# Beyond Incomplete Spanning: Convenience Yields and Exchange Rate Disconnect

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# Outline

# Exchange Rates and Short Yields

- in large class of models, exchange rates are forward-looking

**Exchange Rate Valuation Equation**

The nominal exchange rate in FC/USD is:

$$s_t^{\frac{*}{\$}} = \mathbb{E}_t \sum_{\tau=0}^{\infty} (r_{t+\tau}^{\$} - r_{t+\tau}^{*}) - \mathbb{E}_t \sum_{\tau=0}^{\infty} rx_{t+\tau}^{\frac{*}{\$},FX} + \mathbb{E}_t [\lim_{\tau \to \infty} s_{t+\tau}].$$

Campbell-Clarida (1987); Froot-Ramadorai (2005)

- dollar exchange rate today reflects future
  1. short rate differences $(r_{t+\tau}^{\$} - r_{t+\tau}^{*})$
  2. FXRP $\mathbb{E}_t rx_{t+\tau}^{\frac{*}{\$},FX} = r_{t+\tau}^{\$} - r_{t+\tau}^{*} + \mathbb{E}_{t+\tau} \Delta s_{t+\tau+1}$ earned by foreign investors going long in USD
- dollar appreciates when $y_{t+\tau}^{\$} \nearrow$ and dollar FXRP $rx_{t+\tau}^{\frac{*}{\$},FX} \searrow$

# Stationary Exchange Rates

- substitute out the short rates to obtain:

$$s_t^{\frac{*}{\$}} - s_0^{\frac{*}{\$}} = \lim_{N\to\infty} N(y_t^{\$,N} - y_t^{*,N})$$

$$+ \underbrace{\lim_{N\to\infty} \mathbb{E}_t^* \sum_{j=1}^{N} (rx_{t+j}^{*,N-j+1} - rx_{t+j}^{\$,N-j+1})}_{\text{deviations from EH}} \underbrace{- \lim_{N\to\infty} \mathbb{E}_t \sum_{\tau=0}^{N} rx_{t+\tau}^{\frac{*}{\$},FX}}_{\text{deviations from short-run UIP}}$$

- over long holding periods, foreign and domestic bonds are equally risky with stationary exchange rates.

**Long-Run U.I.P.**

$$(s_t^{\frac{*}{\$}} - s_0^{\frac{*}{\$}}) = \lim_{N\to\infty} N(y_t^{\$,N} - y_t^{*,N})$$

# Bond Risk Premium Channel for QE.

- GRV (2020) and GHSS (2020) bring equilibrium models of term structure with market segmentation (Vayanos and Villa, 2019) to FX markets: arbitrageurs in bond markets and FX markets (GM (2015))
- downward sloping demand curves for Treasurys.
  - decrease in net US supply of long bonds (QE): US arbs demand smaller BRP on long USD bonds

$$\mathbb{E}_t rx_{t+j}^{US,N} = \mathbb{E}_t[hpr_{t+1}^{US,N} - y_t^{\$}] \searrow$$

  - Fed controls long yields $y_t^{\$,N}$ and hence the exchange rate

# Outline

1. Exchange Rates and Yields
2. **Exchange Rates and Convenience Yields**
3. Three Exchange Rate Puzzles
4. Incomplete markets Model with Convenience Yields
5. Convenience Yield Channel for QE

# Exchange Rates and Convenience Yields

- future convenience yields are priced into USD exchange rate (Jiang, Krishnamurthy and Lustig, 2021).

**Exchange Rate Valuation Equation**

$$
\begin{aligned}
s_t^{\frac{*}{\$}} &= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\lambda_{t+\tau}^{\$,*}) + \mathbb{E}_t \sum_{\tau=0}^{\infty} (r_{t+\tau}^{\$} - r_{t+\tau}^{*}) \\
&- \mathbb{E}_t \sum_{\tau=0}^{\infty} rx_{t+\tau}^{\frac{*}{\$},FX} + \ldots
\end{aligned}
$$

- measured by CIP deviation in Treasury markets $\lambda_t^{\$,*} = -\frac{x_t}{(1-\beta)}$, *where*

$$
x_t^{Treasury} \equiv \underbrace{y_t^{\$}}_{cash\ Treasury} - \underbrace{\left(y_t^{*} - (f_t^{1} - s_t)\right)}_{synthetic\ Treasury}.
$$

