The Impact of Liquidity Constraints on Bank Lending Policy*

By

David C Webb

DISCUSSION PAPER 299

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The Impact of Liquidity Constraints on Bank Lending Policy*

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Abstract

This paper examines the linkages between banks' ability to renegotiate loans and their provision of demand deposits which allow agents to smooth consumption. On the asset side, banks provide value to firms by standing ready to renegotiate loan terms. On the liability side, banks provide a way for consumers with liquidity needs to smooth consumption. The problem identified is that banks may not be able to provide new funds for borrowers when they themselves are short of cash, because of either the return on their investments being poor, or because depositors are asking to withdraw more funds than expected. Banks subject to liquidity shortages may ration loans to good borrowers. This problem is shown to depend upon the nature of the deposit contract and banks' inability to issue new deposits which are subordinated to existing deposits. An important implication of the existence of rationing is that borrowers may accept to partly insure the bank when there is rationing through making higher payments on their loans if they can. In return for this, they make lower payments in the event that there is no rationing. A further implication is that entrepreneurs may prefer to undertake projects which yield higher returns at the intermediate date and so insulate themselves from being rationed and having to renegotiate loans. This latter phenomena exhibits some characteristics

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of short-termism. Finally, the role of an inter-bank market in alleviating these problems is briefly discussed.

1 Introduction

This paper is concerned with banks' dual role in supplying loans to firms and taking deposits from households. Lending arrangements are such that in some circumstances loans need to be renegotiated. On the liability side of their balance sheets, banks provide a way for consumers with liquidity needs to smooth consumption. The paper integrates these two functions of banks and in an environment with no deposit insurance or lender of last resort, illustrates a problem for the banking system. The problem is that banks may not be able to provide new funds for borrowers, particularly on renegotiated terms, if withdrawals by depositors are too high. Supplying liquidity on the liability side of the balance sheet may conflict with supplying liquidity on the asset side of the balance sheet. Thus the paper shows how, in a financial system where firms are primarily financed by bank debt and banks are primarily financed by demand deposits, there can be a knock-on effect whereby depositors' liquidity demands can lead to a debt overhang problem and generally under-investment relative to the first-best. The paper also shows how this affects entrepreneur project choice, showing that it can cause a bias towards less risky short term projects. The focus of the paper is on the first round effect of this problem so that possible contagion effects are not examined.[1]

Because state contingent payments are not verifiable, finance takes the form of debt, and because of the timing of project returns this is long term debt. However, at some intermediate date after the arrival of information, a decision to continue or liquidate the firm must be made. Under some circumstances there may be a debt overhang problem requiring that the debt contract be renegotiated. The nature of the information about the firm at this date and the need to renegotiate the loan contract makes a bilateral relationship with a single bank appropriate. Renegotiation is done on a bilateral basis and will not just reflect the firm's prospects but also the strength of the bank's balance

[1] Rajan (1994) analyses a similar problem but from a different perspective. He examines the interdependence of lending policies in an environment where banks have short horizons along the lines examined by Stein (1989) and shows that if relative performance affects bank reputation and hence stock price, banks may 'jam' their earnings signal. Bank lending can be inefficiently high or low. The present paper in contrast examines the co-ordination problem of banks as suppliers of liquidity to borrowers and depositors.

sheet.

Turning to the behaviour of banks in our model. Diamond and Dybvig (1983) and Bryant (1980) provide the classic treatment of the deposit contract, in which banks pool risks in an environment where privately observed consumption shocks are uncorrelated across agents. Optimal insurance contracts cannot be written on privately observed consumption shocks, but a bank can provide insurance to depositors by arranging deposit contracts which support a pareto optimal allocation. However, their model has multiple Nash Equilibria. One is the equilibrium which supports the ex-ante welfare maximising allocation and, given the important institutional feature that the deposit contract satisfies a sequential service constraint, the other is a bank run. The run is triggered by events exogenous to the models fundamentals. The investment technology which banks invest in is deterministic but illiquid so that a bank is unable to honour all of its liabilities at par if all depositors withdraw and hence ultimately is liquidated. A second and perhaps more satisfactory series of models concentrate on explaining the conditions under which depositors might rationally change their beliefs about the riskiness of banks. Gorton (1985), Chari and Jagganathan (1988) and Jacklin and Bhattacharya (1988) all offer information-based explanations of bank runs. In these models depositors are incompletely informed about the true value of the banks' portfolio and make inferences from available information about this performance. Runs occur because a subset of depositors receive interim information signalling a low future return on the bank's investments.

The present paper models the interaction of banks, firms and depositors along similar lines to Jacklin and Bhattacharya (1988). The liability side of our model is the same as in their paper, the banks' role being to provide liquidity to depositors. In Jacklin and Bhattacharya banks can invest in a riskless short-term technology or in a risky long-run technology that cannot be liquidated. However, in the first part of this paper we assume that bank's income is only from financing an overlapping sequence of risky long-run investments, which require investment at two dates, but which can be liquidated at the interim date when the quality of the investment is realised. Only later do we examine the role of riskless investments. Firms' project selection and financing decisions play a crucial role in our model. In particular, intermediate financing decisions including liquidation and loan renegotiation are of central importance.

Banks offer a menu of contracts to depositors and loans to firms which are tailored to fit with the expected liquidity needs of these agents. However, in some circumstances, depositors who receive poor information about the performance of the bank's loan book, choose to withdraw at a higher rate than they would have in the absence of such information. If the withdrawal rate is sufficiently high, there is a run on the bank and the bank faces a liquidity shortfall, in the sense that it cannot raise sufficient financing to take all positive net present value investments, while repaying depositors. The problem is the deposit contract itself. This contract is not contingent upon the performance of the bank's balance sheet or the state underlying it. Alternatively, given depositors' withdrawals, the bank may suffer from low income relative to withdrawals because of "bad luck", income is less than expected. The paper explores the implications of these two types of liquidity shortfall for banks' lending. The impact on lending policy is high because deposits are unsubordinated and new deposits issued to finance loans may dilute existing depositors holdings, causing early withdrawal. This effect is more acute the larger the liquidity shortfall. Moreover, the impact of a liquidity shortfall caused by a run is always worse than one caused by bad luck.

An important feature of the model is that if the bank suffers a liquidity shortfall of high magnitude, it rations even high-quality entrepreneurs, and renegotiates loans with those that it does finance. In this case the bank obtains the full surplus from renegotiation. However, this is passed back to entrepreneurs through more favourable terms when they are high quality and are granted finance on unrationed terms. In other words, we have the following result: banks suffer liquidity shortages which because of the way they are financed (the deposit contract) have real effects. The consequences of this are borne by bank's borrowers who partially insure the bank through loan renegotation.

The paper goes on to consider the effects of the above problem on entrepreneurs' project choice. We show that in equilibrium, in order to offset the impact of the liquidity shortfall, some entrepreneurs, but not all, will choose to undertake safe projects of lower value which insure them against being affected by the bank's potential liquidity problem. The results here complement those of Dewatripont and Maskin (1995) and Von Thadden

(1995) that illustrate short termism in project choice arising from inefficiencies in financial intermediation. Finally we draw conclusions and discuss the implications of the relaxation of some assumptions, such as there being no interbank market.

