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Delegation Chains

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Abstract

We ask why we observe multiple layers of decision-making in fund management with investors, sponsors, fund managers, and consultants, even if additional decision-makers are costly and do not contribute to superior performance. In our model, an investor hires a wealth manager ("sponsor"), who can delegate asset allocation decisions to a fund manager with investing abilities inferior to her own. Delegation results in lower performance but may be chosen because it reduces the sponsor's reputational risk: Offloading decisions to fund managers creates an additional decision-maker who may be responsible for inferior performance and garbles inferences about the sponsor's ability. We characterize when excessive delegation arises and the properties of delegation chains.

Keywords: Career concerns; delegated portfolio management; money management; pension funds; mutual funds

JEL Classifications: G23, G34, G10, G11.

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"Giving greater discretion to outside money managers always leaves the treasurer's office with an extra layer of people to blame." (Lakonishok et al., 1992, p. 374.)

1 Introduction

The $47-trillion institutional money management industry is characterized by significant delegation of ownership and decision making.[1] Corporations hire pension funds to manage employee retirement contributions but these pension fund managers—instead of investing directly—often select and invest in mutual funds to access traded securities. For example, in recent years, US pension funds have invested up to 20% of their assets in mutual funds.[2] Indeed, in subcontracting to other fund managers, pension funds may even use the services of additional intermediaries and experts such as consultants.[3] In a similar vein, institutional investors sometimes invest in funds of funds, which then invest in individual hedge funds, which in turn trade in financial assets.[4] Incremental layers of ownership and delegated decision-making are costly in terms of additional fees and are likely to foster agency problems.[5] Thus, their presence should be justified on the grounds of improved performance. Yet, the majority of scholarly studies on institutional money management document inferior or insignificant performance.[6] Why does such significant delegation of ownership and decision making persist in institutional asset management in the absence of any clear performance benefits?

Our simple answer to this puzzle is that multi-layered structures of ownership and delegated decision making help wealth managers to better manage their reputational risk. In institutional money management, wealth managers compete in a labor mar-

[1]The estimated size of the institutional asset management industry is taken from Gerakos et al. [2019] and is for the year 2012 (see their Table 1). This was 29% of $173 trillion of worldwide investable assets for that year.

[2]See Dasgupta et al. [2021], Figure 3.1.

[3]A small but longstanding literature beginning with Lakonishok et al. [1992] describes the institutional details of the institutional money management industry. Notable contributions include Coggin et al. [1993], Christopherson et al. [1998], Ferson and Khang [2002], Goyal and Wahal [2008], Busse et al. [2010], Jenkinson et al. [2016], and Gerakos et al. [2019].

[4]See Agarwal and Naik [2005], Section 5 and Brown et al. [2012] on funds of funds. Multiple layers of ownership also occur with venture capital, private equity, and special-purpose acquisition companies (Riemer, 2007; Kolb and Tykvova, 2016).

[5]For example, Dasgupta and Piacentino [2015] show that the presence of equity blockholders who are simultaneously principals and agents can weaken corporate governance.

[6]The majority of studies cited in footnote 3 above focus on US equities and document inferior or insignificant performance in the instutional asset management industry. The recent study of Gerakos et al. [2019] considers a broader class of assets and finds some evidence of positive performance.

ket that constantly assesses their abilities based on observed portfolio performance.[7] However, portfolio performance depends on luck as well as wealth managers' abilities, which creates reputational risk beyond their control. In such a context, delegation to another agent creates an inference problem for the labor market: Whereas direct investing by the wealth manager offers the labor market a signal about her individual ability, delegating decisions to an agent forces the labor market to jointly assess the ability of the wealth manager as well as that of her agent. This dual inference makes the signal about the wealth manager herself less precise, since it adds noise from the uncertain ability of the agent. Hence, the additional layer of delegation makes the wealth manager's reputation less volatile, and—if she is risk averse—delegation increases her expected utility even if it is costly, e.g., by way of worse performance or additional fees. Put differently, delegation reduces reputational risk since information is garbled by the "additional layer of people to blame" mentioned in the opening vignette. We show that this mechanism may be effective simply because wealth managers have career concerns, and does not require psychological needs for blaming or scapegoating.

Our argument covers a wide range of institutional arrangements, such as pension funds delegating asset allocation decisions to mutual funds, fund managers' subcontracting hiring choices to consultants, or assets being held in ownership chains involving pension funds, funds of funds, and hedge funds, as in the examples described above. Clearly, other rationales for the existence of multiple layers of delegated decision making and ownership exist, e.g., intermediaries may provide liquidity, diversification, or specialize in the management of specific types of assets (e.g., in venture capital, private equity, or real estate). In this paper, we abstract from these conventional explanations for delegation and promote the notion of excessive delegation. Given the questionable performance documented in institutional money management, it is legitimate to search for explanations that emphasize market failure rather than efficiency considerations. Our goal is to understand why forms of delegation that have been consistently associated with poor performance may persist. While our principal focus is on institutional money management, our argument is generic and applies to other contexts as well.

We analyze a model with three types of actors: a principal, who represents ultimate investors, (e.g., the beneficiaries of a corporate pension plan); a skilled wealth manager (e.g., a pension fund manager), who we call the sponsor; and a large group of other skilled individuals (e.g., mutual fund managers) who we refer to as fund managers. The

[7] See Lakonishok et al. [1992], Del Guercio and Tkac [2002], and Goyal and Wahal [2008].

principal lacks investment management skills, but can observe and evaluate portfolio performance. The sponsor and the fund managers all have investment management skills. The sponsor is also skilled in choosing good fund managers. The sponsor's asset management skills are superior to those of fund managers, even conditional on using her selection skills to choose a good fund manager. Hence, there is no reason to delegate portfolio management to a fund manager for performance considerations; as discussed above, we focus on delegation that is not driven by efficiency considerations. The abilities of sponsors and fund managers are uncertain and the market for wealth managers makes inferences about their abilities based on their portfolio performance. Sponsors and fund managers are career concerned because their utility depends on the labor market's reputational assessments. Such career concerns are the sole source of agency conflicts in our model.

The sponsor can either choose to trade directly or delegate trading decisions to a fund manager she selects from a pool. Motivated by our interest in excessive delegation, we construct our model to ensure that delegation is always undesirable for the principal. If the principal could simply write forcing contracts to prevent delegation, he would always do so. Then excessive delegation would never arise, obviating our analysis. Accordingly, we assume that such forcing contracts cannot be written and the principal can only provide incentives in the form of profit sharing to the sponsor to influence the sponsor's decision whether to delegate.

Within the remit of any profit sharing incentives provided by the principal, the sponsor's reputational concerns influence her incentive to delegate as follows. The sponsor's reputational risk is lower with delegation to a fund manager than with direct investing because of the additional layer of inference we discuss above. The market for wealth managers observes only the portfolio performance. Under delegation,the market needs to disentangle the uncertain abilities of the fund manager from those of the sponsor. This renders the market's inference about the sponsor's ability less precise and, accordingly, the sponsor's reputation is less volatile under delegation than under direct investing; in the latter case, the market can infer the quality of the sponsor more precisely from her portfolio performance. If the plan sponsor is risk averse over her reputation, she will be tempted to delegate excessively to reduce her reputational risk. We provide microfoundations for such risk aversion over reputation by showing that it arises whenever the sponsor has standard concave preferences over wealth, and if fund flows are not too convex in fund performance. The latter assumption rules out winner-take-all markets in which investors allocate assets to a small number of star performers, but it is consistent with empirical research on pension funds, our leading interpretation for the sponsor: Del Guercio and Tkac [2002] document that the flow-performance relationship faced by pension funds is approximately linear, meaning that even modest degrees of risk aversion over wealth would generate risk aversion over reputation as in our model.

Delegation is undesirable for the principal and he can prevent it only indirectly by conceding a larger share of trading profits to the sponsor, which provides her with sufficiently high-powered incentives to compensate for the benefits from reducing reputational risk. However, if the trading profits the principal needs to give up to the sponsor are too large, then it is better for him to tolerate lower performance and accept excessive delegation than to write a contract that would prevent it. This can arise, for example, if the profitability achievable from trade in the asset class is not large, if the sponsor's career concerns are very strong, or if the performance advantage of the sponsor over a fund manager is low. Under any of these conditions, excessive delegation arises in equilibrium. Thus, our model generates empirical predictions on the conditions under which excessive delegation becomes more likely. In particular, we show how excessive delegation depends on the characteristics of wealth managers (e.g., their information quality or career concerns) as well as on those of the assets they trade (e.g., market depth).Our model also has implications with regard to the identities of those who are most likely to engage in excessive delegation. As discussed above, the key driver of excessive delegation is the desire to reduce reputational risk which, in turn, depends on the convexity of the flow-performance relationship. If wealth managers are risk averse over wealth and if the flow-performance relationship is not too convex, wealth managers will be averse to reputational risk, which creates incentives for excessive delegation. By contrast, if the flow-performance relationship is sufficiently convex, preferences over reputation will also be rendered convex, generating the opposite attitude towards delegation. Thus, our model predicts that fund managers such as equity mutual funds who have been shown to face highly convex flow performance relationships (see, e.g., Chevalier and Ellison, 1997) are unlikely to wish to subcontract asset management, whereas pension funds who face relatively linear flow performance relationships (Del Guercio and Tkac, 2002) will be more likely to do so. In effect, our model suggests that mutual funds are relatively likely to be located at the bottom of delegation chains, whereas pension funds are relatively likely to be located in the middle, a finding that resonates with the stylized facts discussed above.

Finally, we explore the limits of our argument by relaxing some of our key assumptions and ask whether we can construct cases in which delegation leads to more reputational risk. We show this to be possible, albeit only for a restrictive set of parameters in which the market's uncertainty about the abilities of fund managers is much larger than the uncertainty about plan sponsors' abilities, *and* in which sponsors have a sufficiently high ability to screen fund managers. Importantly, the conditions for this case are restrictive and not symmetric to those that lead to a reduction of reputational risk through delegation. The reason is that delegation creates a two-layer inference problem, which is always biased towards a reduction of reputational risk, and the opposite case obtains only in the presence of sufficiently strong countervailing forces. Finally, we show that it is possible to even obtain a case of too little delegation if these countervailing effects are sufficiently strong, and if, in addition, fund managers are also assumed to have superior asset management skills relative to sponsors. Again, we show that constructing such a case requires stringent assumptions to overcome the tendency towards excessive delegation inherent in the model.

2 Related literature

We contribute to several strands of the literature. First, we contribute to the large and diverse literature on delegated portfolio management and on optimal contracting with fund managers (see Stracca [2006] and Bhattacharya et al. [2008] for surveys). However, while this literature has generated important insights into the frictions associated with delegated portfolio management and the structure of optimal contracts in fund management, it has not posed the question on whether delegation to fund managers itself is optimal, and why multiple layers of decision making should arise in wealth management. The empirical literature on institutional asset management discussed in the Introduction formulates a puzzle by showing that delegation to fund managers and consultants is pervasive and costly, but often not associated with superior performance, and our model sheds light on why such delegation may arise.

Second, we also contribute to the literature on the endogenous emergence of financial intermediaries (e.g., Diamond and Dybvig, 1983) and of the structure of the asset management industry. The agent labeled "fund manager" in our model could be a separate fund management company, a consultant, or an investment adviser. These intermediaries emerge endogenously in our model in order to address sponsors' need to manage their reputational risk, and this mechanism is novel with respect to the literature. Two prior papers—Glode and Opp [2016] and Glode et al. [2019]—have also modeled chains of financial intermediaries, but our approach is very different. Their models analyze the efficiency properties of an exogenously given intermediation chain and show that such chains (in particular, longer chains) may improve efficiency because intermediation reduces the asymmetry of information between any pair of buyers and sellers. By contrast, delegation chains emerge endogenously in our model despite the fact that they are inefficient.

Third, we contribute to the literature on career concerns. Stemming from the seminal work of Holmstrom [1999], this literature asks how the incentives of economic agents are affected if they take into account how the labor market in the future infers their ability from decisions they make today. Career concerns have been applied widely in asset management, e.g., to model herd behavior and interpret portfolio churning, but never to the issue of endogenous delegation and ownership chains.[8] Thus, our study represents a new application of career concerns within asset management. Interestingly, several of these applications, e.g, the herding models of Scharfstein and Stein [1990] and Dasgupta and Prat [2008], have the characteristic that the fund manager hides information by taking an action that does not reveal his signal, i.e., by signal jamming in the tradition of Holmstrom and Ricart i Costa [1986]. Notwithstanding the fact that, in our model, the sponsor chooses to delegate in order to dampen market inferences, our theoretical mechanism is distinct from signal jamming models of career concerrns. This is beause in those models, the career-concerned agent receives a signal and then—influenced by the prior—takes an action to obfuscate or "jam" it, whereas in our case the sponsor's delegation decision occurs before she has received a signal.

Finally, we contribute to the literature on multi-layered hierarchies and centralized versus decentralized decision-making (see Poitevin, 2000 and Mookherjee, 2006 for surveys). This literature addresses the question of when it is optimal to delegate decisions to an agent with a lower rank in a hierarchy rather than have the principal collect all relevant information and then make all decisions based on it. For example, in Aghion and Tirole [1997], principals may motivate agents by giving them more control over decisions. This literature is complementary to ours, since it takes the existence of an organizational structure and delegation to an agent for granted, whereas we derive delegation as an outcome.

