Co-ordination Failure and the Role of Banks in the Resolution of Financial Distress*

Spyros Pagratis
London School of Economics (FMG)

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Abstract

Despite a steady accumulation of empirical work, there has been no theoretical work attempting to shed light on the role of banks in facilitating public debt exchange offers when creditors face co-ordination problems. In this paper we develop a simple model of financial distress, consistent with institutional characteristics of out-of-court renegotiation of debt. We use an asymmetric, sequential-move global game framework. The model contains three sectors: a firm in financial distress, a well (though not perfectly) informed bank creditor and a diffuse set of small claimants to the firm that interact in an environment of asymmetric information about the firm's solvency condition. We show that when the bank accepts restructuring, it injects a degree of *strategic solidity* in the market and contract-revision offers become successful for lower levels of the firm's fundamentals than when the bank does not interact with other creditors. However, it is shown that making the bank's concession contingent on high minimum tendering rates undermines the positive information externality of its restructuring action.

1 Introduction

Empirical evidence suggests that banks play a potentially important role in facilitating the resolution of financial distress. Inspired by Corsetti et al (2001), one possible explanation for this is that the actions of large creditors, such as banks, simply allow small creditors to co-ordinate more efficiently. In particular, if a bank lender chooses to restructure, this may be taken by smaller, possibly

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less informed creditors such as public debt holders or suppliers, to imply that the going concern value of the firm, and thus the value of new claims offered, exceeds the liquidation value of the firm. In this way, we could argue that banks and other large well-informed creditors act to facilitate the resolution of financial distress by injecting a degree of *strategic solidity* in credit markets.

In the literature, there has been no theoretical work directed at examining this proposition. Rather, this has tended to concentrate mainly on firms' optimal choices between public and private debt, and the agency and other costs associated with diffused versus concentrated ownership of debt when the firm is *out* of financial distress. This is despite a steady accumulation of empirical work that has examined the role of banks in facilitating public debt exchange offers (out-of-court resolution of financial distress) when creditors face co-ordination problems and banks are assumed to own some proprietary, though not necessarily superior, information about the going concern value of the firm.

James (1995, 1996), for example, finds evidence consistent with the hypothesis that bank participation in debt restructuring transactions facilitates public debt exchange offers. In particular, he finds evidence that forgiveness of principal by banks induces public-debt holders to accept a debt exchange offer more easily and to reduce principal more aggressively[1]. Also the likelihood of achieving minimum tendering rates - which is a typical prerequisite in debt exchange offers - increases[2]. He further finds evidence that transactions in which banks forgive principal typically involve firms with more severe financial distress (e.g. higher leverage), which suggests that banks make concessions only when their claims are likely to be impaired. James (1996) also reports that, in all cases where banks make concessions, they make their offers contingent upon the successful completion of the public debt exchange offer.

Asquith, Gertner & Scharfstein (1994) analyse how financially distressed firms try to avoid bankruptcy through public/private debt restructuring, asset sales, mergers and capital expenditure reductions. Using a sample of companies with high-yield, *junk* bond issues with financial difficulties, they find evidence that the firm's debt structure affects the way financially distressed firms restructure their claims. In particular, a combination of secured private debt and numerous public debt issues seems to impede out-of-court restructuring and the firm's debt structure affects the way financially distressed firms restructure their claims.

In contrast to James (1996), AG&S find that banks almost never loosen financial constraints by forgiving principal, while loosening financial constraints does not reduce the probability of bankruptcy[3]. They argue, however, that their sample is very specific as it focuses on the high-yield bond market, and

[1] In fifteen debt restructuring transactions where banks took no action the average reduction in public debt was 19%, while in 14 cases where the bank reduced principle the average reduction in public debt was 56%.

[2] In all cases where banks offer to scale down their loans actual tendering rates are above the minimum specified for success compared to 30% when banks do not make concessions.

[3] 59% of firms whose banks loosen financial constraints still went bankrupt vs. 68% of the firms whose banks tighten the constraints, though there are differences in restructuring periods until bankruptcy.

the results should not be generalised.

Gilson, John and Lang (1990) find evidence that the likelihood of out-of-court debt restructuring is positively related to the firm's reliance on bank debt.

In the theoretical front, Bolton and Freixas (2000) discuss a model of corporate finance where both supply and demand influence the availability of finance within an equilibrium set-up with asymmetric information. They argue that banks can help firms in times of distress because they can exploit their superior information/borrower screening skills. In addition, an important feature of their model is banks' ability to securitise senior portions of *rescue* finance they extend to firms in distress (e.g. in a *debtor-in-pocession* situation) and avoid the incentive to liquidate inefficiently a firm in financial distress. In equilibrium, banks choose to increase their supply of loans, provided that they can price effectively for the extra risk and they are not capital constrained. That way bank loans may substitute for other forms of finance and facilitate the resolution of financial distress.

Diamond (1993) argues that, because bank lenders are generally secured, they have little incentive to make concessions. Gertner and Scharfstein (1991) provide a model that illustrates how bank participation in the restructuring transaction can mitigate *holdout* problems among public debt-holders. In doing so, however, they assume common knowledge about the firm's economic fundamentals which allows perfect co-ordination of creditors' actions.

Jaffee and Shleifer (1990) consider a model where they examine how investment banks *protect* firms from financial distress due to self-fulfilling failure of calls of convertible bonds. They provide an analogy to Diamond and Dybvig's (1983) bank runs model by arguing that, by underwriting the *forced conversion* of convertible bonds, investment banks essentially provide insurance (a put option) to the firm in the same way that deposit insurance provides protection against bank runs.

Yet Jaffee and Shleifer assume that the economic fundamentals of the firm i.e. the value of firm's assets, is common knowledge among creditors and there is no uncertainty about equilibrium behaviour of creditors. This allows perfect co-ordination of creditors' actions and results in multiple Nash equilibria[4]. Moreover, it implies that there is only risk shifting from the firm to the bank and there is no information content in an investment bank's action to accept the underwriting. It is exactly that information content that is central to our analysis.

Giammarino (1989) models the resolution of financial distress under Chapter 11 proceedings as a non-cooperative game of incomplete information played by a firm and a single creditor. He considers a model of financial distress of a firm with equity entirely owned by a single risk-neutral individual and debt outstanding which is entirely owned by a perfectly co-ordinated group of risk-neutral debt-holders. He shows that, despite the possibility of costless, out-of-court reorganisation, it may be rational for firms to incur significant financial

[4] For a discussion on this issue see Morris and Shin (2000)

costs in the resolution of financial distress due to the existence of asymmetric information and judicial discretion.

In this paper, we develop a simple model of financial distress which is consistent with the institutional characteristics of out-of-court renegotiation of a firm's contractual obligations (e.g. debt exchange offers). The framework we adopt is an asymmetric, sequential-move global game.

A firm, a bank creditor and a continuum of small claimants to the firm interact in an environment of asymmetric information about the firm's solvency condition. We investigate the extent to which acceptance by a well-informed bank creditor to commit further to a financially distressed firm (e.g. via a new loan) facilitates contract revision offers made by the firm to a diffuse set of claimants (e.g. public-debt holders, suppliers etc.). In our model the bank is a *large* creditor by virtue of its non-negligible financial size. Although the size of the bank may be very small, compared to the balance sheet of the financially distressed firm, and insufficient to *manufacture* a bail-out of the firm, it is not of measure zero.