2 The Basic Model

Firms with multiperiod investment projects obtain finance from banks. At each date information is distributed symmetrically between borrowers and lenders. Banks themselves compete to supply loan contracts and for depositors' money to fund them. In the basic model we think of banks as simple intermediaries between lenders and borrowers. The key features we wish to capture are: a continuation-liquidation decision for entrepreneurs and partial debt forgiveness. This latter feature takes the form of renegotiation of the terms of the contract. Given the specialised nature of information and the problem of free-riding in renegotiation, this type of financial arrangement will most naturally be implemented as a bilateral relationship with a given lender.

2.1 Entrepreneurs

Entrepreneurs are risk neutral wealth maximisers. They have a three date horizon, $t = 0, 1, 2$. They have no initial wealth, but each is endowed with a single project which requires an investment of I_0 at date $t = 0$ and I_1 at date $t = 1$. For simplicity, the project only yields a return at date $t = 2$. If the investment I_1 is not made at date $t = 1$ the project is liquidated for $0 \leq L_1 < I_0$. Throughout the paper and without loss to the main points we assume that $L_1 = 0$. At each date, if the entrepreneur is inactive he obtains an alternative return on his human capital of $u > 0$.[2]

At date $t = 0$, the entrepreneur's project type is uncertain but will be publicly observable at date $t = 1$ before the investment decision at that date. However, project type is not verifiable. There are two project types: good and bad (poorer); denoted respectively by g and b. These projects have the same investment needs but differ in their returns: a

[2]The utility from inactivity means that in some states unless the burden of debt is reduced an entrepreneur with high debt will prefer inactivity to undertaking some positive value investments.

type g project returns Y_2^g at date $t = 2$ with probability one, a type b project returns Y_2^b with probability $0 < p_2^s < 1$ and zero otherwise with $p_2^s \gamma_2^b < p_2^g$ At date $t = 1$, the returns on the two different types of project satisfy the following inequalities:

$$Y_2^g - I_1 > u \text{ for type } g \tag{1a}$$

$$p_2^s Y_2^b - I_1 > u \text{ for type } b \tag{1b}$$

At date $t = 0$, the entrepreneur assigns prior probabilities of each type of project being realised, denoted, respectively, by $p_1^g > 0$ and $p_1^b = \left(1 - p_1^b\right) > 0$.

2.2 Unconstrained Banks

Our entrepreneurs must obtain finance from banks. Initially, banks are assumed to have access to a perfectly elastic supply of deposits. We refer to these banks as unconstrained. Throughout our banks are risk neutral value maximisers, who are engaged in Bertrand competition. Banks receive applications for finance from identical entrepreneurs at date $t = 0$. At this time they, like entrepreneurs, do not know the project type but will discover this at date $t = 1$.

2.3 The Contracting Problem with an Unconstrained Bank

We assume that finance takes the form of debt contracts. Because projects generate no income at date $t = 1$, if the entrepreneur is to be financed at date $t = 0$ the bank must have a claim against date $t = 2$ income. Let the contractual payment be denoted by D_2. This fixed payments can be enforced, if for example, in the event that it is not paid, the borrower loses everything. An important assumption we make is that participation in the borrower-lender relationship at each date, $t = 0$ and $t = 1$, is voluntary. Hence participation at each date requires that the parties have the incentive to do so.

There follows a summary of events:

Date $t = 0$: Banks offer finance to entrepreneurs so as to undertake a project whose type is unknown to both the bank and entrepreneur. Type contingent contracts cannot be written. The borrower promises to repay D_2 at date $t = 2$. The bank supplies I_0 at date $t = 0$.

Date $t = 1$: Nonverifiable information about project type is observed by both borrower and lender. Both parties can quit the relationship at this date, in the event of which the bank recovers the liquidation value of the project. The bank and entrepreneur can, however, renegotiate the loan terms. This happens if the entrepreneur is type b and threatens to quit if the debt payment is not cut so that he receives a return at least equal to his outside option $u > 0$. However, the entrepreneur cannot threaten switch to another bank because the costs of switching are too high. We, assume the bank makes a take-it or leave-it offer, so the surplus from renegotiation goes to the bank, and the entrepreneur receives not more than $u > 0$. If the loan is not renegotiated, the original contract stands. If the entrepreneur's project is not liquidated he is supplied with additional finance of I_1.

Date $t = 2$: The returns from projects are realised and shared between the bank and entrepreneurs.

2.4 The Contract

First we examine the entrepreneur's problem at date $t = 1$. At this date he discovers his type, then:

if he is type g assuming $Y_2^g - D_2 > u$, the entrepreneur will want to participate and make the investment I_1;

if he is type b, $p_2^s \left(Y_2^b - D_2\right) < u$ but if $p_2^s Y_2^b > u$, there exists a reduced value of D_2 denoted by $\widehat{D}_2$ for which the entrepreneur participates;

Now examine the bank's problem at date $t = 1$. Having granted a contract at date $t = 0$, and having observed the entrepreneur's type, the bank chooses a value maximising response to borrowers loans requests. If the entrepreneur is type g it funds $I_1$and obtains a return of $D_2 > I_1$. If the entrepreneur is type b and is to make the investment, D_2 must be renegotiated. Since the bank gets the full surplus in renegotiation, D_2 is cut to $\widehat{D}_2$, which satisfies $p_2^s\left(Y_2^b - \widehat{D}_2\right) = u$. Let $\widehat{p}_2^s$ denote the actual proportion of type b entrepreneurs refinanced at date $t = 1$ who turn out to be successful at date $t = 2$. The distribution of $\widehat{p}_2^s$ has an unconditional mean of p_2^s and a positive, finite, variance. The necessary and sufficient condition for an unconstrained bank to participate and supply I_1 is that $E_1\left(\widehat{p}_2^s\right)\widehat{D}_2 - I_1 > 0$ where $E_1\left(\widehat{p}_2^s\right) = p_2^s$.

Given the above behaviour in the subgames at date $t = 1$, we can now examine the problem at date $t = 0$. The net return to the entrepreneur at date $t = 0$ if he does the project is then given by

$$p_1^g\left[Y_2^g - D_2\right] + p_1^b\left[p_2^s\left(Y_2^b - \widehat{D}_2\right)\right] - 2u \geq 0 \tag{2}$$

with

$$Y_2^g - D_2 \geq u \text{ and } p_2^s\left(Y_2^b - \widehat{D}_2\right) = u$$

Let α_1^g and $\alpha_1^b = (1 - \alpha_1^g)$ denote the respective proportions of type g and b entrepreneurs realised at date $t = 1$, who were financed by the bank at date $t = 0$. The payments D_2 and $\widehat{D}_2$ satisfy the bank's profitability conditions:

$$E_0\left\{\alpha_1^g\left[D_2 - I_1\right] + \alpha_1^b\left[p_2^s\widehat{D}_2 - I_1\right] - I_0\right\} \geq 0 \tag{3a}$$

and

$$E_1\left(\widehat{p}_2^s\right)\widehat{D}_2 - I_1 > 0 \tag{3b}$$

We assume that the bank's expectations of α_1^g and α_1^b satisfy the following consistency conditions:

$$E_0\left(\alpha_1^g\right) = p_1^g \tag{3c}$$

$$E_0\left(\alpha_1^b\right) = p_1^b$$

where the distributions of α_1^g and α_1^b both have positive, finite variances. Finally, Bertrand competition at date $t = 0$ ensures that given $\widehat{D}_2$, D_2 satisfies (3a) as an equality.