[8] A discussion of several prior asset management applications of career concerns models can be found in Bhattacharya et al. [2008].

3 Model

The model has a wealthy individual, corporation, or group of individuals ("principal" or "investor," he) who hire a skilled wealth manager ("the sponsor," she) to manage their wealth. The sponsor may, in turn, delegate wealth management to another skilled individual ("fund manager," also he). As discussed in the introduction, the leading interpretation of our model is that the sponsor is a pension fund while the fund manager is a mutual fund.

The sponsor can be of two types, which determine the quality of her information (as specified below): $\tau_S \in \{G, B\}$ with prior probability of being good $\gamma_S = Pr(\tau_S = G) \in (0, 1)$. All variables related to sponsors are subscripted S. The sponsor does not know her type.

There is a countable infinity of fund managers. As with the sponsor, each fund manager can be of two types $\tau_F \in \{G, B\}$ with prior probability of being good $\gamma_F = Pr(\tau_F = G) \in (0, 1)$. The type τ_F is independent across fund managers and no fund manager knows his type. All variables related to fund managers are subscripted F.

There is a single risky asset, with ultimate cash flow of $\widetilde{v} \in \{0, 1\}$ with prior probability of high cash flows $\gamma_A = Pr(\widetilde{v} = 1)$. Throughout the paper, wherever relevant, we follow the convention of referring to realized values of the random variable $\widetilde{v}$ by v. All variables related to the asset are superscripted A. The variables $\tau_S, \tau_F, \widetilde{v}$ are independent of each other.

The sponsor can either choose direct investment or delegation. Under direct investment the sponsor trades directly. Under delegated investment, the fund manager makes trading decisions.

Trade occurs in a competitive market that is modeled as in Kyle and Vila [1991]. Uninformed noise traders either buy u units or sell u units with equal probability. Informed trades are initiated either by the sponsor or the fund manager, based on signals they observe as described below. To camouflage their trades they also either buy u units and sell u units. A competitive market maker observes the aggregate order flow and sets a price at which market makers make zero profits. The market maker knows whether informed trading occurs under direct or delegated investment.

The sponsor and—under delegation—fund managers make their choices at some date t. Asset payoffs are publicly realized at some date $T > t$, when trading profits and losses are realized.

Direct investment. If the sponsor chooses direct investment, then she receives a signal $s_S^A \in \{0,1\}$ about the asset payoff. The precision of the signal is type-dependent and is determined as follows:

$$Pr(s_S^A = v|\widetilde{v} = v, \tau_S) = \sigma_{S,\tau_S}^A, \tag{1}$$

where $v \in \{0,1\}$, and

$$1 \geq \sigma_{S,G}^A > \sigma_{S,B}^A \geq \frac{1}{2}. \tag{2}$$

Here and throughout, subscripts on variables with multiple indices indicate the agent, here the sponsor, and superscripts indicate further information, e.g., the item to which the variable refers, here the asset. Types of agents may be added as a second subscripts if applicable. We define

$$\sigma_S^A \equiv \gamma_S \sigma_{S,G}^A + (1-\gamma_S)\,\sigma_{S,B}^A = Pr\left(s_S^A = v|\widetilde{v} = v\right) = Pr\left(s_S^A = \widetilde{v}\right) \tag{3}$$

as the unconditional probability that the sponsor observes a correct signal about the asset if she invests directly. Conditional on the signal, the sponsor either buys u units or sells u units of the asset, as noted above.[9]

Delegated investment. If the sponsor chooses delegated investment, she is then sequentially matched with fund managers. For each fund manager she is matched with, she receives a signal $s_S^F \in \{G,B\}$. The precision of the signal is type-dependent and is determined as follows:

$$Pr(s_S^F = \tau_F^*|\tau_F = \tau_F^*, \tau_S) = \sigma_{S,\tau_S}^F \tag{4}$$

where $\tau_F^* \in \{G,B\}$ and

$$1 \geq \sigma_{S,G}^F > \sigma_{S,B}^F \geq \frac{1}{2}. \tag{5}$$

We define:

$$\sigma_S^F \equiv \gamma_S \sigma_{S,G}^F + (1-\gamma_S)\,\sigma_{S,B}^F = Pr\left(s_S^F = \tau_F^*|\tau_F = \tau_F^*\right) = Pr\left(s_S^F = \tau_F\right), \tag{6}$$

[9]Given the presence of noise traders, it is always profit maximizing for a privately informed sponsor (or fund manager) to trade, and thus we do not consider the possibility of no trade.

which is the unconditional probability that the sponsor observes a correct signal about the fund manager. The sponsor may, upon observing the signal, either hire the fund manager or reject him. If the fund manager is rejected, the sponsor is then matched with the next fund manager in the sequence and receives a signal about him. If the sponsor hires the fund manager, the hiring game ends.

The hired fund manager then receives a signal about the asset payoff. The precision of the signal is type-dependent and is determined as follows:

$$Pr(s_F^A = v|\widetilde{v} = v, \tau_F) = \sigma_{F,\tau_F}^A \tag{7}$$

with

$$1 \geq \sigma_{F,G}^A > \sigma_{F,B}^A \geq \frac{1}{2}. \tag{8}$$

Conditional on the signal, the fund manager either buys u units or sells u units of the asset, as noted above.

Action choices. The investor can offer the sponsor a profit sharing contract, $\kappa_S \in [0,1)$ where $\kappa_S = 0$ implies the absence of explicit incentive provision. The sponsor chooses whether to invest directly, $a_S = DI$, or to delegate, $a_S = DE$. If $a_S = DI$, the sponsor must choose her own trading strategy, which we denote by $\widetilde{\theta}_S : s_S^A \rightarrow \{-u, u\}$, denoting the realized trade by $\theta_S \in \{-u, u\}$. If $a_S = DE$, the sponsor must first choose her hiring strategy, $\widetilde{h}_S : s_S^F \rightarrow \{0, 1\}$, where 1 denotes hiring and 0 denotes waiting for the next fund manager, and we denote the realized hiring decision by $h_S \in \{0, 1\}$. Describing the strategy in this static way is without loss of generality, since the sponsor is costlessly matched with a countably infinite sequence of managers, but can match with at most one of them.[10] Then the sponsor offers the fund manager a profit sharing contract, $\kappa_F \in [0, \kappa_S]$, such that $\kappa_F = 0$ again implies the lack of explicit incentive provision. Further, when $a_S = DE$, the hired fund manager must choose his own trading strategy $\widetilde{\theta}_F : s_F^A \rightarrow \{-u, u\}$, again denoting the realized trading outcome by $\theta_F \in \{-u, u\}$.

Observation and inferences. At date T the labor market observes whether the sponsor chose direct investment or delegation, what trade the relevant agent (the spon-

[10]Each potential manifestation of the strategy h_S effectively generates a "stopping rule." For example, the static strategy $h_S(G) = 1$ and $h_S(B) = 0$ is equivalent to the stopping rule "the sponsor hires the first fund manager for whom she observes a good signal."

sor in case of direct investment and the fund manager in case of delegation) chose, and also the realized value of the asset payoff. The market then updates its belief about the sponsor's and, if relevant, also the fund manager's type. Thus, formally, at date T the labor market observes:

1. whether delegation took place ($a_S \in \{DI, DE\}$), which establishes the identity of the trading party, denoted by the function,

$$i(a_S) = \begin{cases} S & \text{if } a_S = DI \\ F & \text{if } a_S = DE \end{cases},$$

2. the trading action chosen ($\theta_{i(a_S)} \in \{-u, u\}$), and

3. the observed final value of the asset's payoff ($v \in \{0, 1\}$).

We define the (realized) *reputation* of each agent $a \in \{S, F\}$ as the market's posterior of the probability that her type is good and write it as a function of the delegation decision, the trading action, and the final asset payoff as

$$\gamma_a^T \left(a_S, \theta_{i(a_S)}, v\right) = Pr\left(\tau_a = G \left| a_S, \theta_{i(a_S)}, v\right.\right). \tag{9}$$

The sponsor's reputation is defined under both direct investment ($\gamma_S^T\left(a_S = DI, \theta_S, v\right)$) and delegated investment ($\gamma_S^T\left(a_S = DE, \theta_F, v\right)$), since the labor market can make relevant inferences about her skills whether she uses her private information to trade directly ($a_S = DI$) or whether she uses her private information to select a fund manager ($a_S = DE$), who then uses his private information to trade. However, the fund manager's reputation is defined (and relevant) only under delegated investment ($\gamma_F^T\left(a_S = DE, \theta_F, v\right)$) because in the case of direct investment, the fund manager plays no role in the model.

Payoffs. In our model, agents are motivated via a combination of monetary and reputational rewards. The latter component reflects agents' career concerns. In particular, we assume that they have quasi-linear utility functions of the form:

$$U_a\left(m, \gamma_a^T\right) = m + \alpha V_a\left(\gamma_a^T\right), \tag{10}$$

where m is any monetary payoff, γ_a^T is labor market's realized time T-posterior belief about the relevant agent for $a \in \{S, F\}$, $V_a(\cdot)$ is increasing, and $\alpha > 0$ is a scale parameter measuring the importance of career concerns relative to direct monetary payoffs. We assume that $V_S(\cdot)$ is strictly concave but impose no assumption on the curvature of $V_F(\cdot)$.

Microfounding utility over reputation. The payoff function V_a given in (10) is a reduced form, which maps agent a's reputation into utility for each $a \in \{S, F\}$. Since the first part of the discussion of our microfoundation applies to *both* sponsors and fund managers, for ease of exposition, in the next three paragraphs we use the term "asset managers" whenever we wish to refer to either the sponsor or the fund manager.

We can understand the reduced form utility V_a as a combination of two features: (a) asset managers enjoy concave utility $U_a^T(W_{\hat{T}})$ over their future wealth $W_{\hat{T}}$, consumed in some date $\hat{T} > T$ in the future (after this game is over and earned on the basis of the final reputation generated from this game at date T); (b) their future time-$\hat{T}$ wealth results from their continued employment as asset managers which, in turn, is a function of their date T reputation. The first part is standard and the second part can be microfounded by conceiving of the market for asset managers as a simple matching market. We describe such a market briefly here and provide more details in the appendix.

Assume that asset manager a in our model competes against a continuum of incumbent asset managers with measure one who are pre-matched before period T with a continuum of investors of equal measure. Then, in period T investors can reallocate some of their capital to the asset manager in our model as follows. Consider an investor i who currently employs an incumbent asset manager with ability $\widetilde{\gamma}_{i,a}$, which is distributed with cdf $F_{i,a}(\widetilde{\gamma}_{i,a})$ and becomes known at period T. Note that we include "a" in the subscript for indexing incumbent managers to indicate like for like replacements, i.e., sponsors compete against sponsors, funds managers against fund managers. Then, if the reputation of asset manager a exceeds the ability of the incumbent asset manager, $\gamma_a^T > \widetilde{\gamma}_{i,a}$, investor i will reallocate one unit of capital from her current asset manager i to asset manager a. Otherwise, no reallocation of funds will take place. Assume also that the abilities $\widetilde{\gamma}_{i,a}$ are independently and identically distributed, so we can omit the subscript i, and asset manager a will obtain $F_a\left(\gamma_a^T\right)$ units of capital. Thus, the F_a-function describes the flow-performance relationship.[11] Finally, we as-

[11] Note that for the purposes of our argument, it is only important what distribution F of future

sume that managing one unit of capital is worth w to asset manager a.[12] Hence, asset manager a with reputation γ_a^T has a payoff $wF_a\left(\gamma_a^T\right)$, from which she derives utility $V_a\left(\gamma_a^T\right) \equiv U_a^T\left(wF_a\left(\gamma_a^T\right)\right)$.

The curvature of the V_a function depends on the curvatures of U_a^T and F_a. If we impose some mild regularity conditions on F_a, and if U_a^T has the usual properties of a utility function, then the V_a-function is concave in γ_a^T whenever F_a is not too convex in γ_a^T relative to the coefficient of absolute risk aversion associated with U_a^T. We state and derive this result more formally in the appendix.[13] Essentially, V_a is concave if risk aversion is high enough to compensate for the potential convexity of F_a, and the only cases in which reputational utility is convex are those in which the fund flows are a sufficiently convex function of prior performance.

Building on our microfoundation, we now return to some stylized facts discussed in the Introduction to support the assumptions made above on V_S and V_F. Prior research suggests that flows in the market for pension fund managers are approximately linear in performance (Del Guercio and Tkac, 2002). Thus, in our leading interpretation of the sponsor as a pension fund manager, F_S is approximately linear, and therefore even modest concavity in U_S^T will result in a V_S that is concave, justifying the assumption made above. In contrast, the literature on equity mutual funds suggests that flow performance relationships are quite convex (e.g., Chevalier and Ellison, 1997). Thus, in our leading interpretation of the fund manager as a mutual fund manager, F_F is likely to be significantly convex, and it is thus a priori unclear whether the concatenation of U_F^T and F_F will result in a V_F which is overall concave or convex. Accordingly, we make no assumption about the curvature of V_F.

Parameter restrictions.

To abstract away from delegation that is driven by efficiency considerations, we make two assumptions. First, we require that fund managers are equally good at asset management as the sponsor:

Assumption 1. *The fund manager and the sponsor are equally skilled in wealth man-*

fund flows fund managers *believe* in.

