FINANCIAL MARKETS GROUP

AN ESRC RESEARCH CENTRE

LONDON SCHOOL OF ECONOMICS

E·S·R·C ECONOMIC & SOCIAL RESEARCH COUNCIL

ISSN 0956-8549-362

LIQUIDITY AND CREDIT RISK

Jan ERICSSON[a] and Olivier RENAULT[b] [1]

a Faculty of Management - McGill University, 1001 Sherbrooke Street West, Montreal, PQ, H3A1G5 Canada. ericsson@management.mcgill.ca.

b IRES, Département des Sciences Economiques, Université Catholique de Louvain, 3 Place Montesquieu, 1348 Louvain-la-Neuve, Belgium, renault@ires.ucl.ac.be

First version February 1998
This version February 2000

Abstract

We develop a simple binomial model of liquidity and credit risk in which a bondholder has the option to time the sale of his security, given a distribution of potential buyers, bids and liquidity shocks. We examine as a benchmark the case without default and find that our model predicts a decreasing term structure of liquidity premia, in accordance with the empirical findings of Amihud and Mendelson (1991). Then, we study the default risky case and show that credit risk influences liquidity spreads in a non-trivial way. We find that liquidity spreads are an increasing function of the volatility of the firm's assets and leverage - the key determinants of credit risk. Furthermore we show that bondholders are more likely to sell their holdings voluntarily when bond maturity is distant and when default becomes more probable. Finally, in a sample of US corporate bonds, we find support for the time to maturity effect and the positive correlation between credit and liquidity risks.

Key words : credit risk, liquidity, structural models.

[1] Financial support from Marie Curie Fellowships ERB4001GT965223 and ERB4001GT970319 is gratefully aknowledged. This research was also supported by the Belgian government grant (Poles d'Attraction inter-universitaires PAI P4/01).
We have received helpful comments from Ronald Anderson, Wolfgang Bühler, Riccardo Calcagno, Darrell Duffie, Xiaofei Li, Franck Moraux, Olivier Scaillet, and Richard Stapleton. We thank seminar participants at Université Catholique de Louvain, McGill University, Inquire Europe and Crédit Commercial de France for their comments.

1 Introduction

Credit risk and liquidity risk have been put forward as two of the main justifications for the existence of yield spreads above benchmark treasury notes or bonds (see FISHER (1959)). While a rapidly growing body of literature has focused on credit risk[2] since MERTON (1974), liquidity has remained a relatively unexplored topic, in particular for defaultable securities. The purpose of this paper is to develop a simple structural model of liquidity and credit risk in an attempt to better understand the interaction between these two sources of risk.

Throughout the paper, we define liquidity as the ability to sell a security quickly and at a price close to its value in frictionless markets. We thus think of an illiquid market as one in which a sizeable discount may have to be paid for a position to be liquidated.

Structural credit risk models along the lines of MERTON (1974) are often criticized on the following two counts. First it is argued that the levels of yield spreads generated by the models are too low to be consistent with observed spreads.[3] This may very well be a result of specification errors. However it should be noted that such casual empiricism fails to take into account that these models only price credit risk. If prices on corporate bonds reflect compensation for other sources of risk such as illiquidity then one should expect to find that these models overprice bonds.

A second criticism often brought up is that the levels of credit spreads obtained with most structural models are negligible for very short maturities and that this is inconsistent with empirical evidence.[4] Our model implies non trivial liquidity premia for short maturities and can thus resist this line of criticism.

In recent empirical work on credit spreads, ANDERSON and SUNDARESAN (2000) show that although extensions of the BLACK and COX (1976) model are often able to capture large parts of the variation in spread indices, during some sub-samples they fail to do so. This suggests that another factor exogenous to their model is at play. DUFFEE (1999) estimates a two factor[5] model of bond prices on corporate debt data. His findings suggest that his factors are not sufficient to model yield spreads. We posit that a natural candidate for an additional factor should be one that captures the illiquidity of corporate debt markets.

We model credit risk in a modified MERTON (1974) framework. Although interest rate risk is an important determinant of corporate bond prices we abstract from it in order to allow any interaction between credit and liquidity risk to be analyzed in isolation. Structural models with stochastic interest rates have been proposed for example by SHIMKO, TEJIMA and VAN DEVENTER (1993), NIELSEN, SAA-REQUEJO and SANTA-CLARA (1993) and WANG (1999).

We introduce two distinct sources of liquidity risk. First when the firm is solvent, the

[2]See for example BLACK and COX (1976), SHIMKO, TEJIMA and VAN DEVENTER (1993), LONGSTAFF and SCHWARTZ (1995a), ANDERSON and SUNDARESAN (1996), JARROW and TURNBULL (1995).

[3]Such an argument can be found for example in JONES, MASON and ROSENFELD (1984), LONGSTAFF and SCHWARTZ (1995a) and MELLA-BARRAL and PERRAUDIN (1996).

[4]This argument is one of the motivations for the article by DUFFIE and LANDO (1997).

[5]One of the factors captures term structure risk and the other is meant to capture credit risk.

bearer of a bond is subjected to random liquidity shocks. Such shocks can for example reflect cash constraints or rebalancing of the investor's portfolio because the specific bond is no longer appropriate for hedging or diversification purposes. With a certain probability he may have to sell his bond immediately. The price he would have to sell at is assumed to be a random fraction of the price in a perfectly liquid market. The distribution of this fraction is modelled as a function of the number of traders active in the market for a particular bond.

The supply side of the market is an endogenous function of the state of the firm and the probability of liquidity shocks. When there is no liquidity shock, the bondholder still has the option to sell if the price he can obtain is good enough. Although if a bondholder could hold the bond until maturity he would avoid accepting a discount altogether, he will sell if the price is better than the expected value of waiting and exposing himself to the risk of being forced to sell at a less favorable price.

The second important assumption we make is that heterogeneity in liquidity is maintained in the market for distressed debt. Bonds that were relatively illiquid before default remain less liquid than other distressed bonds. This is supported by empirical evidence. WAGNER (1996) studies the market for distressed debt and finds that medium and small defaulted issues outperform the returns on larger (and hence more liquid) issues. The relative size of issues is likely to be invariant to whether a firm is solvent or in default (at least prior to the issue of restructured assets). Hence we would expect the liquidity of an issue to be positively related to the volume outstanding, before and after financial distress.

We finally allow for market-wide liquidity shocks where the mean number of traders suddenly falls due to an external event. Russia's recent default for example triggered a flight to quality in many credit risky markets including the US corporate bond market where liquidity plummeted.

Our model implies that liquidity spreads are decreasing and convex functions of time to maturity. This is consistent with empirical evidence on markets for Treasury securities. AMIHUD and MENDELSON (1991) examine the yield differentials between US Treasury notes and bills - securities with differing liquidity and find that term structures of liquidity premia indeed have this particular shape. KEMPF and UHRIG (1997) study liquidity effects on German government bonds and their findings support the conclusions of AMIHUD and MENDELSON (1991) as to the shape of the term structure of liquidity premia.

In addition to the shape of the term structure, our model implies that the level of liquidity premia is correlated to the probability of financial distress. The premia will be higher when default is likely as bondholders will have less time to look for attractive offers and may have to liquidate their positions on unfavorable terms.

Furthermore, we show that the optimal trading behavior of the bondholder is a function of time to maturity, firm risk and leverage. The bondholder's behavior is summarized by the discount that he is willing to sell his security at. When time to maturity is short there is a relatively small chance that he will be forced to sell his bond at an unfavorable price. He will thus be unwilling to accept a large discount. However when the bond maturity is distant he will be willing to sell at a relatively lower price. Hence our model implies that the market for recent vintages should be more active than seasoned ones and it predicts that trading should be more active the riskier the issue. When the firm is near default the probability of a sudden shortening of a bond's effective maturity may be substantial and the willingness of bondholders to sell will increase.

Reduced form models of credit risk such as DUFFIE and SINGLETON (1999) or LANDO (1998) are typically able to include liquidity as a component of their total spreads. In this class of models, default occurs "by surprise" at a random date and with random intensity. However in these models, only aggregate spreads can be derived from actual data and one cannot distinguish a situation with a high liquidity premium and little default risk from one of a very liquid but risky bond. Separating liquidity from credit spreads is not only theoretically interesting, it is also necessary for hedging and portfolio management. In order to set up effective strategies for these purposes it is not sufficient to merely know how much compensation one receives or pays for different risks, but also to which risks one is exposed to and to what extent.

The LONGSTAFF (1995) model lies close to ours in spirit. He measures the value of liquidity for a security as value of the option to sell it at the most favorable price for a given time window. Although our results are not directly comparable because the author derives upper bounds for liquidity discounts for a given sales-restriction period, his definition of liquidity comes close to our own.

