Pareto-improving Asymmetric Information in a Dynamic Insurance Market

By

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DISCUSSION PAPER 266

June 1997

FINANCIAL MARKETS GROUP

AN ESRC RESEARCH CENTRE

LONDON SCHOOL OF ECONOMICS

E·S·R·C ECONOMIC & SOCIAL RESEARCH COUNCIL

ISSN 0956-8549-266

Competition through Contracts, Incomplete Information and Learning in a Dynamic Insurance Market with one-sided Commitment

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June 13, 1997

[1]I am very grateful to participants to the Applied Microeconomics Seminar at UCL, to whom I had the opportunity to present the first version of this paper, and particularly to Costas Meghir. I want to thank, too, those who made useful remarks at séminaire Fourgeaud in Paris, as well as at student seminars at LSE, CORE, and a Journée DELTA in Paris — specially Roger Guesnerie and Alan Manning. I also emphasize my debt to participants to the 23rd Seminar of the European Group of Risk and Insurance, in particular Keith Crocker, Georges Dionne, Ray Rees and Arthur Snow, who gave me interesting comments on an earlier version. Last but not least, I thank Paul Beaudry, Alberto Bisin, Markus Brunnermeier, Pierre-André Chiappori, Leonardo Felli, John Moore, Thomas Piketty, Kevin Roberts, Jean-Charles Rochet and David Webb from whom I benefited interesting discussions, insightful comments and encouragements. Support from the French Federation of Insurance Companies (FFSA) is gratefully acknowlegded. All errors are mine.

Contents

Abstract

This paper explores the dynamics of insurance markets under incomplete information. Various information structures are examined, according to the degree of communication between companies. We get equilibrium existence even when adverse selection arises through differentiated learning. This and the Pareto-dominance of private information structures seem to mitigate the prevalent view that adverse selection and competition do not match well in insurance markets ; moreover, it provides a new scope for empirical studies. Technically, we extend to dynamics Rothschild-Stiglitz' equilibrium concept, and get to reconsider the "no-malus" property, which we prove to result from the non-consideration of feed-back effects of future on present.

Keywords : *bonus/malus, information transmission, learning, one-sided commitment, switching.*

1 Introduction

Contract theory has much expanded since the seminal contributions in the late seventies, and comprehensive studies have been led in the Principal-Agent setting ; however, competition in dynamic insurance markets has not received much attention so far, and many important questions remain unanswered : we have little to say on the shape of equilibrium contracts and on optimal informational structures. On a more theoretical ground, we do not quite know how the famous Rothschild-Stiglitz (from now on : R-St) no-existence result [21] mixes with dynamics.

This article aims to give some pieces of answer to these questions. We consider a world of pure incomplete information, leaving aside issues of incentives ; this assumption and the simplicity of our model will allow us to understand quite clearly some mechanisms of dynamic markets.

In short, our model deals with competition in dynamic insurance markets under one-sided commitment and incomplete but initially symmetric information [1]. That is, we take the view that agents differ through their probabilities of accident — their "types" — and cannot engage in type-improving activities. In addition, each agent together with his initial insurer learns about his own type through the history of his accidents ; however, it needs not be the case of the other insurers, depending on the information structure. As a result, an asymmetry of information may arise endogenously after one period has elapsed. Our main results are that (i) an equilibrium always exists, even when adverse selection arises, (ii) in that case, equilibrium contracts do not display the usual "no-malus" property, but rather exhibit bonus *and* malus, and (iii) no-communication structures are strictly Pareto-dominant. They rely mainly on the *two-way* interaction between present and future, which had not been taken into account in previous studies.

(i) leads to reconsider the scope of R-St' result [2], for it states that provided adverse selection comes from differentiated learning, dynamics restores the existence of an equilibrium in insurance markets. Moreover, creating adverse selection in such an endogenous way is strictly Pareto-improving (iii) ; thus, adverse selection and competition in insurance markets match fairly well here, as opposed to the conventional wisdom.

Policy implications include fostering commitment, and more importantly making information about accident claims private [3]. The model also provides a rationale for

[1] A strong incentive to build such a model is that car insurers do *not* believe in adverse selection a priori, but rather in learning. Moreover, recent empirical results [2] [6] also push towards such a framework, as developed in a few pages.

[2] A more technical analysis proves that the "Nash" and the "monopolistic competition" interpretations of the R-St equilibrium concept are no more equivalent in a dynamic setting — details available on request.

[3] This issue of optimal information transmission is particularly relevant nowadays in Europe, in the prospect of a possible uniformization of insurance systems across countries.

existing bonus/malus contracts.

Our work takes it roots in different streams of the literature. In what follows we review them briefly.

- **Equilibrium existence in insurance markets with adverse selection.** A very prolific literature has followed the problematic no-existence result of R-St ; rather than reviewing it extensively [4], let us pick up in the literature the contributions that are most relevant for our purposes.
  - Wilson [22] derived simultaneously with R-St the inexistence result, and proposed an alternative concept : an *anticipative* equilibrium, in which companies are allowed, after each agent has choosed a contract, to withdraw any contract that makes losses ; then, existence is restored.
    This new concept has however been much criticized, on the grounds that (1) real insurance markets do not behave like that, and (2) according to that withdrawal process, agents may be left without any insurance policy.
  - Riley's [20] bargaining game also restores existence (for companies are allowed to react by adding more contracts), but does not lead to the same output as Wilson's anticipative concept. Again, it does not seem very close to the insurance market's behaviour ; however, in a dynamic model of managerial labour markets, it could be thought as a good modelization of the bargaining process between managers and outside firms after one period has elapsed, as it has been argued by Ricart i Costa [19] [5].
  - Hellwig [14] rationalized these two concepts in terms of refined [6] sequential equilibria of some three stages games.
  - More recently, some dynamic models of insurance markets under pure adverse selection — Nilssen [17] (no-commitment), and Dionne-Doherty [5] (commitment and renegotiation) — aimed to give insights on the shape of equilibrium contracts. Unfortunately, their approach was basically static, for the second-period outcome was specified *exogenously.* To be able to fix the second-period outcome equal to the static R-St equilibrium, they had on the one hand to assume the latter to exist — so that their models give no clues as to the existence of equilibrium — and on the other hand to restrict to no-communication structures [7], leaving aside the possibility

[4] See Henriet-Rochet [12] or Crocker-Snow [3] for more details.

[5] In [7], we use this bargaining modelization in a simple labour market model to prove that the usual "downward-rigid wages" result of this literature may fail to hold.

[6] Using Kohlberg and Mertens [15] "stability" refinement criterion.

