Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity*

NICOLAS CARAMP — UC Davis

DEJANIR H. SILVA — Purdue University

June 20, 2023

Abstract

We study the role of the revaluation of real and financial assets in the monetary policy transmission mechanism. We build an analytical heterogeneous-agents model with two main ingredients: i) rare disasters; and ii) heterogeneous beliefs. The model captures time-varying risk premia and precautionary savings in a linearized setting that nests the textbook New Keynesian model. Quantitatively, the model matches the empirical response of asset prices. When households' consumption equal dividends on real assets, changes in risk premia affect asset prices, but have no effect on output and inflation. With long-term and risky government debt, changes in risk premia caused by monetary policy have large real effects, and they account for the majority of the economy's response to changes in nominal interest rates.

JEL Codes: E21, E52, E44

Keywords: Monetary Policy, Wealth Effects, Asset Prices, Heterogeneity

*Caramp: UC Davis. Email: ncaramp@ucdavis.edu. Silva: Krannert School of Management, Purdue University. Email: dejanir@purdue.edu.

We would like to thank Francesco Bianchi, Francisco Buera, Colin Cameron, James Cloyne, Bill Dupor, Gauti Eggertsson, Rustam Jamilov (discussant), Òscar Jorda, Rohan Kekre (discussant), Narayana Kocherlakota, Steffan Lewellen (discussant), Dmitry Mukhin, Benjamin Moll, Sanjay Singh, Jón Steinsson, Gianluca Violante (discussant) and seminar participants at EFA Annual Meeting, NBER Summer Institute, UC Berkeley, UC Davis, the Federal Reserve Bank of St. Louis, SED, WFA Annual Meeting, Paul Wooley Centre Conference for their helpful comments and suggestions.

1 Introduction

A long tradition in monetary economics emphasizes the role of the revaluation of real and financial assets in shaping the economy's response to changes in monetary policy. Its importance can be traced back to both classical and Keynesian economists.[1] Keynes himself described the effects of interest rate changes as follows:

> There are not many people who will alter their way of living because the rate of interest has fallen from 5 to 4 per cent, if their aggregate income is the same as before. [...] Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, *depends on the effect of these changes on the appreciation or depreciation in the price of securities and other assets*.
> - John Maynard Keynes, The General Theory of Employment, Interest, and Money (emphasis added).

These revaluation effects caused by monetary policy have been documented by an extensive empirical literature. Bernanke and Kuttner (2005) study the effects of monetary shocks on stock prices. Gertler and Karadi (2015) and Hanson and Stein (2015) consider the effects on bonds. A robust finding of this literature is that changes in asset prices are explained mainly by fluctuations in future excess returns, related to changes in the risk premia, rather than changes in the risk-free rate.[2]

The extent to which changes in asset prices play a relevant role in the transmission of monetary policy to the real economy, however, has been controversial. One view highlights the importance of *wealth effects*. For instance, Cieslak and Vissing-Jorgensen (2020) show that policymakers track the behavior of stock markets because of their impact on households' consumption, while Chodorow-Reich et al. (2021) study the importance of this channel empirically. An alternative view defends that changes in asset valuations have no real implications. Cochrane (2020) and Krugman (2021) argue that movements in discount rates lead to changes in "paper wealth," without an impact on consumption.

In this paper, we study how monetary policy affects the real economy through changes in asset prices. We provide a new framework that generates rich asset-pricing dynamics and heterogeneous portfolios while preserving the simplicity of the textbook New Keynesian model. In particular, we propose a new solution technique that enable us to obtain time-varying risk premium and precautionary savings motive without having to resort to higher-order approximations.[3] We derive necessary conditions for changes in risk premia

[1] The revaluation of government liabilities was central to Pigou (1943) and Patinkin (1965), while Metzler (1951) considered stocks and money. Tobin (1969) focused on the revaluation of real assets.

[2] For a recent review of this work, see Bauer and Swanson (2023).

[3] As shown in e.g. Schmitt-Grohé and Uribe (2004), a standard perturbation around the non-stochastic steady state can only generate time-varying risk premia with a third-order approximation.

to affect the real economy. Under very special conditions, we obtain a *risk-neutrality result*, where changes in risk premia caused by monetary shocks affect asset prices, but it has no effect on output and inflation. These conditions are, however, very stringent. We then assess quantitatively the importance of this channel and find that changes in risk premia account for a large fraction of the response of output and inflation to changes in monetary policy.

We consider an economy populated by workers and savers with two main ingredients: i) rare disasters, and ii) heterogeneous beliefs. Rare disasters enable us to capture both a precautionary savings motive and realistic risk premia. Barro (2009) and Gabaix (2012) argue that the risk of a rare disaster can successfully explain major asset-pricing facts.[4] Savers invest in stocks, government bonds, and household debt, and have heterogeneous beliefs, as in Caballero and Simsek (2020). As a consequence, they hold heterogeneous portfolios in equilibrium. This allows us to capture *time-variation* in risk premia in response to monetary shocks. Workers are constrained in equilibrium, so borrowers and savers have heterogeneous MPCs. Despite being stylized, the model captures quantitatively central aspects of the monetary transmission mechanism, including the term premium, the equity premium, and corporate spreads, as well as the differential responses of borrowers and savers to monetary shocks observed in the data.

Our first contribution is methodological and consists of an aggregation result. Given investor heterogeneity, we must characterize not only the dynamics of aggregate output and inflation, but also the behavior of portfolios, asset prices, net worth, and individual consumption. This increases the dimensionality of the problem and typically makes deriving analytical results infeasible. We show that our economy satisfies an *as if* result: the economy with heterogeneous savers behaves as an economy with a representative saver, but the probability of disaster, as implied by market prices, is time-varying and responds to monetary policy. This *market-implied disaster probability* is a key determinant of asset prices, and it is the main channel through which investor heterogeneity affects the real economy.

Our second contribution identifies conditions under which time-varying risk premia plays a role in the transmission of monetary policy to the real economy. Consistent with the empirical evidence, a contractionary monetary shock leads to an increase in risk premia and a reduction in the price of risky assets. One could then conclude that this reduction in households' wealth leads to a reduction in consumption. However, as the discount rate increases, the present discounted value of consumption decreases as well. The net ef-

[4] Rare disasters have been widely used to explain a range of asset-pricing "puzzles"; see Tsai and Wachter (2015) for a review.

fect of changes in risk premium is ambiguous and depends on whether households are net buyers or net sellers of risky assets. As recently articulated by Cochrane (2020) and Krugman (2021), a household who just consumes the dividends from their financial assets can still afford the same level of consumption after a change in discount rates. The wealth effect should then be zero in this case.

Formally, we show that the aggregate wealth corresponds to the sum of all households' wealth *net* of the change in the cost of the original consumption bundle. Naturally, the aggregate wealth effect does not depend on private debt. While private debt matters for individual households' consumption, the gross positions cancel out when we aggregate at the household sector level. More interestingly, the aggregate wealth effect does not depend on the equity premium either. It turns out that the difference between the revaluation of the households' assets and liabilities (including consumption) is given by the government's liabilities. The intuition is simple: in a closed economy, only the government is a counterpart to the household sector taken as a whole.[5] Thus, whether risk affects the *aggregate* wealth effect depends on the characteristics of government debt. We show that, in the absence of a precautionary motive, there are three cases in which risk has no impact on aggregate wealth: *i*) when government debt is zero, *ii*) when government debt is short term, and *iii*) when government debt is a consol. In these cases, either the households' net revaluation effect is zero or it is independent of risk premia.

The presence of risk also affects the households' precautionary motives. This effect arises from the redistribution among savers after a monetary shock. Because optimists hold a larger fraction of their wealth in risky assets (long-term bonds and equity), an increase in the interest rate disproportionately reduces their wealth. Holding the *aggregate* wealth effect constant, this redistribution of wealth is then reflected in the market-implied probability of disaster, which increases after the monetary shock as pessimist savers increase their holdings of risky assets. This is the "as-if" result in action: redistribution between optimists and pessimists is akin to an increase in the "objective" probability of disaster risk in a model with a representative agent. Note that the precautionary savings channel changes the *timing* of consumption but not the households' aggregate wealth.

Putting together all these results, we obtain a complete characterization of the consumption channel of monetary policy in this model. We show that the transmission of monetary policy to aggregate consumption has two components, one that affects its *present value* and one that affects its *timing*. The present value of consumption is given by the aggregate wealth effect. The timing of consumption depends in a prominent way on private debt and aggregate risk. For private debt, the intuition is that monetary policy re-

[5] In an open economy, the foreign sector would be an additional counterpart.

distributes between borrowers and savers. Because borrowers and savers have different MPCs with respect to transitory income shocks, a contractionary monetary policy reduces aggregate consumption on impact. However, because all households in the economy have an MPC of one for *permanent* changes in their income, savers eventually increase their consumption so that the present value of the changes cancel out. For aggregate risk, while precautionary savings increase on impact, they gradually decrease as the market-implied risk in the economy transitions back to its steady-state level. The present value of this effect is also zero.

In the absence of an aggregate wealth effect, monetary policy has then only a limited effect on the economy. A reduction in interest rates stimulates the economy in the present at the expense of a more depressed economy in the future. We also show that the central bank is unable to affect inflation when the wealth effect is zero. Moreover, future inflation rates respond *positively* to changes in nominal interest rates in this case. Therefore, the central bank's ability to stimulate the economy and control inflation is tightly connected to its ability to generate aggregate wealth effects.

Finally, our solution method allows us to obtain time-varying risk premia in a linearized setting and provide a complete analytical characterization of the channels involved. The method consists on perturbing the economy around a stationary equilibrium with *positive aggregate risk* instead of adopting the more common approach of approximating around a non-stochastic steady state. By perturbing around the stochastic stationary equilibrium, we are able to obtain time variation in precautionary motives and risk premia using a first-order approximation, while the standard approach would require a third-order approximation (see e.g. Andreasen 2012).[6] This hybrid approach can prove useful in other settings where capturing risk premia is important. It is well known that business cycle fluctuations in TFP cannot generate large risk premia without assuming implausible large risk aversion (see Mehra and Prescott, 1985). Disaster risk has been successful on this front, and our method shows how to incorporate it into rich macroeconomic models without sacrificing tractability.

Our calibration departs from the standard practice in three important ways. First, we set the households' intertemporal elasticity of substitution to 0.25 (which implies a risk aversion coefficient of 4 given our CRRA specification). This choice is lower than the usual value of 1 or 0.5. However, our choice is closer to recent studies using microdata, such as Best et al. (2020) who find a value of 0.1. Second, we need to calibrate the parameters

[6]Moreover, by linearizing around an economy with zero monetary risk, we are able to solve for the stochastic stationary equilibrium in closed form, avoiding the need to compute the risky steady state numerically, as in Coeurdacier et al. (2011).

associated with the disaster risk. For the parameters governing the steady-state levels, we follow Barro (2009). This implies an annual probability of a disaster of 1.7%. For the time-varying component of the risk premium, we calibrate the elasticity of the disaster shock to monetary policy to match the initial response of the term premium in Gertler and Karadi (2015). We show that this calibration generates a conditional equity premium and corporate spread that is consistent with the literature. Finally, for the fiscal response to a monetary shock, we augment the procedure in Christiano et al. (1999) to incorporate fiscal variables. We use the yield on the 5-year government bond to compute the government's intertemporal budget constraint.

To quantify the importance of the channels present in the model, we start with the standard RANK model and add risk and household debt one at a time. We find that the forces in RANK explain less than 20% of the consumption response on impact to a monetary shock, risk explain around 50%, household slightly more than 20%, and the interaction of the two slightly less than 10% Thus, risk and household debt are crucial components of the monetary transmission mechanism.

Literature review. Wealth effects have a long tradition in monetary economics. Pigou (1943) relied on a wealth effect to argue that full employment could be reached even in a liquidity trap. Kalecki (1944) argued that these effects apply only to government liabilities, as inside assets cancel out in the aggregate, while Tobin highlighted the role of private assets and high-MPC borrowers.[7] Recently, wealth effects have regained relevance. In an influential paper, Kaplan et al. (2018) build a quantitative HANK model and find only a minor role for the standard intertemporal-substitution channel, leading the way to a more important role for wealth effects. Much of the literature has focused on the role of heterogeneous marginal propensities to consume (MPCs) in settings with idiosyncratic income risk. Instead, our focus is on aggregate risk and private debt.

Our work is closely related to two strands of literature. First, it relates to the analytical HANK literature, such as Werning (2015), Debortoli and Galí (2017), and Bilbiie (2018). While this literature focuses primarily on how the cyclicality of income interacts with differences in MPCs, we focus instead on how heterogeneous asset positions interact with differences in MPCs. We see these two channels as mostly complementary: even though Cloyne et al. (2020) does not find significant differences in income sensitivity across bor-

[7] Tobin (1982) describes the role of inside assets: "The gross amount of these 'inside' assets was and is orders of magnitude larger than the net amount of the base. Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and debtors. But if the spending propensity were systematically greater for debtors, even by a small amount, the Pigou effect would be swamped by this Fisher effect."

rowers and savers, Patterson (2019) finds a positive covariance between MPCs and the sensitivity of earnings to GDP across different demographic groups, suggesting that the income-sensitivity channel is operative for a different cut of the data. We share with Eggertsson and Krugman (2012) and Benigno et al. (2020) the emphasis on private debt, but they abstract from a precautionary motive and focus instead on the implications of deleveraging. Iacoviello (2005) also considers a monetary economy with private debt but focuses instead on the role of housing as collateral. Our work is also related to Auclert (2019), which studies the redistribution channel of monetary policy arising from portfolio heterogeneity. Our paper emphasizes the redistribution channel in the context of a general equilibrium setting with aggregate risk.

The paper is also closely related to work on how monetary policy affects the economy through changes in asset prices, including models with sticky prices, such as Caballero and Simsek (2020), and models with financial frictions, such as Brunnermeier and Sannikov (2016) and Drechsler et al. (2018). In recent contributions, Kekre and Lenel (2020) consider the role of the marginal propensity to take risk in determining the risk premium and shaping the response of the economy to monetary policy, and Campbell et al. (2020) use a habit model to study the role of monetary policy in determining bond and equity premia. Our model highlights instead the role of heterogeneous MPCs, positive private liquidity, and disaster risk in an analytical framework that preserves the tractability of standard New Keynesian models.

Finally, a recent literature studies rare disasters and business cycles. Gabaix (2011) and Gourio (2012) consider a real business cycle model with rare disasters, while Andreasen (2012) and Isoré and Szczerbowicz (2017) allow for sticky prices. They focus on the effect of changes in disaster probability while we study monetary shocks in an analytical HANK model with rare disasters.

2 D-HANK: A Rare Disasters Analytical HANK Model

In this section, we consider an analytical HANK model with two main ingredients: i) the possibility of rare disasters, ii) heterogeneous beliefs. We first describe the non-linear model and later consider a log-linear approximation around a stochastic stationary equilibrium.

2.1 The Model

Environment. Time is continuous and denoted by $t \in \mathbb{R}_+$. The economy is populated by households, firms, and a government. There is a continuum of households which can be of three types: *workers*, *optimistic savers*, and *pessimistic savers* (denoted by w, o and p, respectively), who differ in their discount rates and beliefs about the probability of aggregate shocks. Households can borrow or lend at a riskless rate subject to a borrowing constraint, and they can invest on government bonds and corporate equity. Workers are the only ones who supply labor, and they are relatively impatient, so their borrowing constraint is binding in equilibrium.

Firms can produce final or intermediate goods. Final-goods producers operate competitively and combine intermediate goods using a CES aggregator with elasticity $\epsilon > 1$. Intermediate-goods producers use labor as their only input and face quadratic (Rotemberg, 1982) pricing adjustment costs. Intermediate-goods producers are subject to an aggregate productivity shock: with Poisson intensity $\overline{\lambda} \geq 0$, they receive a shock that permanently reduces their productivity. This shock is meant to capture the possibility of rare disasters: low-probability, large drops in productivity and output, as in the work of Barro (2006, 2009). We say that periods that predate the realization of the shock are in the *no-disaster state*, and periods that follow the shock are in the *disaster state*. The disaster state is absorbing, and there are no further shocks after the disaster is realized. Assuming an absorbing disaster state simplifies the presentation, but it can be easily relaxed, as shown in the appendix.[8]

The government sets fiscal policy, comprising of transfers to workers and savers, and monetary policy, specified by an interest rate rule subject to a sequence of monetary shocks. The government issues long-term nominal bonds that pay exponentially decaying coupons. We denote by $Q_{L,t}e^{-\psi_L t}$ the nominal price of the bond in the no-disaster state, which pays coupons $e^{-\psi_L s}$ at all dates $s \geq 0$. We denote by $Q^*_{L,t}$ the corresponding (normalized) price of the bond in the disaster state, where the star superscript is used throughout the paper to denote variables in the disaster state. The rate of decay ψ_L is inversely related to the bond's duration, where a perpetuity corresponds to $\psi_L = 0$ and the limit $\psi_L \to \infty$ corresponds to the case of short-term bonds.

Savers' problem. Savers face a portfolio problem where they choose how much to invest in short-term bonds, long-term bonds, and corporate equity. In this section, we as-

[8]Allowing for partial recovery, as in Barro et al. (2013) and Gourio (2012), introduces dynamics in the disaster state, but it does not change the main implications for the no-disaster state, which is our focus.

sume that households issue only short-term risk-free bonds and the government issues only long-term bonds. We study the case of defaultable long-term household debt in Section 5. The nominal return on the long-term bond is given by[9]

$$dR_{L,t} = \left[\frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L\right] dt + \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} d\mathcal{N}_t,$$

where $\mathcal{N}_t$ is a Poisson process with arrival rate $\overline{\lambda}$ (under the objective measure).

The price of a claim on real aggregate corporate profits is denoted by $Q_{E,t}$ and the real return on equities evolves according to

$$dR_{E,t} = \left[\frac{\Pi_t}{Q_{E,t}} + \frac{\dot{Q}_{E,t}}{Q_{E,t}}\right] dt + \frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}} d\mathcal{N}_t,$$

where Π_t denotes real profits and $Q^*_{E,t}$ is the equity price in the disaster state.

Importantly, savers have heterogeneous beliefs regarding the probability of a disaster. Savers' subjective beliefs about the arrival rate of the aggregate productivity shock are given by λ_j, for $j \in \{o, p\}$, where we assume that $\lambda_o \leq \lambda_p$. We follow e.g. Chen et al. (2012) and assume that savers are dogmatic in their beliefs about disaster risk, so we abstract from any learning process. We also assume that $\rho_o - \rho_p = \lambda_p - \lambda_o$, which ensures that both types of savers are unconstrained in the long run.

Savers face a constant hazard rate of death $\xi \geq 0$. Newborn savers inherit the wealth from parents and they are optimistic with probability $\frac{\mu_o}{\mu_o+\mu_p}$ and pessimistic with probability $\frac{\mu_p}{\mu_o+\mu_p}$. Let $C_{j,t}(s)$ denote the time t consumption of a type-j saver born at date $s \leq t$ and $\overline{C}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} C_{j,t}(s) ds$ denote average consumption of type-j savers, where similar notation applies to other variables. For ease of notation, we often drop the dependence on s and simply write $C_{j,t}$ instead of $C_{j,t}(s)$.[10]

Let $B_{j,t} = B^S_{j,t} + B^L_{j,t} + B^E_{j,t}$ denote the net worth of a type-j saver, the sum of short-term bonds ($B^S_{j,t}$), long-term bonds ($B^L_{j,t}$), and equity holdings ($B^E_{j,t}$). A type-j saver chooses consumption $C_{j,t}$, long-term bonds $B^L_{j,t}$, and equity holdings $B^E_{j,t}$, given an initial net worth $B_{j,t} > 0$, to solve the following problem:

$$V_{j,t}(B_{j,t}) = \max_{[C_{j,z}, B^L_{j,z}, B^E_{j,z}]_{z\geq t}} \mathbb{E}_{j,t}\left[\int_t^{t^*} e^{-\rho_j(z-t)} \frac{C_{j,z}^{1-\sigma}}{1-\sigma} dz + e^{-\rho_j(t^*-t)} V^*_{j,t^*}(B^*_{j,t^*})\right],$$

[9]This expression follows from $dR_{L,t} = \frac{e^{-\psi_L t}}{Q_{L,t}e^{-\psi_L t}} dt + \frac{d(Q_{L,t}e^{-\psi_L t})}{Q_{L,t}e^{-\psi_L t}}$ and $dQ_{L,t} = \dot{Q}_{L,t} dt + (Q^*_{L,t} - Q_{L,t}) d\mathcal{N}_t$.

[10]The perpetual youth assumption pins down the long-run wealth distribution among optimistic and pessimistic savers, but it is otherwise not central to our results.

subject to the flow budget constraint

$$dB_{j,t} = \left[(i_t - \pi_t)B_{j,t} + r_{L,t}B^L_{j,t} + r_{E,t}B^E_{j,t} + T_{j,t} - C_{j,t}\right]dt + \left[B^L_{j,t}\frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} + B^E_{j,t}\frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}}\right]d\mathcal{N}_t,$$

as well as borrowing and short-selling constraints

$$B_{j,t} \geq 0, \qquad B^L_{j,t} \geq 0, \qquad B^E_{j,t} \geq 0,$$

where i_t is the nominal interest rate, π_t is the inflation rate, $r_{L,t} \equiv \frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L - i_t$ is the excess return on long-term bonds conditional on no disasters, $r_{E,t} \equiv \frac{\Pi_t}{Q_{E,t}} + \frac{\dot{Q}_{E,t}}{Q_{E,t}} - (i_t - \pi_t)$ is the excess return on equities conditional on no disasters, and $T_{j,t}$ denote government transfers. The random (stopping) time t^* represents the period in which the aggregate shock hits the economy. $V^*_{j,t^*}(\cdot)$ and B^*_{j,t^*} denote, respectively, the value function and net worth in the disaster state. The savers' problem in the disaster state corresponds to a deterministic version of the problem above, as the disaster happens only once. The non-negativity constraint on $B^L_{j,t}$ captures the assumption that only the government can issue long-term bonds. The discount rate for savers can be written as $\rho_j \equiv \tilde{\rho}_j + \xi$, where $\tilde{\rho}_j$ captures subjective discounting and ξ captures the effect of mortality risk.

Savers are unconstrained at all times in equilibrium. The Euler equation for short-term bonds, derived in Appendix B, is given by

$$\frac{\dot{C}_{j,t}}{C_{j,t}} = \sigma^{-1}(i_t - \pi_t - \rho_j) + \frac{\lambda_j}{\sigma}\left[\left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma} - 1\right], \tag{1}$$

where $C^*_{j,t}$ is the consumption of a type-j saver in the disaster state. The first term captures the usual intertemporal-substitution force present in RANK models. The second term captures the *precautionary savings motive* generated by the disaster risk, and it is analogous to the precautionary motive that emerges in HANK models with idiosyncratic risk.

The Euler equation for long-term bonds is given by

$$r_{L,t} = \underbrace{\lambda_j\left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma}}_{\substack{\text{price of}\\ \text{disaster risk}}}\underbrace{\frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}}}_{\substack{\text{quantity of}\\ \text{risk}}}. \tag{2}$$

This expression captures a risk premium on long-term bonds, which pins down long-term interest rates in equilibrium. The premium on long-term bonds is given by the product of the *price of disaster risk*, the compensation for a unit exposure to the risk factor, and the *quantity of risk*, the loss the asset suffers conditional on switching to the disaster state.

Similarly, the Euler equation for equities is given by

$$r_{E,t} = \lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}} \right)^\sigma \frac{Q_{E,t} - Q^*_{E,t}}{Q_{E,t}}. \tag{3}$$

The expression above pins down the (conditional) equity premium in equilibrium. As stocks and long-term bonds are exposed to the same aggregate shock, average returns by unit of risk (the price of risk) is the same for both assets. Differences in expected returns are then driven by differences in the quantity of risk.

Workers' problem. In contrast to savers, workers supply labor and have GHH preferences (Greenwood et al., 1988) over consumption and labor. Their problem is given by

$$V_{w,t}(B_{w,t}) = \max_{[C_{w,z}, N_{w,z}]_{z \geq t}} \mathbb{E}_{w,t} \left[\int_t^{t^*} \frac{e^{-\rho_w(z-t)}}{1-\sigma} \left(C_{w,z} - \frac{N_{w,z}^{1+\phi}}{1+\phi} \right)^{1-\sigma} dz + e^{-\rho_w(t^*-t)} V^*_{w,t^*}(B_{w,t^*}) \right],$$

subject to the flow budget constraint

$$dB_{w,t} = \left[(i_t - \pi_t) B_{w,t} + \frac{W_t}{P_t} N_{w,t} + T_{w,t} - C_{w,t} \right] dt,$$

and the borrowing constraints $B_{w,t} \geq 0$, where W_t is the nominal wage, P_t is the price level, and $T_{w,t}$ denotes fiscal transfers to workers.

We focus on the case where the initial condition is $B_{w,0} = 0$ and ρ_b is sufficiently large, so borrowers are constrained at all periods. For simplicity, we have already imposed that $B^S_{w,t} = B^E_{w,t} = 0$, given that short-selling constraints would otherwise be binding if borrowers could choose $B^S_{w,t}$ and $B^E_{w,t}$. As borrowers are constrained, their beliefs about the disaster probability play no role in the determination of equilibrium.

The labor supply is determined by the standard condition:

$$\frac{W_t}{P_t} = N^\phi_{w,t}.$$

GHH preferences imply that there is no income effect on labor supply, roughly in line with the evidence (see e.g. Auclert et al., 2021), and simplifies the model aggregation.[11]

[11] GHH preferences also avoid the counterfactual implications caused by income effects on labor supply in heterogeneous-agent models with sticky prices emphasized by Broer et al. (2020).

Market-implied probabilities and the SDF. From Equations (2) and (3), we can see that, even though savers disagree on the probability of a disaster, they agree on the *value* of a unit of consumption in that state.[12] We can then price any cash flow using the beliefs and marginal utility of either optimistic or pessimistic savers. Instead of using the beliefs of a specific saver, it is convenient to define the economy's stochastic discount factor (SDF) using the aggregate consumption of savers, $C_{s,t} \equiv \frac{\mu_o}{\mu_o+\mu_p}\overline{C}_{o,t} + \frac{\mu_p}{\mu_o+\mu_p}\overline{C}_{p,t}$, and the corresponding disaster probability implied by asset prices, as shown in Proposition 1.[13]

Proposition 1 (Market-implied disaster probability)**.** *Define the market-implied disaster probability λ_t as follows:*

$$\lambda_t \equiv \left[\frac{\mu_o\overline{C}_{o,t}}{\mu_o\overline{C}_{o,t} + \mu_p\overline{C}_{p,t}}\lambda_o^{\frac{1}{\sigma}} + \frac{\mu_p\overline{C}_{p,t}}{\mu_o\overline{C}_{o,t} + \mu_p\overline{C}_{p,t}}\lambda_p^{\frac{1}{\sigma}}\right]^{\sigma}, \tag{4}$$

and let $\mathbb{E}_t[\cdot]$ denote the expectation operator associated with the arrival rate λ_t for the disaster shock. Then, $\eta_t = e^{-\int_0^t \rho_{s,z}dz}C_{s,t}^{-\sigma}$ is a valid stochastic discount factor, i.e., η_t correctly prices all tradeable assets given the disaster probability λ_t and the process $\rho_{s,t} \equiv \rho_j + \lambda_j - \lambda_t$.

Proof. To ensure that η_t correctly prices long-term bonds and equities, consistent with Equations (2) and (3), the market-implied disaster probability must satisfy the condition:

$$\lambda_t\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma} = \lambda_j\left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma} \Rightarrow C^*_{j,t} = \left(\frac{\lambda_j}{\lambda_t}\right)^{\frac{1}{\sigma}}\frac{C^*_{s,t}}{C_{s,t}}C_{j,t}.$$

Plugging $C^*_{j,t}$ into the definition of savers' average consumption in the disaster state, $C^*_{s,t} \equiv \frac{\mu_o}{\mu_o+\mu_p}\overline{C}^*_{o,t} + \frac{\mu_p}{\mu_o+\mu_p}\overline{C}^*_{p,t}$, and rearranging gives Equation (4). By setting $\rho_{s,t} = \rho_j + \lambda_j - \lambda_t$, we ensure that η_t correctly prices risk-free bonds, i.e., $\mathbb{E}_t[d\eta_t]/\eta_t = -(i_t - \pi_t)dt$. □

The market-implied probability λ_t is a CES aggregator of individual probabilities, weighted by the corresponding consumption share. Expression (4) is reminiscent of the complete-markets formula with heterogeneous beliefs in e.g. Varian (1985). However, under complete markets, the consumption shares of optimistic and pessimistic savers would be constant. In contrast, consumption shares can potentially move over time in our setting, which leads to endogenous time-variation in the perceived probability of a disaster,

[12] The value of a consumption unit in the disaster state for saver j is $\lambda_j(C^*_{j,t}/C_{j,t})^{-\sigma}$, the continuous-time version of the standard expression for state prices, which is equalized for all savers from Equations (2)-(3).