The solution to the contract problem illustrated above implements the first best. This follows, because at each date all positive net present value investments are undertaken. Given the inequalities in (1), investment should take place at date $t = 1$ for firms of types g and b. Therefore investment should take place at date $t = 0$ if,

$$p_1^g\left[Y_2^g - I_1\right] + p_1^b\left[p_2^s Y_2^b - I_1\right] - I_0 - 2u \geq 0 \tag{4}$$

Implementation of the contract ensures that at date $t = 1$ both type g and b firms apply for and receive finance. Then substituting from condition (3) into (2) yields (4), so we have the first best.

3 The Contracting Problem with Constrained Banks

The unconstrained bank of the last section is an intermediary with access to a perfect capital market. It has both the access to capital and incentives to implement the terms of the first-best contract. We now change the model by modifying bank behaviour. We assume that banks have the dual role of financing entrepreneurs and providing liquidity services to depositors. Our purpose then is a straightforward one of investigating the implications of this dual function of banks for their lending policy and firms investment behaviour. In particular, we wish to identify potential sources of inefficiency in lending policy.

Before embarking on this exercise we list a number of assumptions about the market environment in which banks operate. There is no lender of last resort, there is no deposit insurance, there is no secondary market for bank loans and banks do not issue equity. Thus we restrict banks to be pure intermediaries between the commercial loan market and depositors. Our banks are owned by risk neutral wealth maximisers whose only initial asset is a banking licence. These banks compete in attracting deposits and in making loans to entrepreneurs. The only agents in our model who have any initial wealth are depositors.

3.1 Depositor Behaviour

Banks attract funds from depositors by offering deposit contracts [3]. The model of depositor behaviour is in the spirit of Diamond and Dybvig (1983), although the particular formulation is based on Jacklin and Bhattacharya (1988). The details of the depositor model are in the **Appendix.** The idea is that depositors deposit at say date $t = 0$ and have consumption needs at dates $t = 1$ and $t = 2$. However, they are subject to shocks to their time preference parameter at date $t = 1$, so their liquidity needs are stochastic. Banks pool different investments to generate income at different dates to supply depositors with higher returns than under autarky and provide insurance against liquidity shocks. But depositors can withdraw when they want. The deposit contract has to satisfy incentive constraints, which ensure that different types of depositor on discovering their type (which is private information) make withdrawals consistent with their type. This means that the deposit contract can offer only partial liquidity shock insurance. In a non-run equilibrium at date $t = 0$ depositors forecast that they will be high time preference, type 1 with probability π^1 and low time preference, type 2, with probability $\pi^2 = (1 - \pi^1)$; and the actual proportion of depositors depositing at date $t = 0$, who stick to the withdrawal pattern for type 1 at date $t = 1$, denoted by f_0, equals the expected proportion, $f_0 = \pi^1$

However, there is a problem with the above contract. The contract is assumed not to be contingent on the realisation of f_0 at date $t = 1$ or the variables which determine f_0. Hence, as pointed out by Jacklin and Bhattacharya, if some type 2 depositors receive

[3] In this paper we restrict attention to deposit contracts. Jacklin and Bhattacharya show for particular economies how for certain distributions of shocks, in terms of ex ante utility, deposit contract allocations dominate equity contracts, and for other distributions the reverse is true

interim information, σ, at date $t = 1$ this may lead them to revise down the return on the bank's investments at date $t = 2$; which in turn causes a violation of the date $t = 1$ incentive constraints (given in the **Appendix** by (A4)). This leads to a higher rate of withdrawal than expected so that $f_0 > \pi^1$,which is called a run equilibrium.

There are four key points to note about deposit contracts. Firstly, deposits are not contingent on the return from the bank's investments. They are claims that are like debt, in the sense that they have a fixed prior claim on the bank's income. But this claim cannot be renegotiated. Secondly, deposit contracts supply depositors with liquidity insurance; the depositors bear no risk when the bank is solvent but share the bank's assets equally if it is insolvent. Hence, unlike in the model in Hellwig (1994), date $t = 0$ depositors cannot share the risk of returns at date $t = 2$ contingent upon date $t = 1$ information. Thirdly, depositors are free to withdraw early, on terms independent of their type, thereby exposing the bank to further risk. Finally, we make the important distinction between subordinated and non-subordinated deposits. If deposits made at date $t = 1$ are subordinated, they can only be paid at date $t = 2$ after date $t = 0$ deposits have been paid. If deposits made at date $t = 1$ are non-subordinated, they have the same claim at date $t = 2$ as those made at date $t = 0$, thereby diluting the income claim of the earlier deposits.

At each date banks attract two period depositors, with contracts of the type described above. Our model has only four dates $t = -1, 0, 1, 2$. At date $t = 0$ new deposits per typical loan made at date $t = 0$, are denoted by B_0. Similarly, at date $t = 1$ new deposits per date $t = 0$ loan, are given by B_1. At date $t = 1$, a bank's deposits consist of two-period old deposits made at date $t = -1$ and, one-period old deposits made at date $t = 0$. In the general case new deposits made at date $t = 1$ can be withdrawn at dates $t = 2$ and $t = 3$ (although our example ends at date $t = 2$). We define aggregate deposit withdrawals at dates $t = 1, 2$ as the total value of depositor withdrawals per typical date $t = 0$ loan. At date $t = 2$ the total withdrawal from the bank, denoted by C_2, is

$$C_2 = f_0 C_0^{12} + (1 - f_0)\, C_0^{22} + f_1 C_1^{11} + (1 - f_1)\, C_1^{21} \qquad (5)$$

where $f_0 C_0^{12}$ is the withdrawal by date $t = 0$ type 1 depositors at date $t = 2$, and so on. Here the aggregate quantities C_0^{12} etc., are determined by the contract described in the Appendix but the fraction f_0 is determined by the actions of date $t = 0$ depositors at date $t = 1$, based on their information, σ. However, because our model ends at date $t = 2$ deposits made at date $t = 1$ must be withdrawn at date $t = 2$ so that for modelling reasons we set $f_1 = 1$.

The total withdrawal at date $t = 1$ is given by

$$C_1 = f_{-1}C_{-1}^{12} + (1 - f_{-1})\, C_{-1}^{22} + f_0 C_0^{11} + (1 - f_0)\, C_0^{21} \tag{6}$$

3.2 The Modified Contract Problem

Banks supply capital to borrowers of the types examined earlier. To finance their loans, banks supply two-period deposit contracts of the type outlined. At each date information about entrepreneurs' projects and banks' balance sheets is symmetrically distributed between entrepreneurs and banks. New depositors know the structure of the bank's balance sheet, but deposit contracts cannot be contingent upon the realisation of the banks investments. However, at each date some one-period old type 2 depositors receive a signal about the performance of the banks portfolio and can withdraw early. The bank can renegotiate with borrowers at date $t = 1$ but deposit contracts are not renegotiable.