[7] From a practical point of view, assuming no communication of accident claims is not problematic : many countries use such a device. However, supposing information about past contracts private seems less realistic (at least it would require some regulation), and to my knowledge there are no countries in which it has been implemented.

to compare informational structures. The exogenous specification does not lead to inconsistent results, but in Dionne-Doherty the final menu of contracts is not always renegotiation-proof, due to the private information assumption, which limits the scope of their result on the shape of equilibrium contracts[8]. Thus we get no answer to our questions here.

- **Optimal labour contracts.**
  This literature, mainly based on Harris-Holmstrom [11], aims to characterize the shape that equilibrium long-term labour contracts should exhibit, assuming information incomplete but initially symmetric as well as perfect observability of outputs by all the market ; in the paper quoted just above, the authors isolate the effect of the desire for insurance on the shape of contracts, given that only firms can credibly commit to long-term contracts. They find out an option against bad outputs, that is to say downward-rigid wages or equivalently no-malus. Much research has arised from this initial result. In particular, and among other directions, effort has been introduced in the setting, in order to explore the effects of incentives and career concerns (see for instance Gibbons-Murphy [10]). Another idea was to extend it to the case where information is not transmitted between companies (Ricart i Costa [19], who also studies the joint effects of desire for insurance and managerial task assignments[9]). See Gibbons [9] for a very good review of both theoretical and empirical work on the whole subject. As noted in footnotes, our paper reconsiders some results on the shape of equilibrium contracts.

- **The value of information.**
  Initiated by J.Hirshleifer in 1971 [13] and Marshall [16], this literature could be summarized as follows (this is of course highly simplifying !) : when information is incomplete but initially symmetric, and without production[10], the *social* value of having information revealed to *everyone* is negative ; however, when information is privately revealed to only *some* agents, the social value of

[8]In a note on dynamic adverse selection, I give some results on the shape of equilibrium contracts under *public* information with an *endogenous* equilibrium specification and various assumptions on commitment — details available on request.

[9]Although providing an excellent intuition on the relationship between intra-task and across-tasks wage differentials, this paper has somehow overlooked the derivation of the shape of equilibrium contracts within a task ; to be more precise, it considers a bargaining approach *à la* Riley but assumes the outcome of this bargaining process independant from the reservation long-term contracts issued at the beginning of the initial period. This *exogenous* approach is quite similar to that used later on by Nilssen and Dionne-Doherty, but here it leads to a time-inconsistent result — see [7] for an alternative result.

[10]Then, most of the papers consider an economy with production, which mitigates the results ; for instance it has been pointed out that, in an insurance market, risk-categorizing may not be purely detrimental, for knowing one's risk allows for engaging in a productive actvity (care) that reduces your risk. Another line of research has argued that when agents have already some private information *prior to contracting*, the additional information revealed posterior to contracting could well have social value (Crocker-Snow [4]).

having it made public is either negative or positive, depending on the values of parameters.
Thus, it could be thought that our result of Pareto-dominance of private information structures is in contradiction with this literature ; for in our model, information is privately revealed to only some agents, and having it made public is always strictly Pareto-worsening[11], whatever the values of parameters. This is however not inconsistent, as we will soon show.

- **Insurance literature.**
  Some papers, related to the literature quoted above, have explored the dynamics of contracts in an insurance market where information is initially symmetric (Boyer-Dionne-Kihlstrom [1], Palfrey-Spatt [18]), but they confined their study to the case in which information is public and nobody (or everybody) is able to commit. Both state that experience-rating is welfare-decreasing in this context. The former explores what one can say in an insurance market with production. The latter incorporate care in its study. We depart from these analyses by considering one-sided commitment[12], which corresponds best to the reality and has remained unexplored so far.

- **Empirical Contract Theory**
  There is a more recent and growing literature on empirical contract theory, among which we pick two papers particularly relevant for our study. In Chiappori-Salanié [2], the authors test the existence of adverse selection using data from french car insurance companies, and reject the presence of adverse selection. On the other hand, Dionne, Gourieroux and Vanasse [6] get the opposite conclusion from Canadian data ! However, these two results, although apparently incompatible with one another, can both be understood and recovered in our framework, which provides a rationale for such a symmetric information (rather than adverse selection a priori) modelization of car insurance markets. Indeed, suppose that information is initially symmetric, and consider what happens after one period has elapsed : if information is shared by companies — as it is the case in France — then information will remain symmetric, and no adverse selection will arise[13], but if there is no information transmission between companies (as in Quebec), then adverse selection will arise endogenously, which may explain why Dionne, Gourieroux and Vanasse do find evidence of adverse selection.

  But if empirics are present at the basis of this article by providing a strong rationale for learning models, the converse is also true ! For we will see in

[11]This implies that the social value of information is negative.

[12]In a small note, we also derive the much easier case of no-commitment and private information, details available on request.

[13]Of course, in the real life, policyholders may learn more than insurers about their own type. But, as we will prove later on, the crucial issue is to know whether this other differentiated learning is significant or not.

these pages the crucial necessity to be able to know the *nature*[14] of adverse selection in such and such insurance market in order to make predictions on optimal regulation policies and on the efficiency of competition.

In section 2, we explicit the model ; the next two sections are devoted to the resolution of the model when information about accident claims is transmitted or not (respectively), and section 5 concludes.

2 Model

We consider a two-period model[15], in which some insurers (at least two of them) compete through contracts to attract potential policyholders. These form a continuum of individuals of mass 1, distributed according to a law of probability F on $[0, 1]$ which reflects the distribution of the probability p of having an accident each period. We refer to p as the *type* of agents. It does not vary across time for a given individual, and we assume that people do not know their own type, but only F which is common knowledge : therefore, as announced in the introduction, the information about types is incomplete *but initially symmetric.*

From now on, let us specify the distribution F as follows :

$$F = \lambda \delta_{p_L} + (1 - \lambda)\delta_{p_H}$$

which means that individuals can either be good risks (with type $p = p_L$) or bad risks (with type $p = p_H > p_L$), and that the proportion of good risks in the population is $\lambda \in]0, 1[$. This assumption entails no loss of generality at all[16], and allows for much more clarity of exposition.

Throughout the paper, we will denote by (A) the event *accident,* by D the amount of damage in case of the event (A), and by (N) the complementary event *no accident*[17]. $\bar{p}$ designs the average probability of accident on the whole population :

$$\bar{p} = \int_0^1 p dF(p) = \lambda p_L + (1 - \lambda) p_H$$

[14] In a sense that I will precise later on.

[15] The restriction to two periods has been widely used by contract-theoretists, for it is often the only way to get a tractable model in which one can give insights about the shape of dynamic contracts. Note, in addition, that in our case, it may even have some direct practical relevance, since in some states of the US, experience-rating is limited by law to 3 years. Remark also that when information is public, it is straightforward to extend the model (and the results) to any finite number of periods ; it is also likely that the results extend in the private information case (work in progress), even though it is then very difficult to characterize the outcome of the insurance market as precisely as in the two-period model.

