Optimal Managerial Remuneration

and Firm-level Diversification

By

Erlend Nier

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Abstract

In a model that exhibits both moral hazard and hidden information on the part of the manager different remuneration schemes are discussed and the optimal contract between financial investor and manager is derived. Assuming the manager is risk-neutral and protected by limited liability, a benefit from diversification is shown to exist even though the projects which the manager develops are technologically unrelated and choices made on one project do not constrain the choices on any other project. (JEL classification D82, G31, G34).

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1. Introduction

1.1. Motivation

The main motivation for this paper is to identify a set of circumstances under which firm-level diversification may be beneficial. It is well known that in a world of risk-averse investors diversification as such creates benefits by reducing portfolio variance. It is less clear however, why diversification should be observed at the firm level. After all, if investors are able to hold well - diversified portfolios themselves, why should firm-level diversification add any further value. Traditional arguments that come to mind rely on economies of scope between projects (synergies) or a reduced probability of incurring bankruptcy costs (Lewellen (1971)). However, there may be scope for firm-level diversification to be beneficial even when these effects are absent or unimportant. We will analyze a model which abstracts from both economies of scope (project returns will be independently distributed) and costs of bankruptcy (all investors will hold identical claims) and instead take an agency problem between financial investors and managers as the starting point. The agency problem will involve both moral hazard and hidden information on the part of the manager. This paper argues that the hidden information component is a particularly important feature of the agency relationship between financial investor and manager. Financial investors not only want managers to work hard. They also want managers, who acquire inside information about projects, to make prudent investment decisions in the light of this information. The presence of hidden information will make the model suitable especially for R&D- intensive industries, such as chemicals, drugs, or oil extraction. In these industries, managers typically gain private information about the likely return of a project in the course of project development. Financial investors will then want the manager to reevaluate the project and make an informed investment decision in their interest. By offering some wage contract to the manager investors may be able to provide the right incentives for both these managerial decisions. Providing incentives will however be costly; the manager will receive an informational rent over and above the compensation for his effort. We will be able to show that if the manager is risk-neutral and protected by limited liability this informational rent will be driven down as one assigns more projects to the manager, thus providing a rationale for firm-level diversification.

The second major motivation for this paper is to examine different remuneration schemes in such a setting and to explore how the optimal incentive scheme changes when the manager is in charge of more than one project. This is an interesting question in its own right in that managers may be assigned several projects for reasons other than rent-reduction. Thus for example fund managers necessarily make joint decisions on a number of assets. In recent years much research has been devoted to the form of optimal managerial remuneration in the presence of agency problems. Most of the results give a justification for contracts that are monotonic in the return of the projects and have led theorists to argue that contracts that involve awarding shares or options may be taken as approximations of the contracts which are derived as optimal. However, there is some issue of whether these contracts are incentive compatible if managers are asked to make investment decisions on the basis of their private information and in which way the optimal incentive scheme should respond to the fact that managers are diversified.

1.2. Literature

Much of the existing Principal-Agent literature assumes that the agent has a single project. Notable exceptions are Holmstrom and Milgrom (1991) on multi-task principal-agent analysis, Aghion and Tirole (1994) on real and formal authority, and Diamond (1984), and Williamson (1986) on diversified financial intermediaries.

More closely related to the arguments in this paper are two strands of literature, one concerned with optimal managerial remuneration schemes and the other concerned with explaining firm-level diversification in a Principal-Agent context.

Some authors (e.g. Haubrich (1994)) have taken the simple moral hazard problem with a risk-averse agent, as analyzed for example in Grossman and Hart (1983), to be informative on optimal managerial remuneration. In this model there is a single-dimensional effort choice to be made by the agent which is stochastically related to observed output. If more effort is put in, output is more likely to be higher in the sense of first-order stochastic dominance. Since the principal wants to elicit effort, the contract will reward high profit outcomes and thus typically exhibit monotonicity in observed returns. When the principal is risk-neutral but the agent is risk-averse, there is scope for insurance. Providing wage insurance will however blunt incentives and is therefore costly. Holmstrom and Ricart-i-Costa (1986) were the first to argue that this model might not fully capture the incentive problems between manager and financial investor. Financial investors, they argue, may be more worried about how effective managers are at making decisions. Stochastic managerial ability is introduced and the focus is on career concerns rather than effort choice. Lambert (1988) is the first to introduce more-dimensional decisions. The manager expends effort on gathering information and then selects the best project conditional on his private information. A similar route is taken by Huang and Suarez (1996) who derive an option contract assuming risk-neutrality and limited liability, rather than risk-aversion on the part of the manager, as does Lambert (1988). None of these papers however examines remuneration schemes for diversified managers. Whereas Huang and Suarez (1996) is closest to the assumptions made in the present paper, both their focus and their analysis differ from the present paper even for the case of the undiversified manager which they consider.

There are two papers that analyze firm-level diversification in a Principal-Agent context, Aron (1988), and Hermalin and Katz (1996). Aron (1988) analyzes a moral hazard problem with a risk-averse manager. The manager is asked to choose an effort variable, which has a noisy but positive impact on the returns of a production process. Financial investors use realized return as a signal for the effort level chosen. When there are two projects, the manager still chooses a single effort variable which now becomes an input into both processes. This enables financial investors to observe two independent signals of the manager's effort choice, so that the precision of their inference is improved. She obtains an optimal extent of diversification by economies of scale in production. Hermalin and Katz (1996) also couch their analysis in terms of a pure moral hazard problem, Again the manager is asked to choose an effort level. Diversification is thought of as splitting this effort variable and letting the fractions enter two activities. Again this will under certain conditions improve the informativeness of the observed returns. In both Aron (1988) and Hermalin and Katz (1996) diversification is driven by the fact that projects are technologically related. In Aron (1988) the single effort choice becomes a common input into two different processes, whereas in Hermalin and Katz (1996) it is split in a known ratio and then enters both projects.

By contrast, in this paper the projects that the manager is asked to develop are technologically unrelated. Choices made concerning one project do not constrain the choices on any other project. It is only through the fact that the manager solves a joint problem when allocated more than one project under some contract that the projects become linked. Also, we do not study a monotonous stochastic relationship between some unobservable effort variable and the observable project return. Instead we posit a model where hidden information on the part of the manager is a crucial ingredient. The manager is thought of as an insider who comes to have superior information on the likely return of the projects. Financial investors want the manager to make investment decisions in their interest. If the prospects are good they want him do go ahead with the investment. If the inside information is unfavorable, they want the manager to abort the project. Here, the investor wants the manager to make prudent choices, which is an element absent from the cited papers. This paper also argues that insurance issues are not at the heart of the incentive problem between managers and financial investors, and assumes instead that both financial investors and managers are risk-neutral. This assumption can be justified by noting that stakes may well be high both for managers and financial investors. Given decreasing absolute risk-aversion it seems far from clear that financial investors are less risk-averse than managers and should therefore insure managers. I would like to argue instead, that risk-neutrality on both sides may well be a more reasonable approximation.

1.3. Organization of the paper

Section 2 will set up the model and introduce the notation for the general case, where the manager is assigned N projects. In section 3 we will step back from the general case and look at contracts for the undiversified manager, who has a single project. Section 4 will consider the case when the manager is asked to develop two projects at the same time, and section 5 will return to the general case. In section 6 we will examine several extensions of the analysis. Section 7 will conclude.

2. The General Framework, N projects

There is one manager who becomes associated with some number N of indivisible projects, indexed by $p = 1, .., N$. Each project requires a financial outlay of I. We normalize the manager's initial wealth to zero. There is one financial investor who is endowed with funds sufficient to finance the projects. Both the financial investor and the manager are assumed to be risk-neutral. The manager is, however, protected by limited liability; his wage cannot be negative.

The information structure is as follows. The ex ante distribution of the project returns is known to all parties. We assume that the returns are distributed independently and identically across projects. For each project the manager can privately choose to expend nonpecuniary effort and is then privately informed whether the project investigated is good or bad. Since this information remains private, the manager will at an interim stage have more information than the financial investor, if he chooses to investigate the project. Thus there will be interim adverse selection or "hidden information". There is moral hazard as well in that the manager's effort choice is assumed to be unobservable to investors. It turns out to be immaterial, whether the subsequent investment decision itself is assumed to be observable to the financial investor, since for each project she will be able to infer it perfectly from the realized project return, which is both observable and verifiable.

Let us come to the timing of the model. When time starts the financial investor offers at take-it-or-leave-it contract to the manager, which the manager can accept or reject. If he accepts, the financial investor hands over sums of NI to the manager.

Then the manager privately chooses a vector of effort levels $e = (e_1, ..., e_N)$, where it is assumed that for each project the effort level can only take two values, $e_p \in \{0, 1\}$; either the manager investigates a given project, $e_p = 1$, or he does not, $e_p = 0$. The effort cost associated with e is given by $C(e)$

success probability for project p. With probability P the project is promising, $\pi_p = \overline{\pi}$, with probability $1-P$ the project is bad, $\pi_p = \underline{\pi}$.

The manager then receives a signal $s(e) = (s_1, ...s_N)$ of the success probabilities realized. Its precision will depend on the effort expended. In particular, it is assumed that $s_p = \pi_p$ if $e_p = 1$ and $s_p = 0$ if $e_p = 0$, that is, the manager receives a perfect signal on project p if he has expended effort on it, whereas he receives no information pertaining to p, if he did not spend effort on investigating it.

Having observed the signal the manager makes an investment decision on each project , which is summarized by $d = (d_1, ..., d_N)$. From the assumed indivisibility of projects we have $d_p \in \{0, 1\}$. Note that investment, $d_p = 1$, involves spending a financial outlay of I but does not cause further nonpecuniary costs to the manager. Also, it is assumed that the effort choice does not constrain the investment decision, so that "blind" investment is possible[1].

Finally, and observable to both the financial investor and the manager, the vector of project (gross) returns $\widetilde{z} = (\widetilde{z}_1, ..., \widetilde{z}_N)$ realizes. We assume $z_p \in \{0, R\}$ if $d_p = 1$ and $z_p = I$ if $d_p = 0$. In accordance with the earlier interpretation for π, we let $\Pr[z_p = R \mid d_p = 1] = \pi_p$ and $\Pr[z_p = 0 \mid d_p = 1] = (1 - \pi_p)$, where, recall, $\pi_p \in \{\underline{\pi}, \overline{\pi}\}$. Realized project returns z are handed over from the manager to the investor, who in turn pays the manager a wage w , the size of which can depend on the observed vector z of project returns. Note that if the manager did not invest in a project, he will just return I to the investor, so that the investment decision on any project can be perfectly inferred from its return.

As for the profitability of the projects we assume that $\overline{\pi}R > I$, but $\underline{\pi}R < I$, so that it is interim efficient to invest, if and only if the project is good. We also want $P\overline{\pi}R + (1-P)I - c > \max\{I, P\overline{\pi}R + (1-P)\underline{\pi}R\}$, so that, given an interim efficient investment decision is taken, it is efficient to investigate each project. One can then distinguish two cases. If $P\overline{\pi}R + (1-P)\underline{\pi}R > I$, the project could profitably be undertaken without the manager reevaluating the project at the interim stage. On the other hand, if $P\overline{\pi}R + (1-P)\underline{\pi}R < I$, the manager's job of reevaluating the project and aborting it, if it turns out to be bad, is necessary for the project to be

[1] Under the alternative assumption that effort spent is necessary for investment the qualitative results remain the same.

profitable (net of implementation costs) ex ante.

