What is the Central Bank's Game?

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Abstract

In this paper we, first, by explicitly taking account of the private sector's influence and pressure on the monetary authorities, provide a more plausible representation of the motivations of the two main players. We then incorporate persistence into the model and show that the optimal policy of the authorities will be state dependent. Finally and most importantly, we highlight an inconsistency between two strands in the literature of monetary analysis, namely the long lags of monetary policy and the time inconsistency. Such a lag of monetary policy means that the policy will be transparently observed before it affects the economy, consequently the Central Bank cannot fool anybody who has not already bound herself into a contract longer than that lag. Even if such contracts are pervasive, the inflationary bias arising from time inconsistency must be much smaller than that previously assessed.

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Non-technical Summary

This paper makes three contributions toward our understanding of the Central Banker's game. The first involves providing a more plausible representation of the motivations of the two main players, e.g. by explicitly taking account of the private sector's influence and pressure on the monetary authorities. The second is to incorporate persistence into the model. We show that, with persistence, the optimal policy of the authorities will be state dependent, so that a fixed contract for the Central Bank which is optimal on average, will at times lead to an inflation, or a deflation bias. The third, and most important, is that we highlight an inconsistency between two strands in the literature of monetary analysis, namely the existence of long lags between adjustments to the monetary policy instrument and their real effect on the economy, and the game theoretic models of time inconsistency, in which monetary policy is invariably assumed to act instantaneously. Such a (generally accepted) lag of monetary policy means that the policy will be transparently observed before it affects the economy. Consequently the Central Bank cannot fool anybody who has not already bound herself into a contract longer than that lag. Unless such contracts are pervasive, time inconsistency cannot occur. Even if such contracts are pervasive, the bias arising from time inconsistency must be a small fraction of that previously assessed (on a no lag basis), since such a large proportion of such contracts will come up for renewal before the monetary policy action takes effect. On these grounds we doubt that the standard model of time inconsistency provides a fully satisfactory rationale for the inflationary proclivities of the industrialised world in the last half century.

What is the Central Bank's Game?

I. Introduction

In this paper we highlight an inconsistency between two strands in the literature of monetary analysis, which appears to have gone largely unnoticed heretofore.

First, in empirical, descriptive and policy-related accounts of the operation of monetary policy, a key, central feature of these is that there are long lags between (policy-induced) adjustments in the Central Bank's instrument and its effect on the real economy. Friedman's report of such lags as "long and variable" is, perhaps, the locus classicus. But virtually all empirical exercises in this field acknowledge and incorporate such lags. Indeed, in one recent exercise, Gerlach and Smets (1995), following Bernanke and Blinder (1992), the absence of any immediate effect of monetary policy on output was a means of identifying a monetary policy shock from a demand shock in a VAR. In another recent empirical exercise, Clark, Laxton and Rose (1995), began with this quote from Alan Greenspan, (1994, p.609), "Shifts in the stance of monetary policy influence the economy and financial markets with a considerable lag, as long as a year or more. The challenge of monetary policy is to interpret current data on the economy and financial markets with an eye to anticipating future inflationary forces and countering them by taking action in advance."

The second strand is the time inconsistency literature, dating back to Kydland and Prescott (1977). In this literature the Central Bank can use its monetary policy instrument instantaneously to affect output. Hence, with the private sector's inflation expectations given at any time, there will be an incentive for the monetary authorities to expand output, even if the authorities have tried to commit themselves not to do so. Indeed, the problem of time inconsistency is often held to explain much of the inflationary bias in recent decades. A large proportion of the theoretical literature on monetary policy and central banking has been taken up with elaborating this analysis, e.g. Cukierman 1992, and with trying to suggest partial remedies, ranging through independent central banks (Fischer, 1994), to the appointment of conservative central bankers (Rogoff, 1985), to specially designed contracts for central bankers (Walsh, 1995; Persson and Tabellini 1993).

Yet these two strands are mutually inconsistent. If monetary instruments operate with a lag, then a rational public will observe them, (they can hardly fail to notice interest rate changes), and adjust their expectations accordingly. Hence the public cannot be fooled, and the time inconsistency problem argument vanishes.

An obvious response to this is that certain nominal wage/price contracts may be fixed for some considerable length of time, e.g. for a year in the case of wage contracts, so that by the time that monetary policy actions taken now become effective, some previous contracts (fixed before that action was decided) are still binding. If we assume the existence of over-lapping long-term fixed contracts, then the economy will exhibit persistence (Fischer, 1977; Taylor, 1980), and indeed it does.

As we shall demonstrate, in conditions without persistence, but with a lag in the effect of monetary policy actions, the choice of monetary policy actions become trivial. What becomes crucial is the manner in which persistence is modeled. If persistence is modeled solely in terms of an autocorrelated deviation of output from its equilibrium level, e.g. such that:

$$y_t - y_n = \alpha(\pi_t - E\pi_t) + \theta(y_{t-1} - y_n) + u_t, \qquad (1.1)$$

where y and y_n are the logarithms of the real and equilibrium output respectively, π inflation, E the rational expectation operator at time t, α a positive coefficient, θ a positive coefficient between zero and unity, and u_t a random shock with the standard assumption of its mean and covariance being zero and variance equal to σ_u^2, then the discretionarily chosen monetary policy will be time-varying and fairly complex to estimate, but it will not have an inflationary bias.

It is only if some wages/prices have been set prior to the transparently observed monetary action, and remain in place after that monetary action starts to bite, that any inflationary bias can exist. Given the long lags of monetary policy, we question how important such extended wage contracts actually are. But without them, the whole basis of time inconsistency, and with it much of the superstructure of arrangements for central bank independence, etc., either disintegrates or needs to be radically recast. Nevertheless, in much of the earlier literature in this genre, monetary policy was deemed to be effective in part because of the comparative rigidity of wage/price contracts (relative to the speed of operation of monetary policy). With the growing popularity of the Lucas supply function, incorporating full wage/price flexibility, this rationale for time inconsistency was, however, implicitly abandoned. We suggest that it needs reviving.

We can model the putative existence of such long contracts by having a proportion, w, of the population fix their contracts for a period longer than the lag between monetary policy action being observed and taking effect. The reminder, (1 - w), can observe instrument changes in monetary policy before they come into operation and so factor these into their wage/price decision. We shall assume, for the purpose of modelling, that monetary policy has a one period lag and that the proportion, w, fixing their wages, do so for two periods, with w/2 of these population fixing in each period. In this case, only half of the contracts from this proportion of population are subject to the surprising effect of monetary policy. That is, w/2 portion of workers fix their nominal wage price at a time, t-1, prior to the monetary action in that same period, but which contract is still in effect at time t, when monetary policy bites. The rest of population consists of two parts, w/2 renegotiate their contracts at t after observing the monetary action at time t-1, and (1-w) sign their contracts each period which contracts are shorter than the length of the monetary policy lag. Then,

$$y_t - y_n = (1 - w/2)\alpha(\pi_t - E\pi_t) + (w\alpha/2)(\pi_t - E_{t-1}\pi_t) + \theta(y_{t-1} - y_n) + u_t. \quad (1.2)$$

Because monetary policy at time t only becomes effective at time t+1, the authorities cannot use monetary policy now to surprise those who formed their contract at t-1 since these latter will renegotiate their contract next period in full understanding of the lagged effect of today's policy. Putting the same point more formally, $\partial[(1 - w/2)\alpha(\pi_t - E\pi_t)]/\partial i_{t-1} = 0$, which is demonstrated in section IV.3, although it is intuitively obvious.

The importance of time inconsistency as a possible *raison d'etre* for an inflationary bias must depend, among other things, on the comparative length of the monetary policy lag effect and wage contracts. Yet relatively little empirical work has been done on this. Instead, in most theoretical analysis monetary policy has been assumed to work instantaneously, and after all wage contracts have already been set. This is clearly incorrect. Given the well attested long lags of monetary policy, we doubt whether time inconsistency will have been a prime cause of recent inflation in developed countries. Indeed, as Alesina (1989) has shown, there is little, or no, evidence of the existence of political cycles that might be expected to accompany time inconsistency. Moreover, if time inconsistency should lead to endemic, severe inflation, the rational response of the private sector should be to shorten the length of their wage contracts, which, given a fixed monetary lag length, would remove the source of the inflationary bias.

There are, of course, other possible reasons for an inflationary bias to monetary policy, besides the attempt to raise real output above its equilibrium level. These reasons could be from either revenue motive (Alesina and Tabellini, 1987; Cukierman, 1992, Chapter 4), or mercantilist or balance of payment motive (Alesina and Tabellini, 1987; Cukierman, 1992, Chapter 5), or a financial stability motive (Cukierman, 1992, Chapter 7). We would argue that lags in the operation of monetary policy would also blunt the revenue motive. The revenue obtained by the government from inflation compresses the extra seigniorage, plus the unexpected loss suffered by bond-holders from inflation, and hence nominal interest rates being higher than expected. In most western countries, with low monetary base/output and high debt/output ratios, the latter component of an inflation tax is likely to be much higher than the former. In so far as the private sector can observe monetary policy changes before they impact on output and prices, because of lags, then both nominal and real interest rate should rise in the intermediate period before inflation adjusts. During this elapse of time between the monetary expression and its effect on inflation, (depending on whether this is perceived by the private sector and the balance between the currency output and debt output ratios), public sector finances would probably worsen. The longer the lag, and the more transparent the ultimate effect on inflation, the greater such worsening would be.

The longer lag in the effect of monetary policy on output and inflation would not stop the exchange rate from starting to depreciate from the outset, (though there should be no change in its initial level since the rise in the domestic interest rates should offset the worsening in relative future inflation). Although with longer lags, the initial rate of depreciation should be somewhat less, we accept that competitive devaluation could remain a motive for monetary expansion and inflation, even when the private sector can observe such policy changes before they take effect within the domestic economy. Although many observers do remain concerned about the likelihood of such behavior, we remain doubtful, on empirical grounds, whether this syndrome has been responsible for world-wide inflationary pressures. The three major world economic powers are the USA, Germany and Japan. Apart, perhaps, from the USA in the Nixon era, have these countries been motivated by the desire to achieve a competitive devaluation?