To date, TYCHON and VANNETELBOSCH (1997) is, to our knowledge, the only paper which explicitly models the liquidity of corporate bonds endogenously. They use a strategic bargaining setup in which transactions take place because investors have different views about bankruptcy costs. Although some of their predictions are similar to ours, their definition of liquidity risk differs significantly. Notably, as their liquidity premia are linked to the heterogeneity of investors' perceptions about the costliness of financial distress, their model predicts that liquidity spreads in Treasury debt markets should be zero.

Two of the predictions of our model are that young issues should be more liquid than seasoned ones and that liquidity spreads should be correlated with the level of credit risk. Taking credit ratings as measures of default risk and choosing two proxies of liquidity (time elapsed since issuance and volume outstanding), we test these hypotheses on a sample of a thousand US corporate bonds. We find that age affects corporate bond spreads with a positive sign: older issues carry a premium over younger ones. As predicted, we also find evidence of a correlation between default risk and liquidity premia as measured by amount outstanding: the impact of this liquidity proxy on yield spreads is greater for lower-rated securities.

The structure of the paper is the following. Section 2 presents the model and describes the default generating mechanism and the sources of illiquidity. Section 3 analyzes liquidity spreads in the default-free case and reports the general shape of the term structure of liquidity spreads in this context. Section 4 and 5 describe the default risky case and its implications for bond trading respectively while section 6 introduces the possibility of market-wide liquidity shocks. Section 7 reports on our empirical tests on the model's predictions and section 8 concludes.

2 The Model

We assume for simplicity that agents are risk-neutral. We thus avoid the issue of determining a suitable equivalent martingale measure for an illiquid and therefore incomplete market.[6] Although risk aversion and the associated risk premium might affect the quantitative outputs of our model they are unlikely to change the qualitative results.

The uncertainty relating to the firm value $\{v_t\}_{t=0}^{t=T}$ is modelled using a standard binomial formulation[7] in which σ denotes the volatility of the firm's assets and Δt is the time interval between two nodes[8]. The risk-free interest rate r is assumed to be constant. Following MERTON (1974), we assume that the firm is financed by a single issue of discount debt with maturity T and promised principal repayment P. We consider the value of the perfectly liquid bond $B_L(t)$ to be the benchmark against which we will compare the value of an illiquid discount bond $B_I(t)$. The price of a "liquid" security is given by the price that would obtain under the assumptions made in a MERTON (1974)-like setting.

At maturity the holder receives the principal repayment when the firm is solvent or a fraction of the value of the firm when it is in default.

$$B_L(T) = \min(P, v_T - K),$$

where K represents the costs of financial distress. These are taken to include both direct costs (legal fees etc.) and indirect costs arising from suboptimal operating decisions (due to e.g. over or underinvestment incentives in financial distress), lost business relationships, etc.

We assume that at maturity the firm is liquidated and proceeds are distributed to the respective claimants. There is hence no market liquidity problem at this date and the illiquid bond price is

$$B_I(T) = B_L(T).$$

[6] Few papers have addressed the problem of default risk in incomplete markets. Lotz (1997) studies local risk minimization (equivalent to the problem of pricing under the Minimal Martingale Measure) in a reduced form framework of credit risk. MORAUX and VILLA (1999) also work under this measure but derive prices in the MERTON (1974) model when the assumption of asset tradeability is lifted. We believe that the issue of an appropriate martingale measure is important for practical purposes but that dealing with it within the context of our current framework would only serve to obscure the fundamental forces at play.

[7] Thus the probability of an up move in a given time interval Δt is given by

$$p = \frac{e^{r\Delta t} - d}{u - d},$$

where

$$u = \frac{1}{d} = e^{\sigma\sqrt{\Delta t}}.$$

[8] Note that the choice of Δt is not irrelevant to our results. Δt can be seen as the potential trading frequency of the bondholder or the time necessary for the distribution of offers to change. If we were to let $\Delta t \longrightarrow 0$, the liquidity spread would vanish because in any finite interval, the bond holder would receive an infinity of offers and would get an offer with zero discount with probability one.

By looking at the effect of the time step in our binomial model, we may study the impact of trading restrictions on the value of securities along the lines of LONGSTAFF (1995) paper.

We assume that financial distress is triggered when the value of the firm's assets reaches an exogenous lower boundary L. We have specified L as a fraction g of the present value of the debt's principal :

$$L = g \exp(-rT)P.$$

We thus impose an upper bound on the firm's leverage in terms of its "quasi-debt ratio".[9]

When a realization of firm value v_t becomes known we observe if the firm is solvent. If not, the bonds are worth :

$$\begin{aligned} B_L(t) &= \max(L - K, 0), \\ B_I(t) &= \max(L - K - \kappa, 0), \end{aligned}$$

where κ is an additional cost reflecting illiquidity in the distressed debt market. This cost implies a higher expected return for previously illiquid debt in the distressed debt market, as observed in practice.[10] Note that in contrast to Tychon and Vannetelbosch (1997), we do not assume that bankruptcy costs are investor specific.

In our model, the following events occur given that the firm remains solvent, i.e. $v_t > L$ (Figure 1 summarizes the sequence of events). First the bondholder will find out whether he will be forced to sell his bond due to some exogenous liquidity shock. Such shocks may occur as a result of unexpected cash shortages, the need to rebalance a portfolio in order to maintain a hedging or diversification strategy, or to meet capital requirements. The bondholder could for example be an insurance company which may face a sudden jump in claims because of an earthquake. The probability of being forced to sell during a particular period is θ.

Given that he is forced to sell, the discount that he faces is modelled as follows. The price offered by any one particular trader is assumed to be a fraction $\widetilde{\delta}$ of the perfectly liquid price B_L. We assume this fraction to be uniformly distributed on $[0,1]$. He may however obtain several offers and will retain the best one. Given that he calls his broker the latter will obtain N offers for him, where N is assumed to be Poisson with parameter γ

$$N \sim Po(\gamma),$$

so that γ is the expected number of offers[11]. One may also think of γ as the number of active traders in the market for a particular type of bond. This number may differ for institutional reasons. For example, banks are less likely to be active in the market for

[9] The quasi-debt ratio was used as a leverage measure by Merton (1974) and defined as the present value of its debt obligations at the risk free rate in relation to the firm's value:

$$q = \frac{e^{-rT}P}{v_0}.$$

[10] See Wagner (1996).

[11] A constant probability of offers has been used in a different context in the microstructure literature (e.g. Glosten and Milgrom (1985), Easley and O'Hara (1987)).

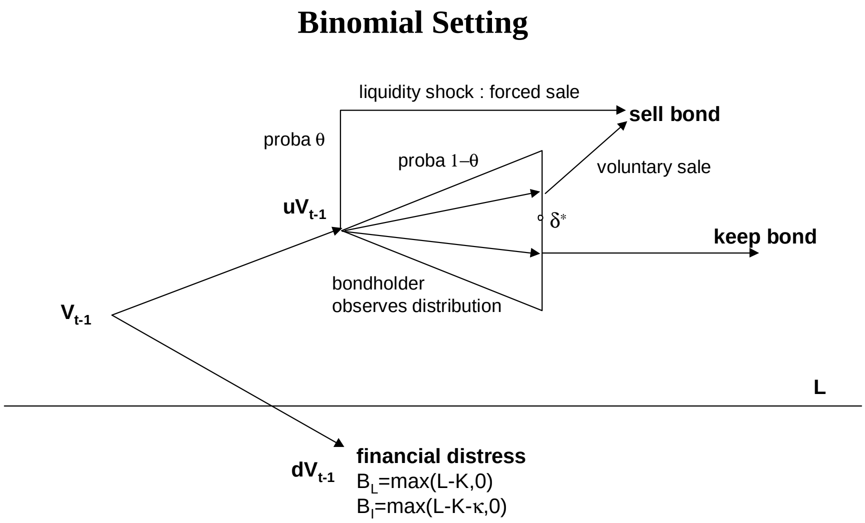


Figure 1: The sequence of events at an arbitrary point in time $t < T$.

highly rated debt as a result of the way that capital requirements are structured.[12] The choice of distribution and support for the individual discounts is admittedly simplistic but we retain it for illustrative purposes.[13] The expected best fraction of the liquid price that he will be offered will thus be[14]

$$\bar{\delta}(\gamma) \equiv E\left[\tilde{\delta}\left(\gamma\right)\right] = \sum_{n=0}^{\infty} e^{-\gamma}\frac{\gamma^n}{n!} \cdot \frac{n}{n+1}.$$

We assume that the distributions of liquidity shock arrivals and discounts are independent of the firm value uncertainty. This would apply for example to our above case of an insurer facing a sudden jump in claims. The expected value of the bond given a forced sale is thus

$$E_t\left[\tilde{\delta}(\gamma)B_L\left(t\right)\right] = B_L\left(t\right)E\left[\tilde{\delta}(\gamma)\right] = B_L\left(t\right)\bar{\delta}(\gamma),$$

where $E_t\left[.\right]$ denotes conditional expectation with respect to information available at date t. If he is not forced to sell (with probability $(1-\theta)$) he still has the option to sell his bond should the best offer made to him be acceptable. If he decides to sell he will receive a random payment of

$$\tilde{\delta}(\gamma)B_L\left(t\right),$$

and if he decides not to sell, the holding value is

$$e^{-r\Delta t}E_t\left[B_I\left(t+1\right)\right].$$

[12]Under the current guidelines, capital requirements do not discriminate across bond ratings. Hence banks are at a competitive disadvantage in the investment-grade corporate debt market.