Resulting from this setup, given any contract between the financial investor and the manager, the manager has a number of strategies available to him. We denote these by $(e, d(\cdot))$. He can choose which, if any, projects to look at, $e \in \{0,1\}^N$, and choose any function mapping the set of possible signals received into the set of investment decisions, $d(\cdot) : \{0, \underline{\pi}, \overline{\pi}\}^N \rightarrow \{0,1\}^N$. Since the financial investor is assumed to have all the bargaining power, he will be interested in implementing the efficient strategy $(e^*, d^*(\cdot))$. This is defined by $e_p^* = 1\ \forall p$ and $d_p^* = 1$ if $s_p = \overline{\pi}$ and $d_p^* = 0$ if $s_p = \underline{\pi}\ \forall p$, i.e. investigate all projects and invest only if the signal is favorable. Let us assume for now that the investor wants to give the manager incentives to choose this strategy and that he wants to do this as cheaply as possible. The investor's problem is then to choose a wage schedule $w(\cdot)$ to maximize return net of wages, making sure that the manager's expected wage compensates for the effort costs incurred, that the wage schedule induces the manager to voluntarily choose the efficient strategy, and finally, that the manager never receives a negative wage.

One can write down the investor's problem as follows:

$$\max_{w(\cdot)} E\left[\widetilde{z} - w(\widetilde{z}) \mid \widetilde{s} = \widetilde{\pi}, d = d^*(\widetilde{\pi})\right]$$

s.t.

(IC_d):

$$E\left[w(\widetilde{z}) \mid s = \pi,\ d = d^*(\pi)\right] \geq$$
$$E\left[w(\widetilde{z}) \mid s = \pi,\ d = d(\pi)\right]\ \forall \pi, \forall d(\cdot)$$

(IC_e):

$$E\left[w(\widetilde{z}) \mid \widetilde{s} = \widetilde{\pi}, d = d^*(\widetilde{\pi})\right] - Nc \geq$$
$$E\left[w(\widetilde{z}) \mid \widetilde{s} = \widetilde{s}(e), d = d(\widetilde{s})\right] - \sum_{p=1}^{N} ce_p\ \ \forall e, \forall d(\cdot)$$

(IR):

$$E\left[w(\widetilde{z}) \mid \widetilde{s} = \widetilde{\pi}, d = d^*(\widetilde{\pi})\right] - Nc \geq 0$$

(NNW):

$$w(z) \geq 0 \; \forall z$$

If the manager chooses the efficient strategy, so that $\widetilde{s} = \widetilde{\pi}$ and $d = d^*(\widetilde{\pi})$, this will induce a certain ex ante distribution over returns and thus over wages. Taking this distribution as given, the financial investor maximizes her payoff by minimizing the expected wage to be paid to the manager. This distribution will however only obtain, if the manager voluntarily chooses the efficient strategy. This is what is ensured by the (IC_d) and (IC_e) constraints. The (IC_d) constraint ensures that, given the manager has investigated all projects, he makes efficient interim investment decisions, whereas the (IC_e) constraint ensures that, ex ante, investigating all projects and then choosing $d^*(\cdot)$ is superior to any other strategy in terms of expected wage net of effort costs. (IR) then makes sure that, again ex ante, the manager gets compensated for his effort cost, if he chooses the efficient strategy. Lastly, the Non-negative wage (NNW)constraint is a limited liability constraint that forces all wages to be non-negative.

Rather than proceeding directly to a derivation of the general solution to this problem, it seems interesting to see what can be learned from the basic case, where the manager is given a single project.

3. One project per manager, N=1

In this case there is a single indivisible project which requires a financial investment I and yields a gross return $z \in \{0, R\}$. With probability P the project is good, i.e. $\Pr(z = R) = \overline{\pi}$ and $\Pr(z = 0) = (1 - \overline{\pi})$, and with probability $(1 - P)$ the project is bad , i.e. $\Pr(z = R) = \underline{\pi}$ and $\Pr(z = 0) = 1 - \underline{\pi}$, where $\overline{\pi} > \underline{\pi}$. The project is profitable, if it is good, $\overline{\pi}R > I$, but unprofitable, if it is bad, $\underline{\pi}R < I$, and worthwhile ex ante, given that investment efficiently conditions on the signal, $P\overline{\pi}R + (1 - P)I - c > \max\{I, P\overline{\pi}R + (1 - P)\underline{\pi}R\}$, so that the financial investor would like the manager to first develop the project, $e^* = 1$, and that, if he sees the favorable signal to go ahead with the investment, $d^*(\overline{\pi}) = 1$, but if sees a bad signal to abort the project, $d^*(\underline{\pi}) = 0$, and to return the outlay I back to the financial investor. To implement this behavior the investor solves the following problem:

$$\max_{w(0),w(I),w(R)} P\left[\overline{\pi}\left(R - w(R)\right) + (1 - \underline{\pi})\left(-w(0)\right)\right] + (1 - P)\left[I - w(I)\right] - I$$

$s.t.$
$(IC_d\ d(\overline{\pi})=1)\ \overline{\pi}w(R)+(1-\overline{\pi})w(0)\geq w(I)$
$(IC_d\ d(\underline{\pi})=0)\, w(I)\geq \underline{\pi}w(R)+(1-\underline{\pi})w(0)$
(IC_e)
$(1)\quad P[\overline{\pi}w(R)+(1-\overline{\pi})w(0)]+(1-P)w(I)-c\geq w(I)$
$(2)\quad P[\overline{\pi}w(R)+(1-\overline{\pi})w(0)]+(1-P)w(I)-c\geq$
$\quad P[\overline{\pi}w(R)+(1-\overline{\pi})w(0)]+(1-P)[\underline{\pi}w(R)+(1-\underline{\pi})w(0)]$
$(IR)\ P[\overline{\pi}w(R)+(1-\overline{\pi})w(0)]+(1-P)w(I)-c\geq 0$
$(NNW)\ \ w(0)\geq 0, w(I)\geq 0, w(R)\geq 0$

In this basic problem the manager has two possible investment decisions available to him. The IC_d-constraints ensures, that interim, given that the manager has spent effort and thus received a signal $s\ \epsilon\ \{\overline{\pi},\underline{\pi}\}$, he makes the right investment decision in response to the two possible signals. Next, if the manager does not spend effort, and accordingly receives $s=0$, he again has two possible options. He can either abstain from investing and return I to the investor or invest blindly. (IC_e) (1) and (2) ensure that both these options are less worthwhile to the manager than following the efficient strategy. Lastly, the expected wage induced by the efficient strategy is ensured by (IR) to be larger than the effort cost c, and all wages have to be non-negative.

The optimal contract will have three constraints binding. First of all note that $w(0)=0$ will be one of the binding constraints. To see why, assume $w(0)>0$. Then one can decrease $w(0)$ by some $\varepsilon\leq w(0)$ and increase $w\ (R)$ by $\frac{(1-\overline{\pi})}{\overline{\pi}}\varepsilon$. This will leave all constraints satisfied, can be done costlessly and relaxes $IC_d\ d(\underline{\pi})=0$, as well as $(IC_e)\,(2)$ since

$$\underline{\pi}\frac{(1-\overline{\pi})}{\overline{\pi}}\varepsilon-(1-\underline{\pi})\,\varepsilon<0$$

which is true, since $\overline{\pi}>\underline{\pi}$. Also, one sees that IR will not be binding, since it is implied by $IC_e\,(1)$ and $w(I)\geq 0$. In fact $w(I)>0$ necessarily, because otherwise $w(R)=w(0)=0$ from $IC_d\ d(\underline{\pi})=0$ and NNW. One also sees that $IC_d\ \ d(\overline{\pi})=1$ must be slack, since if it were binding $IC_e\,(1)$ would be violated. Likewise $(IC_d\ d(\underline{\pi})=0)$will not be binding, since if it were, $IC_e\,(2)$ would be violated. Next, $w(R)>0$, since otherwise one would have $(1-\overline{\pi})\,w(0)\geq w(I)\geq$

$(1-\underline{\pi})\, w(0)$, combining$(IC_d\; d(\overline{\pi})=1)$ and $IC_d\; d(\underline{\pi})=0$. This could be true only if $w(I)=w(0)=0$, contradicting $w(I)>0$. This leaves $IC_e(1)$ and (2) as the only possible further binding constraints. Substituting $w(0)=0$ into these two constraints one finds

$$\begin{aligned} & P\overline{\pi}w(R)+(1-P)\,w(I)-c \\ = \;& w(I) \\ = \;& [P\overline{\pi}+(1-P)\,\underline{\pi}]\, w(R) \end{aligned}$$

which is easily solved for

$$\begin{aligned} w(R) &= \frac{c}{P\overline{\pi}+(1-P)\,A-A} \\ w(I) &= \frac{Ac}{P\overline{\pi}+(1-P)\,A-A} \end{aligned}$$

where $A=P\overline{\pi}+(1-P)\,\underline{\pi}$ is the expected interim success probability. Notice that this contract is monotone in return,

$$0=w(0)<w(I)<w(R).$$

Under this contract the manager's expected wage payment exceeds his effort cost c. This excess payment can be interpreted as his informational rent and it can be read off from the RHS of the binding $(IC_e)\,(1)$ constraint as being equal to $w(I)$. The rent arises both because of the moral hazard and the hidden information component of the agency problem. Notice in particular, that the financial investor has to reward a gross return of I with a positive wage. This is because the financial investor does not know, whether the manager has returned I because the manager efficiently aborted an unprofitable project, or whether the manager did not investigate the project and then also did not invest. If the financial investor knew that the project is bad she would not have to reward a gross return of I, which makes for a net return of zero.

Note that an informational rent would not arise in the benchmark case, where s is publicly observable. One could then pay the manager a wage $w(z,s)$. Clearly then, $w(I,\underline{\pi})=w(R,\overline{\pi})=w(0,\overline{\pi})=c$, $w(I,\overline{\pi})=w(R,\underline{\pi})=w(0,\underline{\pi})=0$, and $w(z,0)=$

0 $\forall z$ would implement the efficient strategy and would involve a zero rent. Since the contract cannot condition on s when s is private information, an informational rent arises. The reader may ask, however, whether the wage contract derived above can be improved upon by letting the manager make explicit and verifiable announcements about the signal he received, so that the contract could then condition on $w(z, \widehat{s})$. It is shown in the appendix that this is not the case. The wage contract derived above remains optimal, when one allows the manager to make those explicit claims to the financial investor and d is viewed as an observable and hence contractible variable. This result establishes, that the wage contract derived above is equivalent to a direct revelation mechanism and hence is optimal in the class of all possible mechanisms implementing the efficient effort and investment choice when s is unknown.

Before we look at implementing other possible strategies, let us briefly discuss whether certain remuneration schemes that are observed in practice, satisfy the set of constraints and may thus potentially be viewed as approximations to the optimal contract derived above.

Consider first offering the manager a flat wage $\overline{w}$ and a bonus b for a return of R. Then the wage schedule will be $w(0) = \overline{w}$, $w(I) = \overline{w}$, $w(R) = \overline{w} + b$. One sees immediately, that under such a contract $\underline{\pi} w(R) + (1 - \underline{\pi}) w(0) > w(I)$, so that both IC_d $d(\underline{\pi}) = 0$ and $(IC_e)(2)$ are violated. The manager will invest even after observing that the project is bad. What is more, the manager will always prefer to invest blindly, rather than spending effort on becoming informed. The bonus scheme can thus be seen as inducing excessive risk-taking by the manager.

One might next consider an option contract. In reality managers are often awarded options with a strike equal to the expected return of the firm. If the option is made exercisable when z has realized, one would have $w(z) = \delta(z - E^*(z))^+$, where $E^*(z) = P\overline{\pi}R + (1 - P)I$. Since $E^*(z) > I > 0$, the manager will only exercise when $z = R$, so that the wage is again flat for realizations other than R. Since $w(I) = w(0) = 0$, but $w(R) > 0$ one again finds both IC_d $d(\underline{\pi}) = 0$ and $IC_e(2)$ violated. Again, under such a contract one would expect to see excessive risk-taking by the manager. Note that both the option contract and the bonus scheme would be incentive compatible in a model where there is a simple increasing relationship between the manager's effort choice and the project return, as would be the case in a pure moral hazard model. One should note therefore, that such a model might be seriously misleading if the manager's task is to make an informed investment decision in the interest of the financial investor, as it is assumed here.

