Structural Breaks, Incomplete Information And Stock Prices

By

Allan Timmermann

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Structural Breaks, Incomplete Information and Stock Prices*

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Abstract

This paper presents new empirical evidence on the existence of structural breaks in the fundamentals process underlying US stock prices and develops an asset pricing model which considers the possibility of such breaks. Three breakpoints are identified: The Great Depression, World War II, and the oil price shocks around 1974. Different hypotheses for how investors form expectations about future dividends after a break are proposed and analyzed. A model in which investors do not have full information about the parameters of the dividend process but gradually update their beliefs as new information arrives is shown to induce volatility clustering and serially correlated stock returns after a break. These patterns are confirmed to exist in US stock returns around the time of the breakpoints.

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1. Introduction

Is the fundamentals process underlying US stock prices stable over several decades? This stability assumption is implicitly made in the vast majority of papers in the empirical asset pricing literature that tests present value models. Recent studies have questioned this assumption, however. Discussing the mean return on US stocks since 1926, Brennan (1997) argues that "... there are good reasons to doubt that this parameter has remained constant for almost three quarters of a century which has witnessed the most dramatic economic, technological and social change of any comparable period in history" (Brennan, page 5). Observations like these suggest that a full understanding of asset prices requires careful consideration of the stability of the underlying fundamentals process.

This paper proposes a new approach to modeling stock market prices which links structural breaks in the underlying dividend process with the assumption that investors have imperfect information about the new dividend growth rate after a break. Our approach is based on new empirical tests which suggest that there are multiple breaks in the fundamentals process underlying US stock prices and the paper considers their importance in the context of a simple asset pricing model.

Structural breaks in the dividend process, if present, can affect stock prices in two important ways.[1] First, like any shock to the endowment process, breaks will affect future dividends. The main difference between breaks and ordinary shocks to dividends is that the former imply rare level shifts and will remain in effect for a long time. This is the 'persistence' effect of breaks.[2]

Breaks also give rise to an important information effect which concerns how much information investors have and how they form expectations about future dividends following a structural break. One possibility, which we name the full information case, is that investors instantaneously observe the new parameters of the dividend process after each break. While this is an important benchmark, it seems far from empirically plausible. Episodes linked to breaks in the dividend process, such as the Great Depression, were associated with substantial uncertainty

[1]Stock prices are symmetrically influenced by breaks in the dividend process and breaks in the discount rate process so breaks in the latter will have similar effects.

[2]This effect is not unique to structural break models and is also observed for Markov switching processes, c.f. Cecchetti, Lam & Mark (1990).

over future prospects of the economy. Such uncertainties, we argue, can better be modeled by assuming that investors have incomplete knowledge of the new dividend growth rate and undertake a recursive updating process which gradually provides them with more precise growth estimates as new data emerges.

This imperfect information hypothesis has important empirical implications. In the period following a structural break investors cannot rely on historical data to produce an estimate of the new mean dividend growth rate. Large revisions in investors' parameter estimates are more likely to occur immediately after such breaks since the 'learning clock' runs fast and this produces a clustering in the volatility of asset prices through their dependence on investors' beliefs.[3] In contrast, under full information a break in the dividend process will only show up as a single outlier in the return distribution in the period where the break occurs. Figure 1 provides a simple illustration of the speed of the learning clock and the resulting difference in stock returns under these two informational assumptions.

Earlier papers have suggested that instabilities in the fundamentals process may be important to asset prices in the context of switches between recurrent states. For example, Cecchetti et al. (1990) model switches between a boom and a recession state in the endowment process underlying US stock prices. Evans & Lewis (1995) also argue that investors in the foreign exchange market anticipate infrequent switches between recurring appreciation and depreciation states and that this can significantly affect the currency risk premium. By assuming that states repeat and that switches do not represent clean breaks with the past, investors in these models can use historical information to update their beliefs. This is a key difference to the informational assumption made in this paper according to which historical information cannot be used to predict future dividends after a break. Under the latter scenario, revisions in investors' parameter estimates will be larger and volatility of asset prices higher immediately after a break as indicated in Figure 1.

Lewis (1989) provides the only previous analysis that explicitly considers the

[3]This effect is related to the general point made by Genotte (1986) that estimation risk adds an extra factor to the fluctuation in asset prices. Genotte conjectures that turbulence in the market return process will continue to have an effect on market prices after the original shock because of the market's learning process. Our analysis shows that this conjecture is correct when asset prices are endogenously determined and depend on investors' beliefs.

market's learning following a break in the fundamentals process.[4] In her seminar paper, Lewis considers the possible occurrence of a single breakpoint, the timing of which is known to investors, and she argues that a break in the US money demand equation during the early 1980s was at least partially responsible for variations in the dollar exchange rate in the subsequent period. Our paper generalizes the idea of a single break to the case with multiple breaks and estimates the dates, magnitudes and number of breaks.

The contribution of our paper is threefold. First, we present new empirical evidence on breaks in the fundamentals process (endowment growth and the discount rate) underlying US stock prices. In their own right these results have important implications for modeling of stock prices. Second, we present a simple asset pricing model which allows for multiple breaks in the fundamentals process and we simulate stock prices generated by this model both under full information and under Bayesian learning. Earlier studies have considered switches between repeated states or single structural breaks but no study has previously considered asset prices when the fundamentals process is subject to multiple breaks and stock prices are determined endogenously with respect to investors' learning following a break.[5] Having identified *ex ante* the particular historical episodes where breaks occurred in the fundamentals process, we finally test the implications and new predictions of our asset pricing model against US stock returns data around these episodes.[6]

The plan of the paper is as follows. Section 2 presents the empirical evidence on breaks in the fundamentals process underlying US stocks while Section 3 develops

[4]Other studies have indicated the existence of breaks in the fundamentals process. Barsky & DeLong (1993) use a Chow test to identify a structural break in the volatility of dividend growth around World War II.

[5]This combination is important. Genotte (1986) calls for a model where the market's learning affects the underlying return process, while Lewis (1989) conjectures that multiple break points are needed to better explain movements in asset prices. Our findings suggest that both components are important to asset prices.

[6]Several studies consider the effect of investors' learning on asset prices. In some models learning effects vanish asymptotically (Bray & Savin (1986), Timmermann (1993)), while in others they are present at all points in time (Barsky & DeLong (1993), Brock & LeBaron (1996), Evans & Lewis (1995), Timmermann (1996)). While this literature has yielded important insights, these approaches may not identify the episodes where learning effects matter the most to stock prices, however. We argue that incomplete information and learning effects are likely to be important to understanding asset prices mainly after a structural break when there is considerable uncertainty about future dividends.

an asset pricing model with breaks in the dividend process. Incomplete information about the parameters of the dividend process and recursive learning effects are introduced in Section 4. Section 5 reports results from simulations of the model under complete and incomplete information and compares the simulations to actual US stock returns data. New predictions generated by our asset pricing model are reported in Section 6, while Section 7 discusses the implications of our findings for empirical tests of asset pricing models.

2. Structural Breaks in Fundamentals: Empirical Evidence

2.1. *Structural Breaks in the Endowment Process*

Standard equilibrium asset pricing models (e.g., Lucas (1978)) assume that non-storable dividends from a single endowment source are the economy's only source of income. Thus these models do not take a stand on whether the best measure of endowments is provided by aggregate dividends, output or consumption. We follow Cecchetti et al. (1990) and investigate the evidence on breaks in all three series measured on a real, annual per-capita basis over the period 1890-1994.[7] Plots of the time series are presented in Figure 2. The most significant feature of the data is the high volatility of the dividend and GDP series during the interwar period and the lower volatility of the series after world war II.

To formally test for breaks in the endowment process we follow the procedure for consistent estimation of multiple breakpoints in linear regression models developed by Bai & Perron (1998). Let y_t be a process which may have multiple breakpoints, let $\mathbf{x}_t$ be a vector of factors whose regression coefficients in the equation for y_t remain constant, and let $\mathbf{z}_t$ be a vector of factors whose coefficients in the linear regression of y_t on $(\mathbf{x}_t' \ \mathbf{z}_t')'$ change at m discrete (break) points in time:

$$\begin{array}{ll} y_t = \mathbf{x}_t'\beta + \mathbf{z}_t'\delta_1 + u_t & t = 1, 2, ...T_1 \\ y_t = \mathbf{x}_t'\beta + \mathbf{z}_t'\delta_2 + u_t & t = T_1 + 1, ...T_2 \\ \cdots\cdots\cdots\cdots & \cdots\cdots\cdots \\ y_t = \mathbf{x}_t'\beta + \mathbf{z}_t'\delta_{m+1} + u_t & t = T_m + 1, ...T. \end{array} \quad (1)$$

Here T is the sample size, $T_1 < T_2 < ... < T_m < T$ and u_t is a disturbance

[7]We use the same data sources as Cecchetti et al, with the one exception that an updated series for the consumption deflator reported in Shiller (1989) is adopted to measure inflation.

term.[8] Bai and Perron develop tests for the consistent estimation of the number and location of breakpoints ($T_1, ..., T_m$) and the parameters (β' $\delta'_1, ..., \delta'_{m+1}$). We use their methods to investigate the presence of breakpoints in two sets of variables that reflect different aspects of the endowment process. The first set is simply the first-differenced logarithm of annual dividends, $\Delta \ln(D_t)$, output, $\Delta \ln(GDP_t)$, and consumption, $\Delta \ln(C_t)$. Currently available econometric techniques do not facilitate consistent estimation of multiple unknown breaks in the variance of a process. To capture the possibility of a break in the volatility of the endowment process we use the following volatility proxy: $y_t = |\Delta \ln(Y_t)|$, the absolute value of the log-change in the endowment process, Y_t.

Ideally one would like to test for breaks in the model that was actually used by investors to forecast future values of the endowment process. However, in the absence of agreements on a structural model for the endowment process, this is difficult to accomplish. Instead we resort to testing for breaks in a simple first-order autoregressive time series representation which captures the essential dynamics of the data series. It is conceivable that this procedure overlooks break points that investors with a larger conditioning information set were aware of historically.

