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Early Versus Late Admittance

By

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Abstract

We develop an incomplete contract model to analyze the enlargement strategy of a club. An applicant is characterized by his wealth and the degree of conformity with the club standard. The club gains only from a fully reformed new member, but reform is costly. The club chooses between early admittance, where it can enforce reform through its partial control power, and late admittance, where entry is conditioned on completed reform. Under the optimal enlargement strategy of the club, wealthy applicants pay an entrance fee and enter early, and poor applicants enter in reversed order: A less advanced is admitted early and a more advanced late. Moreover, poor applicants extract rents that increase in the ratio of reform distance to wealth. If the club can impose a deadline for late entry, it can eliminate all rents with stage financing. In the dynamic game, renegotiation undermines the viability of the late admittance strategy. In the finite game, the applicant's rent from a late offer is non-monotonic in his reform distance and the ability to deteriorate his reform status strategically need not be detrimental to the club.

JEL Classification: D71, G30 Keywords: Club Theory, Incomplete Contracts, Reform Incentives, Governance

1 Introduction

Much economic activity can be attributed to clubs providing goods that are excludable but (partially) nonrival in consumption. Common to clubs as diverse as partnerships, trade unions, international organizations, and more conventional facility sharing organizations (e.g., golf clubs) is the right to admit or reject new members. The literature on the optimal club size focuses on the trade-off between lower per capita costs and increased congestion in the club (e.g., Cornes and Sandler (1996)), and more recently on dynamic voting problems (e.g., Roberts (1999)). We consider a complementary aspect of club enlargement and analyze how the club sets the entry date strategically to influence the applicant's reform incentives. In addition, the paper offers a methodological innovation by applying analytical methods from corporate finance to the question of club enlargement and international integration.

In particular, this paper investigates when a club should admit an applicant who needs to adjust to the club standard in order to become a valuable member. The two options are early and late entry. Under early admittance, the applicant enters prior to having reached the club standard and reforms as newly admitted member, while under late admittance, the applicant enters after having reformed. The key insight of the paper is that the time of entry affects the applicant's incentive to reform which in turn determines the enlargement gains and their distribution between current club members and applicant. More specifically, our main results map the applicant's characteristics to the timing of entry in both the static and the dynamic game. In addition to membership in supranational organizations such as the European Union (EU), our model is applicable to enlargement decisions at large such as hiring employees and naturalizing citizenship applicants.

To focus on the interdependence between admittance date and reform incentives, we analyze an incomplete contract model with incumbent club members acting as a single agent, and without congestion from increased membership. Applicants differ in their observable wealth and reform requirement. An applicant benefits from membership irrespective of whether or not he reforms, but the club gains only if the entrant ultimately reaches the club standard. Reform investments entail an opportunity cost of foregone consumption for the applicant but yield no benefit other than possible entry. In the model, reform investments and reform status are non-verifiable. The early admittance strategy relies on internal enforcement to induce the entrant to undertake reform. The late admittance strategy uses transfer payments and future membership benefits as inducement. The first results in the paper derive the club's optimal admittance strategy in the static game under the restriction that the club can make or demand only a single initial transfer. Under this restriction, sufficiently wealthy applicant types receive an early offer. Entering early is valuable due to discounting, and rich applicants are able and willing to pay more in order to gain entry early. Poor applicants are offered admittance in reversed order: Types who start off close to the club standard enter late and less advanced types enter early. The reversed admittance order is best understood by considering the cost of implementing reform. The most advanced types have a low opportunity cost of reform, and the future membership benefit provides sufficient incentive to reform. For other types, the club must provide funding in excess of the reform cost to induce reform. This overfunding increases with the applicant's initial reform distance. As a result, the early offer is cost efficient for less advanced types because internal control ensures that part of the funds is used for reform. Furthermore, all but the most advanced applicants reap rents that increase in their reform distance.

In the remainder of the paper, we restrict attention to applicant types without wealth and study the late admission offer in more detail. First, we relax the constraint of a single initial transfer and admit a payment also at the end of the reform process. The optimal late offer then includes such an ex post reward for all applicant types who do not reform unless they receive a rent. Since contingent rewards are a more powerful incentive scheme than ex ante overfunding, types who require a small rent receive up-front only reform finance and ex post a reward. For more backward types the optimal late offer combines ex ante overfunding and ex post rewards due to consumption smoothing.

Second, we examine how renegotiation and opportunism affect the viability of the late admittance strategy. If the club can set a deadline, a late offer with stage financing eliminates the rents of all applicants. Disbursing the reform funds in small installments reduces the opportunity cost of reform at any time, thereby making reform incentive compatible. Moreover, the optimal stage financing schedule is robust to renegotiation attempts by the applicant. In the dynamic game where the club is unable to commit to a deadline, opportunistic behavior raises the cost of implementing reform and may even result in a failure to induce reform under the late offer. The reason is that failing to reform in the current period does not preclude reform and entry at some later time. This prevents the club from credibly denying future funding. As a result, a reform inducing transfer has to compensate the applicant for both current opportunity cost and foregone future rents. In the finite game, the rents are nonmonotonic in the reform distance and in the number of remaining periods. Furthermore, the club may gain if the applicant can use its reform funds for strategic deterioration of the current reform status. Both results are driven by the existence of a distant end date that enables the club to credibly commit to zero funding for some periods. The prospect of subsequent periods without financing reduces the opportunity cost in the current period, thereby inducing the applicant to reform.

We believe that our analysis provides a useful starting point to study more complex enlargement decisions. Topical international integration examples are the Eastern Enlargement of the European Union (EU), and China's application to the World Trade Organization (WTO). As regards the EU-enlargement, one central issue in the discussion is the timing of the applicants' entry relative to their reform progress. The admission date and reform incentive are interdependent, and the EU enlargement exhibits several key properties of our model. In particular, reform fulfillment of the multi-dimensional entry conditions and reform efforts are hard (impossible) to verify; there is no enforcement institution with authority over sovereign applicant countries;¹ and some EU reform requirements do not coincide with the applicants' optimal development strategy.² Moreover, the experience from recent International Monetary Fund lending to Russia shows that deviation of development funds into consumption is a real concern. In our view, the present paper also offers insights applicable to organizations at large. First, recruitment and integration of new members is a common event in most organizations. Second, the club's partial control over the entrant's resource allocation can be interpreted as monitoring or training technologies with a deadweight cost. Hence, other possible interpretations of the early versus late strategy are the choice between internal training and incentive schemes or the choice between input and output monitoring.

Our analysis builds on two ideas from the theoretical corporate finance literature; substitutability between wealth and monitoring (e.g., Holmström and Tirole (1997)) and liquidation threat as a disciplining device (e.g., Bolton and Scharfstein (1990)). In moral hazard models of direct and indirect lending, there are typically three regimes (e.g., Diamond (1991), Freixas

¹For example, "Based on the EU-Poland European Agreement, EU rules for public procurement also apply to Poland. However, chief EU antitrust legislator van Miert cannot sue Poland in the European Court of Justice if the country violates the rigid public procurement regulations. In that case the two sides must first negotiate." (Basler Zeitung Jan 4, 1999; own translation)

²For illustration, the burden imposed on Poland by requiring its water quality to meet the standards of the EU water directive is estimated to amount to \$40 bn. Given the Polish level of development, this is a considerable luxury, large enough to depress the country's growth rate. (Financial Times, October 6, 1997.) Citing a World Bank study, the source also points to annual maintenance costs of up to \$200 per citizen, or around 4% of Poland's GDP.

and Rochet (1997)). Entrepreneurs with sufficient wealth can issue cheap direct debt because the relatively low repayment obligations do not induce them to divert borrowed resources. Entrepreneurs with less wealth have to borrow more in order to invest. Because of the larger repayment obligations, monitoring is needed to prevent diversion, and these entrepreneurs can only borrow from banks. Finally, the poorest entrepreneurs cannot raise any outside finance. These three outcomes correspond to the late, early, and no admittance offer in our framework where the reform distance is a measure of the agency problem. Future membership benefits as an incentive mechanism resembles the liquidation threat and the denial of future funding as tools to discipline borrowers without collateral or sufficient pledgeable returns (e.g., Hart and Moore (1994), Gromb (1994)).

Originating in public choice theory, the club literature typically presupposes multiple decision makers and hence decision making through the operation of some voting procedure (see surveys by Sandler and Tschirhart (1980), and Cornes and Sandler (1996)). Accordingly, analyses of the club size discusses the existence of majority voting equilibria and problems associated with endogenous electorates (e.g., Roberts (1999)). These issues do not arise in our model because we assume that the incumbent club members are homogenous. Instead we focus on the relationship between admittance date and applicants' reform incentives. To our knowledge, this aspect of club enlargement has not been analyzed in the literature.

Two papers on the Eastern Enlargement of the EU are also related to our paper. Berglöf and Roland (1998) discuss the advantages of early and late accession for the reform incentives of the Central and Eastern European countries. In contrast to our paper, they focus on political constraints of the reform process and do not allow pre-financing of reform in the late entry scenario. Wallner (1999) argues that a self-interested EU may force applicants to incur specific investments ahead of entry negotiations, and then exploit their lower outside opportunities by setting tougher terms of entry. We abstract from hold-up problems arising from relationshipspecific investments and examine the incentive effects of contingent late admittance.

The paper is organized as follows. We describe the model in Section 2 and characterize the club's optimal enlargement strategy in the static game in Section 3. In Section 4, we enrich the strategy space of the club to include an expost reward to an entrant. Section 5 contains the analysis of renegotiation of an offer. This part is divided in four Subsections, focusing in turn on renegotiation in the one period game (5.1), stage financing (5.2), opportunistic consumption in the dynamic game (5.3), and strategic deterioration (5.4). Section 6 concludes. All technical

proofs are in the Appendix.

2 Model

We consider a club that acts as a single player and faces an applicant for membership.³ The applicant's type is defined by his wealth $w \ge 0$ and the reform requirement d, where $d \equiv x^C - x^0 > 0$ denotes the distance between the applicant's initial status $x^0 \in (0, x^C)$ and the club standard x^C along some reform dimension x. While x is observable, it is not verifiable.

Current members of the club realize a collective enlargement gain $\Pi^R > 0$ from a fully adjusted new member. If the new member fails to reach the standard, the enlargement payoff is $\Pi^U < 0$. The binary payoff is meant to capture the idea that heterogeneity among club members lowers membership benefits. As the club provides a public good to all its members, membership benefits accrue also to a free-rider, i.e., unreformed new member. Reflecting this notion, an applicant realizes a given amount $\pi > 0$ when being admitted, irrespective of whether or not he (ultimately) conforms with the club standard. For simplicity, we ignore that this benefit may well be larger for a conforming member.

Reform is modeled as a costly adjustment of the applicant's initial position towards the club standard.⁴ Our focus is on conformity requirements that constitute a deviation from the applicant's optimal stand-alone resource allocation, rather than reforms that raise the applicant's welfare directly.

Assumption 1 Investment in x yields no direct benefit to the applicant.

By investing F, the applicant moves from x^0 to $x = x^0 + F$. We assume for now that $x > x^0$ but relax this constraint later (Subsection 5.4). The club has financial slack and can finance (part of) the applicant's reform. In fact, the club is the natural source of funding for a wealth-constrained applicant. Since reform and entry do not yield any other returns than the enlargement gains, no other lender can recoup the investment, unless the club reimburses it. (In this case, the club may as well provide the funding directly.) Feasibility of reform requires that $F \leq w + s$, where $s \in \mathbb{R}$ is the financial transfer linked to an admittance offer. A positive

 $^{^{3}}$ Our model of the enlargement process abstracts from several features common to club models, notably negative consumption externalities and the aggregation of heterogenous incumbents' preferences. Furthermore, we abstract from the provision of the club good.

⁴Many club standards are arbitrary or historically determined and need not be inherently optimal. They are valuable because members adhere to them, thereby coordinating their behavior. For instance, when the Channel Tunnel project provided for a railway connection between the UK and Continental Europe, England faced the pure adjustment cost of altering its railway track width.

transfer is a subsidy from the club to the applicant, while a negative one is an entrance fee charged by the club. Initially, we restrict the club to make or demand a single transfer at the beginning of the game.

Investing in the club standard has an opportunity cost, which we model as forgone consumption. The applicant can either invest or consume the disposable resources. The utility of consumption is given by the function u(.), which is twice continuously differentiable and satisfies u(0) = 0, u' > 0, and u'' < 0. The applicant's payoff function is additively separable in the membership benefit π and the utility u(.) from consumption.⁵

The club has all the bargaining power in the enlargement negotiations. In addition to rejecting an applicant, it has two enlargement strategies. The club can offer late admittance conditional on prior reform investment. Alternatively, it can offer early admittance, where any investment in meeting the club standard is undertaken after the applicant has joined. By joining the club, the entrant becomes subject to club rules and institutions.⁶

Assumption 2 The club can control a fraction $\gamma \in (0,1)$ of the resources available to newly admitted members, while it has no such enforcement power over non-members.

