2)[-\phi\theta(i_t - Ei_t) + u_t + \theta v_t)] = 0$ in equation (11).

III.2 The Systematic State-Contingent Part of the Optimal Commitment Policy Rule

Let us start with step one. Substituting Ey_t and $E\pi_t$ into (12), we have

$$Ei_t = \pi_{t-1} + \alpha + \theta(\lambda-\phi\theta)[(k-\beta c_2)\pi_{t-1} - k\pi^* - \beta c_1]/\{\phi[\lambda^2 + \theta^2(k - \beta c_2)]\}. \qquad (13)$$

To determine the values of c_1 and c_2, we apply the envelope theorem into equation (8). Doing so gives: $V'(\pi_{t-1}) = 2(c_1 + c_2\pi_{t-1}) = 2\{\lambda^2/[\lambda^2 + \theta^2(k - \beta c_2)]\}[k\pi^* + \beta c_1 - (k-\beta c_2)\pi_{t-1}]$. Hence,

$$c_1 = \lambda^2(k\pi^* + \beta c_1)/[\lambda^2 + \theta^2(k - \beta c_2)], \qquad (14)$$

$$c_2 = -\lambda^2(k - \beta c_2)/[\lambda^2 + \theta^2(k - \beta c_2)]. \qquad (15)$$

Solving (15) for c_2 gives us two roots, and the only valid root is

$$\frac{\quad}{\sqrt{\quad}} \; . \qquad (16)$$

This is because the other root, $\sqrt{\quad}$,
is always greater than the root defined by (16). As Lockwood and Philippopoulos (1994) and Svensson (1995) have shown, only the smaller root is relevant in these circumstances.

Substituting (16) in (14), we have:

$$\frac{\quad}{\sqrt{\quad}} \; . \qquad (17)$$

Substituting c_1 and c_2 into the monetary authority's expected optimal feedback rule, equation (12), and further substituting Ei_t into (4) and (5), we have:

$$Ei_t = \pi_{t-1} + \alpha - \{k\theta(\lambda-\phi\theta)/\{\phi[(1-\beta)\lambda^2 + (k-\beta c_2)\theta^2]\}\}(\pi^*-\pi_{t-1})$$

$$= \pi_{t-1} + \alpha - [k\theta(\lambda-\phi\theta)/\phi\Delta](\pi^*-\pi_{t-1}), \qquad (18)$$

$$E\pi_t = \pi_{t-1} + \{k\theta^2/[(1-\beta)\lambda^2 + (k-\beta c_2)\theta^2]\}(\pi^*-\pi_{t-1}) = \pi_{t-1} + (k\theta^2/\Delta)(\pi^*-\pi_{t-1}), \qquad (19)$$

$$Ey_t = y_n + \{k\lambda\theta/[(1-\beta)\lambda^2 + (k-\beta c_2)\theta^2]\}(\pi^*-\pi_{t-1}) = y_n + (k\lambda\theta/\Delta)(\pi^*-\pi_{t-1}), \qquad (20)$$

where Δ denotes $(1-\beta)\lambda^2 + (k-\beta c_2)\theta^2$ = [√]/2 for compact notion.

Note first that the systematic behavior of the instrument as well as that of the two endogenous variables is a function of the difference between the inflation target, π^*, and realized inflation last period, π_{t-1}. The interest rate is determined in expectation before the shocks are observed as a positive or negative function of the gap between desired and last period's actual inflation, whereas expected inflation and output are positively related to this gap. The signs and magnitude of the reduced-form adjustment coefficients in front of $(\pi^* - \pi_{t-1})$ are discussed at length below. The target output level, y_n, does not enter the optimal feedback rule because π_t is the only state variable in the model on account of persistence. Equation (20) shows that there is indeed persistence in output, but this in induced via the inertia in inflation rather than arising from the intrinsic dynamics embedded in the structural equation that determines the behavior of output.

From (18), (19) and (20), a number of important properties of the model with the optimal commitment rule can be derived:

Lemma 1: *If 1) goes to zero, i.e., no persistence, or 2) with > 0 and when there are no shocks, then* Ei_t = * + , E_t = *, *and* $Ey_t = y_n$.

Proof: The first result directly follows from Δ = [√]/2 = $k\theta^2$ when λ goes to zero. The second result holds because with $\lambda > 0$ and when there are no shocks, then in the long run $E\pi_t = \pi_{t-1} = \pi^*$. Hence, $Ey_t = y_n$ and $Ei_t = \pi^* + \alpha$.■

Based on this lemma, it is clear that when there is no persistence and in the long-run equilibrium when there are no shocks, both output and inflation targets can be hit each period and the real interest rate is $\alpha > 0$. In these special cases there is no tradeoff between hitting the inflation and output targets. Moreover, for certain parameter values, one target may be hit each period but not the other. Investigating equation (19) reveals that the inflation target can be hit each period when k goes to infinity, a scenario where the monetary authority is only concerned with price stability. Similarly, equation (20) implies that the output target can be hit each period if $\theta = 0$ or $k = 0$. However, in the more general case of persistence and all other parameter values, neither target can be hit each period. That is:

Theorem 1: *With persistence and with shocks, then Ei_t *+ , E_t *, and Ey_t y_n.*

This result implies that when there is persistence, even in expectation the monetary authority cannot hit its target π^* every period even though it can indeed fully commit to the policy rule. This is in sharp contrast with the result from previous studies, e.g. (Svensson, 1995), where in expectation the monetary authority can hit its target π^* every period if it can fully commit to the policy rule, although the realized inflation rate will typically differ from its target π^*. The reason for this difference is that in our model, unlike previous studies, inflation also has persistence which we believe is a fundamental feature of Phillips curve.

Comparing (18), (19) and (20), we also notice that $(\lambda - \phi\theta)$ only appears as a part of the numerator of the optimal feedback rule, equation (18), but it does not appear in the equations for $E\pi_t$ and Ey_t. Recall from (4) and (5) that $(\lambda - \phi\theta)$ does appear as a part of the denominator of $E\pi_t$ and Ey_t, hence it directly affects $E\pi_t$ and Ey_t. In particular, when $(\lambda - \phi\theta) = 0$, both $E\pi_t$ and Ey_t become infinity, i.e., $E\pi_t$ and Ey_t become unstable before the optimal rule on the nominal interest rate is imposed. But the outcome for $E\pi_t$ and Ey_t, after imposing such an optimal rule becomes invariant to the critical condition $(\lambda - \phi\theta)$. Moreover, as a part of the numerator of the optimal feedback rule, $(\lambda - \phi\theta)$ cannot affect the stability of the interest rate even if it goes to zero. This important point can be seen clearly in the analysis below.

We start our analysis by investigating the value of $k\theta^2/\Delta$.

Lemma 2: $0 < k\theta^2/\Delta < 1$.

Proof: Because $\Delta = [\quad \sqrt{\quad} \quad]/2 > \quad > k\theta^2 > 0$, $0 < k\theta^2/\Delta < 1$.■

Using Lemma 2 to investigate Ei_t, we have the following theorem.

