Collateral, Renegotiation and the Value of Diffusely Held Debt

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COLLATERAL, RENEGOTIATION AND THE VALUE OF DIFFUSELY HELD DEBT

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Abstract

Debt with many creditors is analyzed in a continuous-time pricing model of the levered firm. We specifically allow for debtor opportunism vis-a-vis a non-coordinated group of creditors, in form of repeated strategic renegotiation offers and default threats. We show that the creditors' initial entitlement to non-collateralized assets will be expropriated through exchange offers. Exchange offers successively increase the level of collateral until all assets are fully collateralized. The ex ante optimal debt contract is neither fully collateralized nor without any collateral. Diffusely held debt allows for a larger debt capacity and bears lower credit risk premia than privately held debt. We derive simple closed-form solutions for the value of equity and defaultable bonds. Numerical estimates show that the bond valuation is very sensitive to the correct specification of the debt renegotiation model.

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Keywords: Debt Reorganization, Multiple Creditors.

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Introduction

Recently, a growing body of literature has introduced corporate finance concepts into valuation models of defaultable securities, and endogenized variables such as the capital structure choice, the lower reorganization bound and the outcome of bargaining between debtor and creditors in fully dynamic models.[1] Yet in all the existing work incorporating capital structure theory or bargaining models into debt valuation theory, the number of creditors has been ignored and implicitly, a fiction has been invoked that the borrower is confronted with a single "representative" creditor.

The purpose of this paper is to explicitly model the strategic interaction between shareholders and creditors when there are *multiple creditors*. We study dynamic strategies of debt renegotiation and default in this environment and analyze the impact of the optimal opportunistic debtor strategy on the value of defaultable bonds and the efficient financing of projects.

There is little reason to assume that creditors would coordinate their responses to a renegotiation offer or a default threat: An individual creditor will prefer to free-ride on the debt restructuring effort of others, and the larger the number of creditors, the stronger this tendency to hold out.[2] Individual creditors are not inclined to make concessions, although they realize that doing so would be in their collective interest.[3] The importance of the hold-out effect is highlighted by numerous empirical studies showing that out-of-court debt restructurings with many creditors bear a substantial risk of failure.[4]

This does not imply, however, that diffusely held publicly traded debt is immune to

[1]Building on Black and Cox (1976), Leland (1994) and Leland and Toft (1996) endogenize the shareholders' decision to trigger liquidation and determine the optimal capital structure of firm. Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Mella-Barral (1999) extend the analysis to allow for the strategic interaction between shareholders and debtholders in debt renegotiation, prior to liquidation.

[2]More precisely, the theoretical literature shows that individual investors' incentives to hold out depends on the probability of being decisive or "pivotal" for success or failure of the tender offer. This probability depends on the relative size of a creditor's debtholdings - quite similar to the analogous effect emerging in takeover bids studied by Grossman and Hart (1980) and more rigorously by Holmstrom and Nalebuff (1992). Incomplete information is a necessary ingredient to obtain this result, but the prediction is fairly robust with respect to changes in the informational assumptions. For these results, see e.g. Detragiache and Garella (1996) and Hege (1999).

[3]The effective renegotiation-proofness of widely dispersed debt has inspired a number of theories about the choice of the number of creditors or the choice between private and publicly traded debt. In Bolton and Scharfstein (1996) and Berglöf and von Thadden (1994), contracting with two creditors rather than a single one or contracting with a complex debt structure commits the debtor to refrain from strategic default.

[4]Empirical work by Brown, James and Mooradian (1993)(1994), James (1995)(1996) Franks and Torous (1989)(1994), Gilson, John and Lang (1990), Gilson (1997), Asquith, Gertner and Scharfstein (1994), Helwege (1994), Chatterjee, Dhillon and Ramirez (1995) and Hotchkiss (1995) shows evidence in this respect.

renegotiation efforts. But it means that debt restructuring proposals must be engineered so as to spoil the attractiveness of the hold-out option. Offers can be successful if they are designed to dilute the value of creditors rejecting the offer. Opportunistic shareholders, if faced with a non-cohesive group of creditors, have powerful devices at hand to exert such dilution threats which are unavailable vis-a-vis a single creditor, namely strategies which essentially threaten to relocate wealth *between* creditors.

Dilution threats of this sort have in common that they *(i)* impose a scheme of wealth transfers *from creditor to creditor*, and *(ii)* make these transfers implicitly *conditional* on rejection of the debt restructuring proposal. These strategies are coercive since creditors stand to lose if they do not accept the restructuring proposal, relative to those who do. Creditors are made to rush in to tender, in particular if the number of new contracts is limited and they are served on a first-come-first-serve basis.

Debt-for-debt exchange offers proposing *more senior* or *secured* claims are the leading case of such dilution threats. Empirical literature suggests that they are in fact very common.[5] A well-known example are the so-called *"exit consents"*.[6] In an "exit consent", the right to participate in the exchange or tender offer is explicitly tied to a vote approving the exit from a seniority covenant restricting the issuance of more senior debt (Roe (1987)). Bondholders will then first rush in to waive the covenant to secure their right to exchange; once the covenant is stripped, each bondholder prefers to tender because if she were the only one to hold out, the liquidation value of her single junior claim as well as the secondary market value of a severely illiquid bond issue would suffer.

The paper examines the optimal debt renegotiation strategy of an opportunistic debtor facing a non-coordinated group of creditors in a continuous-time model of the levered firm. The set-up of the model closely follows Mella-Barral (1999), but is adapted to allow for multiple creditors. We allow for a rich set of actions at the discretion of the debtor and study strategies of repeated debt exchange offers and dilution threats. The exchange offer strategies available to shareholders follows closely the typical procedure in debt exchange offers, allowing the debtor to offer more senior claims or additional collateral in exchange for concessions in every round, as well as the possibility to default strategically. Importantly, each of these actions can be taken at any time, and as often as the debtor likes.

Our results show that the dynamic dimension of the model is crucial: The debtor is limited

[5]In various samples compiled about US exchange offers, we found the following number indicating fractions where more senior debt is offered: James (1996) 64 %, Gertner and Scharfstein (1991) at least 28 %, Chatterjee et.al. (1995) 76 %, Brown et.al. (1993) 43 % (research questions and sample selection criteria differ widely).

[6]In the sample by Chatterjee et.al. (1995), 50 % of the tender offers and 33 % of the exchange offers contain exit consents.

in obtaining concessions from creditors precisely because subsequent debt renegotiations are possible. We show that the debtor can only obtain concessions if she can credibly limit the value of additional concessions she could obtain later on. We consider the most obvious device to commit to such a limit, which is to offer *additional collateral* in each renegotiation round in order to make the debt exchange offer acceptable.

We solve for the shareholder's ex post optimal exchange offer strategy, and show that the shareholder will successively trade coupon concessions for increases in collateral values, until all assets are pledged as collateral. Creditors' initial entitlement to a share of the non-collateralized assets (by virtue of the Absolute Priority Rule) turns out to have simply no value since it will subsequently be expropriated through an opportunistic debtor's exchange offer strategy.

Moving backwards in time we then analyse the ex ante optimal contract with dispersed creditors. We first observe that if all assets are collateralized in the initial contract, then debt is not renegotiable, and our set-up is then akin to Leland's (1994) model. We establish that with fully collateralized debt contracts, there is a unique initial debt level leading to the first best firm value. We then find that such a design is optimal, whenever the shareholder's needs to raise funds remain *below* the maximal amount compatible with her not having incentives to default earlier than optimal.

Conversely, if the funding needs of the shareholder are *larger* than this threshold amount, then the optimal dispersed debt contract is renegotiable: It essentially provides for a relatively large coupon as long as the firm does well, but embeds the possibility to obtain coupon concessions once the firm's performance deteriorates.

More precisely, the debt contract must then be designed so as to commit the shareholder to trigger her exchange offers in an optimal fashion. Depending on the initial level of the coupon, this involves the design of an optimal level and dynamic evolution of the collateral values, for which we identify the following trade-off: on the one hand, too high a collateralization would not leave enough flexibility to renegotiate debt ex post; on the other hand, if too little of the assets are tied up as collateral when exchange offers are launched early, then the shareholder would have incentives to trigger exchange offers prematurely.

We also examine the desirability of dispersed debt issuance over cohesively held (private) debt: Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Mella-Barral (1999) document the fact that with a single creditor, concessions can be forced with a strategic default threat, i.e. the shareholder's threat to cease debt service payments and walk away. Howev are the only vehicle to engineer debt concessions.

The choice of creditor dispersion therefore involves a comparison of different drivers of contingent coupon concessions. We show that borrowing from dispersed creditors is attractive from an issuer's point of view, as *(i)* her capacity to raise funds is always higher and *(ii)* the credit risk premium is lower than if she borrowed privately.

We analyze the pricing and efficiency implications of these strategies and derive simple closed-form solutions for the value of equity and defaultable bonds. Numerical simulations show that with widely held debt, the default risk premium may be reduced by a large margin as compared to single-creditor debt. Using a wrong debt model can lead to substantial pricing errors: Default risk premia and credit spreads are sensitive to whether the debt model specification correctly accounts for the *multiplicity of creditors* and/or *initial collateralization* of the debt.

We present the set-up in Section I. In Section II, we define exchange offer strategies and explain the mechanism of dilution. In Section III, we solve for the shareholder's ex post optimal strategy. In Section IV, we determine the consequences for the creditors' willingness to lend at entry and for the choice of the collateral structure. In Section V, we provide closed form solutions and study a numerical example. Section VI discusses possible extensions and concludes.

I. The Model

A. Operations and the Abandonment Decision

Consider a firm, set up by a person called the incumbent or shareholder at date $t = 0$, which purchases a set of real assets worth I. The cash generating ability of these real assets is related to a single uncertain state variable, x_t, which summarizes economic fundamentals, and follows a diffusion process:

$$dx_t \quad = \quad \mu(x_t)\,dt \quad + \quad \sigma(x_t)\,dB_t\,, \tag{1}$$

where B is a standard Brownian motion. Once the firm is set up, the incumbent can do the following:

1. She can generate a period income flow, combining her human capital and protected technology with the purchased real assets. Let $\Pi(x_t)$ denote the present value of a perpetual claim on the income flow that results from such *operations*, assuming no limited liability.

2. Although she could operate the firm forever, she can also *abandon* operations, and sell the firm's real assets. Let $V^*(x_t)$ denote the liquidation value of the assets.

We assume that an abandonment decision is *irreversible*. The abandonment decision is thus best viewed as the decision to liquidate the firm or to file for bankruptcy. This set-up, however, allows for a wider interpretation of this irreversible decision. For example, it allows to capture aspects of the property rights view of the firm: abandonment means then that relation-specific investments with a reduced value outside the firm are dismantled and parts of the cash generating ability of the firm is lost, and irreversibility means that the restoring of the combination of human and physical capital, after a period of abandonment, is not costless.

Whatever the preferred interpretation, we assume that there are some states of the world x where other parties, like competitors, have a better use for the assets than the incumbent. In these poor states, $V^*(x)$ is actually greater than $\Pi(x)$ and the abandonment decision is desirable, as formalized by Assumption 1 below. We assume that $\Pi(x)$ is increasing in x; this is not necessarily the case for $V^*(x)$. All assets are assumed to be tangible in the sense that all of $V^*(x)$ can be pledged as collateral.

B. Value of the Firm under the First Best Abandonment Policy

The value of the firm, for a given closure policy, is readily obtained. If operations are abandoned the first time the state variable x_t reaches a lower level y, the firm is worth

$$V(x_t \mid y) \quad \equiv \quad \Pi(x_t) \; + \; [V^*(y) - \Pi(y)] \;\; \mathcal{P}(x_t \rhd y) \; . \tag{2}$$

The first term on the right hand side, $\Pi(x_t)$, is the value of a perpetual entitlement on the current flow of income. The second term is the product of the change in asset value intervening when the irreversible regime switch occurs, $[V^*(y) - \Pi(y)]$, and a probability-weighted discount factor for this event, $\mathcal{P}(x_t \rhd y)$ which we now define.

We assume risk neutrality and a constant safe interest rate, ρ.[7] We denote by $T \equiv \inf\{\, \tau \mid x_\tau = y \,\}$ the first time at which the state variable x_t hits the level y, and by $f_t(T)$ the density of T conditional on information at t. Then the probability-weighted discount factor $\mathcal{P}(x_t \rhd y)$ is just the Laplace transform of $f_t(T)$

$$\mathcal{P}(x_t \rhd y) \quad = \quad \int_t^\infty e^{-\rho(T-t)} f_t(T) \; dT \; . \tag{3}$$

Clearly, the first best policy consists of selecting the abandonment trigger level, y, in order to maximize $V(x_t \mid y)$. The ex ante *optimal* abandonment trigger level, which we denote $\tilde{y}$, must therefore satisfy the first order condition

$$\frac{\partial V(x_t \mid \tilde{y})}{\partial \tilde{y}} \quad = \quad 0 \; . \tag{4}$$

[7] Harrison and Kreps (1979) show how to extend the results of the paper to a world, without risk-neutrality by using an equivalent martingale measure.

Sufficient conditions for the existence and uniqueness of the optimal abandonment trigger level $\tilde{y}$ are guaranteed by:

Assumption 1 *At the entry state x_0, the option value of the decision to trigger liquidation at y,*

$$[V^*(y) - \Pi(y)] \ \mathcal{P}(x_0 \triangleright y), \tag{5}$$

is a strictly concave function in y, maximized at a trigger level $\tilde{y}$ strictly smaller than x_0.

We assume that the project is actually worthwhile undertaking at the entry state x_0, i.e. the initial investment I is less than the value of the firm under the first best policy, $V(x_0 \mid \tilde{y})$. Thus, the incumbent has the best use for the assets in the good states, but in low states of the world it becomes eventually optimal for the incumbent to abandon the firm. *Figure 1* illustrates this set-up.

This structural model, consisting of *(i)* the firm and its project, $\{\Pi(x); I; x_0\}$ and *(ii)* its uncertain environment, $\{x; \mu(x); \sigma(x); \rho; V^*(x)\}$ is expressed in rather general terms. In Section V, we will consider a standard parametrization of the model, which will permit to derive closed-form solutions for the securities values and the key variables.

The set-up so far is identical to Mella-Barral (1999). This is deliberate since it will allow for a direct comparison of the results, and hence for an analysis of the differences between a firm choosing to finance with private debt and a firm issuing publicly traded debt. We will next adapt Mella-Barral's model to allow for multiple creditors.

