

Should Speculators be Taxed?

By

James Dow and Rohit Rahi

DISCUSSION PAPER 291

---

FINANCIAL MARKETS GROUP  
AN ESRC RESEARCH CENTRE

---

LONDON SCHOOL OF ECONOMICS

E · S · R · C  
ECONOMIC  
& SOCIAL  
RESEARCH  
COUNCIL

**Should Speculators be Taxed?**

**By**

**James Dow and Rohit Rahi**

**DISCUSSION PAPER 291**

**LSE FINANCIAL MARKETS GROUP**

**DISCUSSION PAPER SERIES**

**April 1998**

**James Dow is a professor at London Business School and the European University Institute. Rohit Rahi is a lecturer in the Accounting and Finance Department at the London School of Economics and also a member of the Financial Markets Group. Any opinions expressed here are those of the authors and not necessarily those of the Financial Markets Group.**

**SHOULD SPECULATORS BE TAXED?\***

**James Dow**

European University Institute

and

London Business School

and

**Rohit Rahi**

London School of Economics

First draft: February 1997

This version: May 13, 1997

---

\* We thank seminar participants at the University of Warwick for comments. This research is partly funded by the European Commission TMR Network on Financial Market Efficiency and Economic Efficiency, grant no. FMRX-CT96-0054.

## ABSTRACT

A number of economists have supported the taxation of speculation in financial markets. We examine the welfare economics of such a tax in a model of trading in a financial market where some agents have superior information. We show that in some cases a tax on speculators may actually increase speculative profits. This occurs if the speculators' benefit from less informative prices offsets the costs of the tax. The effect on the welfare of other agents depends on how revelation of information changes risk-sharing opportunities in the market. It is possible for the introduction of a tax to cause a Pareto improvement.

*Journal of Economic Literature* Classification Numbers: G14, G18, D82, D60

*Keywords:* Speculation, Hedging, Transactions Tax, Information Revelation

## 1. Introduction

A number of economists have promoted taxes on speculation in financial markets, for example Tobin (1978), Stiglitz (1988), and Summers and Summers (1988). The main thrust of their arguments is that speculation is largely a rent-seeking activity that either has little or no economic benefit, as in Hirshleifer's analysis of private "foreknowledge" of information that will soon become public (Hirshleifer (1971), or is even positively harmful, as in Tobin's critique of "destabilizing speculation" (Tobin has generally concentrated on foreign exchange speculation, but has also criticized equity markets (Tobin (1984)). Of course, the opposite point of view has been articulated as well, for example by Miller (1991) and Schwert and Seguin (1993) who view speculative trading in financial markets as a productive economic activity that promotes efficiency and innovation, and by Scholes (1981) who argues that securities taxes are distortionary and self-defeating as they can be largely avoided by tax-minimizing trading strategies.

Various taxes are levied on securities trading (see Campbell and Froot (1995) and Schwert and Seguin (1993) for a general description). For example, the UK imposes stamp duty of 1/2% on share sales (this is due to be abolished when transactions become dematerialized). Sweden had a 1% transactions tax until 1984, which was increased to 2% in 1986 before being abolished in 1992 (Umlauf (1993)). In the US, mutual funds that derive more than 30% of gross income from securities sold after being held for less than three months are subject to the "short-short rule," a disadvantageous tax regime (Miller (1991)). Although some countries have tended to reduce these taxes (*e.g.* Sweden and the UK), others such as France and the US have recently proposed to introduce or increase them: for example "la Taxe Tobin" was one of the elements of Lionel Jospin's election manifesto "Propositions pour la France" (Jospin (1996)). A similar proposal has been actively considered in the US. Both the US and French proposals were specifically intended to target speculation as opposed to trading for other motives.

Since this is an important practical problem in the economics of financial markets, it is desirable to investigate it analytically using the standard methods of welfare economics. This has not been done before, probably because of the technical complexity of modelling financial market equilibrium with asymmetric information and rational, utility maximizing agents.

However, the problem does not seem intractable. In this paper we investigate it in a very simple framework of a single-period exchange economy. The model is intended to

describe an equity market, although in principle the analysis could be applied to other assets such as foreign exchange. We study the following question: does a tax on speculators always make them worse off in equilibrium? The answer is negative. We show that speculators themselves may be better off as a result of a tax. Next, we extend the analysis to address the related question: can a tax on speculators be beneficial? In fact, it is straightforward to show that a tax may be Pareto improving. Both speculators and agents who trade for other motives may be made better off as a result of the tax. This is true even if the tax revenues are wasted.

We investigate these questions in a standard rational expectations model similar to Kyle (1985) and Rochet and Vila (1991), but with competitive agents. Speculators receive private information about an asset value, and they trade with agents whose initial risk exposures give them a hedging motive to trade the asset. The intuition for the result is as follows: the tax reduces the informativeness of prices as speculators scale back their trades. Speculators, other things being equal, prefer less information revelation; in this case the benefits of reduced informativeness must be weighed against the costs of paying the tax but the balance may be favourable to the tax. Hedgers, on the other hand may prefer either more or less revelation depending on whether the “Hirshleifer effect” (early revelation of the risk that hedgers wish to insure makes hedging impossible, as in Hirshleifer (1971)) dominates the “dynamic spanning” effect (early revelation of an extraneous risk factor makes the asset a better hedging instrument). See Dow and Rahi (1996) and Marin and Rahi (1995) for a description of how these effects operate.

