Equity Finance, Adverse Selection

and Product Market Competition

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Abstract

This paper analyses the effect of asymmetric information between a firm and its outside investors on the firm's competitive position in a model where first-period competition is followed by a financing stage à la Myers and Majluf (1984). In our model, interim profit generated by the competition stage takes the role of financial slack and determines the extent to which external equity finance is required for a new investment opportunity. I consider the full set of equilibria in our version of the Myers and Majluf model and formally analyse financial slack as a comparative statics variable. Using this, I derive the firm's first period objective from first principles. In contrast to models of predatory behavior, I find that in the presence of an adverse selection problem the need to finance externally may provide a strategic benefit rather than a strategic disadvantage. The reason is that the adverse selection problem may induce speculative behavior, which will make the firm more aggressive vis à vis its rival. (JEL classification D82, G30, L13)

1 Introduction

This paper analyses the implications of asymmetric information between a firm and its outside investors on the firm's strategic position in its product market. The model abstracts from issues of precommitment and has a financing stage which occurs after the product market stage. This captures the idea that the firm continuously interacts with its product market competitor, but then at some point in time may have to take recourse to the financial market. The financing stage is a version of the Myers and Majluf (1984) model of equity finance, in which the firm's management is assumed to have superior information on the value of the firm's assets in place. This gives rise to a lemons problem, in that there may be equilibria in which only bad firms may issue and invest. In their paper, Myers and Majluf stress the importance of financial slack to mitigate the adverse selection problem. They do not formally analyse the role of financial slack, however. Also, as has been pointed out by Giammarino and Lewis (1988) as well as Cadsby, Frank, and Maksimovic (1990), they do not formally analyse the full set of equilibria of their model. This paper provides an analysis of both these issues for the particular version of the model. I then argue for an equilibrium selection such that the probability of the good firm investing is increasing in the amount of financial slack available and analyse the implications of this equilibrium play on the first-period competition. The idea is that first-period profit takes the role of financial slack and determines the amount the firm needs to raise externally. Under the assumption that first period profit is stochastic, I show that the firm will no longer maximise the expected value of first-period profit. In addition, it will care about the variance of the profit distribution and will seek to influence it by its output choice[1]. Depending on

[1] In a set-up similar to our own, Raposo (1998) analyses the implications of an adverse selection problem on the firm's optimal risk-management. She does not, however, explore the incentives effects for a firm which competes in a strategic environment, which is the focus of the analysis here.

the severity of the adverse selection problem, this may make the firm a more aggressive or a less aggressive competitor. I identify situations in which the fact that the firm has to finance externally actually confers a strategic benefit on the firm, since it will have an incentive to compete more aggressively in order to increase the probability of investment.

A large part of the literature on adverse selection and product market rivalry has focused on formalising the idea that, if a firm has to finance externally under conditions of asymmetric information, this will make it vulnerable to predatory behaviour by rivals. This issue is explored in Poitevin (1989), Bolton and Scharfstein (1990) and Phillips (1993).

Poitevin (1989) argues that uncertainty about the value of an entrant may be larger than that of the incumbent firm. To signal its quality, the entrant may have to issue debt, whereas the incumbent can finance with equity. Debt financing renders the entrant vulnerable to predation through the possibility of bankruptcy. This can be exploited by the incumbent predator, who may engage in a price war that decreases the entrants cash flow and increases his probability of bankruptcy.

In Bolton and Scharfstein (1990) the underlying agency problem is that first period profit is nonverifiable. To induce the entrant firm to truthfully reveal its profit there is a threat to terminate funding when reported profit is low. This, however, will encourage rivals to ensure that the firm's first period performance is poor. Bolton and Scharfstein model this by assuming that the rival has the option to increase the probability of low firm profit by taking an action that costs the rival some fixed amount. They derive the optimal contract, which, because of the trade-off between deterring predation and mitigating incentive problems, may or may not be designed to deter predation.

In a model set-up close to our own, Phillips (1993) analyses the case of two-period competition between two firms. Firms must make an investment at the end of the first period in order to stay in business in the second period.

One of the firms has a deep pocket, whereas the other firm has to finance the investment through debt. There is asymmetric information regarding the firm's second period prospects, which under the assumption of debt financing creates an incentive to invest, even if the investment has negative net present value. To resolve this problem, some portion of the investment has to be financed by internal cash. This again creates incentives for the rival firm to compete more aggressively in the first period to reduce the firm's cash reserves and to force it to forgo the investment. Just as in our set-up, in the Phillips (1993) model internal cash is at the heart of the analysis. In contrast to our analysis, Phillips focuses on the rival's predatory incentives, from which I abstract. Another key difference to the model of this paper is that the incentive problem which underlies the Phillips analysis comes about only because the firm is restricted to issue debt and would disappear if the firm could issue equity, as is assumed here.

There are two more articles which are related to our analysis. In Rotemberg and Scharfstein (1990) managers maximise a weighted average of expected profits and the stock price. While the authors do not provide a formal argument, they argue that such behaviour may come out of a model in which the firm anticipates to issue equity in the future. They analyse a two period model in which demand and cost conditions are uncertain, but correlated across periods and the stock market tries to infer these from the firm's and its rivals' realised profits. Kovenock and Phillips (1995) invoke the pecking order theory of finance to argue that external finance is more costly than internal finance. In a model in which capacity has to be financed before revenues are earned, they analyse the incentive to reduce financial slack by issuing debt for both price and quantity competition. They find such an incentive to increase financing costs for price competition, but not for quantity competition. Their result can be viewed as complementary to the result obtained by Maksimovic (1990), who has found a strategic value in reducing the variable cost of borrowing, and thus an incentive to decrease financing costs, for the case of quantity competition. In contrast to both Rotemberg and Scharfstein (1990) and Kovenock and Phillips (1995), who argue informally that the objective function they assume may be justified by costs of external funds, I derive the firm's first period objective function from first principles, taking asymmetric information between the manager and the financial investor as a starting point. I consider an asymmetric setting in which only one of the firms, firm i, faces an investment opportunity which needs to be funded externally. This can be justified by saying that firm j either does not happen to have a positive NPV project, or that it has sufficient internal cash to finance it without recourse to the capital market, i.e. it has a deep pocket. Yet another way of justifying this is to say that there is less uncertainty on firm j, perhaps because it is better known by financial investors. In terms of modelling strategy, the main implication of the assumption is that one can be sure that firm j is a straight expected profit maximiser. Thus, issues arising from a possible reversal of the nature of competition, which have been the focus of a previous paper (Nier (1998)), will not arise here, since firm j has a standard downward sloping reaction function.

2 The Model

The model has two periods. At the start of the first period, at $t = 0$, a firm, which is called firm i, competes with another firm, which is labelled firm j, as one of two duopolists in a product market. Each firm has to choose some strategic variable which affects both its own profit and the profit of the rival. This variable can be thought of as the firm's output choice. In addition, profits are affected by a random variable $\widetilde{z}$, which may in general be a vector and has positive support on some set Z. The profit function is identical for both firms. For firm i it is given by $\widetilde{\pi}^i = \pi^i\left(\widetilde{z}^i, q^i, q^j\right)$. The profit function is assumed to satisfy[2]:

$$\forall \left(z^i, q^i, q^j\right) \; with \; z \epsilon Z, \; q^i \geq 0, q^j \geq 0$$

$$(i)\, \pi^i_{ii}\left(z^i, q^i, q^j\right) < 0, (ii)\, \pi^i_j\left(z^i, q^i, q^j\right) < 0, (iii)\, \pi^i_{ij}\left(z^i, q^i, q^j\right) < 0 \qquad \text{(A1*)}$$

$$\begin{aligned}(iv)\, z' &> z \Rightarrow \pi^i\left(z', q^i, q^j\right) > \pi^i\left(z, q^i, q^j\right)\\ (v)\, z' &> z \Rightarrow \pi^i_i\left(z', q^i, q^j\right) \geq \pi^i_i\left(z, q^i, q^j\right)\end{aligned}$$

The random variables $\widetilde{z}^i$ and $\widetilde{z}^j$ realise, and become publicly known at $t = 1$, after q^i and q^j have been chosen.

At $t = 1$ a second random variable realises, which affects the value of firm $i's$ assets in place. These assets are thought of as unrelated to the product market in which the firm competes in the first period. The value of the assets in place is best thought of as the liquidation value of the firm. Firm i can be of two types. If the firm is of type H, the value of its assets in place is s_H. If the firm is of type L, its assets in place are worth s_L, where $s_H > s_L$. Ex ante (at $t = 0$) the firm is of the high type H with probability r and it is of the low type L with probability $1 - r$. I assume that the firm's type is privately revealed to the firm's owner at $t = 1$. At this stage outside investors only know the ex ante probability r.

At $t = 1$ an investment opportunity opens up to the firm which requires an initial outlay of I and returns an expected payoff of x. Both these values are publicly known. Again, I want to assume that the investment opportunity is unrelated to the product market in which the firm competes in the first period. One can think of it as an investment into some new line of business or as funds required for a diversifying acquisition. It is assumed that this

[2] This assumption is slighly more general than that made in Brander and Lewis (1986) and will enable me to analyse both the specification of Brander and Lewis (1988), as well as an alternative profit function, which combines demand uncertainty with some additional additive uncertainty.

investment is profitable, so that $x > I$. Therefore when $\pi^i \geq I$ the firm will always invest, since in this case it need not (and will not) raise any external funds. When first period profits fall short of the required outlay, the firm can raise $I - \pi^i$ externally. Following the original analysis of Myers and Majluf (1984), I assume that the firm is constrained to issue outside equity[3]. Therefore, for any realisation of first period profit π^i such that $\pi^i < I$ a continuation game ensues. In this game the firm can either raise $I - \pi^i$ from outside investors by selling off a fraction α of the firm's equity and invest, or forgo the investment opportunity. For the main part of the analysis it is assumed that financial investors are able to observe the realised profit and thus the financing need of the firm with certainty. Also, the market for outside equity is assumed to be competitive. Thus, in the spirit of Myers and Majluf (1984), it is assumed that there is an auction for the firm's equity, which ensures that the price for the firm's equity is bid down to the point where financial investors just break even.

At $t = 2$ the investment pays off, the value of the assets in place become publicly known, the firm is liquidated and all claims are settled[4].

I assume risk neutrality and a discount rate of zero for simplicity.

[3] It is well known that financing via rights issued to existing shareholders solves the adverse selection problem. To rule out rights issues, one can think of the firm as being owner-managed and that the owner does not have any funds other than π. Allowing the firm to issue risky debt rather than equity would make the financing problem less severe, but would not change the main conclusions of the model, as long as some adverse selection remains and is reflected in the default premium that has to be paid by the firm. Since the focus here is not on solving the financial adverse selection problem, but on the implications of an adverse selection problem for a firm which competes in a product market, I have chosen to consider the market for equity, just as in the original paper by Myers and Majluf (1984).

[4] The timing of the model has been chosen to disentangle the financing stage from the product market stage as much as possible. If one had the firm know its value from the start, for example, the main conclusions of the model would be preserved. One would have an additional issue, however, in that high type and low type firms would behave differently at the product market stage, so that the financial market could try and draw inferences from observed profit.

