Prices, Price Processes, Volume and their Information
- A Survey of the Market Microstructure Literature -

by

Markus K. Brunnermeier

DISCUSSION PAPER NO 270

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This literature survey provides an up to date overview of the recent theoretical developments about price processes. The paper casts in particular light on informational aspects of price changes. A common feature of these models is that the price communicates information, which is dispersed among many traders.

After contrasting the Rational Expectation Equilibrium concept with the Bayesian Nash Equilibrium concept, a connection between market completeness and information revelation is drawn. The No-Speculation Theorem is explained. It states that difference in information alone does not lead to more trade, given that all traders behave rationally. On the other hand differential information combined with possible gains from trade can lead to large trading volumes. No-Trade Theorems describe circumstances where asymmetric information can lead to market breakdown although there are gains from trade. Two different kinds of No-Trade Theorems are covered in this survey. One due to the adverse selection effect and the other one is due to too much information revelation through prices which leads to reduced risk sharing. Lastly situations are described, under which bubbles can occur, although all traders are rational and forward looking.

The following section of the survey classifies the standard market microstructure models into five groups. A distinction is drawn between models where traders can trade conditional on the future prices (limit order models) and models in which traders can only submit market orders. They can be further subdivided into models with strategic and competitive traders, respectively. The main focus of the paper are dynamic models covering a whole price process. By means of these models a new rationale for the technical/chart analysis can be illustrated and stock market crashes can be explained.

The final section deals with various types of herding models. In general, in sequential decision making, herding behaviour can arise although all agents behave rationally. Resulting informational cascades are due to the fact that the decision of someone's predecessors provides a noisy signal of her information. If traders have short horizons, which can be due to their risk aversion, they have an incentive to gather the same information as other traders do.

Prices, Price Processes, Volume and their Information - A Survey of the Market Microstructure Literature -

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This paper provides an up-to-date review and summary of the existing literature on the informational aspects of price processes. A common feature of these models is that prices reflect information that is dispersed among many traders. The paper begins by contrasting the Rational Expectation Equilibrium concept with the Bayesian Nash Equilibrium concept, and then draws a connection between market completeness and information revelation. The No-Speculation Theorem and the No-Trade Theorems are also explained. The No-Trade Theorems describe circumstances where asymmetric information can lead to market breakdown even though there are gains from trade. The paper also examines situations under which bubbles can occur even when all traders are rational and forward looking. The second part of the survey addresses CARA-Gaussian market microstructure models. These models are classified into five groups. A distinction is drawn between limit order models and market order models. These models are further subdivided into models with strategic or competitive traders. Dynamic models are used to illustrate a rationale for technical/chart analysis. The various types of herding models are described in the final section.

JEL Classification: D82, D83, D84, G12, G13, G14

Keywords: technical analysis, asymmetric information, rational expectations, bubbles, herding

Contents

Chapter 1

Introduction

1.1 Motivation

Every day a vast number of assets changes hands. Whether these assets are stocks, bonds, currencies, derivatives, real estate or just somebody's house around the corner, there are common features driving the market price of these assets. Asset prices fluctuate more sharply than the prices of ordinary consumption goods do. We observe emerging and bursting bubbles, bullish markets as well as stock market crashes. There are many questions which fascinate academics, professionals as well as laymen. When do bubbles develop and crashes occur? Can price history provide us some hint about future price developments? Does technical or chart analysis make sense? Why is the trading volume in terms of assets so much higher than real economic activity? Can people's herding behaviour be simply attributed to irrational panic? Going beyond positive theory, some normative policy issues also arise. What are the early warning signals indicating that a different policy should be conducted? Can a different design of exchanges and other financial institutions reduce the risk of crashes and bubbles?

If financial crises and huge changes in assets prices only affect the nominal side of the economy, there would not be much to be worried about. However, as illustrated by the recent experiences of the Southeast Asian tiger economies, stock market and currency turmoil can easily turn into full-fledged economic crises. The unravelling of financial markets can spill over and affect the real side of economies.

A good understanding of price processes can help us foresee possible dangers. In recent years, the academic literature has achieved a great leap forward in understanding price processes of assets. This paper offers a detailed and up to date review of the recent theoretical literature in this area. It provides a framework for understanding price processes and emphasises the informational aspects of asset price dynamics. The survey focuses exclusively on models that assume that all agents are rational and act in their own self-interest. It does not cover models which attribute empirical findings to the irrational behaviour of agents.

The distinguishing feature of assets is that they entail uncertain payments, most of which occur far in the future. The price of assets is driven by expectations about these future payoffs. New information causes market participants to re-evaluate their expectations. For example, investors react to news about a company's future dividend prospects in the case of stocks or bonds, and to news of a country's economic prospects in the case of currencies. Depending on their information, market participants buy or sell the asset. In short, their information affects their trading activity and, thus, the asset price. Information flow is, however, not just a one-way street. Traders who do not receive a piece of new information are still conscious of the fact that the actions of other traders are driven by their information set. Therefore, uninformed traders can infer part of the other traders' information from the movement of an asset's price. The models covered in this literature survey demonstrate that past price processes can be studied to infer even more information of other traders. Therefore, technical analysis might not be as unreasonable as some earlier theoretical papers had suggested.

1.2 Structure of the Survey

Chapter 2 introduces the concept of a Rational Expectations Equilibrium (REE), contrasts it with the Bayesian Nash Equilibrium concept, and then highlights their conceptual problems. A connection between market completeness and information revelation is drawn. We explain why prices in a REE only partially reveal private information in a dynamic model with incomplete equitisation, even though the markets might be "dynamically complete". Moreover, the intuition for two different proofs of the No-Speculation Theorem is provided. This theorem states that difference in information alone does not lead to trade, given that all traders behave rationally. No-Trade Theorems describe circumstances where asymmetric information can lead to market breakdown even when there are gains from trade. Two different types of No-Trade Theorems are covered in this survey. One appeals to the adverse selection effect, while the other argues that too much information revelation through prices leads to reduced risk sharing. Bubbles are defined and the concept of higher order uncertainty is introduced. This allows us to describe situations under which bubbles can occur even when all traders are rational and forward looking.

The third chapter covers models where traders' utility functions exhibit constant absolute risk aversion (CARA) and where all random variables are assumed to be normally distributed. It is possible to find closed form solutions for this class of models. The survey classifies the CARA-Gaussian market microstructure models into five groups. A distinction is drawn between models where traders can trade conditional on the future price (limit order models) and models in which traders can only submit market orders. These models can be further subdivided into models with strategic and competitive traders. The main focus of the survey is on dynamic models that cover a whole price process. A rationale for technical/chart analysis can be illustrated by means of these models. Dynamic models can also explain stock market crashes. In a setting with widely dispersed information, even relatively unimportant news can lead to big price swings and crashes.

The final chapter deals with the various types of herding models. In general, herding behaviour can arise in sequential decision making even though all agents behave rationally. Informational cascades arise because a predecessor's action only provides a noisy signal of her information. If traders have short horizons, perhaps due to their risk aversion, they have an incentive to gather the same information as other traders do.

The survey is closely linked to the original articles and is by no means a substitute for them. Rather, it develops a flavour for the intuition and the main ideas of the original papers, while highlighting their crucial technical steps and assumptions. We use a consistent notation based on He and Wang (1995) throughout this survey to facilitate comparison between papers. We also attempt to indicate the corresponding variable notations in a footnote to ease reference to the original articles. Finally, please note the words 'investor', 'trader' and 'agent' are used interchangeably in the literature. The same is true for 'informed investor' and 'insider', 'uninformed investor' and 'outsider', and 'liquidity trader' and 'noise trader'. If market makers are monopolists, they are sometimes also referred to as 'specialists'. If many market makers are competing against each other they are also called 'dealers'.

This survey is far from exhaustive. It is restricted to price processes and hence it addresses only a specific niche in the market micro structure literature. For a broader overview we refer the reader to O'Hara (1995). Nöldeke (1993) also provides a rigorous introduction to this literature, including its technical aspects.

Chapter 2

Theoretical Results in a General Setting

2.1 Rational Expectations Equilibrium and Bayesian Nash Equilibrium Concept

Hayek (1945) was one of the first to look at the price system as a mechanism for communicating information.[1] In a world with uncertainty where information is dispersed throughout the economy, prices play an informational role. In other words, in a setting of uncertainty where traders have asymmetric or differential information, expectations are important. Muth (1960),(1961) proposed a rational expectations framework, which requires that people's subjective beliefs about probability distributions actually correspond to objective probability distributions. This rules out systematic forecast errors. The advantage of the rational expectations hypothesis over ad hoc formulations of expectations is that it provides a simple, general and plausible way of handling expectations. It also makes it possible to analyse the efficiency of markets as transmitters of information. In a rational expectations equilibrium (REE) agents draw inferences from all available information, i.e. from exogenous and endogenous data, in particular from prices. When available, investors use the information conveyed by the price as well as their private information to choose their demand schedules.

In a Rational Expectation Equilibrium all traders behave competitively, i.e. they are price takers. They take the price correspondence, a mapping from the information sets of all traders into the price space as given. In a Baysesian Nash Equilibrium, however, agents take the strategies of all other players, and not the equilibrium price correspondence, as given. The Bayesian Nash equilibrium concept allows us to analyse strategic interactions in which traders take their price impact into account. Kyle (1989) shows that the Bayesian Nash Equilibrium in demand schedules need not converge to the (competitive) Rational Expectations Equilibrium as the number of traders increases to infinity. A major drawback of the Bayesian Nash Equilibrium concept is the multiplicity of equilibria

[1]This introductory section is based on Bray (1985), Nöldeke (1993), and O'Hara (1995).

and their dependence on out of equilibrium beliefs.

A possible closed form solution of a REE can be derived in the following four steps using the method of undetermined coefficients. First, each individual trader's beliefs about the unknown variables have to be specified. The beliefs are represented by a joint probability distribution over relevant variables and all information sets for all I traders. Second, the investors derive their optimal demand, based on their (parameterised) beliefs and their preferences. In the third step, the market clearing conditions are imposed for all markets and the endogenous price variables are derived. Since individuals' demands depend on traders' beliefs, so do the price variables. In step four, rational expectations are imposed. This means that the joint probability distribution of the variables is equal to the investors' beliefs. The last step allows us to determine the parameter values of the traders' belief structures by equating the coefficients. Viewed more abstractly, the REE is a fixed point of the function $f: \mathbf{P} \rightarrow \mathbf{P}$. The domain $\mathbf{P}$ of f consists of a set of possible price conjectures. These price conjectures are functions which map from the information sets of the I agents, $\{\mathcal{F}^1, ..., \mathcal{F}^I\}$ into the price space, i.e. $\mathbf{P} = \{P \mid P : \{\mathcal{F}^1, ..., \mathcal{F}^I\} \rightarrow \mathbb{R}_+^J\}$. Given a certain price conjecture, which is assumed to be the same for all agents,[2] traders make inference from the price and derive their utility maximising individual demand. The actual relationship between the traders information sets, $\{\mathcal{F}^1, ..., \mathcal{F}^I\}$, and the prices can be derived from the market clearing condition. $\mathbf{P}$ is the set of these actual relationships. $f: \mathbf{P} \rightarrow \mathbf{P}$ is determined by utility maximising behaviour and market clearing. At the fixed point $f(P^*(\cdot)) = P^*(\cdot)$ the conjectured price function is correct and this is a REE. A dynamic REE is defined as a sequence of self-fulfilling forecast functions.

If the price reveals a sufficient statistic for the information dispersed in the economy the REE is said to be *informationally efficient*, Grossman (1978). Moreover, if even all dispersed information, i.e. the join, is revealed by prices, the REE is *fully revealing*.[3] Otherwise, the REE is *partially revealing*. Informationally efficient REE can be derived by considering the corresponding artificial economy, in which all private information is treated as being public. The equilibrium of this artificial economy is a full communication equilibrium. Having solved for this equilibrium, one has to verify that it is a REE of the underlying diverse information economy.

If it is not possible to explicitly solve the REE, the question of existence arises. Existence problems are attacked from two directions, existence theorems and non-existence examples. The crucial question for the existence of fully revealing REE is whether the mapping from signals into prices is invertible. The first non-existence example was provided by Kreps (1977). Four main results provide boundaries on the existence of REE. If there is only a finite number of possible signals (e.g. {high, middle, low}) and prices can be any vector in $\mathbb{R}_+^n$, the invertibility of the mapping from signals into prices fails only in special circumstances. Radner (1979) concluded that a REE exists and is fully

[2] Relaxing this assumption might lead to interesting new insights.

[3] More formally, let $\overline{\mathbf{S}}$ be a sufficient statistic for $\{\mathcal{F}^1, ..., \mathcal{F}^I\}$. $\overline{\mathbf{S}}$ is a sufficient statistic if the knowledge of $\overline{\mathbf{S}}$ leads to the same sequence of action rules (local strategies). $P(\cdot) : \{\mathcal{F}^1, ..., \mathcal{F}^I\} \overset{g(\cdot)}{\rightarrow} \overline{\mathbf{S}} \overset{f(\cdot)}{\rightarrow} P$ is informationally efficient if $f(\overline{\mathbf{S}})$ is invertible. If in addition $g(\{\mathcal{F}^1, ..., \mathcal{F}^I\})$ is bijective (i.e. invertible), then $P(\cdot)$ is bijective and the prices are fully revealing.

revealing, for a generic set of economies. Thus, the example by Kreps is not robust, since a small change in the parameters would destroy non-existence. If the signal structure is more general, in the sense that a signal realisation can take on any value on $\mathbb{R}$, or even $\mathbb{R}^m$, the dimensionality of the signal space plays a crucial role. Allen (1982) showed that if the number of relative prices, which is the number of assets minus one, is larger than the dimensionality of the signal space, then a REE does exist and is fully revealing for a generic set of economies. In the case where the dimension of the signal space is equal to the number of relative prices there exists an open set of economies with no REE, Jordan and Radner (1982). If the dimension of the signal space is higher than the dimension of the relative price space then there exists a generic set of economies with non fully revealing REE, Jordan (1983).

The REE concept requires that traders conduct complicated calculations. The question therefore arises whether it is possible to describe a plausible learning process which ultimately yields rational expectations if traders face the same situation repeatedly. It is shown in Bray and Kreps (1987) that rational learning of REE using a correctly specified Bayesian model is actually a more elaborate and informationally demanding form of REE. In such an extended REE traders learn the "conventional" REE. Alternatively, if agents are boundedly rational in the sense that they are only using ordinary least square regressions to learn the relationship between the price and the underlying information, the outcome converges under certain conditions to the REE, Bray (1982). The speed of this OLS-learning is analysed in more detail in Vives (1993).

A major drawback of the REE concept is that it is only implementable for the case of finitely many traders, if private information satisfies a kind of smallness, Blume and Easley (1990). More precisely, the private information of a single individual alone must not have any impact on the equilibrium. For the case of a continuum of traders Dubey, Geanakoplos, and Shubik (1987) show that no continuous mechanism (including the submission of demand functions to a market maker) can (uniquely) implement the REE correspondence, since the demand function game does not specify a unique outcome in the case of several market clearing prices. The actual trading outcome depends on the trading mechanism, which makes it clear that the market structure matters. Laffont (1985) considers a class of economies in which Rational Expectations Equilibria can be implemented by an incentive compatible mechanism. This mechanism provides the right benchmark for welfare analysis. Laffont (1985) provides an example of an informationally efficient Rational Expectations Equilibrium which is interim inefficient and a partially revealing Rational Expectations Equilibrium which is ex-post inefficient.

There are three forms of allocative efficiency. One has to distinguish between ex-ante, interim and ex-post allocation efficiency. Ex-ante efficiency refers to the time before signals are realised, interim efficiency to the time after signal realisation but before prices are observed and ex-post efficiency refers to the time after the information revelation through prices.[4] For informationally efficient REE ex-post allocation efficiency is a direct implication of the First Welfare Theorem, informationally efficient REE can be ex-ante and interim inefficient if the anticipated information revelation by prices can prevent trade.

[4]The different notions of allocative efficiency are discussed in more detail in Section 2.3

Hirshleifer (1971) provides an example where price reveals the true state. Consequently, knowing the price, nobody has an incentive to trade in order to share ex-ante risk. In other words, price revelation can make ex-ante desirable insurance impossible.

Because of the Hirshleifer effect, REE which only partially reveal the information of traders may be desirable. When prices reveal less information trade can be possible. On the other hand partially-revealing REE lead to a more severe adverse selection problem as uninformed investors can infer less information from prices. The trade-off between Hirshleifer effect and adverse selection effect is formally analysed in Marin and Rahi (1996).

Despite this obstacle, the REE is a useful and tractable device for analysing the informational role of prices.

2.2 Partially-Revealing Equilibria and Incomplete Equitisation

Apart from the conceptual problems associated with REE such as the problem of implementation or the strong rationality requirements, the application of informationally efficient REE faces further hurdles. This section puts forward arguments in favour of partially-revealing REE and shows that they can arise in an incomplete markets framework. In a dynamic setting incomplete equitisation can already lead to only partially-revealing REE.

If prices are informationally efficient, i.e. they are a sufficient statistic for all private signals, no trader will condition her demand on her private signal. But if traders' demand is independent of the signals how can prices be informationally efficient? How do traders know whether the observed price is the rational expectations equilibrium price or an off equilibrium price? The Grossman-Paradox arises. In a model with endogenous information acquisition, informational efficiency precludes any costly information gathering. There is no incentive to gather costly signals, if the sufficient statistic of all signals can be inferred from the prices. The problem that an overall equilibrium with endogenous costly information acquisition does not exist if markets are informationally efficient is known as the Grossman-Stiglitz Paradox. These paradoxes do not arise in a Bayesian Nash equilibrium where the traders take the strategies of others, but not proces, as given. The resolution of these paradoxes is shown in Dubey, Geanakoplos, and Shubik (1987) when traders can only submit market orders and in Jackson (1991) when traders submit demand schedules, i.e. limit and stop orders. In general, a Bayesian Nash equilibrium in mixed strategies also exists in this setting.

In partially-revealing REE, incompletely informed traders face a signal extraction problem which does not allow them to infer the true reasons for the price change. Changes in prices can be due to information about:

- the dividend/liquidation value of securities/assets;
- endowments;

- preference shocks (e.g. cross-sectional changes in risk aversion); and
- private investment opportunities.

Each trader knows the true reason for a price change as long as a it is due to symmetric information. Only the part of the price change due to asymmetric or differential information reveals information to the uninformed traders. Agents can generally only infer the price impact of this asymmetric/differential information, but not the actual information itself. The question uninformed agents face is whether this information is also relevant for their portfolio choice. In other words is asymmetric/differential *information of common interest* or only of *private interest*; more generally: to what extent is information of common interest? To keep the analysis tractable, information which is partially of common interest and partially of private interest is assumed to be decomposable into these two parts. The literature refers to trade due to information of common interest as informational trading, whereas trade due to information of private interest is called uninformed trading or noise trading. For example information about the liquidation value of an asset is of common interest. On the other hand, an endowment shock for a certain group of investors might affect the portfolio choice of all investors via a change in the equilibrium prices, yet it primarily concerns those investors who experience the endowment shock. A further example for information of private interest is provided in Wang (1994). In his model informed investors receive information about a private investment opportunity in which only they can invest. A REE is partially revealing if less informed traders cannot determine whether the unexpected price changes are due to others' information of common interest or information of their private interest. An illustration is provided in Figure 1.

Price Changes due to:

asymmetric/differential information — symmetric information

information of common interest | *information of common and private interest* | *information of private interest*

decompose

informational trading | non-informational noise trading

INFERENCE PROBLEM
for uninformed traders

Figure 1: Inference Problem from Price Changes

If there are only a few assets, informed agents' trading possibilities are quite restricted. Thus, there are many information constellations which can lead to the same trading behaviour, and more precisely, to the same price vector. This need not be true when more (linearly independent) assets are traded, i.e. more states are insurable. Thus, completing markets seems to improve information revelation for two reasons. First, the action space for the (competitive) informed traders increases, and second, the number of prices, which uninformed traders can make inference from also increases. This reasoning holds particularly in a setting where uninformed agents not only observe prices and the aggregate trading volume but also all individual net trade vectors. This is always the case in a two trader setting. It seems obvious that not only the number of traded securities matters, but also which securities are traded is important. The actual security design has a tremendous impact on information and has motivated the optimal security design literature. We do not focus on this strand of literature but direct interested readers to Duffie and Rahi (1995), and Allen and Gale (1994).

However, *dynamically (synthetically) complete* markets are not a sufficient condition for the existence of a informationally efficient REE. Markets are dynamically complete if, through dynamic trading strategies, all payoff relevant states/histories are insurable. In other words, any payoff stream for all different states/histories can be *synthesised*. Markets are *completely equitisable* (or complete in the sense of Debreu (1959)) if there are enough linearly independent securities such that any payoff stream can be generated through a once-and-for-all-trade in $t = 0$. Obviously, markets which are completely equitisable are also dynamically complete.

Whether markets are completely equitisable or only dynamically complete can make an enormous difference in information aggregation and revelation of prices. Grossman (1995) illustrates this point by means of an example. He compares an economy with actively traded (one-period) bonds with an economy of passively held annuities. Both economies can have the same payoff stream, since bonds allow agents to synthesise the payoff stream of the annuity. In a world of asymmetric/differential information, dynamic trading strategies require traders to make inferences about the future path of bond prices at each point in time. On the other hand, only the passive strategy of holding the market portfolio is required, when all payoff streams are equitisable at $t = 0$. Bonds are traded, either for allocative reasons, i.e. in order to synthesise a certain payoff stream or for informational reasons, e.g. to exploit information about the future interest rate. Thus, for uninformed traders, an inference problem arises when there is asymmetric information at $t = 0$ since the extent of allocative trading need not be common knowledge. A second example is provided in Grossman (1988). He points out the important informational difference between a synthesised and a real option. This is analysed within a general equilibrium setting in Grossman and Zhou (1996). If the option is traded, the implied volatility of the underlying asset can be inferred through the option price. This is not possible if the option is synthesised by dynamic trading strategies. In economies where the average risk aversion is decreasing in income, synthesised options lead to higher volatility and mean reversion in returns.

As long as the term "state of the world" is not specified the definition of complete equitisation is somewhat vague. A state of the world comprises a complete description of the fundamentals and agents' knowledge about them as well as the knowledge about the other agents' knowledge and so on ad infinitum. Fundamental, or payoff-relevant, events are the possible endowment processes for each investor, the dividend processes and the price processes for each asset for the entire history from $t = 0$ to $t = T$. The knowledge component describes how the information partitions of each individual investor over the fundamental events evolves over time. In models with higher order uncertainty, this component can also cover higher order knowledge, which is the knowledge about others' knowledge, etc. The price process is part of the fundamentals as well as part of the knowledge component, since the price processes are endogenous signals in REE. The price processes are endogenously determined in the model. An example of a possible state space is given by

$$\{\{endowments\}_{i\in\mathbb{I}}, \{dividend\ of\ asset\ j\}_{j\in\mathbb{J}}, \{price\ of\ asset\ j\}_{j\in\mathbb{J}}, \{\{signals\}_{j\in\mathbb{J}}\}_{i\in\mathbb{I}}\}_{t=0,\dots,T}$$

A *current* state in t for trader i is an event grouping all states which cannot be ruled out by the information provided until time t.

In general, the description of a (current) state can be quite cumbersome. There are two ways of simplifying the (current) state description. First, one can group all states (events) which yield the same fundamentals up to the current time and in addition predict the same future fundamentals. In particular, this is possible if the economy exhibits some symmetry. Second, one can exploit a recursive structure of a system. For example, the state description can be simplified when the past matters only for a certain time. In general, a simpler *"sufficient (current) state description"* can be found. In this case the new (current) state description is a sufficient statistic for the corresponding group of states (events) in the more cumbersome state space.

Complete equitisation is possible when there are traded assets with linearly independent payoffs such that conditional trading on any fundamental event/history in T is possible at time $t = 0$. In other words any fundamental event is insurable at $t = 0$. If any fundamental event is insurable through *dynamic trading strategies* by trading through $t = 0, ..., T$, then the market is said to be dynamically complete.[5] These dynamic trading strategies have to satisfy a measurability condition. The condition states that at any time a trader can only apply different trading strategies for different states, when he can distinguish between them. A sufficient condition for the dynamic completeness of markets is that the number of linearly independent assets is larger than the maximum splitting index for all traders i at any time t. The splitting index at t of trader i indicates in how many subpartitions his information partition can be split into when he receives new additional information at $t + 1$. In the case of symmetric information, the partitions are defined over the fundamental events. In general, the partitions have to be defined over the smallest state space such that the partitions of all traders are common knowledge. In a dynamically complete market setting, any payoff stream can be *synthesised* using dynamic trading strategies. In an incomplete equitisation setting, non-informational trading, possibly over the whole trading horizon, can therefore occur in order to obtain the desired

[5]E.g. if the dividend payments of one asset are normally distributed, then the number of states is already infinite and therefore any market with finitely many assets is incompletely equitisable. In a continuous time model the market can still be dynamically complete.

income stream. Since at the same time, insiders trade to make use of their information, uninformed traders face an inference problem. They do not know the extent to which the price change is due to insider trading and therefore the price is only partially revealing.

As mentioned above, a fully revealing (informationally efficient) REE generically exists as long as the number of relative prices exceeds the dimensionality of the signal space (the dimensionality of the sufficient statistic for the signal space, respectively). In an economy with complete equitisation the number of assets with linearly independent payoffs is equal to number of fundamental events/histories. Taking the current consumption price as numeraire, the number of relative prices also equals this number. Thus there exists generically a competitive REE which reveals all fundamental histories. Since knowing the fundamental histories/events is sufficient for any trading decision, there exists (generically) an informationally efficient REE in economies with complete equitisation.[6] In economies with incomplete equitisation, this need not be the case and thus the actual signal realisations may matter. When the signal space is larger than the number of relative prices, still a partially-revealing REE might exist. Existence proofs for partially-revealing REE are only given for special parameterised economies.

Allen (1981) provides a class of exchange economies where the price is "privately revealing". The traders' private signals combined with the price is a sufficient statistic for the pooled information of all traders. In these economies, there is one relative price and a two dimensional signal space and therefore a fully revealing REE need not exist. The full communication equilibrium of the artificial economy can still be used in such a setting for proving the existence of a REE. In a more general environment where the asymmetry of information persists in equilibrium, a different proof has to be found. In order to apply a Fixed Point Theorem expected utility functions and thus the excess demand functions must be continuous in prices. Ausubel (1990) presents a set of economies where every trader gets two signals. The first signal is a real number and the second signal is binary. The imposition of some differentiability conditions on marginal utility allows him to construct a partially revealing REE.

Most partially-revealing REE are achieved by making the total supply of assets noisy. Since this is the standard method in the CARA-Gaussian environment, we will discuss this approach in greater detail in the next chapter.

A different approach is applied in Allen (1985). In this model traders do not know the true market clearing price. All that investors observe is a noisy price. Since individuals' demands are based on this noisy price, markets clear only approximately. According to the Dominated Convergence Theorem, the noisy component smoothes out discontinuities in the excess demand function. This allows her to apply the Fixed Point Theorem on excess demand functions (instead of on the price mappings) and show the existence of a partially-revealing REE. In Allen's model, traders know the equilibrium relationships between prices and parameters describing the uncertain environment precisely, but the prevailing price vector is not completely accurate. In other words, agents' models (=beliefs) coincide with the true model. This rationality assumption is relaxed in Anderson and Sonnenschein

[6] A formal proof of this claim still needs to be shown!

(1982) and McAllister (1990), where agents' beliefs are not only based on the state of the world but also on the realised price. Their approach incorporates elements of bounded rationality and goes beyond the scope of this literature survey.