[12]If fund fees increase less than proportionally with fund size, then we would need to introduce a concave function $w(F)$ here, which would make the case for a concave $V-$function even stronger. Hence, we abstract from this case.

[13]There we also provide a couple of examples. If the utility function over future wealth has constant relative risk aversion with risk aversion parameter ρ, and the cdf F is exponential such that $F(\gamma) = \frac{exp\{\alpha\gamma\}-1}{exp(\alpha)-1}$, we show that V is concave if and only if $\rho > F''/F' = \alpha$, i.e., if absolute risk aversion exceeds the convexity of F.

agement, conditional on type: $\sigma^A_{F,G} = \sigma^A_{S,G}$ *and* $\sigma^A_{F,B} = \sigma^A_{S,B}$.

Second, we also assume that the pool of available fund managers is of strictly lower quality than the average quality of the sponsor, even conditional on the sponsor using her private information to select the best possible fund manager.

Assumption 2. *The likelihood of a good fund manager conditional on the sponsor observing a good signal is lower than the unconditional probability of a good sponsor:*

$$\overline{\gamma}_F \equiv Pr\left(\tau_F = G \middle| s^F_S = G\right) = \frac{\gamma_F \sigma^F_S}{\gamma_F \sigma^F_S + \left(1 - \sigma^F_S\right)\left(1 - \gamma_F\right)} < \gamma_S, \tag{11}$$

where σ^F_S is the probability that the sponsor observes a good signal about the fund manager and defined in (6). The probability $\overline{\gamma}_F$ is the likelihood that the type of the fund manager is $\tau_F = G$, conditional on the sponsor observing a good signal about the manager. It is clear that the sponsor can maximize her chances of matching with a skilled fund manager by only hiring a manager about whom she observes a good signal. Assumptions 1 and 2 jointly ensure that, even if the sponsor uses her own private information to maximize the probability of matching with a good fund manager, the expected quality of the hired fund manager will be lower than the unconditional average quality of the sponsor. This assumption rules out delegation for efficiency reasons, i.e. for increasing the likelihood of making better asset allocation decisions.

4 Delegation and Reputation Risk

Our formal analysis proceeds in two steps. First, in this section, we derive a key result regarding the properties of the reputation profiles of the sponsor, depending on whether she invests directly or whether she delegates (Proposition 1). Proposition 1 plays a key role in the subsequent analysis and holds whenever trading and hiring strategies are *informationally optimal* in the sense defined below. For this section, we assume that the sponsor and the fund manager follow such strategies. In Section 5, we then verify that these strategies are indeed equilibrium outcomes of the trading and hiring subgames and explore the implications of Proposition 1 for inefficient delegation.

Informationally optimal trading and hiring strategies. To begin, we call those strategies that make maximal use of available private information *informationally optimal* trading and hiring strategies, The hiring strategy in which the sponsor hires the first fund manager for whom she observes $s_S^F = G$ is informationally optimal and we denote it by $\widetilde{h}_S = \widetilde{h}^{opt}$. Thus, $\overline{\gamma}_F$, defined in equation (11) above, represents the expected quality of the fund manager conditional on being hired under the informationally optimal hiring rule. Similarly, the informationally optimal trading profile involves buying when the signal is positive ($\widetilde{\theta}_a(1) = u$) and selling when the signal is negative ($\widetilde{\theta}_a(0) = -u$). We denote such trading profiles by $\widetilde{\theta}_a = \widetilde{\theta}^{opt}$ for $a = \{S, F\}$.

With this definition of the informationally optimal hiring strategy and the definition of $\overline{\gamma}_F$ in (11), we can define σ_F^A analogously to σ_S^A (see (3)) as

$$\sigma_F^A \equiv \overline{\gamma}_F \sigma_{F,G}^A + (1 - \overline{\gamma}_F)\,\sigma_{F,B}^A = Pr\left(s_F^A = v^* | \widetilde{v} = v^*\right) = Pr\left(s_F^A = v^* | \widetilde{v} = v^*, \widetilde{h}_S = \widetilde{h}^{opt}\right), \tag{12}$$

which is the probability that the fund manager observes a correct signal about the asset if the sponsor has delegated the investment decision and adopted the informationally optimal hiring strategy, so that conditional on being hired the fund manager is good with probability $\overline{\gamma}_F$. Assumptions 1 and 2 together imply that $\sigma_F^A < \sigma_S^A$, i.e. the probability of observing the correct signal about the asset is lower with delegated investing under $\widetilde{h}_S = \widetilde{h}^{opt}$ compared to direct investing.

Reputation profiles with direct investing. The sponsor initially decides on whether to trade directly or to delegate to a fund manager. If she chooses to invest directly, she will receive a signal about the asset payoff which—given a trading strategy—will induce her to trade. Following her trade, the asset payoff will realize and her posterior reputation will be formed in the labor market. Thus, by choosing to invest directly, the sponsor effectively selects a stochastic reputational profile. The characteristics of the stochastic reputation profile, in turn, depend on her trading strategy. We first characterize the sponsor's reputation profile under direct investment when her trading strategy is informationally optimal, as defined above.

Under direct investment, the labor market bases its inferences about the sponsor on her trade (a purchase u or a sale $-u$) and the realized asset payoff (1 or 0). Under the informationally optimal trading strategy, $\widetilde{\theta}^{opt}$, there is a simple mapping between signals and trades ($\widetilde{\theta}^{opt}(1) = u$, $\widetilde{\theta}^{opt}(0) = -u$) and the observation of the trade is equivalent to the observation of the sponsor's signal. In turn, the symmetry in the signal structure (see equations (2) and (5)) implies that the labor market's inferences are fully determined by whether the signal was correct or not, i.e., whether $s_S^A = v$ or $s_S^A \neq v$.

Denote the *reputation profile* for the sponsor under direct investment and trading strategy $\widetilde{\theta}_S$ by $\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}_S, \widetilde{v}\right)$ by analogy to (9), replacing realized trades θ_S by the strategy $\widetilde{\theta}_S$ and realized asset cash flow v by the random variable $\widetilde{v}$. For the case in which $\widetilde{\theta}_S = \widetilde{\theta}^{opt}$, the discussion above indicates that $\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)$ can take exactly two possible values, depending on whether $s_S^A = v$ or $s_S^A \neq v$. These are described in Lemma 1:

Lemma 1. *If the sponsor invests directly using strategy $\widetilde{\theta}^{opt}$, her realized reputation can take on exactly two possible values as follows:*

$$\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right) = \begin{cases} \gamma_S \frac{\sigma_{S,G}^A}{\sigma_S^A} > \gamma_S & \text{if } s_S^A = v \\ \gamma_S \frac{1-\sigma_{S,G}^A}{1-\sigma_S^A} < \gamma_S & \text{if } s_S^A \neq v \end{cases}, \tag{13}$$

and expected reputation is $E\left[\widetilde{\gamma}_S^T\right] = \gamma_S$.

Hence, if the sponsor trades directly on her own signal, her reputation increases to $\gamma_S \frac{\sigma_{S,G}^A}{\sigma_S^A} > \gamma_S$ if she trades correctly, i.e, if she buys and the asset value is high *ex post*, or if she sells and the asset value is low *ex post*. The probability that the sponsor observes a correct signal is σ_S^A and given from (3). Recall that $\sigma_{S,G}^A$ is the same probability for a good sponsor. If she trades incorrectly, because she buys (sells) when the signal is high (low), but the signal is incorrect and the asset value is low (high), her reputation falls to $\gamma_S \frac{1-\sigma_{S,G}^A}{1-\sigma_S^A} < \gamma_S$, where $1 - \sigma_{S,G}^A$ is the probability that a good sponsor observes an incorrect signal. The sponsor's reputation does not change in expectation, which follows from the law of iterated expectations.

Reputation profiles with delegation. If the sponsor chooses to delegate, she will then receive information about potential fund managers and choose one on the basis of her hiring strategy. This fund manager will then receive a signal, trade on the basis of his strategy, leading to a posterior reputation for both himself and for the sponsor. Thus, by choosing to delegate investment decisions, the sponsor selects a different stochastic reputational profile. Unlike the reputational profile under direct investment given in (13), the reputational profile with delegation also depends on the sponsor's hiring strategy $\widetilde{h}_S$, which becomes an additional argument in $\widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}_S, \widetilde{\theta}_F, \widetilde{v}\right)$. We characterize this profile below for the case in which the sponsor's hiring strategy and the fund manager's trading strategy are both informationally optimal.

Lemma 2. *If the sponsor delegated the investment decision, and hires a fund manager using hiring strategy $\widetilde{h}^{opt}$ who then trades using trading strategy $\widetilde{\theta}^{opt}$, her realized reputation can take on exactly two possible values as follows*

$$\widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right) = \begin{cases} \gamma_S \frac{\Sigma_{S,G}}{\Sigma_S} > \gamma_S & \text{if } s_F^A = v \\ \gamma_S \frac{1-\Sigma_{S,G}}{1-\Sigma_S} < \gamma_S & \text{if } s_F^A \neq v \end{cases}, \tag{14}$$

with

$$\Sigma_{S,G} = \sigma_{F,G}^A \gamma_{F,G} + \sigma_{F,B}^A \left(1 - \gamma_{F,G}\right) \text{ where } \gamma_{F,G} = \frac{\gamma_F \sigma_{S,G}^F}{\gamma_F \sigma_{S,G}^F + \left(1 - \gamma_F\right)\left(1 - \sigma_{S,G}^F\right)} \tag{15}$$

and

$$\Sigma_{S,B} = \sigma_{F,G}^A \gamma_{F,B} + \sigma_{F,B}^A \left(1 - \gamma_{F,B}\right) \text{ where } \gamma_{F,B} = \frac{\gamma_F \sigma_{S,B}^F}{\gamma_F \sigma_{S,B}^F + \left(1 - \gamma_F\right)\left(1 - \sigma_{S,B}^F\right)}, \tag{16}$$

and

$$\Sigma_S = \gamma_S \Sigma_{S,G} + \left(1 - \gamma_S\right) \Sigma_{S,B}. \tag{17}$$

Her expected reputation is $E\left[\widetilde{\gamma}_S^T\right] = \gamma_S$.

Under the informationally optimal hiring strategy, the sponsor hires a fund manager after observing that $s_S^F = G$. Good sponsors observe the correct signal about the fund manager's type with higher probability, and hence have a higher probability than bad sponsors of hiring good fund managers ($\gamma_{F,G} > \gamma_{F,B}$). The hired fund manager trades on the basis of the signal he receives. A good fund manager, in turn, observes the correct signal about the asset payoff more often than a bad fund manager. Thus, since the quality of the sponsor's information determines the quality of the fund manager selected, and—in turn—the quality of trading information and the probability of a correct trade, the labor market can make inferences about the quality of the sponsor based on the trading outcomes of the fund manager. In particular, the probability of a correct *delegated* trade by the fund manager is $\Sigma_{S,G}$ ($\Sigma_{S,B}$) for a good (bad) sponsor, and Σ_S is the average probability of a correct delegated trade. Hence, the sponsor's reputation rises to $\gamma_S \frac{\Sigma_{S,G}}{\Sigma_S} > \gamma_S$ in the event that the fund manager trades correctly, and falls to $\gamma_S \frac{1-\Sigma_{S,G}}{1-\Sigma_S} < \gamma_S$ if the fund manager trades incorrectly. Again, the reputation of the fund manager does not change on average.

The conjunction of the role of two layers of skill gives rise to the more involved expressions in (14) compared to (13). Under direct investment, the sponsor's realized reputation depends on whether her own signal about the asset payoff is correct, which is determined by terms of the form $Pr(s_S^A = v|\tau_S = G) = \sigma_{S,G}^A$ and $Pr(s_S^A = v) = \sigma_S^A$. Under delegated investment, the sponsor's realized reputation depends on whether her chosen fund manager's signal about the asset payoff is correct, which in turn is determine by terms of the form $Pr(s_F^A = v|\tau_S = G) = \Sigma_{S,G}$ and $Pr(s_F^A = v) = \Sigma_S$.

Comparing reputation with direct investment and with delegation. We are now in a position to compare the reputation outcomes for the sponsor with direct investing (Lemma 1) and delegated investing (Lemma 2), which gives rise to our first main result.

Proposition 1. *Under informationally optimal trading and hiring strategies, the sponsor's reputation under direct investment,* $\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)$*, is a mean preserving spread of her reputation under delegated investment,* $\widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)$.

The underlying intuition is grounded in the law of iterated expectations: The sponsor does not affect her reputation *on average* by delegating or investing directly – her expected posterior reputation under either choice is simply her prior reputation. However, delegation enables her to *garble* inferences about herself by involving the skills of a second agent. Forcing market participants to combine two layers of information when assessing the reputation of the fund manager makes this inference noisier, so that the sponsor's reputation after observing trading success (failure) increases (decreases) less compared to the case with direct investing. In particular, as is clear from Lemmas 1 and 2, in the delegated case reputation rises from γ_S to $\gamma_S \frac{\Sigma_{S,G}}{\Sigma_S}$ in case of a correct trade (and falls to $\gamma_S \frac{1-\Sigma_{S,G}}{1-\Sigma_S}$ in case of an incorrect trade) instead rising from γ_S to $\gamma_S \frac{\sigma_{S,G}^A}{\sigma_S^A}$ (or falling to $\gamma_S \frac{1-\sigma_{S,G}^A}{1-\sigma_S^A}$) in the case of direct investment; the proof of Proposition 1 shows that $\gamma_S \frac{\Sigma_{S,G}}{\Sigma_S} < \gamma_S \frac{\sigma_{S,G}^A}{\sigma_S^A}$ and $\gamma_S \frac{1-\Sigma_{S,G}}{1-\Sigma_S} > \gamma_S \frac{1-\sigma_{S,G}^A}{1-\sigma_S^A}$. This reduces the reputational risk the sponsor is exposed to. In the next section, we build on this insight by using the risk aversion of the sponsor, which creates a strategic advantage from delegation by providing protection from reputational risk.

