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HOUSING MARKET FLUCTUATIONS IN A LIFE-CYCLE ECONOMY WITH CREDIT CONSTRAINTS*

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Abstract

This paper presents a first step towards a new theory of housing market fluctuations. We develop a life-cycle model where agents face credit constraints and their housing consumption is restricted to a discrete set of possibilities. The market interaction of young credit constrained agents climbing the property ladder with old agents trading down, generates co-movements of aggregate house prices, volume of transactions and income, consistent with the patterns observed in the U.S. and the U.K. Under plausible assumptions, the model reproduces the slight lead of transaction volume over the other two series as documented in the data. Our theory asserts that the fluctuations in housing prices depend crucially on fluctuations in the current income of young households (the first-time buyers). Thus, it sheds light on why housing prices are more volatile than GDP, and why they exhibit some degree of predictability in a market where households optimize over the timing of their transactions.

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In both the U.S. and the U.K., housing prices and the volume of residential property transactions move with GDP. Housing prices display greater fluctuations than GDP; the volume of property transactions displays the highest volatility. We propose a theory of the housing market that is consistent with these co-movements and relative volatilities.

Our theory builds on two important features of observed household behaviour. First, the typical life-cycle pattern of housing consumption involves lumpy adjustments along the property ladder with jumps toward bigger dwellings when young, followed by downsizing later in life for some households. The U.S. Census data shows the median price of first-time buyer purchases at about 75 percent of the median price of repeat buyer purchases. First-time buyers tend to be in their early thirties, whereas repeat buyers are generally in their early forties. Thus, at least some part of the population moves to a more expensive property within a few years of their first purchase.[1] There is also strong evidence that housing consumption declines with age for the elderly.[2]

Second, credit constraints in the form of down-payment or collateral requirements significantly affect the housing consumption of individuals. This is well documented in the literature.[3] Such constraints force households to delay their purchase while saving for a down payment. A minority of first-time buyers receive significant help from their family.[4] In addition, down-payment requirements amplify the effects of capital gains when the owner is considering moving to a bigger place. For example, a gain of $\$5,000$ on a property allows the owner to invest in another property worth an extra $\$50,000$ if the down-payment requirement is 10%.

Given these two observations, we design a life-cycle model with the following characteristics. Agents are born in their parents' house. As soon as they have accumulated enough savings to afford a small dwelling (a flat, hereafter), they purchase one. A few years later, having saved more, they buy a bigger dwelling (a house, hereafter). Finally, in old age, some agents reduce their consumption of housing services by moving back into a flat before they die. In each period, as many old agents die as young agents are born, so the overall population size stays constant. Since the focus of this paper is on the demand side of the market, we assume a fixed total supply of flats and houses. We further assume that each dwelling accommodates exactly one agent (with the exception

[1] Lumpy adjustments can be explained by transaction costs that have to be paid when a dwelling is traded; cf. Grossman and Laroque (1990).

[2] Mankiw and Weil (1989), Green and Hendershott (1996), Megbolugbe, Sa-Aadu, and Shilling (1997), and Jones (1997).

[3] Linneman and Wachter (1989), Jones (1989), Ioannides (1989), Zorn (1989), Duca and Rosenthal (1994), Engelhardt (1996), and Haurin, Hendershott and Wachter (1997).

[4] Engelhardt (1996) reports that only one-fifth of U.S. first-time buyers receive some help from relatives in accumulating their down payment.

of young agents who can live with a parent). This in turn implies that the number of people living with their parents is fixed.

The life-cycle structure implies the following interaction of agents on the housing market. Two types of agents buy flats. The first are young agents who have been saving for a down payment while living at home. Their decision to move depends on the price of flats, their ability to accumulate sufficient savings, and their current income. The second type of flat buyers are older people who are downsizing their housing stock from a house to a flat. Declining wealth and/or declining preferences for houses over flats motivate their decision. On the other side of the flat market, the sellers are either old agents leaving the housing market, or middle aged people who have been saving for a house down payment while living in a flat. The moving decision of the latter depends on their ability to accumulate a sufficient down payment, as well as the price of houses. Another factor is the evolution of the flat price from the time when these agents purchased their own.

Since the number of agents living with their parents is fixed in equilibrium, the price of flats must be such that a constant fraction of the young cohorts cannot move out of their parents' home. As a consequence, the price of flats is a function of the accumulated wealth and current income of the young agents. On the other hand, the old agent who is at the downsizing margin between house and flat ownership faces no credit constraint. He is indifferent between the two types of properties, so the difference between the user cost (i.e., cost of holding the property for one period) of a house and that of a flat just equals this marginal agent's utility premium for a house. Therefore, the equilibrium house price consists of two components: the price of flats and the utility premium of houses for the marginal downsizing old.

These price fundamentals imply that the volatility of housing prices depends on the volatility of the income of young households (the first-time buyers), not on that of *per capita* income. Moreover, to the extent that the income of young households is predictable, so are housing prices.

The reason why transaction volume increases with price in the model is that the young flat owners' demand for housing shifts more than the demand of older agents in response to changes in income. For example, let us consider the model's impulse response to an income shock; i.e., suppose that starting from the steady state equilibrium, agents are surprised by a one-period income increase. From the day of the surprise onward, assume perfect foresight. The income shock first affects everyone's demand through the traditional income effect. This is the only effect on the housing demand of older agents. Second, the income shock enables agents living with their parents to afford a higher down payment and, therefore, a bigger loan toward the purchase of a property. This implies that flat prices must increase in order to keep the number of agents living with their parents constant. Third, the capital gain enjoyed by flat owners reinforces the effect of the income increase on their borrowing capacity. These capital gains can be used as down payment for a bigger property. Consequently, the shock has stronger effects on the demand for houses of young flat owners than on that of the elderly. In equilibrium, the number of agents living in houses must remain constant. So house prices rise in order to induce some elderly house owners to sell their house to young flat owners; more young flat owners buy houses from elderly house owners than would have happened without the shock. This means a younger population of house owners and an increase in the volume of transactions at the time of the income increase. The symmetric argument applies for a negative shock, which triggers a decrease in price and transaction volume, and an increase in the average age of house owners.[5]

The results are preserved when we extend the model to include the possibility of renting a flat. The extra margin between renting and owning a flat generates additional fluctuations in the volume of transactions. In response to a positive income shock, for example, the owner occupancy rate increases, so some properties owned by investors who rented them out are sold to new owner occupiers.

Our theory can explain the observed lead of transaction volume over the business cycle. As the economy slips into recession, prices start to fall. Young flat owners suffer capital losses, which slow them down in their move up the property ladder. The volume of transaction decreases immediately. Towards the end of a recession, most agents who own a flat have bought it at a low price. Having suffered no or only small capital losses, these agents can afford the down payment on a house earlier than their predecessors could, and transaction volume rises. This is then reinforced by the capital gains flat owners enjoy as the economy picks up, leading to a further rise in transaction volume at the beginning of the boom. Towards the end of the boom, most agents who own a flat have bought it at a high price. Having enjoyed no or only small capital gains, these agents cannot afford the down payment on a house as early as their predecessors could, so transaction volume falls. This fall is reinforced by flat price decreases as the economy enters the recession.

The aggregate housing market regularities that this paper attempts to explain concern fluctuations of housing prices and transaction volume over the business cycle. In both the U.S. and the U.K., aggregate housing prices and transaction volume move with GDP,

[5]In our model, therefore, transaction volume in the housing markets moves with agents' wealth. By contrast, Grossman and Laroque (1990, p.46) suggest that both large increases and large decreases in agents' (stock market) wealth should trigger a spike in the volume of housing transactions.

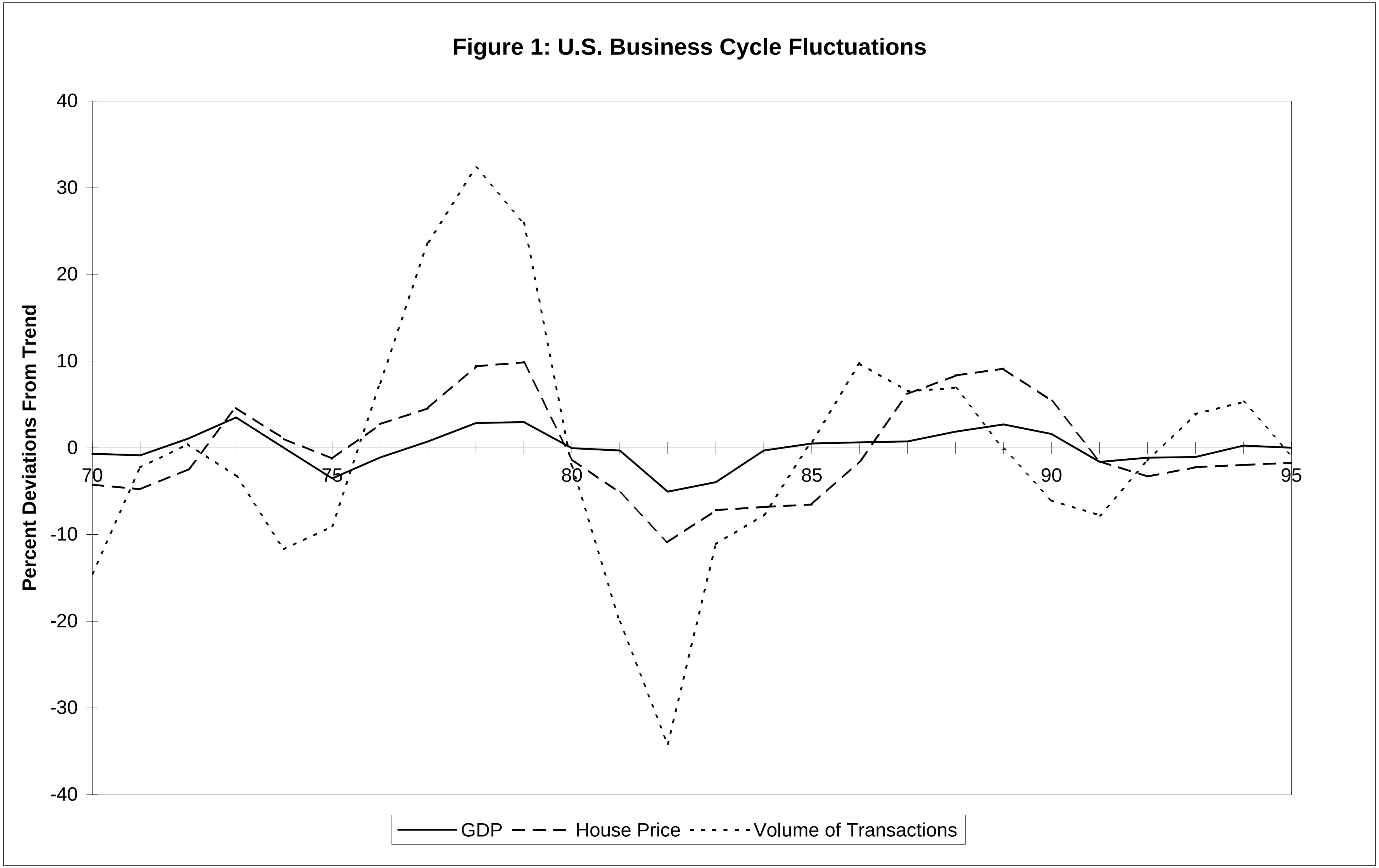


Figure 1: U.S. Business Cycle Fluctuations
Percent Deviations From Trend
40
30
20
10
0
-10
-20
-30
-40
70
75
80
85
90
95
GDP
House Price
Volume of Transactions

