Contrasting Different Forms of Price Stickiness: An Analysis of Exchange Rate Overshooting and the Beggar Thy Neighbour Policy

By

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E·S·R·C ECONOMIC & SOCIAL RESEARCH COUNCIL Contrasting Different Forms of Price Stickiness: An Analysis of Exchange Rate Overshooting and the Beggar Thy Neighbour Policy

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Contrasting Different Forms of Price Stickiness: An Analysis of Exchange Rate Overshooting and the Beggar Thy Neighbour Policy¹

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Abstract

This paper considers a two country economy similar to that in Obstfeld and Rogoff (1995). We build on their model by distinguishing between sticky retail prices, sticky wholesale prices and sticky wages. We find that conclusions about whether monetary shocks lead to exchange rate overshooting and spillovers on foreign production and consumption depend crucially on the form of price stickiness assumed in the economy. Sticky retail prices not only allow for a profitable 'Beggar Thy Neighbour Policy' but also lead to exchange rate overshooting. Although the outcome is similar to the seminal work by Dornbusch (1976), the driving force in our model is quite different. With sticky retail prices, the exchange rate overshoots even though the interest rate parity may not even hold in equilibrium. These results are in sharp contrast to the outcomes under the sticky wholesale prices scenario wherein prices are fixed in the producers' currency. Contrary to the spirit of the 'Beggar Thy Neighbour Policy', an unexpected money expansion benefits inhabitants in the other country as well. The interest rate parity always holds in equilibrium and there is no exchange rate overshooting.

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1 Introduction

Monetary policy has real effects in a world with sluggish price adjustments. Furthermore, in an open economy setting, domestic monetary policy also impacts economies abroad. The foreign economies are affected not only by exchange rate movements but also through other spillover effects. The effect of monetary policy on exchange rate volatility is of considerable interest. Macroeconomists have long discussed how domestic monetary policy affects foreign production, consumption and inflation rates. The violation of the uncovered interest rate parity and of the purchasing power parity (PPP) in the short-run are other puzzles in international macroeconomics. In this paper we argue that exchange rate volatility, spillovers, interest rate parity and PPP depend strongly on the type of nominal rigidity that allows monetary shocks to have an effect on real variables. The importance of different forms of price stickiness can then be weighted given the existing empirical evidence.

Most arguments that were concerned with the effect of monetary policy in an open economy were based on a static Mundell Flemming type analysis. This has changed substantially over the last years. A growing body of recent research looks at these effects in a dynamic framework. Most of these papers use elements of the framework that has been developed by Obstfeld and Rogoff (1995) and Svensson and van Wijnbergen (1989). They combine the new intertemporal approach to the current account with rational expectations and the traditional Keynesian setting of sluggish price adjustment. In all these models, monetary policy has real effects by stimulating demand to which supply adjusts. While prices/wages are only sticky in the short-run, asymmetric monetary policy leads to permanent effects through perfectly integrated international bond markets. Our model extends this framework in various ways.

The model describes a two country world, home and foreign, that is populated by workers that provide labour to firms. There is a complete home bias in the ownership of firms. Each firm produces in only one of the two countries and is in monopolistic competition with firms both abroad and at home. Unlike Obstfeld and Rogoff (1995) we assume that firms are able to price discriminate between countries. While we believe that this is realistic, we also need this assumption to study different sorts of price stickiness. There is a substantial amount of evidence that borders have a much bigger effect on price disparities than for example transport costs, (Engel and Rogers 1996). Furthermore, we assume that firms are monopsonists on the labour market. We need to introduce an imperfection to study wage stickiness following positive and negative monetary shocks. We believe that allocating the market power to the firm is more realistic than allocating it to the workers. This is in contrast to monopolistic competition between different trade unions which is assumed in a series of papers that use a similar framework, e.g. Obstfeld and Rogoff (1996) and Hau (1998). While monopsonistic market power of firms is certainly an extreme assumption, labour economists have previously argued that positive output effects following the introduction of minimum wages are a sign of firms with monopsonistic market power (Manning 1995). In comparison to the social optimum, prices are too high in our model due to monopolistic competition and wages are too low because of the monopsonistic market power of firms. If there is money expansion in an economy with sticky prices, nominal wages will adjust while real prices decrease. This leads a priori to more production in the country that expands its money supply and suggests a current account surplus. On the other hand, if wages are sticky, a money supply increase leads to higher prices and thus to lower real wages. This leads to lower production and a current account deficit. The monopsonistic market structure allows us to study both mirror images.

We distinguish between three different forms of price stickiness. We start with retail prices being fixed, by which we mean that prices are fixed in the consumers' currency. This is the sort of price stickiness that is traditionally assumed in Keynesian models such as Mundell (1961), (1963) and Dornbusch (1976). We then go on to analyse the implications of wholesale price stickiness, which we define as prices being fixed in the producers' currency. We show, this formulation is actually equivalent to the formulation in Obstfeld and Rogoff (1995) where producers can not price discriminate between countries. Finally, we compare these types of price stickiness with sticky wages.

The link between monetary policy and exchange rate volatility has drawn new attention. Mussa (1976) and (1986) first argued that the increased volatility of the real exchange rate in the post Bretton Wood period has to be explained by sluggish price adjustment and increased volatility of monetary disturbances. In contrast, Stockman (1988) proposed that the increased volatility is due to increased volatility of productivity shocks. Monetary models of exchange rates were further discredited when Meese and Rogoff (1983) showed that these models could not explain exchange rate movements. Recent research using VAR techniques has drawn attention back to monetary shocks. It has been shown that monetary disturbances explain a significant part of nominal and real exchange rate volatility, see e.g. Clarida and Gali (1994) and Eichenbaum and Evans (1995).

In our theoretical model the nominal exchange rate moves immediately no matter whether wages of whether wages, wholesale or retail prices are sticky. Under sticky wholesale prices, it jumps by less than the magnitude of the monetary expansion and immediately reaches its new steady state value. In contrast, under sticky retail prices the exchange rate jumps by more than the monetary expansion and returns to the old steady state level in the long-run. If wages are sticky, the exchange rate moves more than the money supply. The exchange rate immediately reaches its new steady state as in the case of sticky wholesale prices. Given sticky retail prices, the volatility of the real exchange rate, as measured by the relative price of a consumption basket in the two countries, displays the same volatility as the nominal exchange rate. This is in line with the empirical findings of Rogoff (1996). Under sticky wholesale prices and under sticky wages, the real exchange rate does not move at all because the law of one price always holds.

Empirical evidence about the spillover effects appears to be inconclusive. McKibbin and Sachs (1991) argue that the spillover effects of monetary policy on real variables are small while Canzoneri and Minford (1986) claim that they are reasonably big and negative. It is important to understand the size and direction of spillover effects before

one can discuss the need for international monetary coordination. Traditional Keynesian models predict a negative response of foreign output to domestic monetary expansions (e.g. Mussa (1979)). A depreciation in the home currency raises the price of foreign goods. This leads to a substitution away from foreign goods and to a reduction in production abroad. This is not necessarily true in the Obstfeld and Rogoff (1995) model because the income effect can potentially dominate the substitution effect. However, in their model welfare always increases in both countries no matter which country expands its money supply.

We show in this paper that both the size of spillover effects on foreign consumption and production and their direction depends crucially on the type of nominal stickiness assumed. Under sluggish wholesale prices Obstfeld and Rogoff's (1995) result is confirmed even though we do not assume the law of one price. On the other hand, if retail prices are sticky, the foreign country's welfare is unambiguously negatively affected by monetary expansions at home. The traditional Keynesian notion of "beggar thy neighbour" policies is reinstated. Foreign consumption is negatively correlated with a money expansions at home whilst the equilibrium labour input is positively correlated. Under sticky wholesale, prices the correlations of both consumption and production change from the short to the long-run. While consumption is initially positively affected by a foreign money expansion, it is negatively correlated in the long-run. The opposite is true for production. Spillover effects under sticky wages are very different from the effects under sticky prices. The effect is almost the mirror image of what happens under sticky wholesale prices. Foreign production is negatively correlated in the long-run to home money expansions. Consumption abroad declines in the short-run but increases in the long-run.

The empirically established J-curve effect shows that the trade balance is negatively correlated with current and future exchange rates while it is positively correlated with past exchange rates. In our model, the current account is initially positive if either of the two prices are sticky but turns out to be negative under wage stickiness. In the long-run the sign of the current account is reversed turning negative under sticky prices and positive under sticky wages. It is worthwhile noting that while the cross-correlation of the trade balance with the current exchange rate has different signs under sticky wages and sticky prices, the cross-correlation of the terms of trade and the trade balance is always positive. Even under sticky wages, where the exchange rate is negatively correlated the prices move far enough to allow the terms of trade to be positively correlated with the trade balance. Our findings extends the findings of Backus, Kehoe, and Kydland (1994) to monetary shocks. They found that while the J-curve effect can be reconciled with permanent productivity shocks, it is not possible to reconcile the negative correlation with fiscal shocks. In our model the efficiency gain of monetary disturbances is also only short-term even though they lead to permanent effects due to international lending.

The remaining paper is organised as follows. Section 2 introduces the model and Section 3 analyses the steady state. Section 4 discusses the effects of monetary disturbances under different kinds of price stickiness. Section 5 summarises the results and compares the effects of different types of price stickiness and real imperfections. Conclusions are

presented in Section 6. Proofs not presented in the text are in the appendix.