The following is a summary of the sequence of events:

Date $t = 0$: There are a sufficiently large number of banks in the economy, that no bank finds itself in a unique position at this date. Hence, any given bank is engaged in Bertrand competition with at least one other bank. A bank has an initial position resulting from making loans in the past and taking in deposits. We assume it has serviced current withdrawals by depositors and has refinanced all existing borrowers. Clearly there could be rationing at date $t = 0$ of date $t = -1$ borrowers but at no loss we assume not. Hence, at this date the bank's initial portfolio is taken as given and will generate a random income of R_1 at $t = 1$. Banks offer finance to new entrepreneurs. New entrepreneurs apply for finance. At this date neither the entrepreneur nor the banks know the entrepreneur's project type which is made known to both parties at date $t = 1$. Banks, do, however, forecast the proportions of each type that they finance and entrepreneurs estimate their probability of being each type. The bank supplies I_0 to the entrepreneur at this date and stands ready to supply I_1 at date $t = 1$. The entrepreneur promises to repay D_2 at date $t = 2$ if he can, but this amount may be renegotiated at date $t = 1$. However, as will be shown, there is the possibility of banks rationing finance at date $t = 1$ according to a rationing scheme, so at date $t = 0$ both banks and entrepreneurs must estimate the probability of each type of entrepreneur being rationed, and, moreover, in the event of a type being rationed the proportion who are denied finance. The parties also anticipate that the terms of any renegotiated finance will depend upon whether there is rationing or not. To finance these loans the bank takes in new two-period deposits, B_0, with claims (C_0^{11}, C_0^{21}) and(C_0^{12}, C_0^{22}).

Date $t = 1$: The value of $R_1 = \alpha_0^g D_1 + \alpha_0^m \widehat{p}_1^s \widehat{D}_1$ is realised; where D_1 is the payment made by date $t = -1$ type g borrowers and $\widehat{D}_1$ is the payment made by date $t = -1$ type b borrowers. R_1 plays an important role in our model and depends upon the realisation of two types of variable. It depends upon the values of α_0^g, and α_0^b determined at date $t = 0$ and the realisation of $\widehat{p}_1^s$ at date $t = 1$. Note that the proportion of type b entrepreneurs who are successful at date $t = 1$ is the only uncertainty about R_1 actually resolved at this date. Type 2 depositors born at date $t = 0$ receive a signal, σ. This signal includes (precise) information on the bank's lending policy at date $t = 1$. If σ is high, the bank's return at date $t = 2$ is forecast to be relatively high. σ also includes information used to re-estimate $\widehat{p}_2^s$ as $E_1(\widehat{p}_2^s \mid \sigma)$; for this to have any effect on behaviour, σ must include information leading to a revision of the forecast of $\widehat{p}_2^s$ made at date $t = 0$. Depositors then submit their withdrawal requests and the bank accommodates requests to withdraw C_1 or is liquidated. One period old borrowers, initially financed at date $t = 1$, who find out that they are either type g or b, apply for further finance. The bank may have a liquidity shortfall. It decides how many new deposits to take in, B_1, with claim C_{11}^1 and may ration finance to type g and b entrepreneurs. Renegotiation of loans takes place, the form of which depends upon whether there is rationing or not. Borrowers cannot switch to another bank.

At this date, a liquidity shortfall can occur for one of two reasons:

1. The outcome of R_1 given α_0^g and α_0^b, is assumed to be affected by the actual proportion, $\widehat{p}_1^s$ of date $t = -1$ borrowers who are successful at date $t = 1$, which can differ from the expected value. If at date $t = 1$ the return from the bank's earlier investments, R_1, is low because of "bad luck"; then even if C_1 is as expected , with no type 2 depositors withdrawing money early, $f_0 = \pi^1$, the bank may have a liquidity problem:

$$R_1 < f_{-1}C_{-1}^{12} + (1 - f_{-1})\, C_{-1}^{22} \tag{7}$$

In particular, income from past investments, R_1, will not pay-off date $t = -1$ depositors. The probability of this occurring is higher, the higher the variance of R_1. The bank cannot make the depositors return at date $t = 1$ contingent upon R_1 and cannot renegotiate with depositors. Hence, income from new deposits, B_1, will first used to pay this shortfall.

2. Now consider the alternative possibility that the bank has a liquidity problem at date $t = 1$ because there is a run, that is some date $t = 0$, type 2 depositors, receive poor signals, σ. They use σ to estimate (precisely) a low value of α_1^g or a low value of $\widehat{p}_2^s$ given by $E_1\left(\widehat{p}_2^s \mid \sigma\right)$. Thus they forecast a poor return on the bank's portfolio at date $t = 2$. Then if they withdraw early $f_0\left(\sigma\right) > \pi^1$. We could have $R_1 \geq f_{-1}C_{-1}^{12} + (1 - f_{-1})\, C_{-1}^{22}$ but $f_0 C_0^{11} + (1 - f_0)\, C_0^{21}$ is high.

Note that the first type of liquidity problem depends critically upon a high proportion of type b entrepreneurs turning out to have done poorly at date $t = 1$. The second type of problem can have two sources in our model (although one will do). It can arise because at date $t = 1$ the bank is found to have a low proportion of good borrowers, or because the proportion of type b loans that are expected to perform well at date t=2 is low.

Date $t = 2$: Income from projects is realised and shared between depositors, the bank's owners and entrepreneurs. At this date the only uncertainty to be resolved is the proportion of type b entrepreneurs who are successful $\widehat{p}_2^s$.

It is important to emphasise here that the problems of liquidity shortfalls at date $t = 1$ arise, in both cases, from realisations of the bank's portfolio differing from expectations.

3.3 Equilibrium

We develop the model by assuming that banks suffer from liquidity shortfalls at date $t = 1$ and that the banks ration loans at this date. We first show the properties of an equilibrium with rationing at date $t = 1$ and then show that rationing can exist in equilibrium. The contracting problem is examined recursively. Thus we begin at date $t = 1$ and examine the sub-game at this stage. Entrepreneurs' types are realised and those of types g and b apply for further finance. If the bank is not subject to a liquidity shortfall, it grants all loan requests, with type g paying D_2 at date $t = 2$ and type b renegotiating their loan and promising to pay $\widehat{D}_2$ at date $t = 2$. In the absence of rationing at date $t = 1$, the bank's return at date $t = 2$ is

$$Z_2 = \max \left[\alpha_1^g D_2 + \alpha_1^b \widehat{p}_2^s \widehat{D}_2 - C_{2,} 0\right] \tag{8a}$$

Now suppose that the bank is subject to a liquidity shortfall and rations loans. Clearly loans to type b are of lower value to the bank than loans to type g entrepreneurs, so that this type will all be denied finance before any type g are rationed.