2.3 No-Speculation Theorems and No-Trade Theorems

In the Aumann structure, information is represented by partitions over different possible states of the world. The state space can be extended such that the partitions of all traders are common knowledge. This survey does not cover non-partitional information dealing with issues of bounded rationality. Representing information as partitions allows us to define knowledge operators which report the states of the world in which a certain event is known. There are two equivalent notions of common knowledge. In the usual terminology an event in a certain state is common knowledge if all agents know that the true state lies in this event and all know that all know and so on ad infinitum. The more tractable notion is that an event in a certain state is common knowledge if and only if the finest common coarsening of all traders' partitions, (i.e. the meet) is a subset of this event. This formal notion of common knowledge allowed Aumann to show that rational players cannot "agree to disagree" about the probability of a given event. In other words, if the posterior probability of a rational player about a certain event is common knowledge, then the other player must have the same posterior probability, Aumann (1976). This result requires that all players use the Bayesian updating rule (i.e. they are rational) from a common prior distribution. This latter assumption of common priors is also known as the Harsanyi doctrine and is in conflict with the Axioms of Savage (1954). However, it acts as a scientific discipline on possible equilibrium outcomes, Aumann (1987). The common prior doctrine states that differences in probability assessments must be due to differences in information. Aumann's (1976) agreement result says intuitively that if some rational trader, A, has a different probability assessment than trader, B, then trader B must conclude that this can only be due to the fact that trader A has information trader B has not considered yet and/or vice versa. It is important to note that the fact that posterior probabilities are equal, does not mean that all traders followed the same reasoning to get this common posterior. They need not have the same information. The formal proof makes use of the sure-thing principle which states that if the expected value of a random variable conditional on event E is the same as the expected value conditional on event F, then the expected value conditional on event $E \cup F$, where $E \cap F = \emptyset$, is the same. Geanakoplos (1994) uses this sure-thing principle to generalise Aumann's result and show that common knowledge of actions negates asymmetric information about events. If action rules,[7] which are mappings from players' partitions into the action space, are common knowledge, then there exists an environment with symmetric information that would lead to the same action. From this "Agreement Theorem" it follows that two rational agents

[7]An action rule is a mapping from the partition into the action space at a certain point in time. A strategy is a sequence of action rules for every possible partition.

never bet against each other. The same carries over to a situation with many traders. If the net trade vectors of the traders are common knowledge no trader will speculate.[8]

But even when the net trade vectors are not common knowledge, i.e. each trader only knows his trading activity and observes the price and maybe the aggregate trading volume, No-Speculation Theorems may still apply. The No-Speculation Theorem in Milgrom and Stokey (1982) states that if it is common knowledge that the current allocation is ex-ante Pareto efficient, new asymmetric information will not lead to trade, given traders are strictly risk averse and hold concordant beliefs.[9] If the current allocation is known to be ex-ante Pareto efficient, then there is no incentive to trade in order to share risk. In general, an allocation is Pareto efficient if there does not exist an alternative allocation which yields a strictly higher expected utility for at least one agent without reducing the expected utility level of all others. One has to distinguish between ex-ante, interim and ex-post Pareto efficiency. If an allocation satisfies the Pareto criteria with respect to the expected utility of agents before receiving any signal then it is ex-ante efficient. An allocation is interim Pareto efficient if it satisfies the Pareto criteria with respect to the expected utility conditional on knowing the private signal. This has to be true for any realisation of the signal. Similarly, an allocation is ex-post efficient, if there can be no possible Pareto improvement in the expected utilities of agents even after the private signals and public price signals are observed. If an allocation is ex-post inefficient, i.e. an ex-post Pareto improvement for any signal realisation can be made then an ex-ante Pareto improvement is also possible. Therefore ex-ante efficiency implies interim efficiency, which in turn implies ex-post efficiency.

No-Speculation theorems can either be proved using the sure-thing principle as in Milgrom and Stokey (1982) or by using the fact that more knowledge cannot hurt a Bayesian optimiser in a non-strategic environment.[10] The latter approach was used by Kreps (1977) and Tirole (1982) and makes use of the fact that, with common priors, pure speculation, i.e. trade caused by asymmetric information at a Pareto optimal allocation, is a zero-sum game. Anyone who receives a trading offer can infer that her opponent wants to make money by using her superior information. Since the opponent can only gain if somebody else loses, nobody will be willing to trade except at prices that already incorporate her information. In other words, passive investment is a (weakly) dominant strategy. The No-Speculation Theorems can be applied to Bayesian games as well as to REE models. A crucial assumption is that it is common knowledge that the current allocation is ex-ante Pareto optimal.

No-Trade Theorems describe situations which lead to a no trade outcome although the current allocation is not ex-ante Pareto efficient. Asymmetric information does not generate trade in such circumstances. Worse still, it inhibits trade that would otherwise

[8] We refer to speculation as trading where it is common knowledge that agents trade purely for reasons of asymmetric/differential information.

[9] A more detailed discussion can be found in Section 4.1, wherein we discuss Grundy and McNichols (1989).

[10] In a strategic dynamic environment, ignorance may allow an agent to commit himself to a (subgame perfect) action which is Pareto improving.

have occured. If the net trade vector is common knowledge the no-trade result extents to REE where the current allocation is not ex-ante Pareto efficient, Geanakoplos (1994). Another kind of No-Trade Theorem arises from the Hirshleifer effect, as mentioned above, Hirshleifer (1971). In this case the anticipated information revelation through prices prevents agents from risk sharing trade. In a world with uncertainty where one group of risk averse traders is better off in one state and the other group in the other state, trading provides a means for ex-ante Pareto improving risk sharing. After the uncertainty is resolved, the group of traders which is better off is not willing to trade anymore, since any allocation is ex-post Pareto efficient. Consider an information structure such that no trader can distinguish between both states, but the combined information, i.e. the join, provides knowledge about the true state. Now, if the price reveals the true state, knowledge of the price prevents trading. Trade will not take place in the first place in anticipation of the information revelation of the price.

Another group of No-Trade theorems is related to Akerlof's market for lemons, Akerlof (1970). They relate to situations where the current allocation is not ex-ante Pareto efficient and agents want to trade for informational and non-informational reasons, e.g. for risk sharing. As explained in Section 2.2, non-informational trading demand also arises in incomplete equitisation settings. A price change can be due to high/low informed or uninformed demand, which then leads to a signal extraction problem. Uninformed traders face an adverse selection problem, which allows the informed traders to extract an information rent from the uninformed. If the number of informed traders or the informational advantage of the insiders is too large, then the loss uninformed traders incur through the information rent for the insiders can outweigh their hedging gains. In these cases they are unwilling to trade and one observes a market break down. Bhattacharya and Spiegel (1991) analyse market breakdowns for the case of a single information monopolist who trades with infinitely many competitive uninformed investors. In this model the information monopolist trades strategically, i.e. he takes into account the fact that his order will have an impact on information revelation through prices. The authors conclude their analysis by providing some justifications for insider trading laws.

No-Speculation theorems and No-Trade theorems even arise in a setting with heterogeneous prior beliefs. Morris (1994) show that incentive compatibility considerations can preclude trading.

He and Wang (1995) show that new asymmetric information need not lead to a no-trade outcome if the information is dispersed among many traders. Dispersed information can even lead to a higher trading volume than the volume that would result under symmetric information. Their model is discussed in more detail in Section 4.3. The difference is that the initial allocation is not ex-ante Pareto efficient, or at least it is not common knowledge that it is ex-ante Pareto optimal. Also, since prices are only partially revealing, the Hirshleifer effect does not preclude trade.

2.4 Bubbles

If the stock price exceeds its fundamental value a bubble occurs.[11] The literature dealing with bubbles is huge. Famous historical bubbles are described in Garber (1990). The major quarrel is concerned with the question of whether large changes in prices are due to shifts in the fundamentals or just bubbles. The empirical finding of excess volatility literature starting with LeRoy and Porter (1981) and Shiller (1989) are arguments in favour of the existence of bubbles. The difficulty lies in determining the fundamental value of an asset. It seems plausible to consider the fundamental value of an asset in normal use as opposed to some value it may have as a speculative instrument. This description of a fundamental value depends on the context of a particular equilibrium and is therefore not exogenous. A problem arises when there are multiple equilibria. The same repeated economy can have different equilibria in different periods, if e.g. sunspots are used by the agents as a co-ordination device. Sunspots are random variables whose realisation in each period is common knowledge but have no economic relevance except that they emerged as convention to be used as co-ordination devices.

In the case of certainty, the fundamental value is just the present value of the equilibrium market value of the dividend stream. Taking expectations of the possible fundamental values in the case of uncertainty where all agents have the same information is problematic since this is only correct if all investors are risk neutral and there are complete markets. If traders have asymmetric information about the uncertain dividend stream additional problems arise. It is not clear which probability beliefs one should take into account in order to derive the expected value. Determining the fundamental value is therefore quite problematic. In Allen, Morris, and Postlewaite (1993) a clear upper bound on any reasonable fundamental value is used. If the price exceeds this upper bound of fundamental values a "strong bubble" occurs. More formally, a "strong bubble" occurs if there exists a state of the world in which every agent knows with probability one that the price is strictly above this upper bound. An "expected bubble" exists if there is a state of the world in which the price of the asset exceeds every agent's marginal valuation of the asset. This is in line with the notion that the right to resell an asset makes traders willing to pay more for it than they would pay if they were forced to hold the asset forever. This definition was used by Harrison and Kreps (1978) who attributed it to Keynes (1936). In the case of incomplete equitisation this definition does not make much sense since dynamic trading strategies can lead to 'expected bubbles'. The same price need not be an expected bubble if there is complete equitisation. Thus the definition of expected bubbles hinges on the degree of equitisation.

Economists have used different frameworks to explain the existence of bubbles. One way to generate bubbles is to introduce traders who behave irrationally, De Long, Shleifer, Summers, and Waldmann (1990a). If all trades are rational, backward induction rules out bubbles. There are, however, situations where the backward induction argument fails. First, this is the case in models with infinite horizon combined with myopic investors or an overlapping generation framework. With an infinite horizon, backward induction does not have a determined starting point and bubbles can occur because of a "lack of market

[11] This section is mostly based on Allen, Morris, and Postlewaite (1993).

clearing at infinity." Fiat money is the classical example of a bubble. Tirole (1982) showed that expected bubbles can exist and follow a Martingale if traders are myopic. Second, bubbles may exist in a finite time horizon model, provided there are infinitely many trading opportunities in the remaining trading rounds, Bhattacharya and Lipman (1995). Third, Allen, Morris, and Postlewaite (1993) show that higher order uncertainty can also lead to bubbles. Higher order uncertainty refers to uncertainty about others' information. In almost all standard economic models information is concerned with "pay-off relevant", or fundamental, events. The partitions other agents have are common knowledge. This allows us to model asymmetric information about fundamentals but not about others' knowledge. Higher order uncertainty deals with the case where one does not know the information structure of others, i.e. there is uncertainty about the information partitions others have. Without loss of generality this can be dealt with using an extended state space covering different possible information structures.

In Allen, Morris, and Postlewaite (1993) an example of a strong bubble is provided. In this example informed traders know the liquidation value of the asset, but they do not know whether other traders know it too. Since Tirole (1982) showed that if the ex-ante allocation is Pareto efficient there cannot even be an expected bubble in a fully dynamic REE, it is necessary for the existence of a bubble that there are gains from trade. The example provided in Allen, Morris, and Postlewaite (1993) illustrates four different ways to generate gains from trade. A further necessary condition for the existence of an expected bubble is that short sale constraints are strictly binding with a positive probability, Harrison and Kreps (1978). For strong bubbles to occur two further requirements are necessary. First, each agent has to have private information. This rules out cases in which prices are fully revealing. Second, agents' net trade vectors must not be common knowledge. As discussed above, even at an ex-ante Pareto inefficient allocation this could rule out any trade activity by the theorem that common knowledge of action negates asymmetric information. Therefore, in their example there are at least three traders. Although every trader knows the true value of the asset in their example, i.e. the true value of the asset is mutual knowledge, its price is higher. This is the case, because each trader thinks that he can resell the asset at a price above the true value.

The notion of higher order knowledge captures not only information about fundamentals and other agents' information, but also information about others' information about information, etc. ad infinitum. By extending the state space, the description of a state has many components capturing the payoff and all the higher orders of knowledge. The state space grows fast with the order of knowledge considered. Given a certain state space and a set of events, Morris, Postlewaite, and Shin (1995), define "depth of knowledge" as the number of iterations of knowledge necessary to distinguish all those states which are distinguishable on the basis of fundamentals and iterated knowledge of fundamentals. The state space in the example of Allen, Morris, and Postlewaite (1993) has depth of knowledge one since it is sufficient to distinguish between the states in which others have or do not have information in addition to the fundamental events. It is important to note that the whole state space and partitions of the state space are assumed to be common knowledge. Therefore agents know much more in state spaces with a lower depth of knowledge.

It is shown in Morris, Postlewaite, and Shin (1995) that a strong bubble can only exist in period t if it is not mutual knowledge that at time $(t+1)$ it will be mutual knowledge, that ..., that in $(T-1)$ the true asset value is mutual knowledge. Because information is revealed with the price process, mutual knowledge in period t already incorporates the information that can be inferred from the price in period t. Since knowledge can only improve through time, it follows that a bubble can only exist at or after time t if the true asset value is not $(T-t)$th order mutual knowledge at time t. Furthermore, if the state space exhibits only a depth of knowledge of order n, then n order mutual knowledge is sufficient to rule out bubbles. If this is the case, the true value is common knowledge anyway. In Tirole (1982) the assumed depth of knowledge is zero and therefore any bubble can be ruled out. In the case where the depth of knowledge of the state space is higher than the order of mutual knowledge of the true asset value, some bounds for the size of the bubble can still be provided. For this purpose a subset of the state space is taken, which is common p-belief. In other words this subset is believed to be true with at least probability p by each agent and each agent believes that each agent believes that this subset is true with probability p and so on ad infinitum. Using reasoning similar to the knowledge case (which is similar to the $p=1$ case[12]) bounds for the size of bubbles can be derived. The minimum bound can be found by varying the subset of the state space and its associated p.

[12] Dekel and Gul (1997) discuss the distinction between p=1-beliefs and knowledge.

Chapter 3

Classification of CARA-Gaussian Closed-Form Solution Models

In order to go beyond pure existence proofs, one has to specify the economy in more detail. The loss of generality is compensated by the finding of closed-form solutions which allow some comparative statics. In this chapter we restrict ourselves to a class of economies where all random variables are normally distributed.

These models can be classified in the following manner:

- Limit Order Models
  - Competitive Limit Order Models
  - Strategic Limit Order Models
- Market Order Models
  - Competitive Market Order Models Without a Market Maker
  - Strategic Market Order Models With a Market Maker in Which Traders Move First
  - Sequential Trade Market Order Models in Which the Market Maker Moves First

Two tools facilitate our analysis. The Projection Theorem can be used to address the signal extraction problem, while the Certainty Equivalence argument can be used for simplifying the utility maximising problem.

The final wealth can be written as the inner product $W = \mathbf{\Pi}'\mathbf{x}$, where the vector $\mathbf{\Pi}$ represents the (random) liquidation values of the J securities and the vector $\mathbf{x}$ describes a portfolio. W is also normally distributed, since the sum of normally distributed random variables is also normally distributed.

All traders have a constant absolute risk aversion (CARA) utility function of the exponential form

$$U^i(W) = -\exp(-\rho^i W)$$

where the absolute risk aversion measure $\rho^i = -U^i{}''(W)/U^i{}'(W)$ is independent of agent i's income. Taking the expectation, a function resembling a moment generating function is obtained.[1]

$$E[U^i(W) \mid \mathcal{F}^i] = \int_{-\infty}^{+\infty} -\exp(-\rho^i W) dW_{\mathcal{F}^i} = -\exp[\underbrace{-\rho^i(E[W \mid \mathcal{F}^i] - \frac{\rho^i}{2} Var[W \mid \mathcal{F}^i])}_{Certainty\ Equivalent}]$$

Therefore, maximising the expected utility conditional on the appropriate information set $\mathcal{F}^i$. is equivalent to maximising the certainty equivalence. The imposed assumptions result in linear demand functions.

Before conducting the maximisation procedure, the conditional expected value and variance have to be derived. The Projection Theorem helps us derive the conditional expectations and the conditional covariance and variance terms. Given our assumption that random variables are normally distributed, the first two moments are sufficient to describe the whole conditional distribution. The Projection Theorem provides the *linear* projection of S on the space of quadratic integrable functions. For normally distributed variables, the linear projection is also the optimal one. The Projection Theorem in any Hilbert space is:

$$E[\Pi \mid \vec{S}] = E[\Pi] + (\vec{S} - E[\vec{S}])'\mathbf{Var}^{-1}[\vec{S}]\mathbf{Cov}[\Pi, \vec{S}]$$

$$Var[E[\Pi \mid \vec{S}]] = \mathbf{Cov}[\Pi, \vec{S}]'\mathbf{Var}^{-1}[\vec{S}]\mathbf{Cov}[\Pi, \vec{S}]$$

If Π can be decomposed in $\Pi = E[\Pi \mid \vec{S}] + e$, such that the error term e is orthogonal to $E[\Pi \mid \vec{S}]$, i.e. $Cov[E[\Pi \mid \vec{S}], e] = 0$, we have $Var[\Pi] = Var[E[\Pi \mid \vec{S}]] + Var[e]$ and $Cov[\Pi, E[\Pi \mid \vec{S}]] = Var[E[\Pi \mid \vec{S}]]$. Since $Var[\Pi \mid \vec{S}] = Var[e]$,[2]

$$Var[\Pi \mid \vec{S}] = Var[\Pi] - \mathbf{Cov}[\Pi, \vec{S}]'\mathbf{Var}^{-1}[\vec{S}]\mathbf{Cov}[\Pi, \vec{S}]$$

For the special case with signals of the form,

$$S_t^i = \Pi + \epsilon_{S,t}^i,$$

where the distribution of the liquidation value $\Pi \sim \mathcal{N}(0, \sigma_\Pi^2)$ and the error terms $\epsilon_{S,t}^i \sim \mathcal{N}(0, \sigma_{S,t}^2)$ are mutually independent, the Projection Theorem simplifies to:

$$E[\Pi \mid \underbrace{S_1^i, \ldots, S_T^i}_{:=\mathcal{F}_T^i}] = E[\Pi] + \underbrace{\frac{1}{\frac{1}{\sigma_\Pi^2} + \sum_t \frac{1}{\sigma_{S,t}^2}}}_{Var[\Pi \mid S_1^i, \ldots, S_T^i]} \sum_t [\frac{1}{\sigma_{S,t}^2}(S_t^i - E[\Pi])].$$

---

[1] The certainty equivalent for multinormal random variables is

$$E[\exp(\mathbf{w}^\mathbf{T}\mathbf{A}\mathbf{w} + \mathbf{b}^\mathbf{T}\mathbf{w} + d)] = |\mathbf{I} - 2\mathbf{\Sigma}\mathbf{A}|^{-1/2} \exp(\underbrace{\frac{1}{2}\mathbf{b}^T(\mathbf{I} - 2\mathbf{\Sigma}\mathbf{A})^{-1}\mathbf{\Sigma}\mathbf{b} + d}_{Certainty\ Equivalent}],$$

where $\mathbf{w} \sim \mathcal{N}(0, \mathbf{\Sigma})$ and the (co)variance matrix is $\mathbf{\Sigma}$ positive definite. $\mathbf{A}$ is a symmetric $m \times m$ matrix, $\mathbf{b}$ is an m-vector and d is a scalar. Note the left-hand side is only well-defined if $(\mathbf{I} - 2\mathbf{\Sigma}\mathbf{A})$ is positive definite. See also the discussion of Brown and Jennings (1989) in Section 4.1 or Anderson (1984, Chapter 2).

[2] Note, if Π is a vector $Var[\Pi \mid \vec{S}]$ is the conditional covariance matrix.

The conditional variance $Var[\Pi \mid S_1^i, ..., S_T^i] =: V^c$ is linked to the original variances through the following expression:[3]

$$\frac{1}{V^c} = \frac{1}{\sigma_\Pi^2} + \sum_t \frac{1}{\sigma_{S,t}^2}.$$

The information set of an agent i, $\mathcal{F}_T^i$ is given by $\{S_1^i, ..., S_T^i\}$.

Note, the Kalman filter is also derived from the Projection Theorem. The Kalmam filter technique is especially useful for steady state analysis of dynamic models. The problem has to be brought in state-space form

$$z_{t+1} = Az_t + Bx_t + \epsilon_{t,1}$$

$$S_t = Cz_t + \epsilon_{t,2},$$

where, the error terms are i.i.d. normally distributed. The first equation is the transition equation, which determines how the state vector z_t moves depending on the control vector x_t. The second equation is the measurement equation, which describes the relationship between the signal S_t and the current state z_t

3.1 Limit Order Models

Most of the REE models and all the models discussed so far are limit order models. In these models each trader submits a whole demand schedule. Such a demand schedule can be achieved by combining many stop and limit orders. Investors can therefore trade conditional on future prices. Therefore a trader's information set contains not only her signal but also current prices, which influence the optimal individual demand. Within the class of limit order models one can distinguish between competitive and strategic models.

3.1.1 Competitive Limit Order Models

In competitive models traders take the price as given when forming their optimal demand. Investors neglect that their trading activity influences the price, which in turn serves as a signal. In these models traders do not attempt to manipulate prices. To justify such behaviour one could assume that each trader is only a point in a "continuum of clones" with identical private information.[4]

Grossman (1976) presents one of the first models with a closed form REE solution, where information about the return on a single risky asset is dispersed among traders. Every trader receives a noisy signal about the true payoff Π, $S^i = \Pi + \epsilon_S^i$, where $(\epsilon_S^i)_{i=1}^I$ are mutually independent and identically normally distributed. The riskless asset is traded at an exogenously fixed price with perfectly elastic supply. Using her signal and the price, each trader is solving the signal extraction problem described above to derive her optimal demand. The market clearing condition then provides the market clearing price.

[3] The reciprocal of the variance is called the precision.

[4] An excellent overview of limit order models is given in Admati (1989).

In Grossman's example, the equilibrium is informationally efficient, i.e. the equilibrium price is a sufficient statistic for all signals $\{S^i\}_{i=1}^I$. Therefore individual demand schedules do not depend on their private signal which leads to the Grossman-Paradox discussed earlier. A further consequence of the information revelation of prices is that no one has an incentive to acquire costly information. Thus an overall equilibrium with costly endogenous information acquisition does not exist (Grossman-Stiglitz-Paradox). It seems plausible that individual demand does not depend on traders' incomes since all traders have CARA utility functions. At first sight it is surprising that traders' demands do not even depend on the equilibrium price itself, although it serves as a sufficient statistic for all information in the economy. A price change in a REE has not only an income and substitution effect but also an information effect. A price increase signals a higher expected payoff of this asset in an economy with a single risky asset. With CARA utility functions and one risky asset, the income effect should not play any role. In a symmetric equilibrium with fixed aggregate supply, all traders have to demand the same number of assets, independent of the price, in order to satisfy the market clearing condition. Therefore, in Grossman's model the substitution effect and information effect cancel each other out. Finally as discussed in Section 2.3, informationally efficient REE can lead to a no-trade outcome.

Noisy REE models were developed to address these conceptual problems. A random variable whose unobserved realisation affects the equilibrium price is introduced in these models. This noise term makes prices only partially revealing because traders cannot infer whether a price change is due to the noise component or due to informed trading. The informational content of prices can be measured by a signal to noise ratio. In Grossman and Stiglitz (1980) the aggregate supply of the risky asset is random. In their model there are only two groups of traders: the informed (those who bought an identical signal) and the uninformed. Since the supply of risky assets is random, uninformed traders can only partially infer the signal of the informed. Each trader decides whether to acquire information at a certain cost. Grossman and Stiglitz (1980) derive the overall equilibrium with endogenous information acquisition and determine the fraction of informed traders in equilibrium. Their model captures the partial information transmission role of prices, but not the information aggregation role since information is not dispersed among the traders.

This additional aspect is analysed in Hellwig (1980) and Diamond and Verrecchia (1981). Similar to Grossman (1976) the signals are conditionally independent of each other given the true payoff. Whereas in Hellwig (1980) the aggregate supply of the risky asset is assumed to be random, in Diamond and Verrecchia (1981) each investor's endowments are i.i.d. and therefore aggregate supply is random as long as the number of traders does not converge to infinity.[5] For both models a closed-form solution is found where prices are only partially revealing. Hellwig shows that the REE in the "high noise limit" (where the variance of aggregate endowments goes to infinity) corresponds to the equilibrium in which market participants do not try to learn something from the equilibrium price. On the other hand the REE at the "low noise limit" corresponds to an informationally

[5]Non-i.i.d. endowments are mathematically untractable and are excluded in our discussion.

efficient REE as in Grossman (1976). The same is true when investors are almost risk neutral, since investors do not try to insure against the randomness of aggregate supply.

Incorporating noisy aggregate supply or noisy excess demand through random endowments can be thought of as a simplified reduced form for modelling liquidity traders. Liquidity traders trade for reasons exogenous to the model or due to information of private interest. This is in general the case in a setting with incomplete equitisation. In Wang (1994) informed investors trade not only for informational reasons but also in order to invest in private investment opportunities. These private investment opportunities are not equitisable, i.e. conditional trade on their dividend streams is not possible. There will always be some components of trade that are not perfectly predictable by others and not perfectly correlated with the future payoff of the traded assets. The latter is conducted by liquidity/noise/"life-cycle" traders.

Admati (1985) extended Hellwig's setting to a model with multiple risky assets and infinitely many agents. In this model, the price of an asset does not necessarily increase with its payoff or decrease with its actual supply. This is the case because a price change in one asset can provide information about other risky assets. Admati's model illustrates that not only the correlation between financial assets' returns (which is the focus in CAPM), but also the correlation between the prediction errors in traders' information is important for determining equilibrium relations.

The main focus in Pfleiderer (1984) is the role of volume and variability of prices. He analyses how a change in the signal's precision alters expected volume. His results are extended in He and Wang (1995) which is discussed in further detail in Section 4.3.

Admati and Pfleiderer (1986), (1990) analyse how an information monopolist should sell his information to competitive limit order traders. The more this information is revealed by the price, the lower is the traders' incentive to pay for this information. Admati and Pfleiderer show that it is optimal for a seller to add noise to his information when his information is very precise. This increases the fraction of market participants that would be willing to pay to become better informed. When the number of traders is large, it is better to sell personalised signals, i.e. signals with an idiosyncratic noise term. In this case, the information monopolist sells identically distributed signals to all traders and not only to a fraction of the market participants. The information monopolist can also sell his information indirectly by using it to create an investment fund. Admati and Pfleiderer (1990) show that the fund manager always makes full use of his information. They also illustrate that the degree to which information is revealed by the market price determines whether an indirect sale or direct sale of information leads to higher revenue for the information seller.

Subsection 5.4.2 will discuss competitive limit order models with market makers. In these models the group of risk neutral market makers observe only the limit order book, i.e. the noisy aggregate demand schedule. Since they are risk neutral they act as a competitive fringe and thus their information set determines the equilibrium price.

3.1.2 Strategic Limit Order Models

Traders behave "schizophrenically" in a competitive REE. On the one hand each traders' signal can influence her demand, but her demand has no impact on the price. On the other hand, this price reveals her signal. The competitive environment can be justified when the number of investors becomes very large and each investor is infinitesimal small.

It is, however, a stylised fact that the best-informed traders are large. The traders take into account the effect that their trading has on prices in strategic limit order models. Each trader knows that when she trades larger quantities *prices will move against her.* She therefore incorporates this effect in forming her optimal demand correspondence. Thus, strategic models allow us to analyse market price manipulation by some large traders.

In Kyle (1989) a strategic REE is derived as a symmetric Bayesian Nash equilibrium in demand schedules. Each trader's strategy is a demand schedule which is submitted to an auctioneer. The auctioneer collects all individual demand schedules and derives the market clearing price. Kyle's model is actually a uniform price auction of a divisible asset, whose supply is random. Given CARA-utility functions and normally distributed random variables all excess demand functions are linear. In Kyle's model the informed traders' information set consists of a private individual signal about the true value of the asset and the price. Each demand correspondence of an informed investor is linear in her individual signal as well as in the price. The demand function of the uninformed traders is also linear in price. The fractions of informed investors and uninformed investors is common knowledge in equilibrium. In a Nash equilibrium each player takes the strategies of all others as given.[6] Therefore each trader faces a residual supply curve. Each informed trader $i \in \{1, ..., I\}$ acts like a monopsonist with respect to the residual supply curve

$$p = p_I^i + \lambda_I x^i$$

where p_I^i is random and λ_I is constant. Since informed traders observe p_I^i they choose their demand x^i to maximise their expected utility conditional on p_I^i and on their private individual signal S^i. The reciprocal of λ_I can be viewed as "market depth", the liquidity of the market. Whereas in competitive models the aggressiveness of informed traders is only restricted by their risk aversion, in strategic models risk aversion and consequences of strategic behaviour on price cause agents to react less aggressively. They try to avoid trading their informational advantage away. It is also worth noticing that individuals' demand is no longer independent of their initial endowment in strategic models.

In strategic models the prices reveal less information than in a competitive REE which facilitates costly information acquisition. Even in the limit, when noise trading vanishes or traders become almost risk neutral, prices do not become informationally efficient. However, this does not mean that the profit derived from private information (the information rent) is not driven down to zero. Kyle (1989) also shows that as the number of informed speculators increases to infinity the model converges to a monopolistic competition outcome which need not be the same as in a competitive environment. The case with a single information monopolist and many competitive outsiders is analysed in more

[6] Equilibrium strategies are mutual knowledge in a Nash equilibrium, Brandenburger (1992).

detail and a tractable closed-form solution is derived. Bhattacharya and Spiegel (1991) derived the No-Trade Theorem discussed in Section 2.3 for this special case.

Jackson (1991) shows that the Grossman-Stiglitz Paradox depends crucially on the price taking behaviour of the traders. He develops a strategic limit order model in which a finite number of risk neutral traders submit demand functions. Thereby he models explicitly the price formation process, which illustrates how the signal is incoporated into the price. For specific parameters, in the Bayesian Nash equilibrium of this game costly information acquisition occurs although the price is informationally efficient. In other words, although some agents bear information acquisition costs, they do not have any informational advantage. In this situation, they acquire information because they are driven by the beliefs of the other agents about their information acquisition. Allowing for mixed strategies in a Bayesian Nash equilibrium also resolves the Grossman-Stiglitz Paradox.

Madhavan (1992) compares this setting with a two stage game developed in Glosten (1989) where dealers first quote a price (schedules) and in the second stage investors submit their orders. He wants to illustrates the difference between a quote-driven market such as NASDAQ or SEAQ and an order-driven market, capturing some features of the NYSE.