Theorem 2: *Under the optimal control, the ratio between the systematic adjustment in the nominal interest rate between t and t-1 and the change in inflation between t-1 and t-2, $(Ei_t - E_{t-2}i_{t-1})/(\ _{t-1} - \ _{t-2})$,*[17] *is respectively between (1, /) if > and (/ , 1) if < .*

Proof: From equation (18), $(Ei_t - E_{t-2}i_{t-1})/(\pi_{t-1} - \pi_{t-2}) = 1 + [k\theta^2(\lambda - \phi\theta)/\phi\theta\Delta]$. Notice that $0 < k\theta^2/\Delta < 1$, hence,

if $\lambda > \phi\theta$, then $1 < 1 + [k\theta^2(\lambda - \phi\theta)/\phi\theta\Delta] < 1 + (\lambda - \phi\theta)/\phi\theta = \lambda/\phi\theta$;

and if $\lambda < \phi\theta$, then $1 > 1 + [k\theta^2(\lambda - \phi\theta)/\phi\theta\Delta] > 1 + (\lambda - \phi\theta)/\phi\theta = \lambda/\phi\theta$. ■

Theorem 2 provides the upper and lower bounds of the systematic adjustment in the nominal interest rate between t and t-1 with respect to the change in inflation between t-1 and t-2. Because the systematic state-contingent part of the optimal policy rule at t is state-contingent on π_{t-1}, the change in inflation can only be measured lagged one period. The Theorem says that if $\lambda > \phi\theta$, then the systematic adjustment in the nominal interest rate is greater than the change in inflation itself but less than $\lambda/\phi\theta$ times the change in inflation. Otherwise, i.e., if $\lambda < \phi\theta$, then the systematic adjustment in the nominal interest

[17] E_{t-2} denotes the rational expectations taken at the end of period t - 2.

rate is less than the change in inflation itself but greater than $\lambda/\phi\theta$ times the change in inflation. In the special case where $\lambda = \phi\theta$, Ei_t is not a function of π^* but set equal to π_{t-1} alone. Therefore, whether λ is greater or less than $\phi\theta$ becomes a very critical condition.

Indeed, such a condition is in accord with economic intuition. The condition that $\lambda > \phi\theta$ corresponds to situations of high persistence (high λ), hence high inflationary pressure is carried over to the next period; and/or low sensitivity of inflation to excess demand (low θ), hence the less effective is the nominal interest rate as the policy instrument to affect inflation through aggregate demand; and/or low sensitivity of aggregate demand to the real interest rate (low ϕ), hence the less effective is the nominal interest rate as the policy instrument to affect aggregate demand. With high persistence relative to the parameters affecting the monetary authorities' ability to control inflation, a more active adjustment in the nominal interest rate is called for. By symmetry, smaller adjustment in the control instrument is needed as λ approaches $\phi\theta$.

As pointed out above, the possibility that the magnitude of θ may itself be a function of the state of the business cycle can have important implications for the conduct of monetary policy. In particular, Laxton, Meredith and Rose (1995) and Clark, Laxton and Rose (1996) have presented empirical evidence that the value of θ is larger for a positive output gap, $y_t - y_n$, than for a negative output gap of the same absolute size. Moreover, they showed that this nonlinear inflation-output tradeoff implies that the greater the variability in output, the further the average level of output will lie below capacity output, y_n. The magnitude of this difference between average and capacity output was found to depend on the type of monetary policy response function, but those used in these papers were admittedly rather ad hoc.

Bean (1996) has also recently explored the implications of such nonlinearity for the optimal policy feedback rule. In other words, he has derived the optimal policy response using a dynamic programming model with a loss function similar to that employed in this paper. In order to keep the problem manageable, Bean uses a simple model where the current output gap is a function of the current value of the monetary authority's instrument and a random disturbance term which is imperfectly observed by the monetary authority, and where the change in inflation depends only on the current output gap. His model incorporates the accelerationist Phillips curve in which the inflation rate expected this period is equal to last period's inflation rate, and thus embodies persistence in inflation but not in output.

In order to obtain a closed form expression for the optimal feedback rule, Bean uses a first-order approximation to a nonlinear Phillips curve. His theoretical results agree with those in the two papers cited above, namely, that in the presence of nonlinearity and uncertainty, the optimal policy rule implies that the average level of output will lie below capacity output. Moreover, he also finds that the greater the degree of nonlinearity, the slower is the rate of disinflation because of the declining marginal effectiveness in reducing inflation of a large negative output gap. In future work, we plan to explore the extent to which Bean's results carry over to our model with rational expectations.

Using Lemma 2 to investigate $E\pi_t$ and Ey_t, we have the following results.

Theorem 3: *Under the optimal control, the ratio between the expected adjustment* is stable and the optimal control improves the equilibrium outcome in terms of inflation.

Theorem 4: *Under the optimal control, the ratio between the expected adjustment in output between t and t-1 and the inflation change between t-1 and t-2,* $(Ey_t - E_{t-1}y_{t-1})/(\ _{t-1} - \ _{t-2})$*, is between (- / , 0), regardless of whether   is greater or less than .*

Proof: From equation (20), $(Ey_t - E_{t-1}y_{t-1})/(\pi_{t-1} - \pi_{t-2}) = - k\lambda\theta^2/\Delta\theta$, which is between $(- \lambda/\theta, 0)$, due to $0 < k\theta^2/\Delta < 1$. ■

Theorem 4 shows that the expected adjustment in output between t and t-1 and the inflation change between t-1 and t-2 always move in opposite directions. And in terms of the absolute value, the ratio between the expected adjustment in output between t and t-1 and the inflation change between t-1 and t-2 is less than λ/θ. The economic intuition of this theorem is clear: the larger the most recent increase in inflation, the larger the expected decline in output in order to reduce inflation in the current period via the Phillips curve. Moreover, dividing the adjustment coefficient in the expression for $E\pi_t$, i.e., $k\theta^2/\Delta$ as shown in (19), by that for Ey_t, i.e., $k\lambda\theta/\Delta$ as in (20), we find that the ratio is $\theta/\lambda > 0$. That is:

Theorem 5: *Under the optimal control, the ratio between the adjustment coefficients for* $E\ _t$ *and* Ey_t *is / > 0.*

Theorem 5 indicates that under the optimal control, the adjustment coefficients for $E\pi_t$ and Ey_t move in the same direction. Hence, for a given deviation of inflation from its target, the larger the adjustment of $E\pi_t$ to the target level of inflation, the larger the adjustment of expected output away from its target, and vice versa. This result is due to the effect of the Phillips curve which governs the relationship between the inflation and output adjustment; that is, the larger the adjustment in inflation in order to reach to its target, the larger the expected adjustment in output in the current period, which implies a larger deviation of output from its target. Except in the cases we have specified in Lemma 1, both inflation and output targets cannot be hit simultaneously, and as Theorem 5 indicates, there is a tradeoff between reaching these two targets.

We have seen that without imposing the optimal feedback rule, the value of $(\lambda - \phi\theta)$ affects both $E\pi_t$ and Ey_t, and in particular, when the value of $(\lambda - \phi\theta)$ goes to zero $E\pi_t$ and Ey_t become unstable. However, with the optimal feedback rule, theorems 3 and 4 show that the value of $(\lambda - \phi\theta)$ does not affect either $E\pi_t$ or Ey_t. Furthermore, the ratio between the expected adjustment in inflation and the change in inflation and the ratio between the expected adjustment in output between t and t-1 and the inflation change between t-1 and t-2 all are bounded within a narrow band as a result of the optimal feedback rule. Therefore the optimal feedback rule does indeed stabilize $E\pi_t$ and can stabilize Ey_t, but, according to Theorem 5 there is a tradeoff between reaching the inflation and output targets. We state this result as a proposition below.