Rationing Scheme

Let $0 \leq \gamma_1 \leq 1$ *and* $0 \leq \delta_1 \leq 1$ *be, respectively, the proportions of loan requests granted to type* b *and* g *entrepreneurs. Then if rationing occurs, first* $0 \leq \gamma_1 < 1$, *then if* $\gamma_1 = 0$ *and further rationing is necessary set* $0 \leq \delta_1 < 1$.

Since all type b entrepreneurs are identical and at date $t = 1$ need D_2 to be cut to $\widehat{D}_2$(at a maximum) if they are to continue, there is no scope for a type b to manipulate the rationing scheme by offering to pay more than $\widehat{D}_2$. However, this is not so for a type g. If this group is rationed, a type g entrepreneur will obtain finance with probability δ_1. In the absence of rationing he would pay D_2 at date $t = 2$ and get a return of $Y_2^g - D_2$, but with rationing he expects $\delta_1 \left(Y_2^g - D_2\right)$. But then he could offer to pay $D_2 + \varepsilon$ to the bank and be better-off if $-\delta_1 \varepsilon + \vartriangle \delta_1 \left(Y_2^g - D_2\right) > 0$. Moreover, the bank will accept the offer. There will be competition between type g entrepreneurs for the available funds. The bank will then appropriate all the type g entrepreneurs' surplus, with D_2 rising to

$D_2 = Y_2^g - u$. Thus, if $0 \leq \gamma_1 < 1$, and $\delta_1 = 1$ the bank's return at date $t = 2$ is

$$Z_2 = \max \left[\alpha_1^g D_2 + \alpha_1^b \gamma_1 \widehat{p}_2^s \widehat{D}_2 - C_2, 0\right] \tag{8b}$$

and if $\gamma_1 = 0$ but $0 \leq \delta_1 < 1$,

$$Z_2 = \max \left[\alpha_1^g \delta_1 D_2' - C_2, 0\right] \tag{8c}$$

The bank's owners net return at date $t = 1$ depends upon whether the bank continues at this date. If it does, then

$$Z_1 = R_1 - \alpha_1^g \delta_1 I_1 - \alpha_1^b \gamma_1 I_1 + B_1 - C_1 \geq 0 \tag{9}$$

where is R_1 the bank's net income at date $t = 1$ from earlier investments; the next two terms are the cost of financing investments; B_1 is the value of new deposits at date $t = 1$ and C_1 is withdrawals by depositors. Note that at date $t = 1$ the bank may have positive net earnings. This arises when, after having financed all deposit withdrawals and loan requests out of income, so no new deposits need to be taken in, $B_1 = 0$, we have $Z_1 > 0$. In this case it is assumed that Z_1 is paid out as a dividend. Although not shown formally here, this assumption is justified in this model, since when there is no rationing at date $t = 1$ retained earnings can only raise depositor welfare at the expense of bank's owners. On the other hand, if the bank continues and new deposits are issued to finance loans, $B_1 > 0$, then $Z_1 = 0$. So the bank does not issue deposits to increases retained earnings or to pay a dividend

If on the other hand the bank is liquidated at date $t = 1$, then $B_1 = 0$ and R_1 is distributed to depositors, with each depositor receiving $R_1 / (C_1 + C_2)$ with C_1^{11} set equal to zero.

The bank's problem at date $t = 1$ is to choose γ_1,δ_1 and B_1 to maximise

$$J_1 = \max \ [Z_1 + E_1 J_2] \tag{10}$$

where expectations are taken at date $t = 1$ and $J_2 = Z_2$ as given by either (8a) or (8b) or (8c), depending upon the regime.

Given the above behaviour in the subgames at date $t = 1$, we now examine the problem at date $t = 0$. At this date, the entrepreneur estimates the probability that no type b projects are rationed at date $t = 1$ as y_1; and of no type g being rationed as z_1. The entrepreneur also estimates that if type b are rationed, a proportion $\overline{\gamma}_1 = E_0\gamma_1$ will receive finance, and if type g are rationed, a proportion $\overline{\delta}_1 = E_0\delta_1$ will receive finance. Therefore, the probability, when his project is type b, that he is funded is $q_1^b = y_1 + (1 - y_1)\overline{\gamma}_1$, and if he is type g, $q_1^g = z_1 + (1 - z_1)\overline{\delta}_1$. Then the entrepreneur's expected net return at date $t = 0$ is

$$p_1^g\left[z_1\left(Y_2^g - D_2\right) + (1 - z_1)\overline{\delta}_1\left(Y_2^g - D_2'\right)\right] + p_1^b q_1^b\left(p_2^s Y_2^b - \widehat{D}_2\right) + p_1^g\left(1 - q_1^g\right)u + p_1^b\left(1 - q_1^b\right)u - 2u \tag{11}$$

The bank's expected return at date $t = 0$ is

$$J_0 = \ \max \ \{-I_0 + B_0 + E_0 J_1\} \tag{12a}$$

where B_0 are deposits raised at date $t = 0$ and J_1 is given in (10); where $\widehat{D}_2$ and D_1', respectively, satisfy

$$p_2^s\left(Y_2^b - \widehat{D}_2\right) = u \tag{12b}$$

and

$$Y_2^g - D_2' = u \tag{12c}$$

Using (8), (9) and (10) and evaluating (12a) over the different subgames, we obtain

$$E_0 \left\{ \max \left[\alpha_1^g \left(z_1 D_2 + (1 - z_1) \overline{\delta}_1 D_2' \right) + \alpha_1^b q_1^b \widehat{p}_2^s \widehat{D}_2 - C_2, 0 \right] + \left[R_1 - \alpha_1^g q_1^g I_1 - \alpha_1^b q_1^b I_1 - C_1 + B_1 \right] \right\} \tag{13}$$

$$+ B_0 - I_0 < 0$$

where α_1^g and α_1^b satisfy the consistency conditions in (3c) and $E_0 (\widehat{p}_2^s) = p_2^s$. Finally, Bertrand competition at date $t = 0$ ensures that (13) is set equal to zero.

Lemma 1 *Given that there is rationing at date $t = 1$, entrepreneur participation at date $t = 0$ requires that there is a positive probability that type g projects are not rationed at date $t = 1$.*

<u>Proof</u> Suppose to the contrary, then type b are never given credit at date $t = 1$, $y_1 = 0$ and $\overline{\gamma}_1 = 0$. Also for type g, $z_1 = 0$ and $D_2' = Y_2^g - u$ with the bank gaining the full surplus from continuation. It follows by substitution into (11), that the entrepreneur cannot be compensated for his human capital and will not participate at date $t = 0$. Hence, there must be a positive probability that the entrepreneur is an unrationed type g.■

The following proposition is immediate:

Proposition 2 *If there is a positive probability that type g are not rationed at date $t = 1$, and there is a sufficiently high return to type g entrepreneurs when this type is not rationed, then participation at date $t = 0$ occurs. Moreover, competition between banks at date $t = 0$ ensures that the surplus from renegotiation at date $t = 1$ is captured by borrowers at date $t = 0$.*

<u>Proof</u> Consider the second part of the proposition first. If the entrepreneur is financed at date $t = 0$, competition between banks means that (13) is set equal to zero. As we have seen, if the borrower is type b he receives a return at date $t = 1$ equal to the opportunity

cost of his human capital. If he is type g and there is rationing, renegotiation again gives the entrepreneur a return equal to the opportunity cost of his human capital. Inspection of (13) shows that zero profit at date $t = 0$ requires that $z_1 D_2$ takes the appropriate value. It is through the adjustment of this value in (11) that the full benefit of renegotiation is passed back to entrepreneurs. Inspection of (11) then shows that if z_2 is sufficiently high and D_2 sufficiently low, participation occurs at date $t = 0$.■

This proposition illustrates an important property of our model. A constrained bank does not fully insure its borrowers. For as we have seen, the borrowers bear some of the costs of liquidity shortfalls, by either being rationed or in renegotiating loans. Entrepreneurs are compensated by obtaining better returns at date $t = 1$ when they are type g and there is no rationing. They are also compensated by a reduced probability of being rationed if they are type g.