Figure 2: U.K. Business Cycle Fluctuations

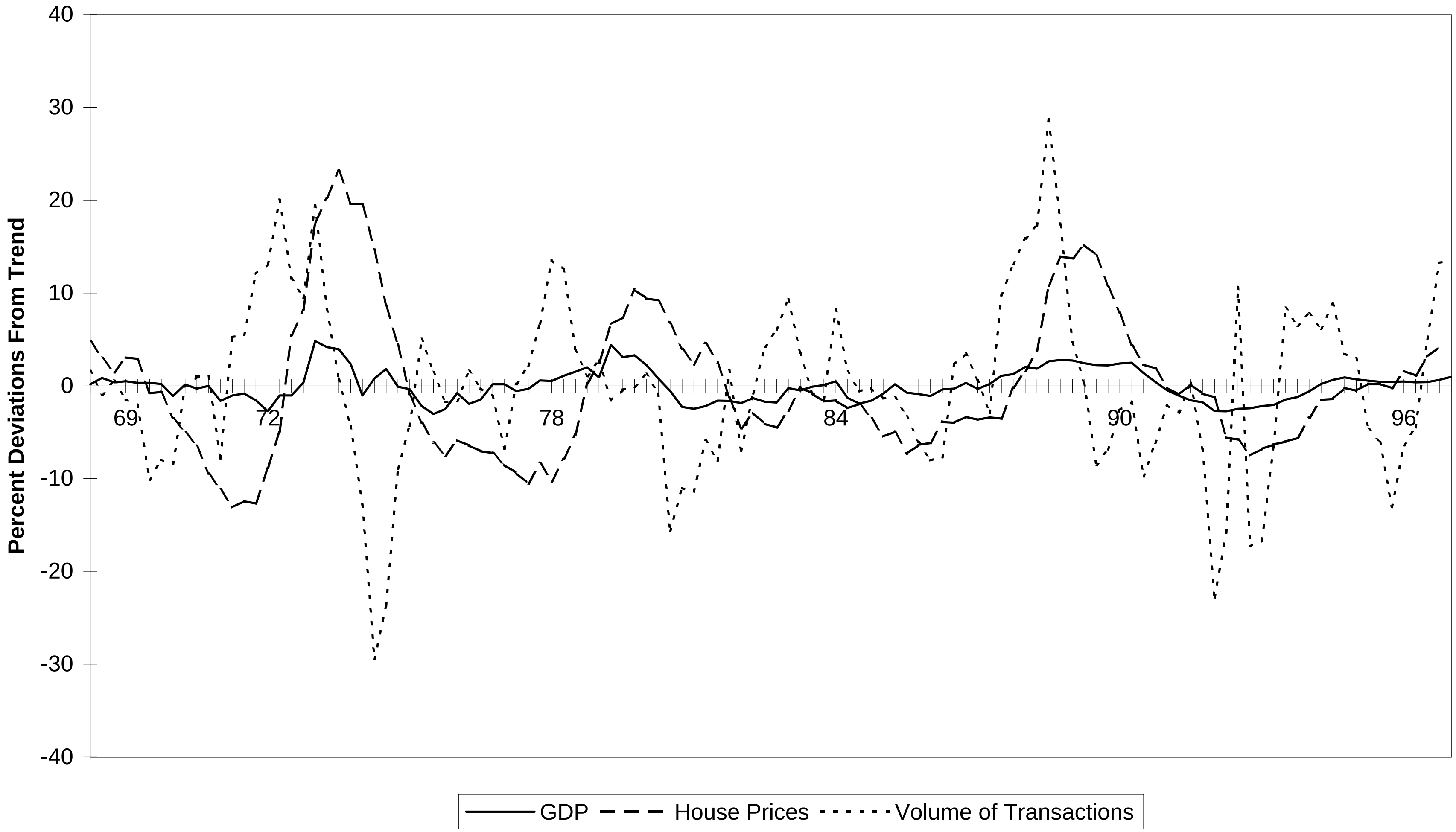


Table 1: Cyclical behaviour of the U.S. housing market, yearly data 1970-95

Variable	Volatility relative to GDP	Correlation of GDP with Variable at: t-3	t-2	t-1	t	t+1	t+2	t+3
		Years						
Housing Price	2.77	-.36	.03	.49	.78	.50	.14	-.16
Vol. of Trans.	6.74	-.26	.35	.74	.65	.10	-.30	-.48

The *volatility relative to GDP* is calculated as the ratio of two standard deviations. The standard deviation in the numerator is that of the percentage deviations of the variable from its trend. The standard deviation in the denominator is that of the percentage deviations of GDP from its trend.

Table 2: Cyclical behaviour of the U.K. housing market, quarterly data 1965:1-1996:1

Variable	Volatility relative to GDP	Correlation of GDP with Variable at: t-5	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+5
		Quarters									
Housing Price	4.45	-.2	0	.2	.4	.55	.65	.68	.67	.60	.39
Vol. of Trans.	5.35	.6	.6	.52	.45	.35	.26	.11	-.06	-.21	-.41

See Table 1 for the definition of *volatility relative to GDP*.

with the transaction volume leading both GDP and housing prices; cf. Figures 1 and 2.[6] For the U.S., Figure 1 suggests a lead of approximately one year, which is confirmed by the cross-correlation analysis of Table 1. This is similar to the U.K. where the same computations show a lead of four to five quarters; cf. Table 2. In both countries, housing price fluctuations, as percentage deviations from trend, are larger than those of GDP but smaller than those of the volume of transactions; cf. the volatility relative to GDP reported in Tables 1 and 2.[7] These regularities are not unique to the U.K. and the U.S. For example, a similar pattern of housing price and GDP co-movements can be observed in France and Japan.

In addition to the above regularities, U.S. data exhibits negative correlation between the average age of repeat buyers and housing prices; cf. Figure 3. This is predicted by our model. In fact, it is a distinctive feature of our theory that young agents climb the property ladder faster during booms, and slower during recessions.

[6] Both figures, as well as Tables 1 and 2, are based on H-P filtered time series data. We use standard values for λ; i.e., 100 for the U.S. yearly data and 1,600 for the U.K. quarterly series. Prices and GDP are converted in real terms using each country's CPI. The U.S. data is from the Statistical Abstract of the U.S. (various years). The U.K. data is from Datastream. The reported business cycle facts are very robust to the type of filtering used in constructing the series. For example, the same facts would emerge had we detrended linearly.

[7] Note that relative to GDP, housing prices are more volatile in the U.K. than in the U.S. According to our theory, one possible explanation could be that the income of first-time buyers is more volatile (relative to GDP) in the U.K. than in the U.S.

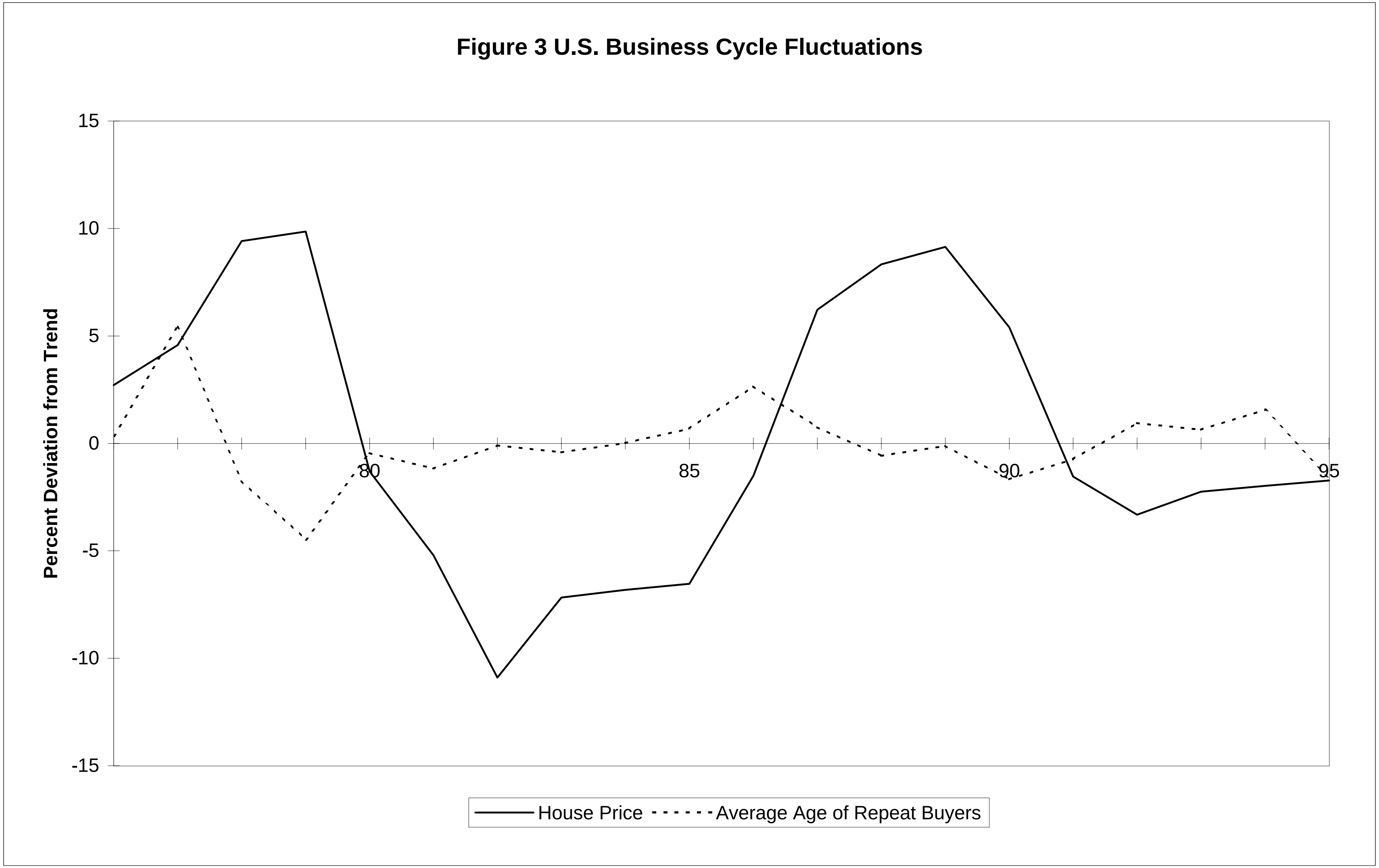


Figure 3 U.S. Business Cycle Fluctuations
Percent Deviation from Trend
15
10
5
0
-5
-10
-15
80
85
90
95
House Price
Average Age of Repeat Buyers

Further support for our theoretical framework comes from the literature which estimates reduced form housing price equations. This literature points to current income and demographic factors as significant determinants of short run housing market dynamics.[8] As to demographics, the size of the 20-34 year old cohort matters most in the U.S. (Mankiw and Weil, 1989); Muellbauer and Murphy (1997) report that studies on U.K. data reveal the same finding for the 25-29 year old cohort. This is again predicted by our model. As mentioned earlier, these agents are first-time buyers. Our model predicts that shocks to their current income have determinant effects on flat and house prices. Although33.99980TD130.00010T6a3(earlier,)-3001TDfl(Ad000103fl(rev)Tjfl129no0TDfl7Dfl(to)Tjflarry

The rest of the paper is organised as follows. Section 1 presents the model. Section 2 provides an analytical characterization of equilibrium prices and transaction volume in a particular configuration of our model. Section 3 examines the response of the model to transitory and permanent income shocks; numerical examples clarify the discussion. Section 4 extends our model to incorporate a rental sector. Section 5 discusses an alternative form of the down-payment requirement which helps explain the lead of transaction volume over price. Section 6 contains concluding remarks.