Finally, consider the possibility to promise the manager a certain fraction α of the ex post return, that is, to award the manager shares in the company. Then the return contingent payment to the manager is just $w(z) = \alpha z$. Given that $\overline{\pi}R > I$ and that $\underline{\pi}R < I$, one sees immediately, that both IC_d- constraints are satisfied for any $\alpha \in [0,1]$. The IC_e-constraints reduce to

(1) $\alpha\left[P\overline{\pi}R + (1-P)I\right] - c \geq \alpha I$

(2) $\alpha\left[P\overline{\pi}R + (1-P)I\right] - c \geq \alpha\ \left[P\overline{\pi}R + (1-P)\underline{\pi}R\right]$

implying a lower bounds on α, which is given by

$$\alpha \geq \frac{c}{\min\left\{P\left(\overline{\pi}R - I\right),\ (1-P)\left(I - \underline{\pi}R\right)\right\}}$$

If α is larger than this lower bound, the share contract will be incentive compatible. Observe, however, that under a share contract the financial investor can expect a net return of at most $(1-\alpha)E^*[z] - I$ .While a share contract is thus a possible tool to implement the efficient strategy, it will, since the payment to the manager is not being minimized, in general be an unnecessarily costly one.

Even when the optimal contract is used to implement the efficient strategy, the rent accruing to the manager may be so large that the financial investor will not find it in his interest to implement the efficient strategy. It is easy to see that this will be the case whenever

$$P\overline{\pi}R + (1-P)I - c - \frac{Ac}{P\overline{\pi} + (1-P)A - A} < \max\left\{I, P\overline{\pi}R + (1-P)\underline{\pi}R\right\}$$

Since $\underline{\pi}R < I$ implies $P\underline{\pi}R + (1-P)I < I$, it clearly does not make sense to implement $d(\underline{\pi}) = 1$ and $d(\overline{\pi}) = 0$. One is therefore left with implementing $d(s) = 0\ \forall s$ or $d(s) = 1\ \forall s$. It is worth observing that none of $d(s) = 0\ \forall s$ and $d(s) = 1\ \forall s$ can be implemented in conjunction with $e = 1$, since there is no way of telling whether the manager has chosen $e = 1$ or $e = 0$. Since neither $d = 1$ nor $d = 0$ involves any cost to the manager, both of these two possibilities are optimally implemented by offering the manager a flat wage of zero. Therefore, if $P\overline{\pi}R + (1-P)\underline{\pi}\dot{R} > I$ and implementing the efficient strategy is not viable, the financial investor will implement blind investment, $e = 0$ and $d = 1$, whereas if $P\overline{\pi}R + (1-P)\underline{\pi}\dot{R} < I$ the financial investor will implement $e = 0$ and $d = 0$. The latter possibility is of course equivalent to not financing the project. In both cases there is an inefficiency arising from the fact that the manager receives an informational rent.

4. Two projects per manager, N=2

Let us now examine the case where the manager is assigned two identical projects. The contract can then condition on the vector of observable returns, so that the following matrix of wages needs to be determined:

$$\begin{matrix} w(0,0) & w(0,I) & w(0,R) \\ w(I,0) & w(I,I) & w(I,R) \\ w(R,0) & w(R,I) & w(R,R) \end{matrix}$$

However, since the projects are *iid*, it is clear that the optimal wage schedule will exhibit symmetry, in that $w(x,y) = w(y,x) \equiv w(x+y)$, so that only the following six wages need to be found: $\{w(0), w(I), w(R), w(2I), w(I+R), w(2R)\}$.

If the manager chooses the efficient strategy, this will induce a probability distribution over returns and thus over wages. The financial investor will receive an expected return and will have to pay an expected wage which is given by

$$\begin{aligned} &E^*(w) \equiv \\ &P^2\left[(1-\overline{\pi})^2 w(0) + 2\overline{\pi}(1-\overline{\pi}) w(R) + \overline{\pi}^2 w(2R)\right] \\ &+2P(1-P)\left[(1-\overline{\pi}) w(I) + \overline{\pi} w(I+R)\right] \\ &+(1-P)^2 w(2I) \end{aligned}$$

The financial investor's problem is to minimize this expression by choice of

$\{w(0), .., w(2R)\}$. This minimization is subject to a number of constraints, which we will introduce as we go along.

A first subset of constraints is given by the IC_d-constraints. Given that the manager has spent effort on both projects, these constraints make sure that the manager makes the right investment decision for each possible vector of signals received, $(\overline{\pi},\overline{\pi})$ $(\overline{\pi},\underline{\pi})$ $(\underline{\pi},\overline{\pi})$ $(\underline{\pi},\underline{\pi})$. Given a vector of signals, exactly one of the possible four strategies $(0,0), (0,1), (1,0), (1,1)$ is efficient. To ensure efficient investment one must have that for each signal the expected wage under the efficient investment strategy is weakly larger than under any other strategy [2]:

IC_d

$d(\overline{\pi},\overline{\pi}) = (1,1):$

$$(1-\overline{\pi})^2 w(0) + 2\overline{\pi}(1-\overline{\pi}) w(R) + \overline{\pi}^2 w(2R)$$

$(1) \geq w(2I)$

$(2) \geq (1-\overline{\pi}) w(I) + \overline{\pi} w(I+R)$

$d(\overline{\pi},\underline{\pi}) = (1,0):$

$$(1-\overline{\pi}) w(I) + \overline{\pi} w(I+R)$$

$(1) \geq w(2I)$

$(2) \geq (1-\underline{\pi}) w(I) + \underline{\pi} w(I+R)$

$(3) \geq (1-\underline{\pi})(1-\overline{\pi}) w(0) + [(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi}] w(R) + \overline{\pi}\underline{\pi} w(2R)$

$d(\underline{\pi},\underline{\pi}) = (0,0):$

$$w(2I)$$

$(1) \geq (1-\underline{\pi}) w(I) + \underline{\pi} w(I+R)$

$(2) \geq (1-\underline{\pi})^2 w(0) + 2(1-\underline{\pi})\underline{\pi} w(R) + \underline{\pi}^2 w(2R)$

Let us now come to the set of IC_e-constraints. These ensure that the wage net of effort costs the manager can expect when he chooses to spend effort on both projects, $e = (1,1)$, and then invests efficiently is larger than the expected wage net of effort costs under any other effort choice $(0,0),(0,1),(1,0)$ and all possible subsequent investment strategies. Let us start with the set of constraints that discourage $e = (0,0)$. Given this effort choice, the manager does not receive any information on the projects, $s = (0,0)$, and may choose among

$$(d(0,0)) \ \epsilon \ \{(0,0)(0,1)(1,0)(1,1)\}$$

Since the wage contract is symmetric $d(0,0) = (0,1)$ and $d(0,0) = (1,0)$ will result in the same expected payoff, so that one can write down the following three constraints:

$IC_e(e \neq (0,0)):$

[2] It is easily checked that the constraint for $(\underline{\pi},\overline{\pi})$ is exactly the same as for $(\overline{\pi},\underline{\pi})$ and it is therefore omitted. As for the notation, inequality signs are understood to relate to the top line.

$E^*(w) - 2c$

$(1) \geq w(2I)$

$(2) \geq P[(1-\overline{\pi})w(I) + \overline{\pi}w(I+R)] + (1-P)[(1-\underline{\pi})w(I) + \underline{\pi}w(I+R)]$

$(3) \geq P^2\left[(1-\overline{\pi})^2 w(0) + 2\overline{\pi}(1-\overline{\pi})w(R) + \overline{\pi}^2 w(2R)\right]$
$+2P(1-P)[(1-\underline{\pi})(1-\overline{\pi})w(0) + [(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi}]w(R) + \overline{\pi}\underline{\pi}w(2R)]$
$+(1-P)^2\left[(1-\underline{\pi})^2 w(0) + 2(1-\underline{\pi})\underline{\pi}w(R) + \underline{\pi}^2 w(2R)\right]$

One will also have to discourage strategies that involve the manager becoming informed on one of the two projects only, that is, the manager being lazy on one project, $(L=1)$. Clearly, the constraints discouraging $e=(0,1)$ and $e=(1,0)$ will be identical and we can focus on $e=(1,0)$. Given this effort choice the manager will receive a signal $s \in \{(\underline{\pi},0)(\overline{\pi},0)\}$ and can therefore condition his investment decision on the signal received. Thus, if $e=(1,0)$, say, the following strategies are possible : $\binom{d(\overline{\pi},0)}{d(\underline{\pi},0)} \epsilon$

$$\binom{0,0}{0,0}, \binom{0,0}{1,0}, \binom{1,0}{0,0}, \binom{1,0}{1,0}$$
$$\binom{0,0}{0,1}, \binom{0,0}{1,1}, \binom{1,0}{0,1}, \binom{1,0}{1,1}$$
$$\binom{0,1}{0,0}, \binom{0,1}{1,0}, \binom{1,1}{0,0}, \binom{1,1}{1,0}$$
$$\binom{0,1}{0,1}, \binom{0,1}{1,1}, \binom{1,1}{0,1}, \binom{1,1}{1,1}$$

To illustrate, and for ease of reference, let us write out the constraints associated with deviations to these strategies explicitly.

The first row translates into the following four constraints.

$(IC_e)\,(e \neq (1,0))$

$E^*(w) - 2c$

$(1) \geq w(2I) - c$

$(2) \geq P^2 w(2I)$
$+P(1-P)w(2I)$
$+(1-P)P[(1-\underline{\pi})w(I) + \underline{\pi}w(I+R)]$

$+(1-P)^2\left[(1-\underline{\pi})\,w(I)+\underline{\pi}w(I+R)\right]-c$

$(3)\geq P^2\left[(1-\overline{\pi})\,w(I)+\overline{\pi}w(I+R)\right]$
$+P(1-P)\left[(1-\overline{\pi})\,w(I)+\overline{\pi}w(I+R)\right]$
$+(1-P)\,Pw(2I)$
$+(1-P)^2\,w(2I)-c$

$(4)\geq P^2\left[(1-\overline{\pi})\,w(I)+\overline{\pi}w(I+R)\right]$
$+P(1-P)\left[(1-\overline{\pi})\,w(I)+\overline{\pi}w(I+R)\right]$
$+(1-P)\,P\left[(1-\underline{\pi})\,w(I)+\underline{\pi}w(I+R)\right]$
$+(1-P)^2\left[(1-\underline{\pi})\,w(I)+\underline{\pi}w(I+R)\right]-c$

The constraints associated with the second row are
$(IC_e)\,(e\neq(1,0))$
$E^*(w)-2c$
$(5)\geq P^2w(2I)$
$+P(1-P)\,w(2I)$
$+(1-P)\,P\left[(1-\overline{\pi})\,w(I)+\overline{\pi}w(I+R)\right]$
$+(1-P)^2\left[(1-\underline{\pi})\,w(I)+\underline{\pi}w(I+R)\right]-c$

$(6)\geq P^2w(2I)$
$+P(1-P)\,w(2I)$
$+(1-P)\,P\left[(1-\underline{\pi})(1-\overline{\pi})\,w(0)+\left[(1-\underline{\pi})\,\overline{\pi}+(1-\overline{\pi})\,\underline{\pi}\right]w(R)+\overline{\pi}\underline{\pi}w(2R)\right]$
$+(1-P)^2\left[(1-\underline{\pi})^2\,w(0)+2\,(1-\underline{\pi})\,\underline{\pi}w(R)+\underline{\pi}^2w(2R)\right]-c$