As for the financial stability motive we hope to discuss this in a forthcoming paper on interest rate smoothing. We rather doubt whether this is one of the main reasons either for interest rate smoothing, or for systematic inflation, but we would prefer to deal it in our own future work. In such future work we hope to comment on other causes of inflationary bias, which we shall show to be positively related to the monetary policy lag, (not negatively related as in the case of time inconsistency).

In this paper, however, we shall stick with the standard game theoretic models of the interaction between the monetary authorities and the private sector, which have been such a fertile and fruitful field in recent years.

Besides the key problem of lags in the operation of monetary policy, there remain a number of other respects in which the current generation of models outlining this game are implausible and unrealistic. To some extent such shortcomings have been inevitable given the need to keep such models relatively simple and mathematically tractable. Nevertheless, we believe that it should be possible to align several features of the structure of such models more closely to reality without making the whole exercise impossibly complex.

We have three main concerns about the present structure of these models. First, the utility functions of both the monetary authorities and the private sector are not motivated with sufficient realism. Second, the structure of the models, with Lucas-type supply functions, perfectly clearing markets and stochastic shocks, means that in most cases each game lasts for one-period only. While the games can be turned into multi-period exercises, for example by the introduction of asymmetric information, learning and credibility, (Cukierman, 1992), it would seem more sensible to incorporate the persistence, auto-correlation, which is observed in fluctuations of output and employment around their trend levels, directly into the model. Third, and most important, virtually all these models assume that monetary policy measures affect the economy instantaneously, albeit sometimes with imperfect precision. This abstracts entirely from a major, probably the main, problem that the monetary authorities face in reality. This is that there are long and variable lags between adjusting the instrument of monetary policy, in practice short-term interest rates (<u>not</u> the monetary base), and that having its full effect on the economy. Consequently the monetary authorities have to predict the likely future state of the economy, and to do so in a world which, because of persistence, cannot be expected to have real variables in equilibrium subject only to some purely stochastic shock.

So, the purpose of this paper is to provide a more realistic motivational basis for the actions of the main players, i.e., the monetary authorities and the private sector, and to incorporate both persistence and operational lags into the structure of the game, while remaining within the context of the standard game-theoretic analysis.

First, let us reconsider the roles, and utility functions of the two main players, the private sector and the monetary authorities. The private sector's sole function in most of the current games in the literature is to select a level of nominal wages and prices, conditioned on its expectations of future inflation, which in turn depend on its perceptions of what the monetary authority will subsequently want to do. In so far as this sector's utility function is represented at all, it is usually only a (symmetric) function of the deviation of actual from expected inflation. Usually (though not in all such exercises) the private sector is assumed not to care about unemployment, but, even when it does do so, such concern is immaterial for the game, because the private sector's only role lies in setting nominal contracts, prior to the monetary authority's determination of its monetary instrument.

In contrast to the private sector which usually either does not care about, or cannot influence, the level of employment, the monetary authority is supposed to wish to achieve a level of employment above the equilibrium natural rate. But since the private sector either does not care about unemployment, or cannot punish the authorities for high unemployment, the motivation for this preference is rarely clear. In most cases, the "critical ingredient responsible for the inflationary bias result is the notion that due to the existence of distortionary labor taxes, the natural level of employment... is lower than the desired rate", (Cukierman, 1992, p. 35). The artificiality of this argument is evidenced by the paucity of studies trying to give an empirical basis to a relationship between the inflationary bias and the extent of such tax-produced distortion. Moreover, Cukierman argues that "a competitive equilibrium view of the labor market does not deliver an inflationary bias even in the presence of distortionary labor taxes and nominal contracts", p. 38, and retreats to a more complicated labour market involving both unionized and competitive sectors. Even here, employment is only a decreasing function of the real wage rate under some quite complex conditions, which Cukierman assumes rather than tests. Thus the 'social welfare' basis for the inflationary bias remains somewhat tenuous and obscure.[1] In much of the literature it gets

[1] Other economists, e.g. Tabellini (1988) and Persson and Tabellini (1990), however, have argued that the structure of the labor market, e.g. with a monopolistic trade union and intensive insiders power, can provide a sufficient basis for this argument.

slipped in early as an assumption, which serves to allow the modelling exercise to proceed, without much, or any, justification, certainly not empirical justification.

In the more descriptive accounts of the basis for an inflationary bias and for time inconsistency, if not in most of the formal models, the reason for that bias is clearly stated as being to please the electorate, the 'feel-good' factor, in the short-run. Time inconsistency is thus related to the political business cycle, though even here empirical evidence for the actual existence of this latter (Alesina, 1989) is much more sparse than is widely believed. But if politicians want to expand employment to become more popular among voters, then a desire for employment above the natural rate must enter into the utility function of the private sector.

There are, indeed, several reasons why this might be so, e.g. connected with the fact that most agents cannot choose their optimal amount of work effort along a continuously differentiable function. Moreover, a decrease in unemployment is consistently treated as 'good news'. It is, we assert, a stylized fact of modern economies that less unemployment is always preferred to more. Thus, we express the private sector's (P) utility function as:

$$UP_t = -\phi(y_t - y^*)^2 - (\pi_t - \pi^*)^2, \qquad (1.3)$$

where y^* is maximum attainable output in logarithm, (and can also be expressed as $y^* = ky_n$, where k is a positive, possibly constant, coefficient greater than unity), and the second term represents the standard formulation whereby the costs of inflation are represented by the deviation of inflation, symmetrically given the quadratic form, from the desired level, π^*, which may be zero[2].

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[2] An alternative, and possibly preferable, way of setting up this utility function would be to express the costs of inflation in two parts. The first would relate to the deviation of

The private sector's level of utility is, however, irrelevant to the monetary authorities in most of the social welfare explanations of the inflationary bias, since the private sector's only function in such models is to set nominal wages and prices. In the more realistic political models, however, the private sector's utility does enter the utility function of the political authorities, since at the time of elections the politicians will be removed from power if the electorate becomes unhappy.

Thus, if the monetary authorities are dominated by the decisions of the political authorities, Central Bank subservience, then their utility function will depend on the satisfaction rating which they are receiving from the private sector. Hence the monetary authorities' utility function will respond to, and in a sense incorporate, the utility function of the private sector (Goodhart, 1994). This does not, of course, prevent the monetary authorities from placing a differing weighting on employment and inflation depending on differences in position, information and tastes. Hence we may write for the monetary authorities (M), a utility function:

$$UM_t = -\gamma\phi(y_t - y^*)^2 - \xi(\pi_t - \pi^*)^2 - 2\psi\pi_t, \tag{1.4}$$

where γ and ξ are positive coefficients, which may be greater, or less, than unity, depending on the differences between the politicians and the private sector's objective weightings. γ and ξ may be time varying, e.g. over the electoral cycle, as a result of political pressure from the private sector. The additional term, $\psi\pi_t$, is entered to account for the possibility, explored by

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actual from expected, rather than desired, inflation, and would relate to unanticipated inflation. The second term, perhaps linear in inflation, could then account for the costs of anticipated inflation. We did not attempt this in this exercise, because it would both have involved extra complications and taken our formulation of the model further from the mainstream of the existing literature in this field. We leave this for future work.

Walsh (1995) and Persson and Tabellini (1993), of penalising the monetary authorities for inflationary outcomes, in such a way as to counteract their inflationary bias. Again ψ is a positive coefficient, whose size depends on the incentive structure imposed on the authorities.

Granting the Central Bank independence, with a mandate to achieve price stability, appointing a conservative Central Bank governor (Rogoff), and/or penalising the monetary authorities for inflationary outcomes through an incentive contract (Walsh, 1995; Persson and Tabellini, 1993) are institutional ways to lower γ towards zero and to raise ξ and ψ. Even with full independence, however, γ is never likely to reach zero. There remain several routes through which the feeling of the general public will influence Central Bankers. At the minimum no one likes to be criticized and reviled, and such criticism is likely to lead to some reconsideration. Beyond that, no institutional change can ever be irrevocable, however much that may be denied. What one generation has established and promulgated, a succeeding generation can reverse and undo. The Central Bank has no more divisions than the Pope. The Central Bank's position and influence ultimately depends on persuading those who have delegated power to it, (in a democracy that is the public at large), that it is undertaking its delegated responsibilities in a satisfactory manner. However independent the Central Bank may be through Act of Government, Treaty, Statute, or whatever, it cannot ignore its various constituencies, e.g. the public at large, the financial sector, the political parties, etc. So, constitutional changes to the status of the Central Bank may well, and are so intended to, change the coefficients in the monetary authority's utility function, but do not, in our view, change its functional form.

Having thus derived the functional form of the utility functions for the public, and the private, sectors we then imbed this, in Section II, into a model in which the other elements, a Lucas supply function, and a function instantaneously linking the expected rate of inflation to the expected setting of the monetary instrument, so that the short run Phillips curve becomes:

$$y_t - y_n = \alpha(\pi_t - E\pi_t) + u_t. \tag{1.5}$$

Substituting this into the monetary authorities' utility function gives:

$$UM_t = - \gamma\phi[y_n + \alpha(E\pi_t - \pi_t) + u_t - y^*]^2 - \xi(\pi_t - \pi^*)^2 - 2\psi\pi_t. \tag{1.6}$$

This formulation is very close to the standard format of the Barro-Gordon, or Cukierman, game structure, with the addition of a Walsh-type inflation penalty imposed on the authorities. As shown in Section II, without persistence or lags, the basic results of this game include an inflationary bias, unless the weight ψ on the Central Bank's disutility from inflation is raised sufficiently to offset the inflationary bias. This latter is the key finding of the recent work by Walsh and by Persson and Tabellini. In this respect our results remain identical to those of recent work in this field. However, the basic causation of the inflationary bias here is totally different. It arises not because the authorities want to fool the people for their own good into working harder, but because the general public wants a boom, an expanding economy, and the politicians do not have the strength to resist what the public wants. As we note further in Section II, this throws a somewhat different light on the contra-inflationary success of the Bundesbank, and implies that constitutional change towards Central Bank independence may be more problematical than is sometimes suggested by the more standard exercises.