Under the late offer, a candidate retains full control over all resources w + s, even if the club has provided funds (s > 0). In contrast, a newly admitted member has full discretion only over a fraction $(1 - \gamma)$ of its resources, where γ reflects the strength of the club's internal enforcement power. For simplicity, we abstract from the abuse of power by the club. The club allocates all resources under its control to reform until the investment amounts to d. Once this level is reached, the club refrains from interfering with the entrant's resource allocation decision.

Besides the partial control γ over the resources w + s, the club has no other instrument to induce a new member to reform. In particular, the club has no discretion over the benefit π . Instead, π is predetermined by applying the club rules, as the club cannot discriminate against its new (or any other) member.⁷ We also assume that a newly admitted member cannot be

⁵The additively separable payoff function can be motivated in different ways. First, π may be a non-monetary, non-transferable private benefit. Second, the applicant may be a large group of individuals, e.g., a country, and different subgroups benefit from (foregone) consumption and membership. Third, club rules may prohibit to withhold membership benefits in response to insufficient reform. For instance, the EU could not coerce Greece into implementing environmental safety measures with the threat of withholding Structural Funds or Common Agricultural Policy transfers.

⁶Prominent formal club enforcement institutions are the European Court of Justice and the arbitration mechanism of the WTO, while peer pressure is an example of an informal disciplining tool.

⁷ For example, the EU faces high (possibly prohibitive) costs of altering the sections of the *acquis communautaire* that detail the agricultural and regional support funds for which new members qualify.

expelled against its will. Both these features obtain if changes of club rules and expulsion are subject to unanimity.

To focus on the link between admittance and reform, we assume that it is not profitable for the club to admit an applicant who never reaches the club standard. This requires that the sum of the club's and applicant's gain from unreformed entry is small. The precise condition is derived in Lemma 14 in the Appendix.

Assumption 3 $\Pi^{U} + u^{-1}(\pi) < 0.$

Assumption 3 ensures that the club strictly prefers no enlargement to admitting an applicant who will not reform. Henceforth, unless explicitly stated, we abstract from offers that do not induce reform.

While all parameters and variables are observable, only the receipts of payments and the entry into the club are verifiable.⁸ Hence, contracts on payoffs (π and Π^R) or reform (x) are not enforceable and a conditional late entry offer must be self-enforcing.⁹

First we consider a single period enlargement game. For simplicity, we set the length of a period equal to the time necessary to achieve full reform. The sequence of moves in the period unfolds as follows (see Figure 1). At date 0, the club makes a take-it-or-leave-it admittance offer to an applicant of type (d,w). More precisely, the club chooses the triple (\bar{x}, s, j) where $\bar{x} \leq x^C$ is the reform requirement, $s \in \mathbb{R}$ is the financial transfer, and j = L, E, N is the timing of enlargement (late, early, and no offer). Then the applicant either accepts or rejects the offer. If no entry was offered, or if the applicant rejects an offer, the game ends and the parties get their reservation payoffs that are normalized to zero. Upon acceptance, the amount s is transacted and the consumption and reform investment decisions are taken. At date 1, $x = x^0 + F$ is observed and the late conditional offer is executed.

There is a common discount factor $\delta < 1$, and the date 0 value of the late enlargement benefits is $\delta \pi$ and $\delta \Pi^k$, k = U, R. For simplicity, we abstract from the difference between late entry and initially unreformed early entry with subsequent reform. Hence, the payoff to the club from early entry is also $\delta \Pi^R$ if the entrant reaches x^C by date 1, but Π^U otherwise.

⁸ If π were verifiable, the club could contractually impose a penalty (withholding π) on an early entrant failing to reform. As a result, an early offer would weakly dominate a late one.

⁹The entrant's share π of the total (gross) enlargement surplus $\Pi^R + \pi$ can be interpreted as reflecting his bargaining power. We assume that π is either non-verifiable or non-transferable, and that the club has all bargaining power. Alternatively, $\pi + \Pi^R$ could be transferable and the applicant's bargaining power could be such that he extracts a share $\frac{\pi}{\Pi^R + \pi}$ of $\pi + \Pi^R$ in the expost negotiations. This framework results in the same incentive structure.



Figure 1: Timing

3 Early versus Late Admittance

We solve for the optimal admittance strategy by backward induction. First, we derive the minimum necessary transfer s to implement reform, given that the applicant has accepted an early or late offer. Second, we solve for the minimum necessary transfer such that an applicant accepts an early or a conditional late offer and subsequently reforms. Finally, we compare the cost of the these two offers as a function of the type.

The club maximizes $\delta \Pi^R - s$ by choosing a reform threshold \overline{x} , a transfer $s \in \mathbb{R}$, and the type of offer $j = \{E, L, N\}$, subject to the applicant's optimal response.¹⁰ At date 1, x is observed and the decisions left to the club depend on whether it has made an early or late admittance offer at date 0. In the case of an early offer, the applicant has already been admitted and the enlargement gains materialize mechanically. For $x = x^C$, the date 0 value of the enlargement is $\delta \Pi^R - s$, and $\Pi^U - s$ otherwise. The entrant gets $\pi + u (w + s - F)$ independent of x.

In case of a late offer, the club has to take the final admittance decision. The date 0 value of the enlargement payoff to the club is $\delta \Pi^R - s$ if $x = x^C$, and $\delta \Pi^U - s$ otherwise. If the club refuses admittance, its payoff is -s. Because the parties cannot contract upon x, admittance of an applicant that meets the entry condition needs to be self-enforcing (subgame perfect). Hence, the club admits the applicant if $x \ge x^C$ and rejects him otherwise. For the time being, we assume that the club has set $\overline{x} = x^C$ and show later that this is optimal. Given this admittance rule, the applicant gets a payoff with a date 0 value of $\delta \pi + u (w + s - F)$ if $x \ge \overline{x}$,

¹⁰In fact, the club's payoff range includes $\Pi^U - s$. As we show in Appendix G, unreformed entry is strictly dominated by Assumption 3, and we restrict the analysis here to reformed entry.

and u(w + s - F) otherwise.

Upon acceptance of an offer, s is transferred, and the reform investment decision is taken.

Lemma 1 (Reform Implementation) Under both early and late admittance, full reform can be implemented for any type (d, w).

i) With an early admittance offer, the minimum necessary transfer is

$$s^{E\prime} = \frac{d}{\gamma} - w$$

ii) With a late admittance offer, the minimum necessary transfer is

$$s^{L\prime} = \begin{cases} d-w & if \quad d < \hat{d}; \\ \hat{s}(d,w) & otherwise, \end{cases}$$

where $\hat{d} = u^{-1}(\delta \pi)$ and $\hat{s}(d, w)$ solves $\delta \pi + u(w + s - d) = u(w + s)$. The threshold \hat{d} increases in δ , and the transfer \hat{s} decreases in δ and increases in d.

In the case of early admittance, the entrant already enjoys the membership benefits and has no incentive to reform. Instead, he spends all its discretionary resources $(1 - \gamma) (w + s)$ on consumption. Relying on the club's limited internal enforcement, full reform is feasible only if the entrant's total resources after entry are no less than $\frac{d}{\gamma}$. Hence, the club has to set s such that $\gamma (w + s) \ge d$. While the club can induce reform for any early entrant, the cost $\frac{d}{\gamma} - w$ becomes prohibitive for types with a sufficiently large reform requirement. The borderline above which the club prefers no enlargement to the early offer is given by $d^{NE} = \gamma (\delta \Pi^R + w)$.

In the case of late admittance, investment in F is of value to the applicant only if it leads to entry, but comes at the opportunity cost of forgone consumption. Hence, the applicant either does not reform (F = 0) or invests exactly the amount needed to meet the entry condition (F = d). For full reform to be feasible, the club must leave the applicant at least s = d - w. Having the necessary funds at their disposal, only the most advanced applicant types $(d \le \hat{d})$ reform. For all other types $(d > \hat{d})$, the utility from diverting d - w exceeds the future membership benefits. Nonetheless, the club can induce these types to reform by giving them a larger amount. Such overfunding renders reform incentive compatible, because the marginal utility of consuming w + s - d is larger when d is invested in reforms than when the entire w + s is used for consumption. The minimum late offer transfer that provides reform incentives is \hat{s} . This transfer increases in d but decreases in δ . A larger d raises the opportunity cost of reform, while a larger δ raises the benefit of reform. As with the early offer, there is a critical reform distance d above which the late offer ceases to be profitable for the club.



Figure 2: Acceptance Of An Early Offer

The borderline is given by $\delta \Pi^R = \hat{s}$, which defines an increasing and concave curve $d^{NL} = w + \delta \Pi^R - u^{-1} \left[u \left(\delta \Pi^R + w \right) - \delta \pi \right]^{.11}$

The club's optimal admittance strategy does not follow directly from the lowest implementation cost of reform. In addition, an applicant must also accept an early or a late offer. The minimum necessary transfer that is both accepted and implements reform obtains from comparing implementation and individual rationality constraints in each case.

Lemma 2 (Acceptance Early) An applicant accepts a reform-implementing early offer with a minimum transfer

$$s^{E} = \begin{cases} d - w + u^{-1} \left[u \left(w \right) - \pi \right] & \text{if } w \ge u^{-1} \left[\pi + u \left(\frac{1 - \gamma}{\gamma} d \right) \right]; \\ \frac{d}{\gamma} - w & \text{otherwise.} \end{cases}$$

Figure 2 shows how the applicant types are separated according to the binding constraint. In Region I, applicants are poor relative to their reform distance and the feasibility constraint

¹¹Concavity of u is crucial for overfunding to improve reform incentives. If the marginal utility of consumption were constant, say unity, the incentive constraint for reform under a late offer would be $\delta \pi + \hat{s} = d + \hat{s}$. In this case overfunding could not reduce the applicant's opportunity cost of reform. As a result, the late admittance strategy would implement full reform only for applicant types $d < \hat{d}$, and d^{NL} would be equal to \hat{d} . While concavity of u is necessary for overfunding to be effective, further restrictions are needed to determine the curvature of s^L in d. (See the discussion at the end of this section and in Appendix H.)



Figure 3: Acceptance Of A Late Offer

 $w + s \ge d/\gamma$ determines the transfer s^E . Applicants in Region II are relatively wealthy, and the entrance fee is constrained by their outside option of not joining, u(w). All types for which the feasibility (FC^E) and the individual rationality constraint (IR^E) simultaneously bind constitute the curve $(IR^E - FC^E)$, separating Regions I and II. Finally, for any given w, types from Region I require a larger transfer than those from II.¹²

Lemma 3 (Acceptance Late) An applicant accepts a reform-implementing late offer with a minimum transfer

$$s^{L} = \begin{cases} \hat{s} & \text{if } d > u^{-1} \left(\delta \pi \right) \text{ and } d > w - u^{-1} \left[u \left(w \right) - \delta \pi \right]; \\ d - w + u^{-1} \left[u \left(w \right) - \delta \pi \right] \text{ if } d > u^{-1} \left(\delta \pi \right) \text{ and } d \le w - u^{-1} \left[u \left(w \right) - \delta \pi \right]; \\ d - w \quad \text{if } d \le u^{-1} \left(\delta \pi \right) \text{ and } w < u^{-1} \left(\delta \pi \right); \\ d - w + u^{-1} \left[u \left(w \right) - \delta \pi \right] \text{ if } d \le u^{-1} \left(\delta \pi \right) \text{ and } w \ge u^{-1} \left(\delta \pi \right). \end{cases}$$

Figure 3 illustrates the binding constraint and consequent transfers for each applicant type. Region I contains applicant types that are relatively poor and have a large reform requirement. For those types, the incentive constraint (IC^L) binds. The types in Regions II and IV are rich relative to their reform distance, and the minimum accepted transfer is determined by their

The IR^E requires $\pi + u\left(\frac{1-\gamma}{\gamma}d\right) \ge u(w)$, where $\frac{1-\gamma}{\gamma}d$ is the minimum retained after reforming. Manipulation yields $\frac{d}{\gamma} - w \ge d - w + u^{-1}\left[u(w) - \pi\right]$.

outside option u(w). That is, the individual rationality constraint (IR^{L}) binds. The dividing line between Regions I and II, $(IR^L - IC^L)$ is given by the points where both constraints simultaneously bind. This implies that the transfer is zero along this curve. For applicants in Region III, the membership benefit outweight the utility from consuming the amount d. Subsidized types (d > w) do not divert any resources, while types with d < w are willing to pay an entrance fee. Thus, the minimum accepted transfer is determined by the feasibility constraint (FC^L) . For any given w, types from Region I require the largest transfer.¹³

In addition to the transfer s and the timing of admittance, an offer made in the beginning of the period specifies a threshold \overline{x} . Setting $\overline{x} = x^C$ is immediate. In a late offer, the club will admit an applicant at date 1 only if $x = x^C$. Hence, a choice $\bar{x} < x^C$ is not time consistent and will simply be ignored by the club at the time of the final admission decision. In an accepted early offer, $F = d \leq \gamma (w + s)$ by Lemma 2. That is, the reform investment comes from the club controlled fraction of w + s and the choice of \bar{x} is inconsequential. Thus, in either offer it is a weakly dominant strategy for the club to set $\bar{x} = x^{C}$. While this threshold is implicitly understood by a rational applicant, we assume that the club formally announces it.