$(7)\geq P^2\left[(1-\overline{\pi})\,w(I)+\overline{\pi}w(I+R)\right]$
$+P(1-P)\left[(1-\overline{\pi})\,w(I)+\overline{\pi}w(I+R)\right]$
$+(1-P)\,P\left[(1-\overline{\pi})\,w(I)+\overline{\pi}w(I+R)\right]$
$+(1-P)^2\left[(1-\underline{\pi})\,w(I)+\underline{\pi}w(I+R)\right]-c$

$(8)\geq P^2\left[(1-\overline{\pi})\,w(I)+\overline{\pi}w(I+R)\right]$
$+P(1-P)\left[(1-\overline{\pi})\,w(I)+\overline{\pi}w(I+R)\right]$
$+(1-P)\,P\left[(1-\underline{\pi})(1-\overline{\pi})\,w(0)+\left[(1-\underline{\pi})\,\overline{\pi}+(1-\overline{\pi})\,\underline{\pi}\right]w(R)+\overline{\pi}\underline{\pi}w(2R)\right]$
$+(1-P)^2\left[(1-\underline{\pi})^2\,w(0)+2\,(1-\underline{\pi})\,\underline{\pi}w(R)+\underline{\pi}^2w(2R)\right]-c$

The constraints associated with the third row are

$(IC_e)\,(e \neq (1,0))$

$$E^*(w) - 2c$$

$$(9) \geq P^2\left[(1-\overline{\pi})\,w(I) + \overline{\pi}w(I+R)\right]$$
$$+P(1-P)\left[(1-\underline{\pi})\,w(I) + \underline{\pi}w(I+R)\right]$$
$$+(1-P)\,Pw(2I)$$
$$+(1-P)^2\,w(2I) - c$$

$$(10) \geq P^2\left[(1-\overline{\pi})\,w(I) + \overline{\pi}w(I+R)\right]$$
$$+P(1-P)\left[(1-\underline{\pi})\,w(I) + \underline{\pi}w(I+R)\right]$$
$$+(1-P)\,P\left[(1-\underline{\pi})\,w(I) + \underline{\pi}w(I+R)\right]$$
$$+(1-P)^2\left[(1-\underline{\pi})\,w(I) + \underline{\pi}w(I+R)\right] - c$$

$$(11) \geq P^2\left[(1-\overline{\pi})^2\,w(0) + 2\overline{\pi}(1-\overline{\pi})\,w(R) + \overline{\pi}^2 w(2R)\right]$$
$$+P(1-P)\left[(1-\underline{\pi})(1-\overline{\pi})\,w(0) + \left[(1-\underline{\pi})\,\overline{\pi} + (1-\overline{\pi})\,\underline{\pi}\right]w(R) + \overline{\pi}\underline{\pi}w(2R)\right]$$
$$+(1-P)\,Pw(2I)$$
$$+(1-P)^2\,w(2I) - c$$

$$(12) \geq P^2\left[(1-\overline{\pi})^2\,w(0) + 2\overline{\pi}(1-\overline{\pi})\,w(R) + \overline{\pi}^2 w(2R)\right]$$
$$+P(1-P)\left[(1-\underline{\pi})(1-\overline{\pi})\,w(0) + \left[(1-\underline{\pi})\,\overline{\pi} + (1-\overline{\pi})\,\underline{\pi}\right]w(R) + \overline{\pi}\underline{\pi}w(2R)\right]$$
$$+(1-P)\,P\left[(1-\underline{\pi})\,w(I) + \underline{\pi}w(I+R)\right]$$
$$+(1-P)^2\left[(1-\underline{\pi})\,w(I) + \underline{\pi}w(I+R)\right] - c$$

Finally, there are four constraints associated with deviations to strategies given by the last row.

$(IC_e)\,(e \neq (1,0))$

$$E^*(w) - 2c$$

$$(13) \geq P^2\left[(1-\overline{\pi})\,w(I) + \overline{\pi}w(I+R)\right]$$
$$+P(1-P)\left[(1-\underline{\pi})\,w(I) + \underline{\pi}w(I+R)\right]$$
$$+(1-P)\,P\left[(1-\overline{\pi})\,w(I) + \overline{\pi}w(I+R)\right]$$
$$+(1-P)^2\left[(1-\underline{\pi})\,w(I) + \underline{\pi}w(I+R)\right] - c$$

$$(14) \geq P^2\left[(1-\overline{\pi})\,w(I) + \overline{\pi}w(I+R)\right]$$
$$+P(1-P)\left[(1-\underline{\pi})\,w(I) + \underline{\pi}w(I+R)\right]$$

$+(1-P)P[(1-\underline{\pi})(1-\overline{\pi})w(0)+[(1-\underline{\pi})\overline{\pi}+(1-\overline{\pi})\underline{\pi}]w(R)+\overline{\pi}\underline{\pi}w(2R)]$
$+(1-P)^2\left[(1-\underline{\pi})^2w(0)+2(1-\underline{\pi})\underline{\pi}w(R)+\underline{\pi}^2w(2R)\right]-c$

$(15)\geq P^2\left[(1-\overline{\pi})^2w(0)+2\overline{\pi}(1-\overline{\pi})w(R)+\overline{\pi}^2w(2R)\right]$
$+P(1-P)[(1-\underline{\pi})(1-\overline{\pi})w(0)+[(1-\underline{\pi})\overline{\pi}+(1-\overline{\pi})\underline{\pi}]w(R)+\overline{\pi}\underline{\pi}w(2R)]$
$+(1-P)P[(1-\overline{\pi})w(I)+\overline{\pi}w(I+R)]$
$+(1-P)^2[(1-\underline{\pi})w(I)+\underline{\pi}w(I+R)]-c$

$(16)\geq P^2\left[(1-\overline{\pi})^2w(0)+2\overline{\pi}(1-\overline{\pi})w(R)+\overline{\pi}^2w(2R)\right]$
$+P(1-P)[(1-\underline{\pi})(1-\overline{\pi})w(0)+[(1-\underline{\pi})\overline{\pi}+(1-\overline{\pi})\underline{\pi}]w(R)+\overline{\pi}\underline{\pi}w(2R)]$
$+(1-P)P[(1-\underline{\pi})(1-\overline{\pi})w(0)+[(1-\underline{\pi})\overline{\pi}+(1-\overline{\pi})\underline{\pi}]w(R)+\overline{\pi}\underline{\pi}w(2R)]$
$+(1-P)^2\left[(1-\underline{\pi})^2w(0)+2(1-\underline{\pi})\underline{\pi}w(R)+\underline{\pi}^2w(2R)\right]-c$

To ensure that the manager accepts the contract, there will again be an $IR-$constraint.
(IR) :
$E^*(w)-2c\geq 0$

It remains to state the limited liability constraints
(NNW) :
$w(0)\geq 0, w(I)\geq 0, w(R)\geq 0, w(2I)\geq 0, w(I+R)\geq 0, w(2R)\geq 0$

The optimal contract implementing the efficient strategy has six constraints binding. These are $w(0)=0$, $w(I)=0, w(R)=0$ and $(IC_e)(e\neq(0,0))(1),(2),$ and (3). Substituting the binding $NNW-$constraints into $(IC_e)(e\neq(0,0))(1),(2),$ and (3) one has:

$$\begin{aligned}
& P^2\overline{\pi}^2w(2R)+2P(1-P)\overline{\pi}w(I+R)+(1-P)^2w(2I)-2c \\
= \; & w(2I) \\
= \; & [P\overline{\pi}+(1-P)\underline{\pi}]w(I+R) \\
= \; & \left[P^2\overline{\pi}^2+2P(1-P)\overline{\pi}\underline{\pi}+(1-P)^2\underline{\pi}^2\right]w(2R)
\end{aligned}$$

which one can solve for the closed form solution of the remaining wages as

$$
\begin{aligned}
w(2R) &= \frac{2c}{[P\overline{\pi}+(1-P)A]^2-A^2} \\
w(I+R) &= \frac{A2c}{[P\overline{\pi}+(1-P)A]^2-A^2} \\
w(2I) &= \frac{A^2 2c}{[P\overline{\pi}+(1-P)A]^2-A^2}
\end{aligned}
$$

where again, $A = P\overline{\pi} + (1-P)\underline{\pi}$.

For a formal proof of the optimality of this contract the reader is referred to the proof of the general case, which is given in the appendix. Let us here give some intuition on how this contract works. Since $(IC_e)\,(e \neq (0,0))\,(1)\,,(2)\,,$ and (3) all hold with equality, the manager is made indifferent between not investing at all, blindly investing into one project, and blindly investing into both projects, when he did not investigate any of the two projects. One can also check that when $w(0) = 0$, $w(I) = 0, w(R) = 0$, and $w(2I) = Aw(I+R) = A^2\ w(2R)$ holds for the remaining wages, the manager is made indifferent as to whether to blindly invest or not, given that he investigated one of the two projects, but did not investigate the other; in terms of the matrix of possible investment strategies for $e = (1,0)\,,$ one can show that each investment strategy in the same column of that matrix will give the manager the same expected wage. Thus, looking at the constraints associated with the first column, the RHSs of $(IC_e)\ (e \neq (1,0))\,(1)\,,(5)\,,(9)\,,$ and (13) all reduce to the same expected utility. Likewise the RHSs of $(IC_e)\ (e \neq (1,0))\,(2)\,,(6)\,,(10)\,,$ and (14) are the same, and the same is true for the third column and its associated constraints $(IC_e)\ (e \neq (1,0))\,(3)\,,(7)\,,(11)\,,$ and $(15)\,.$ Finally, all of $(IC_e)\,(e \neq (1,0))$ $(4)\,,(8)\,,(12)\,,$ and (16) share the same value on the RHS. Thus, whatever strategy the manager is planning to follows for the project he becomes informed on, he is indifferent as to investing or not investing into the project he did not look at. Going on from there, one can see, that among the deviations in the matrix of investment strategies the strategies in the third column yield the highest payoff, which is intuitive, since those are the ones that specify efficient investment on the project the manager did investigate. By substituting the expressions for the positive wages, one can finally show that these deviations leave the manager with a lower expected utility then if he does not investigate any of the two projects, $e = (0, 0)$, and hence also with a lower ex ante expected utility than the manager can obtain if he follows the efficient strategy. Next, if $w(0) = 0$, $w(I) = 0, w(R) = 0$, and $w(2I) = Aw(I + R) = A^2\, w(2R)$, it is easy to see that given that the manager becomes informed on both projects he interim has an incentive to invest efficiently, so that the (IC_d) constraints are satisfied.

Turning to the $NNW-$constraints, one can see why $w(0) = 0\ w(I) = 0$, $w(R) = 0$, at the optimum by noting that whenever these wages are paid, the manager has returned a zero return on one of the projects. A return of zero is more likely to occur when the manager has deviated from the efficient strategy than under the efficient strategy. Therefore, if, say, $w(0) > 0$ one can decrease $w(0)$ and at the same time increase $w(2R)$ in such a way as to leave the manager's equilibrium payoff unchanged, but making deviations from the efficient strategy less attractive. This can be achieved by reducing $w(0)$ by some $\varepsilon \leq w(0)$ and increasing $w(2R)$ by $\varepsilon\frac{(1-\overline{\pi})^2}{\overline{\pi}^2}$. The reader can easily check that this will leave all constraints satisfied, but will relax all those constraints, in which $w(0)$ enters as multiplied by off-equilibrium conditional probabilities $(1-\underline{\pi})^2$or $(1-\underline{\pi})(1-\overline{\pi})$. Likewise, if $w(R) > 0$ one can reduce $w(R)$ by some $\varepsilon \leq w(R)$ and increase $w(2R)$ by $\varepsilon\frac{2\overline{\pi}(1-\overline{\pi})}{\overline{\pi}^2}$ to leave all constraints satisfied, but relaxing all constraints that contain $w(R)$ premultiplied by off-equilibrium conditional probabilities of $2(1-\underline{\pi})\underline{\pi}$ or $[(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi}]$. Finally, if $w(I) > 0$ one can reduce $w(I)$ by some $\varepsilon \leq w(I)$ and at the same time increase $w(I + R)$ by $\varepsilon\frac{(1-\overline{\pi})}{\overline{\pi}}$. Again this operation will leave expected wages unchanged if the manager chooses the efficient strategy, but will make deviations less worthwhile, relaxing all constraints which contain $w(I)$ as premultiplied by the off-equilibrium path probability $(1-\underline{\pi})$ on their RHS.