3 Second-Stage Equilibria

I solve the game backwards and start at the beginning of the second period, $t = 1$. For any realisation of first period profit π such that $\pi < I$ the firm may either decide to raise $I - \pi$ from an outside investor by selling off a fraction α of the firm's equity and invest, or forgo the investment opportunity[5]. Denote the firm's decision by $d \epsilon \{1, 0\}$, where $d = 1$ when the firm decides to raise $I - \pi$ and invests and $d = 0$ when the firm does not invest. In order to capture that the firm can make this decision conditional on its type let $\delta_H = \Pr[d = 1 \mid H]$ and $\delta_L = \Pr[d = 1 \mid L]$. Outside investors can expect to break even, given their beliefs about the firm. Let ρ be the probability attached to the possibility that the firm is of the high type.

An equilibrium of this game is defined as follows.

Definition 1 *For any realized π such that $\pi < I$ an equilibrium of the game is a quadruple $(\delta_H, \delta_L, \rho, \alpha)$ such that*

1. Beliefs are updated using Bayes' rule

$$\rho = \frac{r\delta_H}{r\delta_H + (1 - r)\,\delta_L}$$

2. α is defined by the equation

$$I - \pi = \alpha\left[\rho s_H + (1 - \rho)\, s_L + x\right]$$

ensuring that the financial investors just break even, given equilibrium beliefs.

3. For any $k \epsilon \{H, L\}$

$$\begin{array}{rl} \delta_k = 1 \; if & s_k + \pi < (1 - \alpha)\left[s_k + x\right] \\ \delta_k \epsilon \left[0, 1\right] \; if & s_k + \pi = (1 - \alpha)\left[s_k + x\right] \\ \delta_k = 0 \; if & s_k + \pi > (1 - \alpha)\left[s_k + x\right] \end{array}$$

Analysing the set of equilibria of this game, one arrives at the following

[5] In both this and the next section I focus on firm i and drop the superscript throughout.

Lemma 1 *There is a cut-off level of realised first period profits $\widehat{\pi}$, such that*

a) for $\pi < \widehat{\pi}$ the unique equilibrium of the continuation game is a separating equilibrium in which $\delta_H = 0$, $\delta_L = 1, \rho = 0$, and

$$\alpha = \frac{I - \pi}{s_L + x}$$

b) for $\pi \geq \widehat{\pi}$ there exists a pooling equilibrium in which $\delta_H = 1$, $\delta_L = 1, \rho = r$, and

$$\alpha = \frac{I - \pi}{[r s_H + (1 - r)\, s_L + x]}$$

The cut-off $\widehat{\pi}$ is implicitly defined as the solution to

$$s_H + \pi = \left(1 - \frac{I - \pi}{[r s_H + (1 - r)\, s_L + x]}\right)[s_H + x]$$

A proof can be found in Appendix 1.

It is useful at this point to analyse the cut-off profit level $\widehat{\pi}$. One can find an explicit expression for it by rearranging the condition under which the high type will invest. This yields

$$s_H + \pi \leq \left(1 - \frac{I - \pi}{[r s_H + (1 - r)\, s_L + x]}\right)[s_H + x]$$

$\iff$

$$I - (x - I)\left(\frac{[s_H + x]}{[r s_H + (1 - r)\, s_L + x]} - 1\right)^{-1} \leq \pi$$

For $\pi = \widehat{\pi}$ this holds as an equality and one arrives at

$$I - (x - I)\left(\frac{[s_H + x]}{[r s_H + (1 - r)\, s_L + x]} - 1\right)^{-1} = \widehat{\pi}$$

By inspection, one sees that the profit level required for a pooling equilibrium to exist is increasing in the required investment outlay I. One also finds that it is decreasing in the net present value of the project[6]. For lower net present value, a higher amount of internal funds is required for it to be optimal to go

[6]This result is derived algebraically in the later section 5.

ahead with the project. This is because in the pooling equilibrium the high type faces a dilution cost. Since the firm receives $(I-\pi)$, but pays

$$\alpha\left[s_H+x\right]=\frac{I-\pi}{\left[rs_H+(1-r)\,s_L+x\right]}\left[s_H+x\right]$$

this cost is equal to the difference which can be written as

$$(I-\pi)\left(\frac{\left[s_H+x\right]}{\left[rs_H+(1-r)\,s_L+x\right]}-1\right)$$

The dilution cost is decreasing in π. Therefore, for low net present value projects to be acceptable, the dilution cost has to be low, which requires a higher π.

Finally, the required profit level is increasing in the dilution factor

$$\left(\frac{\left[s_H+x\right]}{\left[rs_H+(1-r)\,s_L+x\right]}-1\right)$$

The larger is this factor, the higher is the level of internal funds needed to make investment attractive.

It is worth pointing out that, as long as the project has a strictly positive net present value, $\widehat{\pi}<I$. Recall also that for $\pi\geq I$ both types of firm will invest, since then there is no need to raise external funds, Therefore, given that the pooling equilibrium is played whenever it exists, one has that for $\pi<\widehat{\pi}$ only the low type invests and for $\pi\geq\widehat{\pi}$ both types will invest.

For the main part of the analysis I am going to assume that the pooling equilibrium is played whenever it exists. Before doing so, let us explore how much generality is lost by such an assumption[7]. One finds

Lemma 2 *There is a cut-off level of realised first period profits* $\widehat{\widehat{\pi}}$ *such that*

[7]Myers and Majluf (1984) do not provide a formal characterisation of the full set of equilibria of their model. In a model set-up similar to ours the issue of multiple equilibria has first been addressed by Cadsby, Frank, and Maksimovic (1990). The main difference between their analysis and the analysis here is that they assume financial slack to be zero, whereas I focus on first-period profit as the main variable of interest.

a) for $\pi \leq \hat{\hat{\pi}}$ *a separating equilibrium exists in which* $\delta_H = 0,\ \delta_L = 1, \rho = 0,$ *and*

$$\alpha = \frac{I - \pi}{s_L + x}$$

b) for $\pi > \hat{\hat{\pi}}$ *the unique equilibrium is a pooling equilibrium in which* $\delta_H = 1,\ \delta_L = 1, \rho = r,$ *and*

$$\alpha = \frac{I - \pi}{[r s_H + (1 - r)\, s_L + x]}$$

The cut-off $\hat{\hat{\pi}}$ *is implicitly defined as the solution to*

$$s_H + \pi = \left(1 - \frac{I - \pi}{s_L + x}\right)[s_H + x]$$

A proof is provided in Appendix 1.

Putting Lemma 2 and Lemma 3 together it is immediate that $\hat{\hat{\pi}} \geq \hat{\pi}$. In fact one can show that $\hat{\hat{\pi}}$ is strictly larger than $\hat{\pi}$. This can be easily established by deriving an explicit expression for $\hat{\hat{\pi}}$. The condition for the separating equilibrium to exist is

$$s_H + \pi \geq \left(1 - \frac{I - \pi}{s_L + x}\right)[s_H + x]$$

This can be rearranged to give

$$I - (x - I)\left(\frac{s_H + x}{s_L + x} - 1\right)^{-1} \geq \pi$$

For $\hat{\hat{\pi}}$ this holds as an equality and one has

$$I - (x - I)\left(\frac{s_H + x}{s_L + x} - 1\right)^{-1} = \hat{\hat{\pi}}$$

To see that $\hat{\pi} < \hat{\hat{\pi}}$ note that

$$\left(\frac{[s_H + x]}{[r s_H + (1 - r)\, s_L + x]} - 1\right)^{-1} > \left(\frac{s_H + x}{s_L + x} - 1\right)^{-1}$$

$\iff$

$$\left(\frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]} - 1\right) < \left(\frac{s_H + x}{s_L + x} - 1\right)$$

It remains to characterise semiseparating equilibria. These equilibria are such that the low type always invests, whereas the high type is just indifferent between investing and not investing, given beliefs. The high type randomises and beliefs are consistent with the probability of the high type investing. One finds

Lemma 3 *A semiseparating equilibrium exists if and only if* $\pi \epsilon \left[\widehat{\pi}, \widehat{\widehat{\pi}}\right]$. *In a semiseparating equilibrium* $\delta_L = 1$, $\delta_H \epsilon \, [0, 1]$, *and*

$$\rho\left(\delta_H\right) = \frac{r\delta_H}{r\delta_H + (1-r)}$$

For given parameters δ_H *is the solution to*

$$s_H + \pi = \left(1 - \frac{I - \pi}{[\rho\left(\delta_H\right) s_H + \left(1 - \rho\left(\delta_H\right)\right) s_L + x]}\right)[s_H + x]$$

A proof is given in Appendix 1.

Across the semiseparating equilibria one finds that

$$\frac{d\delta_H}{d\pi} < 0$$

That is, as the high type has more cash on hand, the probability of the high type investing falls. This may seem counterintuitive, in particular since, as one moves from separating equilibria to pooling equilibria, an increase in profit is associated with an increase in δ_H. The intuition for the case of the semiseparating equilibria is that for these an indifference condition has to hold. As one lets δ_H increase ρ increases, so that dilution costs decrease. Dilution cost are decreasing in π. To keep the high type indifferent between investing and not investing, dilution costs have to increase through a decrease in π.

One can summarise Lemmas 2-4 in the following

Corollary 1 *For $\pi < \widehat{\pi}$ the unique equilibrium is a separating equilibrium. For $\pi \epsilon \left[\widehat{\pi}, \widehat{\widehat{\pi}}\right]$ separating, semiseparating and pooling equilibria exist. For $\widehat{\widehat{\pi}} < \pi$ the unique equilibrium is a pooling equilibrium.*

For $\pi \epsilon \left[\widehat{\pi}, \widehat{\widehat{\pi}}\right]$ one has multiple equilibria[8]. Figure 1 gives a graphical representation of the equilibrium correspondence. Which equilibrium is played for each π will affect the first period objective of firm i and will therefore affect our conclusions on the outcome of the first period competition. I would like to motivate an equilibrium selection such that $\rho(\pi)$ is weakly increasing in π. This condition rules out semiseparating equilibria being played on $\left[\widehat{\pi}, \widehat{\widehat{\pi}}\right]$. It also rules out an equilibrium selection such that the equilibrium moves from a pooling equilibrium to a separating equilibrium as π increases. The condition has some intuitive appeal since one can view lack of internal funding as the source of the inefficiency that arises as the high type forgoes investment. Intuitively, this problem should become less severe and the market should place a higher probability on the firm being of the high type as the amount of external funding which the firm asks for becomes smaller.

In deriving the set of equilibria for the financing game, it was assumed that the value of the assets in place is the only source of asymmetric information and that the financial investor knows all other variables including the realisation of first-period profit with certainty. As one moves away from this assumption and introduces some uncertainty as regards the first-period

[8] Standard equilibrium refinements like the Intuitive Criterion (Cho and Kreps(1987)) do not help to reduce the number of equilibria here. These refinements are based on the notion that players should have reasonable beliefs on off-equilibrium behaviour. The idea is that in a given equilibrium some off-equilibrium action may not be undertaken because it is assigned an unreasonable belief. There are two reasons, why such arguments do not work in the Myers and Majluf model. First, the action space is not rich enough: there are only two possible actions, to issue equity, and not to issue equity. Second, the belief assigned to not issuing equity is not payoff- relevant, precisely because no equity is being issued. In the separating and the semiseparating equilibria both possible actions are on the equilibrium path. In the pooling equilibrium, the profitability of an off-equilibrium move to not issuing equity does not vary with the belief that is assigned to this action. Standard refinements therefore do not have any bite.

profit, one would expect financial investors to take a larger equity issue as a sign of the firm being more likely of the bad type. The reason is that the bad type has a dilution gain which is increasing in the issue size, whereas the high type has a dilution cost which is increasing in the issue size. Bad firms will therefore have a stronger incentive to overstate their financing needs and should therefore be thought of as more likely to issue larger amounts.