5 Excessive Delegation

In this section, we build on Proposition 1 to delineate conditions under which excessive delegation will arise in equilibrium. Proposition 1 suggests that – if trading and hiring decisions are informationally optimal– the sponsor may be tempted to delegate excessively: since sponsors are career concerned and risk averse over reputation, *all else equal*, they will be tempted to delegate to other agents even when such agents are *less* able and generate lower trading profits than themselves. To translate such a strategic motive into observable implications we need to ensure that excessive delegation can arise *in equilibrium*, i.e., given the strategic choices of the principal, the sponsor, and (if relevant) fund managers. It is, a priori, not at all obvious that this can happen; after all, excessive delegation is unambiguously harmful for the principal, and he – in turn – is able to incentivize the sponsor to potentially forego delegation by giving her a sufficient stake in trading profits.

A further complication is that career concerns, in addition, to fostering incentives to over-delegate, may also *directly* affect the manner in which private information is utilized for trade, making trade itself (over and above delegation) sub-optimal. The effects of career concerns on incentives to under-utilize information in trade has been analyzed, for example, by Dasgupta and Prat [2008]. In order to isolate our analysis from such previously identified effects of career concerns, we impose a restriction on asset payoff distributions, essentially limiting the skewness of traded assets:

Assumption 3. *Available assets are not skewed, i.e.,* $\gamma_A \in \left(1 - \sigma_F^A, \sigma_F^A\right)$.

We shall show below that Assumption 3 implies that the career concerns of sponsors (under direct investment) and fund managers (under delegated investment) do not affect how they trade, thus isolating our analysis from previously identified strategic effects.

By backward induction, we start with the *trading subgames* induced (i) for the sponsor when $a_S = DI$ and (ii) for the fund manager when $a_S = DE$. We show that under Assumption 3, *trade is informationally optimal* in both of those subgames, for (i) *any* incentives the principal may provide to the sponsor, $\kappa_S \in [0, 1]$, (ii) *any* incentives the sponsor may provide to the fund manager, $\kappa_F \in [0, \kappa_S]$ and (iii) and given the optimal hiring strategy $h_S = \widetilde{h}^{opt}$.

Lemma 3. *For any* $\kappa_S \in [0, 1], \kappa_F \in [0, \kappa_S]$ *and for* $h_S = \widetilde{h}^{opt}$*, if* $a_S = DI$ *then* $\theta_S = \widetilde{\theta}^{opt}$ *is an equilibrium in the sponsor's trading subgame, and if* $a_S = DE$ *then* $\theta_F = \widetilde{\theta}^{opt}$ *is an equilibrium in the fund manager's trading subgame.*

Under $a_S = DE$, the fund manager is incentivized by a combination of profit sharing (when $\kappa_F > 0$) and reputation. Given the existence of noise traders, profits are clearly maximized by buying when $s_F^A = 1$ and selling when $s_F^A = 0$, i.e., by choosing $\theta_F = \widetilde{\theta}^{opt}$. The reputational incentive of fund managers is more subtle: his goal is to "look informed," i.e., to not choose a trade that reveals him to have incorrect information ex post (and thus diminishes his reputation). When the prior probability of $\widetilde{v} = 1$ is high or low, the fund manager's perceived probability of ending up with the ex post incorrect trades by following a private signal that "disagrees" with the prior (e.g., selling because $s_F^A = 0$ when γ_A is close to 1) will be high. The fund manager will wish to deviate to buying unconditionally for sufficiently high γ_A and selling unconditionally for sufficiently low γ_A, and it will thus not be possible to sustain $\widetilde{\theta}^{opt}$ as an equilibrium strategy.[14] However, as we show in the proof of Lemma 3, when $\gamma_A \in \left(1 - \sigma_F^A, \sigma_F^A\right)$, such conformism will not arise even if $\kappa_F = 0$, and the fund manager will always wish to trade according to his private information, and hence $\theta_F = \widetilde{\theta}^{opt}$ will be the equilibrium outcome.

It is noteworthy that the relevant bounds on γ_A to guarantee that even a purely reputationally concerned fund manager will choose $\theta_F = \widetilde{\theta}^{opt}$ depend on the quality of the fund manager's information, σ_F^A. This is intuitive, because the more precise is the fund manager's private information, the more extreme must be the prior on $\widetilde{v} = 1$ in order to induce him to ignore his own information.

Under $a_S = DI$, similar arguments apply, but now to the sponsor instead of the fund manager. By Assumptions 1 and 2, the sponsor is, on average, better informed than the fund manager, i.e., $\sigma_S^A > \sigma_F^A$. Thus, under ranges of γ_A for which even the less well-informed fund manager will trade on the basis of his information in equilibrium, the sponsor will certainly trade on the basis of her's. Thus, even for $\kappa_S = 0$, the sponsor will choose $\theta_S = \widetilde{\theta}^{opt}$ in equilibrium for $\gamma_A \in \left(1 - \sigma_F^A, \sigma_F^A\right) \subset \left(1 - \sigma_S^A, \sigma_S^A\right)$. Hence, $\kappa_S > 0$ can only encourage informative trading in the presence of noise traders.

Given that we now know $\theta_S = \widetilde{\theta}^{opt}$ under direct investment and $\theta_F = \widetilde{\theta}^{opt}$ under delegated investment, we can characterize trading profits.

Lemma 4. *1. Under $\theta_a = \widetilde{\theta}^{opt}$ for $a \in \{S, F\}$ and $h_S = \widetilde{h}^{opt}$, expected trading profits*

[14] For a full analysis of such herding incentives, see Dasgupta and Prat [2008].

are given by

$$E\left[\pi\left(\sigma_a^A, \theta_a = \widetilde{\theta}^{opt}\right)\right] = u\gamma_A\left(1-\gamma_A\right)\left(2\sigma_a^A - 1\right). \tag{18}$$

2. *Trading profits are (i) increasing in u and (ii) increasing in σ_a^A.*

3. *Trading profits are always lower with delegation under informationally optimal trading strategies:*

$$\Delta\pi \equiv E\left[\pi\left(\sigma_F^A, \widetilde{\theta}^{opt}\right)\right] - E\left[\pi\left(\sigma_S^A, \widetilde{\theta}^{opt}\right)\right] < 0, \tag{19}$$

4. *$\Delta\pi$ is decreasing in u, $\gamma_S - \overline{\gamma}_F$, and $\sigma_{a,G}^A - \sigma_{a,B}^A$.*

The trading game has the standard structure of binary Kyle-models (e.g., Kyle and Vila, 1991). Hence, trading profits π increase in the liquidity of the market, which is here represented by the amount u traded by noise traders; in the initial uncertainty about the asset payoff $\gamma_A(1-\gamma_A)$; and in the informativeness of the signal, which is measured by $2\sigma_a^A - 1$ and approaches zero as $\sigma_a^A \to 1/2$. Delegation leads to strictly lower trading profits because the expected ability of the fund manager under the informationally optimal hiring strategy, $\overline{\gamma}_F$, is strictly lower than the ability of the sponsor in direct trading by Assumption 2. Accordingly, the reduction in profit from delegation, $\Delta\pi$, increases with the difference $\gamma_S - \overline{\gamma}_F$ in abilities between the sponsor and the fund manager. It also increases with liquidity, which leverages the difference in abilities, and with the differences in ability between good agents ($\sigma_{a,G}^A$) and bad agents ($\sigma_{a,B}^A$).

Next, we turn to the hiring subgame in which the sponsor chooses her hiring strategy and decides how to incentivize the fund manager she hires. These decisions are relevant to determine the overall costs and benefits from delegating investment decisions to a fund manager.

Lemma 5. *For any $\kappa_S \in [0,1]$, if the sponsor chooses $a_S = DE$, then it is an equilibrium action to set $\kappa_F = 0$ and $h_S = \widetilde{h}^{opt}$.*

When the sponsor chooses to delegate, her continuation payoff depends on the profits generated by the fund manager she hires and on the sponsor's reputation generated by such trades. Since the quality of the hired fund manager affects trading outcomes—and thus profits and (sponsor) reputation—the sponsor's hiring strategy affects both continuation profits and reputation. By choosing $h_S = \widetilde{h}^{opt}$, the sponsor maximizes her profits since the fund manager subsequently uses her information optimally by choosing $\theta_F = \widetilde{\theta}^{opt}$ (Lemma 3) and $h_S = \widetilde{h}^{opt}$ maximizes the chance of obtaining a well informed fund manager. Further, since the fund manager chooses the informationally optimal trading strategy $\widetilde{\theta}^{opt}$ for any incentive scheme with $\kappa_F \geq 0$, and since profit sharing with $\kappa_F > 0$ is costly for the sponsor, she optimally chooses $\kappa_F = 0$. Finally, if the labor market believes that the sponsor hires according to $\widetilde{h}^{opt}$, then, given that the fund manager subsequently chooses $\theta_F = \widetilde{\theta}^{opt}$ (Lemma 3), we know from Lemma 2 that the sponsor's continuation reputation from an ex post correct trade by the fund manager is $\gamma_S \frac{\Sigma_{S,G}}{\Sigma_S}$ and from an ex post incorrect trade is $\gamma_S \frac{1-\Sigma_{S,G}}{1-\Sigma_S}$. Since $\gamma_S \frac{\Sigma_{S,G}}{\Sigma_S} > \gamma_S \frac{1-\Sigma_{S,G}}{1-\Sigma_S}$, the sponsor will wish to maximize the ex post probability of correct trades by the fund manager, and can do so by choosing the highest quality fund manager, which is a fund manager about whom she observes signal $s_S^F = G$, i.e., to set $h_S = \widetilde{h}^{opt}$. This, in turn, is consistent with the labor market's belief, thus completing the equilibrium argument.

Lemmas 3 and 5 pin down equilibrium trading and hiring strategies $\theta_F = \widetilde{\theta}^{opt}$ and $h_S = \widetilde{h}^{opt}$. We can now utilize Proposition 1 to establish that delegation has strictly positive implications for the reputation of the sponsor:

Lemma 6. *The sponsor's net reputational benefits from delegation with the optimal hiring strategy over investing directly are always positive:*

$$\Delta V_S \equiv E\left[V_S\left(\widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)\right)\right] - E\left[V_S\left(\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)\right)\right] > 0. \quad (20)$$

Lemma 6 is stated without proof as it follows directly from Proposition 1 and the concavity of V_S.

The results so far show that delegation involves a trade-off from the point of view of the sponsor. Hiring a fund manager involves delegation to an agent with lower ability who delivers lower trading profits. At the same time, hiring a fund manager reduces reputational risk, since it makes trading profits a noisier signal about the sponsor's ability. The sponsor's utility function (10) combines career concerns and trading profits.

To see how the sponsor resolves this trade-off, we calculate her expected utility from direct investment, which follows directly from equation (10) and Lemmas 3 and 4:

$$E\left[U_S\left(\kappa_S \pi_{DI}, \widetilde{\gamma}_S^T\right)\right] = \kappa_S E\left[\pi\left(\sigma_S^A, \theta_S = \widetilde{\theta}^{opt}\right)\right] + \alpha E\left[V_S\left(\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)\right)\right]. \quad (21)$$

Similarly, her expected utility from delegation can be derived from (10), using Lemmas

3, 4, and 5:

$$E\left[U_S\left(\kappa_S \pi_{DE}, \widetilde{\gamma}^T_{S,DE}\right)\right] = \kappa_S E\left[\pi\left(\sigma^A_F, \theta_F = \widetilde{\theta}^{opt}\right)\right] + \alpha E\left[V_S\left(\widetilde{\gamma}^T_S\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)\right)\right]. \quad (22)$$

Hence, using (19) and (20), the net gain in expected utility from delegation can be expressed as:

$$\begin{aligned} \Delta U_S &\equiv E\left[U_S\left(\kappa_S \pi_{DE}, \widetilde{\gamma}^T_{S,DE}\right)\right] - E\left[U_S\left(\kappa_S \pi_{DI}, \widetilde{\gamma}^T_{S,DI}\right)\right] \\ &= \kappa_S \Delta\pi + \alpha \Delta V_S \end{aligned}, \quad (23)$$

which represents the net benefit from delegation to the sponsor. Note that ΔU_S is decreasing in κ_S, since $\Delta\pi < 0$ from (19), and increasing in α, since $\Delta V_S > 0$ from (20). Hence, we observe:

Lemma 7. *(i) For any u and κ_S, there exists a critical value for α, α_D, such that the sponsor is indifferent between direct investment and delegation for $\alpha = \alpha_D$. For $\alpha > \alpha_D$, the sponsor strictly prefers delegation to direct investing and for $\alpha < \alpha_D$ the sponsor prefers direct investment. (ii) α_D increases in u and κ_S and decreases in $\overline{\gamma}_F$.*

Lemma 7 is intuitive. Simply put, how the sponsor resolves the trade-off between the reputational benefits ΔV_S and the foregone trading profits from delegation depends on her share κ_S in the trading profits and the weight α career concerns have in her utility function. Since profits are bounded, for any κ_S, there exists α large enough such that $\Delta U_S > 0$ and the sponsor prefers delegation. Similarly, for α sufficiently small, (19) implies that $\Delta U_S < 0$ and the sponsor prefers direct investment. Hence, by continuity, a critical value α_D such that $\Delta U_S = 0$ must exist. Part (ii) of Lemma 7 follows from the second part of Lemma 4 and observing that reputational concerns $\widetilde{\gamma}^T_S$ do not depend on u and $\overline{\gamma}_F$, whereas $\widetilde{\gamma}^T_S\left(a_S = DE, \theta_F = \theta^{opt}\right)$ increases in $\overline{\gamma}_F$.