1 The Model

Consider a life-cycle economy where agents live J periods. Each period, a measure one of agents is born, each of them identified by a name $i \in [0,1]$. This index determines the size of the endowment of the numeraire consumption good, food, that the agent receives. More precisely, the endowment at date t for agent i of age $j = 1, \ldots, J$ is

$$e_t(i,j) = (1+i)\, w_t(j), \tag{1}$$

so the poorest age j agent receives $w_t(j)$, and the richest agent of this cohort receives $2w_t(j)$. We assume that there are no bequests or, equivalently for our results, that no bequest is received until agents have advanced a few steps along the property ladder.[11]

Other than food, there are two additional commodities in the economy: flats and houses. They are both available in fixed amounts, S^F and S^H, respectively. While young, agents may live with their parents although at a utility cost for themselves, but not for their parents. Each agent maximizes a time-separable utility function over bundles of food, c, and the type of housing, h, with $h \in \{P, F, H\}$, where P, F, and H stand for parents, flat and house, respectively. The instantaneous utility is assumed additively separable in food and housing, $U(c) + G(h)$. Utility from future consumption of food and housing is discounted at a fixed rate $\beta < 1$, common to all agents. To clarify our argument we assume $U(.)$ to be linear. The linear utility assumption yields a trivial optimal food consumption plan as long as the interest rate equals or exceeds $1/\beta - 1$: save everything for the end.

The most important feature of our model is the interaction of young constrained buyers and old unconstrained house sellers/flat buyers. To obtain such an interaction on the housing market,[12] we provide the old with a utility motive for downsizing their

[11] The crucial assumption for our results is that the housing consumption path of at least some agents in the economy is constrained by their lack of liquidity during the early stages of their life. Such an assumption is widely supported by the empirical evidence, e.g. Engelhart (1996).

[12] By *housing market* we mean the market for both flats and houses. Similarly, we shall refer to a price index for the housing market as the *housing price*.

housing stock. We assume that, as agents age, they progressively lose their taste for a house and some of them eventually prefer a flat. Specifically, we assume the following function for the utility of housing services:

$$\begin{aligned} G(P;i,j) &= u^P < 0 \quad \forall i \; \forall j, \\ G(F;i,j) &= 0 \quad \forall i \; \forall j, \\ G(H;i,j) &= \overline{u}^H > 0 \quad \forall i \text{ if } j < j^H, \\ G(H;i,j) &= \underline{u}^H + i \cdot \Delta(j) \quad \forall i \text{ if } j^H \le j \le J \end{aligned} \tag{2}$$

with $\underline{u}^H < 0$ and $\Delta(j) > 0$ strictly decreasing in j. Thus, j^H is a threshold age. Starting at that age, each agent's utility premium for houses over flats depends on her name and age: it increases with the index i and decreases with the age j. In particular, agents with names close to zero prefer to live in a flat from age j^H on. The parameters $\Delta(j)$ measure the dispersion of agents' housing preferences.[13]

Agents may save at the interest rate r_t, exogenously given. They can borrow at the same interest rate, but only up to a limit. We assume dwellings are the only collateralizable assets. Agents are born with no assets. Living with one's parents is free and has zero collateral value. Lenders have the right to seize assets in the case of default. Due to some implicit transaction costs, lenders do not allow borrowers to hold a debt higher than a fixed proportion, γ, of the discounted expected next period value of the dwelling purchased. We assume perfect foresight and thus formulate this condition as

$$-s_t \le \gamma \frac{q_{t+1}^h}{1+r_t} \tag{3}$$

where s_t denotes savings (hence $-s_t$ is the amount borrowed), and q_t^h denotes the time t price of a dwelling of type $h \in \{F, H\}$.[14] At the time of purchase, this constraint amounts to a down-payment requirement which depends on the value of the dwelling and the anticipated price change; the required down payment equals $q_t^h - \gamma q_{t+1}^h/(1+r_t)$. The rest of the time, this constraint limits the amount of home equity loans. An advantage of this forward looking borrowing constraint is that it captures the observation that in periods of booms, lenders tend to allow higher loans in proportions to the price of the purchase. In periods of house price decline, lenders tend to require more down payment

[13] In the equilibrium constructed below, all agents of a given cohort own a house at some age $j^* < j^H$. We can think of the housing preferences of older agents as being determined by a random draw at age j^*. Since incomes do not matter any more for housing choices after age j^*, it is then convenient (and without loss of generality) to use one and the same index to describe agents' incomes while young and their preferences while old.

[14] As formulated here, the constraint supposes that banks make margin calls on mortgages. Alternatively, we could have assumed that the constraint needs to hold only when the size of the loan is increased. The major difference between such a specification and the one above is that some agents may be forced to remain in their current dwelling due to negative equity, as happened recently in the U.K. following a negative shock.

as protection in case they need to seize the collateral. Later, we present what happens if loans are restricted to a fixed proportion of contemporaneous price.

The timing of the model is such that at the end of each period, agents decide which type of accommodation to occupy in the following period, execute the corresponding transactions on the housing and credit markets, and last, consume food. Agent i of age j chooses consumption c, savings s and next period's dwelling h' optimally given the state of the economy, her current savings, and her current dwelling h. The budget constraint is

$$c_t + s_t \leq (1 + r_{t-1})s_{t-1} + e_t(i, j) + q_t^h - q_t^{h'} \tag{4}$$

– today's consumption plus end-of-period savings must not exceed the beginning-of-period savings plus the endowment, plus the net receipts from a possible housing market transaction.

Given an exogenous sequence of interest rates $\{r_t\}_t$, an equilibrium in this economy is a sequence of flat and house prices $\{q_t^F, q_t^H\}_t$ together with a sequence of allocations $\{c_t(i, j), h_t(i, j), s_t(i, j)\}_t$ for all names $i \in [0, 1]$ and all ages $j = 1, ..., J$, such that in each period, the allocation solves each agent's constrained optimisation problem and the flat and house markets clear. A steady state equilibrium of our economy is a time independent equilibrium. For the economy to be in steady state, the interest rate and the endowment profile must be constant.

The assumed heterogeneity of agents will yield a variety of housing consumption paths in equilibrium. Agent $i + \epsilon$ (for $\epsilon > 0$) will spend more on housing over her lifetime than agent i. In general, this does not imply that agent $i + \epsilon$ occupies a dwelling at least as big and expensive as agent i in *every* period.[15] For the sake of tractability, however, we will specify the parameters of the utility function $G(.)$ and the endowment profile $e_t(.)$ such that the resulting equilibrium exhibits a monotonic relationship, within each cohort, between the agent's name and the type of dwelling she owns. We can then introduce cutoff indices $i_t^F(j)$ and $i_t^H(j)$ such that, in the cohort of age j at time t, all agents with indices in $[0, i_t^F(j))$, $[i_t^F(j), i_t^H(j))$ and $[i_t^H(j), 1]$ live with their parents, in a flat, and in a house, respectively.

[15] When agent i moves into a flat at age j, for example, agent $i + \epsilon$ could find it optimal to remain with her parents in order to accumulate savings faster and then move to a house earlier than agent i.

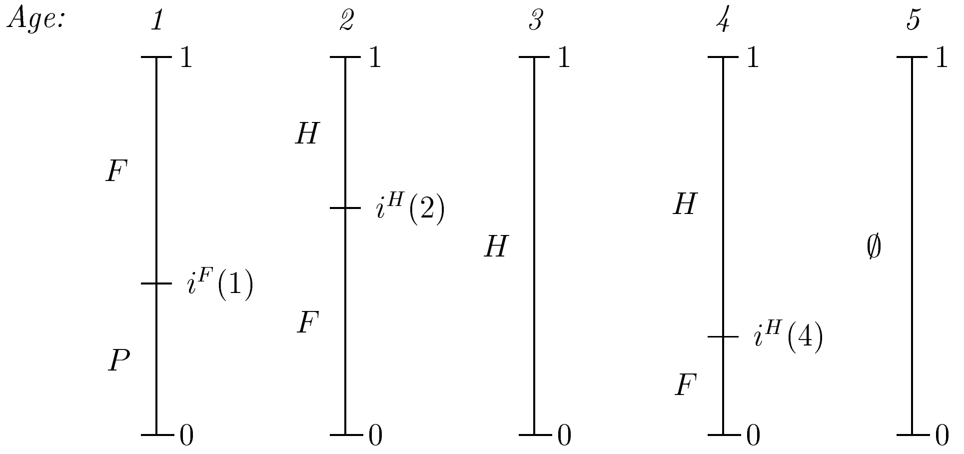


Figure 4: End of period steady state housing choices by age groups

2 Equilibrium

To highlight the effects at play in our model, we study the reaction of a simple configuration to various shocks. Suppose agents live five periods ($J = 5$). We will focus on a configuration of our model where the pattern of housing choices in steady state equilibrium is as in Figure 4. This figure shows the results of the end-of-period trade in dwellings.[16] In this equilibrium, young agents move out of their parents' home as soon as they can afford a flat. Similarly, young agents buy a house as soon as they can afford one. Some agents can afford a flat at the end of their first period of life. The others stay with their parents, waiting for their second period endowment to help them pay the down payment on the flat. At age 3, everyone has accumulated enough wealth to purchase a house. Some agents sell the house and move into a flat at the end of their age 4 period because their utility premium for a house is not high enough any more to justify the extra expense ($J^H = 5$). At the end of their life, all agents sell their dwellings, consume their wealth and die.

