Corporate Walkout Decisions

and the Value of Default

By

Tom Dahlstom
and
Pierre Mella-Barral

DISCUSSION PAPER 325
May 1999

FINANCIAL MARKETS GROUP
AN ESRC RESEARCH CENTRE

LONDON SCHOOL OF ECONOMICS

ISSN 0956-8549-325

CORPORATE WALKOUT DECISIONS AND THE VALUE OF DEFAULT

Tom Dahlström* Pierre Mella-Barral†

May 1999

LSE/FMG Working Paper, 1999

Abstract

We present a continuous-time asset pricing model of the levered firm where shareholders select not only the *timing* but also the *form* of control transfers. Owners are allowed to *walk out* of the firm either by (i) *defaulting* on their debt obligations or (ii) selling the firm with its debt obligations, as in a *corporation sale*. The structural model relates shareholders' ex-post choice to both technological and financial factors. We obtain that the likelihood of default being chosen instead of a corporation sale increases with (i) the degree of leverage displayed by the firm and (ii) its technological supremacy in the industry. Moreover, whereas default necessarily involves inefficient timing of ownership transfers, corporation sales eliminate agency costs and achieve the correct allocation of resources. By ignoring such direct sales of ownership rights, existing defaultable bond pricing models thus often exaggerate risk premia and underestimate the borrowing ability (debt capacity) of firms.

*Department of Economics, Research Unit on Economic Structures and Growth (RUESG), P.O. Box 10 (Snellmaninkatu 12 B), 00014 University of Helsinki, Finland; tel.: (358) 9 191 7958; fax: (358) 9 191 7980; e-mail: dahlstro@valt.helsinki.fi. Financial support from the Yrjö Jahnsson Foundation, the Finnish Cultural Foundation, and the Foreign and Commonwealth Office is gratefully acknowledged.

†London School of Economics, Houghton Street, London WC2A 2AE, UK; tel.: (44) 171 955-73.19; fax: (44) 171 955-74.20; e-mail: p.mella@lse.ac.uk. Financial support from the Financial Markets Group at the LSE is also gratefully acknowledged.

In the case of a levered firm, ownership rights over productive assets may be passed from one set of shareholders to another either indirectly or directly:

1. The transfer is indirect when the existing shareholders' creditors take control of the firm before selling ownership to another set of shareholders.

   This occurs when existing shareholders exercise their limited liability option, defaulting on their debt obligations, or more generally *repudiate* the outstanding contract they have with their creditors. Creditors are then entitled to force a bankruptcy procedure, resulting ultimately in the liquidation of the firm.

2. Conversely, the transfer is direct when the creditors never take control of the firm.

   This occurs when the existing shareholders simply

The magnitude of credit spreads observed in debt markets are known to be substantial.[4] This suggests that decisions taken by shareholders when the firm is in financial distress deserve particular attention as they greatly affect the value of corporate assets. However, this evidence does not imply that the shareholders' terminal decision is necessarily related solely to the limited liability option. Empirical evidence actually suggests that direct sales of ownership rights occur much more frequently than indirect transfers.[5] Default only constitutes one form of voluntary abandonment available to shareholders, and one might want to speak more generally of the "*walkout*" decision, where in addition to default, shareholders can also choose to sell the firm with its debt.

This paper explicitly studies the value of the shareholders' entitlement to relinquish ownership either directly or indirectly, i.e., the value of their *walkout option*. It also examines the implications of this choice of exit form for a firm's ability to efficiently raise the outside finance required to undertake worthwhile projects.

We present a continuous-time pricing model of the levered firm in a competitive industry where shareholders select not only the *timing* but also the *form* (indirect or direct) of ownership transfers. They choose between (i) reneguing or (ii) selling their ownership rights, as means of relinquishing control. That is, whenever they wish, they can either (i) *default* on their debt obligations or (ii) sell the firm to competitors with its debt obligations, as in a *corporation sale*. Shareholders are throughout assumed to behave non-cooperatively, so as to maximize the value of their holdings.

The paper contributes to both corporate finance and valuation theory, incorporating insights of the corporate finance literature into a valuation framework. Our structural model enables us to jointly obtain efficiency and pricing results with directly testable implications. The set-up explicitly considers the technological characteristics of the firm as well as of its competitors and we derive simple closed-form solutions for the value of shares and bonds throughout the firm's existence.

We obtain that the shareholders' non-cooperative optimum policy bears direct relations to the firm's (i) capital structure and (ii) relative technological characteristics: First, shareholders of firms with low levels of leverage tend to exit by means of corporation sales, whereas those of highly levered firms tend to default. Secondly, the likelihood of default being chosen instead of a corporation sale increases with the technological supremacy of the firm.

Whereas default necessarily involves inefficient timing of ownership transfers (too early), corporation sales achieve the correct allocation of resources because the interests of all asset holders are realigned. One implication for valuation theory is that, given that existing defaultable bond pricing models relate shareholders' walkout decisions exclusively to default, they often exaggerate risk premia and underestimate the borrowing ability (debt capacity) of firms. Such models are likely to perform at their worst when applied to (i) industries where firms possess quite similar access-protected technologies and (ii) sectors where the need for

[4] Kim, Ramaswamy and Sundaresan (1993) report 77 basis points as the average spread for investment-grade corporate bonds, and Litterman and Iben (1991) report historical ranges for par spreads between 20 and 130 basis points.

[5] In the Compustat data set used by Pastena and Ruland (1986), for instance, 531 manufacturing firms merged during 1970-1983, whereas only 56 filed for bankruptcy in the same period.

credit finance is relatively modest.

We present numerical simulations, showing how changes in firm parameters affect risk premia and credit spreads. Not surprisingly, the direct transfer of ownership alternative has a major influence on corporate debt pricing, as for typical parameter values, the difference with a "default-only" optimization can reach 50%.

The paper is structured as follows. Section I presents the model, derives asset values and the optimal policy in the first-best context, when the firm is unlevered. Section II introduces debt and discusses the moral hazard problem that thereby arises. Section III determines shareholders' ex-post behavior and examines the resulting ex-ante behavior of creditors. Our results on the pricing and efficiency implications of walkout decisions are then aggregated in two main Propositions. Section IV consolidates these results providing (i) closed-form pricing solutions, (ii) measures of the relative price impact of our analysis and (iii) numerical estimates. Section V concludes highlighting the main testable implications.

I. The Model.

A. Main Components and Random Time Line of the Model

Our continuous-time pricing model is intended to capture in a structural fashion the following features of firms operating in a competitive environment:

- *Firms* are created when worthwhile *projects* exist and the requisite finance can be obtained. Implementing new projects requires the combination of *human capital*, which is inalienable by nature, and *physical assets* that need to be acquired against an initial *investment*.[6]

- In addition to the firm responsible for originating the project - the *innovative firm* - the project could also be operated by a number of *competitors*. Each competitor's technology is defined by an *access protected* set of opportunities.[7]

- In a competitive industry, if initially a project has a positive NPV to an innovative firm, acquiring at that time the physical assets involved and operating them with their own technology is most likely to yield a negative NPV to other firms.

- Although the innovative firm's technology is initially superior to all rival technologies, it is most likely that there exist other economic circumstances under which either (i) the physical assets are best scrapped or (ii) a competitor's technology is superior. There are, therefore, circumstances under which it would be ex-ante optimal to see the innovative firm's operations abandoned.

- Owners can *abandon* projects at any time, i.e., give up control over the assets. This can be followed either by (i) actual *termination*, whereby the physical assets cease to

[6]We will construct our pricing model in terms related to the inalienable human capital theory of debt of Hart and Moore (1994).

[7]Access (and not having access) to a technology is at the heart of Rajan and Zingales (1998).

serve a productive purpose and are scrapped, or by (ii) *continuation*, whereby these assets are utilized by competitors.

- Finally, reversing an abandonment decision is *costly* because any transfer of ownership avoids access to some of the operating owners' opportunity set. This is, for example, the case when there is an element of uniqueness to the technological abilities of the innovators: In such circumstances, abandonment involves the withdrawal of the innovator's human capital from the project and competitors do not gain access to the innovator's know-how as part of an ownership transfer.

To capture these elements, the random time line of the model, illustrated in Figure 1, is as follows:

1. In the beginning, a project becomes available to the innovative firm and is considered worthwhile implementing, i.e., it offers a positive NPV. The firm is however cash constrained, and it issues debt to finance the initial investment.[8] We use ex-ante and ex-post with respect to the date the debt is issued.

2. Ex-post, when the innovative firm has started operating the project, shareholders have *ownership* rights. In the terminology of Grossman and Hart (1986) and Hart and Moore (1990), these rights consist of (i) residual control rights and (ii) income rights:

   - *Residual control rights* imply that shareholders have the right to decide all usages of the physical assets in a way not inconsistent with the debt contract.
   - Both shareholders and bondholders have *income rights*: These are provisions of the contract which stipulate the sharing of the revenues generated by the project between the two classes of claimants.

   While operating the project, the shareholders are constantly aware that (i) the physical assets have some value to competitors and (ii) can always be scrapped. These two technological alternatives define the *reservation value* of the firm. The model is constructed to reflect the fact that there are always circumstances when a firm should abandon its operations.

3. Shareholders eventually decide to *walk out* of the firm. They have the right to abandon the project whenever they want, and importantly, they are the only ones who control this decision. The walkout occurs at a random time which is optimized over by the shareholders, ex-post.

   Shareholders also control the *form* of their walkout: They can relinquish ownership either indirectly or directly. That is, shareholders can abandon operations either

   (a) *reneguing* their ownership rights. Practically, shareholders repudiate debt contracts, *defaulting* on their debt service obligations.

[8] Simple debt contracts will be justified considering that the state is observable but not verifiable to outsiders, hence cannot form part of an enforceable contract. Alternatively, debt could be justified by the presence of a tax advantage.

(b) or *selling* their ownership rights. This involves selling the firm with its debt obligations to competitors, as in a *corporation sale*.

4. (a) Default may then be followed either by (i) a *continuation* of the project, because competitors are interested in purchasing it, or (ii) a *termination* of the project.

   (b) A corporation sale can only be followed by a *continuation* of the project: Termination only occurs if competitors are not interested in pursuing the project, i.e. if their willingness to buy the firm and its debt obligations is zero.

5. Where shareholders' walkout has been followed by the competitor taking over operations (whether through a repudiation or corporation sale), the latter will then control the assets until, in turn, deciding at some later date to abandon the project, which necessarily results in termination.

In Figure 1, the dashed lines highlight the fact that the (i) continuation and (ii) termination alternatives completely determine the *reservation value* of the firm. Steps 4 and 5 are therefore only there to feed backwards in time the alternative available to shareholders during operations in step 2.

B. Basic Assumptions and Exogenous Parameters

The project we consider becomes available to the innovative firm at date $t = 0$. The factors influencing the project's profitability are summarized by an uncertain state variable which follows a diffusion process:

$$dx_t \quad = \quad \mu(x_t)\, dt \quad + \quad \sigma(x_t)\, dB_t\,, \tag{1}$$

where B_t is a standard Brownian motion. To implement the project it is necessary to acquire a set of physical assets against an initial investment I.

Combining the tangible assets with human capital, the innovative firm can generate indefinitely a period income flow which is state-dependent. If, however, the firm chooses not to implement the project at date $t = 0$, access to this income flow is lost. In other words, we are dealing with a now-or-never investment opportunity.[9] We will denote by $\Pi(x)$ the unlimited liability value of a perpetual claim (and obligation) on the project's income flow, which only depends on the level of the state variable x. We assume that $\Pi(x)$ is a continuous and twice differentiable function of x. This is a reasonable assumption, given that $\Pi(x)$ is the integral over time of a perpetual stream of discounted period income flows.

Competitors can also purchase and operate this set of physical assets. They can actually do so at any time $t \geq 0$, but are unable to strategically prevent the innovative firm from accessing $\Pi(x)$ at $t = 0$ by purchasing all available copies of the assets. Their technology differs from that of the innovator, implying that a transfer of the assets from the innovator to the competitor entails some form of *restructuring* of productive activities. We similarly

[9] Assuming that the option to invest expires immediately after its inception enables us to ignore the optimisation across entry triggers which would result in a two-dimensional problem without the possibility of deriving closed-form solutions.

denote $\Pi^*(x)$ the unlimited liability value of a perpetual claim on the competitors' income flow. We also assume that $\Pi^*(x)$ is a continuous and twice differentiable function of x.

Operating losses are not ruled out; consequently $\Pi(x)$ and $\Pi^*(x)$ may be negative over some range. It may therefore (and will by construction) become worthwhile *terminating* operations altogether at some point, scrapping the physical assets. To simplify, we normalize the scrap value of the assets to zero.

Abandoning a project, whether it leads to actual termination or not, is assumed to be an *irreversible* decision. This allows an interpretation in the spirit of Hart and Moore (1994), who consider that liquidation avoids future use of the incumbents' inalienable human capital. Furthermore, asset sales often involve a partial dismantlement of the technology.

The implicit cost of ownership transfer that a corporation can inflict on its creditors is based on the corporations' ability to prevent others from capturing its opportunity set. Ownership transfers can actually be viewed as groups of *asset redeployment* decisions. From a property rights perspective, operations involve a *limited*, *access protected* set of opportunities, and a transfer of ownership replaces the incumbents' production opportunity set by the competitors' access protected set.

The irreversibility assumption also allows us to structurally account for asset redeployment costs. The model implicitly captures them in the calibration of $\Pi(x)$, $\Pi^*(x)$, and zero. The set of opportunities that are lost when a firm abandons operations is measured by the cost that reconstituting this firm's working combination would entail. The asset redeployment costs incurred when the innovative firm abandons are therefore measured by $\Pi(x) - \Pi^*(x)$, precisely because we assume that $\Pi(x)$ is not available afterwards.

The difference between $\Pi(x)$ and Π^* also captures the degree to which the initial investments are inalienably specific to current operations. When the specialization of an investment is inalienable (as human capital), the specialization implies a relative reduction of the outside value of the physical assets. $\Pi(x)$ is then very different from $\Pi^*(x)$. This is particularly the case with start-up firms or firms at an early stage of their life. Very often these firms' technological superiority is due to their human capital, and this essential human capital would leave (as it is always entitled to) if another firm were to assume control. Consequently competitors are not willing to pay much for the physical assets of such firms.

We assume risk neutrality and a constant identical borrowing and lending safe interest rate, ρ.[10] Trading of assets is assumed to occur continuously in perfect and frictionless markets with no asymmetry of information or transaction costs. Firms' management act in the best interest of their shareholders, ignoring the insiders-outsiders principal-agent conflict of interest discussed in Hart (1993).