C. Financial Contracts

In order to finance the initial investment, I, the incumbent needs to seek external financing since she has only limited wealth. Denote by $I_D \leq I$ the amount that needs to be financed externally. Only debt contracts are available[8], and they are restricted to the form $\mathcal{D}_0 \equiv \{\delta_0; C_0^*(x)\}$:

1. A promise of a perpetual flow of coupon payments, δ_0.

2. The right, if the incumbent repudiates the contract, to impose a prespecified sharing of the liquidation proceeds $V^*(y)$ (invoking debt collection law). The details of this sharing rule are as follows:

[8]This assumption should be thought of as being the consequence of the following implicit assumptions: x_t, $\Pi(x_t)$ and $V^*(x_t)$ are only verifiable in the case of liquidation, and it is impossible (or prohibitively costly) to write contracts conditional on x_t, $\Pi(x_t)$, $V^*(x_t)$ or past repayments. It is beyond the scope of this paper to derive the optimality of debt contracts, and we refer instead to the security design literature showing when debt contracts are optimal, including the costly state verification model pioneered by Townsend (1979) and incomplete contract models as e.g. in Hart and Moore (1998).

(a) A portion $C_0^*(x)$ of the proceeds of the liquidation sale, $V^*(x)$, is secured by collateral.

(b) Each debtholder is entitled to a par value, $P = \delta_0/\rho$, before the incumbent receives anything. The contract is subject to the Absolute Priority Rule.

The only element of this contract which is more general than a standard debt contract is the function $C_0^*(x)$ which is referred to as the *initial collateral.* $C_0^*(x)$ specifies each creditor's collateral value as a function of the total liquidation proceeds, $V^*(x)$.

We assume that there are N bonds issued and that each creditor holds only one bond. The number of creditors N is so large that each creditor will behave atomistically, and in particular completely neglect his impact on success or failure of a debt restructuring proposal.[9]

Trading of assets occurs continuously in perfect and frictionless markets with no asymmetry of information. Furthermore, we abstract from the insider-outsider agency conflict between shareholders and management and assume that the incumbent maximizes the shareholder value.

II. Multiple Creditors and Debt Renegotiation

A. Shareholder Opportunism

The need to issue debt opens up a basic conflict of interest between incumbent and outside investors. The incumbent has residual control rights,[10] i.e. the right to freely decide on the use of the assets as long as she meets her contractual obligations. The final control decision appertaining to the shareholder is the selection of the abandonment trigger level, y.

Related to this final choice, the shareholder decides in continuous time whether and when to renegotiate the debt contract. She decides on a sequence of offers launched to obtain concessions from the creditors, possibly supported by (credible) strategic default threats. The decisions on the renegotiation offers and the final abandonment decision are interdependent since the total amount of concessions on the debt services determines when the shareholder will find it optimal to trigger the abandonment decision.

The shareholder maximizes solely the value of her equity and acts in a purely opportunistic fashion vis-a-vis the bondholders. She anticipates fully the impact of her renegotiation offers on her choice of abandonment trigger level, y. If the incumbent could finance the entire

[9]This is a standard assumption since Grossman and Hart (1980).

[10]This terminology follows Grossman and Hart (1986) and Hart and Moore (1990).

initial investment, then the first best abandonment trigger level, $\tilde{y}$, would easily be implemented. An agency problem arises, however, because the incumbent has limited wealth, implying that the resulting value of the firm, $V(x_t \mid y)$, could be below the first best value $V(x_t \mid \tilde{y})$.

B. Exchange Offer Strategies and Dilution Threats

Dispersed creditors (bondholders) act as a non-coordinated group in our model. This has two important effects on debt renegotiation: first, in contrast to a large creditor, a small individual creditor will not necessarily accept a Pareto-improving exchange offer since if enough of the other creditors accept, he might be better off by holding out. Second, the shareholder has the possibility to exploit the *non-cohesiveness* of the creditors by attaching *dilution threats* to an exchange offer.

To account as much as possible for the shareholder's options, we endow the shareholder with a rich set of strategies to pursue coercive strategies via dilution threats which we call the set of *exchange offer strategies.* A single *exchange offer* is a proposal of a limited number of new debt contracts in exchange for voluntary surrender of old contracts. This follows closely the typical procedure in debt exchange offers. We define the set of exchange offer strategies formally as follows:

Definition 1 *An exchange offer strategy,* $\mathsf{s} \equiv \{(\underline{x}_k, n_k, \mathcal{D}_k) \mid k \in \{1; \ldots; K\}\}$*, is a collection of sequential debt exchange offers* $(\underline{x}_k, n_k, \mathcal{D}_k)$*, where* $\underline{x}_k$ *is the timing (trigger level) of the* k^{th} *offer,* $n_k \leq n_{k-1}$ *is the number of new contracts offered,* $\mathcal{D}_k = \{\delta_k; C_k^*(x)\}$ *is the new debt contract replacing the contract* $\mathcal{D}_{k-1}$ *and* K *is the number of offers.*

The shareholder will choose the exchange offer strategy that maximizes her equity value. The restriction to a finite number of offers is without loss of generality since it turns out that the last offer is well defined. Therefore, any exchange offer strategy can be represented as a finite sequence. The idea behind an exchange offer strategy can be explained as follows:

1. When the state variable reaches the k^{th} threshold level, $\underline{x}_k$, the shareholder proposes the n_{k-1} creditors who hold contracts $\mathcal{D}_{k-1}$ to exchange their existing debt contract for a new one, $\mathcal{D}_k$. As before, the new contract is characterized by a coupon, δ_k, and the collateralized portion of the liquidation value, $C_k^*(x)$. We only consider exchange offers to those creditors who hold the contract offered in the previous round; we will argue in Section III.F that this is sufficient to describe the shareholders' options.

2. In practice, a vast majority of exchange offers are *conditional* on some sort of minimum acceptance rate, and they often also try to incite creditors to "rush in" by limiting the number of contracts available for exchange or by drastically limiting the time

window where exchange is guaranteed. Because all creditors will tender in the equilibria described below, we can without loss of generality assume that the k^{th} exchange offer is made conditional on n_k creditors tendering.[11] We will make use of the accounting convention $n_0 = N$.

3. If the initial contract is protected by some covenant against further issuance of debt it clearly must be removed for an offer to be valid. Typically, bond indentures require some majority or super-majority of $m \geq 0.5$ in order to alter any covenant. In practice, the covenant can be removed using an exit consent solicitation[12], i.e. the right to tender is tied to approval to exiting from the protective covenant. To ensure the uniqueness of the equilibrium analyzed below, we formally assume that in the initial debt contract $\mathcal{D}_0$, there is no seniority covenant and therefore $m = 0$.

4. The number of exchange offers, K, is endogenously determined by the game between shareholder and creditors; K could be finite or infinite. The shareholder cannot ex ante commit to a certain number K.

5. We allow the shareholder to enforce every exchange offer with a *strategic default threat.* A strategic default threat means that the shareholder can commit to cease debt service payments and walk away if the offer is not accepted (if not at least n_k creditors tender for the k^{th} offer). As a consequence, all what creditors can do if a strategic default treat is attached to an exchange offer and the offer is rejected, is to seize the court and to distribute the liquidation value $V^*(x)$ according to seniority.

The size of acceptable coupon reductions will be limited by incentive compatibility conditions of the creditors. One important consequence of these incentive compatibility conditions is well-known: Gertner and Scharfstein (1991), among others, have shown that *pari passu* offers (equal seniority) will not be accepted. To see the reason, recall that every debt value can be decomposed into two components, the value of debt service payments (only coupons in this model) on the one hand, and the value of the residual claim rights on the other hand. Since a hold-out can assure himself the initial coupon without any negative consequences, he cannot be made to accept a lower coupon unless the value of his liquidation right is higher when accepting than when rejecting.

[11] The assumption that the number of available new contracts is shrinking with each exchange offer simplifies the calculations greatly: It allows to analyze the strategy choice of an individual creditor without any strategic spillovers, i.e. the value functions of a creditor for its various options vis-a-vis an exchange offer are independent of the other creditors' choices.

[12] Section 316(b) of the US Trust Indenture Act of 1939 requires that each individual bondholder agrees to any change in a core term of a bond issue such as principal amount, interest rate, or maturity. However, protective covenants that limit the firm's capacity to issue senior debt can be altered through a majority or super-majority vote.

Since all liquidation proceeds will belong to the creditors anyway (by virtue of the Absolute Priority Rule), the increase in the residual claim *must* come at the expense of other creditors. Therefore, a successful exchange offer must threaten to relocate wealth *between creditors*, or in other words contain *dilution threats* against the creditors' residual claim rights.

Such a dilution threat essentially implies a reduction in the expected liquidation rights value of those creditors who decline the offer. Notice that multiple creditors are essential since a redistribution of wealth between creditors can only be engineered if creditors cannot coordinate their strategies.

Clearly, if the shareholder proposes residual claims of *higher seniority* than the seniority of all claims issued before, other things being equal, this makes the offer more attractive *relative* to the position of those creditors who do not exchange. Since the dilution threat is purely based on a mechanism relocating wealth between creditors, offering higher seniority is a costless, but valuable device to use for the shareholder. Therefore, in an optimal exchange offer strategy, the shareholder will always make full use of this option to *senioritize* the residual claims. Henceforth, we consider that in every exchange offer, the par value is *strictly senior to all claims issued earlier* and that the liquidation value is *strictly impaired* by the contracts on offer.

C. Dilution Threats in Practice

Throughout this paper, we use debt-for-debt exchanges offering more senior debt claims and notably increases in collateral as the leading case for dilution threats. Empirical work suggests that other techniques with the same economic effect are also common in practice. We wish to emphasize that the economic mechanism behind these alternative strategies is much the same as in the debt-for-debt exchange cum collateral increase on which we focus in this model, viz. it is based on dilution threats.

Many debt-for-debt exchange offers *shorten the maturity* of the debt claims. Because the debt is risky, this increases the expected value of a tendering creditor's residual claim. The extreme form of maturity shortening is a *cash payment*.[13] The form of such bond workouts is more like a tender offer in that, rather than offering new securities, the bonds are bought back for cash.[14]

A very common alternative for firms seeking debt restructuring consists of *selling assets*

[13] In samples compiled about US exchange offers, we found the following about fractions proposing maturity shortening (including cash offers): Gertner and Scharfstein (1991) 67 %, Chatterjee et.al. (1995) 27 %.

[14] Companies seeking debt restructurings are typically companies in financial distress. The fact that so many of them are willing to spend cash, typically a precious resource for distressed firms, to buy back long-term securities may appear less puzzling once it is put in the perspective of dilution.

at the same time. Again, this reduces the value of existing liquidation rights: The cash proceeds of asset sales may be redistributed to tendering bondholders if they are used to finance a cash tender offer. They may also be used to sweeten a debt-for-equity exchange offer, e.g. if they are set aside for future dividend payouts. So even if debt is exchanged for equity or other junior claims, it can still be the case that the offer is based on a dilution threat, in that if higher cash flow promises are made, the implicit liquidation value of those holding out is lowered.

Similarly, it is possible to spin off valuable assets into a different legal entity beyond the reach of existing debtholders.[15] Other options of dilution include *risk-shifting investments.*[16] Finally, increasing the par value of some creditors without any change in the seniority also dilutes the value of existing liquidation rights.[17]

In general, exchange offer strategies of the kind analyzed here can be viewed as transfers from pre-repudiation income rights to increased liquidation rights. Our analysis applies to *any* restructuring package offering this combination in order to overcome the hold-out effect.

The repeated nature of possible dilution threats implies that they can only be successful if accompanied by a (credible) pledge that the newly extended, more senior liquidation right is *irreversible.* This is formally shown in Lemma 1 below.

In this paper, we discuss additional *collateral* as the most obvious candidate to make such an irreversible pledge. Our use of the term collateral should be understood in a wide sense, as encompassing all contractual designs where the gap in the residual value between a tendering creditor and a hold-out is *irreversible.* It includes other mechanisms, like the ones just discussed, that offer explicitly or implicitly the same guarantee to creditors that their liquidation rights cannot be expropriated in successive renegotiation rounds.

D. Exchange Trigger Points

It will not be optimal for the shareholder to trigger new offers unless conditions worsen, so the asset valuation problem will be path-dependent only as far as the minimum state is concerned. Therefore, one additional state variable, $\check{x}_t$, is sufficient to keep track of the path-dependence. $\check{x}_t$ denotes the historical minimum reached by the state v viousnot(the)-0(will)-150 *"regime" k*. Given that these offers are respectively triggered the first time x_t reaches the levels $\underline{x}_k$ and $\underline{x}_{k+1}$, regime k corresponds to $\check{x}_t \in (\underline{x}_{k+1}; \underline{x}_k]$. Immediately after entry the firm is in regime 0, after the first offer in regime 1, and so on until the last regime K which is maintained until abandonment.

For all $\check{x}_t \in (\underline{x}_{k+1}; \underline{x}_k]$, the value of the shareholder's claim will be denoted by $S^{(k)}(x_t)$ where the superscript (n) designates the regime k. The $K+1$ regimes give a sufficiently fine information partition for our purposes and the actual historic lows $\check{x}_t$ can be omitted from the notation. After K debt exchanges are completed, the final decision that the shareholder will take is the abandonment decision, by repudiating debt contracts when x_t reaches the abandonment level, y.

We denote by $\mathcal{T}_k$ the set of successfully tendering debtholders in the k^{th} exchange offer, and by $\mathcal{H}_k$ the set of debtholders that are holding out (or being held out) in the k^{th} round for the first time. Note that creditors in the set $\mathcal{H}_k$ have by definition successfully tendered in all previous rounds. Therefore, the set of successfully tendering debtholders corresponds to $\mathcal{T}_k \equiv \{1; \ldots; n_k\}$ and the set of debtholders being held out to $\mathcal{H}_k \equiv \{n_k+1; \ldots; n_{k-1}\}$. In regime k, the value of the claim of each debtholder who tendered and succeeded in obtaining the new contract in the most recent offer (the k^{th} offer) will be denoted by $D^{(k)}_{i\in\mathcal{T}_k}(x_t)$. The value of the claim of each debtholder who was held out in the most recent offer (hence succeeded in all prior offers) will be denoted by $D^{(k)}_{i\in\mathcal{H}_k}(x_t)$. In regime k, the total value of debt outstanding is $\sum_{i=1}^{N} D^{(k)}_{i\in\{\mathcal{T}_k \cup \mathcal{H}_{j\leq k}\}}(x_t)$.