In this paper we analyze the costs and benefits of taxes on speculation in a static model without a production sector. Clearly, if asset prices can influence productive activities (for example, by improving the allocation of investment resources), this would tend to offset the effects studied here. Depending on the parameters of the economy this effect could be arbitrarily large or arbitrarily small (see Dow and Rahi (1996)). The more interesting question concerns the extension to a model with long-term and short-term informed trading. A tax on short-term trading may encourage speculators to focus on long-term information which may have superior economic value. An analytically complete study of this question remains an open problem (but see Subrahmanyam (1995)).

In this paper we study the impact of a tax on speculators. Although in practice it is impossible to distinguish perfectly between speculators and other traders, the short-short rule in the US, Jospin's proposed tax, and others are designed to target speculators by

using short holding periods as a proxy for speculative motives. This is one reason why in this paper we study the effects of a tax on speculators only. The other reason is to separate analytically the effect of a tax on speculation from a more general tax.

The paper proceeds as follows. To study the welfare effects of informed trading, we consider the rational expectations equilibrium of a parametric model with price-taking agents. We start by showing the result in a simple case where an informed trader interacts with uninformed liquidity traders, but the uninformed trade is treated as an exogenous random variable. This device for modelling uninformed trade is standard in the literature and allows the results to be derived simply. However, since the exogenous liquidity trader device is unsatisfactory for a proper welfare analysis, we then consider the case where uninformed traders have an initial risk exposure that can be hedged with the security.

## 2. A Simple Noise-Trader Model

We consider a risky security whose value is given by the random variable  $v$ . In addition there is a riskless bond whose interest rate is normalized to zero.

There is a speculator with von Neumann-Morgenstern utility function  $U_S$  exhibiting constant absolute risk aversion  $r_S$ . He behaves competitively and should be thought of as representing a continuum of infinitesimal speculators. He privately observes a signal  $s$  that is correlated with the asset value  $v$ . He is subject to a tax on his trades. For analytical tractability, the tax payment is assumed to be proportional to the square of his trade. Taking a position  $\theta_S$  at the market price  $p$  incurs a tax payment of  $\frac{\tau}{2}\theta_S^2$  and leaves him with terminal wealth

$$w_S = \theta_S(v - p) - \frac{\tau}{2}\theta_S^2. \quad (1)$$

We assume that  $v$  and  $s$  are joint normal. Without loss of generality we can take them to be of mean zero and write:

$$v = s + \epsilon,$$

where  $\epsilon$  is a normally distributed "residual" with mean zero, independent of  $s$ . The variances of  $s$  and  $\epsilon$  are denoted by  $V_s$  and  $V_\epsilon$  respectively.

There is also a random quantity  $\eta$  of exogenous liquidity trade. The random variable  $\eta$  is normal with mean zero and variance  $V_\eta$ , and is independent of  $s$  and  $\epsilon$ .

Prices are set by a risk-neutral, competitive market-maker who observes

$$\theta = \theta_S + \eta,$$

the net aggregate order flow from the speculators and liquidity traders, and sets the price equal to the expected value of the asset, conditional on  $\theta$ .

DEFINITION 1. *An equilibrium is a price function  $p(\theta)$  and a trade  $\theta_S$  such that:*

(a)  $\theta_S \in \arg \max E[U_S(w_S)|s, p]$ , and

(b)  $p = E(v|\theta)$ .

Before computing the equilibrium and proceeding to the analysis of the model, we make two remarks on the features of the model. First, the quadratic form of the tax payment may not appear natural as a description of actual taxes which are generally approximately linear. However, a linear function of the magnitude of the trade would not be differentiable at zero, since trading positions can be positive or negative. The quadratic form of the tax payment that we have used here is the standard way to circumvent this problem, since it preserves differentiability and linearity of the solution. See, for example, Subrahmanyam (1995) for a similar application to a tax problem.

Secondly, the model presented here is similar to the standard models that have been used in the literature to analyze security market trading with asymmetric information. On the one hand, it is the same model as in Kyle (1985), except that the informed trader can condition his demand on the price and behaves competitively. Rochet and Vila (1991) also modified Kyle's model to allow for conditioning on the price, although retaining a monopolistic informed trader. Allowing conditioning on price (generalized limit orders) seems desirable as, in practice, traders do not face a significant amount of execution risk (the risk that their orders will be executed at a price different from the current market price); and in any case they are able to use limit orders to prevent execution risk. Furthermore the execution risk in Kyle's framework is capable of significantly influencing traders' optimal strategies. This is particularly relevant when exogenous liquidity trades are replaced by optimal endogenous trades, as we do later in the paper. As shown in Spiegel and Subrahmanyam (1992), execution risk can distort hedging demands perversely.

On the other hand, the model used here is precisely a rational expectations model of the kind studied by Grossman (1977) and Grossman and Stiglitz (1980), with an additional agent who is uninformed and risk-neutral.