[13] A long tradition in asset-pricing relates the consumption of stockholders, savers in our economy, and asset prices. See e.g. Mankiw and Zeldes (1991) and Parker (2001).

even though the objective disaster probability is constant. We can then price assets as-if the economy has a representative saver with (endogenous) time-varying beliefs.

Firms' problem. Intermediate-goods producers are indexed by $i \in [0, 1]$ and operate in monopolistically competitive markets. Final good producers are price takers and combine intermediate goods to produce the final good. Their demand for variety i is given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$, and the equilibrium price level is given by $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$.

Intermediate-goods producers operate the linear technology $Y_{i,t} = A_t N_{i,t}$. Productivity in the no-disaster state is given by $A_t = A$, and productivity in the disaster state is given by $A_t = A^*$, where $0 < A^* < A$. Intermediate-goods producers choose the rate-of-change of prices $\pi_{i,t} = \dot{P}_{i,t}/P_{i,t}$, given the initial price $P_{i,0}$, to maximize the expected discounted value of real profits subject to Rotemberg quadratic adjustment costs:

$$Q_{i,t}(P_{i,t}) = \max_{[\pi_{i,z}]_{z \geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_z}{\eta_t} \left(\frac{P_{i,z}}{P_z} Y_{i,z} - \frac{W_z}{P_z} \frac{Y_{i,z}}{A} - \frac{\varphi}{2} \pi_{i,t}^2 \right) dz + \frac{\eta_{t^*}}{\eta_t} Q_{i,t^*}^*(P_{i,t^*}) \right], \tag{5}$$

the demand $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$, and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, where $Q_{i,t}^*(P_i)$ denotes the firms' value function in the disaster state. The price $P_{i,t}$ is a state variable and $\pi_{i,t}$ is a control variable. The parameter φ controls the magnitude of the pricing adjustment costs. These costs are rebated to households, so they do not represent real resource costs. Profits are discounted using the economy's SDF, and expectations are computed using the market-implied probability λ_t, consistent with savers' valuation of the firm.

Combining the first-order condition and the envelope condition for problem (5), we obtain the non-linear New Keynesian Phillips curve:

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t} \right) \pi_t - \frac{\epsilon}{\varphi A} \left(\frac{W_t}{P_t} - (1 - \epsilon^{-1}) A \right) Y_t, \tag{6}$$

assuming a symmetric initial condition $P_{i,0} = P_0$, for all $i \in [0, 1]$, and $\pi_{i,t}^* = 0$.

Government. The government is subject to a flow budget constraint

$$\dot{D}_{G,t} = (i_t - \pi_t + r_{L,t}) D_{G,t} + \sum_{j \in \{w,o,p\}} \mu_j T_{j,t},$$

and a No-Ponzi condition $\lim_{t \to \infty} \mathbb{E}_0[\eta_t D_{G,t}] \leq 0$, where $D_{G,t}$ denotes the real value of government debt, $D_{G,0} = D_G$ is given, and analogous conditions hold in the disaster state. Transfers to workers are given by the policy rule $T_{w,t} = T_w(Y_t)$. We assume $T_{o,t} = T_{p,t}$, and the government adjusts transfers to savers such that the No-Ponzi condition is satisfied.

In the no-disaster state, monetary policy is determined by the policy rule

$$i_t = r_n + \phi_\pi \pi_t + u_t, \tag{7}$$

where $\phi_\pi > 1$, u_t is a monetary shock, and r_n denotes the real rate when $\pi_t = u_t = 0$ at all periods. We assume that in the disaster state there are no monetary shocks, that is, $i_t^* = r_n^* + \phi_\pi \pi_t^*$. By abstracting from the policy response after a disaster, we isolate the impact of changes in monetary policy during "normal times."

Market clearing. The market-clearing conditions for goods, bonds, and equities are given by

$$\sum_{j\in\{w,o,p\}} \mu_j \overline{C}_{j,t} = Y_t, \qquad \sum_{j\in\{w,o,p\}} \mu_j \overline{B}_{j,t}^S = 0, \qquad \sum_{j\in\{w,o,p\}} \mu_j \overline{B}_{j,t}^L = D_{G,t}, \qquad \sum_{j\in\{w,o,p\}} \mu_j \overline{B}_{j,t}^E = Q_{E,t},$$

and labor market clearing is $\mu_b N_{b,t} = N_t$, where $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}} di\right)^{\frac{\epsilon-1}{\epsilon}}$ and $N_t = \int_0^1 N_{i,t} di$.

2.2 Equilibrium dynamics

Stationary equilibrium. We define a stationary equilibrium as an equilibrium in which all variables are constant in each aggregate state. In particular, the economy will be in a stationary equilibrium in the absence of monetary shocks, that is, $u_t = 0$ for all $t \geq 0$. Since variables are constant in each state, we drop time subscripts and write, for instance, $C_{j,t} = C_j$ and $C_{j,t}^* = C_j^*$. For ease of exposition, we follow Bilbiie (2018) and assume that T_w implements $C_w = Y$ and $C_w^* = Y^*$, and discuss the general case in Appendix B.

The natural interest rate, the real rate in the stationary equilibrium, is given by

$$r_n = \rho_s - \lambda \left[\left(\frac{C_s}{C_s^*} \right)^\sigma - 1 \right],$$

where ρ_s and λ are the values of $\rho_{s,t}$ and λ_t in the stationary equilibrium, and $0 < C_s^* < C_s$. We assume that the natural rate is positive, $r_n > 0$. The precautionary motive depresses the natural interest rate relative to the one that would prevail in a non-stochastic economy.

From Equation (2), we can pin down the term spread, the difference between the yield on the long-term bond and the short-term rate, in this economy:

$$r_L = \lambda \left(\frac{C_s}{C_s^*} \right)^\sigma \frac{Q_L - Q_L^*}{Q_L},$$

and $Q_L^* < Q_L$. It can be shown that $r_L = i_L - r_n$, where i_L is the yield on the long-term bond in the stationary equilibrium.[14] Thus, our model generates an upward-sloping yield curve, where long-term yields exceed the short rate, consistent with the data.[15]

Similarly, the equity premium (conditional on no-disaster) is given by[16]

$$r_E = \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma} \frac{Q_E - Q_E^*}{Q_E},$$

and $Q_E^* < Q_E$. Therefore, the equity premium is positive in the stationary equilibrium.

Households have heterogeneous portfolios in equilibrium. Workers are against the borrowing constraint and hold no equities or long-term bonds. Optimistic savers are more exposed to disaster risk than pessimist investors. The exact composition of their portfolio is indeterminate, as we have one redundant asset. For concreteness, we focus on the case $B_o^E = B_p^E$, so optimists hold more long-term bonds, i.e. $B_o^L > B_p^L$. This leads to a simpler presentation in the analysis that follows.

Log-linear dynamics. Following the monetary policy literature, we focus on a log-linear approximation of the equilibrium conditions. However, instead of linearizing around the non-stochastic steady state, we linearize the equilibrium conditions around the (stochastic) stationary equilibrium described above. Formally, we perturb the allocation around the economy where $u_t = 0$ and $\lambda > 0$, while the standard approach would perturb around the economy where $u_t = \lambda_t = 0$. This enables us to capture the effects of (time-varying) precautionary savings and risk premia in a linear setting, as shown below.[17]

Let lower-case variables denote log-deviations from the stationary equilibrium, e.g., $y_t \equiv \log Y_t/Y$ and $c_{w,t} \equiv \log C_{w,t}/C_w$. Workers' consumption is given by

$$c_{w,t} = \frac{WN_w}{PY}(w_t - p_t + n_{w,t}) + T_w'(Y) y_t \Rightarrow c_{w,t} = \chi_y y_t, \tag{8}$$

using $w_t - p_t = \phi y_t$ and $n_{w,t} = y_t$, where $\chi_y \equiv \frac{WN_w}{PY}(1+\phi) + T_w'(Y)$.

The coefficient χ_y controls the cyclicality of income inequality among workers and

[14] The yield on the long-term bond is given by $i_{L,t} = Q_{L,t}^{-1} - \psi_L$ and, in a stationary equilibrium, $\dot{Q}_{L,t} = 0$, so the expected excess return conditional on no disaster r_L equals the term spread $i_L - r_n$.

[15] The upward-sloping yield curve is caused by the lack of precautionary savings in the disaster state. We would obtain similar results by introducing expropriation and inflation in a disaster, as in Barro (2006).

[16] The unconditional equity premium equals r_E minus the expected loss on a disaster. Using λ to compute the expected loss, the (unconditional) equity premium would be given by $\lambda \left[(C_s/C_s^*)^{\sigma} - 1 \right] (Q_E - Q_E^*)/Q_E$.

[17] This method differs from the procedure considered by Coeurdacier et al. (2011) or Fernández-Villaverde and Levintal (2018), as we linearize around a stochastic steady state of an economy with no monetary shocks, instead of the stochastic steady state of the economy with both shocks.

savers. We focus on the case $0 < \chi_y < \mu_w^{-1}$, such that the consumption of savers, which is given by $c_{s,t} = \frac{1-\mu_w \chi_y}{1-\mu_w} y_t$ from the market clearing for goods, is also increasing in y_t.

Linearizing Equation (1), aggregating savers, and defining $\hat{\lambda}_t \equiv \log \frac{\lambda_t}{\lambda}$, we obtain

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \frac{\lambda}{\sigma} \left(\frac{C_s}{C_s^*} \right)^{\sigma} \left[\hat{\lambda}_t + \sigma c_{s,t} \right]. \tag{9}$$

Combining condition (8) for borrowers' consumption, equation (9) for savers' Euler equation, and the market-clearing condition for goods, we obtain the evolution of aggregate output. Proposition 2 characterizes the dynamics of aggregate output and inflation, given the paths of i_t and $\hat{\lambda}_t$. Proofs omitted in the text are provided in Appendix A.

Proposition 2 (Aggregate dynamics)**.** *Given $[i_t, \hat{\lambda}_t]_{t \geq 0}$, the dynamics of output and inflation is described by the conditions:*

i. ***Aggregate Euler equation:***

$$\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + \chi_\lambda \hat{\lambda}_t, \tag{10}$$

where $\tilde{\sigma}^{-1} \equiv \frac{1-\mu_w}{1-\mu_w \chi_y} \sigma^{-1}$, $\delta \equiv \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma}$, and $\chi_\lambda \equiv \tilde{\sigma}^{-1} \delta$.

ii. ***New Keynesian Phillips curve:***

$$\dot{\pi}_t = \rho \pi_t - \kappa y_t, \tag{11}$$

where $\rho \equiv \rho_s + \lambda$ and $\kappa \equiv \varphi^{-1}(\epsilon - 1)\phi Y$.

Condition (10) represents the *aggregate Euler equation*. This equation has three terms, capturing the effects of heterogeneous MPCs, aggregate risk, and heterogeneous beliefs. The first term is the product of the aggregate elasticity of intertemporal substitution (EIS), $\tilde{\sigma}^{-1}$, and the real interest rate. The aggregate EIS depends on the cyclicality of inequality among workers and savers, as captured by χ_y. As in the work of Werning (2015) and Bilbiie (2017), heterogeneous MPCs amplify the effect of changes in interest rates if workers' consumption share is procyclical (i.e., $\chi_y > 1$), as it implies that $\tilde{\sigma}^{-1} > \sigma^{-1}$.

The second term, δy_t, captures the effect of aggregate risk. In the absence of belief heterogeneity, so $\hat{\lambda}_t = 0$, we can write output as $y_t = -\tilde{\sigma}^{-1} \int_t^\infty e^{-\delta(s-t)}(i_s - \pi_s - r_n)ds$. Hence, a positive δ dampens the effect of future real interest rates, as in the discounted Euler equation of McKay et al. (2017). In our setting, this is the result of a precautionary motive in response to aggregate disaster risk instead of idiosyncratic income risk.

The third term in the aggregate Euler equation, $\chi_\lambda \hat{\lambda}_t$, captures the effect of heterogeneous beliefs. An increase in the market-implied disaster probability implies that pessimistic investors have a higher consumption share, as shown in Proposition 1. This increase in average pessimism triggers a stronger precautionary motive in the aggregate. Notice this effect is not present without investor heterogeneity, as $\hat{\lambda}_t = 0$ when savers have homogeneous beliefs.

Finally, Proposition 2 derives the New Keynesian Phillips curve. The linearized Phillips curve coincides with the one obtained from models with Calvo pricing. As in a textbook New Keynesian model, inflation is given by the present discounted value of future output gaps, $\pi_t = \kappa \int_t^\infty e^{-\rho(s-t)} y_s ds$. One distinction relative to the standard formulation is that future output gaps are not discounted by the natural rate r_n but by a higher rate $\rho > r_n$, as firm value is risky in our setting, which requires a higher discount rate.

2.3 Monetary policy and risk premia

Asset prices. The response of asset prices to monetary policy depends crucially on the behavior of the price of disaster risk, as shown in Equations (2) and (3). In its log-linear form, the price of disaster risk is given by

$$p_{d,t} \equiv \sigma(c_{s,t} - c^*_{s,t}) + \hat{\lambda}_t. \tag{12}$$

This expression has two terms. The first term captures the increase in the savers' marginal utility of consumption if the disaster shock is realized. The second term represents the change in the market-implied disaster probability after a monetary shock.

Given the price of risk, we can price any financial asset in this economy. For example, the (linearized) price of the long-term bond in period zero is given by[18]

$$q_{L,0} = -\underbrace{\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n)dt}_{\text{path of nominal interest rates}} - \underbrace{\int_0^\infty e^{-(\rho+\psi_L)t} r_L p_{d,t} dt}_{\text{term premium}}. \tag{13}$$

The yield on the long-term bond, expressed as deviations from the stationary equilibrium, is given by $-Q_L^{-1} q_{L,0}$, which can be decomposed into two terms: the path of nominal interest rates, as in the expectations hypothesis, and a *term premium*, capturing variations in the compensation for holding long-term bonds. The term premium depends on the

[18]Linearizing Equation (2) and rearranging, we obtain $\dot{q}_{L,t} - (\rho + \psi_L) q_{L,t} = i_t - r_n + r_L p_{d,t}$. Integrating this condition forward gives us Equation (13).

price of risk, $p_{d,t}$, and the asset-specific loading r_L. Because the term premium responds to monetary shocks, the expectation hypothesis does not hold in this economy.

The pricing condition for equities is analogous to the one for long-term bonds:

$$q_{E,0} = \underbrace{\frac{Y}{Q_E}\int_0^\infty e^{-\rho t}\hat{\Pi}_t dt}_{\text{dividends}} - \underbrace{\int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_E p_{d,t}\right] dt}_{\text{discount rate}}, \tag{14}$$

where $\hat{\Pi}_t = y_t - \frac{WN}{PY}(w_t - p_t + n_t)$. Equity prices respond to changes in monetary policy through two channels: a *dividend channel*, capturing changes in firms' profits, and a *discount rate channel*, capturing changes in real interest rates and risk premia. Risk premia depends on the price of risk, $p_{d,t}$, and the asset-specific loading r_E.

Market-implied disaster probability. Log-linearizing Equation (4), we obtain

$$\frac{1}{\sigma}\lambda^{\frac{1}{\sigma}}\hat{\lambda}_t = \mu_{c,o}\mu_{c,p}\left(\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}\right)\left[c_{p,t} - c_{o,t}\right], \tag{15}$$

where $\mu_{c,j} \equiv \frac{\mu_j \overline{C}_j}{\mu_o \overline{C}_o + \mu_p \overline{C}_p}$, for $j \in \{o, p\}$.

The market-implied disaster probability increases when the monetary shock redistributes consumption towards pessimistic investors. We show in Appendix B.3 that the relative consumption of the two types of savers evolves according to

$$\dot{c}_{p,t} - \dot{c}_{o,t} = -\tilde{\xi}(b_{p,t} - b_{o,t}), \tag{16}$$

where $\tilde{\xi} \geq 0$ is proportional to the mortality parameter, and the law of motion of relative net worth $b_{p,t} - b_{o,t}$ is given by

$$\dot{b}_{p,t} - \dot{b}_{o,t} = -\chi_{b,c}(c_{p,t} - c_{o,t}) + \chi_{b,b}(b_{p,t} - b_{o,t}) + \chi_{b,c_s}c_s, \tag{17}$$

where the coefficients $\chi_{b,c}, \chi_{b,b}$, and χ_{b,c_s} are a function of portfolios and equilibrium returns in the stationary equilibrium. Hence, the dynamics of relative consumption depends on relative net worth $b_{p,t} - b_{o,t}$, while the dynamics of relative net worth $b_{p,t} - b_{o,t}$ depends on relative consumption and savers' aggregate consumption. As c_s depends on y_t, we must simultaneously solve for $[c_{p,t} - c_{o,t}, b_{p,t} - b_{o,t}]_0^\infty$ and $[i_t, y_t, \pi_t]_0^\infty$, which involves a relatively large dynamic system. In this case, obtaining analytical results would be infeasible. In the next proposition, we show that this system actually satisfies an *approximate block recursivity* property, where we can solve for $c_{p,t} - c_{o,t}$ (or $\hat{\lambda}_t$) and $b_{p,t} - b_{o,t}$

independently of (y_t, π_t), provided that the effect of $c_{s,t}$ on risk premia is small.

Proposition 3 (Approximate block recursivity)**.** *Suppose $r_k c_{s,t}$ is small for $k \in \{L, E\}$, i.e. $r_k c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$. Then, the market-implied probability of disaster $\hat{\lambda}_t$ and relative net worth $b_{p,t} - b_{o,t}$ can be solved independently of the aggregate variables (y_t, π_t), and they are given by*

$$\hat{\lambda}_t = e^{-\psi_\lambda t} \hat{\lambda}_0, \tag{18}$$

and $b_{p,t} - b_{o,t} = e^{-\psi_\lambda t}(b_{p,0} - b_{o,0})$, where $\psi_\lambda \geq 0$ is strictly increasing in ξ, it is equal to zero if $\xi = 0$ and it approaches infinity if $\xi \to \infty$. If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, then the initial value of $\hat{\lambda}_t$ is given by

$$\hat{\lambda}_0 = \epsilon_\lambda (i_0 - r_n), \tag{19}$$

where $\epsilon_\lambda \geq 0$ and the inequality is strict if $\lambda_p > \lambda_o$.

Note that the effect of changes in the price of risk on risk premia is given by $r_k p_{d,t} = r_k(\sigma c_{s,t} + \hat{\lambda}_t)$, using $c^*_{s,t} = 0$. If $r_k c_{s,t}$ is second-order on the size of the monetary shock, then this term can be ignored, and Proposition 3 shows that we can solve for $\hat{\lambda}_t$ independently of (y_t, π_t). As the dynamics of (y_t, π_t) depends on $\hat{\lambda}_t$, but $\hat{\lambda}_t$ does not depend on (y_t, π_t), we say the system is (approximately) block recursive. We show in the appendix that the solution ignoring the terms $r_k c_{s,t}$ tracks very closely the numerical solution where these terms are taken into account.

An important implication of Equation (19) is that the price of risk increases after a contractionary monetary shock. A monetary tightening redistributes wealth away from optimistic investors, as they are more exposed to risky assets. The economy becomes on average more pessimistic, which raises the required compensation for holding risky assets. The increase in risk premia in response to contractionary monetary shocks is consistent with the evidence in, e.g., Gertler and Karadi (2015) and Hanson and Stein (2015). Notice that investor heterogeneity is necessary for this result, as $\hat{\lambda}_t = 0$ when $\lambda_o = \lambda_p$.

Market incompleteness is necessary for monetary policy to affect risk premia in our setting. A first form of market incompleteness comes from the lack of hedging instruments against monetary shocks. If savers could hedge monetary surprises, their consumption share would not react to monetary shocks, and λ_t would be constant. A second form of market incompleteness comes from mortality risk. Monetary shocks have a permanent effect on the wealth distribution when $\xi = 0$. Mortality risk implies that the wealth distribution reverts to its long-run level, so the effects of monetary policy on risk premia eventually die out.

3 Monetary Policy and Wealth Effects

In the previous section, we considered the effects of monetary policy on risk premia and asset prices through its impact on $\hat{\lambda}_t$. We study next how the revaluation of real and financial assets induced by monetary policy affects the real economy.

3.1 The dynamic system

A monetary policy shock triggers two types of policy response: i) changes in the path of nominal interest rates i_t; and ii) changes in the fiscal backing τ_t. Output and inflation respond to the joint change in policy variables. To isolate the impact of nominal interest rates on asset prices, it is important to disentangle the role of i_t and τ_t. This motivates the following two-step procedure. First, we express output and inflation in terms of the path of policy variables $\{i_t, \tau_t\}$. Second, we derive an *implementability result* that shows how to map policy variables to the underlying monetary shock u_t in the interest rate rule (7).

We start by considering the system of differential equations in Proposition 2:

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \delta & -\tilde{\sigma}^{-1} \\ -\kappa & \rho \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \nu_t \\ 0 \end{bmatrix}, \tag{20}$$

where $\nu_t \equiv \tilde{\sigma}^{-1}(i_t - r_n) + \chi_\lambda \hat{\lambda}_t$ depends only on the path of nominal interest rates. The eigenvalues of the system are given by

$$\overline{\omega} = \frac{\rho + \delta + \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}, \qquad \underline{\omega} = \frac{\rho + \delta - \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}.$$

The following assumption, which we assume holds for all subsequent analysis, guarantees that the eigenvalues are real-valued and have opposite signs, i.e., $\overline{\omega} > 0$ and $\underline{\omega} < 0$.

Assumption 1. *The following condition holds: $\rho\delta < \tilde{\sigma}^{-1}\kappa$.*

Assumption 1 implies that the system lacks exactly one boundary condition.[19] The missing boundary condition can be provided by an aggregate intertemporal budget constraint (IBC). From savers' transversality condition, combined with workers' budget constraint, we obtain the (non-linear) aggregate IBC:

$$\mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0} C_t dt\right] = D_{G,0} + Q_{E,0} + \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\left(\frac{W_t}{P_t} N_t + T_t\right) dt\right],$$

[19] Assumption 1 implies that the equilibrium is indeterminate under an interest-rate peg. As shown in Section 3.5, local determinacy requires $\phi_\pi \geq 1 - \frac{\rho\delta}{\tilde{\sigma}^{-1}\kappa} \equiv \overline{\phi}_\pi$, and $\overline{\phi}_\pi > 0$ under Assumption 1.

where $C_t \equiv \mu_w C_{w,t} + (1-\mu_w)C_{s,t}$ and $T_t = \mu_w T_{w,t} + (1-\mu_w)T_{s,t}$. See Appendix C for a derivation. This expression says that the present value of aggregate consumption equals the value of assets held by the household sector: government bonds, stocks, and human wealth (i.e., the value of labor income after transfers).

To linearize the expression above, it is convenient to define $Q_{C,t} \equiv \mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t} C_z dz\right]$, the value of the consumption claim, and $Q_{H,t} \equiv \mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t}\left(\frac{W_z}{P_z}N_z + T_t\right)dz\right]$, the value of human wealth. We can compute the price of these two claims in the same way as we priced stocks and bonds (see Equations 13 and 14). For instance, the value of the consumption claim satisfies the condition:

$$q_{C,0} = \frac{C}{Q_C}\int_0^\infty e^{-\rho t} c_t dt - \int_0^\infty e^{-\rho t}\left(i_t - \pi_t + r_C p_{d,t}\right)dt, \tag{21}$$

where $q_{C,0} \equiv \log Q_{C,0}/Q_C$ and $r_C \equiv \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_C - Q_C^*}{Q_C}$.

The linearized intertemporal budget constraint can then be written as follows:

$$Q_C q_{C,0} = D_G q_{L,0} + Q_E q_{E,0} + Q_H q_{H,0}. \tag{22}$$

From Equation (22), we have that the aggregate IBC is a necessary equilibrium condition. The next lemma establishes the sufficiency of the aggregate IBC for pinning down the equilibrium. That is, it shows that if $[y_t, \pi_t]_0^\infty$ satisfies system (20) and the IBC (in its log-linear form), then we can determine the consumption, portfolio, labor supply as well as wages and prices such that all equilibrium conditions are satisfied.

Lemma 1. *Suppose that, given a path for the nominal interest rate and fiscal backing $[i_t, \tau_t]_0^\infty$, $[y_t, \pi_t]_0^\infty$ satisfy system (20) and the aggregate intertemporal budget constraint (22). Then, $[y_t, \pi_t]_0^\infty$ can be supported as part of a competitive equilibrium.*

Therefore, the equilibrium dynamics can be characterized as the solution to the dynamic system (20), subject to the boundary condition (22). Importantly, changes in asset prices can affect output and inflation through its impact on the IBC.

3.2 Aggregate wealth effect and risk-premium neutrality

We define the *aggregate wealth effect* as (minus) the total compensation required for households' initial consumption bundle to be just affordable. Thus, a monetary policy shock generates a negative wealth effect if a positive compensation is required for households to afford their pre-shock consumption level. Formally, we define the aggregate wealth effect, normalized by Y, as follows:

$$\Omega_0 \equiv -\frac{1}{Y}\sum_{j\in\{w,o,p\}}\left(\mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\mu_j C_j dt\right] - \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\mu_j C_{j,t} dt\right]\right). \tag{23}$$

We show in Appendix C.4 that this definition corresponds to (minus) the sum of the Slutsky wealth compensation, as defined in Mas-Colell et al. (1995), which justifies referring to Ω_0 as a wealth effect.

Linearizing Equation (23), we obtain

$$\Omega_0 = \int_0^\infty e^{-\rho t} c_t dt, \tag{24}$$

so the aggregate wealth effect determines the present discounted value of consumption. Combining the intertemporal budget constraint (22) and the pricing condition for consumption (21), we obtain

$$\Omega_0 = \frac{D_G}{Y} q_{L,0} + \frac{Q_E}{Y} q_{E,0} + \frac{Q_H}{Y} q_{H,0} + \frac{Q_C}{Y}\int_0^\infty e^{-\rho t}\left(i_t - \pi_t + r_C p_{d,t}\right) dt. \tag{25}$$

The expression above shows that the aggregate wealth effect equals the revaluation of real and financial assets net of the discount rate effect on consumption. This implies that the wealth effect is not simply given by the change in financial wealth, a fact that plays an important role in understanding the impact of changes in risk premia on the real economy. The next lemma shows how the net discount rate effect depends on the level and riskiness of government debt.

Lemma 2. *The aggregate wealth effect Ω_0 is given by*

$$\Omega_0 = \int_0^\infty e^{-\rho t}\left[\hat{\Pi}_t + \frac{WN}{PY}(w_t - p_t + n_t) + \hat{T}_t\right] dt + \bar{d}_G q_{L,0} + \bar{d}_G \int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_L p_{d,t}\right) dt,$$

where $\hat{T}_t \equiv \frac{T_t - T}{Y}$.

Proof. Using the pricing condition for $q_{k,0}$, $k \in \{C, H, E\}$, and Equation (22), we obtain

$$\int_0^\infty e^{-\rho t} c_t dt - \frac{Q_C}{Y}\int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_C p_{d,t}\right] dt = \int_0^\infty e^{-\rho t}\left[\hat{\Pi}_t + \frac{WN}{PY}(w_t - p_t + n_t) + \hat{T}_t\right] dt$$
$$- \frac{Q_H + Q_E}{Y}\int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n\right] dt - \left[\frac{Q_H}{Y} r_H + \frac{Q_E}{Y} r_E\right]\int_0^\infty e^{-\rho t} p_{d,t} dt + \frac{D_G}{Y} q_{L,0}.$$

Using the fact that $Q_C = Q_H + D_G + Q_E$ and $Q_C^* = Q_H^* + D_G \frac{Q_L^*}{Q_L} + Q_E^*$, we obtain $\frac{Q_C}{Y} -$

$\frac{Q_H+Q_E}{Y} = \frac{D_G}{Y} \equiv \overline{d}_G$ and $\frac{Q_C}{Y} r_C - \frac{Q_H r_H + Q_E r_E}{Y} = \overline{d}_G r_L$, given $r_k = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_k - Q_k^*}{Q_k}$. Combining these expressions with the equation above, we obtain (24) after some rearrangement. □

The first term in Ω_0 captures the effect of changes in cash flows, namely (after-tax) profits and wages. Naturally, households become wealthier if profits and wages increase in response to a monetary shock, everything else constant. The last two terms capture the net effect of changes in discount rates, i.e. interest rates and risk premia, which depends on the level of government debt.

Risk-premium neutrality. Asset revaluations caused by monetary policy have received significant attention recently. For instance, Cieslak and Vissing-Jorgensen (2020) show that policymakers pay attention to the stock market due to its potential (consumption) wealth effect. In contrast, Cochrane (2020) and Krugman (2021) argue that wealth gains on "paper" are not relevant for households who simply consume their dividends. The next proposition isolate the necessary conditions under which the latter view is correct.