Throughout this paper, we assume that the bank creditor has an information advantage over other creditors. This is consistent with the literature on the importance of banks' monitoring abilities and how banks might get access to better information compared to other types of creditors. That literature includes, among others, Bolton and Freixas (1998, 2000), Holstrom and Tirole (1997), Houston and James (1996), Rajan (1992), Lummer and McConnell (1989), James (1987), Fama (1985), to mention just a few.

In particular, we assume that the relative precision of the information that is possessed by a bank relative to that of other creditors tends to infinity. That assumption might seem over-restrictive and it might also be considered as eliminating any practical relevance of our results. This is because, in the real-world, the relative precision of information among different parties may be difficult, if not impossible, to evaluate. However, that assumption is without loss of generality and it is solely imposed in order to facilitate the analysis and allow the derivation of a closed form solution. In light of Corsetti et al (2001), it can be shown numerically that the direction of our results is robust to any level of relative precision of agents' private information. In other words, assuming that bank creditors have (infinitely) more accurate information than other creditors is *not* a prerequisite for claiming that banks facilitate the resolution of financial distress and inject a degree of *strategic solidity* in credit markets.

In fact, what drives our results and allows us to argue in favour of such a claim is that public debt exchange offers exhibit *full strategic complementarities.* That is, the expected payoff of an agent who accepts a tender offer increases with the proportion of other agents that co-ordinate in the same direction. This allows us to use the methods of Carlsson and van Damme (1993) on *global games* and to focus on *trigger strategies*, which, under full strategic complementarities, can be shown to be the only *dominant solvable* equilibrium strategies.

The intuition that underlies our analysis is that, in large-scale debt renegotiations, which typically involve the restructuring of both public and private claims, the actions taken by banks are usually observable and a bank's response to a debt renegotiation offer might influence, to a greater or lesser extent, the equilibrium strategies of other creditors.

This is, firstly, due to bank's non-zero financial mass and the non-negligible amount of funds it is able to inject/rollover. But it is also because a bank's action is expected to affect, through Bayesian updating, the beliefs of other creditors about the outcome of the debt renegotiation and the fundamentals of the financially distressed entity. Finally, a bank, by accepting a restructuring offer conditional upon acceptance of the debt exchange offer by a minimum proportion of public-debt-holders (*minimum tendering rate*), it may allow both itself and other creditors to make better informed decisions. This is, conditional offers may permit aggregation of individual information and allow individual creditors to base their actions on the *collective knowledge* of other creditors.

Our results are consistent with empirical evidence and show that acceptance by a bank to commit more funds to the financially distressed firm facilitates contract revision. In particular, contract revision offers become *successful* at lower values of the firm's fundamentals compared to the situation where the bank has no role in the debt restructuring. This implies, lower deadweight costs of inefficient liquidation. Yet it can be shown that rejection of the offer by the bank results in liquidation of the firm at higher levels of the firm's fundamentals compared to the situation where the bank is out of the game.

Moreover, the analysis suggests that a bank's concession to commit more funds to a financially distressed firm may even exacerbate the *run* by small claimants should that acceptance be made contingent upon acceptance of the contract revision offer by a *high* proportion of claimants. This implies that *excessive conditionality*, in the form of *high* minimum tendering rates, undermines the positive information externality of bank's acceptance and the signaling effect of a bank's action may become at best irrelevant to the decisions by small claimants.

Throughout the paper we adopt a fairly generic characterisation of the financially distressed entity and we do not make any specific reference to the ownership structure of that entity, the role for equity capital, or potential conflict of interests between shareholders and bondholders in the spirit of Jewnsen and Meckling (1976). Thus, our analysis by-passes possible conflicts of interest between different classes of security holders and concentrates on the workouts of financial distress, the potential inefficiencies that may arise from creditors' co-ordination problems and how those inefficiencies can be alleviated via the appropriate involvement of a bank creditor.

A consequence of that is the analysis not to permit the simultaneous treatment of both the co-ordination problems among creditors, when the firm is *in* financial distress, and the potential moral-hazard problems associated with the large-creditor/debtor relationship, when the debtor is *out* of financial distress. This could be a subject of future research.

However, the generic characterisation of the firm's balance sheet allows us to add some thoughts that stretch beyond the resolution of financial distress in the corporate sector and relate to the resolution of international financial crisis. In particular, we could draw a parallel between the balance sheet of the financially distressed firm in our model, and the capital account of a country during the onset of a financial crisis, We could then discuss the implications of our findings on the *doctrine of catalytic finance* and the scope and rationale for IMF lending.

In September 2003, for example, the Brazilian government has authorised the negotiation of a new, one-year deal with the IMF. The government's Treasury secretary, Joaquim Levy, was then quoted as saying[5]

> *...Obviously, our objective is to walk alone and not depend on the fund. But a one-year renewal could be an "important mechanism of information" to investors who did not follow Brazil's progress closely.*

The above statement, other than being striking given the strong criticism of the IMF by president da Silva for more than twenty years in opposition, it suggests that there is something more than money in the involvement of a large, informed creditor in the resolution of financial distress. This is, such a creditor's involvement may act as *mechanism of information* that allows less informed – and possibly *small* – creditors to co-ordinate better in the right direction. This is consistent with the findings of this paper that large creditors may act as *gate keepers* to the system and, should debtors' fundamentals justify it, inject a degree of *strategic solidity* among other creditors.

We begin by presenting the model in section 2. The solution proceeds in steps in sections 3, 4 and 5. We conclude in section 6 by summarizing our results and by adding some thoughts on the possible implications of our findings on the debate about the role of catalytic finance in the resolution of international financial crisis.

2 The Model

We consider a three-period setting $\{\tau = 0, 1, 2\}$ in which a firm with a risky project, a large creditor and a continuum of small claimants (suppliers) to the firm interact in an environment of asymmetric information. To model strategic interactions among agents we use the methods of Carlsson and van Damme (1993) on *global games* as applied by Morris and Shin (2000, 2001), Rochet and Vives (2001) and Goldstein and Pauzner (2000).

Instead of focusing explicitly on debt exchange offers to a diffuse set of public-debt holders, we consider the case where a set of asymmetrically informed suppliers to the firm are asked, through a take-it-or-leave-it offer, to make concessions about the timing of their payment and the delivery of inputs to the firm. This assumption is without loss of generality and, as it will become clear latter, it is made solely in order to simplify agents' payoff functions.

[5] FT, September 12, 2003.

2.1 Agents

There are three types of risk-neutral agents: the firm's owners (the firm) that run a risky project, the firm's banker (the bank), whose financial resources are limited, and a continuum of firm's non-equity stakeholders (suppliers). At date $\tau = 0$ the firm has equity capital E and long-term secured bank debt (loan) with face value B and maturity at $\tau = 2$. The firm also signs identical contracts with the suppliers that promise to deliver inputs for the project at date $\tau = 1$[6]. Inputs are project-specific and if suppliers will not deliver at date $\tau = 1$ they have to sell the inputs at a discount elsewhere. We also assume that at $\tau = 1$ the firm has a number of obligations to other parties (e.g. employes) of total amount equal to C.