C. Asset Valuation

Asset prices are time-homogeneous, because (i) the uncertain state variable and the safe interest rate are time independent, and (ii) the problem is parametrized in terms of unlimited liability values of perpetual claims on different income flows available - $\Pi(x)$, $\Pi^*(x)$ and zero

[10]Harrison and Kreps (1979) show how to extend the results of the paper to a world without risk-neutrality by using an equivalent martingale measure.

- that are just state dependent. Pricing formulas can therefore be expressed in extremely simple fashion, introducing a single pricing operator, $\mathcal{P}(x_t \rhd y)$:

Let $T_y \equiv \inf\{\, \tau \mid x_\tau = y \,\}$ denote the first time at which the state variable x_t hits the level y, and $f_t(T_y)$ the density of T_y conditional on information at t. The Laplace transform of $f_t(T_y)$,

$$\mathcal{P}(x_t \rhd y) \;\equiv\; \int_t^\infty e^{-\rho(T_y - t)} f_t(T_y)\; dT_y \;, \tag{2}$$

is then very intuitively a probability-weighted discount factor for events that will occur the first time the state variable, currently equalling x_t, reaches the level y. This operator allows a direct derivation of all the pricing expressions contained in this paper, and will crucially help us to develop an intuitive understanding of their dynamics.

D. Reservation Value of the Firm

As we mentioned earlier, competitors can purchase the set of physical assets at any time $t \geq 0$. If they do so, they can operate them using their own technology (which is the support of $\Pi^*(.)$), with the option to later *abandon* these operations. Because at any time $t > 0$, access to $\Pi(x_t)$ is lost, a competitors' abandonment *terminates* the life of the physical assets by scrapping them. The value of the physical assets in the hands of competitors can therefore readily be obtained as follows:

If competitors ultimately abandon their operations the first time the state variable, currently at x_t, reaches a lower level y, the assets are worth

$$V^*(x_t \mid y) \;\equiv\; \Pi^*(x_t) \;-\; \Pi^*(y)\; \mathcal{P}(x_t \rhd y) \;. \tag{3}$$

In the expression of $V^*(x_t \mid y)$, the first term on the right-hand side is the value of a perpetual entitlement on the competitors' flow of income, $\Pi^*(x_t)$. The second term is the product of *(i)* the change in asset value intervening when the irreversible regime switch occurs, $[0 - \Pi^*(y)]$, and *(ii)* the probability-weighted discount factor for this event, $\mathcal{P}(x_t \rhd y)$.

Clearly, competitors' first-best policy consists of selecting their irreversible abandonment trigger level, y, in order to maximize the value of their claim, $V^*(x_t \mid y)$. The *optimal* competitor termination trigger level, which we denote $\underline{x}^*$, is therefore simply obtained by solving the first-order condition

$$\frac{\partial V^*(x_t \mid \underline{x}^*)}{\partial \underline{x}^*} = 0. \tag{4}$$

Here, nothing guarantees the actual desirability of the competitors' abandonment decision. The existence and uniqueness of an optimal competitor termination trigger level is ensured as follows:

Assumption 1 *At the entry state x_0, the option value of the competitors' decision to trigger termination at y,*

$$-\,\Pi^*(y)\; \mathcal{P}(x_0 \rhd y)\,,$$

is a strictly concave function in y, maximized at a trigger level, $\underline{x}^*$, strictly smaller than x_0.

Under Assumption 1, if a competitor acquires the assets of the firm when the economic fundamental x_t is greater than $\underline{x}^*$, he will initially find it worthwhile operating the firm but will then eventually prefer to abandon it when x_t has fallen low enough (to $\underline{x}^*$). That is, there are good states where the competitors' use for the assets dominates scrapping. However, it becomes eventually optimal for them to abandon the firm, in poor states of the world.

We denote $U^*(x_t) \equiv V^*(x_t \mid \underline{x}^*)$ the (first-best) value of the physical assets in the hands of competitors. In accordance with what was stated earlier, we rule out the possibility of competitors finding it worthwhile to implement the project at the outset by assuming the following:

Assumption 2 *At the entry state x_0, competitors are not interested in acquiring the set of physical assets. The initial investment, I, required is greater than $U^*(x_0)$.*

In a competitive industry where there is a fairly large number of identical competitors, each firm's willingness to pay for the physical assets must equal the value it expects to generate by owning those assets; $U^*(x_t)$ then represents the *bid* made by competitors for ownership of the assets at any time $t \geq 0$.[11] As such, it can be used to derive the value of the innovative firm itself, as a compound option: While operating the assets, the innovative firm knows that it can at any time abandon these operations and sell its assets for $U^*(x_t)$. $U^*(x_t)$ is therefore also the *reservation value* of the innovative firm implementing the project.

E. Value of the Innovative Firm, and First-Best Policy

The value of the physical assets in the hands of the innovative firm can thus be obtained directly as follows: If after implementation of the innovative project, operations are abandoned the first time the state variable x_t reaches a lower level y, the assets are worth

$$V(x_t \mid y) \equiv \Pi(x_t) + [U^*(y) - \Pi(y)] \; \mathcal{P}(x_t \rhd y) . \tag{5}$$

In the expression of $V(x_t \mid y)$, the first term on the right-hand side is the value of a perpetual entitlement on the innovative firm's flow of income, $\Pi(x_t)$. The second term is the product of *(i)* the change in asset value intervening when these operations are abandoned and the assets of the firm are either purchased by competitors or scrapped, $[U^*(y) - \Pi(y)]$, and *(ii)* the probability-weighted discount factor for this event, $\mathcal{P}(x_t \rhd y)$.

The innovative firm's first-best policy consists of selecting the irreversible abandonment trigger level, y, in order to maximize the value of the firm, $V(x_t \mid y)$. The *ex-ante optimal* firm abandonment trigger level, which we denote $\underline{x}$, therefore solves

$$\frac{\partial V(x_t \mid \underline{x})}{\partial \underline{x}} = 0. \tag{6}$$

Again, nothing guarantees the actual desirability of the firm's abandonment decision. The existence and uniqueness of an optimal (innovative) firm abandonment trigger level is ensured as follows:

[11] Here we assume not only that the secondary market for the assets is competitive but that the competitors face no cash constraints, or, alternatively, that the innovative firm's operations are sufficiently small in comparison with the acquiring firm's not to affect its capital structure adversely.

Assumption 3 *At the entry state x_0, the option value of the firm's decision to trigger abandonment at y,*

$$[\,U^*(y) \;-\; \Pi(y)\,] \;\; \mathcal{P}(x_0 \rhd y)\,,$$

is a strictly concave function in y, maximized at a trigger level, $\underline{x}$, strictly smaller than x_0.

Under Assumption 3, if the innovative firm acquires the physical assets at entry, its owners' ex-ante optimal policy from then onwards is as follows: They should first operate the project, but will eventually also find it preferable to abandon these operations when the economic fundamental x_t falls low enough (the first time the level $\underline{x}$ is reached). We denote $U(x_t) \equiv V(x_t \mid \underline{x})$ the value of the innovative firm under this first-best policy. We proceed ddy(pr)-30Dfl(-31)-30fl(t(31)-30((asseth)-320fort(31)-30com[(p)-300titors,est)]Tjfl/T171Tf46

II. Controlling the Timing and Form of Walkouts

A. Introducing Debt

Let us assume the shareholders of the innovative firm have limited wealth and can only contribute to the initial investment up to a level W_S which is strictly less than I. They must therefore at date $t = 0$ seek external financing in order to acquire the set of physical assets. Denote I_D the amount creditors are asked to lend. Clearly, I_D must be greater or equal to $I - W_S$ for the project to be realised. To finance the initial investment, the innovative firm issues infinite-maturity debt contracts offering the following:

1. A promise to an instantaneous flow of coupon paymenpa4-14.9995TD3'T171T29593.000T2(equa01cg:)

9980TDfl(t

drive our analysis, the model is developed setting τ to zero.

B. Moral Hazard Problems Associated with Debt

In terms of corporate governance, the main feature differentiating equity from debt is *ownership*: Although both shareholders and bondholders have *income rights*, the shareholders are the only ones entitled to decide all usages of the physical assets in a way not inconsistent with the contract: They hold *residual control rights*. Shareholders forfeit these rights only upon declaring default; any decisions concerning the allocation and use of the assets are thereafter made solely by the creditors.

The right of shareholders to exercise their limited liability option whenever they wish is an important one in the theory of asset pricing, but what we wish to underline here is that default only constitutes one form of voluntary abandonment available to shareholders. One ought to speak more generally of a "*walkout*" decision, where in addition to default, shareholders can also choose to sell the firm *with* its debt, i.e., through a *corporation sale*, which is arguably the form of abandonment most commonly observed. In such a sale, control remains throughout in the hands of different shareholders; creditors can neither influence the timing nor the direction (identity of buyer) of the sale.

More precisely, shareholders' walkout decisions consist of the following two dimensions:

1. Shareholders control the *timing* of their walkout, selecting non-cooperatively the abandonment trigger level, y. As they can do so at any time, they choose y in the whole state space.

2. Shareholders also control the *form* of their walkout, selecting between *two* possible *equity reservation value functions*:

   (a) They can decide to *renegue* their ownership rights, *defaulting* on their debt obligation. The assets of the firm are then sold and the proceeds from the sale equal $U^*(y)$. Debtholders are paid first, and it is only if creditors' collateral is fully paid that shareholders will receive anything. In this case the equity reservation value equals $U^*(y)$ minus $U^*(y) \wedge P$.

   (b) They can alternatively decide to *sell* their ownership rights, selling the physical assets of the firm with its debt obligations, as in a *corporation sale*. This option is clearly only available if competitors have an interest in purchasing these physical assets for a strictly positive price. That is, if y is greater than $\underline{x}^*$, otherwise the assets are just worth scrapping.

   In this scenario, after the shareholders' walkout, the debt is not immediately repudiated by the competitor who takes over ownership. Competitors, who are not assumed to be wealth constrained, hence ultimately abandon at the ex-ante optimal $\underline{x}^*$. Therefore the equity reservation value at the time of the corporation sale is equal to $U^*(y)$ minus $\delta/\rho\,[\,1 - \mathcal{P}(y \triangleright \underline{x}^*)\,]$.

We can already relate these two forms of abandonment and the resulting reservation value functions:

Lemma 1 *When shareholders choose to default, bondholders are not compensated up to their par value. Consequently, shareholders' ex-post optimal equity reservation value function, which we denote $S^*(y)$, is therefore*

$$S^*(y) \;=\; \max\left\{\, 0\;;\; U^*(y) \;-\; \frac{\delta}{\rho}\,[\,1 \;-\; \mathcal{P}(y \triangleright \underline{x}^*)\,]\,\right\}\;. \tag{7}$$

Lemma 1 states that it is only optimal for shareholders to default at y when in doing so their claim becomes worthless, i.e. $S^*(y) = 0$. It rules out the possibility that shareholders could receive part of the residual value in conjunction with default. The significance of this result is that it captures the fundamental incentive problem facing shareholders: whereas the value of the assets reacts to the state even within the default region, shareholders' rewards do not.

C. Valuing Shares and Bonds

In order to price shareholders' and debtholders' claims, we first establish the values they would have if the innovative firm's operations were abandoned the first time the state variable x_t reached a *given* level y. Although they are conditional on the walkout occuring at y, these calculations will nevertheless embed the shareholders' optimization over the form of walkout. Here, we use the fact that the optimal walkout is implicitly given by the shareholders' ex-post optimal equity reservation value function, $S^*(y)$, and that this is just a function of the (single) control variable y (Lemma 1).

We therefore directly obtain that the values of the shares and the debt prior to abandonment, for a given couple y and δ, are respectively

$$S(x_t \mid y, \delta) \;=\; \Pi(x_t) - \frac{\delta}{\rho} \;+\; \left[S^*(y) - \Pi(y) + \frac{\delta}{\rho}\right]\;\mathcal{P}(x_t \triangleright y)\;, \tag{8}$$

$$D(x_t \mid y, \delta) \;=\; \frac{\delta}{\rho} \;+\; \left[U^*(y) - S^*(y) - \frac{\delta}{\rho}\right]\;\mathcal{P}(x_t \triangleright y)\;. \tag{9}$$

The structure of these expressions is similar to equations (3) and (5): the first term on the right-hand side is the value of a perpetual entitlement on the flow of income to the particular asset class; $\Pi(x) - \delta/\rho$ for equity and δ/ρ for bonds. The second term is the product of (i) the change in asset value intervening when the (innovative firm's) shareholders abandon operations, which is the expression in the square brackets, and (ii) the probability-weighted discount factor for this event, $\mathcal{P}(x_t \triangleright y)$.

Shareholders choose the timing and form of their walkout in order to maximize the value of their claim. Assuming they select it in an unconstrained fashion,[13] the *shareholders' ex-post optimal* walkout trigger level, $\underline{x}_{\mathcal{S}}$, therefore solves

$$\underline{x}_{\mathcal{S}} \;=\; \arg\max_{y}\; S(x_t \mid y, \delta) \tag{10}$$

[13]This is the Endogenous Closure Rule assumed in Leland (1994, 1998), Leland and Toft (1996), Fries, Miller and Perraudin (1997), Mella-Barral and Perraudin (1997) and Mella-Barral (1999).