# Convenience Yield Channel for QE

our paper develops a convenience yield channel

$$
\begin{aligned}
s_t^{\frac{*}{\$}} &= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\lambda_{t+\tau}^{\$,*}) + \mathbb{E}_t \sum_{\tau=0}^{\infty} (r_{t+\tau}^{\$} - r_{t+\tau}^{*}) \\
&- \mathbb{E}_t \sum_{\tau=0}^{\infty} rx_{t+\tau}^{\frac{*}{\$},FX} + \ldots
\end{aligned}
$$

- dollar appreciates when future US Treasury CY $\lambda_{t+\tau}^{\$,*} \nearrow$
- CY channel also creates a role for flows/quantities.
    1. QE: when Fed buys long-dated Treasurys, $\lambda_{t+\tau}^{\$,*} \nearrow \rightarrow s_t \nearrow$
    2. QE: when Fed buys MBS and issues reserves, $\lambda_{t+\tau}^{\$,*} \searrow \rightarrow s_t \searrow$
    3. QE: when Fed buys long-dated Treasurys and issues reserves??

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# Puzzles

$$\Delta s^{\frac{*}{\$}}_{t+1} = m^{\$}_{t+1} - m^{*}_{t+1}$$

1. **Volatility puzzle**: Brandt, Cochrane, and Santa-Clara (2006)

$$\begin{aligned} var_t(\Delta s_{t+1}) &= var_t(m^*_{t+1}) + var(m_{t+1}) \\ &- 2\rho_t(m_{t+1}, m^*_{t+1}) std_t(m_{t+1}) std_t(m^*_{t+1}). \end{aligned}$$

2. **Counter-cyclicality puzzle** (correlation puzzle, exchange rate disconnect): Kollmann (1991), Backus and Smith (1993)

$$Corr_t\left(\Delta s_{t+1}, m^{\$}_{t+1} - m^{*}_{t+1}\right) = 1$$

3. **Risk premium puzzle** (Forward premium / UIP puzzle): Tryon (1979), Hansen and Hodrick (1980), Fama (1984)

# Market Incompleteness: Not enough

$$\Delta s_{t+1}^{\frac{*}{\$}} = m_{t+1}^{\$} - m_{t+1}^{*} + \eta_{t+1}$$

Lustig and Verdelhan (2018)

1. **Volatility puzzle**:

$$\begin{aligned} var_t(\Delta s_{t+1}) &= var_t(m_{t+1}^{*}) + var(m_{t+1}) \\ &- 2\rho_t(m_{t+1}, m_{t+1}^{*}) std_t(m_{t+1}) std_t(m_{t+1}^{*}) - var_t(\eta_{t+1}). \end{aligned}$$

2. **Counter-cyclicality puzzle** (correlation puzzle, exchange rate disconnect):

$$Corr_t\left(\Delta s_{t+1}, m_{t+1}^{\$} - m_{t+1}^{*}\right) >> 0$$

3. **Risk premium puzzle** worsens

# Market Incompleteness+Convenience Yields

our paper brings incomplete markets wedges and Euler equation wedges (convenience yields), no market segmentation

- foreign investors earn convenience yield on US bonds :

$$E_t\left(M^*_{t+1}R^{f,*}_t\right) = E_t\left(M^*_{t+1}e^{\eta_{t+1}}\frac{S_t}{S_{t+1}}\right) = 1,$$

$$E_t\left(M^*_{t+1}\frac{S_{t+1}}{S_t}R^f_t\right) = E_t\left(M_{t+1}e^{\eta_{t+1}}\right) = \exp(-\tilde{\lambda}_t),$$

where the IM wedges are:

$$\widehat{M}_{t+1} = M_{t+1}e^{\eta_{t+1}} = \frac{S_{t+1}}{S_t}M^*_{t+1},$$

and the Euler equation wedges $\lambda_t$ are the convenience yields.

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# Bond Investor Euler equations

- Let $P_t$ denote the cumulative return on the US bond, and $P_t^*$ denote the cumulative return on the foreign bond:

$$dP_t = r_t P_t dt, dP_t^* = r_t^* P_t^* dt.$$

- Investors can trade both home and foreign risk-free assets. Let $S_t = \exp(s_t)$, $M_t = \exp(m_t)$ and $M_t^* = \exp(m_t^*)$.
  - domestic investors don't earn convenience yields:

$$0 = \mathbb{E}_t[d(M_t P_t)], 0 = \mathbb{E}_t[d(M_t S_t^{-1} P_t^*)]$$

  - foreign investors earn convenience yields on US bonds:

$$0 = \mathbb{E}_t[d(M_t^* P_t^*)], 0 = \mathbb{E}_t[M_t^* S_t P_t \tilde{\lambda}_t dt + d(M_t^* S_t P_t)].$$

- flow convenience yield $S_t P_t \tilde{\lambda}_t dt$ in the foreigner investors' pricing condition for the home (dollar) risk-free asset.