[16] This is because the average probability of accident $\bar{p}$ is the only relevant information contained in the entire distribution F, as one can check easily.

[17] In the real life, there may be more than two relevant events (consider for instance car insurance) ; but, in that case, we can reinterpret (A) as *many accidents,* and (N) as *few accidents,* which tends to show that the commonly used *two events assumption* is not very restrictive.

After one period has elapsed, each individual, as well as his insurer, updates his beliefs about p according to Bayes' rule ; and only when information about accident claims is public do other insurers also update their priors. Bayesian updating yields a new distribution, characterized by the value of λ for each event that may happen during the first period, that is to say :

$$\lambda^A = \lambda \frac{p_L}{\bar{p}} < \lambda$$

$$\lambda^N = \lambda \frac{1 - p_L}{1 - \bar{p}} > \lambda$$

λ^A (respectively λ^N) is the proportion of good risks among the sub-population A (respectively N) of individuals who had an accident (respectively no accident) at t=1. λ^A and λ^N fully summarize the learning process, and in turn give us the average probability of accident for each sub-population :

$$\bar{p}^A = \lambda^A p_L + (1 - \lambda^A) p_H > \bar{p}$$

$$\bar{p}^N = \lambda^N p_L + (1 - \lambda^N) p_H < \bar{p}$$

$\bar{p}$, $\bar{p}^A$ and $\bar{p}^N$ are linked through the "martingale property" : $\bar{p} = \bar{p}\bar{p}^A + (1 - \bar{p})\bar{p}^N$, which states that the expectation of tommorow's type conditional to today's information is equal to today's expected type [18].

All insurers are assumed to be risk-neutral, whereas individuals are risk-adverse; we represent their preferences by a time-separable von Neumann-Morgenstern utility function U, that we suppose to be twice continuously differentiable, strictly increasing and strictly concave ; thus : $U" < 0 < U'$. We denote by δ the discount factor, which is the same for everyone.

At the beginning of each period, all individuals are paid the same wage W [19], and can sign a contract with any insurer. At t=0, contracts can cover both periods, in which case we speak of *long-term contracts*. We assume that each agent can get insurance from one insurer at a time, and that there is a perfect equivalence between accidents and claims [20]. Insurers, on their side, compete through contracts to attract policyholders. We extend the Rothschild-Stiglitz equilibrium concept to dynamics :

definition : *an equilibrium of the two-period insurance market is a set $\mathcal{E}$ of long-term contracts such that*

(a) *no long-term contract(s) can be added to $\mathcal{E}$ at t=0 and make positive profits,*

[18] This is a general property of Bayesian learning processes.

[19] One can view W as what is left after all purchases other than insurance premiums have been paid.

[20] We view the issue of hiding accidents as something important per se and worth studying, but leave it aside in this paper ; relaxing this assumption is part of our agenda for future research.

and

(b) *no short-term contract*[21] *can be added to $\mathcal{E}$ at t=1 and make positive profits.*

As stressed in the introduction, we focus on one-sided commitment and renegotiation : insurers can commit to long-term contracts, but cannot precommit not to renegotiate them at t=1. On the contrary, individuals are assumed to be unable to commit not to switch to another insurance company after the end of the first period, which means that, at t=1, they will choose either to stick to their long-term contract or to switch to a short-term contract (covering the remaining period). We also make the following two technical assumptions:

(A1) Individuals who are indifferent between sticking to their initial long-term contract and switching to a short-term contract do not switch.

(A2) When agents from the sub-population A are indifferent between two contracts including one exhibiting full coverage, they choose the latter.

A contract is characterized by a couple $C = (\alpha, \beta)$ for each relevant event of each period concerned : α designs the reimbursment net of premium when an accident occurs, and β represents the premium paid by the policyholder to the insurer. Thus, given that he has signed a contract (α, β), an agent's wealth is $W - D + \alpha$ in the event (A), and $W - \beta$ else. In particular, a contract (α, β) exhibits full coverage if and only if $\alpha + \beta = D$; overinsurance (ie : $\alpha + \beta > D$) is discarded by assumption. A long-term contract is a sextuplet $(\alpha_1, \beta_1; \alpha_2^N, \beta_2^N; \alpha_2^A, \beta_2^A)$ — with evident notations.

We denote by

- $\{C_{RS}^A = (\alpha_{RS}^A, \beta_{RS}^A); C_{RS}^N = (\alpha_{RS}^N, \beta_{RS}^N)\}$ the R-St separating pair of contracts which corresponds to the two sub-populations A and N (at t=1). C_{RS}^A is the "full-insurance at fair-odds" contract for the sub population A, while C_{RS}^N breaks-even on the sub-population N and makes the A just indifferent between both contracts.

- $\bar{C}$ (respectively $\bar{C^A}$,$\bar{C^N}$) the full-insurance contract that breaks even on the whole population (respectively : the sub-population A, the sub-population N):

$$\bar{C} = (\bar{p}D, (1 - \bar{p})D),\ \bar{C^A} = (\bar{p^A}D, (1 - \bar{p^A})D),\ \bar{C^N} = (\bar{p^N}D, (1 - \bar{p^N})D)$$

- C^* the contract most preferred by the sub-population N among all contracts that break even on the whole population.

[21] We deliberately ruled out pairs of contracts here : this will make the exposition much more clear. It must be stressed that it is not a restrictive assumption, for the shape of results would remain unchanged else — details available on request.

We can view these particular contracts on the following diagram :

FIGURE 1 ABOUT HERE

Note that for a given sub-population, characterized by an average probability of accident p, each indifference curve is upward-sloping, with a minimal slope equal to $\frac{p}{1-p}$ at the point where it crosses the full-insurance line ; at that point too, it is tangent to an isoprofit line. Utility (respectively, profit) increases when contracts go to the South-East (respectively, North-West), as summarized on figure 2 :

FIGURE 2 ABOUT HERE

3 Perfect communication : public information

Throughout this section, we assume that information about accident claims submitted during the first period by individuals is shared by all insurance companies[22]. Therefore, any individual wishing to switch to another insurer at t=1 will be known by the latter as belonging to the sub-population A or N — depending on whether he had an accident during the first period or not —, which means that the past history of accidents of an agent switches with him. As a result, the information remains symmetric during the second period : no firm has any informational advantage over the others, and — more crucially — no agent has more information than a company at any moment.