2.2 Investment

Initial investments are made at date $\tau = 0$ when the bank loan (B) and equity capital (E) are used to finance firm's liquid asset reserves (L) and firm's investment (I_0) in an illiquid risky project. At date $\tau = 0$ the firm also places orders to the suppliers for an aggregate quantity Q of inputs with payment taking place upon delivery at $\tau = 1$. Firm and suppliers agree on a price per unit of inputs produced equal to unity[7].

Bank-debt is held by a *large* well-informed bank while supply contracts are signed by a diffuse set of small, poorly informed suppliers. The bank is a large creditor by virtue of the face value of the loans it extends to the firm compared with the individual credit lines extended by non-equity stakeholders which individually are considered negligible as a proportion of the whole (i.e. of measure zero).

At date $\tau = 1$ the firm requires a minimum quantity of new inputs rQ (where, $0 < r < 1$) in order for the project to reach its final stage and generate a return (R) at date $\tau = 2$. Otherwise the project must be abandoned and the firm is liquidated.

The minimum proportion of inputs (r) that has to be delivered in order for the project to continue could be interpreted as the *minimum tendering rate* in a debt exchange offer if instead of suppliers we were considering a continuum of public debt holders (e.g. short-term commercial paper investors). The quantity $(1 - r)$ could also be regarded as the maximum contraction of firm's operations before the firm becomes unable to operate as a going concern.

Moreover, at $\tau = 1$ the firm needs an amount of cash (C) in order to cover a number of necessary operating expenses (e.g. labour costs). Failure to meet those obligations at $\tau = 1$ would result in severe disruption of firm's operations, abandonment of the project and liquidation.

We distinguish between *insolvency* and *illiquidity* by defining solvency in

[6] As in Berlin and Saunders (1996) our setting assumes suppliers cannot be paid up front.

[7] Although we do not intend to derive explicitly the optimal loan level and quantity of inputs agreed to be supplied, the liability structure of the firm allows us to capture strategic interactions between the claimants of the firm.

terms of firm's ability to meet its contractual obligations at the final date $(\tau = 2)$ out of project's payoff. We adopt the following definition:

Definition 1 *At date $\tau = 1$ the firm is considered to be solvent if and only if it is considered capable of meeting all its contractual obligations (i.e. both to the bank and to its diffused set of claimants) at the final date* $(\tau = 2)$.

Now, let (L) be the book value of the firm's liquid assets at date $\tau = 0$. This implies the following accounting identity.

$$I_0 + L = B + E$$

As of date $\tau = 0$, the liquidation value $\left(\widetilde{L}\right)$ of firm's liquid assets at the intermediate date $(\tau = 1)$ is a random variable with the following probability distribution:

$$\widetilde{L} = \begin{cases} L_H & w.p. \quad p \\ L_L & w.p. \quad 1-p \end{cases}$$

where, subscripts H and L stand for *high* and *low* respectively and $L_H > Q + C$ and $L_L \leq Q + C$. In other words, $1 - p$ is the ex-ante probability of financial distress at the intermediate date[8]. In this model liquidity shocks are considered exogenous and relate to the marketability of firm's liquid assets, for example due to general market conditions.

At date $\tau = 1$ the firm has to pay its suppliers (Q) and cover its operating expenses (C). Firm's liquid asset reserves can then be used as a means for payment. The liquidation value of the project is small relative to the size of the firm's balance sheet and we normalise it to zero. Moreover, in case of liquidation of the firm we assume that priority rules are enforced for secured lenders.

At date $\tau = 1$, supply contracts may be cancelled (foreclosed) by suppliers at a cost (c) should the firm claim that it is *unable* to pay the initially agreed price per unit of supplied inputs[9]. This formulation is in line with Berlin and Saunders (1996) who consider a perfectly co-ordinated set of suppliers (i.e. a representative supplier) that may choose, at the intermediate date, to terminate a supply relationship that has been established with a firm at a previous date. Berlin and Saunders assume that if the supply relationship is severed, the supplier's next best market is less profitable than if no such supply relationship had been established.

[8] Assuming common knowledge of the parameters, a necessary condition for investment to take place at the initial date is:

$$pL_H + (1-p)\,L_L > Q + C$$

[9] We proceed by assuming that the claim, by the firm, that it faces liquidity problems is truthful and reveals the fact that $L = L_L$. In other words, there is no gaming from the firm in order to extract value from its creditors.

3 The Problem

We impose the following structure on the problem. At date $\tau = 1$ the firm is in financial distress (i.e. $L = L_L$) and has not enough resources to pay in full its suppliers (Q) and to cover its operating costs (C). Moreover, we assume that the liquidation value of firm's financial slack is not even enough to repay in full its debt to the bank (B) in case of *acceleration* of debt at $\tau = 1$ (i.e. $L_L < B$). For notational convenience and without loss of generality we set $L_L = 0$. The firm is also unable to raise money by selling new securities to outside investors.

Yet, the firm needs at least proportion r of the aggregate input quantity (Q) and an amount of cash equal C in order to meet its operating costs and continue with the project until the final period. In order to avoid liquidation and pursue a value enhancing project the firm requests the bank to provide a capital injection $\left(B^1 = C\right)$ in the form of senior unsecured loan. Should the bank agree to provide the new loan the firm also has to offer a new contract to its suppliers in exchange of the old one. The exchange offer should allow the firm to receive the necessary amount of inputs in order to carry on with the project

3.1 The Debt Restructuring Offer

The debt restructuring offer by the firm takes the form of a take-it-or-leave-it offer. The offers to suppliers are identical and provide the delivery of inputs at date $\tau = 1$ in exchange of an unsecured debt claim to the firm payable at $\tau = 2$ (e.g. bill of trade, promissory note etc.) for every unit of inputs delivered. The debt claims are payable at $\tau = 2$ and each one has a face value equal to α_s. In order to deal with *hold out* problems we assume that $\alpha_s > 1$.

Definition 2 *The renegotiation of supply contacts is considered successful if and only if at least proportion r of firm's suppliers accept to deliver at $\tau = 1$ in exchange of unsecured debt claims payable at $\tau = 2$.*

We assume that suppliers' responses to the contract revision offer are pooled together and inputs are released in exchange of debt contracts only if the renegotiation of supply contracts is *successful*[10].

Moreover, in line with empirical evidence (James (1996)), we assume that acceptance by the bank to extend new credit is made contingent upon *successful* completion of the renegotiation of supply contracts. In other words, a necessary condition for the bank to extend the new loan and for *tendering* suppliers to deliver the inputs is that a minimum proportion (r) of suppliers accept the new contract terms.

[10] An alternative interpretation of the contract revision offer is to consider it a debt exchange offer to a diffuse set of public debt holders with minimum tendering rate r. The Trust Indenture Act of 1939 prohibits any change in the timing or amount of public debt payments and forces public debt restructurings to take the form of exchange offers. Under debt exchange offers firms offer new claims to debt holders that accept to tender with the offer typically made contingent on the acceptance of a minimum proportion of the public debt (see, for example, Gertner and Scharfstein (1991)).