# Flight-to-Treasurys

- pricing kernel dynamics:

$$dm_t = -\mu dt - \sigma dZ_t, dm_t^* = \phi s_t dt - \sigma dZ_t^*,$$

- convenience yield dynamics:

$$\tilde{\lambda}_t = \ell \frac{\exp(\lambda_t)}{\exp(\lambda_t) + 1}, d\lambda_t = -\theta \lambda_t dt + \nu dX_t$$

- flight-to-Treasurys: positive correlation between $m^*$ shocks and $\lambda$ shocks

$$\begin{aligned} [dZ_t, dX_t] &= \rho << 0, \\ [dZ_t^*, dX_t] &= \rho^* << 0. \end{aligned}$$

# FR Decomposition of Exchange Rates

- forward looking expression: FR-JKL decomposition.

**Exchange Rate Valuation Equation**

The log of the exchange rate is given by

$$s_t = \bar{s} + \mathbb{E}_t \int_t^{\infty} (r_u - r_u^*) du - \mathbb{E}_t \int_t^{\infty} \pi_u du,$$

where $\mathbb{E}_t \int_t^{\infty} \pi_u du$ encodes future currency risk premia and the convenience yields:

$$\begin{aligned} \pi_t &= \mathbb{E}_t[d \log(P_t S_t / P_t^*)] = \mathbb{E}_t[ds_t] + r_t - r_t^* \\ &= -\frac{1}{2}\tilde{\lambda}_t + \frac{1}{2}\sigma\gamma_t\nu(\rho + \rho^*). \end{aligned}$$

# Equilibrium Exchange Rates with IM and CY Wdges

- solve for exchange rate dynamics with incomplete markets and convenience yield wedges

**Exchange Rate Dynamics**

The equilibrium exchange rate process is given by:

$$ds_t = \alpha_t dt + \beta_t \sigma(dZ_t^* - dZ_t) + \gamma_t \nu dX_t.$$

  - $\beta$ (degree of incompleteness) controls the size of $\eta$ (*incomplete markets wedges*)
  - $\gamma$ controls the impact of the convenience yields $\tilde{\lambda}$ (*Euler equation wedges*)
  - incomplete markets: many equilibria indexed by $\beta, \gamma$
    - $\beta_t = 1, \gamma = 0$: complete markets.

$$ds_t^{cm} = (-\mu - \phi s_t^{cm})dt + \sigma(dZ_t^* - dZ_t).$$

.

# Calibration

- HJ bound: $\sigma = 0.5$
- foreign flight to Treasurys: $\rho = 0, \rho^* = -0.50$
- CY parameters: $\tilde{\lambda}_t$ has mean of 2.5% and stdev of 2.1%

# Simulation Results

| $\beta$ | (2) FX-Conv Yield Coef | (3) FX Vol (%) | (4) FX-SDF Coef | (5) Exp.Log Return (%) |
|---|---|---|---|---|
| *data* | $1.00 - 1.50$ | 10.00 | $<< 0$ | $-1.89$ |
| **0.10** | **1.70** | **11.93** | **0.10** | **-1.97** |
| 0.14 | 2.77 | 16.01 | 0.14 | -2.71 |
| 0.19 | 3.79 | 21.55 | 0.19 | -3.66 |
| 0.25 | 4.70 | 27.16 | 0.25 | -3.16 |
| 0.32 | 5.77 | 33.72 | 0.32 | -4.41 |
| 0.50 | 7.32 | 47.51 | 0.50 | -6.67 |

(2) reports the slope coefficient in regression of $\Delta s_t$ on $\Delta\tilde{\lambda}_t$. (3) reports FX vol. (4) reports slope coefficient in regression of $\Delta s$ on $m - m^*$. (5) reports the exp. log excess return on long position in USD.

# The Vol Puzzle: Shock Absorbers

**Exchange Rate Vol**

The conditional variance of the IM exchange rate is

$$Var_t = \gamma_t^2 \nu^2 + 2\beta^2\sigma^2 + 2\gamma_t \nu \beta \sigma (\rho^* - \rho)$$

- CM benchmark: $Var_t^{CM} = 2\sigma^2$ when $\beta = 1$ and $\gamma = 0$
- IM model: when $\beta << 1$ and $\gamma >> 0$
  - USD appreciates less when $m$ is higher than $m^*$
  - $\beta$: incomplete market wedges absorb SDF shocks (not exchange rates)
  - $\gamma > 0, \rho^* < 0$: CY Euler equation wedges also absorb shocks
    - reverse flight to Treasurys when $m$ is higher than $m^*$: lower CY causes USD to appreciate less