For a continuous distribution of first-period profit it is difficult to state these ideas formally. To motivate the equilibrium selection condition that $\rho(\pi)$ is weakly increasing in π, I move to a situation where first period profit can be either high or low. Consider a profit distribution such that $\pi \epsilon \{\underline{\pi}, \overline{\pi}\}$, where the probability of the high realisation is $\Pr[\pi = \overline{\pi}] = p$, independent of the firm's type. I want both profit realisations to be in the region of multiple equilibria, that is $\hat{\hat{\pi}} > \overline{\pi} > \underline{\pi} > \hat{\pi}$. I assume now that financial markets are unable to observe profits. When the profit realisation is $\overline{\pi}$, the firm has three possibilities. It can issue $I - \overline{\pi}$ worth of equity, or it can understate its profit realisation and issue a larger amount of equity equalling issue size $I - \underline{\pi}$, or it can forgo investment. When the firm issues the larger amount, it is assumed that it is able to pay the original owners a dividend equalling the surplus cash. Equivalently one can assume that when $\overline{\pi}$ realises the owners are able to eat up the amount $\overline{\pi} - \underline{\pi} = \varepsilon$ without the financial investors being able to observe this. When the profit realisation is $\underline{\pi}$, the firm has two possibilities, as before: it can either issue $I - \underline{\pi}$ or forgo the investment[9]. Denote by $\delta_H(\overline{\pi})$ the probability that the high type issues $I - \overline{\pi}$, given a profit realisation of $\overline{\pi}$ and by $\mu_H(\overline{\pi})$ the probability that the high type issues the larger amount $I - \underline{\pi}$, given a profit realisation of $\overline{\pi}$. Likewise, denote by $\delta_L(\overline{\pi})$ the probability that the low type issues $I - \overline{\pi}$, given a profit realisation of $\overline{\pi}$ and by $\mu_L(\overline{\pi})$ the probability that the low type understates its profit and issues the larger

[9]There is no point in assuming that the firm has a third possibility of issuing a smaller amount than it needs for investment since financial investors are able to observe whether the investment is undertaken or not, and would be able to demand their money back if the firm did not invest after having issued equity.

amount $I-\underline{\pi}$, given a profit realisation of $\overline{\pi}$. Finally, let $\delta_H(\underline{\pi})$ and $\delta_L(\underline{\pi})$be the probabilities that the high type and the low type issue $I-\underline{\pi}$, respectively, given that the profit realisation is $\underline{\pi}$. These probabilities satisfy

$$\begin{aligned}\delta_H(\overline{\pi})+\mu_H(\overline{\pi}) &\leq 1\\ \delta_L(\overline{\pi})+\mu_L(\overline{\pi}) &\leq 1\end{aligned}$$

and

$$\begin{aligned}\delta_H(\underline{\pi}) &\leq 1\\ \delta_L(\underline{\pi}) &\leq 1\end{aligned}$$

For any action the firm takes, financial investors have a belief on the type of the firm. Let $\rho(\overline{\pi})$ be the probability that the firm is of the high type when the issue size $I-\overline{\pi}$ is observed and $\rho(\underline{\pi})$ be the probability attached to the firm being of the high type, given that the larger issue size of $I-\underline{\pi}$ is observed. In an equilibrium in which the amount $I-\underline{\pi}$ is issued with positive probability one will have

$$\rho(\underline{\pi})=\frac{r\left[(1-p)\,\delta_H(\underline{\pi})+p\mu_H(\overline{\pi})\right]}{r\left[(1-p)\,\delta_H(\underline{\pi})+p\mu_H(\overline{\pi})\right]+(1-r)\left[(1-p)\,\delta_L(\underline{\pi})+p\mu_L(\overline{\pi})\right]}$$

In an equilibrium in which the amount of $I-\overline{\pi}$ is issued with positive probability one will have

$$\rho(\overline{\pi})=\frac{rp\delta_H(\overline{\pi})}{rp\delta_H(\overline{\pi})+(1-r)\,p\delta_L(\overline{\pi})}=\frac{r\delta_H(\overline{\pi})}{r\delta_H(\overline{\pi})+(1-r)\,\delta_L(\overline{\pi})}$$

While destroying some equilibria, the introduction of additional asymmetric information regarding the realisation of first period profit will in general generate further equilibria that are supportable by particular choices of out-of-equilibrium beliefs. To justify the equilibrium selection criterion I want to focus on equilibria in which, as in the case of symmetric information with respect to profit, both issue sizes occur with positive probability on the equilibrium path. To illustrate how the introduction of uncertainty changes the set of equilibria of the game, let us first ask whether the semiseparating equilibrium on both $\underline{\pi}$ and $\overline{\pi}$ survives the introduction of uncertainty. This would be so if for neither type it pays to understate its profit, given the equilibrium beliefs. Let us first check the high type. Given $\overline{\pi}$ the high type would want to deviate to $\mu_H(\overline{\pi}) = 1$, if

$$\begin{aligned} & \left(1 - \frac{I - \overline{\pi} + \varepsilon}{\left[\rho\left(\underline{\pi}\right) s_H + \left(1 - \rho\left(\underline{\pi}\right)\right) s_L + x\right]}\right) \left[s_H + x\right] + \varepsilon \\ > & \left(1 - \frac{I - \overline{\pi}}{\left[\rho\left(\overline{\pi}\right) s_H + \left(1 - \rho\left(\overline{\pi}\right)\right) s_L + x\right]}\right) \left[s_H + x\right] \end{aligned}$$

But

$$\begin{aligned} & \left(1 - \frac{I - \overline{\pi} + \varepsilon}{\left[\rho\left(\underline{\pi}\right) s_H + \left(1 - \rho\left(\underline{\pi}\right)\right) s_L + x\right]}\right) \left[s_H + x\right] + \varepsilon \\ = & \left(1 - \frac{I - \underline{\pi}}{\left[\rho\left(\underline{\pi}\right) s_H + \left(1 - \rho\left(\underline{\pi}\right)\right) s_L + x\right]}\right) \left[s_H + x\right] + \varepsilon \\ = & s_H + \underline{\pi} + \varepsilon \\ = & s_H + \overline{\pi} \\ = & \left(1 - \frac{I - \overline{\pi}}{\left[\rho\left(\overline{\pi}\right) s_H + \left(1 - \rho\left(\overline{\pi}\right)\right) s_L + x\right]}\right) \left[s_H + x\right] \end{aligned}$$

so that for the high type, understating its profit does not increase its payoff, given equilibrium beliefs. The high type is just indifferent as to whether to play its equilibrium strategy or to deviate to a larger issue size. Intuitively, the high type gains from issuing a larger amount, since this will lead to a more favourable belief. This is so since $\rho(\underline{\pi}) > \rho(\overline{\pi})$ in the semiseparating equilibrium. On the other hand, the dilution cost incurred by the high type is larger for larger issue sizes. These two effects exactly cancel out. The high type therefore does not have an incentive to deviate to larger issue size, but is just as happy playing the equilibrium strategy. For the low type, on the other hand, a deviation to $\mu_L(\overline{\pi}) = 1$ is profitable, since

$$\left(1 - \frac{I - \overline{\pi} + \varepsilon}{\left[\rho\left(\underline{\pi}\right) s_H + \left(1 - \rho\left(\underline{\pi}\right)\right) s_L + x\right]}\right) \left[s_L + x\right] + \varepsilon$$

$$> \left(1 - \frac{I - \overline{\pi}}{\left[\rho\left(\overline{\pi}\right) s_H + \left(1 - \rho\left(\overline{\pi}\right)\right) s_L + x\right]}\right) \left[s_L + x\right]$$

$\Longleftrightarrow$

$$\varepsilon \left(1 - \frac{\left[s_L + x\right]}{\left[\rho\left(\underline{\pi}\right) s_H + \left(1 - \rho\left(\underline{\pi}\right)\right) s_L + x\right]}\right)$$

$$> \frac{\left[I - \overline{\pi}\right]\left[s_L + x\right]}{\left[\rho\left(\underline{\pi}\right) s_H + \left(1 - \rho\left(\underline{\pi}\right)\right) s_L + x\right]} - \frac{\left[I - \overline{\pi}\right]\left[s_L + x\right]}{\left[\rho\left(\overline{\pi}\right) s_H + \left(1 - \rho\left(\overline{\pi}\right)\right) s_L + x\right]}$$

which is satisfied since $\rho\left(\underline{\pi}\right) > \rho\left(\overline{\pi}\right)$ so that the LHS is strictly positive and the RHS is strictly negative. Intuitively, again the low type gains from more favourable equilibrium beliefs associated with the larger issue size. In addition, its dilution gain is increasing in the issue size. Both effects work in the same direction here and make it profitable for the low type to choose the larger issue size. The low type, therefore, does have an incentive to understate its profit. The semiseparating equilibrium does not survive the introduction of additional uncertainty since beliefs are such that $\rho\left(\underline{\pi}\right) > \rho\left(\overline{\pi}\right)$. In fact, one can show

Lemma 4 *In any equilibrium in which both issue sizes occur with positive probability,* $\rho\left(\underline{\pi}\right) \leq \rho\left(\overline{\pi}\right)$.

Proof: Assume $\rho\left(\underline{\pi}\right) > \rho\left(\overline{\pi}\right)$. Then $\mu_L\left(\overline{\pi}\right) = 1$, since

$$\left(1 - \frac{I - \overline{\pi} + \varepsilon}{\left[\rho\left(\underline{\pi}\right) s_H + \left(1 - \rho\left(\underline{\pi}\right)\right) s_L + x\right]}\right) \left[s_L + x\right] + \varepsilon$$

$$> \left(1 - \frac{I - \overline{\pi}}{\left[\rho\left(\overline{\pi}\right) s_H + \left(1 - \rho\left(\overline{\pi}\right)\right) s_L + x\right]}\right) \left[s_L + x\right]$$

as shown above. $\mu_L\left(\overline{\pi}\right) = 1$ implies $\delta_L\left(\overline{\pi}\right) = 0$, so that one has

$$\rho\left(\underline{\pi}\right) > \frac{r\delta_H\left(\overline{\pi}\right)}{r\delta_H\left(\overline{\pi}\right) + \left(1 - r\right)\delta_L\left(\overline{\pi}\right)} = 1$$

This completes the proof.

Notice that the set of equilibria in which both issue sizes occur with positive probability includes all those equilibria in which there is no incentive to understate, i.e. in which $\mu_L(\overline{\pi}) = \mu_H(\overline{\pi}) = 0$. The latter set of equilibria is a subset of all possible combinations of the equilibria with perfect information regarding profit. One can easily show that of these combinations, only four survive the introduction of uncertainty. These are separating on both $\underline{\pi}$ and $\overline{\pi}$, separating on $\underline{\pi}$ and semiseparating on $\overline{\pi}$, semiseparating on $\underline{\pi}$ and pooling on $\overline{\pi}$, and separating on $\underline{\pi}$ and pooling on $\overline{\pi}$. As we have seen, semiseparating on both $\underline{\pi}$ and $\overline{\pi}$ does not survive since under the equilibrium beliefs the low type has an incentive to $\mu_L(\overline{\pi}) = 1$, whereas the high type has no such incentive.