We now proceed to solve and analyze the model. We look for a linear equilibrium of the form

$$p = \lambda s + \mu \eta, \tag{2}$$



for some parameters  $\lambda$  and  $\mu$  that will be determined below. Since the speculator has constant absolute risk aversion and normally distributed wealth (conditional on knowing  $s$  and  $p$ ), his optimization problem reduces to choosing a position  $\theta_S$  to maximize the mean-variance criterion  $E(w_S|s, p) - \frac{r_S}{2}\text{Var}(w_S|s, p)$ . His optimal trade is given by

$$\begin{aligned}\theta_S &= \frac{s - p}{r_S V_\epsilon + \tau} \\ &= \frac{(1 - \lambda)s - \mu\eta}{r_S V_\epsilon + \tau}.\end{aligned}\tag{3}$$

Hence the total order flow is

$$\theta = \frac{(1 - \lambda)s + (r_S V_\epsilon + \tau - \mu)\eta}{r_S V_\epsilon + \tau}.$$

We conjecture that the order flow is proportional to  $(\lambda s + \mu\eta)$ . Then the coefficients on  $s$  and  $\eta$  in the above expression must lie in the proportion

$$\frac{\lambda}{\mu} = \frac{1 - \lambda}{r_S V_\epsilon + \tau - \mu}.$$

Therefore,

$$\frac{\mu}{\lambda} = r_S V_\epsilon + \tau.\tag{4}$$

Furthermore,

$$\begin{aligned}p &= E(v|\theta) \\ &= E(v|\lambda s + \mu\eta) \\ &= \frac{\lambda V_s}{\lambda^2 V_s + \mu^2 V_\eta} \cdot (\lambda s + \mu\eta).\end{aligned}$$

Given the conjectured form of the price function (2), it follows that

$$\lambda^2 V_s + \mu^2 V_\eta = \lambda V_s.\tag{5}$$

From (4) and (5), one can solve directly for  $\lambda$  and  $\mu$ . This shows:

PROPOSITION 2.1. *There exists a unique linear equilibrium. The price function is*

$$p = \lambda s + \mu\eta,$$

*the equilibrium holding of the speculator is*

$$\theta_S = \frac{(1 - \lambda)s - \mu\eta}{r_S V_\epsilon + \tau},$$

*and the order flow is*

$$\theta = (r_S V_\epsilon + \tau) V_\eta V_s^{-1} (\lambda s + \mu \eta),$$

where

$$\lambda = \frac{V_s}{V_s + (r_S V_\epsilon + \tau)^2 V_\eta}$$

and

$$\mu = \frac{V_s (r_S V_\epsilon + \tau)}{V_s + (r_S V_\epsilon + \tau)^2 V_\eta}.$$

The market-maker learns  $(\lambda s + \mu x)$  from observing the order flow and sets the price equal to it. The ratio  $\frac{\lambda}{\mu}$  is strictly decreasing in  $\tau$ : a tax on informed trading makes the order flow less revealing.

We now consider the effect of the tax on the speculator's equilibrium expected utility. The speculator's wealth is not normally distributed *ex ante* since it is a quadratic form of normal random variables: in (1), the value of the asset  $v$ , the price  $p$ , and the optimal trade  $\theta_S$  are all normal random variables. Notice that when the agent chooses his portfolio  $\theta_S$ , he conditions on his signal and he takes price as given, so his wealth is normally distributed given his information and given any choice of  $\theta_S$ . Hence when deriving  $\theta_S$  we were able to use the standard mean-variance certainty-equivalent as the objective. When calculating the agent's *ex ante* expected utility, however, we must regard the price and trading position as random. The speculator's expected utility is

$$\begin{aligned} EU_S(w_S) &= E[-\exp(-r_S w_S)] \\ &= -E\left[E[\exp(-r_S w_S) | s, p]\right] \\ &= -E\left[\exp\left(-r_S \left[E(w_S | s, p) - \frac{r_S}{2} \text{Var}(w_S | s, p)\right]\right)\right]. \end{aligned} \quad (6)$$

Substituting (1) and (3) we obtain:

$$EU_S(w_S) = -E\left[\exp\left(-\frac{r_S \theta_S^2}{2} (r_S V_\epsilon + \tau)\right)\right],$$

which, by a standard formula (given in the Appendix as Lemma A.1), is an increasing monotonic transformation of  $(r_S V_\epsilon + \tau) \text{Var}(\theta_S)$ . Now, using Proposition 2.1 and (5),

$$\begin{aligned} (r_S V_\epsilon + \tau) \text{Var}(\theta_S) &= \frac{(1 - \lambda)^2 V_s + \mu^2 V_\eta}{r_S V_\epsilon + \tau} \\ &= \frac{(1 - \lambda) V_s}{r_S V_\epsilon + \tau} \\ &= \frac{(r_S V_\epsilon + \tau) V_s V_\eta}{V_s + (r_S V_\epsilon + \tau)^2 V_\eta}. \end{aligned}$$

Hence

$$\text{sgn} \left( \frac{\partial EU_S}{\partial \tau} \right) = \text{sgn} [V_s - (r_S V_\epsilon + \tau)^2 V_\eta].$$

Evaluating this at  $\tau = 0$ , we get the following result:

**PROPOSITION 2.2.** *Introducing a small tax on speculators will make them better off if and only if*

$$V_s > r_S^2 V_\epsilon^2 V_\eta.$$

We can also solve for the tax rate that is optimal for the speculators. Setting  $\frac{\partial EU_S}{\partial \tau} = 0$  and solving for  $\tau$ , we obtain

$$\tau = \sqrt{\frac{V_s}{V_\eta}} - r_S V_\epsilon.$$

The intuition underlying the result is not difficult to see. If one regards the information of all agents in the economy as given, then a tax on any one class of agents will tend to make them worse off. However, in this case there is the offsetting effect that taxing the speculators makes them reduce the scale of their trades, resulting in a less informative price in equilibrium and increasing their informational advantage over other agents. The condition on the parameters in Proposition 2.2 can be interpreted as follows: if the order flow reveals “too much” information, either because speculators trade aggressively (low risk aversion  $r_S$  or low residual risk  $\epsilon$ ) or because the noise trade is small in magnitude (low  $V_\eta$ ), a tax can benefit speculators by making the order flow less revealing.