3.2 Bank's Payoff Function

At date $\tau = 1$ the firm has no collateral to offer but both the old and the new loan to the firm rank first in the firm's capital structure. Yet, if the bank rejects the offer then the firm will be liquidated immediately (e.g. under Chapter 7 proceedings). In that case, the seniority of the old claim to the firm is of no value given that the claim is severely impaired (actually is worthless) due to zero liquidation value of the firm at $\tau = 1$. Given also that bank's agreement to extend new credit to the firm is made conditional upon successful completion of the exchange offer to suppliers, the loss that the bank will incur if default takes place at $\tau = 1$ is limited only to the old loan amount (B). In case of default at $\tau = 2$ the bank has the first claim on what the project has generated up to the total loan amount $(B + C)$. Yet, if there is no default at all, the bank fully recovers both the new and the old loan amount $(B + C)$. The following table summarises the bank's loss function under different scenarios:

Bank	*Default at* $\tau = 1$	*Default at* $\tau = 2$	*No Default*
Accept	-B	-$LGD \times (B + C)$	0
Reject	-B	-	-

where, LGD is the loss-given-default (e.g. internal-systems-based) associated with the situation where there is default at $\tau = 2$[11]. For convenience we assume that -$LGD \times (B + C) < -B$, or that $LGD > \frac{B}{B+C}$. This assumption intends to capture the non-trivial nature of bank's commitment to extend new credit at $\tau = 1$. This is, the amount of new credit C is not negligible compared with the original amount B. Moreover, banks usually claim that it is their policy when they extend credit to make sure that the firm is solvent. In other words, the provision of extra security (i.e. enhanced seniority, collateral etc.) other than affecting the terms of lending it is not the driving force behind bank's decision to extend credit or not. As a result, it would be conceptually wrong, on an ex-ante basis, to relate explicitly the bank's payoff in case of default at $\tau = 2$ to the firm's liquidation value. This would obstruct us from the original objective which is to capture the effect of bank's belief about the *solvency* status of the firm on small claimants' actions. Furthermore, it would computationally burden our analysis making it very specific to distributional assumptions about agents' signals[12].

3.3 Suppliers' Contingent Payoffs

We assume that inputs are project-specific and if suppliers choose not to deliver at date $\tau = 1$ they have to sell the inputs at a discount (c) elsewhere[13].

[11] The use of a fixed LGD is consistent with the *foundations internal-ratings-based* IRB approach that has been proposed by the Basel Committee on Banking Supervision. The IRB approach requires banks to assign a fixed LGD figure to particular classes of credit exposures.

[12] Even the uniqueness of trigger strategy equilibrium could be lost.

[13] We use that assumption in order to avoid the complication of explicitly building the term structure of credit spreads into the model or arbitrarily assume a gross rate of return $(r > 1)$

Rochet and Vives (2001) use a similar formulation where they interpret a fixed foreclosure cost (c) as a reputation cost of fund managers due to *bad judgement*. Such an interpretation would be applicable to our model should instead of a continuum of suppliers we would assume a continuum of unsecured creditors (e.g. short-term commercial paper investors).

We also assume that, on an ex-ante basis, suppliers expect to receive a small fixed payoff (l) in case of default and liquidation of the firm at $\tau = 2$. For simplicity we set that contingent payoff equal to zero[14]. This assumption is without loss of generality, although one could argue that suppliers' expected pay-off conditional on default at $\tau = 2$ should be determined endogenously as a function of the proportion of suppliers that accept the offer. Yet, this would bring undue complication in the model given that what we intend to capture is suppliers' incentives to avoid the cost of not selling their inputs to the monopsonist firm, or of extending credit to an insolvent firm. This assumption is also consistent with empirical evidence. White (1983), for example, observes that unsecured creditors receive little or no payoff in liquidation[15]. He also argues that some unsecured claims such as trade creditor claims are generally not covered by subordination agreements and rank at the bottom of the seniority ranking.

If there is no default both at $\tau = 1$ and $\tau = 2$, suppliers not only recover the originally contracted amount per unit of inputs supplied, but also a premium ($i.e.\alpha_s - 1$)above that amount. Given the above, suppliers' loss function looks as follows:

Suppliers	*Default at* $\tau = 1$	*Default at* $\tau = 2$	*No Default*
Accept Offer	-c	-*1*	$\alpha_s - 1$
Reject Offer	-c	-c	-c

Where, $0 < c \leq 1$ and $-c < 0 < \alpha_s - 1$ because of $\alpha_s > 1$.

3.4 Information

At date $\tau = 0$, the minimum proportion of required inputs (r), the probability distribution of firm's liquid assets at the intermediate date, the level of bank debt (B) and the aggregate claims by firm's suppliers (Q) as well as the level of operating expenses (C) and the cost (c) of selling the goods in the outside market are common knowledge among agents. We also assume that, as of date $\tau = 0$, the return (R) of the firm's risky project has an improper prior distribution[16].

at $\tau = 1$ for every dollar of credit extended by suppliers to the firm at $\tau = 1$.

[14] In a similar setting, Rochet and Vives (2001) use a payoff equal to zero assuming that this is what a fund manager would get for rollovering a credit exposure to an entity that has subsequently defaulted.

[15] In a sample of 178 liquidated firms White (1983) finds that the average payoff rates to unsecured creditors is approximately 2.5%. Nevertheless, for firms reorganising under Chapter 11 proceedings the payoff rates are above 32%.

[16] Improper priors allow the analysis to focus exclusively on agents' updated beliefs conditional on their private signals, without taking into account the information contained in the

At $\tau = 1$, creditors receive private noisy signals about the return of the firm's risky project. Those signals constitute the only information available to creditors about the economic value of firm's investment. Let y be the signal observed by the bank, which is of the following form:

$$y = R + \nu\varepsilon$$

where, $\nu > 0$ is a constant and ε is a normal random variable with zero measure and unit variance, density function $g(\cdot)$ and is independent of R. We denote by $G(\cdot)$ the cumulative distribution function of $g(\cdot)$. Also at $\tau = 1$ each supplier i privately observes the following signal:

$$x_i = R + \sigma\varepsilon_i$$

where, $\sigma > 0$ is a constant and $\{\varepsilon_i\}$ are independent, identically distributed normal random variables with zero measure, unit variance and density function denoted by $(f(\cdot))$. They are also independent of ε. We denote by $F(\cdot)$ the cumulative distribution function of $f(\cdot)$.