3.2 Market Order Models

When an investor submits a single limit order she faces execution risk. She cannot be sure that her order will be executed, since the price can move beyond the set limit. She can avoid this risk by submitting a menu of limit and stop orders such that at each price a certain quantity of orders will be executed. Alternatively, the execution risk can be avoided by using market orders. However, then only the quantity of trade can be fixed and the agent has to bear the risk of changing prices. This additional price risk can complicate the economic analysis and therefore, in most models, risk neutrality of all traders is assumed. As seen before, risk aversion is not needed in strategic models, since not only risk aversion but also strategic behaviour causes investors to trade less aggressively. It can be shown that limit order and market order models exhibit the same degree of informational efficiency in the case of a single informed risk neutral investor, Nöldeke (1993).

If the trader submits her market order before the market maker sets the price, the price risk is completely borne by the investor. In the case where the market maker first sets the price, he commits himself and therefore bears the risk. We will discuss strategic models in which the investors have to move first before turning to models in which the market maker moves first. Some market order models without an actively trading market maker are briefly summarised, before discussing these models.

3.2.1 Competitive Market Order Models Without a Market Maker

Hellwig (1982) uses a market order model in order to resolve the Grossman-Stiglitz Paradox. In Hellwig's dynamic model traders can only trade conditional on the past prices and not on the current price. Therefore statistical inference and market clearing do not occur simultaneously. In Hellwig (1982) a null set of the continuum of traders receives information in advance. In discrete time this information is only revealed by the price one period later. This gives the insider the possibility to make use of their information to achieve a positive return. Therefore, traders have an incentive to acquire information. Even as the time span between the trading rounds converges to zero, the insider can make strictly positive returns and an information efficient outcome can be reached arbitrary closely. In Hellwig (1982), traders are myopic and the individual demands are exogenously given rather than derived from utility maximisation. Blume, Easley, and O'Hara (1994) analyse the informational role of volume within such a framework. This model is covered in Section 4.4.

3.2.2 Strategic Market Order Models With Market Maker in Which the Investors Move First

The classical reference for this class of models is Kyle (1985).[7] In his batch clearing model there are three groups of risk neutral players, a single informed investor, many liquidity traders and a market maker who sets the price. The liquidity traders trade for reasons exogenous to the model. Their demand is given by the random variable $\Theta \sim \mathcal{N}(0, \sigma_\Theta^2)$. The single, risk neutral, information monopolist is the only one who knows the true value of the risky asset, Π. He trades to maximise his profit which is in the static single auction version of the model, the capital gain $(\Pi - P_1)$ times the quantity of stocks, x, that he holds. Since he acts strategically, he takes into account that his demand x will influence the price, P_1. The informed trader rationally believes that the market maker follows a price setting rule which is linear in the aggregate net order flow $(x + \Theta)$. Formally he maximises his profit $\pi = (\Pi - P_1)x$, where, according to his beliefs, $P_1 = P_0 + \lambda(x + \Theta)$. The single market maker only observes the aggregate net order flow $(x + \Theta)$ and knows that the true value, Π, is distributed $\mathcal{N}(P_0, \sigma_{\Pi,0}^2)$. Since he cannot observe the net trade vector, $\mathbf{x}$, of the informed investors the No-Trade-Theorem, explained in Section 2.3, does not apply. Kyle assumes that the risk neutral market maker acts competitively and thus sets a fair price given his information, i.e. $P_1 = E[\Pi \mid x + \Theta]$. Since Π and Θ are normally distributed, and with insider's demand x is linear in Π, the Projection Theorem can be applied to solve the signal extraction problem. The Bayesian Nash equilibrium is obtained by equating the coefficients and is given by

$$P_1 = P_0 + \lambda(x + \Theta), \ \textit{where} \ \ \lambda = \frac{1}{2}\left(\frac{\sigma_\Theta^2}{\sigma_{\Pi,0}^2}\right)^{(-1/2)}$$

λ, the amount of noise trading, (σ_Θ^2), together with the original variance of Π, $\sigma_{\Pi,0}^2$, determines to what extent the market maker reacts to a higher/lower aggregate net order flow. The reciprocal of λ, $(1/\lambda)$ represents the market depth. If on average a lot of uninformed noise trading is going on, the market maker will not adjust the price so

[7] In comparision to the original article the notation is: $\Pi = v$, $\sigma_{\Pi,0}^2 = \Sigma_0$, $\Theta = u$.

quickly if he observes a high order flow. Therefore in this case markets are deep, i.e. many orders can be absorbed without huge price movements. The expected profit for the insider is given by $E[(\Pi - P_1)x] = \frac{1}{2}(\sigma_\Theta^2 \sigma_{\Pi,0}^2)^{(1/2)}$. The market maker breaks even on average. He loses money to the insider but makes the same amount of money from the noise traders on average. After one trading round information is partly revealed and the new variance of the true value of the stock is only half of the original one. Kyle extended this static model to a series of discrete call markets (a sequential auction). In this dynamic setting, the insider faces the trade-off that taking on a larger position in early periods increases early profits but worsens prices in later trading rounds. She tries not to trade her information advantage away. She therefore exploits her information across time by hiding behind noise trading. A dynamic linear recursive equilibrium is derived in Kyle (1985). The author solves the dynamic problem by porposing an ad hoc value function, which he verifies at a later stage. Note the insider takes the equilibrium λ_t as given, since the market maker can not determine whether the observed aggregate order flow is due to a deviation of the insider or due to a different signal realisation or noise trader demand. As the time intervals converge to zero in the continuous auction equilibrium, noise trading follows a Brownian motion and the informed trader continuously pushes the price towards her price valuation. The speed of price adjustment is equal to the difference between her price valuation and the current price divided by the remaining trading time. The market depth, $(1/\lambda)$, is constant over time and the market is "infinitely tight", i.e. it is extremely costly to turnover a position in a very short period of time. This is the case because the insider can break up her informational trade into many tiny pieces. Prices follow Brownian motion (which is a martingale process).

Biais and Rochet (1997) show that for the case where the value of the stock is not continuously distributed, out of equilibrium beliefs have to be specified and one has to deal with the problem of multiple equilibria.

Back (1992) extended Kyle's continuous time model by modelling strategy spaces and information directly in continuous time. In Holden and Subrahmanyam (1992) there are many informed traders who compete against each other. This speeds up information revelation through prices. As in the Cournot case, insiders who have the same information are more aggressive and, therefore, trade more of their insider information away. In a dynamic setting the information is revealed immediately as time becomes continuous. In Holden and Subrahmanyam (1994) the insiders are risk-averse. This further speeds up information revelation. Risk-averse agents trade more aggressive in early periods since future prices are more uncertain. Foster and Viswanathan (1996) develop a model, where informed traders observe different signals, which will be discussed in Section 4.6. In all these models the focus is on the price process and information revelation. The Bid-Ask Spread is the focus of the next section.

3.2.3 Sequential Trade Models in Which the Market Maker Moves First

In limit order models all traders submit whole demand schedules. In the market order models discussed above the market maker sets the price after observing the total net order flow. In this section we discuss models in which the market maker has to set the price before he observes orders. He will therefore set the price conditional on the magnitude of the market order. In other words, the market maker sets a whole supply schedule and then the investors choose their optimal market order. If some traders want to buy a large number of shares then the market maker asks for a higher "ask price", since the investor could have superior information. Similarily if someone wants to sell[8] a large number of shares he offers a lower "bid price." In the sequential trading model in Glosten and Milgrom (1985)[9] the order size is fixed to one share at a point in time. Therefore, there is a single ask and a single bid, where the spread is defined as the difference between ask and bid. In Glosten and Milgrom's model there is a continuum of traders. A fraction μ is informed and a fraction $(1-\mu)$ is uninformed. Informed traders do want to trade when their expected value of the asset is strictly larger than the ask or strictly smaller than the bid. Uninformed traders trade for reasons exogenous to the model. They buy or sell one stock randomly with equal probability independent of the information. It is further assumed that the market maker and all traders are risk neutral.[10] Note, in this setting, submitting an order does not change the price at which the order will be executed. The market maker had set this price in advance and thus the order can only influence the future price development. Furthermore he does not care about the future price development, since the probability that the same trader has a chance to trade again is zero. Thus each trader would like to trade an infinite amount of the stock or not trade at all if the spread is too large. By assumption each trader is restricted to trade only one share. Moreover, in this simplified version, informed traders know the true value of the asset, which is either 0 or 1.[11] If the true value is 0 they sell when the bid price is larger than zero and accordingly when the true value is 1 they buy when the ask price is smaller than one. The specialist observes the buy and sell orders and consequently updates his beliefs about the asset's value using Bayes' Rule. Since the asset's value is either 0 or 1 his conditional expected value is equal to his probability that the true value of the asset is 1. The Bayesian Rule also exhibits that a "no trade event" will not alter his beliefs. The same is true if he observes the same number of buy and sell orders. Thus the market imbalance (the difference between sell and buy orders), is a sufficient statistic for the whole history of market orders.

Glosten and Milgrom (1985) assume that the market maker sets the ask and bid such that his expected profit on any trade is zero. The existence of at least one potential competitor combined with risk neutrality makes this assumption reasonable. Therefore the specialist sets the price equal to his belief that the true value of the asset is 1. Since he can set the price conditional on the next order he takes it into account. The bid price is therefore his belief, given the current market imbalance plus an additional sell order,

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[8] A sell order is considered as a negative buy order.

[9] In this survey we will follow the version in Nöldeke (1993).

[10] Risk neutrality of the market maker abstracts from inventory models. In inventory models the market makers can end up with a non optimally diversified portfolio at the end of the trading day. They therefore demand a spread Ho and Stoll (1981).

[11] Strictly speaking, these models are not CARA-Gaussian models. Agents have a constant absolute risk aversion coefficient of zero since they are risk neutral, but the true asset return is not normally distributed. The return distribution can be easily modified to a normal distribution.

whereas he sets his ask according to his beliefs given the current imbalance minus one stock, (an additional buy order). The market maker needs this spread to break even since he faces an adverse selection problem. If the fraction of informed traders increases, the adverse selection problem becomes more severe and therefore a wider spread is needed. On the other hand, a higher number of informed traders also increases the speed of information revelation. This analysis implies that the midpoint between ask and bid is not the current expected value for the market maker unless his current expected value is 0.5. Consider the case where the current expected belief is above 0.5. An additional buy order has less informational content than a sell order and, therefore, the midpoint is biased downwards. As the market imbalance increases in absolute terms, i.e. there are more buy orders than sell orders or more sell orders than buy orders, the market maker becomes more certain about the information of the insiders and therefore the size of the spread falls. The transaction price is a Martingale but not the quoted prices (ask and bid). The latter are only Markov but not Martingales since any additional trading round leads in expectation to more information for the market maker which tightens the spread over time.[12] Thus the spread size is not a Martingale and consequently it cannot be the case that bid and ask are Martingales. Moreover this model also exhibits serial correlation of order flows.

Easley and O'Hara (1987) extend Glosten and Milgrom's sequential trading model in two ways. In Glosten and Milgrom (1985) the supply schedule which the market maker posts is reduced to one unit of purchase and one unit of sales. Easley and O'Hara allow two different order sizes, small and large orders. Furthermore they introduce the concept of "event uncertainty." Only with probability α will the information structure be as in Glosten and Milgrom (1985). With probability $(1 - \alpha)$ an information event does not occur and only uninformed traders trade with each other. Neither the market maker nor the uninformed traders know the true value of the stock. They also ignore whether some traders are informed or not. This model incorporates higher order knowledge since the depth of knowledge of the state space is higher by one degree.[13]

In Easley and O'Hara (1987) nature chooses once at the beginning of the trading day whether an information event happens or not. If information is released the pool of infinitely many traders contains a fraction μ of informed and a fraction $(1 - \mu)$ of uninformed traders. In the other case only uninformed traders are in the pool. Uninformed investors trade for exogenous reasons and take no information aspect into account. They submit large and small orders in an ex-ante specified probabilistic way. Informed traders always prefer to trade large quantities if both quantities are traded at the same price. Informed traders do not act strategically concerning the future price path. They do not take into account that trading a large quantity can influence the future price process. This is justified since there are infinitely many informed traders and thus the probability that an individual trader has the chance to trade again is zero. However, they choose their optimal quantity which is exogenously restricted to either 1 or 2 units. Since at

[12] A price process is Markov if a single state, the current price, can represent the whole history. It is a Martingale if the expected future prices are equal to the current price.

[13] Concerning depth of knowledge see Section 2.4.

equal prices informed traders prefer to trade larger quantities, the market maker will set a larger spread for the large trades, e.g. block trades.

Depending on the parameter constellation two types of equilibria can arise. In a separating equilibrium all informed traders prefer to trade two shares, the large quantity, despite the larger spread. Uninformed traders submit market orders for one and two stocks, as exogenously specified. In this separating equilibrium, the spread for market order of size one is zero. In a pooling equilibrium, informed traders submit small and large orders and the market maker requires a spread for both quantities, the larger spread for the block trades. The market maker's uncertainty about whether there was an information release dictates that both the size and the sequence of trades matters. Incorporating this feature can help to explain the partial price recovery that characterises most block trading sequences. The impact of an "event uncertainty" is discussed in more detail in Easley and O'Hara (1992) for the simpler case where the trade size is restricted to one unit as in Glosten and Milgrom (1985). In the case of event uncertainty, the actual trade or no trade is a signal about whether there was some information released to the insiders at the beginning of the trading day. By observing the sequence of market orders the market maker can update his beliefs, not only about the true value of the asset, but also whether the insiders got information about this true value. Absence of trade, therefore, provides a signal and thus the time per se is not exogenous to the price. If the market maker observes a no-trade outcome, he increases his beliefs that nobody has any information and therefore the quotes will be pulled toward 1/2. If the midpoint is at 1/2, observing no trade makes asymmetric information less likely and therefore leads to a lower spread. If the spread is not straddling 1/2 the effect is not so obvious. Further results of their analysis are that the last transaction price is not a sufficient statistic for the past and thus the transaction price process is a Martingale, but is no longer Markov.

Glosten (1989) relaxes the restriction that order sizes are limited to one or two units. Therefore, the market maker quotes a whole price schedule instead of a single bid and ask price. Glosten (1989) compares the dealership market structure consisting of many competitive market makers with the 'specialist system' in which all investors exclusively trade through a monopolistic specialist. He shows that although the monopolistic market maker makes a positive expected profit, under certain circumstances the 'specialist system' provides a higher market liquidity than the competitive system. Glosten (1989) focuses on a one shot interaction between a strategic risk averse trader and the market maker. The trader has a exponential utility function and faces an (normally distributed) endowment shock. In addition, he receives a noisy signal about the liquidation value of the stock, $S = \Pi + \epsilon$, where Π and ϵ are independently normally distributed. He trades, therefore, for liquidity/insurance as well as informational reasons. Given the price schedule set by the market maker(s), the trader submits his utility maximising market order. Before the trader submits his order, the market maker(s) commit(s) himself (themselves) to a price schedule. In the case of competition among market makers they are forced to set a informationally efficient price schedule $P(x) = E[\Pi|x]$. Note, in contrast to Kyle (1985), in Glosten (1989) the market maker sets a price function for a forthcoming *single* transaction which stems from investors who trades for informational as well as for hedging/liquidity reasons. Glosten shows that the more extreme a position the investor wishes to take, the more likely it is that he trades for informational reasons. The market makers, therefore, have to protect themselves by making the price schedule steeper. For extremely large orders, market makers are unable to protect themselves and, therefore, the market closes down. On average, market makers profit from trading with investors with extreme endowment shocks, since they trade for re-balancing reasons and lose to those traders with small endowment shocks who trade for informational reasons. Rothschild and Stiglitz (1976) suggest that an existence problem exists in a setting where there is a continuum of types of investors. Glosten defends his setting because of its tractability and qualitative similarity to a discretised version. Hellwig (1992) shows that the non-existence of fully revealing outcomes in any signalling model arises because of the unbounded type space.

A monopolistic specialist commits himself to a different price schedule, which is determined by

$$\arg \max_{P(\hat{x}(\cdot))} E[P(\hat{x}(\cdot))\hat{x}(\cdot) - \Pi\hat{x}(\cdot)]$$

where, $\hat{x}(\cdot)$ is the optimal order size (function) of the trader depending on his endowment shock and his information. In contrast to the competitive market maker case, the monopolist has the ability to cross-subsidise different order sizes. In equilibrium he earns a larger profit from more likely small trades, but makes losses on unlikely large trades. The large trades are unlikely to occur, but likely to result from information based trading. By keeping the price of large trades relatively low, the specialist guarantees that traders with extreme signals do not reduce their trade size in order to pool with trades with less extreme signals. This is the reason why a market structure with a monopolistic specialist stays open for larger trade sizes than a market with multiple market makers. The problem a single market maker faces can also be viewed as a principal-agent-problem. The principal (specialist) sets a menu of contracts $(x, P(x))$ from which the agent chooses the one which maximises his expected utility. This is a mechanism design problem for the market maker, which was analysed as such in Rochet and Vila (1994) with the difference that they consider exogenous noise trading and allow for more general distributions.

The models considered so far avoid any strategic interaction between market makers. They just assume that competition among market makers leads to zero profit in expectations. Dennert (1994) explicitly analyses this interaction. In the first stage, all market makers set a bid and ask price and commit themselves to trade up to an exogenously specified number of shares. In the second stage, the trader chooses his optimal demand. If he trades for liquidity reasons, he will trade with the market maker(s) who offer(s) the best price. An informed trader, on the other hand, trades with many market makers simultaneously as long as it is profitable for him. An increase in the number of registered market makers leads to an increase in informed trading and, thus, increases the transaction costs for the liquidity trader. In the second part of this paper, market makers set whole price schedules in stage one. The analysis exhibits elements of a first-price auction for divisible goods.

Biais, Martimort, and Rochet (1997) model imperfect competition between market makers within the framework of mechanism design theory and make use of the tool of variation calculus.[14] The risk-neutral market maker(s) set the price schedules in the first stage. The risk-averse insider submits market orders possibly to all market makers. The insider's market order depends on his signal and his endowment shock. While Glosten (1989) analyses only the extreme cases of perfect competition and monopoly, Biais, Martimort, and Rochet (1997) analyse the more general case where the number of market makers is finite. The authors derive a unique equilibrium in which the unit price of the shares is increasing. They show that the equilibrium trading volume is below the optimal risk sharing level but higher than in the monopoly case. Competition among market makers leads to a deeper market, which was not necessarily the case in Dennert (1994). Market makers face limited competition due to adverse selection. Market makers are reluctant to undercut each other since this exposes them to a greater extent to disadvantageous informational trade. The intuition is similar to the winner's curse in auction theory. Even as the number of the market makers tends to infinity, a strictly positive bid-ask spread remains and the sum of market makers' profit is still strictly positive. This limiting case is similar to the analysis of Glosten (1994). Uninformed traders, who submit limit orders to a public limit order book before the insider submits his market orders, face the same problem. Glosten (1994) studies the case of perfect competition in limit orders.

Almost all existing models can be grouped in the five categories outlined above. A nice overview about the existence of linear equilibria in static models is provided by Bagnoli, Viswanathan, and Holden (1994). Most dynamic models have the common feature that the price adjusts instantaneously to public information but only gradually to private information. This gradualism is caused by noisy asset supply, strategic behaviour of informed traders or is exogenously given by assuming a sequential trading mechanism where traders are restrained to trade only a certain quantity.

[14] This paper departs from the normality assumption.

Chapter 4

Further Dynamic Models and Technical Analysis

This chapter covers some dynamic models in more detail. The first section deals with competitive two period models in which all traders are price takers and all traders receive their information at the same time. These models shows that past prices sill have informational value. Section 4.2 introduces the Infinite Regress problem and it demonstrates how learning can lead to serial correlation of variables. Multi-period models are analysed and the informational content of volume data is also illustrated in this Section 4.4. The value of technical analysis in models in which different traders receive information at different times is the focus of Section 4.5. Finally, Section 4.6 covers strategic market order models based on the seminal work of Kyle (1985).

4.1 Competitive Two-Period Limit Order Models, Technical Analysis and Crashes

In Grundy and McNichols (1989) and Brown and Jennings (1989) two simple competitive limit order models are developed in which not only the current price, but also the past price is useful in predicting the value of the asset. In these competitive limit models technical analysis has positive value. Grossman's (1976) model suggests that capital markets are strong-form informationally efficient, i.e. the revelation of any private information will not change the equilibrium. The noisy REE discussed below are not even weak-form informationally efficient, following Fama's definition, Fama (1970) (1976), i.e. the current price is not a sufficient statistic for all past prices. There are, however, alternative definitions of weak-form efficiency whose conditions are satisfied by these REE.

Brown and Jennings (1989) extend a model similar to Hellwig (1980) to two periods.[1] In their model there are infinitely many a priori identical investors, denoted by $i \in \mathbb{I} = \{1, 2, 3, ...\}$ who are endowed with B_0 units of the riskless asset. B_0 can be normalised

[1] As far as possible we will follow the notation in He and Wang (1995) in order to make the papers comparable.

without loss of generality to zero, since all investors have CARA utility functions. All investors start with the same information set, $\mathcal{F}_0$, with beliefs about the liquidation value of $\mathcal{N}(\mu_{\Pi,0}, \sigma^2_{\Pi,0})$. At $t = 1$ and $t = 2$ each investor gets a private signal about the liquidation value, Π, of the risky asset in $T = 3$, i.e.

$$S^i_t = \Pi + \epsilon^i_{S,t},$$

where $\epsilon^i_{S,t}$ is normally i.i.d. with $\mathcal{N}(0, \sigma^2_{S,t})$. As the signals are unbiased, by the Law of Large Numbers, the average signal $S_t = \lim_{I\to\infty} \frac{1}{I}\sum_{i=1}^{I} S^i_t$ equals Π with probability one in each t. Trader i's information set is given by $\mathcal{F}^i_1 = \{\mathcal{F}_0, S^i_1, P_1\}$ in $t = 1$ and $\mathcal{F}^i_2 = \{\mathcal{F}^i_1, S^i_2, P_2\}$ in $t = 2$. The information sets contains the current price P_t since, in a limit order model, traders can trade conditional on the price of the stock P_t. Let trader i's stock holding in t be denoted by x^i_t. His final wealth in period $T = 3$ is then given by

$$W^i_3 = B_0 + x^i_1(P_2 - P_1) + x^i_2(\Pi - P_2)$$

where B_0 is normalised to zero. Traders' expected utility functions are given by

$$E[-\exp(-\rho W^i_3) \mid \mathcal{F}_0],$$

where the constant absolute risk aversion measure ρ is the same for all traders. Each trader maximises his expected utility, given his information set and his price conjecture. Backward induction allows us to break up this optimisation process into two steps. Given a certain x^i_1 the maximum utility value at $t = 2$ is given by

$$J^i_2(x^i_1) = \max_{x^i_2} E\{-\exp[\rho(x^i_1(P_2 - P_1) + x^i_2(\Pi - P_2)] \mid \mathcal{F}^i_2\}.$$

At $t = 1$ the problem is

$$J^i_1 = \max_{x^i_1} E\{J^i_2(x^i_1) \mid \mathcal{F}^i_1\}.$$

A REE is then given by the equilibrium prices P_1 and P_2 which have to coincide with the traders price conjectures as well as the optimal stock holdings (x^i_1, x^i_2) for each investor i. The market clearing condition guarantees that demand equals supply in both periods. Whereas the average per capita demand for the risky asset is given by $x_t = lim_{I\to\infty} \sum_{i=1}^{I} x^i_t / I$, the per capital supply is assumed to be random in this noisy REE. The random supply is given by Θ_1 in $t = 1$ and Θ_2 in $t = 2$, where $\Theta_2 = \Theta_1 + \Delta\Theta_2$.[2] Brown and Jennings assume that Θ_1 and $\Delta\Theta_2$ are normally distributed

$$(\Theta_1, \Delta\Theta_2) \sim \mathcal{N}[(0,0), \begin{pmatrix} \sigma^2_{\Theta_1} & \varrho\sigma_{\Theta_1}\sigma_{\Delta\Theta_2} \\ \varrho\sigma_{\Theta_1}\sigma_{\Delta\Theta_2} & \sigma^2_{\Delta\Theta_2} \end{pmatrix}]$$

where ϱ is the correlation between the supply increments Θ_1 and $\Delta\Theta_2$. Conditioning trade in $t = 2$ on P_1, i.e. technical analysis, has positive value for two reasons. First,

[2]Note, that x_t denotes holdings rather than additional trading demand, whereas Θ_1 and $\Delta\Theta_2$ refer to additonal supply.

$P_1 = \mathbf{L}[\Pi, \Theta_1]$ provides a useful second signal about the true payoff Π. $\mathbf{L}[\cdot]$ is a linear operator. This effect is most pronounced in the case, where Θ_1 is independent of Θ_2. Second, if Θ_1 and Θ_2 are correlated the price P_1 in $t = 1$ helps to get a better prediction of Θ_1. Θ_1, in turn, is useful in predicting Θ_2. A better prediction of Θ_2 reduces the noise in $t = 2$ and thus allows P_2 to reveal more about the liquidation value Π. Furthermore, knowing P_2 also allows to get a better prediction of Θ_1. Thus a joint estimation using both price conjectures P_1 and P_2 enhances information revelation. Grundy and McNichols (1989) show that for the case of perfect correlation, i.e. $\Delta\Theta_2 = 0$, P_1 and P_2 perfectly reveal Π. Therefore P_1 has predictive value even in $t = 2$. Note, Θ_1 and Θ_2 are still correlated, even if $\varrho = 0$, since ϱ is defined as the correlation coefficient between Θ_1 and $\Delta\Theta_2$. In this case Θ_t follows a random walk and the prediction of Θ_1 using P_1 provides the expectation of Θ_2.

The non-myopic REE is derived in the following steps. Since all random variables in this model are normally distributed one can make use of the Projection Theorem. Thus all conditional expected values are linear in their unconditional expected values and the signal surprise component, $S^i - E[S^i]$. Brown and Jennings derive this simple linear relationship for $E_1^i[\Pi]$, $E_1^i[\Theta_1]$, $E_1^i[\Theta_2] = \varrho\frac{\sigma^2_{\Delta\Theta_2}}{\sigma^2_{\Theta_1}}$, $E_1^i[P_2]$, $E_2^i[\Pi]$, where $E_t^i[\cdot]$ is a simplified notation for $E[\cdot \mid \mathcal{F}_t^i]$. They also show that covariance matrices $V_1^i[\Pi, S_2^i, P_2]$ and $V_2^i[\Pi]$ are constants on $\mathcal{F}_0$, where $V_t^i[\cdot]$ denotes $Var[\cdot \mid \mathcal{F}_t^i]$.

The optimal stock holding can be derived by using backward induction. The value function in $t = 2$ given stock holding x_1^i in $t = 1$ is

$$J_2^i(x_1^i) = max_{x_2^i} E_2^i\{-\exp[-\rho[x_1^i(P_2 - P_1) + x_2^i(\Pi - P_2)]]\}$$

The optimal x_2^i in $t = 2$ is

$$x_2^i = \frac{E_2^i[\Pi] - P_2}{\rho V_2^i[\Pi]}$$

as in Hellwig (1980). Therefore

$$J_2^i(x_1^i) = E_2^i\{-\exp[-\rho[x_1^i(P_2 - P_1) + \frac{E_2^i[\Pi] - P_2}{\rho V_2^i[\Pi]}(\Pi - P_2)]]\}.$$

The only random variable at $t = 2$ is Π, which is normally distributed. Therefore, the expectation is given by

$$J_2^i(x_1^i) = -\exp[-\rho[x_1^i(P_2 - P_1)] - \frac{(1/2)(E_2^i[\Pi] - P_2)^2}{V_2^i[\Pi]}].$$

The value function for $t = 1$ can then be rewritten as

$$J_1^i = \max_{x_1^i} E_1^i\{-\exp[-\rho[x_1^i(P_2 - P_1)] - \frac{(1/2)(E_2^i[\Pi] - P_2)^2}{V_2^i[\Pi]}]\}.$$

With respect to the information set, $\mathcal{F}_1^i$, $E_2^i[\Pi]$ and P_2 are normally distributed random variables. In order to take expectations we rewrite the equation given above in matrix form by completing squares.[3]

$$J_1^i = \max_{x_1^i} E_1^i\{-\exp[\rho x_1^i P_1 + \underbrace{(-\rho x_1^i, 0)}_{:=L^{i\prime}} \underbrace{\begin{pmatrix} P_2 \\ E_2^i[\Pi] \end{pmatrix}}_{:=M^i} +$$

$$+ \underbrace{(P_2, E_2^i[\Pi])}_{=M^{i\prime}} \underbrace{\frac{1}{2} \begin{pmatrix} +\frac{1}{V_2^i[\Pi]} & -\frac{1}{V_2^i[\Pi]} \\ -\frac{1}{V_2^i[\Pi]} & +\frac{1}{V_2^i[\Pi]} \end{pmatrix}}_{:=\mathbf{N}} \underbrace{\begin{pmatrix} P_2 \\ E_2^i[\Pi] \end{pmatrix}}_{=M^i}]\}$$

Furthermore, let Q_i be the conditional expected value conditional on $\mathcal{F}_1^i$ of the multinomial random variable M^i and its conditional Covariance matrix W, i.e. $Q^i := E_1^i[M^{i\prime}]$, $\mathbf{W} := V_1^i[M^{i\prime}]$.