The result would not hold if there were a monopolistic informed trader, since unlike competitive speculators he could optimally control the informativeness of prices regardless of taxes. In our model, there is an externality since an individual informed trader does not consider the effect his trade will have on increasing information revelation and thereby lowering the profits of others. With oligopolistic informed traders (for example with Cournot oligopoly) one could presumably derive a similar result. However we have not explored this case since it is well-known that with imperfect competition one can easily obtain effects that would be perverse in the perfectly competitive case.

### 3. A Model with Rational Traders

In this section we describe a modified version of the model in which all traders maximize utility and have rational expectations. As before there is a privately informed speculator

with terminal wealth given by (1). There are two other agents who trade for hedging reasons. The initial endowment of hedger 1 is  $e_1 = xz$ , where  $z$  is a random variable representing a risk factor that is correlated with the asset payoff, and  $x$  is the exposure to this risk factor. We assume that  $x$  is unknown to other agents, hence it is itself a random variable. After privately observing  $x$ , the hedger trades an amount  $\theta_1$  which results in net wealth

$$w_1 = xz + \theta_1(v - p).$$

Hedger 2's endowment is simply  $e_2 = z$ , and he trades  $\theta_2$  to realize terminal wealth

$$w_2 = z + \theta_2(v - p).$$

Agent  $i$  ( $i = S, 1, 2$ ) has a von Neumann-Morgenstern utility function  $U_i$  with constant absolute risk aversion  $r_i$ . All agents take prices as given.

In this model, the “noise” that prevents equilibrium from being fully revealing arises from the trading of hedger 1. This agent trades a random amount which depends on his privately observed endowment shock  $x$ . The endowment shock could be interpreted as a liquidity shock suffered by the agent, resulting in a need to rebalance his portfolio. Unlike the noise trade in the model of the previous section, this hedging trade results from hedger 1 maximizing utility and making inferences like any other rational trader. Hedger 2 also trades rationally.

The model presented here, with two hedgers, one with a stochastic risk exposure and one with a fixed exposure, is similar to the model used by Dow and Rahi (1996) to study the feedback effect of stock prices on real investment. This formulation is chosen because it is the simplest one for computational purposes that is also rich enough analytically. If hedger 2 were dropped from the model, the only hedging trade would come from hedger 1 and he would be perfectly informed in equilibrium. On the other hand if hedger 2's risk exposure were also stochastic, it would be impossible to solve the model in closed form.

We assume that  $s, \epsilon, z$  and  $x$  are jointly normally distributed with mean zero. The endowment shock  $x$  is independent of all the other random variables, while by construction the signal  $s$  and the residual  $\epsilon$  are mutually independent. The endowment risk factor  $z$  is correlated with the asset payoff  $v$ . We take the covariance of  $z$  with the signal  $s$  to be nonnegative (without loss of generality) and its covariance with the residual  $\epsilon$  to be nonzero (otherwise, equilibrium is necessarily fully revealing). To simplify the exposition we assume the latter covariance is positive.

We denote the variance of a random variable  $g$  by  $V_g$ , its covariance with another random variable  $h$  by  $V_{gh}$ , its regression coefficient on  $h$  (the “beta” of  $g$  with respect to  $h$ ) by  $\beta_{gh} := V_{gh}V_h^{-1}$ , and their correlation by  $\rho_{gh}$ . To summarize our assumptions on correlations,  $V_{zs} \geq 0$  and  $V_{ze} > 0$ . We also assume that

$$r_1^2 V_x V_z < 1. \quad (7)$$

This turns out to be a necessary and sufficient condition for the expected utility of hedger 1 to be well-defined.

The market-maker observes the aggregate order flow  $\theta = \theta_S + \theta_1 + \theta_2$ , and sets the price equal to the conditional expectation of the asset payoff given the order flow,

$$p = E(v|\theta).$$

For agents  $i = S, 1, 2$ , we define  $\mathcal{I}_i$  to be the information observed by  $i$ , *i.e.* his private information together with the price  $p$ . Accordingly,  $\mathcal{I}_S = (s, p)$ ,  $\mathcal{I}_1 = (x, p)$  and  $\mathcal{I}_2 = p$ .

**DEFINITION 2.** *An equilibrium is a price function  $p(\theta)$  and a trade  $\theta_i$  for each agent  $i = S, 1, 2$ , such that:*

- (a)  $\theta_i \in \arg \max E[U_i(w_i)|\mathcal{I}_i]$ , and
- (b)  $p = E(v|\theta)$ .