At $\tau = 1$, the bank moves first and decides whether to increase its leverage to the firm. It does so conditional on its private signal (y) and taking into account the effect of its action on suppliers' behaviour . Suppliers then decide unilaterally whether to extend credit to the firm by delivering their goods at $\tau = 1$ for payment at $\tau = 2$. Their actions are conditional upon their private signals (x_i) and the commonly observed action by the bank to extend new credit to the firm or not. We consider the following definition:

Definition 3 *A supplier's and bank's strategy is a rule of action that maps each realization of its signal to one of two actions: to extend credit to the firm by accepting the offer, or to reject the offer.*

Suppliers strategies are determined at equilibrium by balancing the benefit of a particular strategy against the opportunity cost of that strategy, taking into account strategic complementarities. Given that individually they are unable to influence the solvency of the firm, suppliers fail to account for the effect of their individual decisions on the completion of firm's project. Yet, they are able to account for the effect of their actions as a whole. Thus, they foreclose whenever the expected benefit $(1 - c)$ of doing so is higher than the expected benefit of extending credit to the firm via the new contract:

Similarly, the bank accepts to provide new credit to the firm if the total amount it expects to lose from doing so is less than the old loan (B) that it will definitely lose if it will reject the offer.

Let us suppose that suppliers and the bank follow trigger strategies around critical signal levels x^* and y^* respectively. In case where x^* and y^* are uniquely

prior distribution. In any case, our results with the improper prior can be seen as the limiting case as the information in the prior density goes to zero. See Hartigan (1983) for a discussion of improper priors, and Morris and Shin (2000) for a discussion of the latter point.

determined it can be shown that there is a unique, dominance solvable equilibrium where suppliers and the bank follow their respective trigger strategies around x^* and y^*[17].

4 Suppliers' Equilibrium in Trigger Strategies

Let (R^*) be the critical level of actual investment return below which proportion of suppliers higher than $(1-r)$ rejects firm's proposal. We first prove the following two lemmas.

Lemma 1 *Given signal x_i, the critical level of investment return (R^*) below which rejection by suppliers generates default, is: $R^* = x^* - \sigma F^{-1}(1-r)$.*
Proof. *See Appendix.* ■

Lemma 2 *Provided there is no default at the interim date and conditional on signal $x_i = x^*$, supplier i's belief about the proportion (l^*) of other suppliers that receive a signal lower than his is: $l^* = \frac{(1-r)}{2}$.*
Proof. *See Appendix.* ■

Without apology, both for this section and the rest of the paper, we have assumed that the realised sample distribution of suppliers is always the common distribution of suppliers' signals[18].

We are ready now to solve for suppliers' equilibrium in trigger strategies.

4.1 Suppliers' equilibrium

Conditional on bank's acceptance to extend new credit to the firm, the critical value (x^*) of suppliers' $\{i\}$ signal solves the following equation:

$$-c\Pr(R < R^* \mid x_i = x^*, y > y^*) -$$

$$-\Pr(R^* < R < B + C + a_s(1-l^*)Q \mid x_i = x^*, y > y^*) +$$

$$(a_s - 1)\Pr(R > B + C + a_s(1-l^*)Q \mid x_i = x^*, y > y^*) = -c \qquad (1)$$

By setting $R^* = x^* - \sigma F^{-1}(1-r)$, $l^* = \frac{(1-r)}{2}$ and expressing R in terms of x_i, the critical signal level x^* solves the following equation:

$$-c\frac{\Pr\left(\varepsilon_i > F^{-1}(1-r), \varepsilon_i - \frac{v}{\sigma}\varepsilon < \frac{x^*-y^*}{\sigma}\right)}{\Pr\left(\varepsilon_i - \frac{v}{\sigma}\varepsilon < \frac{x^*-y^*}{\sigma}\right)} -$$

[17] See, for example, Corsetti, C., Dasgupta, A., Morris, S., and H. S. Shin (2001).

[18] In lemma (2) for example, one could derive any proportion of suppliers between zero and $(1-r)$ depending on how he extends the Lebesgue measure.

$$-\frac{\Pr\left(\frac{x^*-B-C-a_s\frac{(1+r)}{2}Q}{\sigma}<\varepsilon_i<F^{-1}(1-r),\varepsilon_i-\frac{v}{\sigma}\varepsilon<\frac{x^*-y^*}{\sigma}\right)}{\Pr\left(\varepsilon_i-\frac{v}{\sigma}\varepsilon<\frac{x^*-y^*}{\sigma}\right)}+$$

$$+(a_s-1)\frac{\Pr\left(\varepsilon_i<\frac{x^*-B-C-a_s\frac{(1+r)}{2}Q}{\sigma},\varepsilon_i-\frac{v}{\sigma}\varepsilon<\frac{x^*-y^*}{\sigma}\right)}{\Pr\left(\varepsilon_i-\frac{v}{\sigma}\varepsilon<\frac{x^*-y^*}{\sigma}\right)}=-c \qquad (2)$$

For $\frac{v}{\sigma}\to 0$, equation (2) simplifies as follows:

$$-c\frac{\Pr\left(\varepsilon_i>F^{-1}(1-r),\varepsilon_i<\frac{x^*-y^*}{\sigma}\right)}{\Pr\left(\varepsilon_i<\frac{x^*-y^*}{\sigma}\right)}-$$

$$-\frac{\Pr\left(\frac{x^*-B-C-a_s\frac{(1+r)}{2}Q}{\sigma}<\varepsilon_i<F^{-1}(1-r),\varepsilon_i<\frac{x^*-y^*}{\sigma}\right)}{\Pr\left(\varepsilon_i<\frac{x^*-y^*}{\sigma}\right)}+$$

$$+(a_s-1)\frac{\Pr\left(\varepsilon_i<\frac{x^*-B-C-a_s\frac{(1+r)}{2}Q}{\sigma},\varepsilon_i<\frac{x^*-y^*}{\sigma}\right)}{\Pr\left(\varepsilon_i<\frac{x^*-y^*}{\sigma}\right)}=-c \qquad (3)$$

We consider two cases: 1) $F^{-1}(1-r)<\frac{(x^*-y^*)}{\sigma}$ 2) $F^{-1}(1-r)>\frac{(x^*-y^*)}{\sigma}$.

Proposition 1 *For $F^{-1}(1-r)<\frac{(x^*-y^*)}{\sigma}$ the critical level of small claimants' signal (x^*) when they observe bank's action is given by:*

$$x^*=(B+C)+a_sQ\left(\frac{1+r}{2}\right)+\sigma F^{-1}\left[\frac{(1-r)(1-c)}{a_s}\right]$$

Proof. *See Appendix* ■

Proposition 2 *When small claimants observe bank's action and for $F^{-1}(1-r)>\frac{(x^*-y^*)}{\sigma}$ the critical level of their signal (x^*) solves the following equation:*

$$a_sF\left(\frac{x^*-B-C-a_s\frac{(1+r)}{2}Q}{\sigma}\right)=(1-c)F\left(\frac{x^*-y^*}{\sigma}\right)$$

Proof. *See Appendix* ■

We now turn to solve for bank's equilibrium in trigger strategies.

5 Bank's Equilibrium in Trigger Strategies

Conditional on signal y, return (R) is normally distributed with mean y and standard deviation v. We consider the following lemma:

Lemma 3 *Provided there is no default at $\tau = 1$ and conditional on bank's signal being $y = y^*$, bank's belief $\left(l^b\right)$ about the proportion of suppliers that reject firm's offer at $\tau = 1$ is given by:*

$$l^b = \frac{\int_{\frac{x^*-y^*-\sigma F^{-1}(1-r)}{v}}^{+\infty} F\left(\frac{x^*-y^*-vw}{\sigma}\right) f(w)\, dw}{F\left[\frac{y^*-x^*}{v} + \frac{\sigma}{v} F^{-1}(1-r)\right]}$$

For $\frac{v}{\sigma} \to 0$, $l^b = F\left(\frac{x^*-y^*}{\sigma}\right)$.