The derivation of $\underline{x}_{\mathcal{S}}$ is the result of an optimization over (a) walkout time and (b) walkout form. This double optimization is easiest carried out in two steps: First consider the value of equity *conditional* on one particular form of walkout (default or corporation sale) and optimize over the time of this decision (its trigger level). Secondly compare conditional equity values to determine which form of walkout is actually the optimal one. The procedure is as follows:

1. Define $S_{def}(x_t \mid y, \delta)$ and $S_{corp}(x_t \mid y, \delta)$ to be the *conditional* value of the equity assuming that shareholders choose a *default* and a *corporation sale*, respectively, as the walkout form and set the timing of abandonment as if this walkout form were the only one available. Algebraically, this consists of fixing $S^*(y) = 0$ for the former and $S^*(y) = U^*(y) - \delta/\rho\,[\,1 - \mathcal{P}(y \rhd \underline{x}^*)\,]$ for the latter:

$$S_{def}(x_t \mid y, \delta) \;\equiv\; \Pi(x_t) - \frac{\delta}{\rho} \;+\; \left[-\Pi(y) + \frac{\delta}{\rho}\right] \;\mathcal{P}(x_t \rhd y)\;, \tag{11}$$

$$S_{corp}(x_t \mid y, \delta) \;\equiv\; \Pi(x_t) - \frac{\delta}{\rho} \;+\; [U^*(y) - \Pi(y)]\;\;\mathcal{P}(x_t \rhd y) \;+\; \frac{\delta}{\rho}\,\mathcal{P}(x_t \rhd \underline{x}^*)\;. \tag{12}$$

Derive the shareholders' ex-post optimal *default* and *corporation sale* trigger levels, $\underline{x}_{def}$ and $\underline{x}_{corp}$, solving respectively the first-order optimality conditions:

$$\frac{\partial S_{def}(x_t \mid \underline{x}_{def}, \delta)}{\partial \underline{x}_{def}} \;=\; 0\;, \qquad \text{and} \qquad \frac{\partial S_{corp}(x_t \mid \underline{x}_{corp}, \delta)}{\partial \underline{x}_{corp}} \;=\; 0\;. \tag{13}$$

2. Compare $S_{def}(x_t \mid \underline{x}_{def}, \delta)$ and $S_{corp}(x_t \mid \underline{x}_{corp}, \delta)$ to determine which of the two forms of walkout corresponds to the unconditional optimal form of walkout. The actual walkout trigger level is then

$$\underline{x}_{\mathcal{S}} = \begin{cases} \underline{x}_{def} & \text{when} \quad S_{def}(x_t \mid \underline{x}_{def}, \delta) \;\;\geq\; S_{corp}(x_t \mid \underline{x}_{corp}, \delta) \\ \underline{x}_{corp} & \text{when} \quad S_{corp}(x_t \mid \underline{x}_{corp}, \delta) \;\;>\; S_{def}(x_t \mid \underline{x}_{def}, \delta)\;. \end{cases} \tag{14}$$

Notice that all existing defaultable bond pricing models associate the shareholders' walkout decision exclusively with default (reneging ownership rights), i.e., they ignore the shareholders' option to sell ownership rights. In our set-up, this amounts to imposing $S^*(y) = 0$ and $\underline{x}_{\mathcal{S}} = \underline{x}_{def}$. The values for equity and debt such a restriction would generate are $S_{def}(x_t \mid \underline{x}_{def}, \delta)$ and $D_{def}(x_t \mid \underline{x}_{def}, \delta) \equiv V(x_t \mid \underline{x}_{def}) - S_{def}(x_t \mid \underline{x}_{def}, \delta)$. In Section IV.B, we will introduce simple measures to assess the impact of this restriction on calculations of risk premia and credit spreads. In Section IV.C, we will then carry out several numerical applications. In some cases, we will obtain very substantial differences in asset values.

From now on, to simplify, we denote $S(x_t \mid \delta) \equiv S(x_t \mid \underline{x}_{\mathcal{S}}, \delta)$ and $D(x_t \mid \delta) \equiv D(x_t \mid \underline{x}_{\mathcal{S}}, \delta)$ the ex-post value of the shares and the debt, respectively, under this second-best policy.

The policy resulting from (10) is second best, because it has no reason to yield the ex-ante optimal usage of the assets under the first-best policy derived in Section 1. This double choice of timing and form of exit generally leads to a moral hazard problem, as for most coupon levels, δ, the resulting ex-post value of the firm, $V(x_t \mid \underline{x}_{\mathcal{S}}) = S(x_t \mid \delta) + D(x_t \mid \delta)$, can only be less than the ex-ante optimal one, $U(x_t)$.

III. Second-Best Policy

In this Section, we determine the second-best policy that will actually be chosen: We first study the ex-post behavior of shareholders, assuming a given debt contract. Working backward in time, we then examine the ex-ante behavior of creditors to determine whether, to start with, this debt contract could have been issued at the date of entry, $t = 0$. Finally, we gather our results to describe the second-best sequence of events, in chronological order.

A. Ex-Post Behavior

We analyze the ex-post behavior of shareholders and the resulting second-best usage of assets, considering all *algebraically* possible coupon levels without limiting their range. We assume that the project was financed at time $t = 0$ by issuing a coupon δ paying debt contract, and examine at dates $t > 0$ the timing and form of shareholders' preferred walkout.

We derive the following observations by simply examining the non-cooperative solutions obtained in the unconstrained ex-post optimization problem of equation (8), for a *given* debt coupon δ:

Lemma 2 *If shareholders' walkout occurs through default, the walkout trigger level, $\underline{x}_{\mathcal{S}}$, is strictly increasing in the coupon level, δ. If the walkout takes the form of a corporation sale instead, its trigger level is independent of the coupon level.*

$$\text{If } \; S^*(\underline{x}_{\mathcal{S}}) \;=\; 0\,, \quad \text{then } \; \frac{\partial \underline{x}_{\mathcal{S}}}{\partial \delta} \;>\; 0\,. \tag{15}$$

$$\text{If } \; S^*(\underline{x}_{\mathcal{S}}) \;=\; U^*(\underline{x}_{\mathcal{S}}) \;-\; \frac{\delta}{\rho}\,[\,1 - \mathcal{P}(\underline{x}_{\mathcal{S}} \triangleright \underline{x}^*)\,]\,, \quad \text{then } \; \frac{\partial \underline{x}_{\mathcal{S}}}{\partial \delta} \;=\; 0\,. \tag{16}$$

Increasing the debt has, in the case of default, no effect on the shareholders' stake in the residual value (which remains at zero by Lemma 1), but reduces the current revenue flow, favoring *earlier* abandonment. These dynamics of default are similar to those established in Mella-Barral (1999). However, such dynamics are particular to default: The monotonous relationship between the degree of indebtedness and exit timing breaks down when abandonment occurs through corporation sales; the timing of such sales is entirely *independent* of the capital structure of the firm.

The nesting result that the dynamics of walkout (shareholders' abandonment time versus leverage) differ across *forms* of walkout will be at the heart of the efficiency results we derive. To start with, relating these observations to the firm value leads immediately to the following result:

Corollary 1 *Whereas default results in a suboptimally early abandonment of the firm's operations, corporation sales lead to an ex-ante optimal usage of assets.*

$$\text{If } \; S^*(\underline{x}_{\mathcal{S}}) \;=\; 0\,, \quad \text{then } \; \underline{x}_{\mathcal{S}} > \underline{x} \quad \text{and } \; V(x_t \,|\, \underline{x}_{\mathcal{S}}) \;<\; U(x_t)\,. \tag{17}$$

$$\text{If } \; S^*(\underline{x}_{\mathcal{S}}) \;=\; U^*(\underline{x}_{\mathcal{S}}) \;-\; \frac{\delta}{\rho}\,[\,1 - \mathcal{P}(\underline{x}_{\mathcal{S}} \triangleright \underline{x}^*)\,]\,,$$

$$\text{then } \; \underline{x}_{\mathcal{S}} \;=\; \underline{x} \quad \text{and } \; V(x_t \,|\, \underline{x}_{\mathcal{S}}) \;=\; U(x_t)\,. \tag{18}$$

Corollary 1 establishes the differing welfare implications of the two walkout forms. Default triggers off an agency problem inherent to debt contracts and causes superior technologies to be utilized insufficiently: the innovative firm should operate the assets until x_t hits $\underline{x}$; instead it chooses to abandon the assets at an earlier point in time when x_t reaches $\underline{x}_{\mathcal{S}}$, where $\underline{x} < \underline{x}_{\mathcal{S}}$. Corporation sales, on the other hand, eliminate this moral hazard problem as they realign the interests of all asset holders when contractual or concessionary solutions are not available.

We have seen in Section I.E that for a given (i) firm and its project, $\{\Pi(x), I, x_0\}$, (ii) its competitors, $\{\Pi^*(x)\}$, and (ii) the industry it is evolving in, $\{x; \mu(x); \sigma(x); \rho\}$, one of two post-abandonment scenarios, "termination-abandonment" or "continuation-abandonment", corresponds to the first-best policy. It is convenient to pursue our analysis distinguishing between these two types of situations. The differences in $\partial \underline{x}_{\mathcal{S}} / \partial \delta$ identified in Lemma 2, and the first-order optimality conditions which determine $\underline{x}^*$ and $\underline{x}_{\mathcal{S}}$ yield the following results:

Lemma 3 *When an ex-ante optimal usage of assets involves a termination-abandonment, i.e. $\underline{x} < \underline{x}^*$, shareholders will always choose to walk out by means of default.*

After default, for lower levels of coupons, $\delta \in (0\,;\, \hat{\delta}^*\,]$ where

$$\hat{\delta}^* \;\equiv\; \rho\, \Pi(\underline{x}^*) \;-\; \rho\; \Pi^*(\underline{x}^*)\; \frac{d\Pi(\underline{x}^*)}{d\underline{x}^*} \left[\frac{d\Pi^*(\underline{x}^*)}{d\underline{x}^*} \right]^{-1} , \tag{19}$$

the usage of assets will be terminated, whereas for higher coupon levels, it will be continued by competitors.

Figure 4 gives a graphical representation of the latter case (higher coupon levels) considered in Lemma 3. The values of the exogenous parameters characterizing the setup are exactly those used in Figure 2, where we illustrated the value the firm would take under the first-best policy.

Lemma 4 *When an ex-ante optimal usage of assets involves a continuation-abandonment, i.e. $\underline{x}^* < \underline{x}$, shareholders choose to walk out by means of a corporation sale for lowest coupon levels, $\delta \in (0\,;\, \hat{\delta}\,]$, where $\hat{\delta}$ solves*

$$\hat{\delta} \;=\; \rho\, \frac{\Pi(\underline{x}_{def}) \;+\; [\, U^*(\underline{x}) \;-\; \Pi(\underline{x})\,]\;\; \mathcal{P}(\underline{x}_{def} \rhd \underline{x})}{1 \;-\; \mathcal{P}(\underline{x}_{def} \rhd \underline{x}^*)} \;. \tag{20}$$

and by means of default for higher ones, usage of the assets being continued by competitors in both cases. Shareholders' walkout is thus always followed by continuation when $\underline{x}^* < \underline{x}$.

Figure 5 illustrates the case with a corporation sale (lower coupon levels); the setup corresponds exactly to that of Figure 3, and the graphs gain in being compared. Lemmas 3 and 4 highlight the fact that ex-post not only the (random) time of the walkout may be too early (Corollary 1), but also the form of the walkout may be different from the ex-ante optimal one.

B. Resulting Ex-Ante Behavior

We now examine the ex-ante behavior of creditors to determine which coupon levels, among all the algebraically possible, may have been contracted upon at the date of entry, $t = 0$: Given that shareholders need to raise a total of I_D in external finance, which coupon level δ will creditors require ex ante to be contracted upon? Is it even possible to always find a contract that allows us to implement the project?

We begin simply studying the response of debt values to coupon levels: Inserting the shareholders' ex-post optimal walkout trigger level, $\underline{x}_{\mathcal{S}}$, derived from (10), into the debt value function (9) in place of y, we express the ex-post value of debt, $D(x_t \mid \delta)$, solely as a function of the coupon level. Then, using Lemma 1 we obtain

Lemma 5 *If shareholders' ex-post walkout choice consists of default, the value of debt prior to this event, $D(x \mid \delta)$, is first increasing, then decreasing in the coupon level, δ. Therefore, there exists a coupon*

$$\overline{\delta} \;\equiv\; [\,1 \,-\, \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})\,] \left[\frac{\partial \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})}{\partial \underline{x}_{\mathcal{S}}} \; \frac{\partial \underline{x}_{\mathcal{S}}}{\partial \delta} \right]^{-1} \,, \tag{21}$$

at which the value function of debt reaches a maximum. Consequently, coupon levels issued at entry must be in the interval $\delta \in (\,0\,;\,\bar{\delta}\,]$

Lemma 6 *If shareholders' ex-post walkout choice consists of a corporation sale, the value of debt prior to this event, $D(x \mid \delta)$, is a constantly increasing function of δ.*

We can now proceed to examine the heart of the ex-ante financing problem: At the date of entry, $t = 0$, the shareholders, if possible, wish to issue a debt contract whose market value, $D(x_0 \mid \delta)$, equals the share of the initial outlay they are unable to cover by internal funding, I_D. We assume that debt is not issued at a discount. Sufficient debt financing is then found, if for a given initial state, x_0, there exists a contractual coupon δ, such that

$$I_D = D(x_0 \mid \delta). \tag{22}$$

To establish whether financing is possible, it is actually preferable to study the inverse function

$$\delta = \delta(x_0; I_D); \tag{23}$$

which is a mapping from the financial requirement space to the contract space. The case where termination-abandonment is ex-ante optimal presents us with little difficulty in this respect. Combining Lemmas 3 and 5 yields:

Lemma 7 *When an ex-ante optimal usage of assets involves a termination-abandonment, i.e. $\underline{x} < \underline{x}^*$, a unique coupon level δ and associated debt contract can be found to finance the project as long as the required external finance, I_D, does not exceed $D(x_0 \mid \bar{\delta})$.*

The function relating the coupon to the finance required, $\delta = \delta(x_0; I_D)$, is then strictly increasing in I_D over the interval $I_D \in [0; D(x_0 \mid \bar{\delta}))$.

Figure 6 gives a graphical representation of Lemma 7. Given that an ex-ante optimal usage of assets involves a termination-abandonment, we know that shareholders will ultimately default (Lemma 3), hence $D(x_0 \mid \bar{\delta}) = D_{def}(x_0 \mid \underline{x}_{def}, \delta)$. Then, the upper part of the figure simply shows the value of the debt at entry, $D(x_0 \mid \bar{\delta})$, for different levels of the coupon, δ. The lower part of the figure illustrates the implications for project financing, by simply inverting axis: If I_D is less than $D(x_0 \mid \bar{\delta}))$, the project can be financed. For a given I_D there exists a contractual coupon, $\delta(x_0; I_D)$, which leads to a debt value equal to I_D.

The case where continuation-abandonment is preferred ex ante ($\underline{x}^* < \underline{x}$) is more complex. Combining Lemmas 4, 5 and 6 reveals that, as we consider gradually increased finance requirements, I_D, the chosen contracts may result in a switch from corporation sales to default at a critical coupon threshold $\hat{\delta}$. This shifting between walkout forms is associated with a corresponding shift in the relevant debt value function, causing a *discontinuity* in debt value as a function of coupon level:

Lemma 8 *When an ex-ante optimal usage of assets involves a continuation-abandonment, i.e.,* $\underline{x}^* < \underline{x}$*, a unique coupon level* δ *and associated debt contract can be found to finance the project as long as the required external finance,* I_D*, does not exceed* $D(x_0 \mid \hat{\delta}) \vee D(x_0 \mid \bar{\delta})$.

(i) If $D(x_0 \mid \bar{\delta}) < D(x_0 \mid \hat{\delta})$*, the chosen coupon* $\delta(x_0; I_D)$ *is a continuous and strictly increasing function in* I_D *throughout the interval* $I_D \in (0; D(x_0 \mid \hat{\delta}))$*, with a constant slope* $\partial\delta/\partial I_D = \rho\, [\,1 \,-\, \mathcal{P}(x_0 \rhd \underline{x})\,]^{-1}$.