Taking expectations yields

$$J_1^i = \max_{x_1^i}\{- \mid \mathbf{W} \mid^{-(1/2)} \mid 2\mathbf{N} + \mathbf{W}^{-1} \mid^{-(1/2)} \exp[\rho x_1^i P_1 + L^{i\prime} Q^i - Q^{i\prime}\mathbf{N}Q^i +$$

$$+ (1/2)(L^{i\prime} - 2Q^{i\prime}\mathbf{N}) \underbrace{(2\mathbf{N} + \mathbf{W}^{-1})^{-1}}_{:=\mathbf{G}} (L^i - 2\mathbf{N}Q^i)]\},$$

where the term in exp[] is the certainty equivalent. The FOC w.r.t. x_1^i is given by

$$x_1^i = \frac{E_1^i[P_2] - P_1}{\rho G_{11}} + \frac{E_1^i[x_2^i](G_{12} - G_{11})}{\rho G_{11}}$$

where G_{ij} are the elements of the matrix $\mathbf{G}$, and

$$x_2^i = \frac{E_2^i[\Pi] - P_2}{\rho V_2^i[\Pi]}.$$

Given the price conjectures of the trader, $x_1^i = \mathbf{L}[\mu_{\Pi,0}, S_1^i, P_1]$ and $x_2^i = \mathbf{L}[\mu_{\Pi,0}, S_1^i, S_2^i, P_1, P_2]$, where $\mathbf{L}[\cdot]$ denotes a linear operator.
This allows us to derive the market clearing price as a linear function

$$P_2 = \mathbf{L}[\mu_{\Pi,0}, \Pi, \Theta_1, \Theta_2],$$

$$P_1 = \mathbf{L}[\mu_{\Pi,0}, \Pi, \Theta_1].$$

Brown and Jennings show that technical analysis has value as long as P_2 depends also on Θ_1. This is consistent with the intuition provided earlier.

Since Brown and Jennings only show existence of a non-myopic dynamic REE for the special cases where P_2 or P_1 together with P_2 are informationally efficient they continue their analysis for myopic-investor economies. Myopic dynamic models were first analysed in Singleton (1987).

[3]See also Anderson (1984, Chapter 2).

In a myopic investor economy[4] the first period stock holding simplifies to

$$x_1^i = \frac{E_1^i[P_2] - P_1}{\rho V_1[P_2]}.$$

The second period stock holding is as before

$$x_2^i = \frac{E_2^i[\Pi] - P_2}{\rho V_2[\Pi]}.$$

Brown and Jennings show that under certain parameter restrictions technical analysis has strict positive value. As mentioned above technical analysis has value if Θ_1 helps to predict Θ_2, and Θ_2 has an impact on the information revelation of P_2 and/or $P_1 = \mathbf{L}[\Pi, \Theta_1]$ provides a second noisy observation of Π. The authors provide three equivalent conditions under which technical analysis has no value. Technical analysis has no value, when individual demand in $t = 2$ is independent of P_1, or equivalently P_2 is independent of Θ_1 or equivalently $Cov[\Pi, P_1 \mid P_2, S_1^i, S_2^i, \mathcal{F}_0^i] = 0$.

Vives (1995) is able to derive a closed-form solution for the case $\varrho = 0$ even if investors act non-myopically by adding a risk-neutral competitive market-maker sector. Focus of Vives' work is to contrast the informativeness of the price process in an economy with myopic investors with an economy where investors have long horizons. In this multi-period model, scaplers, floor brokers, etc. of the risk-neutral market-maker sector observe only the limit order books, i.e. the aggregate demand. They set the price equal to the conditional expectation of the liquidations value Π given the information from the limit order books. Introducing this competitive fringe changes the model quite dramatically. Vives (1995) shows that due to the normal distribution the current limit order book (or equivalently the current price) is a sufficient statistic for all data from the past limit-order books. In other words the prices are (semi-strong) informationally efficient and thus technical analysis has no value. The importance of the competitive market-maker sector can be illustrated for the case where private information is only released at $t = 1$. A buy and hold strategy is optimal for the informed traders in this case. At $t = 1$ informed traders buy assets as in the static Hellwig-model and hold it till T. The aggregate demand (limit order book) in $t = 1$ contains the demand of the insiders and the demand of the noise traders, Θ_1. Market-makers set the price equal the conditional expectation of Π given the aggregate demand. At $t = 2$ the holding of informed traders does not change. Therefore the limit order books contains only the additional noise trader demand $\Delta\Theta_2$. Since $\varrho = 0$, $\Delta\Theta_2$ contains no additional information and thus market maker set $P_2 = P_1$ absorbing the additional noise demand. In a model without competitive fringe like in Brown and Jennings (1989) informed traders have to take on the position of the additional noise trading in $t = 2$. Since each demand function of the informed traders depends on his signal, more information is revealed by P_2. Having a competitive fringe the only motive to trade is to exploit the information advantage and not to insure each other. This allows

[4]Interpreting myopic investor economies as OLG models can be misleading, since the agents in $t = 2$ still condition their demand on their signal in $t = 1$.

Vives (1995) to derive a closed form solution even for the case where private information arrives in every period. He shows that the net trading intensity of insiders in period t depends directly on the precision of period t signals.

In Grundy and McNichols (1989) the signals are distorted by a common error term and for most of the paper the random supply in $t = 1$, Θ_1, is kept equal to Θ_2, i.e. the random absolute supply is perfectly correlated. This clarifies the results in Brown and Jennings (1989). Grundy and McNichols' paper not only focuses on technical analysis but also provides a deeper understanding of the No-Speculation Theorem of Milgrom and Stokey (1982).

In their model, exogenous random supply of a single risky asset is given by endowment shocks for each individual trader, which is similar to Diamond and Verrecchia (1981). These shocks are i.i.d. with $\mathcal{N}(\mu_{\Theta_1}, \sigma^2_{\Theta_1} I)$. As $I \to \infty$ the average per capita supply shock, Θ_1, is still random with $\mathcal{N}(\mu_{\Theta_1}, \sigma^2_{\Theta})$, since the variance of individual endowments depends on the number of traders I. Furthermore the overall variance of the total supply shocks goes to infinity and thus the Law of Large Numbers cannot be applied. Moreover, as I converges to infinity the individual endowment shock has almost no impact on the aggregate supply and thus provides no information. Therefore, the only private signal trader i receives is

$$S_1^i = \Pi + \omega + \epsilon^i_{S,1},$$

which has a common error term ω and an idiosyncratic error term $\epsilon^i_{S,1}$. Both error terms are independently normally distributed with mean zero and variance σ^2_ω and $\sigma^2_{\epsilon_{S,1}}$. The average signal is given by $S_1 := lim_{I\to\infty}(\sum S_1^i/I) = \Pi + \omega$. In contrast to Brown and Jennings (1989) traders receive their private signal only at $t = 1$. As before, traders maximise CARA utlity functions. The constant absolute risk aversion coefficient of trader i is $\rho^i \in [\rho_L, \rho_U] \subset (0, \infty)$. Grundy and McNichols derive, as a first step, a one period reference model. In this model traders conjecture a linear price relation

$$P_1 = \alpha_{0,1} + \alpha_{S,1} S_1 + \alpha_{\Theta,1} X,$$

where X is the aggregate demand in equilibrium. The optimal stock holding of trader i is therefore[5]

$$x_1^i = \frac{E_1^i[\Pi] - P_1}{\rho^i V_1[\Pi]},$$

where $E_1[\Pi]$ is linear in P_1 and S_1^i by the Projection Theorem. Notice, that $V_1[\Pi]$ is higher if the variance of the common error term is higher. Averaging over all traders gives the average per capita demand

$$X = \frac{1}{\overline{\rho} V_1[\Pi]}[\beta_{0,1} + \beta_{P,1} P_1 + \beta_{S,1} S_1]$$

where $\overline{\rho}$ is the harmonic mean[6] of all traders' risk aversion coefficients. Rearranging the

[5] I normalise the risk free interest rate $r = 0$, i.e. $R = 1$.

[6] Harmonic mean is defined as $\frac{I}{\sum_i (1/\rho_i)}$.

trader's price conjecture gives

$$X = -\frac{\alpha_{0,1}}{\alpha_{\Theta,1}} + \frac{1}{\alpha_{\Theta,1}} P_1 - \frac{\alpha_{S,1}}{\alpha_{\Theta,1}} S_1 = \Theta_1.$$

By equating the coefficients one gets the REE. The aggregate demand is downward sloping and the supply is vertical. The important coefficient is $(\alpha_{S,1}/\alpha_{\Theta,1})$. Changes in S_1 lead to a parallel shift of the demand curve, whereas changes in Θ_1 shift the vertical supply curve. The size of the demand curve shift as S_1 changes is measured by $\alpha_{S,1}$, whereas the size of the supply curve shift caused by a different Θ_1 is captured by $\alpha_{\Theta,1}$. Since traders cannot make out whether a price change is due to a demand shift or a supply shift (S_1 or Θ_1 change), $\alpha_{S,1}/\alpha_{\Theta,1}$ measures the simultaneous equation problem.

The basic model is then extended to a two period model where, in the second round, no new private information is released and the random supply Θ_2 is the same as in period one. One might expect that no trader will change his stock holding, since no new information arrived. Grundy and McNichols show that his no-trade outcome is indeed an equilibrium. There is, however, a second equilibrium, where the average signal S_1 is revealed. If all trader rationally conjecture

$$P_1 = \alpha_{0,1} + \alpha_{S,1} S_1 + \alpha_{\Theta,1}\Theta_1$$

$$P_2 = \alpha_{0,2} + \alpha_{S,2} S_1 + \alpha_{\Theta,2}\Theta_2,$$

where $\Theta_1 = \Theta_2$ and if both equations are linearly independent, then S_1 can be revealed. This can be the case if $\frac{\alpha_{S,1}}{\alpha_{\Theta,1}} \neq \frac{\alpha_{S,2}}{\alpha_{\Theta,2}}$, since then we have two linearly independent equations with two unknowns.

Grundy and McNichols prove that an informationally efficient REE, which fully reveals S_1, exists as long as the variance of ω is not too large. Their proof proceeds in two steps. First, they show, given a linear pricing relation in round 1, there exists a S_1-revealing equilibrium in round 2. Second, as long as the variance of ω, σ_ω^2, is not too large, traders rationally foresee the existence of a S_1 revealing equilibrium in round 2. When $0 < \sigma_\omega^2 < \overline{\sigma}_\omega^2$ two S_1-revealing REE exist. In these equilibria, there are two sources of uncertainty in the first round: x_2^i and P_2. These equilibria show that even when no new information arrives, prices and stock holdings can change if the additional price P_2 reveals more of the private information. When $\sigma_\omega^2 = 0$, both equilibria, the S_1-revealing and the non-S_1-revealing, are identical for the first round.

In the S_1-revealing REE, trade also occurs in period two, although the only new public information is P_2. This seems striking in light of the No-Speculation Theorem developed in Milgrom and Stokey (1982). The No-Speculation Theorem predicts a null trade outcome in period 2, if the allocation after trade in period 1 is Pareto optimal and the prior beliefs about the signals in $t = 2$ are concordant before the signal becomes known. Beliefs are concordant if traders agree on the conditional likelihood of any given realisation of the signal, i.e.

$$Pr[S_2^i = s \mid \Pi = \pi, \mathcal{F}_1^i] = Pr[S_2^i = s \mid \Pi = \pi, \mathcal{F}_1^1] \; \forall \; i, S, \Pi.$$

Intuitively, beliefs are concordant if traders agree about everything expect the prior probability of payoff-relevant states. Since the only new signal in $t = 2$ is P_2, which is public, it is sufficient that beliefs about P_2 only are "essentially concordant", i.e.

$$\frac{Pr[S_2^i = s \mid \Pi = \pi, \mathcal{F}_1^i]}{Pr[S_2^i = s \mid \Pi = \pi\prime, \mathcal{F}_1^i]} = \frac{Pr[S_2^i = s \mid \Pi = \pi, \mathcal{F}_1^i]}{Pr[S_2^i = s \mid \Pi = \pi\prime, \mathcal{F}_1^i]} \ \forall\, i, S, \Pi$$

Pareto optimality is given if the marginal rate of substitution for consumption across any two states is the same for all investors. Grundy and McNichols show that if the investors behave *myopically* they reach a Pareto optimal location after the first round. However, when P_2 becomes known this allocation is no longer Pareto efficient since traders' beliefs about P_2 are not "essentially concordant" at the end of the first round. Therefore, trade will occur. If traders apply dynamic trading strategies, i.e. they do *not* behave *myopically*, trade can also occur in period 2. This is the case when $\sigma_\omega^2 > 0$, i.e. there is a common unknown noise term in the signal. The trading outcome in round 1 is neither Pareto efficient given information $\mathcal{F}_1^i$, nor are the beliefs about the public signal P_2 concordant. When $\sigma_\omega^2 = 0$, the true liquidation value Π can be inferred from P_2 and trade 1 allocation is Pareto efficient and beliefs about P_2 are concordant. In this case the No-Speculation Theorem applies and the only trade which occurs is a swapping of riskless assets.

Grundy and McNichols continue their study by introducing an additional publically observable signal in $t = 2$

$$Y_2 = \Pi + \epsilon_{Y,2}.$$

In this case a S_1-revealing REE with trade in $t = 2$ exists as before, but also when the new public information Y_2 is informative, i.e. $Cov(\Pi, \omega) \neq Var(\omega)$. The authors also provide necessary and sufficient conditions for the existence for non-S_1-revealing REE, in which no trade occurs in the second round. Finally, they consider the case where the random supply (Θ_1, Θ_2) is not the same in both periods but correlated as in Brown and Jennings (1989). Both types of equilibria exist in this generalised version. In the non-S_1-revealing type, no informational trade will occur; the whole trading volume is determined by the additional noisy supply. In the second type, the sequence of prices $\{P_1, P_2\}$ only partially reveals S_1, since the supply shocks are not perfectly correlated anymore. However, the sequence of prices reveals more about S_1 than P_1 alone. Their paper shows that technical analysis has positive value and that trading can be self-generating. It also makes clear how important the traders price conjectures are in determining the economic outcome.

Romer (1993) provides a rationale for large price movements without news. He shows, within a two period asymmetric information model, that a small commonly known supply shift in the second period can lead to large price movements. The aim of his paper is to give a rational explanation for the stock market crash in 1987. In his model asymmetric information is only partially revealed in the first period, but in contrast to Grundy and McNichols (1989) it incorporates uncertainty about the quality of other investors' signals, i.e. higher order uncertainty. Each investor receives one of possibly three signals about the liquidation value of the single risky asset, $\Pi \sim \mathcal{N}(\mu_\Pi, \sigma^2_\Pi)$.[7]

$$S^j = \Pi + \epsilon_{S^j},$$

where $\epsilon_{S^2} = \epsilon_{S^1} + \delta^2$, $\epsilon_{S^3} = \epsilon_{S^2} + \delta^3$ and ϵ_{S^1}, δ^2, δ^3 are independently distributed with mean of zero and variance $\sigma^2_{\epsilon_{S^1}}$, $\sigma^2_{\delta^2}$, $\sigma^2_{\delta^3}$, respectively. Thus, S^j is a sufficient statistic for S^{j+1}. There are two equally likely possible states of the world for the signal distribution. *Either* half of the traders receive signal S^1 and the other half signal S^2 *or* half of the traders receive signal S^2 and the other half signal S^3. It is obvious that traders who receive signal S^1 (or S^3) can infer the relevant signal distribution, since each investor knows the precision of his own signal. Only trader who receive signal S^2 do not know whether the other half of traders has received the more precise signal S^1 or the less precise signal S^3. Finally, as usual, the random supply in period 1 is given by the independently distributed random variable $\Theta_1 \sim \mathcal{N}(\mu_{\Theta_1}, \sigma^2_{\Theta_1})$.[8]

In contrast to Grundy and McNichols (1989), the supply of the risky asset changes in period two. This change is common knowledge making the no-trade outcome of Grundy and McNichols (1989) very unlikely. The change in price caused by the supply shock allows the S^2-investors to infer more precisely the signal distribution. Thus a small supply change can lead to revelation of 'old' information and can have a huge impact on prices. Alternatively, if in addition an option is traded, the quality of information can be revealed by its price already in $t = 1$. This is only the case as long as the quality of the signals can be summarised in one parameter. The informational difference between traded and synthesised options was discussed in more detail in Section 2 Grossman (1988).

The stock holdings in equilibrium of S^1-traders, $x^1(S^1)$ can be derived directly from the Projection Theorem. They do not make any inference from the price, since they know that their information is sufficient for any other signal. Traders with S^3-signals face a more complex problem. They know the signal distribution precisely but they also know that they have the worst information. In addition to their signal S^3, they try to infer signal S^2 from the price P_1. The equilibrium price in $t = 1$, P_1, is determined by $x^2(S^2, P_1) + x^3(S^3, P_1) = \Theta_1$ (assuming a unit mass of each type of investors). Since an S^3 trader knows $x^2(\cdot)$, $x^3(\cdot)$ and the joint distribution of S^2, S^3 and Θ_1, he can derive the distribution of S^2 conditional on S^3 and P_1. Since $x^2(S^2, P_1)$ is not linear in S^2, $x^3(S^3, P_1)$ is also nonlinear. S^2-investors do not know the signal distribution. Therefore, the $Var[P_1 \mid S^2]$ depends on the higher order information, i.e. whether the other half of traders are S^1- or S^3-investors. S^2-traders use P_1 to predict more precisely the true signal distribution, i.e. information quality of other traders. If they observe an extreme P_1, then it is more likely that other investors got signal S^3. Otherwise, if P_1 is close to the expected price μ_Π then it seems that others are S^1-traders. S^2-investors' demand functions $x^2(S^2, P_1)$ are therefore not linear in P_1, since P_1 changes not only the expectations about Π, but also the variance. This nonlinearity forces Romer to restrict his analysis

[7]The notation in the article is: $\Pi = \alpha, S_j = s_j, \mu_\Pi = \mu, \sigma^2_\Pi = V_\alpha, \Theta_1 = Q, \mu_{\Theta_1} = \overline{Q}, \sigma^2_{\Theta_1} = V_Q$.

[8]Romer claims that he needs this random supply term in order to avoid an informationally efficient REE in $t = 1$. I do not see the necessity for this term since a single price cannot reveal two facts, the signal and the signals quality. In my opinion the structure is rather similar to the partial-revealing REE analysis in Ausubel (1990).

to a numerical example. His simulation shows that S^2-investors' demand functions are more responsive to price changes. Using these results Romer tries to explain the market meltdown in October 19, 1987. In Section II, Romer (1993) develops an alternative model with trading costs and widespread dispersion of information which explains the stock market crash. This model is not covered by this survey. Another model which incorporates uncertainty about the signal precision of other trades is Blume, Easley, and O'Hara (1994). Blume, Easley, and O'Hara (1994) avoid these nonlinearity problems by considering a market order model instead of a limit order model. In their model volume rather the price in the next period reveals the quality of information.

Gennotte and Leland (1990) provide a similar explanation for stock market crashes in a 'one period' model.[9] As in Romer (1993) there are no major news events. Gennotte and Leland (1990) consider two groups of informed traders. Each (price-)informed trader receives an individual private signal $S^i = \Pi + \epsilon^i$ about the liquidation value $\Pi \sim \mathcal{N}(\mu_\Pi, \sigma_\Pi^2)$. Supply-informed traders receive a signal about the total supply. Aggregate supply results from dynamic hedging trades like program trading, stop and loss strategies etc. as well as from noise liquidity supply represented by the random variable Θ. Random supply Θ can be divided into the part $\overline{\Theta}$ which is known to everybody, Θ_S which is only known to the supply-informed traders, and the liquidity supply Θ_L which is not known to anybody. Superior knowledge of the supply-informed traders about Θ_S allows them to infer more information from the equilibrium price p_1. Gennotte and Leland (1990) show that the equilibrium is given by $p_1 = f(\Pi - \mu_\Pi - k_1\Theta_L - k_2\Theta_S)$, where k_1 and k_2 are real constants. Note, that since $\pi(p_1)$ need not be linear, p_1 need not be normally distributed. However, $f^{-1}(p_1)$ is still normally distributed. A "crash" is possible if $f(\cdot)$ is discontinuous, i.e. a small change in the argument of $f(\cdot)$ leads to a large price shift. In the absence of any program trading (i.e. $\pi(p_1) = 0$) $f(\cdot)$ is continuous. This rules out crashes. Nevertheless, an increase in the supply can lead to a large price shift. The price change is small if the change in supply is common knowledge, i.e. change in $\overline{\Theta}$. If the supply shift is only observable by supply-informed traders, the price change is still moderate. This occurs because price-informed and supply-informed traders take on a big part of this additional supply even if the fraction of informed traders is low. Supply-informed traders know that the additional excess supply does not result from different price signals while price-informed traders can partially infer this from their signal. If, on the other hand, the additional supply is not observable at all, a small increase in the liquidity supply Θ_L can have a large impact on the price. Crashes only occur when the program trading is large enough to cause a discontinuous price correspondence $f(\cdot)$. The discontinuity stems from non-linearity of program trading $\pi(p_1)$ in p_1 which can lead to the possibility of multiple equilibria. Crashes are much more likely and prices are more volatile if some investors underestimate the supply due to program trading. Gennotte and Leland (1990) illustrate their point by means of an example of a put replicating hedge strategy (synthetic put). Their analysis is in the spirit of Grossman (1988). However, their paper also explains why stock prices do not immediately rebound after a stock crash.

[9] We adjusted the notation to $S^i = p_i'$, $\Pi = p$, $\mu_\Pi = \overline{p}$, $\sigma_\Pi^2 = \Sigma$, $p_1 = p_0$, $\Theta = m$, $\Theta_L = L$, $\Theta_S = S$.

In order to get a better understanding about price processes, one would like to have models which capture a larger time horizon than essentially two periods. Before discussing dynamic models with differential information, we deal with a simpler information structure, namely asymmetric information. Townsend's (1983) article makes clear what kind of problems can arise from a more general information structure.

4.2 Serial Correlation Induced by Learning and the Infinite Regress Problem

Townsend (1983) laid bare crucial points in his article "Forecasting the Forecast of Others", viewing rational expectation from a macroeconomic angle. Within a rational expectations framework, decision makers solve dynamic decision problems following their objective function and infer information from well specified information sets, taking the aggregate laws of motion as given. These laws are, in turn, those actually generated in the model. The focus of his article is the characteristics of economic time series. He shows that learning can convert serially uncorrelated shocks into serially correlated movements in economic decision variables. Since agents may respond to variables generated by the decisions of others, time series can display certain cross-correlation and may appear more volatile. In the case of disparate but rational expectations, decision makers forecast the forecasts of others. This can lead to relatively rapid oscillations and can make forecasts, as well as forecast errors, serially correlated.

He analysed the behaviour of time series in a dynamic model with a continuum of identical firms in each of two markets. The demand schedule in each market (island) i is given by

$$P_t^i = -\beta_1 Y_t^i + \xi_t^i,$$

where P_t^i is the price in market i, Y_t^i is the aggregate output of all individual production functions $y_t^i = f_0 k_t^i$, and ξ_t^i is a demand shock. This shock consists of a "persistent" economy wide component Θ_t and a "transitory" market specific shock ϵ_t^i, i.e.

$$\xi_t^i = \Theta_t + \epsilon_t^i,$$

where

$$\Theta_t = a_\Theta \Theta_{t-1} + \nu_t \quad -1 < a_\Theta < 1,$$

follows a AR(1) process with ϵ_t^i and ν_t jointly normal and independent. Firms can infer ξ_t^is, but they do not know exactly which part steams from a persistent economywide shock and which part is market specific and transitory. After stating the firm's maximisation problem, Townsend derived the first order conditions using the certainty equivalence theorem. He defines the dynamic *linear* rational expectations equilibrium in laws of motion. Following Sargent (1979) one can derive the law of motion of the aggregate (capital stock) in each market without directly calculating the firm-specific laws of motion. The aggregate law of motions have the advantage that they can be computed without being specific about information sets and forecasting. In Townsend's setting the equilibrium can be found by finding the statistically correct forecasts.

Considering the inference problem, the paper is divided into two parts. In the first part, firms in market 1 cannot observe the price in market 2, whereas market 2 firms observe both prices. Townsend calls this an hierarchical information structure. In the second part, firms in both markets can make inferences from both markets' prices.

In part one the information set of market 1 firms consists of $\mathcal{F}_t^1 = \{\underline{K}_t^1, \underline{P}_t^1, \underline{M}_t^1\}$, i.e. the aggregate capital, the price and the common market 1 mean forecast of Θ_t, M_t where $\underline{Z}_t$ denotes a stochastic process up to and including time t. Using only observations in t of this information set allows firms to infer exactly the total shock to the economy ξ^1. The inference problem for firms in market 2 is similar, except that their information set also contains the price in market 1, i.e. $\mathcal{F}_t^2 = \{\underline{K}_t^2, \underline{P}_t^2, \underline{M}_t^2, \underline{P}_t^1\}$. The price in market 1 provides additional information to what extend the shock is permanent and is, therefore, used in making the forecasts of the shock components. The components of ξ^i, however, cannot be inferred precisely although past data help to get a better forecast. The inference problem of the firms can be solved in two ways. Either one uses the Projection Theorem or one applies Kalman filtering, which derives from the Projection Theorem. Applying the Projection Theorem directly has the disadvantage that the state space increases with the history of time. The latter is a steady state approach and exploits a recursive algorithm. Therefore, it is often assumed that the initial date is $t = -\infty$. It is important to notice that Kalman filtering can only be applied if the state vector[10] in the state space form is of finite dimension. Using these methods one can derive $\underline{\hat{\nu}}_t := E(\nu_t \mid \mathcal{F}_t^i)$, the forecasts of $\underline{\nu}_t$ as a linear combination of $\underline{\nu}_t$, and $\underline{\epsilon}_t^i$. It now becomes obvious that the learning mechanism causes some persistence. Although ν_t, ϵ_t^i are uncorrelated, their forecasts are correlated, since both forecasts $E(\nu_t \mid \mathcal{F}_t^i)$ and $E(\nu_{t-1} \mid \mathcal{F}_{t-1}^i)$ contain $\underline{\nu}_{t-1}$. In other words, all past $\underline{\nu}_{t-1}$ influence the prediction of ν_t. Similarly $E(\Theta_t \mid \mathcal{F}_t^i)$ and $E(\Theta_{t-1} \mid \mathcal{F}_{t-1}^i)$, as well as the forecast errors $[E(\Theta_t \mid \mathcal{F}_t^i) - \Theta]$ are serially correlated. It is important to notice that the forecast error for past Θ_s (s¡t) decreases as time goes on and more and more observations are available.

So far only market 2 firms were forming inference about the components of the demand shock from an endogenous time series, the price in market 1. The price in the first market also depends on the average beliefs in this market, $\underline{M}_t^1$, i.e. the market 1 expectations. These expectations are well defined and can be expressed in terms of a finite number of states. Therefore, the Kalman filter can be applied. In the second part of the paper the information structure is not hierarchical anymore. Firms in market 1 can also draw inferences from P_t^2. Since P_t^2 depends on the common market 2 forecasts, M_t^2, firms in market 1 must have expectations about M_t^2, i.e. $E_t^1(M_t^2)$. But firms in market 2 (firms 2) see P_t^1 also. So firm 2 must have expectations on M_t^1, i.e. $E_t^2(M_t^2)$. Thus firm 1 needs to know expectations $E_t^1(M_t^2)$ and $E_t^1(E_t^2(M_t^1))$. This chain of reasoning can be continued ad infinitum. Therefore, we face an infinite regress problem. One needs, in the

[10] A state in this setting is an element of the "sufficient (current) state description" as desribed in Section 2.2.

space of mean beliefs, infinitely many state variables. This prevents us from applying the standard Kalman filter formulas. Notice that the infinite regress problem arises although the depth of knowledge is only zero. The infinite regress problem is not due to a high depth of knowledge but due to inference of endogenous variables. Townsend then goes on to discuss a related but different infinite regress problem, in which he analyses the case of infinitely many markets.

New methods developed by Marcet and Sargent (1989a) (1989b) in convergence of least squares learning to rational-expectations equilibria allow us to tackle this infinite regress problem differently. Sargent (1991) shows in that a solution can be found in self-referential models by defining the state variables in a different way. The idea is to model agents as forecasting by fitting vector *arma* models for whatever information they have available. The state vector for the system as a whole is defined to include the variables and the innovation in the vector *arma* models fit by each class of agents in the model. This contrasts to the former formulation in Townsend (1983) where the state covers a system of infinitely many orders of expectations about exogenous hidden state variables. This new approach - to my knowledge - has not been applied so far in the finance literature. Most of the literature avoids the infinite regress problem by assuming a hierarchical information structure as in Wang (1993),(1994). In models with differential information the problem is elegantly by-passed. This is the case in He and Wang (1995) a competitive model and in Foster and Viswanathan (1996) a strategic model.

4.3 Competitive Multi-Period Limit Order Models

Wang (1993) is - to my knowledge - the first to use Kalman filters in the financial economics literature. He avoids the infinite regress problem by assuming a hierarchical information structure. In his model the information of the informed investors statistically dominates the information of the uninformed. In other words, all variables the uninformed investors can observe are also known by the informed traders. The main focus of this paper is the impact of information asymmetries on the time series of prices, risk premiums, price volatility and the negative autocorrelation in returns, i.e. the mean reverting behaviour of stock prices. For analysing these questions he uses a dynamic asset-pricing model in continuous time. In his economy, investors can invest either in a riskless bond with constant rate of return $(1+r)$, or in equity which generates a flow of dividends at an instantaneous stochastic growth rate D. D is determined by the following diffusion process:

$$dD = (\Pi - kD)dt + \mathbf{b}_D d\mathbf{w},$$

where the state variable Π follows an Ornstein-Uhlenbeck process,

$$d\Pi = a_\Pi(\overline{\Pi} - \Pi)dt + \mathbf{b}_\Pi d\mathbf{w},$$

and $\mathbf{w}$ is a (3x1) vector of standard Wiener processes, $a_\Pi(> 0)$, $\overline{\Pi}$, $k(\geq 0)$ are constants and $\mathbf{b}_D$, $\mathbf{b}_\Pi$ are (1x3) constant matrices.