Proposition 1: *The optimal commitment policy rule stabilizes the rational expectation model. However, the larger the expected adjustment of inflation* <u>*towards*</u> *its target the larger the expected adjustment of output* <u>*away*</u> *from its target, and vice versa.*

Having obtained the solution for the expected feedback rule, we can move to the second step in our two-step procedure to determine i_t, the shock-dependent feedback rule, as well as π_t and y_t.

III.3 <u>The Random Shock-Dependent Part of the Optimal Commitment Policy Rule</u>

Without loss of generality, we can write:

$$i_t = Ei_t + \gamma_1 u_t + \gamma_2 v_t. \tag{21}$$

As already noted above, we have (6) and (7), i.e., $\pi_t = E\pi_t - \phi\theta(i_t - Ei_t) + u_t + \theta v_t$ and $y_t = Ey_t - \phi(i_t - Ei_t) + v_t$. Using (21), (6), (7) and imposing (12) onto (11), the monetary authority's optimal feedback rule, we have:

$$-\phi\gamma_1 u_t + \gamma_2 v_t) + v_t + \theta(k-\beta c_2)[- \phi\theta(\gamma_1 u_t + \gamma_2 v_t) + u_t + \theta v_t] = 0. \tag{22}$$

Comparing the adjustment coefficients in front of u_t and v_t leads to two conditions, i.e., $[-\phi\gamma_1 + \theta(k-\beta c_2)(- \phi\theta\gamma_1 + 1)]u_t = 0$ and $[- \phi\gamma_2 + 1 + \theta(k-\beta c_2)(-\phi\theta\gamma_2 + \theta)]v_t = 0$. The former condition implies $\gamma_1 = \theta(k-\beta c_2)/\{\phi[1 + \theta^2(k-\beta c_2)]\}$, and the latter implies $\gamma_2 = 1/\phi$. Hence, i_t, π_t and y_t can all be expressed in two parts, the systematic state-contingent part and the random shock-dependent part, as follows.

$$i_t = \pi_{t-1} + \alpha - \{k\theta(\lambda-\phi\theta)/\{\phi[(1-\beta)\lambda^2 + (k-\beta c_2)\theta^2]\}\}(\pi^* - \pi_{t-1})$$
$$+ \{\theta(k-\beta c_2)/\{\phi[1 + \theta^2(k-\beta c_2)]\}\}u_t + (1/\phi)v_t. \tag{23}$$

$$\pi_t = \pi_{t-1} + \{k\theta^2/[(1-\beta)\lambda^2 + (k-\beta c_2)\theta^2]\}(\pi^* - \pi_{t-1}) + \{1/[1 + \theta^2(k-\beta c_2)]\}u_t. \tag{24}$$

$$y_t = y_n + \{k\lambda\theta/[(1-\beta)\lambda^2 + (k-\beta c_2)\theta^2]\}(\pi^* - \pi_{t-1}) - \{\theta(k-\beta c_2)/[1 + \theta^2(k-\beta c_2)]\}u_t. \tag{25}$$

Equation (23) is the complete optimal feedback rule which includes both the systematic state-contingent part and the random shock-dependent part. From equations (23), (24) and (25), we have the following theorem regarding the effect of random shocks, u_t and v_t, on i_t, π_t and y_t.

Theorem 6: *Under the optimal commitment policy rule, the impact of the random shock u_t on the adjustment of the nominal interest rate is between (0, 1/)u_t, between (0, 1)u_t on the inflation level, and between (-1/ , 0)u_t on the output level. The effect of v_t on i_t is (1/)v_t, but this shock to aggregate demand has no effect on either $_t$ or y_t.*

Proof: From equations (23), (24) and (25), u_t affects i_t, π_t and y_t respectively by $\{\theta(k-\beta c_2)/\{\phi[1 + \theta^2(k-\beta c_2)]\}\}u_t$, $\{1/[1 + \theta^2(k-\beta c_2)]\}u_t$ and $-\{\theta(k-\beta d_2)/[1 + \theta^2(k-\beta c_2)]\}u_t$, which are greater than zero but less than $(1/\phi\theta)$, greater than zero but less than 1, and greater than $-(1/\theta)$ but less than zero, respectively, following the results that $0 < 1/[1+\theta^2(k-\beta c_2)] < 1$ and $0 < \theta^2(k-\beta c_2)/[1+\theta^2(k-\beta c_2)] < 1$ due to $k-\beta c_2 > 0$. The second part of the theorem follows from equations (23), (24) and (25). ■

Based on this theorem it is clear that the optimal control shrinks the effect of the random shock u_t on inflation from 100 percent of u_t before the optimal policy is imposed to strictly less than 100 percent of u_t after the optimal policy is imposed. And it furthermore completely eliminates the effect on inflation of the random shock, v_t, on aggregate demand. Therefore, optimal control shrinks the effect of both random shocks on inflation rate. Similarly, the optimal control transmits an effect on output of random inflation shock u_t up to $(1/\theta)u_t$. And it furthermore completely eliminates the effect on output of the random shock v_t. Therefore, optimal control may shrink the effect of random shocks on output if $1/\theta < 1$.

We state these results regarding the effect of random shocks on inflation and output level as:

Proposition 2: *The optimal commitment policy rule completely eliminates the effect on inflation and output of the random shock v_t. It does shrink the effect on inflation of the random shock u_t, but it may or may not shrink the effect on output of u_t.*

Based on Propositions 1 and 2, we have reached a conclusion regarding the effectiveness of active policy with commitment in a rational expectations model: with (slightly) asymmetric information and inflation persistence, an active policy rule improves the equilibrium outcome in the rational expectations model.

Note that the driving force for the above policy effectiveness result is the sluggish wage and price adjustment rather than the asymmetric information. We chose asymmetric information as one possible modelling strategy to rationalize the existence of sluggish price and wage adjustments. As long as such sluggish adjustments exist, that is so long as the private sector cannot adjust wages and prices instantly even if they observe the shocks the same time as the monetary authority, the same policy effectiveness result still holds. Therefore, our assumption on *interim* asymmetric information between the monetary authority and the private sector is not as restrictive as it appears, and it can be relaxed in a number of different ways. In one extreme case, both the monetary authority and the private sector can observe the shocks simultaneously, with policy effective as long as sluggish adjustments exist. At the other extreme, with neither party observing anything in the current period, the monetary authority can still effectively use its instrument to stabilize the economy. The only difference then is that as the monetary authority has no information on shocks, it has to base its control only on the expected optimal feedback rule.

To summarize, we arrive at the following conclusion regarding the effectiveness of active policy rule with commitment in a rational expectations model: *with sluggish adjustments in wages and prices, and therefore inflation persistence, which may also be reflected by asymmetric information, an active policy rule with commitment stabilizes the rational expectations model and improves the model's equilibrium outcome*.