The above analysis allows us to classify the applicant types into recipients of early, late, and no offer. To obtain an unambiguous classification, we make a further assumption.

Assumption 4 i)
$$\delta \pi < u \left(\delta \Pi^R \right) - u \left[(1 - \gamma) \left(\delta \Pi^R \right) \right];$$

 $ii) \frac{u'(x)}{u' \left[(1 - \gamma) x \right]} > 1 - \gamma, \ \forall x \ge 0.$

Part i) of Assumption 4 ensures that the least profitable type to get an admittance offer receives an early one. Part *ii*) implies that the set of types for which late is the preferred offer is connected.¹⁴ After presenting our results, we discuss the robustness of our results with respect to this assumption. Define $w_1 \equiv u^{-1}(\delta \pi)$ and w_2 as satisfying $\delta \pi + u [(1 - \gamma) w] = u(w)$.

Proposition 1 (Optimal Offer) Only types with $w > \frac{d}{\gamma} - \delta \Pi^R$ receive an admittance offer. i) For applicants $d \leq w_1$, the club follows a 'reversed' admittance order.

$$s = d - w + u^{-1} \left[\underbrace{u \left(w + \overbrace{s}^{>0} \right) - \delta \pi}_{>0} \right].$$

For a given w, this is greater than $d - w + u^{-1} [u(w) - \delta \pi]$, the transfer for types from Regions II and IV. For types from Region I, $u(w+s) - \delta \pi > 0$ implies that the transfer to types in III (d-w) is also less. ¹⁴For example, $u(\cdot) = \sqrt{\cdot}$ and $u(\cdot) = \log(\cdot)$ satisfy Part (ii).

¹³For the types in Region I,



Figure 4: The Optimal Offer

ii) For applicants $w_1 < w \le w_2$, the club offers early entry to the most advanced types, and follows a reversed admittance order otherwise.

iii) For applicants $w > w_2$, the club offers only early entry.

Figure 4 illustrates the Proposition. On the one hand, entering early rather than late is of value due to discounting. Rich applicants have a sufficiently low marginal utility of wealth and are willing to pay more in order to gain entry early. This discounting effect dominates for rich types. On the other hand, poor types receive reform funding from the club. They have a high marginal utility of wealth, and hence a strong temptation to consume the funds. An incentive compatible late offer is then more expensive for the club than using its imperfect internal enforcement technology to get reform implemented. For an intermediate range of wealth relative to reform distance, the cheapest way to implement reform is the use of leverage from conditioning entry on prior full reform. In this range, where both d and w are not too large, neither the wealth effect nor the overfunding effect are sufficiently strong to dominate the leverage effect.

For any given wealth level, types with a too large reform distance do not receive an admittance offer. From a social efficiency perspective, too few types receive offers. The socially



Figure 5: Transfer Payments under the Optimal Offer

efficient cut-off rule for the early offer is $\pi + \delta \Pi^R - d = 0$, while the club applies $\delta \Pi^R - s = 0$, where $s \ge d$.

Our notion of 'reversed' admittance order refers to 'reform time' (not calendar time). More advanced types, i.e., low d values, are admitted after they have reformed, while less advanced enter prior to reforming. Thus, the enlargement strategy applies "double standards". Unlike more backward candidates, stronger candidates are asked to prove their willingness to conform with the club standard prior to admittance.

Corollary 1 (*Reform Payments*) Among the entrants, wealthy types pay an entrance fee in addition to the full reform cost, intermediate types pay part of their reform cost, while poor types receive a rent in addition to their reform cost.

Corollary 1 is illustrated in Figure 5. In Region I, applicants are sufficiently poor relative to their reform distance that the club must provide more funds than the reform cost. Under the late offer, such overfunding is necessary to meet the incentive constraint, while under the early offer the club is unable to control all the transferred funds. In Region II, the club and the applicant share the reform costs, while in Region III the applicants are so wealthy that they pay an entrance fee in addition to the full reform cost. The rent that the least advanced types earn under both early and late offers rises in the reform distance d, making reformed entry eventually prohibitively expensive for the club. Thus, the transfer decreases in the ratio of wealth to reform distance.

Corollary 2 (Comparative Statics) i) An increase in γ enlarges the set of early offer candidates, and weakly reduces the transfer to all candidates that receive an offer.

ii) An increase in δ or π enlarges the set of late offer candidates, and weakly decreases the transfer to all candidates. An increase in δ also strictly enlarges the set of types receiving an offer.

Stronger internal enforcement (larger γ values) makes the early offer cheaper for the club and turns some previous recipients of late or no offers into early offer types. The transfer s^E falls in those regions where the FC^E is the binding constraint. For wealthy applicants, the FC^E is slack and the IR^E binds, and hence, their entrance fee is unchanged. A rise in the discount factor makes late entry worth more and hence, increases reform incentives. Furthermore, it relaxes the IR^L . Accordingly, the club substitutes late for early offers for some candidates. It also shifts d^{NE} upwards, and hence, early offers are made to some former no-offer types. A larger π raises the relative attractiveness of the late offer, because it relaxes the IC^L while the FC^E is unaffected. Although it also relaxes the IR^E , the boundaries between early and late offer lie strictly in the set of types where the FC^E determines s^E . Hence, while a larger π lowers s where the IR^E binds, it does not change the type of offer.

Proposition 1 and the Corollaries crucially depend on Assumption 4. While the discounting effect underlying the optimality of early offers for wealthy types only requires concavity, the 'reversed' admittance result for poorer types also depends on the degree of curvature of u.¹⁵ More precisely, for candidates $d < u^{-1}(\delta \pi)$, the late offer transfer s^L is determined by the feasibility constraint (FC^L) , and hence is independent of u and smaller than s^E . For applicants $d \ge u^{-1}(\delta \pi)$, s^L is given by the incentive constraint (IC^L) and by virtue of Assumption 4 increases at a faster rate than γ , the rate at which s^E increases. As a result, there is a unique value d^J , above which early offers are cheaper.

Alternatively, the 'reversed' admittance result also obtains by restricting the function u to the class of hyperbolic absolute risk aversion (HARA) functions that satisfy DARA. Such

¹⁵As pointed out in the discussion following Lemma 1, the effectiveness of overfunding also relies on the concavity of u. If the marginal utility of consumption were unity, the late admittance strategy would be dominant only for $d < \hat{d}$, and d^{LE} would be equal to \hat{d} .

functions imply that s^L is convex in d (see Appendix H). If neither Assumption 4 nor DARA-HARA holds, s^L may be concave in d (for any given w), and two possible cases arise. First, the s^L curve may lie everywhere below the s^E curve, and early offers are dominated by either a late offer or no offer. Second, the concave s^L may intersect s^E twice, generating either the 'reversed' admittance pattern, or a pattern late-early-late, depending on whether the second intersection lies above or below the cut-off line d^{NE} , where early offers cease to be profitable for the club. Common to both Assumption 4 and DARA-HARA is that s^L increases by more than s^E .

Instead of additional restrictions on the function u, a fixed enforcement cost also implies that s^L increases at a faster rate than s^E (which is independent of d). Suppose that at a cost $\Gamma > 0$ the club is able to fully control how a newly admitted member allocates the resources at his disposal. For advanced poor applicants, relying on the incentive of future membership and offering late conditional entry is the cost efficient way of implementing reform, while early entry dominates for all candidates with $s^L - d > \Gamma$. Thus, concavity of u in combination with a lump sum enforcement cost is sufficient for the 'reversed' admittance result.

4 Rewarding Entry

Up to this point, the club was restricted to a single transfer at the beginning of the game. We now relax this constraint and allow for a payment also at the end. This permits the club to offer the applicant a reward for having reformed. Such a reward can be offered under a late admittance offer, because entry is verifiable and occurs after reform; in contrast, this is not feasible when entry precedes reform (early admittance). For now, we assume that the club can commit to pay the promised reward to a late entrant. Henceforth, we consider only the cohort of applicants with zero wealth.

Assumption 5 Applicants have zero wealth (w = 0).

This subset is both analytically tractable and of interest for the trade-off between early and late offers because neither strategy is strictly dominated over the whole range of reform distance.

As the applicant has no money, some prefinance is needed to make reform possible. To increase reform incentives, the club can pay additional money either before or after the applicant reforms. Consider first a late offer that pays only the pure reform cost (s = d) initially, and has a reward p payable upon full reform and entry. This yields the following incentive constraint:

$$\delta \pi + \delta u(p) \ge u(d) \,.$$

Define \hat{p} as the value of p that satisfies this constraint with equality:

$$\hat{p} \equiv u^{-1} \left[\frac{u(d)}{\delta} - \pi \right].$$

It follows immediately that a sufficiently high reward induces any applicant to reform, and that \hat{p} is unique and increases in d. Compared to ex ante overfunding, a reward tempts applicants less to consume, because the amount available at date 0 is limited to d rather than s > d, and reform is directly rewarded with p. Thus, rewards provide more powerful reform incentives. The optimal late offer may, however, be a mixed offer, combining overfunding (s > d) and a reward (p > 0).

Proposition 2 (*Mixed Late Offer*) For applicants with zero wealth, the minimum incentive compatible late offer with full commitment is :

$$(s^L, p^L) = \begin{cases} (d, 0) & \text{for } d \leq \hat{d}; \\ (d, \hat{p}) & \text{for } \hat{d} < d \leq \tilde{d} \\ (s^M, p^M) & \text{for } d > \tilde{d}, \end{cases}$$

where \widetilde{d} satisfies

$$u'(0) - u'(d) = u' \left[u^{-1} \left(\frac{u(d)}{\delta} - \pi \right) \right],$$

and (s^{M}, p^{M}) is the unique pair satisfying both $\delta \pi + \delta u(p) + u(s - d) = u(s)$ and u'(p) = u'(s - d) - u'(s).

The present value of the agency cost to the club, $s^M - d + \delta p^M$, weakly increases in d.

As in Section 3, the most advanced types $(d \leq \hat{d})$ reform without rents $(s^L = d)$, and there is no need to offer a reward. For less advanced applicants, the incentive constraint is binding and the club must offer a rent, that is, pay more than d, to induce reform. While a pure reward is a more powerful incentive scheme than overfunding, its power declines for more backward applicants as additional units of reward yield ever lower marginal utility. Hence, a pure reward offer is optimal for intermediate applicants $(\hat{d} < d \leq \tilde{d})$ that require a small rent to reform. The mixed late offer is optimal for the most backward types $(d > \tilde{d})$, because it smooths consumption over time and reduces the cost of inducing reform. Given that applicant and club have the same discount rate, the levels of overfunding and reward are set such that the marginal utility of overfunding equals that of the reward (u'(p) = u'(s - d) - u'(s)). The thresholds \hat{d} and \tilde{d} increase in δ , since a more patient applicant requires a smaller rent to reform.

Rewarding full reform lowers the cost of implementing the late admittance strategy. Given the unchanged cost of the early offer, the set of candidates that receive late offers becomes larger. A reward, however, is only effective if the club can commit to pay the promised amount upon entry. Absent such commitment, the club will renege on the promised reward. Indeed, having reformed, the applicant accepts a revised offer of entry without reward, given that the club has all bargaining power. As admittance remains discretionary, and reform is non-verifiable, the applicant has no means to avoid such renegotiation.¹⁶

Since there are no sources of commitment within our framework, we do not extend the analysis of using the reward to wealthy candidates. Instead, we analyze how renegotiation affects the trade-off between early and late offer, restricting the club's late offer once again to initial overfunding.

5 Renegotiation and Opportunism

We now allow the applicant to trigger renegotiation, and the club to fund the reform in installments (stage financing). Neither modification affects the cost of the early offer. While we subsequently focus on the late offer, the optimal admittance strategy of the club still results from a comparison of the cost of the early and the late offer.

Renegotiation affects reform incentives in two ways. First, the applicant benefits when he can obtain refinance sooner. Second, he also benefits if he can obtain refinance more often, that is, when there are multiple periods. We address these two effects in turn and start by analyzing renegotiation and stage-financing within one period (Subsections 5.1 and 5.2). The outcome of the one-period problem is equivalent to that in a multi-period game where the club can set (and commit to) a deadline. Thereafter, we consider the dynamic game (Subsection 5.3). This corresponds to a situation where the club is unable to commit to a reform deadline other than the exogenously given end date of the game. Finally, we allow the applicant to lower its reform status strategically (Subsection 5.4).

¹⁶ For the same reasons, the late offer without commitment may not be attractive to wealthy applicants. If they have resources left after reforming, the club can renege on the initial agreement and demand a renewed payment for admittance.

5.1 Renegotiation

Consider the one-period game of Section 2 with zero-wealth applicants and allow for renegotiation after each ε interval, where $\frac{d}{\varepsilon}$ is an integer. To isolate the impact of the applicant's opportunistic behavior on the late admittance strategy, we defer the analysis of stage financing to the next Subsection. For now, the transfer at any time $i\varepsilon$, $i = 0, 1, ..., \frac{d}{\varepsilon} - 1$, must at least cover the remaining reform cost (fully prefinanced offer). Given the one-period time constraint, full reform leaves no slack and requires that the applicant reforms continuously.