The fraction w of informed traders observe in addition to $\underline{D}_t$, $\underline{P}_t$, also $\underline{\Pi}_t$, whereas the uninformed only know $\underline{D}_t$ and $\underline{P}_t$, i.e. $\mathcal{F}^i(t) = \{D_\tau, P_\tau, \Pi_\tau : \tau \leq t\}$ and $\mathcal{F}^u(t) = \{D_\tau, P_\tau : \tau \leq t\}$.[11] It is clear that the informed can infer the expected growth rate $(\Pi - kD)$. When $k = 0$, Π is simply the dividend growth rate. When $k > 0$, Π/k can be interpreted as the short-run steady-state level of the dividend rate D, which fluctuates around a long-run steady-state level $\overline{\Pi}/k$.

In this setting, the rational expectations equilibrium would fully reveal Π to the uninformed. Although the price would adjust, no trading would occur. Under incomplete markets, there can be motivation other than the arrival of new information that cause investors to trade. In the case where the price is not informationally efficient, the irrelevance of heterogeneous information breaks down and investors will trade. In order to have a incomplete markets setting, Wang introduces an additional state variable by assuming that a stochastic quantity of stock supply. The total amount of stocks $(1 + \Theta)$ should be governed by the stochastic differential equation

$$d\Theta = -a_\Theta \Theta dt + \mathbf{b}_\Theta d\mathbf{w}$$

where $\mathbf{b}_\Theta$ is constant (1x3) matrix and $\mathbf{w}$ are the Wiener Processes mentioned above. In this environment the uninformed face following problem. They cannot distinguish whether a change in (P_t, D_t) is due to a change in the dividend growth rate Π_t or due to a change in noise supply Θ_t.

Wang analyses first the benchmark case of perfect information, in which all investors are informed. The equilibrium price takes on the form

$$P^* = \Phi + (p_0^* + p_\Theta^* \Theta)$$

where Φ represents the net present value of expected future cash flows discounted at the risk free rate r and the second term reflects the risk premium. He shows that the expected excess return to one share is independent of the variance of the noise supply. In other words, volatility in prices caused by temporary shocks in supply do not change the risk premium in the case of symmetric information.[12] This is in contrast to De Long, Shleifer, Summers, and Waldmann (1990b) where investors have finite horizons and they face additional risk since the remaining trading periods to rewind their positions are becoming fewer. He and Wang (1995) also consider a finite horizon model, in which the variance of Θ affects the risk premium.

For the case of asymmetric information, we outline all of the major steps, since they are useful for the analysis of later papers. Wang proceeds in the following way to determine a linear rational expectations equilibrium. First, he defines the primary state variables consisting of all known variables for the informed traders. The state space covers also "induced state variables" reflecting the estimates of the uninformed investors. The actual state description should incorporate all signals which the investors receive. Wang simplified the state space by using equivalently the estimates of uninformed investors.

[11] The notation $\underline{Z}_t$ represents a (continuous) process up to and including t.

[12] Θ can be inferred by everybody and Θ describes a economy-wide shock. Together with CARA utility functions this result seems plausible.

As a second step he proposes a linear rational expectations equilibrium price

$$P = (\phi + p_0) + p_D^* D + \underbrace{p_\Pi \Pi + p_\Theta \Theta}_{:=\xi} + p_\Delta \hat{\Pi} = \Phi + (p_0 + p_\Theta \Theta) + p_\Delta \Delta,$$

depending on $\hat{\Pi}(t) := E[\Pi \mid \mathcal{F}_t^u]$, the estimate of $\Pi(t)$ by uninformed investor. $\hat{\Pi}(t)$ depends on the whole history of dividends and prices. The equilibrium price reveals to the uninformed traders the sum $\xi := p_\Pi \Pi + p_\Theta \Theta$. Therefore, $\mathcal{F}_t^{D,P} = \mathcal{F}_t^{D,\xi}$. The equilibrium price does not depend additionally on $\hat{\Theta} := E[\Theta \mid \mathcal{F}_t^u]$, since $p_\Pi \hat{\Pi} + p_\Theta \hat{\Theta} = p_\Pi \Pi + p_\Theta \Theta =: \xi$. In other words, the uninformed investors can derive ξ but do not know exactly whether the price change is due to a change in Π or Θ.

Given the proposed linear REE one can derive in a third step the estimates, $\hat{\Pi}$, and $\hat{\Theta}$. Focusing on a steady state analysis, the uninformed investors apply the Kalman filter on all past data of dividends D and prices P to infer their estimates $\hat{\Pi}$, and $\hat{\Theta}$.[13] Their joint estimation of Π and Θ based on both D and P generates the *induced correlation* between the estimates of $\hat{\Pi}$ and $\hat{\Theta}$.

In the fourth step the process for the estimation error $\Delta := \hat{\Pi} - \Pi$ is derived.[14] It is shown that it follows

$$d\Delta = -a_\Delta \Delta dt + \mathbf{b}_\Delta d\mathbf{w}.$$

This estimation error is mean-reverting to zero and thus only temporary. This is the case since the uninformed investors constantly update their estimates, as in Townsend (1983).

Let us in a fifth step derive the instantaneous excess return process $dQ := (D - rP)dt + dP$. It is well known from static models that, for deriving the demand functions the excess returns are relevant.

In the sixth step the uninformed investors' optimisation problem is derived. As in the static case one, can exploit the CARA utility to derive a mathematically tractable form of expected utility for the Bellman equation. The estimators, $\hat{\Pi}_t$ and $\hat{\Theta}_t$, provide a sufficient statistic for $\mathcal{F}^u(t)$. Therefore, by the Separation Principle, $\hat{\Pi}_t$ and $\hat{\Theta}_t$ can be estimated first and then in a second stage the control problem can be dealt with.[15] The optimal control problem is then solved in a similar manner for the informed investors.

Finally the market clearing conditions are imposed and the above proposed price equation can be derived.

Using simulations, Wang (1993) shows the impact of this information structure on stock prices, the risk premium, price volatility and negative serial correlation in returns. Increasing the number of uninformed traders has two effects in this model. First, there is overall less information in the market and prices become less variable. Second, there exists more uncertainty about future dividend payments. Investors will demand a higher risk premium and, therefore, prices become more sensitive to supply shocks. Asymmetry in information among investors can cause price volatility to increase, because the adverse selection problem becomes more severe. Wang demonstrates that the existence of uninformed investors increases the risk premium, since the risk premium only depends on the

[13] For a more detailed discussion see Lipster and Shiryayev (1977).

[14] Note that the estimation error for Θ is given by $p_\Pi / p_\Theta (\Pi - \hat{\Pi})$.

[15] For a more detailed discussion see Fleming and Rishel (1975).

fundamental risk of the asset perceived by the investors. When the fraction of uninformed investors increases, the price contains less information about future dividend growth. He also shows that the strong mean reversion in $\Theta(t)$ generates negative serial correlation in stock returns even in the case of symmetric information. This correlation can be enhanced as the fraction of uninformed investors increases. Finally it is shown that the optimal investment strategy of the informed investors not only depends on the value of underlying true state variables but also on the reaction of uninformed investors. In other words, the informed investors make use of the estimations errors of the uninformed. Wang also found that the trading strategies for less informed investors can look like *trading chasing*, i.e. they rationally buy when the price rises and sell when the price drops. He and Wang conclude their paper with further comments and possible generalisations. One was that the whole economy can be reduced to an effective two persons setup[16] even if all investors have different risk aversion coefficients.

In a similar, but discrete time version, Wang (1994) analysed the behaviour of volume. The other major difference to the continuous time model is that, although no exogenous noise is introduced, the price is only partially revealing. This is due to the modeled incompleteness of the markets. If markets are incomplete and investors are heterogeneous, prices are not only affected by aggregate risk but also by individual risk. In such an environment volume plays an important role. This paper tries to show the link between volume and heterogeneity of investors. Investors differ in their information as well as in their private investment opportunities. In order to avoid the infinite regress problem informed investors have a strictly statistically dominant information set in comparison to uninformed traders. Markets are incomplete, since only informed investors have an additional private investment opportunity, besides stocks and bonds whose rate of return is $R = (1 + r)$. The dividend of a stock consists of a persistent component F_t and an idiosyncratic component $\epsilon_{D,t}$. F_t, which is only observable by informed investors, follows an AR(1) process:

$$D_t = F_t + \epsilon_{D,t}$$

$$F_t = a_F F_{t-1} + \epsilon_{F,t}, \ 0 \leq a_F \leq 1.$$

Whereas informed investors can observe F_t, all uninformed traders get the same noisy signal S_t about F_t.[17]

$$S_t = F_t + \epsilon_{S,t}$$

Define, for later reference, the excess *share* return as $Q_t := P_t + D_t - RP_{t-1}$. Informed traders can also invest in their private investment opportunity which yields a stochastic excess rate of return of

$$q_t = Z_{t-1} + \epsilon_{q,t}$$

where Z_t follows an AR(1) process

$$Z_t = a_Z Z_{t-1} + \epsilon_{Z,t}, \ 0 \leq a_Z \leq 1$$

[16]This follows from the aggregation theorem (see Rubinstein (1974)).

[17]To avoid the infinite regress problem informed traders observe this signal, too.

Similar to the stock return the process $\underline{Z}_t$ is only known to the informed traders. Besides making use of their information advantage, hedging the risk reflected by $\epsilon_{q,t}$ is the only incentive for informed traders to trade. All ϵ-terms are normally i.i.d. with the exception that $\epsilon_{D,t}$ and $\epsilon_{q,t}$ can be positively correlated. Wang shows in the case of symmetric information that if $\epsilon_{D,t}$ and $\epsilon_{q,t}$ are uncorrelated a change in expected returns on the private investment will not alter the investors' stock holdings. This changes with a positive correlation between $\epsilon_{D,t}$ and $\epsilon_{q,t}$ since the stock and the private investment are becoming substitutes. The problem the uninformed investors face is that they cannot sort out whether a price increase is due to informed trading, i.e. an increase in F_t, or due to uninformed trading, in which case informed traders just want to rebalance their portfolio because of a change in the profatibility of their private investment opportunities. Uninformed traders face, therefore, an adverse selection problem. The analysis of the equilibrium follows the same steps as in Wang (1993). First, the states of the economy $F_t, Z_t, \hat{F}_t = E[F_t \mid \mathcal{F}_t^u]$ are defined. Second, the linear pricing rule

$$P_t = -p_0 + (a - p_F)\hat{F}_t + p_F F_t - p_Z Z_t$$

is proposed. Third, from this equation it is obvious that uninformed traders can infer the sum $\xi_t = p_F F_t - p_Z Z_t$, thus $\xi_t = p_F \hat{F}_t - p_Z \hat{Z}_t$. This explains why $\hat{Z}_t$ is redundant in the state description within the class of linear equilibria. Fourth, using Kalman filtering one derives $\hat{F}_t$, $\hat{Z}_t$ and the estimation errors $\hat{F}_t - F_t =: \Theta_t$.[18] It can be shown that the estimation error Θ_t follow an AR(1) process, i.e.

$$\Theta_t = a_\Theta \Theta_{t-1} + \epsilon_{\Theta,t}, \ 0 \leq a_\Theta < 1.$$

The strict inequality $a_\Theta < 1$ implies that the forecast error is mean reverting. Through time the uninformed traders will learn "old" F_s, Z_s better and better but in every period new F_t, Z_t are appearing. Uninformed investors are *"chasing" a moving target.* The unconditional variance of the estimation error, $Var(\Theta_t) =: \epsilon$ reflects the degree of asymmetry of information.

For determining the optimal stock demand it proves useful to derive the expected excess returns for informed and uninformed traders. The optimal portfolio for each group of investors is a composition of a mean variance efficient portfolio and a hedging portfolio. Investors want to hedge, since expected returns on both the stock and the private investment technology change over time. Since returns on the stock are correlated with changes in expected future returns, it provides a vehicle to hedge against changes in future investment opportunities. Knowing the optimal portfolios, the trading strategies for the informed and uninformed investors can be written as:

$$X_t^i = f_0^i + f_Z^i Z_t + f_\Theta^i \Theta_t$$

$$X_t^u = f_0^u + f_Z^u \hat{Z}_t.$$

This shows that the optimal stock holding of the uninformed traders only changes when their expectation about the others private investment opportunities changes, i.e. $X_t^u -$

[18]Hint: $\hat{Z}_t - Z_t$ is determined by $\xi_t = p_F F_t - p_Z Z_t = p_F \hat{F}_t - p_Z \hat{Z}_t$.

$X^u_{t-1} = f^u_Z(\hat{Z}_t - \hat{Z}_{t-1})$.

$$\hat{Z}_t - \hat{Z}_{t-1} = E^u_t[Z_t] - E^u_{t-1}[Z_{t-1}]$$

can be decomposed into

$$\{E^u_t[Z_{t-1}] - E^u_{t-1}[Z_{t-1}]\} + \{E^u_t[Z_t] - E^u_t[Z_{t-1}]\}$$

where the first component deals with correcting errors of previous periods and the second component induces new positions. Knowing that all trading volume is changes in holding of either the informed or the uninformed investors we can characterise volume by

$$\mathcal{V}_t = (1-w) \mid X^u_t - X^u_{t-1} \mid = (1-w) \mid f^u_t \mid\mid \hat{Z}_t - \hat{Z}_{t-1} \mid$$

$$E[\mathcal{V}_t] = 2(1-w) \mid f^u_Z \mid \sqrt{2/\pi}.$$

Equipped with these formulae, the effects of asymmetric information on volume by increasing the noise of the signal can be analysed. As the signal of the uninformed becomes less precise the asymmetry of information increases and the adverse selection problem becomes more severe. This reduces the trading volume. This need not be the case if a non-hierarchical information structure is assumed as in Pfleiderer (1984) or He and Wang (1995). Trading volume is always accompanied by price changes, since investors are risk averse. If informed traders face high excess return in private investment they try to rebalance their portfolio by selling stocks. In order to make stocks more attractive to uninformed investors they have to reduce the price. This price reduction needs to be even higher if the adverse selection problem is more severe. This shows that the trading volume is positively correlated with absolute price changes and that this correlation increases with information asymmetry. Volume is also positively correlated with absolute dividend changes. In the case of symmetric information public news announcements about dividends change only the current price, but not the expected return or trading volume. In the case of asymmetric information, different investors update their expectations differently. They respond to public information differently since they interpret it differently. Uninformed investors change their estimates for $\underline{F}_{t-1}$ and $\underline{Z}_{t-1}$ and trade to correct previous errors and establish new positions. Volume in conjunction with current change in dividends or returns can also be used to improve the forecast for expected future excess returns. Under symmetric information, public news (like a dividend change announcement) is immediately reflected in the price. Under asymmetric information, public news can lead to corrections previous trading mistakes. Wang shows that an increase in dividends accompanied by high volume implies high future returns. High volume indicates that the change in dividend was unanticipated. A dividend increase should, therefore, increase prices. The second component of excess returns, the price change, is different because it provides information about noninformational trading as well as the stock's future dividends. Under symmetric information, trade is only done to rebalance portfolios and it is always accompanied by changes in the current price in the opposite direction. In the case of asymmetric information, uninformed investors trade for two reasons: to correct previous errors and to take on new positions if the price adjusts to noninformational trading from the informed investors. The correlation between the current volume and the current returns and expected future returns is ambiguous.

One inconsistency of this analysis is although volume can help to predict future returns uninformed investors in this model do make use of it. A model in which investors also take the information content of volume into account is given in Blume, Easley, and O'Hara (1994), which we will discuss in Section 4.4. First we will refer to a model with generalised information structure, which is provided by He and Wang (1995).

In the dynamic models discussed so far information asymmetry was either strict hierarchical or the information was revealed before the next trading round, as in Admati and Pfleiderer (1988). In He and Wang (1995) this unrealistic assumption is relaxed. They develop a model in which investors have differential information concerning the underlying liquidation value of a stock $\Pi + \delta$. The main focus of their model is the relationship between the pattern of volume and the flow and nature of information. They also analyse the link between volume and price volatility. They find that high volume generated by exogenous private or public information is accompanied by high volatility in prices, whereas high volume generated by endogenous information (like prices) is not accompanied by high volatility.

In their model, there are infinitely many investors, represented by the set $\mathbb{I}$.[19] Each investor can either invest in a bond with a certain gross return rate $R = 1$ or in a stock with a liquidation value $\Pi + \delta$ at the final date T. In contrast to Wang (1993) and (1994) this model has a finite horizon and all dividends are paid at the final period T. Each investor $i \in \mathbb{I}$ receives a *private* signal S_t^i about the first component of the stock's liquidation value, Π:

$$S_t^i = \Pi + \epsilon_{S,t}^i,$$

where $\epsilon_{S,t}^i$ is normally i.i.d. with $\mathcal{N}(0, \sigma_{S,t}^2)$ for all investors. They also observe a public signal Y_t about Π:

$$Y_t = \Pi + \epsilon_{Y,t},$$

where $\epsilon_{Y,t} \sim \mathcal{N}(0, \sigma_{Y,t}^2)$. In addition all traders observe the price P_t. The second component of the liquidation value, δ, is never revealed before the terminal date T.

Without noisy supply the true value of Π would be revealed immediately in $t = 1$. To make the model interesting, the supply of asset is 1 plus a noise term Θ_t. This noise term follows a Gaussian AR(1) process

$$\Theta_t = a_\Theta \Theta_{t-1} + \epsilon_{\Theta,t} \ , \ -1 < a_\Theta < 1.$$

This paper provides a way to characterise a linear equilibrium of the above economy in a mathematically tractable way. The vector of state variables of the economy is given by $\Phi_t = (\Pi; \underline{\Theta}_t; \underline{Y}_t; \{\underline{S}_t^i\}_{i \in \mathbb{I}})$ where an underlined capital letter stands for the whole stochastic process up to and including time t.

The main goal is to simplify quite dramatically this state space.[20] For characterising the equilibrium price it is useful to derive expected values for Π_t and Θ_t conditional on

[19] He and Wang make use of charge spaces. For more details about charge spaces see Feldman and Gilles (1985) and Rao and Rao (1983).

[20] In contrast to the former discussed Wang-papers we have included the signals directly in the state space and not the expected values of Π.

different information sets. $\hat{\Pi}_t^c$ and $\hat{\Theta}_t^c$ with superscript c is based on publically available information, $\hat{\Pi}_t^{p,i}$, $\hat{\Theta}_t^{p,i}$ with superscript (p,i) are the expected values based *only* on private information, whereas the same terms with superscript i represent the expected values taking all available private and public information of investor i into account. Instead of the hat, $\hat{\cdot}$, the expected value is also written as $E_t^c[\cdot]$, $E_t^{p,i}[\cdot]$, $E_t^i[\cdot]$ whereas the notation for the variance is given by $V_t^c[\cdot]$, $V_t^{p,i}[\cdot]$, $V_t^i[\cdot]$. He and Wang focus on linear REE. Thus $P_t = \mathbf{L}[\Phi_t] = \mathbf{L}[\Pi; \underline{\Theta}_t; \underline{Y}_t; \{\underline{S}_t^i\}_{i\in\mathbb{I}}]$, where $\mathbf{L}[\cdot]$ expresses a linear functional relationship.

Lemma 1 of He and Wang (1995) reduces the necessary state space to $(\Pi; \underline{\Theta}_t; \underline{Y}_t)$, i.e. $P_t = \mathbf{L}[\Pi; \underline{\Theta}_t; \underline{Y}_t]$. This can be shown by using the Law of Large Numbers, since the mean of infinitely many signals converges with probability 1 to Π.[21] Furthermore, by exploiting the above linear relationship one can replace $\underline{\Theta}_{t-1}$ by $\underline{P}_{t-1}$. We therefore have $P_t = \mathbf{L}[\Pi, \Theta_t, Y_t, \underline{P}_{t-1}, \underline{Y}_{t-1}]$ which can be rewritten as.

$$P_t = a_t \underbrace{(\Pi - \mu_t\Theta_t)}_{=:\xi_t} + \underbrace{b_tY_t + \mathbf{L}[\underline{P}_{t-1}, \underline{Y}_{t-1}]}_{=\mathbf{L}[\hat{\Pi}_t^c]}$$

The sum $\xi_t := \Pi - \mu_t\Theta_t$ can be inferred by every investor. Therefore, in a linear REE following information sets are equivalent $\mathcal{F}^c = \{\mathcal{F}_0, \underline{P_t}, \underline{Y_t}\} \Leftrightarrow \{\mathcal{F}_0, \underline{\xi_t}, \underline{Y_t}\}$. He and Wang show in the following steps that the second term $b_tY_t + \mathbf{L}[\underline{P}_{t-1}, \underline{Y}_{t-1}]$ is equal to $\mathbf{L}[\hat{\Pi}_t^c]$, i.e. it satisfies a specific structure. After this is shown one can conclude that the equilibrium is determined by Π, Θ_t, and $\hat{\Pi}_t^c$. In order to derive this specific structure we make use of the equivalence between $\mathcal{F}^c = \{\mathcal{F}_0, \underline{P_t}, \underline{Y_t}\}$ and $\{\mathcal{F}_0, \underline{\xi_t}, \underline{Y_t}\}$. The authors apply Kalman filtering to derive the *first order* expectations $\hat{\Pi}^c$, $\hat{\Theta}^c$, i.e. conditional on public information $(\underline{\xi}_t, \underline{Y}_t)$, and $\hat{\Pi}^i$, $\hat{\Theta}^i$, i.e. conditional on all information $(\underline{\xi}_t, \underline{Y}_t, \underline{S}_t^i)$. The stochastic difference equations are given by Lemma 2. It is easy to show that $\{\hat{\Pi}_t^i, \hat{\Theta}_t^i, \hat{\Pi}_t^c, \hat{\Theta}_t^c\}$ follows a Gaussian Markov process under filtration $\mathcal{F}_t^i$. Since information is differential in this model, investor i's trading strategy can also depend on *higher-order* expectations, i.e. expectations about the expectations of others etc. He and Wang show in Lemma 3 that higher-order expectations can be reduced to first-order expectations. First, they show that $\hat{\Pi}_t^i$ is a weighted average of $\hat{\Pi}_t^c$ and $\hat{\Pi}_t^{p,i}$. $\hat{\Pi}_t^c$ is given by Lemma using Kalman filtering, where $\hat{\Pi}_t^{p,i}$ follows immediately form the Projection Theorem. The weights α_t and $(1-\alpha_t)$ are independent of i and α_t is given by the ratio $V_t^{p,i}[\Pi_t]/V_t^i[\Pi_t]$. Having represented $\hat{\Pi}_t^i$ as $\alpha_t\hat{\Pi}_t^c + (1-\alpha_t)\hat{\Pi}_t^{p,i}$, one can derive, by integrating over i and by taking conditional expectations, the second-order expectations of Π as a weighted average of two first-order expectations. In general, i's higher-order expectations can be expressed as linear function of his first-order expectations. Therefore, it is sufficient if i's optimal trading strategy depends only on his first-order expectations.

For deriving the optimal stock demand it is useful - as usual - to define excess return on one share of stock as $Q_{t+1} := P_{t+1} - P_t$. He and Wang assume for solving the investors' dynamic optimisation problem in Lemma 4 - for the time being - that Q_t and $\Psi_t^i =: E_t^i[\Psi]$, where Ψ_t^i is a simplified state space, follow the Gaussian process

$$Q_{t+1} = A_{Q,t+1}\Psi_t^i + B_{Q,t+1}\epsilon_{t+1}^i$$

[21] Remember we have infinitely many investors by using charge spaces.

$$\Psi^i_{t+1} = A_{\Psi,t+1}\Psi^i_t + B_{\Psi,t+1}\epsilon^i_{t+1}.$$

They state the Bellman equation, exploit the nice property of CARA utility function in forming expected utility for next period, and derive the optimal stock demand function which is linear in Ψ^i_t. Finally they verify that Q_t and Ψ^i_t follow this Gaussian process.

By imposing the market clearing condition the equilibrium price is determined by

$$P_t = [(1 - p_{\Pi,t})\hat{\Pi}^c_t + p_{\Pi,t}\Pi] - p_{\Theta,t}\Theta_t = (1 - p_{\Pi,t}\hat{\Pi}^c_t) + p_{\Pi,t}\xi_t.$$

The stock price depends only on Π, Θ_t, and $\hat{\Pi}^c_t$, so $\mathbf{L}[\hat{\Pi}^c_t]$ summarises the whole history. This is the result that was required to show. The price P_t follows a Gaussian Markov process.

Since $\hat{\Pi}^c_t$ depends on P_t, P_t is only determined implicitly. However, finding the explicit solution is trivial, since $\hat{\Pi}^c_t$ is linear in P_t. Given the price, one can derive the expected excess return $E^i_t[Q_{t+1}]$ from which it follows that the investor's optimal stock demand is given by

$$X^i_t = d_{\Theta,t}\hat{\Theta}^i + d_{\Delta,t}\underbrace{(\hat{\Pi}^i_t - \hat{\Pi}^c_t)}_{:=\Delta}$$

Given the market clearing condition

$$d_{\Theta,t} = 1$$

$$d_{\Delta,t} = \frac{\alpha_t}{y_t(1-\alpha_t)}.$$

He and Wang also provide a recursive procedure to calculate the equilibrium. Starting with a guess for the conditional variance of Π in $T-1$ they derive coefficients $p_{\Pi,T-1}$, $p_{\Theta,T-1}$, demand, equilibrium price and other parameters. As they proceed backwards they check whether the initial guess of the variance of Π was correct. If not they start the procedure with a new initial guess.

Having derived the equilibrium allows us to examine different patterns of trading volume and how private information is gradually impounded into the price. In the benchmark case with *homogeneous information*, i.e. $\sigma_{S,1} = 0$, the true value $\Pi = \hat{\Pi}^c_t = \hat{\Pi}^i_t$ is known immediately and the only remaining risk lies in δ. The equilibrium price in this case is given by $P_t = \Pi - p_{\Theta,t}\Theta_t$ where the second term represents the risk premium. $1/p_{\Theta,t}$ measures the market liquidity in the sense of Kyle (1985). The risk premium increases with the Variance of δ and over time. The latter increase is due to the fact that the number of trading periods left to unwind speculative positions is decreasing. Furthermore, with only few periods remaining and with $\mid \Theta \mid$ large it becomes less likely that the mean reverting AR(1) process of Θ_t will reach a value of zero. The volume of trade, $\mathcal{V}^*$ is totally determined by noise trading, which is defined by

$$\mathcal{V}^* = \int_i \mid \Theta_t - \Theta_{t-1} \mid = \mid \Theta_t - \Theta_{t-1} \mid$$

with

$$E[\mathcal{V}^*] = \sqrt{2/\pi Var[\Delta\Theta_t]}.$$

In the case of *differential information* the equilibrium price is given by

$$P_t = [(1 - p_{\Pi,t})\hat{\Pi}_t^c + p_{\Pi,t}\Pi] - p_{\Theta,t}\Theta_t$$

The second component is associated with the risk premium as in the homogeneous information case. The first component reflects investors expectations about the stock's future payoff. This is not simply proportional to the average of investors' expectations: $\alpha_t\hat{\Pi}_t^c + (1-\alpha_t)\Pi_t$. This is because dynamic trading strategies generate equilibrium prices that differ from those generated by static/myopic strategies since current state variables depend on the history of the economy. The difference between dynamic and myopic strategies also appeared in Brown and Jennings (1989) and in Grundy and McNichols (1989). In particular, the current state depends on past prices. As investors continue to trade, the sequence of prices reveals more information, as shown in their Corollary 1. This tends to decrease $p_{\Theta,t}$, whereas the reduction in remaining trading rounds tends to increase $p_{\Theta,t}$.

The optimal trading strategies consist of two parts. The first represents the supply shock, the second investors' speculative positions. It is important to notice that the trading activity generated by differential information is *not* the simple sum of each investor's speculative investments. This is because, in the case of heterogeneous information, non-informational trade by one investor could be viewed as an informational trade by another. It is also possible that investors on both sides of the trade think that their trades are non-informational, but the trading is purely due to differential information.[22] The paper focuses on the *additional* trading volume generated by differential information.

$$V_t := \mathcal{V}_t - \mathcal{V}_t^*$$

It's expected value is given by

$$E[V_t] = \frac{1}{\sqrt{2\pi}}(\sqrt{Var[\delta\Theta_t] + Var[\delta x_t^i]} - \sqrt{Var[\delta\Theta_t]})$$

where $x_t^i := X_t^i - \Theta_t = p_{\Pi,t}/p_{\Theta,t}(\hat{\Pi}_t^{p,i} - \Pi)$ is trader i's total trading activity associated with differential information. In Corollary 2, He and Wang provide a closed form solution for the equilibrium volume in the special case where $\sigma_\delta = 0$. It says that informational trading occurs only as long as investors receive new private information. In this case the individual trader does not know whether the other investors trade because of new information or because of liquidity reasons. It is not common knowledge whether the allocation is Pareto efficient. This is the reason why the dynamic version of the No-Speculation theorem given in Geanakoplos (1994) does not apply in this case.[23] If on the other hand investors receive only private information in $t = 1$ the prices will adjust, but no informational trade will occur. This is the difference to the possible no-trade equilibrium in Grundy and McNichols (1989) which can only occur, if $a_\Theta = 1$ and investors receive information only in the first period.