Table 1 reports the number of break points identified by three separate criteria used by Bai and Perron to test for structural breaks.[9] The double maximum test ($D_{\max}$) lets the number of breaks be unknown and tests the null of no breaks against the alternative of one or more breaks, while the two information criteria are based on the penalized likelihood function. The number of breaks identified by the tests varies considerably depending on which test is used and which series is studied.[10] Overall the empirical evidence identifies two breaks in the endowment process, the first associated with the beginning of the Great Depression and the

[8]Bai & Perron (1998) consider two separate break point specifications. If lagged dependent variables are included as regressors in (1), then u_t must be a martingale difference sequence and hence cannot be autocorrelated. However, if no lagged dependent variables are included as regressors u_t can be serially correlated and heteroskedastic.

[9]A Gauss program provided by Bai and Perron was used in the estimations. In these tests the minimum length between breaks was set to ten years in the case of the annual data and five years in the case of the monthly data. The maximum number of breakpoints was set to five, and we allowed for heteroskedasticity in the residuals.

[10]Given the considerable difficulty in estimating precisely the mean of these processes, we would expect structural break tests based on $\Delta \ln(Y_t)$ to have considerably lower power than tests based on the volatility proxy $|\Delta \ln(Y_t)|$.

second associated with World War II.

Table 2 presents the parameter estimates from the breakpoint regressions. The tests identify a single (imprecisely estimated) breakpoint in the drift of the processes around the beginning of the Great Depression (1930), while, in the case of the proxy for the volatility of the endowment process, the parameter estimates suggest the presence of two breaks around 1930 and 1942. With a length of two to four years the 90 percent confidence intervals for the dates of the breaks in the volatility proxy are estimated very precisely for the dividend and output series. In contrast these confidence intervals have a length of 16 and 22 years for the consumption series.

These are full-sample test results and one could argue that *ex ante* investors did not have sufficient information to decide if a break had occurred in-sample.[11] To relax the assumption that investors could use full-sample information, we adopted the procedure for real-time recursive monitoring of structural changes proposed by Chu, Stinchombe & White (1996) (CSW). Let i be the minimum length of the data for which, under the null, the parameters are assumed to be constant: $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = ... = \boldsymbol{\beta}_\iota$, and suppose that interest lies in testing for breaks in the process under consideration (y_t) at some time $\kappa > \iota$. CSW suggest estimating recursive regressions of the type $y_t = \mathbf{x}_t'\boldsymbol{\beta}_\iota + \varepsilon_t$ to obtain the following 'fluctuation detector':

$$\widehat{Z}_\kappa = \kappa D_\iota^{-1/2}(\widehat{\beta}_\kappa - \widehat{\beta}_\iota), \tag{2}$$

where $D_\iota = M_\iota^{-1} V_0 M_\iota^{-1}$, M_ι is a consistent estimator of $\sum_{t=1}^{\iota} x_t x_t'/\iota$ and V_0 is a consistent estimator of the moment matrix $E[S_\iota S_\iota']/\iota$ with $S_\iota = \sum_{t=1}^{\iota} \mathrm{x}_t \varepsilon_t$. By monitoring the stability of the recursive parameter estimates, this real time procedure can tell if a break has occurred. Figure 3 plots the 5 percent critical bound[12] against the recursive breakpoint statistic computed for the intercept and first order autoregressive coefficients in a regression of the log-differenced endowment series against an intercept and a single lag. For all three endowment processes the plots suggest that there is a break in the early 1930s in the simple autoregressive representations.[13] This confirms the earlier finding that break(s) are present in the endowment process and it indicates that these were sufficiently transparent so that,

[11] On this point see also Arthur, LeBaron & Palmer (1997) who argue that investors' perceptions of how stationary a world they live in can significantly affect the dynamics of asset prices.

[12] Asymptotic bounds on $|\widehat{Z}_\kappa|$ provided by CSW are used to compute the critical values.

[13] The fluctuation monitor works best for a reasonably large value of ι, the minimum sample over which the parameters stay constant. In our calculations we set ι =25. This rules out a test

at least in principle, investors could have detected such break(s) in the light of the historically available data.

As an alternative to testing for breaks in the endowment process one can attempt to infer that investors believed a break had occurred by looking for breaks in the dividend-price ratio.[14] Dividend yields are strongly serially correlated so a first-order autoregressive equation is again adopted in the Bai-Perron regressions covering the period 1926-1997:[15]

$$\begin{array}{llll} Yield_t = & \text{-0.00020} & +1.053 Yield_{t-1} & \text{(1926:1-1932:6)} \\ & (0.0010) & (0.019) & [32:3\text{-}32:9] \\ & 0.0117 & +0.697 Yield_{t-1} & \text{(1932:7-1937:7)} \\ & (0.0011) & (0.025) & [37:3\text{-}37:11] \\ & 0.0006 & +0.985 Yield_{t-1} & \text{(1937-1997)} \\ & (0.0004) & (0.009) & \end{array} \quad (3)$$

Figures in square brackets provide 90 percent confidence intervals for the end-point of the preceding period while figures in round brackets report heteroskedasticity consistent standard errors for the estimated coefficients. Two breakpoints are identified by this procedure. During the first period (1926-1932) the dividend yield increases dramatically. The second period (1932-1937) is characterized by a much higher intercept term but also by less persistence in the yield compared to the other two periods. The dates of the breakpoints are very precisely determined with 90 percent confidence intervals shorter than ten months. In conjunction with the earlier findings, the evidence of at least two breaks in the endowment process underlying US stock prices seems strong.

2.1.1. Structural Breaks or Regime Switching?

An obvious alternative to the structural break interpretation of the endowment growth series is to consider the data as the outcome of a finite-state Markov switching process as proposed by Hamilton (1989). To investigate this possibility we es-

for a second break, following the first break in the early thirties, before the mid-fifties. No such break was found in any of the series.

[14]Breaks in the yield could of course also reflect breaks in the discount factor, a point we will have more to say about shortly.

[15]This regression uses monthly data on a twelve-month average of dividends divided by the price of the value-weighted CRSP portfolio.

timated a two-state Markov switching model similar to the one used in Cecchetti et al. (1990) and Evans & Lewis (1995). Suppose that the mean and variance of the growth in the endowment process is driven by a latent state variable, s_t:

$$\Delta \ln(Y_t) = \mu_{s_t} + \sigma_{s_t}\varepsilon_t. \tag{4}$$

Furthermore, suppose that s_t follows a two-state Markov switching process with constant transition probabilities:[16]

$$\begin{aligned} \Pr(s_t &= 1|s_{t-1} = 1) = p_{11} \\ \Pr(s_t &= 2|s_{t-1} = 1) = p_{12} \\ \Pr(s_t &= 2|s_{t-1} = 2) = p_{22} \\ \Pr(s_t &= 1|s_{t-1} = 2) = p_{21}. \end{aligned} \tag{5}$$

Conditional on being in a given state the density of the log-differenced endowment process is assumed to be Gaussian with state-specific mean (μ_j) and volatility (σ_j):

$$f(\Delta \ln(Y_t)|s_t = j) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp(\frac{-(\Delta \ln(Y_t) - \mu_j)^2}{2\sigma_j^2}), j = 1, 2. \tag{6}$$

Summing across states gives the unconditional density which can be used in the estimation of the model. Panel I of Table 3 reports the estimated parameters of this model. A high volatility state ($s_t = 2$) with a volatility estimate at least five times higher than the estimate in the low volatility state ($s_t = 1$) is identified for all three endowment series. The two states are highly persistent and the estimated probability of staying in a state exceeds 0.88.

To facilitate interpretation of the two states, inferred probabilities of being in the low-volatility state are plotted in Figure 4. For the dividend and consumption series the two-state Markov switching model associates the low-volatility state with the post-war decline in volatility, while for the GDP data the high volatility state is associated with the period between the Great Depression and world war II. These state probabilities are indicative of breaks in the dividend and consumption

[16]We follow standard practice and assume a two-state process for s_t. Cecchetti et al. (1990) provide additional evidence on the fit of this two-state model to the data on the endowment process.

processes. Of course the low power in distinguishing between breaks and rarely repeated regime switches means that a recurring regime model cannot be statistically rejected in a sample as small and with as few breaks as ours. However, from the point of view of considering the evolution in the endowment series across several decades, as is required by the present value model, one can reasonably argue that the data does not support imposing the tight structure implied by a Markov switching model.

2.1.2. ARCH Models for the Endowment Process

The very pronounced patterns of heteroskedasticity in the three endowment processes shown in Figure 2 and their very high variability around the Great Depression suggest that an autoregressive conditional heteroskedasticity (ARCH) model may provide a good fit to the endowment series. To investigate this possibility we estimated GARCH(1,1) models of the type

$$\Delta \ln(Y_t) = \mu + \gamma \Delta \ln(Y_{t-1}) + \varepsilon_t, \tag{7}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}, \tag{8}$$

where $\varepsilon_t \sim N(0, h_t)$. Parameter estimates for the three endowment processes are reported in panel II of Table 3, and a plot of the conditional volatility, h_t, is given in Figure 5. Both the dividend and GDP series produce estimates of α_1 and β_1 that add up to a number greater than one, suggesting that the volatility processes are mildly explosive and that the second moments of these series do not exist. This does not appear to be a reasonable description of the data in Figure 2 and the strong persistence in the conditional volatility seems entirely due to the very high volatility in the early thirties. Volatility after the second world war also declines to a level much below what one would expect if the GARCH model provided a good description of the data. Rather, the extremely low post-war volatility and the high volatility in the thirties are suggestive of a structural break in the series.

2.2. *Breaks in the Discount Rate Process*

The present value model implies that stock prices depend symmetrically on future values of the endowment and discount rate processes, a point forcefully made by

Campbell & Shiller (1988). It follows that breaks to the discount rate process would have similar effects on stock prices as breaks in the endowment process and thus have to be investigated.