[13]Given that our model is numerical, it can accommodate any $[0,1]$-distribution without any difficulty.

[14]We have gathered the details of the calculations details in the appendix.

Hence the value of the illiquid bond (if the firm is solvent) is

$$\begin{aligned} B_I(t) &= \theta\bar{\delta}(\gamma)B_L(t) \\ &\quad + (1-\theta)\max\left(B_L(t)\, E\left[\tilde{\delta}(\gamma)\right], e^{-r\Delta t}E_t\left[B_I(t+1)\right]\right). \end{aligned} \tag{1}$$

We denote by δ_t^* the reservation price fraction above which the bondholder will decide to sell at time t and below which he will keep his position until the next period unless he faces a liquidity shock. This allows us to rewrite

$$\max\left(B_L(t)\, E\left[\tilde{\delta}(\gamma)\right], e^{-r\Delta t}E_t\left[B_I(t+1)\right]\right),$$

as[15]:

$$\begin{aligned} & E\left[B_L(t)\,\tilde{\delta}(\gamma)\, I_{\tilde{\delta}>\delta^*} + e^{-r\Delta t}E_t\left[B_I(t+1)\right] I_{\tilde{\delta}\le\delta^*}\right] \\ = \;& B_L(t)\, E\left[\tilde{\delta}(\gamma)\, I_{\tilde{\delta}>\delta^*}\right] + P\left[\tilde{\delta}\le\delta^*\right] e^{-r\Delta t}E_t\left[B_I(t+1)\right], \end{aligned}$$

where I_A is the indicator function taking the value 1 if event A is true and 0 otherwise. The critical value for the offered price fraction $\tilde{\delta}(\gamma)$, above which the bondholder will decide to sell is

$$\delta_t^* = \frac{e^{-r\Delta t}E_t\left[B_I(t+1)\right]}{B_L(t)}.$$

The motivation for the randomness of δ, i.e. the implicit assumption that different prices for the same security can be realized at any one time is the same as for the occurrence of liquidity shocks. Some agents trade for hedging or cash flow reasons and may thus accept to buy at a higher (or sell at a lower) price than other traders.[16]

This is consistent with the structure of the US corporate bond market, an OTC market dominated by a limited number of dealers. This structure can lead to information asymmetries that result in several prices being quoted in a given market at the same time[17]. The variable δ_t^* should be thought of as a "reservation" price : the lowest price acceptable to the bondholder to sell his security.

In order to compute bond prices, we use backward induction as usual in a tree setting. We start at the maturity of the bonds where values are known and roll backward through the tree until the initial date.

Now that we have discussed our assumptions and resulting modelling framework we proceed to present our numerical results. We begin with the case of credit risk-free debt and then proceed to the case of corporate debt.

[15] Details of the calculations of $P\left[\tilde{\delta}(\gamma) > \delta^*\right]$ and $E\left[\tilde{\delta}(\gamma)I_{\tilde{\delta}>\delta^*}\right]$ can be found in appendix.

[16] Our assumption is similar to the concept of liquidity traders in market microstructure.

[17] See for example SCHULTZ (1998) and CHAKRAVARTY and SARKAR (1999).

3 Results in the Default Risk-free Case

We the case of a default risk-free bond first. In doing so we achieve two tasks. First, we define a benchmark case necessary to analyze the interaction of credit and liquidity risk. Without knowing what the term structure of liquidity spreads looks like in the credit risk-free case, it would be hard to see what the full impact of this uncertainty is. Second, we allow the implications of the model to be related to empirical results which are available for the Treasury markets.

For all simulations in the following sections, we define a set of parameters which will be used unless stated otherwise. This is to facilitate comparisons between the various graphs and tables. This base case has the following parameters : the time step is $\Delta t = 1/12$ (one month), the risk-free interest rate is $r = 7\%$, the average number of market-makers active for the security is $\gamma = 7$, the probability of a liquidity shock is $\theta = 0.874\%$ which corresponds to a yearly probability of 10% of having at least one shock and the quasi-debt ratio is $q = 0.6$. In the next section (default risky case), we will supplement the base case with values of bankruptcy costs and of the default boundary.

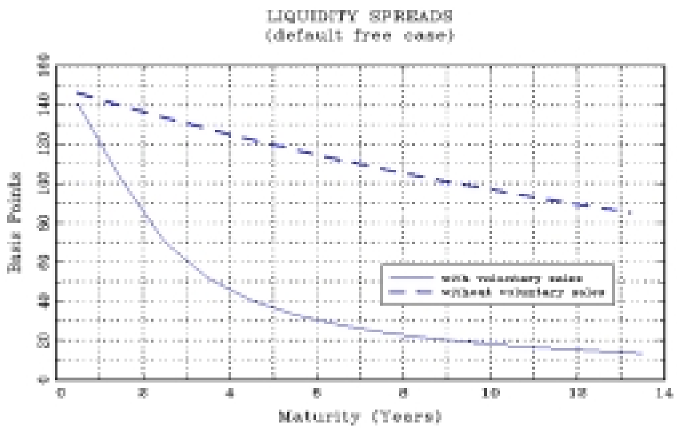


Figure 2

Figure 2 plots two specifications of the liquidity spread. The dashed line, which is almost straight, is the liquidity spread when the bondholder cannot sell his position before maturity unless he is forced to do so by a liquidity shock. The solid line is the liquidity spread when we lift the constraint of no early sale. It is then a decreasing and convex function of time to maturity. This clearly shows that it is the ability to sell voluntarily prior to maturity which yields the shape of the liquidity curve. This particular shape is consistent with empirical evidence of AMIHUD and MENDELSON (1991) and KEMPF and UHRIG (1997) for the US and German government debt markets respectively.

AMIHUD and MENDELSON (1991) study the markets for US Treasury notes and bills of equal maturity, instruments with identical interest rate risk exposure and payoff structure.

The difference between these instruments lies in the liquidity in their secondary market. They find that the yield spreads on these securities differ on average by about 42 basis points and that the differential is a convex and decreasing function of time to maturity.

KEMPF and UHRIG (1997) test for the existence of a liquidity spread in longer government bonds in the German market.[18] The authors find a statistically significant average spread of 17 basis points. This level and that found by AMIHUD and MENDELSON (1991) is well within the range of our results for reasonable parameters as shown in Figure 2.

Note that a decreasing function for *yield spreads* does not mean that liquidity has a smaller impact on *prices* for long bonds. On the contrary, as will be shown in the next section, percentage price differences are larger for long bonds.

The ability of the bondholder to decide whether or not to sell his security voluntarily is valuable as he may be able to avoid unfavorable sales following liquidity shocks. By examining the values of δ_t^* along different nodes in a given binomial tree (Figure 11) we find that δ_t^* decreases in time to maturity. We will examine these issues in more detail in section 5.

The relationship between the critical price fraction and time to maturity can be explained as follows. The longer the time to maturity, the more likely an adverse liquidity shock and associated large price discount. Decreasing the time to maturity makes it more probable that the bondholder can hold his bond to maturity and avoid selling his bond at a discount.

This intuition is compatible with the notion that the liquidity of on-the-run bond issues is considerably better than that of seasoned issues. However, note that since we have taken the demand side of the market to be exogenous, we do not suggest that our model explains this stylized fact in full. However if the willingness to sell is higher early in the life of a bond then it will certainly contribute. For a discussion of the differences between returns of the most recent Treasury auction and off-the-run securities, see WARGA (1992).

Note also that we ignore trading costs: these also tend to generate lower liquidity for bonds closer to maturity because when bonds are expected to be paid back shortly, transaction costs become proportionately higher. Thus transaction costs could further increase the spread at the short end of the term structure.

4 Results in the Default-risky Case

In this section we add credit risk into our framework in order to illuminate the interaction between liquidity risk and the possibility of financial distress.

We first present the shape of liquidity spread curves in the two cases when voluntary sales before maturity are precluded or allowed. We then analyze the respective share of yield spreads explained by liquidity and credit risk and show that liquidity premia can be very large in our model. Bounds are then obtained for price discounts (the difference between the price of a liquid and an otherwise identical illiquid bond) and some comparative

[18] They compare the yields of a sample of large issues (issue size is used as a proxy for liquidity) against those of smaller issues over the period 1992-94. All 143 bonds in their study were issued between 1982 and 1994 with initially 10 years to maturity and had a residual maturity between 0.5 and 10 years.

statics of the liquidity spreads are provided in subsection 4.4. Finally, bondholder trading behavior is analyzed in the last subsection.