2 The model

2.1 Consumers' problem

The world is a 1×1 square in our model. A fraction n of the population lives in the home country and a fraction (1-n) abroad. There is also a continuum of firms on the interval [0,1]. A measure of n firms produce at home and a measure (1-n) in the foreign country. Home firms are symmetrically owned by home citizens and foreign firms by foreign citizens. Each inhabitant works in one firm located in his country but consumes the whole range of home and foreign produced goods. The group of potential workers for each firm is of measure one. All citizens maximise an additively separable utility function with a common discount rate δ ,

$$U = \sum_{t=1}^{\infty} \left(\frac{1}{1+\delta}\right)^t u(C_t^h, \frac{M_t^h}{p_t^h}, L_t^h).$$

As in Obstfeld and Rogoff (1996), the flow utility is Cobb Douglas in money and in the composite consumption good. The marginal disutility of labour is constant κ .

$$u(C_t^h, \frac{M_t^h}{v^h}, L_t^h) = \ln C_t^h(z) + \chi \ln \frac{M_t^h(z)}{v^h} - \frac{\kappa}{2} L_t^h(z)^2$$

The citizens derive positive utility from holding real money in their own currency. Holding more cash saves them trips to their bank. The flow utility exhibits constant elasticity of substitution (CES) of ρ among the different commodities. The composite consumption good is, therefore, given by

$$C_t^h(z) = \left[\int_0^1 c_t^h(k,z)^{\frac{\rho-1}{\rho}} dk\right]^{\frac{\rho}{\rho-1}}$$
 and the price index is defined as

$$p_t^h = \left[\int_0^1 p_t^h(k)^{1-\rho} \right]^{\frac{1}{1-\rho}}.$$

The superscript h refers to the home country and f to the foreign country.

The budget constraint for an individual agent of type z is given by

$$p_t^h C_t^h + p_t^h \frac{1}{1+r_t} B_t^h + M_t^h = L_t^h(z) w_t^h(z) + \pi_t^h + M_{t-1}^h + p_t^h B_{t-1}^h - p_t^h \tau_t^h,$$

where τ_t^h are real government transfers, B_t^h denotes the face value of bond holdings between period t to t+1. Given the interest rate r_t the present value of the bond is $\frac{1}{1+r_t}B_t^h$. w is the nominal wage and π_t^h is the share of profits from home firms that the agent holds stocks of.

As in Obstfeld and Rogoff (1995) (1996) citizens are not allowed to trade their shares of the firms. However they can trade real bonds in order to smooth their consumption. Agents choose their labour supply, their consumption stream, their money holdings and their bond holdings.

The government's revenue results from seigniorage. We will assume throughout this analysis that the government balances its budget in each period.¹

¹We do not really have to assume this. As long as the government spends all its revenue on transfers or buys the same consumption baskets as the economy's agents, Ricardian equivalence in the model ensures that a temporary deficit or surplus has no effect.

$$M_t^h - M_{t-1}^h = p_t^h \tau_t^h$$

The consumption side is identical to the formulation used by Obstfeld and Rogoff (1996).

2.2 Firms' problem

As in the standard framework, we assume that companies are monopolistic competitors in the goods market. Each good k is produced by firm k only. Furthermore we assume that each company is a monopsonist in the labour market. This is one crucial assumption that leads to very different dynamics in our model under sticky wages compared to the standard framework. The dynamics under sticky prices is largely unaffected by this assumption. We believe that there is empirical evidence suggesting that this is a reasonable assumption. The market power is typically with the employers rather than with the employees (Manning 1995). Therefore, it can be misleading to shift the market power to the workers for modelling purposes.

For the price setting, we assume that producers can differentiate between foreign and home markets. The production function for an individual home firm k takes the simple constant returns form

 $y^{hh}(k) = L^{hh}(k)$ for the home market h and

 $y^{hf}(k) = L^{hf}(k)$ for the foreign (export) market f.

The firm k maximises its profit $\pi^h(k)$, which depends not only on the prices it sets but also on the exchange rate E

 $\max_{L^{hh},L^{hf}} \pi^h(\bar{k}) = p^h(k)L^{hh}(k) + Ep^f(k)L^{hf}(k) - w^h \left(L^{hh}(k) + L^{hf}(k)\right),$

subject to

home goods demand: $p^h(k) = p^h(k; L^{hh}(k)),$ foreign goods demand: $Ep^f(k) = Ep^f(k; L^{hf}(k)),$ labour supply: $w^h = w^h(L^{hh}(k) + L^{hf}(k)).$

In the next section we solve the consumers' and producers' optimisation problem by assuming that both prices and wages are flexible.

3 **Steady State Analysis**

We analyse the steady state by assuming that all prices are flexible. Maximising the consumers' utility and the entrepreneurs' profits in this setting leads us to a system of equations that determines the equilibrium.

Proposition 1 The symmetric equilibrium of the economy is fully determined by the following eight equations and their foreign counterparts. (all variables are per capita)

1.
$$C_{t+1}^h(z) = \left(\frac{1+r_t}{1+\delta}\right) C_t^h(z)$$
 (consumption Euler equation),

2.
$$\frac{M_t^h(z)}{p_t^h}=\chi C_t^h \frac{1+i_t^h}{i_t^h}$$
, where $1+i_t^h=\frac{p_{t+1}^h}{p_t^h}\left(1+r_t\right)$ (money demand),

3.
$$L_t^h = \frac{1}{\kappa} \frac{1}{C_t^h} \frac{w_t^h}{p_t^h}$$
 (labour supply),

4.
$$p_t^h = \left[np_t^h(h)^{1-\rho} + (1-n)p_t^h(f)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$
 (price index),

5.
$$C_t^h = \frac{p_t^h(h)}{p_t^h} L^{hh} + \frac{Ep_t^f(h)}{p_t^h} L^{hf} + B_{t-1}^h - \frac{B_t^h}{1+r_t} (budget \ constraint),$$

6.
$$L_t^{hh} = \left[\frac{p_t(h)}{p_t^h}\right]^{-\rho} nC_t^h$$
, $L_t^{hf} = \left(\frac{p_t^f(h)}{p_t^f}\right)^{-\rho} (1-n)C_t^f$ (goods demand for home and export goods market),

7.
$$L_t^h = L_t^{hh} + L_t^{hf}$$
 (total labour demand),

8.
$$L_t^{hh} = \left(2\frac{\rho}{\rho-1}\frac{w_t^h}{p_t^h}\right)^{-\rho}nC_t^h$$
, $L_t^{hf} = \left(2\frac{\rho}{\rho-1}\frac{w_t^h}{E_tp_t^f}\right)^{-\rho}(1-n)C_t^f$ (labour demand for home and export goods market).

This system of equations is almost identical to the system in Obstfeld and Rogoff (1996). The only differences occur in the labour supply and demand equation as well as in the goods supply equation. We give entrepreneurs monopsonistic power in the labour market, thereby reducing the labour demand by a factor of 2^{ρ} . The reduced supply enables the entrepreneurs to charge a mark up that is double the one that Obstfeld and Rogoff (1996) find. Additionally we allow firms to discriminate in prices between home and foreign markets, i.e. they can choose the labour input that serves the domestic and export markets separately. The consumers' CES utility function leads to a simple mark up pricing by firms. A comparison of the goods and the labour demand functions (equation 6 and 8) shows that entrepreneurs always set prices that are higher by a factor of (2^{p-1}) than the production costs. Since the costs of serving the two markets are determined by the home wage, the price firms charge in the two countries is the same. Effectively a Purchasing Power Parity (PPP) or a no arbitrage condition holds even though it has not been assumed $(Ep^f(h) = p^h(h))$. This fact is proven formally in the next lemma.

Lemma 1 Purchasing Power Parity $(p^h = Ep^f)$ holds when prices and wages are flexible, even though firms could price discriminate.

Proof. The firm's profit maximisation problem is given by $\max_{L^h,L^{hh}} L^{hh} p^h(h) + (L^h - L^{hh})(p^f(h)E) - wL^h$ subject to

(1) inverse goods demands in both countries

$$p^{h}(h) = \left(\frac{nC^{h}}{L^{hh}}\right)^{\frac{1}{\rho}} p^{h} \text{ and } p^{f}(h) = \left(\frac{nC^{h}}{L^{hh}}\right)^{\frac{1}{\rho}} p^{f} \text{ and}$$
(2) labour supply function

 $w^{h} = \frac{1}{\kappa} \frac{C^{h}}{L^{h}} p^{h}.$ The first order conditions (FOC) are given by $(p^{h}(h) - p^{f}(h)E) + L^{hh} \frac{\partial p^{h}(h)}{\partial L^{hh}} - L^{hf} E \frac{\partial p^{f}(h)}{\partial L^{hf}} = 0$

 $p^f(h)E - w - L^h \frac{\partial w^h}{\partial L^h} = 0$

The assumption of the constant elasticity utility function ensures that the demand functions are isoelastic.

 $\frac{\partial p^h(h)}{\partial L^{hh}} \frac{L^{hh}}{p^h(h)} = L^{hf} E \frac{\partial p^f(h)}{\partial L^{hf}} \frac{L^{hf}}{Ep^f(h)} = -\frac{1}{\rho}$ Substituting these relations into the second and third terms of the first FOC shows that the relative price that ensures the optimal allocation between foreign and home market, is given by

$$p^h(h) = Ep^f(h).$$

As long as the first FOC holds, firms set the same price in both markets. Since this holds for all individual prices it is also valid for the price indices. Hence, as long as prices are flexible, PPP holds even though it is not assumed.

However, we will see in Lemma 5 presented in the next section that purchasing power parity need not hold if certain prices are sticky.

In an international equilibrium the bond market has to clear, i.e. $nB_t^h = (1-n)B_t^f =$: B_t. The international bond market can be thought of as follows. Consumers submit demand schedules to an international intermediary. These schedules specify how many consumption baskets they are willing to lend or borrow for a given interest rate. The international intermediary determines the interest rate such that the bond market clears and collects and delivers the consumption baskets.

It is difficult to determine the steady state of the economy unless we assume that bond holdings are internationally balanced. Hence, we adopt the strategy of determining the symmetric steady state and later on log-linearise the system of equations of Proposition 1 around this steady state.