A large empirical literature in finance models risk premia on stocks as a function of regressors such as lagged interest rates and default premia, see, e.g., Breen, Glosten & Jagannathan (1989) and Whitelaw (1994).[17] We follow this literature and estimate regressions of the type

$$\rho_t = \alpha + \beta_1 I_{t-1} + \beta_2 Def_{t-1} + \varepsilon_t, \tag{9}$$

where ρ_t is the excess return on stocks over and above the return on a 1-month T-bill, I_{t-1} is the lagged 1-month T-bill rate and Def_{t-1} is the lagged default premium, computed as the difference between the yields on BAA and AAA-rated portfolios of commercial bonds. Post-war data from 1954-1997 rather than data for the longer period 1926-1997 is chosen in order to match our results with those reported in the existing literature and to account for the fact that the T-bill rate process changes after 1953 following the Accord which allowed interest rates to vary freely.[18] Monthly excess returns on the value- and equal-weighted CRSP portfolios are used in the regressions. The former portfolio puts more weight on large firms than the latter does and we use both portfolios to investigate the robustness of our results. For the value-weighted portfolio the regression results were as follows:[19]

$$\begin{array}{lllll} \rho_t = & 0.043 & -22.23 I_{t-1} & +9.46 Def_{t-1} & 1954{:}1\text{-}1962{:}10 \\ & (0.020) & (5.71) & (28.92) & [62{:}5\text{-}63{:}3] \\ & 0.035 & -13.24 I_{t-1} & +29.31 Def_{t-1} & 1962{:}11\text{-}1974{:}9 \\ & (0.013) & (2.68) & (16.17) & [72{:}2\text{-}77{:}4] \\ & 0.012 & -4.00 I_{t-1} & +18.76 Def_{t-1} & 1974{:}10\text{-}1997{:}12 \\ & (0.007) & (1.24) & (7.14) & \end{array} \tag{10}$$

[17] Another common regressor is the dividend yield. Since this variable would also be affected by breaks in the discount rate process and since doubts have been raised about the suitability of using this regressor, we exclude it from the model.

[18] Hence investors would conceivably have used a different forecasting model prior to our sample period. Note that this need not imply a break in the discount rate process at that point in time.

[19] We present the results for models with two break points. The criteria identified between one and four breaks, and the model with two breaks had well-defined estimates and confidence intervals.

while for the equal-weighted portfolio we obtained the following results:

$$\begin{array}{llllll} \rho_t = & 0.011 & -4.03 I_{t-1} & +21.51 Def_{t-1} & \text{1954:1-1969:5} & \\ & (0.018) & (3.54) & (26.69) & [69:2\text{-}69:8] & \\ & -0.164 & +1.20 I_{t-1} & +185.80 Def_{t-1} & \text{1961:11-1975:1} & (11) \\ & (0.044) & (5.01) & (34.10) & [74:10\text{-}75:4] & \\ & 0.011 & -6.08 I_{t-1} & +34.04 Def_{t-1} & \text{1975:2-1997:12} & \\ & (0.009) & (1.56) & (9.09) & & \end{array}$$

For both portfolios a break is identified around the oil price shocks in the mid-seventies, while the timing of the first break differs across the two portfolios. Note that the sensitivity of excess returns with respect to the default premium variable is highest in the interval from the sixties to the mid-seventies. There is also evidence that the sensitivity of returns on the value-weighted portfolio with respect to the short interest rate has declined markedly over time while no such effect is present for the equal-weighted portfolio. This difference between the small and large firms' conditional return equation is not surprising in view of these firms' different sensitivity with respect to changing economic conditions. A conservative interpretation of the data suggests that a strong third candidate for a breakpoint in the fundamentals process is 1974-75.

This evidence is also consistent with notions of a break occurring in the US economy sometime during the early-to-mid seventies. Although the analysis of the endowment data failed to identify such a break, it should be recalled that it relied on annual data and may have missed some breaks. Furthermore, Garcia & Perron (1996) find evidence of a break in the inflation rate around 1973 and they report that the volatility of inflation almost triples at this point.[20] Breaks in the inflation process may be indicative of a break in the endowment process according to Fama (1981) who argues that there is a correlation between shocks to inflation and unobservable shocks to future real economic growth.[21]

[20] Garcia & Perron (1996) also report evidence of a second regime switch in ex-post, real interest rates around 1979-80, but their Table 2 suggests that there is only a single switch in the inflation rate process.

[21] Ultimately what really matters to asset prices is agents' perceptions of whether a break has occurred. Studies of the productivity of the US economy by, e.g., Clark (1978) and Norsworthy, Harper & Kunze (1979) have also reported evidence of a slowdown in the productivity growth of the US economy around 1973 which could not be attributed to standard economic factors

3. Stock Prices Under Full Information and Breaks in the Dividend Process: A Theoretical Model

The empirical evidence in Section 2 suggests that a model of US stock prices must account for a number of important features of the underlying fundamentals process. First and foremost there appear to be multiple breaks in fundamentals. Common to the Great Depression, the second world war and the oil price shocks of the mid-seventies is that these events were largely unforeseen, yet were rapidly recognized once they had occurred. Furthermore, these events appear to be sufficiently unique to make it unlikely that they are repeated draws from the same (two-state) switching process.

In this section we propose a simple asset pricing model which accounts for these aspects of the fundamentals process. Compared with the Markov switching model of Evans & Lewis (1995) we relax the assumption of recurring regimes whose transition probabilities are driven by a first-order Markov process. To acknowledge the uniqueness of the breaks we instead assume that, after each break, the parameters of the dividend process will be drawn from a continuous distribution.[22,23]

The purpose of the model is not to provide an empirically accurate description of dividends. Like the existing models in the literature it should be thought of as a stylized model that allows us to establish some basic properties of stock prices under multiple breaks in the dividend process. Real dividends (D_t) are assumed to follow a geometric random walk process:

$$\ln(D_{t+1}) = \ln(D_t) + \mu_{t+1} + \sigma_{t+1}\varepsilon_{t+1}, \tag{12}$$

where μ_{t+1} is a drift term, σ_{t+1} is the volatility parameter and $\varepsilon_{t+1} \sim N(0,1)$ is a standard normally distributed innovation term. Define s_{t+1} as a 'break indicator' such that $s_{t+1} = 0$ implies that there is no break in the dividend process, while

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and hence indicate that a break was perceived to have occurred in the US economy in the early seventies.

[22] As argued previously, breaks in the discount rate will have symmetric effect on asset prices and can be analyzed accordingly. Alternatively, breaks can be thought of as occurring in the differential between the discount rate and the growth rate, $r - g$.

[23] This also guarantees that parameter uncertainty will not be eliminated even asymptotically. If the data were generated by a finite-state Markov process, investors would eventually learn the parameter values arbitrarily well, although of course they need not know the true state of the economy.

if $s_{t+1} = 1$, a break has occurred in period $t+1$. Also let $\Pr(s_{t+1} = 0|\varepsilon_{t+1}) = \pi$ and $\Pr(s_{t+1} = 1|\varepsilon_{t+1}) = 1 - \pi$, be the probabilities of no break and a break, respectively, for all possible realisations of ε_{t+1}, and assume that the process for s_{t+1} is independently and identically distributed and also independent of the εs. Then the process for $(\mu_{t+1}, \sigma^2_{t+1})$ is given by

$$\begin{array}{lll} \Pr(\mu_{t+1} = \mu_t, \sigma^2_{t+1} = \sigma^2_t | s_{t+1} = 0) & = 1 & \text{(no break)} \\ \Pr(\mu_{t+1} \leq \overline{\mu}, \sigma^2_{t+1} \leq \overline{\sigma}^2 | s_{t+1} = 1) & = F(\overline{\mu}, \overline{\sigma}^2) & \text{(break)} \end{array} \tag{13}$$

where $F(.,.)$ is the bivariate cumulative density function for the new values of μ_{t+1} and σ^2_{t+1}. Under these assumptions we have $E_t[D_{t+1}/D_t|s_{t+1} = 0] = \exp(\mu_t + \sigma^2_t/2)$ $\equiv (1 + g_t)$, one plus the mean growth rate conditional on no break in the dividend process.

If a break occurs in the dividend process we assume that the new mean growth rate, $g_{t+1} \equiv \exp(\mu_{t+1} + \sigma^2_{t+1}) - 1$, is drawn from a uniform density $U(g_{t+1})$ defined on the support $[\underline{g}, \overline{g}]$. Then equation (13) simplifies to

$$\begin{array}{lll} \Pr(\exp(\mu_{t+1} + \frac{\sigma^2_{t+1}}{2}) = \exp(\mu_t + \frac{\sigma^2_t}{2}) | s_{t+1} = 0) & = 1 & \text{(no break)} \\ \Pr(\exp(\mu_{t+1} + \frac{\sigma^2_{t+1}}{2}) \leq 1 + g | s_{t+1} = 1) & = \frac{g - \underline{g}}{\overline{g} - \underline{g}} & \text{(break)} \end{array} \tag{14}$$

for all $g \in [\underline{g}, \overline{g}]$. The possibility of breaks in the mean growth rate is the only non-standard part of the specification of the dividend process and the innovation term is homoskedastic and serially uncorrelated. The sort of changes in the mean dividend growth rate that we have in mind with this dividend specification are rare structural breaks like the ones identified in the empirical section.

To focus on the implications for stock prices of breaks in the dividend process and the associated revisions in investors' expectations about future dividends, we consider the simplest possible asset pricing setup. Stock prices (P_t) are assumed to be determined by a present value relation based on a representative, risk-neutral investor:

$$P_t = \frac{1}{1+r} E_t[P_{t+1} + D_{t+1}] \tag{15}$$

where r is the (constant) discount rate. Hence fluctuations in stock prices are completely driven by shocks to dividends and revisions in investors' beliefs. E_t is the expectation operator conditional on investors' information set at time t $\{\Omega_t\}$

which consists of $\{D_t, D_{t-1}, ..., P_t, P_{t-1}, ..., s_t, s_{t-1}, ..\}$. Thus investors observe if a break has occurred in a given period. In practice investors may either have superior information that allows them to anticipate a break or, conversely, only gradually realize that a break has occurred. The advantage of our model is that it allows us to study in a very 'clean' way the behaviour of stock prices around breaks.