3.1 Characterizing equilibria

The following result states a perfect equivalence between equilibria of the market and solutions to a maximization problem :

Proposition 1 *Given an equilibrium $\mathcal{E}$ of the insurance market, every contract in $\mathcal{E}$ satisfies the following maximization problem :*

$$(\mathcal{M}1) \quad \max_{C_{LT}=(C_1;C_2^N,C_2^A)} \{\mathbf{E}_{\bar{p}}[Ut|C_1] + \delta[(1-\bar{p})\mathbf{E}_{\bar{p}^N}[Ut|C_2^N] + \bar{p}\mathbf{E}_{\bar{p}^A}[Ut|C_2^A]]\}$$

$$\text{s.t.} \quad \begin{cases} \mathbf{E}_{\bar{p}}[\Pi|C_1] + \delta[(1-\bar{p})\mathbf{E}_{\bar{p}^N}[\Pi|C_2^N] + \bar{p}\mathbf{E}_{\bar{p}^A}[\Pi|C_2^A]] = 0 & (1) \\ \mathbf{E}_{\bar{p}^N}[Ut|C_2^N] \geq \mathbf{E}_{\bar{p}^N}[Ut|\bar{C}^N] & (2) \\ \mathbf{E}_{\bar{p}^A}[Ut|C_2^A] \geq \mathbf{E}_{\bar{p}^A}[Ut|\bar{C}^A] & (3) \end{cases}$$

Conversely, any solution of ($\mathcal{M}1$) is an equilibrium of the market.

Remark : $\mathbf{E}_{\bar{p}}[Ut|C_1]$ is a short-hand notation for $(1-\bar{p})U(W-\beta_1) + \bar{p}U(W-D+\alpha_1)$, and so on.

[22]We also assume, throughout the paper, that information about past contracts is public.

Let us interpret the maximization problem ($\mathcal{M}1$) : it says that the market behaviour results into a long-term contract (or a set of long-term contracts) that maximizes individuals' expected discounted utility subject to three constraints : (1) means that acompanies make zero discounted expected profits (which results from competition at t=0, as we will prove in a few lines), and (2) and (3) form a no-switching condition at t=1.

Prior to the proof of proposition 1, let us remark that any equilibrium of the market must display *full-coverage at each date* : if not, it would be dominated by some triplet of contracts giving as much profits to insurers and strictly more expected utility to individuals. As a consequence, renegotiation-proofness follows.

Proof of proposition 1 :

- Consider a (possible) equilibrium $\mathcal{E}$, and proceed by contradiction : let $C_{LT} \equiv (C_1; C_2^N, C_2^A)$ be one of the long-term contracts in $\mathcal{E}$. We assume that C_{LT} does not solve ($\mathcal{M}1$) ; this could be for three different reasons:

  (i) First, C_{LT} could satisfy (1),(2) and (3) but not maximization ; then, there would exist a long-term contract C'_{LT} giving strictly more expected discounted utility to individuals while still making non-negative profits and being robust to entry at t=1 ; and therefore, there would also exist a long-term contract $C"_{LT}$ providing strictly more expected discounted utility to individuals and strictly more profits to any entrant offering it at t=0, while being robust to entry at t=1. This contract would then be preferred to C_{LT} by all individuals, which provides a contradiction.

  (ii) Now, let us assume that C_{LT} violates (1) : either it induces losses — in which case companies would rather offer no contract at all — or it induces positive profits ; in the latter case, it is dominated by a contract $C'_{LT} \equiv (C'_1; C_2^N, C_2^A)$, where C'_1 exhibits full-coverage and is such that overall profits are slightly lower (although still positive) than for C_1. Therefore : $\mathbf{E}_{\bar{p}}[\Pi|C'_1] < \mathbf{E}_{\bar{p}}[\Pi|C_1]$ which implies, given that both C_1 and C'_1 display full-insurance, that $\mathbf{E}_{\bar{p}}[Ut|C'_1] > \mathbf{E}_{\bar{p}}[Ut|C_1]$; this in turn proves that C'_{LT} is strictly preferred to C_{LT} by individuals, which again provides a contradiction, for any entrant at t=0 offering C'_{LT} would make positive profits.

  (iii) Eventually, let us suppose that C_{LT} violates (2) (the case in which (3) is violated is strictly identical to this one) ; then, any entrant at t=1 can make positive profits by offering to the sub-population N a contract C_{ST}^N just between C_{LT}^2 and $\bar{C}^N$, which is absurd.

- Conversely, we must prove that any solution of $(\mathcal{M}1)$ is an equilibrium of the market, which turns out to be quite clear once we have solved $(\mathcal{M}1)$:

  Proposition 2 $(\mathcal{M}1)$ *has a unique solution* $\{C_1; C_2^N, C_2^A\}$ *which is characterized by :* $C_2^N = \bar{C}^N, C_1 = C_2^A$ *and (1). The constraint (3) is not binding at the optimum.*

  Proof of proposition 2 : In a first step, we solve $(\mathcal{M}1)$ without taking (3) into account : afterwards, we will have to check that (3) is satisfied at the optimum.

  Note that $(\mathcal{M}1)$ has only three variables — $\beta_1, \beta_2^N, \beta_2^A$ —, for all contracts exhibit full-coverage.

  The Lagrangian associated to $(\mathcal{M}1)$ writes:

  $\mathcal{L} = \{Exp.disc.utility\} + \mu\{Exp.disc.profits\} + \nu\{\mathbf{E}_{\bar{p}^N}[Ut|C_2^N] - \mathbf{E}_{\bar{p}^N}[Ut|\bar{C}^N]\}$

  As the problem is concave, the first-order conditions are necessary and sufficient conditions. Let us state them:

$$\left\{\begin{array}{lcl}
\frac{\partial \mathcal{L}}{\partial \beta_1} = 0 = -U'(W-\beta_1) + \mu & \iff & U'(W-\beta_1) = \mu \\
\frac{\partial \mathcal{L}}{\partial \beta_2^A} = 0 = -U'(W-\beta_2^A)\bar{p} + \mu\bar{p} & \iff & U'(W-\beta_2^A) = \mu \\
\frac{\partial \mathcal{L}}{\partial \beta_2^N} = 0 = -U'(W-\beta_2^N)(1-\bar{p}) & \iff & U'(W-\beta_2^N) = \frac{\mu}{1+\frac{\nu}{1-\bar{p}}} \\
\qquad +\mu(1-\bar{p}) - \nu U'(W-\beta_2^N) & & \\
\qquad (1) & \iff & (\beta_1 - \bar{p}D) + \bar{p}(\beta_2 A - \bar{p}^A D) \\
 & & +(1-\bar{p})(\beta_2 N - \bar{p}^N D) = 0 \\
\qquad (2) & \iff & U(W-\beta_2^N) \geq U(W-\bar{\beta}^N)
\end{array}\right.$$