An applicant of type d who at date 0 receives $s_d \ge d$ chooses among three strategies. First, he can refrain from renegotiating, consume $s_d - d$, and reform fully. This strategy yields a payoff equal to $\delta \pi + u (s_d - d)$. Second, he can immediately consume the entire transfer s_d . In this case, he will not improve his initial status until the renegotiation. Faced with a refinance request, the club refuses to pay out new funds because the initial slack renders full reform within the period infeasible. As any immediate consumption above $s_d - \varepsilon$ results in the same outcome, the applicant is better off consuming the entire s_d , which yields a payoff equal to $u (s_d)$.

Third, the applicant can invest ε in reform, consume $s_d - \varepsilon$, and request additional funding at the first renegotiation. Given that the applicant has undertaken ε reform under the initial offer, full reform by the end of the period remains possible. Moreover, a strictly lower transfer $s_{d-\varepsilon}$ induces further reform investment during the next ε interval because the reform position has improved and membership comes sooner. Thus, an applicant who reforms continuously can extract further funding at each renegotiation. Discounting the payoffs that accrue after Δ time by $e^{-r\Delta}$, where $e^{-rd} = \delta$, the applicant's payoff from reforming and renegotiating is

$$u(s_d - \varepsilon) + e^{-r\varepsilon}u(s_{d-\varepsilon} - \varepsilon) + e^{-2r\varepsilon}u(s_{d-2\varepsilon} - \varepsilon) + \dots + e^{-r(d-\varepsilon)}u(s_{\varepsilon} - \varepsilon) + \delta\pi.$$

This payoff exceeds $\delta \pi + u (s_d - d)$, the payoff from the first option where the applicant reforms without renegotiation. Whether renegotiating and full reform dominates immediate consumption of s_d (second option) depends on the reform distance, the number of renegotiation possibilities, and the transfer schedule under the renegotiation path. Clearly, for a sufficiently large number of refinancing transfers, the applicant strictly prefers to reform.¹⁷ This holds even if

¹⁷Similarly, for a given ε and consequent number of renegotiations $\frac{d}{\varepsilon} - 1$, full reform can be implemented by a payment profile $s_{d-i\varepsilon}$, $i = 0, 1, ..., \frac{d}{\varepsilon} - 1$, where each component $s_{d-i\varepsilon} \leq \hat{s} (d - i\varepsilon, 0)$, the incentive compatible amount without renegotiation for the type $d - i\varepsilon$ with zero-wealth (Section 3). Since by reforming the applicant extracts further rents in renegotiations, a lower current transfer induces reforms. The above inequality is strict for each component but the last, since there is then no further refunding occasion.

the club provides the minimum payment allowed under the assumption of a fully prefinanced offer, that is, if $s_{d-i\varepsilon} = d - i\varepsilon$.

Proposition 3 (*Renegotiation*) If the fully prefinanced late offer can be renegotiated sufficiently often, early admittance is the dominant enlargement strategy.

Although the club has by assumption all bargaining power, it cannot control the cost of the late offer with renegotiation. Between two renegotiation dates, the applicant reforms only the minimum necessary to keep reformed entry feasible, and consumes the rest. Provided it was optimal for the club to fund the applicant previously, it remains so at the renegotiation, regardless of past opportunistic behavior. Hence, the club cannot commit to refuse refinancing at the renegotiation. As a result, the club pays repeatedly for those parts of the reform schedule that, while prepaid for, have not yet been carried out. Even if the club funds at each time only the remaining reform distance ($s_{d-i\varepsilon} = d - i\varepsilon$), the refinancing costs become arbitrarily large as the number of renegotiation opportunities increases.¹⁸

With every renegotiation opportunity, the applicant extracts further rents. Since renegotiation does not affect the early offer, and the club can choose the offer, the applicant cannot obtain higher rents than those under an early offer $(\frac{1-\gamma}{\gamma}d)$. Thus, as there are more renegotiation dates, there are fewer types who receive a late offer, and ultimately, the club makes only early offers. If the club switches to an early offer for advanced types $(d \leq d^J)$, they enjoy strictly larger rents compared to the game without renegotiation.

The above analysis shows that renegotiation undermines the late admittance strategy in the static game. The framework allows the applicant to trigger renegotiation but prevents the club from adapting its finance schedule to the additional payment opportunities (full prefinance requirement). While this highlights the cost to the club of opportunistic behavior of the applicant, it tilts the playing field in favor of the applicant. Henceforth, we allow the club to make use of repeated payment occasions to control incentives by refining the late offer with stage financing.

5.2 Stage Financing

Consider the one period game of Section 2 with penniless applicants and allow the club to split the funding into slices of any size. More specifically, the club chooses in the beginning the dates

¹⁸ If renegotiation dates were a choice variable, applicants would prefer to renegotiate as soon as they have spent $s_d - \varepsilon$ on consumption, while the club would always want to defer it.

and installments and can commit to this schedule. In contrast to the previous Subsection, we abstract initially from renegotiation after every ε interval.

Formally, an offer with stage financing has a = 1, 2, ...A stages, with stage lengths l^a and transfers s^a . Since the length of the period is restricted to d, full reform requires that $\sum_a l^a = \sum_a d^a \equiv d$, where d^a denotes the stage reform requirement. Consider an applicant with a remaining reform distance $d' \leq d$ who is endowed with an amount $m = l^a$ at the beginning of stage a. Anticipating future installments $s^{a'} = l^{a'}$ for all remaining stages a', he invests the entire amount m into reform if the future membership benefit exceeds the value of current consumption, that is, if $e^{-rd'}\pi \geq u(m)$.

Lemma 4 (Stage Funds) Given a remaining reform distance d', the largest deviation-proof installment is $\hat{s}^a(d') = u^{-1}(e^{-rd'}\pi)$.

The Lemma follows from inverting the above condition (holding with equality) and setting $m = s^a$. At any time, the benefit of reforming is the discounted membership benefit, which is at least $\delta \pi$. The opportunity cost of reform is the foregone consumption value of the current transfer s^a . By lowering the installment s^a , the club can make reform within the stage incentive compatible for any type. A shorter remaining reform distance increases the opportunity cost of diverting the funds, because the current value of future membership becomes larger as it comes sooner. Hence, the largest deviation-proof transfer \hat{s}^a varies inversely with the remaining distance d'.

Feasibility of full reform requires at least a total transfer d. Lemma 4 implies that an offer without overfunding can be made incentive compatible by dividing d into sufficiently many installments.

Lemma 5 (Number of Stages) For any applicant type d, implementation of full reform without overfunding requires at least <u>A</u> stages, where <u>A</u> satisfies

$$\sum_{a=1}^{\underline{A}-1} \hat{s}^a < d \le \sum_{a=1}^{\underline{A}} \hat{s}^a.$$

The minimum number of stages \underline{A} is inversely related to the reform distance d and is finite for any type d. While the club can always choose more than \underline{A} stages, no offer with fewer stages implements reform without overfunding. Fewer stages imply that at least one transfer exceeds the largest deviation-proof installment in that stage. **Proposition 4 (Stage Financing)** The late admittance offer with stage financing implements full reform without overfunding for all types.

The Proposition follows from Lemmata 4 and 5. For the most advanced applicant types $(d \leq \hat{d})$, splitting up the reform finance is not necessary as they reform even when fully prefinanced (Section 3). Less advanced types are induced to reform without overfunding because financing in installments reduces the opportunity cost of reform. By Lemma 4, more backward types require more stages, and the length of early stages is shorter. When the membership is further away, the club can entrust the applicant with less reform funds.

As stage financing eliminates the applicant's rents, it raises the threshold d^{NL} above which the late offer is no longer profitable for the club. In addition, disbursing the reform funds as late as possible reduces the date 0 cost to the club. Hence, the optimal stage financing schedule has an infinite number of stages, that is, a continuous flow of transfers. This discounting effect raises the threshold d^{NL} further. Finally, an applicant receives either the late admittance stage finance offer or no offer, since early admittance leaves entrants with positive rents $\left(\frac{1-\gamma}{\gamma}d\right)$.

The result in Proposition 4 is robust to renegotiation every ε interval as considered in Subsection 5.1. The optimal offer with continuous transfers has instantaneous reform finance. In the absence of prefinancing, the applicant must use any financial inflow for reform to keep reformed entry feasible. Hence, at any renegotiation the club adheres to its instantaneous reform finance, preventing the applicant from extracting rents. More generally, stage financing may be viewed as a renegotiated offer where the club adapts the transfer schedule to the potential opportunism of the applicant by not prefinancing reform beyond the next renegotiation date.

Stage financing crucially depends on the assumption that the club can commit to a final completion date, thus excluding financing thereafter. Above, such commitment derives from the single period game of length d. Due to the time constraint, full reform is no longer possible within the period if the applicant fails to reform during a stage.¹⁹ This commits the club to deny funding renewal to an applicant who lags behind in its reform. Similarly, it must not be possible for the applicant to 'accelerate' reforms, that is, to compensate for lost time by reforming faster later on. Otherwise, the applicant can use reform funds for consumption and extract further funding.

¹⁹Stage financing without overfunding also implements full reform if the period is longer than the time needed for full reform. The club simply assigns all slack time to a first stage without financing and without reform requirement, and then proceeds as discussed.

5.3 The Infinite and Finite Game

We now examine the late admittance strategy in the dynamic framework with zero wealth applicants. For simplicity, we allow for renegotiation and new offers only at the beginning of each period. (That is, we set $\varepsilon = d$). In contrast to the one period game, failure to reform in the current period does not preclude full reform and entry at some later time. Moreover, provided it is optimal for the club to provide funding in this period, it will be optimal in (some) later periods as well. The club's inability to deny future funding enables the applicant to extract future rents, further exacerbating the agency problem. Thus, a reform inducing transfer has to cover the applicant's current opportunity cost of reform and compensate him for foregone future rents.

We begin by analyzing the infinite horizon game, where future rents are largest and the agency problem is most pronounced. Thereafter we examine the role of commitment stemming from a finite end date.

In the infinite horizon game, the incentive constraint is

$$\delta \pi + u (s - d) \geq \frac{u (s)}{1 - \delta},$$

where current and future transfers are equal since the current optimal transfer must also be optimal in any future period under identical conditions. Note that zero reform strictly dominates partial reform in all periods. Partial reform lowers current consumption and future rents, without achieving entry earlier.

Rewriting the above constraint illustrates the two countervailing effects of overfunding on reform incentives:

$$\delta \pi \geq \underbrace{u(s) - u(s - d)}_{\text{Opportunity cost of reform}} + \underbrace{\frac{\delta u(s)}{1 - \delta}}_{\text{Future consumption}}$$

On the one hand, overfunding lowers the opportunity cost of reform in the present period. This is the reason why sufficiently large overfunding induces any type to reform in the static game of Section 3. On the other hand, current overfunding also increases the expected stream of future transfers, making current reform less attractive. This effect arises only in the dynamic game, and implies that overfunding is either strictly larger than in the one period game or even fails to implement reform altogether.

Proposition 5 (Infinite Horizon) In the game with infinitely many periods, applicant types $d \leq \hat{d}_{\infty} = u^{-1} [\delta (1 - \delta) \pi]$ reform with a transfer s = d.

If
$$u'(0) \leq \frac{u'[u^{-1}(\delta(1-\delta)\pi)]}{1-\delta}$$
, all applicant types $d > \hat{d}_{\infty}$ cannot be induced to reform

If
$$u'(0) > \frac{u'[u^{-1}(\delta(1-\delta)\pi)]}{1-\delta}$$
, applicant types $d \in \left(\hat{d}_{\infty}, d_{\infty}^{N}\right]$ reform with overfunding. The value $d_{\infty}^{N} > \hat{d}_{\infty}$ is the first element of the pair $\left(d_{\infty}^{N}, s^{N}\right)$ that solves $\delta\pi + u(s-d) = \frac{u(s)}{1-\delta}$ and $u'(s-d) = \frac{u'(s)}{1-\delta}$. The transfer $s(d,\delta)$ is given by the smallest s such that $\delta\pi + u(s-d) \ge \frac{u(s)}{1-\delta}$. Overfunding $s(d,\delta) - d$ increases in d . Applicants $d > d_{\infty}^{N}$ cannot be induced to reform.

As in the one period model, the late offer induces reform without overfunding for the most advanced types $(d \leq \hat{d}_{\infty})$. If there are future reform periods, current consumption is more attractive because it allows the applicant to extract future rents. Hence, fewer types $(\hat{d}_{\infty} < \hat{d})$ reform without rents. For the same reason, overfunding may fail to induce reform for all $d > \hat{d}_{\infty}$. In fact, overfunding weakens reform incentives in the case where a larger transfer raises the value of future rents $(\frac{\delta u(s)}{1-\delta})$ by more than it lowers the current opportunity cost of reform [u(s) - u(s - d)]. By concavity of u, this case occurs when the first unit of overfunding for the best type $d > \hat{d}_{\infty}$ raises the present value of future consumption by more than it lowers the opportunity cost of reform $(u' [u^{-1} (\delta (1 - \delta) \pi)] / (1 - \delta) \ge u'(0))$.