Since $(IC_e)(e \neq (0,0))(1)$ is binding one can read off the rent accruing to the manager from its RHS. It is given by

$$rent = w(2I) = \frac{A^2 2c}{[P\overline{\pi} + (1-P)A]^2 - A^2}$$

One can show that

$$\frac{Ac}{P\overline{\pi} + (1-P)A - A} + \frac{Ac}{P\overline{\pi} + (1-P)A - A} > \frac{A^2 2c}{[P\overline{\pi} + (1-P)A]^2 - A^2}.$$

This means that the rent arising from employing two managers, one for each project, is higher than the rent arising from employing one manager for both projects. In fact, one can also show that[3]

$$\frac{Ac}{P\overline{\pi}+(1-P)\,A-A} > \frac{A^2 2c}{\left[P\overline{\pi}+(1-P)\,A\right]^2 - A^2}$$

which says that if one gives an additional project to the manager, his informational rent will decrease. Thus the financial investor can implement project investigation and efficient reevaluation more cheaply by allocating two projects to one manager.

The intuition behind this is that, given any wage contract, the manager faces a joint problem when he is given two projects. The manager will not decide what to do on one project independently of what he does on the second project, collapsing the second project to some expectation and then work out what is optimal for the first. Rather, these decisions are linked. Since the manager will make a joint decision on both projects, the wage contract should take account of this fact and condition not on each of the project returns separately but on the whole vector of returns. The optimal contract does just that. Notice in particular, that under the optimal contract the manager receives a zero wage whenever the return on any one of the two projects is zero, that is whenever the vector of returns contains a 0. On the other hand, wages are positive and increasing in the number of times the vector contains an R.

Thus the manager is punished and rewarded not on the basis of his average performance but using both return observations in a particular way. To see why this helps reducing the rent, recall that in the one-project case the manager was punished and received a wage of zero only if he returned a zero gross return on the project, but was paid positive wages for returns of I or R. If the manager has two projects, he is rewarded more highly when the return vector does not contain a zero, but punished with a zero wage whenever he returns zero on any one of the projects, even if on the other project he returns I or R. Thus with two projects one can push the manager's wage down to the limited liability constraint more often. Another way of seeing this is to say that with two projects, since wages for a return vector containing a $0's$ are zero and wages for a vector containing $I's$ and $R's$ are pushed up to compensate, there is a greater spread between the positive wages and zero. The manager has more to lose

[3] For a proof the reader is referred to section 5 below.

when he deviates, which will make it easier to provide incentives for the manager to choose the efficient strategy.

Even when the optimal contract is used to implement the efficient strategy, the manager still receives a positive rent, so that the financial investor may not find it in his interest to implement the efficient strategy on the two projects. This will be the case, whenever

$$P\overline{\pi}R + (1-P)\,I - c - \frac{A^2 c}{\left[P\overline{\pi} + (1-P)\,A\right]^2 - A^2} < \max\left\{I, P\overline{\pi}R + (1-P)\,\underline{\pi}R\right\}$$

Notice that the financial investor will never implement the efficient strategy on one project only and either blind investment or no investment on the other project. While both blind investment and no investment can be implemented at a zero wage, if efficient investment is implemented on one project only, this is optimally done by using the contract for the one-project case. This will however involve an even larger rent than implementing efficient investment on both projects, since

$$\frac{Ac}{P\overline{\pi} + (1-P)\,A - A} > \frac{2A^2 c}{\left[P\overline{\pi} + (1-P)\,A\right]^2 - A^2}$$

Thus efficient investment is implemented either on both projects or on none of them. In the latter case, either blind investment or no investment is implemented on both projects depending on which is more profitable.

5. General case, N projects per manager

Let us now proceed to analyze the general problem of implementing the efficient strategy when the manager is given N projects. Recall that the financial investor's problem can be written down as

$$\max_{w(\cdot)} E\left[\widetilde{z} - w\left(\widetilde{z}\right) \mid \widetilde{s} = \widetilde{\pi}, d = d^*\left(\widetilde{\pi}\right)\right]$$

s.t.

(IC_d) :

$$E\left[w\left(\widetilde{z}\right) \mid s = \pi,\ d = d^*\left(\pi\right)\right] \geq$$
$$E\left[w\left(\widetilde{z}\right) \mid s = \pi,\ d = d\left(\pi\right)\right]\ \forall \pi, \forall d\left(\cdot\right)$$

(IC_e) :

$$E\left[w\left(\widetilde{z}\right) \mid \widetilde{s}=\widetilde{\pi}, d=d^{*}\left(\widetilde{\pi}\right)\right]-Nc \geq$$
$$E\left[w\left(\widetilde{z}\right) \mid \widetilde{s}=\widetilde{s}\left(e\right), d=d\left(\widetilde{s}\right)\right]-\sum_{p=1}^{N} ce_p \quad \forall e, \forall d\left(\cdot\right)$$

(IR) :

$$E\left[w\left(\widetilde{z}\right) \mid \widetilde{s}=\widetilde{\pi}, d=d^{*}\left(\widetilde{\pi}\right)\right]-Nc \geq 0$$

(NNW) :

$$w\left(z\right) \geq 0 \; \forall z$$

Note first that since all projects are identical, the optimal contract will exhibit *symmetry* in the following sense. For two return vectors z and z' such that z contains k times R, $N-k-J$ times 0, and J times I, and z' also contains k times R, $N-k-J$ times 0, and J times I, wages will be equal, i.e. $w\left(z\right)=w\left(z'\right)$. We will henceforth use the notation $w\left(z\right)=w\left(z'\right)\equiv w\left(k,J\right)$, where the first argument denotes the number of R's, the second argument the number of I's, and the number of $0'$s are being suppressed. Given that the manager faces such a symmetric contract, the manager's optimal strategy will also be symmetric, in the sense that, for any investment strategy $d\left(\cdot\right)$, and for any signal $s\left(e\right)=\left(s_1,...s_N\right)$, any permutation of s will induce the same permutation on investment decisions $d\left(s\right)$. Using this, one can rewrite (IC_d) and (IC_e) in terms of the explicit distribution over wages induced by any strategy (these expressions are shown in the appendix). Having spent effort on all projects, and having received a signal $s=\pi$, only their expected success probability. The financial investor wants the manager to become informed on all projects, $L = 0$, and then to invest efficiently. The manager's expected utility from this will therefore have to be greater than the utility from any other effort choice $L = 1, .., N$ and any subsequent investment strategy, (IC_e). Lastly, this expected utility must be greater than zero, (IR). The wage contract that optimally implements the efficient strategy can then be characterized as follows.

Proposition 1. *$\forall N, \forall P, c, \overline{\pi}, \underline{\pi}$, s.t. $\overline{\pi} > \underline{\pi}$, c the following contract optimally implements the efficient strategy:*

a)

$$w(k, N-K) = 0 \ \ \forall k < K, \ \forall K = 0, .., N$$

b)

$$w(K, N-K) = \frac{A^{N-K} Nc}{\left[P\overline{\pi} + (1-P) A\right]^N - A^N} \ \ \forall K = 0, ..N$$

where $A = P\overline{\pi} + (1-P)\underline{\pi}$.

Part a) of the proposition says that the manager optimally receives a zero wage whenever the return on any of the projects is zero. The intuition is that such an event is more likely to occur when the manager deviated than when he invested efficiently. In particular, assume that the manager has decided to invest on K projects but not to invest on $N-K$ projects. On the equilibrium path all of the K projects will have had a conditional success probability of $\overline{\pi}$, that is the $K-$vector of success probabilities for those projects would have been $\pi_K^* = (\overline{\pi}, ..., \overline{\pi})$. Off the equilibrium path, however, the manager may sometimes invest into K projects even though the vector of success probabilities for the K projects is $\pi'_K \neq \pi_K^*$, that is when this vector contains one or more $\underline{\pi}'s$. This will occur when the manager invests into projects he knows to be bad, but also when the manager invests blindly into bad projects. Both on and off the equilibrium path the manager will receive a wage $w \in \{w(0, N-K), w(1, N-K) ..., w(K, N-K)\}$, depending on how many $R's$ he returns. It is shown in the appendix, that

$$\frac{a^*}{b^*} = \frac{\Pr\left[w = w(K, N-K) \mid \pi_K^*\right]}{\Pr\left[w = w(k, N-K) \mid \pi_K^*\right]} > \frac{\Pr\left[w = w(K, N-K) \mid \pi'_K\right]}{\Pr\left[w = w(k, N-K) \mid \pi'_K\right]} = \frac{a'}{b'}$$

for all $k < K$, for any such π'_K and for any fixed $N-K$. Hence, one can costlessly relax constraints by reducing $w(k, N-K)$ by ε and increasing $w(K, N-K)$ by$\varepsilon \frac{b^*}{a^*}$, since

$$-b^*\varepsilon + \frac{b^*}{a^*}a^*\varepsilon = 0,$$

so that on-equilibrium path conditional expected wages are left unchanged, but

$$-b'\varepsilon + \frac{b^*}{a^*}a'\varepsilon < 0,$$

so that conditional expected wages from deviating from efficient investment are reduced.

Apart from the binding (NNW) −constraints given in part a) the solution has the set of constraints $IC_e, (L = N)$ binding. Incorporating the binding (NNW) −constraints, these constraints are given by

$$\sum_{K=0}^{N} \binom{N}{K} P^K (1-P)^{N-K} \overline{\pi}^K w(K, N-K) - Nc$$

$$\geq A^B w(B, N-B)$$

$$\forall B = 0, 1, .., N$$

For N projects,this set comprises $N+1$ constraints: when the manager did not investigate any project he still has the choice of investing blindly into any number $B = 0, 1, ..., N$ of projects. Under the optimal contract, all of these constraints hold with equality, so that, given that he did not investigate any project, the manager is made indifferent as to the number of projects, he invests in blindly, that is, given $L = N$, for any B, the manager expected wage is the same. This

Also, given part a) of the proposition the (IC_d) constraints reduce to

$$\begin{aligned} w(K, N-K) &\geq \underline{\pi} w(K+1, N-K-1) \ \forall K = 0, .., N-1 \\ \overline{\pi} w(K, N-K) &\geq w(K-1, N-K+1) \ \forall K = N, .., 1 \end{aligned}$$

where the first constraint discourages overinvestment while the second forestalls underinvestment. Both are clearly met when $w(K, N-K) = Aw(K+1, N-K-1)$ as under the schedule given in part b) of the proposition, so that the manager interim has an incentive to invest efficiently.

From the binding IC_e-constraints one can read off the size of the rent accruing to the manager. It is

$$r_N = w(0, N) = \frac{A^N Nc}{[P\overline{\pi} + (1-P)A]^N - A^N}$$

Before we analyze this expression, let us make the following

Remark 1. : *For $N \geq 2$, the optimal wage schedule will in general not be monotone and will thus be neither concave nor convex in aggregate (or average) returns.*

Proof:$w(N-1, 0) = 0 < w(0, N)$, but $(N-1)R > NI$ for N large enough.