In the appendix we prove that, under full information and with breaks in the dividend process, the stock price is given by the following proposition:

Proposition

Suppose that each period the mean growth rate of the dividend process breaks with probability $1-\pi$ and that, after a break, the new mean growth rate is drawn from a uniform density with support $[\underline{g}, \overline{g}]$, where $\overline{g} < r$. Then, under full information, the rational expectations stock price formed according to the present value model (15) is given by

$$P_t = \left(\frac{a + \pi(1+g_t)}{1 + r - \pi(1+g_t)} \right) D_t \tag{16}$$

where a is a constant defined by

$$a = \frac{(1-\pi)\left((1+\frac{\underline{g}+\overline{g}}{2})(\overline{g}-\underline{g}) + \pi A\right)}{\overline{g} - \underline{g} - (1-\pi)B},$$

and

$$\begin{aligned} A &= \frac{-(\overline{g}^2 - \underline{g}^2 + 2(\overline{g}-\underline{g}))}{2\pi} - \frac{(1+r)(\overline{g}-\underline{g})}{\pi^2} + \frac{(1+r)^2}{\pi^3} \ln\left(\frac{1+r-\pi(1+\underline{g})}{1+r-\pi(1+\overline{g})}\right) \\ B &= \frac{1}{\pi}\left((\frac{1+r}{\pi}) \ln\left(\frac{1+r-\pi(1+\underline{g})}{1+r-\pi(1+\overline{g})}\right) + \overline{g} - \underline{g} \right) \end{aligned}$$

In the special case where $\pi = 1$ (fixed mean growth rate) the formula for stock prices simplifies to $P_t = \frac{(1+g_t)D_t}{r-g_t}$, while when $\pi = 0$ we have $P_t = \frac{(1+(\overline{g}+\underline{g})/2)D_t}{r-(\overline{g}+\underline{g})/2}$.

Notice the tradeoff involved in the choice of π, the parameter determining the breakpoint frequency. If π is low, breaks occur frequently but their effect tends to be smaller since they are expected to influence dividends over a shorter future horizon. If π is close to one, breaks will be rare but they also have a much larger effect when they do occur.

4. Stock Prices Under Incomplete Information and Recursive Learning

The solution to the stock price in Section 3 was derived under the assumption that, at each point in time, investors know the true mean of the dividend growth rate (g_t). This assumption becomes less plausible in the presence of breaks in the dividend process. After a break investors no longer have access to a large sample of historical data points that can provide them with a precise estimate of the new parameters of the dividend process. If investors do not know the true parameter values it is plausible to assume that they attempt to learn the current mean of the dividend growth rate through efficient use of information after the break. To make investors' estimation problem tractable we assume that only the drift of the dividend process (μ_t) is unknown and subject to breaks, i.e. $\sigma_t^2 = \sigma^2$ is a constant known by investors.[24]

We consider the learning process of a single, representative investor. As argued by Arthur, LeBaron & Palmer (1997) this makes the results stronger since they do not depend on arbitrary assumptions about differences in beliefs across agents. These authors go on to call for simpler asset pricing models with learning that only depend on a minimum of parameters whose values can be easily interpreted and our model is designed to do just this: Investors only have to learn about a single parameter (the dividend growth rate) whose value is drawn from a known (uniform) distribution.

Assuming that investors have full information about μ_t or that they only have information about this parameter through past dividend observations are equally implausible. To account for any additional information investors may have that provides them with a more precise estimate of μ_t, we simply assume that they observe the dividend drift with less noise than if they only observed the recorded dividends. Suppose that the noise in the temporary component of the dividend growth rate can be decomposed as follows:

$$\Delta \ln(D_t) \;=\; \mu_t + \sigma(\psi \varepsilon_{1,t} + \sqrt{1-\psi^2}\varepsilon_{2,t})$$

[24]There are two reasons for making this assumption. First, as argued by Merton (1980) and Brennan (1997), the mean parameter of the fundamentals process is typically imprecisely estimated in small samples, while the volatility can be more precisely estimated by, e.g., sampling data more frequently. Secondly, this assumption allows us to derive an explicit solution to the estimation problem under recursive learning.

$$\equiv \quad \xi_t + \sigma\sqrt{1-\psi^2}\varepsilon_{2,t} \tag{17}$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are two standard normally distributed variables that are mutually independent. Investors are assumed to observe ξ_t so that $\psi \in [0;1]$ is a constant that reflects the precision of investors' information about the dividend process. Values of ψ strictly less than one increase the precision of investors' estimate of μ_t, the variance of which is $\psi^2\sigma^2/n$, where n is the number of observations on dividends since the most recent break. The full information model of section 3 is a special case of this setup obtained when $\psi = 0$.

Investors use an efficient Bayesian updating procedure and are interested in calculating the mean dividend growth rate, $\lambda(\mu_t) = \exp(\mu_t + \sigma^2/2)$ to predict future dividends. Let $\boldsymbol{\xi}_t = (\xi_t, \xi_{t-1}, ..., \xi_{t-n+1})$, where n is the number of observations from the dividend process since the most recent break. Using the assumption that $\boldsymbol{\xi}_t$ is normally distributed, the likelihood function for μ_t conditional on $\boldsymbol{\xi}_t$ is given by $L(\mu_t; \boldsymbol{\xi}_t) = (1/\sqrt{2\pi\psi^2\sigma^2/n})\exp(\frac{-(\overline{\xi}_t-\mu_t)^2}{2\psi^2\sigma^2/n})$, where $\overline{\xi}_t = (1/n)\sum_{i=0}^{n-1}\xi_{t-i}$. Let $p(\mu_t)$ be the prior distribution for μ_{t+1}. Then the expected value of the dividend growth rate follows from Bayes' rule:

$$E[\lambda(\mu_t)|\boldsymbol{\xi}_t] = \frac{\int \lambda(\mu_t)L(\mu_t;\boldsymbol{\xi}_t)p(\mu_t)d\mu_t}{\int L(\mu_t;\boldsymbol{\xi}_t)p(\mu_t)d\mu_t} \tag{18}$$

This expression accounts for estimation error which in turn, and as pointed out by Genotte (1986), will affect asset returns over any discrete-time interval.[25] Since investors know the structure of the problem, the prior in our model equals the unconditional density for the mean dividend growth rate, i.e. the indicator function $I_{[\underline{g};\overline{g}]}$ scaled by $1/(\overline{g}-\underline{g})$. The mean growth rate is bounded between $\underline{g}$ and $\overline{g}$; $1+\underline{g} \leq \exp(\mu_t + \sigma^2/2) \leq 1+\overline{g}$, so that the true value of the unknown drift, μ_t, lies between the following bounds: $\underline{l} \equiv \ln(1+\underline{g}) - \sigma^2/2 \leq \mu_t \leq \ln(1+\overline{g}) - \sigma^2/2 \equiv \overline{l}$. It follows that the expression in the numerator of (18) is given by

$$\frac{1/(\overline{g}-\underline{g})}{\sqrt{2\pi\psi^2\sigma^2/n}}\int_{\underline{l}}^{\overline{l}} \exp(\frac{-(\overline{\xi}_t-\mu_t)^2}{2\psi^2\sigma^2/n} + \mu_t + \frac{\sigma^2}{2})d\mu_t$$

[25]This Bayesian learning rule is equivalent to a rational expectations scheme in which investors use information efficiently but do not observe the true growth rate after a break: investors start with an unbiased prior and update their beliefs efficiently, c.f. Bossaerts (1997).

$$= \frac{\exp(\frac{\sigma^2}{2} + \overline{\xi}_t + \frac{\psi^2\sigma^2}{2n})}{\overline{g} - \underline{g}} \left(\Phi(\frac{\overline{l} - \overline{\xi}_t - \psi^2\sigma^2/n}{\sigma\psi/\sqrt{n}}) - \Phi(\frac{\underline{l} - \overline{\xi}_t - \psi^2\sigma^2/n}{\sigma\psi/\sqrt{n}}) \right) \quad (19)$$

while the expression in the denominator of (18) reduces to

$$\begin{aligned} & \frac{1/(\overline{g} - \underline{g})}{\sqrt{2\pi\psi^2\sigma^2/n}} \int_{\underline{l}}^{\overline{l}} \exp(\frac{-(\overline{\xi}_t - \mu_t)^2}{2\psi^2\sigma^2/n}) d\mu_t \\ = & \frac{1}{\overline{g} - \underline{g}} \left(\Phi(\frac{\overline{l} - \overline{\xi}_t}{\sigma\psi/\sqrt{n}}) - \Phi(\frac{\underline{l} - \overline{\xi}_t}{\sigma\psi/\sqrt{n}}) \right) \end{aligned} \quad (20)$$

and the expected dividend growth rate is given by the ratio of (19) over (20). The simplicity of the estimation problem confronting investors guarantees an explicit solution to (18) and is important for the later simulations which are based on a large number of computations of the expected growth rate. Stock prices under Bayesian learning are formed as the present value of the predicted dividends along the lines leading to the Proposition in Section 3.

Our model is closely related to the analysis of Lewis (1989) which considers the market's forecast error process arising from a once-and-for-all break in the drift parameter of the first-differenced fundamentals process. Investors learn gradually about the shift through a Bayesian updating rule and, as in our model, also know the time where the fundamentals process may have changed. Lewis analyses separate scenarios depending on whether the new drift parameter is known or unknown to investors. Compared to the case where the market knows the drift after the switch, Lewis finds that learning evolves much more slowly when investors have to estimate this parameter. This observation will be important to our simulation results.

It is also instructive to compare our model to the dividend growth rate process and the learning problem analyzed by Barsky & DeLong (1993). These authors present evidence that the long-run movements in the price-dividend ratio of US stocks can be explained by investors' projections of future dividends modeled as a long moving average of their own past with geometrically declining weights. To project future dividends, investors in our model in stead put the same weight on dividend observations following the most recent break and zero weight on observations prior to a break.

5. Simulations of the Model

Structural breaks introduce non-linearities in dividends and stock prices and recursive learning effects introduce non-stationarities in returns. This rules out standard econometric tests of our model, c.f. Bossaerts (1995).[26] For this reason we evaluate the model by simulating dividends according to equations (12) - (14) and by forming stock prices according to the formulae in sections 3 and 4. The purpose of this analysis is not to calibrate the moments of stock returns but rather to study some of the qualitative features associated with breaks and different models for investors' expectation formation. To focus on one of the most commonly studied time horizons, we simulate dividends and stock prices at the monthly frequency. The following set of dividend parameter values are assumed in the experiments

$$r = 0.075, \overline{g} = 0.06, \underline{g} = -0.02, \sigma = 0.11, \pi = 0.995, \psi = 0.10. \qquad (21)$$

The first four parameter values are annualized so that the annual real discount rate is 7.5 percent, and the minimum and maximum values of the dividend growth rate are -2 and 6 percent, respectively, yielding an average growth rate of 2 percent per annum and a volatility of 11 percent. In real terms this matches the endowment data over the period 1890 - 1994.[27] The choice of interval for the dividend growth rate is based on our assumption that the dividend growth rate is drawn from a uniform distribution with support $[\underline{g}; \overline{g}]$, such that $r < \overline{g}$. The value of π means that the drift of the dividend process on average changes about once every twenty years. In equation (17) ψ is set equal to 0.10 and hence we assume that investors have additional information that allows them to reduce the standard error of their drift estimate by 90 percent. This makes the informational assumptions - that investors have quite precise, though not perfect, information about the drift of the dividend process - more plausible. Results from 5000 simulations for a sample size

[26] Bossaerts provides a comprehensive analysis of the effects of learning on econometric tests of market efficiency when returns either follow an exogenous process or investors believe they are drawn from a stationary distribution. In our model learning effects die out gradually, only to re-emerge after a subsequent break.