If the first unit of overfunding improves the reform incentives of the best type $d > \hat{d}_{\infty}$, there exists an intermediate range of types $d \in (\hat{d}_{\infty}, d_{\infty}^N]$ who can be induced to reform with s > d, and overfunding strictly increases in d in this range. Additional units of overfunding, however, reduce the opportunity cost of reforming by less than they increase the future rents. Hence, overfunding fails to induce the least advanced types $(d > d_{\infty}^N)$ to reform.

For the most advanced types $(d < \hat{d}_{\infty})$, discounting has an ambiguous effect on reform incentives. On the one hand, a higher discount factor raises the present value of entry $(\delta \pi)$, which makes reform more attractive. On the other hand, it raises the present discounted value of all rents $(\frac{u(d)}{1-\delta})$, weakening the incentives. At low levels of δ , the first effect dominates, and reform incentives, and hence \hat{d}_{∞} increase in δ . At high values of δ , the reverse holds, and \hat{d}_{∞} decreases.²⁰

For intermediate types $d \in (\hat{d}_{\infty}, d_{\infty}^N]$, patience unambiguously increases the value of the future rents by more than the present value of the membership benefits. The former increases by $\frac{u(s)}{(1-\delta)^2}$ in δ , while the latter increases by π . Since these applicants receive a transfer s > d,

²⁰Differentiating both sides of the incentive constraint yields π and $\frac{u(s)}{(1-\delta)^2}$, respectively. While π is constant, $\frac{u(s)}{(1-\delta)^2}$ strictly increases in δ . The definition of \hat{d}_{∞} implies that $u\left(\hat{d}_{\infty}\right) < \pi$. Thus, initially \hat{d}_{∞} rises in δ .

 $\pi < u(s)$, which implies that $\frac{u(s)}{(1-\delta)^2} > \pi$. Thus, d_{∞}^N decreases in δ , while overfunding s - d increases for all types who can still be induced to reform.

We now analyze the finite horizon game and the role of commitment that the end date Tlends to the club. The incentive constraint for reform in a given period $t \leq T$ is

$$\delta \pi + u \left(s_t - d \right) \ge \sum_{\tau=t}^{T} \delta^{\tau-t} u \left(s_{\tau} \right).$$

Unlike in the infinite horizon game, the club can now vary the transfer over time. Thus, a larger current transfer does not imply that the stream of an applicant's future rents increases. This tends to reduce the cost of implementing reform.

Proposition 6 (Finite Horizon) Given a finite number T - t of remaining reform periods, an applicant of type

i) $d \leq \hat{d}_t = u^{-1} \left(\frac{1-\delta}{1-\delta^{T-t+1}} \delta \pi \right)$ reforms in period t with a transfer $s_t = d$. The threshold \hat{d}_t decreases in T-t and is larger than \hat{d}_{∞} .

ii) $d \in \left(\hat{d}_t, d_t^N\right]$ reforms in period t

The types $d > d_t^N$ may or may not reform depending on the date t value of all subsequent transfers. By the definition of d_t^N , the sequence of $\hat{s}_{\tau}(d)$ does not extend from the end date until t. For the best type $d > d_t^N$, the sequence $\hat{s}_{\tau}(d)$ ends in t + 1. As the present value of the sequence $(\hat{s}_{\tau}(d))_{t+1}^T$ exceeds the membership benefit, this type cannot be induced to reform in t. Hence, the club does not provide reform finance $(s_t = 0)$.

As the applicant's reform distance increases, his sequence $\hat{s}_{\tau}(d)$ breaks off closer to the end date T. Suppose the club provides zero funding until the starting period of the sequence $\hat{s}_{\tau}(d)$. As the number of consecutive zero transfer periods increases, the current value of the sequence of \hat{s}_{τ} falls. Hence, for sufficiently many zero transfer periods, the membership benefit again exceeds the discounted future stream of transfers, and overfunding then induces reform. As some but not the best types $d > d_t^N$ can be induced to reform in period t, overfunding is non-monotonic in the reform distance d. The existence of zero transfer periods implies that a type $d > d_t^N$ who cannot be induced to reform in t, would reform again in some earlier period t'. This illustrates that even a distant end date T can discipline an applicant to reform.

As a result of the higher agency cost in the dynamic game, fewer types receive a late offer. In the finite game, the club may also delay the late offer instead of not making one. In either case, the late admittance strategy is more costly to the club. In consequence, the threshold between late and early offers (d^{EL}) falls, and more types receive an early offer.²¹ The applicant only benefits from subsequent reform periods if he continues to receive a late offer with overfunding, or if the club switches to an early offer. By contrast, he may be worse off if the club postpones the late offer in the finite game.

5.4 Strategic Deterioration

In any period, an applicant's rents depend on his current reform status. Hence, he may have an incentive to lower the reform status early on to extract higher rents in later periods. Deterioration is a costly downward adjustment of the reform status which we have previously excluded. Given that the applicant has no wealth, such strategic deterioration must be funded through an initial transfer from the club.

We show by example that strategic deterioration may transform a period with a viable late offer into one without a viable late offer. More surprisingly, deterioration may also have the opposite effect, making a late offer in a given period viable which would otherwise not be viable

 $^{^{21}}$ Farell and Maskin (1989) show that a renegotiation-proof threat can be constructed in repeated prisoner's dilemma games. This threat involves zero rents for the punishing player. In our case, this credible threat is strictly dominated for all types $d < d^{NE}$, because the club can profitably employ the early admittance strategy.

in that period. Formally, we introduce deterioration by allowing the investment F to either raise or lower the reform status: $x = x^0 + F$, with $x \ge x^0$ and the cost of spending $F \ge 0$ is given by |F|.

Consider first a two period game with an applicant type $d > \hat{d}_2$. For simplicity, we ignore the early admittance option which amounts to assuming that γ is sufficiently small. In the absence of deterioration, the second period transfer $s_2(d)$ solves $\delta \pi + u[s_2(d) - d] = u[s_2(d)]$, and the first period transfer s_1 solves $\delta \pi + u(s_1 - d) = u(s_1) + \delta u[s_2(d)]$. (The restriction $d > \hat{d}_2$ ensures that both $s_1(d)$ and $s_2(d)$ are larger than d.)

An applicant who receives this first period transfer s_1 deteriorates if

$$u(s_1 - F) + \delta u[s_2(d + F)] \ge u(s_1) + \delta u[s_2(d)]$$

$$\iff \delta (u[s_2(d + F)] - u[s_2(d)]) > u(s_1) - u(s_1 - F).$$

This condition shows the basic trade-off determining the deterioration incentive. Deterioration raises the rents in period 2 but lowers consumption in period 1. While the marginal opportunity cost of deteriorating increases in F, the benefit accrues one period later and is subject to two opposing effects. Given the convexity of s(d) (Section 3), more deterioration yields ever increasing period 2 rents that, however, yield ever less marginal utility. Rather than fully characterizing the optimal extent of deterioration, we provide a simple condition for when the transfer s_1 is not deterioration-proof. Differentiating both sides of the above condition shows that the applicant sets F < 0 if

$$\delta u' \left[s_2 \left(d + F \right) \right] \frac{u' \left[s_2 \left(d + F \right) - \left(d + F \right) \right]}{u' \left[s_2 \left(d + F \right) - \left(d + F \right) \right] - u' \left[s_2 \left(d + F \right) \right]} > u' \left(s_1 - F \right)$$

As for F = 0, $s_1(d) > s_2(d)$, and the condition holds for δ sufficiently large, that is, for δ larger than the inverse of the fraction term.

If the applicant has an incentive to deteriorate, it is a strictly dominant strategy for the club to defer a late offer rather than finance the deterioration. Both the club and the applicant are worse off if the late offer is deferred.²² Once the game has more than a single remaining period, however, the club is not necessarily worse off if strategic deterioration is possible.

Proposition 7 (Deterioration) In a given period of a finite game, a late offer may be viable only if deterioration is part of the applicant's strategy set.

 $^{^{22}}$ If, contrary to our simplifying assumption, the club were to prefer the early over the deferred late offer, the possibility of deterioration would still lower the club's enlargement payoff (though by less) and consequently raise the applicant's rents.

The Appendix provides a numerical example of a three period game with strategic deterioration. In the absence of deterioration, a viable late offer only exists in periods 2 and 3. That is, the example considers a type $d > d_3^N$ whose sequence of declining transfers $\hat{s}_{\tau}(d)$ ends with period 2 (Proposition 6). With deterioration, the applicant has an incentive to deteriorate in period 2 to extract more rents in period 3. The club responds by denying any funding is period 2. The zero transfer in period 2 increases the reform incentive in period 1, and brings into existence a reform inducing period 1 transfer. Conversely, period 2 of the example illustrates that deterioration can also prevent a viable late offer. Furthermore, partial reform need no longer be a dominated course of action, once deterioration is feasible. The reason is that it can commit the applicant not to deteriorate in subsequent periods, thereby granting him access to further funding.

The above example critically depends on the finite horizon of the game, as in Subsection 5.3 where the finite end date enables the club to commit to zero transfers in some periods. In both cases, the anticipated zero transfer periods reduce the applicant's payoff from opportunistic behavior in preceding periods.

6 Conclusion

This paper analyzes a club's choice of admitting an applicant before or after he conforms to the club standard. The early admittance offer relies on internal enforcement for reform, while the late offer uses transfers and contingent admittance to set reform incentives. The club's optimal admittance strategy obtains by comparing the minimum transfer that induces the applicant to accept and reform under each offer. In the static enlargement game future membership benefits and moderate overfunding provide sufficient reform incentives for advanced applicant types with relatively little wealth. For backward types with relatively little wealth, early admittance is optimal because internal control achieves reform at lower cost than does overfunding with a late offer. Wealthy applicants enter early as this allows the club to charge a higher entrance fee.

The viability of the late admittance strategy depends on whether the club can credibly threaten to deny further funding if the applicant were to consume the reform funds. If the club can set and commit to a deadline for entry, the late offer with stage financing induces all applicant types to reform with zero rents. By contrast, overfunding may fail to implement reform in the infinite horizon game. In the finite horizon game, overfunding induces some applicant types to reform but not others, and the amount of incentive compatible overfunding is non-monotonic in the reform distance and in the number of remaining periods. Lastly, we show that the club may be better off if the applicant can use reform funds for strategic deterioration of his initial reform status.

To our knowledge, the timing of admittance and its effect on reform incentives is an issue that the club literature has not yet addressed. Furthermore, our paper also contributes by introducing concepts from corporate finance into the club literature. As the integration of new members is a commonplace in many organizations, the present model provides a starting point for analyzing a range of enlargement decisions, most notably perhaps the Eastern Enlargement of the EU or the admittance of China to the WTO.

The analysis can be extended in a variety of directions. Commitment not to refinance could stem from the club's limited wealth or from congestion in the consumption of the club good. Partial rivalry may enable the club to commit not to refinance by letting several applicants compete for a limited number of slots. Asymmetric information about the applicant's initial reform status may also solve or mitigate the commitment problem. Once the club associates failure to reform with a reform distance that makes enlargement unprofitable, advanced applicant types may be better off reforming because they cannot count on further transfers. Another extension is to allow the club to affect the membership benefits by modifying the club standard. Applied to the EU context, this extension allows to analyze the much debated question of 'widening' versus 'deepening', that is, whether internal EU reform should precede enlargement or vice versa. Finally, and again motivated by the EU example, heterogeneous incumbent club members and constraints on the applicant's reform ability, say due to political pressure groups, may also affect the club's enlargement strategy.

APPENDIX

A Proof of Lemma 1 (Reform Implementation)

i) Once admitted, an applicant has no incentives to reform. Hence, $\gamma(s^E + w) \ge d$ must hold for full reform to be feasible, giving a minimum transfer of $s^E = \frac{d}{\gamma} - w$.

ii) Provided an applicant has accepted a late offer, the club minimizes s subject to

$$w + s \ge d \tag{FC^L}$$

and

$$\delta \pi + u \left(w + s - d \right) \ge u \left(w + s \right). \tag{IC}^L$$

For types $\{(d, w) : d < u^{-1}(\delta \pi) \text{ and } w \in (0, \infty)\}$, the IC^L is slack, given that reform is feasible. Hence, the minimum incentive compatible transfer is $s^L = d - w$.

For types $\{(d, w) : d > u^{-1}(\delta \pi) \text{ and } w \in (0, \infty)\}$, s = d - w violates the IC^L . Thus, the minimum incentive compatible transfer is such that $\delta \pi + u(w + s - d) = u(w + s)$. Finally, total differentiation of the late offer transfer $s = d - w + u^{-1}[u(w + s) - \delta \pi]$ yields

$$\frac{d\hat{d}}{d\delta} = \frac{\pi}{u'(\hat{d})} > 0$$

and

$$\frac{d\hat{s}}{d\delta} = -\frac{\pi}{u'(w+s-d) - u'(w+s)} < 0 \qquad \frac{d\hat{s}}{dd} = \frac{u'(w+s-d)}{u'(w+s-d) - u'(w+s)} > 0.$$

by concavity of u.