IV. The Optimal Discretionary Policy Rule

IV.1 General Considerations

As discussed above, in order to provide a fair test of the value added of the self-imposed constraint of commitment in comparison with a policy rule based on discretion, we have chosen to examine the effects of alternative assumptions about the conduct of monetary policy where time inconsistency does not arise. In this section we solve for the optimal discretionary policy rule and compare it with the optimal commitment policy rule.

When the central bank does not commit on Ei_t, it cannot internalize the impact of its decision rule on the expectations of the private sector. With this approach to monetary policy there is only one decision variable or instrument: the actual *ex post* interest rate, i_t, each period. The central bank loses one policy instrument, which is the expectation of the interest rate, so that it is no longer bound by this constraint. In this case the optimal rule is derived by minimizing the loss function of the monetary authority only with respect to the actual *ex post* interest rate. Consequently, the new dynamic problem in this case is:

$$\max_{i_t} E \quad \beta^t U_t$$
subject to (4) and (5)

This is a dynamic programming problem with one state-variable, π_{t-1}, and one control variable, i_t. As in the commitment case, the solution can be obtained by solving the following problem:

$$V(\pi_{t-1}) = \max_{i_t} E\{-(y_t - y_n)^2 - k(\pi_t - \pi^*)^2 + \beta V(\pi_t)\}, \tag{26}$$
subject to (4) and (5).

For the linear-quadratic problem such as ours, $V(\pi_t)$ must also be quadratic. Without loss of generality, we can write $W(\pi_t) = d_0 + 2d_1\pi_t + d_2\pi_t^2$, so that $W'(\pi_t) = 2(d_1 + d_2\pi_t)$. Using this condition together with (4) and (5), we obtain the first-order conditions from (26) with respect to i_t:

$$(y_t - y_n) + k\theta(\pi_t - \pi^*) - \beta\theta(d_1 + d_2\pi_t) = 0. \tag{27}$$

Taking the expectation of the above equation, we have:

$$(Ey_t - y_n) + k\theta(E\pi_t - \pi^*) - \beta\theta(d_1 + d_2E\pi_t) = 0. \tag{28}$$

Equation (27) defines the optimal feedback rule and (28) defines the expected optimal feedback rule. As in the commitment case, we can assume that the only information asymmetry occurs in the *interim* of each period, and both the monetary authority and the private sector have exactly the same information *ex ante* and *ex post* in each period. Furthermore, we can, as in the commitment case, pursue our analysis in the two-step procedure and start with the systematic state-contingent part.

IV.2 The Systematic State-Contingent Part of the Optimal Discretionary Policy Rule

Substituting Ey_t and $E\pi_t$ into (28), we have

$$Ei_t = \pi_{t-1} + \alpha + \theta(\lambda-\phi\theta)[(k-\beta d_2)\pi_{t-1} - k\pi^* - \beta d_1]/\{\phi[\lambda + \theta^2(k - \beta d_2)]\}. \qquad (29)$$

To determine the values of d_1 and d_2, we can apply the envelope theorem into equation (28). Notice that in the discretionary case Ei_t is not a control variable, hence we have to take into consideration the fact of $\partial Ei_t/\partial\pi_{t-1} = \lambda[\phi + \theta(k - \beta d_2)]/\{\phi[\lambda + \theta^2(k - \beta d_2)]\} \neq 0$, in contrast with the commitment case where $\partial Ei_t/\partial\pi_{t-1} = 0$ because Ei_t is a control variable. Applying the envelope theorem into equation (28) gives:

$$W'(\pi_{t-1}) = 2(d_1 + d_2\pi_{t-1})$$

$$= 2\{\lambda^2[1-\theta^2(k-\beta d_2)]\}/[\lambda + \theta^2(k-\beta d$$

system stability now.

Lemma 3: *The necessary and sufficient condition for the system to be stable is k- $d_2 > 0$.*

Proof: $E\pi_t$, given in (34), converges if and only if its adjustment coefficient is less than 1, which is assured if and only if $k - \beta d_2 > 0$.■

If the system is stable in the discretionary case, then as in the commitment case, the systematic behavior of the instrument as well as that of the two endogenous variables in the discretionary case is also a function of the difference between the inflation target, π^*, and realized inflation last period, π_{t-1}.

Moreover, we notice that: *1) When goes to zero, i.e., no persistence, or 2) with > 0, if the system is stable and there are no shocks, then Ei_t = *+ , E_t = * and $Ey_t = y_n$.* These are the identical results we have obtained in the commitment case and stated there as Lemma 1. Based on these two results, it is also clear that *when there is persistence and when the system is stable, in expectation the monetary authority cannot hit its target * every period even though it follows consistently with the optimal discretionary policy rule*. This is the same as Theorem 1 in commitment.

As in the commitment case, we notice that $(\lambda - \phi\theta)$ only appears as part of the numerator of the optimal feedback rule, equation (33), but it does not appear in the equations for $E\pi_t$ and Ey_t. Again recall from (4) and (5) that $(\lambda - \phi\theta)$ does appear as part of the denominator of $E\pi_t$ and Ey_t, hence it directly affects $E\pi_t$ and Ey_t before the optimal rule for the nominal interest rate is imposed. But the outcome for $E\pi_t$ and Ey_t after imposing such an optimal rule becomes invariant to the critical condition $(\lambda - \phi\theta)$. Moreover, as part of the numerator of the optimal feedback rule, $(\lambda - \phi\theta)$ cannot affect the stability of interest rate even if it goes to zero.

As d_2 has three roots that are extremely complicated, the closed-form solution for the discretionary case is not transparent and the comparisons between the analytical solutions of the commitment and discretionary cases become very difficult. A more fruitful approach is to use numerical methods involving a wide range of plausible values for the parameters of the model to compare the coefficients of adjustment to the gap between the target and the lagged inflation rate. By choice of units we set $\phi = 1$ and choose the benchmark case at $\lambda = 0.5$, $\theta = 0.2$, $k = 1$ and $\beta = 0.9$. We then vary λ from 0 to 1, θ from 0 to 10, k from 0 to 100 and β from 0 to 1. From the numerical results presented in Table I.1 to I.4 below, we find that the coefficients of adjustment to the gap between the target and the lagged inflation rate are uniformly larger with commitment than discretion. This implies that in the case of commitment, the expected deviation of inflation from its target level is always smaller, but the expected deviation of output from its target level is always larger than in the case of discretion.

Tables I.1, I.2, I.3 and I.4 are about here

The above comparisons of the expected equilibrium outcome between the commitment and discretionary cases are summarized as follows:

Proposition 3: *Under the optimal control the expected equilibrium outcome with discretion is qualitatively similar to that with commitment and can be described as function of the difference between the inflation target,* *, *and realized inflation last period,* $_{t-1}$. *Quantitatively, the expected deviation of inflation from its target level is always larger, while the expected deviation of output from its target level is always smaller with discretion than with commitment.*

The fact that commitment generates smaller expected deviation of inflation from its target but simultaneously larger expected deviation of output from its target provides a clear demonstration of the tradeoff, at least in the sense of *ex ante* expectation, between the optimal commitment and discretionary monetary policy rules. This result holds because the ratio between the adjustment coefficients for $E\pi_t$ and Ey_t with discretion, as with commitment, is λ/θ. While commitment generates a larger adjustment coefficient for $E\pi_t$, which implies a smaller expected deviation of inflation from its target, it simultaneously generates a larger adjustment coefficient for Ey_t, which implies larger expected deviation of output from its target.