Proposition 4 (Risk-premium neutrality)**.** *Suppose the government uses a consumption tax to neutralize the precautionary motive induced by $\hat{\lambda}_t$, that is, consider τ_t^c satisfying $\dot{\hat{\tau}}_t^c = \lambda \left(\frac{C_s}{C_s^*}\right) \hat{\lambda}_t$, where $\hat{\tau}_t^c \equiv \log(1+\tau_t^c)$, $\tau_t^c = \tau_t^{c,*}$, and the revenue is rebated back to households. Then, $[y_t, \pi_t]_0^\infty$ is independent of $\hat{\lambda}_t$ if one of the following conditions are satisfied: i) $\overline{d}_G = 0$; ii) $\overline{d}_G > 0$ and $\psi_L = \infty$; iii) $\overline{d}_G > 0$ and $\psi_L = 0$.*

Proof. Savers' Euler equation for the riskless bond is now given by $\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n - \dot{\hat{\tau}}_t^c) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma} \left[\hat{\lambda}_t + \sigma c_{s,t}\right]$, which is independent of $\hat{\lambda}_t$ if $\dot{\hat{\tau}}_t^c = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \hat{\lambda}_t$. As $\tau_t^c = \tau_t^{c,*}$, Euler equations for risky assets are not affected. The aggregate Euler equation then takes the same form as in Equation (10), but with $\chi_\lambda = 0$. As the revenue is rebated back to households, workers are not affected. If $\overline{d}_G = 0$, the last two terms in Ω_0 are equal to zero. If $\overline{d}_G > 0$ and $\psi_L = 0$, $\hat{\lambda}_t$ in the last two terms in Ω_0 exactly cancel out. If $\psi_L = \infty$, government bonds are safe and $r_L = 0$. Ω_0 is independent of $\hat{\lambda}_t$ in all three cases. □

Proposition 4 provides conditions under which time variation in the market-implied disaster probability λ_t does not impact the monetary transmission mechanism. Under such conditions, heterogeneity in portfolios among savers may help improve the model's asset-pricing implications, but they have no bearing on how monetary shocks ultimately affect the real economy. In particular, the solution is independent of $\lambda_p - \lambda_o$. Due to the increase in risk premium, an economy with heterogeneous beliefs would have a larger drop in asset prices after a monetary contraction than an economy where $\lambda_p = \lambda_o$. Despite the larger decline in the value of stocks and bonds, the response of output and inflation would be the same as in the economy without belief heterogeneity.

But why do households in the economy that suffered a larger drop in asset prices consume the same as households in the economy where asset prices did not drop as much? Take for instance the case $\bar{d}_G = 0$, so savers only hold stocks in equilibrium. One could expect that, as stock prices fall more sharply in the economy with $\hat{\lambda}_t > 0$, households would feel poorer and cut consumption relative to the economy with $\hat{\lambda}_t = 0$. However, this intuition does not take into account the fact that households can afford the same level of consumption with less wealth now. As households do not need more resources to afford their initial consumption bundle, this decline in asset prices do not create a negative wealth effect. The fact that changes in financial wealth may translate into no wealth effect provides a precise sense in which these changes may reflect "paper wealth."[20]

A similar point emerges in the discussion of taxation of capital gains. For instance, discussing the impact of a drop in interest rates for an investor whose consumption equals dividends every period, Cochrane (2020) says

> "When the interest rate goes down, it takes more wealth to finance the same consumption stream. The present value of liabilities – consumption – rises just as much as the present value of assets, so on a net basis Bob is not at all better."

In our terms, the increase in financial wealth does not translate into a positive wealth effect, as the drop in the price of stocks exactly cancels out the drop in the value of the consumption claim when consumption equals dividends.

Proposition 4 shows that the impact of changes in risk premia on Ω_0 can be zero even when $\bar{d}_G > 0$. When government bonds are a perpetuity, $\psi_L = 0$, then savers' consumption equal dividends plus coupons from government bonds. In this case, the drop in the value of stocks and long-term bonds exactly cancel out the drop in the value of the consumption claim. When government bonds are short-term, $\psi_L \to \infty$, stocks and the consumption claim are equally risky, so the discount rate effect cancels out again. Risk premia has a non-zero effect on Ω_0 only in the intermediate case $0 < \psi_L < \infty$.

We have focused so far on the impact of $\hat{\lambda}_t$ on Ω_0. However, $\hat{\lambda}_t$ also enters the aggregate Euler equation (10), as the redistribution between optimistic and pessimistic investors affect the average precautionary motive in the economy. For changes in $\hat{\lambda}_t$ to be neutral, in the sense of not affecting output and inflation, the government would have to offset the movements in precautionary motive. Proposition 4 shows the required change

[20]For instance, Fagereng et al. (2022) says "For such an individual [who only consumes dividends], rising asset prices are merely "paper gains," with no corresponding welfare implications."

in taxes to exactly offset this precautionary motive.

One implication of Proposition 4 is that, even though the logic of Cochrane (2020) and Krugman (2021) is present in our setting, it requires very stringent conditions to hold. In the empirically relevant case, $\overline{d}_G > 0$ and $0 < \psi_L < \infty$, movements in risk premia create a wealth effect, i.e., they do not represent only "paper wealth." Moreover, changes in $\hat{\lambda}_t$ may lead to movements in precautionary motive. Therefore, in general, both the aggregate wealth effect and the time-varying precautionary motive affect the real economy.

3.3 Intertemporal substitution, risk, and wealth effect

The next proposition characterizes the output response to a sequence of monetary policy shocks for a given value of the aggregate wealth effect Ω_0. We provide a full characterization of Ω_0 in Section 3.4. For ease of exposition, we focus on the case of exponentially decaying nominal interest rates; that is, we assume $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where ψ_m determines the persistence of the path of interest rates.

Proposition 5 (Aggregate output in D-HANK)**.** *Suppose that $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $\psi_k \neq -\underline{\omega}$, for $k \in \{m, \lambda\}$. The path of aggregate output is then given by*

$$y_t = \underbrace{\tilde{\sigma}^{-1}\hat{y}_{m,t}}_{\text{ISE}} + \underbrace{\chi_p \hat{y}_{\lambda,t}}_{\substack{\text{time-varying}\\ \text{precautionary motive}}} + \underbrace{(\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0}_{\substack{\text{GE multiplier}\times\\ \text{aggregate wealth effect}}}, \tag{26}$$

where $\chi_p \equiv \chi_\lambda \epsilon_\lambda$, $\hat{y}_{k,t}$ is given by

$$\hat{y}_{k,t} = \frac{(\rho - \underline{\omega})\, e^{\underline{\omega}t} - (\rho + \psi_k)\, e^{-\psi_k t}}{(\overline{\omega} + \psi_k)\,(\underline{\omega} + \psi_k)}(i_0 - r_n), \tag{27}$$

and satisfies $\int_0^\infty e^{-\rho t}\hat{y}_{k,t}dt = 0$, $\frac{\partial \hat{y}_{k,0}}{\partial i_0} < 0$, for $k \in \{m, \lambda\}$.

Proposition 5 shows that output can be decomposed into three terms: an intertemporal-substitution effect (ISE), a time-varying precautionary motive, and the aggregate wealth effect. These effects encompass some of main channels of transmission considered by the literature. By setting $\lambda_p = \lambda_o = 0$, the model behaves as a TANK model with zero liquidity, as in Bilbiie (2019) and Broer et al. (2020). Positive disaster probability $\lambda_p = \lambda_o > 0$ introduces a precautionary motive, analogous to HANK models (Kaplan et al. 2018), while $\lambda_p > \lambda_o > 0$ enable us to capture the effect of time-varying risk premia, as in Caballero and Simsek (2020) and Kekre and Lenel (2020).

The first term captures the standard intertemporal substitution channel present in RANK models. It depends on the aggregate EIS $\tilde{\sigma}^{-1} = \frac{1-\mu_w}{1-\mu_w\chi_y}\sigma^{-1}$ and $\hat{y}_{m,t}$ given in (27). Notice that, even though only a fraction $1-\mu_w$ of agents substitute consumption intertemporally, the ISE does not necessarily gets weaker as we reduce the mass of savers in the economy. As we reduce $1-\mu_w$, less agents are capable of intertemporal substitution, but the amplification from hand-to-mouth agents gets stronger. The two effects exactly cancel out when $\chi_y = 1$. Another important property of the ISE is that it is equal to zero on average, i.e. $\int_0^\infty e^{-\rho t}\hat{y}_{m,t} = 0$. An increase in interest rates shifts demand from the present to the future, but it does not change by itself the overall level of aggregate demand.

The second term captures the effect of the time-varying precautionary motive. It is equal to zero in the absence of belief heterogeneity, i.e. $\lambda_o = \lambda_p$. As with the EIS, the precautionary motive shifts demand from the present to the future without changing its overall level, that is, $\int_0^\infty e^{-\rho t}\hat{y}_{\lambda,t}dt = 0$. In contrast to the EIS, the persistence of the precautionary effects is controlled by ψ_λ instead of ψ_m, as it depends on the rate at which the balance sheet of optimistic investors recover after a contractionary shock.

The third term in expression (26) plays an important role, as the aggregate wealth effect determines the average response of output to the monetary shock. The GE multiplier captures the fact that an increase in Ω_0 has a disproportionate effect on initial output. Everything else constant, an increase in Ω_0 would tend to raise output in all periods by $\rho\Omega_0$, creating a parallel shift in output over time. In general equilibrium, a positive aggregate wealth effect leads to inflation on impact, which reduces the real rate and shift consumption to the present. The GE multiplier shows that the effect of Ω_0 on y_0 exceeds the effect on average consumption, $\rho\Omega_0$, by the factor $\frac{\rho-\omega}{\rho} > 1$.

Inflation. The next proposition characterizes the behavior of inflation.

Proposition 6 (Inflation in D-HANK). *Suppose $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $\psi_k \neq -\underline{\omega}$ for $k \in \{m, \lambda\}$. The path of inflation is given by*

$$\pi_t = \tilde{\sigma}^{-1}\hat{\pi}_{m,t} + \chi_p\hat{\pi}_{\lambda,t} + \kappa e^{\underline{\omega}t}\Omega_0, \tag{28}$$

where $\hat{\pi}_{k,t} = \frac{\kappa(e^{\underline{\omega}t} - e^{-\psi_k t})}{(\underline{\omega}+\psi_k)(\overline{\omega}+\psi_k)}(i_0 - r_n)$, $\hat{\pi}_{k,0} = 0$ and $\frac{\partial\hat{\pi}_{k,t}}{\partial i_0} \geq 0$, for $k \in \{m, \lambda\}$.

Inflation can be analogously decomposed into three terms. The first two terms capture the impact of the ISE and time-varying precautionary motive, while the last term captures the impact of the aggregate wealth effect. Because $\hat{\pi}_{k,0} = 0$, the first two terms are initially zero. This implies that initial inflation is determined entirely by the aggregate wealth effect. Moreover, π_t is actually *increasing* in i_0 if $\Omega_0 = 0$.

In a nutshell, Proposition 5 and 6 imply that monetary policy has a very limited impact on the economy in the absence of an aggregate wealth effect, i.e. if $\Omega_0 = 0$. In this case, a stimulus on output in the short run would come at the expense of a more depressed economy in the future, while the central bank would lose its ability to affect initial inflation. Therefore, the aggregate wealth effect plays a key role in the central bank's ability to control inflation or stimulate the economy.

3.4 The determination of the aggregate wealth effect

We consider next the determination of the aggregate wealth effect Ω_0. The aggregate wealth effect can be written as

$$\Omega_0 \equiv \int_0^\infty e^{-\rho t}\left[(1-\chi_\tau)y_t - \tau_t\right]dt + \overline{d}_G q_{L,0} + \overline{d}_G \int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_L p_{d,t}\right)dt,$$

where $\chi_\tau \equiv -\mu_w T'_w(Y)$ is the cyclicality of tax revenues.

The expression above shows that Ω_0 depends on the path of policy variables $[i_t, \tau_t]_0^\infty$ as well as the path of output and inflation $[y_t, \pi_t]_0^\infty$. Propositions 5 and 6 show that y_t and π_t depend on both policy variables and Ω_0. By combining Equations (26) and (28) with the expression for Ω_0, we can solve for Ω_0 in terms of policy variables.

Proposition 7. *Suppose $\chi_\tau + \frac{\overline{d}_G \kappa}{\rho - \underline{\omega}} > 0$. Then, Ω_0 is a function of $[i_t, \tau_t]_0^\infty$ given by*

$$\Omega_0 = \frac{\rho - \underline{\omega}}{(\rho - \underline{\omega})\chi_\tau + \overline{d}_G \kappa}\left[-\int_0^\infty e^{-\rho t}\tau_t dt + \overline{d}_G\left(q_{L,0} + \int_0^\infty e^{-\rho t}(i_t - \hat{\pi}_t - r_n + r_L \hat{\lambda}_t)dt\right)\right], \tag{29}$$

where $\hat{\pi}_t \equiv \tilde{\sigma}^{-1}\hat{\pi}_{m,t} + \chi_p \hat{\pi}_{\lambda,t}$ is a function of $[i_t]_0^\infty$.

Proof. Using $\int_0^\infty e^{-\rho t} y_t dt = \Omega_0$ and $\int_0^\infty e^{-\rho t}\pi_t dt = \int_0^\infty e^{-\rho t}\hat{\pi}_t dt + \frac{\kappa}{\rho - \underline{\omega}}\Omega_0$, we obtain

$$\left(\chi_\tau + \frac{\overline{d}_G \kappa}{\rho - \underline{\omega}}\right)\Omega_0 = -\int_0^\infty e^{-\rho t}\tau_t dt + \overline{d}_G q_{L,0} + \overline{d}_G \int_0^\infty e^{-\rho t}\left(i_t - \hat{\pi}_t - r_n + r_L p_{d,t}\right)dt,$$

after rearranging the expression for Ω_0. Given our assumption, we can divide both sides by $\chi_\tau + \frac{\overline{d}_G \kappa}{\rho - \underline{\omega}}$. This gives Equation (29), using the fact that $r_L p_{d,t} = r_L \hat{\lambda}_t$ up to a first-order approximation. From Equation (13) and $r_L p_{d,t} = r_L \hat{\lambda}_t$, $q_{L,0}$ is a function of only $[i_t]_0^\infty$. □

Proposition 7 shows that Ω_0 is uniquely pin down by $[i_t, \tau_t]_0^\infty$, given $\chi_\tau + \frac{\overline{d}_G \kappa}{\rho - \underline{\omega}} > 0$. This assumption simply states that monetary policy affects the fiscal authority either through tax revenues or through the cost of servicing the debt (or both). This proposition has an important implication: there is only *two* ways through which monetary policy impacts the aggregate wealth effect. First, monetary policy affects Ω_0 through its fiscal backing. Second, monetary policy affects Ω_0 through a *net discount rate effect*, similar to the one discussed in the context of Proposition 4. Importantly, this net revaluation effect is only present when $\overline{d}_G > 0$.

Net discount rate effect. Suppose $\int_0^\infty e^{-\rho t}\tau_t dt = 0$. If we also assume that $\overline{d}_G = 0$, then the household sector consumes the dividends on stocks and human wealth (i.e., profits and wages) every period. The intuition in Cochrane (2020) and Krugman (2021) then applies in this case and changes in discount rates generate no aggregate wealth effect. In the empirically relevant case $\overline{d}_G > 0$, changes in nominal interest rates create a wealth effect. Moreover, it can be shown that one obtains $\frac{\partial \Omega_0}{\partial i_0} < 0$ when government debt is sufficiently long, and this effect gets stronger with ϵ_λ. Therefore, heterogeneous beliefs amplify the effect of changes in nominal rate on the aggregate wealth rate.

Fiscal backing. Suppose $\overline{d}_G = 0$. In this case, a monetary tightening creates a negative wealth effect, and ultimately reduces π_0, if and only if $\int_0^\infty e^{-\rho t}\tau_t dt > 0$. A monetary tightening must necessarily be followed by a fiscal tightening. Notice that the fiscal backing can in principle amplify or dampen the impact of changes in nominal interest rates, which depends on $\overline{d}_G$. This illustrates the importance of disciplining monetary policy's fiscal backing empirically, as otherwise the model can generate an arbitrarily large response to monetary shocks based on a (potentially counterfactual) fiscal response.

The case $\chi_\tau + \frac{\overline{d}_G \kappa}{\rho - \underline{\omega}} = 0$. The analysis above relied on the assumption $\chi_\tau + \frac{\overline{d}_G \kappa}{\rho - \underline{\omega}} > 0$. In the commonly assumed case $\chi_\tau = \overline{d}_G = 0$, this implies that the fiscal backing is zero. Moreover, Ω_0 would be independent of $[i_t, \tau_t]_0^\infty$. In the case $\chi_\tau = \overline{d}_G = 0$, monetary policy can effectively choose Ω_0 in the absence of a net discount rate effect even without the help of the fiscal authority. Proposition 7 shows this is possible *only* in this knife-edge case. If $\chi_\tau > 0$ and/or $\overline{d}_G > 0$, the empirically relevant case, then monetary policy requires the help of fiscal policy in the absence of a net discount rate effect.

3.5 Implementability condition

Propositions 5 to 7 demonstrate how policy variables $[i_t, \tau_t]_0^\infty$ affect output and inflation. However, both the nominal interest rate and the associated fiscal backing are endogenous variables. The next proposition shows how the monetary policy shock u_t uniquely pins down the equilibrium path of nominal interest rates and fiscal backing.

Proposition 8 (Determinacy and implementability)**.** *Consider a given monetary shock $[u_t]_0^\infty$.*

i. (Determinacy) If $\phi_\pi \geq \overline{\phi}_\pi \equiv 1 - \frac{\rho\delta}{\tilde{\sigma}^{-1}\kappa}$, then there exists a unique bounded solution to the system comprised of the Taylor rule (7), the aggregate Euler equation (10), the New Keynesian Phillips curve (11), the market-implied disaster probability (15), and the law of motion of relative consumption (16) and relative net worth (17). We denote this solution by $[i_t^\star, y_t^\star, \pi_t^\star, \hat{\lambda}_t^\star, c_{p,t}^\star - c_{o,t}^\star, b_{p,t}^\star - b_{o,t}^\star]$ and the associated path of taxes by $\tau_t^\star$.

ii. (Implementability) For a given path of nominal interest rates $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, $\psi_m \neq -\underline{\omega}$, and fiscal backing $\int_0^\infty e^{-\rho t}\tau_t dt$, let $\hat{\lambda}_t$ be given by (18), y_t be given by (26), and π_t be given by (28), where Ω_0 is given by (29). If the monetary shock u_t is given by

$$u_t = i_t - r_n - \phi_\pi \pi_t, \tag{30}$$

then $i_t^\star = i_t$ and $\int_0^\infty e^{-\rho t}\tau_t^\star dt = \int_0^\infty e^{-\rho t}\tau_t dt$. Moreover, $y_t^\star = y_t$, $\pi_t^\star = \pi_t$, and $\hat{\lambda}_t^\star = \hat{\lambda}_t$.

The first part of Proposition 8 shows that there is a unique bounded solution to the equilibrium conditions if $\phi_\pi \geq \overline{\phi}_\pi$. As in Bilbiie (2018) and Acharya and Dogra (2020), the threshold for determinacy satisfies $\overline{\phi}_\pi < 1$ due to a precautionary motive, so uniqueness is obtained under a weaker condition than in the textbook model.

The second part of Proposition 8 shows that there is no loss in generality involved in our two-step procedure. Given any path of policy variables, one can find the monetary shock that implements $[i_t, \tau_t]_0^\infty$ in equilibrium. Intuitively, one can easily back out the value of u_t necessary to implement a given equilibrium from the policy rule. Therefore, one can equivalently express the solution in terms of policy variables or in terms of u_t. Equivalence results as this one are well-known in the literature on fiscal-monetary interactions (see e.g. Chapter 22 in Cochrane 2023). In our context, this approach is useful because the fiscal backing can either amplify or dampen the impact of the net revaluation of real and financial assets. By expressing the solution directly in terms of policy variables, we are able to isolate the role of asset revaluations from the response of fiscal policy.[21]

[21] Another advantage of this approach is that we can readily obtain $\hat{\lambda}_t$ in terms of i_t, given the block recursivity property, while solving for $\hat{\lambda}_t$ in terms of u_t requires solving the entire dynamic system.

4 The Quantitative Importance of Wealth Effects

In this section, we study the quantitative importance of wealth effects in the transmission of monetary shocks. We calibrate the model to match key unconditional and conditional moments, including asset-pricing dynamics and the fiscal response to a monetary shock. We find that household heterogeneity and time-varying risk are the predominant channels of transmission of monetary policy.

4.1 Calibration

The parameter values are chosen as follows. The discount rate of savers is chosen to match a natural interest rate of $r_n = 1\%$. We assume a Frisch elasticity of one, $\phi = 1$, and set the elasticity of substitution between intermediate goods to $\epsilon = 6$, common values adopted in the literature. The fraction of workers is set to $\mu_w = 30\%$. The parameter $\overline{d}_G$ is chosen to match a public debt-to-GDP ratio of 66%, and we assume a duration of five years, consistent with the historical average for the United States. The parameter $T_w'(Y)$ is chosen such that $\chi_y = 1$, which requires countercyclical transfers to balance the procyclical wage income. A value of $\chi_y = 1$ is consistent with the evidence in Cloyne et al. (2020) that the net income of mortgagors and non-mortgagors reacts similarly to monetary shocks. The pricing cost parameter φ is chosen such that κ coincides with its corresponding value under Calvo pricing, and it consistent with an average period between price adjustments of three quarters. The half-life of the monetary shock is set to three and a half months to roughly match what we estimate in the data. We set the Taylor rule parameter to $\phi_\pi = 1.5$.

We calibrate the disaster risk parameters in two steps. For the stationary equilibrium, we choose a calibration mostly based on the parameters adopted by Barro (2006). We set λ (the steady-state disaster intensity) to match an annual disaster probability of 1.7%. For our quantitative exercise, we assume that the size of the disaster shock, $\frac{Y^*}{Y}$ is stochastic, and calibrate the distribution of disaster shocks to match the empirical distribution estimated by Barro (2006).[22] The risk-aversion coefficient is set to $\sigma = 4$, a value within the range of reasonable values according to Mehra and Prescott (1985), but substantially larger than $\sigma = 1$, a value often adopted in macroeconomic models. Our calibration implies an equity premium in the stationary equilibrium of 6.1%, in line with the observed equity premium of 6.5%. Moreover, by setting $\sigma = 4$ we obtain a micro EIS of $\sigma^{-1} = 0.25$,

[22]The model is virtually unchanged under this extension, except that $\mathbb{E}[(C_s/C_s^*)^\sigma]$ replaces $(C_s/C_s^*)^\sigma$ in all expressions. Using $C_s/C_s^* = Y/Y^*$, we can calibrate $\mathbb{E}[(C_s/C_s^*)^\sigma]$ using the distribution estimated by Barro (2006).

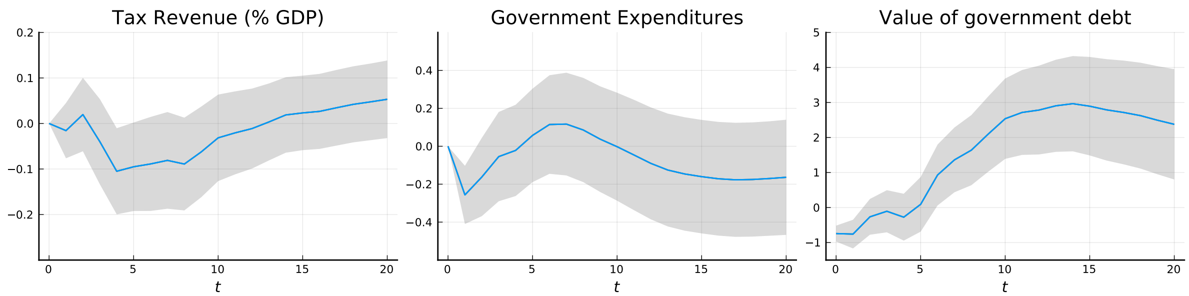

Figure 1: Estimated fiscal response to a monetary policy shock

Note: IRFs computed from a VAR identified by a recursiveness assumption, as in Christiano et al. (1999). Variables included: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, federal funds rate, 5-year constant maturity rate and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The real value of government debt and the 5-year rate are ordered last, and the fed funds rate is ordered third to last. Gray areas are bootstrapped 95% confidence bands. See Appendix D for the details.

in the ballpark of an EIS of 0.1 as recently estimated by Best et al. (2020). We discuss the calibration of ϵ_λ, which determines the elasticity of asset prices to monetary shocks, in the next subsection.

For the policy variables, we estimate a standard VAR augmented to incorporate fiscal variables and compute empirical IRFs applying the recursiveness assumption of Christiano et al. (1999). From the estimation, we obtain the path of monetary and fiscal variables: the path of the nominal interest rate, the change in the initial value of government bonds, and the path of fiscal transfers. We provide the details of the estimation in Appendix D. Figure 1 shows the dynamics of fiscal variables in the estimated VAR in response to a contractionary monetary shock. Government revenues fall in response to the contractionary shock, while government expenditures fall on impact and then turn positive, likely driven by the automatic stabilizer mechanisms embedded in the government accounts. The present value of interest payments increases by 69 bps and the initial value of government debt drops by 50 bps.[23] In contrast, the present value of transfers T_t drops by 12 bps.[24] Moreover, we cannot, at the 95% confidence level, reject the possibility that the present discounted value of the primary surplus does not change in response to monetary shocks and that the increase in interest payments is entirely compensated by the

[23]The present discounted value of interest payments is calculated as $\sum_{t=0}^{\mathcal{T}} \left(\frac{1-\lambda}{1+\rho_s}\right)^{\frac{t}{4}} \left[\bar{d}_t^g (\hat{i}_{L,t} - \hat{\pi}_t)\right]$, where $\mathcal{T}$ is the truncation period, $\hat{i}_{L,t}$ is the IRF of the 5-year rate estimated in the data, and $\hat{\pi}_t$ is the IRF of inflation. We choose $\mathcal{T} = 60$ quarters, when the main macroeconomic variables, including government debt, are back to their pre-shock values. Other present value calculations follow a similar logic.

[24]In the data, expenditures also include the response of government consumption and investment. When run separately, however, we cannot reject the possibility that the sum of these two components is equal to zero in response to monetary shocks.

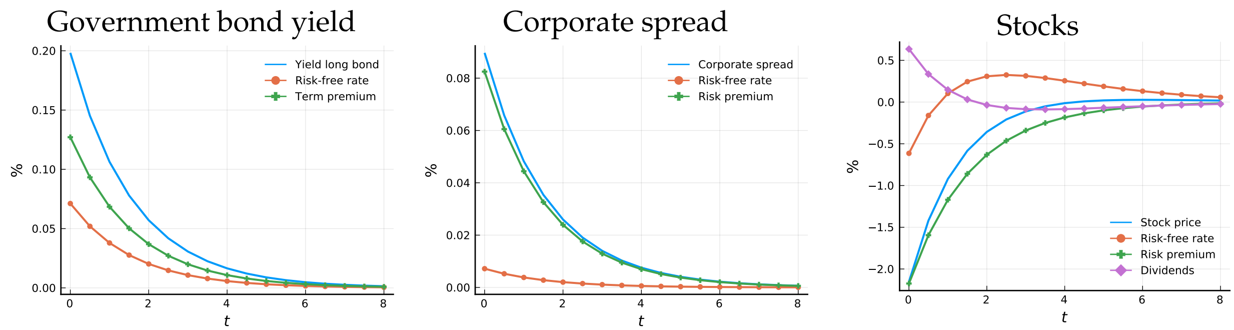

Figure 2: Asset-pricing response to monetary shocks with time-varying risk.

initial reaction in the value of government bonds.

4.2 Asset-pricing implications of time-varying risk

Recall that the price of the long-term government bond is given by

$$q_{L,0} = -\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n + r_L p_{d,t})dt,$$

where $p_{d,t} = \sigma c_{s,t} + \epsilon_\lambda(i_t - r_n)$ is the price of the disaster risk. We use this expression and calibrate ϵ_λ to match the initial response of the 5-year yield on government bonds. Consistent with Gertler and Karadi (2015) and our own estimates reported in Appendix D, we find that a 100 bps increase in the nominal interest rate leads to an increase in the 5-year yield of roughly 20 bps. This procedure leads to a calibration of ϵ_λ of 2.25, which implies an annual increase in the probability of disaster of roughly 95 bps after a 100 bps increase in the nominal interest rate. Figure 2 shows the response of the yield on the long bond and the contributions of the path of future interest rates and the term premium. We find that the bulk of the reaction of the 5-year yield reflects movements in the term premium, a finding that is consistent with the evidence.

The model is also able to capture the responses of asset prices that were not directly targeted in the calibration. Consider first the response of the *corporate spread*, the difference between the yield on a corporate bond and the yield on a government bond (without risk of default) with the same promised cash flow. This corresponds to how the GZ spread is computed in the data by Gilchrist and Zakrajšek (2012). Let $e^{-\psi_F t}$ denote the coupon paid by the corporate bond. We assume that the monetary shock is too small to trigger a corporate default, but the corporate bond defaults if a disaster occurs, where lenders recover the amount $1 - \zeta_F$ in case of default. We calibrated ψ_F and ζ_F to match a duration of 6.5 years and a credit spread of 200 bps in the stationary equilibrium, which is consistent with the estimates reported by Gilchrist and Zakrajšek (2012). Note that the calibration targets the *unconditional* level of the credit spread. We evaluate the model on its ability to generate an empirically plausible *conditional* response to monetary shocks.