[27] There is some evidence of left skew and excess kurtosis in the endowment data. However, the strength of this evidence varies greatly across the three endowment series analyzed in Section 2 and appears mainly to be the result of their changing volatility. For simplicity our simulations assume no breaks in the volatility and we do not attempt to calibrate the higher order moments of the endowment process.

of 800 observations are reported. The historical returns used as a benchmark for the simulations comprise 72 years of monthly data or 864 observations.

Figure 6 plots excess returns generated by a particular simulation. In the upper row the first window presents excess returns under full information while the second window shows excess returns under Bayesian learning. Since dividends are identical for the two returns series the difference between the plots is entirely due to differences in investors' growth estimates, plots of which are provided in the second row of Figure 6. For this particular simulation there were two changes in the dividend growth rate which ranged from minus one to a little over two percent. At the beginning of the sample the growth estimates under Bayesian learning are well above the true growth rate but they quickly decline towards the actual growth rate of just below zero. Following the changes in the actual growth rate around observations 150 and 200 the estimated growth rates become very volatile. Volatility clustering in returns is visibly present in the simulations under learning around these periods. In contrast, when an outlier is observed in the excess return series under full information there is no tendency for this to carry over as higher volatility in subsequent periods.[28]

Table 4 reports moments for the monthly excess returns and the simulated data. Using data on the value-weighted NYSE portfolio over the period 1926(1) - 1997(12) we obtain the results reported in the first column labeled 'Data'. As documented in many previous studies, monthly stock returns are characterized by high volatility, skewness, fat tails, a small degree of serial correlation and strong volatility clustering, c.f. the significant ARCH effects.

Consider next the simulated data under full information and no breaks ($\pi = 1$). This model is unable to match the high volatility, skewness and fat tails observed in the data. This simply reflects the common finding in the asset pricing literature that - in the context of a stationary dividend growth model - dividend variations alone do not seem to fully explain movements in observed stock prices.

Introducing breaks, but maintaining the full information assumption, the volatility of stock returns increases from 3.2 to 3.9 percent and the skewness and kurtosis

[28]Notice that investors' growth estimates may stay on one side of the true mean growth rate for sustained periods of time, as evidenced by the growth estimates below the true mean growth rate between observations 250 and 500. This in turn gives rise to mean reversion in returns measured over long horizons.

also go up dramatically, even exceeding the estimates for US returns. This happens because of the outliers in stock returns observed after a break in the dividend growth rate, c.f. Figure 6. Since isolated outliers is the opposite of volatility clustering, this model cannot replicate the ARCH effects observed in the data. However, the comparison of the full information model with and without breaks clearly demonstrates the importance to the distribution of stock prices of allowing for breaks in fundamentals.

Next consider stock returns under Bayesian learning. Under this scenario the model generates average volatility of 4.4 percent, close to the sample estimate of around 5.5 percent. Compared with the full information case, Bayesian learning decreases the skewness and kurtosis to a level closely in line with the data. It is easy to understand why: Under full information a jump in the dividend growth rate is instantly recognized by investors and shows up as a major revision in the stock price. In contrast, under Bayesian learning, new dividend information after a break will only gradually be incorporated into the price and is weighted against investors' prior beliefs. This gives a more gradual price adjustment and hence decreases the skewness and kurtosis of returns. Despite this gradual adjustment, the Bayesian learning model does not seem to generate much full-sample serial correlation in the *level* of returns.[29]

Importantly, the Bayesian learning model also seems capable of generating substantial volatility clustering. Between 60 and 70 percent of the simulations generate significant ARCH effects. To measure the persistence in the conditional volatility of excess returns we sum the coefficients of the squared residuals in an ARCH(12) regression of squared residuals on a constant and twelve lags. Under full information and breaks the median value of the persistence estimate is -0.03, while under Bayesian learning this figure increases to 0.31. This compares with an estimated persistence of the conditional volatility in the value weighted returns data of 0.76 for the period 1926 - 1997, and 0.47 for the period 1933-1997.

Although we consider the learning process of a single, representative investor, there are some clear parallels between our simulation results and those of Arthur, Holland, LeBaron, Palmer & Tayler (1997) (AHLPT). In the context of simulations

[29]There is some evidence of first order serial correlation in the actual returns data. However, this is likely to reflect non-synchronous trading effects as opposed to genuine predictable patterns in returns. For this reason we do not attempt to replicate this feature of the data.

from an artificial financial market inhabited by heterogenous traders, each of which chooses from a collection of linear forecasting rules but switches between rules over time, these authors demonstrate how two very different regimes can arise. A regime where investors only revise their beliefs very slowly closely resembles a rational expectations state. In strong contrast, in the 'complex' regime where investors revise their beliefs at a faster rate, endogenous expectation effects become important and volatility clustering may occur.

Similar effects arise through an entirely different mechanism in our model. After a structural break occurs, investors cannot use historical information and hence will need to revise their beliefs more frequently. At this point in time, the "learning clock" runs fast as new information leads to large changes in dividend growth rate estimates. This resembles the 'complex' regime. However, after further information has cumulated, the precision of investors' growth estimate has increased so the marginal benefits from updating declines and the equilibrium resembles the rational expectations state in the analysis of AHLPT.[30]

6. New Empirical Predictions

Two conclusions can be drawn from the simulations in the previous section. First, independent of how much information investors' hold, breaks in fundamentals affect the distribution of stock returns in important ways and may provide the key to understanding the kurtosis and fat tails in the observed data. Furthermore, imperfect information and gradual updating of investors' beliefs after a break seem important components to an explanation of the clustering of volatility observed in US stock returns.

Ultimately new insights from our modeling approach can only be gained if it results in new, testable predictions. New predictions from the asset pricing model with breaks originate from the observation that, under a recursive learning scheme, investors' growth estimates are more volatile immediately after a break and become less volatile as more information arrives. Since the stock price is a convex function of the growth estimate, the recursive learning model predicts volatility clustering

[30] Notice also that while in AHLPT's model the equilibrium is determined by the frequency at which agents change their forecasting model, in our setting the type of equilibrium varies over time as a result of exogenous shocks to the dividend process and is determined by how far away from a break point the market is.

and possibly also serial correlation in stock returns after a break in the dividend process has occurred. Serial correlation in returns in the aftermath of breaks in the dividend process may arise because of the possibility of large revisions in investors' growth estimates immediately after a break.[31] While this serial correlation can be detected ex-post by a researcher with access to the complete sample of stock returns, conditional on investors' parameter estimates at a given point in time stock returns are not predictable in-sample.

These propositions are illustrated in the upper row of Figure 7 which, for the simulation used to construct Figure 6, plots the time series of a twelfth-order ARCH test. To track evicence of local volatility clustering in returns we use rolling regressions with a window length of 120 months, or ten years of data. There is no evidence of ARCH effects in the simulated returns from the full information model shown in the first window of Figure 7. A very different picture emerges from the second window in Figure 7 which plots the estimated conditional volatility under Bayesian learning against the five percent critical value of the test statistic. After the occurence of the breaks around observations 150 and 200 there are strong indications of volatility clustering.

To track local serial correlation in returns we calculate Ljung-Box statistics for twelfth order serial correlation, again using rolling regressions with a window length of 120 months. In this particular simulation local serial correlation is not detectable. Most likely this can be attributed to the use of a high order test for serial correlation and the resulting loss in statistical power.[32]

The novelty of the first prediction, concerning the timing of the volatility clustering in stock returns, is that it provides an *ex ante* identification of the point in time, namely after a break in the dividend process, where ARCH effects can be expected to occur in stock returns. The second prediction, serially correlated returns after a break, is also novel. This implication of the model is not driven by the usual risk premium story but is a consequence of large parameter revisions following a break.

To test these predictions on US data, Figure 8 plots monthly excess returns for the value-weighted portfolio well as the twelfth-order LM and Ljung-Box statistics

[31] These are intuitive arguments. Timmermann (1997) proves formally that parameter revisions can generate serial correlation and volatility clustering in stock returns.

[32] A fourth-order Ljung-Box test identifies no serial correlation under full information, but shows strong serial correlation between observations 150 and 300 under Bayesian learning.

for ARCH and serial correlation based on a 120 month rolling window adopted to the returns data. Again we use a high order (12) of the diagnostic test since in practice investors' knowledge of breaks are likely to be less precise than what was assumed in the theoretical model in Sections 3 and 4. The conditional volatility of excess returns is very high around 1932 and 1942 and again in 1975.[33] There is evidence of serial correlation in excess returns over the period 1933-38 and again in the early seventies. The coincidence of the periods for which the analysis in Section 2 identified breaks in the endowment process and periods with either high serial correlation or volatility clustering in stock returns is quite remarkable.[34]

We finally pursue a testing strategy which inspects the data in "event time" and tests for systematic cross-sectional effects. In our case the event is a break in the fundamentals process so we line up the data in 'break point time' or, equivalently, in 'learning time' since a structural break also restarts the learning clock. A significant implication of our asset pricing model is the high but declining volatility in returns after a break point. To test if this implication is confirmed in the data we estimate volatility in 'learning time'. Let $\widehat{\sigma}_t$ be an estimate of the volatility of stock price changes in month t based on daily observations within the month. Also let $\tau_i = t - T_i$, $(T_i < t < T_{i+1})$ be the time since the most recent break date, T_i. τ is then a learning clock and $\tau = 1$ one period after a break, $\tau = 2$ two periods after a break and so on. A simple estimator of the squared volatility of asset prices after τ learning periods is given by

$$\widehat{\sigma}^2_\tau = \frac{1}{m}\sum_{i=1}^{m}\widehat{\sigma}^2_{\tau_i}, \tag{22}$$

where $m = 3$ is the number of breaks and the average is computed across the three break points 1932:1, 1943:1, 1974:9 identified in our sample.[35]

Figure 9 provides a plot of the volatility estimate along with a smooted cubic

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[33] We also computed the conditional volatility from a GARCH(1,1) model fitted to excess returns and found high volatility during the early thirties and the period after 1974.