After the k^{th} offer, the value of the claim of a creditor $i \in \mathcal{H}_j$, a creditor held out or holding out in the j^{th} round, is easily determined: Once he is held out, a creditor also knows that the residual claim value of his bond is $C^*_{j-1}(y)$. If the shareholder will ultimately abandon in the state y, then this creditor's claim is worth

$$D^{(k)}_{i\in\mathcal{H}_j}(x_t) \quad = \quad \frac{\delta_{j-1}}{\rho} \; + \; \left[C^*_{j-1}(y) \; - \; \frac{\delta_{j-1}}{\rho}\right] \mathcal{P}(x_t \rhd y) \qquad \text{where } j \in \{1, \ldots, k\} \qquad (7)$$

We can also write the value of the n_k most senior debt contracts, when the $k+1^{th}$ offer will be made, the first time x_t reaches $\underline{x}_{k+1}$. Bondholders will rush in to tender their old contracts, but know that they will succeed in getting the new one with probability n_{k+1}/n_k, and fail with probability $(n_k - n_{k+1})/n_k$. The value of the claim of a *tendering* creditor $i \in \mathcal{T}_k \equiv \{1; \ldots; n_k\}$ is

$$D^{(k)}_{i\in\mathcal{T}_k}(\underline{x}^+_{k+1}) \; = \; \frac{n_k - n_{k+1}}{n_k} \; D^{(k)}_{i\in\mathcal{H}_{k+1}}(\underline{x}_{k+1}) + \frac{n_{k+1}}{n_k} \; D^{(k)}_{i\in\mathcal{T}_{k+1}}(\underline{x}_{k+1}) \; . \qquad (8)$$

Therefore, the value of these n_k most senior contracts, *before* the $k+1^{th}$ offer occurs can be expressed in the following recursive form

$$D^{(k)}_{i\in\mathcal{T}_k}(x_t) \quad = \quad \frac{\delta_k}{\rho} \; + \; \left[D^{(k)}_{i\in\mathcal{T}_k}(\underline{x}^{+}_{k+1}) \; - \; \frac{\delta_k}{\rho}\right] \; \mathcal{P}(x_t \rhd \underline{x}_{k+1}) \; . \tag{9}$$

Finally, after the K^{th} offer, the value of a creditor who successfully tenders in this last round is given by:

$$D^{(K)}_{i\in\mathcal{T}_K}(x_t) \quad = \quad \frac{\delta_K}{\rho} + \left[C^*_K(y) + \frac{V^*(x_t) - \sum_{j=1}^{K}(n_{j-1} - n_j)C^*_j(y)}{n_K} - \frac{\delta_K}{\rho}\right] \mathcal{P}(x_t \rhd y) \tag{10}$$

Expression (10) takes into account that any non-collateralized assets after the last offer, worth $V^*(x_t) - \sum_{j=1}^{K}(n_{j-1} - n_j)C^*_j(y)$, would be equally distributed among the n_K holders of the most senior claims, that is the creditors successfully tendering in the last round. The value in the last regime of a creditor held out in the K^{th} offer is as stated in expression (7).

III. Ex Post Optimal Exchange Offer Strategy

In this section we study the ex post behavior of the shareholder, *assuming* that the project is financed with K debt contracts $\mathcal{D}_0 \equiv \{\delta_0; C^*_0(x)\}$. That is, we examine the most opportunistic exchange offer strategy she can implement, *once the debt is issued.*

We begin deriving a lower boundary for the ex post value of the debt, and an upper boundary for the ex post value of the equity. We then solve for the shareholder's ex post optimal exchange offer strategy, by showing that the shareholder is actually able to attain this upper boundary level.

A. Limit Values

Once the debt is issued, the most the shareholder could possibly obtain when she makes an offer consists of *(i)* minimizing the debt value while *(ii)* maximizing the total value of the firm. At the time she proposes a new contract, the maximum she can hope to achieve consists of *(i)* minimizing to *zero* the relative surplus given to tendering creditors for debt exchanges to occur, while *(ii)* maximizing the firm value to its first-best value.

Now, a bondholder can always decide never to tender, and his claim would at least yield a coupon flow δ_0 until operations are abandoned. If debt contracts initially carry a collateral $C^*_0(x)$, then in spite of all of the shareholder's dilution efforts, the debt value cannot possibly be reduced below, as viewed from the point of entry x_0,

$$\underline{D}(x_0 \mid y) \quad \equiv \quad \frac{\delta_0}{\rho} \; + \; \left(C^*_0(y) \; - \; \frac{\delta_0}{\rho}\right) \mathcal{P}(x_0 \rhd y) \; . \tag{11}$$

This is the minimal possible value *ex ante* of the claim of a creditor who decides to hold out in the first offer, and the value of each creditor's claim is bounded from below by this *creditor reservation value.* Consequently, at the point of entry the *upper bound on the equity value* can be determined as

$$\overline{S}(x_0) \equiv \max_y \{ V(x_0 \mid y) - N \, \underline{D}(x_0 \mid y) \} . \tag{12}$$

Let $\hat{y}$ denote the abandonment trigger level that maximizes $\overline{S}(x_0)$,

$$\hat{y} \equiv \arg\max_y \left\{ V(x_t \mid y) - N \left[\frac{\delta_0}{\rho} + \left(C_0^*(y) - \frac{\delta_0}{\rho} \right) \mathcal{P}(x_t \triangleright y) \right] \right\}. \tag{13}$$

$\hat{y} = \hat{y}(\delta_0, C_0^*(x))$ is a function of both components of the initial debt contract, the coupon δ_0 and the collateral function, $C_0^*(x)$.

B. The Shareholder's Optimization Problem

We solve next for the shareholder's ex post optimal exchange offer strategy. When determining her optimal exchange offer strategy, s, the shareholder works backwards in time, evaluating the entire sequence of decisions available to her, from the final abandonment to the point of entry. Therefore, the shareholder's ex post optimization problem will be broken down into a recursive sequence of constrained optimization problems.

For any given exchange offer strategy, $\mathsf{s} = \{ (\underline{x}_k, n_k, \mathcal{D}_k) \mid k \in \{1; \ldots; K\} \}$, she first calculates the optimal abandonment trigger level, y_{s}, which occurs *after all* exchange offers have been played out. This trigger level y_{s}, solves

$$y_{\mathsf{s}} \equiv \arg\max_y \left\{ V(x_t \mid y) - \sum_{i=1}^{N} D^{(K)}_{i \in \{\mathcal{T}_K \cup \mathcal{H}_{j \leq K}\}}(x_t) \right\}. \tag{14}$$

Proceeding backwards, the shareholder then calculates the sequence of optimal offers, from the last exchange offer to the point of entry. She optimizes recursively each one of the K offers, for a *given prior* exchange offer strategy. She does this for all $k \in \{1; \ldots; K\}$, starting at $k = K$ and finishing at $k = 1$. The result of previous optimizations $k \in \{j+1; \ldots; K\}$ are fed back in the $k = j$ exchange offer optimization problem.

The characteristic parameters, $(\underline{x}_k, n_k, \mathcal{D}_k)$, of a shareholder's *optimal* k^{th} exchange offer, maximize the value of the equity in regime $k-1$,

$$S^{(k-1)}(x_t) = \max_{\underline{x}_k, n_k, \mathcal{D}_k} \left\{ V(x_t \mid y_{\mathsf{s}}) - \sum_{i=1}^{N} D^{(k-1)}_{i \in \{\mathcal{T}_{k-1} \cup \mathcal{H}_{j \leq k-1}\}}(x_t) \right\} , \tag{15}$$

$$\text{subject to:} \quad n_{k-1} \geq n_k \geq 1 , \tag{16}$$

$$D^{(k)}_{i \in \mathcal{T}_k}(\underline{x}_k) \geq D^{(k)}_{i \in \mathcal{H}_k}(\underline{x}_k) . \tag{17}$$

We will denote by $\mathcal{S}$ the set of exchange offer strategies s which are optimal ex post from the shareholder's perspective, i.e. which solve the above recursive optimization problem.

Equation (16) is called the "k^{th} rationing constraint", as it reflects the condition that the number of new contracts will be (weakly) lower to the number of contract in the most senior class.

Equation (17) is called the "k^{th} tendering constraint", guaranteeing that tendering the old debt contract is better than holding out. It states that, considering the rationing involved, the proposed new contract, $\mathcal{D}_k$, must be more desirable than the current one, $\mathcal{D}_{k-1}$, at the time of the offer, so that tendering debtholders $i \in \mathcal{T}_k$ are better off than hold-outs, debtholders $i \in \mathcal{H}_k$.

Since the problem is recursive in nature, satisfying the k^{th} tendering constraint in equation (17) is less straightforward than it might appear: This condition contains value functions which depend on possible subsequent exchange offers. Determining a feasible exchange offer strategy must also take into account the potential time consistency problems of such a sequence.

Loosely speaking, for a bondholder to tender in state x_t, it must not only be the case that *(i)* the expected payoff from holding out is smaller than the value from tendering, but *(ii)* the value from tendering must also take into account that the bondholder may be exposed to further strategic exchange offers in the future. We will show next that the recursive structure of the tendering constraints yields considerable cutting power regarding the set of feasible exchange offer strategies.

C. The Commitment Problem and the Role of Collateral

Consider the subset of exchange offer strategies where the fraction of the liquidation value of the firm which is collateralized does not evolve, i.e. $C_k^*(x) = C_{k-1}^*(x)$ for some $k \in \{1, \ldots, K\}$, in other words the only reward given to tendering creditors is seniority. Under such strategies, debtholders held-out in earlier rounds are always better off than those held-out in later rounds. This is because the former will ultimately have accepted less reductions in coupon than the latter. Therefore, in any subsequent regime $k' \in \{k+1, \ldots, K\}$:

$$D^{(k')}_{i \in \mathcal{H}_j}(x_t) \quad < \quad D^{(k')}_{i \in \mathcal{H}_l}(x_t) \qquad (18)$$
$$\text{for all } j > l, \qquad \text{where } j \text{ and } l \in \{1, \ldots, k\} \ .$$

In this case, the shareholder's dynamic optimization problem exhibits the following feature: If the tendering condition is *binding* in the k^{th} round, then the tendering condition in the $k - 1^{th}$ round *cannot* be satisfied.

Consequently, such repeated offers suffer from a time consistency problem: Debtholders always reject a first exchange offer, because, if the shareholder has later the possibility to make a second offer, this offer will be such that holding out was actually preferable in the first place. The repeated nature of the problem imposes an interesting *credibility constraint* on feasible strategies of the shareholder. Creditors will not tender in the k^{th} offer if the total residual claim value handed out to the creditors tendering in the $k+1^{th}$ offer can be as high as the same value portion in the k^{th} offer, $V^*(x) - n_k\, C_k^*(x) - \sum_{j=1}^{k-1}(n_{j-1} - n_j)\, C_j^*(x)$. The shareholder has to *refine* her offer and to give tendering bondholders *more* than just higher seniority: She must *commit* not to dilute the rewards again in subsequent offers.

Lemma 1 *Bondholders will never accept an exchange offer which only offers higher seniority. For an offer to be acceptable, there must be a value increase in residual claim rights which are immune to further dilution.*

Proof: A proof is given in the Appendix.

Recall that the number of offers, K, is endogenous and that the shareholder can always propose yet another offer. Therefore, as long as the liquidation rights are not secure, the shareholder can and will launch a subsequent offer which expropriates the liquidation rights through the attribution of more senior claims.

According to Lemma 1, the shareholder must provide a guarantee that the value gain in residual claims of tendering creditors cannot be expropriated in subsequent renegotiation rounds. Any such guarantee must set some of the firm's assets aside and exclude them from further dilution. We consider additional pledges of *collateral* as the device to offer such a guarantee, but refer to our discussion in Section II.C that other techniques could be used as well.

In the k^{th} exchange offer, a commitment against further dilution consists then of a pledge of a new collateral $C_k^*(x)$ replacing the old collateral $C_{k-1}^*(x)$ for each tendering creditor. Even if held out in future renegotiations, each tendering creditor is then assured to receive at least $C_{k-1}^*(x)$, if abandonment occurs in state x.

An exchange offer strategy, $\{\, (\underline{x}_k, n_k, \mathcal{D}_k) \mid k \in \{1; \ldots; K\}\, \}$, must therefore involve new contracts, $\mathcal{D}_k$, that specifically increase the level of collateralized residual claims, $C_k^*(x)$, from the level attained earlier, $C_{k-1}^*(x)$.

We can now also clarify how the number of exchange offers, K, is determined. The last or K^{th} exchange offer is the offer where the last part of the assets is fully collateralized, i.e. when $V^*(x) = \sum_{j=1}^{K}(n_{j-1} - n_j)C_j^*(x)$. Subsequent offers will be rejected, according to Lemma 1, and are irrelevant for the equilibrium outcome.

The question is then how much new collateral must be added at every round for the exchange offer to be *dynamically incentive-compatible*, i.e. to be acceptable for creditors rationally anticipating that further exchange offers are possible. We find that:

Lemma 2 *The k^{th} tendering condition, $D^{(k)}_{i\in\mathcal{T}_k}(\underline{x}_k) \;\geq\; D^{(k)}_{i\in\mathcal{H}_k}(\underline{x}_k)$, can be written*

$$C^*_k(y_\mathsf{s}) \;-\; C^*_{k-1}(y_\mathsf{s}) \quad\geq\quad \frac{(\delta_{k-1}-\delta_k)}{\rho}\;\frac{[1 \;-\; \mathcal{P}(\underline{x}_k \rhd y_\mathsf{s})]}{\mathcal{P}(\underline{x}_k \rhd y_\mathsf{s})}\;. \tag{19}$$

Proof: A proof is given in the Appendix.

Lemma 2 says that in order to get bondholders' approval, an exchange must contain an irrevocable pledge of *more* collateral. If there are K consecutive offers, then each of these offers must offer sufficient new collateral to meet condition (19). After the k^{th} successful offer, the remaining claims to the liquidation value that the shareholder can still redistribute strategically in subsequent offers is bounded by the value of not yet collateralized assets, $V^*(x) \,-\, n_k\, C^*_k(x) \,-\, \sum_{j=1}^{k-1}(n_{j-1}-n_j)\, C^*_j(x)$.

D. Uniqueness of Equilibrium

We turn our attention to the creditors' strategies and the condition when the equilibrium outcome is unique. Throughout, the equilibrium outcome that is analyzed is as follows: once the incentive constraint, $D^{(k)}_{i\in\mathcal{T}_k}(\underline{x}_k) \;\geq\; D^{(k)}_{i\in\mathcal{H}_k}(\underline{x}_k)$, is satisfied, enough of the remaining n_{k-1} creditors tender in order to pick all of the n_k new contracts on offer. This certainly is a (subgame perfect) equilibrium since the sequence of dynamic incentive constraints ensures that the creditors' strategies are best responses.

This leaves the question of uniqueness of this outcome since in renegotiation games with many parties like ours, the multiplicity of equilibria is often endemic. For example, if $N-1$ creditors were to always reject every offer in all renegotiation rounds, then always rejecting could well constitute a (subgame perfect) equilibrium response for the last creditor even if the incentive constraint (19) holds strictly and the outcome is independent of the last creditor's response.