The price of the corporate bond can be computed analogously to the computation of the long-term government bond:

$$q_{F,0} = -\int_0^\infty e^{-(\rho+\psi_F)t}(i_t - r_n)dt - \int_0^\infty e^{-(\rho+\psi_F)t}\left[\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_F - Q_F^*}{Q_F} p_{d,t}\right]dt,$$

where Q_F and Q_F^* denote the price of the corporate bond in the stationary equilibrium in the no-disaster and disaster states, respectively. Given the price of the corporate bond, we can compute the corporate spread. Figure 2 shows that the corporate spread responds to monetary shocks by 8.9 bps. We introduce the excess bond premium (EBP) in our VAR and find an increase in the EBP of 6.5 bps and an upper bound of the confidence interval of 10.9 bps, consistent with the model's prediction. Thus, even though this was not a targeted moment, time-varying risk is able to produce quantitatively plausible movements in the corporate spread.

Another moment that is not targeted by the calibration is the response of stocks to monetary shocks. We find a substantial response of stocks to changes in interest rates, which is explained mostly by movements in the risk premium. In contrast to the empirical evidence, we find a *positive* response of dividends to a contractionary monetary shock. This is the result of the well-known feature of sticky-prices models that profits are strongly countercyclical. This counterfactual prediction could be easily solved by introducing some form of wage stickiness. Despite the positive response of dividends, the model generates a decline in stocks of 2.15% in response to a 100 bps increase in interest rates, which is smaller than the point estimate of Bernanke and Kuttner (2005) but is still within their confidence interval.[25] Fixing the degree of countercyclicality of profits would likely bring the response of stocks closer to their point estimate.

4.3 Wealth effects in the monetary transmission mechanism

Figure 3 (left) presents the response of output and its components to a monetary shock in the New Keynesian model with heterogeneous agents and time-varying risk. We find that output reacts by -1.05% to a 100 bp increase in the nominal interest rate, which is consistent with the empirical estimates of e.g. Miranda-Agrippino and Ricco (2021).

[25] We follow standard practice in the asset-pricing literature and report the response of a levered claim on firms' profits, using a debt-to-equity ratio of 0.5, as in Barro (2006).

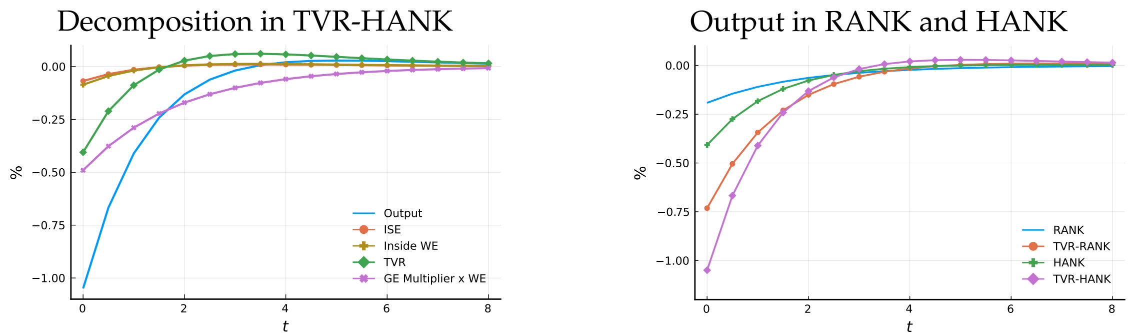

Figure 3: Output in RANK and HANK.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1.

In terms of its components, time-varying risk (TVR) and the outside wealth effect are the two main components determining the output dynamics, representing 39% and 47% of the output response, respectively. In contrast, the ISE accounts for only 6.5% of the output response, indicating that intertemporal substitution plays only a minor role in the monetary transmission mechanism.

These findings stand in sharp contrast to the dynamics in the absence of heterogeneity and time-varying risk. Figure 3 (right) plots the response of output for different combinations of heterogeneity ($\mu_b > 0$ and $\mu_b = 0$) and time-varying risk ($\epsilon_\lambda > 0$ and $\epsilon_\lambda = 0$). By shutting down the two channels, denoted by "RANK" in the figure, the initial response of output would be -0.14%, a more than a sevenfold reduction in the impact of monetary policy. There are two reasons for this result. First, our calibration of $\sigma = 4$ implies an EIS that is one fourth of the standard calibration. This significantly reduces the quantitative importance of the ISE, even if the intertemporal substitution channel represents a large fraction of the output response in the RANK model. Second, our estimate of the fiscal response is substantially lower than the one implied by a standard Taylor equilibrium that imposes an AR(1) process for the monetary shock. We discuss the role of fiscal backing and the implications for the New Keynesian model in Section 4.5 below.

Figure 3 (right) also plots the response of output when there is household heterogeneity but not time-varying risk ("HANK" in the figure), and the response of output when there is time-varying risk but not household heterogeneity ("TVR-RANK" in the figure). We find that heterogeneity increases the response of output by 22 bps while time-varying risk increases it by 54 bps. Notably, by combining both features, we get an increase in the response of output of 86 bps, which is 10 bps larger than the sum of the individual effects. Thus, heterogeneity and time-varying risk reinforce each other. In terms of the fraction of the response of output that can be attributed to each channel, we find that 20.5% can

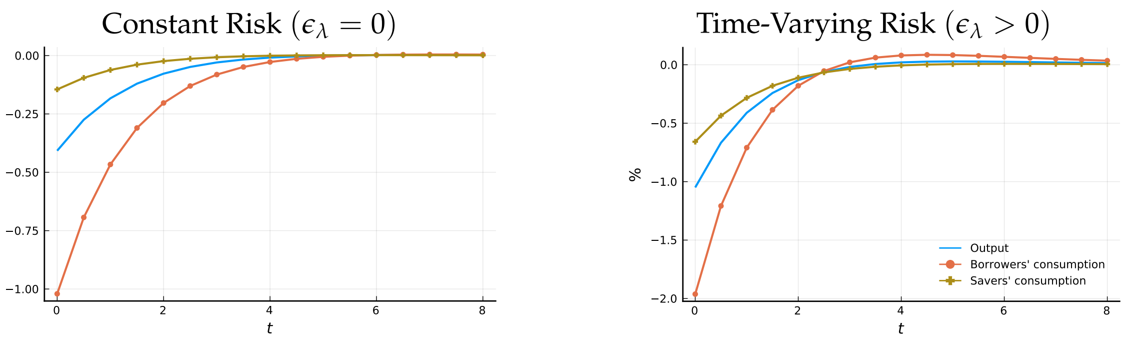

Figure 4: Consumption of borrowers and savers with constant risk and time-varying risk.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1.

be attributed to household heterogeneity, 51.5% corresponds to time-varying risk, and 9.7% is the amplification effect of heterogeneity together with time-varying risk (which is around 50% larger than the contribution of the ISE), while the remainder represents the channels in the RANK model.

Finally, time-varying risk is essential for properly capturing the heterogeneous response of borrowers and savers to monetary policy. Figure 4 shows that borrowers are disproportionately affected by monetary shocks. However, the magnitude of the relative response of borrowers and savers is too large in the economy without time-varying risk. The drop in borrowers' consumption is 7 times greater than the decline in savers' consumption with a constant disaster probability, while it is 3 times greater in the economy with time-varying risk. Cloyne et al. (2020) estimate a relative peak response of mortgagors and homeowners of roughly 3.6. Therefore, allowing for time-varying risk is also important if we want to capture the heterogeneous impact of monetary policy.

4.4 The limitations of the constant disaster risk model

Consider the response of asset prices to a monetary shock in an economy that features constant disaster risk (i.e. $\lambda > 0$ but $\epsilon_\lambda = 0$). Figure 5 (left) shows that the yield on the long bond increases by 6.5 bps, which implies a decline of the value of the bond of 32 bps (given a 5-year duration), less than half of the response estimated by the VAR in Section 4.1. Moreover, movements in the long bond yield are almost entirely explained by the path of nominal interest rates, while the term premium is indistinguishable from zero. This stands in sharp contrast to the evidence reported in Gertler and Karadi (2015) and Hanson and Stein (2015). Similarly, it can be shown that most of the response of stocks in the model is explained by movements in interest rates instead of changes in risk premia,

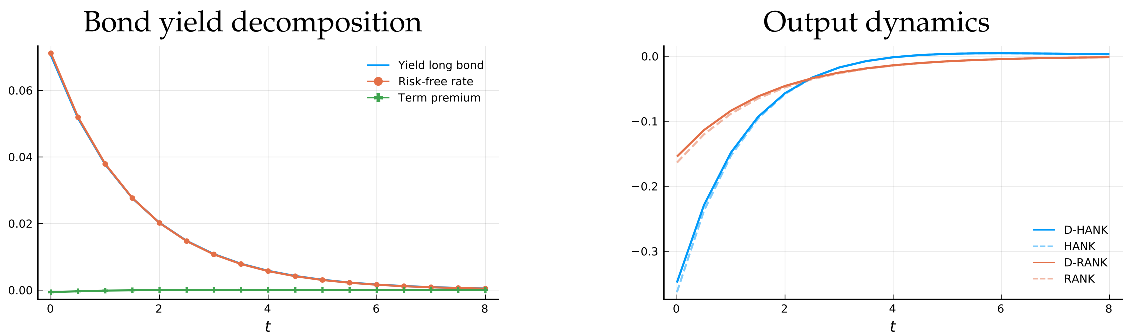

Figure 5: Long-term bond yields and output for economies with and without risk.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1. D-HANK and D-RANK correspond to heterogeneous-agent and representative agent economies with constant disaster risk (i.e. $\lambda > 0$ and $\epsilon_\lambda = 0$). HANK and RANK correspond to economies with no disaster risk (i.e. $\lambda = 0$).

a finding that is inconsistent with the evidence documented in e.g. Bernanke and Kuttner (2005).

Figure 5 (right) shows how the presence of constant disaster risk affects the response of output to monetary shocks for the HANK and RANK economies. We find that risk has only a minor impact on the response of output. Aggregate risk increases the value of the discounting parameter δ, which reduces the GE multiplier and dampens the initial impact of the monetary shock. Given that the term premium barely moves, disaster risk plays only a small role in determining the outside wealth effect. In contrast, the important role of heterogeneity can be seen by comparing the response of the D-HANK and D-RANK economies.

Therefore, while introducing a constant disaster probability allows the model to capture important *unconditional* asset-pricing moments, such as the (average) risk premium or the upward-sloping yield curve, the model is unable to match key *conditional* moments, in particular, the response of asset prices to monetary policy. The limitations of the model with constant disaster probability in matching conditional asset-pricing moments were recognized early on in the literature, leading to an assessment of the implications of *time-varying disaster risk*, as in Gabaix (2012) and Gourio (2012). This justifies our focus on time-varying disaster risk and how it affects the asset-pricing response to monetary shocks and, ultimately, its impact on real economic variables.

4.5 The role of fiscal backing and the EIS

We have found that time-varying risk and heterogeneity substantially amplify the impact of monetary policy on the economy. To properly assess the importance of these two

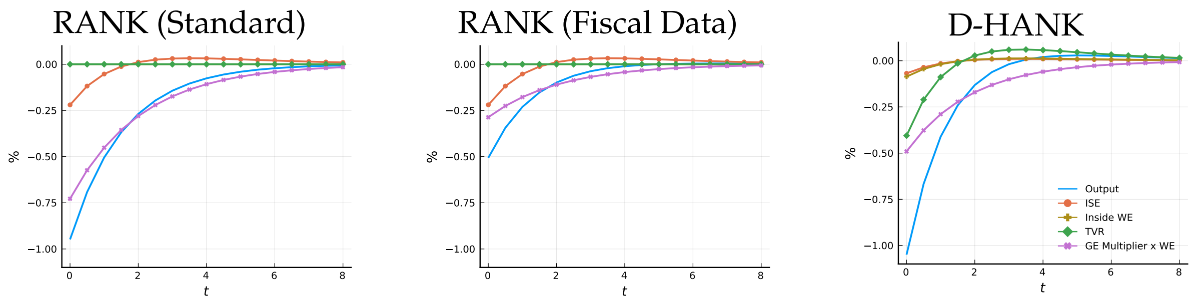

Figure 6: Output in RANK vs D-HANK with time-varying risk.

Note: The first two panels show output in RANK ($\mu_b = \lambda = 0$) with unit EIS ($\sigma^{-1} = 1$). In the left panel, fiscal backing is determined by a Taylor rule, while in the middle panel fiscal backing corresponds to the value estimated in the data. The right panel corresponds to the D-HANK economy with time-varying risk and the estimated fiscal backing.

channels, however, it was crucial to control for the implicit fiscal backing, as discussed in Section 3.4.

Figure 6 illustrates this point. In the three panels, we show the impact of a monetary shock that leads to an increase in nominal interest rates on impact of 100 bps. In the left panel, we consider a RANK economy ($\mu_b = \lambda = 0$) with the standard value for the EIS ($\sigma^{-1} = 1$) and fiscal backing implicitly determined by a Taylor rule with a monetary shock that follows a standard AR(1) process, corresponding to the textbook New Keynesian model. In the middle panel, we consider the same economy but the fiscal backing is set to the value estimated in the data, corresponding to a Taylor equilibrium with a monetary shock that follows the more general specification from equation (30). The right panel shows our D-HANK model with time-varying risk and the calibrated value of the EIS, $\sigma^{-1} = 0.25$.

The response of the textbook economy is only slightly smaller than that of our D-HANK economy despite the lack of time-varying risk or heterogeneous agents. An important reason for this is the difference in the value of the (implicit) fiscal backing, which is almost ten times larger in the textbook economy compared with the one we estimated in the data. When the fiscal backing is the same as in the data, the response of output drops by almost half. The EIS also plays an important role. Even with fiscal backing directly from the data, the response of output is still significant, only slightly less than that in our D-RANK with time-varying risk (see Figure 3). But this same response comes from very different channels. In the RANK economy, the ISE accounts for roughly 40% of the output response, while in our D-RANK the ISE accounts for less than 7% of that response.

These results suggest that the quantitative success of the RANK model is likely the result of a counterfactually large fiscal backing in response to monetary shocks and a strong intertemporal-substitution channel, which compensate for missing heterogeneous agents and risk channels. Once we discipline the fiscal backing with data and calibrate the EIS to the estimates obtained from microdata, our model suggests that heterogeneous agents and, in particular, time-varying risk are crucial for generating quantitatively plausible output dynamics. However, it is important to note that our model made several simplifications to incorporate indebted agents and time-varying aggregate risk without sacrificing the tractability of standard macro models. A natural extension would be to incorporate these channels into a medium-sized DSGE model to better assess the quantitative properties of the New Keynesian model.

5 The Effect of Risk and Maturity of Household Debt

We have focused so far on how monetary policy affects the value of households' assets, such as stocks and bonds. However, movements in risk premia induced by monetary policy can also affect the real economy through its impact on household debt. In this section, we extend the baseline model to allow workers to borrow a positive amount using long-term risky debt. In this case, the effect of monetary policy on workers depends on how the term spread and credit spread, the compensation for holding interest rate and default risk, respond to changes in the short-term interest rate.

5.1 The model with long-term risky household debt

We describe next the model with long-term risky household debt. We highlight only the main differences with respect to the model described in Section 2. Households issue long-term debt that promises to pay exponentially decaying coupons given by $e^{-\psi_P t}$ at period $t \geq 0$, where $\psi_P \geq 0$. Importantly, households cannot commit to always repay their debts. In response to a large shock, i.e. the occurrence of a disaster, households default and lenders receive a fraction $1 - \zeta_P$ of the promised coupons, where $0 \leq \zeta_P \leq 1$. We assume that fluctuations in the no-disaster state are small enough such that they do not trigger a default. Thus, households default only in the disaster state.

We denote the price of household (or private) debt in the no-disaster state by $Q_{P,t}$, and $Q^*_{P,t}$ in the disaster state, so the nominal return on household debt is given by

$$dR_{P,t} = \left[\frac{1}{Q_{P,t}} + \frac{\dot{Q}_{P,t}}{Q_{P,t}} - \psi_P\right] dt + \frac{Q^*_{P,t} - Q_{P,t}}{Q_{P,t}} d\mathcal{N}_t,$$

where $i_{P,t} \equiv \frac{1}{Q_{P,t}} - \psi_P$ is the yield on the bond.

In a stationary equilibrium, the spread between the interest rate on household debt and the short-term interest rate controlled by the central bank is given by

$$r_P = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_P - Q_P^*}{Q_P},$$

where $r_P = i_P - r_n$. Note that the interest rate on household debt incorporates both a credit and a term spread.[26] Households can borrow up to $\overline{D}_{P,t} = Q_{P,t}\overline{F}$, which effectively puts a limit on the face value of private debt $\overline{F}$.[27] In a log-linear approximation of the economy around a zero-inflation stationary equilibrium, workers are constrained at all periods, and their consumption is given by

$$c_{w,t} = \chi_y y_t - \left(\frac{\psi_P}{i_P + \psi_P}(i_{P,t} - i_P) - \pi_t\right)\overline{d}_P, \tag{31}$$

where $\overline{d}_P \equiv \frac{\overline{D}_P}{Y}$ is the debt-to-income ratio in the stationary equilibrium. Equation (31) generalizes the expression for workers' consumption given in Section 2. Monetary policy affects workers indirectly through its effect on the yield on private debt $i_{P,t}$. If we assume that debt is short-term, $\psi_P \to \infty$, and riskless, $\zeta_P = 0$, we obtain $i_{P,t} = i_t$. At the other extreme, we have the case of a perpetuity, $\psi_P = 0$. In this case, households simply pay the coupon every period and there is no need to issue new debt. Therefore, they are completely insulated from movements in nominal interest rates. For intermediate values of maturity and risk, $0 < \psi_P < \infty$ and $0 < \zeta_P < 1$, monetary policy affects borrowers through changes in the nominal interest rate i_t and the spread $r_{P,t}$.

The price of household debt evolves according to

$$\dot{q}_{P,t} = (\rho + \psi_P)q_{P,t} + i_t - r_n + r_P p_{d,t}.$$

The price of the bond depends on the future path of short-term interest rates and the price of disaster risk, $p_{d,t} = \sigma c_{s,t} + \hat{\lambda}_t$. Quantitatively, fluctuations in $p_{d,t}$ are dominated by movements in $\hat{\lambda}_t$, while $\sigma c_{s,t}$ gives a negligible contribution, as shown in Figure 5. Therefore, we assume that $r_P \sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$, analogous to the assumption used in

[26] Let $i_{P,t}^{ND}$ denote the yield on a non-defaultable bond with coupons decaying at rate ψ_P. The term spread corresponds to $i_{P,t}^{ND} - i_t$ and the credit spread to $i_{P,t} - i_{P,t}^{ND}$, so $r_P = (i_P^{ND} - r_n) + (i_P - i_P^{ND})$.

[27] This formulation guarantees that, after an increase in nominal rates, the value of household debt and the borrowing limit decline by the same amount. This specification of the borrowing constraint, combined with the assumption of impatient borrowers, guarantees that borrowers are constrained at all periods.

Proposition 3. In this case, we can solve for the price of the bond in closed-form:

$$q_{P,t} = -\frac{1}{\rho + \psi_P + \psi_m}(i_t - r_n) - \frac{r_P}{\rho + \psi_P + \psi_\lambda}\hat{\lambda}_t, \tag{32}$$

where the nominal interest rates is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$.

Combining workers' consumption with savers' Euler equation (9) and the Phillips curve (11), we obtain the response of aggregate output to monetary shocks. In particular, we can extend the decomposition in Proposition 5 to the case of long-term risky debt.

Proposition 9 (Aggregate output with long-term risky household debt)**.** *Suppose that $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $r_D \sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$. Aggregate output is then given by*

$$y_t = \underbrace{\tilde{\sigma}^{-1}\hat{y}_{m,t}}_{\text{ISE}} + \underbrace{\chi_P \hat{y}_{\lambda,t}}_{\substack{\text{time-varying}\\\text{precautionary motive}}} + \underbrace{\frac{\mu_w \bar{d}_P}{1 - \mu_w \chi_y}\frac{\psi_P\left[\tilde{\psi}_m \hat{y}_{m,t} + r_P \tilde{\psi}_\lambda \hat{y}_{\lambda,t}\right]}{\rho + \psi_P + \psi_m}}_{\text{household-debt effect}} + \underbrace{(\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0}_{\substack{\text{GE multiplier}\times\\\text{aggregate wealth effect}}},$$

where $\tilde{\psi}_k = \psi_k + \rho - r_n$ for $k \in \{m, \lambda\}$.

Proposition 9 shows that household debt effectively amplifies the ISE and the time-varying precautionary motive effect. If household debt is safe and short term (i.e, $\zeta_P = 0$ and $\psi_P \to \infty$), then the household-debt effect loads on $\hat{y}_{m,t}$, amplifying the ISE. When debt is long-term or when households can default, then $r_P > 0$ and the household-debt effect also loads on $\hat{y}_{\lambda,t}$, amplifying the precautionary motive effect.

An important implication of Proposition 9 is that default risk amplifies the household-debt effect, while an increase in the duration of household debt weakens the effect. The spread r_P is increasing in ζ_P, so the interest rate on private debt responds more strongly to an increase in $\hat{\lambda}_t$ when debt is riskier. In contrast, an increase in the duration of household debt (i.e., a reduction in ψ_P) means that households issue less debt at the new rates, so the impact of the change in the cost of serving the debt gets attenuated. In the limit case of a perpetuity, $\psi_P = 0$, the household-debt effect goes to zero. Given that households do not issue new debt, they are not affected by changes in interest rates.

5.2 Quantitative implications

We consider next the quantitative implications of default risk and maturity on household debt. As shown in Proposition 9, these two features have opposing effects on the response of output to monetary policy. To assess the quantitative impact of risk and maturity,

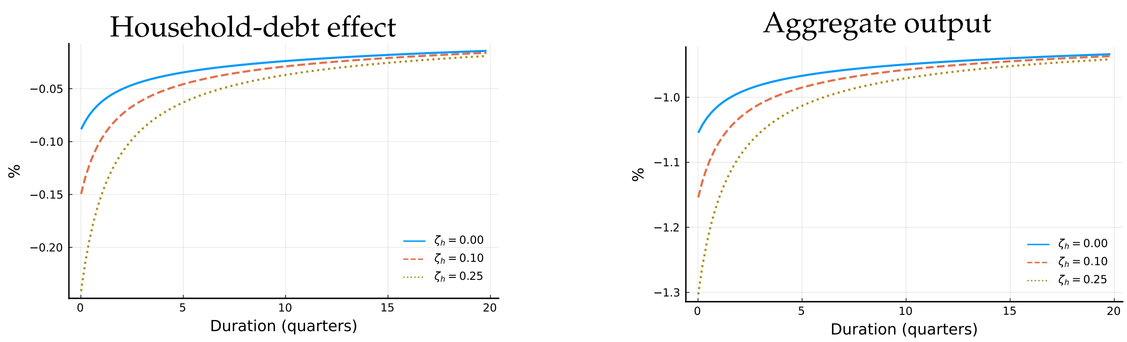

Figure 7: Household-debt effect and output at $t = 0$ as a function of duration.

Figure 7 shows the initial response of the household-debt effect (left panel) and aggregate output (right panel) as a function of the duration of private debt for different values of the haircut parameter ζ_P. Greenwald et al. (2021) estimate the duration of mortgage debt as 5.2 years, the duration of student debt as 4.50, and the duration of consumer debt as 1.0 year. Therefore, we focus on values of duration up to five years. We consider three different values for the haircut parameter: riskless debt ($\zeta_P = 0$); risky debt with a spread in the stationary equilibrium of roughly 4.0% with a 5-year duration ($\zeta_P = 0.10$); risky debt with a spread of 5.0% with a 5-year duration ($\zeta_P = 0.25$).

Default risk substantially amplifies the effect of monetary policy on output when debt is short term. The household-debt effect is almost three times larger in the case of $\zeta_P = 0.25$ compared to $\zeta_P = 0.0$, which corresponds to an increase in the initial response of output of almost 25%. However, this effect is strongly attenuated when household debt is long term. For even relatively small values of duration, the household-debt effect with long-term risky debt is smaller than in the case of short-term riskless debt. For instance, in the case of a five-year duration, the response of output is roughly 10% smaller than the response in the case of short-term riskless debt. The response of output when household debt is zero is roughly 35% smaller than in the economy with (positive) riskless debt, a much larger drop relative to the one caused by introducing long-term bonds.

6 Conclusion

In this paper, we provide a novel unified framework to analyze the role of heterogeneity and risk in a tractable linearized New Keynesian model. The methods introduced can be applied beyond the current model. For instance, they can be applied to a full quantitative HANK model with idiosyncratic risk, extending the results of Ahn et al. (2018) to allow for time-varying risk premia. Alternatively, one could introduce a richer capital structure for firms and study the pass-through of monetary policy to households and firms. These methods may enable us to bridge the gap between the extensive existing work on heterogeneous agents and monetary policy and the emerging literature on the role of asset prices in the transmission of monetary shocks.

References

Acharya, Sushant and Keshav Dogra, "Understanding HANK: Insights from a PRANK," *Econometrica*, 2020, *88* (3), 1113–1158.

Ahn, SeHyoun, Greg Kaplan, Benjamin Moll, Thomas Winberry, and Christian Wolf, "When inequality matters for macro and macro matters for inequality," *NBER macroeconomics annual*, 2018, *32* (1), 1–75.

Andreasen, Martin M, "On the effects of rare disasters and uncertainty shocks for risk premia in non-linear DSGE models," *Review of Economic Dynamics*, 2012, *15* (3), 295–316.

Auclert, Adrien, "Monetary policy and the redistribution channel," *American Economic Review*, 2019, *109* (6), 2333–67.

_, Bence Bardóczy, and Matthew Rognlie, "MPCs, MPEs, and multipliers: A trilemma for New Keynesian models," *The Review of Economics and Statistics*, 2021, pp. 1–41.

Barro, Robert, Emi Nakamura, Jón Steinsson, and José Ursúa, "Crises and recoveries in an empirical model of consumption disasters," *American Economic Journal: Macroeconomics*, 2013, *5* (3), 35–74.

Barro, Robert J, "Rare disasters and asset markets in the twentieth century," *The Quarterly Journal of Economics*, 2006, *121* (3), 823–866.

_, "Rare disasters, asset prices, and welfare costs," *American Economic Review*, 2009, *99* (1), 243–64.

Bauer, Michael D and Eric T Swanson, "A reassessment of monetary policy surprises and high-frequency identification," *NBER Macroeconomics Annual*, 2023, *37* (1), 87–155.

Benigno, Pierpaolo, Gauti B Eggertsson, and Federica Romei, "Dynamic Debt Deleveraging and Optimal Monetary Policy," *American Economic Journal: Macroeconomics*, 2020.

Bernanke, Ben S and Kenneth N Kuttner, "What explains the stock market's reaction to Federal Reserve policy?," *The Journal of Finance*, 2005, *60* (3), 1221–1257.

Best, Michael Carlos, James S Cloyne, Ethan Ilzetzki, and Henrik J Kleven, "Estimating the elasticity of intertemporal substitution using mortgage notches," *The Review of Economic Studies*, 2020, *87* (2), 656–690.

Bilbiie, Florin O., "The New Keynesian Cross: Understanding Monetary Policy with Hand-to-Mouth Households," 2017.

— , "Monetary Policy and Heterogeneity: An Analytical Framework," *CEPR Discussion Paper No. DP12601*, 2018.

— , "The New Keynesian cross," *Journal of Monetary Economics*, 2019.

Blanchard, Olivier J, "Debt, deficits, and finite horizons," *Journal of political economy*, 1985, *93* (2), 223–247.

Broer, Tobias, Niels-Jakob Harbo Hansen, Per Krusell, and Erik Öberg, "The New Keynesian transmission mechanism: A heterogeneous-agent perspective," *The Review of Economic Studies*, 2020, *87* (1), 77–101.

Brunnermeier, Markus K and Yuliy Sannikov, "The I theory of money," 2016.

Caballero, Ricardo J and Alp Simsek, "A risk-centric model of demand recessions and speculation," *The Quarterly Journal of Economics*, 2020, *135* (3), 1493–1566.

Campbell, John Y, Carolin Pflueger, and Luis M Viceira, "Macroeconomic drivers of bond and equity risks," *Journal of Political Economy*, 2020, *128* (8), 000–000.

Chen, Hui, Scott Joslin, and Ngoc-Khanh Tran, "Rare disasters and risk sharing with heterogeneous beliefs," *The Review of Financial Studies*, 2012, *25* (7), 2189–2224.

Chodorow-Reich, Gabriel, Plamen T Nenov, and Alp Simsek, "Stock market wealth and the real economy: A local labor market approach," *American Economic Review*, 2021, *111* (5), 1613–57.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans, "Monetary policy shocks: What have we learned and to what end?," *Handbook of Macroeconomics*, 1999, *1*, 65–148.

Cieslak, Anna and Annette Vissing-Jorgensen, "The economics of the Fed put," 2020.

Cloyne, James, Clodomiro Ferreira, and Paolo Surico, "Monetary policy when households have debt: new evidence on the transmission mechanism," *The Review of Economic Studies*, 2020, *87* (1), 102–129.

Cochrane, John H, "Wealth and taxes," *Cato Institute, Tax and Budget Bulletin*, 2020, (86).

— , *The fiscal theory of the price level*, Princeton University Press, 2023.