We look for an equilibrium with a linear price function

$$p = \lambda s + \mu x.$$

Note that (provided  $\lambda$  and  $\mu$  are both nonzero) the speculator and hedger 1 have the same information in equilibrium: knowing  $p$  and his own signal  $s$ , the speculator can infer hedger 1's risk exposure  $x$ , and similarly hedger 1, who knows  $x$ , can infer  $s$ . The market-maker and hedger 2, on the other hand, are unable to isolate  $s$  from  $x$ .

Analogous to (6) agent  $i$ 's expected utility is

$$E[-\exp(-r_i w_i)] = -E[\exp(-r_i \mathcal{E}_i)],$$

where

$$\mathcal{E}_i := E(w_i|\mathcal{I}_i) - \frac{r_i}{2} \text{Var}(w_i|\mathcal{I}_i).$$

In general we can write

$$w_i = e_i + \theta_i(v - p) - \frac{\tau_i}{2}\theta_i^2,$$

where the endowment  $e_i$  is zero for the speculator,  $xz$  for hedger 1, and  $z$  for hedger 2; and the tax rate  $\tau_i$  is  $\tau$  for the speculator and zero for the hedgers. Then,

$$\mathcal{E}_i = E(e_i|\mathcal{I}_i) + \theta_i \left[ E(v|\mathcal{I}_i) - p \right] - \frac{\tau_i}{2}\theta_i^2 - \frac{\tau_i}{2} \left[ \text{Var}(e_i|\mathcal{I}_i) + \theta_i^2 \text{Var}(v|\mathcal{I}_i) + 2\theta_i \text{cov}(v, e_i|\mathcal{I}_i) \right].$$

Differentiating with respect to  $\theta_i$  we obtain the optimal portfolio:

$$\theta_i = \frac{E(v|\mathcal{I}_i) - p - r_i \text{cov}(v, e_i|\mathcal{I}_i)}{r_i \text{Var}(v|\mathcal{I}_i) + \tau_i}. \quad (8)$$

PROPOSITION 3.1. *There exists a unique linear equilibrium. The price function is*

$$p = \lambda s + \mu x,$$

the equilibrium holdings of the agents are given by

$$\begin{aligned} \theta_S &= \frac{(1 - \lambda)s - \mu x}{r_S V_\epsilon + \tau}, \\ \theta_1 &= \frac{(1 - \lambda)s - (\mu + r_1 V_{z\epsilon})x}{r_1 V_\epsilon}, \\ \theta_2 &= -\frac{(1 - \lambda)V_{zs} + V_{z\epsilon}}{(1 - \lambda)V_s + V_\epsilon}, \end{aligned}$$

and the order flow is

$$\theta = \theta_2 - (1 - \lambda)\mu^{-1}\beta_{z\epsilon}(\lambda s + \mu x),$$

where

$$\lambda = \frac{V_s[(r_S + r_1)V_\epsilon + \tau]^2}{V_s[(r_S + r_1)V_\epsilon + \tau]^2 + r_1^2 V_{z\epsilon}^2 V_x(r_S V_\epsilon + \tau)^2}$$

and

$$\mu = -\frac{r_1 V_s V_{z\epsilon}(r_S V_\epsilon + \tau)[(r_S + r_1)V_\epsilon + \tau]}{V_s[(r_S + r_1)V_\epsilon + \tau]^2 + r_1^2 V_{z\epsilon}^2 V_x(r_S V_\epsilon + \tau)^2}.$$

The proof is in the Appendix. Since  $\theta_2$  is nonstochastic, the order flow is linear in  $(\lambda s + \mu x)$ . The market-maker learns  $(\lambda s + \mu x)$  from observing the order flow and sets the price equal to it. The price and the order flow are informationally equivalent, so that the uninformed hedger has the same information in equilibrium as does the market-maker. Note that  $|\frac{\lambda}{\mu}|$  is strictly decreasing in  $\tau$ : a tax on informed trading makes the price less revealing.

For convenience, we study agents' welfare in equilibrium in terms of their certainty-equivalent wealth, *i.e.* the certain amount of money that gives the same expected utility as their equilibrium *ex ante* distribution of terminal wealth:

$$\begin{aligned} \mathcal{U}_i &:= -\frac{1}{r_i} \ln[-EU_i(w_i)] \\ &= -\frac{1}{r_i} \ln[E[\exp(-r_i w_i)]]. \end{aligned}$$

Notice that this does not require wealth to be normally distributed *ex ante*.

PROPOSITION 3.2. *The payoffs of the agents are:*

$$\begin{aligned} \mathcal{U}_S &= \frac{1}{2r_S} \ln \left[ 1 + \frac{r_S(1-\lambda)V_s}{r_S V_\epsilon + \tau} \right] \\ \mathcal{U}_1 &= \frac{1}{2r_1} \ln \left[ (1 - r_1^2 V_x V_z) [1 + (1-\lambda)^2 V_s V_\epsilon^{-1}] + (\mu + r_1 [(1-\lambda)V_{zs} + V_{z\epsilon}])^2 V_x V_\epsilon^{-1} \right] \\ \mathcal{U}_2 &= \frac{r_2}{2} \left[ \frac{[(1-\lambda)V_{zs} + V_{z\epsilon}]^2}{(1-\lambda)V_s + V_\epsilon} - V_z \right]. \end{aligned}$$

This result mirrors Proposition 4.1 in Dow and Rahi (1996), and the proof is a straightforward adaptation. We are now in a position to carry out comparative statics with respect to the tax rate  $\tau$ , and prove the main result of this section.