Technically speaking, the necessary and sufficient condition for the uniqueness of our equilibrium is that the minimum number of accepting creditors in the first offer, $m\,N$, is not larger than one. Note that if $m\,N > 1$, then if $N-1$ creditors were to reject, the decision of the last creditor would be irrelevant for the allocation since the number of rejecting creditors would exceed the minimum acceptance rate. Rejecting would then be an equilibrium response for every creditor, and always rejecting could be sustained as equilibrium. To exclude this unwanted outcome, the shareholder can always use the following exchange offer strategy: *(i)* Propose in all offers just a single contract,[18] $n_k = 1$ for all $k = 1, ..., K$ and *(ii)* let the inequality (19) hold strictly in all K offers. Tendering is then the unique equilibrium

[18]Since there is no non-negativity restriction on δ_k, the shareholder can get the same aggregate coupon reduction from a single creditor that she can get from a large number of creditors.

response for all N creditors in the first offer, and tendering is the unique response for the remaining creditor in all subsequent offers.

With this strategy, the only point where multiple equilibrium outcomes could possibly arise is during the first exchange offer, and only if the initial contract contained a minimum acceptance rate $m\,N > 1$, for example a seniority covenant which can only be removed by a majority or super-majority. Since this is not the case by assumption, the equilibrium outcome is indeed unique.

E. Fully Collateralized Debt

An important insight of Lemma 2 refers to the particular case where all of the firm's assets are already fully collateralized. This is the case when the initial debt contract, $\mathcal{D}_0 \equiv \{\delta_0; C_0^*(x)\}$, involves an initial collateral function $C_0^*(x) = V^*(x)/N$, for all x:

Corollary 1 *Fully collateralized debt or debt not backed by collaterizable assets cannot be renegotiated.*

Corollary 1 sheds light on a prominent special case in the structural pricing literature, the valuation of *non-renegotiable* debt claims, as they in particular assumed in Leland (1994) and Leland and Toft (1996). In other words, our model can explain the two joint conditions which make the assumption of non-renegotiability realistic: if *(i)* debt claims are widely dispersed and *(ii)* the debtor has no latitude to make irreversible dilution threats. The latter is true when all separable or pledgeable assets are already collateralized, or when the firm's assets are completely intangible.

F. Optimal Exchange Offers

The shareholder's optimization problem can now be rewritten in terms that are more directly related to the variables she actually controls, after replacing the k^{th} tendering constraint with the more specific condition (19). The characteristic parameters $(\underline{x}_k, n_k, \delta_k, C_k^*(x))$ of a shareholder's *optimal* k^{th} exchange offer maximize the value of the equity in regime $k-1$,

$$S^{(k-1)}(x_t) \quad = \quad \max_{\underline{x}_k, n_k, \delta_k, C_k^*(x)} \left\{ V(x_t \mid y_\mathsf{s}) \; - \; \sum_{i=1}^{N} D^{(k-1)}_{i \in \mathcal{T}_{k-1} \cup \mathcal{H}_{j \leq k-1}}(x_t) \right\} , \tag{20}$$

$$\text{subject to:} \qquad n_{k-1} \;\geq\; n_k \;\geq\; 1 \,, \tag{21}$$

$$C_k^*(y_\mathsf{s}) \;-\; C_{k-1}^*(y_\mathsf{s}) \;\geq\; \frac{(\delta_{k-1} - \delta_k)}{\rho} \, \frac{[1 \;-\; \mathcal{P}(\underline{x}_k \rhd y_\mathsf{s})]}{\mathcal{P}(\underline{x}_k \rhd y_\mathsf{s})} \,. \tag{22}$$

This formulation of the shareholder's recursive optimization problem enables us to characterize more precisely the set of optimal exchange offer strategies, by first establishing the following crucial Lemma:

Lemma 3 *If exchange offer strategy* s *is optimal, then the tendering constraint (22) is binding for every exchange offer* $k \in \{1; \ldots; K\}$.

Proof: A proof is given in the Appendix.

The intuition behind Lemma 3 is that in every exchange offer, reducing the new coupon on offer promises the shareholder a twofold gain. First, it reduces the debt service payments value over the expected time horizon until the firm is liquidated. Second, since the abandonment trigger level y is monotonic in the final aggregate coupon value, it prolongs the life expectancy of the firm, and over the additional life span, the equity value must be positive. Thus, the shareholder will reduce the new coupon on offer until the tendering constraint binds.

The fact that the tendering constraint must be binding at every exchange turns out to be powerful in this model: Taking the expression for $D^{(k)}_{i\in\mathcal{H}_j}(x_t)$ given in equation (9), it means that

$$D^{(k)}_{i\in\mathcal{T}_k}(\underline{x}_k) \quad = \quad D^{(k)}_{i\in\mathcal{H}_k}(\underline{x}_k) \quad = \frac{\delta_{k-1}}{\rho} + \left[C^*_{k-1}(y_\mathsf{s}) \;-\; \frac{\delta_{k-1}}{\rho}\right] \mathcal{P}(\underline{x}_k \rhd y_\mathsf{s}), \tag{23}$$

for all $k \in \{1; \ldots; K\}$. In particular, this is true for $n = 1$,

$$D^{(1)}_{i\in\mathcal{T}_1}(\underline{x}_1) \quad = \quad D^{(1)}_{i\in\mathcal{H}_1}(\underline{x}_1) \quad = \frac{\delta_0}{\rho} + \left[C^*_0(y_\mathsf{s}) \;-\; \frac{\delta_0}{\rho}\right] \mathcal{P}(\underline{x}_1 \rhd y_\mathsf{s}). \tag{24}$$

Therefore replacing in the value of a bond in the initial regime,

$$\begin{aligned} D^{(0)}(x_t) \quad &= \quad \frac{\delta_0}{\rho} + \left[D^{(1)}_{i\in\mathcal{T}_1}(\underline{x}_1) - \frac{\delta_0}{\rho}\right] \mathcal{P}(x_t \rhd \underline{x}_1)\ , & (25)\\ &= \quad \frac{\delta_0}{\rho} \;+\; \left[C^*_0(y_\mathsf{s}) - \frac{\delta_0}{\rho}\right]\; \mathcal{P}(x_t \rhd y_\mathsf{s})\ . & (26) \end{aligned}$$

Consequently, the ex post equity value under an optimal exchange offer strategy

$$S^{(0)}(x_t) \quad = \quad V(x_t \mid y_\mathsf{s}) \;-\; N\left(\frac{\delta_0}{\rho} \;+\; \left[C^*_0(y_\mathsf{s}) \;-\; \frac{\delta_0}{\rho}\right] \mathcal{P}(x_0 \rhd y_\mathsf{s})\right)\ . \tag{27}$$

Importantly, since equations (27) and (12) are identical, the shareholder will choose the same abandonment point, $y_\mathsf{s} = \hat{y}$ (where $\hat{y}$ is defined in (13)). Therefore, the solution of (27) corresponds exactly to what we established to be the upper limit on the shareholder's value function,

$$S^{(0)}(x_t) = \overline{S}(x_t) = V(x_t \mid \hat{y}) \;-\; N\left(\frac{\delta_0}{\rho} \;+\; \left[C^*_0(\hat{y}) \;-\; \frac{\delta_0}{\rho}\right] \mathcal{P}(x_0 \rhd \hat{y})\right)\ . \tag{28}$$

That is, the shareholder's ability to renegotiate ex post limits the initial value of diffusely held debt to exactly the creditors' reservation value.

$$D^{(0)}(x_t) \quad = \quad \underline{D}(x_t \mid \hat{y}) \ . \tag{29}$$

In essence, creditor's initial entitlement to a share of the non-collateralized assets is worthless since it will be expropriated through an exchange offer strategy, before repudiation. The ex ante value of non-collateralized assets is ex post fully internalized by the opportunistic shareholder.

We can now also see why it is unimportant that the k^{th} exchange offer is only addressed to the k_{n-1} creditors in the most senior class, $\mathcal{T}_{k-1}$, as we assumed all along. At the time of the offer, the market has already fully priced in the fact that the creditors' entitlement to a part of the non-collateralized assets is worthless (see Eq. (23)), and there is no structural difference in the debt valuation expression between the most senior and more junior creditors. The dollar amount of coupon reduction that one additional dollar of pledged collateral can obtain is purely driven by the tendering constraint (21), irrespective whether a creditor is in the most senior class or not. The shareholder would get exactly the same dollar amount of concessions if addressing the offer to more junior creditors.

We have not yet discussed the efficiency of this ex post solution. If $y_{\mathsf{s}} = \hat{y} \neq \tilde{y}$, then the value of equity plus debt, $S^{(0)}(x_t) + N\, D^{(0)}(x_t) = V(x_t \mid \hat{y})$, is below the first best firm value $V(x_t \mid \tilde{y})$, and value is destroyed. We turn next to the question of maximizing the firm value.

IV. Ex Ante Financing and Contract Design

In Section III, we studied the ex post behavior of the shareholder, assuming the project to be financed with K *given* debt contracts $\mathcal{D}_0 \equiv \{\delta_0; C_0^*(x)\}$. Working backwards in time, we will now drop this assumption. Taking the opportunistic ex post optimization into account, we determine which debt contract shareholder and creditors will find feasible and optimal at the date of entry.

A. Ex Ante Optimal Debt Contract

At the date of entry, the incumbent chooses a debt contract, $\mathcal{D}_0 \equiv \{\delta_0; C_0^*(x)\}$, which maximizes the value of equity net of her investment, $S^{(0)}(x_0) - [I - D^{(0)}(x_0)]$. She therefore solves the following problem:

$$\max_{\delta_0, C_0^*(x)} \quad \{\ V(x_t \mid \hat{y}) \ - \ I\ \} \tag{30}$$

$$\text{subject to:} \quad I_D \ < \ N\ D^{(0)}(x_0)\ , \tag{31}$$

$$0 \ < \ N\, C_0^*(x) \ < \ V^*(x)\ , \quad \text{for all } x\ . \tag{32}$$

The best she can achieve is a value of equity net of her investment equal to the first best firm value minus the overall investment needed to implement the project, $V(x_t \mid \tilde{y}) - I$. Therefore, ex ante the shareholder seeks to maximize firm value. Her problem vis-a-vis the debtholders is to establish credibly, from the point of view of the debtholders, that she will stay the course ex post, i.e. that she is *committed* to an exchange offer strategy s which brings $y_{\mathsf{s}} = \hat{y}(\delta_0, C_0^*(x))$ as close as possible to $\tilde{y}$.

The ex ante problem in the contract design is essentially about establishing this commitment. The two instruments to engineer such a commitment are *(i)* the choice of debt coupon obligation, δ_0, and *(ii)* the choice of the initial collateral, $C_0^*(x)$, which specifies the liquidation value of the assets collateralized as a function of the abandonment state, x. To understand the required relationship between δ_0 and the function $C_0^*(x)$ for the first best to be attained, contrast the equations defining $\tilde{y}$ and $\hat{y} = \hat{y}(\delta_0, C_0^*(x))$:

(a) On the one hand, the ex ante optimal abandonment trigger level,

$$\tilde{y} \quad \equiv \quad \arg\max_{y} \ \{ V(x_t \mid y) \} . \tag{33}$$

(b) On the other hand, the shareholders' ex post abandonment trigger level,

$$\hat{y} \quad \equiv \quad \arg\max_{y} \left\{ V(x_t \mid y) \ - \ N \left[\frac{\delta_0}{\rho} \ + \ \left(C_0^*(y) \ - \ \frac{\delta_0}{\rho} \right) \mathcal{P}(x_t \rhd y) \right] \right\} . \tag{34}$$

Our first question concerns the right balance between *(i)* pre-abandonment income rights (the coupon obligation) and *(ii)* post-abandonment income rights (the collateralized assets), such that the pair of instruments $(\delta_0; C_0^*(x))$, induces an efficient abandonment decision, $\hat{y} = \tilde{y}$. Inspection of the maximization problem (34) provides the following answer:

Condition 1 *An optimal debt contract $\mathcal{D}_0$ involves a pair $(\delta_0, C_0^*(x))$ of coupon obligation and debt collateral such that*

$$\left[V^*(y) \ - \ \Pi(y) \ + \ N \left(\frac{\delta_0}{\rho} - \ C_0^*(\tilde{y}) \right) \right] \ \mathcal{P}(x_0 \rhd y) \, , \tag{35}$$

is a quasi-concave function in y, with maximum at the efficient abandonment level, $\tilde{y}$.

Condition 1 expresses necessary and sufficient conditions on the initial contract $(\delta_0, C_0^*(x))$ ensuring that the firm attains its first best value, $V(x_0 \mid \tilde{y})$. Rewriting expression (35) as $(V^*(y) \ - \ \Pi(y))\,\mathcal{P}(x_0 \rhd y) + \ N \ (\delta_0/\rho \ - \ C_0^*(\tilde{y})) \ \ \mathcal{P}(x_0 \rhd y)$, we know that the first term $(V^*(y) \ - \ \Pi(y))\,\mathcal{P}(x_0 \rhd y)$ is maximized at $\tilde{y}$, by virtue of Assumption 1. Therefore, sufficient (but not necessary) for Condition 1 to hold is that $(\delta_0, C_0^*(x))$ be chosen such that $(\delta_0/\rho \ - \ C_0^*(\tilde{y})) \ \mathcal{P}(x_0 \rhd y)$ is concave with maximum at $\tilde{y}$.

We can immediately gain some useful insights from Condition 1. If Condition 1 holds, then the first order optimality condition with respect to y can be written as:

$$\left[\frac{dV^*(\tilde{y})}{d\tilde{y}} - \frac{d\Pi(\tilde{y})}{d\tilde{y}} - N\frac{dC_0^*(\tilde{y})}{d\tilde{y}}\right] \mathcal{P}(x \rhd \tilde{y}) + \left[V^*(\tilde{y}) - \Pi(\tilde{y}) - N\, C_0^*(\tilde{y}) + N\frac{\delta_0}{\rho}\right] \frac{\partial \mathcal{P}(x \rhd \tilde{y})}{\partial \tilde{y}} = 0 \,. \tag{36}$$

The first order optimality condition implied by Assumption 1 is:

$$\left[\frac{dV^*(\tilde{y})}{d\tilde{y}} - \frac{d\Pi(\tilde{y})}{d\tilde{y}}\right] \mathcal{P}(x \rhd \tilde{y}) + [V^*(\tilde{y}) - \Pi(\tilde{y})] \frac{\partial \mathcal{P}(x \rhd \tilde{y})}{\partial \tilde{y}} = 0 \,. \tag{37}$$

Combining both conditions, we obtain:

$$\frac{dC_0^*(\tilde{y})}{d\tilde{y}} \, [V^*(\tilde{y}) - \Pi(\tilde{y})] = \left[\frac{\delta_0}{\rho} - C_0^*(\tilde{y})\right] \left[\frac{d\Pi(\tilde{y})}{d\tilde{y}} - \frac{dV^*(\tilde{y})}{d\tilde{y}}\right] . \tag{38}$$

Inspection of condition (38) reveals the following relationship between coupon and value of initially collateralized assets. To engineer an increase in the initial market value of a debt issue, $D^{(0)}(x_0)$, the necessary increase in the coupon level δ_0 must be accompanied by either *(i)* a corresponding increase in the value associated of collateralized assets, $C_0^*(\tilde{y})$, or *(ii)* a higher value sensitivity to state, $d\,C_0^*(\tilde{y})/d\,\tilde{y}$, around $\tilde{y}$, or a combination of both alternatives.