4 The First-Period Objective

Let us now go on to construct the objective of firm i in the first-period competition. This will depend on which equilibrium is played for given first-period profit. In line with the arguments in the last section, I will assume that the equilibrium selection is such that $\rho(\pi)$ is weakly increasing in π. This implies that the equilibrium moves from the separating equilibrium to the pooling equilibrium at some cut-off. For concreteness, let us start with the assumption that a pooling equilibrium is played whenever it exists. This means that for $\pi < \widehat{\pi}$ the equilibrium will be separating and for $\pi \geq \widehat{\pi}$ the equilibrium will be pooling as in Lemma 3.

With probability r the firm is type H.

In this case it will not invest for realisation of π, such that $\pi < \widehat{\pi}$ and its payoff will be

$$s_H + \pi$$

For realisations of π, such that $i \geq \pi \geq \widehat{\pi}$ the firm invests and has payoff

$$\left(1 - \frac{I - \pi}{[r s_H + (1 - r) s_L + x]}\right) [s_H + x]$$

$$= s_H + x - \frac{I - \pi}{[rs_H + (1-r)\, s_L + x]} [s_H + x]$$

$$= s_H + x - (I - \pi) \frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]}$$

$$= s_H + x - (I - \pi) \left(\frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]} - 1 \right) + [-(I - \pi)]$$

$$= s_H + \pi + x - I - (I - \pi) \left(\frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]} - 1 \right)$$

The firm receives $s_H + \pi$. It also receives the net present value of the project, but loses the dilution cost of

$$(I - \pi) \left(\frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]} - 1 \right)$$

Finally, for realisation of π such that $\pi > I$ the firm's payoff is

$$s_H + \pi + x - I$$

With probability $1 - r$ the firm is the low type L.

In this case the firm always invests.

For realisations such that $\pi < \widehat{\pi}$ its payoff will be

$$\left(1 - \frac{I - \pi}{s_L + x}\right) [s_L + x]$$

$$= s_L + \pi + x - I$$

For realisations such that $I > \pi \geq \widehat{\pi}$ its payoff will be

$$\left(1 - \frac{I - \pi}{[rs_H + (1-r)\, s_L + x]}\right) [s_L + x]$$

$$s_L + x - (I - \pi)\frac{s_L + x}{rs_H + (1-r)s_L + x}$$

$$s_L + x - (I - \pi)\left(\frac{s_L + x}{rs_H + (1-r)s_L + x} - 1\right) - (I - \pi)$$

$$s_L + \pi + x - I - (I - \pi)\left(\frac{s_L + x}{rs_H + (1-r)s_L + x} - 1\right)$$

$$s_L + \pi + x - I + (I - \pi)\left(1 - \frac{s_L + x}{rs_H + (1-r)s_L + x}\right)$$

It gets $s_L + \pi$ and the net present value of the project. In addition, it gets a dilution gain of

$$(I - \pi)\left(1 - \frac{s_L + x}{rs_H + (1-r)s_L + x}\right) > 0$$

Finally, for realisations of π such that $\pi > i$ the firm's payoff is

$$s_L + \pi + x - I$$

It is important to realise that for any realisation of π the expected dilution cost is zero. This follows directly from the fact that the financial investor breaks even in equilibrium. It can be verified algebraically by noting that

$$\begin{aligned}
& -r(I-\pi)\left(\frac{[s_H + x]}{[rs_H + (1-r)s_L + x]} - 1\right) \\
& + (1-r)(I-\pi)\left(1 - \frac{s_L + x}{rs_H + (1-r)s_L + x}\right) \\
= \; & r(I-\pi) + (1-r)(I-\pi) \\
& - (I-\pi)\left[r\frac{[s_H + x]}{[rs_H + (1-r)s_L + x]} + (1-r)\frac{s_L + x}{rs_H + (1-r)s_L + x}\right] \\
= \; & 0
\end{aligned}$$

For given π the expected payoff is therefore

$$(1-r)[s_L+\pi+x-I]+r[s_H+\pi]\ if\ \pi<\widehat{\pi}$$

and

$$(1-r)[s_L+\pi+x-I]+r[s_H+\pi+x-I]\ if\ \pi\geq\widehat{\pi}$$

Across realisations of π the expected payoff can then be written down as

$$\int_{-\infty}^{\widehat{\pi}}\{(1-r)[s_L+\pi+x-I]+r[s_H+\pi]\}f(\pi)\,d\pi$$
$$+\int_{\widehat{\pi}}^{\infty}\{(1-r)[s_L+\pi+x-I]+r[s_H+\pi+x-I]\}f(\pi)\,d\pi$$

$$=\int_{-\infty}^{\widehat{\pi}}\{(1-r)s_L+rs_H+\pi+(1-r)(x-I)\}f(\pi)\,d\pi$$
$$+\int_{\widehat{\pi}}^{\infty}\{(1-r)s_L+rs_H+\pi+(1-r)(x-I)+r(x-I)\}f(\pi)\,d\pi$$

$$=(1-r)s_L+rs_H+(x-I)+\int_{-\infty}^{\infty}\pi f(\pi)\,d\pi-F(\widehat{\pi})\,r\,(x-I)$$

$$=E[\widetilde{s}]+(x-I)+E[\widetilde{\pi}]-F(\widehat{\pi})\,r\,(x-I)$$

Discarding constants, the firm's first period objective is therefore

$$E[\widetilde{\pi}]-F(\widehat{\pi})\,r\,(x-I)$$

In addition to the expected value of profit, there is a second term. It is the loss in net present value, which occurs when the firm is of the high type and the profit realisation is too low for the pooling equilibrium to exist, so that the high type will not invest. This is multiplied by the probability that the profit realisation is below the cut-off $\widehat{\pi}$, above which the pooling equilibrium is played.

It is easy to see that for any other assumption on equilibrium selection satisfying $\rho(\underline{\pi})\leq\rho(\overline{\pi})$ there will again be a cut-off $\pi\epsilon\left[\widehat{\pi},\widehat{\widehat{\pi}}\right]$ such that the high type invests for profit realisations larger than the cut-off. The first period objective can then be found by replacing $\widehat{\pi}$ with the cut-off chosen.

5 First-Period Competition

Let us now go on to analyse the first-stage game in which each of the two firms chooses a strategic variable to maximise its first period objective. In the general framework profits are given by

$$\widetilde{\pi}^i = \pi\left(\widetilde{z}, q_i, q_j\right)$$

I want to make this more specific in two different ways.

5.1 Profit function à la Brander and Lewis

In line with Brander and Lewis (1986), let us first assume that uncertainty can be represented by a scalar variable, i.e.

$$\widetilde{\pi}^i = \pi^i\left(\widetilde{z}^i, q^i, q^j\right) = \pi^i\left(\widetilde{\theta}^i, q^i, q^j\right)$$

where $\widetilde{\theta}^i \epsilon \left(\underline{\theta}, \overline{\theta}\right)$. The distribution of $\widetilde{\theta}$ is given by $F(\theta)$ which is assumed to have a density $f(\theta)$. Further, the following assumptions hold

$$\forall \; \left(\theta^i, q^i, q^j\right) \;\; with \; \theta^i \in \left(\underline{\theta}, \overline{\theta}\right), q^i \geq 0, q^j \geq 0$$

$$(i)\, \pi^i_{ii}\left(\theta^i, q^i, q^j\right) < 0, (ii)\, \pi^i_j\left(\theta^i, q^i, q^j\right) < 0, (iii)\, \pi^i_{ij}\left(\theta^i, q^i, q^j\right) < 0 \qquad \text{(A1)}$$

$$(iv)\, \pi^i_\theta\left(\theta^i, q^i, q^j\right) > 0, (v)\, \pi^i_{i\theta}\left(\theta^i, q^i, q^j\right) > 0,$$

Here assumptions (iv) and (v) are versions of the more general assumptions in A1*, for the case of a profit function which is differentiable with respect to a scalar random variable $\widetilde{z}^i = \widetilde{\theta}^i$. I also make two additional assumptions. First, I assume that $\widetilde{\theta}$ is uniformly distributed. This guarantees that $f'(\theta) = 0$. Second, I want to assume that $\pi^i_{\theta\theta} = 0$[10].

[10] These assumptions, as well as the formal analysis in this section, are similar to Brander and Lewis (1988). They assume an objective function which is similar to the one we derive by speculating that the firm has debt in its capital structure and that the firm maximises profits minus a fixed exogenous bankruptcy cost, which is incurred whenever profits fall short of the debt obligation.

Consider first the benchmark of single stage competition. Equivalently, assume that $x - I$ is negative, so that neither the low type nor the high type will invest in the second stage. Then firm $i's$ objective is to maximise

$$E\left[\widetilde{\pi}^i\right] = \int_{\underline{\theta}}^{\overline{\theta}} \pi^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i$$

The equilibrium is given by the unique intersection of the firms' reaction functions. These are implicitly defined by

$$\begin{aligned} \int_{\underline{\theta}}^{\overline{\theta}} \pi_i^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i &= 0 \\ \int_{\underline{\theta}}^{\overline{\theta}} \pi_j^j\left(\theta^j, q^i, q^j\right) f\left(\theta^j\right) d\theta^j &= 0 \end{aligned}$$

The intersection will yield equilibrium quantities $(q^i, q^j) = (q^c, q^c)$which will give equilibrium profit

$$\int_{\underline{\theta}}^{\overline{\theta}} \pi^i\left(\theta^i, q^c, q^c\right) f\left(\theta^i\right) d\theta^i \equiv \pi^c$$

Now assume that $x - i$ is positive. Then firm $i's$ objective is

$$\begin{aligned} V^i &= E\left[\widetilde{\pi}^i\right] - F\left(\widehat{\pi}\right) r\left(x - I\right) \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \pi^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i - F\left(\widehat{\theta}\right) r\left(x - I\right) \end{aligned}$$

where $\widehat{\theta}$ is implicitly defined by

$$\pi^i\left(\widehat{\theta}, q^i, q^j\right) - \widehat{\pi} = 0$$

By assumption, only firm i faces an investment opportunity. Firm $j's$ objective therefore is

$$V^j = E\left[\widetilde{\pi}^j\right] = \int_{\underline{\theta}}^{\overline{\theta}} \pi^j\left(\theta^j, q^i, q^j\right) f\left(\theta^j\right) d\theta^j$$

as before.