Given the results shown so far in this section, we are now in a position to solve for how the principal influences the choice of the sponsor. From Lemma 7, the principal can provide sufficient incentives κ_S to the sponsor to prevent her from delegating investment decisions to a fund manager. In this case, the net payoff of direct investment to the principal is

$$\left(1 - \kappa_S\right) E\left[\pi\left(\sigma^A_S, \theta_S = \widetilde{\theta}^{opt}\right)\right].$$

By contrast, if she sets κ_S below the value for which the sponsor foregoes delegation, then it is optimal to set $\kappa_S = 0$ and receive the entire benefits from delegated investment, which are

$$E\left[\pi\left(\sigma_F^A, \theta_F = \widetilde{\theta}^{opt}\right)\right].$$

Hence, the principal chooses between receiving all of the trading profits from delegation, or only a fraction $1 - \kappa_S$ from the higher trading profits from direct investing by the more competent sponsor. Based on these considerations and the Lemmas proved in this subsection, we obtain our main result.

Proposition 2. *(i) For a given value of u, and α large enough, or for a given value of α and u small enough, the principal prefers delegation to direct investment; then the principal chooses $\kappa_S = 0$. (ii) If the principal chooses direct investment, he chooses $\kappa_S = \kappa_S^*$ such that the sponsor is indifferent between direct investment to delegation and chooses direct investment. (iii) κ_S^* decreases in u and increases in α.*

Hence, the principal either chooses to set the sponsor's incentives very low, i.e. $\kappa_S = 0$, and allow delegation, or he chooses κ_S just high enough for the sponsor to prefer direct investment. As α, the importance of reputation to the sponsor rises, it becomes more expensive to incentivize the sponsor to invest directly: the principal would need to provide an ever higher profit share κ_S to ensure direct investment. So, fixing u, there will be α high enough such that it will be optimal for the principal to allow delegation. Similarly, fixing α, as u shrinks the profit pool with which the sponsor can be incentivized to invest directly also shrinks. Thus, for sufficiently low u, the principal will again permit delegation.

6 Discussion and empirical implications

The predictions for excessive delegation depend critically on two parts of the model. First, the shape of the V_S-function, which is important for the impact of reputational risk (Section 6.1). The second part is the size of expected profits $E[\pi]$, because excessive delegation obtains whenever expected profits are too low to make it worthwhile for the principal to foreclose delegation by providing the sponsor with sufficient incentives (Section 6.2). We discuss both parts in this section with a view to exploring the empirical implications of our analysis.

6.1 Convex preferences over reputation and the length of delegation chains

If the principal chooses delegation over direct investment given the conditions in Proposition 2, then the fund manager may in turn choose to either manage the assets himself or delegate to yet another fund manager. Consider an extension of the model above in which each fund manager can decide to either invest directly or to delegate to another fund manager. For simplicity, assume that the skills of each fund manager to identify the skills of another fund manager are equal to those of the sponsor in the baseline model and parameterized by σ^F_{S,τ_S} (see equation (4)), and that the distribution of asset management skills of all fund managers is given by σ^A_{F,τ_F} (see equation (7)). Each fund manager chooses whether to manage the assets himself or whether to delegate the task further and have his reputation evaluated based on his delegation decision. Then the logic of Proposition 2 would apply to all fund managers, who would delegate to other fund managers, set the incentive parameter $\kappa = 0$, and benefit from the "garbling" effect described in Proposition 1.[15] The chain of delegation decisions would stop whenever a fund manager has $\Delta V < 0$ (see conditions (20) and (20)), i.e., with a fund manager whose career concerns derive from a winner-takes-all market so that his $V-$function becomes convex, most likely a mutual fund manager based on our discussion above. (See the discussion of the microfoundations of the V_S-function in Section 3.)

6.2 Expected profits

Our model generates empirical predictions on when to expect excessive delegation. It is noteworthy that these represent joint predictions on the characteristics of wealth managers and of the assets they trade. For example, the profitability achievable from trade in a particular asset is a function of both asset characteristics (e.g., market depth) and of the information quality of those trading the asset (e.g., the average information quality of wealth managers), a wealth manager characteristic. Similarly, the profit share that must be given to the sponsor to disincentivize delegation depends on the degree to which she is career concerned, a characteristic specific to the individual wealth manager (e.g., how young or old she may be), but also affected by the overall labor market for asset managers (e.g., how often and how carefully such managers are evaluated).

[15]Note that the first fund manager delegated to has $\kappa_S = 0$ from Proposition 2. Hence, he would likely also lack the resources to provide incentives.

In our model, the expected profits $E[\pi]$ may also be interpreted as profits per fund. Assume we can conceive of the market for fund management as a Grossman and Stiglitz [1980] economy in which the number of funds is given from the condition that the marginal fund just recovers its fixed costs F of information acquisition, so that for the marginal fund, $E[\pi] = F$. From this perspective, inefficient delegation is more likely to obtain in markets in which fixed costs, and hence, profits, are low. These are likely to be markets dominated by a potentially large number of small funds. Similarly, inefficient delegation should be less likely in markets dominated by a potentially small number of large funds, because in these markets funds can better afford the contracting costs required to prevent inefficient delegation. Under this interpretation of our analysis, technical progress in fund management that increases returns to scale in fund management should lead to a reduction in inefficient delegation. For example, Philippon [2019] argues that robo-advisors increase fixed entry costs.

7 The limits of excessive delegation

In the discussion so far, we require that sponsors and fund managers are *ex ante* equally good at wealth management (Assumption 1), and that, even conditional on the sponsor using her signal to choose fund managers, the pool of fund managers is of lower quality than the average sponsor (Assumption 2). We now explore the importance of these assumptions by first dropping Assumption 1 and subsequently dropping Assumption 2. Hence, to begin, we permit that fund managers can be superior or inferior to sponsors and ask under which conditions delegation may lead to *more* reputational risk than direct investment. Specifically, we want to investigate whether it is possible that the realizations of the sponsor's reputation with direct investment, $\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)$, are more extreme than those from delegation, $\widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)$. From Lemmas 1 and 2, this is equivalent to asking whether

$$\Sigma_{S,G} > \sigma_{S,G}^A \text{ and } \Sigma_{S,B} < \sigma_{S,B}^A, \tag{24}$$

which would invert the rankings established in Proposition 1. The next result shows that this is indeed the case.

Lemma 8. *Under informationally optimal trading and hiring strategies, the sponsor's reputation under delegated investment,* $\widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)$*, is a mean preserving* *spread of her reputation under direct investment,* $\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)$ *if*

$$\sigma_{F,G}^A > \sigma_{S,G}^A + \frac{1-\gamma_F}{\gamma_F}\frac{1-\sigma_{S,G}^F}{\sigma_{S,G}^F}\left(\sigma_{S,G}^A - \sigma_{F,B}^A\right) > \sigma_{S,G}^A \tag{25}$$

and if

$$\sigma_{F,B}^A < \sigma_{S,B}^A + \frac{\gamma_F}{1-\gamma_F}\frac{\sigma_{S,B}^F}{1-\sigma_{S,B}^F}\left(\sigma_{S,B}^A - \sigma_{F,G}^A\right) < \sigma_{S,B}^A. \tag{26}$$

(ii) Conditions (25) and (26) can be satisfied for a generic set of parameters. This set is increasing in $\sigma_{S,G}^F$ *and decreasing in* $\sigma_{S,B}^F$*.*

Conditions (25) and (26) imply that there is greater dispersion in the abilities of fund managers than in the abilities of sponsors, i.e., good fund managers are better than good sponsors, and bad fund managers are also worse than bad sponsors. Hence, delegation creates reputational risk if fund managers' skills are sufficiently more heterogeneous than those of sponsors. If fund managers have more dispersed skills, then the market can make more accurate inferences about the fund manager's skills from observing trading outcomes. However, conditions (25) and (26) also imply that good fund managers have to be better, and bad fund managers worse by a sufficient margin, i.e., there must be a sufficient wedge between σ_{F,τ_F}^A and σ_{S,τ_S}^A. The reason is that the case of reducing reputational risk through delegation (Proposition 1) and the case of increasing reputational risk (Lemma 8) are not symmetric: Delegation always garbles information and, therefore, always has a tendency to reduce reputational risk. The opposite result requires a sufficiently strong countervailing effect, which obtains only if the fund manager is drawn from a sufficiently heterogeneous pool.

How big the required wedge between σ_{F,τ_F}^A and $\sigma_{S,\tau_S}^A$is depends on the skills of sponsors to identify good fund managers. More precisely, if the sponsor's delegation decisions does not reveal much about her type ($\sigma_{S,G}^F \approx \sigma_{S,B}^F$), then fund managers' skills may be very diverse without creating reputational risk. Conversely, if sponsors differ a lot in their abilities to pick good fund managers ($\sigma_{S,G}^F$ is close to 1 and $\sigma_{S,B}^F$ is close to 1/2), then the required wedge between σ_{F,τ_F}^A and σ_{S,τ_S}^Abecomes small, as differences in estimated fund manager skills lead to better inferences about the plan sponsor. Hence, the dispersion of fund managers' asset management skills and those of sponsors' hiring skills are substitutes. Note that Lemma 8 does not depend on whether Assumption 2 holds or not, because reputational risk depends only on the dispersion of skills and not on their average. In our model, the consequences for the agent (sponsor) take the form of a reputation update and are always neutral from the martingale properties of updating reputation (Lemmas 1 and 2). However, the agent prefers less information to be released because of risk aversion.[16]

If conditions (25) and (26) are satisfied, then delegation creates reputational risk rather than avoiding it, and ΔV as defined in Lemma 6 becomes negative. Such a scenario would only become a concern for efficiency if trading profits would be higher with delegation than without, i.e., if the left-hand side of (19) would become positive, because this would open the door to the possibility of too *little* delegation. For this scenario to obtain, we now also drop Assumption 2 and consider the possibility that $\gamma_S \leq \overline{\gamma}_F$, i.e., that fund managers have on average higher asset management skills than sponsors, which creates benefits from delegation through specialization.

Proposition 3. *(i) If $\gamma_S \leq \overline{\gamma}_F$, then there exists a generic set of parameters such that conditions (25) and (26) are satisfied and the trading profits are higher with delegation than with direct investment ($E\left[\pi\left(\sigma_F^A, \theta_F = \widetilde{\theta}^{opt}\right)\right] > E\left[\pi\left(\sigma_S^A, \theta_S = \widetilde{\theta}^{opt}\right)\right]$).*

(ii) For any parameterization that satisfies conditions (25) and (26) and for which trading profits are higher with delegation than with direct investment, and for any u and κ_S, there exists a critical value for α, α_D, such that the sponsor is indifferent between direct investment and delegation for $\alpha = \alpha_D$. For $\alpha < \alpha_D$, the sponsor strictly prefers delegation to direct investing and for $\alpha > \alpha_D$ the sponsor prefers direct investment.

Part (i) of Proposition 3 in itself is predictable, since it shows only that, for some parameters, there is an upside from delegating to a superior fund manager even if this manager is drawn from a very heterogeneous pool. The second part of Proposition is more surprising: Delegation may not be chosen even if it increases trading profits. The reason is that the assumed pool of fund managers is not only superior but also significantly more heterogeneous. As a result, trading profits increase, but so does reputational volatility, and if the latter concern has sufficient weight, then delegation may not be chosen.

Note that the conditions for delegation to be optimal, but not chosen (Proposition 3(ii)) are much more restrictive than those for delegation to be chosen when it is not optimal (Proposition 2). The reason is, again, that these cases are not symmetric. Delegation by itself garbles information and helps sponsors to camouflage their unknown asset management skills and thus reduce their reputational risk. Hence, reputational risk creates a tendency towards excessive delegation, as discussed in Section

[16] As such, our model is closer to Breeden and Viswanathan [2016].

5. The opposite, a suboptimal lack of delegation obtains from combining three conditions: (1) a pool of fund managers with superior asset management skills; (2) a sufficiently heterogeneous pool of fund managers; and (3) a pool of plan sponsors with sufficiently heterogeneous hiring skills. The first conditions leads to higher trading profits from delegation, while the last two conditions together generate reputational risk from delegation. Some combination of all three assumptions is required to counteract the tendency of reputational risk to create excessive delegation.

8 Conclusion

We study a model in which a principal hires a wealth manager, who may in turn delegate asset allocation decisions to a fund manager. In the baseline model, the fund manager has lower investing abilities than the sponsor herself, but delegation may still occur in equilibrium because it makes the reputation of the sponsor less volatile. Since the market attributes inferior performance partially to the lower ability to the fund manager, and hence, only partially to the lower ability of the sponsor to screen fund managers, delegation introduces "another layer of people to blame," which reduces the sponsor's reputational risk. As a result, we may observe inefficient delegation that serves no productive purpose and emerges only to help sponsors to manage their career concerns.