In our analysis of the ex ante optimal contract, we turn now to the following questions: First, are there circumstances under which the incumbent can do as well issuing debt contracts which are either fully or not collateralized at all? Second, can she do as well borrowing from a single creditor, or when would she actually prefer dispersed debt?

B. Debt Collateralization and the Value of Renegotiable Debt

Turning to the first of these questions, we will now discuss when and why the optimal collateral choice must be strictly interior to the extreme cases of either pledging *all* collateralizable assets or *no* assets at all. We refer to this as partial collateral.

Consider first the extreme case where the debt is initially *fully* collateralized, i.e the initial debt contract $\mathcal{D}_0 \equiv \{\delta_0; C_0^*(x)\}$ involves $C_0^*(x) = V^*(x)/N$ for all x. We have already discussed this particular case in Section III.E. Since renegotiation is then impossible (Corollary 1), the values of each bond and the equity, which for clarity we will denote $D_f^{(0)}(x_t)$ and $S_f^{(0)}(x_t)$, respectively, can be derived immediately.

$$D_f^{(0)}(x_t) = \frac{\delta_0}{\rho} + \left[\frac{V^*(\hat{y}_f)}{N} - \frac{\delta_0}{\rho}\right] \mathcal{P}(x_t \rhd \hat{y}_f) \,, \tag{39}$$

$$S_f^{(0)}(x_t) = V(x_t \mid \hat{y}_f) - N\, D_f^{(0)}(x_t) \,. \tag{40}$$

Here, $\hat{y}_f$, the shareholder's non-cooperatively optimal abandonment trigger level corresponds to $\hat{y}(\delta_0; V^*(x)/N)$ and maximizes equity value:

$$\hat{y}_f \quad \equiv \quad \arg\max_y \left\{ V(x_t \mid y) \; - \; N \; \left[\frac{\delta_0}{\rho} \; + \; \left(\frac{V^*(y)}{N} - \frac{\delta_0}{\rho} \right) \; \mathcal{P}(x_t \rhd y) \right] \right\}. \tag{41}$$

Now, an increase in the coupon obligation precipitates shareholder's abandonment, hence increases $\hat{y}_f$. That is, since $-N[1 - \mathcal{P}(x_t \rhd y)]\delta_0/\rho$ is negative and strictly increasing in y, for all $y < x_t$, Assumption 1 implies

$$\frac{\partial}{\partial \delta_0} \; [\, \hat{y}\, (\delta_0,\, V_0^*(x)/N)\,] \quad > \quad 0\;. \tag{42}$$

It follows that, given full collateralization, there is a unique threshold amount of debt service obligations, $\tilde{\delta}_0$, giving rise to the efficient abandonment decision, $\hat{y}_f = \tilde{y}$. Consequently, only if the required funding level happens to be precisely equal to

$$\tilde{I}_D \quad \equiv \quad N\, \frac{\tilde{\delta}_0}{\rho} \; + \; \left(V^*(\tilde{y}) - N\, \frac{\tilde{\delta}_0}{\rho} \right) \; \mathcal{P}(x_0 \rhd \tilde{y}) \;, \tag{43}$$

will issuing fully collateralized debt ensure that the combined value of equity and debt, $S_f^{(0)}(x_t) + N\, D_f^{(0)}(x_t \mid \hat{y}_f)$, equals the first best value of the firm, $V(x_t \mid \tilde{y})$.

On the one hand, if the amount of outside funding I_D is smaller than $\tilde{I}_D$ and fully collateralized debt is issued, the incumbent will default *"later"* than would be first best, i.e. she will abandon operations at a state y strictly smaller than $\tilde{y}$. Conversely, if I_D is larger than $\tilde{I}_D$ and fully collateralized debt is issued, the incumbent will default *"earlier"* than would be first best, i.e. at a state y larger than $\tilde{y}$.

If I_D is *smaller* or equal to $\tilde{I}_D$, then issuing *renegotiable debt*, debt where renegotiation and coupon concessions will occur in poor performances, is not desirable because the debt coupon concessions would imply that the shareholder defaults even later than if the same coupon had been issued with full collateral attached, thus worsening the inefficiency problem.

There is, however, a simple solution in this case. The following simple capital structure policy is optimal and always feasible: Just issue fully collateralized bonds, $\mathcal{D}_0 \equiv \{\tilde{\delta}_0; V^*(x)/N\}$, with an aggregate initial value of $N\, D_f^{(0)}(x_0) = \tilde{I}_D$, that is a leverage exactly equal to the debt level where full collateralization is efficient. Any surplus of funds, $\tilde{I}_D - I_D$, can be used to either reduce the initial equity contribution, $I - I_D$, or to increase the dividend payout.

On the other hand, if the required outside funding I_D is *larger* than $\tilde{I}_D$, then the renegotiation option can *add value* to the firm and to the shareholder's equity: Creditor concessions can increase the ex ante firm value if they postpone the implied abandonment point towards the efficient point, $\tilde{y}$. In this case, whenever the debt contract $(\delta_0, C_0^*(x))$ gives rise to an exchange offer strategy where the firm does not fully renegotiate unless the firm conditions deteriorate, then including renegotiation options in the contract design adds value. A debt contract $(\delta_0, C_0^*(x))$ satisfying Condition 1 is then optimal.

If $I_D > \tilde{I}_D$, then giving the shareholder incentives for the optimal timing of exchange offers and abandonment point implies that[19]

$$N \frac{dC_0^*(\tilde{y})}{d\tilde{y}} \quad > \quad \frac{dV^*(\tilde{y})}{d\tilde{y}} \; . \tag{44}$$

Expression (44) says that the value of remaining non-collateralized assets, left for the shareholder to engineer strategic debt exchange offers, $V^*(\tilde{y}) - N\, C_0^*(\tilde{y})$, is locally *decreasing* in x, around the point $\tilde{y}$. That is, it increases as the state x deteriorates and approaches the abandonment point. This means that it is worthwhile for the shareholder to postpone the exchange offers so as to capture this additional value of her bargaining chip, which will only accrue if the effective abandonment point is close enough to $\tilde{y}$.

Overall, the trade-off that determines the optimal choice of the instruments is as follows. On the one hand, the *possibility to renegotiate* debt terms efficiently when the firm approaches the lower reorganization bound (low x_t) must be assured, by leaving a sufficiently large portion of assets free from initial collateral pledges, $V^*(x) - N\, C_0^*(x)$. On the other hand, creditors must be given *protection from the premature exercise* of these imbedded debt renegotiation options, and this is achieved through a sufficiently high level and steep slope of $C_0^*(x)$. A steep slope of $C_0^*(x)$ means that the shareholder is rewarded with in increase in the value of her bargaining chip, but only if she shows patience in proposing exchange offers. Therefore, many simple specifications of the initial debt collateral lead to an ex ante loss of value, because ex post, the shareholder would choose an exchange offer strategy leading to an inefficient abandonment point at $\hat{y}$ different from $\tilde{y}$. We can summarize:

Proposition 1 *(i) If at the date of entry the required level of outside financing, I_D, is smaller or equal to $\tilde{I}_D$, the first best firm value can be realized by issuing dispersed debt which is fully collateralized and has an initial value of $N\ D^{(0)}(x_0) = \tilde{I}_D$. The debt is not renegotiated after it is issued, and the shareholder abandonment trigger level is the ex ante optimal one, $\tilde{y}$. Once issued, the values of each bond and the equity are, respectively,*

$$D_f^{(0)}(x_t) \quad = \quad \frac{\tilde{\delta}_0}{\rho} + \left[\frac{V^*(\tilde{y})}{N} - \frac{\tilde{\delta}_0}{\rho} \right] \mathcal{P}(x_t \triangleright \tilde{y}) \; , \tag{45}$$

$$S_f^{(0)}(x_t) \quad = \quad V(x_t \mid \tilde{y}) \; - \; K\ D_f^{(0)}(x_t) \; . \tag{46}$$

[19]Condition (44) is obtained by comparing the first-order condition resulting from (41) and condition (36).

(ii) If $I_D > \tilde{I}_D$, then issuing a debt contract satisfying Condition 1 is the optimal form of dispersed debt. After the debt is issued, the shareholder follows a non-cooperative optimal exchange offer strategy, $\mathsf{s} \in \mathcal{S}$, and her abandonment trigger level is the ex ante optimal one, $\tilde{y}$. Once issued, the values of each bond and the equity are, respectively,

$$D^{(0)}(x_t) = \frac{\delta_0}{\rho} + \left[C_0^*(\tilde{y}) - \frac{\delta_0}{\rho}\right] \mathcal{P}(x_t \rhd \tilde{y}) , \tag{47}$$

$$S^{(0)}(x_t) = V(x_t \mid \tilde{y}) - N\, D^{(0)}(x_t) . \tag{48}$$

Thus, a clear case distinction in the optimal debt design emerges as to whether optimally designed debt should be *state-contingent* or not. We say that the debt contract $(\delta_0, C_0^*(x))$ is state-contingent if it induces exchange offers which are contingent on the firm performing poorly and the state variable deteriorating below the entry state, x_0, i.e. an exchange offer strategy s such that $\underline{x}_K < x_0$. In the presence of dispersed creditors, state-contingent debt is both feasible and value-increasing if and only if the required funding level, I_D, exceeds the highest level that can be efficiently managed with a non-renegotiable contract as in Leland (1994).

Consider now the other extreme case, where the debt is initially *not* collateralized at all, i.e. the initial debt contract $\mathcal{D}_0 \equiv \{\delta_0; C_0^*(x)\}$ involves $C_0^*(x) = 0$ for all x. Such a bond could only satisfy Condition 1 if it were a zero coupon bond, $\delta_0 = 0$, implying a zero debt value, $D^{(0)}(x_0) = 0$. Therefore, any debt issue with a positive market value but without collateral, $C_0^*(x) = 0$, cannot attain the first best firm value. For any bond without initial collateral, irrespective of the coupon δ_0, the shareholder's optimal strategy would be to make a single exchange offer leading to full collateralization of the debt *immediately* at the date of entry, x_0. This would be fully priced in, and nothing is gained compared to the issue of fully collateralized debt.

Proposition 2 *Dispersed debt with zero collateral, $C_0^*(x) = 0$, is never preferred over the issue of fully collateralized debt.*

This result simply says that zero collateral debt will never lead to a firm value above the firm value attainable by issuing non-renegotiable (fully collateralized) debt. It follows that if state-contingent debt is optimal, then the shareholder will optimally issue debt with a collateral value evolving between the two extreme cases. Moreover, Condition 1 ties down the optimal form of $C_0^*(x)$ rather rigidly, and many forms of *partial collateral* will not lead to the desired state-contingency: If the shareholder were to issue collateral with a value evolution which is strictly *proportional* to the liquidation value of the firm's assets, then the shareholder would deploy a non-contingent exchange offer strategy and fully collateralize all assets at the date of entry, x_0, just as in the case of zero collateral.

C. Debt Capacity and the Role of Creditor Dispersion

At entry, the project can be financed if there exists a feasible debt contract, $\mathcal{D}_0 \equiv \{\delta_0; C_0^*(x)\}$, such that the aggregate market value of debt equals I_D, the required funding level:

$$I_D \quad = \quad N \; D^{(0)}(x_0) \; . \tag{49}$$

The *debt capacity*, the absolute limit to the amount dispersed creditors are willing to lend, is equal to the highest feasible aggregate value of bonds issued at entry. We denote by $\Lambda(x_0)$ the debt capacity of the firm,

$$\Lambda(x_0) \quad \equiv \quad N \max_{\delta_0, C_0^*(x)} \left\{ \frac{\delta_0}{\rho} + \left[C_0^*(\hat{y}) - \frac{\delta_0}{\rho} \right] \mathcal{P}(x_0 \triangleright \hat{y}) \right\} . \tag{50}$$

Suppose the required level of financing, I_D, is greater than the debt capacity, $\Lambda(x_0)$. Since we consider a project with positive NPV, i.e. $I_D \leq I < V(x_0 \mid \tilde{y})$, the project will then not find financing although it is worthwhile undertaking it. In this case, the agency conflict between the shareholder and outside investors leads to a *financial constraint*.

To better understand the relationship between creditor dispersion and debt capacity, we have to compare to the case where there is just a single creditor, i.e. $N = 1$. In Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997), concessions consist of temporary debt service holidays. The shareholder makes take-it-or-leave-it offers to her creditor, strategically paying less than the originally contracted coupon. In Mella-Barral (1999), the shareholder asks for permanent reductions of debt obligations, forcing her creditor repeatedly to forgive part of her debt. In all models, the debtor has all bargaining power and exercises a *strategic default threat*: that is if the creditor were to reject her renegotiation offer, the shareholder would cease her debt service obligations and force the creditor to call for bankruptcy protection and have the firm liquidated, leaving the creditor a value of $V^*(x)$. As a result, the blackmailed creditor will accept any concession giving him a new debt value of exactly $V^*(x)$, his outside option.

The same threat does not work, however, with dispersed creditors: recall that any single creditor is so small that her acceptance/rejection decision is not decisive for the outcome. Hence, if all other creditors were to accept the offer reducing their aggregate value to $V^*(x)$, the best strategy of a single creditor would be to hold out. It follows that with dispersed debt, strategic default threats cannot be successfully employed. The way to get concessions from dispersed creditors is by pledging additional collateral in the way described earlier.

Thus, there are two differences between single creditor debt and dispersed debt which matter for the debt capacity. First, when facing a single creditor, an opportunistic incumbent shareholder can wring concessions by repeatedly using *strategic default threats*, while the same device has virtually no power against dispersed creditors. Second, if all debt is held by a single creditor, then the degree of collateral is obviously *irrelevant* like any distinction between senior and junior claims.