In Section II we also present several other interesting findings. If the Central Banker is subject to an optimal compensation contract, then it should be no more or less conservative than the private sector and more importantly, although its utility increases proportionally with the relative weight the private sector places on employment, as a result of political pressure, this political pressure factor is actually irrelevant to its decision on monetary policy. On the other hand, without an optimal compensation contract, the Central Banker should always be more conservative than the private sector, regardless of whether the inflationary penalty on the Central Banker is too strong or too weak. In this case the divergence between the private sector's and the Central Banker's objectives increases if the private sector cares deeply about unemployment, and/or puts more political pressure on the Central Banker to counter unemployment, or if monetary policy has more effect on unemployment, or if the output/employment and inflationary targets are very high.

With a Lucas supply-side function, the real economy reverts to its equilibrium level of output each period, unless disturbed by an unpredictable stochastic shock. This condition means that the selection of the monetary instrument in each period will not influence real output in subsequent periods, (but may, under conditions of asymmetric information and learning, influence the private sector's inflation expectations in future periods). This functional form strikes us as far too restrictive and highly unrealistic as a representation of the practical problems facing the monetary authorities.

So in Section III we build persistence into the model by extending the Lucas supply function in a simple way, as follows:

$$y_t - y_n = \alpha(\pi_t - E\pi_t) + \theta(y_{t-1} - y_n) + u_t, \tag{1.7}$$

(we assume $y_n = y_{n\ t-1} = y_{n\ t-i}$). Depending on the value of θ, the expected change in y is no longer constant, even when there are no inflationary surprises. With constant velocity, and assuming for simplicity that the demand for money is homogeneous of degree 1 in both prices and real output, there would need to be a non-zero anticipated change in the money supply to accompany the non-zero anticipated change in real income, consistently with no inflationary surprises. Hence the anticipated change in money supply, (obtained by setting an anticipated level of interest rates), must equal the sum of the anticipated changes in output and prices, or:

$$E\Delta m_t = E\pi_t + E\Delta y_t, \tag{1.8}$$

where m is the logarithm of money supply.

This, of course, considerably complicates both the task of the monetary authorities in trying to optimize its objective function over time, and for the private sector in trying to learn what the monetary authorities are seeking to achieve and thereby to come to a rational expectation of inflation in the current period. We explore these complications in Section III.

The optimal discretionary monetary policy is solved from a dynamic programming model with one control variable, the monetary instrument i_t, and one state variable, the output/employment y_{t-1}. In equilibrium the monetary instrument moves in the opposite direction to the state variable because there exists an optimal feedback rule for i_t on y_{t-1} which ensures that the system is stable. So, when persistence allows above average employment to continue, the authorities will place more weight on holding inflation down, and vice versa.

A second interesting finding in Section III is that the inflationary bias cannot easily be eliminated by offering a Walsh-type (linear and non-state contingent) penalty contract since such a contract can only eliminate the non-state-dependent part of inflationary bias. Another result in this section is that the speed of convergence for the monetary instrument, output, and inflation all critically depend only on the strength of persistence. The weaker the persistence, the greater the speed of convergence.[3]

Besides the assumed absence of persistence, the second serious implausible simplification incorporated in most of these models is that adjustments to the monetary instrument affect the economy instantaneously, though not always with precision. This is clearly not so, since the existence of long and variable lags is again a stylized fact. We can embody this 'fact' into our model by building in a one period lag between the change in the monetary instrument and its effect on inflation (and output).

As it happens, this actually in some ways simplifies the version of the model without persistence. It is no longer possible for the authorities to offset concurrent supply shocks, since these transient shocks are over before the monetary authorities can cause any stabilising reaction in the economy. The advantages of a constant behaviour pattern, which is endogenously derived from an optimal discretionary policy but looks identical as the Friedman's K percent rule, are demonstrated. In so far as expectations about inflation react to the immediate monetary change rather than to the subsequent change in inflation, then monetary expansion now will raise inflation expectations before it can bring about any surprise increase in output. Hence the inflationary bias is reduced, or even eliminated, depending on how the private sector is perceived as adjusting its expectations. This is set out in Section IV.1.

Thus, if you accept lags in the monetary transmission process, then you really have to model an economy with persistence, in order to explain an active policy. The key results of this exercise, with auto-correlation in output, but not at this stage with fixed wage contracts, is reported in Section IV.2. They are: First, in equilibrium there is a state-contingent monetary policy but a constant expected inflation. The constant expected inflation is at its optimal level if a

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[3] Our analysis in Section III is in generally close to Svensson (1995) but with some notable differences. See Section III on these differences.

Walsh-type contract is not implemented. This is simply because a Walsh-type contract creates either a deflationary bias if there is an inflationary penalty or an inflationary bias if there is an inflationary bonus. Since both persistence and monetary policy lags exist in the real world, our result may shed some light why we have not observed a Walsh contract in practice. Second, the instrument needs to be set according to the coefficient of persistence and the gap between the realized and the equilibrium output. Hence, the deviation of monetary instrument from that which would achieve long term price stability can be positive or negative. The need for a time varying monetary policy is caused here by persistence alone, because the lag effect guarantees that there is no time inconsistency problem. Unlike in the case of persistence alone, with persistence and lag effects the money growth depends on the gap between the realized and equilibrium output, (rather than the realized output), and the speed of adjustment relies only on the value of persistence, θ, (rather than many other parameters in addition to θ). Third, such active policy, endogenously derived from the Central Bank's discretionary preferences, can ensure that the expected inflation rate is always π^*; hence, there is no need for any kind of commitment or implementation mechanism to achieve the optimal outcome. The private sector has no influence in the selection of monetary policy simply because discretionary monetary policy is always optimal; there is no need for the private sector to put extra political pressure or other means of influence on the Central Bank's selection of monetary policy.

Finally, when overlapping wage contracts are considered together with persistence and lag effects in Section IV.3, the time inconsistency problem returns and the monetary instrument, output and inflation all become state dependent. The money supply will always converge back to an equilibrium level, as the economy gradually absorbs a big shock due to the negative feedback rule. But in this new case, for a plausible proportion of sticky contract the monetary policy instrument is only about one fourth to, at most, two fifths as effective as in the case of persistence alone. Knowing this, the authorities may have a very small incentive on average towards inflation.

II. Optimal Policy without Persistence or Lags

II.1 The Model

Let us restate the basic model, already described:

Private sector's utility function,

$$UP_t = -\phi(y_t - y^*)^2 - (\pi_t - \pi^*)^2. \qquad (2.1)$$

Monetary authorities' utility function,

$$UM_t = -\gamma\phi(y_t - y^*)^2 - \xi(\pi_t - \pi^*)^2 - 2\psi\pi_t. \qquad (2.2)$$

Lucas supply function,

$$y_t - y_n = \alpha(\pi_t - E\pi_t) + u_t. \qquad (2.3)$$

Effect of instrument on inflation,

$$i_t - \pi_t = \delta_t, \qquad (2.4)$$

where $i_t = \Delta m_t$, and δ_t is assumed to be zero.[4] We could have included an

[4] The effect of the instrument on inflation in equation, (2.4), can be derived from the quantity equation (1.8). Without persistence $E\Delta y = 0$, hence, $i_t = \Delta m_t = \pi_t$. See Section III for more discussion.

error term, (lack of precision), in this equation, but we felt that this was unnecessary for the purposes of this exercise.

So, by substitution,

$$UM_t = -\gamma\phi[y_n + \alpha(i_t - Ei_t) + u_t - y^*]^2 - \xi(i_t - \pi^*)^2 - 2\psi i_t, \qquad (2.5)$$

which transforms into,

$$UM_t = -\alpha^2\gamma\phi[i_t - Ei_t - (y^* - y_n)/\alpha + u_t/\alpha]^2 - \xi(i_t - \pi^*)^2 - 2\psi i_t, \qquad (2.6)$$

Maximising utility with respect to the monetary instrument, i_t, gives,

$$-2\alpha^2\gamma\phi[i_t - Ei_t - (y^* - y_n)/\alpha + u_t/\alpha] - 2\xi(i_t - \pi^*) - 2\psi = 0. \qquad (2.7)$$

For this to be so,

$$(\alpha^2\gamma\phi + \xi)i_t = \xi\pi^* + \alpha^2\gamma\phi Ei_t + \alpha\gamma\phi(y^* - y_n - u_t) - \psi. \qquad (2.8)$$

Taking expectations on both sides leads to,

$$Ei_t = \pi^* + [\alpha\gamma\phi(y^* - y_n) - \psi]/\xi, \qquad (2.9)$$

$$\pi_t = i_t = \pi^* + [\alpha\gamma\phi(y^* - y_n) - \psi]/\xi - [\alpha\gamma\phi/(\alpha^2\gamma\phi + \xi)]u_t. \qquad (2.10)$$

The inflationary bias is higher the greater α, the effect on output of the surprise expansion; γ, the monetary authorities' dislike of unemployment; ϕ, the private sector's dislike of unemployment; and $y^* - y_n$, the size of the gap between the maximum possible and equilibrium output. The bias is lower, the greater is ξ, the monetary authorities' dislike of inflation, and ψ, the extra penalty imposed on them for allowing inflation. Clearly, by setting this penalty accurately, so that

$$\psi^* = \alpha\gamma\phi(y^* - y_n) = \alpha\gamma\phi(k - 1)y_n, \tag{2.11}$$

the inflationary bias could be eliminated, as in Walsh.[5] Thus, all that would seem necessary to eliminate inflation would be an institutional mechanism to give the monetary authorities, e.g. an independent central bank, the right incentive structure in their contract. While we do believe that establishing a good incentive structure is desirable, we shall try to show in the subsequent Sections why in reality things are not quite that simple.

The other point that we want to emphasize is that, once the private sector's utility is properly imbedded into that of the monetary authorities, then γ and ϕ always appear in joint form. It is impossible for any outsider to distinguish whether low inflation is due to the priority that the private sector gives to price stability (relative to unemployment), or to the relative preferences of the authorities, because the two are so closely interrelated.