Proposition 2 The symmetric steady state in which the bond holdings are internationally balanced is given by

1.
$$\bar{L}_0^h = \bar{L}_0^f = \bar{C}_0^h = \bar{C}_0^f = \sqrt{\frac{1}{\kappa} \frac{1}{2} \frac{\rho - 1}{\rho}}$$

2.
$$\bar{r}_0 = \delta$$
,

3.
$$\bar{p}_0^h = \frac{\bar{M}_0^h}{\chi} \frac{1}{\bar{L}^h} \frac{\delta}{1+\delta} = \frac{\bar{M}_0^h}{\bar{M}_0^f} \bar{p}_0^f$$

4.
$$\bar{w}_0^h = \frac{1}{2} \frac{\rho - 1}{\rho} \bar{p}_0^h = \frac{\bar{p}_0^h}{\bar{\sigma}^f} \bar{w}_0^f$$
,

5.
$$\hat{E}_0 = \frac{\bar{p}_0^h}{\bar{p}_0^f} = \frac{\bar{M}_0^h}{\bar{M}_0^f}$$
.

Proof. see annex.

The scale of production is reduced and the real wage is depressed due to the market imperfections inherent in monopolistic goods market and monopsonistic labour markets. The real interest rate is entirely determined by the exogenous time preference of the agents and the exchange rate solely depends on the relative money supply. Money is neutral in this economy and does not have any effect on real variables.

The mark up $\bar{p}_0^h = 2 \frac{\rho}{\rho - 1} \bar{w}_0^h$ in our model is twice as high as in Obstfeld and Rogoff (1996). Because companies are able to use their market power to set wages, they set them too low. This in turn leads to a lower scale of production by a factor of $\sqrt{2}$.

As mentioned earlier we log-linearise the model around the symmetric steady state. \hat{x} approximates the percentage change from the symmetric steady state. We drop the subscript t from all equations which apply only within a period.

Lemma 2 The log-linearized system of equations around the symmetric steady state with B=0 is given by

1.
$$\widehat{\overline{C}}_{t+1}^h = \widehat{\overline{C}}_t^h + \frac{\delta}{1+\delta} \hat{r}_t$$
 (consumption Euler equation),

2.
$$\hat{M}_t^h - \frac{\hat{p}_t^h}{\hat{p}_t} = \frac{\hat{C}}{\hat{C}_t}^h - \frac{\hat{r}_t}{1+\delta} - \frac{\hat{p}_{t+1}^h - \hat{p}_t^h}{\delta}$$
 (money demand),

3.
$$\widehat{\overline{L}}^h = -\widehat{\overline{C}}^h + \hat{w}^h - \hat{p}^h$$
 (labour supply),

4.
$$\widehat{\overline{p}}^h = n\widehat{\overline{p}}^h(h) + (1-n)\widehat{\overline{p}}^h(f)$$
 (price index),

5.
$$\widehat{\overline{C}}^h + \widehat{\overline{p}}^h = \widehat{\overline{L}}^h + n\widehat{\overline{p}}^h(h) + (1-n)\left(\widehat{\overline{p}}^h(f) + \widehat{\overline{E}}\right) + \frac{1}{n}\frac{\delta}{1+\delta}\frac{dB}{\widehat{C}_h^h}$$
 (budget constraint),

6.
$$\widehat{\overline{L}}^{hh} = -\rho \left(\widehat{\overline{p}}^h(h) - \widehat{\overline{p}}^h\right) + \widehat{\overline{C}}^h$$
, $\widehat{\overline{L}}^{hf} = -\rho \left(\widehat{\overline{p}}^f(f) - \widehat{\overline{p}}^f\right) + \widehat{\overline{C}}^f$ (goods demand for home and export market),

7.
$$\widehat{\overline{L}}^h = n\widehat{\overline{L}}^{hh} + (1-n)\widehat{\overline{L}}^{hf}$$
 (total labour demand),

8.
$$\widehat{\overline{L}}^{hh} = -\rho \left(\widehat{\overline{w}}^h - \widehat{\overline{p}}^h\right) + \widehat{\overline{C}}^h$$
, $\widehat{\overline{L}}^{hf} = -\rho \left(\widehat{\overline{w}}^h - \widehat{\overline{p}}^f - \widehat{\overline{E}}\right) + \widehat{\overline{C}}^f$ (labour demand for home and export market).

The log-linearisation allows us to understand the reaction of the economy to exogenous wealth and money shocks. We will use the equations later in order to determine the long-run effects of monetary expansions if either wages or prices are sticky in the short-term. For convenience we first determine the difference in the growth rates of domestic and foreign variables and only later determine the growth rates of individual countries' consumption and production.

Proposition 3 A one time redistribution of the bond holdings by dB does not affect aggregate world consumption or production but leads to the following permanent changes in home consumption, home employment, exchange rate and terms of trade.

1.
$$\widehat{\overline{L}}^w = \widehat{\overline{C}}^w = 0$$
.

$$2. \ \widehat{\overline{C}}^h = \widehat{\overline{C}}^w + (1-n)(\widehat{\overline{C}}^h - \widehat{\overline{C}}^f) = \frac{1+\rho}{2\rho} \frac{1}{n} \frac{\delta}{1+\delta} \frac{dB}{\overline{C}_0^h},$$

3.
$$\widehat{\overline{L}}^h = \widehat{\overline{L}}^w + (1-n)(\widehat{\overline{L}}^h - \widehat{\overline{L}}^f) = -\frac{1}{2}\frac{1}{n}\frac{\delta}{1+\delta}\frac{dB}{\overline{C}_0^h},$$

4.
$$\widehat{\overline{E}} = \left[\hat{M}^h - \hat{M}^f \right] - \frac{1+\rho}{2\rho} \frac{1}{n(1-n)} \frac{\delta}{1+\delta} \frac{dB}{\overline{C}_0^h},$$

$$5. \ \widehat{\overline{p}}^h - \widehat{\overline{E}} - \widehat{\overline{p}}^f = \widehat{\overline{w}}^h - \widehat{\overline{E}} - \widehat{\overline{w}}^f = \frac{1}{2\rho} \frac{1}{n(1-n)} \frac{\delta}{1+\delta} \frac{dB}{G_n^h}.$$

Proof. see annex.

Home agents consume more as a reaction to an exogenous wealth transfer towards the home country. The extent of the increase in consumption depends positively on the substitutability of home and foreign goods. Consumption does not change as much as the income from bond holdings since agents also choose to work less. The home wage rises relative to the foreign wage and the exchange rate falls to lower the price of foreign goods at home and to increase the price of home goods abroad. Thus the foreign country is able to repay its debt. Not surprisingly, an exogenous change in the money supply does not affect any real variables. The exchange rate moves according to the relative money supply in the two countries.

4 Nominal rigidities

So far we have kept prices and wages flexible and have found that a money supply shock has no real effect. It only alters the nominal prices, wages and the exchange rate. In other words, with flexible prices and wages, money is "neutral" and since a money shock does not change the dynamics, it is even "super-neutral".

This changes fundamentally if we assume a sluggish price adjustment. With sticky prices a money shock will not only affect the short-run real variables but will also cause the economy to settle in a different steady state. We will look at a situation where in period zero the economy is in the symmetric steady state as described by Proposition 2. In period one a monetary supply shock occurs but nominal wages/prices are held fixed for that period. In period two all nominal prices and wages adjust and the economy reaches its new steady state. The new steady state can be characterised by the new levels of bond holdings and money supplies (B, M^h, M^f) .

We distinguish between three different types of price stickiness:

- nominal retail price stickiness,
- nominal wholesale price stickiness and
- nominal wage stickiness.

Retail prices are the prices that are paid by the consumers in the two countries. By wholesale prices we mean the prices the producers charge in their own currency.

We follow the methodology developed in Obstfeld and Rogoff (1995) in deriving the dynamic equilibrium with nominal rigidities. We log-linearise the system around the symmetric steady state to find out the short-term dynamics and take into account the fact that certain prices are fixed between period zero and one. We denote the first order percentage change of a variable x in the shock period by \hat{x} .

The economy reaches its new steady state in period two. As in the previous section we denote the percentage deviation between the new steady state and the original symmetric steady state by \widehat{x} . After the money shock at the beginning of period one, agents adjust their net international bond holdings B immediately. From period two onwards all variables stay constant. Bond holdings do not change from period one to period two because agents hold their net wealth constant.² Any steady state of the economy is fully characterised by the money supply and the international bond holdings (the only real state variables). Therefore, the steady state from period two onwards is the same as the steady state under flexible prices if

- (1) the money supply changes in the same way, and
- (2) the bond holdings are exogenously changed to the levels that endogenously arise under price stickiness.

If one knows the money shock and the endogenous redistribution of bonds, the change in period two can be fully characterised by the long-run relationships in Proposition 3.

Because of intertemporal nature of the model, solving for the short-run involves also the long-run changes in the variables consumption \hat{c} , the price index \hat{p}^h and the interest rate \hat{r} . The money demand depends on future price levels and agents want to smooth their consumption path. To determine the short-run changes we will hence need in addition to the equations in Lemma 3 the long-run budget constraint and the linearised long-run money demand equation from Lemma 2.

Lemma 3 For a given form of price/wage stickiness the log-linearized system of equations around the symmetric steady state with B=0 is given by

1.
$$\widehat{\overline{C}}^h = \hat{C}^h + \frac{\delta}{1+\delta} \hat{r}$$
 (consumption Euler equation),

2.
$$\hat{M}^h - \hat{p}^h = \hat{C}^h - \frac{\hat{r}}{1+\delta} - \frac{\hat{p}^h - \hat{p}^h}{\delta}$$
 (money demand),

3.
$$\hat{L}^h = -\hat{C}^h + \hat{w}^h - \hat{p}^h$$
 (labour supply),

4.
$$\hat{p}^h = n\hat{p}^h(h) + (1-n)\hat{p}^h(f)$$
 (price index),

5.
$$\hat{C}^h + \hat{p}^h = \hat{L}^h + n\hat{p}^h(h) + (1-n)\left(\hat{E} + \hat{p}^f(h)\right) - \frac{1}{n}\frac{dB}{Ch}$$
 (budget constraint),

²Unlike Obstfeld and Rogoff (1995) we define B_t as the face value of the bond. Obstfeld and Rogoff (1995) denote the bond price by F_t . In their formulation F_t would jump twice since the interest paid out in period 2 differs from the steady state interest payments. Nevertheless, log-linearisation around $\bar{F} = 0$ makes the difference of the interest payments in the first two steady state periods of second order. Hence, it does not enter the calculations in Obstfeld and Rogoff (1995).

