Monetary Policy, Segmentation, and the Term Structure

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Wealth dynamics in a term structure model

- **Preferred habitat** term structure models
  - Market price of risk depends on arbitrageurs' risk aversion and the amount of risk they bear
  - Term structure is affine when absolute risk aversion is exogenous
    - Vayanos/Vila (2021)
- **Intermediary asset pricing** literature
  - Investment incentives depend on wealth (risk aversion or incentive constraints)
  - Wealth is an endogenous state variable
    - He/Krishnamurthy (2013), Brunnermeier/Sannikov (2014), Bernanke/Gertler/Gilchrist (1999)
- **This paper:** Endogenizes wealth dynamics, and hence risk aversion, in a preferred-habitat model of the term structure
  - Minimalist extension of the Vayanos/Vila model
  - But offers a powerful propagation mechanism, with scope for richer dynamics

Propagation via endogenous wealth helps explain term premia

- Using high-frequency data, paper offers further empirical evidence that **monetary tightening raises the term premium**, especially at longest maturities

- **Monetary tightening** when wealth is **endogenous**:
  - $\rightarrow$ **Yields rise** at all maturities
    - . . . this is the **expectations channel**
  - $\rightarrow$ Wealth of leveraged investors falls
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- This feedback loop **amplifies** impact of monetary shocks on yields

- May also **propagate** effects on yields over time

- Can therefore increase unconditional **volatility** of yields

- And implies **state-dependent** impact of shocks

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  - $\rightarrow$ Therefore **preferred-habitat** investors hold more bonds
  - $\rightarrow$ By market clearing, arbitrageurs hold less
  - $\rightarrow$ Therefore **price of risk decreases**
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Extending Vayanos/Vila to endogenize wealth

- Bond market structure is taken from Vayanos/Vila (2021)
  - **Bond market**: Bonds of tenors $\tau \in (0, 1]$ are traded at price $P_t^{(\tau)}$
  - **Risk-averse arbitrageurs** maximize utility of wealth by choosing to hold bonds of any tenor τ, or to receive riskless short rate r_t
  - **Preferred-habitat investors** demand bonds of a given tenor τ; their demand decreases with price
  - **Exogenous stochastic factors**:
    - Riskless short rate r_t
    - Preferred habitat demand shocks β_t

- But **arbitrageurs are long-lived**, and their **risk aversion varies with wealth**
  - Arbitrageurs are born with wealth $\overline{W}$
  - They maximize **log utility** over **wealth at end of life**
  - **"Perpetual youth"** with death probability ξ per unit of time (Blanchard/Yaari)
  - **Limiting case $\xi \to 1$ nests Vayanos/Vila (2021)**

Two versions of the model

- **Simplified two-asset model** can be solved analytically
  - Discrete time
  - Government offers riskless one-period bonds with return r_t
    - Price $P_t^{(1)} = \exp(-r_t)$ at time t for riskless face value 1 at time $t+1$
  - Preferred-habitat investors demand two-period bonds with price $P_t^{(2)}$
  - Arbitrageurs trade one-period and two-period bonds
  - Shocks to the short rate r_t affect arbitrageurs' wealth because they revalue holdings from $t-1$, which were purchased at price $P_{t-1}^{(2)}$
- **Full model** with a continuum of assets requires numerical solution
  - Continuous time
  - Government offers riskless short return r_t
  - Each class of preferred-habitat investors demands a specific tenor τ, for $\tau \in (0, T]$
  - Arbitrageurs trade across all bonds, at prices $P_t^{(\tau)}$ and the riskless short rate
  - Arbitrageurs' wealth evolves over time as shocks revalue their portfolios

Analytical results of the discrete-time model

- Can solve forward in time, with **one equation in one unknown** each period.
  - One-period bond price is $P_t^{(1)} = \exp(-r_t)$
  - Therefore shock to r_t affects arbitrageurs' wealth W_t derived from portfolio purchased at $t-1$
  - Two-period bond price $P_t^{(2)}$ must satisfy **arbitrageurs' first-order condition**, and clear markets
  - Next, shock to r_{t+1} determines arbitrageurs' wealth W_{t+1} ...

- Implications of the analytical solution:
  - **Prop. 1.** In the limit with **exogenous wealth** ($\xi \to \infty$ and $W_t = \overline{W}$), the response $df_t^{(\tau-1,\tau)}$ of the **forward rate** to a monetary policy shock $d\epsilon_{r,t}$ is less than the change in the expected short rate.
    - In other words, **the term premium falls** in response to a **monetary tightening**
  - **Prop. 2.** When wealth is endogenous ($\xi < \infty$), **arbitrageurs' wealth falls** in response to monetary tightening if they are long in longer bonds ($X_t^{(2)}/W_t > 0$)
  - **Prop. 3.** Falling wealth increases the reaction of the forward rate to a monetary policy shock.
    - Therefore, when wealth is endogenous ($\xi < \infty$), **the term premium may rise** in response to monetary tightening if arbitrageurs are sufficiently long in longer bonds ($X_t^{(2)}/W_t$ sufficiently large).