Differentiating firm $i's$ objective with respect to q^i now yields the first-order condition

$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_i^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i - f\left(\widehat{\theta}\right) \frac{\partial \widehat{\theta}}{\partial q_i} r\left(x - I\right) = 0$$

The second term of the derivative can be analyzed further. Using the implicit function theorem one finds

$$-f\left(\widehat{\theta}\right) \frac{\partial \widehat{\theta}}{\partial q_i} r\left(x - I\right) = f\left(\widehat{\theta}\right) \frac{\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)} r\left(x - I\right)$$

The sign of this expression is ambiguous. It will be positive when $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right) > 0$ and negative when $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right) < 0$. Notice however, that the sign of $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)$ will depend on the position of $\widehat{\pi}$. For higher $\widehat{\pi}$ it is more likely to be positive. To see this note that $\pi_{i\theta}^i\left(\widehat{\theta}, q^i, q^j\right) > 0$ and that

$$\frac{\partial \widehat{\theta}}{\partial \widehat{\pi}} = \frac{1}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)} > 0$$

Therefore the sign of $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)$ will be positive for high $\widehat{\pi}$ and negative for low $\widehat{\pi}$. Moreover, when, as assumed $f'(\theta) = 0$ and $\pi_{\theta\theta}^i = 0$, one can show a monotone relationship between the size of the second term of the derivative and the position of $\widehat{\pi}$.

$$\begin{aligned}
& \frac{\partial}{\partial \widehat{\pi}}\left(-f\left(\widehat{\theta}\right) \frac{\partial \widehat{\theta}}{\partial q^i} r\left(x - I\right)\right) \\
= \; & -f\left(\widehat{\theta}\right) r\left(x - I\right) \frac{\partial \widehat{\theta}}{\partial q^i \partial \widehat{\pi}} - f'\left(\widehat{\theta}\right) r\left(x - I\right) \frac{\partial \widehat{\theta}}{\partial \widehat{\pi}} \frac{\partial \widehat{\theta}}{\partial q^i} \\
= \; & f\left(\widehat{\theta}\right) r\left(x - I\right) \frac{\pi_{i\theta}^i\left(\widehat{\theta}, q^i, q^j\right) \frac{\partial \widehat{\theta}}{\partial \widehat{\pi}} \pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right) - \pi_{\theta\theta}^i\left(\widehat{\theta}, q^i, q^j\right) \frac{\partial \widehat{\theta}}{\partial \widehat{\pi}} \pi_i^i\left(\widehat{\theta}, q^i, q^j\right)}{\left(\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)\right)^2} \\
= \; & f\left(\widehat{\theta}\right) r\left(x - I\right) \frac{\pi_{i\theta}^i\left(\widehat{\theta}, q^i, q^j\right) \frac{\partial \widehat{\theta}}{\partial \widehat{\pi}} \pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)}{\left(\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)\right)^2} > 0
\end{aligned}$$

Thus under the assumptions made, the size of the second term is increasing in $\widehat{\pi}$.

The second order condition is

$$\frac{\partial}{\partial q^i}\left\{\int_{\underline{\theta}}^{\overline{\theta}} \pi_i^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i + f\left(\widehat{\theta}\right) \frac{\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)} r\left(x-I\right)\right\}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \pi_{ii}^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i + \frac{\partial}{\partial q^i}\left\{f\left(\widehat{\theta}\right) \frac{\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)} r\left(x-I\right)\right\} < 0$$

The first term is negative since $\pi_{ii}^i < 0$. Given that $f'\left(\widehat{\theta}\right) = 0$ the second term is equal to

$$f\left(\widehat{\theta}\right) \frac{\left[\pi_{ii}^i\left(\widehat{\theta}, q^i, q^j\right) + \pi_{i\theta}^i \frac{\partial\widehat{\theta}}{\partial q^i}\right] \pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)^2} r\left(x-I\right)$$

$$-f\left(\widehat{\theta}\right) \frac{\left[\pi_{\theta i}^i\left(\widehat{\theta}, q^i, q^j\right) + \pi_{\theta\theta}^i\left(\widehat{\theta}, q^i, q^j\right) \frac{\partial\widehat{\theta}}{\partial q^i}\right] \pi_i^i\left(\widehat{\theta}, q^i, q^j\right)}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)^2} r\left(x-I\right)$$

Assume that $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right) > 0$ at the point at which the first order condition holds. This implies that $\frac{\partial\widehat{\theta}}{\partial q^i} < 0$ so that given $\pi_{\theta\theta}^i = 0$ this expression has a negative sign and the second-order condition holds. When $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right) < 0$, so that $\frac{\partial\widehat{\theta}}{\partial q^i} > 0$ at the point at which the first order condition holds, this expression cannot be signed. In order for the second-order condition to hold, one has to make appropriate assumptions on the relative sizes of the first and the second term. One can guarantee that the second order condition holds, by assuming that $f\left(\widehat{\theta}\right)$ is small.

Next consider equilibrium quantities. When $\widehat{\pi}$ is high enough, such that $\pi_i^i\left(\widehat{\theta}, q^c, q^c\right) > 0$ one will have

$$V_i^i = \int_{\underline{\theta}}^{\overline{\theta}} \pi_i^i\left(\theta^i, q^c, q^c\right) f\left(\theta^i\right) d\theta^i + f\left(\widehat{\theta}\right) r\left(x-I\right) \frac{\pi_i^i\left(\widehat{\theta}, q^c, q^c\right)}{\pi_\theta^i\left(\widehat{\theta}, q^c, q^c\right)} > 0$$

Provided that the intersection of the reaction functions is stable, which requires that $V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji} > 0$, this implies that $q^i > q^c$ and $q^j < q^c$ in equilibrium. On the other hand, when $\widehat{\pi}$ is low, such that $\pi^i_i\left(\widehat{\theta}, q^c, q^c\right) < 0$ one will have

$$V^i_i = \int_{\underline{\theta}}^{\overline{\theta}} \pi^i_i\left(\theta^i, q^c, q^c\right) f\left(\theta^i\right) d\theta^i + f\left(\widehat{\theta}\right) r\left(x - I\right) \frac{\pi^i_i\left(\widehat{\theta}, q^c, q^c\right)}{\pi^i_\theta\left(\widehat{\theta}, q^c, q^c\right)} < 0$$

so that given reaction function stability one has $q^i < q^c$ and $q^j > q^c$ in equilibrium.

For a proof assume first that $\widehat{\pi}$ is such that $\pi^i_i\left(\widehat{\theta}, q^c, q^c\right) = 0$ and then consider a variation in $\widehat{\pi}$. Totally differentiating the system of first-order conditions one has

$$\begin{aligned} V^i_{ii}dq_i + V^i_{ij}dq_j + V^i_{i\widehat{\pi}}d\widehat{\pi} &= 0 \\ V^j_{ji}dq_i + V^j_{jj}dq_j + V^j_{j\widehat{\pi}}d\widehat{\pi} &= 0 \end{aligned}$$

Note that $V^j_{j\widehat{\pi}} = 0$. Then one can solve for comparative statics effects by using Cramer's rule to get

$$\begin{aligned} \frac{dq_i}{d\widehat{\pi}} &= -\frac{V^i_{i\widehat{\pi}}V^j_{jj}}{V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji}} > 0 \\ \frac{dq_j}{d\widehat{\pi}} &= \frac{V^i_{i\widehat{\pi}}V^j_{ji}}{V^i_{ii}V^j_{jj} - V^i_{ij}V^j_{ji}} < 0 \end{aligned}$$

using

$$V^i_{i\widehat{\pi}} = \frac{\partial}{\partial\widehat{\pi}}\left(-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q_i} r\left(x - I\right)\right) > 0$$

as shown above.

Let us summarize these findings in the following

Proposition 1 *When $\widehat{\pi}$ is high enough such that $\pi^i_i\left(\widehat{\theta}, q^c, q^c\right) > 0$ one will have $q^i > q^c$ and $q^j < q^c$ in equilibrium. When $\widehat{\pi}$ is such that $\pi^i_i\left(\widehat{\theta}, q^c, q^c\right) = 0$ one will have $\left(q^i, q^j\right) = \left(q^c, q^c\right)$. When $\widehat{\pi}$ is low enough such that $\pi^i_i\left(\widehat{\theta}, q^c, q^c\right) <$ 0 one will have $q^i < q^c$ and $q^j > q^c$ in equilibrium.*

This is the main result of this paper and is shown graphically in Figure 2. The intuition for this result is the following. First, observe that, when $\pi^i_\theta\left(\theta^i, q^i, q^j\right) > 0$ and $\pi^i_{\theta i}\left(\theta^i, q^i, q^j\right) > 0$, an increase in q_i induces an increase in the spread or variance of $\widetilde{\pi}^i$. For larger q^i, a given swing in θ^i will translate into a larger swing in $\widetilde{\pi}^i$. This is true under the assumptions made on the way uncertainty enters the profit function, and is plausible. It says that, as the firm increases output, it exposes itself more to the underlying demand or cost uncertainty. To see how this follows from the assumptions, note that

$$\begin{aligned} var\left[\widetilde{\pi}^i\right] &= E\left[\left(\widetilde{\pi}^i - E\left[\widetilde{\pi}^i\right]\right)^2\right] \\ &= E\left[\left(\widetilde{\pi}^i\right)^2\right] - \left[E\left[\widetilde{\pi}^i\right]\right]^2 \end{aligned}$$

so that

$$\begin{aligned} \frac{\partial var\left[\widetilde{\pi}^i\right]}{\partial q^i} &= 2E\left[\widetilde{\pi}^i\widetilde{\pi}^i_i\right] - 2E\left[\widetilde{\pi}^i\right]E\left[\widetilde{\pi}^i_i\right] \\ &= 2cov\left[\widetilde{\pi}^i, \widetilde{\pi}^i_i\right] \end{aligned}$$

Both $\pi^i\left(\widehat{\theta}, q^i, q^j\right)$ and $\pi^i_i\left(\widehat{\theta}, q^i, q^j\right)$ are increasing in θ since by assumption $\pi^i_\theta\left(\theta^i, q^i, q^j\right) > 0$ and $\pi^i_{i\theta}\left(\theta^i, q^i, q^j\right) > 0$. Therefore $cov\left(\widetilde{\pi}^i, \widetilde{\pi}^i_i\right) > 0$, which implies that the variance of $\widetilde{\pi}^i$ is increasing in q^i.

The firm's objective function can be written as

$$E\left[\widetilde{\pi}^i\right] - F\left(\widehat{\pi}\right) r\left(x - I\right)$$

or equivalently as

$$(1-r)\left[E\left[\widetilde{\pi}^i\right] + x - I\right] + r\left[E\left[\widetilde{\pi}^i\right] + \left(1 - F\left(\widehat{\pi}\right)\right)\left(x - I\right)\right]$$

The firm obtains the benefit of being able to invest as a high type for realisations of $\widetilde{\pi}^i$ such that $\pi^i > \widehat{\pi}$.

When $\widehat{\pi}$ is in the right tail of the distribution of $\widetilde{\pi}^i$, the firm can increase the probability of investment by increasing the variance of $\widetilde{\pi}^i$. This creates an incentive to increase q^i.