II.2 Optimal Degree of Conservativeness

Because the private sector's utility function is an integral part of our model, wherein the private sector also cares about unemployment, we can use our formal model to assess its preference for the optimal degree of central bank conservativeness, i.e. the value of ξ that maximised its own utility. In this respect we can extend the model of Rogoff, in which the private sector does not

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[5] In a repeated game, as set out in Barro and Gordon (1983), the inflationary bias can also be eliminated if the private sector can employ strategies, e.g. trigger strategies, that penalize the authorities for any such bias.

have an explicit utility function in which concern for employment enters.

From equations (2.3) and (2.10), we have,

$$y_t = y_n + \frac{\xi}{\alpha^2\gamma\phi+\xi}u_t. \tag{2.12}$$

Substituting into equation (2.1) and manipulating leads to,

$$UP_t = -\phi[(k-1)y_n - \frac{\xi}{\alpha^2\gamma\phi+\xi}u_t]^2 - [\frac{\alpha\gamma\phi(k-1)y_n-\psi}{\xi} - \frac{\alpha\gamma\phi}{\alpha^2\gamma\phi+\xi}u_t]^2. \tag{2.13}$$

Taking expectations leads to,

$$EUP_t = -\phi[(k-1)^2y_n^2 + \frac{\xi^2}{(\alpha^2\gamma\phi+\xi)^2}\sigma_u^2] - [\frac{\alpha\gamma\phi(k-1)y_n-\psi}{\xi}]^2 - (\frac{\alpha\gamma\phi}{\alpha^2\gamma\phi+\xi})^2\sigma_u^2. \tag{2.14}$$

Then $\partial EPU_t/\partial\xi = 0$, if and only if,

$$\frac{\alpha^2\gamma\phi^2(\xi-\gamma)}{(\alpha^2\gamma\phi+\xi)^3}\sigma_u^2 = \frac{[\alpha\gamma\phi(k-1)y_n-\psi]^2}{\xi^3} \tag{2.15}$$

From this, the optimal level of ξ for the private sector can be obtained, which becomes,

$$\xi^* = \gamma + \frac{[\alpha\phi(k-1)y_n\gamma-\psi]^2}{\alpha^2\phi^2\sigma_u^2\gamma}(1 + \frac{\alpha^2\phi\gamma}{\xi^*})^3. \tag{2.16}$$

This somewhat messy equation can be depicted more easily in diagrammatic form in Figure 1 below:[6]

Figure 1 is about here

If $\psi = \psi^*$, then the second term in equation (2.16) becomes zero, and the optimal level of conservativeness for the private sector, $\xi^* = \gamma$, i.e. that the authorities additional weighting on unemployment (additional that is to the

[6] This graphic method is adopted from Eijffinger and Schaling (1995).

private sector's), should be exactly equal to their additional weighting on inflation. In other words, if the contract enables zero inflation on average to be delivered, the private sector would like their own balance of preferences between unemployment and inflation to be respected.

In this case, with $\xi^* = \gamma$ and $\psi = \psi^*$, the utility function of the monetary authorities becomes,

$$UM_t = -\gamma\phi(y_t - y^*)^2 - \gamma(\pi_t - \pi^*)^2 - 2\psi^*\pi_t = \gamma[UP_t - 2\alpha\phi(k - 1)y_n\pi_t]. \quad (2.17)$$

This result has two implications: First, the Central Banker, whose characteristics optimise the welfare of the private sector, should be no more or less conservative than the private sector if it is subject to an optimal compensation contract; and second and more important, although the Central Banker's utility increases proportionally with γ, the relative weight it places on employment, (perhaps as a result of political pressure from the private sector electorate), that is actually irrelevant to the decision of monetary policy.

For $\psi \neq \psi^*$, we have to revert to equation (2.16), which leads to the inequality,

$$\infty > \xi^* > \gamma + \frac{[\alpha\gamma\phi(k-1)y_n - \psi]^2}{\alpha^2\gamma\phi^2\sigma_u^2}. \quad (2.18)$$

Note that the second term on the right hand side is always positive for $\psi \neq \psi^*$, thus we will always want a (finitely) conservative Central Banker if the Central Banker does not have an optimal (linear) deflationary incentive. This is the same result as Rogoff (1985) obtains. The need for conservativeness is increased by a higher variance in supply shocks, σ^2_u.

As in Rogoff, the upper bound of the optimal degree of conservativeness is infinite, while unlike Rogoff our lower bound is a positive number defined in (2.18). Another difference from Rogoff is that when $\alpha\gamma\phi(y^* - y_n) > \psi$, i.e. the penalty disincentive on inflation for the Central Banker is too small, (probably the usual case), then ξ^* the optimal degree of conservativeness will rise with α, ϕ, and k, and fall with ψ, (the effect of γ is ambiguous). When the penalty disincentive is too great, the signs are reversed, but now ξ^* rises with γ.

If $\psi = 0$,

$$\xi^* = \gamma + \gamma \frac{(k-1)^2 y_n^2}{\sigma_u^2} (1 + \frac{\alpha^2 \gamma \phi}{\xi^*})^3. \tag{2.19}$$

As in Rogoff (1985), ξ^* increases when γ, α, ϕ, and k all increase but decreases with σ_u.

The implication of this exercise is that there is no preferred level of conservativeness, should we be able to choose an optimal disincentive, ψ, for Central Bankers. Otherwise, a conservative Central Banker will always be desired and the optimal degree of conservativeness critically depends on whether the Central Banker's penalty disincentive on inflation is too large or too small.

II.3 The Divergence Between UM_t and UP_t

In our model the private sector also cares about unemployment and it can influence the monetary authorities' policy conduct in three different ways, i.e. through political pressure, by selecting a conservative Central Banker and by offering the Central Banker an incentive contract. The private sector's utility function, although not identical to that of the monetary authorities, becomes an integral part of that of the monetary authorities. An interesting exercise is to examine the divergence between the two utility functions.

We can examine the divergence between UM_t and UP_t at $\xi = \xi^*$.

Substituting $\xi = \xi^*$ into the monetary authorities' utility function (2.2), we have

$$UM_t = -\gamma\phi(y_t - y^*)^2 - \{\gamma + \frac{[\alpha\gamma\phi(k-1)y_n - \psi]^2}{\alpha^2\gamma\phi^2\sigma_u^2}(1 + \frac{\alpha^2\gamma\phi^2}{\xi^*})^3\}(\pi_t - \pi^*)^2 - 2\psi\pi_t$$

$$= -\gamma[\phi(y_t - y^*)^2 - (\pi_t - \pi^*)^2] - \frac{[\alpha\gamma\phi(k-1)y_n - \psi]^2}{\alpha^2\gamma\phi^2\sigma_u^2}(1 + \frac{\alpha^2\gamma\phi^2}{\xi^*})^3(\pi_t - \pi^*)^2 - 2\psi\pi_t.$$

Hence, the divergence between UP_t and UM_t is:

$$UM_t - \gamma UP_t = - \frac{[\alpha\gamma\phi(k-1)y_n - \psi]^2}{\alpha^2\gamma\phi^2\sigma_u^2}(1 + \frac{\alpha^2\gamma\phi^2}{\xi^*})^3(\pi_t - \pi^*)^2 - 2\psi\pi_t. \quad (2.20)$$

The first case is $\psi = \psi^*$, hence, $\xi^* = \gamma$. The expected divergence between UP_t and UM_t in this case is:

$$E[UM_t - \gamma UP_t] = - 2\gamma\alpha\phi(k - 1)y_n\pi^*.$$

Obviously the absolute value of this expected divergence increases, if ϕ is very large, i.e. the private sector cares deeply about unemployment; if γ is very large, i.e. the private sector puts more political pressure on the Central Banker on unemployment; if α is very large, i.e. the monetary policy has more effect on unemployment; if k or y* is very large, i.e. the output/employment target is very high; and if π^* is very large, i.e. the inflation target is very high. The absolute value of this expected divergence is irrelevant to σ_u^2, the variance of the stochastic supply shock.

This condition, that $\psi = \psi^*$ and $\xi^* = \gamma$, is the first best possible for the private sector because it cannot improve on that institutional arrangement. Consequently, the expected divergence between the absolute value of the expected divergence reaches its unique minimum point at $\psi = \psi^*$ and $\xi = \xi^*$.

Intuitively this is because at $\psi = \psi^*$ and $\xi = \xi^*$, $|E[UM_t - \gamma UP_t]| = 2\gamma E\pi_t$; and $E\pi_t$ reaches its minimum value π^*, since at $\psi = \psi^*$ there is no inflationary bias.

The second case is $\psi = 0$ but $\xi = \xi^*$. The (expected) divergence between UP_t and UM_t in this case is:

$$UM_t - \gamma UP_t = - \alpha^2\gamma^3\phi^2(k-1)^2 y_n^2 \left(1 + \frac{\alpha^2\gamma\phi}{\xi^*}\right)^3 \left[\frac{(k-1)^2 y_n^2}{\xi^{*2}\sigma_u^2} + \frac{1}{(\alpha^2\gamma\phi+\xi^*)^2}\right].$$

The absolute value of this divergence increases, as before when ϕ, γ, α, k and y* are larger. But now the absolute value of this divergence decreases, if σ_u^2 is large, i.e. the variance of the stochastic supply shock is big, and it is not affected by π^*, the inflation target.[7]

Basically the expected divergence between UM_t and UP_t is caused by two sources. The first is that the authorities get an extra penalty, $\psi\pi$, from the Walsh-type contract even if $\psi = \psi^*$. The second is that the authorities get a (one period) benefit from either surprise in inflation if $\psi < \psi^*$, or surprise in deflation if $\psi > \psi^*$, while neither surprise helps the private sector. So those factors which raise the penalty on, and the benefit of, such surprise

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[7] In the general case of $\xi = \xi^*$ but $\psi \neq \psi^*$ and $\psi \neq 0$, the expected divergence between UP_t and UM_t is:

$$E[UM_t - \gamma UP_t] = - \frac{[\alpha\gamma\phi(k-1)y_n - \psi]^2}{\alpha^2\gamma\phi^2\sigma_u^2}\left(1 + \frac{\alpha^2\gamma\phi^2}{\xi^*}\right)^3(\pi_t - \pi^*)^2 - 2\psi E\pi_t$$

$$= - \frac{[\alpha\gamma\phi(k-1)y_n - \psi]^2}{\alpha^2\gamma\phi^2}\left(1 + \frac{\alpha^2\gamma\phi^2}{\xi^*}\right)^3\left[\frac{[\alpha\gamma\phi(k-1)^2 y_n^2 - \psi]^2}{\xi^{*2}\sigma_u^2} + \frac{\alpha^2\gamma^2\phi^2}{(\alpha^2\gamma\phi+\xi^*)^2}\right]$$

$$- 2\psi\left[\pi^* + \frac{\alpha\gamma\phi(k-1)y_n - \psi}{\xi^*}\right]$$

Once again this is increasing in ϕ, γ, α, k and y* and decreasing in σ_u^2. Unlike the case when $\psi = 0$, it is also increasing in π^*, i.e. when the inflation target is higher.

inflation/deflation, a higher ϕ, γ, α, k and y*, cause a bigger divergence. When $\psi \neq \psi^*$, the monetary authorities do not have an optimal incentive to stabilize supply shocks, so the divergence falls as σ_u^2 rises.