Comparison with Mella-Barral (1999) is straightforward since our set-up closely follows his model. Therefore, Mella-Barral's (1999) Proposition 3 applies directly to our model: When there is a *single* creditor, and the shareholder is in a position to make strategic default threats (take-it-or-leave-it offers after defaulting) to this creditor, then the debt is first renegotiated the first time the state variable, x_t, hits a certain threshold level x_s (i.e when $\check{x}_t = x_s$), which solves

$$\frac{\partial D_s^{(0)}(x_s)}{\partial x_t} = \frac{\partial V^*(x_s)/N}{\partial x_t} . \tag{51}$$

The values of each bond and the equity are, respectively

$$D_s^{(0)}(x_t) \equiv \frac{\delta_0}{\rho} + \left[\frac{V^*(x_s)}{N} - \frac{\delta_0}{\rho}\right] \mathcal{P}(x_t \triangleright x_s) \tag{52}$$

$$S_s^{(0)}(x_t) \equiv V(x_t \mid \tilde{y}) - N\ D_s(x_t) . \tag{53}$$

When facing a single creditor, the shareholder will always ultimately abandon at $\tilde{y}$, the efficient point. Therefore, it is as efficient to issue single creditor debt as it is to issue dispersed debt with an optimal contract satisfying Condition 1, simply because for all $x_0 > x_s$

$$S_s^{(0)}(x_0) + N\ D_s^{(0)}(x_0) = S^{(0)}(x_0) + N\ D^{(0)}(x_0) = V(x_0 \mid \tilde{y}) . \tag{54}$$

The interesting difference between a single creditor and dispersed creditors emerges when considering the debt capacity. Because of the presence of the strategic default threat, the absolute limit to the amount a single creditor is willing to lend at the entry state x_0, is $V^*(x_0)$, the liquidation value of the firm.[20] With dispersed debt, we obtain a strikingly different result:

Lemma 4 *The debt capacity, $\Lambda(x_0)$, is always strictly larger than $V^*(x_0)$.*

Proof: A proof is given in the Appendix.

Therefore, by issuing widely dispersed debt, the shareholder can always borrow more than by borrowing from just one lender:

Proposition 3 *If at the date of entry the required level of financing, I_D, is not larger than $V^*(x_0)$, then the project can be financed with either dispersed debt or with debt held by a single creditor.*

If I_D is larger than $V^*(x_0)$ but smaller than the debt capacity, $\Lambda(x_0)$, then the project can only be financed with dispersed debt.

[20] See Equation (44), Section 6.1., of Mella-Barral (1999).

D. Limits of the Analysis

We have analyzed strategies of default and renegotiation when the debt is publicly held by a large group of non-cohesive creditors. We have seen that for the debt to be renegotiated ex post, a gradual collateralization of the debt needs to be allowed for ex ante.

It is important to recognize that in many cases, collateral consists of a collection of assets whose liquidation values is determined by their *physical* characteristics. Therefore, in practice there might be physical restrictions on the shareholder's ex ante choice of the initial collateral, $C_0^*(x)$, that are imposed by the value evolution of the assets that are actually collateralizable.

That is, our theory has only regarded the case where the shareholder is always able to find a particular combination of collateralizable assets, within the assets of the firm about to be constituted, that commits her to the efficient abandonment decision, $\hat{y}(\delta_0, C_0^*(x)) = \tilde{y}$. The optimal dispersed debt contract $\mathcal{D}_0$ gives the first best equity value to the shareholder if and only if there exist collateralizable assets with an aggregate value $C_0^*(x)$ satisfying Condition 1.

What is the shareholder's best contract design if a collection of collateralizable assets satisfying Condition 1 does not exist? In this case, the shareholder will look for the *second best* combination of assets, $C_0^*(x)$, which leads to a constrained efficient $\hat{y} > \tilde{y}$ that is as close as possible to the efficient abandonment point $\tilde{y}$. We certainly believe that in practice, the shareholder's ability to offer the right debt collateral is often so limited that there is no dispersed debt contract with optimal collateral design which can improve upon issuing either *(i)* fully collateralized (non-renegotiable) debt or *(ii)* privately held debt with a single or few creditors.

V. Implementing the Model

In this section, we provide conditions under which closed-from solutions can be obtained for all the concepts and results of the paper. The closed-form solution allows for a *quantitative* appraisal of the effects presented here, notably as to the potentially important role played by *(i)* debt creditor dispersion and *(ii)* the debt collateral dimensions.

A. Closed-Form Solutions

To obtain closed-form solutions, additional structural assumptions are required in order to *(i)* express the Laplace transform, $\mathcal{P}(x_t \triangleright y)$, in simple fashion and to *(ii)* solve explicitly for the different optimal decision trigger levels, using the relevant first order optimality conditions. We propose a structure, namely Geometric Brownian Motion plus linear income processes, which is reasonably general[21] and simple. There also exist alternative model specifications allowing to implement closed-form solutions.

Assumption 2 (GBM-Linear Structure) *: (i) The uncertain state variable, x_t, describing the current status of the firm follows a geometric Brownian motion,*

$$dx_t \quad = \quad \mu\, x_t\, dt \quad + \quad \sigma\, x_t\, dB_t\,, \tag{55}$$

where $\mu < \rho$ and σ are constants, and B_t is a standard Brownian motion.
(ii) There exists four constants Θ_0, Θ_1, Θ_0^*, and Θ_1^*, where $\Theta_0 > \Theta_0^*$ and $\Theta_1 < \Theta_1^*$, such that

$$\Pi(x) \;=\; \Theta_0 \;+\; \Theta_1\, x\;, \qquad \textit{and} \qquad V^*(x) \;=\; \Theta_0^* \;+\; \Theta_1^*\, x\;. \tag{56}$$

Notice that the parameter assumptions $\Theta_0 > \Theta_0^*$ and $\Theta_1 < \Theta_1^*$ guarantee that Assumption 1 is satisfied. Under Assumption 2:

1. $\mathcal{P}(x \rhd y)$ can be expressed as

$$\mathcal{P}(x \rhd y) = \left(\frac{x}{y}\right)^{\lambda}.$$

   where $\lambda \equiv \sigma^{-2}[-(\mu - \sigma^2/2) - ((\mu - \sigma^2/2)^2 + 2\rho\sigma^2)^{1/2}]$.

2. Solving for the decision trigger levels yields simple expressions:

   (a) The ex ante optimal abandonment trigger level,

$$\tilde{y} \quad = \quad \frac{-\lambda}{1-\lambda}\left(\frac{\Theta_0^* - \Theta_0}{\Theta_1 - \Theta_1^*}\right)\,. \tag{57}$$

   (b) Assuming that the incumbent chooses the time of her default in an *unconstrained* fashion,[22] the shareholder's ex post optimal abandonment trigger level, $\hat{y} = \hat{y}(\delta_0, C_0^*(\cdot))$, obtained solving the first order optimality condition is,

$$\hat{y} \quad = \quad \frac{-\lambda}{1-\lambda}\left(\frac{N\delta_0/\rho \;-\Theta_0 \;+\; \Theta_0^*\, N\, C_0^*(\hat{y})}{\Theta_1 \;-\; \Theta_1^* \;+\; N(1-\lambda)^{-1}\,\partial C_0^*(\hat{y}/\partial\hat{y})}\right)\,. \tag{58}$$

[21]This structure actually encompasses that of many existing corporate debt valuation models, including Merton (1974), Black and Cox (1976), Brennan and Schwartz (1984), Fischer, Heinkel and Zechner (1989), Mello and Parsons (1992), Kim, Ramaswamy and Sundaresan (1993), Longstaff and Schwartz (1995), Leland (1994), Leland and Toft (1996), Fries, Miller and Perraudin (1997) and Mella-Barral and Perraudin (1997). They either take the total value of the firm's assets or the price of the commodity produced as the driving process, and all assume x_t to follow a geometric Brownian motion.

[22]This is the Endogenous Closure Rule assumed in Leland (1994), Leland and Toft (1996), Fries, Miller and Perraudin (1997), Mella-Barral and Perraudin (1997) and Mella-Barral (1999).

In particular, when initially the debt is fully collateralized, i.e. $C_0^*(x) = V^*(x)/N$, this trigger level, $\hat{y}_f = \hat{y}(\delta_0, V^*(x)/N)$ becomes

$$\hat{y}_f \quad = \quad \frac{-\lambda}{1-\lambda} \left(\frac{N\delta_0/\rho \ - \Theta_0}{\Theta_1} \right) . \tag{59}$$

3. All asset pricing formulas have a simple functional form:
For $A(x_t) \in \{ S^{(0)}(x_t)\,;\, D^{(0)}(x_t)\,;\, S_f^{(0)}(x_t)\,;\, D_f^{(0)}(x_t)\,;\, S_s^{(0)}(x_t)\,;\, D_s^{(0)}(x_t)\,;\, V(x_t \mid y) \}$,

$$A(x_t) \quad = \quad a_A \ + \ b_A\, x_t \ + \ c_A\, x_t^{\lambda} \ , \tag{60}$$

where (a_A, b_A, c_A) are constants. Table 1 contains the explicit expressions for the constants (a_A, b_A, c_A) for all asset values.

B. Pricing Impact of the Debt Model Specification

The practitioner's question will be: How important is the pricing impact of *(i)* creditor dispersion and *(ii)* the collateralization of the debt analyzed in this paper? Is the potential error due to a misspecification of the debt pricing model important or negligible from the asset valuation point of view, i.e. the pricing error that occurs if a widely held debt security is valued with a debt pricing model that does not allow for the specifics of debt renegotiation with dispersed creditors?

Arguably, the most commonly used measures of the impact of a given risk on debt value are the risk premium investors require to compensate them for being exposed, and the associated credit spread. Accounting for creditor dispersion and debt collateral design, the *default risk premium* that our model generates is given by

$$p(x_t) \quad \equiv \quad \delta_0 \ - \ \rho\, D^{(0)}(x_t) \ , \tag{61}$$

and the *credit spread* is given by

$$s(x_t) \quad \equiv \quad \frac{\delta_0}{D^{(0)}(x_t)} \ - \ \rho \ . \tag{62}$$

To inspect the importance of the debt model specification, we compare our model to the two earlier introduced models developed for *(i) single creditor debt* (tantamount to the Mella-Barral (1999) model) *(ii) fully collateralized debt* (tantamount to an adaptation of the Leland (1994) model). We denote the default risk premium and the credit spread for these models in a similar fashion as *(i)* $(\, p_s(x_t)\,;\, s_s(x_t)\,)$ and *(ii)* $(\, p_f(x_t)\,;\, s_f(x_t)\,)$, respectively:

$$p_s(x_t) \quad \equiv \quad \delta_0 \ - \ \rho\, D_s^{(0)}(x_t) \ , \tag{63}$$

$$s_s(x_t) \equiv \frac{\delta_0}{D_s^{(0)}(x_t)} - \rho \ , \tag{64}$$

$$p_f(x_t) \equiv \delta_0 - \rho \, D_f^{(0)}(x_t) \ , \tag{65}$$

$$s_f(x_t) \equiv \frac{\delta_0}{D_f^{(0)}(x_t)} - \rho \ . \tag{66}$$

To measure the pricing impact of the debt contract choice, we consider the *relative* differences in default risk premia and credit spreads between the model proposed in this paper and these two alternative models. Comparison with these two models allows us to measure the valuation error due to model misspecification, and we measure the error along the two dimensions which are important in our analysis:

1. The *creditor dispersion* dimension, comparing renegotiable debt with a single creditor and with dispersed creditors, is measured by:

$$\Delta p_s \equiv \frac{p_s(x_t) - p(x_t)}{p(x_t)} \quad \text{and} \quad \Delta s_s(x_t) \equiv \frac{s_s(x_t) - s(x_t)}{s(x_t)} \ . \tag{67}$$

2. The *state-contingency* dimension, capturing to what degree the initial collateralization allows for debt to be renegotiable, is measured by:

$$\Delta p_f \equiv \frac{p_f(x_t) - p(x_t)}{p(x_t)} \quad \text{and} \quad \Delta s_f(x_t) \equiv \frac{s_f(x_t) - s(x_t)}{s(x_t)} \ . \tag{68}$$

Conveniently, the default risk premium measures, Δp_s and Δp_f, turn out to be independent of the current state x_t.

For this analysis, we use the fact that in the GBM-Linear Structure, the first renegotiation trigger level when there is a single creditor is:

$$x_s = \frac{-\lambda}{1-\lambda}\left(\frac{N\delta_0/\rho - \Theta_0^*}{\Theta_1^*}\right) \ . \tag{69}$$

B. Numerical Example

We now give some numerical estimates carrying out a simple numerical application, under the "GBM-Linear" structure which yields closed-form pricing formulas.

Example 1: *The income generating process, x_t, fluctuates with $\mu = 2\,\%$ and $\sigma = 20\,\%$. For the value of the firm's initial mode of operation, $\Pi(x_t) = \Theta_0 + \Theta_1 x_t$, we assume $\Theta_0 = 0$ and $\Theta_1 = 1$. After abandonment, the new parameters are $V^*(x_t) = \Theta^* + \Theta_1^* x_t$,* where $\Theta_0^* = 0.5$ *and $\Theta_1^* = 0.5$. The interest rate is $\rho = 5\,\%$. There are $N = 10$ bonds issued, each carrying a coupon $\delta_0 = 0.015$ and a collateral of $C^*(\tilde{y}) = 0.04$.*

In this example, the incumbent's initial advantage in using the assets is fairly large, but the competitors' low value of Θ_1^* nonetheless ensures that eventual abandonment is optimal (Assumption 1). Optimal abandonment will then occur at $\tilde{y} = 0.6126$, implying that the value of the collateral at the abandonment point $C_0^*(\hat{y}) = 0.04$ is about half of the liquidation value, $\frac{V^*(\tilde{y})}{N} = 0.08$. The slope of the debt collateral function, $dC_0^*(\hat{y})/d\hat{y}$, is fixed sufficiently steep in order to satisfy Condition 1 (so that $\hat{y}(\delta_0, C_0^*(x)) = \tilde{y}$). These assumptions also mean that the level of outside financing, $I_D = N\, D^{(0)}(x_0)$, is relatively low. Notice that we would get the same numerical estimates for any linear shift in the parameters Θ_0, Θ_0^*, Θ_1 and Θ_1^*, such that the differences $\Theta_1 - \Theta_1^* = 1/2$ and $\Theta_0 - \Theta_0^* = -1/2$ remain unchanged.

Table 2 and Figure 2 exhibit the results obtained with these input parameters. The figures are impressive: Notice in particular the impact of misspecifying for creditor dispersion, with a relative measure of the default risk premium of $\Delta p_s = 375$ %. In plain English, if the debt model estimates default risk premia by wrongly assuming a single creditor when in reality there are many creditors, the default premium would be almost four times overestimated! Misspecifying the state-contingency dimension leads to similarly large errors: the relative measure for the debt risk premium comes out as $\Delta p_f = 245$ % which means that if the debt model wrongly assumes non-renegotiable (fully collateralized) debt, then the default premium is almost two and a half times overestimated.

Clearly, the magnitude of these differences is also determined by the numerical input values for the firm, $\{x; \Theta_0; \Theta_1; \Theta_0^*; \Theta_1^*\}$, and its economic environment, $\{\mu; \sigma; \rho\}$. We do not extend our numerical simulations, since our intention is merely to convey a qualitative insight, which comes out rather strongly in Example 1: Default risk premia and credit spreads can depend very substantially on whether the debt model specification correctly accounts for the *multiplicity of creditors* and/or *initial collateralization* of the debt.