Notice that this result is obtained in our model where there is no inflationary bias and the discretionary approach is defined by the optimal discretionary monetary policy rule. The main intuition behind this result is that with commitment rule, the monetary authority, with its additional instrument on Ei_t, internalizes the impact of its decision rule on the expectations of the private sector, but it also faces an additional constraint determined by equality between the value of the additional instrument and the private sector's rational expectation of such a value. As a result, commitment does not come free. This sheds new light on the debate over commitment versus discretion, that was first analyzed in Kydland and Prescott (1977), in the absence of time inconsistency or credibility problems.

It should also be noted that, as can be seen in Table I.1 to I.4, this result involves adjustments in the control variable, Ei_t, that are uniformly larger in the case of commitment than discretion. Thus the former strategy involves a more active use of interest rate policy to achieve smaller deviations of inflation from target (but larger deviations of output from target). The model considered here does not take account of possible costs in adjusting interest rates, which have been analyzed by Goodhart (1996). If the model were extended to incorporate these costs, the difference between the two strategies would probably be less sharp, but the qualitative results described above would still appear to hold.

IV.3 <u>The Random Shock-Dependent Part of the Optimal Discretionary Policy Rule</u>

The shock-dependent parts in this discretionary case are derived as in the commitment case. Substituting for γ_1 and γ_2 in (21) with δ_1 and δ_2 respectively, we have:

$$i_t = Ei_t + \delta_1 u_t + \delta_2 v_t. \qquad (36)$$

Simply by substituting d_2 for c_2 in the expression for γ_1 one obtains $\delta_1 = \theta(k-\beta d_2)/\{\phi[1 + \theta^2(k-\beta d_2)]\}$, where $\delta_1 > 0$ if the stability condition $k-\beta d_2 > 0$ holds, whereas $\delta_2 = \gamma_2 = 1/\phi$. Hence, i_t, π_t and y_t can all be expressed in two parts, the systematic state-contingent part and the random shock-dependent part, as follows.

$$i_t = \pi_{t-1} + \alpha - \{\theta(\lambda-\phi\theta)/\phi[\theta^2 + \lambda/(k-\beta d_2)]\}(\pi^*-\pi_{t-1}) + \{\theta(k-\beta d_2)/\{\phi[1 + \theta^2(k-\beta d_2)]\}\}u_t + (1/\phi)v_t. \qquad (37)$$

$$\pi_t = \pi_{t-1} + \{\theta^2/[\theta^2 + \lambda/(k-\beta d_2)]\}(\pi^* - \pi_{t-1}) + \{1/[1 + \theta^2(k-\beta d_2)]\}u_t. \qquad (38)$$

$$y_t = y_n + \{\lambda\theta/[\theta^2 + \lambda/(k-\beta d_2)]\}(\pi^* - \pi_{t-1}) - \{\theta(k-\beta d_2)/[1 + \theta^2(k-\beta d_2)]\}u_t. \qquad (39)$$

Equation (37) is the complete optimal feedback rule for the discretionary case which includes both the systematic state-contingent part and the random shock-dependent part.

Comparing the discretionary solutions, i.e., equations (37), (38) and (39), with the commitment solutions, i.e., equations (23), (24) and (25), we notice that, first, *the shock v_t has the exactly same effect on both solutions*, and second, *the adjustment coefficients for i_t, $_t$ and y_t are analytically the same in both cases but numerically different because c_2 is generally not equal to d_2.*

Furthermore, from simple comparative statics of the coefficients in front of u_t for i_t, π_t and y_t in equations (37), (38) and (39) with respect to d_2, we notice that as d_2 increases, the adjustment coefficient for i_t, π_t and y_t decreases, increases and increases, respectively. From Tables II.1 to II.4 below, it is clear that as λ, θ, k and β change, d_2 (the root for the discretionary case) may be larger or smaller than c_2 (the root for the commitment case). As a result, the adjustment coefficients for i_t, π_t and y_t in the discretionary case will be larger, smaller and smaller, respectively, than those in the commitment case when $d_2 < c_2$, and vice versa when $d_2 > c_2$. Notice that the adjustment coefficients for i_t and π_t in both rules are positive, but the adjustment coefficient for y_t in both rules is negative, which implies that an decrease in its value means that its absolute value increases. Therefore, when $d_2 < c_2$, the adjustment coefficient for i_t, π_t and y_t in the discretionary case is larger (increasing its absolute value), smaller (decreasing its absolute value) and smaller (increasing its absolute value), respectively, the discretionary rule adjusts the nominal interest rate more and it is more effective in reducing the effect of the random shock u_t on inflation but less effective in reducing the effect of the random shock on output, than the commitment rule, and vice versa if $d_2 > c_2$.

Tables II.1, II.2, II.3 and II.4 are about here

Based on equations (37), (38) and (39) and Tables II.1 to II.4, we have the following further results regarding the random shock, u_t, on i_t, π_t and y_t, respectively. We shall compare these results directly with those in the commitment case.

Theorem 7: *If the control system is stable, then as in the commitment case, under the optimal discretionary policy rule the effect of the random shock u_t on the nominal interest rate adjustment is between (0, 1/)u_t, between (0, 1)u_t on inflation, and between (-1/ , 0)u_t on output. If $d_2 < c_2$, then u_t in this discretionary case has a larger effect on i_t, smaller effect on $_t$, and larger effect on y_t, respectively, than that in the commitment case, and vice versa if $d_2 > c_2$.*

Proof: This first part of this theorem holds because the coefficients in front of u_t for i_t, π_t and y_t, respectively $\{\theta(k-\beta d_2)/\{\phi[1+\theta^2(k-\beta d_2)]\}$, $\{1/[1+\theta^2(k-\beta d_2)]\}$ and -

$\{\theta(k-\beta d_2)/[1+\theta^2(k-\beta d_2)]\}$, is greater than zero but less than $(1/\phi\theta)$, greater than zero but less than 1, and greater than $-(1/\theta)$ but less than zero, due to $0 < 1/[1+\theta^2(k-\beta d_2)] < 1$ and $0 < \theta^2(k-\beta d_2)/[1+\theta^2(k-\beta d_2)] < 1$ when the stability condition $k > \beta d_2$ holds. The second part of this theorem holds due to the joint facts that the comparative statics of the coefficients in front of u_t for i_t, π_t and y_t in equations (37), (38) and (39) with respect to d_2 are respectively negative, positive and positive, and d_2 may be larger or smaller than c_2. ■

As in the commitment case, the optimal control shrinks the effect of the random shock u_t on inflation from 100 percent of u_t before the optimal policy is imposed to strictly less than 100 percent of u_t after the optimal policy is imposed. And it furthermore completely eliminates the effect on inflation of the random shock, v_t, on aggregate demand. Therefore, optimal control shrinks the effect of both random shocks on inflation rate. Similarly, the optimal control transmits an effect on output of random inflation shocks, u_t, up to $(1/\theta)u_t$. And it furthermore completely eliminates the effect on output of the random shock v_t. Therefore, optimal control may shrink the effect of random shocks on output, if $1/\theta < 1$.