[22]For example there is no additional noise in t, but half of the traders think $\Theta_t = +0.1$ and the other half thinks $\Theta_t = -0.1$

[23]Similar to Grundy and McNichols (1989) the beliefs about future signals need not be concordant when $\sigma_\delta > 0$.

He and Wang then go on to analyse the behaviour of trading volume after $t = 2$ for the case where only the signal in $t = 1$ is informative. The main findings are that trading persists throughout the whole trading horizon. This is due to the fact that investors establish their speculative position, when they receive their private information in $t = 1$ and then gradually try to unwind their positions. This generates peaks in the volume of trade in the middle of the trading horizon. In the case of public announcements, investors increase their positions right before and close them right after the announcement. Therefore, volume and total amount of information revealed through trading depends on the timing of the announcement. Market liquidity drops right before the announcement and bounces back afterwards. They also find that new information, private or public, generates both high volume and large price changes, while existing private information can generate high volume with little price changes.

4.4 Inferring Information from Trading Volume in a Competitive Market Order Model

One drawback of the models, discussed so far, is that investors do not extract the predictive power of volume. In Blume, Easley, and O'Hara (1994) a group of traders make explicit use of volume data to improve their prediction of the liquidation value of an asset. In contrast to the models discussed so far (with the exception of Romer (1993)) the quality or precision of each traders' signal is not common knowledge. In the sense of Morris, Postlewaite, and Shin (1995) this model exhibits a higher depth of knowledge by one degree. Information about the fundamentals, i.e. payoff relevant events, is dispersed among the traders, but there is also asymmetry in the information about the quality of the traders' signals. In other words, this model incorporates asymmetric second order information. Every investor receives a private signal about the liquidation value, Π of the asset. Each investor knows the quality of his signal, but only a subset of investors, investors in group 1, knows the precision of all signals. The higher order uncertainty about the precision of other investors' signals is the source of noise in their model. This higher order uncertainty provides ground for the predictive power of volume and technical analysis.

Blume, Easley, and O'Hara (1994) start their analysis by showing why the models in Brown and Jennings (1989) and Grundy and McNichols (1989) are not appropriate for analysing the role of volume in predicting the value of an asset.

In the framework of Brown and Jennings (1989) there always exists an informationally efficient REE if trade conditional on price and volume is possible. In this case, trader i's information set is $\mathcal{F}_t^i = (P_t, S_t^i, \mathcal{V}_t, \chi_t)$ where $\mathcal{V}_t$ is the per capita volume

$$\mathcal{V}_t = \frac{1}{2}\frac{1}{I}[\sum_{i=1}^{I} \mid x_t^i \mid + \mid \Theta_t \mid]$$

and χ_t^i is an indicator function indicating whether the trader i is a buyer or seller. In equilibrium traders demand the same amount of the risky assets $x_t^i = x_t^j =: x_t$. By the market clearing condition $x_t = \Theta_t$. Thus each trader can infer from his demand and χ_t^i the noisy supply term Θ_t. As the equilibrium price depends only on Θ_t and the average signal $\overline{S}_t$ can be inferred. Thus, in each period t, the tuple $(P_t, \mathcal{V}_t)$ fully reveals $\overline{S}_t$ and Θ_t and thus technical analysis has no value.

In Grundy and McNichols (1989) each individual is endowed with an i.i.d. random number of risky assets. The variance of this random endowment, Θ_t^i, is given by $Var[\Theta_t^i] = I\sigma_\Theta^2$. In the analysed limit economy with infinitely many traders, i.e. $I \rightarrow \infty$, each individual endowment itself has no information content and the variance of the endowments is also infinite. Since the expected trading volume per capital is $\mathcal{V}_t = \frac{1}{2}lim_{I\rightarrow\infty}\frac{1}{I}\sum_{i=1}^{I} \mid x_t^i - \Theta_t^i \mid$, it is infinite as well and no inference can be drawn from it.

In contrast to Brown and Jennings (1989) and Grundy and McNichols (1989), Blume, Easley, and O'Hara (1994) develop a market order model with a generalised information structure. More precisely the information set of each trader i contains the whole price and volume process up to but *excluding* the current time t. This distinguishes these models, first developed in Hellwig (1982), from limit order models. The normally used limit order models exhibit two drawbacks. First, since volume is not normally distributed, the inference of current volume can be quite cumbersome. Second, revealing REE in a Grundy-McNichols or Brown-Jennings limit order model always exist, since traders can always condition on the information contained in their own net trade: its direction and its magnitude Jordan (1983).

Blume, Easley, and O'Hara (1994) assume the following information structure. The common priors for all traders about the liquidation value are $\Pi \sim \mathcal{N}(\mu_{\Pi,0}, \sigma_{\Pi,0}^2)$. Each trader in group 1 receives a signal

$$S_t^i = \Pi + \omega_t + e_t^i,$$

where $\omega_t \sim \mathcal{N}(0, \sigma_\omega^2)$, $e_t^i \sim \mathcal{N}(0, \sigma_{e,t}^2)$ and ω_t and all e_t^i are independent. Note $\sigma_{e,t}^2)$ varies over time.

Each trader in group 2 receives a signal

$$S_t^i = \Pi + \omega_t + \epsilon_t^i,$$

where all ϵ_t^i are i.i.d. $\mathcal{N}(0, \sigma_\epsilon^2)$. It is common knowledge that $I_1 = \nu I$ traders are in group 1 and $I_2 = (1-\nu)I$ traders are in group 2. The asymmetry about the second order information is captured by asymmetric knowledge about the precision of the signals. Traders in group 1 know the precision of group-1 signals, $(1/\sigma_{e,t}^2)$ in each t and in addition the precision of the signals, group 2 traders get, $(1/\sigma_\epsilon^2)$. Group 2 trades only know the signal precision of their own group. In contrast to $(1/\sigma_\epsilon^2)$ the precision of group 1 signals, $\sigma_{e,t}^2$, varies randomly over time. This rules out the possibility that traders in group 2 learn over time the group 2 signal precision.

The distribution of the signals is, therefore, given by:

for group 1 signals: $S_t^i \sim \mathcal{N}(\Pi, \sigma_{S_{1,t}}^2)$, where $\sigma_{S_{1,t}}^2 = \sigma_\omega^2\sigma_{e,t}^2[\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_{e,t}^2}] =: V[S_{1,t}]$

for group 2 signals: $S_t^i \sim \mathcal{N}(\Pi, \sigma_{S_2}^2)$, where $\sigma_{S_2}^2 = \sigma_\omega^2\sigma_\epsilon^2[\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\epsilon^2}] =: V[S_2]$.

It is obvious from the Strong Law of Large Numbers that the average of the signals in each group, $\overline{S}_{1,t}$ and $\overline{S}_{2,t}$ converges almost surely to $\Pi + \omega_t =: \theta_t$.

Blume, Easley, and O'Hara (1994) restrict their analysis to myopic REE. The individual demand for traders with a constant absolute risk aversion coefficient of unity, i.e. $\rho = 1$, is *approximated* by,
for group 1 traders:

$$x_{1,t}^i = \frac{E_{t-1}^{i,1}[\Pi] - P_t}{V_{t-1}[\Pi]} + \frac{S_{1,t}^i - P_t}{V_{t-1}[S_{1,t}]};$$

and for group 2 traders:

$$x_{2,t}^i = \frac{E_{t-1}^{i,2}[\Pi] - P_t}{V_{t-1}[\Pi]} + \frac{S_{2,t}^i - P_t}{V_{t-1}[S_2]}.$$

In contrast to limit order models, there is an additional second term and the expectations are taken with respect to $\mathcal{F}_{t-1}^i$. Adding the demand functions and imposing the market clearing condition gives the equilibrium price. For the limit economy, P_1 is:

$$P_1 = \frac{\frac{1}{\sigma_{\Pi,0}^2}\mu_{\Pi,0} + [\nu\frac{1}{\sigma_{S_{1,1}}^2} + (1-\nu)\frac{1}{\sigma_{S_2}^2}]\theta_1}{\frac{1}{\sigma_{\Pi,0}^2}\nu\frac{1}{\sigma_{S_{1,1}}^2} + (1-\nu)\frac{1}{\sigma_{S_2}^2}}.$$

Group 1 trader can infer θ_1 from P_1, since they know $\sigma_{S_{1,1}}^2$ and $\sigma_{S_2}^2$. P_1, however, does not reveal θ_1 for group 2 traders, since they do not know $\sigma_{S_{1,1}}^2$. Note that the conditional distribution of θ_1 given P_1 is not normal. Traders in group 2 can infer more information about θ_1 if they include trading volume in their inference calculation. The per capita trading volume in $t = 1$ is:

$$\mathcal{V}_1 = \frac{1}{2}\frac{1}{I}\left(\sum_{i=1}^{I_1} \mid x_{1,t}^i \mid + \sum_{i=I_1}^{I} \mid x_{2,t}^i \mid\right)$$

Volume is not normally distributed, because volume is the absolute amount of normally distributed random variables. Blume et al. explicitly characterise the expected per capita volume $\mathcal{V}_1$ in their Proposition 1.

$$\mathcal{V}_1 = \mathcal{V}_1(\theta_1 - P_1, \sigma_{S_{1,1}}^2, \sigma_{S_2}^2, \ldots)$$

Using the equilibrium price relation, one can substitute for $(\theta_1 - P_1)$ a term depending on the signal precisions. The resulting equation shows that volume conveys information about the signal quality of group 1 traders $(1/\sigma_{S_{1,1}}^2)$.

Plotting the derived expression for $\mathcal{V}_1$ with P_1 on the abscissa yields a V-shape relationship between price and volume, for any given $(1/\sigma_{S_{1,1}}^2)$. The minimum volume is reached at a price level $P_1 = \mu_{\Pi,0}$. The average traders' posterior means coincide with the prior mean. As P_1 deviates from $\mu_{\Pi,0}$, posterior means are changed and the first term of the individual demand functions x_t^i increases the trading volume on average. This results in a strong correlation between volume and price change. The V-shape is very robust. As the signal precision (information quality of group 1 signals) decreases the V-shape becomes more pronounced. The same is true when the quantity of information, i.e. fraction of group 1 traders, decreases.

Keeping the price fixed and differentiating expected capital volume with respect to the precision of trader 1 signals $(1/\sigma^2_{S_{1,1}})$ yields that volume is increasing in the precision of group 1's signals if $(1/\sigma^2_{S_{1,1}}) < (1/\sigma^2_\omega)$ and decreasing if $(1/\sigma^2_{S_{1,1}}) > (1/\sigma^2_\omega)$ (provided $(1/\sigma^2_{S_{1,1}}) > (1/\sigma^2_{S_2})$). Intuitively, if trader 1s' signals are very imprecise, their signals are very dispersed and they place little confidence in their signal. They do not trade very aggressively and thus the expected trading volume is low. If on the other hand the signals are very precise, all group 1 traders receive highly correlated signals and thus the trading volume is low again, since trade occurs only between the groups. Therefore, high volume can be a signal for very precise signals but also for very imprecise signals. Volume is first increasing and then decreasing in the signal precision for a given price P_1 and, therefore, for a observed price volume pair $(P_t, \mathcal{V}_t)$ two outcomes, high or low precision are feasable. In other words, the functional relationship is not invertible. Therefore, Blume et al. restrict their analysis to the increasing branch of $\mathcal{V}$, i.e. $1/\sigma^2_{S_{1,1}} \in (1/\sigma^2_{S_2}; 1/\sigma^2_\omega)$. If this is the case the tuple $(P_t, \mathcal{V}_t)$ is revealing $(\theta_1, \sigma^2_{S_{1,1}})$. Since all signals incorporate the common error term ω_t, the liquidation value $\Pi = \theta_1 - \omega_1$ is not known.

In a dynamic setting more realisations of $\theta_t = \Pi + \omega_t$ can be inferred and, therefore, a better estimate about the true liquidation value, Π can be made. In each period the precision of the signals for traders in group 1 is drawn randomly and the analysis is similar to the static case. One difference is that priors in period t are not exogenous, but derived from the market statistics up to time $t-1$. Second the volume expression is slightly different, since traders' endowments in t are the equilibrium demands in $t-1$. By the Strong Law of Large Numbers the equilibrium price converges almost surely to Π, since in each period traders can infer a new θ_t. As time proceeds trade does not vanish, because although traders' beliefs are converging, their precision is diverging at the same rate. Intuitively, in the early trading round traders beliefs are widely dispersed and, therefore, they trade less aggressively. In later trading periods the beliefs are much closer to each other. Since traders are more confident, they take on larger positions. Both effects are offsetting each other and, therefore, volume does not decline with the number of trading rounds.

In the last section Blume, Easley, and O'Hara (1994) compare the utility of a trader who makes use of past market statistics in interpreting the current market statistics with a trader who bases his trading activity only on current market statistics and his priors in $t = 0$. The value of technical analysis is then defined by the amount of money the latter trader, who forgets all past market data, would be willing to pay to know the forgotten past market statistics. Past market data have value because of the common error term ω_t in the signals. Blume et al. show that the value of technical analysis is decreasing in σ^2_ω and increasing in $\sigma^2_{\Pi,0}$. They conclude that technical analysis has higher value for small, less widely followed stocks.

Note, in Blume, Easley, and O'Hara (1994) all traders trade purely for informational reasons. Nobody faces liquidity shocks and there are no noise traders. The No-Trade Theorem should apply since there are no gains from trade given that agents' endowments and preferences are identical. Hsu and Orosel (1997) show that the trading volume is always zero and thus Blume, Easley, and O'Hara (1994) has a serious shortcoming. One can only hope that their results can by revived by adding some non-informational trading motives, e.g. by introducing liquidity traders.

4.5 Sequential Information Arrival Models and Technical Analysis

In the models with differential information covered so far, all traders receive their information at the same time. Likewise, in the asymmetric information models we discussed informed investors received the same private information at the same time. In this section we will discuss asset trading in situations in which traders receive their information sequentially.

The impact of sequential information arrival on price process and volume was first analysed in Copeland (1976). In his model traders not only take the price of the single risky asset as given but also neglect that the price can convey information. In each period one trader receives a signal. After receiving his signal he alters the intercept of his linear demand function. Hence, in each period of time one trader adjust his demand curve and thus the aggregate demand curve changes. In every period (temporary) "incomplete" equilibria determine the price until a new "complete equilibrium" is reached when all traders have received their signal. Copeland introduces short sales constraints which influence price and volume.

In Section 1 of Copeland (1976) all traders get the same signal (or equivalently they interpret the signal equally) one after the other. The price and volume changes are the same in each period as long as the short sales constraint is not binding for any trader. When the short sales constraint becomes binding, price change and volume decrease over time. After how many trading rounds short sale constraint starts binding, depends on the number of traders, the strength of information, i.e. the intercept shift of the individual demand caused by the new information, and on the supply of the assets. Copeland (1976) generalises the setting to one in which traders can receive different, i.e. either a positive or a negative signal. Stated equivalently they interpret the received signal either optimistically or pessimistically. In this case it is less likely that short sales constraints become binding, since optimists increase their demand, whereas pessimists decrease it. The price change in this case depends crucially on the random order of the signal arrival. There are many different possible ordering of positive and negative signals. Copeland shows that there is still a strong correlation between volume and absolute price change. He also compares the price adjustment process in the sequential information arrival model with one in which information is revealed simultaneously to all traders, a tâtonnement model.

Jennings, Starks, and Fellingham (1981) analyse the relationship between price-change and volume by introducing margin requirements for traders who want to sell short instead of prohibiting short sales altogether. They demonstrate that margins have a significant impact on the relationship between price changes and volume. As in Copeland (1976) the relationship depends on the arrival of signals, on the number of investors and in addition on the implicit cost of the imposed margin requirement.

Treynor and Ferguson (1985) analyse the decision problem of an individual investor who receives information about the value of a stock. The investor roughly knows the price impact of his information, but not whether it is really new information or not, i.e. if other traders have already received it before him. In other words, the order of the sequential information arrival is random and not known. If all other traders, i.e. the markets has already received his information, then it is already incorporated in the price and, therefore, he should not trade. If, on the other hand, he is the first to receive this signal, he should buy or sell the relevant assets.

Treynor and Ferguson (1985) consider a stylised situation in which events are becoming public quite fast. More formally, the event gets known to trader i and to all other traders before a new event occurs. Let t_E be the time of the event, t_i the time when trader i receives the signal and t_M the time when the market knows the information. The authors consider only cases in which $(t_M - t_E)$ and $(t_i - t_E)$ are very short in comparision to the time between events, $(t_{E_j} - t_{E_{j-1}})$.

The investor wants to know the probability of $t_i > t_M$ versus $t_i \leq t_M$. For deriving these probabilities he makes use of his

(1) prior distribution about the information dissemination and
(2) the observed price process, combined with his knowledge about the
  (a) underlying stochastic process of the price path
  (b) price impact of information at t_M.

In Treynor and Ferguson (1985) the prior probability that an event occurred in t_E is uniformly distributed within a certain time span with length δ, i.e. the densitiy is $(1/\delta)$. γ is the probability that all other traders will receive the information in the next period, provided they have not received it so far. Similarly, α is the probability that investor i will receive the information in the next period. This determines the transition probabilities for the Markov process with the following four possible states: ω_1 nobody, ω_2 only trader i, ω_3 only all other traders, and ω_4 all traders, received this signal.

Investor i makes use of his knowledge about the underlying price process governed by $P_t = (1 + r_t)P_{t-1}$, where all r_t are i.i. normally distributed with mean zero and variance σ^2. The price changes by a multiplicative factor $\exp(V)$, at t_M, the time when the event becomes known to all traders. If t_i and t_M are known the distribution of possible price paths is given by

$$\Pr(\underline{r}_{t_i} \mid t_M, t_i) = \prod_{t=-\infty}^{t_i} \{\frac{1}{\sqrt{2\pi}\sigma} \exp[\frac{-r_t^2}{2\sigma^2}]\} \quad \forall t_M > t_i$$

$$\Pr(\underline{r}_{t_i} \mid t_M, t_i) = \prod_{t=-\infty}^{t_i} \{\frac{1}{\sqrt{2\pi}\sigma} \exp[\frac{(-r_t - \delta_t t_M V)^2}{2\sigma^2}]\} \quad \forall t_M \leq t_i$$

where $\underline{r}_{t_i}$ denotes the whole process of returns $r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ up to and including t_i.

Investor i is interested in the probability distribution $\Pr(t_M \mid \underline{r}_{t_i}, t_i)$, which by Bayes' Rule is

$$\Pr(t_M \mid \underline{r}_{t_i}, t_i) = \frac{\Pr(t_M \mid t_i)\ \Pr(\underline{r}_{t_i})}{\sum_{t_m=-\infty}^{\infty} \Pr(t_M \mid t_i)\ \Pr(\underline{r}_{t_i} \mid t_M, t_i)}$$

All terms are known except $\Pr(t_M \mid t_i)$. $\Pr(t_M \mid t_i)$ can be rewritten as

$$\Pr(t_M \mid t_i) = \sum_{t_E} \Pr(t_M \mid t_i, t_E) \Pr(t_E \mid t_i),$$

where $\Pr(t_M \mid t_i, t_E) = \Pr(t_M \mid t_E)$, since, given an event occurred, the probability that all other agents get the information only at t_M is independent of when agent i (will or) has received the information. All these probabilities can be directly derived from the given prior information structure.

Treynor and Ferguson provide a numerical example where the trader i infers from the past price process that, with probability of 70 percent, all other traders have not yet received the same information.

In their last section, the authors deliver an optimal portfolio strategy which allows the investor i to capitalise on his information. Their article shows that technical analysis, i.e. inferring information from past prices, helps in the evaluation of new private information.

4.6 Strategic Multi-Period Market Order Models with a Market Maker

Admati and Pfleiderer (1988) analyse a strategic dynamic market order model.[24] Their model is essentially a dynamic repetition of a generalised version of the static model in Kyle (1985). However, their focus is on intraday price and volume patterns. They attempt to explain the U-shape of the trading volume and price changes, i.e. the abnormal high trading volume and return variability at the beginning and at the end of a trading day. In their model the value of a single risky asset follows the exogenous process

$$\Pi = \overline{\Pi} + \sum_{t=1}^{T} \delta_t$$

where δ_t is a i.i.d. random variable, whose realisation becomes common knowledge only at t. As usual there are two motives for trading: information and liquidity. All I_t informed traders observe the same signal $S_t = \delta_{t+1} + \epsilon_t$ at time t, where $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. In other words, informed traders observe a noisy version of the public information one period in advance. Since δ_{t+1} is known publically in $t+1$ the informational advantage is only *short-lived*. Informed traders have, therefore, no incentive to restrict their trading in order

[24] To be consistent with our notation we denote: $\Pi = F$, $\overline{\Pi} = \overline{F}$, $I_t = n_t$, $J_t = m_t$, $Y_t = \omega_t$, $z_t^j = y_t^j$, $\Theta_t = z_t$, and $\sigma_\epsilon^2 = \phi_t$.

to have a larger informational advantage in the next period. This simplifies the analysis dramatically. However, analysing only short-lived information neglects interesting aspects. In Admati and Pfleiderer (1988) there are two types of liquidity traders, whose demand depends neither on the price nor on their information. Whereas J_t *discretionary liquidity traders* can choose a period within $[T', T'']$ in which to trade, *nondiscretionary liquidity traders*, must trade a given amount at a specific time. For simplicity it is assumed that the market maker as well as all traders are risk neutral. As in Kyle (1985) the market maker observes the total net order flow Y_t, in addition to $\underline{\delta}_{t-1} := (\delta_0, \delta_1, ..., \delta_{t-1})$. The total net order flow in t is given by

$$Y_t = \sum_{i=1}^{I} x_t^i + \sum_{j=1}^{J_t} z_t^j + \Theta_t$$

where the first component represents the aggregated demand from informed traders, the second the aggregated demand from discretionary liquidity traders and the third the aggregated demand from nondiscretionary liquidity traders. The market maker tries to infer the information of the insiders from Y_t. The variance of total liquidity trading, $\Psi_t = Var(\sum_{j=1}^{J_t} y_t^j + \Theta_t)$, is in contrast to Kyle (1985) endogenously determined, as it depends on the strategic decision of the discretionary traders. As in Kyle (1985) the zero profit condition together with risk neutrality implies that the market maker sets the price equal to his expected value. A linear pricing rule is, therefore, given by

$$P_t = \overline{\Pi} + \sum_{\tau=1}^{t} \delta_\tau + \lambda_t Y_t,$$

where $\frac{1}{\lambda_t}$ again measures the market depth. Given this pricing rule the equilibrium value of λ_t is decreasing with the number of informed traders I_t. As in Kyle (1985) the market depth $\frac{1}{\lambda_t}$ is increasing with Ψ, the variance in liquidity traders demand. Admati and Pfleiderer assume that discretionary liquidity traders take λ_t as given, although their trading intensity affects λ_t. The costs of trading for the liquidity traders, which equals the profit for insiders, is the difference between what the liquidity traders pay and the expected value, i.e. $E[(P_t(\underline{\delta}_t, \underline{Y}_t) - \Pi)(\sum_{j=1}^{J_t} z_t) \mid \underline{\delta}_t, \underline{Y}_{t-1}, \sum_{j=1}^{J_t} z_t]$ which is equal to $\lambda_t(\sum_{j=1}^{J_t} z_t)^2$. Therefore, discretionary liquidity traders would trade when λ_t is smallest, i.e. when the market is deepest. This is the case when Ψ_t is high and thus it is optimal for them to "clump" together, which increases Ψ_t even more. High variance in noise trading, Ψ_t allows insiders to hide more of their trade behind noise trade. Their demand in equilibrium is given by $x_t^i = \beta_t^i S_t$, where S_t is the signal about δ_{t+1} and $\beta_t = \sqrt{\frac{\Psi_t}{Var(S_t)}}$. Thus, at times when liquidity traders clump together, informed traders also trade more aggressively. This increases the overall trading volume in this trading period. The problem is, however, that discretionary traders have to coordinate when to trade and, therefore, many equilibria can arise. It is plausible that the convention arose that they all trade in the beginning and at the end of the trading day. Admati and Pfleiderer (1988) show that equilibria in which discretionary traders clump together exist. They also apply a refinement criterium, which shows that these equilibria are the only ones that are robust to small perturbations in the vector of variances of the discretionary liquidity demands. As in Kyle (1985), the amount of information revelation by prices is independent of the total variance of liquidity trading. More noise trade would suggest less informative prices. On the other hand, more noise allows insiders to be more aggressive in their trade which making the price more informative. The aggressiveness of the insiders is such that both effects will balance out.

Admati and Pfleiderer (1988) then incorporate endogenous information acquisition. Traders can buy the signal S_t at a fixed costs c. This makes the number of informed traders, I_t, endogenous. The authors apply two different equilibrium concepts. In the second concept each insider's strategy depends on I_t. When I_t is high, insiders compete with each other and therefore their profits will be lower, or equivalently, the trading costs for liquidity traders will be lower. At times when discretionary traders clump together, Ψ_t is high and, therefore, many insiders will enter the market. This reduces the trading costs for liquidity traders even more since the insiders are competing against each other. Thus, endogenous information acquisition intensifies the effects explained above and one would expect large trading volume at certain times.[25]

Admati and Pfleiderer's model is a very simplified picture of reality. First, information is only asymmetric for one period, i.e. it is short-lived. Long-lived asymmetric information was considered in Foster and Viswanathan (1990) and in Holden and Subrahmanyam (1994) for risk averse traders. The second simplification is that information is only asymmetric but not differential, i.e. all insiders observe the same signal. This assumption is relaxed in the next paper discussed.

Foster and Viswanathan (1996) extend the model in Kyle (1985) to the case with I risk neutral informed investors.[26] Each investor gets a long-lived individual private signal at $t = 0$. In contrast to Admati and Pfleiderer (1988), there are no discretionary liquidity traders. (Nondiscretionary) liquidity traders demand $\Theta_t \sim \mathcal{N}(0, \sigma_\Theta^2)$ shares in period $t \in [1, ..., T]$. As in Kyle (1985) the market maker only observes the total net order flow $Y_t = \sum_{i=1}^{I} x_t + \Theta_t$ and sets the price at time t according to

$$P_t = E[\Pi \mid \underline{Y}_t],$$

where his prior distribution of Π is given by $\mathcal{N}(P_0, \sigma_\Pi^2)$ and $\underline{Y}_t$ denotes the whole process $(Y_1, ..., Y_t)$. Informed traders $i \in \mathbb{I} = \{1, 2, ..., I\}$ have to submit their market orders x_t^i before Y_t becomes known. Since each trader i knows his individual demand, $\underline{x}_t^i$ and the whole history of $\underline{Y}_{t-1}$ he can infer the net order flow of *all other* traders $\underline{z}_{t-1} = \underline{Y}_{t-1} - \underline{x}_{t-1}^i$.

[25]Pagano (1989a) provides a model which illustrates the negative correlation between trading volume and market thinness as well as volatility. In this model risk-averse investors' value the stock for hedging reasons differently and have to pay a fixed transaction cost to enter the market. Each additional trader who enters the market reduces the market thinness and thus volatility. This generates for the other risk-averse traders a positive externality. Pagano (1989a) shows that there are multiple "bootstrap" equilibria, some with low trading volume and high price volatility, and others with high trading volume and low volatility. The latter are Pareto superior. Pagano (1989b) shows that, in presence of different transaction costs, traders may be unable to co-ordinate on a single market.

[26]For consistency we adjust the notation to $I = M$, $t = n$, $Y_t = y_n$, $\Pi = v$, $S_0^i = s_{i,0}$, $\overline{S} = \hat{v}$, $P_t = p_n$, $\hat{S}_{0,t}^i = t_{in}$, $S_t^i = s_{in}$ and $\hat{\cdot}^i = \cdot^i\prime$.

Each informed trader receives an individual signal S_0^i at the starting point of trading. The joint distribution of all individual signals with the asset's true value is given by

$$(\Pi, (S_0^1, ..., S_0^I)) \sim \mathcal{N}((P_0, \vec{0})), \begin{pmatrix} \sigma_\Pi^2 & \Delta_0 \\ \Delta_0 & \Psi_0 \end{pmatrix}),$$

where Δ_0 is a vector with I identical elements, i.e. $\Delta_0' = (c_0, c_0, ..., c_0)$ and Ψ_0 is the variance-covariance matrix of the signals given by

$$\Psi_0 = \begin{pmatrix} \Lambda_0 & \Omega_0 & ... & \Omega_0 \\ \Omega_0 & \Lambda_0 & ... & \Omega_0 \\ ... & ... & ... & ... \\ \Omega_0 & \Omega_0 & ... & \Lambda_0 \end{pmatrix}.$$

This signal structure imposes a strong symmetry assumption, since (a) all signals have the same covariance c_0 with the true asset value, (b) all signals have the same variance Λ_0 and (c) the cross variance between signals is Ω_0 for all signals. It covers also the special cases $\Omega_0 = \Lambda_0$ where all insiders get the same signal and $\Omega_0 = 0$ where all signals are independent.