6.
$$\hat{L}^{hh} = -\rho \left(\hat{p}^h(h) - \hat{p}^h\right) + \hat{C}^h$$
, $\hat{L}^{hf} = -\rho \left(\hat{p}^f(h) - \hat{p}^f\right) + \hat{C}^f$ (goods demand for home and foreign market),

- 7. $\hat{L}^h = n\hat{L}^{hh} + (1-n)\hat{L}^{hf}$ (total labour demand),
- 8. (labour demand equations are replaced by equations which vary with the form of price stickiness).

The labour demand equation in lemma 2 is replaced by $\hat{p}^h(h) = \hat{p}^h(f) = 0$ in the case of sticky retail prices. Under sluggish wholesale prices, i.e. when prices are sticky in the producers' currency, the additional equation is given by $\hat{p}^h(h) = \hat{p}^f(f) = 0$. Similarly, if wages are sticky, it is given by $\hat{w}^h = \hat{w}^f = 0$.

The labour demand equation also varies depending on the form of price stickiness. With both forms of price stickiness, the monopolists always supply the goods demand as long as they earn a positive mark up. The monopolists need not be concerned that additional supply reduces the price. The labour demand, therefore, results directly from the goods demand equation. In the case of sticky prices, the labour demand is determined by the labour supply at this fixed wage.

Note that the budget constraint in the short-run differs from the long-run budget constraint. Fixing the prices or wages leads to a temporary change in real income which agents smooth by saving or dissaving in the international bond market.

Before we go on to Sections 4.1 - 4.3 to explicitly analyse the effect of monetary shocks under the three forms of price stickiness, we derive some qualitative result.

The nominal interest rates are the same in period one regardless of the form of price stickiness. Lemma 4 also shows that the inflation rate from period one to period two has to be the same in both countries.

Lemma 4 Both countries always face the same ex ante nominal interest rate $i^h=i^f$. Furthermore, they experience the same inflation rates between period one and period two. Thus $\left(\widehat{\overline{p}}^h-\widehat{p}^h\right)=\left(\widehat{\overline{p}}^f-\widehat{p}^f\right)$.

Proof. In the steady state, the nominal interest rate coincides with the real interest rate. Both countries always face the same real interest rate. This is also true in the shock period. Hence, taking the difference between the home and foreign consumption Euler equations, we conclude that the consumption differentials are constant in time. Thus it is

 $\widehat{\overline{C}}^h - \widehat{\overline{C}}^f = \hat{C}^h - \hat{C}^f.$

Subtracting the difference of the home and foreign long-run money demands from the short-term money demand differential, we find that the short-run differential of nominal interest rates is given by

 $\hat{\imath}^h - \hat{\imath}^f = (1 + \delta) \left[(\hat{p}^h - \hat{p}^f) - \left(\widehat{\overline{p}}^h - \widehat{\overline{p}}^f \right) \right].$

Given the definition of the nominal interest rate, the differential of interest rate changes is given by

 $\hat{i}^h - \hat{i}^f = -\frac{(1+\delta)}{\delta} \left[(\hat{p}^h - \hat{p}^f) - \left(\widehat{\overline{p}}^h - \widehat{\overline{p}}^f \right) \right].$

A comparison of the last two equations gives the result.

Since real interest rates are the same in both countries, the ratio of the home to foreign consumption levels $\frac{C_l^h}{C_l^t}$ is the same in the short-run and in the long-run. The same is true for the ratio of money supplies $\frac{M_l^h}{M_l^t}$. Lemma 4 illustrates that both countries experience the same price changes from period one to period two. Suppose this were not the case and inflation were higher at home than abroad. This would affect the money equilibrium both in the short-run and in the long-run. On the one hand, the short-run money demand at home would be depressed relative to the foreign one due to the higher opportunity costs of money holding. On the other hand, inflation would increase the price level in period two. This loss in the value of money leads to a higher nominal money holding in the long-run. The proof of Lemma 4 shows that these two opposing effects can only be reconciled with the constant relative consumption and money ratios if the inflation from period one to period two is the same in both countries.

The next lemma analyses whether PPP, which holds under flexible prices, still applies when price stickiness is assumed.

Lemma 5 In the long-run, purchasing power parity $(p^h = Ep^f)$ holds under any form of price stickiness. In the short-run, it still holds under sticky wholesale prices and under sticky wages but not under sticky retail prices.

Proof. In the long-run, firms can adjust their prices and the result that PPP holds under flexible prices applies (Lemma 1). If prices are not flexible, the first order condition becomes irrelevant in the short-term. Nevertheless, it is true that PPP holds under sticky wholesale prices. The argument is as follows. PPP holds in the initial steady state because prices and wages are flexible. In the shock period, the relative price of the same goods in the home and the foreign market moves only with the exchange rate. Hence, the no arbitrage condition continues to hold for each good and, therefore, also for the price levels.

This is obviously not true under fixed retail prices because the exchange rate moves in the shock period $(\hat{E} \neq 0)$. It is intuitively easy to understand why the exchange jumps under sticky retail prices. Under sticky retail prices, the price of consumption stays constant in the shock period. There is no substitution between home and foreign goods. Hence, production is the same in both countries. Now, suppose the exchange rate would not move. This would imply that home and foreign agents have the same real income and, therefore, there is no international borrowing. Consequently, they both consume the same amount. Both also face the same nominal interest rates (Lemma 4). Given all these symmetries, they would demand the same amount of real money. This cannot be an equilibrium because the money supply differs. (For an explicit proof see Proposition 6).

These two lemmas allow us to show that whether interest rate parity holds and the exchange rate overshoots depends on the price stickiness assumed.³

Proposition 4 Uncovered nominal interest rate parity holds under sticky wholesale prices and sticky wages but is violated under sticky retail prices.

Proof. The log-linearised interest rate parity $(1+i_t^h=\frac{E_{t+1}}{E_t}\left(1+i_t^f\right))$ in the shock period is given by

 $\hat{\imath}^h - \hat{\imath}^f = \frac{(1+\delta)}{\delta} \left[\widehat{\overline{E}} - \hat{E} \right].$

Given the definition of the nominal interest rate, the differential of interest rate changes is given by

 $\hat{\imath}^h - \hat{\imath}^f = \frac{(1+\delta)}{\delta} \left[\left(\widehat{\vec{p}}^h - \widehat{\vec{p}}^f \right) - \left(\hat{p}^h - \hat{p}^f \right) \right].$

Under sticky whole sale and sticky wages, we replace the price differentials by the exchange rate changes because PPP holds (Lemma 1 and 5). This proves the first part of the proposition.

Under sticky retail prices, the nominal interest rate differential can be written as

 $\hat{\imath}^h - \hat{\imath}^f = \frac{(1+\delta)}{\delta} \left[\widehat{\overline{E}} \right]$, since PPP is valid in the long-run. This shows that interest parity would only hold if the exchange rate does not change in the first period $(\hat{E}=0)$. This is not the case as the proof of the previous lemma shows.

Proposition 5 While the exchange rate overshoots its long-run value under sticky retail prices, it immediately reaches its new steady state value under sticky wholesale prices as well as under sticky wages.

Proof. Since the interest rate parity holds under sticky wholesale prices and wages (Proposition 4) and the nominal interest rates are the same (Lemma 4), it must be true that in these cases $\vec{E} = \hat{E}$.

From the proof of Lemma 5 for sticky retail prices, we know that the exchange rate jumps in the shock period $(\hat{E} \neq 0)$. Additionally, we know that the nominal interest rate differential and the long-run exchange rate is given by (Lemma 4, Proposition 4)

$$0 = \hat{\imath}^h - \hat{\imath}^f = \frac{(1+\delta)}{\delta} \left[\widehat{\overline{E}} \right].$$

 $0=\hat{\imath}^h-\hat{\imath}^f=\frac{\langle 1+\delta \rangle}{\delta}\left[\widehat{\overline{E}}\right].$ The long-run exchange rate coincides with the initial exchange rate. This completes the proof.

Intuitively, under sticky retail prices the exchange rate has to return to its original level since PPP holds in both steady states and inflation from period zero to period two is the same in both countries. From period zero to period one, inflation is zero due to retail price stickiness. Lemma 4 shows that both countries experience the same inflation

³Betts and Devereux (1996) also consider a model in which firms price discriminate between home and foreign markets. However, their model is de facto static since they do not allow for international bond trading. They find a one-off jump in the exchange rate but no overshooting. The increase in the exchange rate exceeds the one in Obstfeld and Rogoff (1995). The authors claim that the difference is due to pricing to market while we show that it is due to different forms of price stickiness.

rate from period one to period two.

In the following three subsections we analyse the dynamics of the model in more detail assuming in turn one of the three prices to be sticky.

4.1 Sticky retail prices

In this subsection we assume that prices are sticky in the consumers' currency $(\hat{p}^h(h))$ $\hat{p}^f(h) = \hat{p}^f(f) = \hat{p}^h(f) = 0$. These four equation together with Lemma 3 allow us to calculate explicitly the dynamics of the two countries' economies if one or both of them expand their money supply. Specifically, we can analyse spillovers of one country's money expansion on production and consumption abroad.