(ii) If $D(x_0 \mid \hat{\delta}) < D(x_0 \mid \bar{\delta})$*, the chosen coupon* $\delta(x_0; I_D)$ *is a continuous and strictly increasing function in* I_D *over the intervals* $I_D \in (0; D(x_0 \mid \hat{\delta}))$ *and* $I_D \in (D(x_0 \mid \check{\delta}); D(x_0 \mid \bar{\delta}))$*. At* $I_D = D(x_0 \mid \hat{\delta})$*, the chosen coupon function,* $\delta(x_0; I_D)$*, makes a discrete jump from* $\hat{\delta}$ *to* $\check{\delta}$*, where* $\check{\delta}$ *solves* $D_{corp}(x_0 \mid \underline{x}_{corp}, \hat{\delta}) = D_{def}(x_0 \mid \underline{x}_{def}, \check{\delta})$.

The distinction between cases (i) and (ii) of Lemma 8 is best understood graphically:

Figure 7(i) depicts the situation in case (i): The (conditional) debt value associated with default, $D_{def}(x_0 \mid \underline{x}_{def}, \delta)$, fails to reach the level $D_{corp}(x_0 \mid \underline{x}_{corp}, \hat{\delta}) = D(x_0 \mid \hat{\delta})$ even at its maximum point, $\bar{\delta}$. It therefore never pays to issue a debt contract that would result in shareholders' walkout occurring through default, i.e., setting δ above $\hat{\delta}$. In the lower part of the figure we can can see the implications for project financing: The inverse function $\delta(x_0; I_D)$ is cut off above at $\hat{\delta}$.

Figure 7(ii) tells the more complicated case (ii): Over the interval $\delta \in (0; \hat{\delta}]$, we observe the usual straight-line relationship associated with corporation sales. Over the interval $\delta \in (\hat{\delta}; \check{\delta})$, default is optimal for shareholders, but because the market value of such contracts is less than $D(x_0 \mid \hat{\delta})$, they are dominated by the $\hat{\delta}$ coupon contract. Therefore, debt contracts offering a coupon in $\delta \in (\hat{\delta}; \check{\delta})$ will never be issued. At the high end, coupons $\delta \in (\check{\delta}; \bar{\delta}]$ yield higher debt values and therefore serve a purpose. Again, the lower part of the figure shows the implications for the inverse function $\delta(x_0; I_D)$.

C. Combining Results

We now combine the results of Lemmas 1 to 8 in two Propositions, to describe the second-best sequence of events in its more understandable chronological order. Proposition 1 is obtained combining Lemmas 3, 5 and 7. Proposition 2 combines Lemmas 4, 5, 6 and 8.

We continue to distinguish between the two first-best situations that can arise: For a given set of exogenous parameters, one of two post-abandonment scenarios, "termination-abandonment" or "continuation-abandonment", corresponds to the first-best policy, depending on whether $\underline{x} < \underline{x}^*$ or the opposite.

Proposition 1 *Let us consider the case where, (i) the firm* $\{\Pi(x), I, x_0\}$*, (ii) its competitors,* $\{\Pi^*(x)\}$*, and (iii) the industry it is evolving in,* $\{x; \mu(x); \sigma(x); \rho\}$*, are such that an ex-ante optimal usage of assets involves a termination-abandonment, i.e.* $\underline{x} < \underline{x}^*$*.*

1. *At the date of entry,* $t = 0$*:*
   (a) *If the required external financing* $I_D \in (D(x_0 \mid \bar{\delta}); V(x_0 \mid \underline{x})]$ *the positive NPV project cannot find financing.*
   (b) *Conversely, if the required external financing* $I_D \in [0; D(x_0 \mid \bar{\delta}))$*, a debt contract with associated coupon level* δ *is found to finance the project. The function relating the coupon to the finance required,* $\delta = \delta(x_0; I_D)$*, is strictly increasing in* I_D*.*
2. *In case 1(b), shareholders will always walk out by means of default.*
3. *After shareholders' walkout, the usage of the assets will be as follows:*
   (a) *If* $\bar{\delta} < \hat{\delta}^*$*, it will always be terminated.*
   (b) *If, on the other hand,* $\hat{\delta}^* < \bar{\delta}$*,*
      i. *it will be terminated for lower coupon levels* $\delta \in (0; \hat{\delta}^*]$
      ii. *it will be continued by competitors for higher coupon levels* $\delta \in (\hat{\delta}^*; \bar{\delta}]$*.*

$D(x_0 \mid \bar{\delta})$ is the endogenous absolute limit to the amount the firm is able to borrow, often referred to as the debt capacity of the firm. The lack of funding available for projects that are intrinsically worthwhile (item *1(a)*) is the ultimate manifestation of the moral hazard problem associated with shareholders' option to default. Corporation sales provide no relief here, as they are never chosen by the shareholders (item *2*).

Proposition 2 *Let us consider the alternative case where, (i) the firm* $\{\Pi(x), I, x_0\}$*, (ii) its competitors,* $\{\Pi^*(x)\}$*, and (iii) the industry it is evolving in,* $\{x; \mu(x); \sigma(x); \rho\}$*, are such that an ex-ante optimal usage of assets involves a continuation-abandonment, i.e.* $\underline{x}^* < \underline{x}$*.*

1. *At the date of entry,* $t = 0$*:*
   (a) *If the required external financing* $I_D \in (D(x_0 \mid \hat{\delta}) \vee D(x_0 \mid \bar{\delta}); V(x_0 \mid \underline{x})]$ *the positive NPV project cannot find financing.*

(b) *Conversely, if the required external financing* $I_D \in [0; D(x_0 \mid \hat{\delta}) \vee D(x_0 \mid \bar{\delta}))$ *a debt contract with associated coupon level* δ *is found to finance the project.*

   i. *If* $D(x_0 \mid \bar{\delta}) < D(x_0 \mid \hat{\delta})$, *the chosen coupon* $\delta(x_0; I_D)$ *is a continuous and strictly increasing function in* I_D *throughout, with a constant slope* $\partial\delta/\partial I_D = \rho\,[\,1 - \mathcal{P}(x_0 \rhd \underline{x})\,]^{-1}$.

   ii. *If* $D(x_0 \mid \hat{\delta}) < D(x_0 \mid \bar{\delta})$, *the chosen coupon* $\delta(x_0; I_D)$ *is a continuous and strictly increasing function in* I_D *over the intervals* $I_D \in (0; D(x_0 \mid \hat{\delta}))$ *and* $I_D \in (D(x_0 \mid \check{\delta}); D(x_0 \mid \bar{\delta}))$. *At* $I_D = D(x_0 \mid \hat{\delta})$, *the chosen coupon function,* $\delta(x_0; I_D)$, *makes a discrete jump from* $\hat{\delta}$ *to* $\check{\delta}$, *where* $\check{\delta}$ *solves* $D_{corp}(x_0 \mid \underline{x}_{corp}, \hat{\delta}) = D_{def}(x_0 \mid \underline{x}_{def}, \check{\delta})$.

2. *In case 1(b), shareholders will walk out*

   (a) *by means of a corporation sale for lower coupon levels,* $\delta \in (\,0\,;\,\hat{\delta}\,]$, *and*

   (b) *through default for higher coupon levels* $\delta \in (\,\hat{\delta};\,\bar{\delta}\,]$.

3. *After shareholders' walkout, the project will always be bought and operated by the competitors.*

The endogenous absolute limit to the amount the firm is able to borrow or the debt capacity of the firm is here $D(x_0 \mid \hat{\delta}) \vee D(x_0 \mid \bar{\delta})$: The fact that shareholders cannot commit to relinquish ownership through the ex-ante optimal method results in project financing inefficiencies at entry (item *1(a)*).

Figure 8 provides a helpful graphical reading of Propositions 1 and 2: The bottom half of the vertical axis always corresponds to Proposition 1, whereas the top one depicts Proposition 2. This figure summarizes all our results so far: It contrasts, for all possible situations, the difference between the ex-ante optimal behavior (which is a characteristic of the firm), and the actual sequence of events occurring under the second-best policy. In the top graph, the dotted area represents projects that do not find financing, because the required external financing is above the debt capacity (item *1(a)*). In the middle and bottom graphs, the dotted area represents the set of coupon levels that are therefore never agreed upon at entry.

Notice that Figure 8 can be related to the previous ones as follows: Point A in Figure 8 corresponds to the situation portrayed in Figures 2 and 4. For such a point, the firm's second-best policy follows items *1(b)*, *2* and *3(b)ii* in Proposition 1. A point like C in Figure 8, corresponds to the situation portrayed in Figures 3 and 5. Then, the firm's second-best policy follows items *1(b)*, *2(a)* and *3* in Proposition 2.

IV. Implementing the Model

A. Obtaining Closed-Form Pricing Solutions

For this fairly general continuous-time model to yield closed-form pricing solutions, additional structure is needed. The parametrization of (i) the firm and its project, $\{\Pi(x), I, x_0\}$,

(ii) its competitors, $\{\Pi^*(x)\}$, and (iii) the industry it is evolving in, $\{x; \mu(x); \sigma(x); \rho\}$, must permit the following:

First, the specific type of diffusion process we assume to be driving the uncertainty, $\{x; \mu(x); \sigma(x); \rho\}$, must enable us to express the Laplace transform, $\mathcal{P}(x_t \rhd y)$, introduced in equation (2). Secondly, the functional form chosen for the perpetuity-values of the firm's alternatives, $\Pi(x)$ and $\Pi^*(x)$, must allow us to solve explicitly for the different optimal decision trigger levels, as well as threshold levels, using the relevant first-order optimality conditions we have detailed. The following structure performs this and is easy to implement:

"GBM-Linear Structure: *The uncertain state variable, x_t, describing the current status of the firm follows a geometric Brownian motion,*

$$dx_t \;=\; \mu\, x_t\, dt \;+\; \sigma\, x_t\, dB_t\,, \tag{24}$$

where $\mu < \rho$ and σ are constants, and B_t is a standard Brownian motion.
Both the unlimited liability value of a perpetual claim on the income flow from the innovative firm's operations, $\Pi(x)$, and the unlimited liability value of a perpetual claim on the competitors' income flow, $\Pi^*(x)$, are linear in x. That is, there exist four constants Θ_0, Θ_1, Θ_0^*, and Θ_1^*, where $\Theta_0 < \Theta_0^* < 0$ and $0 < \Theta_1^* < \Theta_1$, such that

$$\Pi(x) \;=\; \Theta_0 \;+\; \Theta_1\, x\;, \qquad \text{and} \qquad \Pi^*(x) \;=\; \Theta_0^* \;+\; \Theta_1^*\, x\;.$$

Under this structure:

1. The probability weighted discount factor for future events $\mathcal{P}(x \rhd y)$ becomes simply[14]

$$\mathcal{P}(x \rhd y) \;=\; \left(\frac{x}{y}\right)^{\lambda}, \tag{25}$$

$$\text{where}\;\; \lambda \;\equiv\; \sigma^{-2}[-(\mu-\sigma^2/2) - ((\mu-\sigma^2/2)^2 + 2\rho\sigma^2)^{1/2}]\;. \tag{26}$$

2. *All* asset pricing formulas encountered in the paper, $A(x_t) \in \{\, S(x_t \mid y,\delta)\,;\, D(x_t \mid y,\delta)\,;\, V(x_t \mid y)\,;\, V^*(x_t \mid y)\,\}$, take the following simple form

$$A(x_t \mid \underline{x}) \;=\; a_A \;+\; b_A\, x_t \;+\; c_A\, x_t^{\lambda}\;,$$

   where (a_A, b_A, c_A) are constants.

3. *All* decision trigger levels have very simple closed-form solutions:

   (a) The optimal competitor termination trigger level, $\underline{x}^*$, seen in equation (4) is

$$\underline{x}^* \;=\; \frac{-\lambda}{1-\lambda}\left(\frac{-\Theta_0^*}{\Theta_1^*}\right)\;.$$

[14]See Karlin and Taylor (1975).

(b) The ex-ante optimal firm abandonment trigger level, $\underline{x}$, seen in equation (6) is

$$\underline{x} \;=\; \frac{-\lambda}{1-\lambda}\left(\frac{-\Theta_0}{\Theta_1}\right) \qquad \text{if } \; 0 \;<\; \Theta_0\,\Theta_1^* \;-\; \Theta_0^*\,\Theta_1$$

(and "termination-abandonment" is ex-ante optimal) ,

$$\;=\; \frac{-\lambda}{1-\lambda}\left(\frac{\Theta_0^* - \Theta_0}{\Theta_1 - \Theta_1^*}\right) \qquad \text{if } \; \Theta_0\,\Theta_1^* \;-\; \Theta_0^*\,\Theta_1 \;<\; 0$$

(and "continuation-abandonment" is ex-ante optimal) . (27)

(c) The shareholders' ex-post optimal walkout trigger level, $\underline{x}_{\mathcal{S}}$, seen in equation (10) is

$$\underline{x}_{\mathcal{S}} \;=\; \underline{x}_{def} \;=\; \frac{-\lambda}{1-\lambda}\left(\frac{\delta/\rho \;-\Theta_0}{\Theta_1}\right) \qquad \text{if } \; 0 \;<\; \Theta_0\,\Theta_1^* \;-\; \Theta_0^*\,\Theta_1$$

($\delta \in (\hat{\delta}\,;+\infty)$ and shareholders default) ,

$$\;=\; \underline{x}_{corp} \;=\; \frac{-\lambda}{1-\lambda}\left(\frac{\Theta_0^* - \Theta_0}{\Theta_1 - \Theta_1^*}\right) \qquad \text{if } \; \Theta_0\,\Theta_1^* \;-\; \Theta_0^*\,\Theta_1 \;<0$$

($\delta \in (0\,;\hat{\delta}\,]$ and shareholders sell the corporation) . (28)

4. Of the three threshold coupon levels we introduced, only one has a simple closed-form solution. There are no direct expressions for the other two, but finding them numerically is easy:

(a) The threshold level of debt obligations $\hat{\delta}^*$ in Lemma 3 equation (19) is

$$\hat{\delta}^* \;=\; \rho\,\frac{\Theta_0\,\Theta_1^* - \Theta_0^*\,\Theta_1}{\Theta_1^*} \;.$$

(b) The threshold level of debt obligations $\hat{\delta}$ in Lemma 4 equation (20) solves

$$\left((1-\lambda)\,\frac{\hat{\delta}}{\rho} \;-\Theta_0^*\right)\, f(\underline{x}^*) \;+\; (\Theta_0^* - \Theta_0)\; f(\underline{x}) \;=\; \left(\frac{\hat{\delta}}{\rho} \;-\Theta_0\right)^{1-\lambda} \;, \quad (29)$$

$$\text{where } \; f(x) \;\equiv\; \left(\frac{-\lambda}{(1-\lambda)\,\Theta_1\,x}\right)^{\lambda} \;. \quad (30)$$

(c) The coupon level $\overline{\delta}$ in Lemma 5 equation (21) clearly depends on the entry state, x_0. It is found solving

$$(1-\lambda)\,\frac{\overline{\delta}}{\rho} \;-\Theta_0 \;=\; \left(\frac{\overline{\delta}}{\rho} \;-\Theta_0\right)^{1+\lambda} \; f(x_0) \;.$$

Notice that requiring $\Theta_0^* < 0$ and $0 < \Theta_1^*$ suffices for Assumption 1 to be satisfied. Similarly, requiring $\Theta_0 < \Theta_0^*$ and $\Theta_1^* < \Theta_1$ suffices for Assumption 3 to be satisfied.