Note, however, that the divergence, $UM_t - \gamma UP_t$, is a function of institutional conditions, the level of ψ and ξ. If the divergence becomes too large, so that average expected inflation becomes too high, one would expect institutional changes, i.e. raising ψ and ξ towards ψ^* and ξ^*, that would restrain such behaviour. Put another way, if this time inconsistency model is correct, why have rational private agents not already insisted on institutional changes that would mitigate the problem? Or perhaps the current enthusiasm for independent central banks is an indication of such a response.

What we are doing here is to regard the institutional settings, notably ψ and ξ, as also partly endogenous within the model, rather than exogenously assumed.

II.4 The Optimal Strength of Political Pressure

Within the same framework we can examine what the private sector would consider to be the optimal degree of direct concern among the monetary authorities for employment, by maximising EUP_t with respect to γ, i.e., $\partial EUP_t / \partial\gamma = 0$. This requires

$$\frac{\alpha^2\phi^2\xi\sigma_u^2(\gamma - \xi)}{(\alpha^2\phi\gamma+\xi)^3} = \frac{\alpha\phi(k-1)y_n[\psi - \alpha\phi(k-1)y_n\gamma]}{\xi^2}. \tag{2.21}$$

If $\psi = \psi^*$, then $\gamma^* = \xi$, so γ^* increases if ξ increases. This implies that the private sector wants the authorities to have the same preference as they do, i.e. no more or less conservative than the private sector if it is subject to an optimal compensation contract. This result is not only consistent with the above result shown in (2.17) but also a symmetric reflection of conservativeness in the new contact of political pressure.

Otherwise when $\psi \neq \psi^*$, we have

$$\gamma^* = \xi + \frac{(k-1)y_n[\psi - \alpha\phi(k-1)y_n\gamma^*]}{\alpha\phi\sigma_u^2}(1 + \frac{\alpha^2\phi\gamma^*}{\xi})^3.$$

Hence, γ^* and ψ moving in the same direction. That is, $\gamma^* > \xi$ if and only if $\psi > \psi^*$ and vice versa.

Intuitively, this is because that the political pressure (to increase output/employment) coming out of the private sector is a counter-balance to the Central Banker's incentive for disinflation caused by the penalty contract, ψ. Since a high output/employment comes at the cost of high inflation according to the Lucas supply function, and the private sector cares about both output/employment and inflation, in equilibrium the optimal strength of political pressure, γ, should endogenously increase with the Central Banker's disincentive for inflation, ψ.

III. Optimal Policy with Persistence

Whereas the utility functions of the private sector and the monetary authorities remain the same as in Section II (defined as equations (2.1) and (2.2)), the economy is now different from that in Section II, due to persistence. The first difference is that the Lucas supply function now becomes:

$$y_t - y_n = \alpha(\pi_t - E\pi_t) + \theta(y_{t-1} - y_n) + u_t. \qquad (3.1)$$

And the second difference lies in the effect of the instrument on inflation. Without persistence, the effect of the instrument on inflation equation is simply $i_t - \pi_t = 0$, as we have derived in Section II and described by equation (2.4).

With persistence, however, a portfolio balance effect should be included in the demand for money. Note that the quantity equation, $MV = PY$, can also be expressed as $m + v = p + y$, if we take logarithm on both sides, (where, of course, m, v, p and y are respectively the logarithms of total demand for money, velocity, price and real output). With constant velocity, and assuming for simplicity that the demand for money is homogeneous of degree 1 in both prices and real output, we would need a non-zero anticipated change in the money supply to accompany the non-zero anticipated change in real income, consistently with no inflationary surprises. Hence the anticipated change in money supply, (obtained by setting a transparently observed level of interest rates), must equal the sum of the anticipated changes in output and prices. That is, $m_t - m_{t-1} = p_t - p_{t-i} + y_t - y_{t-1}$, or:

$$i_t = \pi_t + y_t - y_{t-1}. \tag{3.2}$$

Note that it is necessary to distinguish between π_t and i_t. π_t is the inflationary outcome. Although it can be a target, as treated in Svensson (1995), π_t cannot be a monetary policy instrument. i_t is the only instrument in this economy.[8] Without persistence such distinction is not mathematically obvious, simply because $i_t = \pi_t$. With persistence, however, such distinction becomes very important mathematically, since $i_t \neq \pi_t$.

From (3.2), we have $\pi_t = i_t - y_t + y_{t-1}$. Substituting it into (3.1) and after rearrangement, we obtain:

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[8] Our mathematical analysis in this section is close to Svensson (1995) due to the similarity of the problem. A notable difference between his analysis and ours is that we have considered the portfolio balance effect in our monetary effect equation, i.e. equation (3.2), hence, we use i_t as the monetary policy instrument. Svensson simply uses π_t as a policy instrument.

$$y_t = y_n + \alpha'(i_t - Ei_t) + \theta(y_{t-1} - y_n) + u_t/(1+\alpha). \tag{3.3}$$

where we denote $\alpha' = \alpha/(1+\alpha)$ for simplicity. And using this equation, we have

$$\pi_t = (i_t + \alpha Ei_t)/(1+\alpha) + (1 - \theta)(y_{t-1} - y_n) - u_t/(1+\alpha). \tag{3.4}$$

We know that the central bank's problem is:

$$\max_{i_t} E\, \Sigma_{t=1}^{t=\infty} \beta^t UM_t$$
subject to (3.3)

This is a dynamic programming problem with one state-variable, y_{t-1}, and one control variable, i_t. The solution can be obtained by solving the following problem:

$$V(y_{t-1}) = E\max_{i_t}\{-\gamma\phi(y_t-y^*)^2 - \xi(i_t-y_t+y_{t-1}-\pi^*)^2 - 2\psi(i_t-y_t+y_{t-1}) + \beta V(y_t)\}, \tag{3.5}$$

subject to (3.3). The first order condition of the discretionary solution is:

$$-2\alpha'\gamma\phi(y_t-y^*) - 2\xi(1-\alpha')(i_t-y_t+y_{t-1}-\pi^*) - 2\psi(1-\alpha') + \alpha'\beta V'(y_t) = 0,$$

or, $$\alpha\gamma\phi(y_t - y^*) + \xi(i_t - y_t + y_{t-1} - \pi^*) + \psi - 0.5\alpha\beta V'(y_t) = 0, \tag{3.6}$$

where $V'(y_t)$ is the first order derivative of $V(y_t)$ respective to y_t.

For the linear-quadratic problem as ours, $V(y_t)$ must also be quadratic. Without losing generality, we can write $V(y_t) = a_0 + a_1 y_t + 0.5a_2 y_t^2$, so that

$$V'(y_t) = a_1 + a_2 y_t. \qquad (3.7)$$

Substituting (3.7) and (3.3) into (3.6) and then taking expectations, we have

$$Ei_t = \pi^* + \{\alpha\gamma\phi y^* + 0.5\alpha\beta a_1 - \psi - (\alpha\gamma\phi - 0.5\alpha\beta a_2 - \xi)(1-\theta)y_n - [\xi + (\alpha\gamma\phi - 0.5\alpha\beta a_2 - \xi)\theta]y_{t-1}\}/\xi.$$

For short expression, we denote

$$Ei_t = \pi^* + b + cy_{t-1}, \qquad (3.8)$$

hence,

$$i_t = Ei_t + du_t = \pi^* + b + cy_{t-1} + du_t, \qquad (3.9)$$

with $b = [\alpha\gamma\phi y^* + 0.5\alpha\beta a_1 - \psi - (\alpha\gamma\phi - 0.5\alpha\beta a_2 - \xi)(1-\theta)y_n]/\xi,$ (3.10)

$$c = -1 - (\alpha\gamma\phi - 0.5\alpha\beta a_2 - \xi)\theta/\xi, \qquad (3.11)$$

and $d = -(\alpha\gamma\phi - 0.5\alpha\beta a_2 - \xi)/(\alpha^2\gamma\phi - 0.5\alpha^2\beta a_2 + \xi).$ (3.12)

The value of a_1 and a_2 can be solved by applying the envelope theorem to (3.5). Doing so gives us:

$$\begin{aligned} &V'(y_{t-1}) \\ &= a_1 + a_2 y_{t-1} \\ &= E\{[-2\gamma\phi(y_t - y^*) + 2\xi(i_t - y_t + y_{t-1} - \pi^*) \\ &\qquad + 2\psi + \beta(a_1 + a_2 y_t)](\theta - \alpha' c) - 2\xi(i_t - y_t + y_{t-1} - \pi^*) - 2\psi\}. \end{aligned}$$

Substituting y_t and regrouping gives us:

$$a_2 = \theta(\theta - \alpha' c)(-2\gamma\phi + \beta a_2) + 2\xi(\theta - \alpha' c - 1)(c - \theta + 1), \qquad (3.13)$$

$$a_1 = [(\theta-\alpha'c)(-2\gamma\phi+\beta a_2-2\xi)+2\xi](1-\theta)y_n$$
$$+(\theta-\alpha'c)(2\gamma\phi y^*+\beta a_1+2\xi b+2\psi)-2\xi b-2\psi. \tag{3.14}$$

a_2 and a_1 can be solved by substituting b and c into (3.13) and (3.14). The solution is quite complicated. A simpler solution is to substitute a_2 in (3.13) by its expression in c, then solve for c. Doing so gives us:

$$Ac^2 + Bc + \Gamma = 0, \tag{3.15}$$

where $A = \alpha\beta\theta\xi$,

$B = \xi[1 - \beta\theta^2(1 + 2\alpha)]$,

and $\Gamma = \xi + \{-\xi + \alpha\gamma\phi - \beta(1-\theta)[\alpha + (1 + \alpha)\theta]\}\theta$.