3.4 Existence of Rationing

We have examined the model with rationing at date $t = 1$. Our task now is to show that there will be rationing and to understand the properties of the model at date $t = 1$ when rationing occurs.

If the bank does not have a liquidity problem at date $t = 1$, then depositors are paid C_1 and all type g and b entrepreneurs are granted further finance. However, this is not so if there is a liquidity shortfall. We therefore investigate what happens if the bank suffers bad luck or is subject to a run triggered by a low signal, σ. In particular, we investigate the determination of γ_1 and δ_1. For γ_1 or $\delta_1$to be set less than one is inefficient. However. because withdrawal of deposits used to finance earlier investments may not be paid for by the income of these investments, our case of bad luck, the bank may face a type of debt overhang problem (see Myers (1977)). Also the bank may be constrained because of a run.

Bad Luck

First suppose the bank faces a liquidity shortage because of bad luck. Consider the possibility that the bank issues either subordinated or non-subordinated deposits to finance loans. Then we have the following two propositions

Proposition 3 *Suppose that at date $t = 1$, the bank has a liquidity shortfall because of bad luck. If the bank could issue subordinated deposits, then loans will be made if the bank's owners return increases. In this case, either all loan applications from entrepreneurs of types g and b will be granted, or none.*

Proof Issuing subordinated deposits of value B_1 at date $t = 1$ with a claim of C_1^{11} to finance entrepreneurs of types g and b, can only raise the value of type 2 deposits made at date $t = 0$. Hence, $f_0(\sigma) = \pi^1$. Thus, if the bank issues deposits, the optimal deposit contract for date $t = 0$ depositors is implemented. The bank will only grant further loans if there exists a subset of type g and b entrepreneurs who, if financed, raise the value of the bank given in (10). Suppose such a subset exists, then loans to these entrepreneurs will cover the banks' liquidity shortfall. It then follows that all other type g and b entrepreneurs will be financed. Hence γ_1 and δ_1 are both set equal to one. On the other hand, if there does not exist a subset of type g and b entrepreneurs who, if financed at date $t = 1$, will add to bank owners' wealth, then all loan applications will be denied. If the bank makes no further loans at date $t = 1$ it is wound up.■

In reality banks do not issue subordinated deposits, reflecting this we assume that they cannot discriminate between depositors (new and old) at the time of withdrawal. Issuing new, non-subordinated deposits, at date $t = 1$ with claim C_1^{11} dilutes existing deposits.

Proposition 4 *Consider a bank with the same liquidity shortfall at date $t = 1$ as in Proposition 3. Assume that, type g and b entrepreneurs apply for loans. Then a bank which issues non-subordinated deposits may supply more or fewer loans than a bank that issues subordinated deposits.*

Proof a) Suppose that under the conditions of Proposition 3, if the bank issues subordinated deposits at date t=1, then γ_1 and δ_1 are set equal to zero. Now, suppose the bank was instead able to issue non-subordinated deposits, which dilute the claims of date $t = 0$ depositors. Then, if this dilution effect is sufficiently large and the incentive constraint for type 2 depositors born at date $t = 0$ given by (A4) is not violated, the bank can increase its owners wealth, given in (10), by financing some type g and b entrepreneurs. This will be done according to the rationing scheme.

b) Now suppose that if the bank were able to issue subordinated deposits, all type g and b entrepreneurs are financed at date $t = 1$, $\gamma_1 = 1$ and $\delta_1 = 1$. If instead the bank needs to obtain a substantial number of new non-subordinated deposits to finance these loans, there will be a significant degree of deposit dilution. But given the signal, σ, dilution of depositors may mean that the incentive constraint (A4) is violated for some date $t = 0$, type 2 depositors. This means that $f_0(\sigma) > \pi^1$, which means, using (5) and (6), that C_1 rises relative to C_2. To mitigate this effect, the bank may restrict deposit issue and thereby not finance some type g and b entrepreneurs according to the rationing scheme.

Bank Run

Now suppose the bank suffers a liquidity shortage because of a run. Then we have two propositions which parallel the above:

Proposition 5 *Suppose that at date $t = 1$ the bank has a liquidity shortfall because of the behaviour of depositors. Then if the bank can issue subordinated deposits as in Proposition 3, either the bank will finance all type g and b borrowers, or none. In the latter case there will be a full run and the bank is liquidated.*

Proof This proposition is analogous to Proposition 3. The key difference is that drawing in new deposits to finance type g and b entrepreneurs with subordinated deposits has to induce date $t = 0$ type 2 depositors not to withdraw early, so that $f_0(\sigma) = \pi^1$. If refinancing a subset of type g and b entrepreneurs has this effect, then all such requests will be granted. But otherwise none will be financed.■

Now consider what happens if the bank can only issue non-subordinated deposits. Then we have the following proposition which can be contrasted with Proposition 4.

Proposition 6 *Suppose the bank is faced with the same liquidity shortfall as in Proposition 5. Types g and b entrepreneurs apply for continuation loans. Then, if the bank can only issue non-subordinated deposits, it will supply the same or fewer loans than a bank that issues subordinated deposits.*

Proof a) Suppose that if the bank can issue subordinated deposits at date $t = 1$, γ_1 and $\delta_1$and are set equal to zero. Hence with no dilution of depositors, the value of the continuation loans is insufficient to overcome the liquidity shortfall. Thus, if the bank is to finance any continuation loans it must dilute existing depositors claims. Issuing non-subordinated deposits does this but worsens the violation of the incentive constraint for date $t = 0$, type 2 depositors, so that $f_0(\sigma) > \pi^1$. Hence γ_1 and δ_1 are set equal to zero here also.

b) On the other hand, if the bank could issue subordinated deposits it sets γ_1 and δ_1 equal to one. Then, if instead, it could issue non-subordinated deposits, and at date $t = 1$ sets $\gamma_1$and δ_1 equal to one, the dilution of depositors may preclude the date $t = 0$, type 2 depositors incentive constraint from being satisfied, so that $f_0(\sigma) > \pi^1$. If this happens the bank extends fewer loans than if it could issue subordinated deposits, with rationing according to the rationing scheme.■

In comparing the two types of liquidity shortfall we note that at date $t = 1$ a run occurs because depositors receive a signal of poor returns at date $t = 2$. This information will raise the return required by new depositors thereby increasing the impact on bank lending relative to a shortfall due to bad luck.