The pattern in Figure 4 requires that the utility parameters and endowments satisfy certain conditions. For example, we need a sufficiently low u^P and a sufficiently high u^H so that young agents leave their parents and climb the property ladder as soon as they can afford to do so. Endowments at age 1 must be such that only some agents can afford a flat at the end of the first period of life, no agent can afford a house, and so on. Once we have calculated the steady-state equilibrium prices corresponding to Figure 4, it will be straightforward to formulate these parameter restrictions.

[16]For example, $i^F(1)$ is the name of the poorest agent of age 1 who buys a flat at the end of the current period in order to move into it at the beginning of next period.

There is nothing peculiar about the particular model configuration above. In order to capture the mechanism described in the introduction, our model must allow the interaction of two types of agents: those who respond to changes in the degree to which the collateral constraint binds, and those who behave as the usual deep-pocket agents. In addition, it is critical that the marginal house buyer in the second period be a flat owner. This is necessary if capital gains on flats are to influence housing demand in equilibrium. The above configuration produces these features.

To solve for the equilibrium, we first characterize the relevant cutoff indices. By definition, the endowment of agent $i_t^F(1)$ at age 1 is just enough to pay the down payment on the flat:

$$(1 + i_t^F(1))w_t(1) = q_t^F - \gamma \frac{q_{t+1}^F}{1 + r_t}. \tag{5}$$

For given flat prices, the measure of agents who are able to move into a flat at age 1 depends positively on agents' age 1 earnings and negatively on the size of the required down payment. A similar condition determines the value of $i_t^H(2)$. The only difference is that we must account for the fact that agent $i_t^H(2)$ owned a flat in the previous period:

$$(1 + i_t^H(2))W_t(2) - (1 + r_{t-1})q_{t-1}^F + q_t^F = q_t^H - \gamma \frac{q_{t+1}^H}{1 + r_t} \tag{6}$$

where

$$W_t(2) = (1 + r_{t-1})w_{t-1}(1) + w_t(2) \tag{7}$$

is the date t capitalized value of the endowments received in periods $t-1$ and t by the poorest agent of age 2 at t. The accumulated endowments of agent $i_t^H(2)$ at age 1 and 2 plus any net gains on the flat just equals the down payment for the house. For given flat and house prices, the measure of agents in the age 2 cohort who are able to buy a house depends positively on these agents' earnings, and negatively on the required down payment. In addition, capital gains on a flat can help agents move up the housing ladder; conversely, some may find themselves stuck in a flat due to a decrease in its price.

While the cutoff indices $i_t^F(1)$ and $i_t^H(2)$ are determined by the binding collateral and budget constraints, the cutoff $i_t^H(4)$ depends on preferences and user costs:

$$\underline{u}^H + i_t^H(4)\,\Delta(5) = \left[(1 + r_t)q_t^H - q_{t+1}^H\right] - \left[(1 + r_t)q_t^F - q_{t+1}^F\right]. \tag{8}$$

So, $i_t^H(4)$ is the name of the age 4 individual at time t for whom the utility premium of living in a house at time $t+1$ (left-hand side) is equal to the difference between the house and flat user costs, expressed in time $t+1$ terms (right-hand side). The higher the user cost difference between house and flat, and the smaller the utility gain of living in a house, the more agents of the age 4 cohort move to a flat.

The market clearing conditions are

$$(1 - i_t^F(1)) + i_t^H(2) + i_t^H(4) = S^F \tag{9}$$

for the flat market and

$$(1 - i_t^H(2)) + 1 + (1 - i_t^H(4)) = S^H \tag{10}$$

for the house market. Combining equations (9) and (10), we obtain

$$i_t^F(1) = 4 - S^H - S^F. \tag{11}$$

This condition states that the measure of agents who live with their parents at age 2 must equal the total population size minus the measure of agents who live with their parents at age 1, minus the measure of dwellings: $5 - 1 - (S^H + S^F)$.

Replacing $i_t^F(1)$ in (5) by the right-hand side of (11), we obtain the law of motion of the price of flats:

$$q_t^F - \gamma \frac{q_{t+1}^F}{1 + r_t} = (5 - S^F - S^H) w_t(1). \tag{12}$$

In equilibrium, the down payment on flats (left-hand side) is thus proportional to the current income of the youngest agents, and depends negatively on the supply of flats and houses.[17]

Solving for $i_t^H(2)$ and $i_t^H(4)$ in (6) and (8), and inserting these cutoffs in the market clearing condition (10), we obtain the equation

$$\begin{aligned} &\left(\frac{1}{W_t(2)} + \frac{1 + r_t}{\Delta(5)}\right) q_t^H - \left(\frac{\gamma}{W_t(2)} + \frac{1 + r_t}{\Delta(5)}\right) \frac{q_{t+1}^H}{1 + r_t} \\ &= \quad 4 - S^H + \frac{\underline{u}^H}{\Delta(5)} + \frac{q_t^F - (1 + r_{t-1}) q_{t-1}^F}{W_t(2)} - \frac{q_{t+1}^F - (1 + r_t) q_t^F}{\Delta(5)} . \end{aligned} \tag{13}$$

Equations (12) and (13) constitute a system of second order difference equations which characterizes the equilibrium price sequences for flats and houses.

These equations highlight the *fundamental determinants of housing prices* in our model economy. The current income of the poorest first-time buyer determines the price

[17] If it took the poorest agent of each cohort longer to move out of her parents' home, a longer history of income levels would appear in the law of motion for q_t^F. For example, if the poorest agent were not able to buy a flat at age 2, i.e., if $i_t^F(2) > 0$, we would obtain

$$q_t^F - \gamma \frac{q_{t+1}^F}{1 + r_t} = (6 - \ {}^F - \ {}^H) \Big/ \left(\frac{1}{w_t(1)} + \frac{1}{W_t(2)}\right).$$

More generally, if n periods are required for all members of any cohort who will eventually acquire a flat to actually do so, the law of motion of the flat price involves young agents' earnings over the past n periods.

of the first-time buyer property, the flat (equation (12)). The price of houses is such that the difference in user costs between houses and flats is equal to the utility premium of houses over flats for the relevant marginal old agent (equation (8)). This marginal old agent is not subject to any credit constraint and thus behaves as the typical forward-looking asset market operator. The value of a house can therefore be thought of as being made up of two components: the value of the flat plus a premium for the higher utility services that a house generates. So, the level of house prices depends on flat prices, and flat prices depend on the income of young households. This explains why, in spite of the presence of forward-looking unconstrained agents, housing prices depend on current income.

If the interest rate and the endowment profile are constant over time ($r_t = r$ and $w_t(j) = w(j)$ for all t and j), then the economy is in steady state equilibrium at the prices

$$\overline{q}^F = \frac{1+r}{1+r-\gamma}(5 - S^F - S^H)\, w(1)\,, \tag{14}$$

and

$$\overline{q}^H = \left[4 - S^H + \frac{\underline{u}^H}{\Delta(5)} - r\left(\frac{1}{W(2)} - \frac{1}{\Delta(5)}\right)\overline{q}^F\right] \Big/ \left(\frac{1+r-\gamma}{(1+r)W(2)} + \frac{r}{\Delta(5)}\right) \tag{15}$$

with $W(2) = (1+r)w(1) + w(2)$.[18]

Since we are concerned with the predictions of our model regarding transaction volume, we report in Table 3 the market activities of each age group in each period. Summing along the columns, we obtain the total volume of transactions per period.

Table 3: Market activities

	Buy		Sell	
Age	Flat	House	Flat	House
1	1-$i_t^F(1)$			
2	$i_{t-1}^F(1)$	$1 - i_t^H(2)$	$1 - i_t^H(2)$	
3		$i_{t-1}^H(2)$	$i_{t-1}^H(2)$	
4	$i_t^H(4)$			$i_t^H(4)$
5			$i_{t-1}^H(4)$	$1 - i_{t-1}^H(4)$

Since $i_t^F(1) = i_{t-1}^F(1)$, there are $1 + i_t^H(4) = 1 - i_t^H(2) + i_{t-1}^H(2) + i_{t-1}^H(4)$ flats which change owner at the end of period t, and $1 - i_t^H(2) + i_{t-1}^H(2) = 1 + i_t^H(4) - i_{t-1}^H(4)$ houses. In steady state, these figures reduce to $1 + i^H(4)$ for flats and 1 for houses.

[18]With these expressions, the reader will find it straightforward to formulate restrictions on the parameters r, γ, $w(1)$, $w(2)$, $w(3)$, u^P, $\overline{u}^H$, $\underline{u}^H$ and $\Delta(5)$ that guarantee the steady state housing pattern of Figure 4.

3 Dynamics

To gain insights in the dynamic properties of our model economy, we analyse its response to unanticipated impulse and permanent shocks to income. Both types of shock lead to co-movements of housing prices and transaction volume with income.

3.1 Impulse Response

Let us assume that the economy is at a steady state equilibrium as in Figure 4. Then, suppose that the income of all agents increases by a factor σ at date s; i.e., $w_s(j) = \sigma\, w(j)$ for all j, with $\sigma > 1$, but sufficiently close to 1.[19] This increase is unexpected. The income of all agents is back to its steady state level at date $s+1$, where it remains forever after; i.e., $w_t(j) = w(j)$ for all $t \geq s+1$ and all j. The following four properties characterize the response of our economy to this positive income shock.

Property 1 *The price of flats increases for one period.*

From equation (12) we know that the price of flats increases for one period only, at date s, and then returns to its initial steady state level. More precisely, we have

$$q_s^F = \gamma \frac{\overline{q}^F}{1+r} + (5 - S^F - S^H)\sigma w(1). \tag{16}$$

It is easy to see that the increase of the flat price is less than proportional to the income shock. Indeed, at the steady state, the elasticity of the flat price with respect to the incomes of the youngest agents is

$$\frac{w(1)}{\bar{q}^F} \frac{dq^F}{dw_s(1)} = \frac{1+r-\gamma}{1+r} < 1. \tag{17}$$

The stronger the down-payment constraint (i.e. the lower γ), the larger is the fluctuation of the flat price in response to an income shock.

Property 2 *The house price exceeds its steady state level for two periods with an initial increase larger than that of flats.*

[19]A large σ may imply an equilibrium pattern of housing consumption qualitatively different from the one assumed throughout this section; cf. Figure 4. The model is perfectly capable of handling large shocks, but the analytics of large shocks would be more cumbersome without generating additional insights.

From equation (13), we know that at time $s+2$ the economy will be back in steady state.[20] In the Appendix, we show that

$$\left(\frac{1}{W_{s+1}(2)}+\frac{1+r}{\Delta(5)}\right)\frac{dq^H_{s+1}}{d\sigma} \quad = \quad \frac{(1+r)w(1)}{W_{s+1}(2)}\left(i^H_{s+1}(2)-i^F_{s+1}(1)\right), \tag{18}$$

which is positive since $i^H_{s+1}(2) > i^F_{s+1}(1)$. So q^H_{s+1} is strictly increasing in σ; in particular, $q^H_{s+1} > \bar{q}^H$. We further derive that

$$\begin{aligned}\left(\frac{1}{W_s(2)}+\frac{1+r}{\Delta(5)}\right)\frac{dq^H_s}{d\sigma} \quad &= \quad \left(\frac{1}{W_s(2)}+\frac{1+r}{\Delta(5)}\right)\frac{dq^F_s}{d\sigma} \\ &\quad + \frac{w(2)(1+i^H_s(2))}{W_s(2)}+\left(\frac{\gamma}{W_s(2)}+\frac{1+r}{\Delta(5)}\right)\frac{1}{1+r}\frac{dq^H_{s+1}}{d\sigma}\ ,\end{aligned} \tag{19}$$

which shows that the time s house price also rises in response to the income shock, and by more (in absolute terms) than the flat price.