4.1 Shape of the Term Structure of Liquidity Spreads

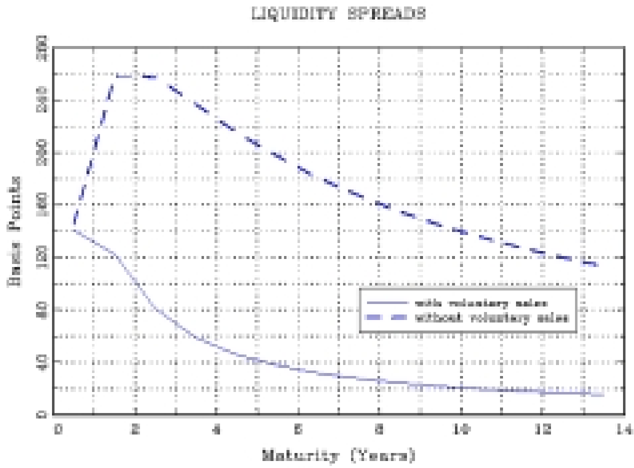


Figure 3

Let us first review some notation. Recall that θ denotes the probability of a liquidity shock in a given time interval Δt. The parameter γ can be thought of as the number of traders active in the market for a particular corporate bond. Leverage is measured by the quasi-debt ratio q which relates the risk-free present value of debt commitments to the current asset value, while κ is a reduced form measure for the cost of illiquidity in the market for distressed debt.

As before, the base case for simulations will be $\Delta t = 1/12, r = 7\%$, $\gamma = 7$, $\theta = 0.874\%$, $q = 0.6$. We now add parameters specific to default risk : the volatility of assets is $\sigma = 0.3$, bankruptcy costs are $K = 10$ and the supplementary costs for illiquid securities are $\kappa = 10$. Finally, the default boundary is $L = g \exp(-rT)P$, where $g = 1$.

We note that spreads are still decreasing in debt maturity so that the qualitative shape of the liquidity term structures does not change with the introduction of credit risk into the analysis (Fig. 3). This is in line with the results of LONGSTAFF (1994) who studied the Japanese market and found similar patterns for credit risky bonds issued by the Japan Finance Corporation of Municipal Enterprise, those of the Tokyo Metropolitan Government and debentures of the Industrial Bank of Japan.[19]

The decreasing term structure of liquidity can help to explain a stylized fact for high grade corporate bond spreads. Structural Merton-type models cannot explain the flat

[19]The author also finds jumps in spreads around six to seven years to maturity but these are due to specificities of the Japanese market.

shape of term structures of spreads above Treasury benchmarks for high grade bonds as reported by DUFFEE (1998). These models produce increasing term structures of credit spreads for low risk securities. However if we add the liquidity component, one can reconcile a structural model with a flat term structure of spreads, the increase in credit spreads being offset by the decrease in liquidity spreads (see Figure 4[20], obtained using $\sigma = 0.2$).

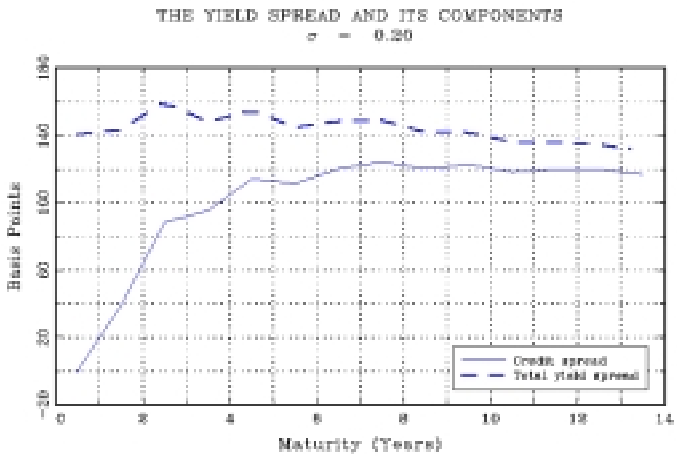


Figure 4

4.2 The Components of Yield Spreads

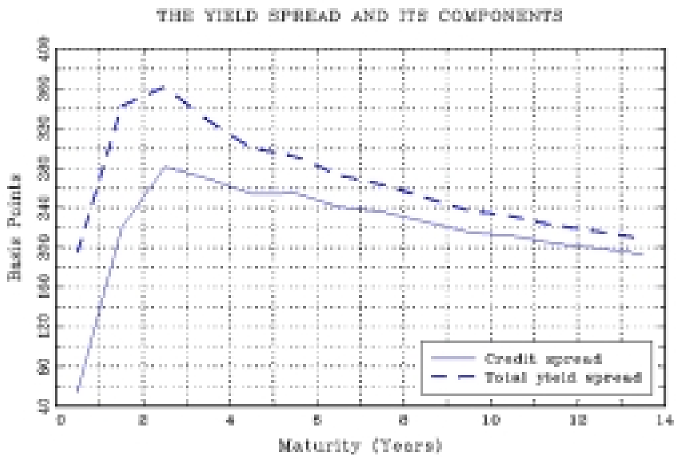


Figure 5

[20]The lack of smoothness of the curves is due to the presence of a barrier in the tree.

Marketability premia are an important part of the total yield spread for short maturities, less so for longer bonds (figures 5-6, obtained using the base case parameters[21]). This follows from our model of credit risk which predicts that default is unlikely for very short maturities. This qualitative result is consistent with the results of LONGSTAFF (1994) who finds a similar split for Japanese data.

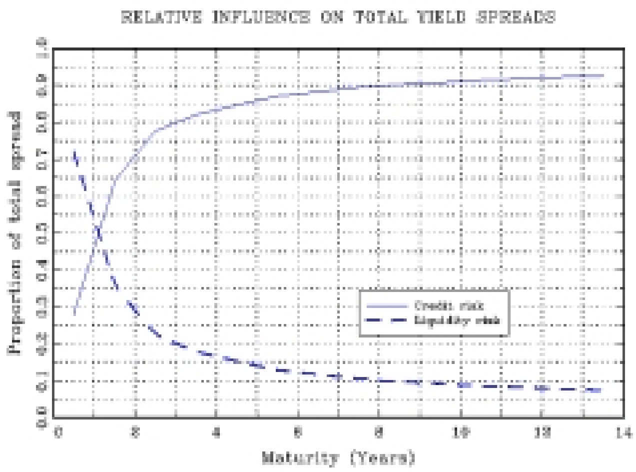


Figure 6

Table 1 reports credit spreads and liquidity spreads (in basis points) for varying levels of asset risk (σ) and maturities, using the base case parameters. The table uses the same values for σ^2 an T as in MERTON (1974).

σ^2	Maturity	Credit	Liquidity
0.03	2	31	85
0.03	5	74	36
0.03	10	85	18
0.10	2	251	99
0.10	5	275	41
0.10	10	225	21
0.20	2	532	134
0.20	5	379	55
0.20	10	264	28

Table 1 : Credit and Liquidity spreads (bps.)

These results show that credit and liquidity risk cannot be treated independently. One cannot simply add a liquidity premium above credit spreads. This is especially important when testing corporate debt models. Spreads above benchmarks will be wider

[21] $\Delta t = 1/12, r = 7\%,\ \gamma = 7,\ \theta = 0.874\%,\ q = 0.6, \sigma = 0.3,\ K = 10,\ \kappa = 10,\ g = 1.$

for speculative-grade debt not only to compensate for credit risk but also because default risk impacts on liquidity spreads.