[34] We would not necessarily expect these points to coincide exactly since investors could either have anticipated a breakpoint (if they have superior information) or failed to immediately identify a break in real time since historically they did not have access to the full sample information.

[35] Although the confidence intervals for the timing of the break points identified in Section 2 were reasonably narrow, some uncertainty remains about these dates. Fortunately the pattern in the relationship between return volatility and learning time is quite robust with respect to the exact timing of the break dates.

spline. The standard error of investors' estimate of the mean dividend growth rate after a break declines at rate $1/\tau$. If learning effects are important to stock prices after a break we would expect to find a similarly declining pattern in the volatility of returns. Although clearly our simple volatility estimate displays considerably variation from month to month, the evidence suggests that, consistent with theory, return volatility declines systematically as the learning clock increases.

7. Conclusion

Present value models have not fared well in empirical tests of US stock prices and are typically rejected in formal tests, c.f. Campbell & Shiller (1987). Our paper offers two reasons for these findings. First, present value models are usually tested jointly with specific assumptions about the process generating dividends and discount factors. For example, dividend growth and discount rates are often modeled as stationary, low-order auto regressions without breaks. Specifications which do not require that the same fundamentals process stays in effect over several decades may do better empirically. Moreover, once less structure is imposed on dividends, forecasting future dividends becomes more difficult for investors and our simulations indicate that this enhances the role in explaining stock returns played by variations in investors' beliefs. Both effects pull towards explaining Campbell and Shiller's finding that the price-dividend ratio implied by the present value model without breaks is less volatile than the observed price-dividend ratio.

Since structural breaks appear to be present in the fundamentals process underlying US stock prices and since such breaks can significantly alter the dynamics of prices, our results bring into question the practice of testing asset pricing models based on full-sample information. Instead it may be necessary to separately consider asset prices around break points and during times further away from such events. Indeed, when we adopted procedures from the event study literature and tested properties of asset prices lined up in 'learning time', we found interesting patterns in stock price volatility related to the distance from the most recent break.

Our paper also has implications for the finance literature which analyses the effect of learning and parameter estimation risk on optimal asset allocation. For example, Klein & Bawa (1977) extend the standard mean-variance optimization problem to cover the case where asset returns are joint normally distributed but have unknown parameters which investors must estimate. More recently Kandel &

Stambaugh (1996) derive the effects of learning on asset allocation when returns on the risky assets are partially predictable. Likewise, in the context of a simple continuous time model where the drift parameter of the underlying asset price process is unknown to investors, Brennan (1997) shows that learning effects can lead investors to act in a more risk averse manner than in the case without learning. Those studies conclude that learning effects can significantly affect investors' choice of optimal portfolio weights even when the process generating returns on the risky asset is exogenous. Our simulation results suggest that it is also important to consider the equilibrium effects of learning as investors' learning after a break can alter the risk and return characteristics of the underlying assets very considerably.

8. Appendix

Proof of the Proposition

Suppose that the solution for P_t takes the form $P_t = \gamma(g_t)D_t$ for some univariate function $\gamma(.)$. Taking expectations conditional on information at time t it follows from (12) - (15) that

$$\begin{aligned}
(1+r)\gamma(g_t)D_t &= \sum_{i=0}^{1} E_t[P_{t+1}+D_{t+1}|s_{t+1}=i]\Pr(s_{t+1}=i) = \\
&\pi D_t \int_{-\infty}^{\infty} (1+\gamma(g_t))(1+g_t)\exp(\varepsilon_{t+1}-\frac{\sigma_t^2}{2})\phi(\varepsilon_{t+1}|\sigma_t^2)d\varepsilon_{t+1} + \\
&+(1-\pi)D_t \int_{\underline{g}}^{\overline{g}}\int_{-\infty}^{\infty} (1+\gamma(g_{t+1}))(1+g_{t+1}) \\
&\exp(\varepsilon_{t+1}-\frac{\sigma_{t+1}^2}{2})\phi(\varepsilon_{t+1}|\sigma_{t+1}^2)d\varepsilon_{t+1}dU(g_{t+1}) \\
&= \pi D_t(1+g_t)(1+\gamma(g_t)) + (1-\pi)D_t \int_{\underline{g}}^{\overline{g}} (1+g_{t+1})dU(g_{t+1}) \\
&+(1-\pi)D_t \int_{\underline{g}}^{\overline{g}} (1+g_{t+1})\gamma(g_{t+1})dU(g_{t+1}), \qquad \text{(A1)}
\end{aligned}$$

where $U(g_{t+1})$ is the uniform distribution with support $[\underline{g};\overline{g}]$, $\phi(.|\sigma_t^2)$ is the normal density function with mean zero and variance σ_t^2, and the last equality follows by using the independence of ε_{t+1} and g_{t+1} and integrating out ε_{t+1}. We remind the reader that $g_t = \exp(\mu_t + \sigma_t^2/2)$. The first term after the last equality sign in (A1) accounts for the expected value of $(D_{t+1} + P_{t+1})$ conditional on no break in the parameters of the dividend process while the final two terms are the conditional expectations of D_{t+1} and P_{t+1} conditional on a break in the dividend process ($s_{t+1} = 1$). Dividing through by (D_t) in equation (A1) and simplifying we get

$$\begin{aligned}(1 + r - \pi(1 + g_t))\,\gamma(g_t) &= \pi(1 + g_t) + (1 - \pi)(1 + E_t[g_{t+1}|s_{t+1} = 1]) \\ &\quad + (1 - \pi)\int_{\underline{g}}^{\overline{g}} (1 + g_{t+1})\gamma(g_{t+1})dU(g_{t+1}). \qquad \text{(A2)}\end{aligned}$$

Next multiply by $(1+g_t)dU(g_t)/\,(1 + r - \pi(1 + g_t))$ and integrate over the interval $[\underline{g}; \overline{g}]$

$$\begin{aligned}&\int_{\underline{g}}^{\overline{g}} (1 + g_t)\gamma(g_t)dU(g_t) \\ &= \int_{\underline{g}}^{\overline{g}} \frac{\pi(1 + g_t) + (1 - \pi)(1 + E_t[g_{t+1}|s_{t+1} = 1])}{1 + r - \pi(1 + g_t)}(1 + g_t)dU(g_t) + \\ &\quad (1 - \pi)\int_{\underline{g}}^{\overline{g}} \frac{1 + g_t}{1 + r - \pi(1 + g_t)}dU(g_t)\int_{\underline{g}}^{\overline{g}} (1 + g_{t+1})\gamma(g_{t+1})dU(g_{t+1}). \qquad \text{(A3)}\end{aligned}$$

Under the assumption that the underlying density $U(.)$ does not vary through time, we must have that

$$\int_{\underline{g}}^{\overline{g}} (1 + g_t)\gamma(g_t)dU(g_t) = \int_{\underline{g}}^{\overline{g}} (1 + g_{t+1})\gamma(g_{t+1})dU(g_{t+1}). \qquad \text{(A4)}$$

This gives an equation which can be used to assess the integral in (A3):

$$\int_{\underline{g}}^{\overline{g}} (1 + g_t)\gamma(g_t)dU(g_t) = \frac{\int_{\underline{g}}^{\overline{g}} \left(\frac{\pi(1+g_t)^2 + (1-\pi)(1+E_t[g_{t+1}|s_{t+1}=1])(1+g_t)}{1+r-\pi(1+g_t)}\right) dU(g_t)}{1 - (1 - \pi)\int_{\underline{g}}^{\overline{g}} \frac{1+g_t}{1+r-\pi(1+g_t)}dU(g_t)} \qquad \text{(A5)}$$

Using that $\int\limits_{\underline{g}}^{z} dU(g_t) = \frac{z}{\overline{g}-\underline{g}}$, (A3) simplifies to

$$\int\limits_{\underline{g}}^{\overline{g}} (1+g_t)\gamma(g_t)dU(g_t) = \frac{\int\limits_{\underline{g}}^{\overline{g}} \left(\frac{\pi(1+g_t)^2+(1-\pi)(1+(\overline{g}+\underline{g})/2)(1+g_t)}{1+r-\pi(1+g_t)}\right) dg_t}{\overline{g}-\underline{g}-(1-\pi)\int\limits_{\underline{g}}^{\overline{g}} \frac{1+g_t}{1+r-\pi(1+g_t)}dg_t}. \tag{A6}$$

We need to evaluate $\int\limits_{\underline{g}}^{\overline{g}} \frac{(1+g_t)^2}{1+r-\pi(1+g_t)}dg_t$ and $\int\limits_{\underline{g}}^{\overline{g}} \frac{1+g_t}{1+r-\pi(1+g_t)}dg_t$. To do so change variables by defining $y_t = (1+g_t)$, so $dy_t = dg_t$, and notice that

$$\begin{aligned} \int \frac{x}{a+bx}dx &= \frac{x}{b} - \frac{a}{b^2}\ln(a+bx), \\ \int \frac{x^2}{a+bx}dx &= \frac{x^2}{2b} - \frac{ax}{b^2} + \frac{a^2}{b^3}\ln(a+bx), \end{aligned}$$

c.f. Gradshteyn & Ryzhik (1994). Letting $a=(1+r)$, $b=-\pi$, we get

$$\begin{aligned} A &\equiv \int\limits_{\underline{g}}^{\overline{g}} \frac{(1+g_t)^2}{1+r-\pi(1+g_t)}dg_t = \frac{-y_t^2}{2\pi} - \frac{(1+r)y_t}{\pi^2} - \frac{(1+r)^2\ln(1+r-\pi y_t)}{\pi^3}\Bigg|_{1+\underline{g}}^{1+\overline{g}} \\ B &\equiv \int\limits_{\underline{g}}^{\overline{g}} \frac{1+g_t}{1+r-\pi(1+g_t)}dg_t = \frac{-y_t}{\pi} - \frac{(1+r)\ln(1+r-\pi y_t)}{\pi^2}\Bigg|_{1+\underline{g}}^{1+\overline{g}}. \end{aligned} \tag{A7}$$