PROPOSITION 3.3. *There is an open set of parameters for which a tax on speculative transactions leads to a Pareto improvement.*

The proof, which appears in the Appendix, proceeds by identifying restrictions on the parameters under which each of the agents, the speculator and the two hedgers, are individually better off in equilibrium when a small tax is introduced on the speculator's transactions. These restrictions are then shown to be consistent.

The speculator's welfare can be improved for the same reason as in Proposition 2.2. If the speculator and hedger 1 (who perfectly infers the signal  $s$  from the price) are not very risk-averse, they speculate too aggressively. A tax on the speculator ameliorates this externality. A necessary condition for the speculator to benefit from a tax is that he be less risk-averse than hedger 1 (see (11)).

The uninformed hedger benefits from the tax if (and only if) he prefers to be less informed in equilibrium, which is the case when  $\beta_{z\epsilon} \leq 2\beta_{zs}$ . Observing a signal that is highly informative about endowments reduces risk-sharing opportunities in the market (the

Hirshleifer effect). This occurs when  $\beta_{zs}$  is large. Conversely, information about  $s$  is valuable when  $\beta_{z\epsilon}$  is large, because it allows the trader to hedge endowment risk more accurately. If  $\beta_{z\epsilon}$  is relatively small compared to  $\beta_{zs}$ , the Hirshleifer effect dominates: imposing a tax is favourable for the hedger as it reduces informed trading and makes the price less informative, mitigating the Hirshleifer effect.

#### 4. Conclusions

In this paper, we have suggested a simple framework for analyzing the consequences of a tax on financial market speculation. We have studied the comparative-statics effects of a change in the tax rate on the welfare of speculators and hedgers in the market. In some cases, the tax can make all agents better off. Of course, we do not suggest that in reality such a tax would actually benefit the speculators themselves. The main contribution of the paper is applying a rigorous analytical framework for assessing the effects of a tax.

The analysis here considers only the impact of the tax on speculative profits and on risk-sharing opportunities for hedgers. We ignore all other economic effects of the tax, among them doubtless many that are as important as, if not more important than, the ones considered here. The most important extension would seem to be to consider also the effect of a tax on incentives to produce long-term and short-term information, and the implications for economic investment and production. We are pursuing this extension in our current research.



APPENDIX

We first state a useful result (see, for example, Dow and Rahi (1996)):

LEMMA A.1. Suppose  $\mathbf{A}$  is a symmetric  $m \times m$  matrix,  $\mathbf{b}$  is an  $m$ -vector,  $c$  is a scalar, and  $\mathbf{w}$  is an  $m$ -dimensional normal variate:  $\mathbf{w} \sim N(0, \Sigma)$ ,  $\Sigma$  positive definite. Then  $E[\exp(\mathbf{w}^\top \mathbf{A} \mathbf{w} + \mathbf{b}^\top \mathbf{w} + c)]$  is well-defined if and only if  $(\mathbf{I} - 2\Sigma\mathbf{A})$  is positive definite, and

$$E[\exp(\mathbf{w}^\top \mathbf{A} \mathbf{w} + \mathbf{b}^\top \mathbf{w} + c)] = |\mathbf{I} - 2\Sigma\mathbf{A}|^{-\frac{1}{2}} \exp\left\{\frac{1}{2} \mathbf{b}^\top (\mathbf{I} - 2\Sigma\mathbf{A})^{-1} \Sigma \mathbf{b} + c\right\}.$$

*Proof of Proposition 3.1.* Using (8) we get

$$\begin{aligned} \theta_S &= \frac{(1-\lambda)s - \mu x}{r_S V_\epsilon + \tau}, \\ \theta_1 &= \frac{(1-\lambda)s - (\mu + r_1 V_{z\epsilon})x}{r_1 V_\epsilon}, \end{aligned}$$

and

$$\theta_2 = -\frac{\text{cov}(z, s|p) + V_{z\epsilon}}{\text{Var}(s|p) + V_\epsilon}. \quad (9)$$

We conjecture that the stochastic part of the order flow  $(\theta_S + \theta_1)$  is proportional to  $(\lambda s + \mu x)$ . Then it follows from the above expressions that

$$\frac{\lambda}{\mu} = -\frac{(1-\lambda)[(r_S + r_1)V_\epsilon + \tau]}{\mu[(r_S + r_1)V_\epsilon + \tau] + r_1 V_{z\epsilon}(r_S V_\epsilon + \tau)}.$$

Cross-multiplying and simplifying, we get

$$\frac{\lambda}{\mu} = -\frac{(r_S + r_1)V_\epsilon + \tau}{r_1 V_{z\epsilon}(r_S V_\epsilon + \tau)}.$$

Also, analogous to (5), we have

$$\lambda^2 V_s + \mu^2 V_x = \lambda V_s. \quad (10)$$

Now the desired expressions for  $\lambda$  and  $\mu$  can readily be obtained.