To summarize, we arrive the following statement regarding the effects of shocks on inflation and output in the discretionary case:

Proposition 4: *The optimal discretionary policy rule completely eliminates the effect on inflation and output of the random shock v_t. It does shrink the effect on inflation of the random shock u_t, but it may or may not shrink the effect on output of u_t. If $d_2 < c_2$, then u_t in this discretionary case has a larger effect on i_t, smaller effect on $_t$, and larger effect on y_t, respectively, than that in the commitment case, and vice versa otherwise. Only in the case where the discretionary rule is more effective in reducing the random shock on inflation and less effective in reducing the random shock on output would it be possible that one rule is superior; otherwise there is always a tradeoff between these two rules.*

The first part of the above proposition implies that, as in the commitment case, with (slightly) asymmetric information and inflation persistence, an active policy rule with discretion improves the equilibrium outcome in our rational expectations model. Furthermore, as in the commitment case, this conclusion only relies on the feature of the Phillips curve, i.e., the sluggish adjustments in wages and prices, and therefore inflation persistence. While such persistence may reflect asymmetric information, the assumption of asymmetric information is not critical for our result.

More importantly, the second part of the above proposition is a new contribution to the debate on commitment versus discretion. It indicates that there is always a tradeoff between these two rules, except in the case where the discretionary rule is more effective in reducing the random shock on inflation and less effective in reducing the random shock on output. In this case the "comparative advantage" of the commitment rule in keeping inflation closer to target in the systematic part may be offset by the greater effectiveness of the discretionary rule in targeting inflation in response to shock. In this exceptional case, one rule may be superior to the other because the aggregate effect of inflation and output deviating from their targets caused by both the systematic and the random shock components becomes ambiguous.

Our result implies that if the monetary authority is concerned with both inflation and output, and the rule governing monetary policy is optimally designed, then except in a special case which may lead one rule to be superior, it will always face a tradeoff between choosing the optimal commitment and discretionary rules. Commitment does not necessarily bring more benefits than discretion. Moreover, when the discretionary rule is optimally designed, as it should be, even in the presence of a large shock there may be no need for an "escape clause" for a conservative central bank, an idea initially proposed by Flood and Isard (1989) and further extended to a "flexible" central bank by Lohmann (1992). This is so because even if u_t is a large shock, its impact in the discretionary case may be stronger on inflation and weaker on output, respectively, than in the commitment case, then the tradeoff relationship between these two rules still exists and, as a result, there will be no obvious advantage for the monetary authority to switch from the commitment to the discretionary rule. That is, depending on the economic environment, the "escape clause" may not be needed even in the presence of a large shock because the authorities are by construction responding optimally.

V. Concluding Remarks

This paper has derived the optimal monetary policy feedback rules with both commitment and discretionary cases in a simplified economy that is characterized by what we believe to be a more realistic version of the Phillips curve than that used in previous analysis of this topic. As persistence in prices and inflation is a feature of all empirical versions of the Phillips curve, it is important that such persistence be captured in theoretical discussions of optimal stabilization policy. Indeed, we would argue that policy questions and issues are relatively uninteresting in a model where a short-run tradeoff between output and inflation exists only in the current period, as such an economy is essentially self-stabilizing.

Moreover, our characterization of inflation control appears to us to have captured at least some of the challenges faced by monetary authorities in achieving their objectives. While ultimately the inflation level depends on the rate of growth of the money stock, short-run stabilization of output and inflation depends on adjusting the (real) interest rate to affect the level of aggregate demand relative to output capacity, and thereby inflation. Thus inflation control is achieved only indirectly via changes in demand.

We have two main findings in the paper. First, with inflation persistence, both commitment and discretion strategies lead to state-contingent and shock-dependent feedback rules. Thus the overall stance of policy is important and our analysis is consistent with that contained in the literature (see Goodhart and

Huang (1995) and Svensson (1995) for recent examples) that has examined the implications of output persistence on the optimal policy rule. Moreover, the form of the feedback rules are economically plausible in that they are expressed in terms of adjustment coefficients times the gap between the target level and lagged inflation, and both the signs and the variations in the coefficients in response to changes in parameter values are also in accord with economic intuition.

Second, our numerical results show that in the sense of *ex ante* expectation there always exists a tradeoff relationship between the two optimal monetary policy rules. A commitment rule takes full account the effects of expectations on the behavior of the economy and in this sense is forward looking, whereas a discretionary rule is myopic in the sense that it only concerned with the impact of the realized interest rate on the economy. The commitment rule results in higher absolute values for the adjustment coefficients for output and inflation, and consequently leads to expected inflation that is closer to its target, but simultaneously to expected output that is further away from its target. Moreover, a discretionary rule may or may not be more effective in reducing the effect of the random shock on inflation and less effective in reducing the effect of the random shock on output than the commitment rule. Except where the discretionary rule is more effective in reducing the effect of the random shock on inflation and less effective in reducing the effect of the random shock on output than the commitment rule, which may lead to the possibility of one rule being superior, there always exists a tradeoff between the use of these two optimal rules.

It is useful to compare the optimal feedback rules derived here with that described by Taylor (1993). In what he calls a representative monetary policy rule, the nominal interest rate that is the instrument of the central bank is set equal to the lagged inflation rate (plus the real steady-state growth rate of 2.2 percent to give a positive real interest rate) and is specified as responding to deviations of inflation from a target of two percent and deviations of output from trend GDP. It is noteworthy that this admittedly *ad hoc* policy rule is quite similar in form to the optimal rules in equations (18) and (33), where the expected interest rate is equal to the lagged inflation rate plus three other items, namely, the real steady-state growth rate of α (which gives a positive real interest rate), a coefficient times the deviation of lagged inflation from the target level (the state-contingent item), and the effect of random shocks (the shock-dependent item). As the optimal rule takes full account of preferences regarding deviations of output from target, such deviations do not appear in the policy rule itself. Taylor's policy rule can be viewed as a kind of reduced form that in principle combines the preferences of the policymaker embodied in the loss function as well as the behavioral parameters and structural relationships in the model, all of which are explicitly incorporated in the coefficients in the optimal policy rules.

Taylor describes his policy rule as having the general properties of rules that have been examined in recent research, e.g., Bryant, Hooper and Mann (1993). Moreover, he finds that it explains remarkably well the actual behavior of the federal funds rate controlled by the Federal Reserve over the period 1987-1992. Clarida and Gertler (1996) estimate a modified Taylor rule over the period 1974-1992 for the short-term interest rate used by the Bundesbank as its policy instrument. They also find that it has considerable explanatory power. While it would be far too strong to conclude anything about the optimality of the policy reaction functions of these two central banks, the theoretical results in this paper suggest that a Taylor-type rule does at least embody certain aspects of an optimal feedback rule. Alternatively, one can regard this empirical evidence as providing some support for the type of theoretical analysis pursued in this paper.