Proposition 6 Under sticky retail prices, money supply shocks give rise to an endogenous change in international net bond holdings given by

$$\frac{dB}{Ch} = \frac{2\rho(1+\delta)}{(1+\rho)\delta} n (1-n) \left[\hat{M}^h - \hat{M}^f \right]$$

 $\frac{dB}{C_0^h} = \frac{2\rho(1+\delta)}{(1+\rho)\delta} n \ (1-n) \left[\hat{M}^h - \hat{M}^f \right].$ Changes in each country's consumption, production, exchange rates and terms of trade are given by

• in the short-run

$$\begin{split} \hat{C}^h &= \hat{M}^h, \\ \hat{L}^h &= \hat{M}^w = n\hat{M}^h + (1-n)\hat{M}^f, \\ \hat{E} &= \left(1 + \frac{2\rho}{(1+\rho)\delta}\right) \left[\hat{M}^h - \hat{M}^f\right], \\ \hat{w}^h - \hat{E} - \hat{w}^f &= -\frac{2\rho}{(1+\rho)\delta} \left[\hat{M}^h - \hat{M}^f\right], \\ \hat{r} &= -\left(\frac{1+\delta}{\delta}\right) \hat{M}^w, \end{split}$$

• in the long-run

$$\begin{split} &\widehat{\overline{C}}^h = (1-n) \left[\hat{M}^h - \hat{M}^f \right], \\ &\widehat{\overline{L}}^h = -\frac{\rho}{(1+\rho)} (1-n) \left[\hat{M}^h - \hat{M}^f \right], \\ &\widehat{\overline{E}} = 0, \\ &\widehat{\overline{p}}^h(h) - \widehat{\overline{E}} - \widehat{\overline{p}}^f(f) = \widehat{\overline{p}}^h(h) - \widehat{\overline{p}}^f(f) = \frac{1}{1+\rho} \left[\hat{M}^h - \hat{M}^f \right], \\ &\widehat{\overline{w}}^h - \hat{E} - \widehat{\overline{w}}^f = \frac{1}{1+\rho} \left[\hat{M}^h - \hat{M}^f \right], \\ &\widehat{p}^h = n \hat{M}^h + (1-n) \hat{M}^f. \end{split}$$

Proof. see annex.

To grasp the intuition more easily, let us consider the special case that there is a money expansion at home while the foreigners keep their money supply constant.

Money holding and consumption at home increase by the same degree since consumers' preferences are homothetic between real money holding and consumption. Note the relative price between real money and consumption is equal to one by definition. Consumers do not substitute between different products since the retail prices stay the same. The additional income which is necessary to afford the higher consumption comes from two sources. First, a positive money shock reduces real prices. At lower real prices, consumers demand more goods and producers, having lost their price setting power, are willing to meet the demand. This leads to lower deadweight losses and higher consumer surplus. Second, the exchange rate jump allows domestic exporters to earn more from their exports. They sell their products at the same foreign retail price and convert the revenues into the home currency at a more favourable exchange rate. Their income increases in real terms since the domestic consumer prices do not change.

A money shock at home affects the foreign economy as well. Whereas the reduction of monopolistic distortions generates some additional consumer surplus, the second source of income is just a redistribution from foreign consumers to home consumers. For foreigners, who export to the home country, an increase in the exchange rate reduces their returns in the foreign currency. Consumers in the home country do not only demand more homeproduced goods but also more foreign made products (by the same degree). They do not substitute between home-made goods and imported goods since the retail prices are fixed. Higher demand for foreign goods combined with sticky prices reduces the monopolistic deadweight loss abroad as well. Consequently, production increases abroad too. The percentage increase in production is the same in both countries. This is due to the absence of substitution between the goods. More production at home and abroad might suggest higher income for foreigner too. However, as indicated above, the large jump in the exchange rate diminishes their real revenues from exporting to the home country. Their exports measured in terms of the number of goods increases but their revenues in their own foreign currency decline. This redistributional effect makes the foreigners worse off. In equilibrium they have to work harder in order to export more goods but their real revenues decline. In summary, an unexpected money expansion at home is beneficial for home citizens but it reduces welfare for foreigners. Therefore, in a world with sticky retail prices, a central bank always has an incentive to increase money supply. This explains the well known strategy "beggar thy neighbour" conducted by many industrialised countries in the beginning of this century. All countries increased their money base in order to profit from the others (Nurkse 1944).

The better off home citizens try to smooth their additional income and, therefore, buy bonds from foreigners at a low real interest rate. This allows foreigners to keep their consumption constant. From period two onwards the prices adjust and, hence, each monopolist will restrain its output in order to achieve higher prices. The trade balance surplus of the home country in period one leads to a trade balance deficit from period two onwards since foreigners have to pay interest for the borrowed amount. Consequently,

⁴Note that this effect is mitigated if agents expect the central bank to increase the money supply.

foreigners have to produce more and consume less in the long-run, whereas home citizen enjoy lower production and higher consumption.

Reduced production at home makes home-produced goods relatively more scarce and, thus, improves the terms of trade for the home country in the long-run. The exchange rate displays very strong short-term volatility. It jumps up in the short-run but comes back to its original level in period two. After a money expansion at home, the change in the exchange rate exceeds the change in the money supply. In other words, the exchange rate increase is larger than in the case of flexible prices. This overshooting is in line with the seminal work of Dornbusch (1976). And indeed Dornbusch (1976) also assumes sticky retail prices. In period two the exchange rate bounces back to its original level. This seems surprising given the fact that the home money supply is higher in the new steady state. Sticky prices, therefore, explain the excess volatility observed for exchange rates.

4.2 Sticky wholesale prices

Whereas in the former section the prices were fixed for the consumers, with sticky wholesale prices the prices are fixed in the producer's currency. Just like in the last section, entrepreneurs, knowing that they have no influence on the price, are happy to meet the additional demand. Nevertheless, the implications of monetary shocks are fundamentally different. Similar to the case of sticky retail prices, the labour demand is replaced by a fixed nominal prices in producers' currency $(\hat{p}^h(h) = \hat{p}^f(f) = 0)$. As the proof of Lemma 5 shows, PPP holds both for the price indices $(\hat{p}^h = \hat{E} + \hat{p}^f)$ and for individual goods prices $(\hat{p}^h(h) = \hat{E} + \hat{p}^f(h), \hat{p}^h(f) = \hat{E} + \hat{p}^f(f))$. This implies a very different reaction of the economy to a monetary shock.

Proposition 7 Under sticky wholesale prices money supply shocks give rise to an endogenous change in international net bond holdings given by

$$\frac{dB}{\hat{C}_o^h} = \frac{2(\rho-1)(1+\delta)}{(\rho+1)\hat{\delta}+2}n(1-n)\left[\hat{M}^h - \hat{M}^f\right].$$

 $\frac{dB}{C_0^h} = \frac{2(\rho-1)(1+\delta)}{(\rho+1)\delta+2}n(1-n)\left[\hat{M}^h - \hat{M}^f\right].$ Changes in each country's consumption, production, exchange rates and terms of trade are given by

$$\hat{C}^h = \underbrace{\frac{\rho \left[\delta(\rho+1) + 2n\right] - (1-n) \left[(1+\rho)\delta\right]}{\rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2n\right] - (1-n) \left[(1+\rho)\delta\right]}{\sqrt{\rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2n\right] + (1-n) \left[2\rho^2\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2n\right] + (1-n) \left[2\rho^2\right] \\ > 1}}} \hat{M}^h \underbrace{-(1-n)\frac{2(\rho-1)}{(\rho+1)\delta + 2}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(1+\rho)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(1+\rho)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta + 2\right]}}_{\substack{f \in \mathcal{C}^h = \frac{\rho \left[\delta(\rho+1) + 2\rho\right] \\ \rho \left[(\rho+1)\delta +$$

$$\begin{split} \hat{C}^w &= \hat{L}^w = \hat{M}^w, \\ \hat{r} &= -\left(\frac{1+\delta}{\delta}\right) \hat{M}^w, \end{split}$$

• in the long-run

$$\begin{split} \widehat{\overline{C}}^h &= \frac{1+\rho}{\rho} \frac{(\rho-1)\delta}{(\rho+1)\delta+2} (1-n) \left[\hat{M}^h - \hat{M}^f \right], \\ \widehat{\overline{L}}^h &= -\frac{(\rho-1)\delta}{(\rho+1)\delta+2} (1-n) \left[\hat{M}^h - \hat{M}^f \right], \\ \left[\widehat{\overline{p}}^h(h) - \widehat{\overline{E}} - \widehat{\overline{p}}^f(f) \right] &= \left[\widehat{\overline{w}}^h - \widehat{\overline{E}} - \widehat{\overline{w}}^f \right] = \frac{(\rho-1)\delta}{\rho((\rho+1)\delta+2)} \left[\hat{M}^h - \hat{M}^f \right], \\ \widehat{\overline{E}} &= \left(\frac{(\rho+1)\delta+2\rho}{\rho((\rho+1)\delta+2)} \right) \left[\hat{M}^h - \hat{M}^f \right] &= \hat{E}, \\ \widehat{\overline{C}}^w &= \widehat{\overline{L}}^w &= 0. \end{split}$$

Proof. see annex.■

Providing home country consumers with more money stimulates their demand for home-produced and foreign-produced goods. Whereas the consumer price for the home-made products is fixed for one period, the retail price for imported good changes with the exchange rate. The exchange rate goes up because the increased demand of foreign products raises the demand of foreign currency as well. This makes imported foreign products for home consumers more expensive and, thus, they will substitute them partly with home-made products.

An increase in home money supply affects home consumers' income in three ways. First, the higher demand for home-produced goods combined with fixed prices reduces the monopoly distortions and, thus, increases production and real income for consumers. Second, the increase in the exchange rate leads to higher export revenues. For given fixed wholesale prices, it makes home-produced goods relatively cheaper for foreigners. This boosts the number of exported goods. Third, the increased exchange rate also makes imported goods more expensive which not only leads to the above described substitution effect but also to a negative real income effect. The overall income effect on home consumption is positive.