B Proof of Lemma 2 (Acceptance Early)

The club minimizes s subject to

$$w + s \ge \frac{d}{\gamma} \tag{FC^E}$$

and

$$\pi + u\left(w + s - d\right) \ge u\left(w\right) \tag{IR}^{E}$$

Feasibility of reform requires that $w + s \geq \frac{d}{\gamma}$ (Lemma 1), and the new entrant retains $w + s - d \geq \frac{1-\gamma}{\gamma}d$ after reforming. Thus, the IR^E requires $\pi + u(w + s - d) \geq \pi + u\left(\frac{1-\gamma}{\gamma}d\right) \geq u(w)$. Hence, for $w < u^{-1}\left[\pi + u\left(\frac{1-\gamma}{\gamma}d\right)\right]$ (Region I), the FC^E binds and $s^E = \frac{d}{\gamma} - w$. In Region II, the IR^E binds and $s^E = d - w + u^{-1}\left[u(w) - \pi\right]$.

Derivation of $(IR^E - FC^E)$: Substituting $w + s = \frac{d}{\gamma}$ from the FC^E into the IR^E directly yields $\pi + u\left(\frac{1-\gamma}{\gamma}d\right) = u(w)$ as the equation defining the $(IR^E - FC^E)$ curve. This curve is concave. Total differentiation yields

$$\frac{dd}{dw} = \frac{\gamma}{1-\gamma} \frac{u'(w)}{u'\left(\frac{1-\gamma}{\gamma}d\right)} > 0,$$

and hence,

$$\frac{d^2d}{dw^2} = \frac{\gamma}{1-\gamma} \frac{u''(w)}{u'\left(\frac{1-\gamma}{\gamma}d\right)} < 0.$$

C Proof of Lemma 3 (Acceptance Late)

The club minimizes s subject to

$$\delta \pi + u \left(w + s - d \right) \ge u \left(w + s \right), \tag{IC^L}$$

$$\delta \pi + u \left(w + s - d \right) \ge u \left(w \right), \tag{IR}^{L}$$

and

$$w + s \ge d. \tag{FC^L}$$

From Lemma 1 it follows that for types $\{(d, w) : d < u^{-1}(\delta \pi) \text{ and } w \in (0, \infty)\}$ (Regions III and IV), the IC^L is always slack. By the same reasoning, the IR^L is slack for $w < u^{-1}(\delta \pi)$ (Region III), and s^L is determined by the FC^L . Conversely, for $w \ge u^{-1}(\delta \pi)$ (Region IV) the IR^L determines s^L .

Lemma 1 further implies that for types $\{(d, w) : d > u^{-1}(\delta \pi) \text{ and } w \in (0, \infty)\}$ (Regions *I* and *II*), the *IR^L* binds for $s \leq 0$ (Region *II*) and the *IC^L* binds for s > 0 (Region *I*), while the *FC^L* is always slack. Solving the *IC^L*(or *IR^L*) for s = 0 yields the $(IR^L - IC^L)$ curve, $d = w - u^{-1} [u(w) - \delta \pi]$. Being the *IC^L* for s = 0, the $(IR^L - IC^L)$ is concave in *w*. Totally differentiating the *IC^L* for $s \geq 0$ yields

$$\frac{dd}{dw} = \frac{u'(w+s-d) - u'(w+s)}{u'(w+s-d)} \\ = 1 - \frac{u'(w+s)}{u'(w+s-d)} \in (0,1),$$

and

$$\frac{d^{2}d}{dw^{2}} = \frac{-u''(w+s) u'(w+s-d) + u'(w+s) u''(w+s-d)}{u'(w+s-d)^{2}}$$

Hence, $\frac{d^2d}{dw^2} < 0$ if and only if -u''(w+s)u'(w+s-d) < -u'(w+s)u''(w+s-d), which amounts to assuming DARA.



Figure 6: Early Versus Late Offers

D Proof of Proposition 1 (Optimal Offer)

We first compare the cost of making an early and a late offer, and then analyze the choice between making an offer and making no offer. Lemmata 2 and 3 together divide the space of applicant types into five regions (Figure 6). The club chooses between an early and a late offer by comparing for each region the respective transfers.

Lemma 6 (Regions 1 and 2) For all types with $w \ge u^{-1}(\pi)$ and $d \le u^{-1}[u(w) - \pi] \frac{\gamma}{1-\gamma}$, $s^{E} \le s^{L}$.

Proof. For the above types, $s^E = d - w + u^{-1} [u(w) - \pi]$ from Lemma 2, while s^L is either equal to $d - w + u^{-1} [u(w) - \delta \pi]$ (Region 1) or implicitly defined by $s = d - w + u^{-1} [u(w + s) - \delta \pi] > 0$ (Region 2) from Lemma 3. Since $s^L > 0$ in Region 2 and $\delta \pi < \pi$, the early offer is more profitable in either case.

For all types $w < u^{-1} \left[\pi + u \left(\frac{1-\gamma}{\gamma} d \right) \right]$, the IR^E is slack. Hence, in the remaining part of the proof we only need to compare the FC^E with the transfer under the late offer.

 $\textbf{Lemma 7} \ \textit{(Region 3)} \ \textit{For types} \left\{ (d, w) : d \in \left[0, u^{-1} \left(\delta \pi\right)\right], w \in \left[0, u^{-1} \left(\delta \pi\right)\right] \right\}, \ s^L < s^E.$

Proof. By Lemmas 2 and 3, $s^L = d - w < \frac{d}{\gamma} - w = s^E$, which holds for any $\gamma \in (0, 1)$.

Lemma 8 (Region 4) For types with $d > u^{-1}(\delta \pi)$ for $w < u^{-1}(\delta \pi)$ and types $d \ge w - u^{-1}[u(w) - \delta \pi]$ for $w \ge u^{-1}(\delta \pi)$, there exists a unique \tilde{d} defined by $\delta \pi = u\left(\frac{d}{\gamma}\right) - u\left[(1-\gamma)\frac{d}{\gamma}\right]$ such that for $d < \tilde{d}$, $s^L < s^E$, and $s^L \ge s^E$ for $d \ge \tilde{d}$. Moreover, $\tilde{d} < d^{NE}$.

Proof. For these types, s^L as defined by $s = d - w + u^{-1} [u(w+s) - \delta\pi]$ is compared to $\frac{d}{\gamma} - w = s^E$. Setting $s^L = s^E$ yields the definition of \tilde{d} . Late admittance is cheaper if $\frac{d}{\gamma} - w > d - w + u^{-1} [u(w+s) - \delta\pi]$, or $\delta\pi > u(w+s^L) - u\left[\frac{(1-\gamma)}{\gamma}d\right]$, which holds for $d > \tilde{d}$, while early is (weakly) cheaper otherwise.

Existence and uniqueness of \tilde{d} , and $\tilde{d} < d^{NE}$ all follow from Assumption 4. The difference $u\left(\frac{d}{\gamma}\right) - u\left[(1-\gamma)\frac{d}{\gamma}\right]$ increases monotonically in d, and $d^{NE} = \gamma \Pi^R$. Hence, $\tilde{d} < d^{NE}$ is implied by $\delta \pi < u\left(\Pi^R\right) - u\left[(1-\gamma)\Pi^R\right]$. Existence of \tilde{d} follows from the fact that $u\left(\frac{d}{\gamma}\right) - u\left[(1-\gamma)\frac{d}{\gamma}\right]$ equals zero for d = 0, that this difference increases monotonically, and that $\tilde{d} < d^{NE}$. Finally, uniqueness follows directly from the monotonicity of $u\left(\frac{d}{\gamma}\right) - u\left[(1-\gamma)\frac{d}{\gamma}\right]$ in d.

Lemma 9 (Region 5) For types with $w \in (u^{-1}(\delta\pi), u^{-1}(\pi))$ and $d < w - u^{-1}[u(w) - \delta\pi]$ and types with $w \ge u^{-1}(\pi)$ and $d \in \left[u^{-1}[u(w) - \pi]\frac{\gamma}{1-\gamma}, w - u^{-1}[u(w) - \delta\pi]\right]$, $s^L > s^E$ iff $d < u^{-1}[u(w) - \delta\pi]\frac{\gamma}{1-\gamma}$, and $s^L \le s^E$ otherwise.

Proof. For these types, the club compares $s^E = \frac{d}{\gamma} - w$ and $d - w + u^{-1} [u(w) - \delta \pi] = s^L$. Hence, $s^E < s^L$ if $\frac{d}{\gamma} - w < d - w + u^{-1} [u(w) - \delta \pi]$. Rearranging yields $d < u^{-1} [u(w) - \delta \pi] \frac{\gamma}{1 - \gamma}$. Equating s^E and s^L defines the $(FC^E - IR^L)$ curve, $d = u^{-1} [u(w) - \delta \pi] \frac{\gamma}{1 - \gamma}$. This curve is concave. Total differentiation yields

$$\frac{dd}{dw} = \frac{u'(w)}{u'\left(\frac{1-\gamma}{\gamma}d\right)} \frac{\gamma}{1-\gamma} > 0 \quad \text{and} \quad \frac{d^2d}{dw^2} = \frac{u''(w)}{u'\left(\frac{1-\gamma}{\gamma}d\right)} \frac{\gamma}{1-\gamma} < 0.$$

Lemma 10 (Point J) The $(FC^E - IR^L)$ and $(IR^L - IC^L)$ curves have a unique intersection (Point J), with d^J implicitly defined by $\delta \pi + u \left(\frac{1-\gamma}{\gamma}d\right) = u \left(\frac{d}{\gamma}\right)$. Moreover, $d^J > u^{-1}(\delta \pi)$.

Proof. The $(FC^E - IR^L)$ curve is defined by $s^E = s^L$, while on the $(IR^L - IC^L)$ curve the transfer $s^L = 0$. Hence, at any intersection $s^E = s^L = 0$ must hold, and this point also must lie on $d = \gamma w$ (the iso-transfer line with $s^E = 0$). Substituting $w = \frac{d}{\gamma}$ into $(FC^E - IR^L)$ (or $(IR^L - IC^L)$) yields $\delta \pi + u \left(\frac{1-\gamma}{\gamma}d\right) = u \left(\frac{d}{\gamma}\right)$. The expression defining d^J is identical to that defining \tilde{d} . Thus, existence, uniqueness, and $d^J < d^{NE}$ all follow from Lemma 8. Moreover, $\tilde{d} = d^J > u^{-1} (\delta \pi)$ because d^J is unique, and the $(IR^L - IC^L)$ curve is increasing, concave, and passes above γw at $w = u^{-1} (\delta \pi)$.

Note that the curve $(FC^E - IR^L)$ as given by $\delta \pi + u\left(\frac{1-\gamma}{\gamma}d\right) = u(w)$ is everywhere above the curve $(IR^E - FC^E)$, $\pi + u\left(\frac{1-\gamma}{\gamma}d\right) = u(w)$, and has the same slope. Hence, the latter intersects the $(IR^L - IC^L)$ (Point P) to the right of Point J. This completes the comparison of an early and a late offer. Although full reform is feasible under either enlargement strategy, the cost of providing the applicant with sufficient acceptance and reform incentives may exceed the benefit of reformed enlargement to the club.

Lemma 11 (No Offer) Under Assumption 4, a profitable late admittance offer implies a profitable early offer, but the reverse does not hold.

Proof. The inequality $d^{NE} > d^{NL}$ requires $\delta \pi < u \left(\delta \Pi^R + w \right) - u \left[(1 - \gamma) \left(\delta \Pi^R + w \right) \right]$. Part (i) of Assumption 4 implies that for w = 0, $d^{NE} > d^{NL}$. By Part (ii), $u \left(\delta \Pi^R + w \right) - u \left[(1 - \gamma) \left(\delta \Pi^R + w \right) \right]$ increases monotonically in w. Hence, the d^{NL} curve lies everywhere below the d^{NE} curve.

Note that $d^{NL} > d^J$. From $d^{NL} = \delta \Pi^R - u^{-1} \left[u \left(\delta \Pi^R \right) - \delta \pi \right]$ at w = 0, it follows that $\delta \pi = u \left(\delta \Pi^R \right) - u \left(\delta \Pi^R - d^{NL} \right)$. Equating this expression with $\delta \pi = u \left(\frac{d}{\gamma} \right) - u \left(\frac{1 - \gamma}{\gamma} d \right)$ (definition of d^J), we obtain $u \left(\frac{d^J}{\gamma} \right) - u \left(\frac{d^J}{\gamma} - d^J \right) = u \left(\delta \Pi^R \right) - u \left(\delta \Pi^R - d^{NL} \right)$. Rearranging yields $u \left(\delta \Pi^R \right) - u \left(\frac{d^J}{\gamma} \right) = u \left(\delta \Pi^R d^{NL} \right)$

Hence, the two curves meet on the horizontal d^J line. The corresponding w coordinate follows from $w = \frac{1-\gamma}{\gamma} d^J \equiv w_3$.