B. Price Impact of Allowing for Corporation Sales

Arguably, the most commonly used measures of the impact of a given risk on debt value are the risk premium investors require to compensate them for being exposed, and the associated credit spread. In this paper, the bonds' walkout-risk premium is given by

$$p(x_t) \quad \equiv \quad \delta \,-\, \rho\, D(x_t \mid \delta)\ , \tag{31}$$

and the credit spread is given by

$$s(x_t) \quad \equiv \quad \frac{\delta}{D(x_t \mid \delta)} \,-\, \rho\ . \tag{32}$$

As mentioned in the introduction, all existing defaultable bond pricing models associate the shareholders' walkout decision exclusively with default. The consequence of this assumption for prices is easily derived here: Technically, it just involves imposing a restriction on the more general optimization we carried out. The value of the debt if we ignored the shareholders' option to sell ownership rights was actually derived in Section II.C. when we considered asset values *conditional* on a form of walkout. Given our set-up, the value of the debt such a restriction would lead to is $D_{def}(x_t \mid \underline{x}_{def}, \delta) \equiv V(x_t \mid \underline{x}_{def}) \,-\, S_{def}(x_t \mid \underline{x}_{def}, \delta)$. Therefore, imposing shareholders' walkout to necessarily consist of default generates a bond's walkout-risk premium (default) equal to

$$p_{def}(x_t) \;\equiv\; \delta \,-\, \rho\, D_{def}(x_t \mid \underline{x}_{def}, \delta)\ ,$$

and an associated credit spread

$$s_{def}(x_t) \;\equiv\; \frac{\delta}{D_{def}(x_t \mid \underline{x}_{def}, \delta)} \,-\, \rho\ .$$

Good measures of the impact of this omission on debt pricing consist of taking the relative differences between risk premia and credit spreads obtained allowing for corporation sales or not:

$$R|p_{def} \;\equiv\; \frac{p(x_t) \,-\, p_{def}(x_t)}{p_{def}(x_t)} \quad \text{and} \quad R|s_{def}(x_t) \;\equiv\; \frac{s(x_t) \,-\, s_{def}(x_t)}{s_{def}(x_t)}\ .$$

The first measure, $R|p_{def}$, is interestingly independent of the current state x_t. It is therefore valid at any time of the firms' life. It is also particularly convenient in that replacing the above expressions of $p(x_t)$ and $p_{def}(x_t)$ yields the following simple formula,

$$R|p_{def} \;=\; \frac{[\,U^*(\underline{x}_{\mathcal{S}}) - S^*(\underline{x}_{\mathcal{S}}) - \delta/\rho\,]\ \mathcal{P}(\underline{x}_{def} \rhd \underline{x}_{\mathcal{S}})}{U^*(\underline{x}_{def}) - \delta/\rho} \,-\, 1\ .$$

Clearly, $R|p_{def} \neq 0$ (and $R|s_{def}(x_t) \neq 0$) whenever in our general model, shareholders' ex-post optimal walkout form consists of a corporation sale: In this case, relative to a default-only model, allowing for this alternative has an impact on prices. Conversely, $R|p_{def} = 0$ (and $R|s_{def}(x_t) = 0$) whenever shareholders' ex-post optimal walkout form consists of default, as allowing for the corporation sale alternative has no impact on prices relative to a default-only model.

The occurrence of one or the other situation is probably best understood looking at Figure 8: $R|p_{def} \neq 0$ (and $R|s_{def}(x_t) \neq 0$) for situations corresponding to a point like C, whereas $R|p_{def} = 0$ (and $R|s_{def}(x_t) = 0$) for situations corresponding to points like A, B and D. We will illustrate situations precisely like A, B, C and D in the numerical results Section IV.C.

When an ex-ante optimal usage of assets involves a continuation-abandonment, i.e., $\underline{x}^* < \underline{x}$, and for lower levels of borrowings, i.e. throughout the interval $\delta \in (0; \hat{\delta})$, existing defaultable bond pricing models, allowing for walkouts to occur only through default, will underestimate the value of the debt for a given coupon, i.e., exaggerate the risk premium demanded by creditors. They will consequently also underestimate the borrowing ability (debt capacity) of the firm.

Overall, the importance of allowing for corporation sale decisions is crucially related to the critical upper boundary, $\hat{\delta}$, at which shareholders are indifferent between the two walkout forms. The conventional restriction (walkout=default) is likely to perform at its worst when applied to industries where firms possess quite similar technological abilities and opportunities for corporation sales abound (high $\hat{\delta}$). Pricing errors should also be significant in sectors where the need for credit finance is modest, e.g., where access to internal funding is ample and the required initial outlays are small in relation to future revenues.

C. Numerical Results

We are now ready to *quantitatively* get a feel for the importance of the debt walkout premium. This is to assess whether allowing for a non-default option is at all important in corporate debt pricing, and show how the answer is related to the conditions under which the question is posed. To do this we carry out two simple numerical applications, under the "GBM-Linear" structure which yields closed-form pricing formulas.

Example 1: *The firm's gross income under the innovative firm's operations,* $\Pi(x_t)$, *fluctuates with* $\mu = 0$ *and* $\sigma = 15\,\%$. *Competitors' gross income would be half of this (normalizing the current productivity to* $\Theta_1 = 1$, *implies* $\Theta_1^* = 1/2$*), but their fixed cost of production would be reduced by a quarter (normalizing costs to* $\Theta_0 = -1$, *then* $\Theta_0^* = -3/4$*). The interest rate is* $\rho = 5\,\%$.

In this example, $\Theta_1 - \Theta_1^* = 1/2$, hence the owners' technological supremacy in the industry is fairly large. Competitors' reduced fixed cost nevertheless ensures a situation where an eventual transfer of ownership is ex-ante optimal (Assumptions 1 and 3). The reduction in fixed cost is not too substantial, as $\Theta_0 - \Theta_0^* = -1/4$, but in all, abandonment has an associated cost of more or less $40\,\%$, which is probably much.

Overall, $\Theta_0\,\Theta_1^* - \Theta_0^*\,\Theta_1 = 1/4$ which is positive. Therefore $\underline{x}^* > \underline{x}$, and a "termination-abandonment" is ex-ante optimal. This situation corresponds to points like A or B in Figure 8. By Lemma 3, shareholders' ex-post optimal walkout choice consists of reneging ownership rights (default). Here, not accounting for the corporation sale alternative does not modify the calculated debt risk premium.

Example 2: *The firm's gross income under the innovative firm's operations,* $\Pi(x_t)$, *fluctuates with* $\mu = 0$ *and* $\sigma = 15\,\%$. *Competitors' gross income would be a quarter less than this* *(normalizing the current productivity to $\Theta_1 = 1$, implies $\Theta_1^* = 3/4$), but their fixed cost of production would be half (normalizing costs to $\Theta_0 = -1$, then $\Theta_0^* = -1/2$). The interest rate is $\rho = 5\,\%$.*

In this example, $\Theta_1 - \Theta_1^* = 1/4$, therefore the owners' technological supremacy in the industry is not as large as in the previous example, and industry participants have access to more similar technologies. Competitors' reduced fixed cost nevertheless ensures a situation where transfer of ownership is eventually ex-ante optimal. Competitors have a more reduced fixed cost, as $\Theta_0 - \Theta_0^* = -1/2$, but in all the figures considered are fairly conservative, as abandonment has an associated cost of more or less 20 %.

Overall, $\Theta_0\,\Theta_1^* - \Theta_0^*\,\Theta_1 = -1/4$ which is negative, so $\underline{x}^* < \underline{x}$, hence a "continuation-abandonment" is ex-ante optimal. By Lemma 4, shareholders' ex-post optimal walkout choice can either consist of (i) selling (corporation sale) or (ii) reneging ownership rights (default), depending on leverage:

(i) Let us first consider a coupon level $\delta = 0.025$ which is lower than $\hat{\delta}$, the threshold coupon level which determines shareholders' ex-post behavior. We therefore consider a situation where shareholders' ex-post optimal walkout choice consists of selling ownership rights (corporation sale), as $\delta \in (\,0\,;\,\hat{\delta}\,])$. In this case, the expressions for $p(x_t)$ and $p_{def}(x_t)$ become

$$p(x_t) \;=\; \rho\,\left[\Theta_0^* + \Theta_1^*\,\underline{x}^* \;-\; \left(\Theta_0^* + \Theta_1^*\,\underline{x}_{def} \;-\; \frac{\delta}{\rho}\right)\,\left(\frac{\underline{x}^*}{\underline{x}_{def}}\right)^{\lambda}\right]\,\left(\frac{x_t}{\underline{x}^*}\right)^{\lambda}, \tag{33}$$

$$p_{def}(x_t) \;=\; \delta\,\left(\frac{x_t}{\underline{x}^*}\right)^{\lambda},\quad \underline{x}^* \;=\; \frac{-\lambda}{1-\lambda}\left(\frac{-\Theta_0^*}{\Theta_1^*}\right),\quad \underline{x}_{def} \;=\; \frac{-\lambda}{1-\lambda}\left(\frac{\delta/\rho\;-\Theta_0}{\Theta_1}\right). \tag{34}$$

Table 1 and Figure 9 exhibit the results obtained with these input parameters. They correspond to a situation such as that depicted in Figures 3, 5, and a point like C in Figure 8. Notice in particular how accounting for the corporation sale alternative modifies the debt risk premium by $R|p_{def} = -47.88\ \%$, irrespective of the state. The risk premium is half of what a "default-only" model would predict, even when the firm is not at all in financial distress! Here, the direct transfer of ownership alternative influences calculated asset values and credit spreads very substantially.

(ii) Let us now consider a coupon level $\delta = 0.10$ which is greater than $\hat{\delta}$. We therefore consider a situation where shareholders' ex-post optimal walkout choice consists of reneging ownership rights (default), as $\delta \in (\,\hat{\delta}\,;\,+\infty\,)$. This corresponds to a point like D in Figure 8. In this case, the expressions of $p(x_t)$ and $p_{def}(x_t)$ are equal. Here, then, not accounting for the corporation sale alternative does not affect calculated asset values.

V. Conclusion

This paper studied the relation between a firm's technological characteristics, its capital structure and the form of walkout shareholders will eventually select. We derived simple closed-form solutions for the value of shares and bonds as well as the debt capacity of the firm. We conclude highlighting the main testable implications of our results.

1. Shareholders of firms with low levels of leverage tend to transfer ownership rights directly, by means of corporation sales. Conversely, shareholders of highly levered firms tend to transfer ownership rights indirectly, by default. This latter case implies an ownership transfer following a bankruptcy procedure. What precisely constitutes "low" and "high" depends on industry and firm-specific factors, which determine a threshold level of coupon obligations $\hat{\delta}$:

   (a) Where opportunities for alternative usages of the physical assets are scarce, $\hat{\delta}$ takes on a low value. Hence, even quite modest debt levels will result in walkouts occurring through default.

   (b) Conversely, in industries where technological ability is more evenly distributed across firms, $\hat{\delta}$ takes on a high value. Hence, even quite highly indebted firms may abandon operations through corporation sales.

2. (a) Corporation sales provide an efficient means of ownership transfer between shareholders: Shareholders choosing this form of walkout form will surrender control over the firm's assets to competitors at the correct time, thereby eliminating agency costs of debt.

   (b) Default, on the other hand, necessarily involves an inefficient timing of ownership transfer: Shareholders choosing this walkout form will decide to relinquish control when the firm's technology is still superior to competitors' technologies, purely for financial reasons.

3. The walkout option also has implications for the pricing of the firm's debt and the extent to which it can expect to finance its operations with this debt. Existing defaultable bond pricing models associate the shareholders' walkout decision exclusively with default. Consequently, (a) for industries where participants possess quite similar technologies and (b) in the lower end of the leverage spectrum, they (i) exaggerate the risk premia demanded from firms and (ii) underestimate their borrowing ability.

   Simple numerical simulations suggest that even for fairly conservative input parameter values, the risk premium may be exaggerated by a factor of 2. Importantly however, this pricing error vanishes as leverage exceeds the critical threshold, $\hat{\delta}$, at which shareholders switch to default as their ex-post optimal choice of walkout form.

Although we have been able to study here the relation between walkout decisions and capital structure in terms of a small number of firm- and industry-related characteristics, we believe it would be worthwhile (but certainly difficult) to extend our results in future research by (i) altering the relative bargaining power between shareholders and creditors; (ii) altering the relative bargaining power between the acquiring and acquired firms; (iii) allowing both shareholders and competitors to face wealth constraints; and (iv) introducing conflicts of interest between managers and owners.

Appendix

Proof of Lemma 1: The equity residual value function is

$$S^*(y) \;=\; \max\left\{\; U^*(y) \;-\; [\,U^*(y) \wedge P\,]\;;\; U^*(y) \;-\; \frac{\delta}{\rho}\,[\,1 \;-\; \mathcal{P}(y \rhd \underline{x}^*)\,]\;\right\}\;. \tag{35}$$

If shareholders default at y, their residual claim value is $U^*(y) \;-\; [\,U^*(y) \wedge P\,] = 0 \vee [\,U^*(y) - P\,]$. However, the par value P is greater or equal to δ/ρ, otherwise debtholders' collateral is not credible in the first place. Consequently,

$$U^*(y) \;-\; P \;\leq\; U^*(y) \;-\; \frac{\delta}{\rho} \;<\; U^*(y) \;-\; \frac{\delta}{\rho}\,[\,1 \;-\; \mathcal{P}(y \rhd \underline{x}^*)\,]\;. \tag{36}$$

For shareholders to actually default, it must be the case that $0 \vee [\,U^*(y) - P\,] = 0$. □

Proof of Lemma 2: If shareholders' walkout takes the form of default, triggered at y, their residual claim value is $S^*(y) \;=\; 0$ (by Lemma 1). Denote $H(y)$ the derivative of the value of the shareholders' option to default at y, with respect to this trigger level,

$$H(y) \;\equiv\; \frac{\partial}{\partial y}\,[\,(\,-\,\Pi(y) \;+\; \delta/\rho\,)\;\; \mathcal{P}(x_0 \rhd y)\,]\;. \tag{37}$$

Denote $h(y)$ the derivative of $H(y)$ with respect to δ,

$$h(y) \;\equiv\; \frac{\partial}{\partial \delta}\left[\,\frac{\partial}{\partial y}\,[\,(\,-\,\Pi(y) \;+\; \delta/\rho\,)\;\; \mathcal{P}(x_0 \rhd y)\,]\,\right] \;=\; \frac{1}{\rho}\,\frac{\partial}{\partial y}\,[\mathcal{P}(x_0 \rhd y)] \;\;>\;\; 0\;. \tag{38}$$

If $\underline{x}_{\mathcal{S}}$ is the shareholders' optimal abandonment trigger level, and abandonment occurs through default, then $H(\underline{x}_{\mathcal{S}}) = 0$. But because $h(\underline{x}_{\mathcal{S}}) > 0$, the root of $H(y) = 0$ increases with δ. In other words, $\partial \underline{x}_{\mathcal{S}}/\partial\delta \,>\, 0$.