While we focus our analysis on the institutional money management industry, the argument is, in fact, broader. The financial industry features many structures such as funds of funds, independent advisors, special-purpose acquisition companies (SPACs), and similar intermediaries in which multiple layers of financial institutions intermediate the ownership of primary securities by ultimate investors. While such intermediaries may have many legitimate purposes, such as providing liquidity, diversification, and improved corporate governance through concentrating ownership, our theory highlights a potential dark side of such ownership chains: Even absent efficiency considerations, excessively long ownership chains may occur if the decision to offload decisions to other agents itself is non-contractible.

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A Appendix

A.1 Microfounding the V_S-function

To see the claim made in the text more formally, recall that $V_S\left(\gamma_S^T\right) \equiv U_S^T\left(wF\left(\gamma_S^T\right)\right)$. In the following, we omit subscripts and superscripts for ease of exposition. Then

$$\frac{dV}{d\gamma} = \frac{\partial U}{\partial W} w \frac{\partial F}{\partial \gamma}$$

and

$$\frac{d^2V}{d\gamma^2} = \frac{\partial^2 U}{\partial W^2} w \frac{\partial F}{\partial \gamma} + \frac{\partial U}{\partial W} w \frac{\partial^2 F}{\partial \gamma^2} < 0 \Leftrightarrow \qquad (27)$$

$$-\frac{\partial^2 U}{\partial (W)^2} \Big/ \frac{\partial U}{\partial W} > \frac{\partial^2 F}{\partial \gamma^2} \Big/ \frac{\partial F}{\partial \gamma}.$$

The left-hand side is the familiar coefficient of absolute risk aversion, whereas the ratio on the right measures the convexity of the cdf F. Hence, if the sponsor is sufficiently risk averse relative to the convexity of F, then the $V-$function is concave. It is sufficient if condition (27) holds for the interval $\left[\gamma_S \frac{1-\sigma_{S,G}^A}{1-\sigma_S^A}, \gamma_S \frac{\sigma_{S,G}^A}{\sigma_S^A}\right]$ given by condition (13); the first two derivatives of F must exist for this argument to hold. For example, if F is uniform, then $\frac{\partial^2 F}{\partial \gamma^2} = 0$ and V is concave for any concave utility function U.

Example 1: exponential functions. Assume the utility function has constant absolute risk aversion, $U(W) = exp\{-\rho W\}$ with absolute risk aversion parameter $\rho > 0$, and the cdf is $F(\gamma) = \frac{exp\{\alpha\gamma\}-1}{exp(\alpha)-1}$ for some $\alpha \neq 0$. (We could set $F(\gamma) = \gamma$, the uniform distribution, for $\alpha = 0$.) The cdf F is convex for $\alpha > 0$ and concave for $\alpha < 0$. With $W = wF(\gamma)$, condition (27) becomes

$$\rho > \alpha.$$

This condition will always be satisfied and V is concave if the convexity of F, parameterized by α is sufficiently low. Conversely, V is convex if $\alpha > \rho$ is large, i.e. if most of the probability mass of F is shifted to the right, which reflects a winner-takes-all contest in which only very high-reputation sponsors will be successful in gaining future employment as pension fund managers.

Example 2: power functions. Assume the utility function has constant relative risk aversion, $U(W) = \frac{W^{1-\rho}}{1-\rho}$,and the cdf F is a power function, $F(\gamma) = \gamma^\alpha$ for some

$\alpha > 0$. F is convex for $\alpha > 1$ and concave for $\alpha < 1$. ($\alpha = 1$ is the uniform distribution.) With $W = wF(\gamma)$, condition (27) becomes

$$\rho > w(\alpha - 1)\gamma^{\alpha-1}.$$

This condition will always be satisfied if F is concave. If F is convex, the right-hand side is increasing in γ and maximized for $\gamma = 1$, hence, $\rho > w(\alpha - 1)$ is sufficient for V to be concave. As in the previous example, V is convex if α is sufficiently large and the cdf reflects a winner-takes-all contest.

A.2 Proofs

Proof of Lemma 1:

First, we derive the reputation under direct investment $\gamma_S^T\left(a_S = DI, \theta_S = \tilde{\theta}^{opt}, \tilde{v}\right)$ under informationally optimal strategies $\theta_S = \tilde{\theta}^{opt}$. Then we can write

$$Pr\left(Buy\,|v = 1,\ \tau_S\right) = Pr\left(s_S^A = 1\,|v = 1,\ \tau_S\right) = \sigma_{S,\tau_S}^A, \tag{28}$$

and, analogously, $Pr\left(Sell\,|v = 0,\ \tau_S\right) = \sigma_{S,\tau_S}^A$ for sells. Reputation depends on whether the sponsor chose correctly or incorrectly, i.e., on whether $s_S^A = v$ or $s_S^A \neq v$. Given trading success, the sponsor attains the following reputation:

$$\gamma_S^T\left(a_S = DI, \tilde{\theta}^{opt}, \tilde{v}\right) = \frac{Pr\left(s_S^A = v|\tau_S = G\right) Pr\left(\tau_S = G\right)}{Pr\left(s_S^A = v\right)}, \tag{29}$$

where all probabilities are evaluated for the case with direct investing. Hence, use (28) to rewrite the first part of the numerator of (29) as

$$\begin{aligned} Pr\left(s_S^A = v|\tau_S = G\right) =& Pr\left(s_S^A = 1|v = 1, \tau_S = G\right) Pr\left(v = 1\right) \\ &+ Pr\left(s_S^A = 0|v = 0, \tau_S = G\right) Pr\left(v = 0\right) \\ =& \sigma_{S,G}^A\gamma_A + \sigma_{S,G}^A\left(1 - \gamma_A\right) = \sigma_{S,G}^A. \end{aligned} \tag{30}$$

and analogously for $\tau_S = B$, $Pr\left(s_S^A = v|\tau_S = B\right) = \sigma_{S,B}^A$. Then rewrite the denominator of (29) as

$$\begin{aligned} Pr\left(s_S^A = v\right) =& Pr\left(s_S^A = v|\tau_S = G\right) Pr\left(\tau_S = G\right) \\ &+ Pr\left(s_S^A = v|\tau_S = B\right) Pr\left(\tau_S = B\right) \\ =& \sigma_{S,G}^A \gamma_S + \sigma_{S,B}^A \left(1 - \gamma_S\right) = \sigma_S^A \end{aligned} \tag{31}$$

from equation (3). Insert (30), (31) and $Pr\left(\tau_S = G\right) = \gamma_S$ into (29) to obtain the first line of (13).

Given failure, the sponsor attains the following reputation:

$$\gamma_S^T\left(a_S = DI, \tilde{\theta}^{opt}, \tilde{v}\right) = \frac{Pr\left(s_S^A \neq v|\tau_S = G\right) Pr\left(\tau_S = G\right)}{Pr\left(s_S^A \neq v\right)}. \tag{32}$$

Rewrite the first part of the numerator of (32) as

$$\begin{aligned} Pr\left(s_S^A \neq v|\tau_S = G\right) =& Pr\left(s_S^A = 1|v = 0, \tau_S = G\right) Pr\left(v = 1\right) \\ &+ Pr\left(s_S^A = 0|v = 1, \tau_S = G\right) Pr\left(v = 0\right) \\ =& \left(1 - \sigma_{S,G}^A\right) \gamma_A + \left(1 - \sigma_{S,G}^A\right) \left(1 - \gamma_A\right) = 1 - \sigma_{S,G}^A. \end{aligned} \tag{33}$$

and analogously for $\tau_S = B$. Then rewrite the denominator of (32) as

$$Pr\left(s_S^A \neq v\right) = 1 - Pr\left(s_S^A = v\right) = 1 - \sigma_S^A \tag{34}$$

from (31). Reinserting both expressions into (32) gives the second line of (13). Since $\sigma_{S,G}^A > \sigma_S^A > \sigma_{S,B}^A$ from equations 2 and (3), it is immediate that $\gamma_S \frac{\sigma_{S,G}^A}{\sigma_S^A} > \gamma_S$ and $\gamma_S \frac{1-\sigma_{S,G}^A}{1-\sigma_S^A} < \gamma_S$.

Finally, we show that $E\left[\gamma_S^T\left(a_S = DI, \tilde{\theta}^{opt}, \tilde{v}\right)\right] = \gamma_S$. The sponsor either observes a correct signal about the asset, with probability σ_S^A, resulting in a reputation of $\gamma_S \frac{\sigma_{S,G}^A}{\sigma_S^A}$; or she observes an incorrect signal, with probability $1 - \sigma_S^A$, resulting in a reputation of $\gamma_S \frac{1-\sigma_{S,G}^A}{1-\sigma_S^A}$. Thus, her expected reputation is:

$$E\left[\gamma_S^T\left(a_S = DI, \tilde{\theta}^{opt}, \tilde{v}\right)\right] = \underbrace{Pr\left(s_S^A = v\right)}_{\sigma_S^A} \underbrace{\gamma_S \frac{\sigma_{S,G}^A}{\sigma_S^A}}_{Pr\left(\tau_S = G|s_S^A = v\right)} + \underbrace{Pr\left(s_S^A \neq v\right)}_{1-\sigma_S^A} \underbrace{\gamma_S \frac{1 - \sigma_{S,G}^A}{1 - \sigma_S^A}}_{Pr\left(\tau_S = G|s_S^A \neq v\right)} = \gamma_S. \tag{35}$$

This completes the proof of the lemma.■

Proof of Lemma 2:

Inferences about the sponsor based on the observed trade of the fund manager can take only two possible values depending on whether $s_F^A = v$ or $s_F^A \neq v$. For the former case, the realized reputation of the sponsor is:

$$Pr\left(\tau_S = G | s_F^A = v\right) = \frac{Pr\left(s_F^A = v | \tau_S = G\right) Pr\left(\tau_S = G\right)}{Pr\left(s_F^A = v\right)}, \tag{36}$$

and similarly for $s_F^A \neq v$. To simplify notation, let the probability of a correct delegated trade for a good (bad) sponsor be $\Sigma_{S,G} := Pr\left(s_F^A = v | \tau_S = G\right)$ ($\Sigma_{S,B} := Pr\left(s_F^A = v | \tau_S = B\right)$). Similarly, let the probability of a good (bad) sponsor to hire a good fund manager be $\gamma_{F,G} := Pr\left(\tau_F = G | s_S^F = G, \tau_S = G\right)$ ($\gamma_{F,B} := Pr\left(\tau_F = G | s_S^F = G, \tau_S = B\right)$). We make two observations. First, the effect of the sponsor's type enters into the conditional probability $Pr\left(s_F^A = v | \tau_S = G\right)$ only via the effect of the sponsor's ability to select a good fund manager. Second, under the informationally optimal hiring strategy, if a fund manager has been hired, then the sponsor must have observed signal $s_S^F = G$. Together, these two observations give us the probability of a correct delegated trade for a good sponsor:

$$\begin{aligned}\Sigma_{S,G} = Pr\left(s_F^A = v | \tau_S = G\right) =& Pr\left(s_F^A = v | \tilde{v}, \tau_F = G\right) Pr\left(\tau_F = G | s_S^F = G, \tau_S = G\right) \\ &+ Pr\left(s_F^A = v | \tilde{v}, \tau_F = B\right) Pr\left(\tau_F = B | s_S^F = G, \tau_S = G\right) \\ =& \sigma_{F,G}^A \gamma_{F,G} + \sigma_{F,B}^A \left(1 - \gamma_{F,G}\right).\end{aligned} \tag{37}$$

Analogously, for a bad sponsor:

$$\begin{aligned}\Sigma_{S,B} = Pr\left(s_F^A = v | \tau_S = B\right) =& Pr\left(s_F^A = v | \tilde{v}, \tau_F = G\right) Pr\left(\tau_F = G | s_S^F = G, \tau_S = B\right) \\ &+ Pr\left(s_F^A = v | \tilde{v}, \tau_F = B\right) Pr\left(\tau_F = B | s_S^F = G, \tau_S = B\right) \\ =& \sigma_{F,G}^A \gamma_{F,B} + \sigma_{F,B}^A \left(1 - \gamma_{F,B}\right).\end{aligned} \tag{38}$$

Consider any stage of the hiring subgame in which the sponsor evaluates a fund manager and either observes a good signal, in which case a match is established and the hiring subgame ends, or the sponsor observes a bad signal, and the game continues. Any stage of the hiring game ends with a good sponsor hiring a good fund manager with probability

$$
\begin{aligned}
Pr\left(s_S^F = G, \tau_S = G, \tau_F = G\right) =& Pr\left(s_S^F = G | \tau_S = G, \tau_F = G\right) Pr\left(\tau_S = G\right) Pr\left(\tau_F = G\right) \\
=& \sigma_{S,G}^F \gamma_S \gamma_F .
\end{aligned}
\tag{39}
$$

Similarly, the probability of a good sponsor hiring a bad fund manager is:

$$
\begin{aligned}
Pr\left(s_S^F = G, \tau_S = G, \tau_F = B\right) =& Pr\left(s_S^F = G | \tau_S = G, \tau_F = B\right) Pr\left(\tau_S = G\right) Pr\left(\tau_F = B\right) \\
=& \left(1 - \sigma_{S,G}^F\right) \gamma_S \left(1 - \gamma_F\right) .
\end{aligned}
\tag{40}
$$