4.3 Spread Bounds

In order to gain some intuition for the behavior of price discounts, we will now turn to Figure 7 which plots these discounts if no early sale is allowed, those in the unrestricted case and finally, their bounds. Price discounts cannot be greater than the immediate payment of the additional bankruptcy discount κ if the bond is subject to default risk. In the default risk-free case, it cannot exceed the average expected discount irrespective of traders quotes $1-\overline{\delta}$. The price discount is thus bounded above by $max\left(\kappa, 1-\overline{\delta}\right)$

The discount is bounded below by $1-\delta^*$, otherwise the bondholder would be willing to immediately sell off his position. Within these bounds, the discount increases and is concave, reflecting the higher probability of an adverse liquidity shock. Naturally, the probability of a liquidity shock during the life of a very long bond tends to 1 and if no early sale is allowed, the spread converges to its upper asymptote. However this is not the case when we can sell during the life of the bond because the probability of a voluntary sale also increases with time to maturity and thus offsets some of the impact of the increasing likelihood of a shock. In fact, it converges to the lower bound, because for longer maturities the probability that a bondholder sells in anticipation of a liquidity shock approaches one.[22]

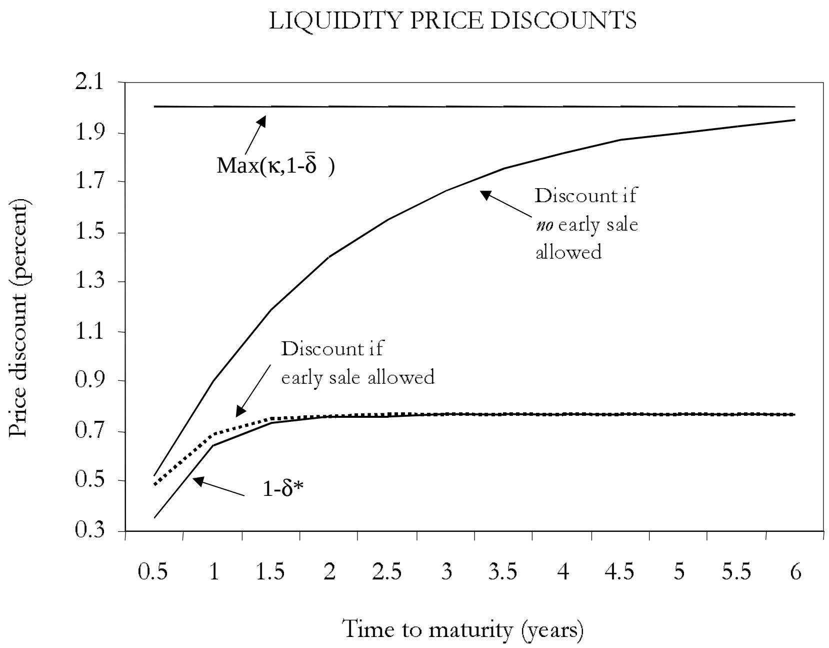


Figure 7

[22]The reason that there is no discounting effect follows from our specification of quotes in illiquid markets as fractions of their value in perfectly liquid markets.

4.4 Comparative Statics

Our main findings so far are the decreasing shape of the term structure of liquidity spreads and that they increase in credit risk as measured by leverage (quasi-debt ratio) and firm asset risk (see table 1 and 2). We now proceed to study in greater detail how the compensation for liquidity risk depends on other model parameters.

The following figures show how three of the most important parameters of our model impact on liquidity spreads for different maturities. We first consider θ which is the probability of facing a liquidity shock. Figure 8 reports the term structures of liquidity spreads for various values of θ corresponding respectively to an annual probability of at least one liquidity shock of 5% $(\theta = 0.426\%)$, 10% $(\theta = 0.874\%)$, 20% $(\theta = 1.842\%)$ and 50% $(\theta = 5.613\%)$. As expected, the greater this probability, the greater the liquidity spread because the more likely the bondholder is to be forced to sell at an unfavorable price.

As we already mentioned, liquidity premia increase in credit riskiness. Figure 9 plots liquidity spreads for various levels of volatility. However, credit risk is not only a matter of default probability (mainly influenced by leverage and volatility), but is also determined by the loss realized in default. The last figure (fig. 10) plots the relationship between the additional loss incurred by illiquid securities upon default and the liquidity premium. Once again a positive relationship is found.

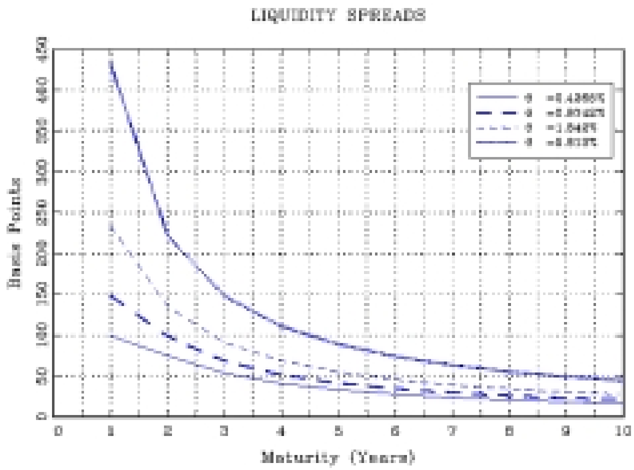


Figure 8

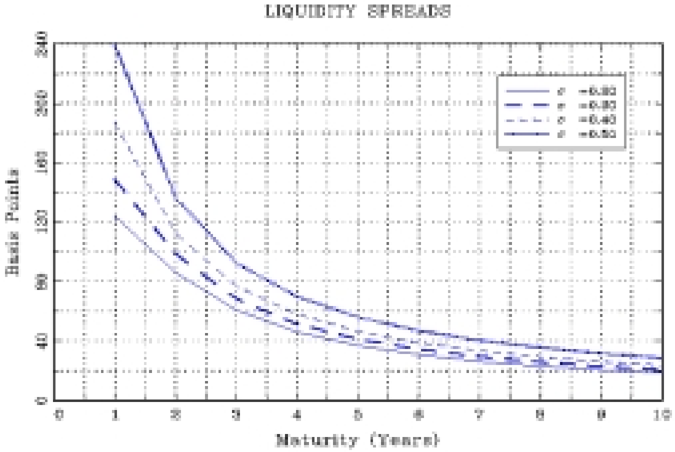


Figure 9

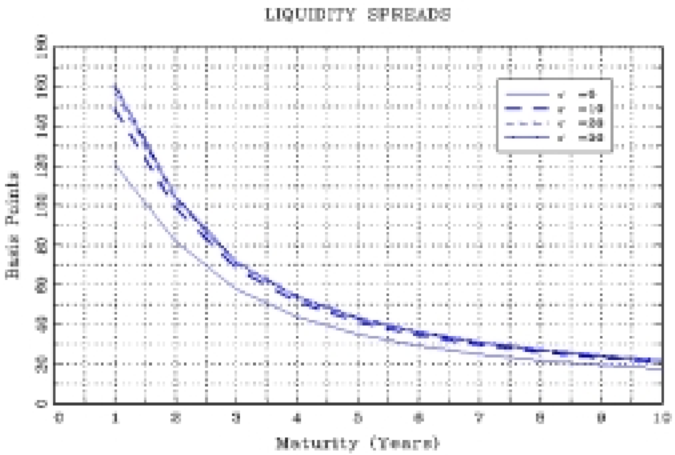


Figure 10

5 Bondholder Trading Behavior

In this section we analyze the willingness of a bondholder to sell his security at a discount to the price that would prevail in a perfectly liquid market. When financial distress approaches, the bondholder realizes that there is an increased likelihood that his option to seek out favorable offers prior to maturity will become worthless. Hence he will be willing to sell at a lower price. Again this would suggest that trading volume increases as default becomes more likely. An empirical study on high-yield debt by SCHULMAN, BAYLESS and PRICE (1993) supports the idea that anticipation of default results in greater trading activity.

Figure 11 illustrates in a binomial tree the percentage price discounts[23] that are acceptable to the bondholder as a function of time to maturity and proximity of default. We see that the closer we are to maturity, the lower the acceptable discount because the bondholder only has a short time to wait before the bond is redeemed and the likelihood of a forced sale at an unfavorable price is smaller. On the other hand, when the issuer of the bonds approaches financial distress, the bondholder will be more willing to sell his security at a greater discount to avoid the risk of facing reorganization costs.

$t=0$	1	2	3	4	5	6	7	8	9	T
									0.00	
								1.42		
							2.63		0.00	
						3.58		1.42		
					4.27		2.63		0.00	
				4.74		3.58		1.42		
			5.08		4.27		2.63		0.00	
		5.37		4.85		3.58		1.42		
	5.80		5.48		4.65		2.63		0.00	
7.05		6.79		6.27		4.98		1.42		
	12.30		11.73		10.82		8.69		0.00	
		def.		*def.*		*def.*		*def.*		
			def.		*def.*		*def.*		*def.*	
				def.		*def.*		*def.*		
					def.		*def.*		*def.*	
						def.		*def.*		
							def.		*def.*	
								def.		
									def.	

Figure 11 : Percentage discounts $(1-\delta_t^* \times 100)$, with $\Delta t = 1$, $T = 10$.

Table 2 focuses more specifically on the impact of default risk on liquidity discounts. Default risk is measured by leverage (quasi-debt ratio: $P\exp(-rT)/v$) and the volatility of assets : σ. The discount the bondholder is ready to sell his bond at clearly increases in both leverage and volatility and can reach considerable levels. These results are robust to the choice of the other model parameters $(\gamma, \theta, K, \kappa, r)$.

[23] defined as $((1-\delta_t^*) \times 100)$.