Notice that this already pertains to the case of $N = 2$, where we found $w(R) = 0$ and $w(2I) > 0$, whereas it may well be that $R > 2I$. This is in contrast to the basic problem, where we found a monotone wage schedule, and is in contrast also to most results found in the literature (exceptions include Innes (1990)). Here, the non-monotonicity results from the interplay between parts a) and b) of the proposition. As long as the manager presents a vector that contains only $I's$ and $R's$, the wage is increasing more than proportionately in the number of $R's$. However, whenever the vector contains one or more $0's$, the wage drops down to zero. While this scheme will provide the right incentives for any given number of projects, one can show that it works better as the number of projects rises. Thus one can define $\{C_N\}_{N=1}^{\infty}$ to be the sequence of optimal contracts as specified in Proposition 1 and then define$\{r_N\}_{N=1}^{\infty}$ as the sequence of associated rents. Analyzing this sequence one arrives at the following

Corollary 2. : *The sequence of rents $\{r_N\}_{N=1}^{\infty}$ is*

a) *strictly decreasing*, $r_N > r_{N+1}$ *and*

b) *converging to zero*, $\lim_{N\to\infty} r_N = 0$

Proof: To prove part a) one writes down the inequality in terms of its explicit expressions,

$$\frac{A^N Nc}{\left[P\overline{\pi} + (1-P)A\right]^N - A^N} > \frac{A^{N+1}(N+1)c}{\left[P\overline{\pi} + (1-P)A\right]^{N+1} - A^{N+1}},$$

Letting $D \equiv \left[P\overline{\pi} + (1-P)A\right]$, cancelling common terms and rearranging one gets

$$N\left(D^{N+1} - A^{N+1}\right) > (N+1)\left(D^N - A^N\right)A$$

or

$$K(D) \equiv N\left(D^{N+1} - A^{N+1}\right) - (N+1)\left(D^N - A^N\right)A > 0$$

To see that this must hold, one needs to note only that $K(A) = 0$ and that

$$\frac{\partial K(D)}{\partial D} = (N+1)ND^N - (N+1)ND^{N-1}A > 0$$

since $D > A$.

To prove part b) one proves $\lim_{N\to\infty} \frac{1}{r_N} = \infty$, which is equivalent to $\lim_{N\to\infty} r_N = 0$, since $r_N > 0\ \forall N$. But since

$$\frac{1}{r_N} = \frac{D^N - A^N}{A^N Nc} = \frac{1}{Nc}\left(\frac{D}{A}\right)^N - \frac{1}{Nc}\left(\frac{A}{A}\right)^N$$

one immediately has $\lim_{N\to\infty} \frac{1}{r_N} = \infty + 0 = \infty$, again using that $D > A$.

The result that the rent accruing to the manager is decreasing as he is assigned more and more projects and that it will vanish in the limit can be taken to provide a rationale for firm-level diversification. One immediate implication of the fact that the rent is strictly decreasing with N we want to state as the following

Corollary 3. *For N large enough the financial investor will want to implement the efficient strategy.*

To see this, note that since

$$P\overline{\pi}R + (1-P)I - c > \max\left\{I, P\overline{\pi}R + (1-P)\underline{\pi}R\right\}$$

there exists a smallest number of projects $\underline{N}$, such that for all $N \geq \underline{N}$

$$P\overline{\pi}R + (1-P)\,I - c - \frac{A^N c}{D^N - A^N} \geq \max\left\{I, P\overline{\pi}R + (1-P)\,\underline{\pi}R\right\}$$

so that it will eventually become profitable to implement the efficient strategy, rather than no investment or blind investment on all projects. Notice that, for any N either the efficient strategy, or no investment or blind investment is implemented on all projects. Given the manager has N projects, the financial investor will never implement the efficient strategy on a subset of $N^{'} < N$ projects and blind investment or no investment on the remaining projects, since this is optimally done by using the contract $C_{N^{'}}$ for the projects on which the efficient strategy is to be implemented and by paying a zero wage on the remaining projects. This will involve a higher rent then implementing the efficient strategy on all projects using C_N. Thus, for any N,the efficient strategy will be implemented on all projects or on none of them. For $N \geq \underline{N}$ it will be implemented on all projects. If $\underline{N} > 1$, then for $N < \underline{N}$, either blind investment or no investment is implemented on all projects depending on which of $P\overline{\pi}R + (1-P)\,\underline{\pi}R$ and I is larger. Both can be implemented at a zero wage. Implementing no investment is of course equivalent to not financing the projects. Notice therefore, that it is possible in this model that small firms will not be financed, whereas large firms always will.

In fact, of course, since the rent from implementing the efficient strategy on all projects is decreasing and converging to zero, in this basic model the optimal firm size is infinite. Among other things, this is due to the simplifying assumption, that the manager's effort endowment is unbounded and that effort costs increase linearly with the number of projects. These issues will be looked at more closely in the next section.

First, however, let us reiterate the basic intuition for why firm-level diversification is beneficial in our setup. It is not, as in Aron (1988) due to the fact that diversification provides additional independent signals of the manager's effort choice. Also, it should be noted that it does not exploit the law of large numbers as in the theory of financial intermediation do the models of Diamond (1984) and Williamson (1986). There the result that the optimal size of an intermediary is infinite is derived by using the fact that if the bank exerts monitoring effort, the monies received per firm cease to be stochastic as the bank takes on an infinite number of firms. Here, while a contract that did exploit the law of large numbers in a simple fashion (e.g. by giving the manager a certain return target) may work well enough in the limit, it would not be able to implement efficient choices by the manager for a finite number of projects. This, however, is achieved throughout by the optimal contract which we derived above. The reason why the rent decreases when the manager is given more projects is that given any wage contract the manager makes joint decision on what to do with the projects. What he does on one project is not independent of what he does on the second. The optimal contract takes this into account and conditions on the vector of project returns in a particular way. In our case we have assumed that limited liability is the source of the contracting problem, i.e. the fact that one cannot impose unbounded punishment on the manager to give him incentives. Thus, when the manager is given one project, he is punished with a zero wage only if he returns zero on the project and he is paid more than zero if he returns I. Looking back at this basic problem one sees that the Non-negative-wealth constraint is indeed responsible for the manager receiving a rent. Without it, one could find $w(0) < 0$, $w(R) > 0$, and $w(I) = 0$ that satisfy all constraints and leave the manager with an expected wage equal to his effort cost. When the manager has more projects, he still is protected by the $NNW-$ constraint, so that a wage of zero is still the worst one can do to the manager. However, this punishment can now be used more and more often, as the manager gets zero whenever he returns zero on any one of the projects he supervises. This allows $w(0, N)$ to shrink down to zero as N becomes large. Thus, one can think of the diversification effect as coming about through "relaxing" the limited liability constraint. Another way of viewing it, would be to say, that diversification relaxes the assumption that the manager is endowed with zero wealth. The incentive problem would not arise, if the manager could finance the project himself, and would be mitigated if he could put in at least some inside equity. When the manager has more projects, the manager's expected compensation for his effort cost can act as a substitute for inside equity, since he stands to lose it if he deviates from efficient decisions..

6. Extensions

6.1. Bounded effort endowment

The assumption made throughout in the analysis is that the manager has an unbounded effort endowment so that one manager can handle any number of projects. This may be a rather bold assumption[4]. One might think that relaxing it will give us an upper bound on the firm size. Let any manager's endowment be $\overline{E}$. Then if one is constrained to employ one manager, the firm size will be bounded by $\overline{N} = \overline{E}$. However, one might also be able to offer a contract as specified above to a coalition of manager. Consider the extreme case where $\overline{E} = 1$, so that one manager can at most handle one project. If one were to offer the wage contract for N projects to a coalition of N managers, and the aggregate wage comes subsequently to be shared equally by all managers, then if one assumes that each manager can costlessly monitor the effort choice, information and investment decision of one other manager, the coalition will accept the contract and enforce the efficient strategies on each of the N projects. It is easy to see then, that under this arrangement the optimal size of a firm is again infinite. Note however, that while we implicitly assumed that it is prohibitively costly for a financial investor to monitor a manager, the optimality of the arrangement described above relies on monitoring being costless inside the firm. While one can think of theoretical reasons why this might be so, a discussion of them is beyond the scope of this extension[5].

6.2. Effort cost a function of N

A straightforward extension of the basic model is to make the effort cost per project a function of N. Thus instead of a per project cost c one now has $c(N)$. An assumption that $c(N)$ is increasing in N may be justified by overheads increasing more than proportionately with N. One may also think that, if there is a coalition of managers, as N rises, it may become more and more costly to enforce an internal monitoring scheme as outlined in the previous section. For any of these reasons let us assume

[4] It has of course been made in the literature before, cf. e.g. Diamond (1984)

[5] An empirical case in point may be German corporate law, under which for large companies a group of chief managers is annually held responsible by shareholders for the performance of the firm.

that $c(N)$ is an increasing sequence which is unbounded above. Since the wage schedule offered in the main text is linear in c, for any N the optimal wage schedule implementing $e_p = 1$ and $d_p(\underline{\pi}) = 0$ and $d_p(\overline{\pi}) = 1$ can be found by replacing c everywhere by $c(N)$. Assuming that the manager's task of reevaluating the project is necessary for the project to be profitable, the condition for it to be profitable to implement efficient investment is

$$[P\overline{\pi}R + (1-P)I] - I - c(N) - \frac{A^N c(N)}{D^N - A^N} \geq 0$$

Clearly, then under the assumptions made the optimal firm size must be finite: even if $\frac{A^N c(N)}{D^N - A^N}$ were to converge to zero, the increasing costs ensure that the LHS will eventually become negative and stay negative for larger N.

7. Conclusion

In an environment that exhibits both moral hazard and adverse selection on the part of the manager we have been able to characterize the optimal contract implementing the efficient investment rule when the number of projects with respect to which both information asymmetries pertain is arbitrary. We have found that increasing the number of projects helps to alleviate the incentive problem between financial investor and manager. With more projects the financial investor is able to provide the right incentives more cheaply. The reason is that the optimal contract conditions on the vector of returns rather than each return observation separately, exploiting the fact that the manager makes joint decisions. The optimal contract changes such that when the manager is assigned more projects, he is more and more unlikely to receive larger and larger positive rewards and more and more likely to be punished with a zero wage. For any fixed N, the manager is rewarded highly when all projects turn out I or R, while he receives nothing when he returns a 0. While this may seem a very stark prediction, and may at first sight not seem to square with contracts we see in reality, one can argue that what the optimal contract is suggesting has some intuitive appeal. If a manager is in charge of project selection and his role is to spot bad projects, then we would expect the manager to receive relatively little when one project turns out badly, even though overall things are going fine. A similar idea can be found in the theory of teams, where team members are paid with reference to their relative performance as compared with the average[6]. There, however, the result is predicated on the assumption that the noise is correlated across team members, whereas our result holds in a world of stochastically independent projects that become linked only because a decision is made on all of them jointly. Our results also suggests that managers of large firms ought to be rewarded more highly, not only because their job presumably takes more effort, but also because they bear more "responsibility" in choosing among a large number of potential projects. Then, however, if things go wrong with one of the projects, the manager ought also be forced to take that responsibility and be given little if any rewards. Notice that shares and options may well come back into the picture when one tries to implement such a scheme. A package of shares and options can easily be used to generate a salary that is more than proportionately increasing in company returns. This package has, however, to be bundled with the threat of being sacked and losing all these benefits altogether to keep the manager from overinvesting into bad projects. One empirical implication of such a reinterpretation would be that we ought to observe large and diversified firms to have a higher turnover of chief executive officers than small and undiversified firms.

[6] cf. Holmstrom (1982)

Appendix

A Direct Revelation Mechanism

This section will analyze a direct revelation mechanism implementing efficient investment for $N = 1$.