Proof. *See Appendix* ■

We now turn to solve for the bank's equilibrium in trigger strategies. We focus on the limiting case where $\frac{v}{\sigma} \to 0$.

5.1 Bank's equilibrium

Given that rejection by the bank would result in default by the firm and the bank would lose the original loan amount (B), bank's trigger point (y^*) solves the following equation:

$$-B \Pr\left(R < R^* \mid y = y^*\right) - D \Pr\left(0 < R < B + C + \alpha_s \left(1 - l^b\right) Q\right) = -B \quad (4)$$

where, $D \equiv LGD \times (B + C)$.

The first term on the LHS captures the conditional feature of banks acceptance; should the bank agrees to provide a new loan (C) but proportion of suppliers higher than the critical level $(1 - r)$ rejects the offer there is default but the bank loses only the original amount (B). The second term captures the bank loss at the 'bad' situation where the bank losses more than B due to default at $\tau = 2$ and the additional loan it extended $\tau = 2$. In case of no-default at $\tau = 2$ the bank loses nothing. Should the bank refuse to extend new loan then it bears a loss of $-B$, which is the term in the RHS.

Substituting R^* from lemma (1) into equation (4) and expressing R in terms of bank's signal (y) we get the following equation:

$$-B \Pr\left(\varepsilon > \frac{y^* - x^*}{v} + \frac{\sigma}{v} F^{-1}(1-r)\right) -$$

$$-D \Pr\left(\frac{y^* - B - C - a_s\left(1 - l^b\right) Q}{v} < \varepsilon < \frac{y^*}{v}\right) = -B \quad (5)$$

The critical signal levels x^* and y^* are found by solving simultaneously equations (2) and (5). Obviously neither of these equations can be solved in closed form in the general case, though they can be solved numerically. We consider, however, the limiting case where $\frac{v}{\sigma} \longrightarrow 0$. In that case, equation (5) becomes as follows:

$$F\left(\frac{y^*}{v}\right) - F\left(\frac{y^* - B - C - a_s\left(1 - l^b\right)Q}{v}\right) = \frac{B}{D} \tag{6}$$

where, from lemma (3), $1 - l^b = F\left(\frac{y^* - x^*}{\sigma}\right)$.

5.2 Non observability of Bank's Action

Suppose now that, *ceteris paribus,* there is no signalling effect in bank's action at $\tau = 1$. We may think of it as a situation where at date $\tau = 0$ the bank extends a senior and unsecured loan to the firm of total amount $B^{'} = B + C$ for repayment at $\tau = 2$, instead of $\tau = 1$, and the fixed operating cost is incurred at $\tau = 0$ (e.g. labour and other costs are paid up front). In that case, and similar to the above scenario where the bank provides a new loan at the intermediate date, suppliers who agree to extend credit to the firm will receive claims junior to the bank loan $\left(B^{'}\right)$.

We assume, as previously, that suppliers follow trigger strategies around critical signal level (x^{**}). There is no reason to expect x^{**} to be the same as x^* given than now suppliers are not able to learn from the action of the bank. Our objective is to compare x^{**} with x^*. The critical signal level (x^{**}) in that case solves the following equation:

$$-c\Pr\left(R < R^{**} \mid x_i = x^{**}\right) -$$

$$-\Pr\left(R^{**} < R < B' + a_s Q\left(1 - l^{**}\right) \mid x_i = x^{**}\right) +$$

$$\left(a_s - 1\right)\Pr\left(R > B' + a_s Q\left(1 - l^{**}\right) \mid x_i = x^{**}\right) = -c \tag{7}$$

where, $B' = B + C$, R^{**} is defined as in lemma (1) and l^{**} is defined as in lemma (2).

Proposition 3 *When there is no learning from bank's action at the intermediate date and ceteris paribus, the critical level of small claimants' signal* (x^{**}) *is given by:* $x^{**} = (B + C) + a_s Q\left(\frac{1+r}{2}\right) + \sigma F^{-1}\left[\frac{(1-r)(1-c)}{a_s}\right]$

Proof. *See Appendix.* ■

From proposition (3) it becomes obvious that the critical signal level x^{**} is increasing in $(B + C)$, Q (leverage factors) and r, though decreasing in c. The effects of a_s and σ are ambiguous. We are now ready to prove the main two propositions of our paper:

Proposition 4 *For values of the minimum tendering rate* (r) *sufficiently small so as* $F^{-1}(1-r) > \frac{x^*-y^*}{\sigma}$, *and conditional on observability of bank's actions, acceptance by the bank to commit more funds to the financially distressed firm facilitates contract revision offers. In particular, contract revision offers become successful at lower values of firm's fundamentals* (R^*) *compared to the situation where bank's actions are unobservable* (R^{**}).

Proof. *See Appendix* ■

Surprisingly, however, proposition (4) does not hold for *large* minimum tendering rates (r), i.e. for tendering rates such that $F^{-1}(1-r) < \frac{x^*-y^*}{\sigma}$. Intuitively, this should be attributed to the conditional character of bank's offer. High values of the minimum tendering rate (r) effectively *destroy* any information conveyed in bank's action to (conditionally) accept to commit more funds to the financially distressed firm[19]. This leads to the following proposition:

Proposition 5 *For (very) high values of the minimum tendering rate* (r) *(i.e.* $F^{-1}(1-r) < \frac{x^*-y^*}{\sigma}$*) the information content of bank's acceptance to (conditionally) commit more funds to the financially distressed firm is totally destroyed by the conditional character of bank's acceptance.*

6 Concluding Remarks

In this paper, we developed a simple model of financial distress consistent with the institutional characteristics of out-of-court renegotiation of a firm's contractual obligations. We investigated the extent to which acceptance by a well informed bank creditor to commit further to the financially distressed firm (i.e. via a new loan) facilitates contract revision offers that are made by the firm to a diffuse set of claimants.

Our results are consistent with empirical evidence, which suggests that banks play a potentially important role in facilitating the resolution of financial distress. In particular, when a bank participates in the debt restructuring, contract revision offers become successful at lower values of the firm's fundamentals compared to the situation where the bank has no role in the restructuring. In that sense, participation by a bank to a debt restructuring reduces the extent of inefficient liquidation due to potential co-ordination problems among creditors. This is proved in proposition (4).

However, the analysis suggests that a bank's action to commit more funds to a financially distressed firm may even exacerbate the *run* by small claimants should that acceptance be made contingent upon acceptance of the contract revision offer by a *high* proportion of claimants. This implies that *excessive conditionality*, in the form of *high* minimum tendering rates, undermines the positive information externality of bank's acceptance. That externality may even become negative and the signaling effect of bank's action at best irrelevant

[19] Note that this result does not depend at all to our assumption about fixed *LGD*.

to the decisions by small claimants. This result is summarised in proposition (5).