Our relative estimate of the default risk premia, Δp_f, indicates that default premia are *lower* with dispersed debt compared to single creditor debt. This finding should not come as a surprise: Concerning the creditor dispersion dimension, recall that on the one hand, dispersed creditors are vulnerable to dilution threats, but on the other hand, they are protected from strategic default threats. This indicates that creditors' exposure to the dilution threat is less important than their exposure to strategic default. Intuitively, the strategic default option allows the shareholder to obtain earlier and/or larger coupon concessions, since in both cases, the aggregate coupon value at the efficient abandonment point will be the same, $N\,\tilde{\delta}_0$. As a consequence, using dispersed debt rather than privately held debt may allow to *reduce the default premium by a large margin*, since it credibly commits the debt issuer against strategic default threats, without causing any distortion in the underlying real behavior - in both cases, there is efficient abandonment at $\tilde{y}$.

VI. Possible Extensions and Conclusion

This paper presents a fully dynamic explanation for the conventional wisdom that creditors enjoy some protection against opportunistic default threat if debt is dispersed among many investors. This is explained by the lack of intertemporal consistency of concessions which are not backed up by guarantees that an extended part of their liquidation right is safe from continued expropriation. The natural candidate for such a guarantee is the addition of collateral, but other forms of commitment are conceivable.

The paper identifies in fact a "double commitment problem" for the debtor: First, to make debt renegotiation possible, the creditor needs the possibility to commit to inexpropriable liquidation rights (collateral). Second, once the creditor has discretion over how and when to attribute these inexpropriable liquidation rights, she needs to commit not to exercise this option prematurely. With regard to the second commitment problem, our analysis was confined to the study of a single possible solution, collateral design.

The choice of average maturity determines what fraction of debt is expected to be refinanced between renegotiation and abandonment: If maturity is very long, none of the debt is expected to be refinanced, with the incentive consequences studied earlier. If maturity is very short, almost all of the debt is fully and instantaneously refinanced, so the debtor has no incentive at all to propose exchange offers, just as with fully collateralized debt.

Appendix

Proof of Lemma 1: Assume that n_1 debtholders have accepted one offer, even though the shareholder can make a second offer in the future. These senior creditors are receiving a coupon δ_1 since accepting the first offer, at $\underline{x}_1$. Now, they are aware that the shareholder's optimal second offer will be such that tendering is marginally better than holding-out,

$$D^{(2)}_{i\in\mathcal{T}}(\underline{x}_2) \quad = \quad D^{(2)}_{i\in\mathcal{H}}(\underline{x}_2) \ .$$

Therefore, just before this second offer, the value of these debtholders' claim will be

$$D^{(1)}_{i\in\mathcal{T}_1}(\underline{x}_2^+) \quad = \quad \frac{\delta_1}{\rho} + \left[C_0^*(y_\mathsf{s}) - \frac{\delta_1}{\rho}\right] \mathcal{P}(\underline{x}_2 \triangleright y_\mathsf{s}) \ .$$

The value of their claim in this current regime 1, i.e for $\check{x}_t \in [\underline{x}_2, \underline{x}_1)$, is therefore

$$D^{(1)}_{i\in\mathcal{T}_1}(x_t) \quad = \quad \frac{\delta_1}{\rho} + \left[C_0^*(y_\mathsf{s}) - \frac{\delta_1}{\rho}\right] \mathcal{P}(x_t \triangleright y_\mathsf{s}) \ .$$

However, bondholders who did not tender during the first offer currently hold a claim worth

$$D^{(1)}_{i\in\mathcal{H}_1}(x_t) \quad = \quad \frac{\delta_0}{\rho} + \left[C_0^*(y_\mathsf{s}) - \frac{\delta_0}{\rho}\right] \mathcal{P}(x_t \triangleright y_\mathsf{s}) \ .$$

Therefore, when the first offer was made, the tendering condition was clearly violated

$$D^{(1)}_{i\in\mathcal{T}_1}(\underline{x}_1) \quad < \quad D^{(2)}_{i\in\mathcal{H}_1}(\underline{x}_1, \underline{x}_1) \ .$$

This contradicts the initial assumption, that n_1 bondholders have accepted one offer. **QED.**

Proof of Lemma 2: Consider that the k^{th} tendering condition is binding:

$$\begin{aligned} D^{(k)}_{i\in\mathcal{T}_k}(\underline{x}_k) \quad &= \quad D^{(k)}_{i\in\mathcal{H}_k}(\underline{x}_k) \\ &= \quad \frac{\delta_{k-1}}{\rho} + \left[C_{k-1}^*(y_\mathsf{s}) - \frac{\delta_{k-1}}{\rho}\right] \mathcal{P}(\underline{x}_k \triangleright y_\mathsf{s}) \ . \end{aligned}$$

In regime k, i.e for $\check{x}_t \in (\underline{x}_{k+1}, \underline{x}_k]$, debtholders who have tendered receive a coupon δ_k until the $k+1^{th}$ offer, which occurs at $\underline{x}_{k+1}$. The value of their claim is

$$D^{(k)}_{i\in\mathcal{T}_k}(x_t) \quad = \quad \frac{\delta_k}{\rho} + \left[D^{(k)}_{i\in\mathcal{H}_k}(\underline{x}_{k+1}^+, \underline{x}_{k+1}^+) - \frac{\delta_k}{\rho}\right] \mathcal{P}(x_t \triangleright \underline{x}_{k+1}) \ .$$

We have two expressions for the value of this claim, just after the k^{th} offer, i.e for $(x_t) = (\underline{x}_k^+)$,

$$\frac{\delta_k}{\rho} + \left[D^{(k)}_{i\in\mathcal{T}_k}(\underline{x}^+_{k+1}) - \frac{\delta_k}{\rho}\right] \mathcal{P}(\underline{x}_k \triangleright \underline{x}_{k+1}) = \frac{\delta_{k-1}}{\rho} + \left[C^*_{k-1}(y_\mathsf{s}) - \frac{\delta_{k-1}}{\rho}\right] \mathcal{P}(\underline{x}_k \triangleright y_\mathsf{s}) \ .$$

Therefore the value of tendering debtholders' claim just before the $k{+}1^{th}$ offer is already determined,

$$D^{(k)}_{i\in\mathcal{T}_k}(\underline{x}^+_{k+1}, \underline{x}^+_{k+1}) = \frac{\delta_k}{\rho} + \left(\frac{\delta_{k-1} - \delta_k}{\rho} + \left[C^*_{k-1}(y_\mathsf{s}) - \frac{\delta_{k-1}}{\rho}\right] \mathcal{P}(\underline{x}_k \triangleright y_\mathsf{s})\right) \frac{1}{\mathcal{P}(\underline{x}_k \triangleright \underline{x}_{k+1})} \ .$$

Now, when the $k+1^{th}$ exchange offer is triggered at $(x_t) = (\underline{x}_{k+1})$, the most opportunistic offer the shareholder could make is such that the $k+1^{th}$ tendering constraint is binding

$$\begin{aligned} D^{(k+1)}_{i\in\mathcal{T}_{k+1}}(\underline{x}_{k+1}, \underline{x}_{k+1}) &= D^{(k+1)}_{i\in\mathcal{H}_{k+1}}(\underline{x}_{k+1}) \\ &= \frac{\delta_k}{\rho} + \left[C^*_k(y_\mathsf{s}) - \frac{\delta_k}{\rho}\right] \mathcal{P}(\underline{x}_{k+1} \triangleright y_\mathsf{s}) \ . \end{aligned}$$

To be time consistent, the k^{th} exchange offer must guarantee that the future value of tendering debtholders' claim just before the $k+1^{th}$ offer is greater or equal to its value if the shareholder decides then to make the most opportunistic $k+1^{th}$ offer possible. The k^{th} exchange offer must ensure that

$$\begin{aligned} &\frac{\delta_k}{\rho} + \left(\frac{\delta_{k-1} - \delta_k}{\rho} + \left[C^*_{k-1}(y_\mathsf{s}) - \frac{\delta_{k-1}}{\rho}\right] \mathcal{P}(\underline{x}_k \triangleright y_\mathsf{s})\right) \frac{1}{\mathcal{P}(\underline{x}_k \triangleright \underline{x}_{k+1})} \\ &\qquad\qquad \geq \frac{\delta_k}{\rho} + \left[C^*_k(y_\mathsf{s}) - \frac{\delta_k}{\rho}\right] \mathcal{P}(\underline{x}_{k+1} \triangleright y_\mathsf{s}) \ . \end{aligned}$$

Therefore, the k^{th} exchange offer must involve an increase in collateral at least equal to

$$C^*_k(y_\mathsf{s}) - C^*_{k-1}(y_\mathsf{s}) \geq \left[\frac{\delta_{k-1} - \delta_k}{\rho}\right] \frac{[1 - \mathcal{P}(\underline{x}_k \triangleright y_\mathsf{s})]}{\mathcal{P}(\underline{x}_k \triangleright y_\mathsf{s})} \ . \qquad \textbf{QED.}$$

Proof of Lemma 3: The proof is by backwards induction. First, we show that the tendering constraint must be binding in the last (the K^{th}) exchange offer. Then, we show that by induction, the tendering constraint must also be binding in all previous exchange offers.

To simplify notation, we consider strategies where $n_k = K$, for all $k \in \{1; ...; K\}$, i.e. the debtor offers K new contracts in each round. This is without loss of generality.

The optimal abandonment trigger level, y_s, solves (with $n_K = N$)

$$y_\mathsf{s} = \arg\max_y \left\{ \left[-\Pi(y) + N\frac{\delta_K}{\rho}\right] \mathcal{P}(x \triangleright y) \right\} \ .$$

The value of all debt claims *right after* the K^{th} offer (with $n_K = N$) is

$$N\frac{\delta_K}{\rho} + \left[V^*(y_\mathsf{s}) - N\frac{\delta_K}{\rho}\right] \mathcal{P}(\underline{x}_K \triangleright y_\mathsf{s}) \ ,$$

and equals the value of all debt claims *right before* the K^{th} offer

$$\sum_{i=1}^{N} D^{(K-1)}_{i\in\mathcal{T}_{K-1}\cup\mathcal{H}_{j\leq K-1}}(\underline{x}^+_K) \ .$$

Therefore, the value of all debt claims before this K^{th} offer, i.e in regime $K-1$ for $\check{x}_t \in [\underline{x}_K, \underline{x}_{K-1})$,

$$\sum_{i=1}^{N} D^{(K-1)}_{i\in\mathcal{T}_{K-1}\cup\mathcal{H}_{j\le K-1}}(x_t) \;=\; N\frac{\delta_{K-1}}{\rho} + \left[\sum_{i=1}^{N} D^{(K-1)}_{i\in\mathcal{T}_{K-1}\cup\mathcal{H}_{j\le K-1}}(\underline{x}_K^{+}) - \frac{\delta_{K-1}}{\rho}\right]\mathcal{P}(x_t \triangleright \underline{x}_K)\ ,$$

$$= K\frac{\delta_{K-1}}{\rho} \;+\; \left[N\frac{\delta_K}{\rho} \;+\; \left(V^*(y_{\mathsf{s}}) - N\;\frac{\delta_K}{\rho}\right)\;\mathcal{P}(\underline{x}_K \triangleright y_{\mathsf{s}}) \;-\; N\frac{\delta_{K-1}}{\rho}\right]\;\mathcal{P}(x_t \triangleright \underline{x}_K)\ .$$

Next, we develop an argument which holds for *any given* sequence of collateral changes, $C_k^*(x)$, for $k = 0, 1, ..., K$. This argument must then also be valid for any *optimal* sequence of collateral values. Replacing in the shareholder's problem in regime $K-1$, for $\check{x}_t \in [\underline{x}_K, \underline{x}_{K-1})$:

$$\max_{(\delta_K, \underline{x}_K)} \quad \Pi(x_t) - \Pi(y_{\mathsf{s}})\mathcal{P}(x_t \triangleright y_{\mathsf{s}}) \;-\; N\;\frac{\delta_{K-1}}{\rho}[1 - \mathcal{P}(x_t \triangleright \underline{x}_K)]$$

$$- \;N\;\frac{\delta_K}{\rho}[\mathcal{P}(x_t \triangleright \underline{x}_K) - \mathcal{P}(x_t \triangleright y_{\mathsf{s}})] \tag{70}$$

$$\text{s.t.} \quad C_K^*(y_{\mathsf{s}}) \;-\; C_{K-1}^*(y_{\mathsf{s}}) \ge \frac{(\delta_{K-1} - \delta_K)}{\rho}\;\frac{[1 \;-\; \mathcal{P}(\underline{x}_K \triangleright y_{\mathsf{s}})]}{\mathcal{P}(\underline{x}_K \triangleright y_{\mathsf{s}})}, \tag{71}$$

$$x_t \ge \underline{x}_K \ge y_{\mathsf{s}} \tag{72}$$

$$y_{\mathsf{s}} = \arg\max\left\{\left[V^*(y) - N\,C_K^*(y) - \Pi(y) + N\frac{\delta_K}{\rho}\right]\mathcal{P}(x \triangleright y)\right\}, \tag{73}$$

We know that the second part of the constraint (72) cannot be binding, because in regime $K-1$, $\delta_{K-1} < \delta_K$. We can disregard this constraint when setting up the (Kuhn-Tucker) Lagrangean of this problem, which we write as:

$$\max_{\delta_K, \underline{x}_K} L \;=\; \Pi(x_t) - \Pi(y_{\mathsf{s}})\mathcal{P}(x_t \triangleright y_{\mathsf{s}}) \;-N\frac{\delta_{K-1}}{\rho}[1 - \mathcal{P}(x_t \triangleright \underline{x}_K)] - N\frac{\delta_K}{\rho}[\mathcal{P}(x_t \triangleright \underline{x}_K) - \mathcal{P}(x_t \triangleright y_{\mathsf{s}})]$$

$$-\mu\left[\frac{(\delta_{K-1} - \delta_K)}{\rho}\;\frac{[1 \;-\; \mathcal{P}(\underline{x}_K \triangleright y_{\mathsf{s}})]}{\mathcal{P}(\underline{x}_N \triangleright y_{\mathsf{s}})} - \left(C_K^*(y_{\mathsf{s}}) \;-\; C_{K-1}^*(y_{\mathsf{s}})\right)\right] \;-\; \nu\,[\underline{x}_K - x_t]$$

The proof is by contradiction. Suppose to the contrary that the K^{th} tendering constraint (71) is *not* binding, i.e. $\mu = 0$. We distinguish two cases:

Case A: $\nu = 0$, i.e. the first part of (72) is *not* binding. Then $\mu = \nu = 0$. The (Kuhn-Tucker) Lagrangean becomes:

$$\max_{\delta_K, \underline{x}_K} L \;=\; \Pi(x_t) - \Pi(y_{\mathsf{s}})\mathcal{P}(x_t \triangleright y_{\mathsf{s}}) \;-N\frac{\delta_{K-1}}{\rho}[1 - \mathcal{P}(x_t \triangleright \underline{x}_K)] - N\frac{\delta_K}{\rho}[\mathcal{P}(x_t \triangleright \underline{x}_K) - \mathcal{P}(x_t \triangleright y_{\mathsf{s}})]$$

Maximizing gives the FOC:

$$\frac{\partial L}{\partial \underline{x}_K} = -N\left[\frac{\delta_{K-1} - \delta_K}{\rho}\right]\frac{\partial \mathcal{P}(\underline{x}_K \triangleright y_{\mathsf{s}})}{\partial \underline{x}_K} \tag{74}$$

By construction, $\delta_K < \delta_{K-1}$. Hence $\partial L / \partial \underline{x}_K > 0$, contradicting the assumption that $\mu = \nu = 0$.