Conversely, if shareholders' walkout takes the form of a corporation sale, triggered at y, their residual claim value is

$$S^*(y) \;=\; U^*(y) \;-\; \frac{\delta}{\rho}\,[\,1 \;-\; \mathcal{P}(y \rhd \underline{x}^*)\,]\;. \tag{39}$$

Denote $G(y)$ the derivative of the value of the shareholders' option to sell the corporation at y, with respect to this trigger level,

$$G(y) \;\equiv\; \frac{\partial}{\partial y}\,[\,(\,U^*(y) \;-\; \Pi(y)\,)\;\; \mathcal{P}(x_0 \rhd y) \;+\; \delta/\rho\;\mathcal{P}(x_0 \rhd \underline{x}^*)\,] \tag{40}$$

$$\;=\; \frac{\partial}{\partial y}\,[\,(\,U^*(y) \;-\; \Pi(y)\,)\;\; \mathcal{P}(x_0 \rhd y)]\;. \tag{41}$$

Denote $g(y)$ the derivative of $G(y)$ with respect to δ,

$$g(y) \;\equiv\; \frac{\partial}{\partial \delta}\left[\,\frac{\partial}{\partial y}\,[\,(\,U^*(y) \;-\; \Pi(y)\,)\;\; \mathcal{P}(x_0 \rhd y)\,]\,\right] \;=\; 0\;. \tag{42}$$

If $\underline{x}_{\mathcal{S}}$ is the shareholders' optimal trigger level and the walkout occurs through a corporate sale, then $G(\underline{x}_{\mathcal{S}}) = 0$. But because $g(\underline{x}_{\mathcal{S}}) = 0$, the root of $G(y) = 0$ is independent of δ, hence $\partial \underline{x}_{\mathcal{S}}/\partial\delta \;=\; 0$. □

Proof of Lemma 3: Assume that shareholders walk out through a corporation sale. Then, $\underline{x}_{\mathcal{S}} = \underline{x}$ (Corollary 1). When $\underline{x} < \underline{x}^*$, the reservation value of the firm at the time of the walkout $U^*(\underline{x}_{\mathcal{S}}) = U^*(\underline{x}) = 0$. But then $S^*(\underline{x}_{\mathcal{S}}) = 0$, contradicting the assumption that shareholders walk out through a corporation sale.

Express the respective first-order optimality conditions which determine $\underline{x}^*$ and $\underline{x}_{\mathcal{S}}$:

$$\frac{d\Pi^*(y)}{\partial y}\,\mathcal{P}(x_t \rhd y) \;+\; \Pi^*(y)\,\frac{\partial \mathcal{P}(x_t \rhd y)}{\partial y} \;=\; 0\,, \tag{43}$$

and

$$-\;\frac{d\Pi(y)}{\partial y}\,\mathcal{P}(x_t \rhd y) \;+\; \left[-\Pi(y) + \frac{\delta}{\rho}\right]\frac{\partial \mathcal{P}(x_t \rhd y)}{\partial y} \;=\; 0\,. \tag{44}$$

If $\underline{x}^*$ equals $\underline{x}_{\mathcal{S}}$, both conditions have the same solution. Replacing the first one in the second yields

$$\Pi^*(y)\left[-\frac{d\Pi(y)}{\partial y}\right]\mathcal{P}(x_t \rhd y) \;-\; \frac{d\Pi^*(y)}{\partial y}\left[-\Pi(y) + \frac{\delta}{\rho}\right]\mathcal{P}(x_t \rhd y) \;=\; 0\,. \tag{45}$$

After simplification, we see that for $\underline{x}^*$ to equal $\underline{x}_{\mathcal{S}}$, the coupon obligation, $\hat{\delta}^*$, must be such that

$$\hat{\delta}^* \;=\; \rho\,\Pi(\underline{x}^*) \;-\; \rho\,\Pi^*(\underline{x}^*)\,\frac{d\Pi(\underline{x}^*)}{d\underline{x}^*}\left[\frac{d\Pi^*(\underline{x}^*)}{d\underline{x}^*}\right]^{-1}\,. \tag{46}$$

For coupons $\delta \in (0\,;\,\hat{\delta}^*\,]$, default occurs at $\underline{x}_{\mathcal{S}}$ which is lower than $\underline{x}^*$, hence the usage of the assets will be terminated. □

Proof of Lemma 4: Corporation sales occur for δ such that $S_{corp}(x \mid \underline{x}_{corp}, \delta) > S_{def}(x \mid \underline{x}_{def}, \delta)$. Given that $\underline{x} < \underline{x}_{def}$ (lemma 2), expanding the inequality $S_{corp}(x \mid \underline{x}_{corp}, \delta) > S_{def}(x \mid \underline{x}_{def}, \delta)$ for $x = \underline{x}_{def}$ gives that corporation sales occur for $\delta \in (0\,;\,\hat{\delta}\,]$, where $\hat{\delta}$ solves the expression given in the lemma. □

Proof of Lemma 5: If $S^*(\underline{x}_{\mathcal{S}}) \;=\; 0$, then

$$\frac{\partial D(x_0 \mid \delta)}{\partial \delta} \;=\; \frac{1}{\rho}\,[\,1 \,-\, \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})\,] \;-\; \frac{\delta}{\rho}\,\frac{\partial \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})}{\partial \underline{x}_{\mathcal{S}}}\,\frac{\partial \underline{x}_{\mathcal{S}}}{\partial \delta}\,. \tag{47}$$

The coupon, $\bar{\delta}$, solves $\partial D(x_0 \mid \delta)/\partial\delta = 0$ and therefore

$$\overline{\delta} \;=\; [\,1 \,-\, \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})\,]\left[\frac{\partial \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})}{\partial \underline{x}_{\mathcal{S}}}\,\frac{\partial \underline{x}_{\mathcal{S}}}{\partial \delta}\right]^{-1}\,.\;\Box \tag{48}$$

Proof of Lemma 6: If $S^*(\underline{x}_{\mathcal{S}}) \;= U^*(\underline{x}_{\mathcal{S}}) \,-\, \delta/\rho\,[\,1 \,-\, \mathcal{P}(\underline{x}_{\mathcal{S}} \rhd \underline{x}^*)\,]$, then

$$\frac{\partial D(x_0 \mid \delta)}{\partial \delta} \;=\; \frac{1}{\rho}\,[\,1 \,-\, \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})\,] \;>\; 0\,.\;\Box \tag{49}$$

References

Aghion, Philippe, and Patrick Bolton, 1992, An Incomplete Contracts Approach to Financial Contracting, *Review of Economic Studies*, 59, 473-494.

Anderson, Ronald W., and Suresh Sundaresan, 1996, Design and Valuation of Debt Contracts, *Review of Financial Studies*, 9, 37-68.

Black, Fischer, and John C. Cox, 1976, Valuing Corporate Securities: Some Effects of Bond Indenture Provisions, *Journal of Finance*, 31, 351-367.

Bulow, Jeremy I. and John B. Shoven, 1978, The Bankruptcy Decision, *Bell Journal of Economics and Management Science*, 9, 437-456.

Decamps, Jean-Paul and Antoine Faure-Grimaud, 1997, The Asset Substitution Effect: Valuation and Reduction through Debt Design, Working Paper, GREMAQ, University of Toulouse.

Ericsson, Jan, 1997, Asset Substitution, Debt Pricing, Optimal Leverage and Maturity, Working Paper, Stockholm School of Economics.

Fries, Steven M., Marcus Miller, and William R.M. Perraudin, 1997, Debt Pricing in Industry Equilibrium, *Review of Financial Studies*, 10, 39-68.

Grossman, Sanford J. and Oliver D. Hart, 1986, The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration, *Journal of Political Economy*, 94, 691-719.

Harrison, Michael J., and David M. Kreps, 1979, Martingales and Arbitrage in Multiperiod Securities Markets, *Journal of Economic Theory*, 20, 381-408.

Hart, Oliver, 1993, Theories of Optimal Capital Structure: The Managerial Discretion Perspective, in Margaret Blair (ed.) *The Deal Decade: What Takeovers and Leveraged Buyouts Mean for Corporate Governance*, Washington, D.C: The Brookings Institution, 19-53.

Hart, Oliver and John Moore, 1989, Default and Renegotiation: A Dynamic Model of Debt, Manuscript, Cambridge: Massachusetts Institute of Technology; London: London School of Economics.

Hart, Oliver and John Moore, 1990, Property Rights and the Nature of the Firm, *Journal of Political Economy*, 98, 1119-1158.

Hart, Oliver and John Moore, 1994, A Theory of Debt Based on the Inalienability of Human Capital, *Quarterly Journal of Economics*, 109, 841-879.

Higgins, Robert C. and Lawrence D. Schall, 1975, Corporate Bankruptcy and Conglomerate Merger, *Journal of Finance*, 30, 93-113.

Jensen, Michael C. and William H. Meckling, 1976, Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure, *Journal of Financial Economics*, 3, 305-360.

Karlin, Samuel, and Howard M. Taylor, 1975, A First Course in Stochastic Processes, Second Edition, New York: Academic Press.

Kim, In Joon, Krishna Ramaswamy, and Suresh Sundaresan, 1993, Does Default Risk in Coupons Affect the Valuation of Corporate Bonds?: A Contingent Claims Model, *Financial*

Management, Autumn, 117-131.

Leland, Hayne E., 1994, Risky Debt, Bond Covenants and Optimal Capital Structure, *Journal of Finance*, 49, 1213-1252.

Leland, Hayne E., 1998, Agency Costs, Risk Management, and Capital Structure, *Journal of Finance*, 53, 1213-1243.

Leland, Hayne E., and Klaus B. Toft, 1996, Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, *Journal of Finance*, 51, 987-1019.

795-802. Litterman, R., and T. Iben, 1991, Corporate Bond Valuation and the Term Structure of Credit Spreads, *Journal of Portfolio Management*, 17, 52-64.

Mella-Barral, Pierre, 1999, The Dynamics of Default and Debt Reorganization, *Review of Financial Studies*, 12, 535-578.

Mella-Barral, Pierre, and William R.M. Perraudin, 1997, Strategic Debt Service, *Journal of Finance*, 52, 531-556.

Merton, Robert C., 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance*, 29, 449-470.

Miller, Merton H., 1977, Debt and Taxes, *Journal of Finance*, 32, 261-275.

Myers, Stuart C. 1977, Determinants of Corporate Borrowing, *Journal of Financial Economics*, 5, 147-175.

Pastena, Victor and William Ruland, 1986, The Merger/Bankruptcy Alternative, *The Accounting Review*, 61, 288-301.

Rajan, R., and L. Zingales, 1998, Power in the Theory of the Firm, CEPR Discussion Paper 1777.

Stiglitz, Joseph E., 1972, Some Aspects of the Pure Theory of Corporate Finance: Bankruptcies and Takeovers, *Bell Journal of Economics and Management Science*, 3, 458-482.