  The first two equations imply that $\beta_1 = \beta_2^A$, ie:

$$\boxed{C_1 = C_2^A}$$

  If (2) were not binding, we would then get $\beta_2^N < \bar{\beta}^N$ and $\nu = 0$, which in turn would imply, due to the first three equations : $\bar{\beta}^N > \bar{p}D$, which is absurd (recall that $\bar{\beta}^N = \bar{p}^N D$ and that $\bar{p}^N < \bar{p}$). Therefore, (2) is binding, that is to say:

$$\boxed{C_2^N = \bar{C}^N}$$

  To put the final touch to this proof, we need to check that (3) is not binding ; using $C_1 = C_2^A$ and $C_2^N = \bar{C}^N$, we can get from (1) the exact value of $\beta_1 = \beta_2^A$:

$$\beta_1 = \beta_2^A = \frac{(\bar{p}D) + \bar{p}(\bar{p}^A D)}{1+\bar{p}} \in]\bar{p}D; \bar{p}^A D[$$

  $\beta_2^A < \bar{p}^A D$ precisely states that (3) is not binding. *Q.e.d.*

3.2 Interpretation of the result

In a first step, let us characterize the situation in which both the insurer and the policyholder are able to commit to long-term contracts : it will be useful to take it as a benchmark during our attempt to interpret the results. The full-commitment equilibrium is no more than the repetition of the optimal static contract ($C_1 = C_2^N = C_2^A = \bar{C}$) : it achieves perfect insurance, by transfering utility from the state (N) to the state (A).

Let us now give some intuition for the fact that (2) is binding, while (3) is not : allowing agents to switch implies, for each insurance company, the threat of an entrant creaming-off all the sub-population N at t=1 ; and only this sub-population has an incentive to switch, for they are (in average) more likely to be good risks and — thanks to the transmission of information — are known as such. On the contrary, individuals having submitted a claim during the first period have nothing to gain from switching, since short-term contracts cannot insure them against the event (A) once it has already occured, whereas long-term contracts could have provided this insurance at t=0.

This also allows us to understand why, contrary to the full-commitment case, companies are no more able to transfer utility from the state (N) to the state (A), but rather *are bound to offer a bonus in the event (N)* (ie : $\beta_2^N \leq \bar{\beta}^N$). However, they are still able to transfer utility from [t=1] to [t=2,(A)], which means that at t=0, individuals can buy insurance against the risk of having an accident, by paying a little more than for the optimal static contract at the first period ; but they cannot any more get insured against the "risk" of having no accident. In particular, we observe that the "no-malus" property is satisfied here.

We can view the equilibrium contract on figure 3:

FIGURE 3 ABOUT HERE

Following the intuition above, we can reinterpret this long-term contract as an option [23] : on figure 4, we can view the price of this option (in terms of utility) ; paying this price at t=0 enables the individuals to preserve themselves against the risk of having an accident.

FIGURE 4 ABOUT HERE

From this resolution of ($\mathcal{M}1$), we can infer that its only solution is actually an equilibrium of the market, which completes the proof of proposition 1.

To conclude this section, let us remark that all its results are robust to the

[23] This interpretation is similar to that in Harris and Holmström [11].

assumptions we can make on information about contracts ; even if contracts were supposed not to be transmitted to rival companies, there would be a unique equilibrium of the market : the very same as when information about past contracts is public.

4 No communication : private information

In this section, we assume on the contrary that information about past accident claims is *not* shared by insurance companies [24]. And, as in the previous section, we try to explore equilibria of the insurance market.

The (new) crucial feature of the model, here, is that at t=1, an asymmetry of information arises between on the one hand each agent together with the company he choosed at t=0, and on the other hand the other companies — including potential entrants —, for the latter do not learn about the characteristics of the agent through accident claims, whereas the former do.

4.1 Characterizing equilibria

As in last section, we try in a first step to restate the problem in terms of a maximization problem. Let us introduce some additional notations :

Condition ($\mathcal{A}$) : *the indifference curve of the sub-population N which includes the point C_{RS}^N does not cross the zero-profit line for the whole population* [25].

[24]At this stage, we must examine the practical feasibility of such an assumption, that is check that policyholders will not be able to credibly reveal their information to prospective employers. Those from the sub-population (A) could indeed, by providing some evidence of their accident, but they would be worse-off afterwards On the other hand, the N's would like very much to be known as such, but they have no means to do so.

Note that the driving force for all this is the *nature* of the transfers, namely : a premium β, and a gross reimbursment $\alpha + \beta$ in case of accident. As β is paid in any case, the only way to distinguish yourself from the other sub-population is to prove to have been paid $\alpha + \beta$ (which makes you worse-off) or *not* to have been paid $\alpha + \beta$ (which is unfeasible).

It is worth noting that it would no more be true in a labour market specification of the model : then, transfers are one-way (wages) in any case, and may be used to reveal oneself to prospective employers — see [7] for such a labour market model.

[25]Note that condition ($\mathcal{A}$) is quite similar to R-St existence condition, once we replace p_L, p_H, λ by $\bar{p}^N, \bar{p}^A, \bar{p}$ respectively. But, as $\bar{p}^N, \bar{p}^A$ and $\bar{p}$ are linked by the formula:

$$\bar{p} = \bar{p}\bar{p}^A + (1 - \bar{p})\bar{p}^N$$

(martingale property of Bayesian updating processes), it is a rather intricate problem to know under which conditions on parameters condition ($\mathcal{A}$) is satisfied. This question can be shown to be equivalent to the following one : in the Rothschild and Stiglitz' static model, is $\frac{p_L}{1-p_H}$ larger or lower than δ_1^{RS} ?

For a generic long-term contract (C_1, C^N, C^A), we denote by (IC^N) (respectively, (IC^A)) the indifference curve for the sub-population N (respectively, A) which contains the point C^N (respectively, C^A). $(\Pi = 0)$ designs the zero-profit line when all the population is concerned ; in the same way, we define $(\Pi^N = 0)$ and $(\Pi^A = 0)$.

Let (IC^{N*}) be the indifference curve (for N) which is tangent to the line $(\Pi = 0)$. (IC^{N*}) and $(\Pi^N = 0)$ intersect at $C^{N*} = (\alpha_N^*, \beta_N^*)$ from which one can draw an indifference curve for A : we call it (IC^{A*}).

FIGURE 5 ABOUT HERE

All this allows to state more clearly our results and, to begin with, proposition 3, which characterizes equilibria of the market.