The analysis in this paper could be extended in a number of ways. Allowing for lags in the effect of interest rates on aggregate demand and for persistence in output would add greater realism, but is likely to add to the complexity of the analysis without affecting the basic finding that with persistence, an optimal monetary policy must be active in the sense of being state-contingent and shock-dependent. In this case there would be a second state variable -lagged output- and it would be of interest to explore whether it would be possible to derive an optimal policy rule that would have the same symmetry as Taylor's rule, i.e., in which the equation for the control variable would involve adjustment coefficients for deviations of both inflation and output from their respective targets. Another extension is to relax the assumption of linear relationship between excess demand and inflation. As noted above, some preliminary work suggests that there are important implications for stabilization policy arising from this type of nonlinearity. We plan to analyze these questions in our future research.

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Table I: Comparisons between the commitment and discretionary cases of the adjustment coefficients in the expressions for Ei_t, $E\pi_t$ and Ey_t

Table I.1: Simulation Results on λ

λ	c_2	i_c	π_c	y_c	d_2	i_d	π_d	y_d
0	0	1	1	0	0	1	1	0
0.01	-0.00249	0.9476	0.9975	0.0499	-0.03758	0.7650	0.8053	0.0403
0.1	-0.20647	0.4129	0.8259	0.4129	-0.55047	0.1871	0.3743	0.1871
0.2	-0.60731	0	0.6073	0.6073	-0.90615	0	0.2663	0.2663
0.3	-1.04083	-0.2313	0.4626	0.6939	-1.15873	-0.1070	0.2141	0.3211
0.5	-1.88280	-0.4519	0.3012	0.7531	-1.51286	-0.2384	0.1589	0.3973
0.7	-2.65479	-0.5418	0.2167	0.7585	-1.75851	-0.3215	0.1286	0.4501
0.9	-3.35177	-0.5793	0.1655	0.7448	-1.94267	-0.3810	0.1089	0.4898
1	-3.67326	-0.5877	0.1469	0.7347	-2.01900	-0.4051	0.1013	0.5064

Table I.2: Simulation Results on θ

θ	c_2	i_c	π_c	y_c	d_2	i_d	π_d	y_d
0	-10	0	0	0	-10	0	0	0
0.01	-9.62779	-0.1887	0.0038	0.1926	-9.54179	-0.0938	0.0019	0.0957
0.1	-3.67326	-0.5877	0.1469	0.7347	-3.22967	-0.2899	0.0725	0.3624
0.2	-1.88280	-0.4519	0.3012	0.7531	-1.51286	-0.2384	0.1589	0.3973
0.4	-0.82348	-0.1318	0.5270	0.6588	-0.51995	-0.0799	0.3196	0.3995
0.5	-0.60732	0	0.6073	0.6073	-0.32293	0	0.3922	0.3922
0.7	-0.36890	0.2065	0.7230	0.5165	-0.11678	0.1485	0.5199	0.3714
1	-0.20647	0.4129	0.8259	0.4129	0	0.3333	0.6667	0.3333
10	-0.00249	0.9476	0.9975	0.0499	0.00245	0.9453	0.9950	0.0496

Legend

"c" and "d" denote the commitment and discretionary cases respectively.

$i_c = -k\theta(\lambda-\phi\theta)/\{\phi[(1-\beta)\lambda^2+(k-\beta c_2)\theta^2]\}$, the adjustment coefficient in the expression for Ei_t;

$\pi_c = k\theta^2/[(1-\beta)\lambda^2+(k-\beta c_2)\theta^2]$, the adjustment coefficient in the expression for $E\pi_t$;

$y_c = k\lambda\theta/[(1-\beta)\lambda^2+(k-\beta c_2)\theta^2]$, the adjustment coefficient in the expression for Ey_t;

$i_d = -\theta(\lambda - \phi\theta)/\phi[\theta^2 + \lambda/(k-\beta d_2)]$, the adjustment coefficient in the expression for Ei_t;

$\pi_d = \theta^2/[\theta^2 + \lambda/(k-\beta d_2)]$, the adjustment coefficient in the expression for $E\pi_t$;

$y_d = \lambda\theta/[\theta^2 + \lambda/(k-\beta d_2)]$, the adjustment coefficient in the expression for Ey_t.

Table I.3: Simulation Results on k

k	c_2	i_c	π_c	y_c	d_2	i_d	π_d	y_d
0	0	0	0	0	0	0	0	0
0.25	-0.91832	-0.2204	0.1469	0.3672	-0.80742	-0.1087	0.0725	0.1812
0.5	-1.34041	-0.3217	0.2145	0.5362	-1.13620	-0.1629	0.1086	0.2715
1	-1.88280	-0.4519	0.3012	0.7531	-1.51286	-0.2384	0.1589	0.3973
1.5	-2.25607	-0.5415	0.3610	0.9024	-1.73300	-0.2950	0.1966	0.4916
15	-4.71790	-1.1312	0.7549	1.8872	-1.10836	-0.8420	0.5614	1.4334
25	-5.16177	-1.2388	0.8559	2.0647	0	-1	0.6667	1.6667
35	-5.40289	-1.2967	0.8645	2.1612	0.90078	-1.0984	0.7323	1.8307
100	-5.89985	-1.4160	0.9340	2.3600	3.634590	-1.3283	0.8856	2.2139

Table I.4: Simulation Results on β

β	c_2	i_c	π_c	y_c	d_2	i_d	π_d	y_d
0	-0.86207	-0.2069	0.1379	0.3448	-0.82304	-0.1111	0.0741	0.1852
0.01	-0.86852	-0.2084	0.1390	0.3474	-0.82856	-0.1120	0.0746	0.1866
0.1	-0.93033	-0.2233	0.1489	0.3721	-0.88072	-0.1201	0.0801	0.2002
0.3	-1.09577	-0.2630	0.1753	0.4383	-1.01353	-0.1417	0.0945	0.2362
0.5	-1.30783	-0.3139	0.2093	0.5231	-1.16892	-0.1687	0.1125	0.2812
0.7	-1.57213	-0.3773	0.2515	0.6289	-1.34005	-0.2013	0.1342	0.3356
0.9	-1.88280	-0.4519	0.3012	0.7531	-1.51286	-0.2384	0.1589	0.3973
0.99	-2.03264	-0.4878	0.3252	0.8131	-1.58669	-0.2559	0.1706	0.4265
1	-2.04951	-0.4919	0.3279	0.8198	-1.59464	-0.2578	0.1719	0.4297

Legend

"c" and "d" denote the commitment and discretionary cases respectively.

$i_c = -k\theta(\lambda-\phi\theta)/\{\phi[(1-\beta)\lambda^2+(k-\beta c_2)\theta^2]\}$, the adjustment coefficient in the expression for Ei_t;

$\pi_c = k\theta^2/[(1-\beta)\lambda^2+(k-\beta c_2)\theta^2]$, the adjustment coefficient in the expression for $E\pi_t$;

$y_c = k\lambda\theta/[(1-\beta)\lambda^2+(k-\beta c_2)\theta^2]$, the adjustment coefficient in the expression for Ey_t;

$i_d = -\theta(\lambda - \phi\theta)/\phi[\theta^2 + \lambda/(k-\beta d_2)]$, the adjustment coefficient in the expression for Ei_t;

$\pi_d = \theta^2/[\theta^2 + \lambda/(k-\beta d_2)]$, the adjustment coefficient in the expression for $E\pi_t$;

$y_d = \lambda\theta/[\theta^2 + \lambda/(k-\beta d_2)]$, the adjustment coefficient in the expression for Ey_t.