As a corollary to the above derivations, we note that the size of housing price fluctuations depends on that of the fluctuations of young households' income, not *per capita* income. Only the income of age 1 and 2 agents appears in the system of difference equations which characterizes flat and house price dynamics. We will use this result later to explain the relative volatility of housing prices and GDP.

For the next two properties, we make the assumption that $w(2) \geq (1-\gamma)w(1)$.[21]

Property 3 *The volume of transactions increases with prices in the period of the shock.*

To show that the volume of transactions increases at time s, it is sufficient to show that $i^H_s(4)$ increases, or equivalently, that $i^H_s(2)$ decreases. Differentiating (8) and replacing the price derivatives with the expressions derived above, we obtain

$$\begin{aligned}&\left(W_s(2)+\frac{\Delta(5)}{1+r}\right)\frac{di^H_s(4)}{d\sigma} \\ &= \quad w(2)(1+i^H_s(2))-\frac{(1-\gamma)w(1)}{1+(1+r)W_{s+1}(2)/\Delta(5)}\left(i^H_{s+1}(2)-i^F_{s+1}(1)\right),\end{aligned} \tag{20}$$

which is positive when $w(2) \geq (1-\gamma)w(1)$.

This derivative can also be written in a more intuitive form as

$$\frac{di^H_s(4)}{d\sigma}=-\frac{di^H_s(2)}{d\sigma}=\frac{w(2)(1+i^H_s(2))+(1-\gamma)\frac{dq^F_s}{d\sigma}-(1-\gamma)\frac{dq^H_s}{d\sigma}}{W_s(2)+\gamma\Delta(5)/(1+r)}\ . \tag{21}$$

[20]More generally, the number of periods it takes for our model economy to return to its steady state after a shock depends on the number of periods it takes for agents to climb the property ladder.

[21]It is possible to formulate other sufficient conditions for these results. The one adopted here is very weak for any reasonable value of γ.

It reveals the three effects of the income shock on trading volume as described in the introduction. The first term in the numerator represents the effects of the rise in income itself. The second term represents the effect of capital gains on the flat. Both effects enable age 2 agents to afford a bigger down payment in the period of the shock. They contribute to an increase in the number of age 2 agents who acquire a house. The third term moderates the effects of the first two: the rise of the house price diminishes the number of age 2 agents who can afford a house. As shown above, the first two effects dominate in equilibrium.

Property 4 *The average age of repeat buyers falls with the increase in transaction volume.*

The average age of repeat buyers is equal to

$$\frac{2\,(1 - i_s^H(2)) + 3\,i_{s-1}^H(2) + 4\,i_s^H(4)}{(1 - i_s^H(2)) + i_{s-1}^H(2) + i_s^H(4)}.$$

We have shown above that $i_s^H(2)$ exceeds its steady state value; at the same time $i_s^H(4)$ is below its steady state value. Therefore, the average age of repeat buyers decreases at the time of the shock.

We have carried out the above analysis for a positive income shock. Since the proofs of properties 1 to 3 rely on signing derivatives with respect to an income coefficient, they also prove that a negative shock (i.e., a drop in that coefficient) produces an instantaneous decrease in house price, flat price, and transaction volume, and an increase in the average age of repeat buyers. Our model therefore produces a positive correlation between the volume of transaction and price changes.[22]

3.2 Permanent Shocks

Given the observed persistence of business cycle shocks, we are interested in the response of our economy to permanent shocks. With the algebra developed in the previous section, it is not hard to verify that similar results hold; i.e., positive (negative) shocks imply an increase (decrease) in house and flat prices, accompanied by a similar movement in the volume of transactions in the period of the shock.

[22] As mentioned in the introduction, this is in accordance with the empirical evidence in various housing markets, and it is different from financial markets where rises in volume seem to be concomitant with both price increases and decreases.

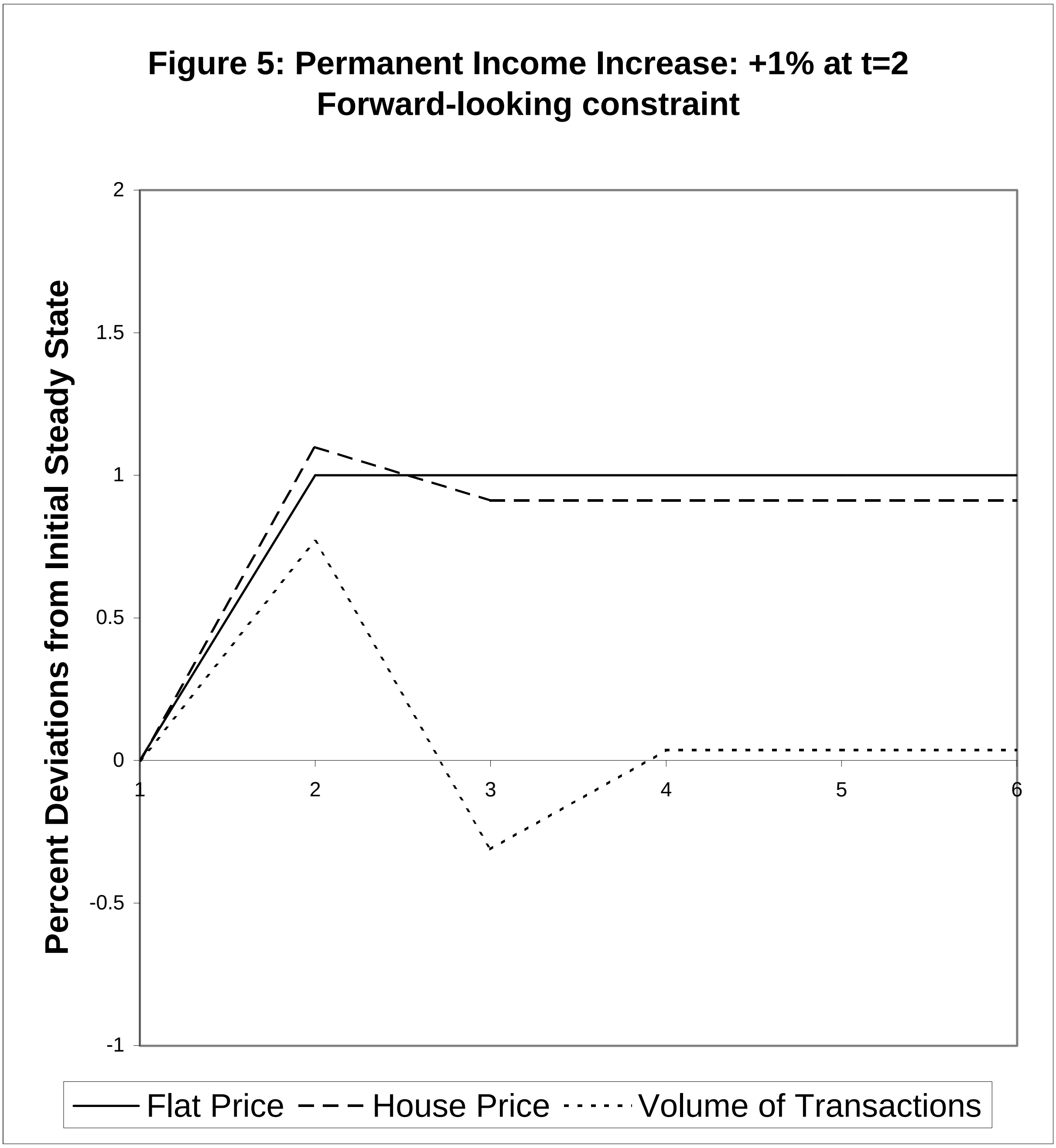


Figure 5: Permanent Income Increase: +1% at t=2
Forward-looking constraint
Percent Deviations from Initial Steady State
2
1.5
1
0.5
0
-0.5
-1
1
2
3
4
5
6
Flat Price
House Price
Volume of Transactions

Figure 5 shows the effects of a one percent permanent income increase.[23] The flat price rises immediately to its new steady state level, one percent higher than its initial level. The house price initially overshoots the income rise. This is due to the increase in the demand for houses, caused by both the income rise and the capital gains realized by agents who own flats at the time of the shock. In the next period, the house price drops to its new steady state level, which is less than one percent higher than its initial level. These fluctuations are accompanied by a rise in $i^H(4)$: from a steady state level of 0.615, it increases to 0.625 in the first period of the income increase before returning to 0.616 in the new steady state. This confirms the mechanism behind the co-movement of transaction volume and prices explained above. Note that the fluctuations of transaction volume are small relative to those of price and income, in contrast to what we observe in the data. The next sections present two modifications of the model which, among other things, generate a higher volatility of transaction volume.

4 Adding a Rental Sector

The co-movement of price and volume is preserved when we include a rental sector for flats. Most interestingly, allowing agents to choose between renting and owning a flat introduces an additional margin of fluctuations in the model. Income shocks now affect the proportion of flats that are owner occupied and therefore generate extra fluctuations in volume of transactions.

Suppose that flats can be rented as well as owned, and that agents have preferences over housing of the following type: owning a house is preferred to owning a flat, owning a flat is preferred to renting a flat, and renting a flat is preferred to living with one's parents.[24] Keeping the linear utility function for consumption, extending the utility function for housing services in (2) to include a suitable utility of renting, $G(R; i, j)$, and choosing suitable parameters for agents' endowments as specified in (1), we can obtain a pattern of housing choices as in Figure 6.

In equilibrium, deep-pocket investors (e.g. wealthy old agents or banks) must be indifferent between holding flats and lending money, so the rental price of flats, R_t, must equal the user cost of flats:

$$R_t = q_t^F - \frac{q_{t+1}^F}{1 + r_t} \ . \tag{22}$$

[23]We used the following parameters for this experiment: $^F = 1.7$, $^H = 2.2$, $\underline{u}^H = -8$, $\Delta(5) = 16$, $w(1) = 8$, $w(2) = 8$, $r = 5\%$, $\gamma = 0.8$.

[24]From the consumer's point of view, a rented flat is a different good than an owned flat.