After some algebra we see that

$$\begin{aligned} A &= \frac{-(\overline{g}^2-\underline{g}^2+2(\overline{g}-\underline{g}))}{2\pi} - \frac{(1+r)(\overline{g}-\underline{g})}{\pi^2} + \frac{(1+r)^2}{\pi^3}\ln\left(\frac{1+r-\pi(1+\underline{g})}{1+r-\pi(1+\overline{g})}\right) \\ B &= \frac{1}{\pi}\left(\frac{1+r}{\pi}\ln\left(\frac{1+r-\pi(1+\underline{g})}{1+r-\pi(1+\overline{g})}\right)+\underline{g}-\overline{g}\right) \end{aligned} \tag{A8}$$

and hence from (A3)

$$\int\limits_{\underline{g}}^{\overline{g}} (1+g_t)\gamma(g_t)dU(g_t) = \frac{\pi A + (1-\pi)(1+\frac{\overline{g}+\underline{g}}{2})B}{(\overline{g}-\underline{g})-(1-\pi)B}, \tag{A9}$$

such that the price-dividend ratio $\gamma(g_t)$ is given by

$$\gamma(g_t) = \frac{\pi(1+g_t) + (1-\pi)(1+\frac{\overline{g}+\underline{g}}{2})}{1+r-\pi(1+g_t)} + \frac{(1-\pi)(\pi A + (1-\pi)(1+\frac{\overline{g}+\underline{g}}{2})B)}{(\overline{g}-\underline{g}-(1-\pi)B)(1+r-\pi(1+g_t))} \tag{A10}$$

which is the expression we were seeking.

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Table 1

Estimated Number of Break Points

Process	Break Point Criterion		
	D_{max}	AIC	BIC
$\Delta \ln(D_t)$	2	2	1
$\Delta \ln(GDP_t)$	1*	2	1
$\Delta \ln(C_t)$	1***	1	1
$\lvert\Delta \ln(D_t)\rvert$	2***	2	2
$\lvert\Delta \ln(GDP_t)\rvert$	2***	2	2
$\lvert\Delta \ln(C_t)\rvert$	1*	2	1
$Yield_t$	2***	3	2
Value-weighted return	4***	2	1
Equal-weighted return	1***	3	1

Note: this table reports the number of breakpoints detected by three different test procedures discussed in Bai and Perron (1996). The results are based on univariate auto-regressive specifications with an intercept term and a single lag of the dependent variable as regressors. D_t, GDP_t and C_t are the annualized, real per-capita dividends, GDP and consumption in the US measured over the period 1890-1994. $Yield_t$ is the monthly dividend yield measured over the period 1926-1954. The excess return regressions use an intercept, the 1-month T-bill rate and the default premium as regressors over the period 1954-1997. D_{max} is the double maximum test statistic which lets the number of breaks be unknown and tests the null of no breaks against the alternative of one of more breaks. AIC and BIC are the Akaike and Bayesian penalized likelihood model selection criteria which account for the automatic improvement in fit resulting from adding increased numbers of parameters in the model

* indicates significance at the 10 percent critical level while *** indicates significance at the 1 percent critical level. These critical levels apply to the reported number of breaks chosen by the D_{max} criterion.

Table 2

Parameter Estimates from Breakpoint Regressions

I. Growth in Endowment

$\Delta \ln(D_t) =$	0.002	$+0.280\Delta \ln(Div_{t-1})$	from 1890 to 1937
	(0.016)	(0.125)	[1890-1994]
	-0.006	$-0.051\Delta \ln(Div_{t-1})$	after 1937
	(0.015)	(0.156)	
$\Delta \ln(GDP_t) =$	0.022	$-0.255\Delta \ln(GDP_{t-1})$	from 1890 to 1930
	(0.008)	(0.183)	[1917-1943]
	0.012	$+0.450\Delta \ln(GDP_{t-1})$	after 1930
	(0.006)	(0.105)	
$\Delta \ln(C_t) =$	0.027	$-0.444\Delta \ln(C_{t-1})$	from 1890 to 1930
	(0.006)	(0.130)	[1919-1941]
	0.013	$+0.341\Delta \ln(C_{t-1})$	after 1930
	(0.005)	(0.132)	

II. Volatility Proxy

$\lvert\Delta \ln(D_t)\rvert =$	0.096	$-0.215\lvert\Delta \ln(Div_{t-1})\rvert$	from 1890 to 1931
	(0.017)	(0.158)	[1929-1933]
	0.246	$-0.394\lvert\Delta \ln(Div_{t-1})\rvert$	1932-1943
	(0.032)	(0.133)	[1942-1944]
	0.016	$+0.561\lvert\Delta \ln(Div_{t-1})\rvert$	after 1943
	(0.014)	(0.234)	
$\lvert\Delta \ln(GDP_t)\rvert =$	0.043	$-0.226\lvert\Delta \ln(GDP_{t-1})\rvert$	from 1890 to 1929
	(0.007)	(0.165)	[1928-1930]
	0.106	$-0.212\lvert\Delta \ln(GDP_{t-1})\rvert$	1930-1943
	(0.016)	(0.164)	[1942-1944]
	0.028	$-0.007\lvert\Delta \ln(GDP_{t-1})\rvert$	after 1943
	(0.006)	(0.166)	
$\lvert\Delta \ln(C_t)\rvert =$	0.069	$-0.636\lvert\Delta \ln(C_{t-1})\rvert$	from 1890 to 1908
	(0.009)	(0.166)	[1900-1916]
	0.036	$+0.156\lvert\Delta \ln(C_{t-1})\rvert$	1909-1946
	(0.007)	(0.145)	[1935-1957]
	0.024	$-0.049\lvert\Delta \ln(C_{t-1})\rvert$	after 1946
	(0.005)	(0.180)	

Note: these results are based on the breakpoint estimation procedure provided by Bai and Perron (1996). D_t, GDP_t and C_t are the real, annualized per-capita dividend, GDP and consumption series for the US over the period 1890-1994. Standard errors appear in brackets under the parameter estimates. Square brackets provide 90 percent confidence intervals for the end point of the preceding interval.

Table 3

Estimation Results for Markov Switching and Garch(1,1) Model Specifications

I. Markov Switching Model

$$\Delta \ell n(Y_t) = \mu_{s_t} + \sigma_{s_t}\epsilon_t,\ S_t = 1,2.$$
$$\Pr(S_t = i|S_{t-1} = i) = P_{ii},\ i = 1,2.$$

Process	μ_1	μ_2	σ_1	σ_2	P_{11}	P_{22}	L.L.
Dividends	0.0016	-0.0068	0.0013	0.0257	0.918	0.889	110.7
	(0.0055)	(0.0236)	(0.0003)	(0.0063)	(0.059)	(0.089)	
GDP	0.0187	0.0177	0.0010	0.0063	0.986	0.937	187.7
	(0.0035)	(0.0169)	(0.0002)	(0.0025)	(0.015)	(0.058)	
Consumption	0.0193	0.0171	0.0004	0.0022	0.983	0.989	205.7
	(0.0031)	(0.0061)	(0.0001)	(0.0004)	(0.022)	(0.014)	

II. GARCH(1,1) Model

$$\Delta \ell n(Y_t) = \mu + \gamma \Delta \ell n(Y_{t-1}) + \epsilon_t, \quad \epsilon_t \sim N(0, h_t)$$
$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}.$$

Process	μ	γ	α_0	α_1	β_1	L.L.
Dividends	-0.0019	0.396	1.8E-5	0.301	0.761	102.6
	(0.0055)	(0.147)	(8.8E-5)	(0.368)	(0.104)	
GDP	0.0120	0.180	1.3E-5	0.178	0.824	186.6
	(0.0040)	(0.111)	(7.3E-5)	(0.153)	(0.076)	
Consumption	0.0193	0.0110	2.9E-5	0.116	0.854	203.4
	(0.0031)	(0.0851)	(2.7E-5)	(0.122)	(0.091)	

Note: The models are fitted to real annual, per-capita endowment growth over the period 1890-1994. L.L. provides the value of the log-likelihood function. Numbers in brackets under the parameter estimates report the estimated standard errors.

Table 4

Statistical Properties of Monthly Stock Returns

(US Data (1926-1997) and Simulated Data)

	DATA	SIMULATIONS		
		Sample Size = 800		
Moments of Excess Returns	Val. wht Ptf.	Full Info. No Breaks	Full Info. Breaks	Bayesian Learning
Standard deviation	0.055	0.032	0.039	0.044
Skewness	0.29	0.10	0.78	0.34
Kurtosis	11.23	2.99	24.81	9.59
Serial Correlation	9.38	0.42	0.42	0.92
R^2 in Yield Regression	0.010	0.000	0.000	0.002
ARCH(1)	73.65	0.47	0.05	5.91
ARCH(4)	108.50	3.43	0.33	20.53
ARCH(12)	220.07	11.38	1.39	31.70
Percentage of Simulations with Significant Value of the Diagnostic Test				
Serial Correlation		3.9	4.4	16.5
ARCH(1)		5.7	1.5	58.0
ARCH(4)		5.6	2.5	70.9
ARCH(12)		5.5	4.3	64.1