Note (3.15) has the feature of $c \rightarrow -1$ for $\theta \rightarrow 0$. Obviously, if $B^2 - 4A\Gamma > 0$, there are two real roots for c in (3.15). The only meaningful solution is:

$$c = \frac{-B + \sqrt{B^2 - 4A\Gamma}}{2A} < 0, \tag{3.16}$$

because only this root has the feature of $c \rightarrow -1$ for $\theta \rightarrow 0$ while the other root, with $c \rightarrow -\infty$ for $\theta \rightarrow 0$, violates this feature implied by (3.15).

Obviously after obtaining c, we can substitute it into (3.11) and (3.12) respectively to find out a_2 and a_1.

The critical implication of this is that when the monetary authorities' utility is maximised with respect to the monetary instrument, i_t, then the state variable, i.e. the last period level of employment y_{t-1}, becomes a determinant of i_t. Note that the value of c is always negative, which implies that the monetary instrument, i_t, moves in the opposite direction to the state variable, y_{t-1}. Since equation (3.9) states the optimal feedback rule for i_t on y_{t-1}, a negative coefficient c implies that the system is stable; therefore, the money supply will always converge to an equilibrium level after the economy gradually absorbs a big shock. Intuitively, as persistence allows above average employment to continue, the authorities will place more weight on holding inflation down, and vice versa. Recall that in equation (2.10) above, without persistence, the sole current variable to which the authorities responded was u_t, the immediate shock. With persistence, the authorities have to take notice of whether the economy is currently above, or below, its equilibrium value. We would insist that any model without this latter feature is simply unrealistic.

A second implication is that one cannot easily set ψ at an optimal constant value ψ^* which will eliminate the inflationary bias. From (3.4),

$$\pi_t = (i_t + \alpha E i_t)/(1+\alpha) + (1-\theta)(y_{t-1} - y_n) - u_t/(1+\alpha)$$

$$= \pi^* + b' + c' y_{t-1} + d' u_t, \qquad (3.17)$$

with $b' = b - (1 - \theta) y_n, \qquad (3.18)$

$c' = c + (1 - \theta), \qquad (3.19)$

and $d' = (d - 1)/(1 + \alpha). \qquad (3.20)$

By setting

$$\psi = \psi^* = \alpha\gamma\phi y^* + 0.5\alpha\beta a_1 - (\alpha\gamma\phi - 0.5\alpha\beta a_2)(1 - \theta) y_n, \qquad (3.21)$$

only the non-state-dependent part of inflationary bias can be eliminated. Since the state variable y_{t-1} enters the π_t term in equation (3.4) above, the inflationary bias is a continuously shifting value of y_{t-1}. Unless the Central Banker is offered a state-contingent incentive contract and such a contract is implemented, there will always be a bias away from zero inflation. Theoretically, such a state-contingent contract does exist, such as shown in Svensson (1995), but the implementation of such a contract may not be practically feasible.

Let us assume that $y_t = \bar{y}_t < y_n$ is a starting point, we can then work out both the current and expected future trajectory of both $i_t \rightarrow i_{t+m}$ and $y_t \rightarrow y_{t+m}$, with $Eu_{t+m} = 0$. By substituting Ei_t, as defined in (3.9), into the Lucas equation (3.3), we have

$$y_t = (1 - \theta)y_n + \theta y_{t-1} + [(\alpha d + 1)/(1 + \alpha)]u_t. \tag{3.22}$$

Therefore,

$$y_{t+m} = \Sigma_{j=0}^{j=m-1}\theta^j(1 - \theta)y_n + \theta^m\bar{y}_t + \frac{\alpha d + 1}{1 + \alpha}\Sigma_{j=0}^{j=m-1}\theta^j u_{t+m-j}$$

$$= (1 - \theta^m)y_n + \theta^m\bar{y}_t + \frac{\alpha d + 1}{1 + \alpha}\Sigma_{j=0}^{j=m-1}\theta^j u_{t+m-j},$$

$$Ey_{t+m} = (1 - \theta^m)y_n + \theta^m\bar{y}_t. \tag{3.23}$$

It is clear that the speed of convergence for y_t is determined by the value of θ.

Let us assume that the authorities choose a constant optimal $\psi = \psi^*$, hence, $b' = 0$, and $\pi_{t+m} = \pi^* + c'y_{t+m-1} + d'u_t$, i.e.,

$$\pi_{t+m} = \pi^* + c'[(1 - \theta^{m-1})y_n + \theta^{m-1}\bar{y}_t + (\alpha d' + 1)\Sigma_{j=0}^{j=m-2}\theta^j u_{t+m-1-j}] + d'u_{t+m}.$$

Substituting $c' = c + (1 - \theta)$ leads to:

$$E\pi_{t+m} = \pi^* + (c + 1 - \theta)[(1 - \theta^{m-1})y_n + \theta^{m-1}\bar{y}_t]. \tag{3.24}$$

Hence the speed of convergence for π_t is not only determined by the value of θ but also by c, which depends on many parameters including α, β, γ, ϕ, θ and ξ. Similarly, the speed of convergence for $i_t = \pi_t + y_t - y_{t-1}$ is also determined by the value of θ and c. Consequently, the speed of convergence for output/employment, determined by θ alone, will be greater, the lower value θ; while the speed of convergence for monetary instrument and inflation depends on the value of many parameters in addition to that of θ.

IV. Inflationary bias

We introduced persistence first, in Section III, before taking account of the other stylized fact, that monetary policy only operates with a long and variable lag, for a good reason; this is that, as we shall demonstrate below, the introduction of lags without persistence means that the exercise is essentially trivial. In this section we further consider the policy lag effect. The critical finding from this exercise is that the time inconsistency problem largely disappears when there is such a lag. Remember that we are using, for the time being, a Lucas supply function, wherein all wages and prices are fully flexible. Hence any lag in monetary policy implies that wage contracts can be revised in the light of the previous monetary action. When there is a lag between the exercise of a monetary policy instrument and its effect on the economy, the private sector, by successfully observing the conduct of monetary policy before it forms its expectation of inflation, cannot be fooled any more. Although this is intuitively obvious, we believe that we are the first to attempt to model this idea formally. We start by focusing attention on the policy lag effect alone in the first sub-section; then we present a full analysis of both the lag effect and persistence.

IV.1 Optimal Monetary Policy with a Lag Effect but without Persistence

Without persistence, such operational lags make the whole exercise of monetary policy essentially trivial. Since the authorities cannot know the future value of shocks, $Eu_{t+1} = 0$, hence they cannot smooth them; nor can they fool the public by creating surprises; hence there is no inflationary bias. So the time inconsistency problem does not exist, and the authorities cannot do better than choosing a constant growth rate for money, a K percent rule. If there are costs to anticipated inflation, the authorities should choose K so as to make inflation, $\pi = 0$. This is the Friedman proposal, but note that in such circumstances the monetary authorities will always <u>want</u> to follow this policy. It will be a rule which is chosen by the discretionary choice of the authorities. No commitment mechanism of any kind would be needed.

This is set out more formally below:

Private sector's utility function,

$$UP_t = - \phi(y_t - y^*)^2 - (\pi_t - \pi^*)^2. \tag{4.1}$$

Monetary authorities' utility function,

$$UM_t = -\gamma\phi(y_t - y^*)^2 - \xi(\pi_t - \pi^*)^2 - 2\psi\pi_t. \tag{4.2}$$

Lucas supply function,

$$y_t - y_n = \alpha(\pi_t - E\pi_t) + u_t. \tag{4.3}$$

Effect of instrument on inflation,

$$i_{t-1} - \pi_t = 0. \tag{4.4}$$

By substitution,

$$UM_t = -\gamma\phi[y_n + \alpha(i_{t-1} - Ei_{t-1}) + u_t - y^*]^2 - \xi(i_{t-1} - \pi^*)^2 - 2\psi i_{t-1}, \tag{4.5}$$

Note that $Ei_{t-1} = i_{t-1}$ due to the lag effect. Therefore, $y_t - y_n = \alpha(i_{t-1} - Ei_{t-1}) + u_t = u_t$, and,

$$UM_t = -\gamma\phi(y_n + u_t - y^*)^2 - \xi(i_{t-1} - \pi^*)^2 - 2\psi i_{t-1}. \tag{4.6}$$

Maximising utility with respect to the monetary instrument, i_{t-1}, gives,[9]

$$-2\xi(i_{t-1} - \pi^*) - 2\psi = 0. \tag{4.7}$$

For this to be so,

$$i_{t-1} = \pi^* - \psi/\xi. \tag{4.8}$$

If $\psi^* = 0$, i.e., no Walsh-type contract is needed, there is no inflationary bias. This is owning to the lag effect, $Ei_{t-1} = i_{t-1}$, which further guarantees that there is no time inconsistency problem. If $\psi > \psi^* = 0$, although there is no time inconsistency problem, a deflationary bias caused by extra (dis)incentive from Walsh-type contract does exist, and it is equal to ψ/ξ. This deflationary bias is higher the greater the weight on Central Bank's penalty on inflation, ψ, or the lower is the Central Bank's degree of conservativeness, ξ. The other parameters, i.e., α, the effect on output of the surprise expansion; γ, the monetary authorities' dislike of unemployment; ϕ, the private sector's dislike of unemployment; and $y^* - y_n$, the size of the gap between the maximum possible and equilibrium output, all have no effect on the deflationary bias.

[9] This calculation is based on the assumption that the Central Banker is forward looking, which we think is plausible. Otherwise, a myopic Central Banker caring only about the current period utility could choose any arbitrary monetary policy because the monetary policy has no effect on his current period utility due to the lag effect.

With no incentive contract, the optimal money supply is always π^*. It is not time dependent unless π^* becomes a time-dependent variable. Here, although this looks like a rule, it is really endogenously derived from the Central Banker's discretionary policy. So the discretionary optimal policy is identical to the Friedman proposal -- a monetary policy rule.