We may draw a parallel between the simple balance sheet of the financially distressed firm in our model, and the capital account of a country during the onset of a financial crisis and discuss possible implications of our analysis on the *doctrine of catalytic finance*. That doctrine rests on the premise that, "under the right conditions, official assistance and private sector funding are strategic complements"[20] and until before the Argentine crisis in 2001, it was the cornerstone of the official community's strategy towards capital account crisis[21]. The main idea was that official assistance to a country that experiences a financial crisis could encourage other creditors to act in a way that mitigates the crisis. Since the Argentine crisis, the doctrine of catalytic finance is less appealing among the G7. In particular, with respect to IMF interventions, there are voices nowadays arguing that the IMF's assistance to a country is exploited by private creditors and, in a sense, the two sources of funding become strategic substitutes during periods of financial crisis, instead of complements. Those voices are further reinforced by a moral-hazard story, according to which, the inability of the IMF to commit not to intervene always exacerbates the moral hazard problem on the part of the debtor country.

Given that our analysis relates to the work outs of financial distress, rather than to the prevention of financial crisis, we could set aside the moral-hazard issue and conclude our discussion by noting the following:

First, the presumption that underlies the doctrine of catalytic finance, namely that official assistance to a country in financial crisis could encourage other creditors to act in a way that mitigates the crisis, is in line with our result that the appropriate involvement of a *large* creditor may alleviate inefficiencies that possibly arise from co-ordination problems among other creditors.

Second, our analysis indicates that *excessive conditionality* in a large creditor's acceptance of a restructuring offer could negate the effectiveness of catalytic finance. In our model, that conditionality was captured through the minimum tendering rate. But it can also take other forms, such as assignment of *preferred creditor status (PCS)* to a large creditor, high tendering rates in *collective action clauses (CACs)* etc. But, it can also be the case that some degree of conditionality may permit creditors to make better informed decisions by allowing them to base their actions on the knowledge of other creditors. In any case, in the context of the resolution of financial distress, conditionality on the provision of financial assistance should be a balancing act.

[20] See Morris and Shin (2003) for an elaborate analysis on catalytic finance..

[21] The September 2000 communique of the International Monetary and Finance Committee states that "the combination of catalytic official financing and policy adjustment should allow the country to regain full market access quickly" .

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7 Appendix

7.1 Proof of Lemma1

Conditional on signal x_i, return (R) is normally distributed with mean x_i and variance σ^2. Given that suppliers' signals $\{x_i\}$ are *iid*, the critical level of investment return (R^*), below which rejection by suppliers generates default, is defined as follows:

$$\Pr\left(x < x^* \mid R = R^*\right) = 1 - r$$

This is,

$$F\left(\frac{x^* - R^*}{\sigma}\right) = 1 - r$$

or

$$R^* = x^* - \sigma F^{-1}\left(1 - r\right)$$

which proves the lemma.

7.2 Proof of Lemma2

Given that suppliers' signals are *iid*, conditional on no default at $\tau = 1$ (i.e. $R > R^*$) and on signal x_i, i supplier's belief about the proportion (l) of other suppliers receiving a signal lower than his is defined as follows:

$$l = \Pr\left(x_j < x_i \mid R > R^*\right) \tag{8}$$

For $x_i = x^*$ equation (8) gives the rejection rate (l^*) that one expects to occur when he observes a signal equal to the critical signal level (x^*) :

$$\begin{aligned} l^* &= \Pr\left(x_j < x^* \mid R > R^*\right) = \\ &= \frac{\Pr\left(x_j < x^*, R > R^*\right)}{\Pr\left(R > R^*\right)} \end{aligned} \tag{9}$$

Conditional on signal $x_i = x^*$, signal x_j is normal with mean x^* and variance $2\sigma^2$. Similarly (R) is also normal with mean x^* and variance σ^2. Moreover, conditional on $x_i = x^*$, x_j and R are correlated with covariance equal to σ^2. Thus, (x_j, R) is a bivariate normal distribution with mean $\boldsymbol{\mu} = (x^*, x^*)'$ and variance/covariance matrix $\mathbf{\Sigma} = \begin{bmatrix} 2\sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2 \end{bmatrix}$. From the definition of the multivariate normal distribution and given that $\mathbf{\Sigma}^{-1} = \frac{1}{\sigma^2}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ and $|\mathbf{\Sigma}| = \sigma^4$, it is easy to show that l^* in equation (9) is given by the following expression:

$$l^* = \frac{\frac{1}{2\pi\sigma^2}\int_{-\infty}^{x^*}\int_{x^*-\sigma F^{-1}(1-r)}^{+\infty} \exp\left[-\frac{(x_j-R)^2+(x^*-R)^2}{2\sigma^2}\right] dR dx_j}{1-r} \tag{10}$$

By changing the order of integration in equation (10) and by applying the transformation $z = \frac{x_j - R}{\sigma}$, we get the following expression:

$$l^* = \frac{\frac{1}{\sqrt{2\pi\sigma^2}} \int_{x^* - \sigma F^{-1}(1-r)}^{+\infty} \left\{ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{x^*-R}{\sigma}} \exp\left[-\frac{z^2}{2}\right] dz \right\} \exp\left[-\frac{(R-x^*)^2}{2\sigma^2}\right] dR}{1-r}$$

Let $w = \frac{R-x^*}{\sigma}$,

$$l^* = \frac{\frac{1}{\sqrt{2\pi}} \int_{-F^{-1}(1-r)}^{+\infty} \exp\left[-\frac{w^2}{2}\right] F(-w)\, dw}{1-r}$$

or

$$l^* = \frac{\int_{-F^{-1}(1-r)}^{+\infty} F(-w) f(w)\, dw}{1-r}$$

Let $w = -t$ and applying the fact that f is symmetric we finally get:

$$l^* = \frac{1}{2} \frac{\int_{-\infty}^{F^{-1}(1-r)} d\left[F(t)\right]^2}{1-r}$$

or

$$l^* = \frac{(1-r)}{2}$$

which proves the lemma.

7.3 Proof of Proposition1

For $F^{-1}(1-r) < \frac{(x^*-y^*)}{\sigma}$ equation (3) becomes:

$$-c\left[F\left(\frac{x^*-y^*}{\sigma}\right) - (1-r)\right] - \left[(1-r) - F\left(\frac{x^* - B - C - a_s\frac{(1+r)}{2}Q}{\sigma}\right)\right] +$$

$$+ (a_s - 1) F\left(\frac{x^* - B - C - a_s\frac{(1+r)}{2}Q}{\sigma}\right) = -cF\left(\frac{x^*-y^*}{\sigma}\right)$$

or

$$x^* = (B + C) + a_s Q\left(\frac{1+r}{2}\right) + \sigma F^{-1}\left[\frac{(1-r)(1-c)}{a_s}\right]$$

which proves the proposition.