Hence, the probability that the hiring subgame ends with a good sponsor hiring a fund manager, good or bad, is

$$
\begin{aligned}
Pr\left(s_S^F = G, \tau_S = G\right) =& Pr\left(s_S^F = G, \tau_S = G, \tau_F = G\right) + Pr\left(s_S^F = G, \tau_S = G, \tau_F = B\right) \\
=& \gamma_S \left(\sigma_{S,G}^F \gamma_F + \left(1 - \sigma_{S,G}^F\right) \left(1 - \gamma_F\right)\right) .
\end{aligned}
\tag{41}
$$

The corresponding probabilities with a bad sponsor are:

$$
\begin{aligned}
Pr\left(s_S^F = G, \tau_S = B\right) =& Pr\left(s_S^F = G, \tau_S = B, \tau_F = G\right) + Pr\left(s_S^F = G, \tau_S = B, \tau_F = B\right) \\
=& \left(1 - \gamma_S\right) \gamma_F \sigma_{S,B}^F + \left(1 - \gamma_S\right) \left(1 - \gamma_F\right) \left(1 - \sigma_{S,B}^F\right) \\
=& \left(1 - \gamma_S\right) \left(\gamma_F \sigma_{S,B}^F + \left(1 - \gamma_F\right) \left(1 - \sigma_{S,B}^F\right)\right) .
\end{aligned}
\tag{42}
$$

Hence, conditional on the type of the sponsor, the probability of hiring a good fund manager is:

$$
\gamma_{F,G} = Pr\left(\tau_F = G | s_S^F = G, \tau_S = G\right) = \frac{\gamma_F \sigma_{S,G}^F}{\gamma_F \sigma_{S,G}^F + \left(1 - \gamma_F\right) \left(1 - \sigma_{S,G}^F\right)} \tag{43}
$$

for a good sponsor, respectively,

$$
\gamma_{F,B} = Pr\left(\tau_F = G | s_F^S = G, \tau_S = B\right) = \frac{\gamma_F \sigma_{S,B}^F}{\gamma_F \sigma_{S,B}^F + \left(1 - \gamma_F\right) \left(1 - \sigma_{S,B}^F\right)} \tag{44}
$$

for a bad sponsor. It is easy to see that, $\sigma_{S,G}^F > \sigma_{S,B}^F \Leftrightarrow \gamma_{F,G} > \gamma_{F,B}$. Then, given $\sigma_{F,G}^A > \sigma_{F,B}^A$, $\gamma_{F,G} > \gamma_{F,B} \Leftrightarrow \Sigma_{S,G} > \Sigma_{S,B}$.

Combining (15) and (16), the unconditional probability of observing a correct trade (signal) is:

$$\begin{aligned} Pr\left(s_F^A = v\right) &= Pr\left(s_F^A = v|\tau_S = G\right) Pr\left(\tau_S = G\right) + Pr\left(s_F^A = v|\tau_S = B\right) Pr\left(\tau = B\right) \\ &= \Sigma_{S,G}\gamma_S + \Sigma_{S,B}\left(1-\gamma_S\right) = \Sigma_S. \end{aligned} \tag{45}$$

We can now calculate the reputation of the sponsor after a correct trade as the probability that the sponsor is good, conditional on a correct trade:

$$\begin{aligned} \widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, s_F^A = v\right) &= Pr\left(\tau_S = G|s_F^A = v\right) \\ &= \frac{Pr\left(s_F^A = v|\tau_S = G\right) Pr\left(\tau_S = G\right)}{Pr\left(s_F^A = v\right)} \\ &= \gamma_S \frac{\Sigma_{S,G}}{\gamma_S \Sigma_{S,G} + \left(1-\gamma_S\right)\Sigma_{S,B}} = \gamma_S \frac{\Sigma_{S,G}}{\Sigma_S}. \end{aligned}$$

Since $\Sigma_{S,G} > \Sigma_{S,B}$, we have that $\Sigma_{S,G} > \Sigma_S$ and $\gamma_S \frac{\Sigma_{S,G}}{\Sigma_S} > \gamma_S$. The establishment of

$$\begin{aligned} \widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, s_F^A \neq v\right) &= Pr\left(\tau_S = G|s_F^A \neq v\right) \\ &= \gamma_S \frac{1-\Sigma_{S,G}}{\gamma_S\left(1-\Sigma_{S,G}\right) + \left(1-\gamma_S\right)\left(1-\Sigma_{S,B}\right)} \\ &= \gamma_S \frac{1-\Sigma_{S,G}}{1-\Sigma_S} < \gamma_S \end{aligned}$$

follows analogously from (15) and (16) (observe that $Pr\left(s_F^A \neq v|\tau_S\right) = 1 - \Sigma_{S,\tau_S}$) and is not repeated here.

Finally, we show that $E\left[\gamma_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)\right] = \gamma_S$. The sponsor either hires a fund manager who observes a correct signal about the asset, with probability Σ_S, resulting in a reputation of $\gamma_S \frac{\Sigma_{S,G}}{\Sigma_S}$; or she hires a fund manager who observes an incorrect signal, with probability $1-\Sigma^S$, resulting in a reputation of $\gamma_S \frac{1-\Sigma_{S,G}}{1-\Sigma_S}$. Thus, her expected reputation is:

$$E\left[\gamma_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)\right] = \Sigma_S \gamma_S \frac{\Sigma_{S,G}}{\Sigma_S} + \left(1-\Sigma^S\right)\gamma_S \frac{1-\Sigma_{S,G}}{1-\Sigma_S} = \gamma_S. \tag{46}$$

■

We prove Proposition 1 by first showing the following Lemma:

Lemma 9. *Consider two random variables* $\widetilde{x}_1 \equiv (p_1, x_1; 1-p_1, x_4)$ *and* $\widetilde{x}_2 \equiv (p_2, x_2; 1-p_2, x_3)$ *with the same expectation* $E[\widetilde{x}_1] = E[\widetilde{x}_2] = \mu$. *Assume that the realizations of* $\widetilde{x}_1$ *are more extreme than those of* $\widetilde{x}_2$ *such that* $x_1 < x_2 < x_3 < x_4$. *Then* $\widetilde{x}_1$ *is a mean-*

preserving spread of $\widetilde{x}_2$ and $\widetilde{x}_2$ second-order stochastically dominates $\widetilde{x}_1$.

Proof of Lemma 9:

To show second-order stochastic dominance, denote the cdf of $\widetilde{x}_1$ by F_1 and that of $\widetilde{x}_2$ by F_2. We have

$$F_1(x_1) = F_1(x_2) = F_1(x_3) = 1 - p_1 \text{ and } F_1(x_4) = 1,$$

and

$$F_2(x_1) = 0, F_2(x_2) = 1 - p_2, \text{ and } F_2(x_3) = F_2(x_4) = 1.$$

We need to show that

$$\sum_{j=1}^{i} \left(F_1(x_j) - F_2(x_j)\right)\left(x_{j+1} - x_j\right) \geq 0 \text{ for } i = 1, 2, 3. \tag{47}$$

We use the following conditions:

$$E\left[\widetilde{x}_1\right] = p_1 x_4 + \left(1 - p_1\right) x_1 = \mu \tag{48}$$

and

$$E\left[\widetilde{x}_2\right] = p_2 x_3 + \left(1 - p_2\right) x_2 = \mu. \tag{49}$$

Case $i = 1$:

$$\left(1 - p_1\right)\left(x_2 - x_1\right) \geq 0.$$

This case is trivially satisfied since $x_2 > x_1$ by assumption.

Case $i = 3$:

$$\left(1 - p_1\right)\left(x_2 - x_1\right) + \left(p_2 - p_1\right)\left(x_3 - x_2\right) - p_1\left(x_4 - x_3\right) \geq 0. \tag{50}$$

Rewrite the left hand side as

$$\begin{aligned} &\left(1 - p_1\right) x_2 + \left(p_2 - p_1\right) x_3 - \left(1 - p_1\right) x_1 - \left(p_2 - p_1\right) x_2 - p_1 x_4 + p_1 x_3 \\ =&\left(1 - p_2\right) x_2 + p_2 x_3 - \left(1 - p_1\right) x_1 - p_1 x_4 \\ =&\mu - \mu = 0. \end{aligned} \tag{51}$$

Hence, condition (50) is always satisfied.

Case $i = 2$**:**

$$(1 - p_1)(x_2 - x_1) + (p_2 - p_1)(x_3 - x_2) \geq 0. \tag{52}$$

From (51), we can rewrite the left hand side of (52) as

$$(1 - p_1)(x_2 - x_1) + (p_2 - p_1)(x_3 - x_2) = p_1(x_4 - x_3),$$

which is positive since $x_4 > x_3$ by assumption. Hence, condition (47) is satisfied for $i = 1, 2,$and 4. ■

Proof of Proposition 1:

The expected reputation of the sponsor under direct investing ($\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)$) and with delegation ($\widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)$) are both equal to γ_S from Lemmas 1 and 2 and therefore identical.

Next, we establish that $\gamma_S \frac{\Sigma_{S,G}}{\Sigma_S} < \gamma_S \frac{\sigma_{S,G}^A}{\sigma_S^A}$ and $\gamma_S \frac{1-\sigma_{S,G}^A}{1-\sigma_S^A} < \gamma_S \frac{1-\Sigma_{S,G}}{1-\Sigma_S}$. To see the former, note that from the definitions of σ_S^A and Σ_S (equations (12) and (17)):

$$\frac{\Sigma_{S,G}}{\Sigma_S} < \frac{\sigma_{S,G}^A}{\sigma_S^A} \Longleftrightarrow \frac{\Sigma_{S,G}}{\Sigma_{S,B}} < \frac{\sigma_{S,G}^A}{\sigma_{S,B}^A}. \tag{53}$$

From Assumption 1, $\sigma_{F,G}^A = \sigma_{S,G}^A$. Then we have $\Sigma_{S,G} < \sigma_{S,G}^A$ and $\Sigma_{S,B} > \sigma_{S,B}^A$ since $\Sigma_{S,G} = \gamma_{F,G}\sigma_{F,G}^A + (1 - \gamma_{F,G})\sigma_{F,B}^A$ and $\Sigma_{S,B} = \gamma_{F,B}\sigma_{F,G}^A + (1 - \gamma_{F,B})\sigma_{F,B}^A$, which implies (53). The condition for $\frac{1-\sigma_{S,G}^A}{1-\sigma_S^A} > \frac{1-\Sigma_{S,G}}{1-\Sigma_S}$ follows symmetrically from $\sigma_{F,B}^A = \sigma_{S,B}^A$ and is not repeated here.

Hence, the reputation with direct investment, $\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)$, and with delegation, ($\widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)$, satisfy the conditions of Lemma 9 with $\widetilde{x}_1 = \widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)$ and $\widetilde{x}_2 = \widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)$. Therefore, $\widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)$ second-order stochastically dominates $\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)$ and $\widetilde{\gamma}_S^T\left(a_S = DI, \widetilde{\theta}^{opt}, \widetilde{v}\right)$ is a mean-preserving spread of $\widetilde{\gamma}_S^T\left(a_S = DE, \widetilde{h}^{opt}, \widetilde{\theta}^{opt}, \widetilde{v}\right)$.■

Proof of Lemma 3:

We start with the case where $a_S = DE$ and $\kappa_F = 0$. Hence incentives are given purely by reputation. Now, utilizing the fact that $h_S = \widetilde{h}^{opt}$ and the expressions for reputation computed earlier, we have:

$$E\left[V_F(u)|s_F^A=1\right]=Pr(v=1|s_F^A=1)V_F\left(\overline{\gamma}_F\frac{\sigma_{F,G}^A}{\sigma_F^A}\right)+Pr(v=0|s_F^A=1)V_F\left(\overline{\gamma}_F\frac{1-\sigma_{F,G}^A}{1-\sigma_F^A}\right),$$

and

$$E\left[V_F(-u)|s_F^A=1\right]=Pr(v=1|s_F^A=1)V_F\left(\overline{\gamma}_F\frac{1-\sigma_{F,G}^A}{1-\sigma_F^A}\right)+Pr(v=0|s_F^A=1)V_F\left(\overline{\gamma}_F\frac{\sigma_{F,G}^A}{\sigma_F^A}\right).$$

Thus, the fund manager will buy conditional on observing $s_F^A=1$ if and only if:

$$\Big(Pr(v=1|s_F^A=1)-Pr(v=0|s_F^A=1)\Big)\left(V_F\left(\overline{\gamma}_F\frac{\sigma_{F,G}^A}{\sigma_F^A}\right)-V_F\left(\overline{\gamma}_F\frac{1-\sigma_{F,G}^A}{1-\sigma_F^A}\right)\right)\geq 0.$$

Since $\frac{\sigma_{F,G}^A}{\sigma_F^A}>\frac{1-\sigma_{F,G}^A}{1-\sigma_F^A}$ and $V_F(\cdot)$ is strictly increasing, the last statement holds if and only if $Pr(v=1|s_F^A=1)\geq Pr(v=0|s_F^A=1)$, i.e., if and only if

$$\frac{\gamma_A\sigma_F^A}{\gamma_A\sigma_F^A+(1-\gamma_A)(1-\sigma_F^A)}\geq\frac{(1-\gamma_A)(1-\sigma_F^A)}{\gamma_A\sigma_F^A+(1-\gamma_A)(1-\sigma_F^A)},$$

which holds if and only if $\gamma_A\geq 1-\sigma_F^A$, which is guaranteed by Assumption 3. It is easy to see, by symmetry, that the fund manager will sell conditional on observing $s_F^A=0$ if and only if $\gamma_A\leq\sigma_F^A$, also guaranteed by Assumption 3. Thus, for $\kappa_F=0$, the fund manager chooses $\theta_F=\theta^{opt}$. Given the presence of noise traders, it is strictly optimal to trade in the direction of one's private information, and thus if the fund manager chooses $\theta_F=\theta^{opt}$ for $\kappa_F=0$, then he will certainly choose $\theta_F=\theta^{opt}$ for $\kappa_F>0$.