Abroad, an increase in the exchange rate makes products from the home country cheaper as well. Therefore, even abroad foreign-produced commodities become less popular. This combined with the decline in export explains why production goes down. The higher the elasticity of substitution ρ , the larger the impact on foreign production. A lower level of production reduces their real income. On the other hand, foreigners' profit from lower import prices, resulting in lower inflation. In period one foreigners want to enjoy the low import prices reflected by the favourable terms of trade. Therefore, they sell real bonds to the home citizens, consume more, and work less in period one. In the long-run they have to pay interest to the home citizens. Therefore, in the new steady state they have to produce more and consume less in comparison to the original steady state. The terms of trade increase since home-produced goods are more scarce in the long-run. Nominal prices increase at home in period two and the exchange rate increase

is smaller than in the case of flexible prices. Note that the higher the elasticity of substitution between the goods, the smaller is the adjustment in the terms of trade through the exchange rate.

Even though we do not assume the law of one price, sticky wholesale prices lead to similar results as Obstfeld and Rogoff (1995).

4.3 Sticky wages

The labour demand equation is replaced by an assumption of fixed wages ($\hat{w}^h = \hat{w}^f = 0$). These equations together with Lemma 3 allow us to determine the dynamics explicitly. As under sticky wholesale prices, PPP holds again. In contrast to the fixed prices scenarios the scale of production is determined by the labour supply rather than by the goods demand. This has important implications specifically for the current account dynamics.

Proposition 8 Under sticky wages money supply shocks give rise to an endogenous change in international net bond holdings given by

$$\frac{dB}{Ch} = -\frac{2(\rho-1)(1+\delta)}{(1+\rho)\delta+2}n(1-n)\left[\hat{M}^h - \hat{M}^f\right].$$

 $\frac{dB}{C_0^h} = -\frac{2(\rho-1)(1+\delta)}{(1+\rho)\delta+2}n(1-n)\left[\hat{M}^h - \hat{M}^f\right].$ Changes in each country's consumption, production, exchange rates and terms of trade are given by

• in the short- run

$$\hat{C}^{h} = \underbrace{-\left(n + (1-n)\frac{\rho-1}{\rho}\frac{\left(\rho+1\right)\delta}{\left(\rho+1\right)\delta+2}\right)}_{<0}\hat{M}^{h}\underbrace{-\left(1-n\right)\left(1 - \frac{\rho-1}{\rho}\frac{\left(\rho+1\right)\delta}{\left(\rho+1\right)\delta+2}\right)}_{<0}\hat{M}^{f},$$

$$\begin{split} \hat{L}^h &= -\hat{M}^h, \\ \hat{E} &= \left(1 + \frac{\rho - 1}{\rho} \frac{(\rho + 1)\delta}{(\rho + 1)\delta + 2}\right) \left[\hat{M}^h - \hat{M}^f\right] = \widehat{\overline{E}}, \\ \hat{p}^h(h) - \hat{E} - \hat{p}^f(f) &= \frac{1}{\rho} \left[\hat{M}^h - \hat{M}^f\right], \\ \hat{w}^h - \hat{E} - \hat{w}^f &= \left(1 + \frac{\rho - 1}{\rho} \frac{(\rho + 1)\delta}{(\rho + 1)\delta + 2}\right) \left[\hat{M}^h - \hat{M}^f\right], \\ \hat{L}^w &= \hat{c}^w = -\hat{M}^w, \\ \hat{r} &= \left(\frac{1 + \delta}{\delta}\right) \hat{M}^w, \end{split}$$

• in the long-run

$$\begin{split} &\widehat{\overline{C}}^h = -\frac{\rho-1}{\rho} \frac{(\rho+1)\delta}{(\rho+1)\delta+2} (1-n) \left[\hat{M}^h - \hat{M}^f \right], \\ &\widehat{\overline{L}}^h = \frac{(\rho-1)\delta}{(\rho+1)\delta+2} (1-n) \left[\hat{M}^h - \hat{M}^f \right], \\ &\widehat{\overline{E}} = \left(1 + \frac{\rho-1}{\rho} \frac{(\rho+1)\delta}{(\rho+1)\delta+2} \right) \left[\hat{M}^h - \hat{M}^f \right] = \hat{E}, \\ &\left[\widehat{\overline{p}}^h(h) - \widehat{\overline{E}} - \widehat{\overline{p}}^f(f) \right] = \left[\widehat{\overline{w}}^h - \widehat{\overline{E}} - \widehat{\overline{w}}^f \right] = -\frac{(\rho-1)}{\rho(\rho+1)\delta+2\rho} \left[\hat{M}^h - \hat{M}^f \right], \\ &\widehat{\overline{L}}^w = \widehat{\overline{C}}^w = 0. \end{split}$$

Proof. see annex.

Increasing home money supply causes upward price pressure at home. Due to the stickiness of nominal wages, higher consumer prices result in lower real wages. Workers, therefore, substitute consumption for leisure and work fewer hours. The resulting contraction in the production of home-made products has at least two effects. First, it reduces the income for home citizens. In expectation of higher future income, they try to borrow from abroad and, therefore, push up the interest rate. Second, home-produced goods become more expensive. Consumers substitute them for imported foreign products. More demand for foreign products and, hence, foreign currency results in a higher exchange rate.

Though a high exchange rate should make imported home-produced goods cheaper abroad, the opposite happens because the price $p^h(h)$ skyrockets. The calculations of the terms of trade highlight this. Consequently, foreign consumers also substitute home-produced goods with foreign goods. Higher demand for their foreign products and higher prices for the imported goods increases their price index too. Foreigners reduce their consumption in favour of more savings. They lend a larger amount to the home citizens. The high real interest rates in period one makes it worthwhile for them to reduce their consumption but to keep their production constant, even though the real wages decline abroad too.

From period two onwards, foreigners will receive interest payments in the form of home-produced goods. Therefore, in the long-run production at home has to increase whereas consumption declines. The opposite is true abroad. Note that the setting with sticky wages replicates the empirical regularity known as the J-curve effect. It is often claimed that after an exchange rate appreciation the trade balance becomes negative for a while before bouncing back and leading to a long-run trade balance surplus. In period one the exchange rate and the trade balance are negatively correlated. However, the terms of trade and the trade balance are positively correlated.

5 Comparing different forms of price stickiness

The formal analysis in Section 4 demonstrates that different forms of price stickiness lead to strikingly different economic outcomes. In this section we compare the implication of a monetary shock for the case of sticky retail prices, sticky wholesale prices and sticky wages. Empirical evidence might then suggest which form of stickiness seems most plausible.

We restrict ourselves to the case of a positive money expansion in the home country. Due to the symmetry between both countries, the effects of a positive money supply shock abroad would mirror the effects. Similarly, a contraction of the money supply leads to the opposite effects. Given different price stickiness, a monetary shock does not only affect nominal variables differently but also affects real variables and the whole dynamics of the economy. The following figure illustrates the impulse response functions triggered by an unexpected money expansion at home.

A monetary expansion under sticky retail prices leads to more production at home

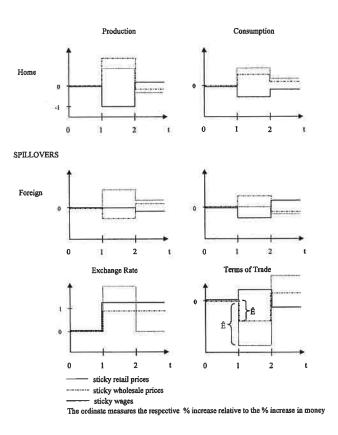


Figure 1: Impulse Response Functions

as well as to an equal increase in production abroad. This is the case since sticky retail prices prevent any substitution between home and foreign goods. With sticky wholesale prices foreign-made goods become more expensive relative to home-produced ones in both countries. The resulting substitution leads to a sharp increase in production at home and a reduction abroad.

The absolute level of production is not only affected by the substitution effect but also by the real income effects due to the unexpected money expansion at home. In both cases, consumers profit from a reduction in the monopolistic distortion due to price stickiness. The crucial distinction between both settings is that with sticky retail prices the large spike in the exchange rate boosts home citizens' revenue from exporting goods, while the consumer prices stay constant. This increases the real income of home citizens at the expense of the foreigners. With sticky wholesale prices nominal export revenues increase too, but so do the consumer prices for the important goods. This explains why in period one consumption by the home citizens increases more when retail prices are sluggish. They are also able to build up a larger trade balance surplus under sticky retail prices. This guarantees that the long-run consumption rise also exceeds the increase in the case of sticky wholesale prices.

The second row of graphs illustrates how the monetary expansion spills over to the foreign country. In the case of sticky retail prices, foreign producers meet the increased home demand. Nevertheless, their export revenue declines in their own foreign currency since the exchange rate increases. The money expansion has a negative wealth impact to foreigners. Since the interest rate is lower due to higher world production, foreigners sell international bonds in order to keep up with the consumption level that they are used to. In short, foreigners have to work harder, become debtors and consequently consume less in the long-run. The 'beggar thy neighbour' strategy is surely optimal in a setting with sluggish retail prices. On the other hand, sticky wholesale prices allow the foreigners to work less and consume more in period one. They can enjoy part of the additional consumer surplus due to reduced monopolistic distortions.

Whereas the effects for the two different forms of price stickiness are demand driven, the effects due to sticky wages are governed by the supply side. The economic implications of sticky wages are in sharp contrast to the outcomes of the other two settings. As outlined in Section 4.3, a money expansion and its resulting price increase leads to lower real wages. Workers work less and production declines. Consequently, they have to reduce their consumption in the short-run and in the long-run. Fewer home-made products and, thus, higher consumer prices reduce consumption abroad too. Nevertheless, foreigners keep up with their production stimulated by higher interest rates. They achieve a current account surplus which leads to a long-run increase in consumption. Interestingly, the current account surplus for the foreigners is exactly the same size as in the case of sticky wholesale prices where foreigners suffer a current account deficit.

In all three forms of price sluggishness, the size of the spillovers effects depends on the size of the home country. The model predicts that smaller countries are more vulnerable than larger countries to money supply shocks of neighbouring large countries.