Proposition 3

- *Let $\mathcal{E}$ be a possible equilibrium of the market ; then, each long-term contract C_{LT} in $\mathcal{E}$ is a solution of the maximization problem ($\mathcal{M}2$) which consists in maximizing the expected discounted utility of individuals subject to (i), (ii), (iii) and (iv) :*

  (i) (IC^A) and (IC^N) intersect on the line $(\Pi^N = 0)$.

  (ii) C_{LT} breaks-even.

  (iii) (IC^N) and $(\Pi = 0)$ do not intersect.

  (iv) (IC^A) and $(\Pi^A = 0)$ do not intersect.

- *Conversely, any solution of ($\mathcal{M}2$) is an equilibrium of the market.*

Note that a necessary condition for (iii) to bind is : $(\mathcal{A})$ unsatisfied.

Proof of proposition 3 : first of all, remark that all contracts still exhibit full-coverage, so that renegotiation-proofness follows. Then consider a long-term contract $C_{LT} = (C_1, C_2^N, C_2^A)$ in a possible equilibrium $\mathcal{E}$; the same proof as in the previous chapter ensures that maximization and conditions (ii) and (iv) hold. Only (i) and (iii) remain to be proved ; again, we proceed by contradiction:

- First, suppose that (IC^N) and (IC^A) intersect over the line $(\Pi^N = 0)$: we can view this situation on figure 6:

FIGURE 6 ABOUT HERE

Thus, any entrant offering a contract C' inside the shaded area would make positive profits (by creaming-off the sub-population N). This provides a contradiction [26].

- Consider now the case in which (IC^N) and (IC^A) intersect under the line $(\Pi^N = 0)$. Then : let C_1 unchanged, decrease a little ${\beta_2}^A$ (${\beta'_2}^A = {\beta_2}^A - \delta\beta$) and increase ${\beta_2}^N$ so that (ii) — and (iii) if needed — remains satisfied : ${\beta'_2}^N = \frac{\bar{p}}{1-\bar{p}}\delta\beta + {\beta_2}^N$. This leads to a long-term contract C'_{LT} providing strictly more expected discounted utility than C_{LT}, as we shall prove it now:

  let us write a first-order approximation of the ex-ante expected utility differential δU between C'_{LT} and C_{LT}:

$$\delta U \simeq -(1-\bar{p})U'(W-\beta_2^N) \times \frac{\bar{p}}{1-\bar{p}}\delta\beta + \bar{p}U'(W-\beta_2^A)\delta\beta \quad ,$$

$$ie \;\; : \quad \delta U \simeq [U'(W-\beta_2^A) - U'(W-\beta_2^N)]\bar{p}\delta\beta$$

  As U' is strictly decreasing, we need to show that $\beta_2^N < \beta_2^A$ in order to get $\delta U > 0$, therefore a contradiction and the first part of the proof completed.

  Then derive a second-order approximation of the ex-ante utility differential ΔU between $C"_{LT}$ and C_{LT} , using a second-order Taylor expansion:

$$\begin{aligned}
\Delta U \;\; \simeq & \quad (1-\bar{p})U'(W-\beta_2^N) \times \Delta\beta - \bar{p}U'(W-\beta_2^A) \times \tfrac{\bar{p}}{1-\bar{p}}\Delta\beta \\
& + \tfrac{1}{2}(1-\bar{p})U"(W-\beta_2^N)(\Delta\beta)^2 + \tfrac{1}{2}\bar{p}U"(W-\beta_2^A)(\tfrac{\bar{p}}{1-\bar{p}}\Delta\beta)^2 \\
\simeq & \quad \{[U'(W-\beta_2^N) - U'(W-\beta_2^A)](1-\bar{p})\Delta\beta\} \\
& + \{\tfrac{1}{2}(1-\bar{p})U"(W-\beta_2^N)(\Delta\beta)^2 + \tfrac{1}{2}\bar{p}U"(W-\beta_2^A)(\tfrac{\bar{p}}{1-\bar{p}}\Delta\beta)^2\} \quad (**)
\end{aligned}$$

  The first term of the right-hand side of (**) is nonnegative (because U' is strictly decreasing and $\beta_2^N \geq \beta_2^A$), and the second term is positive ; thus : $\Delta U > 0$, QED.

  Therefore, C'_{LT} provides strictly more utility to agents than C_{LT}, which implies, by an argument of continuity, that any entrant (at t=0) which offers a contract $C"_{LT}$ "between" C_{LT} and C'_{LT} makes positive profits, which again provides a contradiction.

- Last, suppose that (iii) is not satisfied. Then, any entrant offering at t=1 a contract in the area between (IC^N) and $(\Pi = 0)$ makes positive profits. This, once more, is absurd.

[26] By the way, note that if we had taken the exogenous specification of the literature, we would have derived an outcome (C_1, C_2^N, C_2^A) as in figure 6, which we just proved to lead to a contradiction.

Conversely, consider a possible solution C_{LT} of $(\mathcal{M}2)$, and suppose that $\{C_{LT}\}$ is not an equilibrium of the market. Then, either **(a)** or **(b)**[27] is not satisfied by $\{C_{LT}\}$; but (i), (iii) and (iv) precisely ensure **(b)**. Thus, **(a)** does not hold, and there exists a long-term contract C'_{LT} which people like at least as much as C_{LT} , that gives strictly superior profits to insurers — that is : positive profits, thanks to (ii) — and satisfies **(b)**.

It is worth having a closer look about this point before we go on : when we examine long-term contracts that could destabilize a candidate equilibrium, we must remind that such contracts must be robust to competition at t=1, that is to say must satisfy **(b)**. In other words, due to the dynamic structure of the model, destabilizing contracts are themselves subject to a possible reaction. That is to say that dynamic markets necessarily generate an anticipative behaviour by their structure. This is one of the reasons why we get a general existence result, as stated in the next theorem.

Now, if we come back to the proof : C'_{LT} would satisfy (i) and (iii) as well as a stronger condition than (ii), while providing no less utility than C_{LT}. From this, we can derive the existence of a long-term contract $C"_{LT}$ satisfying (i), (ii), (iii), and giving to agents strictly more expected discounted utility than C_{LT}. Contradiction.

Consequently, any solution of $(\mathcal{M}2)$ is an equilibrium of the market. *Q.e.d.*

4.2 The central result

We now turn to present the final result of this section:

Theorem *There always exists an equilibrium of the market ; moreover, any such equilibrium strictly Pareto-dominates the (unique) equilibrium of the market under public information.*

Proof :

- The existence of an equilibrium of the market can be proved easily, once it has been noted that the objective function in $(\mathcal{M}2)$ as well as its constraints can be rewritten as *continuous*[28] functions of β only — where β designs the premium assotiated to the contract C (which is located at the intersection

[27] cf. page 6.