On the other hand, when $\widehat{\pi}$ is low enough to be in left tail of the distribution of $\widetilde{\pi}^i$, the firm benefits from reducing the variance of $\widetilde{\pi}^i$, since in this case it is a reduction in variance that will increase the probability of realisations larger than $\widehat{\pi}$. Therefore, in this case there is an incentive to reduce q^i

5.1.1 Comparative statics

In our setup $\widehat{\pi}$ is an endogenous variable. It is therefore interesting to explore how the product market equilibrium is influenced by the factors determining $\widehat{\pi}$. Recall that

$$I-(x-I)\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)^{-1}=\widehat{\pi}$$

It was pointed out before that $\widehat{\pi}$ is increasing in the dilution factor

$$\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)\equiv T$$

Note that the size of the dilution factor depends on the uncertainty associated with the value of the assets in place. Consider subjecting the distribution of $\widetilde{s}$ to a mean preserving spread such that s_H is increased by some ε and s_L is decreased by $\frac{r}{(1-r)}\varepsilon$. This will leave $E[\widetilde{s}]=\left[r(s_H+\varepsilon)+(1-r)\left(s_L-\frac{r}{(1-r)}\varepsilon\right)+x\right]$ unchanged but will increase the numerator to $[s_H+x]+\varepsilon$. Larger uncertainty in the sense of a mean preserving spread will therefore increase $\widehat{\pi}$. The cut-off will move towards the right tail of the profit distribution, creating a stronger incentive to increase the variance. This will cause firm i to compete more aggressively and will result in a larger q^i and a smaller q^j. Thus one can see that larger uncertainty may actually benefit firm i, in that it leads to a lower rival quantity. Whether this competitive benefit of increased uncertainty is outweighed by the reduction in the probability of investment will depend on the exact parameter specification of the model.

Next consider an increase in the net present value of the project. The net present value is $x-I$. It can increase through an increase in x or a decrease in I. These effects are best analysed separately. Consider first an increase in x.

$$\frac{\partial}{\partial x}\left\{I-(x-I)\left(\frac{[s_H+x]}{[rs_H+(1-r)s_L+x]}-1\right)^{-1}\right\}=\frac{\partial}{\partial x}\left\{I-(x-I)T(x)^{-1}\right\}$$
$$= -T(x)^{-1}+xT(x)^{-2}\frac{\partial}{\partial x}T(x)$$
$$= -T(x)^{-1}+xT(x)^{-2}\frac{rs_H+(1-r)s_L-s_H}{[rs_H+(1-r)s_L+x]^2}<0$$

Next

$$\frac{\partial}{\partial I}\left\{I-(x-I)\left(\frac{[s_H+x]}{[rs_H+(1-r)s_L+x]}-1\right)^{-1}\right\}$$
$$= 1+\left(\frac{[s_H+x]}{[rs_H+(1-r)s_L+x]}-1\right)^{-1}>0$$

Therefore

$$\Delta(x-I)>0\Longrightarrow\Delta\widehat{\pi}<0$$

which is intuitive. When the net present value of the project is larger, it will exceed the dilution cost for a smaller cut-off. Whether an increase in the net present value of the project will lead firm i to compete more or less aggressively will not only depend on the effect on the cut-off. Recall that the term causing deviations from the Cournot equilibrium is

$$-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^i}r(x-I)$$

There will therefore be a direct and an indirect effect.

$$\frac{\partial}{\partial x}\left\{-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^i}r(x-I)\right\}=-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^i}r-f\left(\widehat{\theta}\right)r(x-I)\frac{\partial\widehat{\theta}}{\partial q^i\partial\widehat{\pi}}\frac{\partial\widehat{\pi}}{\partial x}$$

We know that

$$-f\left(\widehat{\theta}\right)r(x-I)\frac{\partial\widehat{\theta}}{\partial q^i\partial\widehat{\pi}}>0$$

Hence

$$-f\left(\widehat{\theta}\right)r\left(x-I\right)\frac{\partial\widehat{\theta}}{\partial q^{i}\partial\widehat{\pi}}\frac{\partial\widehat{\pi}}{\partial x}<0$$

When

$$-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^{i}}r<0$$

which will be the case when $q^i < q^c$ both effects go in the same direction. An increase in x will cause a further reduction in q^i. Intuitively, the cut-off moves further into the left tail of the profit distribution and the benefit from being above the cut-off is greater. Both give an incentive to reduce the variance at the expense of expected profit so that firm i will compete less aggressively in equilibrium.

When

$$-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^{i}}r>0$$

which will be the case when $q^i > q^c$ the net effect is ambiguous. Start from a situation where the cut-off is in the right tail of the distribution. As the net present value of the project increases, the benefit from being above the cut-off increases, which will give an incentive to increase the variance and cause firm i to compete more aggressively. On the other hand, as x increases the cut-off moves in and there is a reduced incentive to increase the variance so that firm i will have an incentive to compete less aggressively.

Finally, let us look at an increase in r. One finds

$$\begin{aligned}&\frac{\partial}{\partial r}\left\{I-(x-I)\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)^{-1}\right\}\\=\;&(x-I)\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)^{-2}\frac{\partial}{\partial r}\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)<0\end{aligned}$$

which says that an increase in r causes the cut-off $\widehat{\pi}$ to shift in. Again, however, there is a competing direct effect, so that the comparative static results are qualitatively the same as for an increase in x.

I have derived comparative statics results under the assumption that the pooling equilibrium is played whenever it exists. Let us briefly move to the assumption that the separating equilibrium is played whenever it exists. This means that up to a profit level of $\widehat{\widehat{\pi}}$, only the bad type invests. Recall that

$$I-(x-I)\left(\frac{s_H+x}{s_L+x}-1\right)^{-1}=\widehat{\widehat{\pi}}$$

Since we know that $\widehat{\widehat{\pi}}>\widehat{\pi}$ it is clear that firm $i's$ equilibrium quantity will be larger and firm $j's$ equilibrium quantity will be smaller than under the original assumption. Notice that this effect will work against the usual notion that the pooling equilibrium is superior to the separating equilibrium. In our framework, lost investment efficiency will be partly recovered by more aggressive product market behaviour, causing a reduction in rival output and thus benefiting firm i.

The comparative static effects with respect to an increase in the uncertainty regarding s and with respect to an increase in the net present value are qualitatively unchanged. Notice that $\widehat{\widehat{\pi}}$ is not a function of r. Intuitively this comes about since $\widehat{\widehat{\pi}}$ is the profit level such that the high type would invest, even if the market believed that the firm was low type with probability one. An increase in r would therefore only have a direct effect, which would reinforce the deviation of firm $i's$ quantity from the Cournot level.

5.2 An alternative profit function

To explore further the intuition that the cut-off profit level creates incentives to manipulate the variance of the profit distribution, I want to explore a different specification for the profit function of both firms. In particular, I want to assume that profit is represented by

$$\pi^i\left(\tilde{z}^i,q^i,q^j\right)=\pi^i\left(\tilde{a},\widetilde{\theta}^i,q^i,q^j\right)=\left(\tilde{a}-b\left(q^i+q^j\right)\right)q^i-cq^i+\widetilde{\theta}^i$$

There is demand uncertainty, which is assumed to have a two-point distribution. Demand can be high or low, $\tilde{a}\epsilon\left\{\underline{a},\overline{a}\right\}$ and the probability that it is high is denoted by $\Pr[a=\overline{a}]=p$. In addition, there is some additive noise which I assume to have a zero mean normal distribution, $\widetilde{\theta}^i \sim N(0,\sigma)$ [11].

Firm $i's$ objective is to maximise

$$V^i = E\left[\widetilde{\pi}^i\right] - F(\widehat{\pi})\, r(x-I)$$

Denote the conditional means of the profit distribution given a high demand and a low demand realisation, respectively by

$$\begin{aligned}\overline{\pi} &= \left(\overline{a} - b\left(q^i+q^j\right)\right)q^i - cq^i \\ \underline{\pi} &= \left(\underline{a} - b\left(q^i+q^j\right)\right)q^i - cq^i\end{aligned}$$

One can then write down $F(\widehat{\pi})$ explicitly as

$$\begin{aligned}F(\widehat{\pi}) &= pF(\widehat{\pi}\mid\overline{\pi}) + (1-p)\,F(\widehat{\pi}\mid\underline{\pi}) \\ &= p\int_{-\infty}^{\widehat{\pi}} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\overline{\pi})^2}{2\sigma^2}}dv + (1-p)\int_{-\infty}^{\widehat{\pi}} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\underline{\pi})^2}{2\sigma^2}}dv\end{aligned}$$

The first-order condition for firm $i's$ problem is

$$V_i^i = \frac{\partial}{\partial q^i}\left\{E\left[\widetilde{\pi}^i\right] - F(\widehat{\pi})\, r(x-I)\right\} = 0$$

which can be written as

$$p\left(1 - \frac{\partial F(\widehat{\pi}\mid\overline{\pi})}{\partial\overline{\pi}} r(x-I)\right)\frac{\partial\overline{\pi}}{\partial q^i} + (1-p)\left(1 - \frac{\partial F(\widehat{\pi}\mid\underline{\pi})}{\partial\underline{\pi}} r(x-I)\right)\frac{\partial\underline{\pi}}{\partial q^i} = 0$$

One finds

$$\frac{\partial F(\widehat{\pi}\mid\overline{\pi})}{\partial\overline{\pi}} = \int_{-\infty}^{\widehat{\pi}} \frac{1}{\sigma\sqrt{2\pi}} \frac{(v-\overline{\pi})}{\sigma^2} e^{\frac{-(v-\overline{\pi})^2}{2\sigma^2}}dv$$

Since $\widetilde{\pi}^i = \overline{\pi} + \widetilde{\theta}^i$ one can change the variable of integration to $\theta = v - \overline{\pi}$ to get

$$\frac{\partial F(\widehat{\pi}\mid\overline{\pi})}{\partial\overline{\pi}} = \int_{-\infty}^{\widehat{\pi}-\overline{\pi}} \frac{1}{\sigma\sqrt{2\pi}} \frac{\theta}{\sigma^2} e^{\frac{-\theta^2}{2\sigma^2}}d\theta$$

[11] It is easiliy seen that this specification satisfies A1*.

$$
\begin{aligned}
&= -\int_{-\infty}^{\widehat{\pi}-\overline{\pi}} \frac{1}{\sigma\sqrt{2\pi}} \frac{-\theta}{\sigma^2} e^{\frac{-\theta^2}{2\sigma^2}} d\theta \\
&= \left| -\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\theta^2}{2\sigma^2}} \right|_{-\infty}^{\widehat{\pi}-\overline{\pi}} \\
&= -\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\left(\widehat{\pi}-\overline{\pi}\right)^2}{2\sigma^2}} = -f\left(\widehat{\pi} \mid \overline{\pi}\right)
\end{aligned}
$$

Similarly, one finds

$$
\frac{\partial F\left(\widehat{\pi} \mid \underline{\pi}\right)}{\partial \underline{\pi}} = -\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\left(\widehat{\pi}-\underline{\pi}\right)^2}{2\sigma^2}} = -f\left(\widehat{\pi} \mid \underline{\pi}\right)
$$

so that one has

$$
\begin{aligned}
V_i^i &= p\left(1 + \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\left(\widehat{\pi}-\overline{\pi}\right)^2}{2\sigma^2}} r\left(x-I\right)\right) \frac{\partial \overline{\pi}}{\partial q^i} \\
&\quad + \left(1-p\right)\left(1 + \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\left(\widehat{\pi}-\underline{\pi}\right)^2}{2\sigma^2}} r\left(x-I\right)\right) \frac{\partial \underline{\pi}}{\partial q^i} \\
&= p\left(1 + f\left(\widehat{\pi} \mid \overline{\pi}\right) r\left(x-I\right)\right) \frac{\partial \overline{\pi}}{\partial q^i} + \left(1-p\right)\left(1 + f\left(\widehat{\pi} \mid \underline{\pi}\right) r\left(x-I\right)\right) \frac{\partial \underline{\pi}}{\partial q^i}
\end{aligned}
$$

Similarly, one can show that second-order condition can be written as

$$
\begin{aligned}
V_{ii}^i &= p\left(1 + f\left(\widehat{\pi} \mid \overline{\pi}\right) r\left(x-I\right)\right) \frac{\partial^2 \overline{\pi}}{\partial q^{i2}} + \left(1-p\right)\left(1 + f\left(\widehat{\pi} \mid \underline{\pi}\right) r\left(x-I\right)\right) \frac{\partial^2 \underline{\pi}}{\partial q^{i2}} \\
&\quad - p f'\left(\widehat{\pi} \mid \overline{\pi}\right) r\left(x-I\right)\left(\frac{\partial \overline{\pi}}{\partial q^i}\right)^2 - \left(1-p\right) f'\left(\widehat{\pi} \mid \underline{\pi}\right) r\left(x-I\right)\left(\frac{\partial \underline{\pi}}{\partial q^i}\right)^2 < 0
\end{aligned}
$$

Again, it is assumed that this is satisfied.