However, the size of the countries has however no impact on the dynamics of the exchange rate or the terms of trade. The third row of graphs illustrates that with sticky

retail prices, the exchange rate skyrockets in period one and surprisingly comes back to its original level in period two. For the other two forms of stickiness, the exchange rate changes only once. Under flexible prices and wages, the exchange rate moves by the same degree as the money supply and the terms of trade are not affected. The terms of trade $\left[\widehat{\overline{p}}^h(h) - \widehat{\overline{E}} - \widehat{\overline{p}}^f(f)\right]$ represent the number of foreign goods one would receive in exchange for one home-produced good. Sticky wages lead to a larger exchange rate movement than sticky wholesale prices. The reason is that the exchange rate does not only accommodate the relative increase in the money supplies but, since money is not neutral, it also helps the terms of trade to adjust. In the case of sticky wholesale prices the terms of trade increase in the long-run, whereas they decrease under sticky wages in the long-run. Under sticky wholesale prices, the home country becomes the net creditor. Its terms of trade have to deteriorate and the exchange rate jumps by less than the money supply. Under sticky wages, the terms of trade have to move in the home country's favour. The nominal exchange rate jumps more than the money supply. An alternative definition of the terms of trade - which measure the competitiveness of domestic products abroad is best understood by the relative scarcity of the products. Hence, this definition follows immediately from the production activities in both countries. Both definitions coincide only as long as PPP holds. With fixed retail prices, this is not the case in period one.

6 Conclusion

The main message of this paper is that the form of price stickiness matters. Given the empirical regularities like the violation of PPP in the short-run and of the uncovered interest rate parity etc., it seems plausible that the stickiness of retail prices is very important. Retail price stickiness leads to the large spillover effects and reinstates the "beggar thy neighbour" policy. The sticky retail price analysis also suggests that there should be an international coordination of monetary policy.

Some further extensions are left for future research. It would be interesting to extend the analysis to a setting where monetary shocks occur with positive probabilities. An analysis along the lines of Obstfeld and Rogoff (1999) seems promising. We did not cover the case of asymmetric forms of price stickiness, such as when whole sale prices are sticky in the home country while abroad retail prices do not adjust. Some interesting insights might emerge from such an analysis. Introducing productivity shocks bundled with a certain form of price stickiness might lead to slightly different results, especially when the monetary policy cannot adjust immediately and lags the productivity shocks. Another worthwhile extension would be to find an appropriate empirical test that allows us to discriminate between different forms of price stickiness and to empirically estimate their relative importance.

Appendix A

A.1**Proof of Proposition 2**

Let us assume that labour and consumption are identical in the two countries. The consumption Euler equation as usual determines the real interest rate $r = \delta$.

The budget constraint in the symmetric steady state is given by

$$C^{h} = \frac{p^{h}(h)}{L}L^{hh} + \frac{Ep^{f}(h)}{L}L^{hf}$$

 $C^h = \frac{p^h(h)}{p^h} L^{hh} + \frac{Ep^f(h)}{p^h} L^{hf}$. Since the no arbitrage condition holds, it simplifies to

$$C^h = \frac{p^h(h)}{p^h} L^h$$
.

The labour market equilibrium and the world goods market equilibrium imply

$$L^{hh} + L^{hf} = L^h = L^f = L^{ff} + L^{fh}$$

$$L^{hh} + \frac{(1-n)}{n}L^{fh} = C^h = C^f = L^{ff} + \frac{n}{(1-n)}L^{hf}.$$

The last two equations imply that

$$nL^{hf} = (1-n)L^{fh}.$$

Since the capital account is balanced by assumption the current account has to be balanced $nL^{hf}Ep^{f}(h) - (1-n)L^{fh}p^{h}(f) = 0.$

which implies that the terms of trade are zero

$$p^h(h) - p^f(f)E = 0.$$

This implies for the price index that

$$p^h = p^h(h).$$

The labour supply equation together with the mark up formula and the budget constraint implies the scale of production

$$L^h = \sqrt{\frac{1}{\kappa} \frac{1}{2} \frac{\rho - 1}{\rho}} = L^f.$$

$$p^h = \frac{M^h}{x} \frac{1}{L^h} \frac{\delta}{1+\delta}$$

 $\begin{array}{l} L^h = \sqrt{\frac{1}{\kappa}\frac{1}{2}\frac{\rho-1}{\rho}} = L^f. \\ \text{The money demand equation is given by} \\ p^h = \frac{M^h}{\ell}\frac{1}{L^h}\frac{\delta}{1+\delta} \\ \text{Dividing this by the foreign equivalent leads to} \end{array}$

$$E = \frac{p^h}{p^f} = \frac{p^h(h)}{p^f(f)} = \frac{M^h}{M^f}.$$

Proof of Proposition 3 A.2

Taking the differences of the linearised equations of home and foreign variables allows us to write these as a function of the exogenous wealth transfer dB.

1.
$$\widehat{\overline{p}}^h - \widehat{\overline{E}} - \widehat{\overline{p}}^f = \widehat{\overline{w}}^h - \widehat{\overline{E}} - \widehat{\overline{w}}^f = \frac{1}{2\rho} \frac{1}{n(1-n)} \frac{\delta}{1+\delta} \frac{dB}{\overline{c}^h}$$

$$2. \ \ \widehat{\overline{C}}^h - \widehat{\overline{C}}^f = \frac{1+\rho}{2\rho} \frac{1}{n(1-n)} \frac{\delta}{1+\delta} \frac{dB}{\overline{C}_0^h},$$

3.
$$\widehat{\overline{L}}^h - \widehat{\overline{L}}^f = -\frac{1}{2} \frac{1}{n(1-n)} \frac{\delta}{1+\delta} \frac{dB}{\overline{C}_o^h}$$

$$4. \ \ \widehat{\overline{E}} = \left[\hat{M}^h - \hat{M}^f\right] - \frac{1+\rho}{2\rho} \frac{1}{n(1-n)} \frac{\delta}{1+\delta} \frac{dB}{\overline{C}_0^h}.$$

Adding the labour supply functions weighted by the country size and using the price

$$\widehat{\overline{L}}^{w} := n\widehat{\overline{L}}^{h} + (1-n)\widehat{\overline{L}}^{f} = -n\widehat{\overline{C}}^{h} - (1-n)\widehat{\overline{C}}^{f} = -\widehat{\overline{C}}^{w}.$$

Since world production and world consumption has to be equal it follows that $\widehat{\overline{L}}^w = \widehat{\overline{C}}^w = 0$.

The changes of consumption and labour are derived from

$$\widehat{\overline{C}}^h = \widehat{\overline{C}}^w + (1 - n)(\widehat{\overline{C}}^h - \widehat{\overline{C}}^f) = \frac{1 + \rho}{2\rho} \frac{1}{n} \frac{\delta}{1 + \delta} \frac{dB}{d\widehat{C}^h},$$

$$\widehat{\overline{L}}^h = \widehat{\overline{L}}^w + (1-n)(\widehat{\overline{L}}^h - \widehat{\overline{L}}^f) = -\frac{1}{2} \frac{1}{n} \frac{\delta}{1+\delta} \frac{dB}{\overline{C}_0^h}.$$

A.3 Proof for short-term world changes

Adding the consumption Euler equations weighted by the country size leads to

$$\hat{C}^w = -\frac{\delta}{1+\delta}\hat{r}$$
.

Calculate the world long-term and short-term money demand functions

$$\hat{M}^w := n\hat{M}^h + (1-n)\hat{M}^f = \widehat{\overline{C}}^w + n\widehat{\overline{p}}^h + (1-n)\widehat{\overline{p}}^f$$
 (long-term),

$$\hat{M}^w + \frac{1}{\delta} \left(n \widehat{\overline{p}}^h + (1-n) \widehat{\overline{p}}^f \right) - \hat{C}^{tw} = \left(\frac{\delta+1}{\delta} \right) \left(n \widehat{p}^h + (1-n) \widehat{p}^f \right) - \frac{\hat{r}}{1+\delta} \text{ (short-term)}.$$

Substituting the long-term relationship into the short-term one leads to

$$(\frac{\delta+1}{\delta}) \, \hat{M}^w - \hat{c}^w = (\frac{\delta+1}{\delta}) \left(n \hat{p}^h + (1-n) \hat{p}^f \right) - \frac{\hat{r}}{1+\delta}.$$

This relationship can be used to determine the short-term growth rates of world consumption in the three cases.

• sticky wages

Use the labour supply to replace the short-term price changes

$$\left(\frac{\delta+1}{\delta}\right)\hat{M}^w - \hat{C}^w = \left(\frac{\delta+1}{\delta}\right)\left(-\hat{C}^w - \hat{L}^w\right) + \frac{\hat{C}^w}{\delta},$$

and finally since $\hat{C}^w = \hat{L}^w$,

$$\hat{C}^w = -\hat{M}^w.$$

• sticky retail prices

retail prices do not change in the short-term, hence

$$\left(\frac{\delta+1}{\delta}\right)\hat{M}^w - \hat{C}^w = \frac{\hat{C}^w}{\delta}$$
 or

$$\hat{C}^w = \hat{M}^w.$$

sticky wholesale prices

$$\left(\frac{\delta+1}{\delta}\right)\hat{M}^w - \hat{C}^w = \left(\frac{\delta+1}{\delta}\right)\left(n\hat{p}^h + (1-n)\hat{p}^f\right) + \frac{\hat{C}^w}{\delta},$$

and, hence, again

$$\hat{C}^{w} = \hat{M}^{w}.$$

$\mathbf{A.4}$ Proof of Proposition 6

We first subtract the foreign short-term equilibrium equations from their home counterparts using Lemma 3. We do not impose sticky retail prices at this stage because we will use these equations in the proofs for sticky wholesale prices and sticky wages. Therefore,

$$\begin{pmatrix} \hat{L}^h - \hat{L}^f \end{pmatrix} = -\rho \left[\left(\hat{p}^h - \hat{p}^f \right) + \left(\hat{p}^f(h) - \hat{p}^h(f) \right) \right] \text{ (demand)},$$

$$\begin{pmatrix} \hat{C}^h - \hat{C}^f \end{pmatrix} - \left(\hat{L}^h - \hat{L}^f \right) + \frac{1}{n(1-n)} \frac{1}{1+\delta} \frac{dB}{\hat{C}^h_0} = \left[-\hat{p}^h(f) + \hat{p}^f(h) + \hat{E} \right] \text{ (budget constraint)},$$

$$\begin{pmatrix} \hat{M}^h - \hat{M}^f \end{pmatrix} - \left(\hat{p}^h - \hat{p}^f \right) = \left(\hat{C}^h - \hat{C}^f \right) - \frac{1}{\delta} \left(\widehat{\overline{p}}^h - \widehat{\overline{p}}^f \right) + \frac{1}{\delta} (\hat{p}^h - \hat{p}^f) \text{ (money demand)},$$

$$\begin{pmatrix} \hat{C}^h - \hat{C}^f \end{pmatrix} = \left(\widehat{\overline{C}}^h - \widehat{\overline{C}}^f \right) \text{ (consumption Euler equation)},$$

$$\begin{pmatrix} \hat{p}^h - \hat{p}^f \end{pmatrix} = -\left(\hat{C}^h - \hat{C}^f \right) - \left(\hat{L}^h - \hat{L}^f \right) + \left(\hat{w}^h - \hat{w}^f \right) \text{ (labour supply)}.$$