# The Cyclicality Puzzle: Shock Absorbers

**Exchange Rate Vol**

The slope coefficient in regression of $\Delta s$ on $m - m^*$:

$$\beta + \frac{(\rho^* - \rho)}{2\sigma}$$

$$\frac{(\rho^* - \rho)\sigma(1 - 2\beta) + \sqrt{(\rho^* - \rho)^2\sigma^2(1 - 2\beta)^2 + 4(k - \tilde{\lambda}_t)}}{2}.$$

- CM benchmark $\beta = 1$: slope is 1
- in IM: $\beta < 1$
- flight to Treasury: $(\rho^* - \rho) << 0$: lowers slope coefficient
- slope could be negative (in theory)

# Currency Risk Premium

**Exchange Rate Vol**

The expected return on a long position in USD:

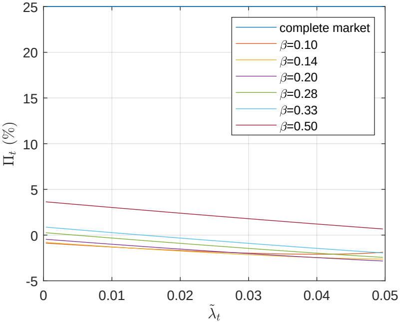
$$\begin{aligned}\Pi_t &= \mathbb{E}_t\left[\frac{dS_t}{S_t}\right] + r_t - r_t^* = \pi_t + \frac{1}{2}[ds_t, ds_t] \\ &= -\tilde{\lambda}_t + \beta\sigma^2 + \sigma\gamma_t\nu\rho^*.\end{aligned}$$

- CM benchmark $\beta = 1, \gamma = 0$: CRP

$$\Pi_t = \sigma^2$$

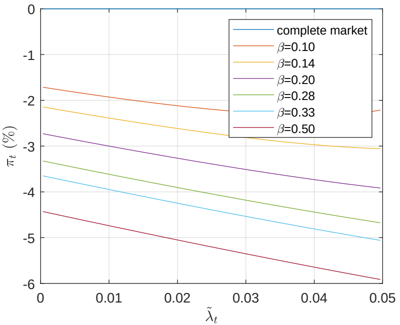
- IM: CRP declines one-for-one with convenience yield
- additional risk premium effect from flight to Treasury $\rho < 0$

# The Dollar's Expected Excess Return




$$
\begin{aligned}
\Pi_t &= \mathbb{E}_t\left[\frac{dS_t}{S_t}\right] + r_t - r_t^* = \pi_t + \frac{1}{2}[ds_t, ds_t] \\
&= -\tilde{\lambda}_t + \beta\sigma^2 + \sigma\gamma_t\nu\rho^*.
\end{aligned}
$$

# The Dollar's Expected Log Excess Return




$$\pi_t = \mathbb{E}_t[d\log(P_t S_t/P_t^*)] = \mathbb{E}_t[ds_t] + r_t - r_t^*$$
$$= -\frac{1}{2}\tilde{\lambda}_t + \frac{1}{2}\sigma\gamma_t\nu(\rho + \rho^*)$$

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# QE

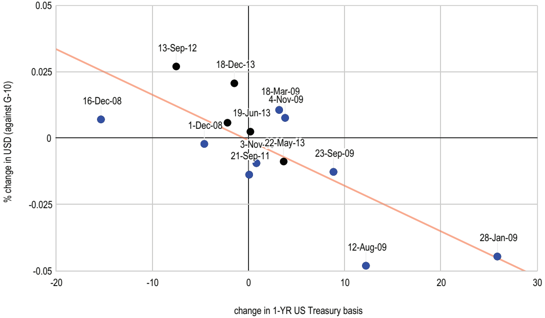


Figure: G-10 Dollar appreciation against change in basis around QE event dates. Sample of 14 QE event dates. 2-day window after QE-event dates.

# IRF to a Convenience Yield Shock

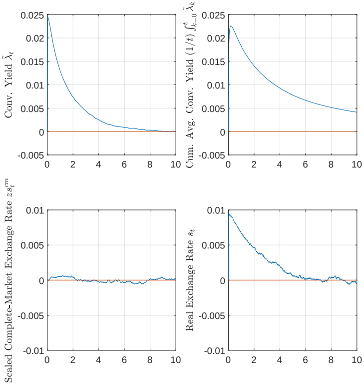


Figure: CY $\lambda_t$ jumps up by 1 standard deviation in the period 0 and simulations in which all shocks have zero means.