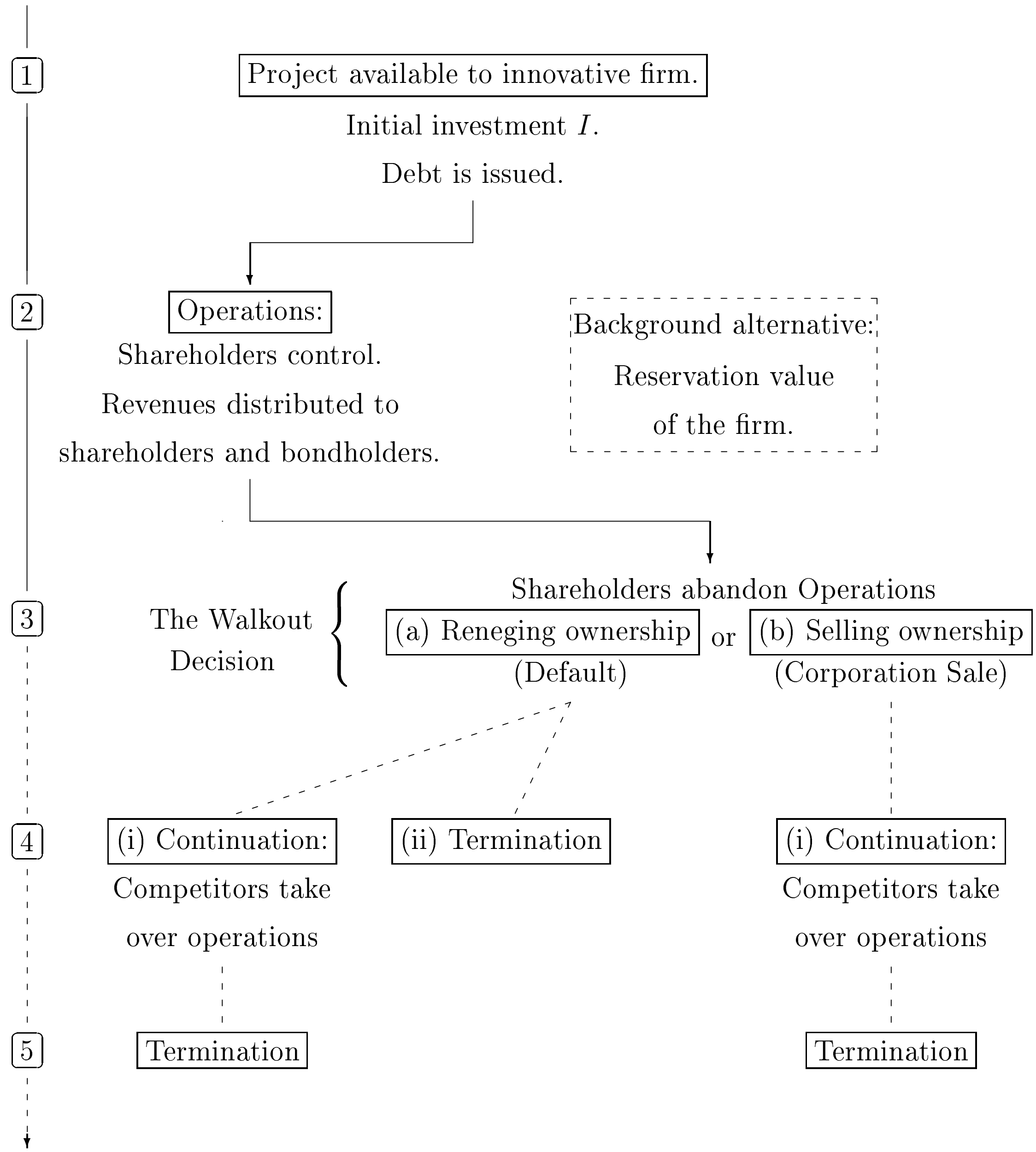


Figure 1: Random time line of the Model

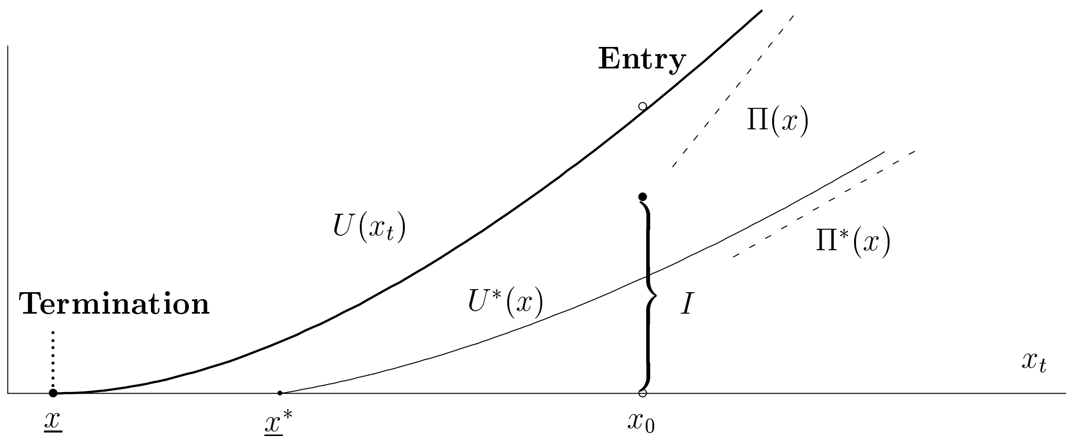


Figure 2: First-best value of the firm consisting of a "termination-abandonment" ($\underline{x} \leq \underline{x}^*$)

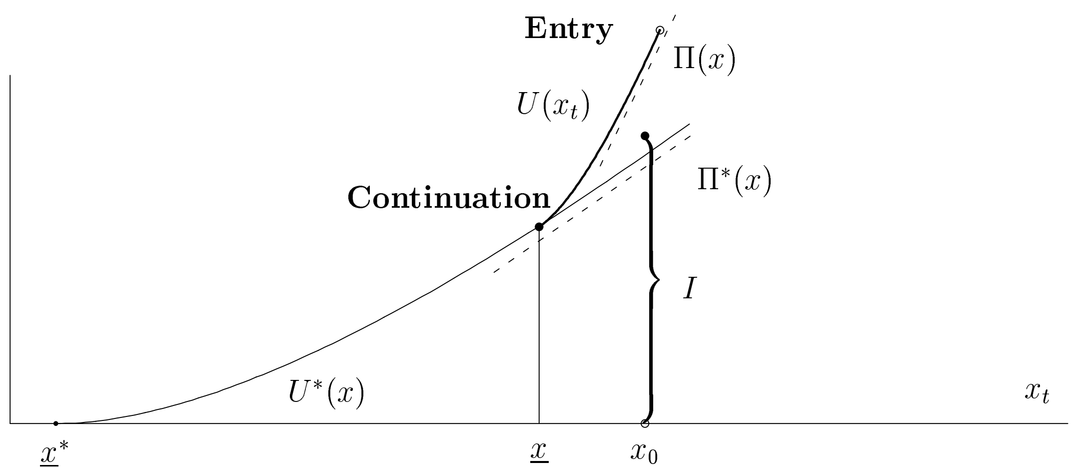
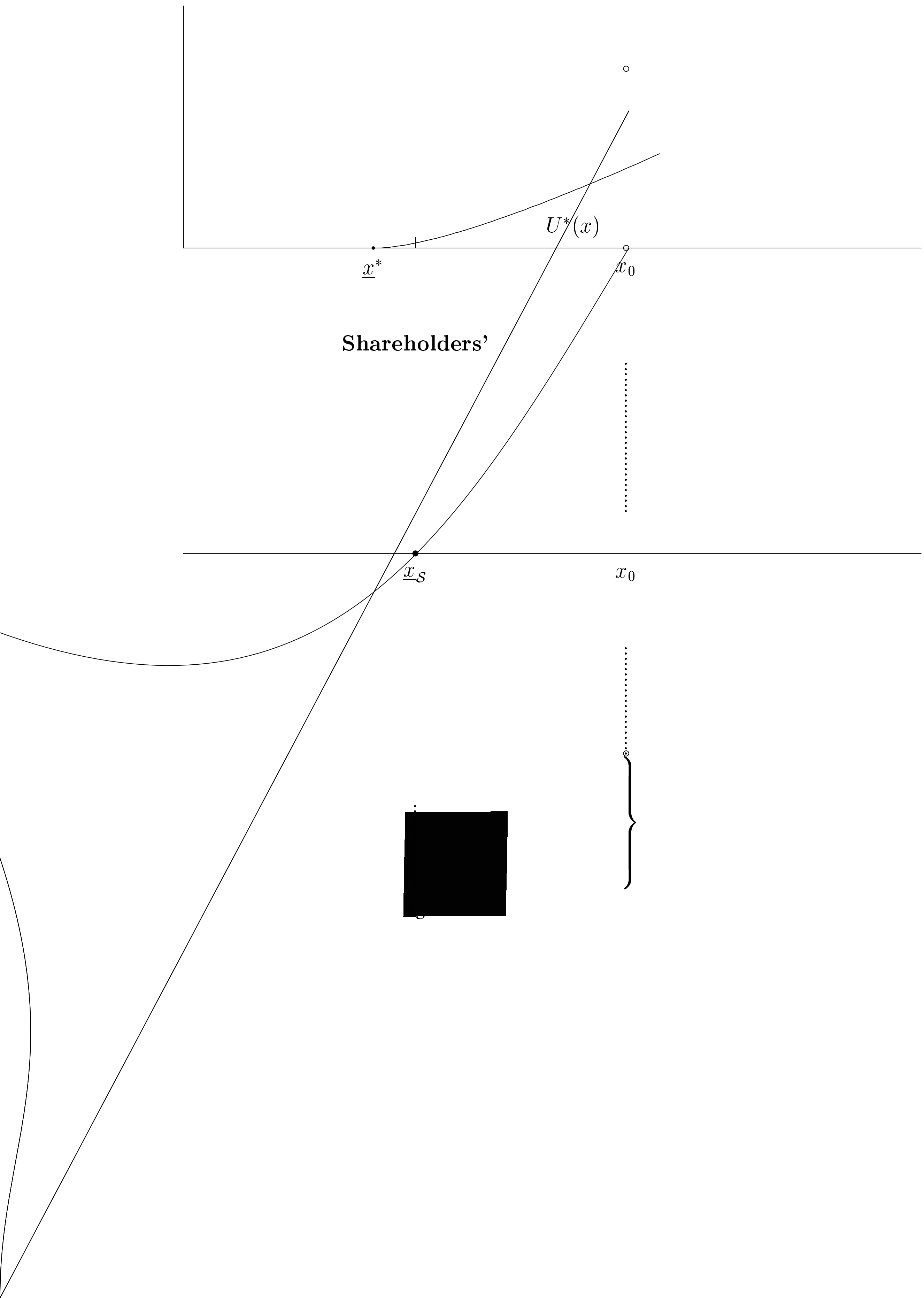


Figure 3: First-best value of the firm consisting of a "continuation-abandonment" ($\underline{x}^* \leq \underline{x}$)



$U^*(x)$
$\underline{x}^*$
x_0
Shareholders'
$\underline{x}_S$
x_0
[redacted]

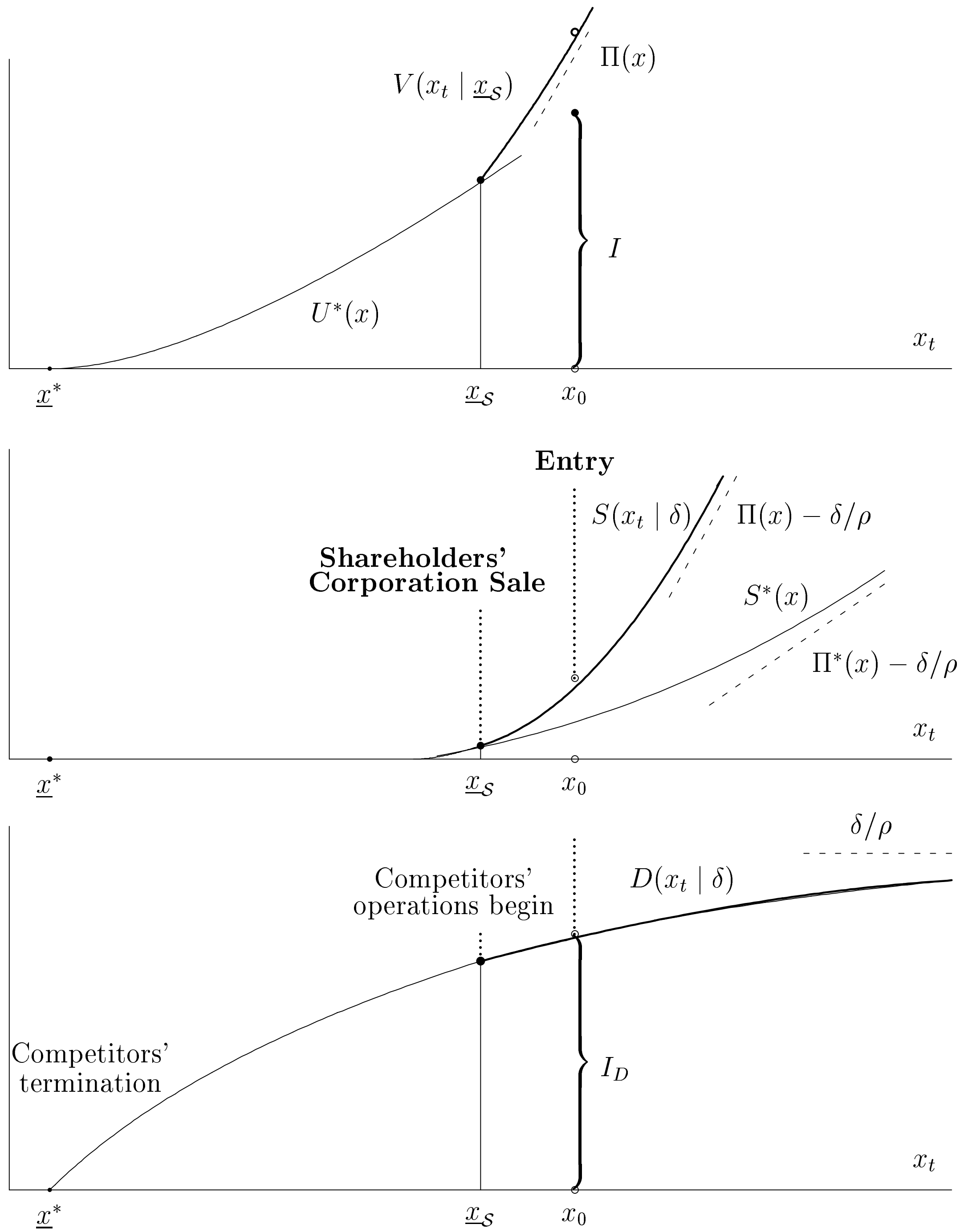
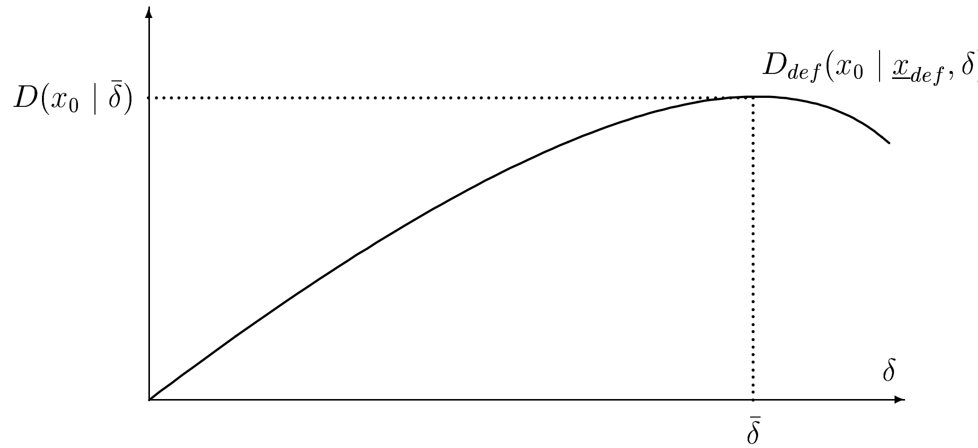


Figure 5: Second-best Firm, Share and Debt values, when (i) the first-best usage of the firm's assets involves a "continuation-abandonment" ($\underline{x}^* \leq \underline{x}$), and (ii) for lower levels of borrowings ($\delta < \hat{\delta}$)



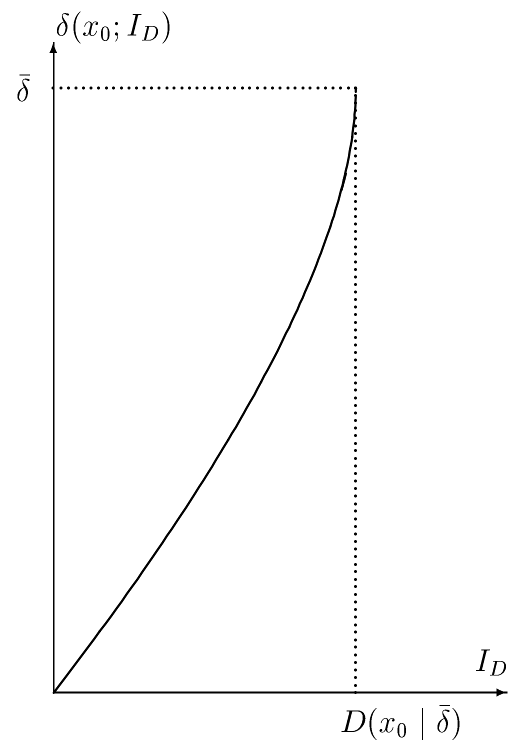


Figure 6: Project Financing when the first-best usage of the firm's assets involves a "termination-abandonment" ($\underline{x} \leq \underline{x}^*$).