Coeurdacier, Nicolas, Helene Rey, and Pablo Winant, "The risky steady state," *American Economic Review*, 2011, *101* (3), 398–401.

Debortoli, Davide and Jordi Galí, "Monetary policy with heterogeneous agents: Insights from TANK models," 2017.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, "A model of monetary policy and risk premia," *The Journal of Finance*, 2018, *73* (1), 317–373.

Eggertsson, Gauti B and Paul Krugman, "Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach," *The Quarterly Journal of Economics*, 2012, *127* (3), 1469–1513.

Fagereng, Andreas, Matthieu Gomez, Emilien Gouin-Bonenfant, Martin Holm, Benjamin Moll, and Gisle Natvik, "Asset-Price Redistribution," Technical Report, Working paper, LSE 2022.

Fernández-Villaverde, Jesús and Oren Levintal, "Solution methods for models with rare disasters," *Quantitative Economics*, 2018, *9* (2), 903–944.

Gabaix, Xavier, "Disasterization: A simple way to fix the asset pricing properties of macroeconomic models," *American Economic Review*, 2011, *101* (3), 406–09.

_ , "Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance," *The Quarterly Journal of Economics*, 2012, *127* (2), 645–700.

Gertler, Mark and Peter Karadi, "Monetary policy surprises, credit costs, and economic activity," *American Economic Journal: Macroeconomics*, 2015, *7* (1), 44–76.

Gilchrist, Simon and Egon Zakrajšek, "Credit spreads and business cycle fluctuations," *American Economic Review*, 2012, *102* (4), 1692–1720.

Gourio, Francois, "Disaster risk and business cycles," *American Economic Review*, 2012, *102* (6), 2734–66.

Greenwald, Daniel L, Matteo Leombroni, Hanno Lustig, and Stijn Van Nieuwerburgh, "Financial and Total Wealth Inequality with Declining Interest Rates," 2021.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman, "Investment, capacity utilization, and the real business cycle," *The American Economic Review*, 1988, pp. 402–417.

Hall, George, Jonathan Payne, and Thomas Sargent, "US Federal Debt 1776-1960: Quantities and Prices," 2018.

Hanson, Samuel G and Jeremy C Stein, "Monetary policy and long-term real rates," *Journal of Financial Economics*, 2015, *115* (3), 429–448.

Iacoviello, Matteo, "House prices, borrowing constraints, and monetary policy in the business cycle," *American Economic Review*, 2005, *95* (3), 739–764.

Isoré, Marlène and Urszula Szczerbowicz, "Disaster risk and preference shifts in a New Keynesian model," *Journal of Economic Dynamics and Control*, 2017, *79*, 97–125.

Kalecki, Michal, "Professor Pigou on 'The Classical Stationary State' A Comment," *The Economic Journal*, 1944, *54* (213), 131–132.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante, "Monetary policy according to HANK," *American Economic Review*, 2018, *108* (3), 697–743.

Kekre, Rohan and Moritz Lenel, "Monetary policy, redistribution, and risk premia," 2020.

Krugman, Paul, "Pride and Prejudice and Asset Prices," 2021.

Mankiw, N Gregory and Stephen P Zeldes, "The consumption of stockholders and non-stockholders," *Journal of financial Economics*, 1991, *29* (1), 97–112.

Mas-Colell, Andreu, Michael Dennis Whinston, Jerry R Green et al., *Microeconomic theory*, Vol. 1, Oxford university press New York, 1995.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson, "The discounted euler equation: A note," *Economica*, 2017, *84* (336), 820–831.

Mehra, Rajnish and Edward C Prescott, "The equity premium: A puzzle," *Journal of Monetary Economics*, 1985, *15* (2), 145–161.

Merton, Robert C, "Continuous-time finance," 1992.

Metzler, Lloyd A, "Wealth, saving, and the rate of interest," *Journal of Political Economy*, 1951, *59* (2), 93–116.

Miranda-Agrippino, Silvia and Giovanni Ricco, "The Transmission of Monetary Policy Shocks," *American Economic Journal: Macroeconomics*, July 2021, *13* (3), 74–107.

Parker, Jonathan A, "The consumption risk of the stock market," *Brookings Papers on Economic Activity*, 2001, *2001* (2), 279–333.

Patinkin, Don, *Money, interest, and prices; an integration of monetary and value theory*, Harper & Row, 1965.

Patterson, Christina, "The Matching Multiplier and the Amplification of Recessions," 2019.

Pigou, Arthur C, "The classical stationary state," *The Economic Journal*, 1943, *53* (212), 343–351.

Rotemberg, Julio J, "Sticky prices in the United States," *Journal of political economy*, 1982, *90* (6), 1187–1211.

Schmitt-Grohé, Stephanie and Martın Uribe, "Solving dynamic general equilibrium models using a second-order approximation to the policy function," *Journal of economic dynamics and control*, 2004, *28* (4), 755–775.

Tobin, James, "A general equilibrium approach to monetary theory," *Journal of Money, Credit & Banking*, 1969, *1* (1), 15–29.

_ , *Asset accumulation and economic activity: Reflections on contemporary macroeconomic theory*, University of Chicago Press, 1982.

Tsai, Jerry and Jessica A Wachter, "Disaster risk and its implications for asset pricing," *Annual Review of Financial Economics*, 2015, *7*, 219–252.

Varian, Hal R, "Divergence of opinion in complete markets: A note," *The Journal of Finance*, 1985, *40* (1), 309–317.

Werning, Iván, "Incomplete markets and aggregate demand," 2015.

Appendix: For Online Publication

A Proofs

Proof of Proposition 2. Consider first the New Keynesian Phillips curve

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t}\right)\pi_t - \frac{\epsilon}{\varphi A}\left(\frac{W}{P}e^{w_t - p_t} - (1-\epsilon^{-1})A\right)Ye^{y_t}.$$

Linearizing the above expression, and using $\frac{W}{P} = (1-\epsilon^{-1})A$, we obtain

$$\dot{\pi}_t = \left(r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\right)\pi_t - \varphi^{-1}(\epsilon - 1)Y(w_t - p_t).$$

Using the fact that $w_t - p_t = \phi y_t$, we obtain $\dot{\pi}_t = (\rho_s + \lambda)\,\pi_t - \kappa y_t$, where $\kappa \equiv \varphi^{-1}(\epsilon - 1)\phi Y$ and we used that $r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} = \rho_s + \lambda$.

Consider next the generalized Euler equation. From the market-clearing condition for goods and borrowers' consumption, we obtain $c_{s,t} = \frac{1-\mu_b\chi_y}{1-\mu_b}y_t$. Combining this condition with the Phillips Curve and savers' Euler equation, and using the fact that $r_n = \rho - \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}$, we obtain $\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi - r_n) + \delta y_t + \chi_\lambda \hat{\lambda}_t$, where the constants $\tilde{\sigma}^{-1}$, δ, and χ_λ are defined in the proposition.

□

Proof of Lemma 1. Suppose $[y_t, \pi_t]_0^\infty$ satisfies system (20) and the intertemporal budget constraint (22) in the no-disaster state. We will show that $[y_t, \pi_t]_0^\infty$ can be supported as an equilibrium, that is, we can find the remaining variables such that all equilibrium conditions are satisfied. Consider first the disaster state. The savers' budget constraint implies $T_{s,t} = -\rho_s b_s b_{s,t^*}$. All remaining variables are equal to zero in the disaster state.

Consider now the no-disaster state. The real wage is given by $w_t - p_t = \phi y_t$ and workers' labor supply is given by $n_{w,t} = \phi^{-1}(w_t - p_t)$. Workers' consumption is given by $c_{w,t} = \chi_y y_t$, while savers' consumption is given by $c_{s,t} = \frac{1-\mu_w\chi_y}{1-\mu_w}y_t$. Consumption and net worth of optimistic and pessimistic savers is given by the expressions in Appendix B.3. By construction, the market-clearing conditions for goods and labor are satisfied. Because y_t satisfies the aggregate Euler equation, the savers' Euler equation is also satisfied. Because π_t satisfies the New Keynesian Phillips curve, the optimality condition for firms is satisfied.

We set $b_{s,t}^E = q_{E,t}$ and $b_{s,t}^S = 0$, which implies that the market clearing condition for stocks and short-term bonds hold. It remains to check that the market-clearing conditions for long-term bonds hold. Savers' financial wealth and government debt evolve according to

$$B_s\dot{b}_{s,t} = r_n B_s b_{s,t} + (T_{s,t} - T_s) + (i - \pi_t - r_n)B_s + (r_{L,t} - r_L)B_s^L + r_L B_s^L b_{s,t}^L \\ + (r_{E,t} - r_E)B_s^E + r_E B_s^E b_{s,t}^E - Yc_{s,t},$$

$$D_G\dot{d}_{G,t} = D_G(r_n + r_L)d_{G,t} + (T_t - T) + (i_t - \pi_t - r_n + r_{L,t} - r_L)D_G,$$

where $b_{s,0} = \frac{B_s^L}{B_s}q_{L,0} + \frac{B_s^E}{B_s}q_{E,0}$ and $d_{G,0} = q_{L,0}$.

Aggregating the budget constraint of workers and savers, using the market-clearing conditions, and the expression for equity returns, we obtain

$$(1-\mu_w)B_s^L\dot{b}_{s,t}^L \;=\; (T_t - T) + (1-\mu_w)\left[(i - \pi_t - r_n + r_{L,t} - r_L)B_s^L + (r_n + r_L)B_s^L b_{s,t}^L\right],$$

where we used $B_s b_{s,t} = B_s^S b_{s,t}^S + B_s^L b_{s,t}^L + B_s^E b_{s,t}^E$. Subtracting the government's flow budget constraint from the condition above, we obtain

$$(1-\mu_w)B_s^L\dot{b}_{s,t}^L - D_G\dot{d}_{G,t} = (r_n + r_L)((1-\mu_w)B_s^L b_{s,t}^L - D_G d_{G,t}),$$

using $(1-\mu_w)B_s^L = D_G$. Integrating this expression, we obtain $(1-\mu_w)B_s^L b_{s,t}^L - D_G d_{G,t} = e^{(r_n+r_L)t}\left[(1-\mu_w)B_s^L b_{s,0}^L - D_G d_{G,0}\right] = 0$, where the equality uses the market clearing condition in period 0. Therefore, the market clearing condition for long-term bonds is satisfied in all periods. The only condition that remains to be checked is the No-Ponzi condition for the government or, equivalently, the aggregate intertemporal budget constraint. Because condition (22) is satisfied, the No-Ponzi condition for the government is also satisfied. □

Proof of Propositions 5 and 6. We can write dynamic system (20) in matrix form as $\dot{Z}_t = AZ_t + B\nu_t$, where $B = [1, 0]'$. Applying the spectral decomposition to matrix A, we obtain $A = V\Omega V^{-1}$ where $V = \begin{bmatrix} \frac{\rho-\overline{\omega}}{\kappa} & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & 1 \end{bmatrix}$, $V^{-1} = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\begin{bmatrix} -1 & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & -\frac{\rho-\overline{\omega}}{\kappa} \end{bmatrix}$, and $\Omega = \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix}$. Decoupling the system, we obtain $\dot{z}_t = \Omega z_t + b\nu_t$, where $z_t = V^{-1}Z_t$ and $b = V^{-1}B$.

Solving the equation with a positive eigenvalue forward and the one with a negative eigenvalue backward, and rotating the system back to the original coordinates, we obtain

$$y_t = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{11}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{12}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz$$
$$\pi_t = V_{22}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{21}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{22}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz,$$

where $V^{i,j}$ is the (i,j) entry of matrix V^{-1}. Integrating $e^{-\rho t}y_t$ and using the intertemporal budget constraint,

$$\Omega_0 = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)\frac{1}{\rho-\underline{\omega}} - \frac{1}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty\left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt + \frac{1}{\rho-\underline{\omega}}V_{12}V^{21}\int_0^\infty e^{-\rho t}\nu_t dt.$$

Rearranging the above expression, we obtain

$$V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right) = (\rho-\underline{\omega})\Omega_0 + \frac{\rho-\underline{\omega}}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty e^{-\overline{\omega}t}\nu_t dt,$$

where we used the fact $\frac{V_{11}V^{11}}{\rho-\overline{\omega}} + \frac{V_{12}V^{21}}{\rho-\underline{\omega}} = 0$. Output is then given by $y_t = \tilde{y}_t + (\rho-\underline{\omega})e^{\underline{\omega}t}\Omega_0$, where $\tilde{y}_t = -\frac{\overline{\omega}-\rho}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\overline{\omega}-\delta}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\rho-\underline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$. Inflation is given by $\pi_t = \tilde{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0$, where $\tilde{\pi}_t = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\kappa}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$.

If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, then $\nu_t = \tilde{\sigma}^{-1}e^{-\psi_m t}(i_0 - r_n) + \chi_\lambda\epsilon_\lambda e^{-\psi_\lambda t}(i_0 - r_n)$. We then obtain $\tilde{y}_t = \tilde{\sigma}^{-1}\hat{y}_{m,t} + \chi_p\hat{y}_{\lambda,t}$ and $\tilde{\pi}_t = \tilde{\sigma}^{-1}\hat{\pi}_{m,t} + \chi_p\hat{\pi}_{\lambda,t}$, where $\chi_p \equiv \chi_\lambda\epsilon_\lambda$, $\hat{y}_{k,t} = \left[-\frac{\psi_k+\rho}{(\psi_k+\underline{\omega})(\psi_k+\overline{\omega})}e^{-\psi_k t} + \frac{\rho-\underline{\omega}}{(\psi_k+\overline{\omega})(\psi_k+\underline{\omega})}e^{\underline{\omega}t}\right](i_0 - r_n)$, and $\hat{\pi}_{k,t} = \frac{\kappa(e^{\underline{\omega}t}-e^{-\psi_k t})}{(\underline{\omega}+\psi_k)(\overline{\omega}+\psi_k)}(i_0 - r_n)$. Note that $\int_0^\infty e^{-\rho t}\hat{y}_{k,t}dt = 0$, $\frac{\partial\hat{y}_{k,0}}{\partial i_0} = -\frac{1}{\psi_k+\overline{\omega}} < 0$, and $\lim_{t\to\infty}\hat{y}_{k,t} = 0$. Moreover, $\hat{\pi}_0 = 0$, $\frac{\partial\hat{\pi}_{k,t}}{\partial i_0} \geq 0$ with strict inequality if $t > 0$.

□

Proof of Proposition 8. We divide this proof in three steps. First, we derive the condition for local uniqueness of the solution under the policy rule (7). Second, we derive the path of $[y_t, \pi_t, \hat{\lambda}_t, b_{p,t} - b_{o,t}, i_t]_0^\infty$ for a given path of monetary shocks. Third, we show how to implement a given path of nominal interest rates $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and a given value of fiscal backing $\int_0^\infty e^{-\rho t}\tau_t dt$.

Equilibrium determinacy. First, using the Taylor rule, we can write ν_t in Equation 20 as $\nu_t = \tilde{\sigma}^{-1}(\phi_\pi - 1)\pi_t + \chi_\lambda\hat{\lambda}_t + \tilde{\sigma}^{-1}u_t$. Combining the Phillips curve (11) with the system

(B.62), we obtain a dynamic system in the variables $[y_t, \pi, \hat{\lambda}_t, b_{p,t} - b_{o,t}]$:

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \\ \dot{\hat{\lambda}}_t \\ \dot{b}_{p,t} - \dot{b}_{o,t} \end{bmatrix} = \begin{bmatrix} \delta & \tilde{\sigma}^{-1}(\phi_\pi - 1) & \chi_\lambda & 0 \\ -\kappa & \rho & 0 & 0 \\ 0 & 0 & 0 & -\tilde{\xi}\chi_{\lambda,\Delta c} \\ 0 & 0 & -\chi_{\Delta b,\lambda} & \chi_{\Delta b,\Delta b} \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ \hat{\lambda}_t \\ b_{p,t} - b_{o,t} \end{bmatrix} + \begin{bmatrix} \tilde{\sigma}^{-1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u_t,$$

where $\hat{\lambda}_t = \chi_{\lambda,\Delta c}(c_{o,t} - c_{p,t})$ and $\chi_{\Delta b,\lambda} = \chi_{\Delta b,\Delta c}/\chi_{\lambda,\Delta c}$, given the boundary condition

$$b_{p,0} - b_{o,0} = -\left(\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o}\right) \int_0^\infty e^{-(\rho+\psi_L)t}(\phi_\pi \pi_t + u_t + r_L \hat{\lambda}_t)dt.$$

There is a unique bounded solution of the system above if the matrix of coefficients above has three eigenvalues with positive real parts and one eigenvalue with a negative real part. Denote the matrix of coefficients by A and consider the eigendecomposition of the matrix $A = V\Omega V^{-1}$, where Ω is the diagonal matrix of eigenvalues and V the matrix of eigenvectors. The eigenvalues are given by

$$\omega_1 = \frac{\rho + \delta + \sqrt{(\rho+\delta)^2 - 4\left(\tilde{\sigma}^{-1}(\phi_\pi - 1)\kappa + \rho\delta\right)}}{2}, \qquad \omega_3 = \frac{\chi_{\Delta b,\Delta b} + \sqrt{\chi^2_{\Delta b,\Delta b} + \tilde{\xi}\chi_{\lambda,\Delta c}\chi_{\Delta b,\lambda}}}{2}$$

$$\omega_2 = \frac{\rho + \delta - \sqrt{(\rho+\delta)^2 - 4\left(\tilde{\sigma}^{-1}(\phi_\pi - 1)\kappa + \rho\delta\right)}}{2}, \qquad \omega_4 = \frac{\chi_{\Delta b,\Delta b} - \sqrt{\chi^2_{\Delta b,\Delta b} + \tilde{\xi}\chi_{\lambda,\Delta c}\chi_{\Delta b,\lambda}}}{2}.$$

Notice that $\omega_3 > 0$ and $\omega_4 < 0$. Therefore, equilibrium determinacy requires $\omega_1 > 0$ and $\omega_2 > 0$. A necessary condition is $\phi_\pi > 1 - \frac{\rho\delta}{\tilde{\sigma}^{-1}\kappa} \equiv \overline{\phi}_\pi$, as otherwise the first two eigenvalues are real-valued and $\omega_2 \leq 0$. For ϕ_π sufficiently large, the eigenvalues are complex, but their real part is still positive. So, the condition $\phi_\pi > \overline{\pi}_\pi$ is sufficient to ensure determinacy.

Solution of the dynamic system. In matrix form, the dynamic system is given by $\dot{Z}_t = AZ_t + Bu_t$, where $Z_t = [y_t, \pi_t, \hat{\lambda}_t, b_{p,t} - b_{o,t}]'$ and $B = [\tilde{\sigma}^{-1}, 0, 0, 0]'$. Let $z_t = V^{-1}Z_t$ and $b = V^{-1}B$, which gives the system $\dot{z}_t = \Omega z_t + bu_t$. For $i = 1, 2, 3$, we can solve the equation forward, $z_{i,t} = -b_i \int_t^\infty e^{-\omega_i(s-t)} u_s ds$, and for $i = 4$ we solve it backwards: $z_{4,t} = e^{\omega_4 t} z_{4,0} + b_4 \int_0^t e^{\omega_4(t-s)} u_s ds$. Rotating to the original coordinates, we obtain:

$$Z_t = v_4 e^{\omega_4 t} z_{4,0} - \sum_{i=1}^{3} v_i b_i \int_t^\infty e^{-\omega_i(s-t)} u_s ds + v_4 b_4 \int_0^t e^{\omega_4(t-s)} u_s ds,$$

where v_i denotes the ith eigenvector, which are given by

$$v_1 = \left[\kappa^{-1}(\rho-\omega_1), 1, 0, 0\right]', \qquad v_3 = \left[v_{3,1}, \frac{\kappa v_{3,1}}{\rho-\omega_3}, \frac{\chi_{\Delta b,\Delta b}-\omega_3}{\chi_{\Delta b,\lambda}}, 1\right]'$$
$$v_2 = \left[\kappa^{-1}(\rho-\omega_2), 1, 0, 0\right]', \qquad v_4 = \left[v_{4,1}, \frac{\kappa v_{4,1}}{\rho-\omega_4}, \frac{\chi_{\Delta b,\Delta b}-\omega_4}{\chi_{\Delta b,\lambda}}, 1\right]',$$

$v_{i,1} = -\frac{(\rho-\omega_i)\chi_\lambda}{(\delta-\omega_i)(\rho-\omega_i)+\kappa\tilde{\sigma}^{-1}(\phi_\pi-1)}\frac{\chi_{\Delta b,\Delta b}-\omega_i}{\chi_{\Delta b,\lambda}}$ for $i = 3, 4$, and $b = [-\frac{\kappa\tilde{\sigma}^{-1}}{\omega_1-\omega_2}, \frac{\kappa\tilde{\sigma}^{-1}}{\omega_1-\omega_2}, 0, 0]'$. Using the fact that $\psi_\lambda = -\omega_4$, we obtain $b_{p,t} - b_{o,t} = e^{-\psi_\lambda t}(b_{p,0} - b_{o,t})$ and $\hat{\lambda}_t = \frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}} e^{-\psi_\lambda t}(b_{p,0} - b_{o,0})$, which coincides with the results from Proposition 3. y_t and π_t are given by

$$y_t = \sum_{i=1}^{2}(-1)^i \frac{\tilde{\sigma}^{-1}(\omega_i-\rho)}{\omega_1-\omega_2}\int_t^\infty e^{-\omega_i(s-t)}u_s ds - \frac{\chi_\lambda(\rho+\psi_\lambda)}{(\omega_1+\psi_\lambda)(\omega_2+\psi_\lambda)}\hat{\lambda}_t$$
$$\pi_t = \sum_{i=1}^{2}(-1)^{i-1} \frac{\tilde{\sigma}^{-1}}{\omega_1-\omega_2}\int_t^\infty e^{-\omega_i(s-t)}u_s ds - \frac{\kappa\chi_\lambda}{(\omega_1+\psi_\lambda)(\omega_2+\psi_\lambda)}\hat{\lambda}_t.$$

If $u_t = \sum_{k=1}^K \varphi_k u_{k,t}$, where $u_{k,t} = e^{-\psi_k t}u_{k,0}$, then

$$y_t = -\sum_{k=1}^{K}\varphi_k \frac{\rho+\psi_k}{(\omega_1+\psi_k)(\omega_2+\psi_k)}\tilde{\sigma}^{-1}u_{k,t} - \frac{\chi_\lambda(\rho+\psi_\lambda)}{(\omega_1+\psi_\lambda)(\omega_2+\psi_\lambda)}\hat{\lambda}_t$$
$$\pi_t = -\sum_{k=1}^{K}\varphi_k \frac{\kappa}{(\omega_1+\psi_k)(\omega_2+\psi_k)}\tilde{\sigma}^{-1}u_{k,t} - \frac{\kappa\chi_\lambda}{(\omega_1+\psi_\lambda)(\omega_2+\psi_\lambda)}\hat{\lambda}_t.$$

The nominal interest rate is given by

$$i_t - r_n = \sum_{k=1}^{K}\varphi_k \frac{(\delta+\psi_k)(\rho+\psi_k)-\tilde{\sigma}^{-1}\kappa}{(\omega_1+\psi_k)(\omega_2+\psi_k)}u_{k,t} - \frac{\phi_\pi\kappa\chi_\lambda}{(\omega_1+\psi_\lambda)(\omega_2+\psi_\lambda)}\hat{\lambda}_t.$$

The initial value of $\hat{\lambda}_0$ satisfies the condition:

$$\hat{\lambda}_0 = -\frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}}\left(\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o}\right)\left[\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n)dt + \frac{r_L}{\rho+\psi_L+\psi_\lambda}\hat{\lambda}_0\right],$$

solving for $\hat{\lambda}_0$, we obtain

$$\hat{\lambda}_0 = \frac{\frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}}\left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right)}{1 - \frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}}\left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right)\frac{r_L}{\rho+\psi_L+\psi_\lambda}}\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n)dt.$$

Combining the expression above with the expression for i_t, we obtain

$$\hat{\lambda}_0 = \frac{\frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}}\left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right)\sum_{k=1}^K \varphi_k \frac{(\delta+\psi_k)(\rho+\psi_k)-\tilde{\sigma}^{-1}\kappa}{(\omega_1+\psi_k)(\omega_2+\psi_k)}\frac{u_{k,0}}{\rho+\psi_L+\psi_k}}{1+\frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\lambda}}\left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right)\left[\frac{\phi_\pi\kappa\chi_\lambda}{(\omega_1+\psi_\lambda)(\omega_2+\lambda)}\frac{1}{\rho+\psi_L+\psi_\lambda} - \frac{r_L}{\rho+\psi_L+\psi_\lambda}\right]}.$$

Implementability condition. Take $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $\int_0^\infty e^{-\rho t}\tau_t dt$ as given, let Ω_0 be given by (29), y_t be given by (26) and π_t be given by (28). Define u_t as follows:

$$u_t = i_t - r_n - \phi_\pi \pi_t. \tag{A.1}$$

Let $[y_t^\star, \pi_t^\star, \hat{\lambda}_t^\star, b_{p,t}^\star - b_{o,t}^\star]_0^\infty$ be the solution to the four-dimensional dynamic system discussed above and $[i_t^\star, \tau_t^\star]$ the associated interest rate and fiscal backing. We show next that $y_t^\star = y_t$, $\pi_t^\star = \pi_t$, $\hat{\lambda}_t^\star = \hat{\lambda}_t$ and $i_t^\star = i_t$. First, notice that $u_t = \sum_{i=1}^3 \varphi_k e^{-\psi_k t} u_{k,0}$, where $u_{k,0} = i_0 - r_n$, and φ_k and ψ_k are given by

$$\varphi_1 = 1 + \frac{\phi_\pi\tilde{\sigma}^{-1}\kappa}{(\underline{\omega}+\psi_m)(\overline{\omega}+\psi_m)}, \qquad \psi_1 = \psi_m, \qquad \varphi_2 = \frac{\phi_\pi \frac{1-\mu_w}{1-\mu_w\chi_y}\chi_p\kappa}{(\underline{\omega}+\psi_\lambda)(\overline{\omega}+\psi_\lambda)}, \qquad \psi_2 = \psi_\lambda,$$

$$\varphi_3 = -\kappa\phi_\pi\left[\frac{\tilde{\sigma}^{-1}\kappa}{(\underline{\omega}+\psi_m)(\overline{\omega}+\psi_m)} + \frac{\frac{1-\mu_w}{1-\mu_w\chi_y}\chi_p\kappa}{(\underline{\omega}+\psi_\lambda)(\overline{\omega}+\psi_\lambda)} + \frac{\Omega_0}{i_0 - r_n}\right], \qquad \psi_3 = -\underline{\omega}.$$

As $u_{k,0}$ is proportional to $i_0 - r_n$, for $k = 1,2,3$, and $\hat{\lambda}_0^\star$ is proportional to a linear combination of the $u_{k,0}$, then $\hat{\lambda}_t^\star = \epsilon_\lambda^\star e^{-\psi_\lambda t}(i_0 - r_n) = \epsilon_\lambda^\star u_{2,t}$, for some constant $\epsilon_\lambda^\star$. If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n) =$, then $\epsilon_\lambda^\star = \epsilon_\lambda$. We guess that $\epsilon_\lambda^\star = \epsilon_\lambda$ and verify that nominal interest rates are exponentially decaying with rate ψ_m. The nominal interest rate is given by

$$i_t^\star - r_n = \sum_{k=1}^3 \frac{(\delta+\psi_k)(\rho+\psi_k)-\tilde{\sigma}^{-1}\kappa}{(\omega_1+\psi_k)(\omega_2+\psi_k)}\varphi_k u_{k,t} - \frac{\phi_\pi\kappa\chi_\lambda}{(\omega_1+\psi_\lambda)(\omega_2+\psi_\lambda)}\epsilon_\lambda u_{2,t}.$$

Notice that $(\delta+\psi_k)(\rho+\psi_k) - \tilde{\sigma}^{-1}\kappa = (\underline{\omega}+\psi_k)(\overline{\omega}+\psi_k)$, so the term multiplying $u_{3,t}$ is equal to zero, as $\psi_3 = -\underline{\omega}$. Using the fact that $\chi_\lambda\epsilon_\lambda = \frac{1-\mu_w}{1-\mu_w\chi_y}\chi_p$, the term multiplying $u_{2,t}$ is also equal to zero. Finally, the term multiplying $u_{1,t}$ is equal to one, so $i_t^\star - r_n = i_t - r_n$. From the Taylor rule we have that $u_t = i_t - r_n - \phi_\pi\pi_t = i_t^\star - r_n - \phi_\pi\pi_t^\star$, so $\pi_t^\star = \pi_t$. If the nominal interest rate and $\hat{\lambda}_t$ coincide in the two equilibria, then we must have