The s = 0 line: For $\{(d, w) : d \in [u^{-1}(\delta \pi)], w \in [u^{-1}(\delta \pi)]\}$, the transfer is given by $s^{L} = d - w$. Hence, the s = 0 line has d = w. For $w \in (w_1, w_2)$ the s = 0 line is given by $(IR^{L} - IC^{L})$. For $w > w_2$, the transfer is $s^{E} = \frac{d}{\gamma} - w$, and hence, the s = 0 line is $d = \gamma w$.

F Proof of Corollary 2 (Comparative Statics)

The no-offer separating line is $d^{NE} = \gamma \left(\delta \Pi^R + w \right)$ with $\frac{dd^{NE}}{d\gamma} > 0$ and $\frac{dd^{NE}}{d\delta} > 0$. From Lemma 10, the upper separating line between late and early offer, d^J , is defined by $\delta \pi = u \left(\frac{d^J}{\gamma} \right) - u \left(\frac{1-\gamma}{\gamma} d^J \right)$. By Assumption 4,

$$\frac{dd^{J}}{d\gamma} = \frac{\frac{d}{\gamma} \left[u'\left(\frac{d}{\gamma}\right) - u'\left(\frac{1-\gamma}{\gamma}d\right) \right]}{\left[u'\left(\frac{d}{\gamma}\right) - (1-\gamma)u'\left(\frac{1-\gamma}{\gamma}d\right) \right]} < 0, \qquad \frac{dd^{J}}{d\delta} = \frac{\gamma\pi}{\left[u'\left(\frac{d}{\gamma}\right) - (1-\gamma)u'\left(\frac{1-\gamma}{\gamma}d\right) \right]} > 0,$$

and

$$\frac{dd^{J}}{d\pi} = \frac{\gamma \delta}{\left[u'\left(\frac{d}{\gamma}\right) - (1-\gamma)u'\left(\frac{1-\gamma}{\gamma}d\right)\right]} > 0$$

The lower separating line is given by the $(FC^E - IR^L)$ curve, $d = u^{-1} [u(w) - \delta \pi] \frac{\gamma}{1-\gamma}$. It follows immediately that

$$\frac{dd}{d\gamma} = u^{-1} \left[u\left(w \right) - \delta \pi \right] \frac{1}{\left(1 - \gamma \right)^2} > 0, \qquad \frac{dd}{d\delta} = -\frac{\pi}{u' \left(\frac{1 - \gamma}{\gamma} d \right)} < 0,$$

and

$$\frac{dd}{d\pi} = -\frac{\delta}{u'\left(\frac{1-\gamma}{\gamma}d\right)} < 0$$

As

$$\frac{d\left[\frac{d}{\gamma}-w\right]}{d\gamma} = -\frac{d}{\gamma^2} \quad \text{and} \quad \frac{d\left[d-w+u^{-1}\left[u\left(w\right)-\pi\right]\right]}{d\pi} = -\frac{1}{u'\left(w+s-d\right)},$$
$$s^E = \max\left[\frac{d}{\gamma}-w, d-w+u^{-1}\left[u\left(w\right)-\pi\right]\right] \text{ weakly decreases in } \gamma \text{ and } \pi. \text{ As}$$
$$d\left(d-w+u^{-1}\left[u\left(w\right)-\delta\pi\right]\right) \qquad \pi$$

$$\frac{d\delta}{d\delta} \qquad u'(w+s-d)'$$

$$\frac{d\left(d-w+u^{-1}\left[u(w)-\delta\pi\right]\right)}{d\pi} = -\frac{\delta}{u'(w+s-d)},$$

$$\frac{d\hat{s}}{d\delta} = -\frac{\pi}{u'(w+s-d)-u'(w+s)}, \quad \text{and} \quad \frac{d\hat{s}}{d\pi} = -\frac{\delta}{u'(w+s-d)-u'(w+s)},$$

$$= \max\left\{d-w, d-w+u^{-1}\left[u(w)-\delta\pi\right], \hat{s}\right\} \text{ also weakly decreases in } \delta \text{ and } \pi.$$

 $s^{L} = \max\left\{d - w, d - w + u^{-1}\left[u\left(w\right) - \delta\pi\right], \hat{s}\right\} \text{ also weakly decreases in } \delta \text{ and } \pi.$

G Admittance Offers Without Reform

In this section we analyze the club's optimal behavior for non-reform implementing offers. First, we show that no offer strictly dominates a late, non-reform implementing offer. Second, we derive the optimal non-reform implementing early offer and identify the set of types that accept such an offer. Third, we show that Assumption 3 implies that the club strictly prefers no offer to a non-reform implementing early offer.

Lemma 12 (Non-Reform Late) Making no offer strictly dominates an accepted, non-reform implementing late offer.

Proof. Without reform, the club never admits a late applicant as $\Pi^U < 0$. Hence, an applicant accepts a non-reform implementing late offer if and only if $s \ge 0$, since then u(w+s) > u(w). Since $\Pi^U - s < 0$, the club strictly prefers to make no offer.

Lemma 13 (Acceptance Non-Reform Early) Applicant types with $w \geq u^{-1} \left[\pi + u \left(\frac{1-\gamma}{\gamma} d \right) \right]$ never accept an early, non-reform implementing offer. For $w < u^{-1} \left[\pi + u \left(\frac{1-\gamma}{\gamma} d \right) \right]$, the optimal non-reform implementing early offer has a transfer

$$s^{EN} = \begin{cases} \frac{u^{-1}[u(w)-\pi]}{1-\gamma} - w & if \quad w \ge u^{-1}(\pi); \\ -w & otherwise. \end{cases}$$

Proof. The condition for an early offer that leaves insufficient funds for reform is 0 < 0 $\gamma(w+s) < d$. The applicant rejects such an offer if and only if $\pi + u \left[(1-\gamma)(w+s) \right] < u(w)$. For $w < u^{-1} \left[\pi + u \left(\frac{1-\gamma}{\gamma} d \right) \right]$, the club minimizes s subject to

$$\pi + u\left[(1 - \gamma)\left(w + s\right)\right] \ge u\left(w\right) \tag{IR}^{EN}$$

and

$$\frac{d}{\gamma} > w + s \ge 0. \tag{FC^{EN}}$$

If $u(w) < \pi$, then the club can extract all the applicant's wealth, i.e., s = -w. Otherwise, the IR^{EN} binds.

We now compare early admittance without reform with no offer.

Lemma 14 (Non-Reform Early) Given Assumption 3, making no offer strictly dominates an accepted, non-reform implementing early offer.

Proof. Lemma 13 implies that the club's payoff from a non-reform implementing early offer is at most $\Pi^U + u^{-1}(\pi)$.

While simple, the condition in Assumption 3 is overly strong, since it would be sufficient that no reform is dominated by either no offer or reformed entry.

H The Curvature of s^L

In general, the curvature of the transfer s^L in d is ambiguous. Let '+' denote the argument w + s in u, and no subscript w + s - d. Differentiation of $\frac{ds}{dd}$ from the proof of Lemma 1 yields

$$\frac{d^2s}{dd^2} = \frac{u''\left(\frac{ds}{dd} - 1\right)\left[u' - u'_{+}\right] - u'\left[u''\left(\frac{ds}{dd} - 1\right) - u''_{+}\frac{ds}{dd}\right]}{\left[u' - u'_{+}\right]^2}$$

Substituting for $\frac{ds}{dd}$ and simplifying, the numerator can be written as $u'^2 u''_+ - u'^2_+ u''$. Hence, $\frac{d^2s}{dd^2} > 0$, i.e., s^L is convex in d, if and only if

$$-\frac{u''}{u'^2} > -\frac{u''_+}{u'^2_+}$$

This condition holds for a diminishing coefficient of absolute risk aversion, weighted by the reciprocal of the marginal utility. Denote this weighted coefficient $\beta(\cdot) = \frac{\lambda(\cdot)}{u'(\cdot)} = -\frac{u''(\cdot)}{u'(\cdot)^2}$, where $\lambda(\cdot)$ is the standard coefficient of absolute risk aversion. While $\beta' < 0$ does not hold for (negative) exponential or logarithmic utility functions, it holds for instance for $u(\cdot) = \sqrt{\cdot}$.

We can show that $\beta' < 0$ is generally satisfied for a subset of DARA-HARA functions. Following Merton (1971), hyperbolic absolute risk aversion (HARA) functions can be written as

$$U(x) = \frac{1-\gamma}{\gamma} \left(\frac{\beta x}{1-\gamma} + \eta\right)^{\gamma},$$

with $\beta > 0$, $\gamma \neq 1$, $\frac{\beta x}{1-\gamma} + \eta > 0$ and $\eta = 1$ if $\gamma = -\infty$. The coefficient of absolute risk aversion of this class of functions is

$$A(x) = \frac{1}{\frac{x}{1-\gamma} + \frac{\eta}{\beta}}$$

which leads to

$$A'(x) = \frac{-1}{(1-\gamma)\left(\frac{x}{1-\gamma} + \frac{\eta}{\beta}\right)^2}$$

Hence, A'(x) < 0 for $\gamma < 1$, which defines a subset DARA-HARA of the general HARA functions. With $U'(x) = \beta \left(\frac{\beta x}{1-\gamma} + \eta\right)^{\gamma-1}$ and $U''(x) = -\beta^2 \left(\frac{\beta x}{1-\gamma} + \eta\right)^{\gamma-2}$, the weighted coefficient of absolute risk aversion of HARA functions is then

$$-\frac{U''}{U'^2} = \left(\frac{\beta x}{1-\gamma} + \eta\right)^{-\gamma}.$$

We have diminishing (weighted) absolute risk aversion if and only if

$$\frac{d}{dx}\left(\frac{\beta x}{1-\gamma}+\eta\right)^{-\gamma} = -\frac{\beta\gamma}{1-\gamma}\left(\frac{\beta x}{1-\gamma}+\eta\right)^{-\gamma-1} < 0.$$

Hence, a necessary and sufficient condition for the above condition to hold is $\gamma \in (0, 1)$ under the restriction to real-valued utility. This defines the subclass of DARA-HARA functions for which the weighted measure of absolute risk aversion is decreasing.

I Proof of Proposition 2 (Mixed Late Offer)

Lemma 1 with w = 0 implies that $(s^L, p^L) = (d, 0)$ for $d \leq \hat{d} = u^{-1}(\delta \pi)$. For $d > \hat{d}$, the incentive constraint $\delta \pi + u(s-d) + \delta u(p) \geq u(s)$ requires either p > 0 or s > d or both. Any offer with p = 0 is strictly dominated, because the marginal effect of the first $\frac{1}{\delta}$ units of reward, $\delta u'(0) \frac{1}{\delta}$, exceeds that of one unit of overfunding, u'(s-d) - u'(s). Thus, for all $d > \hat{d}$, $p^L > 0$. For all d satisfying $u'(\hat{p}) > u'(0) - u'(d)$, the optimal contract is (d, \hat{p}) . As \hat{p} increases in d, $u'(\hat{p})$ declines in d and approaches zero. In contrast, u'(0) - u'(d) is strictly positive and increases in d. Thus, there exists a unique value \tilde{d} that satisfies $u'(\hat{p}) = u'(0) - u'(d)$. Substituting $\hat{p} = \left[u^{-1}\left(\frac{u(d)}{\delta}\right) - \pi\right]$ yields the definition of \tilde{d} stated in the Lemma. From the definition of \tilde{d} and $\hat{p} = 0$ for $d = \hat{d}$ it follows that $\tilde{d} > \hat{d}$. Accordingly, for all $d \in \left(\hat{d}, \hat{d}\right)$, s = d and $p = \hat{p}$, where \hat{p} solves $\delta \pi + \delta u(p) = u(d)$; for $d > \tilde{d}$, $s^M > d$ and $p^M > \hat{p}$ hold.

Among the pairs (s, p) that satisfy $\delta \pi + \delta u(p) + u(s - d) \ge u(s)$ with equality, an optimal pair is defined by u'(p) = u'(s - d) - u'(s). We prove uniqueness by contradiction. Consider a pair (s', p') that satisfies both constraints. Another pair (s'', p'') that also satisfies $\delta \pi + \delta u(p) + u(s - d) = u(s)$ is characterized by s' < s'' and p' > p'' (or vice versa). In either case, the condition u'(p) = u'(s - d) - u'(s) cannot hold given that the pair (s', p') satisfies it.

Consider an optimal pair (s^M, p^M) for a given d. For any larger d, the premium or overfunding or both must increase to satisfy $\delta \pi + \delta u(p) + u(s - d) = u(s)$. The optimality condition u'(p) = u'(s - d) - u'(s) implies that both increase.