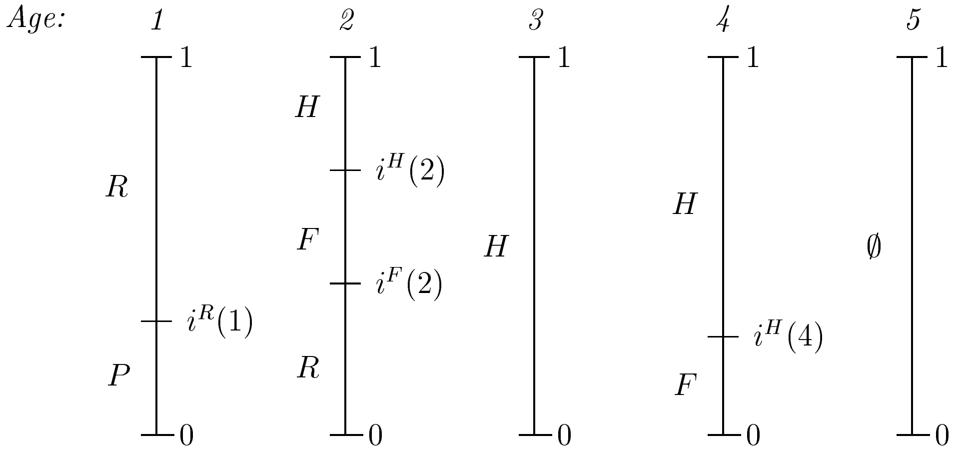


Figure 6: End of period steady state housing choices by age groups in the presence of a rental sector for flats

Given that agents prefer living on their own, they move out of their parent's dwelling as soon as they can afford to do so. By definition, the income of agent $i_t^R(1)$ at age 1 is just sufficient to pay the rent (which we assume to be payable in advance):

$$R_t = (1 + i_t^R(1))w_t(1). \tag{23}$$

Since $i_t^R(1) = 4 - S^F - S^H$ (this is the number of agents living with their parents at age 2), the equilibrium rent is

$$R_t = (5 - S^F - S^H)w_t(1). \tag{24}$$

Equation (22) then yields the law of motion for the flat price,

$$q_t^F - \frac{q_{t+1}^F}{1 + r_t} = (5 - S^F - S^H)w_t(1), \tag{25}$$

which is the same as the law of motion derived without the rental sector, equation (12), when $\gamma = 1$. This implies that at the steady state, the elasticity of the flat price with respect to the income of the youngest agents is smaller when a rental sector is present than when there is no renting; cf. equation (17). In this sense, the rental sector buffers the effects of income shocks on young agents.

Given the rent R_{t-1} and flat prices q_t^F and q_{t+1}^F, we can determine the cutoff index $i_t^F(2)$ from the equation

$$q_t^F - \gamma\frac{q_{t+1}^F}{1 + r_t} = (1 + i_t^F(2))W_t(2) - (1 + r_{t-1})R_{t-1}, \tag{26}$$

with $W_t(2)$ as defined in (7). Similarly, $i_t^H(2)$ satisfies the identity

$$q_t^H - \gamma\frac{q_{t+1}^H}{1 + r_t} = (1 + i_t^H(2))W_t(2) - (1 + r_{t-1})R_{t-1}. \tag{27}$$

Finally, the decision of older agents to move out of their house is the same as in the absence of a rental sector, so the cutoff $i_t^H(4)$ is again given by equation (8).

Inserting $i_t^H(2)$ and $i_t^H(4)$ into the market clearing condition for houses and using the no-arbitrage relation (22), yields the same law of motion for the house price as in the absence of a rental sector, equation (13). This implies that our previous results still apply. The only difference is quantitative. Since the flat price fluctuations are smaller here, so are those of the house prices.

To compute the transaction volume, we need to make some further assumptions. In particular, we assume that no flat that is rented from one period to the next, is sold. We further assume that all owner-occupied flats are sold first to households planning to live in them. This seems plausible when one accounts for the cost of making a property fit for rental. The transaction volumes of flats and houses are then

$$\max\{i_{t-1}^H(2) - i_{t-1}^F(2), i_t^H(2) - i_t^F(2)\} + i_t^H(4) \quad \text{and} \quad 1 + i_t^H(4) - i_{t-1}^H(4),$$

respectively. The transaction volume of houses is the same as in the model without the rental sector. The first term of the transaction volume of flats, however, is different, reflecting the additional margin between flat ownership and rental for young agents. If $i_{t-1}^H(2) - i_{t-1}^F(2) < i_t^H(2) - i_t^F(2)$, more agents of age 2 become owners of a flat at the end of period t, and they buy $i_t^H(2) - i_t^F(2)$ flats in total: $i_{t-1}^H(2) - i_{t-1}^F(2)$ from previous owner-occupants, and the remainder from investors who rented them out. On the other hand, if $i_{t-1}^H(2) - i_{t-1}^F(2) > i_t^H(2) - i_t^F(2)$, then $i_{t-1}^H(2) - i_{t-1}^F(2)$ flats are sold at the end of period t: $i_t^H(2) - i_t^F(2)$ to new owner-occcupants, and the rest to investors who will rent them out.[25]

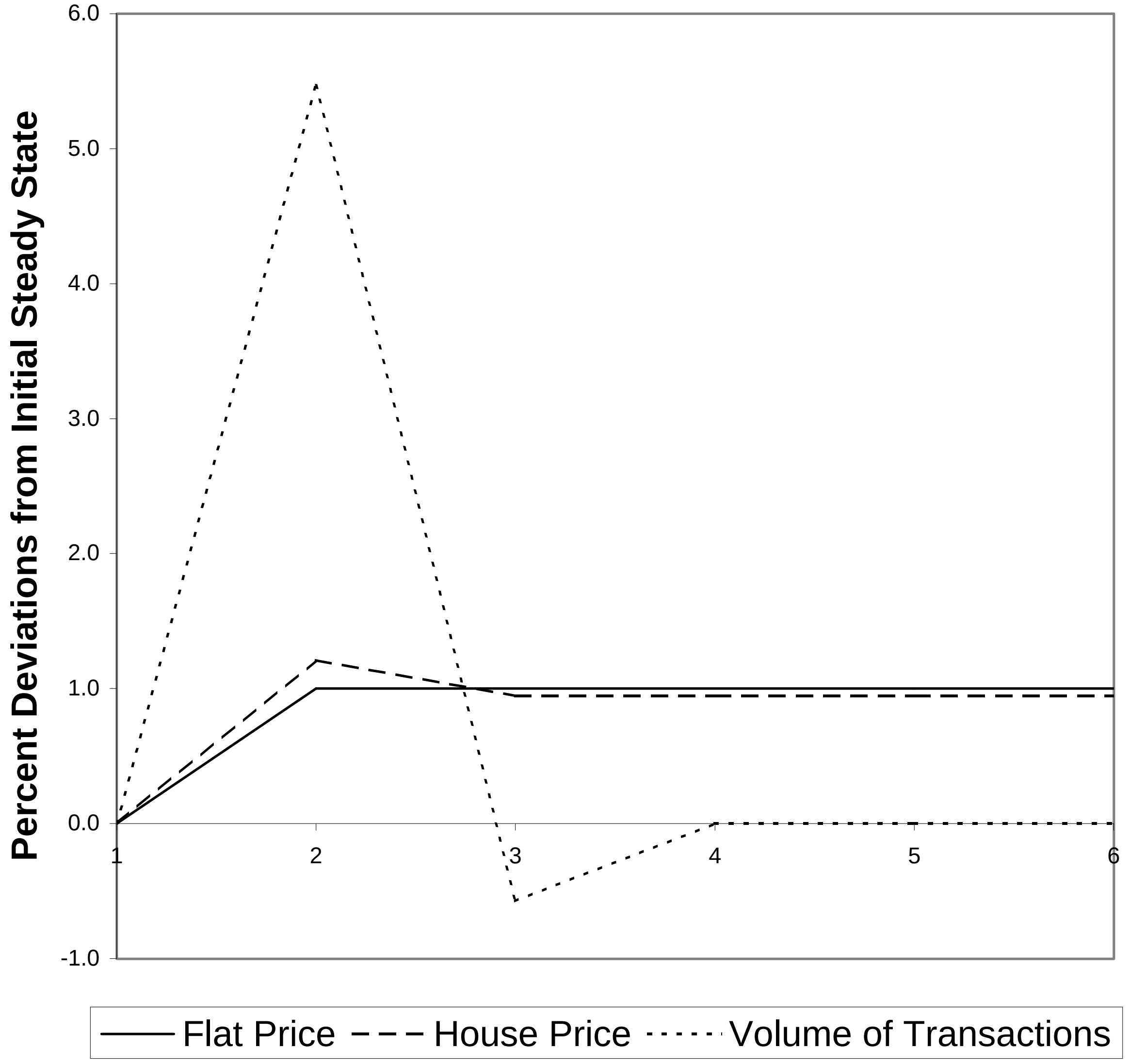
Figure 7 presents the effects of a 1 percent permanent income increase.[26] The effect on prices is similar to what we obtained without the rental sector. However, the fluctuations of transaction volume are much larger now.

To highlight the contribution of the additional margin between owner-occupation and rental, we briefly digress to a model where this margin is the only source of fluctuations in transaction volume. Consider a model economy similar to the one above except that agents only live two periods, and there are no houses. Assume parameters such that the tenure choices in steady state equilibrium are as in Figure 8. Then, following the same

[25] We can compute the owner-occupancy rate for any cohort and for the population. Interestingly, it follows qualitatively the fluctuations in housing prices when the economy is subject to temporary shocks, rising with housing prices during a boom before returning to its initial level after a bust. This is in agreement with the data for the U.K. boom-bust of the late Eighties as reported in Ortalo-Magné and Rady (1998).

[26] The following parameters were used for this experiment: $^F = 1.7$, $^H = 2.2$, $\underline{u}^H = -2$, $\Delta(5) = 16$, $w(1) = 6$, $w(2) = 24$, $r = 5\%$, $\gamma = 0.8$.

Figure 7: Permanent Income Increase: +1% at t=2
Forward-looking constraint and rental sector



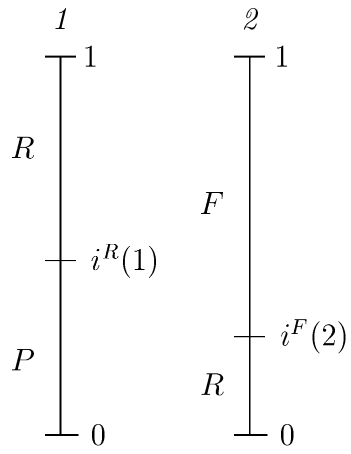


Figure 8: End of period steady state tenure choices by age groups in a simple model with owner occupation and rental of flats, but no houses

approach as before, we derive that

$$R_t = q_t^F - \frac{q_{t+1}^F}{1+r_t} = (3 - S^F)w_t(1) \tag{28}$$

and

$$(1 + i_t^F(2))W_t(2) = R_t + (1-\gamma)\frac{q_{t+1}^F}{1+r_t}. \tag{29}$$

Starting from the steady state, a positive income shock generates an increase in flat prices. It also generates an increase in the volume of transactions if it reduces $i_t^F(2)$ and so raises the owner-occupancy rate, which can be shown to happen if and only if

$$\frac{w(2)}{w(1)} > \frac{(1+r)}{1-\gamma}r, \tag{30}$$

a plausible condition. In this case, the increase in the volume of transactions is equal to the increase in the number of owner occupants. Conversely, a negative income shock leads to an increase in $i_t^F(2)$, hence a decrease in the owner-occupancy rate. But this does not affect transaction volume, since all previously owner-occupied properties are still sold, some of them to investors who will rent them out.