[28] And even twice continuously differentiable

of (IC_2^N), (IC_2^A), and $(\pi^N = 0))$, and that β takes its values in a compact interval:

$$\begin{array}{cl}
\textbf{Max} & \{U(W-\beta_1) + (1-\bar{p})U(W-\beta) + \bar{p}U(W-D+\frac{1-\bar{p}^N}{\bar{p}^N}\beta)\} \\
(\beta \in [\beta*, \bar{\beta}^N]; \beta_1) & \\
\textbf{s.t.} & \left\{\begin{array}{rcl}
-(\beta_1 - \bar{p}D) & = & (1-\bar{p})[W - \bar{p}^N D \\
& - & U^{-1}((1-\bar{p}^N)U(W-\beta) + \bar{p}^N U(W-D+\frac{1-\bar{p}^N}{\bar{p}^N}\beta) \\
& + & \bar{p}[W - \bar{p}^A D \\
& - & U^{-1}((1-\bar{p}^A)U(W-\beta) + \bar{p}^A U(W-D+\frac{1-\bar{p}^N}{\bar{p}^N}\beta)
\end{array}\right.
\end{array}$$

- To prove the last part of the theorem, consider the public information equilibrium $C_{pub} = \{C_1 = C_2^A, C_2^N = \bar{C}^N\}$, and note that (IC_2^N) and (IC_2^A) intersect under the line $(\Pi^N = 0)$. Thus, by increasing a little[29] β_2^N and decreasing β_1 so as to keep satisfied the break-even constraint (ii), we get strictly more expected utility, as one can show easily[30]. *Q.e.d.*

4.3 Comments

We can by now understand better why information should be private rather than public : preventing information transmission at t=1 makes individuals from the sub-population N unrecognizable as such, and therefore enables insurers to propose at t=0 long-term contracts which involve positive profits on the sub-population N in the second period (see figure 7). This in turn implies that $(\beta_2^N - \beta_2^A)$ can be reduced, which means that agents get more insurance : their ex-ante intertemporal expected utility increases, and thus the social welfare increases too[31].

In other words, hiding information has a commiment-enhancing value : it provides to the individuals *a partial commitment device* not to switch to other insurers in case of good news on their type (N).

This intuition must however be mitigated by the effects of competition between insurers, for having information made private — and therefore competition reduced at t=1 — increases the set of long-term contracts that insurers can offer at t=0. Such a situation is good on the one hand (this is another way to restate the intuition above) but it means on the other hand that the set of long-term contracts that can upset a candidate-equilibrium increases too, and therefore competition at t=0 is enhanced[32] ! The overall effect here is unambiguous (and positive), but the idea of a trade-off between reducing competition at t=1 and strenghtening it

[29]So that conditions (i), (iii) and (iv) of $(\mathcal{M}2)$ remain satisfied.

[30]The proof is quite similar to that of proposition 3.

[31]Companies break-even anyway.

[32]This trade-off is of critical relevance in dynamic adverse selection models of competition ; in particular, we think that the existence of an equilibrium may occur more often under semicommitment than under full-commitment (work in progress).

at t=0 is worth noting, for it points out that changing information structures can have quite opposite effects — depending on the degree of competition of the market.

At this stage, one can solve the puzzle we underlined in the introduction, namely the apparent contradiction between the strict Pareto-dominance result and what we know from the litterature on the value of information [16].

Consider an insurance market with incomplete and initially symmetric information, in which each policyholder gets some information about his own type ; then, according to the literature, having it made public may either have a positive or a negative social value, depending on parameter values.

Now, suppose that the initial insurer gets this information too ; this alone would not change anything. However, if in addition the revelation of information arises through the history of accidents — as in our model — having it made public is Pareto-worsening, and thus has a negative social value.

Therefore, the clue is somehow related to the *nature* of information. More precisely, the Pareto-comparison result is driven by the *ex-ante contractibility of posterior information flows* : information arising through the history of accidents is particular in that accidents can be contracted upon ex-ante.

As a corollary, any contractible device that produces an informational advantage ex-post to policyholders and their initial insurer is Pareto-improving[33].

Remark also that we do not always get unicity in terms of contracts offered, since when we express the ex-ante utility as a function of β only, we do not necessarily get a concave function[34]. However, a corollary to theorem 1 is that all equilibria of the market provide the same expected discounted utility to agents. We get unicity in terms of welfare.

Let us come back a short moment on the specificity of dynamics. First, as we said before, anticipation is *structurally* present in dynamic models : it is a necessity, which does not come from an artefact of modelization but rather by nature. And this is why we get existence in any case. Second, it is essential to note that if long-term contracts depend on the threat of a short-term contract one period later, *the converse is also true.* Indeed, the threats of creaming-off the sub-population N at t=1 depend upon the existing long-term contracts, which act as *an endogenous reservation option.* In other words, the interaction between present and future acts in both ways. This point may seem obvious, but it is not, for it had up to now been completely ignored by the literature on optimal contracts, as we saw in the introduction. As a result, we get to reconsider the shape of equilibrium contracts when information is private, and indeed derive quite different results :

[33]This provides a very strong rationale for "boites noires".

[34]But still, we claim that we have *generic* unicity, that is to say unicity for almost all values of parameters.

Proposition 4 *Equilibrium contracts under private information structures involve bonus* and *malus as soon as (iii) is not binding.*

Remark : when (iii) binds, we may or not observe malus, according to the cost of constraint (iii) — measured by its Lagrange multiplier.

Proof : suppose that (iii) does not binds at the optimum.

- First, the constraint $\beta \geq \bar{\beta}^N$ never binds, for having β_1 decreased a little and β_2^A increased (so as to keep the break-even constraint satisfied) leads to strictly higher expected utility.

- Now suppose that $\beta \leq \beta*$ is binding.
  If (IC_1) were to be strictly under (IC_2^N), then increasing β_1 and decreasing β_2^N would be improving.
  Thus, $\beta_1 \geq \beta_2^N$, and generically $\beta_1 > \beta_2^N$ (strict bonus).
  Moreover, $\beta_1 \geq \beta_2^A$ would imply $\beta_2^N < \bar{\beta}^N$ (through the break-even constraint), and thus (ii) would be violated. Thus, we observe strict malus.

- Last, consider the case in which you have an interior solution ; in that case too one can prove that the solution displays both bonus and malus (see appendix).

  The situation is depicted on figure 7 below.