Equation system: the general model

- **Driving processes**: risk-free rate shocks r_t and preferred-habitat demand shifters β_t
- Arbitrageurs' **Euler equation** for bondholdings $x_t^{(\tau)}$ of tenor τ:

$$E_t \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt \;=\; \frac{1}{w_t} \int_0^T x_t^{(s)} \mathrm{Cov}_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}, \frac{dP_t^{(s)}}{P_t^{(s)}} \right) ds$$

- Arbitrageurs' **log utility** implies that their portfolio shares are independent of wealth:

$$\frac{x_t^{(\tau)}}{w_t} \;=\; \frac{X_t^{(\tau)}}{W_t}$$

- **Preferred-habitat demand curve**:

$$Z_t^{(\tau)} \;=\; -\alpha(\tau) \log P^{(\tau)} - \theta_0(\tau) - \theta_1(\tau)\beta_t$$

- **Market clearing**:

$$Z_t^{(\tau)} + X_t^{(\tau)} \;=\; 0$$

- Notice we can simplify by plugging the market clearing condition into the Euler equation ...

Simplifying the equations of the general model

- **Driving processes**: risk-free rate shocks r_t and preferred-habitat demand shifters β_t

- **Aggregating arbitrageurs' Euler equations** for bondholdings:

$$E_t \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt \;=\; \frac{1}{W_t} \int_0^T \left(\alpha(\tau) \log P_t^{(\tau)} + \theta_0(\tau) + \theta_1(\tau)\beta_t \right) \mathrm{Cov}_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}, \frac{dP_t^{(s)}}{P_t^{(s)}} \right) ds$$

- Can also **aggregate wealth dynamics**:

$$dW_t \;=\; W_t r_t dt + \int_0^T \left(\alpha(\tau) \log P_t^{(\tau)} + \theta_0(\tau) + \theta_1(\tau)\beta_t \right) \left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt \right] d\tau \;=\; \xi \left(\overline{W} - W_t \right) dt$$

Solution method

1. Conjecture solution for the prices in terms of the state variables:
$$P_t^\tau = P^\tau(r_t, \beta_t, W_t)$$
2. Ito's Lemma implies that price and wealth dynamics can be written as:
$$\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} = \omega_t^{(\tau)} dt + \eta_{r,t}^{(\tau)} dB_{r,t} + \eta_{\beta,t}^{(\tau)} dB_{\beta,t} \quad (1)$$
$$dW_t = \omega(r_t, \beta_t, W_t) dt + \eta_r(r_t, \beta_t, W_t) dB_{r,t} + \eta_\beta(r_t, \beta_t, W_t) dB_{\beta,t} \quad (2)$$
3. Guess the coefficients in the expansion of dW_t
4. Plug those functional forms into the Euler equation and the wealth evolution equation
5. Solve the resulting system of PDEs, using sparse collocation methods, to determine $P^\tau(r_t, \beta_t, W_t)$
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8. **Comment: Ouch!!!**

- Impact of 1pp tightening on forward rates, instrumented by Jarocinski/Karadi high-frequency monetary shocks
- Consistent with Nakamura/Steinsson evidence that tightenings do not affect 10Y term premium
- But term premium does rise significantly for sufficiently long-duration bonds

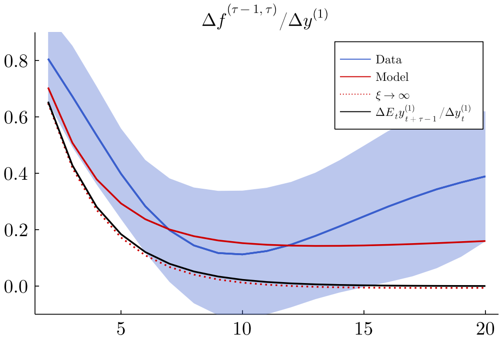

Figure 6: $f_t^{(\tau-1,\tau)}$ on $y_t^{(1)}$ given monetary shock: model vs. data

- Model calibrated to data on duration of arbitrageur portfolios
- Long duration **term premium does not tend to zero** in calibrated model

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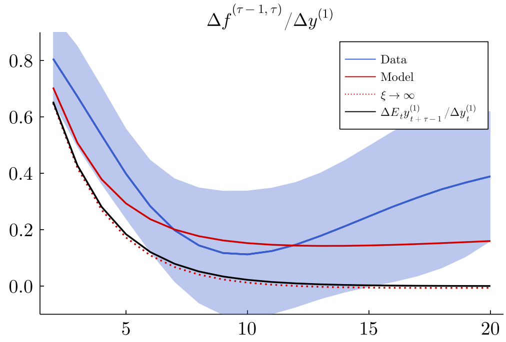

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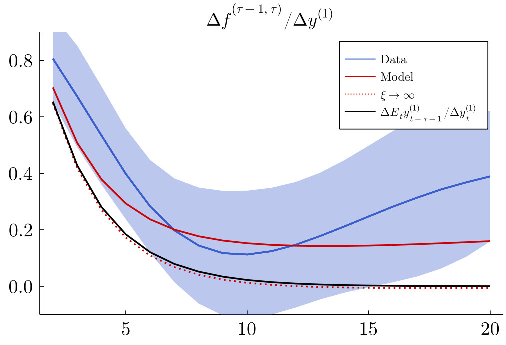

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- **Model term premium is U-shaped** if calibrated duration is slightly increased

Some questions

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- This paper, like related papers, assumes that the slope of preferred habitat demand, as a function of yield, is a hump-shaped function of yields: $\tau\alpha \exp(-\delta\tau)$. What is the motivation for choosing this functional form? Exponential decay is not necessary for convergence when the maximum tenor is finite.

Conclusions

- Great paper, rich results
- Important technical advance
  - But not easy to implement
- Points to an economically important amplification and propagation mechanism
  - Effect of monetary policy on long-run yields goes through term premium, not just expectations component
  - Increased unconditional volatilities
  - Wealth-dependent impact of exogenous shocks

THANKS FOR YOUR ATTENTION!