In summary, as long as condition (30) holds, a positive income shock raises the owner-occupancy rate and thereby the volume of transactions.[27] A negative shock, however, prompts a decrease in owner occupancy with no change in transaction volume. So, introducing a perfectly flexible rental sector for flats into our initial model amplifies the

[27] We only present here a steady state equilibrium such that $i^R(1) > i^F(2)$. In equilibria with $i^R(1) \leq i^F(2)$, the condition for an increase of transaction volume in an upturn is $\frac{w(2)}{w(1)} > \frac{r(1+r)}{1+r-\gamma}$, which is even less restrictive.

volume of transaction increases in upturns, but not the decreases in downturns. The asymmetry distinguishes this mechanism for volume of transaction fluctuations from the one highlighted in the initial model, which is symmetric. The next section modifies the initial model in a way that amplifies the changes in transaction volume both in booms and busts.

5 An Alternative Down-Payment Constraint: Myopic Banks

Mortgage lenders' stated practice is to require a down payment amounting to a fraction of *current* price. To the extent that lenders vary this fraction in anticipation of price changes, the forward-looking constraint of the previous sections can capture their behaviour. In this section, we analyse housing market fluctuations when lenders do *not* make such adjustments, i.e., when they impose a "myopic" constraint that limits debt to a *fixed* fraction of the current property value. We find that the ability of our theory to match the data improves when we assume this myopic constraint. In particular, this formulation generates a higher volatility of housing prices, a volatility of the volume of transactions larger than that of housing prices and income, as well as a lead of transaction volume over the other two series.

Suppose that lenders do not allow borrowers to hold a debt higher than a fixed proportion, γ, of the contemporaneous value of their dwelling, that is, borrowings are limited to $-s_t \leq \gamma q_t^h$. At the time of purchase, this constraint amounts to a down-payment requirement of $(1-\gamma)q_t^h$. Going back to the tenure pattern of Figure 4, we repeat the steps in Section 2 and obtain the following characterisation of the equilibrium prices:

$$q_t^F = \frac{5 - S^F - S^H}{1-\gamma} w_t(1) \tag{31}$$

and

$$\begin{aligned}&\left(\frac{1-\gamma}{W_t(2)} + \frac{1+r_t}{\Delta(5)}\right) q_t^H - \frac{1}{\Delta(5)}\, q_{t+1}^H \\ &= \; 4 - S^H + \frac{u^H}{\Delta(5)} + \frac{q_t^F - (1+r_{t-1})q_{t-1}^F}{W_t(2)} - \frac{q_{t+1}^F - (1+r_t)q_t^F}{\Delta(5)}\,. \end{aligned} \tag{32}$$

Starting with the economy at steady state equilibrium and calculating the effect of an impulse income shock $\sigma > 1$ as in Section 3, we find again the co-movement of prices and transaction volumes, but the effects of temporary shocks are stronger in this version of the model.[28] This is due to lenders allowing an increase in borrowing proportional to the contemporaneous price increase. As a result, prices rise more, which amplifies the effects of the shock on the young marginal house buyers and hence on the volume of transactions. In the model with the forward-looking constraint, by contrast, lenders recognise the temporary nature of the housing price increase, and therefore restrict the amount of extra lending they allow. Under the forward-looking constraint, smaller price movements yield less capital gains for flat owners and therefore fewer extra purchases of houses relative to steady state.

In the case of a permanent positive income shock, the flat price rises permanently under both the forward-looking and the myopic constraints so that the extra amount of borrowing allowed for the purchase of a flat is proportional to its price increase in both cases. Under either type of constraint, the house price initially overshoots its new steady state. With the myopic constraint, house buyers' borrowings in the period of the shock are restricted to a fraction of the current high house price; whereas with the forward-looking constraint, agents' borrowings are restricted to a fraction of next period's lower price. As a consequence, acquiring a house is easier under the myopic constraint. The house price rises more at the date of the shock, convincing more elderly to sell their house to the higher number of young agents who can afford the required down payment. This implies a higher fluctuation in transaction volume under the myopic constraint than under the forward-looking one with no rental sector. Figure 9 illustrates this effect. It shows the same experiment as Figure 5, but with the myopic constraint.[29] As in the data, volume of transactions fluctuates more than price.

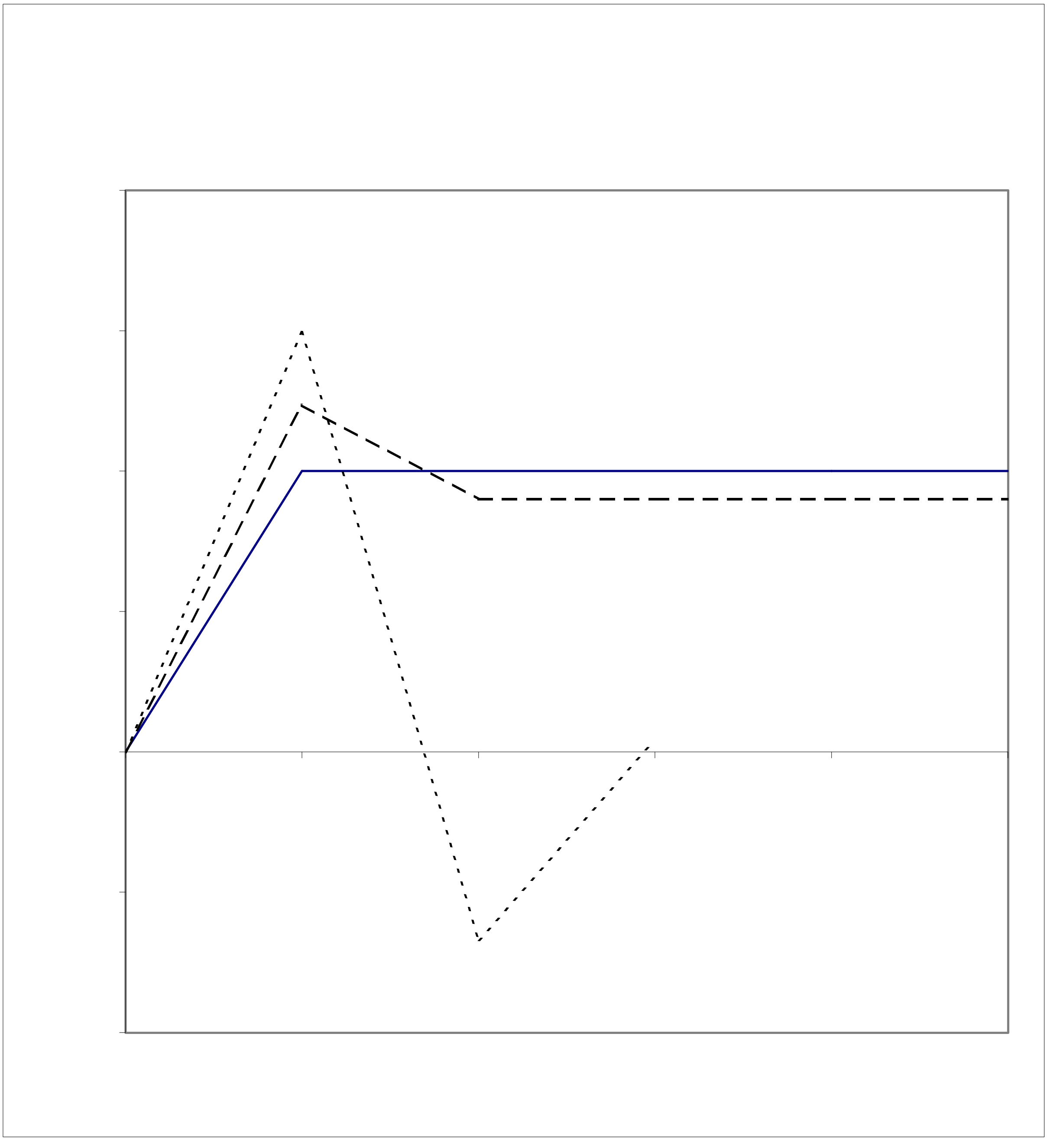
Another interest of this version of the model is its ability to match the observed lead of transaction volume over price movements. Suppose agents know that their income is subject to cyclical fluctuations around some steady state level: income is first 0.1%, then 0.2%, then 0.1% above steady state, then 0.1%, 0.2% and 0.1% below steady state, and so on. Figure 10 shows this income pattern together with the corresponding equilibrium prices and transaction volume.[30] Over the cycles, the correlation of income with contemporaneous transaction volume is 0.30, whereas it is 0.98 with one period lagged transaction volume.[31]

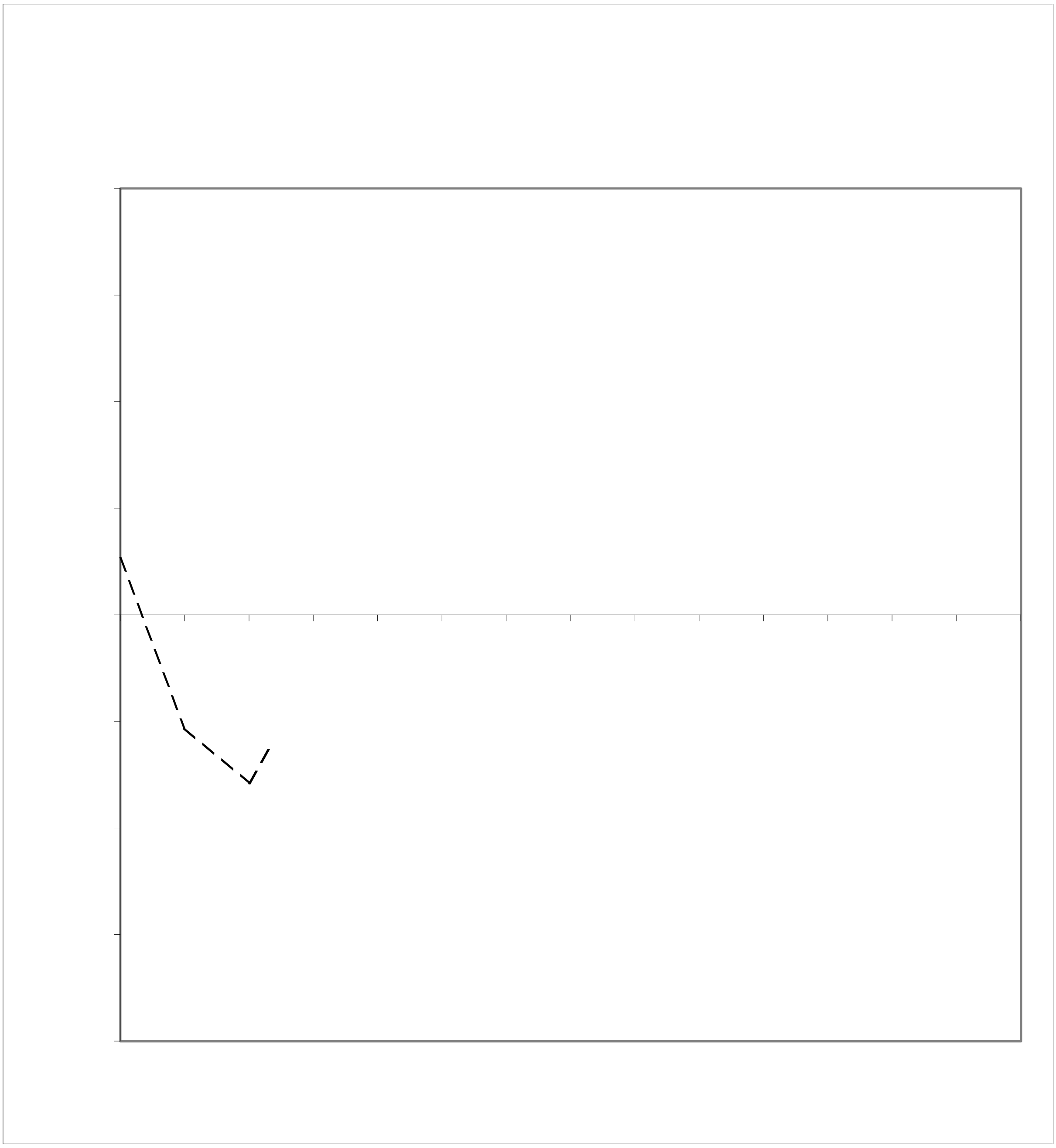
[28]Details of the derivations are available from the authors upon request.