The conditional moments for hedger 2, who observes only the price, are equivalent to the moments conditional on  $(\lambda s + \mu x)$ . Using standard properties of the normal distribution (Anderson (1984)) together with (10), we obtain:

$$\begin{aligned} \text{Var}(s|p) &= (1-\lambda)V_s \\ \text{cov}(z, s|p) &= (1-\lambda)V_{zs}. \end{aligned}$$

Substituting into (9) gives the desired formula for  $\theta_2$ . It is straightforward to compute the equilibrium order flow. ■

*Proof of Proposition 3.3.*

Using the expression for the speculator's payoff in Proposition 3.2, it is easy to show that  $(\frac{\partial U_s}{\partial \tau})_{\tau=0} > 0$  if and only if

$$V_s > \frac{r_1^2 r_S^2 V_x V_{z\epsilon}^2}{r_1^2 - r_S^2}. \quad (11)$$

Similarly for hedger 1,  $(\frac{\partial U_1}{\partial \tau})_{\tau=0} > 0$  if and only if

$$\begin{aligned} & 2r_1 r_S^3 (r_1 + r_S) V_x V_s V_{z\epsilon}^2 (1 - r_1^2 V_x V_z) \\ & + (r_1 r_S^2 V_x V_{z\epsilon} (V_{z\epsilon} + V_{zs}) + V_s (r_1 + r_S)) \\ & \cdot (r_1^2 r_S V_x V_{z\epsilon} [2(r_1 + r_S) V_{zs} + r_S V_{z\epsilon}] - (r_1 + r_S)^2 V_s) > 0. \end{aligned}$$

Recalling that  $V_{z\epsilon}$  and  $V_{zs}$  are positive, and using (7),  $(\frac{\partial U_1}{\partial \tau})_{\tau=0} > 0$  if

$$V_s < \frac{r_1^2 r_S V_x V_{z\epsilon} [2(r_1 + r_S) V_{zs} + r_S V_{z\epsilon}]}{(r_1 + r_S)^2}. \quad (12)$$

Since  $\lambda$  is strictly decreasing in  $\tau$ , we can deduce from Proposition 4.2 in Dow and Rahi (1996) that the uninformed hedger's payoff  $U_2$  is strictly increasing in  $\tau$  if  $|\beta_{zs} - \beta_{z\epsilon}| \leq \beta_{zs}$ , which is equivalent to

$$V_s \leq \frac{2V_{zs} V_{z\epsilon}}{V_{z\epsilon}}. \quad (13)$$

It remains to show that there is an open set of parameters which satisfy the three inequalities above, (11), (12), and (13), as well as (7), while preserving positive definiteness of the covariance matrix of the models's random variables. Positive definiteness is equivalent to requiring that all variances are nonzero and that

$$\rho_{zs}^2 + \rho_{z\epsilon}^2 < 1. \quad (14)$$

There is an open interval of possible values for  $V_s$  consistent with (11) and (12) if and only if

$$\frac{r_1^2 r_S^2 V_x V_{z\epsilon}^2}{r_1^2 - r_S^2} < \frac{r_1^2 r_S V_x V_{z\epsilon} [2(r_1 + r_S) V_{zs} + r_S V_{z\epsilon}]}{(r_1 + r_S)^2}$$

or, equivalently,

$$r_S^2 V_{z\epsilon} < (r_1^2 - r_S^2) V_{zs}.$$

Inequality (13) can simultaneously be satisfied if  $V_{z\epsilon}$  is sufficiently large. Inequality (14) holds for  $V_z$  sufficiently large, and finally (7) can be satisfied by choosing  $V_x$  sufficiently small. ■

## REFERENCES

1. Anderson, T. W. (1984): *An Introduction to Multivariate Statistical Analysis (2nd ed.)*. New York: Wiley.
2. Campbell, J. and K. Froot (1995): "International Experience with Securities Transaction Taxes," working paper, Harvard University.
3. Dow, J. and R. Rahi (1996): "Informed Trading, Investment and Welfare," working paper, Birkbeck College and European University Institute.
4. Grossman, S. J. (1977): "The Efficiency of Futures Markets, Noisy Rational Expectations and Informational Externalities," *Review of Economic Studies*, 44, 431-449.
5. Grossman, S. J. and J. E. Stiglitz (1980): "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, 70, 393-408.
6. Hirshleifer, J. (1971): "The Private and Social Value of Information and the Reward to Inventive Activity," *American Economic Review*, 61, 561-574.
7. Jospin, L. (1996): "Propositions pour la France."
8. Kyle, A. S. (1985): "Continuous Auctions and Insider Trading," *Econometrica*, 53, 1315-1335.
9. Marín, J. M. and R. Rahi (1995): "Information Revelation and Market Incompleteness," working paper, Birkbeck College, University of London.
10. Miller, M. (1991): *Financial Innovations and Market Volatility*. Oxford: Basil Blackwell.
11. Rochet, J.-C. and J.-L. Vila (1994): "Insider Trading without Normality," *Review of Economic Studies*, 61, 131-152.
12. Scholes, M. S. (1981): "The Economics of Hedging and Spreading in Futures Markets," *Journal of Futures Markets*, 1, 265-286.
13. Schwert, G. W. and P. J. Seguin (1993): "Securities Transaction Taxes: An Overview of Costs, Benefits and Unresolved Questions," *Financial Analysts Journal*, 49, 27-35.