Moreover under such circumstances the private sector has no influence in the selection of monetary policy. Indeed, since discretionary monetary policy is always optimal, there is no need for the private sector to put extra political pressure or other means of influence on the Central Bank's selection of monetary policy.

The optimal monetary policy in this case --with a lag but without persistence-- is a discretionary choice of a Friedman-type constant K percent rule. When persistence is considered, as we show in the next section, optimal monetary policy becomes active.

IV.2 Optimal Monetary Policy with a Lag Effect and Persistence

With persistence, however, lags make the exercise of monetary policy more complicated.[10] As in the case of no persistence, the authorities can neither know the future value of shocks, $Eu_{t+1} = 0$, (hence they cannot smooth them), nor can they (usually) fool the public by creating surprises; hence there is no inflationary bias, and the time inconsistency problem does not exist, unless wage/price contracts are longer than the monetary policy lag. But, unlike the case of no persistence, now optimal discretionary policy depends on the state

[10] Our model here treats the dynamics of both lags of monetary policy and persistence as independently determined, although in principle we agree that both are probably reflections of a more general propagation mechanism that converts economic disturbances into highly serially correlated movements in many real variables, eg output and unemployment. Our excuse for retaining our present approach is that we would prefer, at this stage, to take one step at a time.

variable y_{t-1}, because of persistence. Therefore the authorities should choose a time dependent growth rate for money rather than a fixed K percent rule.

This is set out more formally below:

Lucas supply function,

$$y_t - y_n = \alpha(\pi_t - E\pi_t) + \theta(y_{t-1} - y_n) + u_t. \tag{4.9}$$

With persistence and a lag, the portfolio balance effect has to be considered. From the quantity equation, by doing the same exercise as in last section, we have

$$\dot{i}_{t-1} = \pi_t + y_t - y_{t-1}. \tag{4.10}$$

From (4.10), we have $\pi_t = \dot{i}_{t-1} - y_t + y_{t-1}$. Substituting it into (4.9), noticing $\dot{i}_{t-1} = E\dot{i}_{t-1}$ and $y_{t-1} = Ey_{t-1}$, and after rearrangement, we obtain:

$$y_t = y_n + \theta(y_{t-1} - y_n) + u_t/(1+\alpha), \tag{4.11}$$

hence $$\pi_t = \dot{i}_{t-1} + (1 - \theta)(y_{t-1} - y_n) - u_t/(1+\alpha). \tag{4.12}$$

As in last section, the central bank's problem is:

$$\max_{\dot{i}_{t-1}} E \sum_{t=1}^{t=\infty} \beta^t UM_t$$
subject to (4.11)

This looks like a dynamic programming problem with one state-variable, y_{t-1}, and one control variable, $\dot{i}_{t-1}$. But actually it is not. Because y_t, (equal to $y_n + \theta(y_{t-1} - y_n) + u_t/(1+\alpha)$ due to the lag effect), is not a function of $\dot{i}_{t-1}$, $\dot{i}_{t-1}$ cannot be used to smooth y_t over a number of periods. So the problem boils down to a one period issue. This point can be seen clearly from the following exercise.

Let us assume that the above problem is a dynamic programming problem. Then its solution comes from solving the following problem:

$$V(y_{t-1}) = E\max_{i_{t-1}}\{-\gamma\phi(y_t - y^*)^2 - \xi(i_{t-1} - y_t + y_{t-1} - \pi^*)^2 - 2\psi(i_{t-1} - y_t + y_{t-1}) + \beta V(y_t)\}, \quad (4.13)$$

subject to (4.11). The first order condition of the discretionary solution is simply:

$$-2\xi(i_{t-1} - y_t + y_{t-1} - \pi^*) - 2\psi = 0,$$

because the partial derivation of the first item, $-\gamma\phi(y_t - y^*)^2$, and the last item, $\beta V(y_t)$, in (4.13) with respect to i_{t-1} are both zero due to the fact that y_t is not a function of i_{t-1}.

For the above first order condition to be so, $i_{t-1} = \pi^* + y_t - y_{t-1} - \psi/\xi$. Because the monetary authorities at period t-1 have no advance information on the shock in the following period t, u_t, they can only use the expected value of y_t, Ey_t, in their policy instrument i_{t-1}:

$$i_{t-1} = \pi^* + Ey_t - y_{t-1} - \psi/\xi = \pi^* - (1 - \theta)(y_{t-1} - y_n) - \psi/\xi. \quad (4.14)$$

From (4.12), it is obvious that:

$$\pi_t = i_{t-1} + (1 - \theta)(y_{t-1} - y_n) - u_t/(1+\alpha) = \pi^* - \psi/\xi - u_t/(1+\alpha). \quad (4.15)$$

If $\psi = 0$, i.e., no Walsh-type contract, the instrument needs to be set at a level different from that consistent with long term price stability. Depending on the coefficient of persistence, θ, and the gap between the realized and the equilibrium output, $y_{t-1} - y_n$, this deviation can be positive or negative. The need for a time varying monetary policy is caused by persistence alone, because the lag effect, $Ei_{t-1} = i_{t-1}$, guarantees that there is no time inconsistency problem. But unlike in the case of persistence only, where i_{t-1} is a decreasing function of y_{t-1} and its speed of adjustment is determined by the value of c, here i_{t-1} is a decreasing function of $y_{t-1} - y_n$ and its speed of adjustment is only determined by $-(1-\theta)$. Obviously, other parameters, including α, β, γ, ϕ and ξ, which determine the value of c, all have no effect on the speed of adjustment of i_{t-1} now.

The variations over time in i will be larger, the larger the extra penalty imposed on the Central Bank for allowing inflation, ψ; the higher the gap between the realized and equilibrium output, $y_{t-1} - y_n$; the larger the coefficient of persistence, θ; and the lower is the Central Bank's degree of conservativeness, ξ. The other parameters, i.e., α, the effect on output of the surprise expansion; γ, the monetary authorities' dislike of unemployment; and ϕ, the private sector's dislike of unemployment; once again all have no effect on inflationary bias.

From equation (4.15), it is clear that $\pi_t = \pi^* - \psi/\xi - (\alpha'/\alpha)u_t$, hence there will be a deflationary bias if $\psi > 0$ or an inflationary bias if $\psi < 0$. Obviously, at $\psi = 0$, i.e. no Walsh contract, $E\pi_t = \pi^*$, there will be no inflationary/deflationary bias. Since both persistence and lag effect exists in the real world, and we have just shown that in a model with these characteristics a Walsh contract would do harm rather than good to an economy, this perhaps can explain why we have not observed a Walsh contract in practice.[11, 12]

[11] After his careful analysis of New Zealand's Reserve Bank Act of 1989, Walsh himself admits that this Act does not represent an optimal incentive contract (Walsh, 1994).

[12] This should not be read as a criticism of Walsh's work. Within a principal-agent context, the optimal incentive structure will be model dependent. Walsh's linear penalty was

Note that this optimal outcome, (as in the case of a lag effect without persistence analyzed in subsection IV.1), is also derived endogenously from discretionary policy. The monetary authorities again here always want to follow this optimal, (but time varying, in contrast to the fixed K percent), monetary policy and once again, no commitment of any kind would be needed. It is interesting to compare our result with that in Svensson (1995). In his analysis of persistence, the optimal outcome can only be obtained either through commitment (optimal rule) or by delegation with an optimal state-contingent target to a conservative central bank. Although the first solution is hard to implement indeed, the second is even more complicated and difficult to implement. After we consider a monetary policy lag effect, the optimal outcome can always be achieved from discretionary policy choice. There is no need for any complicated implementation mechanism in our case.

As in the case of no persistence, the private sector has no influence on the selection of monetary policy. This is because the private sector's influence is modeled as its political pressure for output/employment stability. When there is no time inconsistency problem, the private sector cannot be fooled by surprises in monetary policy created by the Central Banker. Hence, output and employment become independent of monetary policy. In other words, since discretionary monetary policy is always optimal, there is no need for the private sector to put extra political pressure or other means of influence on the Central Bank's selection of monetary policy.

In short, when monetary policy operates with a time lag (longer than that of wage/price contracts), the time inconsistency problem does not exist because the private sector cannot be fooled by the Central Banker. Regardless of whether, or not, there is persistence, as long as there is a lag effect, no

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indeed a solution to the main policy problem generated by the Barro-Gordon model, which should not necessarily be expected to hold under more general conditions.

incentive contract *a la* Walsh is needed and the expected inflation rate is always π^*. Depending on whether, or not, there is persistence, the Central Bank's optimal monetary policy should be conducted differently. When there is no persistence, the optimal monetary policy is a constant rate of growth in the money supply in every period. This passive, constant policy is endogenously derived from the Central Bank's discretionary preferences, although it looks identical to the Friedman's K percent rule. When there is persistence, the optimal monetary policy is a time varying rate of growth in the money supply in every period. This active policy endogenously derived from the Central Bank's discretionary preferences can ensure that the expected inflation rate is always π^*; hence, there is no need for any kind of commitment or implementation mechanism to achieve the optimal outcome.

IV.3 The Optimal Monetary Policy with the Lag Effect, Persistence and Overlapping Wage Contracts

With operational lags, persistence and overlapping wage contracts, the Lucas supply function becomes equation (1.2), restated as follows:

$$y_t - y_n = (1 - w/2)\alpha(\pi_t - E\pi_t) + (w\alpha/2)(\pi_t - E_{t-1}\pi_t) + \theta(y_{t-1} - y_n) + u_t, \quad (4.16)$$

From (4.10), we have $\pi_t = i_{t-1} - y_t + y_{t-1}$. Applying an expectation operator at t-j on both sides of it, we hence have:

$$E_{t-j}\pi_t = E_{t-j}i_{t-1} - E_{t-j}y_t + E_{t-j}y_{t-1}. \quad (j \geq 0). \qquad (4.17)$$

Substituting π_t and (4.17) into the Lucas function (4.16), we have:

$$y_t - y_n = (1-w/2)\alpha(-y_t+Ey_t) + (w\alpha/2)[i_{t-1}-y_t+y_{t-1}-E_{t-1}(i_{t-1}-y_t+y_{t-1})]$$
$$+ \theta(y_{t-1}-y_n) + u_t. \qquad (4.18)$$

From the above equation, it is obvious that the current monetary policy, due to its lag effect, has no effect to the group of workers who fix their contract at t. But those who fix their contracts at t-1 are stuck if actual prices at t deviate from their expected level $E_{t-1}\pi_t$, hence, the current monetary policy does have an effect on this group of workers.