By applying the Projection Theorem one gets

$$E[\Pi - P_0 \mid S_0^1, ..., S_0^I] = \Delta_0'[\Psi_0]^{-1} \begin{pmatrix} S_0^i \\ ... \\ S_0^I \end{pmatrix}.$$

By the imposed symmetry assumptions, all elements of the vector $\Delta_0'[\Psi_0]^{-1}$ are identical, say to κ. Therefore, the inner product can be rewritten as

$$E[\Pi - P_0 \mid S_0^1, ..., S_0^I] = \underbrace{\kappa I}_{:=\theta} \underbrace{\frac{1}{I}\sum_{i=1}^{I} S_0^i}_{:=\overline{S}} = \theta\overline{S}.$$

$\overline{S}$, the average of all signals S_0^i is a sufficient statistic for all signals. It follows that the market maker and the informed traders need not infer each individual signal S_0^i but only the average signal $\overline{S}$. This allows us to simplify the sufficient state description dramatically.

The market maker's estimate of S_0^i at t is given by

$$\hat{S}_{0,t}^i := E[S_0^i \mid Y_1, ..., Y_t] = E[S_0^i \mid \underline{Y_t}].$$

The market maker sets a competitive price $P_t = E[\Pi \mid \underline{Y}_t]$. Since $(S_0^1, ..., S_0^I)$ is a sufficient statistic for Y_t and for[27] $P_0 = 0$

$$P_t = E[E[\Pi \mid S_0^1, ..., S_0^I] \mid Y_t] = \theta E[\overline{S} \mid \underline{Y}_t] = \theta \frac{1}{I} \sum_{i=1}^{I} \hat{S}_{0,t}^i.$$

[27]In my opinion equations (2) and (5) in the paper are only correct if $P_0 = 0$.

The informational advantage of informed trader i in period t is the difference

$$S_t^i := S_0^i - \hat{S}_{0,t}^i.$$

Foster and Viswnathan further define the following conditional variances and covariances

$$\Sigma_t := Var(\theta\overline{S} \mid \underline{Y}_t) = Var(\theta\overline{S} - P_t) = Var(E[\Pi \mid S_0^1, ..., S_0^I] \mid \underline{Y}_t),$$
$$\Lambda_t := Var(S_0^i \mid \underline{Y}_t) = Var(S_t^i),$$
$$\Omega_t := Cov(S_0^i, S_0^j \mid \underline{Y}_t) = Cov(S_t^i, S_t^j),$$

and derive the following relationships (using the Projection Theorem):

$$\Sigma_t = \frac{\theta^2}{I}[\Lambda_t + (I-1)\Omega_t],$$

which implies

$$\Lambda_{t-1} - \Lambda_t = \Omega_{t-1} - \Omega_t,$$
$$\Sigma_{t-1} - \Sigma_t = \theta^2[\Lambda_{t-1} - \Lambda_t],$$

and therefore

$$\Lambda_t - \Omega_t = \chi \ \ \forall t.$$

Since the market maker will learn the average signal $\overline{S}$ much faster than any individual signal, the correlation between the informational advantage of insiders, Ω_t, must become negative after a sufficient number of trading rounds. This negative correlation between S_t^i will cause the waiting game explained below. Foster and Viswanathan use a Bayesian Nash equilibrium concept given the price setting behaviour of the market maker and restrict their analysis to linear Markov equilibria. As in Kyle (1985) the applied equilibrium concept is Bayesian Nash equilibrium with elements of the Rational Expectations Equilibrium concept. The equilibrium is represented by a tuple $(\mathbf{X}^1, ..., \mathbf{X}^I, \mathbf{P})$ where $\mathbf{X}^i$ is a vector of demand correspondences for trader i for each date, t, i.e.

$$\mathbf{X}^i = (x_1^i, ..., x_T^i), \ where \ x_t^i = x_t^i(S_0^i, \underline{Y}_{t-1}^i, \underline{z}_{t-1}^i),$$

and $\mathbf{P}$ is a vector of price setting functions for each t, i.e.

$$P_t = P_t(\underline{Y}_t) = E[\Pi \mid \underline{Y}_t].$$

$x_t^i(\cdot)$ is the stock holding of trader i at time t which maximises his profits from time t until T. $\mathbf{X}^i(\cdot)$ is optimal by backward induction. Foster and Viswanathan impose a Markov Perfect refinement criterion on the possible set of equilibria. How restrictive this criterion is depends on which state space the (trade) strategies can be based on. There are, therefore, two different state spaces: The first state space is given by the choice of nature, whereas the second covers events of the original state space, on which traders can base their trading strategies. The smaller the latter state space is, the more restrictive is the Markov Perfect refinement criterion. The state space given by the choice of nature is $(\Pi, \{S_0^i\}_{i\in\mathbf{I}}, \underline{\Theta}_T)$. Incorporating the choice of each trader, one can consider the following extended state space$(\Pi, \{S_0^i\}_{i\in\mathbb{I}}, \{\underline{x}_T, \underline{\Theta}_T\}_{i\in\mathbb{I}})$. Knowing $\Theta_t = \sum_{i=1}^I x_t^i - Y_t$, $(\Pi, \{S_0^i\}_{i\in\mathbb{I}}, \{\underline{x}_T, \underline{Y}_T\}_{i\in\mathbb{I}})$. This state space can be rewritten as $(\Pi, \{\underline{S}_T^i\}_{i\in\mathbb{I}}, \{\underline{\hat{S}}_{0,T}^i\}_{i\in\mathbb{I}}, \{\underline{x}_T, \underline{Y}_T\}_{i\in\mathbb{I}})$, as $S_t^i = S_0^i - \hat{S}_{0,t}^i$. All strategies have to satisfy the measurability condition, i.e. traders can condition their strategies only on states they can distinguish, i.e. on partitions. The author focus on linear recursive Markov perfect equilibria which satisfy following conditions:

$$x_t^i = \beta_t S_{t-1}^i,$$

$$\hat{S}_{0,t}^i = \hat{S}_{0,t-1}^i + \zeta_t Y_t,$$

$$P_t = P_{t-1} + \lambda_t Y_t,$$

where $Y_t = \sum_i^I x_t^i + \Theta_t$ and it is shown that $\lambda_t = \theta\zeta_t$ and $\hat{S}_{0,t}^i = \hat{S}_{0,t-1}^i + \zeta_t Y_t$ is necessary to guarantee that the forecasts of the others' forecasts is linear.

Foster and Viswanathan show that the dimensionality of the state space can be reduced, since a sufficient statistic for the past can be found for this equilibrium concept. Trader i bases his strategy on his information set $(S_0^i, \underline{Y}_{t-1}, \underline{x}_{t-1}^i)$. Since in equilibrium his optimal demand is given by $x_\tau^i = x_\tau^i(S_0^i, \underline{Y}_{\tau-1})$ $\forall\tau$ his information set can be simplified to $(S_0^i, \underline{Y}_{t-1})$. This makes it clear that trader i can only manipulate trader j's beliefs about the true value, Π, via $\underline{Y}_t$. Foster and Viswanathan show that S_{t-1}^i, the information advantage at $t-1$, is a sufficient statistic for trader i to predict $E[\Pi - P_{t-1} \mid \mathcal{F}_{t-1}^i] = \eta_t S_{t-1}^i$, since P_{t-1} is common knowledge and all random variables are normal. η and ϕ are constant regression coefficients. As this is true for all traders, it is also sufficient for trader i to forecast S_{t-1}^j in order to forecast the forecasts of others, i.e. $E[S_{t-1}^j \mid \mathcal{F}_{t-1}^i] = \phi_t S_{t-1}^i$. By induction the t^{th}order forecast, the forecast of trader i about the forecast of trader j about the forecast of trader i, etc. is also a linear function of S_{t-1}^i. Thus the hierarchy of forecasts is not history dependent. In other words the infinite regress problem, which we will discuss in detail later, is avoided. Their analysis shows that, in equilibrium, the dimensionality issue can be resolved.

In order to check whether this is really a Nash equilibrium one has to show that no trader has an incentive to deviate. For analysing deviation a larger state space is needed and the dimensionality issue arises again. Suppose only trader i deviates from the equilibrium strategy and submits arbitrary market orders $(x_1^i, ..., x_t^i)$ in the first t periods. Let $Y_t^{i\prime}, P_{t\prime}, \hat{S}_{0,t}^{i,i\prime}, S_t^{j,i\prime}$ with the additional superscript $^{i\prime}$, be the corresponding variables when traders play the equilibrium strategies. By construction $S_t^{i,i\prime}$, the informational advantage, is orthogonal to $(\underline{Y}^{i\prime}_{t-1})$. Note that $(\underline{Y}^{i\prime}_{t-1})$ is in i's information set because i also knows the strategy he would have followed in equilibrium and thus he can also derive the change in other traders' expectations caused by his strategy change. Therefore trader i's information set also captures $S_{t-1}^{i,i\prime}$, $P_{t-1}^{i\prime}$, $S_{0,t-1}^{j,i\prime}$. A sufficient statistic for his information set is given by $S_{t-1}^{i,i\prime}$ together with the deviation from the equilibrium price $(P_{t-1}^{i\prime} - P_{t-1})$. Therefore $E[\Pi - P_{t-1} \mid \mathcal{F}_{t-1}^i] = E[\Pi - P_{t-1}^{i\prime} \mid S_{t-1}^{i,i\prime}] + (P_{t-1}^{i\prime} - P_{t-1})$. Foster and Viswanathan conjecture the value function for trader i. Foster and Viswanathan conjecture the value function for a trader i

$$V^i[S_{t-1}^{i,i'}, P_{t-1}^{i'} - P_{t-1}] = \alpha_{t-1}(S_{t-1}^{i,i'})^2 + \psi_{t-1}S_{t-1}^{i,i'}(P_{t-1}^{i'} - P_{t-1}) - \mu_{t-1}(P_{t-1}^{i'} - P_{t-1})^2 + \delta_{t-1}$$

and derive the optimal market order size for a certain time period. The resulting conditions for the Markov Perfect linear recursive equilibrium allow to verify that the proposed value function was indeed correct. Finally, they relate their results to less general models, like Kyle (1985), Holden and Subrahmanyam (1992) and others.

For calculating numerical examples, Foster and Viswanathan apply a backward induction algorithm for the case of three traders and four trading rounds. They compare four different correlations between the initial signals S_0^i; very high, low positive, zero and low negative correlation. The major findings are that (1) the lower the signal correlation, the less informative is the price process, (2) profit for insiders is lowest with identical information and highest with positive but not perfect correlation, (3) with positive signal correlation λ_t, the market maker's sensitivity falls over time, whereas with negative correlation λ_t rises with the trading rounds, and finally (4) the conditional correlation of the remaining information advantage S_t^i is decreasing over time and becomes negative provided there are enough trading rounds.

These results are the outcome of two effects. First, with heterogeneous signals the competitive pressure is reduced since each trader has some monopoly power. Second, when the S_t^is become negatively correlated traders play a waiting game. This is driven by the fact that the market maker learns more about the average signal than about the individual signals. With negatively correlated S_t^i, traders are more cautious and more reluctant to take on large positions early. Foster and Viswanathan then go to analyse the effects of increasing the number of trading rounds keeping the total liquidity variance, $T\sigma_\Pi^2$, constant. With more trading rounds the speed of information revelation is higher and a U-shape pattern of λ_t arises and becomes more pronounced. This U-shape of the market maker's sensitivity results from the waiting game. Their analysis suggests that dynamic competition with heterogeneously informed traders can be quite distinct. Whereas insiders with identical information trade very aggressively, (i.e. they are in a "rat race") insiders with heterogeneous information trade less aggressively since they play a waiting game.

Back, Cao, and Willard (1997) conduct the same analysis in continuous time. They show that there does not exist a linear equilirium when signals are imperfectly correlated. The reason is that near the announcement date a relatively large amount of information remains private, which causes the market to approach complete illiquidity.

Vayanos (1996) studies a strategic dynamic limit order model à la Kyle (1989). He shows that the forgone gains from trade lost due to strategic behaviour, increase as the time between trades shrinks.

Chapter 5

Herding Models

In the preceeding sections we dealt with situations in which information became known gradually. In many situations some agents could act earlier than others. Sequential decision taking may cause a phenomena like herding even if all agents behave rationally. Herding in sequential decision making can occur for at least three different reasons:

- Payoff Externalities
- Reputational Effects in Principal-Agent Models
- Informational Externalities

The latter two effects can not only lead to herding but also to delays in decision making in a world where the order of decision making is endogenous. No decision may be made at all in the extreme case. Herding models refer to an environment in which each agent makes one irreversible decision. This distinguishes the herding literature from the experimentation literature.

5.1 Herding Due to Payoff Externalities

In an environment in which payoffs are higher if all agents choose the same action, it is obvious that herding occurs in at least one possible equilibrium. Standard coordination failure games provide one possible environment.[1] The model in Admati and Pfleiderer (1988) is an example of herding caused by payoff externatility. All discretionary liquidity traders try to trade at the same time, i.e. clump together. Bank runs are further examples of herding in finance. Froot, Scharfstein, and Stein (1992) and Hirshleifer, Subrahmanyam, and Titman (1994) show that myopic investors try to acquire information about the same event as others. These papers will be discussed in greater detail below. In this class of models herding is defined as "doing the same." However, agents do not neglect their own information. Whether herding is socially optimal depends on the whole payoff structure of the game.

[1] However, mixed strategy equilibria are also possible.

5.2 Reputational Cascades in Principal-Agent-Models

In this section we will refer to models in which herding among agents is induced by reputational effects towards the principal. Herding in this case is only individually optimal and the First Best cannot be achieved. In Scharfstein and Stein (1990) two risk neutral agents (managers) invest sequentially in two identical investment objects. Each manager is either smart or dumb. Neither the principal nor the agents themselves know the types. Each agent receives a binary signal $\{S_H^i, S_L^i\}$ about the true liquidation value $\Pi \in \{\Pi_H, \Pi_L\}$ of the projects. The signal structure satisfies following conditions:

(1) $\Pr(S_H \mid \Pi_H, smart) > \Pr(S_H \mid \Pi_L, smart)$
(2) $\Pr(S_H \mid \Pi_H, dumb) = \Pr(S_H \mid \Pi_L, dumb)$
(3) $\Pr(S_H \mid smart) > \Pr(S_H \mid dumb)$
(4) smart agents' signals are (perfectly) correlated

(1) states that a smart agent gets with higher probability the right signal, whereas (2) says that dumb managers get with equal probability the high signal, independent of whether the project is good or bad. Condition (3) guarantees that the signal is purely about the investment project and cannot be used by a single agent to improve his knowledge about his type. Condition (4) states that smart agents have the same (correlated) forecast error. The payoffs $\{\Pi_H, \Pi_L\}$ are such that the first agent invests if he gets the high signal and does not otherwise. Knowing this, the second agent can infer the signal of the first agent and can base his decision on both signals. First Best, however, is not obtained in this environment, since agent two cares about his reputation with respect to the principal, i.e. he wants to appear as a smart type. Agent two's reputation increases when he makes the right decision. If he makes the wrong choice, it is better for his reputation if the first agent has chosen the wrong alternative, too. Combining this with the assumption that it is more likely that two smart agents get the same wrong signal causes him to follow the others' decisions. Agent two's information set contains both signals. If his signal coincides with the one of agent one, he will make the same investment decision as agent one. If his signal is different from the first one he will still follow the first agent's decision. Intuitively, the reasoning is that if he is wrong so is agent one. This could also be due to the possibility that two smart agents accidentally received a wrong signal. If he would follow his own signal then the principal thinks that it is more likely that at least one agent is dumb. If it turns out that his signal is wrong, he would be considered a dumb manager. The authors show that a separating equilibrium does not exist and agent two always employs a herding strategy in equilbrium given plausible beliefs, Cho and Kreps (1987). This result hinges on the assumption that the prediction error of smart agents is correlated, because only then the principal's updating rule becomes a function of agent one's investment decision. The herding effect disappears for the case where the smart signals are independent.[2] Trueman (1994) generalised this analysis and applied it to analyst forecasting. Another model with reputational cascades is Zwiebel (1995). In this model managers may prefer choosing a

[2] In my opinion herding may also occur if agents are very risk averse, in particular if the principal employs relative performance evaluation.

common action rather than a superior innovation, since the common action provides a more accurate benchmark for relative performance evaluation.

5.3 Statistical Cascades Due to Information Externalities

In models in this section, an agent takes neither reputational effects nor the fact that his true payoff for a given action depends on the actions of others into account. Precluding pre-play communication, herding is only generated by the positive informational externalities a predecessor generates for his successors. Inefficient information cascades can occur if the successor can only partially infer the predecessors' information from his action but he still ignores his own signal.

5.3.1 Exogenous Sequencing

This strand of papers began with Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992). In Banerjee (1992) I risk neutral agents choose an asset $j \in [0, 1]$ on an interval of the real line. All assets' payoffs are zero, with the exception of asset j^*, whose certain payoff is Π. All agents have uniform priors. With probability $\alpha < 1$ an agent gets a signal, which is true with probability β and false with probability $(1 - \beta)$. If the signal is wrong, then it is uniformly distributed on the interval. Agents make their decision sequentially. Successors can observe the predecessors' decisions, but not their signals.

Banerjee derives the Bayesian Nash equilibrium after he has assumed three tie-breaking rules - which are in favour of a non-herding outcome. In equilibrium, the first agent follows his signal or chooses $j = 0$ as assumed decision rule if he has not observed a signal. Given the tie-breaking rule the second decision maker only follows the first agent if he has no signal. Otherwise he follows his signal, which can be identical to agent one's decision. The third decision maker always follow his predecessors if they have chosen the same action $j' \neq 0$, regardless of his signal. This is optimal for him, since both predecessors choose only the same asset $j' \neq 0$ in two cases. Either agent one got signal j' and agent two got no signal and followed agent one or agent one and agent two got the same signal j'. In the former case - which occurs with probability $(1 - \alpha)$ - decision maker three is indifferent between following the predecessors' decisions and his own signal. In the latter case, whose likelihood is α, j' is j^* with probability one, since the event that agent one and agent two get the same wrong signal j' is a zero probability event. Therefore, decision maker three will follow his predecessors and ignore his signal. Agent four knows that agent three's decision carries no information about his signal. Thus, he faces exactly the same situation as decision maker three and herding will occur. In the case where agent one gets a wrong signal and agent two no signal, i.e. he just follows agent one, the whole crowd runs in the wrong direction. This happens, although asset j^* could be found with probability one, if a large enough number of agents could communicate. This inefficiency only occurs (in sequential decision making) if the predecessors' actions are not a sufficient statistic for their information, i.e. the successors can only partially infer the information of the predecessors. In Banerjee (1992) the one dimensional action space on $[0,1]$ cannot reflect the signal which is two-dimensional, a variable on $[0,1]$ and a binary variable $\Xi = \{0,1\}$, which indicates whether the predecessor received a signal or not.

In Bikhchandani, Hirshleifer, and Welch (1992) this is achieved by a discrete action space: adopt or reject. Although the signal in their basic example is also only binary $\{H, L\}$, the action space cannot capture the whole information of a later decision maker.[3] This information consists of his own signal and of information derived from predecessors' actions.[4] The first decision maker follows his signal in their model. If the second agent gets an opposite signal he is indifferent between adopting and rejecting. Bikhchandani, Hirshleifer, and Welch (1992) assume a random tie-breaking rule. In this case the third agent cannot infer the second agent's signal. If the second agent randomly chooses the same action as the first one, the third agent cannot infer the second agent's signal. From the viewpoint of the third agent, it can be that either the second agent got the same signal as the first one or the opposite signal and randomly chose the same action. If the third decision-maker observes that both predecessors have chosen the same action, he will always follow them regardless of his own signal. All following decision makers know that the third decision-maker ignored his own signal and therefore they do not try to infer any information from his action. Actually, they face the same problem as the third decision maker and, therefore, join the crowd. The cascade evolving in this manner prevents the aggregation of information and, therefore, convergence to the correct action need not occur.

The authors show for a special case in their section "Fashion Leaders" that a higher signal precision for the first decision maker can make informationally inefficient cascades sooner and more likely. The reason is that higher signal precision for the first agent makes it more likely that the second agent follows the first one. In a setting with an endogenous decision sequence it seems plausible that agents with the highest precision are willing to decide first. Their analysis about the fragility of cascades to public information releases is also interesting. Public information prior to the first agent's decision can make inefficient cascades even more likely. On the other hand, public information after a cascade has already begun, is always socially beneficial. Already a small amount of public information can shatter a long-lasting cascade. As explained above, a cascade is created by the decision of the first two agents and, thus, the public information need only lift out their information.

Gale (1996) provides a similar example, where in contrast to Bikhchandani, Hirshleifer, and Welch (1992) the *signal space* is continuous, i.e. $S^i \in [-1; +1]$ and the action space is still binary $\{H, L\}$. In this case a cascade can be shattered if an extreme signal arises. For simplicity, lets assume that the payoff of at least I identical investment opportunities is given by the average of all signals, i.e. $\Pi = \frac{1}{I}\sum_{i=1}^{I} S^i$. Given that the signals are uniformly distributed over $[-1, +1]$ the First Best solution is achieved if all

[3] In their generalised version the signals are not binary.

[4] A continuous action space could reveal the posterior of an immediate predecessor which is a sufficient statistic for all past signals. In this case no herding occurs.

agents invest if and only if $\Pi = \frac{1}{I}\sum_{i=1}^{I} S^i > 0$. In sequential decision making, agent one invests if $S^1 > 0$ and agent 2 if $S^2 + E[S^1 \mid action^1] > 0$, etc. If agent 2 observes that agent 1 has invested, he will invest if $S^2 + E[S^1 \mid S^1 > 0] = S^2 + \frac{1}{2} > 0$. If agent 2 also invests, agent 3 will invest if $S^3 + \frac{3}{4} > 0$, and so forth. In other words if agent 1 and 2 have invested then agent 3 needs to receive a really bad signal $S^3 < -\frac{3}{4}$ in order to not invest, i.e. not to follow his predecessors. This means that a cascade becomes more and more stable over time, i.e. the signal necessary to break up a cascade has to be more and more extreme. Although herding behaviour will occur, an informational cascade can never occur in Gale (1996). Smith and Sørensen (1994) define 'herding' as convergence in actions and 'cascades' as convergence in agents' beliefs. In cascades agents ignore their own signal and base the current decision only on historic actions.

Smith and Sørensen (1994) not only consider a continuous signal space but also allow agents' preferences to differ. Incorporating diversity in taste can lead to situations of 'confounded learning'. In such situations the observed history does not provide additional information for decision making and the decision of each type of agent might forever split between two actions.

Smith and Sørensen (1997) relate herding models to the literature of experimentation. This literature stems from Rothschild's (1974) two-armed bandit analysis. Herding models correspond to the experimentation problem faced by single myopic experimenter who forgets his formal signal but remembers his past actions. The incorrect herding outcomes correspond to the familiar failure of complete learning in an optimal experimentation problem.

Lee (1993) shows, how crucial the discretness of the *action space* is. The discretness plays a dual role. (1) It prevents somebody's actions from fully revealing his posteriors and (2) it prevents each agent from fully using of his information. In Lee's model the likelihood of an inefficient cascade decreases as the action space grows. He also claims that Banerjee's model is an exceptional case, since the (degenerated) payoff structure in Banerjee (1992) does not distinguish between small and large errors.[5]

5.3.2 Endogenous Sequencing and Strategic Delay

If each decision maker could decide when to decide, everybody would want to be the last one, in order to profit from the positive information externalities generated by his predecessors' decisions. Strategic delays caused by information externalities were first discussed in Chamley and Gale (1994) and Gul and Lundholm (1995).

In Chamley and Gale (1994) time is discrete $t = 1, 2, ..., \infty$ and each of (randomly) I agents has an investment opportunity, i.e. a real option to invest or not to invest. Each investor knows whether he himself has an investment opportunity, but he does not how

[5] Lee's claim cannot follow from his model, since in his model every agent gets a signal with certainty unlike in Banerjee (1992). It can be shown that the payoff function in Banerjee (1992) can be generalised to a certain degree.

many investors have this opportunity as well. Therefore, he does not know I. The true payoff of the identical investment opportunities is increasing in I, in the number of possible investment opportunities, and not in the number of investments actually undertaken. Agents who invest early reveal that they have an investment opportunity. This positive information externality allows the successors to update their beliefs about the true I. In order to avoid all agents waiting forever, each agents waiting costs are given by a common discount factor $0 < \delta < 1$. Chamley and Gale (1994) focus on *symmetric* Perfect Bayesian Equilibria in which agents apply behavioural strategies.[6] They show that there are three exclusive possible equilibrium continuation paths given a certain history of past investments. If beliefs about the number of people who got an investment opportunity are sufficiently optimistic, all players immediately invest and the game ends. On the other hand, if these beliefs are sufficiently pessimistic no one will invest and hence no information is revealed. In this case the game ends as well, since one period later the situation has not changed. For intermediate beliefs, given a certain investment history, the remaining players with investment opportunity are indifferent between investing and waiting. Hence, they randomise in this period, i.e. employ a behavioural strategy. The remainig investors who have not yet invested try to update their beliefs about the total number of investment opportunities from the random number of investments in this period. It is obvious that information aggregation is inefficient in such an setting. The authors also show that as the period length increases, the possibility of herding disappears. This raises an interesting question concerning continuous trading on stock markets: does continuous trading lead to strategic delays and herding, i.e. to a worse information aggregation than in a single batch auction.[7]

In Gale (1996) the type of an agent is given by his signal $S^i \in [-1, 1]$ about the payoff of the investment opportunities. The payoff of each investment project is $\Pi = \frac{1}{I}\sum_i^I S^i$. Gale considers only the case for two agents, i.e. $I = 2$. With a common discount factor δ, a decision-maker with a higher signal is more impatient to invest than somebody with a lower signal. The aim is to derive the threshold level $\overline{S}$ for the signal value required to motivate an investor into investing in period 1. Whether somebody exercises his real option early depends on the probability that he will regret in the next period that he has invested early. An investor who invests early regrets it if the other investor has not invested and his posterior beliefs about the payoff are $S^i + E[S^{-i} \mid S^{-i} < \overline{S}] < 0$ are negative. The event that the other agent does not invest occurs with probability $\Pr(S^i < \overline{S})$. In equilibrium, an agent with signal $\overline{S}$ is indifferent between waiting and investing in the first period, i.e. the waiting costs are equal to the option value of delay.

$$(1-\delta)\overline{S} = -\delta \Pr(S^i < \overline{S})\{\overline{S} + E[S^i \mid S^i < \overline{S}]\}$$

There exists a unique equilibrium $\overline{S}$ in which information is not fully revealed and the outcome need not be efficient. E.g. if both signals are $0 < S^i < \overline{S}$ nobody will invest

[6] Action rules determine an action at a certain partition/decision knode. Randomising over different action rules at any partition is a behavioural strategy. A strategy is a sequence of action rules. Randomising over *pure* strategies is a mixed strategy.

[7] As seen in Section 4.1, Grundy and McNichols (1989) show that with an increasing number of trading rounds per day, and thus observable prices, the price sequence can reveal the noise term or even higher dimensional signals.

even though it would be socially optimal. Another feature of the equilibrium is that the game ends after two periods. If nobody invested in the first two periods, investment stops forever, i.e. an investment collapse can occur. Similar results carry over to a setting with I agents.

In contrast to Chamley and Gale (1994) in Gul and Lundholm (1995) time is continuous. In Bikhchandani, Hirshleifer, and Welch (1992) and Banerjee (1992) agents (partially) ignore their own information which consequently leads to inefficient information aggregation, i.e. to information cascades. In Gul and Lundholm (1995) the timing when to act as well as when *not* to act improves the information aggregation. In their model, endogenous sequencing leads to information efficient *clustering* as opposed to informationally inefficient information cascades. In Gul and Lundholm's model agents maximise a utility function, which captures a tradeoff between the accuracy of a prediction and how early the prediction is made (waiting costs). Each agent observes a signal $S^i \in [0,1]$, which helps him to forecast $\Pi = \sum_{i=1}^{I} S^i$. The authors show that the strategy of each player can be fully described by a function $t^i(S^i)$. $t^i(S^i)$ reports the latest possible time at which agent i with signal S^i will make his forecast given the other players have not done so already. Since $t^i(S^i)$ is continuous and strictly decreasing, i.e. $t^i(S^i)$ is invertible, the time when the first agent acts fully reveals his signal to the succeeding decision maker. In a two agent setting, the second agent will make his prediction immediately afterwards. Whereas in the former models only the succeeding decision makers profit from positive information externalities, in Gul and Lundholm (1995) the first agent learns from the others' inaction. The first decision maker can *partially* infer the signals of his successors by noticing that they have not acted before him. This biases his decision towards the successor's forthcoming decisions. Consequently agents tend to cluster, i.e. their forecasts are closer together in a setting with endogenous sequencing than in a setting with exogenous ordered forecast. Gul and Lundholm call this effect *'anticipation'*.[8] There is a second source of clustering, labelled *'ordering'*. This occurs because (1) agents with the most extreme signal realisations have higher waiting costs and thus act first and (2) the signals of predecessors are revealed fully, whereas inaction of the successors only partially reveals their signals. More pronounced signals have a larger impact on the true value $\Pi = \sum_{i=1}^{I} S^i$. Since more pronounced signals are fully revealed first, whereas the signals with lower impact are fully revealed later, forecasts are 'on average' closer together than in the case where the less pronounced signals would be fully revealed first.