$b^{\star}_{p,t} - b^{\star}_{o,t} = b_{p,t} - b_{o,t}$. From the aggregate Euler equation, we obtain

$$y^{\star}_t = -\int_0^{\infty} e^{-\delta(s-t)}(i^{\star}_s - r_n - \pi^{\star}_s + \chi_\lambda \hat{\lambda}^{\star}_s)ds = -\int_0^{\infty} e^{-\delta(s-t)}(i_s - r_n - \pi_s + \chi_\lambda \hat{\lambda}_s)ds = y_t,$$

so $\overline{y}_t = y_t$. Finally, if output, inflation, nominal interest rates, and the market-implied disaster probability coincide in the two equilibria, from the intertemporal budget constraint we must have $\int_0^\infty e^{-\rho t}\tau^{\star}_t dt = \int_0^\infty e^{-\rho t}\tau_t dt$. □

Proof of Proposition 9. The workers' financial wealth in the no-disaster state evolves according to $\dot{B}_{w,t} = (i_t - \pi_t + r_{P,t})B_{w,t} + W_t N_{w,t} + T_{w,t} - C_{w,t}$. Using the fact that $B_{w,t} = -Q_{P,t}\overline{F}$ and $q_{P,t} = -\frac{i_{P,t} - i_P}{i_P + \psi_P}$, we obtain Equation 31. From the market clearing condition for goods, we obtain savers' consumption: $c_{s,t} = \frac{1-\mu_w \chi_y}{1-\mu_w} y_t + \frac{\mu_w \overline{d}_P}{1-\mu_w}\left(\frac{\psi_P}{i_P+\psi_P}(i_{P,t} - i_P) - \pi_t\right)\overline{d}_P$. Assuming exponentially decaying interest rates, and using the yield on the private bond $i_{P,t} - i_P = \frac{i_P + \psi_P}{\rho + \psi_P + \psi_m}(i_t - r_n) + \frac{i_P + \psi_P}{\rho + \psi_P + \psi_\lambda} r_P \hat{\lambda}_t$, we can write savers' consumption as follows

$$c_{s,t} = \frac{1-\mu_w\chi_y}{1-\mu_w} y_t + \frac{\mu_w \overline{d}_P}{1-\mu_w}\left[\frac{\psi_P}{\rho+\psi_P+\psi_m}(i_t - r_n) + \frac{\psi_P r_P}{\rho + \psi_P + \psi_\lambda}\hat{\lambda}_t - \pi_t\right]. \tag{A.2}$$

The Euler equation for savers can be written as

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} c_{s,t} + \chi_p \hat{\lambda}_t. \tag{A.3}$$

Combining equations (A.2) and (A.3), we obtain

$$\dot{y}_t = \left[\frac{1-\mu_w}{1-\chi_y\mu_w}\sigma^{-1} - \frac{\mu_w\overline{d}_P}{1-\chi_y\mu_w} r_n\right](i_t - \pi_t - r_n) + \left[\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} - \frac{\mu_w \overline{d}_P}{1-\chi_y\mu_w}\kappa\right] y_t$$
$$+ \left[\frac{1-\mu_w}{1-\chi_y\mu_w}\chi_p + \frac{\mu_w\overline{d}_P}{1-\chi_y\mu_w}\frac{\psi_P r_P(\rho - r_n + \psi_\lambda)}{\rho+\psi_P+\psi_\lambda}\right]\hat{\lambda}_t + \frac{\mu_w\overline{d}_P}{1-\chi_y\mu_w}\left[r_n + \frac{\psi_P(\rho - r_n + \psi_m)}{\rho+\psi_P+\psi_m}\right](i_t - r_n),$$

We can then write the aggregate Euler equation as $\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + v_t$, where $\tilde{\sigma}^{-1} \equiv \frac{1-\mu_w}{1-\chi_y\mu_w}\sigma^{-1} - \frac{\mu_w\overline{d}_P}{1-\chi_y\mu_w} r_n$, $\delta \equiv \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} - \frac{\mu_w\overline{d}_P}{1-\chi_y\mu_w}\kappa$, and $v_t \equiv \frac{\mu_w\overline{d}_P}{1-\chi_y\mu_w}\left[r_n + \frac{\psi_P(\rho-r_n+\psi_m)}{\rho+\psi_P+\psi_m}\right](i_t - r_n) + \left[\frac{1-\mu_w}{1-\chi_y\mu_w}\chi_p + \frac{\mu_w\overline{d}_P}{1-\chi_y\mu_w}\frac{\psi_P r_P(\rho-r_n+\psi_\lambda)}{\rho+\psi_P+\psi_\lambda}\right]\hat{\lambda}_t$. Therefore, following a derivation analogous to the one in Proposition 5, output is given by

$$y_t = \sigma^{-1}\hat{y}_{m,t} + \chi_p \hat{y}_{\lambda,t} + \frac{\mu_w\overline{d}_P}{1-\mu_b}\frac{\psi_P[\tilde{\psi}_m\hat{y}_{m,t} + r_P\tilde{\psi}_\lambda\hat{y}_{\lambda,t}]}{\rho+\psi_P+\psi_m} + (\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0,$$

where $\tilde{\psi}_k \equiv \psi_k + \rho - r_n$ for $k \in \{m, \lambda\}$. □

B Derivations for Section 2

B.1 The non-linear model

Savers' problem. Let $C_{j,t}(s)$ denote the consumption at time t of a saver of type $j \in \{o,p\}$ and denote the aggregate consumption of a type-j saver by $\overline{C}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} C_{j,t}(s) ds$, where a similar notation applies to the other variables. Given that the problem of all type-j savers is the same, except for the value of net worth, we drop the dependence on s and write $C_{j,t}$ instead of $C_{j,t}(s)$ to ease notation.

The HJB for the savers' problem is given by

$$\tilde{\rho}_j V_{j,t} = \max_{C_{j,t}, B^L_{j,t}, B^E_{j,t}} \frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \xi V_{j,t} + \frac{\partial V_{j,t}}{\partial t} + \lambda_j \left[V^*_{j,t} - V_{j,t} \right] + \frac{\partial V_{j,t}}{\partial B_{j,t}} \left[(i_t - \pi_t) B_{j,t} + r_{L,t} B^L_{j,t} + r_{E,t} B^E_{j,t} + T_{j,t} - C_{j,t} \right]. \tag{B.1}$$

where $V^*_{j,t}$ is evaluated at $B^*_{j,t} = B_{j,t} + B^L_{j,t} \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} + B^E_{j,t} \frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}}$ and $B_{j,t} > -\overline{D}_p$.

The corresponding HJB in the disaster state is given by

$$\tilde{\rho}^*_j V^*_{j,t} = \max_{C^*_{j,t}, B^{L,*}_{j,t}, B^{E,*}_{j,t}} \frac{(C^*_{j,t})^{1-\sigma}}{1-\sigma} - \xi V^*_{j,t} + \frac{\partial V^*_{j,t}}{\partial t} + \frac{\partial V^*_{j,t}}{\partial B^*_{j,t}} \left[(i^*_t - \pi^*_t) B_{j,t} + T^*_{j,t} - C^*_{j,t} \right],$$

where we imposed that $r^*_{L,t} = r^*_{E,t} = 0$, as there is no risk in the disaster state.

The first-order conditions are given by[1]

$$C_{j,t}^{-\sigma} = \frac{\partial V_{j,t}}{\partial B_{j,t}}, \qquad \frac{\partial V_{j,t}}{\partial B_{j,t}} r_{k,t} = \frac{\partial V^*_{j,t}}{\partial B^*_{j,t}} \frac{Q_{k,t} - Q^*_{k,t}}{Q_{k,t}}, \qquad (C^*_{j,t})^{-\sigma} = \frac{\partial V^*_{j,t}}{\partial B^*_{j,t}}, \tag{B.2}$$

for $k \in \{L, E\}$, and savers are indifferent about the level of long-term bonds in the disaster state.

Combining the expressions above, we obtain Equations (2) and (3). Differentiating the HJB equation in the no-disaster state with respect to $B_{j,t}$, we obtain the envelope condi-

[1]Formally, the solution is also subject to the state-constraint boundary condition $\frac{\partial V_{j,t}(-\overline{D}_P)}{\partial B} \geq \left(-(i_t - \pi_t)\overline{D}_P + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \widetilde{T}_{j,t} \right)^{-\sigma}$. See Achdou et al. (2017) for a discussion of state-constraint boundary conditions in the context of continuous-time savings problems with borrowing constraints.

tion:[2]

$$\rho_j \frac{\partial V_{j,t}}{\partial B_{j,t}} = \frac{\partial V_{j,t}}{\partial B_{j,t}}(i_t - \pi_t) + \frac{\mathbb{E}_{j,t}[d\left(\frac{\partial V_{j,t}}{\partial B_{j,t}}\right)]}{dt}, \tag{B.3}$$

where $\rho_j \equiv \tilde{\rho}_j + \xi$ captures subjective discounting and mortality risk.

Using the optimality condition for consumption and the condition above, we obtain:

$$i_t - \pi_t - \rho_j = -\frac{\mathbb{E}_t[dC_{j,t}^{-\sigma}]}{C_{j,t}^{-\sigma}dt} = \frac{\sigma C_{j,t}^{-\sigma-1}\dot{C}_{j,t} - \lambda_j\left[(C_{j,t}^*)^{-\sigma} - C_{j,t}^{-\sigma}\right]}{C_{j,t}^{-\sigma}}, \tag{B.4}$$

using the fact that $dC_{j,t} = \dot{C}_{j,t}dt + [C_{j,t}^* - C_{j,t}]d\mathcal{N}_t$ and Ito's lemma. Rearranging the expression above, we obtain Equation (1). A similar envelope condition holds in the disaster state, which gives the Euler equation for the disaster state

$$\frac{\dot{C}_{j,t}^*}{C_{j,t}^*} = \sigma^{-1}(i_t - \pi_t - \rho_j^*), \tag{B.5}$$

where $\rho_j^* \equiv \tilde{\rho}_j^* + \xi$.

Total wealth. Let $Q_{T_j,t}$ denote the present discounted value of transfers:

$$dQ_{T_j,t} = \left[(i_t - \pi_t + r_{T_j,t})Q_{T_j,t} - T_{j,t}\right]dt + \left[\frac{Q_{T_j,t}^* - Q_{T_j,t}}{Q_{T_j,t}}\right]d\mathcal{N}_t. \tag{B.6}$$

where $r_{T_j,t} = \lambda_j\left(\frac{C_{j,t}}{C_{j,t}^*}\right)^{\sigma}\frac{Q_{T_j,t} - Q_{T_j,t}^*}{Q_{T_j,t}}$.

Define total wealth as $\tilde{B}_{j,t} = B_{j,t} + Q_{T_j,t}$ and the sum of equities and the value of transfers as $\tilde{Q}_{E,t} = Q_{E,t} + Q_{T_j,t}$, so we can write the law of motion of $\tilde{B}_{j,t}$ as follows

$$d\tilde{B}_{j,t} = \left[(i_t - \pi_t)\tilde{B}_{j,t} + r_{L,t}\tilde{B}_{j,t}^L + \tilde{r}_{E,t}\tilde{B}_{j,t}^E - C_{j,t}\right]dt + \left[\tilde{B}_{j,t}^L\frac{Q_{L,t} - Q_{L,t}^*}{Q_{L,t}} + \tilde{B}_{j,t}^E\frac{\tilde{Q}_{E,t} - \tilde{Q}_{E,t}^*}{\tilde{Q}_{E,t}}\right]d\mathcal{N}_t, \tag{B.7}$$

where $\tilde{B}_{j,t}^L = B_{j,t}^L$, $\tilde{r}_{E,t} = \frac{Q_{E,t}}{\tilde{Q}_{E,t}}r_{E,t} + \frac{Q_{T_j,t}}{\tilde{Q}_{E,t}}r_{T_j,t}$, and $\tilde{B}_{j,t}^L = \frac{r_{E,t}}{\tilde{r}_{E,t}}B_{j,t}^E + \frac{r_{T_j,t}}{\tilde{r}_{E,t}}Q_{T_j,t}$.

Therefore, the problem where savers receive transfers is equivalent to a problem where savers do not receive transfers, but their wealth is $\tilde{B}_{j,t}$ and instead of investing on equities, they invest on a claim on profits plus fiscal transfers. One implication of this observation

[2] Here we used the fact that $\mathbb{E}_{j,t}[dF(B_{j,t},t)] = \left[F_t + \lambda_j[F^* - F] + F_B\left((i-\pi)B_j + r_L B_j^L + r_E B_j^E - C_j\right)\right]dt$ for any function $F(B_{j,t},t)$, according to Ito's lemma.

is that consumption is proportional to total wealth, $\tilde{B}_{j,t}$, instead of $B_{j,t}$.

Savers' aggregate behavior. Denote aggregate consumption and net worth of type-j savers as follows

$$\overline{C}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} C_{j,t}(s) ds, \qquad \overline{B}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} B_{j,t}(s) ds, \tag{B.8}$$

From the optimality condition for risky assets, we obtain

$$\frac{C_{j,t}(s)}{C^*_{j,t}(s)} = \frac{C_{j,t}(s')}{C^*_{j,t}(s')} \Rightarrow \frac{C_{j,t}(s)}{C^*_{j,t}(s)} = \frac{\overline{C}_{j,t}}{\overline{C}^*_{j,t}}. \tag{B.9}$$

We can then write the optimality condition for risky assets as follows

$$r_{k,t} = \lambda_j \left(\frac{\overline{C}_{j,t}}{\overline{C}^*_{j,t}} \right)^{\sigma} \frac{Q_{k,t} - Q^*_{k,t}}{Q_{k,t}}, \tag{B.10}$$

where $k \in \{L, E\}$.

The evolution of aggregate consumption of a type-j saver conditional on no-disaster is given by

$$\dot{\overline{C}}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} \dot{C}_{j,t}(s) ds + \xi (C_{j,t}(t) - \overline{C}_{j,t}) \tag{B.11}$$

$$= \overline{C}_{j,t} \left[\sigma^{-1} \left(i_t - \pi_t - \rho_j \right) + \frac{\lambda_j}{\sigma} \left[\left(\frac{\overline{C}_{j,t}}{\overline{C}^*_{j,t}} \right)^{\sigma} - 1 \right] + \xi \left(\frac{C_{j,t}(t)}{\overline{C}_{j,t}} - 1 \right) \right]. \tag{B.12}$$

The net worth of newborn savers is given by $B_{j,t}(t) = \frac{\mu_o}{\mu_o + \mu_p} \overline{B}_{o,t} + \frac{\mu_p}{\mu_o + \mu_p} \overline{B}_{p,t}$. As the consumption-total-wealth ratio is the same for all savers of the same type, we obtain

$$\frac{C_{j,t}(t)}{\overline{C}_{j,t}} = \frac{\frac{\mu_o}{\mu_o + \mu_p} \overline{B}_{o,t} + \frac{\mu_p}{\mu_o + \mu_p} \overline{B}_{p,t} + Q_{T_s,t}}{\overline{B}_{j,t} + Q_{T_s,t}} \tag{B.13}$$

Combining the expressions above, we can derive the law of motion of aggregate consumption for savers:

$$\frac{\dot{C}_{s,t}}{C_{s,t}} = \sigma^{-1} \left(i_t - \pi_t - \rho_{s,t} \right) + \frac{\lambda_t}{\sigma} \left[\left(\frac{C_{s,t}}{C^*_{s,t}} \right)^{\sigma} - 1 \right]. \tag{B.14}$$

The parameter ξ does not affect the aggregate Euler equation for savers. However, ξ controls the relative consumption of optimistic and pessimistic savers:

$$\frac{\dot{\overline{C}}_{o,t}}{\overline{C}_{o,t}} - \frac{\dot{\overline{C}}_{p,t}}{\overline{C}_{p,t}} = \xi \left[\overline{B}_{p,t} - \overline{B}_{o,t}\right] \frac{\frac{\mu_o}{\mu_o+\mu_p}\overline{B}_{o,t} + \frac{\mu_p}{\mu_o+\mu_p}\overline{B}_{p,t} + Q_{T_s,t}}{(\overline{B}_{o,t} + Q_{T_s,t})(\overline{B}_{p,t} + Q_{T_s,t})} \tag{B.15}$$

The evolution of aggregate net worth of a type-j saver conditional on no-disaster is given by

$$\dot{\overline{B}}_{j,t} = \int_{-\infty}^{t} \xi e^{-\xi(t-s)} \dot{B}_{j,t}(s) ds + \xi(B_{j,t}(t) - \overline{B}_{j,t}) \tag{B.16}$$

$$= \overline{B}_{j,t} \left[i_t - \pi_t + r_{L,t}\frac{\overline{B}^L_{j,t}}{\overline{B}_{j,t}} + r_{E,t}\frac{\overline{B}^E_{j,t}}{\overline{B}_{j,t}} - \frac{\overline{C}_{j,t} - T_{j,t}}{\overline{B}_{j,t}} + \xi\left(\frac{B_{j,t}(t)}{\overline{B}_{j,t}} - 1\right)\right]. \tag{B.17}$$

This implies that the aggregate net worth of savers evolves according to

$$\frac{\dot{B}_{s,t}}{B_{s,t}} = i_t - \pi_t + r_{L,t}\frac{B^L_{s,t}}{B_{s,t}} + r_{E,t}\frac{B^E_{s,t}}{B_{s,t}} - \frac{C_{s,t} - T_{s,t}}{B_{s,t}}. \tag{B.18}$$

The relative net worth of optimistic and pessimistic savers evolves according to

$$\frac{\dot{\overline{B}}_{o,t}}{\overline{B}_{o,t}} - \frac{\dot{\overline{B}}_{p,t}}{\overline{B}_{p,t}} = \sum_{k\in\{L,E\}} r_{k,t}\left(\frac{\overline{B}^L_{o,t}}{\overline{B}_{o,t}} - \frac{\overline{B}^k_{p,t}}{\overline{B}_{p,t}}\right) - \left(\frac{\overline{C}_{o,t} - T_{s,t}}{\overline{B}_{o,t}} - \frac{\overline{C}_{p,t} - T_{s,t}}{\overline{B}_{p,t}}\right) + \xi\left(\frac{B_{o,t}(t)}{\overline{B}_{o,t}} - \frac{B_{p,t}(t)}{\overline{B}_{p,t}}\right). \tag{B.19}$$

Workers' problem. The HJB for the workers' problem is given by

$$\tilde{\rho}_w V_{w,t} = \max_{\tilde{C}_{w,t}, N_{w,t}, B^L_{w,t}} \frac{\tilde{C}^{1-\sigma}_{w,t}}{1-\sigma} + \frac{\partial V_{w,t}}{\partial t} + \lambda_w\left[V^*_{w,t} - V_{w,t}\right] + \frac{\partial V_{w,t}}{\partial B_{w,t}}\left[(i_t - \pi_t)B_{w,t} + r_{L,t}B^L_{w,t} + \frac{W_t}{P_t}N_{w,t} + \widetilde{T}_{b,t} - \tilde{C}_{w,t} - \frac{N^{1+\phi}_{w,t}}{1+\phi}\right].$$

subject to the state-constraint boundary condition

$$\frac{\partial V_{w,t}(0)}{\partial B_{w,t}} \geq \left(\frac{W_t}{P_t}N_{w,t} - \frac{N^{1+\phi}_{w,t}}{1+\phi} + \widetilde{T}_{w,t}\right)^{-\sigma}, \tag{B.20}$$

where we adopted the change of variables $\tilde{C}_{w,t} \equiv C_{w,t} - \frac{N^{1+\phi}_{w,t}}{1+\phi}$.

For simplicity, we have already imposed that $B^E_{w,t} = 0$. We show below that $B^L_{w,t} = 0$ and a similar argument shows that workers would be against the short-selling constraint for equities when $B^E_{w,t}$ is a choice variable.

The optimality condition for labor supply is given by

$$N_{w,t}^{\phi} = \frac{W_t}{P_t}. \tag{B.21}$$

We focus on an equilibrium where workers are always constrained. To derive the conditions that ensure this is indeed the case, we start by considering a stationary equilibrium where all variables are constant conditional on the state. If workers are constrained in the stationary equilibrium, then they will also be constrained if fluctuations are small enough.

In a stationary equilibrium, net consumption $\tilde{C}_b$ in the no-disaster state is given by

$$\tilde{C}_w = \frac{W}{P}N_w - \frac{N_w^{1+\phi}}{1+\phi} + T_w, \tag{B.22}$$

and an analogous expression holds in the disaster state. Notice that the expression above does not depend on ρ_w or λ_w.

For workers to be unconstrained, the following condition would have to hold:

$$\frac{\dot{\tilde{C}}_{w,t}}{\tilde{C}_{w,t}} = \sigma^{-1}(r_n - \rho_w) + \frac{\lambda_w}{\sigma}\left[\left(\frac{\tilde{C}_{w,t}}{\tilde{C}_{w,t}^*}\right)^{\sigma} - 1\right]. \tag{B.23}$$

For ρ_b sufficiently large, workers would want a declining path of consumption, so current consumption would be above $\frac{W}{P}N_w - \frac{N_w^{1+\phi}}{1+\phi} + T_w$, which would violate the state-constraint. Hence, the constraint must be binding for ρ_w sufficiently large.

If the workers hold a positive amount of the long-term bonds, then the following condition must hold

$$r_L = \lambda_w \left(\frac{\tilde{C}_w}{\tilde{C}_w^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L}. \tag{B.24}$$

As C_w and C_w^* are independent of λ_w, the equation above would hold only if λ_w equals the value $\overline{\lambda}_w \equiv \frac{r_L}{\left(\frac{C_w}{C_w^*}\right)^{\sigma}\frac{Q_L - Q_L^*}{Q_L}}$. For $\lambda_w > \overline{\lambda}_w$, borrowers would want a smaller dispersion between C_w and C_w^*, which requires holding less risky bonds, violating the non-negativity constraint on long-term bonds. Therefore, borrowers will hold zero long-term bonds for λ_w sufficiently large.

Firms' problem. Final goods are produced according to the production function $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}} di\right)^{\frac{\epsilon-1}{\epsilon}}$. The solution to final-good producers problem is a demand for variety i given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$. The price level is given by $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$.

The intermediate-goods producers' problem is given by

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s\geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_s}{\eta_t} \left(\frac{P_{i,s}}{P_s} Y_{i,s} - \frac{W_s}{P_s} \frac{Y_{i,s}}{A_s} - \frac{\varphi}{2} \pi_s^2(j) \right) ds + \frac{\eta_{t^*}}{\eta_t} Q^*_{i,t^*}(P_{i,t^*}) \right],$$

subject to $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, given $P_{i,t} = P_i$.

The HJB equation for this problem is

$$0 = \max_{\pi_{i,t}} \eta_t \left(\frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t} \frac{Y_{i,t}}{A} - \frac{\varphi}{2} \pi_{i,t}^2 \right) dt + \mathbb{E}_t[d(\eta_t Q_{i,t})], \tag{B.25}$$

where $\frac{\mathbb{E}_t[d(\eta_t Q_{i,t})]}{\eta_t dt} = -(i_t - \pi_t) Q_{i,t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial t} + \lambda_t \frac{\eta_t^*}{\eta_t} \left[Q^*_{i,t} - Q_{i,t} \right]$.

The first-order condition is given by

$$\frac{\partial Q_{i,t}}{\partial P_i} P_{i,t} = \varphi \pi_{i,t}.$$

The change in π_t conditional on no disaster is then given by

$$\left(\frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2} \pi_{i,t} P_{i,t} \right) P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} P_{i,t} = \varphi \dot{\pi}_{i,t}. \tag{B.26}$$

The envelope condition with respect to $P_{i,t}$ is given by

$$0 = \left((1-\epsilon) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A} \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_{i,t}} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2} \pi_{i,t} P_{i,t} + \\ \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} - (i_t - \pi_t) \frac{\partial Q_{i,t}}{\partial P_i} + \lambda_t \frac{\eta_t^*}{\eta_t} \left(\frac{\partial Q^*_{i,t}}{\partial P_i} - \frac{\partial Q_{i,t}}{\partial P_i} \right). \tag{B.27}$$

Multiplying the expression above by $P_{i,t}$ and using Equation (B.26), we obtain

$$0 = \left((1-\epsilon) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A} \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t + \varphi \dot{\pi}_t - (i_t - \pi_t) \varphi \pi_{i,t} + \lambda_t \varphi \frac{\eta_t^*}{\eta_t} \left(\pi^*_{i,t} - \pi_{i,t} \right).$$

Rearranging the expression above, we obtain the non-linear New Keynesian Phillips curve

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta^*}{\eta_t} \right) \pi_t - \frac{\epsilon \varphi^{-1}}{A} \left(\frac{W_t}{P_t} - (1 - \epsilon^{-1}) A \right) Y_t,$$

where we have assumed that $P_{i,t} = P_t$ for all $i \in [0, 1]$ and that $\pi_t^* = 0$.

B.2 The stationary equilibrium

Aggregate output. Consider a stationary equilibrium with zero inflation. From the New Keynesian Phillips curve, we obtain

$$\frac{W}{P} = (1-\epsilon^{-1})A, \qquad \frac{W^*}{P} = (1-\epsilon^{-1})A^*. \tag{B.28}$$

Combining the expressions above with the labor supply condition, we obtain

$$Y = \mu_w (1-\epsilon^{-1})^{\frac{1}{\phi}} A^{\frac{1+\phi}{\phi}}, \qquad Y^* = \mu_w (1-\epsilon^{-1})^{\frac{1}{\phi}} (A^*)^{\frac{1+\phi}{\phi}}. \tag{B.29}$$

Disaster state. From the Euler equation for short-term bonds, the type-j saver will be unconstrained in a stationary equilibrium only if $r_n^* = \rho_j^*$. To ensure this is the case, we assume that $\rho_j^* = \rho_s$ for $j \in \{o, p\}$, where ρ_s is the effective discount rate implicit in the SDF, so the real interest rate in the disaster state is given by $i_t^* - \pi_t^* = r_n^* = \rho_s$. The excess return on long-terms bonds and equity are equal to zero, $r_L^* = r_E^* = 0$, so the price of the long-term bond is given by

$$Q_L^* = \frac{1}{r_n^* + \psi_L}, \tag{B.30}$$

and the equity price is given by $Q_E^* = \frac{\Pi^*}{r_n^*}$.

The consumption of borrowers is given by

$$C_w^* = (1-\epsilon^{-1})\frac{Y^*}{\mu_w} + T_w^*. \tag{B.31}$$

We assume that the government chooses fiscal transfers so workers have a given share $0 < \mu_{Y,w} < 1$ of output, so $C_w^* = \mu_{Y,w}\frac{Y^*}{\mu_w}$. The parameter $\mu_{Y,w}$ captures the government's preference for redistribution. This requires that the government sets $T_w^* = \left[\frac{\mu_{Y,w}}{\mu_w} - \frac{1-\epsilon^{-1}}{\mu_w}\right] Y^*$. In the main text, we focus on the case $\mu_{Y,w} = \mu_w$.

Savers' consumption is given by

$$C_j^* = r_n^* B_j^* + T_j^*, \tag{B.32}$$

where $B_j^* = B_j + B_j^L \frac{Q_L^* - Q_L}{Q_L} + B_j^E \frac{Q_E^* - Q_E}{Q_E}$.

Aggregate consumption of savers is given by

$$C_s^* = r_n^* \frac{\overline{D}_G^*}{\mu_s} + \frac{\Pi^*}{\mu_s} + T_s. \tag{B.33}$$

Transfers to savers must satisfy $T_s = (1-\mu_{Y,w}-\epsilon^{-1})\frac{Y^*}{\mu_s} - r_n^*\frac{\overline{D}_G^*}{\mu_s}$ such that the government's budget constraint is satisfied. This implies that the aggregate consumption of savers is given by $C_s^* = (1-\mu_{Y,w})Y^*$.

No-disaster state. The consumption of workers is given by

$$C_w = \left[(1-\epsilon^{-1})A\right]^{\frac{1+\phi}{\phi}} + T_w. \tag{B.34}$$

As in the disaster state, the government chooses fiscal transfers so workers have a given share $0 < \mu_{Y,w} < 1$ of output, so $C_w = \mu_{Y,w}Y$ and $C_s = (1-\mu_{Y,w})Y$. This requires that the government sets $T_w = \left[\frac{\mu_{Y,w}}{\mu_w} - \frac{1-\epsilon^{-1}}{\mu_w}\right]Y$. It remains to determine the relative consumption of optimistic and pessimistic savers.

The consumption of individual savers is given by

$$C_j = r_n B_j + r_L B_j^L + r_E B_j^E - T_j \tag{B.35}$$

The expression above ensures that total bond holdings of each individual saver is constant over time. To ensure that the aggregate bond holdings of optimistic and pessimistic savers is also constant, we must take into account the effect of births and deaths. Each instant a mass $\xi\mu_o$ of optimistic savers dies, which leads to a reduction in wealth for this group of $\xi\mu_o B_o$. Newborns inherit the wealth of their parents and a fraction $\frac{\mu_o}{\mu_o+\mu_p}$ of newborns is optimistic, so the influx of newborns raise the aggregate wealth of optimistic savers by $\frac{\mu_o}{\mu_o+\mu_p}\xi\left[\mu_o B_o + \mu_p B_p\right]$. These two effects cancel each other if the following condition is satisfied

$$\xi\mu_o B_o = \frac{\mu_o}{\mu_o+\mu_p}\xi\left[\mu_o B_o + \mu_p B_p\right] \Rightarrow B_o = B_p, \tag{B.36}$$

so $B_j = B_s$ for $j \in \{o,p\}$, where B_s is the average net worth of savers. We then have $B_j(s) = \overline{B}_j$ and $C_j(s) = \overline{C}_j$.