		Firm risk (σ)				
		0.10	0.15	0.20	0.25	0.30
	0.20	5.45	5.45	5.45	5.45	5.46
Quasi	0.40	5.45	5.45	5.47	5.57	5.62
debt	0.60	5.45	5.47	5.68	5.79	7.05
ratio	0.80	5.46	5.89	6.17	9.38	9.82

Table 2 : The acceptable ($t = 0$) discount as a function of leverage and firm risk. Base case with $\Delta t = 1$ and $T = 10$.

6 Market-wide Shocks

So far we have taken the prevailing market conditions which the bondholder faces to be fixed. In other words the average number of traders willing to trade a given bond was taken to be a constant. This does not allow for adverse shocks to the economy accompanied by worsening credit-market conditions. We now relax this assumption in an attempt to model the effects of "flight to quality" on yield premia. An example of such a sudden liquidity collapse is given by the recent Russian crisis. While US corporate Baa spreads had oscillated in a range of about 120-150 basis points for several years, they suddenly surged to over 200 basis points when Russia announced that it would stop paying its debt obligations. Investors reacted by withdrawing their money invested in credit risky assets worldwide. Many bonds could not even be quoted for weeks because of a lack of demand and transactions.

We assume that normal market conditions are represented by an average number of traders γ^N and that with probability θ^C there is a deterioration in the willingness of traders to deal with corporate bonds to the extent that the Poisson parameter for the expected number of offers drops to $\gamma^C < \gamma^N$.

This extension will not affect the structure of our bond pricing formula (1) at a given node in the binomial tree but will influence the fraction of the fundamental value of a bond realized in a sale. In particular, the probability of obtaining a given number of offers can now be written as

$$P(N = n) = \left(\left(1 - \theta^C\right) e^{-\gamma^N} \frac{\left(\gamma^N\right)^n}{n!} + \theta^C e^{-\gamma^C} \frac{\left(\gamma^C\right)^n}{n!} \right)$$

and this will directly affect the expected fraction that a bondholder could expect to obtain both in the event of a forced sale and a preemptive trade.[24]

Although we do not present the results relating to this extension, we note that it does not alter the qualitative results of the model and that it has limited impact on the quantitative outputs.

[24] A more detailed treatment of these expressions can be found in the appendix.

7 Testing the Model's Predictions

In this section we investigate whether corporate bond data support our model's prediction that liquidity and credit risk should be positively related. Full structural estimation of our model lies beyond the scope of this paper. Rather, we test this implication by looking for evidence of a positive correlation between default risk and liquidity spreads in our data. We are also interested in testing the decreasing shape of the term structure of liquidity spreads.

As a general model for the bond spread S_t^i of issue i at time t,we use

$$\ln S_t^i = \alpha_0 + \alpha_1 X_{1,t}^i + ... + \alpha_n X_{n,t}^i + \gamma_1 Y_{1,t} + ... + \gamma_m Y_{m,t} \quad (2)$$

where X series are firm- or issue-specific variables such as amount outstanding or time after issuance while Y variables are common to all issues and can be macroeconomic or market variables. Such a logarithmic specification guarantees positive spreads.

The data consists of over a thousand US zero-coupon bonds recorded monthly from 1986 to 1996. Zero-coupon bonds are particularly well suited for a study of liquidity because they are not biased by coupon effects.[25] Unless stated otherwise in subsection (7.1) the variables are taken from Datastream.

Summary statistics	
Number of bonds	1096
Number of observations	54163
of which	
Investment grade bonds	49987
Speculative grade bonds	1759
Non rated bonds	2417
Mean spread	76 bp
Mean time to maturity	12.29 years
Mean bond age	5.04 years
Mean volume	\$ 106 M

Spreads are calculated as the difference between the risky bond yield and the risk-free rate obtained by the NELSON and SIEGEL (1987) procedure. A more detailed description of the construction of spreads is provided in appendix.

7.1 The data

We include four common variables (volatility, an aggregate measure of leverage, a credit cycle indicator and the risk-free rate) which should capture global trends in corporate bond spreads. These $Y-$variables are not indexed by the bond issue i as they do not depend on the specific securities.

[25] For a review of these effects, see SUNDARESAN (1997) chapter 5.

1. Volatility

   Asset volatility and leverage are the two most important determinants of default risk in a firm-value based model of credit risk. Asset volatility is however not directly observable for most firms and it is typically proxied in empirical work by the volatility of the stock when the issuing firm has publicly traded equity. In this paper, we will be focussing on an aggregate measure of asset volatility across firms and will use the volatility of the stock market as a proxy for asset volatility. We have chosen to include implied volatility rather than historical volatility[26] because implied volatility is a forward looking measure (the traders' expectation of volatility). The measure of implied volatility we use is the Chicago Board Options Exchange VIX index which is a weighted average of the implied volatilities of eight options with 30 days to maturity. We expect the volatility to enter the regression with a positive sign, since a greater volatility implies a greater risk of default and should be reflected in higher spreads.

2. Leverage

   Similarly, we use an aggregate measure of leverage in our regressions. Leverage is calculated as Debt/(Debt + Equity) from series extracted from the U.S. Federal Reserve's Flow of Funds Accounts.[27]. We expect a positive sign for leverage as highly levered firms have a higher probability of default.

3. The credit cycle

   The credit cycle indicator measures the volume of corporate debt in the market. We have no strong view about the expected sign for this variable. On the one hand, one could argue that it should have a negative link with spreads, as large amounts are issued when spreads are tight and high spreads tend to postpone debt issuance by corporations. On the other hand, a large flow of debt in the market should produce an offer shock which can only be matched by demand at the cost of higher spreads.

4. The risk-free rate

   Corporate yields can be broken down into a risk-free rate component and a spread. How these two components interact has been a matter of debate in the literature. Do spreads increase when the risk-free rate rises or do they decrease? Recent evidence in Duffee (1998) has shown that one could expect a negative sign for the risk-free rate at least for investment grade bonds, i.e. that spreads tend to fall when Treasury yields rise. Our panel is a mixture of high quality and speculative bonds and some include imbedded options which have been shown to lessen the negative impact of the risk-free rate on spreads. We have chosen to include the yield on the 30-year U.S. Treasury bond as the risk-free rate and prudently expect it to carry a negative sign.

[26] We have also carried out the regressions on 30-day and 90-day historical S&P500 volatility and the results were very similar.

[27] We thank Ronald Anderson for providing us the series.

The other independent variables ($X-$variables) in the regression are firm-specific or issue-specific.

5. Ratings

   We include bond rating dummies for credit risk. Each dummy variable takes the value 1 if the bond falls into a particular rating category or 0 otherwise. We include 9 such dummy variables for ratings AAA to C. The value of the parameters should thus be understood as the additional average spread (possibly negative) on a bond of a given rating category compared to a non-rated bond.

We also include two liquidity proxies: a time effect and a size effect.

6. Time after issuance
   It has been reported in the literature that bonds are more liquid immediately after issuance and rapidly lose their marketability as a larger share of the issues become locked into portfolios.[28] Therefore, our first proxy is the time elapsed since the issuance of the bond. We expect that the parameter for this variable will be positive to reflect the fact that older issues bear higher spreads than recently issued bonds.

7. Issue amount outstanding
   The size effect is measured by the amount outstanding of the issue reflecting the hypothesis that larger issues tend to be more liquid than smaller issues. We thus should find a negative sign for this variable in our regressions.

7.2 Results

The two hypotheses we wish to test are a) whether liquidity spreads are higher for more credit-risky securities and b) whether the term structure of liquidity spreads is decreasing. We test them in two separate regressions each including all the regressors above and an additional variable.

In the first regression, we will include a variable VolIG whose value will be the issue amount outstanding times an indicator function taking the value 1 if the bond is investment grade (BBB or better) and 0 otherwise. If the impact of liquidity (as proxied by the amount outstanding) is greater for investment grade than for speculative grade bonds, then the parameter for VolIG should be negative. Conversely, if it is smaller (in absolute value) for investment grades than for speculative grade bonds as predicted by our model, the parameter should be positive.

In the second regression, we follow a similar approach and include the variable VolShort whose value is the amount outstanding times the indicator function taking the value 1 if the bond has less than ten years to maturity and 0 otherwise. Our model predicts that liquidity should have a much greater impact on the spreads of short bonds than those of

[28] See for example chapter 10 in Fabozzi and Fabozzi (1995).

long bonds. If the data supports this prediction, one should find that VolShort carries a negative sign.

Tables 3 and 4 report the results of the regressions. All parameters except volatility and the risk-free rate in the first regression are significant at the 1% confidence level. As expected, the rating dummies decrease in credit quality. The only exception to this rule is the CCC dummy which is smaller than the B parameter. This is probably due to the low number of bonds in this risk class and to an industry effect. An important determinant of spreads which is ignored in our regressions is the expected recovery rate in case of default. This has been shown to vary substantially across industries (see ALTMAN and KISHORE (1996)). Thus, if our CCC sample contains a large proportion of issuers in an industry where expected recovery rates are high, one could have a downward bias on spreads which would be reflected in a lower-than-expected parameter for the dummy variable.