J Proof of Proposition 3 (Renegotiation)

Consider the minimum admissible payment stream $s_{d-i\varepsilon} = d - i\varepsilon$, for $i = 0, 1, \dots, \frac{d}{\varepsilon} - 1$ where each refinancing is contingent on an additional improvement by ε in the reform status. Suppose this payment stream implements full reform. The resulting cost to the club,

$$s^{L}(\varepsilon) = s_{d} + e^{-r\varepsilon} (d - \varepsilon) + e^{-2r\varepsilon} (d - 2\varepsilon) + \dots + e^{-r(d - \varepsilon)}\varepsilon,$$

decreases in ε with $\lim_{\varepsilon \to 0} s^L(\varepsilon) = \infty$. Hence, for any given d, there exists an $\varepsilon^S(d) > 0$ such that for $\varepsilon < \varepsilon^S$, $s^L(\varepsilon) > \frac{d}{\gamma}$ holds. If this payment stream fails to implement reform, the claim is valid, too.

K Proof of Lemma 5 (Number of Stages)

Feasibility of full reform requires $\sum_{a=1}^{A} s^a \ge d$, by Lemma 4, $\hat{s}^a = u^{-1} \left(e^{-rd'} \pi \right)$, where $d' \le d$, and no overfunding implies $s^a = d^a$ for $a = 1, \dots, \underline{A}$. For $A < \underline{A}$, either $\sum_{a=1}^{A} \hat{s}^a < d$ and full reform is not feasible, or $\sum_{a=1}^{A} s^a \ge d$ and for at least one stage $s^a > \hat{s}^a$.

L Proof of Proposition 5 (Infinite Horizon)

For all $d \leq \hat{d}_{\infty}$, the condition $\delta \pi + u (s - d) \geq \frac{u(s)}{1-\delta}$ is satisfied for s = d. If for s = d, the inequality $u'(0) > \frac{u'(\hat{d}_{\infty})}{1-\delta}$ holds, there exist $s > \hat{d}_{\infty}$ such that $\delta \pi + u \left(s - \hat{d}_{\infty}\right) > \frac{u(s)}{1-\delta}$. Hence, by continuity of $\left[\frac{u(s)}{1-\delta} - u \left(s - d\right)\right]$ in d, there is an $\alpha' > 0$ such that the constraint continues to hold with (s - d) > 0 for all types $\hat{d}_{\infty} + \alpha$, where $\alpha \leq \alpha'$ and $\alpha' > 0$. Concavity of u implies that $\frac{u'(s-d)}{u'(s)}$ declines in s and approaches 1 in the limit. Hence, given the initial condition $\frac{1}{1-\delta} < \frac{u'(0)}{u'(d)}$, the term $\frac{u(s)}{1-\delta} - u \left(s - d\right)$ has a unique minimum at s > d. Moreover, this minimum value increases in d, and is strictly larger than $\delta \pi$ for $d > u^{-1} \left[(1 - \delta) \pi \right]$. Hence, there exists a unique $d_{\infty}^{\infty} < u^{-1} \left[(1 - \delta) \pi \right]$ such that the minimum of $\frac{u(s)}{1-\delta} - u \left(s - d\right)$ equals $\delta \pi$, that is, $\delta \pi + u \left(s - d\right) = \frac{u(s)}{1-\delta}$ and $u' \left(s - d\right) = \frac{u'(s)}{1-\delta}$. For all $d < d_{\infty}^{N}$, $\delta \pi + u \left(s - d\right) = \frac{u(s)}{1-\delta}$ has two solutions in s, of which the club prefers the lower. For all $d > d^{N}$, $\delta \pi + u \left(s - d\right) \geq \frac{u(s)}{1-\delta}$ cannot be satisfied with any $s \geq d$. If for s = d, the inequality $u'(0) \leq \frac{u'(\hat{d}_{\infty})}{1-\delta}$ holds, $\delta \pi < \frac{u(s)}{1-\delta} - u \left(s - d\right)$ for all $d > \hat{d}_{\infty}$ and $s \geq d$.

M Proof of Proposition 6 (Finite Horizon)

i) For $d \leq \hat{d}_t = u^{-1} \left(\frac{1-\delta}{1-\delta^{T-t+1}} \delta \pi \right)$, $s_t = d$ satisfies the incentive constraint $\delta \pi + u \left(s_t - d \right) \geq \sum_{\tau=t}^{T} \delta^{\tau-t} u \left(s_{\tau} \right)$. The threshold \hat{d}_t strictly decreases in T-t and approaches \hat{d}_{∞} as T-t tends to infinity.

ii) For the types $\hat{d}_t < d < d_t^N$ we first characterize in a Lemma the transfer necessary to implement reform. Consider a type $d > \hat{d}_t$ who does not reduce his reform distance over time (no partial reform). Denote by $(\hat{s}_{\tau}(d))_t^T$ a sequence of transfers with $\hat{s}_{\tau}(d) \ge d$ that implements reform in each τ , $\tau = t, t + 1, ...T$.

Lemma 15 (Declining Transfers) The transfer \hat{s}_{τ} increases monotonically in the number of remaining periods (T-t).

Proof. Suppose without loss of generality that the incentive constraint may be slack in only the final period. Thus, in period T, $\delta \pi \geq u(d)$ holds, and the incentive constraint in the preceding period $\delta \pi + u(\hat{s}_{-1} - d) = u(\hat{s}_{-1}) + \delta^2 \pi$ is binding with $\hat{s}_{-1} > d$. In the third period from the end, the constraint is $\delta \pi + u(\hat{s}_{-2} - d) = u(\hat{s}_{-2}) + \delta [u(\hat{s}_{-1}) + \delta^2 \pi]$. The transfer \hat{s}_{-2} is larger than \hat{s}_{-1} if $\delta [u(\hat{s}_{-1}) + \delta^2 \pi] > \delta^2 \pi$ holds. The strict inequality $\hat{s}_{-1} > d$ implies that this is the case.

Consider any \hat{s}_{t+1} and assume that the sequence $\hat{s}_{t+1}...\hat{s}_T$ strictly decreases. In period t, the incentive constraint for reform is

$$\delta \pi + u(\hat{s}_t - d) \ge u(\hat{s}_t) + \sum_{\tau = t+1}^T \delta^{\tau - t} u(\hat{s}_{\tau}).$$

Solving for $u(\hat{s}_{t+1})$ yields

$$u(\hat{s}_{t+1}) = \delta \pi + u(\hat{s}_{t+1} - d) - \sum_{\tau=t+2}^{T} \delta^{\tau-t-1} u(\hat{s}_{\tau}).$$

Setting $\hat{s}_t = \hat{s}_{t+1}$ and substituting $u(\hat{s}_{t+1})$ into the constraint of the previous period t, we obtain

$$\delta \pi + u \left(\hat{s}_t - d \right) \ge \left[\delta \pi + u \left(\hat{s}_t - d \right) - \sum_{\tau = t+2}^T \delta^{\tau - t - 1} u \left(\hat{s}_\tau \right) \right] + \sum_{\tau = t+1}^T \delta^{\tau - t} u \left(\hat{s}_\tau \right).$$

This condition cannot be satisfied because the declining sequence $\hat{s}_{t+1}...\hat{s}_T$ implies that

$$0 < \sum_{\tau=t+1}^{T} \delta^{\tau-t} u(\hat{s}_{\tau}) - \sum_{\tau=t+2}^{T} \delta^{\tau-t-1} u(\hat{s}_{\tau}).$$

This follows from collecting all terms that are equally discounted. There is a lone term $\delta^{T-t}u(\hat{s}_T)$, and all terms that are equally discounted are multiplied by $u(\hat{s}_t) - u(\hat{s}_{t+1})$. This factor is strictly positive for t = T - 1, and by backward induction for all preceding periods t < T - 1 as well. Thus, $\hat{s}_t = \hat{s}_{t+1}$ violates the incentive constraint in period t, and increased overfunding $(\hat{s}_t > \hat{s}_{t+1})$ is required.

Now we establish the threshold d_t^N . Since for $d = \hat{d}_t$, $\delta \pi = \sum_{\tau=t}^T \delta^{\tau-t-1} u(d)$, there exists a type $d = \hat{d}_t + \alpha$ where $\alpha < \alpha'$ and $\alpha' > 0$ such that for

N Proof of Proposition 7 (Deterioration)

Proof by numerical example. Consider a three period game with $u(\cdot) = \sqrt{\cdot}$, $\pi = 2.5$, $\delta = .2$, and d = 1.1. The incentive constraint in period 3 is $.5 + \sqrt{s-d} = \sqrt{s}$, which is solved by $s = (.25 + d)^2$ with $\frac{ds}{dd} = .5 + 2d$. Substituting d yields $s_3 = 1.8225$. Using backward induction, the incentive constraint in Period 2 is $.5 + \sqrt{s-1.1} = \sqrt{s} + .2\sqrt{1.8225}$ and $s_2 = 6.2816$.

The incentive constraint in Period 1 is $.5 + \sqrt{s - 1.1} = \sqrt{s} + .04\sqrt{1.8225} + .2\sqrt{6.2816}$. This equation has no solution, because the right-hand side equals $\sqrt{s} + .55526 > .5 + \sqrt{s - 1.1}$.

A sufficient marginal condition for deterioration in period 2 is $u'(s_2) < \delta u'(\hat{s}_d) \frac{d\hat{s}_d}{dd}$. This condition holds, as the following manipulation shows. Substituting $\frac{ds}{dd} = .5 + 2d$ yields $\frac{1}{2}(s_2)^{-\frac{1}{2}} < \delta \frac{1}{2}(\hat{s})^{-\frac{1}{2}}$ (.5 + 2d). Using the values s_1, s_2 , and d, we obtain $\frac{\sqrt{1.8225}}{\sqrt{6.2816}} = .53864 < .54$.

The incentive to deteriorate increases in d, as successive substitution of d = 1.11; 1.2; and 1.3 and recalculation of the period 2 transfer illustrates:

$$d = 1.11: \quad .5 + \sqrt{s - 1.11} = \sqrt{s} + .2(.25 + 1.11), \text{ and } s_2 = 6.4934$$

$$\frac{1.36}{\sqrt{6.4934}} = .53371 < .2(.5 + 2(1.11)) = .544$$

$$d = 1.2: \quad .5 + \sqrt{s - 1.2} = \sqrt{s} + .2(.25 + 1.2), \text{ and } s_2 = 8.7743$$

$$\frac{1.45}{\sqrt{8.7743}} = .48951 < .2(.5 + 2(1.2)) = .58$$

$$d = 1.3: \quad .5 + \sqrt{s - 1.3} = \sqrt{s} + .2(.25 + 1.3), \text{ and } s_2 = 12.363$$

$$\frac{1.55}{\sqrt{12.363}} = .44083 < .2(.5 + 2(1.3)) = .62$$
The incentives to deteriorate in period 2 decreases in the amount

The incentives to deteriorate in period 2 decreases in the amount of deterioration, as substitution of successive discrete values $\Delta = .01; .1; .2$ show (deterioration is only profitable for $\Delta = .01$):

$$\begin{split} \Delta &= .01; \quad \frac{\sqrt{1.8496}}{\sqrt{6.2816} - .01} = .54306 < .2 \left(.5 + 2 \left(1.11 \right) \right) = .544; \\ \Delta &= .1; \quad \frac{\sqrt{2.1025}}{\sqrt{6.2816} - .1} = .5832 > .2 \left(.5 + 2 \left(1.2 \right) \right) = .58; \\ \Delta &= .2; \quad \frac{\sqrt{2.4025}}{\sqrt{6.2816} - .2} = .62853 > .2 \left(.5 + 2 \left(1.3 \right) \right) = .62. \end{split}$$

The lowest reform inducing transfer in period 1 solves $.5 + \sqrt{s - 1.1} = \sqrt{s} + .04\sqrt{1.8225}$, and is given by $s_1 = 2.1205$.

Lastly, we need to confirm that the applicant does not deteriorate in period 1. We showed above that deterioration in 1 strengthens the incentive to deteriorate in 2. Thus, like full consumption, deterioration in 1 carries the consequence of no rents in period 2. The applicant has a marginal incentive to deteriorate in 1 if $u'(s_1) < \delta^2 u'(\hat{s}_d) \frac{d\hat{s}_d}{dd}$. Substitution yields $\frac{1}{2}(s_1)^{-\frac{1}{2}} < \delta^2 \frac{1}{2}(\hat{s})^{-\frac{1}{2}}(.5+2d)$. This condition is violated since $\frac{\sqrt{1.8225}}{\sqrt{2.1205}} = .92707 > .04(2.7) = .108$. On the margin, the cost to deteriorating in period 1 is prohibitive given that it will be followed by a zero transfer in the subsequent period. Moreover, substitution of discrete values show that the marginal incentive further deteriorates in the extent of deterioration:

$$\frac{1.36}{\sqrt{2.1205 - .01}} = .93615 > .04 (.5 + 2 (1.11)) = .1088;$$
$$\frac{1.45}{\sqrt{2.1205 - .1}} = 1.0201 > .04 (.5 + 2 (1.2)) = .116;$$
$$\frac{1.55}{\sqrt{2.1205 - .2}} = 1.1185 > .04 (.5 + 2 (1.3)) = .124.$$

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