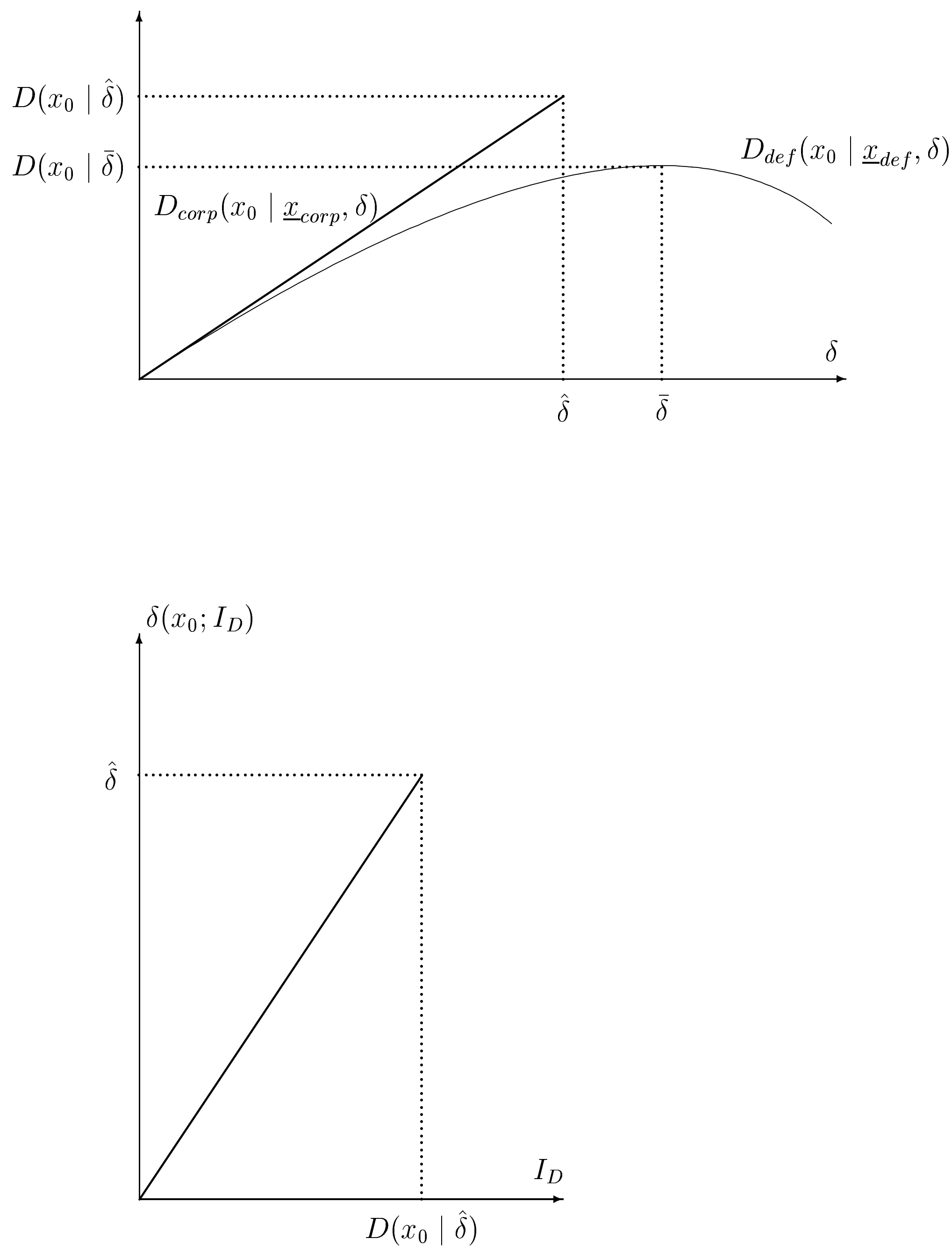
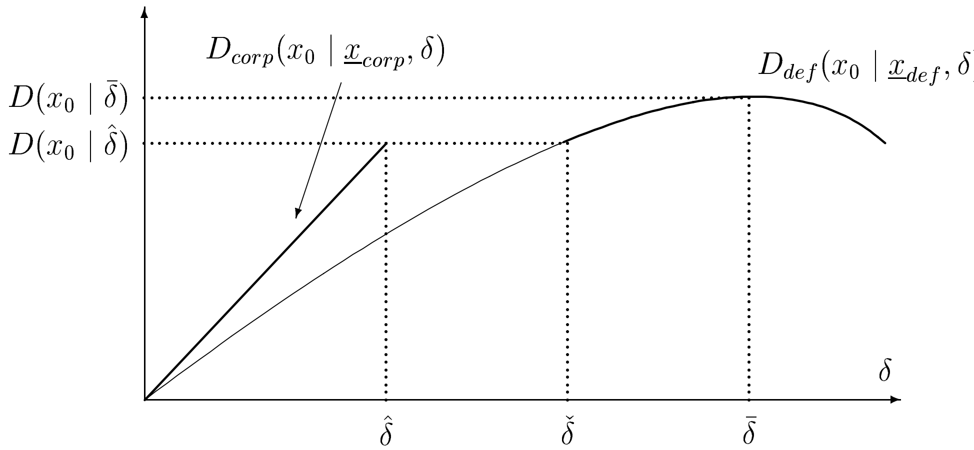


Figure 7: (i): Project Financing when (a) the first-best usage of the firm's assets involves a "continuation-abandonment" ($\underline{x}^* \leq \underline{x}$), and (b) $D(x_0 \mid \bar{\delta}) < D(x_0 \mid \hat{\delta})$



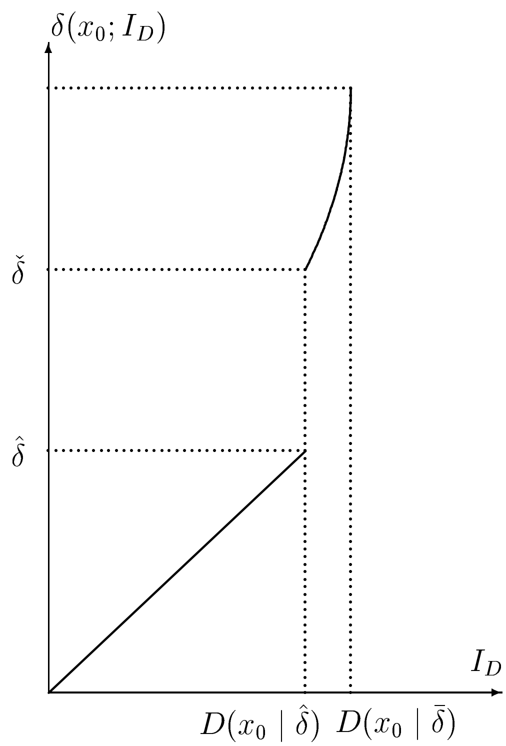


Figure 7: (ii): Project Financing when (a) the first-best usage of the firm's assets involves a "continuation-abandonment" ($\underline{x}^* \leq \underline{x}$), and (b) $D(x_0 \mid \hat{\delta}) < D(x_0 \mid \bar{\delta})$

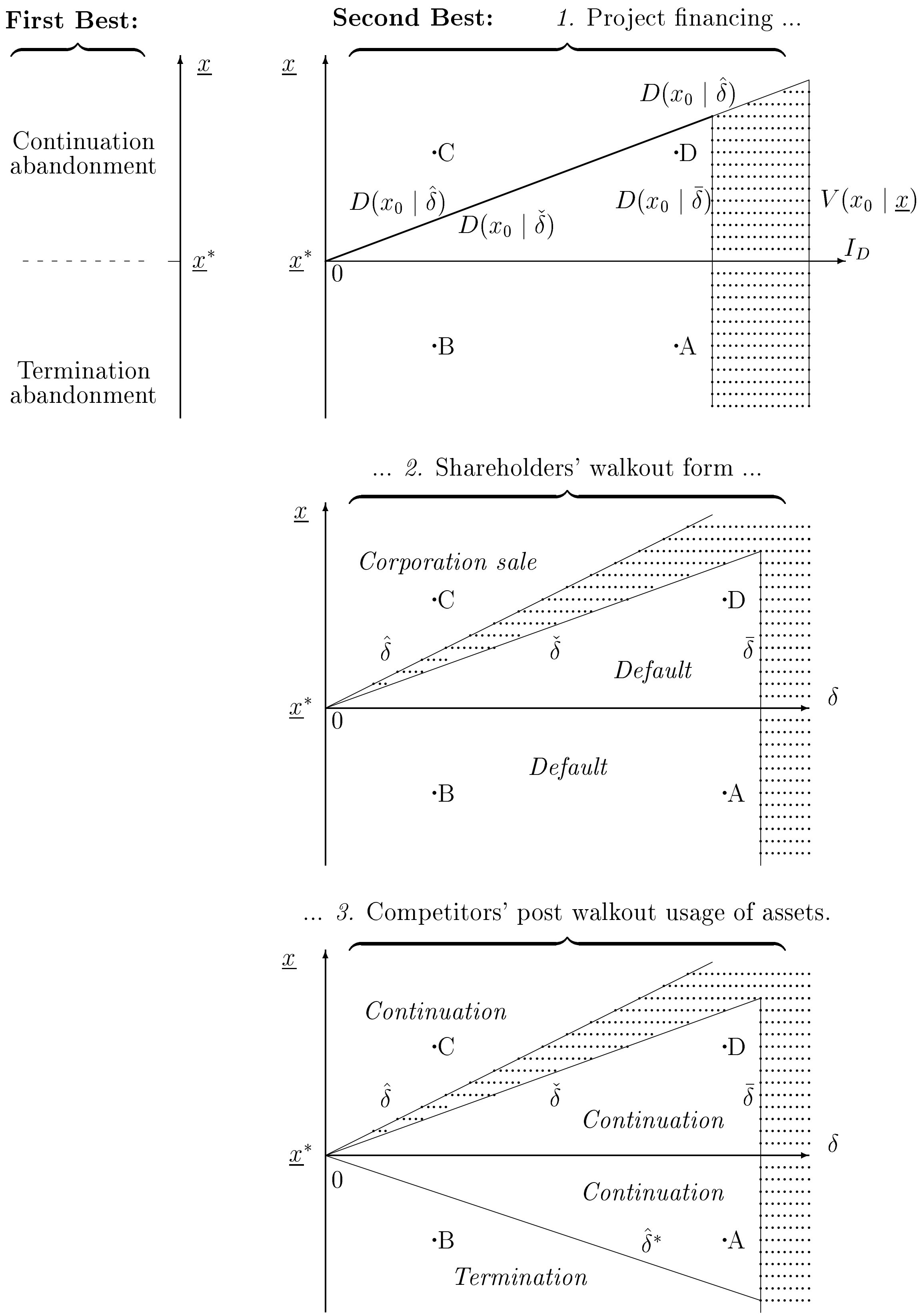


Figure 8: First-best versus second-best policies, function of (i) firm characteristics and (ii) debt service obligations

Table 1: Price Impact of Allowing for Corporation Sales in Example 2(i)

Example 2(i) is such that (i) the first-best usage of the firm's assets involves a "continuation-abandonment" ($\underline{x}^* \leq \underline{x}$), and (ii) for lower levels of borrowings ($\delta < \hat{\delta}$). Input parameters are $\mu = 0$, $\sigma = 0.15$, $\theta_1 = 1$, $\theta_1^* = 3/4$, $\theta_0 = -1$, $\theta_0^* = -0.5$, $r = 0.05$ (Example 2), and $\delta = 0.025$ (case (i)).

Decision trigger level:		Value
Optimal competitor termination	$\underline{x}^*$	0.4167
Optimal ownership transfer	$\underline{x}$	1.2500
Optimal shareholders' walkout	$\underline{x}_{\mathcal{S}}$	1.2500
Optimal shareholders' default	$\underline{x}_{def}$	0.9375
Debt Value when shareholders walkout:		**Value**
Default or Corporation sale	$D(\underline{x}_{\mathcal{S}} \mid \delta)$	0.4675
Default-only	$D_{def}(\underline{x}_{def} \mid \underline{x}_{def}, \delta)$	0.2517
Debt Value at $x_t = 2$:		**Value**
Default or Corporation sale	$D(x_t \mid \delta)$	0.4630
Default-only	$D_{def}(x_t \mid \underline{x}_{def}, \delta)$	0.4292
Credit Spreads at $x_t = 2$:		**Value (bps)**
Default or Corporation sale	$s(x_t)$	39.86
Default-only	$s_{def}(x_t)$	82.53
Relative difference	$R\|s_{def}(x_t)$	-51.70 %
Risk Premium at $x_t = 2$:		**Value/δ**
Default or Corporation sale	$p(x_t)$	7.38 %
Default-only	$p_{def}(x_t)$	14.17 %
Relative difference	$R\|p_{def}$	-47.88 %

Figure 9 : Price Impact of Allowing for Corporation Sales in Example 2(i)

Example 2(i) is such that (i) the first-best usage of the firm's assets involves a "continuation-abandonment" ($\underline{x}^* \leq \underline{x}$), and (ii) for lower levels of borrowings ($\delta < \hat{\delta}$). Input parameters are $\mu = 0$, $\sigma = 0.15$, $\theta_1 = 1$, $\theta_1^* = 3/4$, $\theta_0 = -1$, $\theta_0^* = -0.5$, $r = 0.05$ (Example 2), and $\delta = 0.025$ (case (i)). Figure 9(a) compares the debt values we obtain when shareholders are allowed to choose the form of their walkout, $D(x_t \mid \delta)$, with the values that would be obtained if this optimization was not allowed for and shareholders were assumed to walkout only through default, $D_{def}(x_t \mid \delta)$. The value of the debt contract if it was riskless, δ/ρ and the residual value of the firm, $U^*(x_t)$ are also exhibited. Figure 9(b) compares the associated credit spreads with both forms of walkout and with default-only, $s(x_t)$ and $s_{def}(x_t)$, respectively. Figure 9(c) compares the risk premium with both forms of walkout and with default-only, $s(x_t)$ and $s_{def}(x_t)$, respectively. Here, risk premia are expressed in percentage of debt coupon δ. Figure 9(d) shows the relative difference between risk premia obtained allowing for corporation sales or not, $R|p_{def}$ (expressed in percentage).