The distinctive feature of Zhang's (1997) model is that the precision (quality) of the private signal, and not just its content is private information. His model incorporates higher order uncertainy. The signal is binary, and reports with probability p^i which of the two investment projects is the good one. The quality (precision) of the signal is measured by p^i, where each p^i is drawn from a continuous probability distribution over $[1/2, \overline{p}]$, with $\overline{p} < 1$. The realisation of the signal as well as its quality p^i is only known to agent i. The agents' action space at each point in time is either to wait (which discounts the payoffs

[8]Note the similarity to (descending) Dutch auctions.

by δ) or to invest either in investment project 1 or 2. As in Gul and Lundholm (1995) time is continuous.

Zhang derives the unique equilibrium in pure strategies in closed form. The equilibrium exhibits an initial delay of action till the agent with the highest precision (highest p^i) invests. Given the binary investment choice and binary signal space the second decision maker will always ignore his signal, since it is of worse quality. He will immediately mimic the first movers investment decision. Consequently the second agent's investment choice carries no additional information and therefore all other agents will follow immediately the first mover, as well. In summary, after a certain initial delay one can observe a sudden onset of investment cascades. In contrast to Gul and Lundholm (1995) the outcome is not informationally efficient, since everybody's investment decision depends only on the signal with the highest precision. Moreover, the initial delay incurs waiting costs, which is another source of inefficiency. As the number of agents increases the per capita efficiency loss is bounded away from zero. Furthermore each player tends to wait longer, since it is more likely that someone has a more precise signal and will invest before him.

Gale (1996) discusses the problems which arise in herding models in continuous time. For a more detailed discussion of the 'closure problem' see Harris, Stinchcombe, and Zame (1997).

Neeman and Orosel (1998) analyse a sequential common value auction, where the seller can determine the order in which he will approach potential buyers. Potential buyers submit a bid, i.e. their action space is continuous. The bid of a rival bidder creates a information externality as well as a payoff externality. In contrast to the English common value auction, potential buyers can only bid when they are approached by the seller. This makes the winner's curse in this auction less severe, thus leading to better information aggregation as well as to a (weakly) higher revenue for the seller. Neeman and Orosel's analysis can also be viewed as a search problem for the seller where buyers' bids are correlated for three reasons. First, their signals are correlated. Second, they bid for an object of common value, and third, all buyers condition their bid on the publicly observed history of bids.

Allowing agents to act more than once and/or revising their decision would be a natural extension to the herding literature. This leads us to models of experimentation. See Bolton and Harris (1993), Bergemann and Välimäki (1996) and Leach and Madhavan (1993) for useful expositions.

5.4 Herding in Financial Economics

Prices in financial markets are almost continuous if one neglects the small tick size. The same is true for the possible order size. Given a one dimensional signal space, herding is thus very unlikely to occur, Lee (1993). Traders in financial markets, therefore, do not blindly follow investment decisions of others. On the other hand, herding results can be obtained by introducing higher order uncertainty and fixing the order size for each trader, as in Glosten and Milgrom (1985). Herding also occurs in information acquisition when all investors search for the same piece of information. This is the case when they have short horizons.

5.4.1 Herding, Bubbles and Higher Order Uncertainty

In finance, trading causes not only informational externalities, but also changes in prices. This changes the payoff structure for all successors. Avery and Zemsky (1995) use a information structure similar to Bikhchandani, Hirshleifer, and Welch (1992). They show in a Glosten-Milgrom setting (see Section 3.2.3) that the price adjusts exactly in such a way that it offsets the incentives to herd. This is the case, because the market maker and the insiders learn the same from past trading rounds. Avery and Zemsky (1995) distinguish between herding behaviour and informational cascades. Herding occurs if traders imitate the decision of their predecessors even though their own private signal advises them to take a different action. In informational cascades no additional information is revealed to the market, since the distribution over the observable actions is independent of the state of the world. An informational cascade never occurs in an extended Glosten-Milgrom setting in which insiders get different noisy signals. In addition, traders never engage in herding behaviour provided signals are monotonic. The price converges to the true asset value and the price process exhibits no "excess volatility" given its Martingale property. Similar to Easley and O'Hara (1992), Avery and Zemsky introduce "event uncertainty." Insiders receive either a perfect signal that no new information has arrived or a noisy signal which reports with probability p the true asset value, $\Pi \in \{0,1\}$. In other words, all insiders receive signals with the same precision, $p' = 1/2$ (no information event) or $p' = p \in (1/2, 1]$. p is known to the insiders, but not to the market maker, i.e. the market maker does not know whether an information event occurred or not. This asymmetry in higher order information between insiders and the market maker allows insiders to learn more about the price process (trading sequence) than the market maker. Since the market maker sets the price, the price adjustment is slower. This can lead to herding behaviour. This is consistent with the results in (Bikhchandani, Hirshleifer, and Welch 1992) where prices are esentially 'fixed.' Note, that event uncertainty can lead to herding behaviour, but not to informational cascades, because the market maker can gather information about the occurrance of an information event. Surprisingly, herding increases the market maker's awareness of information events. However, herding does not distort the asset price and thus it does not explain bubbles. In order to explain bubbles and excess volatility a more complex information structure is needed. Avery and Zemsky consider a setting with two types of informed traders. One group of traders receives its signals with low precision p_L, whereas the other receives them with high precision $p_H = 1$, i.e. they receive a perfect signal. This information structure incorporates higher order uncertainty, since it is not known whether the proportion of insiders with the precise signal is high or low. This makes it difficult for the market maker to differentiate between a market composed of well informed traders following their perfect signal and one with poorly informed traders who herd. In the latter case, bubbles[9] can arise.

Gervais (1995) shows that uncertain information precision can lead to a cascade state. In this case, insider's information precision gets never revealed and thus the bid-ask spread does not reflect the true precision. In Gervais (1995) all agents receive a signal with the same precision, p_H, $p_L > 1/2$, or $p_{no} = 1/2$. If $p_{no} = 1/2$, no information event occurs. In contrast to Avery and Zemsky (1995), the signals do not refer to the liquidation value of the asset, Π, directly, but only to a certain aspect π_t of Π. More formally, every trader receives a noisy signal about aspect π_t, which takes on a value $\frac{1}{T}$ or $-\frac{1}{T}$ with equal probability of 1/2. The final liquidation value of the asset is then given by $\Pi = \sum_{t=1}^{T} \pi_t$. Note for each π_t, there is only one signal. As in Glosten and Milgrom (1985) the risk-neutral market maker sets competitive quotes. If the bid-ask spread is high, insiders trade only if their signal precision is high. The trade/no-trade sequence allows the market maker to updates his beliefs about the quality of the insider's signals. Furthermore, he updates his beliefs about the true asset value Π. Therefore, the competitive spread has to decrease over time. Note the trading/quote history is more informative for insiders because they already know the precision of the signal. When the competitive bid-ask spread decreases below a certain level, insiders will engage in trading independent of the precision of their signal. This prevents the competitive market maker to learn more about the signals' precision, i.e. the economy ends up in a cascade state.

5.4.2 Herding in Information Acquisition and Short Horizons

Brennan (1990) noted the strong interdependence of individual information acquisition decisions. In a market with many investors the value of information about a certain (latent) asset may be very small if this asset pays a low dividend and no other investor acquires the same information. If on the other hand many investors collect this information the share price adjusts and rewards those traders who gathered this information first. Coordinating information collection activities can therefore be mutually beneficial.[10] Brennan (1990) formalises his argument using an overlapping generation model where agents live only for three periods.

In Froot, Scharfstein, and Stein (1992) herding in informational acquisition is due to investors' short horizons, i.e. their myopia. This behaviour effects the asset price. In general, backward induction rules out any alteration of the price process caused by the short horizons of traders. This is true in models with exogenous information acquisition and a finite number of time periods. However, as shown in Tirole (1982) myopic behaviour can lead to bubbles in infinite horizon models. In Froot, Scharfstein, and Stein (1992) the asset price is influenced by the endogenous information decision. Each individual trader has to decide whether to receive a signal about event A or event B. All traders

[9]Bubbles are possible, because trade is restricted to one stock at a time in a Glosten-Milgrom setting. Therefore, the results of Section 2.4 for limit order models do not apply.

[10]Note, for stock price manipulation coordination is also required if there are many investors in the market.

worry only about the short-run price development, since they are short-sighted. They can only profit from their information if it is subsequently reflected in the price. Since this is only the case if enough traders observe the same information, each trader's optimal information acquisition depends on the others' information acquisition. The resulting positive information spillovers explain why traders care more about the information of others than about the fundamentals. In Keynes (1936) words, "skilled investment today is to "beat the gun"...". Observing this behaviour of traders led Keynes to compare the stock market with a beauty contest. A judge in a beauty contest who wants to support the winning candidate, has to be more concerned about the opinion of the other judges than about the relative beauty of the contestants. This phenomenon does not arise in the stock market, if all traders take the whole future into account, i.e. if they only care about the final liquidation value. If this is the case, information spillovers are negative. Thus, it is better to have information others do not have. Therefore in this case every investor will try to collect information for different events.

In Froot, Scharfstein, and Stein (1992) traders are assumed to be short sighted and then the picture changes. In the articles Hirshleifer, Subrahmanyam, and Titman (1994) and Holden and Subrahmanyam (1996) short horizons of traders are not exogenously assumed, but result endogenously form the model specification. We will discuss these two articles after providing the intuition of Froot, Scharfstein, and Stein (1992). Froot, Scharstein and Stein base their model on Kyle (1985). The asset's liquidation value is given by two components, ν and δ, i.e.

$$\Pi = \nu + \delta,$$

where $\nu \sim \mathcal{N}(0, \sigma_\nu^2)$ refers to event A and $\delta \sim \mathcal{N}(0, \sigma_\delta^2)$ to the independent event B. Each trader can decide whether to observe either ν or δ, but not both. After observing ν or δ he submits a market order to the market maker at $t = 1$. The authors assume that half of the submitted market orders are executed at $t = 1$ and the second half at $t = 2$. The period in which an order is processed is random. Furthermore, liquidity traders submit market orders of aggregate random size Θ_t in each period. As in Kyle (1985) the risk neutral market maker sets a competitive price in each period based on the observed total net order flow. Thus, the price partially reveals the information collected by the informed traders. At $t = 3$ all traders, i.e. insiders and liquidity traders, unwind their position by assumption. In other words, the risk neutral market maker takes on all risky positions. The trading price at $t = 3$, P_3 is $\Pi = \nu + \delta$ with probability α for the case where ν and δ are publicly announced at $t = 3$. ν and δ are announced only at $t = 4$ with probability $(1 - \alpha)$. Therefore the trading price does not change, i.e. $P_3 = P_2$. Thus, α provides a measure of the degree of short-sightedness of the traders. The traders' horizons are very short if α is close to zero. In that case, traders only care about P_2 and not about Π. On the other extreme, if α is close to one, traders' horizons are long.

The expected profit per share for an insider is $\frac{P_2 - P_1}{2}$ if ν and δ are only announced at $t = 4$, i.e. $P_3 = P_2$. This case occurs with probability α. With probability $(1/2)$ the trader is lucky and his order is processed early, i.e. he gets his shares for P_1. He can then sell it at $t = 3$ for $P_3 = P_2$. With probability α, however, ν and δ are announced already at $t = 3$, i.e. $P_3 = \Pi$. In this case a trader who submitted an order at $t = 1$ also buys a share for P_1 or P_2 with equal probability, but sells it at $t = 3$ for $P_3 = \Pi$. His expected profit in this case is given by $\Pi - \frac{1}{2}[P_1 + P_2]$. Thus, the overall expected profit per share for an informed trader is

$$E\{\alpha[\Pi - \frac{P_1 + P_2}{2}] + (1-\alpha)[\frac{P_2 - P_1}{2}]\}.$$

In both cases the profit is determined by P_3, the price at which the informed trader unwinds his position. $P_3 = \Pi$ with probability α. Thus, ν and δ are equally important, with probability α. With probability $(1-\alpha)$, $P_3 = P_2$. Since P_2 depends on the information set of all informed traders, each insider cares about which information the other traders are collecting. Let's consider for illustrative reasons the extreme case $\alpha = 0$, i.e. ν and δ are only publicly announced in $t = 4$. If in this case all other investors collect information ν, then information δ is worthless, since δ will only enter into the price in $t = 4$, i.e. after the investors have already unwinded their positions. In this case all investors will herd to gather information ν and nobody will collect information δ.[11] Thus, short horizons of traders creates positive informational spillovers which lead to herding in information acquisition.

However, even if all investors herd on some noise term ζ, which is totally unrelated to the fundamental value $\Pi = \nu + \delta$, a rational investor is better off if he also collects information ζ rather than only information about fundamentals. If $\alpha = 0$ and all other investors are searching for ζ, the fundamentals ν and δ are only reflected in P_4. The price at which the traders have to close their position, $P_3 (= P_2)$ depends on ζ, given their strategies. In the case where it is not sure whether ν and δ will only be announced at $t = 4$, i.e. $\alpha > 0$, herding in information acquisition still occurs if α is sufficiently small, i.e. traders are sufficiently short sighted. Note for the case $\alpha = 1$ demands are "strategic substitutes", while for $\alpha = 0$ they are "strategic complements".

One might argue that the above reasoning may break down in an overlapping generations (OLG) framework in which a new generation of short-sighted traders enters the market in each period. Inefficient herding still occurs in the following OLG setting. Generation t speculators can study one of k pieces of information. At the end of period t, one of these pieces will be randomly drawn and publicly announced. In the following period $t + 1$ a new additional piece of information can be studied. Thus, each trader in each generation can study one of k pieces of information. For each generation it pays off to have accidentally studied the information, which gets publicly announced at the end of the period. Since one is only lucky with a probability $(1/k)$ the price movement will be determined more by what the other traders have studied. Hence herding in information acquisition may occur.

In the preceeding papers, the herding behaviour was due to the exogenously assumed short horizons of the traders. In the following two papers short horizons of traders are endogenously derived. In contrast to Froot, Scharfstein, and Stein (1992) in Hirshleifer, Subrahmanyam, and Titman (1994) and Holden and Subrahmanyam (1996) competitive

[11] In a (Nash) equilibrium it is mutual knowledge which information the other traders are collecting, Brandenburger (1992).

limit order models are employed to derive endogenously myopic behaviour from agents' risk aversion.

In Hirshleifer, Subrahmanyam, and Titman (1994) a continuum of competitive risk averse investors search for the *same* information δ about the liquidation value Π of a single risky asset.

$$\Pi = \overline{\Pi} + \delta + \epsilon,$$

where $\overline{\Pi}$ is known and $\delta \sim \mathcal{N}(0, \sigma_\delta^2)$, $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. Some investors, whose mass is M, receive information δ accidentally early, i.e. already in $t = 1$, whereas the others, whose mass is $(N - M)$ are informed later. Both groups of traders receive the same information δ, but at different times. All traders maximise CARA utility functions of the final wealth W_3, i.e. $U = -\exp(-\rho W_3)$. The demand for the risky asset by the early-informed is denoted by $x_t^e(\delta, \cdot)$, whereas that by the late-informed is $x_t^l(\cdot, \cdot)$. The aggregate demand of liquidity traders is modelled by the random variables $\Theta_1 \sim \mathcal{N}(0, \sigma_{\Theta_1}^2)$ in $t = 1$ and $\Delta\Theta_2 \sim \mathcal{N}(0, \sigma_{\Delta\Theta_2}^2)$ in $t = 2$.[12] Finally, there is also a group of risk neutral competitive market makers (scalpers, floor brokers, etc.) who observe the limit order book, i.e. the noisy aggregate demand schedules, but not the information δ. The noisy aggregate demand function is $D_1(\cdot) = Mx_1^e(\delta, \cdot) + (N - M)x_1^l(\cdot) + \Theta_1$ in $t = 1$ and $D_2(\cdot) = Mx_2^e(\delta, \cdot) + (N - M)x_2^l(\delta, \cdot) + \Theta_1 + \Delta\Theta_2$ in $t = 2$. Given risk neutrality and competitiveness of the market makers, the market makers set a semi-strong efficient price with respect to their information sets, i.e. $P_1 = E[\Pi \mid D_1(\cdot)]$ and $P_2 = E[\Pi \mid D_1(\cdot), D_2(\cdot)]$.

In equilibrium investors conjecture the following linear price relations:

$$P_2 = \overline{\Pi} + a\delta + b\Theta_1 + c\Delta\Theta_2$$

$$P_1 = \overline{\Pi} + e\delta + f\Theta_1.$$

The equilibrium is derived by backward induction. At $t = 2$ both groups of investors, early and late informed, know δ and their stock holding is therefore as usual

$$x_2^e(\delta, P_2) = x_2^l(\delta, P_2) = \frac{\overline{\Pi} + \delta - P_2}{\rho\sigma_\epsilon^2}.$$

At $t = 1$ only the group of early-informed investors knows δ. Their stock holding is

$$x_1^e(\delta, P_1) = \frac{E[P_2 \mid \mathcal{F}_1^e] - P_1}{\rho}\left[\frac{1}{Var[P_2 \mid \mathcal{F}_1^e]} + \frac{1}{\sigma_\epsilon^2}\right] + \frac{\overline{\Pi} + \delta - E[P_2 \mid \mathcal{F}_1^e]}{\rho\sigma_\epsilon^2}.$$

The demand of early-informed trades consists of two components. The first term captures the speculative demand due to an expected price change. The second term is the expected final stock holding, which the early-informed traders try to achieve at the "on average" better price P_1. Investors who receive their signal only at $t = 2$, demand nothing at $t = 1$, i.e. $x_1^l = 0$. This is due to the fact that they do not have superior information to the

[12] All demand functions are expressed in stock holdings, therefore, the additional demand in $t = 2$ is given by $\Theta_2 - \Theta_1 = \Delta\Theta_2$.

market makers. Since the market makers are risk neutral (1) no risk premium is offered and (2) expected P_2 is unbiased. In other words, risk averse late-informed traders cannot hedge their date 2 demands already at $t = 1$.

There are five equilibrium configurations for the coefficients of the price relations in this economy. In the fully revealing equilibrium no investor holds any stocks. In addition there are two equilibria where prices do not move, i.e. $P_1 = P_2$. Hirshleifer, Subrahmanyam, and Titman (1994) focus on the remaining two equilibria in which trading occurs and the price is not the same in both periods. In these equilibria both price moves, $P_1 - P_0$ and $P_2 - P_1$ are positively correlated with δ, i.e. P_2 reveals "on average" more about δ than P_1. This is due to the fact that the market makers' information set, which determine the price, is improving by observing two noisy aggregate demand curves. Furthermore, both aggregate demand curves depend on information δ, given that both groups of traders observe the same information. This is supported by the correlation between Θ_1 and Θ_2, since $\Delta\Theta_2$ is independent of Θ_1. The price changes $P_1 - P_0$ and $P_2 - P_1$ themselves, however, are uncorrelated and thus prices follow a Martingale process, given the market makers' filtration.

The trading behaviour of the early-informed investors exhibits speculative features. They take on large positions in $t = 1$ and "on average" unwind partially in $t = 2$ at a more favourable price P_2. More precisely, their trading in $t = 1$, x_2^e is positively correlated with the price change $P_2 - P_1$ in $t = 2$, whereas their trading in $t = 2$ is negatively correlated with this price change. Therefore, they partially unwind their position and realise capital gains "on average." The intuition for this result is as follows. Since no risk premium is paid due to the market makers' risk neutrality, risk averse traders would be unwilling to take on any risky stock in the absence of any informational advantage. Early-informed investors are willing to take on risk since they receive a signal δ in $t = 1$. Their informational advantage, together with the existence of noise traders compensates them for taking on the risk represented by the random variable ϵ. However, the informational advantage of early-informed traders with respect to the late-informed traders vanishes in $t = 2$, since both now receive the same signal δ. Thus, early-informed traders share the risk with late-informed traders in $t = 2$, i.e. $Cov(x_2^l, x_2^e) > 0$. In addition, the informational advantage of the early-informed with respect to the market makers shrinks as well, since at $t = 2$ market makers can observe a second different limit order book. This limit order book carries information for the market makers, especially since the stock holding of the noise traders is correlated in both periods. This allows the market makers to get a better idea about δ and, thus, P_2 should be "on average" closer to $\overline{\Pi} + \delta$ than P_1. Therefore, in period two, both these effects cause early-informed traders to partially unwind the position they built up in the previous preiod. The unwinding behaviour of early-informed traders in this sequential information arrival models also stimulates trading volume.

The fact that early-informed traders on average unwind their position in $t = 2$ is in sharp contrast to models based on Kyle (1985). In these models the risk neutral insider tries to buy the stocks in small pieces in order to hide behind noise trading, i.e. his stock holding over time is positively correlated.

Having analysed the second stage, Hirshleifer, Subrahmanyam, and Titman (1994) show that herding can occur in the information acquisition stage. At the time when traders decide which information to collect they do not know whether they will find the information early or late. Hirshleifer, Subrahmanyam, and Titman (1994) derive expressions for utilities of the early-informed and late-informed individuals. The authors provide a numerical example, in which the ex-ante utility before knowing when one receives the information is increasing in the total mass of informed traders. If this is the case, it is worthwhile for traders to concentrate on the same informational aspects, i.e. gather information about the same stocks. In other words they will herd in information acquisition. Whether a higher mass of informed traders really increases their ex-ante utility depends on the parameters, especially on σ_ϵ^2. More informed traders lead to more late-informed traders, which makes it easier for early-informed traders to unwind larger positions in $t = 2$. Thus, there are more traders in $t = 2$ willing to share the risk resulting from ϵ. This is disadvantageous for the late-informed, since there is tougher competition among them. This is the case since the extent of noise trading does not change. Increasing the mass of informed traders also increases the number of early-informed traders. This decreases the utility of both, early-informed and late-informed traders. In order to obtain herding the first effect has to outweigh the latter three. This requires that σ_ϵ^2 is sufficiently high. The authors try to extend their analysis by introducing some boundedly rational elements which lies outside the scope of this literature survey.

In Hirshleifer, Subrahmanyam, and Titman (1994) all traders search for the same piece of information, which they randomly receive earlier or later. In Holden and Subrahmanyam (1996) traders can decide whether to search for short-term information or for long-term information. They choose between two signals which are reflected in value at different points of time. Holden and Subrahmanyam show that under certain conditions all risk averse traders focus exclusively on the short-term signal.

In their model the liquidation payoff of a single risky asset is

$$\Pi = \overline{\Pi} + \delta + \eta + \nu + \epsilon,$$

where δ, η, ν, and ϵ are mutually independent normally distributed and $\overline{\Pi}$ is normalised to zero without loss of generality. Traders who acquire short-term information observe δ at $t = 1$. At $t = 2$, δ becomes publicly known, as well as η, which no trader has known before. Traders who search for long-term information observe ν already in $t = 1$, which gets known to the public only in date 3. At $t = 3$ ϵ is also realised and known by all traders, i.e. Π is common knowledge at $t = 3$. Note that since the components of Π cannot be traded directly the markets are incomplete. This is essential for this analysis.

A competitive limit order model is employed as in Hirshleifer, Subrahmanyam, and Titman (1994). A mass M of long-term informed traders and a mass of $N = 1 - M$ of short-term traders submit limit orders to the limit order book. The aggregate order size of the liquidity traders is random and given by Θ_1 in $t = 1$ and $\Delta\Theta_2$ in $t = 2$. A group of risk neutral market makers observes only the publicly available information and the noisy aggregate demand schedule, i.e. the limit order book. Since the market makers act competetively and they are risk neutral, their information sets determines the price.[13]

[13]For a similar model with differential information see Vives (1995).

Analysing the overall equilibrium backwards, the mass of short-term traders, N, and of long-term traders M, is kept fixed at the second stage and is endogenised at the first stage. Backward induction is also applied within the second stage for deriving the optimal stock holdings of informed risk averse traders. At $t = 2$, the stock holding demand are standard for the long-term informed traders,

$$x_2^l = \frac{\nu + \delta + \eta - P_2}{\rho\sigma_\epsilon^2}$$

and for the short-term informed,

$$x_2^s = \frac{E[\nu \mid \mathcal{F}_2^s] + \delta + \eta - P_2}{\rho[\sigma_\epsilon^2 + Var[\nu \mid \mathcal{F}_2^s]]} = 0.$$

$x_2^s = 0$, since the market makers have the same information set as the short-term-informed and therefore the numerator in the above equation is zero. In economic terms, it would not make a lot of sense for risk averse short-term investors to hold risky stocks if the risk neutral market makers have the same information. Since x_2^s is zero, x_1^s is standard, i.e.

$$x_1^s = \frac{E[P_2 \mid \mathcal{F}_1^s] - P_1}{\rho Var[P_2 \mid \mathcal{F}_1^s]}.$$

Short-term informed traders try to exploit the expected price change $(P_2 - P_1)$ and at $t = 2$ they close their position. Long-term traders stock holding at $t = 1$ is

$$x_1^l = \frac{E[P_2 \mid \mathcal{F}_1^l] - P_1}{\rho S_1} + \varrho E[x_2^l \mid \mathcal{F}_1^l],$$

where S_1 and ϱ are nonstochastic quantities.

Holden and Subrahmanyam derive the REE only for a special case and continue their analysis with numerical simulations. In equilibrium long-term traders reduce their date 1 demand if the variance of η, whose realisation will be announced at $t = 2$, is very high. They do not want to expose themselves to the announcement risk generated by η (and reflected in P_2). They engage in heavier trading after a large part of uncertainty about the asset's value is resolved.

Holden and Subrahmanyam go to endogenise M and, thus, $N = 1 - M$. The equilibrium mass M can be derived by comparing the ex-ante utilities of short-term informed traders with the utility of long-term informed traders. They show that for certain cases the ex-ante utility from collecting short-term information is higher for $M \in [0, 1]$ than the utility from gathering the long-term signal. Thus, all traders search for the short-term signal in equilibrium. This is the case if the traders are sufficiently risk averse and σ_ϵ^2 is substantially high.[14] Intuitively, short-term informed investors can only make use of their information from the price change $(P_2 - P_1)$, provided there are noise trader in $t = 1$, distorting P_1. Since η makes P_2 risky, high variance in η reduces their aggressiveness. Long-term informed traders can exploit their information from both price changes,

[14]The assumption that σ_ϵ^2 has to be very high is hidden in the legend of figure 3.

$(P_2 - P_1)$ and $(P_3 - P_2)$. As described above, high variance of η makes long-term informed agents delay their purchase. Therefore, they are more active at $t = 2$ and they exploit $(P_3 - P_2)$ to a greater degree. If the variance of ϵ is very high, i.e. speculating at $t = 2$ is very risky, long-term informed traders are very cautious at $t = 2$. Thus, they cannot make as much money out of their information as short-term informed traders can.

Holden and Subrahmanyam further show that as the degree of liquidity trading increases, both types of information are more valuable. Short-term investors profit more from higher variance, at least for the case where the variance of noise trading is the same in both periods.[15]

Another question they address is whether long-term information can be made more valuable by making it short-term. In other words, is it profitable for long-term informed investors to disclose their information already in $t = 2$? The impact of early credible disclosures is discussed in their last section.

Shleifer and Vishny (1990) provide further reasons why investors might be short-sighted. Incomplete markets which prevent complete risk sharing, credit constraints and other market imperfections make arbitrage cheaper for short-term assets than long-term assets resulting in less mispricing in short-term assets. In other words, it leads to systematically less accurate pricing of long-term assets. This, in turn, affects investment decisions of managers in the firms. Managers who are averse to mispricing of their equity because of potential takeovers etc. therefore, tend to conduct more short-term investments whose returns can be verified quickly. Alternatively, short-term behaviour of managers can also arise in agency models. Since this is true not only for the managers of listed firms but also for the managers of investment firms, pension funds etc. even institutional traders behave mypocially. Brandenbruger and Polak (1996) have a model where managers ignore their superior information and follow the opinion of the market. This is strictly less informationally efficient than herding behaviour among profit maximising firms.

[15] In my opinion this is only true if σ_ϵ^2 is sufficiently high.

Chapter 6

Conclusion and Summary

This survey covers a large section of rational models, in which difference in information drives prices. We begin with the concept of rational expectation equilibria and go on to study different partially-revealing REE. These models are summarised and the the limitations of the REE concept is discussed. We provide the intuition for two proofs of the No-Speculation Theorem. This theorem states that at a Pareto-efficient allocation and given concordant beliefs, information will not lead to further trade. Furthermore, two different kinds of No-Trade Theorems are illustrated. They refer to no-trade outcomes even when the current allocation is not Pareto optimal. The possible occurrence of bubbles even in situations where all traders are rational is explained.

The second part of the survey is restricted to the class of CARA-Gaussian models. The main focus is on dynamic REE models and on providing some rationale for technical analysis. The final section covers models of sequential information arrival. We catalog herding models and illustrate the distinct features of these models. The survey closes with herding models in information acquisition caused by short horizons of traders.

Most of this literature is quite recent and further major developments can be expected. The papers covered in this survey are by no means conclusive, nor do I claim that I have chosen the most important ones. Moreover, I have neglected a broad range of areas (like price experimentation, the analysis of disclosure of private information, effects driven by bounded rationality, extensive discussion about endogenous information acquisition etc.). There are many factors affecting the price process of an asset. This is surely one reason why this merits interest and examination.

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