FIGURE 7 ABOUT HERE

To conclude this section, let us consider the other fundamental reason why we get a general existence result. This result clearly relies on our assumption on incomplete information : we had assumed it initially symmetric, ruling out any kind of adverse selection *per se*. Our motivations were (1) evidence sustaining such a model (2) the strong conviction of car insurers, and (3) the tractability of the model. Nevertheless, there are many kinds of insurance markets, of which a number displays adverse selection *per se*. Moreover, even when adverse selection arises endogenously, it may not be totally contractible. For instance, adverse selection arising from the no-communication of accident records to rival companies relies on accident claims, *on which we can contract ex-ante* : this is the clue to the existence result. But it could be quite reasonable to suppose that policyholders do not only learn through accidents, but also through driving experience, on which we cannot contract. And the latter source of adverse selection would verysimilarly be detrimental for equilibrium existence, although it arises endogenously. In addition, it would temperate our comparison of information structures, for we said that private information Pareto-dominated public information, and proved this clear-cut result to derive from the ex-ante contractibility of posterior flows of information.

This short discussion, we think, stresses the importance of knowing the *nature* of adverse selection, and cries out for empirical studies.

5 Concluding comments

The simplicity of this model allowed us to get answers to our introductory questions — or at least a mean to get these answers via empirical studies.

First, we saw that dynamics, by nature, help to restore existence of equilibrium under incomplete information. Our comparison of informational structures even led to assert that in an insurance market under incomplete but initially symmetric information, creating adverse selection leads to a Pareto-improvement, which has strong policy implications. However, we underlined the importance of the *nature* of adverse selection, whether exogenous or endogenous — and in the latter case whether contractible or not.

[redacted] driving experience.

Second, we derived optimal contracts and proved that the "no-malus" result did not need be true under no-communication structures. This is the first rationale for bonus and malus [36] in dynamic insurance markets under semicommitment.

Not surprisingly for a learning model, commitment has a positive social value.

The most obvious limit of that model is the absence of productive activity. Nevertheless, we believe that our intuitions would still hold in a model incorporating care (work in progress).

As many dynamic models of incomplete information, ours neglects access to credit market, which is *a priori* an important shortcoming. However, we were able to solve the model under access to a (perfect or imperfect) credit market, but choosed

[35] This is all the more true that predictions are very likely to be opposite in dynamic models of adverse selection (work in progress).

[36] To be precise, Dionne and Doherty do get experience-rating with semi-commitment, but not bonus-malus.

not to incorporate the results in the present paper, for it does not affect the intuitions[37].

6 Appendix

In order to show the "bonus *and* malus" result in the case of an interior solution, it is sufficient to write the first-order conditions associated with $(\mathcal{M}'2)$ — derived from $(\mathcal{M}'2)$ by forgetting (iii). Let us rewrite it in function of the two parameters (β and β_1) :

$$\underset{(\beta,\beta_1)}{\mathbf{Max}} \quad \{U(W-\beta_1)+(1-\bar{p})U(W-\beta)+\bar{p}U(W-D+\tfrac{1-\bar{p}^N}{\bar{p}^N}\beta)\}$$

$$\mathbf{s.t.} \quad \begin{cases} (\beta_1-\bar{p}D) & \\ \quad + & (1-\bar{p})[W-\bar{p}^N D-U^{-1}((1-\bar{p}^N)U(W-\beta)+\bar{p}^N U(W-D+\tfrac{1-\bar{p}^N}{\bar{p}^N}\beta))] \\ \quad + & \bar{p}[W-\bar{p}^A D-U^{-1}((1-\bar{p}^A)U(W-\beta)+\bar{p}^A U(W-D+\tfrac{1-\bar{p}^N}{\bar{p}^N}\beta))]=0 \end{cases}$$

$$\begin{aligned} First,\ set\ : \quad \mathbf{E}^N(\beta) &\equiv (1-\bar{p}^N)U(W-\beta)+\bar{p}^N U(W-D+\tfrac{1-\bar{p}^N}{\bar{p}^N}\beta) \\ \mathbf{E}^A(\beta) &\equiv (1-\bar{p}^A)U(W-\beta)+\bar{p}^A U(W-D+\tfrac{1-\bar{p}^N}{\bar{p}^N}\beta) \\ \mathbf{E}(\beta) &\equiv (1-\bar{p})U(W-\beta)+\bar{p}U(W-D+\tfrac{1-\bar{p}^N}{\bar{p}^N}\beta)\ ; \end{aligned}$$

thus, the constraint rewrites:

$$\beta_1+(1-\bar{p})\{W-U^{-1}(\mathbf{E}^N(\beta))\}-\bar{p}\{W-U^{-1}(\mathbf{E}^A(\beta))\}=2\bar{p}D.$$

Let μ be the Lagrange multiplier associated with this constraint ; from the first-order conditions, we get:

$$\frac{\frac{d}{d\beta}(\mathbf{E}(\beta))}{(1-\bar{p})\frac{d}{d\beta}(\mathbf{E}^N(\beta))(U^{-1})'(\mathbf{E}^N(\beta))+\bar{p}\frac{d}{d\beta}(\mathbf{E}^A(\beta))(U^{-1})'(\mathbf{E}^A(\beta))}=\mu=\frac{U'(W-\beta_1)}{1}$$

Thus:

$$\frac{1}{U'(W-\beta_1)}=\frac{1}{U'\{U^{-1}(\mathbf{E}^N(\beta))\}}\times(\frac{(1-\bar{p})\frac{d}{d\beta}(\mathbf{E}^N(\beta))}{\frac{d}{d\beta}(\mathbf{E}(\beta))})+\frac{1}{U'\{U^{-1}(\mathbf{E}^A(\beta))\}}\times(\frac{\bar{p}\frac{d}{d\beta}(\mathbf{E}^A(\beta))}{\frac{d}{d\beta}(\mathbf{E}(\beta))})$$

By linearity and martingale property, we now get:

$$(1-\bar{p})\frac{d}{d\beta}(\mathbf{E}^N(\beta))+\bar{p}\frac{d}{d\beta}(\mathbf{E}^A(\beta))+\frac{d}{d\beta}[(1-\bar{p})\mathbf{E}^N(\beta)+\bar{p}\mathbf{E}^A(\beta)]=\frac{d}{d\beta}[(\mathbf{E}(\beta))]\ ,$$

from which we can infer that $\frac{1}{U'(W-\beta_1)}$ is a weighted sum of $\frac{1}{U'\{U^{-1}(\mathbf{E}^N(\beta))\}}$ and $\frac{1}{U'\{U^{-1}(\mathbf{E}^A(\beta))\}}$.

Reminding that $U^{-1}(\mathbf{E}^N(\beta))=U(W-\beta_2^N)$, $U^{-1}(\mathbf{E}^A(\beta))=U(W-\beta_2^A)$, $\beta_2^N<\beta_2^A$ and that $\frac{1}{U'}$ is strictly increasing, we are able to conclude that:

$$\boxed{\beta_2^N<\beta_1<\beta_2^A}$$

Q.e.d.

[37] See [8] for details.

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