From the market clearing condition for assets, we obtain

$$B_s = \frac{\overline{D}_G + Q_E}{1-\mu_w}, \qquad B_s^L = \frac{\overline{D}_G}{1-\mu_w}, \qquad B_s^E = \frac{Q_E}{1-\mu_w}. \tag{B.37}$$

Using the fact that $B_o = B_p$ and $T_o = T_p$ in a stationary equilibrium, we can write the consumption of optimistic and pessimistic savers as follows:

$$C_o = C_s + r_L \frac{\mu_p}{\mu_o + \mu_p}(B_o^L - B_p^L) + r_E \frac{\mu_p}{\mu_o + \mu_p}(B_o^E - B_p^E) \tag{B.38}$$

$$C_p = C_s - r_L \frac{\mu_o}{\mu_o + \mu_p}(B_o^L - B_p^L) - r_E \frac{\mu_o}{\mu_o + \mu_p}(B_o^E - B_p^E). \tag{B.39}$$

We can use the Euler equations for risky assets to eliminate r_L and r_E from the expressions above, which gives us

$$C_o = C_s \left[1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{\mu_p}{\mu_o + \mu_p}\mathcal{R}_o\right], \qquad C_o^* = C_s^* \left[1 - \frac{\mu_p}{\mu_o + \mu_p}\frac{r_n^*\mathcal{R}_o}{1 - \zeta_Y}\right], \tag{B.40}$$

$$C_p = C_s \left[1 - \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{\mu_o}{\mu_o + \mu_p}\mathcal{R}_o\right], \qquad C_p^* = C_s^* \left[1 + \frac{\mu_o}{\mu_o + \mu_p}\frac{r_n^*\mathcal{R}_o}{1 - \zeta_Y}\right], \tag{B.41}$$

where $\mathcal{R}_o \equiv \frac{Q_L - Q_L^*}{Q_L}\frac{B_o^L - B_p^L}{C_s} + \frac{Q_L - Q_L^*}{Q_E}\frac{B_o^E - B_p^E}{C_s}$ represents optimistic relative risk exposure.

Notice that $\mathcal{R}_o$ pins down the share of consumption of optimistic savers:

$$\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p} = \frac{\mu_o}{\mu_o + \mu_p}\left[1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{\mu_p}{\mu_o + \mu_p}\mathcal{R}_o\right], \tag{B.42}$$

which is an implicit function of the share of consumption of optimistic savers, as λ is also a function of $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p}$. The left-hand is strictly increasing in $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p}$ and it is zero if $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p} = 0$. For $\mathcal{R}_o > 0$ and $\lambda_o < \lambda_p$, the right-hand side is decreasing in $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p}$ and it is positive if $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p} = 0$. Then, $\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p}$ is a strictly increasing function of $\mathcal{R}_0$, in the range $\mathcal{R}_0 > 0$. This implies that λ is decreasing in $\mathcal{R}_o$, but $\lambda\mathcal{R}_o$ is strictly increasing.

From the optimality condition for risky assets, we obtain the condition

$$\left(\frac{1 + \lambda\,(1 - \zeta_Y)^{-\sigma}\,\frac{\mu_p}{\mu_o + \mu_p}\mathcal{R}_o}{1 - \frac{\mu_p}{\mu_o + \mu_p}\frac{r_n^*\mathcal{R}_o}{1 - \zeta_Y}}\right)^{\sigma} = \frac{\lambda_p}{\lambda_o}\left(\frac{1 - \lambda\,(1 - \zeta_Y)^{-\sigma}\,\frac{\mu_o}{\mu_o + \mu_p}\mathcal{R}_o}{1 + \frac{\mu_o}{\mu_o + \mu_p}\frac{r_n^*\mathcal{R}_o}{1 - \zeta_Y}}\right)^{\sigma}, \tag{B.43}$$

where we used the fact that $\frac{C_s^*}{C_s} = 1 - \zeta_Y$.

The left-hand side of the expression above is strictly increasing in $\mathcal{R}_o$ and it is equal to 1 for $\mathcal{R}_o = 0$. The right-hand side is strictly decreasing in $\mathcal{R}_o$ and it is equal to $\frac{\lambda_p}{\lambda_o} \geq 1$. Then, there exists a unique value of $\mathcal{R}_o$ solving the equation above and $\mathcal{R}_o \geq 0$, with strict inequality if $\lambda_p > \lambda_o$.

The value of $\mathcal{R}_o$ pins down $\frac{\mu_o C_o}{\mu_p C_p + \mu_p C_p}$ and the market-implied disaster probability:

$$\lambda = \left[\frac{\mu_o C_o}{\mu_p C_p + \mu_p C_p}\lambda_o^{\frac{1}{\sigma}} + \frac{\mu_p C_p}{\mu_p C_p + \mu_p C_p}\lambda_p^{\frac{1}{\sigma}}\right]^{\sigma}. \tag{B.44}$$

From the Euler equations for short-term and long-term bonds, we obtain

$$r_n = \rho_j - \lambda_j\left[\left(\frac{C_j}{C_j^*}\right)^{\sigma} - 1\right], \qquad r_k = \lambda_j\left(\frac{C_j}{C_j^*}\right)^{\sigma}\frac{Q_k - Q_k^*}{Q_k}, \tag{B.45}$$

for $k \in \{L, E\}$, where $r_L = \frac{1}{Q_L} - \psi_L - r_n$, $r_E = \frac{\Pi}{Q_E} - r_n$, and $\Pi = \epsilon^{-1}Y$.

Notice that, given $\lambda_o\left(\frac{C_o}{C_o^*}\right)^{\sigma} = \lambda_p\left(\frac{C_p}{C_p^*}\right)^{\sigma}$, the condition $\rho_o + \lambda_o = \rho_p + \lambda_p$ is necessary for the Euler equation for short-term bonds to be satisfied with constant consumption for both types of savers.

Using the fact that $\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} = \lambda_j\left(\frac{C_j}{C_j^*}\right)^{\sigma}$, we can write the Euler equations in terms of aggregate savers' consumption:

$$r_n = \rho_s - \lambda\left[\left(\frac{C_s}{C_s^*}\right)^{\sigma} - 1\right], \qquad r_k = \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{Q_k - Q_k^*}{Q_k}, \tag{B.46}$$

for $k \in \{L, E\}$, where ρ_s satisfy the condition $\rho_s + \lambda = \rho_j + \lambda_j$ for $j \in \{o, p\}$.

We solve next for the price of risky assets. Given r_L, we can solve for Q_L:

$$\frac{1}{Q_L} - \psi_L - r_n = \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\left(1 - \frac{Q_L^*}{Q_L}\right) \Rightarrow Q_L = Q_L^*\frac{r_n^* + \psi_L + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}}{r_n + \psi_L + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}}, \tag{B.47}$$

where $Q_L > Q_L^*$, as $r_n < r_n^*$ due to the precautionary motive in the no-disaster state.

The loss in long-term bonds in the disaster state is given by

$$\frac{Q_L - Q_L^*}{Q_L} = \frac{r_n^* - r_n}{r_n^* + \psi_L + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}}, \tag{B.48}$$

which is positive as $r_n^* > r_n$. This implies that long-term interest rates are higher than short-term interest rates in the stationary equilibrium, i.e., the yield curve is upward sloping in this economy.

The equity price is given by

$$\frac{\Pi}{Q_E} - r_n = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \left(1 - \frac{Q_E^*}{Q_E}\right) \Rightarrow Q_E = \frac{\Pi + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_E^*}{r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma}}, \tag{B.49}$$

so the loss on equity in the disaster state is given by

$$\frac{Q_E - Q_E^*}{Q_E} = \frac{\Pi - r_n Q_E^*}{\Pi + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_E^*} = \zeta_\Pi + Q_E^* \frac{\frac{\Pi^*}{Q_E^*} - \left(r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \zeta_\Pi\right)}{\Pi + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_E^*}, \tag{B.50}$$

where $\zeta_\Pi \equiv 1 - \frac{\Pi^*}{\Pi}$ is the size of the drop in profits. It can be shown that the second term is positive for $\sigma > 1$. Therefore, the equity premium is positive in the stationary equilibrium. Notice that the drop in the values of equities in the disaster state comes from both the reduction in dividends and the drop in the price-dividend ratio in the disaster state due to higher natural rate.

Finally, given the quantity of risk for stocks and bonds, the value of $\mathcal{R}_o$ pins down a linear combination of $B_o^L - B_p^L$ and $B_o^E - B_o^E$, but it does not pin down their exact values. For instance, we could assume that $B_o^E = B_p^E$, such that differences in beliefs translates in differences in bond holdings. Alternatively, we could set $B_o^L - B_p^L = B_o^E - B_p^E$, so the optimistic investors holds more of stocks and long-term bonds. All these configurations are consistent with equilibrium and they do not affect prices or consumption.

B.3 Log-linear approximation

We consider next the effects of an unexpected monetary shock for an economy starting at the stationary equilibrium described above.

Market-based disaster probability. Linearizing Equation (4) around the stationary equilibrium, we obtain

$$\frac{\lambda^{\frac{1}{\sigma}}}{\sigma} \hat{\lambda}_t = \mu_{c,o} \mu_{c,p} \left(\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}\right) \left[c_{p,t} - c_{o,t}\right], \tag{B.51}$$

where $\mu_{c,j} \equiv \frac{\mu_j \overline{C}_j}{\mu_o \overline{C}_o + \mu_p \overline{C}_p}$ and $c_{j,t} \equiv \log \overline{C}_{j,t} / \overline{C}_j$, for $j \in \{o, p\}$. Note that $c_{j,t}$ denote the log-deviation of *average* consumption of type-j savers.

The expression above implies that changes in the relative consumption of optimistic and pessimistic investors affects the market-based probability of disaster. In particular, shocks that redistribute towards pessimistic investors at time t raise $\hat{\lambda}_t$.

Relative consumption. From the optimality condition for risky assets, we obtain

$$\lambda_o^{\frac{1}{\sigma}} \frac{\overline{C}_{o,t}}{\overline{C}^*_{o,t}} = \lambda_p^{\frac{1}{\sigma}} \frac{\overline{C}_{p,t}}{\overline{C}^*_{p,t}} \Rightarrow c_{p,t} - c_{o,t} = c^*_{p,t} - c^*_{o,t} \tag{B.52}$$

Relative consumption, in the no-disaster and disaster states, is given by

$$\dot{c}_{p,t} - \dot{c}_{o,t} = -\tilde{\xi}(b_{p,t} - b_{o,t}), \qquad \dot{c}^*_{p,t} - \dot{c}^*_{o,t} = -\tilde{\xi}(b^*_{p,t} - b^*_{o,t}), \tag{B.53}$$

where $\tilde{\xi} \equiv \xi \frac{B_s}{B_s + Q_{T_s}}$, and we used Equation (B.15) and the analogous condition in the disaster state.

Relative net worth. Linearizing Equation (B.19), we obtain

$$\begin{aligned} \dot{b}_{p,t} - \dot{b}_{o,t} = & \sum_{k \in \{L,E\}} r_k \left[\hat{r}_{k,t} \left(\frac{\overline{B}^k_p}{\overline{B}_p} - \frac{\overline{B}^k_o}{\overline{B}_o} \right) + \frac{\overline{B}^k_p}{\overline{B}_p}(b^k_{p,t} - b_{p,t}) - \frac{\overline{B}^k_o}{\overline{B}_o}(b^k_{o,t} - b_{o,t}) \right] \\ & - \left(\frac{\overline{C}_p}{\overline{B}_p} c_{p,t} - \frac{\overline{C}_o}{\overline{B}_o} c_{o,t} \right) + \frac{\overline{C}_p - T_p}{\overline{B}_p} b_{p,t} - \frac{\overline{C}_o - T_o}{\overline{B}_o} b_{o,t} - \xi \left(b_{p,t} - b_{o,t} \right), \end{aligned} \tag{B.54}$$

where $\hat{r}_{k,t} = \hat{\lambda}_t + \sigma c_{s,t} + \frac{Q^*_k}{Q_k - Q^*_k} q_{k,t}$.

Using the fact that $\frac{\overline{C}_j - T_j}{\overline{B}_j} = r_n + \sum_{k \in \{L,E\}} r_k \frac{B^k_j}{B_j}$, we can write the expression above as follows

$$\begin{aligned} \dot{b}_{p,t} - \dot{b}_{o,t} = & \sum_{k \in \{L,E\}} r_k \left[\hat{r}_{L,t} \left(\frac{\overline{B}^k_p}{\overline{B}_p} - \frac{\overline{B}^k_o}{\overline{B}_o} \right) + \frac{\overline{B}^k_p}{\overline{B}_p} b^k_{p,t} - \frac{\overline{B}^k_o}{\overline{B}_o} b^k_{o,t} \right] - \left(\frac{\overline{C}_p}{\overline{B}_p} c_{p,t} - \frac{\overline{C}_o}{\overline{B}_o} c_{o,t} \right) \\ & + (r_n - \xi)(b_{p,t} - b_{o,t}). \end{aligned} \tag{B.55}$$

The relative net worth in the disaster state is given by

$$\dot{b}^*_{p,t} - \dot{b}^*_{o,t} = -\xi(b^*_{p,t} - b^*_{o,t}) \Rightarrow b^*_{p,t} - b^*_{o,t} = e^{-\xi(t-t^*)}(b^*_{p,t^*} - b^*_{o,t^*}), \tag{B.56}$$

for $t \geq t^*$, where $b^*_{p,t^*} - b^*_{o,t^*}$ is given by

$$b^*_{p,t^*} - b^*_{o,t^*} = b_{p,t^*} - b_{o,t^*} - \sum_{k \in \{L,E\}} \left[\left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o} \right) \frac{Q^*_k}{Q_k} q_{k,t^*} + \left(\frac{B^k_p}{B_p} b^k_{p,t^*} - \frac{B^k_o}{B_o} b^k_{o,t^*} \right) \frac{Q_k - Q^*_k}{Q_k} \right]. \tag{B.57}$$

Relative risk exposure. Given that the consumption-wealth ratio for savers is constant in the disaster state, we have that $c^*_{j,t} = b^*_{j,t}$, so we obtain that $c^*_{p,t} - c^*_{o,t} = b^*_{p,t} - b^*_{o,t}$. Using the expression above and the fact that $b^*_{p,t} - b^*_{o,t} = c^*_{p,t} - c^*_{o,t} = c_{p,t} - c_{o,t}$, we can solve for the relative risk exposure:

$$\sum_{k\in\{L,E\}} \frac{Q_k - Q_k^*}{Q_k}\left(\frac{B_p^k}{B_p}b_{p,t}^k - \frac{B_o^k}{B_o}b_{o,t}^k\right) = b_{p,t} - b_{o,t} - (c_{p,t} - c_{o,t}) - \sum_{k\in\{L,E\}}\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right)\frac{Q_k^*}{Q_k}q_{k,t}. \tag{B.58}$$

The dynamic system. Using the expression above to eliminate the relative risk exposure, the relative net worth at the no-disaster state is given by

$$\begin{aligned}\dot{b}_{p,t} - \dot{b}_{o,t} = {} & (\hat{\lambda}_t + (\sigma-1)c_{s,t})\sum_{k\in\{L,E\}} r_k\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right) + (\rho_s + \lambda - \xi)(b_{p,t} - b_{o,t}) \\ & - (\rho_s+\lambda)(c_{p,t} - c_{o,t}) - \sum_{k\in\{L,E\}} r_k\left(\frac{\overline{B}_p^k}{B_p}(c_{p,t} - c_{s,t}) - \frac{\overline{B}_o^k}{B_o}(c_{o,t} - c_{s,t})\right).\end{aligned} \tag{B.59}$$

The deviation of consumption from average can be written as

$$c_{p,t} - c_{s,t} = \frac{\mu_o\overline{C}_o}{\mu_o\overline{C}_o + \mu_p\overline{C}_p}(c_{p,t} - c_{o,t}), \qquad c_{o,t} - c_{s,t} = -\frac{\mu_p\overline{C}_o}{\mu_o\overline{C}_o + \mu_p\overline{C}_p}(c_{p,t} - c_{o,t}). \tag{B.60}$$

Combining the expressions above, we can write $\dot{b}_{p,t} - \dot{b}_{o,t}$ as follows

$$\dot{b}_{p,t} - \dot{b}_{o,t} = -\chi_{\Delta b,\Delta c}(c_{p,t} - c_{o,t}) + \chi_{\Delta b,\Delta b}(b_{p,t} - b_{o,t}) + \chi_{\Delta b,c_s}c_{s,t}, \tag{B.61}$$

where $\chi_{\Delta b,c_s} \equiv (\sigma-1)\sum_{k\in\{L,E\}} r_k\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right)$, $\chi_{\Delta b,\Delta b} \equiv \rho_s + \lambda - \xi$, and

$$\chi_{\Delta b,\Delta c} \equiv \sigma\mu_{c,o}\mu_{c,p}\left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda}\right)\sum_{k\in\{L,E\}} r_k\left(\frac{B_o^k}{B_o} - \frac{B_p^k}{B_p}\right) + (\rho_s+\lambda) + \sum_{k\in\{L,E\}} r_k\left(\frac{\overline{B}_p^k}{\overline{B}_p}\frac{\mu_o\overline{C}_o}{\mu_o\overline{C}_o + \mu_p\overline{C}_p} + \frac{\overline{B}_o^k}{\overline{B}_o}\frac{\mu_p\overline{C}_p}{\mu_o\overline{C}_o + \mu_p\overline{C}_p}\right).$$

In general, we would have to simultaneously solve for the aggregate variables and the relative net worth and relative consumption of pessimistic savers, which would increase the dimensionality of the problem relative to the standard New Keynesian. We assume that $r_k c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$, so this term is small and can be ignored in our approximate solution. This implies that the system is now *block recursive*, where we can solve for the dynamics of relative consumption and relative net worth before fully characterizing the behavior of other aggregate variables. Under this assumption, we can write the joint dynamics of $b_{p,t} - b_{o,t}$ and $c_{p,t} - c_{o,t}$ as follows:

$$\begin{bmatrix} \dot{c}_{p,t} - \dot{c}_{o,t} \\ \dot{b}_{p,t} - \dot{b}_{o,t} \end{bmatrix} = \begin{bmatrix} 0 & -\tilde{\xi} \\ -\chi_{\Delta b,\Delta c} & \chi_{\Delta b,\Delta b} \end{bmatrix} \begin{bmatrix} c_{p,t} - c_{o,t} \\ b_{p,t} - b_{o,t} \end{bmatrix} \tag{B.62}$$

Persistence of $\hat{\lambda}_t$. Given that $\chi_{\Delta b,\Delta c} > 0$, the system above has a positive and a negative eigenvalue, so there is a unique bounded solution given by

$$\begin{bmatrix} c_{p,t} - c_{o,t} \\ b_{p,t} - b_{o,t} \end{bmatrix} = \begin{bmatrix} \frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\Delta c}} \\ 1 \end{bmatrix} e^{-\psi_\lambda t}(b_{p,0} - b_{o,0}) \tag{B.63}$$

where

$$\psi_\lambda \equiv \frac{\sqrt{\chi^2_{\Delta b,\Delta b} + 4\tilde{\xi}\chi_{\Delta b,\Delta c}} - \chi_{\Delta b,\Delta b}}{2}, \tag{B.64}$$

where $\psi_\lambda \geq 0$ is strictly increasing in ξ, it is equal to zero if $\xi = 0$ and it approaches infinity as $\xi \to \infty$.

We can then write the market-implied disaster probability as follows:

$$\hat{\lambda}_t = e^{-\psi_\lambda t}\hat{\lambda}_0, \tag{B.65}$$

where

$$\hat{\lambda}_0 \equiv \sigma\mu_{c,o}\mu_{c,p}\left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda}\right)\frac{\chi_{\Delta b,\Delta b}+\psi_\lambda}{\chi_{\Delta b,\Delta c}}(b_{p,0} - b_{o,0}). \tag{B.66}$$

Hence, ψ_λ captures the persistence of $\hat{\lambda}_t$. If $\xi = 0$, then $\psi_\lambda = 0$ and changes in λ_t are permanent. For high values of ψ_λ, the effects on λ_t are transitory and ψ_λ controls the speed of the convergence.

Wealth revaluation and $\hat{\lambda}_0$. The revaluation of the relative net worth is given by

$$b_{p,0} - b_{o,0} = \sum_{k\in\{L,E\}}\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right)q_{k,0}. \tag{B.67}$$

The price of the long-term bond satisfies the condition

$$-\frac{1}{Q_L}q_{L,t} + \dot{q}_{L,t} - (i_t - r_n) = r_L\left[\hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_L^*}{Q_L - Q_L^*}q^*_{L,t}\right] \tag{B.68}$$

Rearranging the expression above, we obtain

$$\dot{q}_{L,t} - (\rho + \psi_L) q_{L,t} = (i_t - r_n) + r_L(\hat{\lambda}_t + \sigma c_{s,t}). \tag{B.69}$$

Solving the differential equation above, we obtain

$$q_{L,0} = -\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n)dt - \int_0^\infty e^{-(\rho+\psi_L)t} r_L(\hat{\lambda}_t + \sigma c_{s,t})dt. \tag{B.70}$$

Suppose $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $r_L \sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$, then

$$q_{L,0} = -\frac{i_0 - r_n}{\rho + \psi_L + \psi_m} - \frac{r_L \hat{\lambda}_0}{\rho + \psi_L + \psi_\lambda}. \tag{B.71}$$

We focus on the case $\frac{B_p^E}{B_p} = \frac{B_o^E}{B_o}$, so the initial relative wealth revaluation is given by

$$b_{p,0} - b_{o,0} = -\left(\frac{B_p^L}{B_p} - \frac{B_o^L}{B_o}\right)\left[\frac{i_0 - r_n}{\rho + \psi_L + \psi_m} + \frac{r_L \hat{\lambda}_0}{\rho + \psi_L + \psi_\lambda}\right]. \tag{B.72}$$

Plugging the expression above into the expression for $\hat{\lambda}_0$

$$\hat{\lambda}_0 \equiv \frac{\sigma \mu_{c,o}\mu_{c,p}\left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda}\right)\frac{\chi_{\Delta b,\Delta b} + \psi_\lambda}{\chi_{\Delta b,\Delta c}}\left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right)}{1 - \sigma \mu_{c,o}\mu_{c,p}\left(\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda}\right)\frac{\chi_{\Delta b,\Delta b} + \psi_\lambda}{\chi_{\Delta b,\Delta c}}\left(\frac{B_o^L}{B_o} - \frac{B_p^L}{B_p}\right)\frac{r_L}{\rho + \psi_L + \psi_\lambda}} \frac{i_0 - r_n}{\rho + \psi_L + \psi_m}. \tag{B.73}$$

Notice that there is an amplification mechanism between the price of the long-term bond and the change in disaster probability. A wealth redistribution towards pessimistic investors tends to increase $\hat{\lambda}_0$. An increase in $\hat{\lambda}_0$ depresses the value of long-term bonds, redistributing towards pessimistic investors, further increasing $\hat{\lambda}_t$.

Workers' consumption. Log-linearizing workers' budget constraint, we obtain

$$c_{w,t} = \frac{WN_w}{PC_w}(w_t - p_t - n_{w,t}) + \frac{Y}{C_w}T_w'(Y)y_t. \tag{B.74}$$

Using the fact that $w_t - p_t - n_{w,t} = (1 + \phi)y_t$, we can write the expression above as follows

$$c_{b,t} = \chi_y y_t. \tag{B.75}$$

where $\chi_y \equiv \frac{WN_w}{PC_w}(1+\phi) + \frac{Y}{C_w}T'_w(Y)$.

Savers' Euler equation. Linearizing the Euler equation for savers, we obtain

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma}\left[\sigma c_{s,t} + \hat{\lambda}_t\right]. \tag{B.76}$$

Phillips curve. Linearizing the Phillips curver, we obtain

$$\dot{\pi}_t = \rho\pi_t - \kappa y_t, \tag{B.77}$$

where $\kappa \equiv \frac{\phi\epsilon}{\varphi}\frac{WN}{P}$.

B.4 Asset prices

Stock prices. Linearizing the expression for $r_{E,t}$, we obtain

$$\frac{\Pi}{Q_E}(\hat{\Pi}_t - q_{E,t}) + \dot{q}_{E,t} - (i_t - \pi_t - r_n) = r_E\left[\hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_E^*}{Q_E - Q_E^*}q_{E,t}\right]. \tag{B.78}$$

Rearranging the expression above, we obtain

$$\dot{q}_{E,t} - \rho q_{E,t} = -\frac{1}{Q_E}\hat{\Pi}_t + (i_t - \pi_t - r_n) + r_E\left[\hat{\lambda}_t + \sigma c_{s,t}\right], \tag{B.79}$$

where $\hat{\tau}_t = -\log\frac{1-\tau_t}{1-\tau}$ and

$$\hat{\Pi}_t = -\hat{\tau}_t\Pi + (1-\tau)(y_t - (1-\alpha)(w_t - p_t - n_t))Y \tag{B.80}$$

Solving the differential equation above, we obtain

$$q_{E,t} = \frac{1}{Q_E}\int_t^\infty e^{-\rho(s-t)}\hat{\Pi}_s ds - \int_t^\infty e^{-\rho(s-t)}\left[(i_s + \pi_s - r_n) + r_E(\hat{\lambda}_t + \sigma c_{s,t})\right] ds. \tag{B.81}$$

C Derivations for Section 3

C.1 Equilibrium determinacy and the Taylor principle

Combining the dynamics of the output and inflation from Proposition 2 and the Taylor rule $i_t = r_n + \phi_\pi + \epsilon_t$, we obtain the dynamic system

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \delta & -\tilde{\sigma}^{-1}(1-\phi_\pi) \\ -\kappa & \rho \end{bmatrix} + \begin{bmatrix} \tilde{v}_t \\ 0 \end{bmatrix}, \tag{C.1}$$

where

$$\tilde{v}_t = \tilde{\sigma}^{-1} u_t + \frac{1-\mu_w}{1-\mu_w \chi_y} \frac{\lambda}{\sigma} \left(\frac{C_s}{C_s^*} \right)^\sigma e^{-\psi_\lambda t} \hat{\lambda}_0. \tag{C.2}$$

The eigenvalues of the system incorporating the Taylor rule are given by

$$\overline{\omega}_T = \frac{\rho + \delta + \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}(1-\phi_\pi)\kappa - \rho\delta)}}{2}, \qquad \underline{\omega}_T = \frac{\rho + \delta - \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}(1-\phi_\pi)\kappa - \rho\delta)}}{2}. \tag{C.3}$$

The two eigenvalues above will be positive, and there will be a unique locally bounded solution, if the following condition is satisfied

$$\tilde{\sigma}^{-1}(1-\phi_\pi)\kappa - \rho\tilde{\delta} < 0 \Rightarrow \phi_\pi \geq 1 - \frac{\rho\delta}{\tilde{\sigma}^{-1}\kappa} \equiv \overline{\phi}_\pi < 1 \tag{C.4}$$

and $\overline{\phi}_\pi > 0$ if Assumption 1 holds. As $c_{s,t}$ increases with y_t, given $(\mu_b \chi_y < 1)$, risk is procyclical for savers in our economy. Bilbiie (2018) and Acharya and Dogra (2020) show that procyclical uninsurable idiosyncratic risk reduces the threshold on the response of monetary policy to inflation required to achieve local determinacy. A similar phenomenon happens in our case with aggregate disaster risk. Notice that the jump in marginal utility in the disaster state is given by $\left(\frac{C_{s,t}}{C_{s,t}^*}\right)^\sigma$, which in log-linear form is given by $\sigma c_{s,t}$. As $c_{s,t}$ is increasing in y_t if $\mu_b \chi_y < 1$, so the jump in marginal utility is procyclical in our economy.

C.2 Solving the dynamic system

We can write dynamic system (20) in matrix form as $\dot{Z}_t = AZ_t + B\nu_t$, where $B = [1, 0]'$. Applying the spectral decomposition to matrix A, we obtain $A = V\Omega V^{-1}$ where $V = \begin{bmatrix} \frac{\rho-\overline{\omega}}{\kappa} & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & 1 \end{bmatrix}$, $V^{-1} = \frac{\kappa}{\overline{\omega}-\underline{\omega}} \begin{bmatrix} -1 & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & -\frac{\rho-\overline{\omega}}{\kappa} \end{bmatrix}$, and $\Omega = \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix}$. Decoupling the system, we obtain $\dot{z}_t = \Omega z_t + b\nu_t$, where $z_t = V^{-1} Z_t$ and $b = V^{-1}B$.