Additionally we need the difference between the long-term budget constraints and the long-term money demand equations for the reasons outlined in section 4. We use the fact that PPP always holds in the long-run (Lemma 1). Thus,

$$\begin{split} &\left(\widehat{\overline{C}}^h - \widehat{\overline{C}}^f\right) - \left(\widehat{\overline{L}}^h - \widehat{\overline{L}}^f\right) - \frac{1}{\mathfrak{m}(1-n)} \frac{\delta}{1+\delta} \frac{dB}{\widehat{C}_0^h} = \left[-\widehat{\overline{p}}^h(f) + \widehat{\overline{E}} + \widehat{\overline{p}}^f(h)\right] \text{ (budget constraint),} \\ &\left(\widehat{M}^h - \widehat{M}^f\right) - \widehat{\overline{E}} = \left(\widehat{\overline{C}}^h - \widehat{\overline{C}}^f\right) \text{ (money demand),} \\ &\left(\widehat{\overline{L}}^h - \widehat{\overline{L}}^f\right) = -\rho \left[-\widehat{\overline{p}}^h(f) + \widehat{\overline{E}} + \widehat{\overline{p}}^f(h)\right] \text{ (long-term demand).} \end{split}$$

Under the sticky retail price scenario, we know from the proof of Proposition 5 that the exchange rate does not change in the long-run ($\overline{E}=0$). From the long-run money demand equation and the consumption Euler equation, we conclude that the change in both periods consumption is proportional to the change in the money supply

$$\left(\widehat{\overline{C}}^h - \widehat{\overline{C}}^f\right) = \left(\widehat{C}^h - \widehat{C}^f\right) = \left(\widehat{M}^h - \widehat{M}^f\right).$$

Substituting this last equation and the long-run demand equation into the long-run budget constraint we arrive at

$$\widehat{\left(\hat{M}^h - \hat{M}^f\right)} - \frac{1}{n(1-n)} \frac{\delta}{1+\delta} \frac{dB}{C_h^h} = (1-\rho) \left[-\widehat{\overline{p}}^h(f) + \widehat{\overline{E}} + \widehat{\overline{p}}^f(h) \right].$$

Using the expression for the long-term change in the terms of trade that is given in Proposition 3, we can derive the change in net international bond holdings.

$$\frac{dB}{\hat{C}_{h}^{h}} = \frac{2\rho(1+\delta)}{(1+\rho)\delta}n\left(1-n\right)\left(\hat{M}^{h} - \hat{M}^{f}\right).$$

 $\frac{dB}{C_0^h} = \frac{2\rho(1+\delta)}{(1+\rho)\delta} n (1-n) \left(\hat{M}^h - \hat{M}^f \right).$ Substituting this equation into the equations of Propostion 3 we can calculate all the long-run changes of the variables.

For the differences in the short-run, we see from the short-term demand function that under sticky retail prices their is no substitution between foreign and home goods. Thus,

$$(\hat{L}^h - \hat{L}^f) = 0.$$

Using the relative short-term changes in consumption, price levels and production it is easy to see from the labour supply that

$$(\hat{w}^h - \hat{w}^f) = (\hat{M}^h - \hat{M}^f).$$

We can now derive the short-term change in the exchange rate given the short-term budget constraint.

Having derived the differences in short-run changes abroad and at home we use the change in world aggregates, given by annex A.3 to calculate the changes in the individual countries. The methodology is the same as in the proof of Proposition 3.

Proof of Proposition 7 A.5

We again use the differences of the short and long-run changes derived at the beginning of the proof for sticky retail prices. Under sticky wholesale prices, we can make use of the results that PPP also holds in the short-run and that the exchange rate immediately reaches its long-term value $(\hat{E} = \overline{\hat{E}})$.

Substituting the goods and money demand equation into the budget constraint, both for the long and short-run we derive

$$\begin{pmatrix} \hat{M}^h - \hat{M}^f \end{pmatrix} - \hat{E} = (\rho - 1)\hat{E} - \frac{1}{n(1-n)} \frac{1}{1+\delta} \frac{dB}{\hat{C}_0^h} \text{ (short-term budget)},$$

$$\begin{pmatrix} \hat{M}^h - \hat{M}^f \end{pmatrix} - \hat{E} = \frac{1+\rho}{2\rho} \frac{1}{n(1-n)} \frac{1}{1+\delta} \frac{\delta dB}{\hat{C}_0^h} \text{ (long-term)}.$$

From these two equations we derive the change in the international bond holdings and the change in the exchange rate.

$$\begin{split} \hat{E} &= \left(\frac{\delta(\rho+1)+2\rho}{\rho((1+\rho)\delta+2)}\right) \left[\hat{M}^{\hat{h}} - \hat{M}^{f}\right], \\ \frac{dB}{C_0^h} &= \frac{2(\rho-1)}{(\rho+1)\delta+2} n(1-n) \left[\hat{M}^h - \hat{M}^f\right] (1+\delta). \end{split}$$

Just like in the sticky retail price scenario we can derive all the long-run changes using Proposition 3.

We can derive the short-term difference in production from the short-term demand equation using the expression for the exchange rate. Thus,

$$\hat{L}^h - \hat{L}^f = \left(\frac{\delta(\rho+1)+2\rho}{(1+\rho)\delta+2}\right) \left[\hat{M}^h - \hat{M}^f\right].$$

The short-term difference in consumption can then be read from the short-term budget constraint.

 $\hat{C}^h - \hat{C}^f = \left(\frac{\rho^2 - 1}{\rho((1+\rho)\delta + 2)}\delta\right) \left[\hat{M}^h - \hat{M}^f\right].$

Finally, the relative change in wages can be calculated using the labour supply equation.

 $\hat{w}^h - \hat{E} - \hat{w}^f = \left(\frac{2\rho^2(\delta+1) + \delta(\rho-1)}{\rho((1+\rho)\delta+2)}\right) \left[\hat{M}^h - \hat{M}^f\right].$ Having derived the differences in short-run changes abroad and at home, we use the change in world aggregates, given by annex A.3 to calculate the changes in the individual countries. The methodology is the same as in the proof of Proposition 3.

Proof of Proposition 8 A.6

We again use the differences in short-term changes that have been derived at the beginning of the proof for changes under sticky retail prices. Just like under sticky wholesale prices, we can make use of the facts that PPP holds in the short-run and that the exchange rate does not overshoot (Proposition 5). The crucial difference under sticky wages is that the scale of production is determined by the labour supply rather than by the demand.

Using the differences in the long-run money demand equation and the short-run labour supply equations, we can derive the short-term change in labour. Thus,

$$(\hat{M}^h - \hat{M}^f) = [\hat{c}^h - \hat{c}^f] + \hat{E} = -(\hat{L}^h - \hat{L}^f).$$

goods demand equation. Thus,

$$(\hat{M}^h - \hat{M}^f) = \rho (\hat{p}^h(h) - \hat{E} - \hat{p}^f(f)).$$

$$(\hat{C}^h - \hat{C}^f) = -\frac{\rho-1}{\rho} (\hat{M}^h - \hat{M}^f) - \frac{1}{n(1-n)} \frac{1}{1+\delta} \frac{dB}{c_0^2}$$

$$\left(\widehat{\overline{C}}^h - \widehat{\overline{C}}^f\right) = \frac{1+\rho}{2\rho} \frac{1}{n(1-n)} \frac{\delta}{1+\delta} \frac{dB}{\overline{c}_0^h}.$$

We derive the change in the bond holdings and the change in consumption, by substituting the last two equations into each other. Thus,

$$\begin{split} &\frac{dB}{\hat{C}_0^h} = -\frac{2(\rho-1)(1+\delta)}{(1+\rho)\delta+2}n(1-n)\left[\hat{M}^h - \hat{M}^f\right],\\ &\widehat{\overline{C}}^h - \widehat{\overline{C}}^f = -\left(\frac{\rho-1}{\rho}\frac{1}{\delta+2\frac{\rho}{\rho+1}}\delta\right)\left[\hat{M}^h - \hat{M}^f\right]. \end{split}$$

Just like in the sticky price scenarios, the long-term changes can now be calculated using Proposition 3.

The change in the exchange rate can be read from the long-run money demand equation using the change in consumption. It is

$$\frac{\widehat{E}}{\widehat{E}} = \left(1 + \frac{\rho - 1}{\rho} \frac{1}{\delta + 2 \frac{\rho}{2 + 1}} \delta\right) \left[\widehat{M}^h - \widehat{M}^f \right].$$

change in world aggregates, given by annex A.3 to calculate the changes in the individual countries. The methodology is the same as in the proof of Proposition 3.

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