4 Management of Liquidity and Project Choice

In the above we have seen how the liquidity of an incompletely diversified bank can affect its ability to implement the efficient lending policy. Diversification of the bank's loan portfolio reduces the risk depositors are exposed to along the lines examined by Diamond (1984). In our model, diversification will reduce the variance of R_1, and, other things equal, reduces the expected value of the liquidity shortfall.

In developing the model, we did not allow banks to hold safe assets to insure against liquidity shortfalls. Suppose that at date $t = 0$, at the expense of making productive investments, the bank can invest in one-period, safe government bonds. The benefit of holding safe bonds is that the expected value of the liquidity shortfall at date $t = 1$ is reduced. Hence, there is a trade-off for the bank, with the bank limiting its holding of bonds and hence its insurance against liquidity shortfalls. The question arises as to whether firms have an incentive to insure against not being financed by banks? To examine this question we consider a simple example.

Assume that banks do not hold bonds. Suppose that at date $t = 0$, entrepreneurs have a simple project choice. They can either choose a project of the type already examined, or a project which yields a return at date $t = 1$ as well as date $t = 2$. This project, indexed by an asterisk, requires some initial investment of I_0^* at date $t = 0$ and a further investment of I_1^* at date 1, which are the same as for the alternative choice. However, the project yields a deterministic income stream of Y_1^*at date $t = 1$and Y_2^* at date $t = 2$. We assume for simplicity that

$$Y_1^* + Y_2^* \geq I_0^* + I_1^* + 2u \tag{14a}$$

$$Y_2^* > I_1^* + u \tag{14b}$$

$$Y_1^* = I_1^* \tag{14c}$$

$$L_1^* = 0 \tag{14d}$$

This project is positive net present value at date $t = 0$ and covers the entrepreneur's opportunity cost (it is actually analogous to riskless productive storage). The project generates enough income at date $t = 1$ to ensure that the entrepreneur can finance I_1^* at date $t = 1$, without a further loan from the bank. The bank will be compensated with a payment of D_2 at date $t = 2$. Because $L_1^* = 0$, it is in neither parties interest to liquidate the project at date $t = 1$. Since the entrepreneur borrows I_0 at date $t = 0$, but will not borrow anymore at date $t = 1$, $D_2^* = I_{0.}^*$

Assume that the uncertain project is better than the deterministic project in the sense that the quantity in (4) exceeds the net return in (14a) given by

$$Y_1^* + Y_2^* - I_0^* - I_1^* - 2u \geq 0 \tag{15}$$

In the first-best, all entrepreneurs are better-off doing the uncertain project. Now consider what happens in our second-best environment to entrepreneurs who undertake this project. In this environment, entrepreneurs of types g and b at date $t = 1$, receive finance for I_1 with respective probabilities of q_1^g and q_1^b. Entrepreneurs' expected net return at date $t = 0$ if they do the uncertain project is given by (11). This can be compared to the expected return from the deterministic project given by

$$Y_2^* - D_2^* - 2u \tag{16}$$

Now consider the deterministic project from the perspective of banks. At date $t = 0$, banks can finance entrepreneurs, who choose to undertake either the uncertain or the deterministic project. In financing some deterministic projects, there will be a reduction in both the expected return and risk of the average project financed. However, as a greater proportion of the projects banks finance are deterministic, the variance of R_1 declines and the probability of liquidity shortfalls (due to differences between actual and expected returns at date $t = 1$) declines. By a similar argument, the greater the proportion of deterministic projects financed, the lower the impact at date $t = 1$ of differences between the actual and expected proportion of type b entrepreneurs on expected returns at date $t = 2$ and hence the lower the probability of runs. Then given a reduced proportion of entrepreneurs who will demand continuation loans at date $t = 1$ the values of q_1^g and q_1^b will be higher.

We can now examine equilibrium when entrepreneurs have the above project choice. Assume that banks do not hold bonds. Assume also that when the bank finances no deterministic (or more generally, lower risk) projects, there is a positive probability of a liquidity shortfall at date $t = 1$. If the expected cost of this liquidity shortfall is sufficiently large, all entrepreneurs would prefer to undertake the deterministic project if it were available. Then we have the following proposition:

Proposition 7 *In equilibrium, if the deterministic project is available, some entrepreneurs will do this project.*

Proof By assumption, when no deterministic projects are financed, entrepreneurs' net return from the uncertain project, as given in (11), will be less than would be obtained from the deterministic project, as given in (16). Now, if the deterministic project is available, entrepreneurs will want to do it with the financial contract described. Banks will offer to supply this contract on competitive terms. However, the greater the proportion of loans to finance deterministic projects, the higher the values of q_1^g and q_1^b in (11). A bank will finance up to the point that its portfolios of borrowers at date $t = 0$ are indifferent between the two types of project, so that (11) and (16) are equal.■

In establishing this proposition we assumed that the bank does not hold bonds. But since our entrepreneurs have access to a riskless investment there is no reason for the bank to hold the alternative riskless asset.

The above result has some similarities to the findings of Von Thadden (1995). He argues that a commitment not to refinance projects may be an optimal screening devise for creditors facing an adverse selection problem, even though it can induce short-termism on the part of good entrepreneurs. He shows that this problem is mitigated by financial intermediation and in particular by economies of scale in monitoring projects. Dewatripont and Maskin (1995) on the other hand, argue in a similar set up to Von Thadden that creditor liquidity may be used as a commitment mechanism not to refinance poor projects but this may put too high a premium on short-term returns. The present paper on the other hand sees the liquidity problem as arising endogenously through the nature of financial intermediation and in particular the deposit contract. The bias towards short-term returns is then an optimal response to this problem.

5 Conclusion

The paper has shown that banks financed with deposit contracts will pass up financing positive net present value investments so there will be credit rationing. Banks are financed by an overlapping sequence of deposit contracts, with deposits returns not being made completely contingent on the loans they were used to finance, as is realistic. The rationing arises for two reasons. The bank can suffer low income, leading to a debt overhang problem in financing positive net present value continuation loans. A similar problem arises if depositors demand for immediate liquidity is higher then normal. The extent of credit rationing depends upon whether new deposits can be issued to finance the liquidity shortfall and new loans request. The problem of rationing for a given liquidity shortfall is greater because new deposits are non-subordinated, and therefore dilute existing depositors, so there issue is restricted by a need to satisfy the incentive constraint of these depositors.

It was shown that if the bank rations credit, it will first cut finance to borrowers who need to renegotiate loans in order to continue. Then if further rationing is necessary, it will renegotiate with good borrowers who will "bribe" the bank to finance them. We showed that in the event that there is no rationing the bank improves loan terms to borrowers who do not need to renegotiate. But when the bank has done poorly, although some good borrowers will be denied credit, those given credit insure the bank through higher loans repayments. Finally we showed that the existence of the above type of rationing will cause some borrowers to switch to projects of lower net present value because they have a lower (in our example, zero) probability of being rationed at the intermediate date. This will take place up to the point that borrowers are indifferent between the different types of project, with the probability of rationing serving as the equilibrating mechanism.