Interestingly, the split between positive and negative parameters for rating dummies corresponds to the limit between investment grade and speculative grade bonds. This shows that non-rated bonds in our sample are perceived by the market as intermediate between those two classes. The credit cycle indicator carries a positive sign thus reflecting increases in spreads when large amounts of debt flow in the market.

Both leverage and volatility enter with the expected positive sign although volatility is not significant in our regressions. The two main credit risk factors are thus compensated for in the yield spread. Our data also supports a contraction of spreads when risk-free rates increase as indicated in the negative sign for the interest rate parameter. Thus, corporate bonds yields do not bear the full impact of a variation in risk-free rates.

We now turn to liquidity. The parameters for amount outstanding and for time after issuance are all significant and carry the expected sign in both tables. The larger the outstanding amount, the greater the liquidity and the lower the spreads (negative relationship). The older the issue, the less liquid and the higher the spreads (positive relationship). We want to determine whether these relationships are stronger for bonds with high credit risk and with short time to maturity.

In Table 3, we see that the sign of VolIG is positive which implies that the impact of our liquidity proxy is less important for investment-grade bonds than for speculative-grade bonds. There is thus some support for the predicted positive correlation between credit risk and liquidity spreads. Furthermore, the parameter is statistically significant at the 1% confidence level and implies a very large difference between the two broad classes of bonds. The same change in amount outstanding has an impact about ten times larger on speculative-grade than on investment-grade spreads. A natural objection to this finding may be to argue that the average issue size is different in the two sub-samples. To rule out the possibility that this may bias our results, we tested other regressions where we introduced the variation of the amount outstanding rather than its value. The results (not reported here) were qualitatively very similar.

Variable	Parameter
Constant	−3.238873*
AAA	−0.785478*
AA	−0.470552*
A	−0.391502*
BBB	−0.249966*
BB	0.354981*
B	0.374426*
CCC	0.305642*
CC	0.890312*
C	1.402787*
Credit Cycle	0.000243*
Leverage	2.421783*
Volatility	0.000333
Amount outstanding	−0.101294*
VolIG	0.090155*
risk-free rate	−0.382692
Time after issuance	0.005583*

* significant at the 1% level.

Table 3 : Testing for higher impact of liquidity in speculative grade debt.

Variable	Parameter
Constant	−3.219900*
AAA	−0.783940*
AA	−0.456147*
A	−0.378221*
BBB	−0.230897*
BB	0.382692*
B	0.418091*
CCC	0.354021*
CC	0.917176*
C	1.447613*
Credit Cycle	0.000248*
Leverage	2.433099*
Volatility	0.000429
Amount outstanding	−0.007966*
VolShort	−0.302195*
risk-free rate	−0.718106*
Time after issuance	0.005263*

* significant at the 1% level.

Table 4 : Testing for higher impact of liquidity at short maturities.

As predicted by our model, the parameter for VolShort reported in Table 4 is negative. The impact of our liquidity proxy is thus much larger for bonds with less than 10 years to maturity. Furthermore, the data is consistent with the hypothesis that liquidity spreads should be higher for short bonds than for long bonds.

8 Concluding Remarks

We have developed a simple model to illustrate the impact of liquidity risk on the yield spreads of corporate bonds. Despite its simplicity, the model has a number of interesting features. Our main qualitative finding is that the level of liquidity spreads should be positively correlated with credit risk and that they should be decreasing functions of time to maturity.

Another finding is that for reasonable parameter inputs the model is able to generate non-negligible yield spreads even for short maturities. This addresses a common criticism of structural bond pricing models and helps to reconcile them with empirical evidence.

Furthermore, our results are consistent with previous empirical research not only as far as the shape of the term structure (decreasing and convex) is concerned but also in terms of the levels. The relative shares of spreads explained by credit risk and liquidity risk in our model are consistent with earlier empirical findings. In addition, our model is consistent with a number of stylized facts about debt securities. For example, it provides some justification for the greater liquidity of young issues and the higher frequency of trades nearer default reported in the literature.

Finally, we find that US corporate bond data support the prediction of our model that liquidity spreads should be positively correlated with the likelihood of default and that they should decrease with time to maturity.

9 Appendix A : Calculation of Expectations

The expected best fraction of the liquid price that the seller will be offered is

$$E\left[\tilde{\delta}\left(\gamma\right)\right]=\sum_{n=0}^{\infty}P\left(N=n\right)\int_{0}^{1}\delta f^{n}\left(\delta\right)d\delta,$$

where the density $f^n(x)$ is the probability that x is the best price fraction obtained given n offers. Given only one offer, for a uniform distribution the probability of getting a fraction of less than x is

$$F\left(x\right)=x,$$

where F is the cumulative distribution. The probability of getting none higher than x with n independent offers is thus

$$\left(F\left(x\right)\right)^{n}=x^{n},$$

and so the desired density function f^n is

$$f^{n}\left(x\right)=\frac{\partial\left(F\left(x\right)\right)^{n}}{\partial x}=nx^{n-1},$$

and thus, given that the number of offers is Poisson with parameter γ,we get

$$\begin{aligned}E\left[\tilde{\delta}\left(\gamma\right)\right] &= \sum_{n=0}^{\infty}e^{-\gamma}\frac{\gamma^{n}}{n!}\cdot\int_{0}^{1}n\delta^{n}d\delta,\\ &= \sum_{n=0}^{\infty}e^{-\gamma}\frac{\gamma^{n}}{n!}\cdot\frac{n}{n+1}.\end{aligned}$$

Now, in order to compute the value of an illiquid bond at a given node in the binomial tree we need to solve

$$E\left[\tilde{\delta}\left(\gamma\right)\cdot I_{\tilde{\delta}>\delta^{*}}\right]$$

and

$$P\left(\tilde{\delta}>\delta^{*}\right)=E\left[I_{\tilde{\delta}>\delta^{*}}\right]$$

We will do this for two cases. First when there are liquidity shocks to bondholders and there is a constant mean number of traders and second when shocks to the demand side are introduced, reflecting a fall in the mean number of traders.

9.1 Constant mean number of traders

Recall that

$$P\left(N=n\right)=e^{-\gamma}\frac{\gamma^{n}}{n!},$$

and that conditional on n offers our assumption of uniformly distributed offers yields the following density for the price fraction δ offered

$$f_{n}\left(\delta\right)=n\delta^{n-1}.$$

Then it follows that

$$
\begin{aligned}
E\left[I_{\tilde{\delta}>\delta^*}\right] &= \sum_{n=0}^{\infty} P\left(N=n\right) E\left[I_{\tilde{\delta}>\delta^*}\left|N=n\right.\right] \\
&= \sum_{n=0}^{\infty} P\left(N=n\right) \int_{\delta^*}^{1} n\delta^{n-1} d\delta \\
&= \sum_{n=0}^{\infty} e^{-\gamma}\frac{\gamma^n}{n!}\left(1-\left(\delta^*\right)^n\right).
\end{aligned}
$$

$$
\begin{aligned}
E\left[\tilde{\delta}(\gamma)\cdot I_{\tilde{\delta}>\delta^*}\right] &= \sum_{n=0}^{\infty} P\left(N=n\right) E\left[\tilde{\delta}(\gamma)\cdot I_{\tilde{\delta}>\delta^*}\left|N=n\right.\right] \\
&= \sum_{n=0}^{\infty} P\left(N=n\right) \int_{\delta^*}^{1} n\delta^{n} d\delta \\
&= \sum_{n=0}^{\infty} e^{-\gamma}\frac{\gamma^n}{n!}\frac{n}{n+1}\left(1-\left(\delta^*\right)^{n+1}\right).
\end{aligned}
$$

9.2 Random shocks to the demand side

We now allow for the occurrence of liquidity crises on a market-wide basis. We assume that with probability θ^C, the number of traders active in the market will fall from γ^N to $\gamma^C < \gamma^N$. The bond valuation formula in (1) remains unchanged. However the expected price fraction obtained and the probability that a voluntary sale takes place will be influenced. We now have

$$
\begin{aligned}
\bar{\delta}(\gamma^N,\gamma^C) &\equiv E\left[\tilde{\delta}\left(\gamma\right)\right] \\
&= \left(1-\theta^C\right)\sum_{n=0}^{\infty} P\left(N=n\left|\gamma=\gamma^N\right.\right)\frac{n}{n+1} \\
&\quad +\theta^C\sum_{n=0}^{\infty} P\left(N=n\left|\gamma=\gamma^C\right.\right)\frac{n}{n+1} \\
&= \sum_{n=0}^{\infty}\left(\left(1-\theta^C\right)e^{-\gamma^N}\frac{\left(\gamma^N\right)^n}{n!}+\theta^C e^{-\gamma^C}\frac{\left(\gamma^C\right)^n}{n!}\right)\frac{n}{n+1}.
\end{aligned}
$$