7.4 Proof of Proposition 2

For $F^{-1}(1-r) > \frac{(x^*-y^*)}{\sigma}$ equation (3) becomes:

$$-F\left(\frac{x^*-y^*}{\sigma}\right) + F\left(\frac{x^* - B - C - a_s\frac{(1+r)}{2}Q}{\sigma}\right) +$$

$$+\left(a_s-1\right)F\left(\frac{x^*-B-C-a_s\frac{(1+r)}{2}Q}{\sigma}\right)=-cF\left(\frac{x^*-y^*}{\sigma}\right)$$

or

$$a_sF\left(\frac{x^*-B-C-a_s\frac{(1+r)}{2}Q}{\sigma}\right)=\left(1-c\right)F\left(\frac{x^*-y^*}{\sigma}\right) \tag{11}$$

which proves the proposition.

7.5 Proof of Lemma 3

Conditional on no default at $\tau=1$ (i.e. $R>R^*$) and on signal y^*, bank's belief about the proportion $\left(l^b\right)$ of suppliers that receive a signal lower than x^* (i.e. reject firm's offer) is defined as follows:

$$\begin{aligned} l^b &= \Pr\left(x_j<x^* \mid R>R^*, y=y^*\right)= \\ &= \frac{\Pr\left(x_j<x^*, R>R^*\right)}{\Pr\left(R>R^*\right)} \end{aligned} \tag{12}$$

Conditional on $y=y^*$ signal x_j is normally distributed with mean y^* and variance $v^2+\sigma^2$. Similarly, return R is also normal with mean y^* and variance v^2. Moreover, x_j and R are correlated with covariance v^2. Thus, conditional on y^*, (x_j,R) is a bivariate normal distribution with mean $\boldsymbol{\mu}=\left(y^*,y^*\right)'$ and variance/covariance matrix $\mathbf{\Sigma}=\begin{bmatrix} v^2+\sigma^2 & v^2 \\ v^2 & v^2 \end{bmatrix}$. From the definition of the multivariate normal distribution and the fact that $\mathbf{\Sigma}^{-1}=\frac{1}{v^2\sigma^2}\begin{bmatrix} v^2 & -v^2 \\ -v^2 & v^2+\sigma^2 \end{bmatrix}$ and $|\mathbf{\Sigma}|=v^2\sigma^2$, it is easy to show that l^b in equation (12) is given by the following expression:

$$l^b=\frac{\frac{1}{2\pi\sigma v}\int_{-\infty}^{x^*}\int_{x^*-\sigma F^{-1}(1-r)}^{+\infty}\exp\left[-\left(\frac{(x_j-R)^2}{2\sigma^2}+\frac{(R-y^*)^2}{2v^2}\right)\right]dRdx_j}{\Pr\left(y^*-v\varepsilon>x^*-\sigma F^{-1}\left(1-r\right)\right)} \tag{13}$$

By changing the order of integration in equation (13) and by applying the transformation $z=\frac{x_j-R}{\sigma}$, we get the following expression:

$$l^b=\frac{\frac{1}{\sqrt{2\pi v^2}}\int_{x^*-\sigma F^{-1}(1-r)}^{+\infty}\left\{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{x^*-R}{\sigma}}\exp\left[-\frac{z^2}{2}\right]dz\right\}\exp\left[-\frac{(R-y^*)^2}{2v^2}\right]dR}{F\left[\frac{y^*-x^*}{v}+\frac{\sigma}{v}F^{-1}\left(1-r\right)\right]}$$

or

$$l^b=\frac{\int_{x^*-\sigma F^{-1}(1-r)}^{+\infty}F\left(\frac{x^*-R}{\sigma}\right)\frac{1}{\sqrt{2\pi v^2}}\exp\left[-\frac{(R-y^*)^2}{2v^2}\right]dR}{F\left[\frac{y^*-x^*}{v}+\frac{\sigma}{v}F^{-1}\left(1-r\right)\right]}$$

Let $w=\frac{R-y^*}{v}\Rightarrow R=vw+y^*$,

$$l^b=\frac{\int_{\frac{x^*-y^*-\sigma F^{-1}(1-r)}{v}}^{+\infty}F\left(\frac{x^*-y^*-vw}{\sigma}\right)\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{w^2}{2}\right]dw}{F\left[\frac{y^*-x^*}{v}+\frac{\sigma}{v}F^{-1}\left(1-r\right)\right]}$$

or

$$l^b = \frac{\int_{\frac{x^*-y^*-\sigma F^{-1}(1-r)}{v}}^{+\infty} F\left(\frac{x^*-y^*-vw}{\sigma}\right) f(w)\,dw}{F\left[\frac{y^*-x^*}{v} + \frac{\sigma}{v} F^{-1}(1-r)\right]} \tag{14}$$

Equation (14) can be simplified a lot by considering the limiting case where $\frac{v}{\sigma} \to 0$ ($\frac{\sigma}{v} \to \infty$).This is,

$$l^b = F\left(\frac{x^* - y^*}{\sigma}\right) \tag{15}$$

which proves the lemma.

7.6 Proof of Proposition 3

By substituting $R^{**} = x^{**} - \sigma F^{-1}(1-r)$ and $l^{**} = \frac{(1-r)}{2}$ in equation (7) we get the following equation:

$$-c + c(1-r) - (1-r) + a_s F\left(\frac{x^{**} - (B+C) - a_s Q\left(\frac{1+r}{2}\right)}{\sigma}\right) = -c$$

or

$$a_s F\left(\frac{x^{**} - (B+C) - a_s Q\left(\frac{1+r}{2}\right)}{\sigma}\right) = (1-c)(1-r) \tag{16}$$

or

$$x^{**} = (B+C) + a_s Q\left(\frac{1+r}{2}\right) + \sigma F^{-1}\left(\frac{(1-c)(1-r)}{a_s}\right) \tag{17}$$

which proves the proposition.

7.7 Proof of Proposition 4.

Conditional on bank's actions being observable and for $F^{-1}(1-r) > \frac{x^*-y^*}{\sigma}$ proposition (2) shows that small claimants' critical signal level (x^*) solves the following equation:

$$a_s F\left(\frac{x^* - (B+C) - a_s Q\left(\frac{1+r}{2}\right)}{\sigma}\right) = (1-c) F\left(\frac{x^* - y^*}{\sigma}\right) \tag{18}$$

Moreover, conditional on non-observability of bank's actions at the intermediate date (i.e. at $\tau = 1$ the bank is not in the game), and proposition (3) we also have that small claimants' critical signal level (x^*) solves the following equation:

$$a_s F\left(\frac{x^{**} - (B+C) - a_s Q\left(\frac{1+r}{2}\right)}{\sigma}\right) = (1-c)(1-r) \tag{19}$$

But given that $F^{-1}(1-r) > \frac{x^*-y^*}{\sigma}$ (or, $F\left(\frac{x^*-y^*}{\sigma}\right) < (1-r)$) from equations (18) and (19) we get the following inequality:

$$F\left(\frac{x^* - (B+C) - a_s Q\left(\frac{1+r}{2}\right)}{\sigma}\right) < F\left(\frac{x^{**} - (B+C) - a_s Q\left(\frac{1+r}{2}\right)}{\sigma}\right) \tag{20}$$

From the monotonicity of $F(\cdot)$, inequality (20) holds if and only if $x^* < x^{**}$. From lemma (1) we have that $R^* < R^{**}$, which proves the proposition.