Our model involved competition between banks at date $t = 0$ but has no competition at date $t = 1$. This meant that we did not need to specify whether the shocks to the bank's returns at date $t = 1$ were idiosyncratic or systemic. However, if one bank is unlucky and has a poor draw of returns at date $t = 1$, others may be lucky. It follows that an interbank market (as in Bhattacharya and Gale (1987)) will potentially improve matters. Then banks agree at date $t = 0$ that, in the light of asymmetric shocks at date $t = 1$, there will be liquidity transfers from lucky to unlucky banks. However, like any insurance mechanism, its efficiency is limited if the bank's returns are unobservable by other banks so the banks' can make false claims and transfers are thereby limited by the need to satisfy a further incentive compatibility constraint. Also, it has the problem of creating a moral hazard, leading to greater risk taking by banks.

Finally, in the paper we have not considered the roles of deposit insurance and of a lender of last resort. These institutions will certainly have an effect on the impact of liquidity shortages (see Rochet and Tirole (1996)). However, their consideration shifts emphasis away from the central role of liquidity, since the problem then becomes one of risk shifting between the banks and the insurer or lender of last resort.

Appendix

Banks finance their loans by supplying deposit contracts. Our model of depositor behaviour is based upon Jacklin and Bhattacharya (1988). First consider the behaviour of a typical depositor, indexed by k. Each depositor is born at a given date, and then has a three date horizon (0,1,2). He has initial wealth at time $t = 0$ of unity. He desires to consume at dates $t = 1$ and $t = 2$. Depositors are either type 1 (early consumers), or type 2 (late consumers). At date $t = 1$, with probability π_k $(k = 1, 2)$ their preferences are:

$$U\left(c^{k1}\right) + \rho^k U\left(c^{k2}\right) \tag{A1}$$

where ρ^k is the discount rate $(\rho^1 < \rho^2)$ indicating that agents of type 1 discount future consumption more heavily, and $\left(c^{k1}, c^{k2}\right)$ is the consumption of agent k at dates $t = 1$ and $t = 2$. We assume the utility function U is twice differentiable with $U'(0) = \infty, U'(c) > 0, U''(c) < 0$.

Individuals find out their type at date $t = 1$. However, this information is not publicly observable, so that there remains asymmetric information. Hence, at date $t = 1$ ´new opportunities to trade may arise, since the information of the agents has increased.

Depositors could keep their wealth in a safe asset, however, higher returns are available through financing (indirectly) productive investment. The contract offered by banks to individuals is a deposit contract. Agents who deposit one unit at date $t = 0$, are entitled to make a withdrawal at date $t = 1$ and a second at date $t = 2$. The amount that a depositor will be allowed to withdraw is contingent upon the number of withdrawals that the bank has already serviced (sequential service constraint). This constraint has been motivated by Wallace (1988) by the assumption that depositors are sufficiently isolated from each other at the intermediate date so that their early withdrawal demands must be accommodated on a first-come first-served basis. This means that banks cannot 'cumulate' withdrawal requests and make payment contingent on the total (Wallace 1988, p.3). This has the following important implication: A bank in facing a withdrawal demand from depositors at date $t = 1$ is obliged to satisfy this demand even if it has to liquidate investments inefficiently (or forego making efficient loans) and the bank may then be unable to fulfil its obligations to depositors at date $t = 2$.

Consider a bank which has a given portfolio of borrowers and which has supplied deposit contracts to a large number of depositors. Let $x_1$and x_2, respectively, be the (uncertain) income of the bank at dates 1 and 2 per unit deposited. This income depends upon the bank's investment policy and determines the maximum amount payable on the average withdrawal at each date. In what follows the investment policy of the bank is taken as given, though it has to be consistent with attracting deposits. Then the optimal choice problem for a deposit contract, in the absence of interim information, is solved by the vector of functions $(c^{11}, c^{12}, c^{21}, c^{22})$ and realisations $x^1$and x^2. For an agent choosing the deposit scheme (c^{11}, c^{12}), the return will depend on the fraction, f, of agents who choose the contract, the remaining fraction $(1 - f)$ choose (c^{21}, c^{21}). Importantly, the bank cannot offer contracts that are contingent on f or the shocks that determine f. The optimal deposit contract is then the solution to the following problem:

$$\max E_0 \sum_{k=1}^{2} \pi^k \left[U\left(c^{k1}\right) + \rho^k U\left(c^{k2}\right)\right] \tag{A2a}$$

subject to:

$$x_1 \geq \pi^1 c^{11} + \pi^2 c^{21} \tag{A2b}$$

$$x_2 \geq \pi^1 c^{12} + \pi^2 c^{22} \tag{A2c}$$

and $U\left(c^{k1}\right) + \rho^k U\left(c^{k2}\right) \geq$ utility obtained from misrepresenting true type for

$$k = 1, 2 \tag{A2d}$$

The expectation in (A2a) is taken with respect to depositors' common prior information on the distributions of x_1 and x_2. The inequalities in (A2b) and (A2c) are the bank's budget constraints for dates $t = 1$ and $t = 2$. The condition in (A2d) is the incentive compatibility constraint, which ensures that type k agents choose type k deposits, provided the bank pays the promised amounts. The solution to this problem has the important property of some preference shock insurance, however, the extent of this insurance is limited if the incentive constraint is binding. It has the property that $c^{11} > c^{12}$and $c^{21} < c^{22}$ but involves less consumption smoothing than when the incentive constraint is relaxed. Finally, the solution to the above optimisation problem is an equilibrium, since the bank is solvent, and each agent is better-off choosing the type of contract that has been designed for it, so that: $f = \pi^1$.

Following Jacklin and Bhattacharya, assume that some (say a proportion β) of type 2 depositors receive information, σ, at date 1 on the performance of the bank's portfolio, including knowledge of x_1, which is used to recalculate the distribution of x_2. Type 2 depositors use the information σ to form a posterior belief about x_2 denoted by prob $(x_2 \mid \sigma)$. These beliefs are assumed to be consistent with the prior beliefs used in so that:

$$\text{prob}\,(x_2) = \sum_{\sigma} \text{ prob}\,(\sigma) \text{ prob}\,(x_2 \mid \sigma) \text{ for all } x_2 \tag{A3}$$

We also assume that the conditional distribution of x_2 for low realisations of σ is first-order stochastically dominated by distributions corresponding to high realisations of σ. The contract solving (A2) is not contingent upon the interim information, σ. However, this information causes a revision at the interim date of the "incentive constraint" for type 2 depositors to:

$$E\left[U\left(c^{21}\right) + \rho^2 U\left(c^{22}\right) \mid \sigma\right] > E\left[U\left(c^{11}\right) + \rho^2 U\left(c^{12}\right) \mid \sigma\right] \tag{A4}$$

For some low enough realisations of σ the revision of the distribution of x_2 leads to a violation of this constraint. If this happens, type 2 agents will prefer the type 1 allocation and the ex-ante efficient allocation will be upset, $f > \pi^1$.

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