Table II: Comparisons between the commitment and discretionary cases
of the adjustment coefficients in the expressions for the random components

Table II.1: Simulation Results on λ

λ	c_2	$i_{c,u}$	$\pi_{c,u}$	$y_{c,u}$	d_2	$i_{d,u}$	$\pi_{d,u}$	$y_{d,u}$
0	0	0.1923	0.9615	-0.1923	0	0.1923	0.9615	-0.1923
0.01	-0.00249	0.1927	0.9615	-0.1927	-0.03758	0.1986	0.9603	-0.1986
0.1	-0.20647	0.2264	0.9547	-0.2264	-0.55047	0.2822	0.9436	-0.2822
0.2	-0.60731	0.2913	0.9417	-0.2913	-0.90615	0.3385	0.9323	-0.3385
0.3	-1.04083	0.3595	0.9281	-0.3595	-1.15873	0.3777	0.9245	-0.3777
0.5	-1.88280	0.4865	0.9027	-0.4865	-1.51286	0.4315	0.9137	-0.4315
0.7	-2.65479	0.5969	0.8806	-0.5969	-1.75851	0.4682	0.9064	-0.4682
0.9	-3.35177	0.6921	0.8616	-0.6921	-1.94267	0.4952	0.9010	-0.4952
1	-3.67326	0.7347	0.8531	-0.7347	-2.01900	0.5063	0.8987	-0.5063

Table II.2: Simulation Results on θ

θ	c_2	$i_{c,u}$	$\pi_{c,u}$	$y_{c,u}$	d_2	$i_{d,u}$	$\pi_{d,u}$	$y_{d,u}$
0	-10	0	1	0	-10	0	1	0
0.01	-9.62779	0.0966	0.9990	-0.0966	-9.54179	0.0958	0.9990	-0.0958
0.1	-3.67326	0.4128	0.9587	-0.4128	-3.22967	0.3760	0.9624	-0.3760
0.2	-1.88280	0.4865	0.9027	-0.4865	-1.51286	0.4315	0.9137	-0.4315
0.4	-0.82348	0.5447	0.7821	-0.5447	-0.51995	0.4755	0.8098	-0.4755
0.5	-0.60732	0.5576	0.7212	-0.5576	-0.32293	0.4879	0.7561	-0.4879
0.7	-0.36890	0.5642	0.6051	-0.5642	-0.11678	0.5018	0.6487	-0.5018
1	-0.20647	0.5425	0.4575	-0.5425	0	0.5	0.5	-0.5
10	-0.00249	0.0990	0.0099	-0.0990	0.00245	0.0990	0.0099	-0.0990

Legend

"c" and "d" denote the commitment and discretionary cases respectively.
$i_{c,u} = \theta(k-\beta c_2)/\{\phi[1 + \theta^2(k-\beta c_2)]\}$, the adjustment coefficient of the random component for i_t;
$\pi_{c,u} = 1/[1 + \theta^2(k-\beta c_2)]$, the adjustment coefficient of the random component for π_t;
$y_{c,u} = -\theta(k-\beta c_2)/[1 + \theta^2(k-\beta c_2)]$, the adjustment coefficient of the random component for y_t;
$i_{d,u} = \theta(k-\beta d_2)/\{\phi[1 + \theta^2(k-\beta d_2)]\}$, the adjustment coefficient of the random component for i_t;
$\pi_{d,u} = 1/[1 + \theta^2(k-\beta d_2)]$, the adjustment coefficient of the random component for π_t;

$y_{d,u} = -\theta(k-\beta d_2)/[1 + \theta^2(k-\beta d_2)]$, the adjustment coefficient of the random component for y_t.

Table II.3: Simulation Results on k

k	c_2	$i_{c,u}$	$\pi_{c,u}$	$y_{c,u}$	d_2	$i_{d,u}$	$\pi_{d,u}$	$y_{d,u}$
0	0	0	1	0	0	0	1	0
0.25	-0.91832	0.2064	0.9587	-0.2064	-0.80742	0.1880	0.9624	-0.1880
0.5	-1.34041	0.3195	0.9361	-0.3195	-1.13620	0.2870	0.9426	-0.2870
1	-1.88280	0.4865	0.9027	-0.4865	-1.51286	0.4315	0.9137	-0.4315
1.5	-2.25607	0.6187	0.8763	-0.6187	-1.73300	0.5452	0.8910	-0.5452
15	-4.71790	2.1749	0.5650	-2.1749	-1.10836	1.9510	0.6097	-1.9510
25	-5.16177	2.7125	0.4575	-2.7125	0	2.5	0.5	-2.5
35	-5.40289	3.0729	0.3854	-3.0729	0.90078	2.8881	0.4224	-2.8881
100	-5.89985	4.0475	0.1919	-4.0475	3.634590	3.9731	0.2054	-3.9731

Table II.4: Simulation Results on β

β	c_2	$i_{c,u}$	$\pi_{c,u}$	$y_{c,u}$	d_2	$i_{d,u}$	$\pi_{d,u}$	$y_{d,u}$
0	-0.86207	0.1923	0.9615	-0.1923	-0.82304	0.1923	0.9615	-0.1923
0.01	-0.86852	0.1939	0.9612	-0.1939	-0.82856	0.1938	0.9612	-0.1938
0.1	-0.93033	0.2094	0.9581	-0.2094	-0.88072	0.2085	0.9583	-0.2085
0.3	-1.09577	0.2523	0.9495	-0.2523	-1.01353	0.2479	0.9504	-0.2479
0.5	-1.30783	0.3103	0.9379	-0.3103	-1.16892	0.2980	0.9404	-0.2980
0.7	-1.57213	0.3875	0.9225	-0.3875	-1.34005	0.3597	0.9281	-0.3597
0.9	-1.88280	0.4865	0.9027	-0.4865	-1.51286	0.4315	0.9137	-0.4315
0.99	-2.03264	0.5377	0.8925	-0.5377	-1.58669	0.4662	0.9068	-0.4662
1	-2.04951	0.5436	0.8913	-0.5436	-1.59464	0.4701	0.9060	-0.4701

Legend

"c" and "d" denote the commitment and discretionary cases respectively.

$i_{c,u} = \theta(k-\beta c_2)/\{\phi[1 + \theta^2(k-\beta c_2)]\}$, the coefficient in front of u_t in the expression of i_t for the commitment case;

$\pi_{c,u} = 1/[1 + \theta^2(k-\beta c_2)]$, the coefficient in front of u_t in the expression of π_t for the commitment case;

$y_{c,u} = -\theta(k-\beta c_2)/[1 + \theta^2(k-\beta c_2)]$, the coefficient in front of u_t in the expression of y_t for the commitment case;

$i_{d,u} = \theta(k-\beta d_2)/\{\phi[1 + \theta^2(k-\beta d_2)]\}$, the coefficient in front of u_t in the expression of i_t for the discretionary case;

$\pi_{d,u} = 1/[1 + \theta^2(k-\beta d_2)]$, the coefficient in front of u_t in the expression of π_t for the discretionary case;

$y_{d,u} = -\theta(k-\beta d_2)/[1 + \theta^2(k-\beta d_2)]$, the coefficient in front of u_t in the expression of y_t for the discretionary case.