Let us go back to the first-order condition and consider equilibrium quantities. Start by evaluating $\overline{\pi}$ and $\underline{\pi}$ at the Cournot-point $\left(q^c, q^c\right)$. Then note that $\left(q^c, q^c\right)$ is such that

$$
p\frac{\partial \overline{\pi}}{\partial q^i} + \left(1-p\right)\frac{\partial \underline{\pi}}{\partial q^i} = 0
$$

Therefore $V_i^i > 0$ and firm i will have an incentive to increase its quantity beyond the Cournot level when $f\left(\widehat{\pi} \mid \overline{\pi}\right) > f\left(\widehat{\pi} \mid \underline{\pi}\right)$ at the Cournot-point.

There will be no incentive to deviate from the Cournot level, $V_i^i = 0$, when $f(\widehat{\pi} \mid \overline{\pi}) = f(\widehat{\pi} \mid \underline{\pi})$ and it will have an incentive to reduce its quantity, $V_i^i < 0$, when $f(\widehat{\pi} \mid \overline{\pi}) < f(\widehat{\pi} \mid \underline{\pi})$. Rearranging the condition one finds

$$f(\widehat{\pi} \mid \overline{\pi}) > f(\widehat{\pi} \mid \underline{\pi})$$

$\iff$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\left(\widehat{\pi}-\overline{\pi}\right)^2}{2\sigma^2}} > \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\left(\widehat{\pi}-\underline{\pi}\right)^2}{2\sigma^2}}$$

$\iff$

$$\frac{-\left(\widehat{\pi}-\overline{\pi}\right)^2}{2\sigma^2} > \frac{-\left(\widehat{\pi}-\underline{\pi}\right)^2}{2\sigma^2}$$

$\iff$

$$\left(\widehat{\pi}-\overline{\pi}\right)^2 < \left(\widehat{\pi}-\underline{\pi}\right)^2$$

$\iff$

$$\left|\widehat{\pi}-\overline{\pi}\right| < \left|\widehat{\pi}-\underline{\pi}\right|$$

When $\widehat{\pi} = \frac{1}{2}\left(\overline{\pi}+\underline{\pi}\right)$ one has $\left|\widehat{\pi}-\overline{\pi}\right| = \left|\widehat{\pi}-\underline{\pi}\right|$, so that $f(\widehat{\pi} \mid \overline{\pi}) = f(\widehat{\pi} \mid \underline{\pi})$. Thus when the cut-off is exactly halfway between $\underline{\pi}$ and $\overline{\pi}$, conditional den-

To complete the proof, one only needs to reemploy Cramer's Rule for the system of first-order conditions, just as we did in the last section.

The intuition is that when $\widehat{\pi} > \frac{1}{2}\left(\overline{\pi} + \underline{\pi}\right)$, it pays to increase the spread or variance of the profit distribution. Since at (q^c, q^c) one has $\frac{\partial \underline{\pi}}{\partial q^i} < 0$ and $\frac{\partial \overline{\pi}}{\partial q^i} > 0$ moving q^i up will decrease $\underline{\pi}$ and increase $\overline{\pi}$, which will make it more likely that $\pi > \widehat{\pi}$. Thus the key property of the profit function under study is again that the variance of the profit distribution is increasing in q^i. This is clear from the fact that the profit distribution satisfies

$$\begin{aligned} z' &> z \Rightarrow \pi^i\left(z', q^i, q^j\right) > \pi^i\left(z, q^i, q^j\right) \\ z' &> z \Rightarrow \pi^i_i\left(z', q^i, q^j\right) \geq \pi^i_i\left(z, q^i, q^j\right) \end{aligned}$$

so that

$$\frac{\partial var\left[\widetilde{\pi}\right]}{\partial q^i} = 2cov\left[\widetilde{\pi}^i, \widetilde{\pi}^i_i\right] \geq 0$$

as was argued above. It can also be seen more directly by noting that

$$\widetilde{\pi}^i = \left(\widetilde{a}^i - b\left(q^i + q^j\right)\right) q^i - cq^i + \widetilde{\theta}^i$$

implies

$$var\left[\widetilde{\pi}^i\right] = var\left[\widetilde{a}^i\right] q^{i2} + var\left[\widetilde{\theta}^i\right]$$

so that the variance of $\widetilde{\pi}^i$ is increasing in q^i. When the cut-off is in the right tail of the distribution, it will pay the firm to increase the variance of the distribution in a speculative attempt to increase the probability of the profit realisation exceeding the cut-off. This speculative behaviour has two effects. First, it will make it more likely that the firm invests in the second stage. Second, it will let the rival firm decrease its quantity in response to the more aggressive behaviour of firm i.

5.2.1 Comparative statics

As for the comparative statics with respect to $\widehat{\pi}$ one finds unambiguous results when $\overline{\pi} \geq \widehat{\pi} \geq \underline{\pi}$. In this case one finds

$$\frac{\partial^2}{\partial q^i \partial \widehat{\pi}} \left\{E\left(\pi\right) - F\left(\widehat{\pi}\right) r\left(x - I\right)\right\}$$

$$
\begin{aligned}
&= p\frac{1}{\sigma\sqrt{2\pi}}\frac{-\left(\widehat{\pi}-\overline{\pi}\right)}{\sigma^2}e^{\frac{-\left(\widehat{\pi}-\overline{\pi}\right)^2}{2\sigma^2}}r\left(x-I\right)\frac{\partial\overline{\pi}}{\partial q^i} \\
&\quad+\left(1-p\right)\frac{1}{\sigma\sqrt{2\pi}}\frac{-\left(\widehat{\pi}-\underline{\pi}\right)}{\sigma^2}e^{\frac{-\left(\widehat{\pi}-\underline{\pi}\right)^2}{2\sigma^2}}r\left(x-I\right)\frac{\partial\underline{\pi}}{\partial q^i} \\
&= pf'\left(\widehat{\pi}\mid\overline{\pi}\right)r\left(x-I\right)\frac{\partial\overline{\pi}}{\partial q^i}+\left(1-p\right)f'\left(\widehat{\pi}\mid\underline{\pi}\right)r\left(x-I\right)\frac{\partial\underline{\pi}}{\partial q^i}>0
\end{aligned}
$$

It is easy to see that the first-order condition implies that $\frac{\partial\overline{\pi}}{\partial q^i} > 0$ and $\frac{\partial\underline{\pi}}{\partial q^i} < 0$. Therefore when $\overline{\pi} \geq \widehat{\pi} \geq \underline{\pi}$ both terms are positive since the density $f\left(\cdot \mid \overline{\pi}\right)$ has positive slope at $\widehat{\pi}$, whereas the density $f\left(\cdot \mid \underline{\pi}\right)$ has negative slope at $\widehat{\pi}$. The overall comparative static effect is therefore positive. Just as in the last section it will therefore again be the case that more severe asymmetric information will lead firm i to compete more aggressively and firm j to respond by competing less aggressively.

6 Dividends

So far I have analysed a model where the firm did not actively seek to commit itself to a particular output strategy. The equilibrium outputs differ from the Cournot output simply because both firms anticipate firm i to face a financing problem in the future[12]. It should be pointed out, however, that an ex ante commitment to reduce financial slack may be valuable to firm i. Such a commitment could be brought about by the firm entering into a debt contract at $t = 0$, before it chooses its quantity. It could also be brought about by a commitment to a certain dividend policy. Thus imagine that the firm can, before it chooses its quantity, commit to pay out an amount d to existing shareholders at $t = 1$. Let us assume that the pooling equilibrium is played whenever it exists. Then, when the firm is of the high type, it will invest only if $\pi - d \geq \widehat{\pi}$, i.e. when $\pi \geq \widehat{\pi} - d$. A dividend payout of d increases the cut-off by that same amount. This will have two effects. First, it will

[12] This is in contrast to Brander and Lewis (1988) where debt financing precedes the product market competition stage and debt is issued for its commitment value only.

reduce the probability that the investment is taken. This negative effect has to be traded off against a positive effect. This comes about since with a higher cut-off, the firm has more of an incentive to increase the variance of the profit distribution by increasing its output. The dividend commits firm i to a more aggressive product market stance, which will lead its rival firm to reduce its quantity and thus benefit firm i. This trade-off may or may not lead to a strictly positive choice of d, depending on the exact parameter specification of the model.

7 Conclusion

I have analysed a two-period model where firms first compete in a product market and one of the firms then finances an investment opportunity under conditions of asymmetric information. Special care has been taken to analyse the full set of equilibria of the financing game and to motivate an equilibrium selection such that the probability that the firm invests is increasing in the amount of financial slack it has on hand. This introduces a cut-off into the firm's objective function, since it is only for profit realisations above the cut-off that the high type firm will issue and invest. Under these conditions, the firm will not simply maximise the expected value of profit. Rather, it will take into account the consequences its choice of the strategic variable has on the probability that the profit generated exceeds the cut-off. This will lead the firm to consider not only the first moment, but also the second moment of the profit distribution. When the cut-off $\widehat{\pi}$ is high, there is an incentive to speculate and to increase the variance of the profit distribution by increasing its output. The rival anticipates this and responds with a lower output, which will benefit the firm. When the cut-off $\widehat{\pi}$ is low, there is an incentive to hedge and to reduce the variance of the profit distribution by lowering output. The rival will take advantage of this and respond with a higher output, which will harm the firm. One of the main insights of this model is that the fact that a firm has to finance externally does not necessarily worsen the firm's competitive position. In contrast to models where a financially constrained firm faces predation by a deep pocketed rival, in our model a particularly severe financing problem may actually help the firm in making it more aggressive vis à vis its rival. This is an implication of the main comparative statics result of the model, which says that a larger degree of uncertainty, and thus a more severe adverse selection problem, will make the firm a more aggressive competitor as the firm strives to increase the probability of investment. This result may also lend itself to empirical testing. To the extent that smaller firms are surrounded by a larger degree of uncertainty than larger firms, so that size can be taken as a proxy for uncertainty, the model suggests that smaller firms should be more aggressive competitors than larger, more established firms. The results also suggest that smaller firms may be able to survive in an environment in which they are competing against larger firms, precisely because they face a more severe adverse selection problem. Finally, the implications of the model seem consistent with the empirical finding that conglomerates are trading at a discount when measured against "focused" firms[13]. One could argue that in contrast to conglomerates, focused firms have to face the external capital market more often, as there is less scope for cross-subsidisation. In addition, there may be a larger degree of uncertainty surrounding the smaller, focused firm than there is surrounding an established conglomerate. This would imply that more focused firms are the more aggressive competitors and may be one reason why they are more valuable.