Let us assume that the manager can make an announcement about the signal he received, $\widehat{\pi} \epsilon \{\overline{\pi}, \underline{\pi}, 0\}$. Let us also view the investment decision as contractible, so that the contract can specify the investment decision to be made given the announced signal. $d(\widehat{\pi}) : \{\overline{\pi}, \underline{\pi}, 0\} \rightarrow \{0, 1\}$. Wages can then condition both on the realized return and on the announced signal, so that we need to find $w(z, \widehat{\pi})$. It is clear that $w(z, 0) = 0$ for any function $d(\widehat{\pi})$ to be implemented, since if the manager claims not to have spent any effort we will not reward him. Since $w(z, \widehat{\pi}) \geq 0$, if we want to implement $e = 1$ and some $d(\widehat{\pi})$ we can therefore restrict attention to $\widehat{\pi} \epsilon \{\overline{\pi}, \underline{\pi}\}$; the manager is never going to admit that he did not spend effort. Defining

$$W(\pi, \widehat{\pi}) = (1 - d(\widehat{\pi}))\, w(I, \widehat{\pi}) + d(\widehat{\pi})\,(\pi w(R, \widehat{\pi}) + (1 - \pi)\, w(0, \widehat{\pi}))$$

the following constraints will have to be satisfied:

$W(\overline{\pi}, \overline{\pi}) \geq W(\overline{\pi}, \underline{\pi})$

$W(\underline{\pi}, \underline{\pi}) \geq W(\underline{\pi}, \overline{\pi})$

$PW(\overline{\pi}, \overline{\pi}) + (1 - P)\, W(\underline{\pi}, \underline{\pi}) - c \geq 0$

$PW(\overline{\pi}, \overline{\pi}) + (1 - P)\, W(\underline{\pi}, \underline{\pi}) - c \geq PW(\overline{\pi}, \underline{\pi}) + (1 - P)\, W(\underline{\pi}, \underline{\pi})$

$PW(\overline{\pi}, \overline{\pi}) + (1 - P)\, W(\underline{\pi}, \underline{\pi}) - c \geq PW(\overline{\pi}, \overline{\pi}) + (1 - P)\, W(\underline{\pi}, \overline{\pi})$

$w(z, \widehat{\pi}) \geq 0\ \forall (z, \widehat{\pi})$

When $d(\overline{\pi}) = 1$ and $d(\underline{\pi}) = 0$ these constraints reduce to

$\overline{\pi} w(R, \overline{\pi}) + (1 - \overline{\pi})\, w(0, \overline{\pi}) \geq w(I, \underline{\pi})$

$w(I, \underline{\pi}) \geq \underline{\pi} w(R, \overline{\pi}) + (1 - \underline{\pi})\, w(0, \overline{\pi})$

$P[\overline{\pi} w(R, \overline{\pi}) + (1 - \overline{\pi})\, w(0, \overline{\pi})] + (1 - P)\, w(I, \underline{\pi}) - c \geq 0$

$P[\overline{\pi} w(R, \overline{\pi}) + (1 - \overline{\pi})\, w(0, \overline{\pi})] + (1 - P)\, w(I, \underline{\pi}) - c \geq w(I, \underline{\pi})$

$P[\overline{\pi} w(R, \overline{\pi}) + (1 - \overline{\pi})\, w(0, \overline{\pi})] + (1 - P)\, w(I, \underline{\pi}) - c \geq$

$P[\overline{\pi} w(R, \overline{\pi}) + (1 - \overline{\pi})\, w(0, \overline{\pi})] + (1 - P)\, [\underline{\pi} w(R, \overline{\pi}) + (1 - \underline{\pi})\, w(0, \overline{\pi})]$

$w(0, \overline{\pi}) \geq 0, w(I, \underline{\pi}) \geq 0, w(R, \overline{\pi}) \geq 0$

which are exactly the constraints for the program given in the main text for $N = 1$.

Proof of Proposition 1

To prove that the contract specified in Proposition 1 is optimal, let us first introduce some further notation. Partition the set $\mathfrak{N}$ of N projects into two subsets, $\mathfrak{N} = \{\mathfrak{C}, \mathfrak{L}\}$, where $\mathfrak{C}$ is the set of projects the manager investigates (is curious on), and $\mathfrak{L}$ is the subset of projects the manager does not investigate (is lazy on). Denote the number of projects in these sets by C and L, respectively, where $C = N - L$. Introduce two partitions of $\mathfrak{C}: \ \mathfrak{C} = \{\mathfrak{J}, \mathfrak{K}\}$ and $\mathfrak{C} = \{\mathfrak{I}, \mathfrak{T}\}$, where $\mathfrak{J}$ is the subset of projects with low success probability, $\mathfrak{K}$ is the number of projects with high success probability, $\mathfrak{I}$ is the subset of $\mathfrak{C}$ the manager invests into, and $\mathfrak{T}$ is the subset of projects in $\mathfrak{C}$ the manager aborts. Denote the number of projects in these subsets by J and K for the first partition and I and T for the second. Next, partition the set of projects $\mathfrak{I}$ into which the manager knowingly invests into two subsets, $\mathfrak{I} = \{\mathfrak{I}_J, \mathfrak{I}_K\}$, where e.g. $\mathfrak{I}_J = \mathfrak{I} \cap \mathfrak{J}$ is the set of projects the manager knowingly invests in even though success probabilities are low. Next denote the number of elements in these sets by I_J and I_K, so that one has $I_J + I_K \equiv I$.

Given the distributional assumptions made one can then write down IC_d as

$$\sum_{k=0}^{K} \binom{K}{k} \overline{\pi}^k (1-\overline{\pi})^{K-k} w(k, N-K)$$

$$\geq \sum_{k=0}^{I} \sum_{j=0}^{k} \binom{I_J}{j} \binom{I_K}{k-j} \underline{\pi}^j (1-\underline{\pi})^{I_J - j} \overline{\pi}^{k-j} (1-\overline{\pi})^{I_K-(k-j)} w(k, N-I)$$

$$\forall \ (I_J, I_K) \ s.t. \ 0 \leq I_K \leq K, \ 0 \leq I_J \leq N-K \ \ \forall \ K \ s.t. \ 0 \leq K \leq N$$

(Note that by definition $\binom{a}{b} = 0$ for $b > a$). In order to be able to state the set of IC_e-constraints we need to introduce one more piece of notation. If the manager did not investigate L projects, let $\mathfrak{B} \subseteq \mathfrak{L}$ be the set of projects the manager invests in blindly and let B the number of these projects. Let K' be the number of projects in $\mathfrak{B}$ which, unknown to the manager, have high success probability and J' be the number of projects in $\mathfrak{B}$ which, unknown to the manager, have low success probability. The set of IC_e- constraints can then be written down as

$$\sum_{K=0}^{N}\binom{N}{K}P^{K}(1-P)^{N-K}\binom{K}{k}\overline{\pi}^{k}(1-\overline{\pi})^{K-k}w(k,J)-Nc$$

$$\geq\sum_{K=0}^{N-L}\binom{N-L}{K}P^{K}(1-P)^{N-L-K}\sum_{K'=0}^{B}\binom{B}{K'}P^{K'}(1-P)^{B-K'}$$

$$\sum_{k=0}^{I}\sum_{j=0}^{k}\binom{I_J}{j}\binom{I_K}{k-j}\underline{\pi}^{j}(1-\underline{\pi})^{I_J-j}\overline{\pi}^{k-j}(1-\overline{\pi})^{I_K-(k-j)}$$

$$\sum_{k'=0}^{B}\sum_{j'=0}^{k'}\binom{B-K'}{j'}\binom{K'}{k'-j'}\underline{\pi}^{j'}(1-\underline{\pi})^{B-K'-j'}\overline{\pi}^{k'-j'}(1-\overline{\pi})^{K'-(k'-j')}$$

$$w(k+k',N-B-I)-(N-L)c$$

$\forall\, L=1,...,N;\forall\, K\longmapsto(I_J,I_K,B)\ s.t.\ 0\leq I_K\leq K,\ 0\leq I_J\leq N-L-K,\ \ 0\leq B\leq L$

Proof of part a):

In order to prove that $w(k,N-K)=0\ \forall k<K,\ \forall K=0,..,N$ as in a) is indeed optimal, fix some $K>0$ and look at the subset of wages $\{w(k,N-K)\}_{k=0}^{N-K}$ .On the equilibrium path, whenever the manager invests into K projects, but does not invest in $N-K$ projects, the $K-$vector of success probabilities for those projects will be $\pi_K^*=(\overline{\pi},...,\overline{\pi})$, .containing K times $\overline{\pi}$. Off the equilibrium path, the manager invests into K projects and does not invest into $N-K$ projects when the vector of success probabilities for the K projects he invests in is $\pi_K'\neq\pi_K^*$, that is, when this vector contains one or more $\underline{\pi}'s$. Let the number of $\underline{\pi}'s$ in this vector to be $I_J+J'\equiv I_{J'}$, and the number of $\overline{\pi}'s$ to be $I_K+K'\equiv I_{K'}$, where $I_{J'}+I_{K'},=K$. On the equilibrium path one has

$$\Pr\left[w=w(K,N-K)\mid I_{J'}=0,I_{K'}=K\right]=\overline{\pi}^{K}$$

whereas for a given $k<K$

$$\Pr\left[w=w(k,N-K)\mid I_{J'}=0,I_{K'}=K\right]=\binom{K}{k}\overline{\pi}^{k}(1-\overline{\pi})^{K-k}$$

Off the equilibrium path one has

$$\Pr\left[w = w\left(K, N-K\right) \mid I_{J'} > 0, I_{K'} < K\right] = \underline{\pi}^{I_{J'}} \overline{\pi}^{I_{K'}} = \underline{\pi}^{I_{J'}} \overline{\pi}^{K - I_{J'}}$$

whereas one can write

$$\Pr\left[w = w\left(k, N-K\right) \mid I_{J'} > 0, I_{K'} < K\right]$$

$$= \sum_{j=0}^{k} \binom{I_{J'}}{j} \binom{I_{K'}}{k-j} \underline{\pi}^{j} \left(1-\underline{\pi}\right)^{I_{J'}-j} \overline{\pi}^{k-j} \left(1-\overline{\pi}\right)^{I_{K'}-(k-j)}$$

$$= \sum_{j=0}^{k} \binom{I_{J'}}{j} \binom{K - I_{J'}}{k-j} \underline{\pi}^{j} \left(1-\underline{\pi}\right)^{I_{J'}-j} \overline{\pi}^{k-j} \left(1-\overline{\pi}\right)^{K-I_{J'}-(k-j)}$$

Given these expressions we are now ready to establish that

$$\frac{\Pr\left[w = w\left(K, N-K\right) \mid I_J = 0, I_K = K\right]}{\Pr\left[w = w\left(k, N-K\right) \mid I_J = 0, I_K = K\right]}$$

$$\geq \frac{\Pr\left[w = w\left(K, N-K\right) \mid I_{J'} > 0, I_{K'} < K'\right]}{\Pr\left[w = w\left(k, N-K\right) \mid I_{J'} > 0, I_{K'} < K'\right]}$$

Substituting and rearranging one finds

$$\frac{\overline{\pi}^{K}}{\binom{K}{k} \overline{\pi}^{k} \left(1-\overline{\pi}\right)^{K-k}}$$

$$> \frac{\underline{\pi}^{I_{J'}} \overline{\pi}^{K-I_{J'}}}{\sum_{j=0}^{k} \binom{I_{J'}}{j} \binom{K-I_{J'}}{k-j} \underline{\pi}^{j} \left(1-\underline{\pi}\right)^{I_{J'}-j} \overline{\pi}^{k-j} \left(1-\overline{\pi}\right)^{K-I_{J'}-(k-j)}}$$

$\Longleftrightarrow$

$$\overline{\pi}^{K} \overline{\pi}^{k} \left(1-\overline{\pi}\right)^{K-k} \sum_{j=0}^{k} \binom{I_{J'}}{j} \binom{K-I_{J'}}{k-j} \underline{\pi}^{j} \left(1-\underline{\pi}\right)^{I_J}$$