[29]The parameters are the same as for Figure 5 (see footnote 23).

[30]For clarity, Figure 10 does not graph the price of each type of dwelling, but rather the housing price index, calculated as the average of flat and house prices, weighted by the respective number of dwellings. The parameters are the same as for Figure 5 (see footnote 23).

[31]In the models with the forward-looking constraint, by contrast, the volume of transactions does not lead income fluctuations.





Recall that the price of flats follows exactly the fluctuations of the current income of the marginal first-time buyer, while the user cost of houses is determined relative to the user cost of flats so as to account for the house utility premium of the marginal old agent (with name $i^H(4)$). At the beginning of the boom, as income and flat prices rise, flat owners benefit from increased income and capital gains, so more of them buy a house, and $i^H(4)$ rises. In the second period of rising income, house buyers are flat owners who paid a higher price for flats than their counterparts in the previous period. Therefore, fewer of them can afford the house compared to the initial period of high income, and $i^H(4)$ falls. However, since income is still high, $i^H(4)$ remains above its steady state level. With this decrease in $i^H(4)$, the volume of transactions decreases, as does the difference between house and flat user costs. This implies that housing prices rise at a lower rate than income while transaction volume is already dropping.

In the following period, income and hence the flat price are low. This implies capital losses for flat owners. Fewer agents can afford moving to a house, so $i^H(4)$ decreases further, prompting another reduction in the volume of transactions. In the second period of decreasing income, the young flat owners are agents who paid a lower price for flats than their counterparts in the previous period, and thus did not incur much of a capital loss. More of them can afford to move into a house. Therefore, the volume of transactions increases, and so does $i^H(4)$. This implies a rise of the user cost difference between houses and flats. The house price therefore does not decrease as much as the flat price. Both prices are still decreasing while the transaction volume is already rising.

Transaction volume fluctuates more than income and house prices in our simulation, as is the case in the data. The fluctuations of the housing price index, however, are slightly smaller than income fluctuations but greater in the data. One way to generate larger housing price fluctuations would be to parameterize larger income fluctuations for young households. As mentioned earlier, and as shown by equations (31) and (32), the amplitude of housing price fluctuations is determined by the amplitude of income fluctuations for those households who are climbing the property ladder, and not *per capita* income.

In all, this version of our model with myopic banks and no rental sector matches the data best. Its major improvements over the previous versions are larger housing price fluctuations and a possible lead of the volume of transactions over income fluctuations. That banks' behavior is closer to our myopic rather than forward looking assumption is supported by casual observation.[32] Of course, we do observe properties being rented.

[32] More zero-percent down-payment mortgages were available at the end of 1997 in the United Kingdom than in the previous two years, although by then, it was widely acknowledged that the boom of 1996-97 was coming to an end. If banks were forward-looking, the opposite should have been observed.

Nevertheless, the results of our myopic model without rental still apply as long as the stock of properties rented is relatively constant over the business cycle, i.e., if the only arbitrage performed is that by households delaying or anticipating their moving decision. This seems plausible given that transaction and search costs, management fees and other market rigidities allow deviations of the rent from the user cost.

6 Concluding Remarks

This paper has developed a dynamic theory of housing market fluctuations based on credit constraints and lumpy adjustments of housing consumption over the life-cycle. We studied two forms of down-payment constraint and the effects of a rental sector. All variants of the model produce qualitatively the same co-movements of transaction volume and prices in response to shocks. The main mechanism by which the volume of transactions fluctuates (i.e., fluctuations of the age of repeat buyers) seems to be in agreement with the empirical evidence. With forward-looking banks, however, the volume of transactions fluctuates less than income. Introducing a rental sector for flats dampens the fluctuations of the flat price, but increases those of the volume of transactions. The model with myopic banks and no rental sector (or a fixed stock of rental properties) matches both the observed co-movement and relative amplitude of housing prices, transaction volume, and income. This model also provides an explanation for the observed lead of transaction volume over the other two series.

The key contribution of our theory is to suggest a new understanding of housing price fundamentals. In particular, we find that the price of smaller properties is determined by the income of the poorest first-time buyers. The price of larger properties adjusts such that the additional user cost of these properties equals their utility premium relative to smaller properties for endogenously determined marginal agents. The characteristics of these marginal agents fluctuate with income in a manner which amplifies the price fluctuations of larger properties. This view of housing price fundamentals implies that the volatility of housing prices is of the same order as that of the income of young households, not GDP. Moreover, to the extent that young households' income is predictable, so are housing prices.

All the derivations presented in this paper are for specific steady state distributions of agents over the available types of dwelling. The key features of the configurations we analyze are that the young marginal house buyers are flat owners and that some older agents reduce their housing consumption before dying. Various steady state distributions of housing consumption can be accommodated. Our results will hold as long as the fluctuating margins remain the ones that we identified in the paper. In order to focus on these margins, we assumed a perfectly inelastic supply of land and construction services. It should be clear, however, that as long as these supply schedules are not perfectly elastic, our qualitative results hold. Finally, we assumed for simplicity that all agents could move out of their parents' dwelling by the end of their second period of life. Our results are robust to the alternative assumption whereby some remain with their parents for more periods. Moreover, such a specification provides a testable hypothesis with regards to the fluctuations of the average age of first-time buyers: the theory predicts that this age decreases during booms and increases during recessions.[33]

Our theory yields testable hypotheses and raises unexplored empirical questions. We hope it will provide guidance for further empirical research. In future work, we will exploit the analytical tractability of our approach to gain further insights into housing market dynamics and macroeconomic fluctuations.

[33] This extension of our model is studied in a companion paper, Ortalo-Magné and Rady (1998), where we analyse the U.K. housing market boom and crash of the Eighties in light of our theory.

Appendix

Equation (13) implies that the house price differs from its steady state level for two periods, so that $q_t^H = \overline{q}^H$ for all $t > s+1$. Evaluating (13) at time s yields

$$\begin{aligned}
&\left(\frac{1}{W_s(2)} + \frac{1+r}{\Delta(5)}\right) q_s^H - \left(\frac{\gamma}{W_s(2)} + \frac{1+r}{\Delta(5)}\right) \frac{q_{s+1}^H}{1+r} \\
&= 4 - S^H + \frac{u^H}{\Delta(5)} - \frac{1+r}{W_s(2)}\overline{q}^F \\
&+ \left(\frac{1}{W_s(2)} + \frac{1+r}{\Delta(5)}\right) q_s^F - \frac{1}{\Delta(5)}\overline{q}^F
\end{aligned} \tag{33}$$

where $W_s(2) = (1+r)w(1) + \sigma w(2)$. At time $s+1$, (13) becomes

$$\begin{aligned}
&\left(\frac{1}{W_{s+1}(2)} + \frac{1+r}{\Delta(5)}\right) q_{s+1}^H - \left(\frac{\gamma}{W_{s+1}(2)} + \frac{1+r}{\Delta(5)}\right) \frac{\overline{q}^H}{1+r} \\
&= 4 - S^H + \frac{u^H}{\Delta(5)} - \frac{1+r}{W_{s+1}(2)} q_s^F \\
&+ \left(\frac{1}{W_{s+1}(2)} + \frac{1+r}{\Delta(5)}\right) \overline{q}^F - \frac{1}{\Delta(5)}\overline{q}^F
\end{aligned} \tag{34}$$

with $W_{s+1}(2) = (1+r)\sigma w(1) + w(2)$.

Differentiating (34) with respect to σ, we obtain

$$\begin{aligned}
&\left(\frac{1}{W_{s+1}(2)} + \frac{1+r}{\Delta(5)}\right) \frac{dq_{s+1}^H}{d\sigma} - \frac{(1+r)w(1)}{W_{s+1}(2)^2} q_{s+1}^H + \gamma \frac{(1+r)w(1)}{W_{s+1}(2)^2} \frac{\overline{q}^H}{1+r} \\
&= \frac{(1+r)^2 w(1)}{W_{s+1}(2)^2} q_s^F - \frac{1+r}{W_{s+1}(2)} \frac{dq_s^F}{d\sigma} - \frac{(1+r)w(1)}{W_{s+1}(2)^2}\overline{q}^F,
\end{aligned} \tag{35}$$

hence

$$\begin{aligned}
\left(\frac{1}{W_{s+1}(2)} + \frac{1+r}{\Delta(5)}\right) \frac{dq_{s+1}^H}{d\sigma} &= \frac{(1+r)w(1)}{W_{s+1}(2)^2}\left[q_{s+1}^H - \frac{\gamma}{1+r}\bar{q}^H + (1+r)q_s^F - \bar{q}^F\right] \\
&- \frac{1+r}{W_{s+1}(2)}(6 - S^F - S^H)w(1).
\end{aligned} \tag{36}$$

Noting that $q_{s+1}^H - \gamma\bar{q}^H/(1+r) + (1+r)q_s^F - \bar{q}^F = (1 + i_{s+1}^H(2))W_{s+1}(2)$ by definition of the cutoff index $i_{s+1}^H(2)$, and recalling that $i_{s+1}^F(1) = 5 - S^F - S^H$, we obtain the expression in (18). Repeating the same steps with (33) yields equation (19).

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