Taking rational expectation on both sides of the above equation at t-1 and t, we have:

$$E_{t-1}y_t = \theta E_{t-1}y_{t-1} + (1 - \theta)y_n, \qquad (4.19)$$
$$Ey_t = [w\alpha/(2+w\alpha)](i_{t-1}-E_{t-1}i_{t-1}+y_{t-1}-E_{t-1}y_{t-1}) + [\theta/(2+w\alpha)](2y_{t-1}+w\alpha E_{t-1}y_{t-1})$$
$$+(1-\theta)y_n. \qquad (4.20)$$

Substituting these two expressions into (4.18), we get:

$$y_t - y_n = [w\alpha/(2+w\alpha)][(i_{t-1}-E_{t-1}i_{t-1}) +(1-\theta)(y_{t-1} -E_{t-1}y_{t-1})] +\theta(y_{t-1}-y_n) +u_t/(1+\alpha).$$

Note that at t-1, i_{t-2} which can change y_{t-1} is already set and observed, and i_{t-1} cannot change y_{t-1} due to lag effect, therefore $y_{t-1} - E_{t-1}y_{t-1} = \tau' u_{t-1}$, where τ' is a coefficient independent of i_{t-1}. Therefore, we have a new Lucas equation in this case:

$$y_t = y_n + \alpha''(i_{t-1}-E_{t-1}i_{t-1}) + \theta(y_{t-1}-y_n) + \tau_1 u_{t-1} + \tau u_t, \qquad (4.21)$$

where $\alpha'' = w\alpha/(2+w\alpha)$, $\tau_1 = (1-\theta)[w\alpha/(2+w\alpha)]\tau'$, and $\tau = 1/(1+\alpha)$ for simple notation.

From equation (4.21), it is clear that the time inconsistency problem returns once overlapping wage contracts are considered. Comparing (4.21) with (3.3), the Lucas equation when there is no lag effect or overlapping wage contracts, there are two differences. First, $\alpha/(1+\alpha) = \alpha' > \alpha'' = w\alpha/(2+w\alpha)$ for $\alpha > 0$. Hence, the monetary policy instrument now is only about α''/α', i.e. $(w+w\alpha)/(2+w\alpha)$, effective as before in influencing real output. This is no surprise, because in this new case monetary policy can only affect half of the workers who fix a contract longer than that of lag. For $w = 1/2$, the effectiveness factor $(w+w\alpha)/(2+w\alpha) = (1+\alpha)/(4+\alpha)$ has a value between 1/4 to, at most, 2/5. Second, $\tau = 1/(1+\alpha)$, so the coefficient of the shock in both cases is the same. And the coefficient of persistence in both cases, θ, is identical. Since the monetary instrument in this new case with overlapping wage contracts and lag is only at most 2/5th as effective while the effect of both persistence and shocks in two cases are the same, we should expect the equilibrium outcome for output in this new case to be less smooth and stable than before.[13] Furthermore, some tedious algebra can show that the inflationary bias, i.e. b in equation (3.10), is an increasing function of α for $y^* > y_n$, hence, with α decreasing to less than 2/5th of its original value in the case of persistence alone (i.e. without overlapping wage contracts), the inflationary bias in this new case should also decrease even if monetary policies are otherwise exactly the same in both cases.[14]

[13] Lags in monetary policy not only make time inconsistency less of a threat, but also make it much harder for monetary policy to have real effects in controlling aggregate demand, and could necessitate greater interest rate volatility (in an uncertain world) to achieve that end. We hope to show in a forthcoming paper why monetary policy has not, in practice, been varied sufficiently aggressively to stabilize nominal incomes and inflation.

[14] The above inflationary bias could be over stated because we have assumed that monetary policy has a one-period lag and wage contracts are fixed for two periods, while in general many believe that monetary policy has a lag of somewhat over one year, whereas wage contracts are normally fixed for less than two years.

By applying the same dynamic programming approach as in Section III (simply substituting α'' for α'), we have

$$i_{t-1} = \pi^* + b + cy_{t-2} + du_t, \tag{4.22}$$

with $b = [w\alpha\gamma\phi y^* + 0.5w\alpha\beta a_1 - 2\psi + (w\alpha\gamma\phi - 0.5w\alpha\beta a_2 - 2\xi)(1-\theta)y_n]/2\xi$, (4.23)

$$c = -1 - (w\alpha\gamma\phi - 0.5w\alpha\beta a_2 - 2\xi)\theta/2\xi, \tag{4.24}$$

and $d = -(w\alpha\gamma\phi - 0.5w\alpha\beta a_2 - 2\xi)/[(w\alpha)^2\gamma\phi - 0.5(w\alpha)^2\beta a_2 + 2\xi]$. (4.25)

Obviously, the value of a_1, a_2 and c can be solved by following the same procedure as in Section III, and the three critical implications we discovered before still hold. These implications are: the monetary instrument, i_{t-1}, moves in the opposite direction to the state variable, y_{t-2}, due to the negative feedback rule; hence, the money supply will always converge to an equilibrium level as the economy gradually absorbs a big shock; the inflationary bias is state contingent; as a result, one cannot easily set ψ at an optimal constant value ψ^* to eliminate the inflationary bias; and the speed of convergence for output or employment will be greater, the lower value θ, while the speed of convergence for monetary instrument and inflation depends on the value of both c and θ. A major difference between this case of persistence, lag <u>and</u> overlapping wage contracts, and the previous case of persistence alone is that the monetary policy instrument in this new case is, at most, only about two fifths effective as before.

What this provides is a picture of a rather <u>small</u> incentive on average towards inflation, combined with a time-varying incentive to shift policy in a deflationary or inflationary direction that depends on the state of the economy and on previous forecast errors. Frankly we regard this as much more realistic than the majority of current models, both in its assumptions and in its predictions.

V. Conclusions

Game theoretic models of time inconsistency have been so popular, because we have wanted to believe them, despite remarkably slight empirical support. Not only are such models elegant, but they pander to our engrained scepticism about politicians. They can explain why politicians' anti-inflationary rhetoric is so rarely matched with actual success in achieving price stability.

But these models have mostly been unrealistic in several respects. In the first part of this paper we considered two possible improvements to the model. The first involved providing a more plausible representation of the motivations of the two main players; the second, which has also been currently examined in a number of other recent contributions,[15] is to incorporate persistence into the model. Such adjustments improve, in our view, the plausibility of the model at the cost of some extra complexity. For example, we demonstrate that it is impossible to distinguish from observation of the data alone whether low inflation has occurred because of the relative priorities of the private sector, a low ϕ, or of the authorities, a low γ, since these always appear in combined form. Again we show that, with persistence, the optimal policy of the authorities will be state dependent, so that a fixed contract (ψ) for the Central Bank which is optimal <u>on average</u>, will at times lead to an inflation, or a deflation bias. Nevertheless these changes represent an amendment to, rather than a rejection of, the basic model.

The real fireworks come with our third innovation, incorporating the (generally accepted) lag of monetary policy into the model. But such a lag means that the policy will be transparently observed <u>before</u> it affects the economy. Consequently it cannot fool anybody who has not already bound

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[15] See Svensson (1995) for more references on analysis of persistence.

herself into a contract longer than that lag. Unless such contracts are pervasive, time inconsistency cannot occur. Even if such contracts are pervasive, the bias arising from time inconsistency must be a fraction of that previously assessed (on a no lag basis), since such a large proportion of such contracts will come up for renewal before the monetary policy action takes effect.

On these grounds we doubt that the standard model of time inconsistency provides an effective building block of a satisfactory theory to rationalize the inflationary proclivities of the industrialised world in the last half century. What we need is a model in which the lags of monetary policy have a positive effect on inflationary pressure, not a negative effect as in the time inconsistency model. We hope to turn to this exercise in our next paper.

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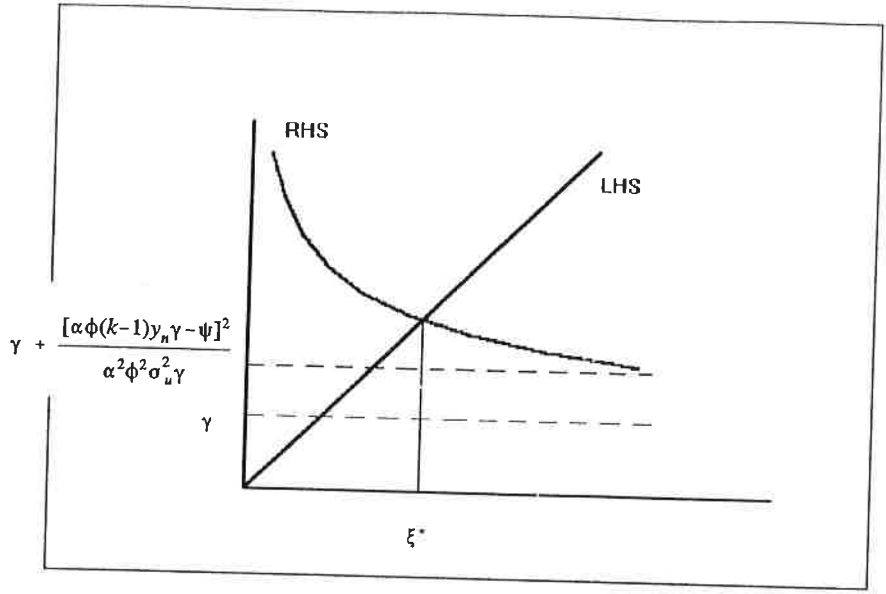
RHS
LHS
$\gamma + \frac{[\alpha\phi(k-1)y_n\gamma - \psi]^2}{\alpha^2\phi^2\sigma_u^2\gamma}$
γ
ξ^*

Figure 1

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