14. Stiglitz, J. E. (1988), "Using Tax Policy to Curb Speculative Trading," *Journal of Financial Services Research*, 3, 101-115.
15. Spiegel, M. and Subrahmanyam, A. (1992): "Informed Speculation and Hedging in a Non-Competitive Securities Market," *Review of Financial Studies*, 5, 307-329.
16. Subrahmanyam, A. (1995), "Transaction Taxes and Financial Market Equilibrium," working paper, UCLA.
17. Summers, L. and V. Summers (1988): "When Financial Markets Work too Well: a Cautious Case for a Securities Transactions Tax," *Journal of Financial Services Research*, 3, 261-286.
18. Tobin, J. (1978): "A Proposal for International Monetary Reform," *Eastern Economic Journal*, 4, 153-159.
19. Tobin, J. (1984): "On the Efficiency of the Financial System," *Lloyds Bank Review*, 153, 1-15.
20. Umlauf, S. R. (1993): "Transactions Taxes and the Behavior of the Swedish Stock Market," *Journal of Financial Economics*, 33, 227-240.

**FINANCIAL MARKETS GROUP**  
**DISCUSSION PAPER SERIES**

- 261 Faure-Grimaud, Antoine, "Product Market Competition and Optimal Debt Contracts: The Limited Liability Effect Revisited", February 1997
- 262 Hart, Oliver and John Moore, "Default and Renegotiation: A Dynamic Model of Debt", February 1997
- 263 de Meza, David and David Webb, "Precautionary Behaviour, Adverse Selection and Excessive Insurance", March 1997
- 264 Henry, Marc and Richard Payne, "An Investigation of Long Range Dependence in Intra-Day Foreign Exchange Rate Volatility", March 1997
- 265 Hartmann, Philipp, "Do Reuters Spreads Reflect Currencies' Differences in Global Trading Activity?", March 1997
- 266 de Garidel, Thomas, "Pareto-improving Asymmetric Information in a Dynamic Insurance Market", June 1997
- 267 León, Angel and Enrique Sentana, "Pricing Options on Assets with Predictable White Noise Returns", July 1997
- 268 Marín, José M. and Rohit Rahi, "Speculative Securities", July 1997
- 269 Nier, Erlend, "Optimal Managerial Remuneration and Firm-level Diversification", July 1997
- 270 Brunnermeier, Markus K., "Prices, Price Processes, Volume and Their Information - A Literature Survey -", July 1997
- 271 Brown, Ward, "R&D Intensity and Finance: Are Innovative Firms Financially Constrained?", July 1997
- 272 Schönbucher, Philipp J., "Term Structure Modelling of Defaultable Bonds", July 1997
- 273 Danielsson, Jon and Casper G. de Vries, "Extreme Returns, Tail Estimation, and Vaule-at-Risk", September 1997
- 274 Sabani, Laura, "Sustainability of Capital Ratios and Regulator Reputation: Discretionary Vs. Binding Legislation", September 1997
- 275 Eijffinger, Sylvester C.W., Harry P. Huizinga and Jan J.G. Lemmen, "Short-Term and Long-Term Government Debt and Non-Resident Interest Withholding Taxes", September 1997

- 276 Board, John, Charles M. S. Sutcliffe and Anne Vila, "Market Maker Performance: The Search for Fair Weather Market Makers", September 1997
- 277 Board, John, Gleb Sandmann and Charles M. S. Sutcliffe, "The Effect of Contemporaneous Futures Market Volume on Spot Market Volatility", September 1997
- 278 Luttmer, Erzo G.J., "What Level of Fixed Costs Can Reconcile Asset Returns and Consumption Choices?" November 1997
- 279 Dessi, Roberta, "Implicit Contracts, Managerial Incentives and Financial Structure", October 1997
- 280 Vitale, Paolo, "Co-ordinated Monetary and Foreign Exchange" November 1997
- 281 Lotz, Christopher, "Locally Minimizing the Credit Risk", January 1998
- 282 Huddart, Steven, John S. Hughes and Markus Brunnermeier, "Disclosure Requirements and Stock Exchange Listing Choice in an International Context", January 1998
- 283 Tonks, Ian and Susanne Espenlaub, "Post IPO Directors' Sales and Reissuing Activity: An empirical test of IPO Signalling Models", March 1998
- 284 Schellekens, Philip, "Caution and Conservatism in Monetary Policymaking", March 1998
- 285 Tonks, Ian, Susanne Espenlaub and Alan Gregory, "Testing the Robustness of Long-Term Under-Performance of UK Initial Public Offerings", March 1998
- 286 Burkhart, Mike, Denis Gromb and Fausto Panunzi, "Block Premia in Transfers of Corporate Control", March 1998
- 287 Ostergaard, Charlotte, Bent E. Sorsen and Oved Yosha, "Permanent Income, Consumption and Aggregate Constraints", April 1998
- 288 Chella, Giles and Antoine Faure-Grimaud, "Dynamic Adverse Selection and Debt", April 1998
- 289 Nier, Erland, "Managers, Debt and Industry Equilibrium", April 1998
- 290 Cerasi, Vittoria, Barbara Chizzolini and Marc Ivaldi, "Sunk Costs and Competitiveness of European Banks after Deregulation", April 1998

Subject to availability, copies of these Discussion Papers can be obtained from the Financial Markets Group (Room G307, 0171 955 7002). Alternatively visit the FMG website <http://cep.lse.ac.uk/fmg/> where papers are available for downloading.