Case B: $\nu \ne 0$, i.e. the first part of (72) is binding. We have then $\mu = 0$ and $\nu \ne 0$ and hence $\underline{x}_K = x_t$. The (Kuhn-Tucker) Lagrangean becomes:

$$\max_{\delta_K, \underline{x}_K} L \;=\; \Pi(x_t) - \Pi(y_{\mathsf{s}})\mathcal{P}(x_t \triangleright y_{\mathsf{s}}) \;-N\frac{\delta_K}{\rho}[1 - \mathcal{P}(x_t \triangleright y_{\mathsf{s}})] - \;\nu\,[\underline{x}_K - x_t]\,.$$

Maximizing gives the FOC:

$$\frac{\partial L}{\partial \delta_K} \;=\; -\frac{N}{\rho}\left[1-\mathcal{P}(x_t \triangleright y_{\mathsf{s}})\right] \;+\; \frac{\partial}{\partial y_{\mathsf{s}}}\left\{\left[-\Pi(y_{\mathsf{s}}) \;+\; N\,\frac{\delta_K}{\rho}\right]\;\mathcal{P}(x \triangleright y_{\mathsf{s}})\right\}\;\frac{\partial y_{\mathsf{s}}}{\partial \delta_K} \tag{75}$$

which simplifies to $\partial L/\partial\delta_K = -N/\rho\,[1-\mathcal{P}(x_t \triangleright y_{\mathsf{s}})]$ by the envelope theorem. Hence $\partial L/\partial\delta_K < 0$, contradicting the assumption that $\mu = 0$.

Thus, in Case A and in Case B there is a contradiction to the assumption that $\mu = 0$. This concludes the proof that the K^{th} tendering constraint is binding.

It remains to develop the induction argument. Consider regime $k < K$, with a coupon of δ_k. The proof by contradiction mirrors the one for the K^{th} regime. Consider Case A where $\mu = \nu = 0$. At any state x_t in the regime k, the first order condition of the Lagrangean yields as in Eq. (74):

$$\frac{\partial L}{\partial \underline{x}_{k+1}} = -N\left[\frac{\delta_k - \delta_{k+1}}{\rho}\right]\frac{\partial \mathcal{P}(x_t \triangleright \underline{x}_{k+1})}{\partial \underline{x}_{k+1}}\ , \tag{76}$$

giving a contradiction as $\frac{\partial L}{\partial \underline{x}_{k+1}} \neq 0$ is implied by $\delta_k > \delta_{k+1}$. Hence if $\mu = 0$, necessarily $\nu \neq 0$. Consider Case B where $\mu = 0$ but $\nu \neq 0$. At any state x_t in the regime k, the first order condition of the Lagrangean becomes in analogy to condition (75):

$$\frac{\partial L}{\partial \delta_k} \;=\; -\frac{N}{\rho}\left[1-\mathcal{P}(x_t \triangleright y_{\mathsf{s}})\right] \;+\; \frac{\partial}{\partial y_{\mathsf{s}}}\left\{\left[-\Pi(y_{\mathsf{s}}) \;+\; N\,\frac{\delta_k}{\rho}\right]\;\mathcal{P}(x \triangleright y_{\mathsf{s}})\right\}\;\frac{\partial y_{\mathsf{s}}}{\partial \delta_k}\ , \tag{77}$$

which simplifies to $\partial L/\partial\delta_k = -N/\rho\,[1-\mathcal{P}(x_t \triangleright y_{\mathsf{s}})] < 0$ by the envelope theorem, contradicting $\mu = 0$. **QED.**

Proof of Lemma 4: Consider a fully collateralized contract $\mathcal{D}_0 = (\delta_0,\, V^*(x)/N)$. Then

$$\hat{y}\,(\delta_0,\, V^*(x)/N) = \arg\max_y\{\Pi(x_0) - \Pi(y)\mathcal{P}(x_0 \triangleright y) - \frac{\delta_0}{\rho}(1-\mathcal{P}(x_0 \triangleright y))\}. \tag{78}$$

To prove that $\Lambda(x_0) > V^*(x_0)$, it is sufficient to show that there exists a feasible $\delta_0 < \infty$ satisfying the condition

$$\frac{\delta_0}{\rho}(1-\mathcal{P}(x_0 \triangleright \hat{y})) + V^*(\hat{y})\mathcal{P}(x_0 \triangleright \hat{y}) > V^*(x_0)\,. \tag{79}$$

A coupon δ_0 satisfying (79) is feasible if it satisfies the feasibility condition

$$\hat{y}\,(\delta_0,\, V^*(x)/N) < x_0. \tag{80}$$

By differentiating (78), (80) is equivalent to the following first-order condition, evaluated locally by setting $\hat{y} = x_0$:

$$\left(\frac{\delta_0}{\rho} - \Pi(\hat{y})\right)\frac{\partial \mathcal{P}(x_0 \triangleright \hat{y})}{\partial \hat{y}} < \frac{\partial \Pi}{\partial \hat{y}}\mathcal{P}(x_0 \triangleright \hat{y})\,. \tag{81}$$

Suppose (81) did not hold. Then

$$\left(\frac{\delta_0}{\rho} - \Pi(\hat{y})\right)\frac{\partial \mathcal{P}(x_0 \triangleright \hat{y})}{\partial \hat{y}} \geq \frac{\partial \Pi}{\partial \hat{y}}\mathcal{P}(x_0 \triangleright \hat{y})\,. \tag{82}$$

Since the RHS of (82) is strictly positive, and $\frac{\partial \mathcal{P}(x_0 \triangleright \hat{y})}{\partial \hat{y}} > 0$, (82) leads to a contradiction if $\frac{\delta_0}{\rho} - \Pi(\hat{x_0}) < 0$. Hence any δ_0 such that $\frac{\delta_0}{\rho} < \Pi(x_0)$ implies that $\hat{y} < x_0$. Therefore, we are left with showing that there exists a δ_0 such that *(i)* (79) holds and *(ii)* $\frac{\delta_0}{\rho} < \Pi(\hat{x_0})$. We demonstrate this by construction. Choose

$$\delta_0 \equiv [\Pi(x_0) - \Pi(\hat{y})\mathcal{P}(\hat{y})]\frac{\rho}{1 - \mathcal{P}(\hat{y})} \tag{83}$$

Since $\Pi(x_0) > \Pi(\hat{y})$, $\frac{\delta_0}{\rho} < \Pi(x_0)$. Finally, by construction of (83):

$$\begin{aligned} \frac{\delta_0}{\rho}(1 - \mathcal{P}(x_0 \triangleright \hat{y})) + V^*(\hat{y})\mathcal{P}(x_0 \triangleright \hat{y}) &= \Pi(x_0) - \Pi(\hat{y})\mathcal{P}(x_0 \triangleright \hat{y}) + V^*(\hat{y})\mathcal{P}(x_0 \triangleright \hat{y}) \\ &= V(x_0 \mid \hat{y}) > V^*(x_0) \ , \end{aligned}$$

where the last inequality follows from our parameter assumptions on $\Pi(\cdot)$ and $V^*(\cdot)$. **QED.**

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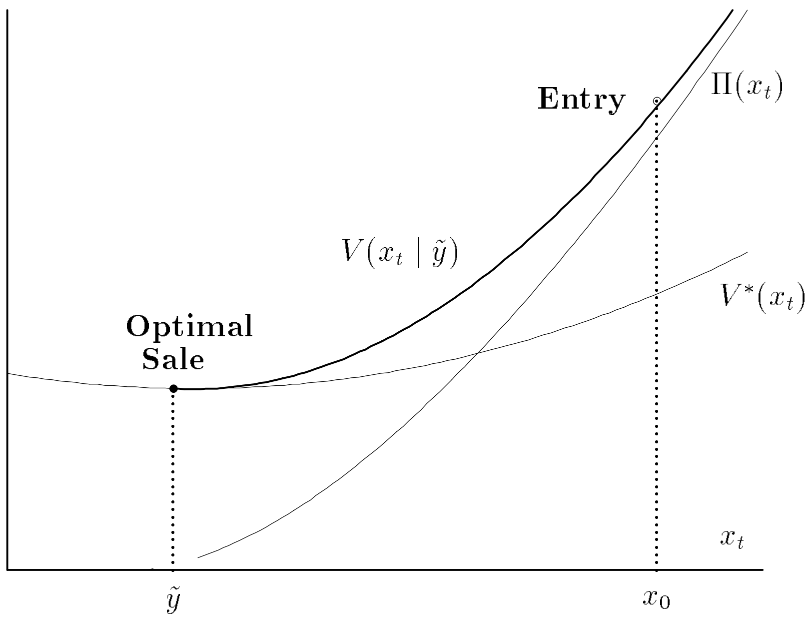

Figure 1: The Firm under the First Best Policy

Table 1: Closed-Form Asset Pricing Formulas in the GBM-Linear Structure.

The table gives the expression of the constants (a_A, b_A, c_A) such that $A(x_t) \; = \; a_A \; + \; b_A \, x_t \; + \; c_A \, x_t^{\lambda}$.

$A(x_t)$	a_A	b_A	c_A
$S^{(0)}(x_t)$	$\Theta_0 - N\delta_0/\rho$	θ_1	$[\Theta_0^* - \Theta_0 + (\Theta_1^* - \Theta_1)\hat{y} - NC_0^*(\hat{y}) + N\delta_0/\rho] \, \hat{y}^{-\lambda}$
$D^{(0)}(x_t)$	δ_0/ρ	0	$[C_0^*(\hat{y}) - \delta_0/\rho] \, \hat{y}^{-\lambda}$
$S_f^{(0)}(x_t)$	$\Theta_0 - N\delta_0/\rho$	θ_1	$[-\Theta_0 - \Theta_1 \hat{y}_f + N\delta_0/\rho] \, \hat{y}_f^{-\lambda}$
$D_f^{(0)}(x_t)$	δ_0/ρ	0	$[(\Theta_0^* - \Theta_1^* \hat{y}_f)/N - \delta_0/\rho] \, \hat{y}_f^{-\lambda}$
$S_s^{(0)}(x_t)$	$\Theta_0 - N\delta_0/\rho$	θ_1	$[-\Theta_0^* - \Theta_1^* x_s + N\delta_0/\rho \, + \{\Theta_0^* - \Theta_0 + (\Theta_1^* - \Theta_1)\tilde{y}\} \, (x_s/\tilde{y})^{\lambda}] \, x_s^{-\lambda}$
$D_s^{(0)}(x_t)$	δ_0/ρ	0	$[(\Theta_0^* - \Theta_1^* x_s)/N - \delta_0/\rho] \, x_s^{-\lambda}$

Table 2: Price Impact of Accounting for (i) Creditor Dispersion and (ii) Collateral Design.

Input parameters are $\mu = 0.02$, $\sigma = 0.20$, $\theta_1 = 1$, $\theta_1^* = 1/2$, $\theta_0 = 0$, $\theta_0^* = 1/2$, $r = 0.05$, $\delta_0 = 0.015$, $C_0^*(\hat{y}) = 0.04$ and $dC_0^*(\hat{y})/d\hat{y}$ is such that $\hat{y}(\delta_0, C_0^*(x))$ equals $\tilde{y}$.

Decision trigger level:		Value
Ex-ante optimal abandonment	$\tilde{y}$	0.6126
Shareholder's ex post optimal abandonment (fully collateralized)	$\hat{y}_f$	1.8377
First renegotiation (single Creditor)	x_s	3.0629

Bond Value at shareholder abandonment:		Value
Multiple Creditors + Optimal Collateral	$D^{(0)}(\tilde{y})$	0.0400
Fully Collateralized Debt (Non-renegotiable)	$D_f^{(0)}(\hat{y}_f)$	0.1419
Single Creditor	$D_s^{(0)}(\tilde{y})$	0.0806

Bond Value at $x_t = 4$:		Value
Multiple Creditors + Optimal Collateral	$D^{(0)}(x_t)$	0.2866
Fully Collateralized Debt (Non-renegotiable)	$D_f^{(0)}(x_t)$	0.2536
Single Creditor	$D_s^{(0)}(x_t)$	0.2365

Risk Premium at $x_t = 4$:		Value/δ
Multiple Creditors + Optimal Collateral	$p(x_t)$	4.46 %
Fully Collateralized Debt (Non-renegotiable)	$p_f(x_t)$	15.41 %
Single Creditor	$p_s(x_t)$	21.16 %
Relative difference	$\Delta p_f(x_t)$	245 %
Relative difference	$\Delta p_s(x_t)$	375 %

Credit Spreads at $x_t = 4$:		Value (bps)
Multiple Creditors + Optimal Collateral	$s(x_t)$	23.34
Fully Collateralized Debt (Non-renegotiable)	$s_f(x_t)$	91.08
Single Creditor	$s_s(x_t)$	134.27
Relative difference	$\Delta s_s(x_t)$	290 %
Relative difference	$\Delta s_f(x_t)$	475 %

Figure 2 : Price Impact of Accounting for (i) Creditor Dispersion and (ii) Collateral Design

Input parameters are $\mu = 0.02$, $\sigma = 0.20$, $\theta_1 = 1$, $\theta_1^* = 1/2$, $\theta_0 = 0$, $\theta_0^* = 1/2$, $r = 0.05$, $\delta_0 = 0.015$, $C_0^*(\hat{y}) = 0.04$ and $dC_0^*(\hat{y})/d\hat{y}$ is such that $\hat{y}(\delta_0, C_0^*(x))$ equals $\tilde{y}$. Figure (a) compares the debt values we obtain when shareholders face dispersed creditors and design the collateral optimally, $D^{(0)}(x_t)$, with the values obtained if (i) the debt is fully collateralized hence non-renegotiable, $D_f^{(0)}(x_t)$, and if (ii) the debt is held by a single creditor, $D_s^{(0)}(x_t)$. The residual value of the firm, $V^*(x_t)$, and the debt value if it was riskless, δ/ρ, are also exhibited. Figure (b) compares the resulting risk premium in each of the three situations depicted above, $p(x_t)$, $p_f(x_t)$ and $p_s(x_t)$, respectively. Here, risk premia are expressed in percentage of debt coupon δ_0. Figure (c) compares the associated credit spreads in each of these three situations, $s(x_t)$, $s_f(x_t)$ and $s_s(x_t)$, respectively.