Noting from above that

$$
E\left[\tilde{\delta}(\gamma)\cdot I_{\tilde{\delta}>\delta^*}\left|N=n\right.\right] = \frac{n}{n+1}\left(1-\left(\delta^*\right)^{n+1}\right),
$$

we immediately obtain

$$
E\left[\tilde{\delta}(\gamma)\cdot I_{\tilde{\delta}>\delta^*}\right] = \sum_{n=0}^{\infty}\left(\left(1-\theta^C\right)e^{-\gamma^N}\frac{\left(\gamma^N\right)^n}{n!}+\theta^C e^{-\gamma^C}\frac{\left(\gamma^C\right)^n}{n!}\right)\frac{n}{n+1}\left(1-\left(\delta^*\right)^{n+1}\right),
$$

and

$$E\left[I_{\tilde{\delta}>\delta^*}\right] = \sum_{n=\mathbf{0}}^{\infty}\left(\left(1-\theta^C\right)e^{-\gamma^N}\frac{\left(\gamma^N\right)^n}{n!}+\theta^C e^{-\gamma^C}\frac{\left(\gamma^C\right)^n}{n!}\right)\left(1-\left(\delta^*\right)^n\right).$$

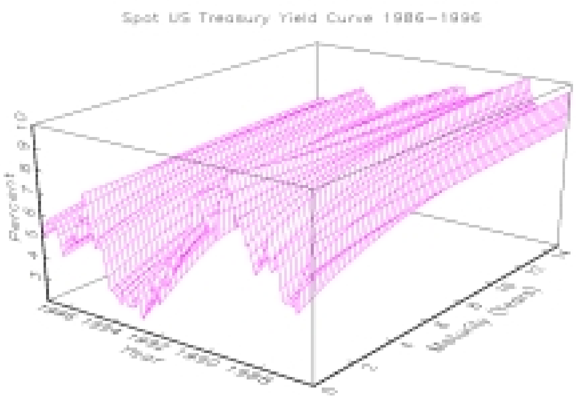
10 Appendix B : Spread Constructions

Spreads are defined as the difference between the yield on a corporate bond and the yield on a U.S. Treasury bond with same maturity. We use zero-coupon bonds only so that our spread calculations are not biased by coupon effects. Given that there does not exist a U.S. Treasury bond for all maturities, we have chosen to construct a whole term structure of risk-free rates from existing bond prices for each month end from January 1986 to December 1996 (132 months).

We use the Nelson and Siegel (1987) algorithm to obtain a smooth yield curve from zero coupon bonds. This procedure is a four parameter yield-curve calibration method whose flexible specification allows us to replicate most term structures shapes usually observed on the market. Formally, the yield at time t on a bond with maturity T is given by

$$R(t,T) = \beta_0 + (\beta_1 + \beta_2)\frac{1-\exp(-T/\beta_3)}{T/\beta_3} - \beta_2 \exp(-T/\beta_3)$$

Using risk-free zero-coupon bonds (mainly strips) to derive the benchmark curves enables us to obtain a nearly perfect fit of observed riskless rates by maximum likelihood (we use the CML tool in Gauss). However we find that the Nelson-Siegel procedure is over-parametrized for zero-coupon bonds and leads to wide differences in the parameter estimates in spite of only mild variations in their initial values. We thus impose a restriction on the first parameter which is the only one with a clear economic interpretation. More precisely, the first parameter represents the yield of a perpetual risk-free bond $R(t,\infty)$. We approximate it by the 30 year U.S. Treasury rate and thus obtain a consistent and robust set of optimal parameters. The constraint also turns out to yield positive forward rates for all maturities and all observation periods, thereby avoiding one of the main criticisms of the algorithm. For each month, we exclude risky bonds whose maturity falls outside the range spanned by the risk-free bonds to avoid the imprecisions of the interpolation procedure outside this range.



REFERE CES :

- Altman E. and V. Kishore (1996) "Almost everything you wanted to know about recoveries on defaulted bonds", *Financial Analysts Journal*, 57-64.
- Amihud Y. and H. Mendelson (1991) "Liquidity, maturity, and the yields on U.S. Treasury securities", *Journal of Finance*, **46**, 1411-1425.
- Anderson R. and S. Sundaresan (1996) "Design and valuation of debt contracts", *Review of Financial Studies*, **9**, 37-68.
- Anderson R. and S. Sundaresan (2000) "A comparative study of structural models of corporate bond yields: an exploratory investigation", *Journal of Banking and Finance*, **24**, 255-269.
- Black F. and J. Cox (1976) "Valuing corporate securities : some effects of bond indenture provisions", *Journal of Finance*, **31**, 351-367.
- Chakravarty S. and A. Sarkar (1999) "Liquidity in U.S. fixed-income markets : a comparison of the bid-ask spread in corporate, government and municipal bond markets", Staff Report, Federal Reserve Bank of New York.
- Duffee G. (1998) "The relation between Treasury yields and corporate bond yield spreads", *Journal of Finance*, **53**, 2225-2242.
- Duffee G. (1999) "Estimating the price of default risk", *Review of Financial Studies*, **12**, 197-226.
- Duffie D. and D. Lando (1997) "Term structure of credit spreads with incomplete accounting information", Working Paper, Stanford University.
- Duffie D. and K. Singleton (1999) "Modeling term structures of defaultable bonds", *Review of Financial Studies*, **12**, 687-720.
- Easley D. and M. O'Hara (1987) "Price, trade size and information in securities markets", *Journal of Financial Economics*, **19**, 60-90.
- Fabozzi F. and T. Fabozzi (1995) "The handbook of fixed income securities", Irwin, New York.
- Fisher L. (1959) "Determinants of the risk premiums on corporate bonds", *Journal of Political Economy*, **67**, 217-237.
- Glosten L. and P. Milgrom (1985) "Bid, ask and transaction prices in a specialist market with heterogeneously informed traders", *Journal of Financial Economics*, **14**, 71-100.

- Jarrow R. and S. Turnbull (1995) "Pricing derivatives on financial securities subject to credit risk", *Journal of Finance*, **50**, 53-85.
- Jones E., S. Mason and E. Rosenfeld (1984) "Contingent claims analysis of corporate capital structures : an empirical investigation", *Journal of Finance*, **39**, 611-627.
- Kempf A. and M. Uhrig (1997) "Liquidity and its impact on bond prices", Working Paper, Mannheim University.
- Lando D. (1998) "On Cox processes and credit risky securities", *Review of Derivatives Research*, **2**, 99-120.
- Longstaff F. (1994) "An analysis on non-JGB term structures", report for Credit Suisse First Boston.
- Longstaff F. (1995) "How much can marketability affect security values", *Journal of Finance*, **50**, 1167-1174.
- Longstaff F. and E. Schwartz (1995) "A simple approach to valuing risky fixed and floating rate debt", *Journal of Finance*, **50**, 789-819.
- Lotz C. (1997) "Locally minimizing the credit risk", Working Paper, London School of Economics.
- Mella-Barral P. and W. Perraudin (1997) "Strategic debt service", *Journal of Finance*, **52**, 531-556.
- Merton R. (1974) "On the pricing of corporate debt : the risk structure of interest rates", *Journal of Finance*, **29**, 449-470.
- Moraux F. and C. Villa (1999) "Market risk premium and structural risky debt pricing in incomplete market", mimeo, Université de Rennes.
- Nelson C. and A. Siegel (1987) "Parsimonious modelling of yield curves", *Journal of Business*, **60**, 473-489.
- Nielsen L., J. Saa-Requejo and P. Santa-Clara (1993) "Default risk and interest rate risk : the term structure of default spreads", Working Paper, INSEAD.
- Schulman J., M. Bayless and K. Price (1993) "Marketability and default influences on the yield premia of speculative-grade debt", *Financial Management*, **22**, 132-141.
- Schultz P. (1998) "Corporate bond trading costs and practices : a peek behind the curtain", Working Paper, University of Notre-Dame.
- Shimko D., N. Tejima and D. Van Deventer (1993) "The pricing of risky debt when interest rates are stochastic", *Journal of Fixed-Income*, **3**, 58-65.
- Sundaresan S. (1997) "Fixed income markets and their derivatives", South Western College Publishing.

- Tychon P. and V. Vannetelbosch (1997) "Debt valuation and marketability risk", Working Paper, Université Catholique de Louvain.
- Wagner H. (1996) "The pricing of bonds in bankruptcy and financial restructuring", *Journal of Fixed-Income*, **6**, 40-47.
- Wang D. (1999) "Pricing defaultable debt : some exact results" *International Journal of Theoretical and Applied Finance*, **2**, 95-100.
- Warga A. (1992) "Bond returns, liquidity and missing data", *Journal of Financial and Quantitative Analysis,* **27**, 605-617.