[13] See for example Lang and Stulz (1994).

Appendix

Proof of Lemma 1:

For part a) note first that for all beliefs $\rho \epsilon [0,1]$ it is optimal for the low type L to set $d = 1$ and invest. To see this, note that the low type will set $d = 1$ whenever

$$s_L + \pi < \left(1 - \frac{I - \pi}{\rho s_H + (1-\rho) s_L + x}\right)[s_L + x]$$

By inspection the RHS of this condition is increasing in ρ. For $\rho = 0$ the condition reduces to

$$s_L + \pi < \left(1 - \frac{I - \pi}{[s_L + x]}\right)[s_L + x]$$

$\Longleftrightarrow$

$$s_L + \pi < s_L + x - (I - \pi)$$

$\Longleftrightarrow$

$$0 < x - I$$

and is satisfied because the investment opportunity has positive net present value by assumption. Hence $\delta_L = 1$ for all beliefs $\rho \epsilon [0,1]$. In any equilibrium the low type will invest.

Next note that this implies an upper bound on equilibrium belief ρ.

$$\rho = \frac{r\delta_H}{r\delta_H + (1-r)\delta_L} = \frac{r\delta_H}{r\delta_H + (1-r)} \leq r$$

Finally, consider the high type H. It is optimal for the high type to invest only if

$$s_H + \pi \leq \left(1 - \frac{I - \pi}{[\rho s_H + (1-\rho) s_L + x]}\right)[s_H + x]$$

Given the upper bound on ρ, and employing the fact that the RHS of this condition is increasing in ρ one has

$$\left(1 - \frac{I - \pi}{[\rho s_H + (1-\rho) s_L + x]}\right)[s_H + x] \leq \left(1 - \frac{I - \pi}{[r s_H + (1-r) s_L + x]}\right)[s_H + x]$$

for all possible equilibrium beliefs $\rho \epsilon [0,r]$. For $\pi < \widehat{\pi}$ one also has

$$s_H + \pi > \left(1 - \frac{I - \pi}{[r s_H + (1-r) s_L + x]}\right) [s_H + x]$$

so that

$$s_H + \pi > \left(1 - \frac{I - \pi}{[\rho s_H + (1-\rho) s_L + x]}\right) [s_H + x]$$

for all possible equilibrium beliefs $\rho \epsilon [0,r]$. This implies that whenever $\pi < \widehat{\pi}$ it will not be optimal for the high type to invest. Hence $\delta_H = 0$ in any equilibrium with $\pi < \widehat{\pi}$. Hence the unique equilibrium has $\delta_H = 0$ and $\delta_L = 1$, which implies $\rho = 0$ and α as shown.

For part b) note first that the argument regarding the equilibrium behaviour of the low type given for part a) goes through regardless of π. When $\pi \geq \widehat{\pi}$, therefore, again the low type will set $\delta_L = 1$ for all $\rho \epsilon [0,1]$. In the proposed pooling equilibrium the high type also always invests, so that $\delta_H = 1$. Given $\delta_L = 1$ and $\delta_H = 1$, the equilibrium belief must be $\rho = r$. When $\pi \geq \widehat{\pi}$ one has

$$s_H + \pi \leq \left(1 - \frac{I - \pi}{[r s_H + (1-r) s_L + x]}\right) [s_H + x]$$

so that it is indeed optimal for the high type to invest given beliefs. This completes the proof.

Proof of Lemma 2

Proof: For part a) recall that $\delta_L = 1$ is optimal for any beliefs. It therefore suffices to show that $\delta_H = 0$ is optimal given $\rho = 0$ and $\pi < \widehat{\widehat{\pi}}$. To see this, one needs to note only that when $\pi < \widehat{\widehat{\pi}}$ one has

$$s_H + \pi \geq \left(1 - \frac{I - \pi}{s_L + x}\right) [s_H + x]$$

For part b) I need to show that the pooling equilibrium is unique. Given that $\delta_L = 1$ is uniquely optimal for all beliefs and all profit levels, one needs to show only that whenever $\pi > \hat{\hat{\pi}}$ holds $\delta_H = 1$ is the only optimal choice for the high type for any belief $\rho \epsilon [0,1]$.

$\delta_H = 1$ is uniquely optimal when

$$s_H + \pi < \left(1 - \frac{I - \pi}{[\rho s_H + (1-\rho) s_L + x]}\right) [s_H + x]$$

When $\rho = 0$ this reduces to

$$s_H + \pi < \left(1 - \frac{I - \pi}{s_L + x}\right) [s_H + x]$$

which is satisfied since $\pi > \hat{\hat{\pi}}$. When $\rho > 0$

$$\left(1 - \frac{I - \pi}{[\rho s_H + (1-\rho) s_L + x]}\right) > \left(1 - \frac{I - \pi}{s_L + x}\right)$$

and the condition for unique optimality is again satisfied. Therefore $\delta_H = 1$ whenever $\pi > \hat{\hat{\pi}}$ which then implies that $\rho = r$ and α as shown. This completes the proof.

Proof of Lemma 3:

Proof: The equation which determines δ_H will deliver a solution $\delta_H \epsilon [0,1]$ if and only if $\pi \epsilon \left[\hat{\pi}, \hat{\hat{\pi}}\right]$. To see this, note first that with $\delta_H = 0$ one has

$$s_H + \pi = \left(1 - \frac{I - \pi}{[s_L + x]}\right) [s_H + x]$$

whereas with $\delta_H = 1$ one has

$$s_H + \pi = \left(1 - \frac{I - \pi}{[r s_H + (1-r) s_L + x]}\right) [s_H + x]$$

We know

$$\rho(\delta_H) = \frac{r\delta_H}{r\delta_H + (1-r)}$$

Hence

$$\rho'(\delta_H) = \frac{r[r\delta_H + (1-r)] - r\delta_H r}{[r\delta_H + (1-r)]^2} = \frac{r(1-r)}{[r\delta_H + (1-r)]^2} > 0$$

Totally differentiating the indifference condition with respect to π and δ_H one has

$$\left(1-\frac{[s_H+x]}{[\rho(\delta_H)s_H+(1-\rho(\delta_H))s_L+x]}\right)d\pi$$

$$-(I-\pi)[s_H+x]\frac{(s_H-s_L)\rho^{'}(\delta_H)}{[\rho(\delta_H)s_H+(1-\rho(\delta_H))s_L+x]^2}d\delta_H=0$$

This implies

$$\frac{d\pi}{d\delta_H}=\frac{(I-\pi)[s_H+x]\frac{(s_H-s_L)\rho^{'}(\delta_H)}{[\rho(\delta_H)s_H+(1-\rho(\delta_H))s_L+x]^2}}{\left(1-\frac{[s_H+x]}{[\rho(\delta_H)s_H+(1-\rho(\delta_H))s_L+x]}\right)}$$

Since the denominator is negative one has

$$\frac{d\pi}{d\delta_H}<0$$

This completes the proof.

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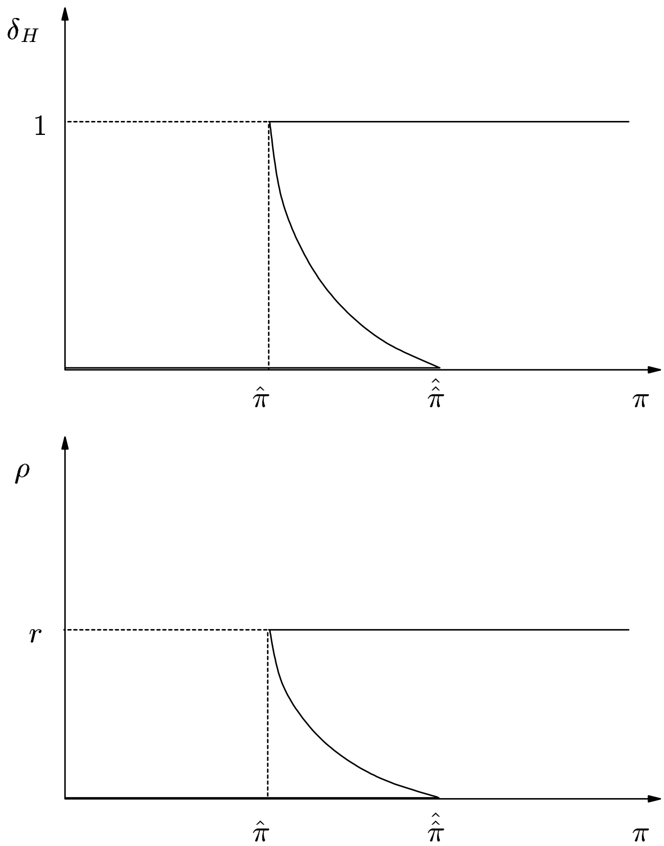

$$\rho = \frac{r\delta_H}{r\delta_H + (1-r)}$$

Figure 1: Second-Stage Equilibria

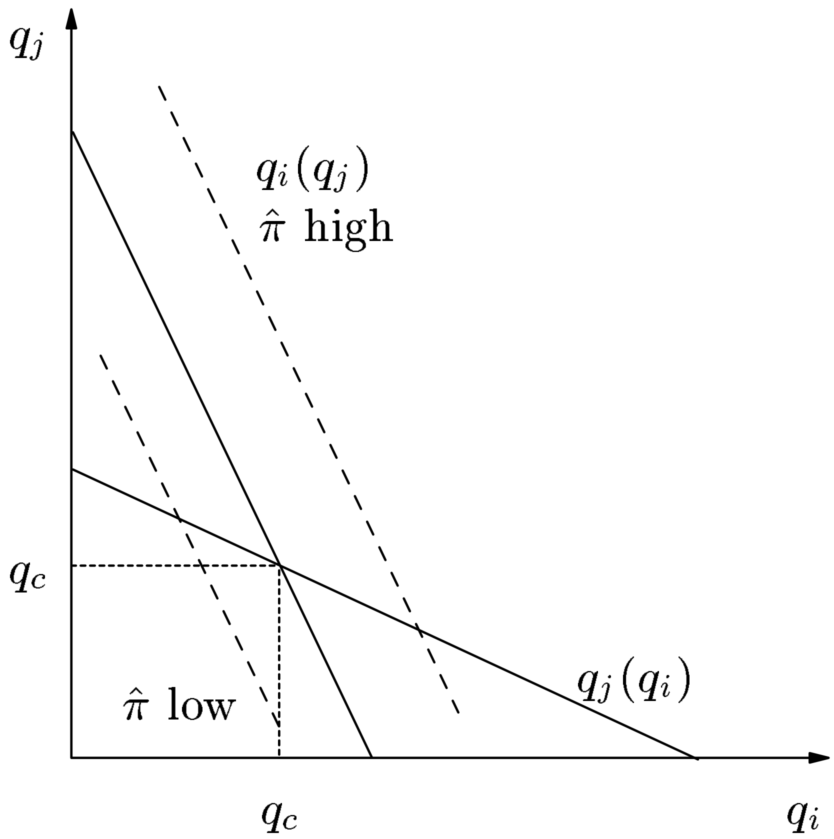

Figure 2: The First-Period Equilibrium

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