Solving the equation with a positive eigenvalue forward and the one with a negative eigenvalue backward, and rotating the system back to the original coordinates, we obtain

$$y_t = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{11}V^{11}\int_t^{\infty} e^{-\overline{\omega}(z-t)}\nu_z dz + V_{12}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz$$

$$\pi_t = V_{22}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{21}V^{11}\int_t^{\infty} e^{-\overline{\omega}(z-t)}\nu_z dz + V_{22}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz,$$

where $V^{i,j}$ is the (i,j) entry of matrix V^{-1}. Integrating $e^{-\rho t}y_t$ and using the intertemporal budget constraint,

$$\Omega_0 = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)\frac{1}{\rho-\underline{\omega}} - \frac{1}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^{\infty}\left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt + \frac{1}{\rho-\underline{\omega}}V_{12}V^{21}\int_0^{\infty} e^{-\rho t}\nu_t dt.$$

Rearranging the above expression, we obtain

$$V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right) = (\rho-\underline{\omega})\Omega_0 + \frac{\rho-\underline{\omega}}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^{\infty}\left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt - V_{12}V^{21}\int_0^{\infty} e^{-\rho t}\nu_t dt.$$

Output is then given by $y_t = \tilde{y}_t + (\rho-\underline{\omega})e^{\underline{\omega}t}\Omega_0$, where $\tilde{y}_t = -\frac{\overline{\omega}-\rho}{\overline{\omega}-\underline{\omega}}\int_t^{\infty} e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\overline{\omega}-\delta}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\rho-\underline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^{\infty} e^{-\overline{\omega}z}\nu_z dz$. Inflation is given by $\pi_t = \tilde{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0$, where $\tilde{\pi}_t = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_t^{\infty} e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\kappa}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^{\infty} e^{-\overline{\omega}z}\nu_z dz$.

C.3 Intertemporal budget constraint

The following lemma characterizes the intertemporal budget constraint faced by savers.

Lemma 3 (Savers' intertemporal budget constraint)**.** *The intertemporal budget budget constraint (IBC) for individual savers and the aggregate of all savers are given by*

i. Individual IBC:

$$\mathbb{E}_0\left[\int_0^{\infty}\frac{\eta_t}{\eta_0}C_{j,t}(s)\right] = B_{j,t}(s). \tag{C.5}$$

ii. Savers' aggregate IBC:

$$\mathbb{E}_t\left[\int_0^{\infty}\frac{\eta_t}{\eta_0}C_{s,t}dt\right] = B_{s,t}, \tag{C.6}$$

where $B_{s,t} = \frac{D_{G,t}+Q_{E,t}}{1-\mu_w}$.

Proof. We consider first the derivation of the individual intertemporal budget constraint. The net worth of a type-j saver born at date s evolves according to

$$dB_{j,t}(s) = (i_t - \pi_t)B_{j,t}(s) + r_{L,t}B^L_{j,t}(s) + r_{E,t}B^E_{j,t}(s) + T_{j,t} - C_{j,t}(s) + \sum_{k\in\{L,E\}} B^k_{s,t}\frac{Q^*_{k,t} - Q_{k,t}}{Q_{k,t}}d\mathcal{N}_t, \tag{C.7}$$

so the expected change in the net worth scaled by SDF is given by

$$\frac{\mathbb{E}_t[d(\eta_t B_{j,t}(s))]}{\eta_t dt} = \left[-(i_t - \pi_t) - \lambda_t \left(\frac{\eta_t^*}{\eta_t} - 1\right)\right] B_{j,t}(s) + (i_t - \pi_t) B_{j,t}(s) + r_{L,t} B^L_{j,t}(s) + r_{E,t} B^E_{j,t}(s)$$
$$T_{j,t} - C_{j,t(s)} + \lambda_t \left[\frac{\eta_t^*}{\eta_t} B^*_{j,t}(s) - B_{j,t}(s)\right], \tag{C.8}$$

using Ito's lemma and $\mathbb{E}_t d\eta_t / \eta_t = -(i_t - \pi_t)dt$.

Integrating the expression above and using the fact that $r_{k,t} = \lambda_t \frac{\eta_t^*}{\eta_t} \frac{Q_{k,t} - Q^*_{k,t}}{Q_{k,t}}$, we obtain

$$\frac{\mathbb{E}_t[\eta_T B_{j,T}(s)]}{\eta_t} - B_{j,t}(s) = \mathbb{E}_t \left[\int_t^T \frac{\eta_z}{\eta_t}(T_{j,z} - C_{j,z}(s))dz\right] \tag{C.9}$$

Given that the household problem with constant mortality rate ξ is identical to the problem of an infinite-horizon household with an additional discount ξ, the standard transversality condition holds[3]

$$\lim_{T\to\infty} \mathbb{E}_{j,t}\left[e^{-\rho_j T} C^{-\sigma}_{j,T}(s) B_{j,T}(s)\right] = 0, \tag{C.10}$$

where $\rho_j \equiv \tilde{\rho}_j + \xi$.

We can change measure and price $B_{j,t}(s)$ using the market-implied probabilities:

$$\lim_{T\to\infty} \mathbb{E}_t \left[\eta_T B_{j,T}(s)\right] = 0, \tag{C.11}$$

Combining the expressions above, we obtain the intertemporal budget constraint:

$$\mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t} C_{j,z}(s) dz\right] = B_{j,t}(s) + \mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t} T_{j,z} dz\right]. \tag{C.12}$$

Notice that $C_{j,z}(s)$ denotes planned consumption for time z for a type-j saver born at date s, conditional on being alive. In particular, this equation implies that, for any date for the household's death $t' \geq t$, we obtain

$$\mathbb{E}_t\left[\int_t^{t'} \frac{\eta_z}{\eta_t}(C_{j,z}(s) - T_{j,z})dz + \frac{\eta_{t'}}{\eta_t} B_{j,t'}(s)\right] = B_{j,t}(s), \tag{C.13}$$

where $B_{j,t'}(s)$ denotes the (involuntary) bequest.

To simplify the aggregation process, it is helpful to index savers in a different way. Let

[3] Merton (1992) provides a general proof of this equivalence for stochastic economies (see Chapter 5) and Blanchard (1985) provides a discussion in the context of an otherwise deterministic model.

$i \in [\mu_w, 1]$ index the *family* (or dynasty) of a given saver. At each point in time, a family has a single member that derives no utility from bequests and faces mortality risk with intensity $\xi \geq 0$. As the member of the family dies, she is replaced by a new member who inherits the wealth, but may have a different type. Let $C_{i,t}$ denote the consumption of family i's member at time t, $T_{i,t}$ the transfer to family i, $B_{i,t}$ the net worth of family i, $j(i,t) \in \{o, p\}$ the type of the member of the family, and $s(i,t)$ the birth date of the current member.

Under this alternative notation, we can write the IBC of family i as follows:

$$\mathbb{E}_t \left[\int_t^{t'} \frac{\eta_z}{\eta_t} (C_{i,z} - T_{i,z}) dz + \frac{\eta_{t'}}{\eta_t} B_{i,t'} \right] = B_{i,t}, \tag{C.14}$$

where t' is the time of death and $B_{i,t'}$ is the involuntary bequest. Integrating this forward, the IBC is then given by

$$\mathbb{E}_t \left[\int_t^{\infty} \frac{\eta_z}{\eta_t} C_{i,z} dz \right] = B_{i,t} + \mathbb{E}_t \left[\int_t^{\infty} \frac{\eta_z}{\eta_t} T_{i,z} dz \right], \tag{C.15}$$

The aggregate consumption and net worth of savers is given by $C_{s,t} = \frac{1}{1-\mu_w} \int_{\mu_w}^1 C_{i,t} di$ and $B_{s,t} = \frac{1}{1-\mu_w} \int_{\mu_w}^1 B_{i,t} di$. Aggregating the equation above across families, we obtain

$$\mathbb{E}_t \left[\int_t^{\infty} \frac{\eta_z}{\eta_t} C_{s,z} dz \right] = B_{s,t} + \mathbb{E}_t \left[\int_t^{\infty} \frac{\eta_z}{\eta_t} T_{s,z} dz \right], \tag{C.16}$$

where $B_{s,t} = \frac{D_{G,t} + Q_{E,t}}{1-\mu_w}$, using the market clearing condition for bonds and equities. □

Aggregate IBC. Applying a similar argument to workers, we obtain

$$\mathbb{E}_t \left[\frac{\eta_T}{\eta_t} B_{w,T} \right] - B_{w,t} = \mathbb{E}_t \left[\int_t^T \frac{\eta_z}{\eta_t} \left(\frac{W_z}{P_z} N_{w,z} + \tilde{T}_{w,z} - C_{w,z} \right) dz \right]. \tag{C.17}$$

Using the fact that $B_{w,t} = 0$, so $\lim_{T \to \infty} \mathbb{E}_t \left[\frac{\eta_T}{\eta_t} B_{w,T} \right] = 0$, we obtain

$$\mathbb{E}_t \left[\int_t^{\infty} \frac{\eta_z}{\eta_t} C_{w,z} dz \right] = \mathbb{E}_t \left[\int_t^{\infty} \frac{\eta_z}{\eta_t} \left(\frac{W_z}{P_z} N_{w,z} + T_{w,z} \right) dz \right] + B_{w,t}. \tag{C.18}$$

Combining the expression above with the IBC for savers, we obtain

$$\mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t} C_z dz\right] = \mathbb{E}_t\left[\int_t^\infty \frac{\eta_z}{\eta_t}\left(\frac{W_z}{P_z}N_z + T_z\right)dz\right] + D_{G,t} + Q_{E,t}, \tag{C.19}$$

where $C_t \equiv \mu_w C_{w,t} + (1-\mu_w)C_{s,t}$ and $T_t = \sum_{j\in\{w,o,p\}} \mu_j T_{j,t}$.

Let $Q_{C,0} \equiv \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0} C_t dt\right]$ denote the value of the aggregate consumption claim and $Q_{H,0} \equiv \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\left(\frac{W_t}{P_t}N_t + T_t\right)dt\right]$ denote the value of human wealth (after transfers). These claims satisfy the following pricing conditions:

$$r_{C,t} = \lambda_t\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^\sigma \frac{Q_{C,t} - Q^*_{C,t}}{Q_{C,t}}, \qquad r_{H,t} = \lambda_t\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^\sigma \frac{Q_{H,t} - Q^*_{H,t}}{Q_{H,t}}, \tag{C.20}$$

where $r_{C,t} \equiv \frac{C_t}{Q_{C,t}} + \frac{\dot{Q}_{C,t}}{Q_{C,t}} - (i_t - \pi_t)$ and $r_{C,t} \equiv \frac{\frac{W_t}{P_t}N_t + T_t}{Q_{H,t}} + \frac{\dot{Q}_{H,t}}{Q_{H,t}} - (i_t - \pi_t)$.

The price of the consumption claim in the stationary equilibrium satisfies the condition

$$\frac{C}{Q_C} - r_n = \lambda\left(\frac{C_s}{C^*_s}\right)^\sigma\left[1 - \frac{Q^*_C}{Q_C}\right] \Rightarrow Q_C = \frac{C + \lambda\left(\frac{C_s}{C^*_s}\right)^\sigma \frac{C^*}{r^*_n}}{\rho} \tag{C.21}$$

Linearizing the pricing condition, we obtain

$$\dot{q}_{C,t} - \rho q_{C,t} = -\frac{C}{Q_C}c_t + i_t - \pi_t - r_n + r_C p_{d,t}, \tag{C.22}$$

where we used the fact that $\frac{C}{Q_C} = r_n + \lambda\left(\frac{C_s}{C^*_s}\right)^\sigma \frac{Q_C - Q^*_C}{Q_C} = \rho - \lambda\left(\frac{C_s}{C^*_s}\right)^\sigma \frac{Q^*_C}{Q_C}$.

Integrating the expression above forward, we obtain

$$q_{C,0} = \frac{C}{Q_C}\int_0^\infty e^{-\rho t}c_t dt - \int_0^\infty e^{-\rho t}\left(i_t - \pi_t + r_C p_{d,t}\right)dt. \tag{C.23}$$

Similarly, the initial price of the claim on human wealth is given by

$$q_{H,0} = \frac{Y}{Q_H}\int_0^\infty e^{-\rho t}\left[(1-\alpha)(w_t - p_t + n_t) + \hat{T}_t\right]dt - \int_0^\infty e^{-\rho t}\left(i_t - \pi_t + r_H p_{d,t}\right)dt, \tag{C.24}$$

where $1 - \alpha \equiv \frac{WN}{PY}$ and $\hat{T}_t = \frac{T_t - T}{Y}$

The linearized intertemporal budget constraint is given by

$$Q_C q_{c,0} = Q_H q_{H,0} + D_G q_{L,0} + Q_E q_{E,0}. \tag{C.25}$$

We can write the expression above as follows

$$\int_0^\infty e^{-\rho t} c_t dt - \frac{Q_C}{Y}\int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_C p_{d,t}\right) dt = \int_0^\infty e^{-\rho t}\left[(1-\alpha)(w_t - p_t + n_t) + \hat{T}_t\right] dt$$
$$- \frac{Q_H}{Y}\int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_H p_{d,t}\right) dt + \frac{D_G}{Y} q_{L,0} + \int_0^\infty e^{-\rho t}\hat{\Pi}_t dt$$
$$- \frac{Q_E}{Y}\int_0^\infty e^{-\rho t}[i_t - \pi_t - r_n + r_E p_{d,t}] dt \tag{C.26}$$

Rearranging the expression above, we obtain

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t}\left[\hat{\Pi}_t + (1-\alpha)(w_t - p_t + n_t) + \hat{T}_t\right] dt + \frac{D_G}{Y} q_{L,0}$$
$$\frac{Q_C - Q_H - Q_E}{Y}\int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n\right) dt + \int_0^\infty e^{-\rho t}\left[\frac{Q_C}{Y} r_C - \frac{Q_H}{Y} r_H - \frac{Q_E}{Y} r_E\right] p_{d,t} dt. \tag{C.27}$$

From the aggregate IBC in the no-disaster and disaster state, we obtain $Q_C = Q_H + D_G + Q_E$ and $Q_C^* = Q_H^* + D_G^* + Q_E^*$, where $D_G^* \equiv D_G \frac{Q_L^*}{Q_L}$. We then obtain the following condition

$$\frac{Q_C}{Y} r_C - \frac{Q_H}{Y} r_H - \frac{Q_E}{Y} r_E = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \left[Q_C - Q_C^* - (Q_H - Q_H^*) - (Q_E - Q_E^*)\right] \frac{1}{Y} = \frac{D_G}{Y} r_L. \tag{C.28}$$

We can then write the discount value of consumption as follows:

$$\int_0^\infty e^{-\rho t} c_t dt = \Omega_0, \tag{C.29}$$

where

$$\Omega_0 \equiv \int_0^\infty e^{-\rho t}\left[\hat{\Pi}_t + (1-\alpha)(w_t - p_t + n_t) + \hat{T}_t\right] dt + \bar{d}_G q_{L,0} + \bar{d}_G \int_0^\infty e^{-\rho t}\left(i_t - \pi_t - r_n + r_L p_{d,t}\right) dt. \tag{C.30}$$

C.4 Wealth effects and Hicksian compensation

In this subsection, we show that Ω_0 corresponds to (minus) the sum of the *Hicksian wealth compensation* for each household. Let $e_j(\eta, U)$ define the expenditure function

$$e_j(\eta, U) = \min_{\{C_j\}} \mathbb{E}_0 \left[\int_0^{t^*} \frac{\eta_t}{\eta_0} C_{j,t} dt + \int_{t^*}^\infty \frac{\eta_t^*}{\eta_0} C_{j,t}^* dt\right], \tag{C.31}$$

subject to $\mathbb{E}_0\left[\int_0^{t^*} e^{-\rho_j t}\frac{C_{j,t}^{1-\sigma}}{1-\sigma}dt + \int_{t^*}^{\infty} e^{-\rho t}\frac{(C_{j,t}^*)^{1-\sigma}}{1-\sigma}dt\right] = U$. The solution to this problem is the Hicksian demand $C_{j,t}^h(\eta, U)$ and $C_{j,t}^{h,*}(\eta, U)$ in the no-disaster and disaster states.

Let η' denote an alternative price process and U' the corresponding utility under the new equilibrium. Mas-Colell et al. (1995) (see page 62) defines the Hicksian wealth compensation as $e_j(\eta', U) - e_j(\eta', U')$. We focus on a first-order approximation, that is, $\eta_t'/\eta_0' = \eta_t/\eta_0 + \tilde{\eta}_t$, where $\tilde{\eta}_t$ is small. Let $\tilde{c}_{j,t} \equiv \log C_{j,t}^h(\eta', U)/C_{j,t}^h(\eta, U)$. Plugging the expression for $C_{j,t}^h(\eta', U)$ into the constraint and linearizing, we obtain

$$\mathbb{E}_0\left[\int_0^{t^*} e^{-\rho_j t}C_{j,t}^h(\eta, U)^{1-\sigma}\tilde{c}_{j,t}dt + \int_{t^*}^{\infty} e^{-\rho_j t}C_{j,t}^{h,*}(\eta, U)^{1-\sigma}\tilde{c}_{j,t}^*dt\right] = 0. \tag{C.32}$$

Notice this implies that $\mathbb{E}_0\left[\int_0^{t^*} \frac{\eta_t}{\eta_0}C_{j,t}^h(\eta, U)\tilde{c}_{j,t}dt + \int_{t^*}^{\infty} \frac{\eta_t^*}{\eta_0}C_{j,t}^{h,*}(\eta, U)\tilde{c}_{j,t}^*dt\right] = 0$. As workers do not engage in intertemporal substitution, we set $\tilde{c}_{w,t} = \tilde{c}_{w,t}^* = 0$, so this equation would hold for them as well. We can then write $e_j(\eta', U)$ as follows

$$\begin{aligned} e_j(\eta', U) &= \mathbb{E}_0\left[\int_0^{t^*} \frac{\eta_t'}{\eta_0'}C_{j,t}^h(\eta, U)dt + \int_{t^*}^{\infty} \frac{\eta_t'}{\eta_0'}C_{j,t}^{*,h}(\eta, U)dt + \int_0^{t^*} \frac{\eta_t}{\eta_0}C_{j,t}^h(\eta, U)\tilde{c}_{j,t}dt + \int_{t^*}^{\infty} \frac{\eta_t^*}{\eta_0}C_{j,t}^{h,*}(\eta, U)\tilde{c}_{j,t}^*dt\right], \\ &= \mathbb{E}_0\left[\int_0^{t^*} \frac{\eta_t'}{\eta_0'}C_{j,t}^h(\eta, U)dt + \int_{t^*}^{\infty} \frac{\eta_t'}{\eta_0'}C_{j,t}^{*,h}(\eta, U)dt\right]. \end{aligned} \tag{C.33}$$

We assume that the initial equilibrium corresponds to the stationary equilibrium, so $C_{j,t}^h(\eta, U) = C_j$ and $C_{j,t}^{h,*}(\eta, U) = C_j^*$. Therefore, the Hicksian wealth compensation is given by

$$e_j(\eta', U) - e_j(\eta', U') = \mathbb{E}_0\left[\int_0^{t^*} \frac{\eta_t'}{\eta_0'}C_j dt + \int_{t^*}^{\infty} \frac{\eta_t'}{\eta_0'}C_j^* dt\right] - \mathbb{E}_0\left[\int_0^{t^*} \frac{\eta_t'}{\eta_0'}C_{j,t}dt + \int_{t^*}^{\infty} \frac{\eta_t'}{\eta_0'}C_{j,t}^*dt\right], \tag{C.34}$$

which corresponds to the definition given in the text after aggregation.

Let $\tilde{Q}_{C,0} \equiv \mathbb{E}_0\left[\int_0^{t^*} \frac{\eta_t'}{\eta_0'}Cdt + \int_{t^*}^{\infty} \frac{\eta_t'}{\eta_0'}C^*dt\right]$ and $Q_{C,0} \equiv \mathbb{E}_0\left[\int_0^{t^*} \frac{\eta_t'}{\eta_0'}C_t dt + \int_{t^*}^{\infty} \frac{\eta_t'}{\eta_0'}C_t^* dt\right]$. In a stationary equilibrium, we have that $\tilde{Q}_C = Q_C$. Linearizing these two expressions, we obtain

$$Q_C\tilde{q}_{C,0} = -Q_C\int_0^{\infty} e^{-\rho t}[i_t - \pi_t - r_n + r_C p_{d,t}]dt \tag{C.35}$$

$$Q_C\tilde{q}_{C,0} = Y\int_0^{\infty} e^{-\rho t}c_t dt - Q_C\int_0^{\infty} e^{-\rho t}[i_t - \pi_t - r_n + r_C p_{d,t}]dt. \tag{C.36}$$

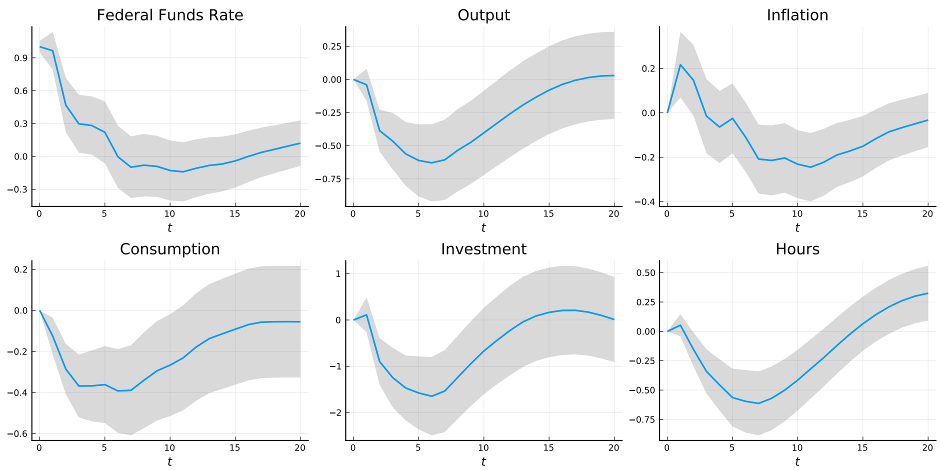

Figure D.1: Estimated IRFs.

This implies that, up to first order, the Hicksian wealth compensation is given by

$$\sum_{j\in\{w,o,p\}} \mu_j \left[e_j(\eta', U) - e_j(\eta', U') \right] = -Y \int_0^\infty e^{-\rho t} c_t dt = -Y\Omega_0. \tag{C.37}$$

Therefore, Ω_0 corresponds to (minus) the sum of the Hicksian wealth compensation for all households.

D Estimation of Fiscal Response to a Monetary Shock

We estimate the empirical IRFs using a VAR identified by a recursiveness assumption, as in Christiano et al. (1999), extended to include fiscal variables. The variables included are: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, the federal funds rate, the 5-year constant maturity rate, and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The identification assumption of the monetary shock is as follows: the only variables that react contemporaneously to the monetary shock are the federal funds rate, the 5-year rate and the value of government debt. All other variables, including tax revenues and expenditures, react with a lag of one quarter.

Data sources. The data sources are: **Nominal GDP:** BEA Table 1.1.5 Line 1; **Real GDP:** BEA Table 1.1.3 Line 1, **Consumption Durable:** BEA Table 1.1.3 Line 4; **Consumption Non Durable:** BEA Table 1.1.3 Line 5; **Consumption Services:** BEA Table 1.1.3 Line 6; **Pri-**

	(1) Revenues	(2) Interest Payments	(3) Transfers & Expenditures	(4) Debt in T	(5) Initial Debt	(1) - (2) - (3) + (4) - (5) Residual
Data	-26	68.88	-12.09	2.91	-49.74	30.13
	[-72.89,20.89]	[30.01,107.75]	[-48.74,24.56]	[-12.79,18.62]	[-68.03,-31.46]	[-4.74,65]

Table D.1: The impact on fiscal variables of a monetary policy shock

Note: Calculations correspond to a a 100 bps unanticipated interest rate increase. Confidence interval at 95% level.

vate Investment: BEA Table 1.1.3 Line 7; **GDP Deflator:** BEA Table 1.1.9 Line 1; **Capacity Utilization:** FRED CUMFNS; **Hours Worked:** FRED HOANBS; **Nominal Hourly Compensation:** FRED COMPNFB; **Civilian Labor Force:** FRED CNP16OV; **Nominal Revenues:** BEA Table 3.1 Line 1; **Nominal Expenditures:** BEA Table 3.1 Line 21; **Nominal Transfers:** BEA Table 3.1 Line 22; **Nominal Gov't Investment:** BEA Table 3.1 Line 39; **Nominal Consumption of Net Capital:** BEA Table 3.1 Line 42; **Effective Federal Funds Rate (FF):** FRED FEDFUNDS; **5-Year Treasury Constant Maturity Rate:** FRED DGS5; **Market Value of Government Debt:** Hall et al. (2018).

All the variables are obtained from standard sources, except for the real value of debt, which we construct from the series provided by Hall et al. (2018). We transform the series into quarterly frequency by keeping the market value of debt in the first month of the quarter. This choice is meant to avoid capturing changes in the market value of debt arising from changes in the *quantity* of debt after a monetary shock instead of changes in *prices*.

VAR estimation. Figure D.1 shows the results. As is standard in the literature, we find that a contractionary monetary shock increases the federal funds rate and reduces output and inflation on impact. Moreover, the contractionary monetary shock reduces consumption, investment, and hours worked.

The Government's Intertemporal Budget Constraint. The fiscal response in the model corresponds to the present discounted value of transfers over an infinite horizon, that is, $\sum_{t=0}^{\infty} \tilde{\beta}^t T_t$, where $\tilde{\beta} = \frac{1-\lambda}{1+\rho_s}$. We next consider its empirical counterpart. First, we calculate a truncated intertemporal budget constraint from period zero to $\mathcal{T}$:

$$\underbrace{b_y b_0}_{\substack{\text{debt}\\\text{revaluation}}} = \sum_{t=0}^{\mathcal{T}} \tilde{\beta}^t \left[\underbrace{\tau y_t + \tau_t}_{\text{tax revenue}} - \underbrace{\tilde{\beta}^{-1} b_y (i_{t-1}^m - \pi_t - r^n)}_{\text{interest payments}} \right] - \underbrace{T_{0,\mathcal{T}} + \tilde{\beta}^{\mathcal{T}} b_y b_{\mathcal{T}}}_{\substack{\text{other transfers/expenditures}\\\text{\& final debt}}} \tag{D.1}$$

The right-hand side of (D.1) is the present value of the impact of a monetary shock on fiscal accounts. The first term represents the change in revenues that results from the real effects of monetary shocks. The second term represents the change in interest payments on government debt that results from change in nominal rates. The last two terms are adjustments in transfers and other government expenditures, and the final debt position at period $\mathcal{T}$, respectively. In particular, $T_{0,\mathcal{T}}$ represents the present discounted value of transfers from period 0 through $\mathcal{T}$. Provided that $\mathcal{T}$ is large enough, such that (y_t, τ_t, i_t) have essentially converged to the steady state, then the value of debt at the terminal date, $b_{\mathcal{T}}$, equals (minus) the present discounted value of transfers and other expenditures from period $\mathcal{T}$ onward. Hence, the last two terms combined can be interpreted as the present discounted value of fiscal transfers from zero to infinity. Finally, the left-hand side represents the revaluation effect of the *initial* stock of government debt.

Table D.1 shows the impact on the fiscal accounts of a monetary policy shock, both in the data and in the estimated model. We first apply equation (D.1) to the data and check whether the difference between the left-hand side and the right-hand side is different from zero. The residual is calculated as

$$\text{Residual = Revenues - Interest Payments - Transfers + Debt in } \mathcal{T} \text{ - Initial Debt}$$

We truncate the calculations to quarter 60, that is, $\mathcal{T} = 60$ (15 years) in equation (D.1). The results reported in Table D.1 imply that we cannot reject the possibility that the residual is zero and, therefore, we cannot reject the possibility that the intertemporal budget constraint of the government is satisfied in our estimation.

The adjustment of the fiscal accounts in the data corresponds to the patterns we observed in Figure 1. The response of initial debt is quantitatively important, and it accounts for the bulk of the adjustment in the fiscal accounts.

EBP. To estimate the response of the corporate spread in the data, we add the EBP measure of Gilchrist and Zakrajšek (2012) into our VAR (ordered after the fed funds rate). Since the EBP is only available starting in 1973, we reduce our sample period to 1973:1-

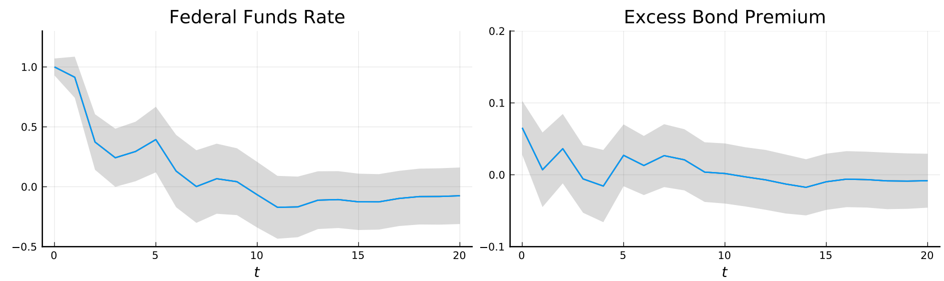

Figure D.2: IRFs for the federal funds rate and excess bond premium.

2007:7. The estimated IRFs are in line with those obtained for the longer sample. We find a significant increase of the EBP on impact, of 6.5 bps, in line with the estimates in the literature.

References

Achdou, Y., Han, J., Lasry, J.-M., Lions, P.-L., and Moll, B. (2017). Income and wealth distribution in macroeconomics: A continuous-time approach.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (1999). Monetary policy shocks: What have we learned and to what end? *Handbook of Macroeconomics*, 1:65–148.