$\Longleftrightarrow$

$$\frac{\sum_{j=0}^{k}\binom{I_{J'}}{j}\binom{K-I_{J'}}{k-j}\underline{\pi}^{j-I_{J'}}\left(1-\underline{\pi}\right)^{I_{j'}-j}\overline{\pi}^{I_{J'}-j}\left(1-\overline{\pi}\right)^{j-I_{J'}}}{\binom{K}{k}}>1$$

But

$$\underline{\pi}^{j-I_{J'}}\left(1-\underline{\pi}\right)^{I_{j'}-j}\overline{\pi}^{I_{J'}-j}\left(1-\overline{\pi}\right)^{j-I_{J'}}=\left(\frac{\left(1-\underline{\pi}\right)\overline{\pi}}{\left(1-\overline{\pi}\right)\underline{\pi}}\right)^{I_{J'}-j}\geq 1$$

with strict inequality for $j<I_{J'}$. Also, as a straightforward application of Vandermond's identity, one has $\sum_{j=0}^{k}\binom{I_{J'}}{j}\binom{K-I_{J'}}{k-j}=\binom{K}{k}$, which establishes the inequality. As explained in the main text, this result allows to set

$w\left(k,N-K\right)=0\;\forall k<K,\;\forall K=1,..,N.$

Proof of part b):

Incorporating a) one can write down a simplified set of IC_e-constraints as

$$\sum_{K=0}^{N}\binom{N}{K}P^{K}\left(1-P\right)^{N-K}\overline{\pi}^{K}w\left(K,N-K\right)-Nc$$

$$\geq\sum_{K=0}^{N-L}\binom{N-L}{K}P^{K}\left(1-P\right)^{N-L-K}\sum_{K'=0}^{B}\binom{B}{K'}P^{K'}\left(1-P\right)^{B-K'}\underline{\pi}^{I_J}\overline{\pi}^{I_K}\underline{\pi}^{B-K'}\overline{\pi}^{K'}$$

$$w\left(I+B,N-B-I\right)-\left(N-L\right)c$$

$\forall\;L=1,...,N;\forall\;K\longmapsto\left(I_J,I_K,B\right)\;s.t.\;0\leq I_K\leq K,\;0\leq I_J\leq N-L-K,\;\;0\leq B\leq L$

$\Leftrightarrow$

$$\sum_{K=0}^{N}\binom{N}{K}P^{K}\left(1-P\right)^{N-K}\overline{\pi}^{K}w\left(K,N-K\right)-Nc$$

$$\geq\sum_{K=0}^{N-L}\binom{N-L}{K}P^{K}\left(1-P\right)^{N-L-K}\underline{\pi}^{I_J}\overline{\pi}^{I_K}A^{B}w\left(I+B,N-I-B\right)-\left(N-L\right)c$$

$\forall\;L=1,...,N;\forall\;K\longmapsto\left(I_J,I_K,B\right)\;s.t.\;0\leq I_K\leq K,\;0\leq I_J\leq N-L-K,\;\;0\leq B\leq L$

The wage schedule stated under b) is derived from the subset of IC_e that has $L=N$ (so that $I=0$), with $B=0,1,..N$. Writing these out one has

$$\sum_{K=0}^{N}\binom{N}{K}P^{K}(1-P)^{N-K}\overline{\pi}^{K}w(K,N-K)-Nc$$

$$\geq A^{B}w(B,N-B)$$

$$\forall B=0,1,..,N$$

from which, after imposing the equality and substituting, the closed form schedule

$$w(K,N-K)=\frac{A^{N-K}Nc}{[P\overline{\pi}+(1-P)A]^{N}-A^{N}}\ \forall K=0,..N$$

is easily obtained. To prove that this wage schedule is optimal we need to show that (i) under this schedule all other constraints are satisfied and that (ii) the choice of binding constraints is optimal.

(i) Start with the (IC_e) constraints. In a first step one can show that under the proposed schedule, for any L and any (I_J,I_K) the manager is indifferent as regards the number of projects he invests into blindly. To see this note that under the proposed schedule

$$w(I+B,N-I-B)=A^{N-I-B}w(N,0)$$

so that IC_e further simplifies to

$$\sum_{K=0}^{N}\binom{N}{K}P^{K}(1-P)^{N-K}\overline{\pi}^{K}w(K,N-K)-Nc$$

$$\geq\sum_{K=0}^{N-L}\binom{N-L}{K}P^{K}(1-P)^{N-L-K}\underline{\pi}^{I_J}\overline{\pi}^{I_K}A^{B}A^{N-I-B}w(N,0)-(N-L)c$$

$$\forall\ L=1,...,N;\forall\ K\longmapsto(I_J,I_K,B)\ \ s.t.\ 0\leq I_K\leq K,\ 0\leq I_J\leq N-L-K,\ \ 0\leq B\leq L$$

from which the claimed indifference of the manager regarding B is immediate. In the following analysis we can therefore let $B=0$ without loss of generality and only consider

$$\sum_{K=0}^{N}\binom{N}{K}P^{K}(1-P)^{N-K}\overline{\pi}^{K}w(K,N-K)-Nc$$

$$\geq\sum_{K=0}^{N-L}\binom{N-L}{K}P^{K}(1-P)^{N-L-K}\underline{\pi}^{I_J}\overline{\pi}^{I_K}A^{N-I}w(N,0)-(N-L)c$$

$$\forall\ L=1,...,N;\forall\ K\longmapsto(I_J,I_K)\ \ s.t.\ \ 0\leq I_K\leq K,\ \ 0\leq I_J\leq N-L-K$$

Note next that since $A>\underline{\pi}$ it cannot be optimal for the manager to have $I_J>0$. Also, since $\overline{\pi}>A$ it cannot be optimal to have $I_K<K$,so that optimally $(I_J,I_K)=(0,K)\ \forall\ K$. Under the proposed wage schedule, given any L and any K the manager will have an incentive to invest efficiently as regards the projects he investigated. It remains to verify that the proposed wage schedule satisfies

$$\sum_{K=0}^{N}\binom{N}{K}P^{K}(1-P)^{N-K}\overline{\pi}^{K}w(K,N-K)-Nc$$

$$\geq\sum_{K=0}^{N-L}\binom{N-L}{K}P^{K}(1-P)^{N-L-K}\overline{\pi}^{K}A^{N-K}w(N,0)-(N-L)c$$

$$\forall\ L=1,...,N$$

Substituting $A^{N-K}w(N,0)$ for $w(K,N-K)$ on the LHS, defining

$$D\equiv P\overline{\pi}+(1-P)A$$

and then substituting

$$w(N,0)=\frac{Nc}{[P\overline{\pi}+(1-P)A]^{N}-A^{N}}=\frac{Nc}{D^{N}-A^{N}}$$

on both sides one obtains

$$\sum_{K=0}^{N}\binom{N}{K}P^K(1-P)^{N-K}\overline{\pi}^K A^{N-K}\frac{Nc}{D^N-A^N}-Nc$$

$$\geq\sum_{K=0}^{N-L}\binom{N-L}{K}P^K(1-P)^{N-L-K}\overline{\pi}^K A^{N-K}\frac{Nc}{D^N-A^N}-(N-L)c$$

$$\forall\ L=1,...,N$$

This can be further simplified to read

$$D^N\frac{Nc}{D^N-A^N}-Nc\geq D^{N-L}A^L\frac{Nc}{D^N-A^N}-(N-L)c\ \ \forall\ L=1,...,N$$

Rearranging one finally obtains

$$D^NN-\left(D^N-A^N\right)N-D^{N-L}A^LN+\left(D^N-A^N\right)(N-L)\geq 0$$

$$\forall\ L=1,...,N$$

$\Longleftrightarrow$

$$\left(A^N-D^{N-L}A^L\right)N+\left(D^N-A^N\right)(N-L)\geq 0$$
$$\forall\ L=1,...,N$$

Note that for $L=N$ (the manager does not investigate any project) one has

$$\left(A^N-A^N\right)N+\left(D^N-A^N\right)(N-N)=0$$

as one would expect given that $L=N$ is binding.

Next define

$$H_L(D)=\left(A^N-D^{N-L}A^L\right)N+\left(D^N-A^N\right)(N-L)$$

and note that

$$H_L(A)=\left(A^N-A^{N-L}A^L\right)N+\left(A^N-A^N\right)(N-L)=0$$

But

$$\frac{\partial H_L}{\partial D} = -\left(N-L\right) N D^{N-L-1} A^L + N\left(N-L\right) D^{N-1} > 0$$

$\forall L = 1,..,N-1$. Hence, since $D > A$ the inequality will be strictly satisfied for $L = 1,..,N-1$, so that the solution satisfies all (IC_e) constraints.

Next one can verify that all IC_d-constraints are satisfied. Incorporating a) these can be written down as

$$\overline{\pi}^K w\left(K, N-K\right)$$

$$\geq \underline{\pi}^{I_J} \overline{\pi}^{I_K} w\left(I, N-I\right)$$

$$\forall \ \left(I_J, I_K\right) \ s.t. \ I_J + I_K = I, \ \ 0 \leq I \leq N \ \ \forall \ K \ s.t. \ 0 \leq K \leq N$$

It is easily seen that these can more succinctly but equivalently be written down as

$$\overline{\pi}^K w\left(K, N-K\right) \geq \underline{\pi}\overline{\pi}^K w\left(K+1, N-K-1\right) \ \forall K = 0,..,N-1$$

$$\overline{\pi}^K w\left(K, N-K\right) \geq \overline{\pi}^{K-1} w\left(K-1, N-K+1\right) \ \forall K = N,..,1$$

The first set of constraints ensures that the manager does not overinvest, while the second set forestalls underinvestment. Both are clearly met when $w\left(K, N-K\right) = Aw\left(K+1, N-K-1\right)$ as under the schedule given in part b).

(ii) To establish optimality of the candidate wage schedule one has to show that Kuhn-Tucker-conditions are satisfied. It is well known that for a linear problem these reduce to the requirement, that the gradient of the objective lies inside the cone generated by the normals of the supposedly binding constraints. Given that $w\left(k, J\right) = 0 \ \forall k < N-J, \ \forall J = 0,..,N$ the minimand can be written as

$$\sum_{K=0}^{N} \binom{N}{K} P^K \left(1-P\right)^{N-K} \overline{\pi}^K w\left(K, N-K\right) - Nc$$

The binding IC_e-constraints are given by

$$\sum_{K=0}^{N} \binom{N}{K} P^K (1-P)^{N-K} \overline{\pi}^K w(K, N-K) - Nc \geq A^B w(B, N-B)$$

$$\forall B = 0, 1, .., N$$

Defining

$$\left[\binom{N}{K} P^K (1-P)^{N-K} \overline{\pi}^K \right]_{K=X} = G_X$$

and stacking the gradient vector and constraint matrix with the top element pertaining to $w(0, N)$ and the first column of the constraint matrix pertaining to $B = 0$ one has

$$\begin{pmatrix} G_0 \\ G_1 \\ G_2 \\ \\ G_N \end{pmatrix} = \begin{pmatrix} G_0 - 1 & G_0 & G_0 & G_0 \\ G_1 & G_1 - A & G_1 & G_1 \\ G_2 & G_2 & G_2 - A^2 & G_2 \\ \\ G_N & G_N & G_N & G_N - A^N \end{pmatrix} * \begin{pmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \\ \lambda_N \end{pmatrix}$$

where we require that $\lambda_l > 0$ for $l = 0, ..., N$.

It is easily checked, that

$$\lambda_l = \frac{\left[\binom{N}{K} P^K (1-P)^{N-K} \overline{\pi}^K A^{N-K} \right]_{K=l}}{\left[P\overline{\pi} + (1-P) A \right]^N - A^N} = \frac{G_l A^{N-l}}{\left[P\overline{\pi} + (1-P) A \right]^N - A^N}$$

so that indeed $\lambda_l > 0$ for $l = 0, ..., N$, q.e.d.

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