The proof for $a_S=DI$ and $\kappa_S\geq 0$ follows symmetrically, as given that $\sigma_F^A<\sigma_A^S$ from Assumptions 1 and 2, Assumption 3 guarantees that $\gamma_A\in\left(1-\sigma_S^A,\sigma_S^A\right)$. ■

Proof of Lemma 4:

First, let σ_a^A, $a\in\{F,S\}$, with σ_S^A defined in (3) and σ_F^A defined in (12). To facilitate notation, write

$$p_a\equiv\gamma_A\sigma_a^A+(1-\gamma_A)\left(1-\sigma_a^A\right)=Pr\left(s_a^A=1\right),\tag{54}$$

which is the probability that agent a observes a good signal and buys; s/he observes a negative signal and sells with probability $1-p_a$. The order flow is fully revealing if noise traders and the agent both buy, then the order flow is equal to $2u$ and the price

$P(2u) = Pr\left(v = 1 \middle| s_a^A = 1\right)$; and when noise traders and the agent both sell, then the order flow equals $-2u$ and the fully-revealing price $P(-2u) = Pr\left(v = 1 \middle| s_a^A = 0\right)$. Hence, conditional on the optimal trading strategy, the market maker sets prices as a function of the fully-revealing order flow as:

$$
\begin{aligned}
P(2u) &= Pr\left(v = 1 \middle| s_a^A = 1\right) \\
&= \frac{Pr\left(s_a^A = 1 \,|v = 1\right) Pr(v = 1)}{Pr\left(s_a^A = 1 \,|v = 1\right) Pr(v = 1) + Pr\left(s_a^A = 1 \,|v = 0\right) Pr(v = 0)} \\
&= \frac{\gamma_A \sigma_a^A}{\sigma_a^A \gamma_A + \left(1 - \sigma_a^A\right)(1 - \gamma_A)} = \frac{\gamma_A \sigma_a^A}{p_a},
\end{aligned}
$$

and

$$
\begin{aligned}
P(-2u) &= Pr\left(v = 1 \middle| s_a^A = 0\right) \\
&= \frac{Pr\left(s_a^A = 0 \,|v = 1\right) Pr(v = 1)}{Pr\left(s_a^A = 0 \,|v = 1\right) Pr(v = 1) + Pr\left(s_a^A = 0 \,|v = 0\right) Pr(v = 0)} \\
&= \frac{\gamma_A \left(1 - \sigma_a^A\right)}{\left(1 - \sigma_a^A\right) \gamma_A + \sigma_a^A (1 - \gamma_A)} = \frac{\gamma_A \left(1 - \sigma_a^A\right)}{1 - p_a}.
\end{aligned}
$$

If either noise traders buy and the agent sells, or if noise traders sell and the agent buys, the order flow equals zero, the market maker assumes that the agent has observed a positive signal with probability p_a, and sets the price as

$$
\begin{aligned}
P(0) &= p_a P(2u) + (1 - p_a) P(-2u) \\
&= \gamma_A.
\end{aligned}
$$

Conditional on observing $s_a^A = 1$, the agent expects that the market maker will execute his or her market order at the price $E[P\,|+u] = \frac{P(2u)+\gamma_A}{2}$, since, from the point of view of the agent, there are equal probabilities that the price will be fully revealing $(= P(2u))$ and that the price will be uninformative $(= \gamma_A)$. Then the agent's expected trading profits from buying u units are

$$
u\left(P(2u) - E[P\,|+u]\right) = u\frac{P(2u) - \gamma_A}{2},
$$

which the agent realizes with probability p_a. Similarly, conditional on observing $s_a^A = 0$, the agent's expected execution price is $E[P|-u] = \frac{P(-2u)+\gamma_A}{2}$ and the trading profits from selling u units are

$$u(E[P|-u] - P(-2u)) = u\frac{\gamma_A - P(-2u)}{2},$$

which the agent realizes with probability $1 - p_a$. Then expected profits are

$$\begin{aligned} E\left[\pi\left(\sigma_a^A, \theta_a = \widetilde{\theta}^{opt}\right)\right] &= \frac{u}{2}\left[p_a\left(\frac{\gamma_A \sigma_a^A}{p_a} - \gamma_A\right) + (1 - p_a)\left(\gamma_A - \frac{\gamma_A\left(1 - \sigma_a^A\right)}{1 - p_a}\right)\right]. \\ &= \frac{u}{2}\left[2\gamma_A\left(\sigma_a^A - p_a\right)\right]. \end{aligned}$$

The result (18) obtains from substituting for p_a from (54) in the last equation. Using (18) in the definition of $\Delta\pi$ in (19) gives

$$\begin{aligned} \Delta\pi &= 2u\gamma_A(1 - \gamma_A)\left(\sigma_F^A - \sigma_S^A\right) \\ &= 2u\gamma_A(1 - \gamma_A)(\overline{\gamma}_F - \gamma_S)\left(\sigma_{a,G}^A - \sigma_{a,B}^A\right) < 0, \end{aligned} \tag{55}$$

where the last inequality follows from Assumptions 1 and 2, which proves the remaining claims. ■

Proof of Lemma 5:

Given that, by Lemma 3, the fund manager chooses $\theta_F = \widetilde{\theta}^{opt}$ for all $\kappa_F \geq 0$, it is a best response for the sponsor to set $\kappa_F = 0$. Now, to determine hiring strategies, start by setting $\kappa_S = 0$, so that the sponsor's incentives are purely reputational. Consider an arbitrary hiring strategy $\widetilde{h}$. In combination with the fund manager's trading strategy, this induces the following expected reputational payoff for the sponsor:

$$\begin{aligned} &Pr(s_F^A = v|\widetilde{h}, \theta_F = \widetilde{\theta}^{opt})V_S\left(\gamma_S^T\left(s_F^A = v|a_S = DE, \widetilde{h}, \theta_F = \widetilde{\theta}^{opt}\right)\right) \\ &+Pr(s_F^A \neq v|\widetilde{h}, \theta_F = \widetilde{\theta}^{opt})V_S\left(\gamma_S^T\left(s_F^A \neq v|a_S = DE, \widetilde{h}, \theta_F = \widetilde{\theta}^{opt}\right)\right). \end{aligned}$$

Now drawing on Lemma 2, if the labor market believes that the sponsor chooses $h_S = \widetilde{h}^{opt}$, then the above expression becomes:

$$\begin{gathered}Pr(s_F^A = v|h_S = \widetilde{h}^{opt}, \theta_F = \widetilde{\theta}^{opt})V_S\left(\gamma_S\frac{\Sigma_{S,G}}{\Sigma_S}\right)\\ +Pr(s_F^A \neq v|h_S = \widetilde{h}^{opt}, \theta_F = \widetilde{\theta}^{opt})V_S\left(\gamma_S\frac{1-\Sigma_{S,G}}{1-\Sigma_S}\right)^{,}\end{gathered}$$

The sponsor wishes to maximize the weight on the first term since

$$\gamma_S\frac{\Sigma_{S,G}}{\Sigma_S} > \gamma_S > \gamma_S\frac{1-\Sigma_{S,G}}{1-\Sigma_S}.$$

He can do this by choosing h_S to maximize $Pr(s_F^A = v|\widetilde{h}, \theta_F = \widetilde{\theta}^{opt})$, which clearly involves hiring only fund managers, and thus the first fund manager, for whom she receives a positive signal, i.e., by setting $h_S = \widetilde{h}^{opt}$. Setting $\kappa_S > 0$ only increases the incentive to hire the best informed fund managers, since then the sponsor gets a slice of trading profits which are increasing in the quality of the chosen fund manager's information, hence if the sponsor chooses $h_S = \widetilde{h}^{opt}$ for $\kappa_S = 0$, she will certainly choose $h_S = \widetilde{h}^{opt}$ for $\kappa_S > 0$. ■

Proof of Lemma 8:

From the proof of Proposition 1, reputational risk is higher with delegation than with direct investment if condition (24) holds. We have

$$\begin{aligned}&\Sigma_{S,G} > \sigma_{S,G}^A\\ \Leftrightarrow&\gamma_{F,G}\sigma_{F,G}^A + (1-\gamma_{F,G})\,\sigma_{F,B}^A > \sigma_{S,G}^A\\ \Leftrightarrow&\sigma_{F,G}^A > \sigma_{S,G}^A + \frac{1-\gamma_{F,G}}{\gamma_{F,G}}\left(\sigma_{S,G}^A - \sigma_{F,B}^A\right),\end{aligned} \tag{56}$$

which is equivalent to (25) after observing that $\frac{1-\gamma_{F,G}}{\gamma_{F,G}} = \frac{1-\gamma_F}{\gamma_F}\frac{1-\sigma_{S,G}^F}{\sigma_{S,G}^F}$. Similarly, we have

$$\begin{aligned}&\Sigma_{S,B} < \sigma_{S,B}^A\\ \Leftrightarrow&\gamma_{F,B}\sigma_{F,G}^A + (1-\gamma_{F,B})\,\sigma_{F,B}^A < \sigma_{S,B}^A\\ \Leftrightarrow&\sigma_{F,B}^A < \sigma_{S,B}^A + \frac{\gamma_{F,B}}{1-\gamma_{F,B}}\left(\sigma_{S,B}^A - \sigma_{F,G}^A\right),\end{aligned} \tag{57}$$

which is equivalent to (25) after observing that $\frac{\gamma_{F,B}}{1-\gamma_{F,B}} = \frac{\gamma_F}{1-\gamma_F}\frac{\sigma^F_{S,B}}{1-\sigma^F_{S,B}}$. Hence, conditions (56) and (57) jointly imply that

$$\sigma^A_{F,B} < \sigma^A_{S,B} < \sigma^A_{S,G} < \sigma^A_{F,G}. \tag{58}$$

(ii) To find parameters that satisfy these conditions, we first define some number $a \geq \frac{1}{2}$ and three numbers η, ε, and δ such that $\eta < \varepsilon < \delta$ and $a + \delta \leq 1$. Then let

$$\sigma^A_{F,B} = a, \sigma^A_{S,B} = a + \eta, \sigma^A_{S,G} = a + \varepsilon, \sigma^A_{F,G} = a + \delta. \tag{59}$$

Then (56) becomes

$$\begin{aligned} &\gamma_{F,G}(a+\delta) + (1-\gamma_{F,G})\, a > a + \varepsilon \\ \Leftrightarrow &\gamma_{F,G}\delta > \varepsilon \end{aligned}$$

and (57) becomes

$$\begin{aligned} &\gamma_{F,B}(a+\delta) + (1-\gamma_{F,B})\, a < a + \eta \\ \Leftrightarrow &\gamma_{F,B}\delta < \eta. \end{aligned}$$

Hence, we can always satisfy conditions (56) and (57) by picking a number $a \in \left[\frac{1}{2}, 1\right)$ and three parameters $\eta, \varepsilon, \delta$ such that

$$0 < \gamma_{F,B}\delta < \eta < \varepsilon < \gamma_{F,G}\delta \leq \delta \leq 1 - a. \tag{60}$$

Such parameters can always be found if $\gamma_{F,B} < \gamma_{F,G} \Leftrightarrow \sigma^F_{S,B} < \sigma^F_{S,G}$, which shows that a generic set of parameters exists. Since the right hand side of (25) is decreasing in $\sigma^F_{S,G}$ and the right hand side of (26) is increasing in $(\sigma^F_{S,B})$, the parameter set has the claimed properties.

Proof of Proposition 8:

(i) To calculate trading profits, use (18) and observe that the critical parameter is σ^A_a. Direct computation using the parameters in (59) gives:

$$\begin{aligned} \sigma^A_F &= a + \delta\overline{\gamma}_F, \\ \sigma^A_S &= a + \eta(1-\gamma_S) + \varepsilon\gamma_S \end{aligned}.$$

Hence,

$$E\left[\pi\left(\sigma_F^A, \theta^F = \widetilde{\theta}^{opt}\right)\right] > E\left[\pi\left(\sigma_S^A, \theta^S = \widetilde{\theta}^{opt}\right)\right] \Leftrightarrow \delta\overline{\gamma}_F > \eta\left(1-\gamma_S\right) + \varepsilon\gamma_S.$$

We have chosen $\eta < \varepsilon < \delta$, hence the second inequality always holds if $\overline{\gamma}_F \geq \gamma_S$.

(ii) Use the parameters constructed in equations (59) and (60). Then delegation creates additional profits, so $\Delta\pi > 0$ in equation (19), and additional reputational risk, so $\Delta V < 0$ in equation (20). Then the claim follows analogously to Lemma 7 and Proposition 2.■