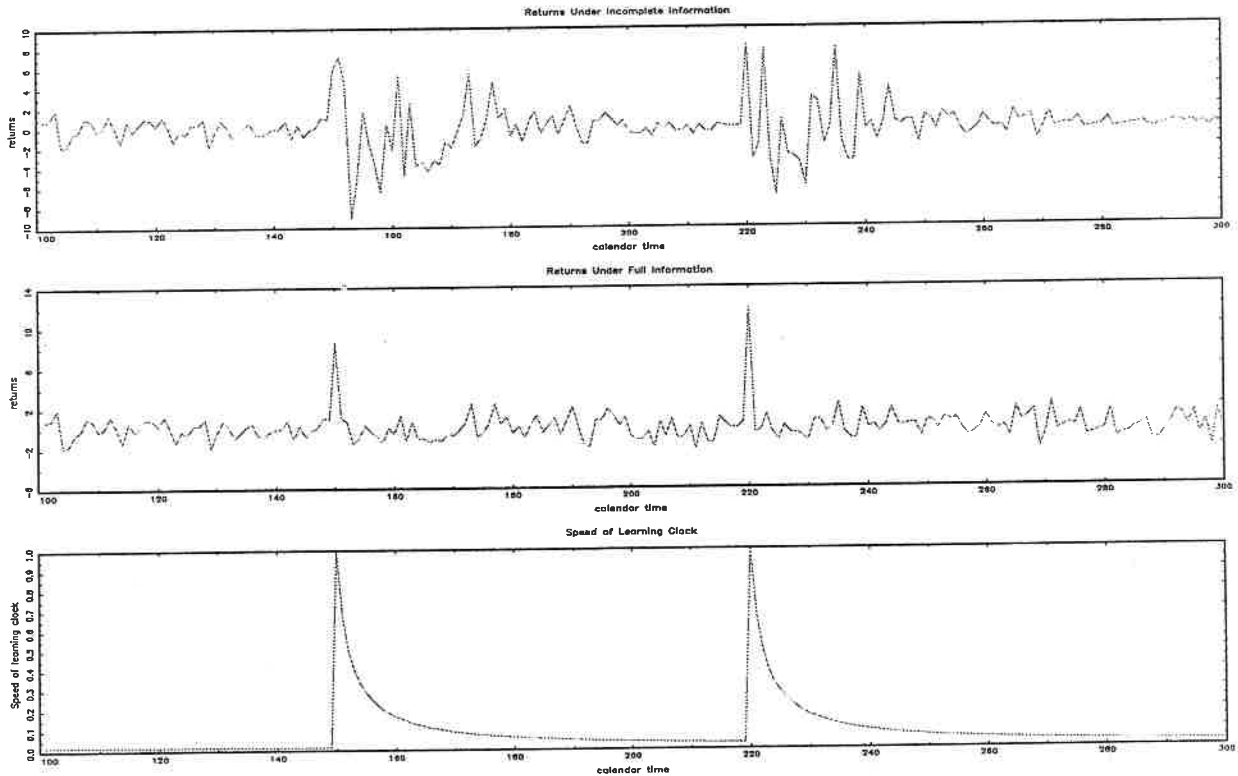
Note: In the section headed "DATA", the first three rows (standard deviation, skewness and kurtosis of excess returns) give the estimates of the first three (centered) moments of the data. Serial correlation is the estimate of the first order Ljung-Box test statistic. R^2 in yield regression is the estimated R^2 from a regression of excess returns on a constant and the lagged dividend yield. The ARCH statistics give the values of the LM test for ARCH suggested by Engle (1982). These are chi-squared distributed with degrees of freedom equal to the order of the test.

In the section labelled "SIMULATIONS" the first three rows present the median estimates of the second to fourth centered moments. Serial correlation gives the median value of the first order Ljung-Box test for serial correlation. R^2 in yield regression provides the median value of the estimated R^2 in the regression of excess returns on a constant and the lagged dividend yield. All simulation results are based on 5000 simulations.
The ARCH statistics provide the median values of the LM test for ARCH in the simulations.
The last four rows provide the percentage of simulations that generate values of a given diagnostic test that are significant at the 5 percent critical level.

The simulation results in the columns labelled "Full Information" assume that stock prices are formed according to Proposition 1 of the paper, with and without structural breaks, while the simulation results in the column labelled "Bayesian Learning" assume that agents project dividends according to equations (17) - (19).
The dividend process is given by equations (13) - (16) and the following parameter values are used in the simulations: $\pi = 0.995$, $\underline{g} = -0.02$, $\overline{g} = 0.06$, $r = 0.075$, $\psi = 0.10$.

Figure 1

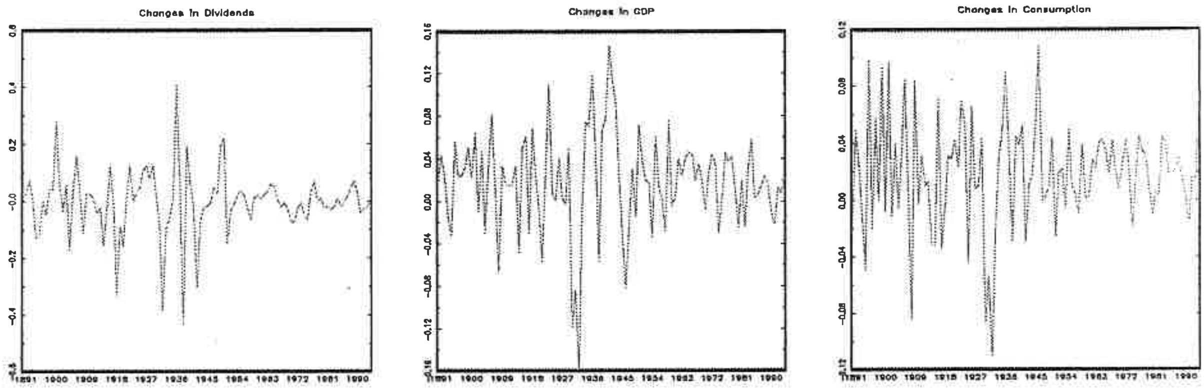
Calendar Time, Speed of the Learning Clock and Volatility of Stock Returns



Note: This Figure assumes that breaks occur in the dividend growth rate at observations 150 and 220 (in calendar time). Under full information the contemporaneous parameters of the dividend growth rate are known at all points in time, while they have to be estimated under incomplete information.

Figure 2

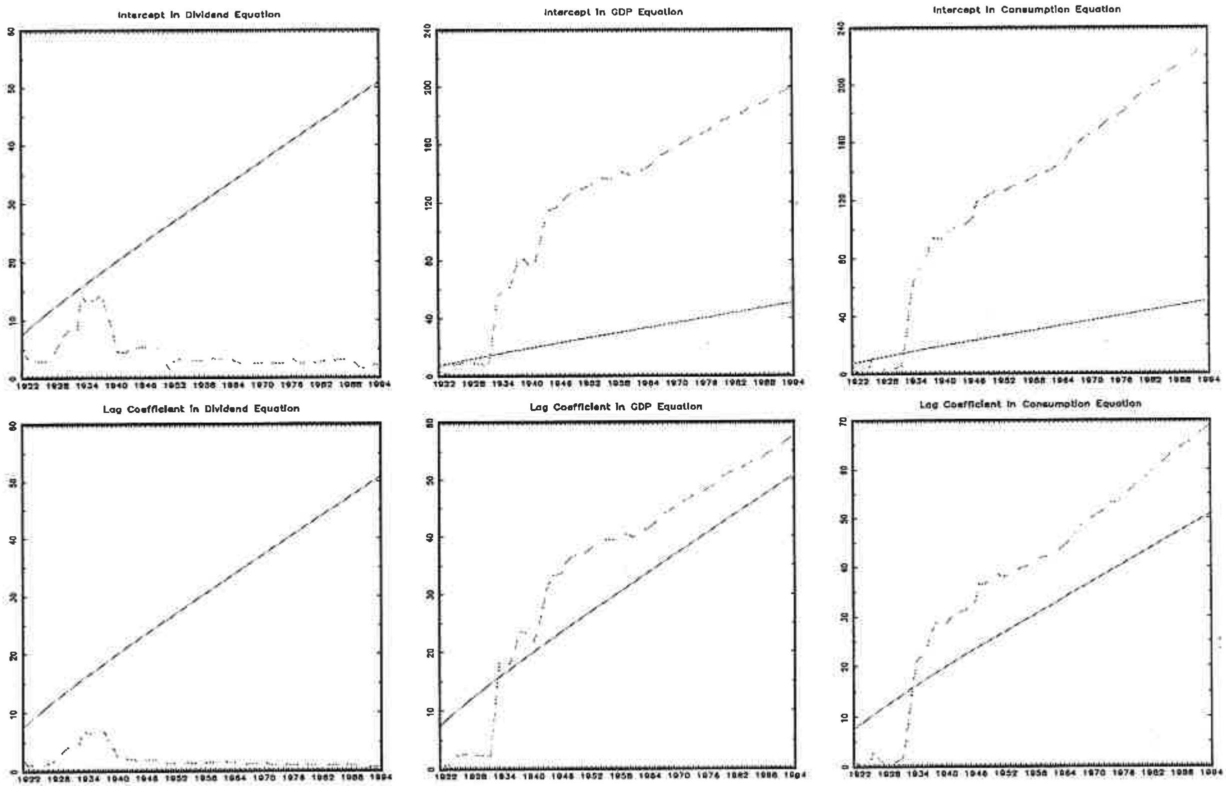
Log-Differences of Endowment Processes (1890-1994)



Note: The plots show changes in the logarithm of real per-capita dividends, GDP, and consumption.

Figure 3

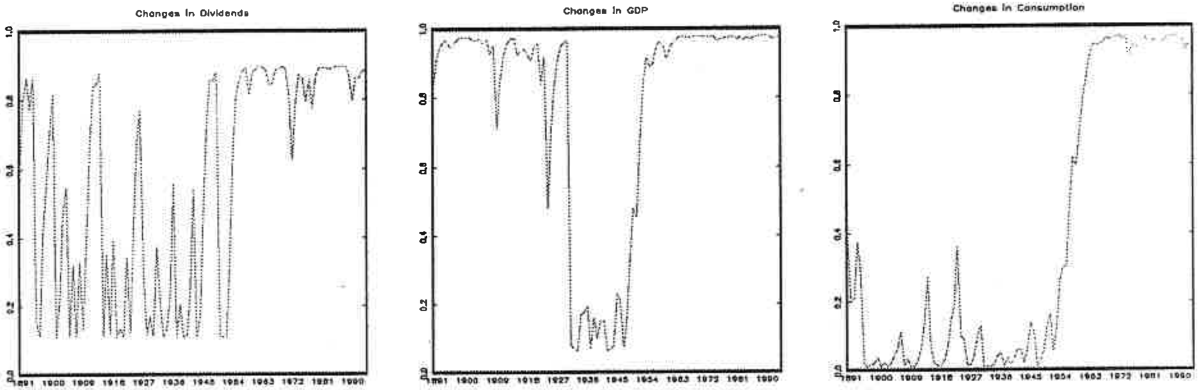
Recursive Estimates of the Fluctuation Detector



Note: Dashed lines show the estimated values of the fluctuation detector for structural breaks proposed by Chu, Stinchcombe, and White (1996). Solid lines plot the 10 percent critical value of the statistic. The test was applied to the coefficient estimates of a first-order autoregressive equation containing an intercept.

Figure 4

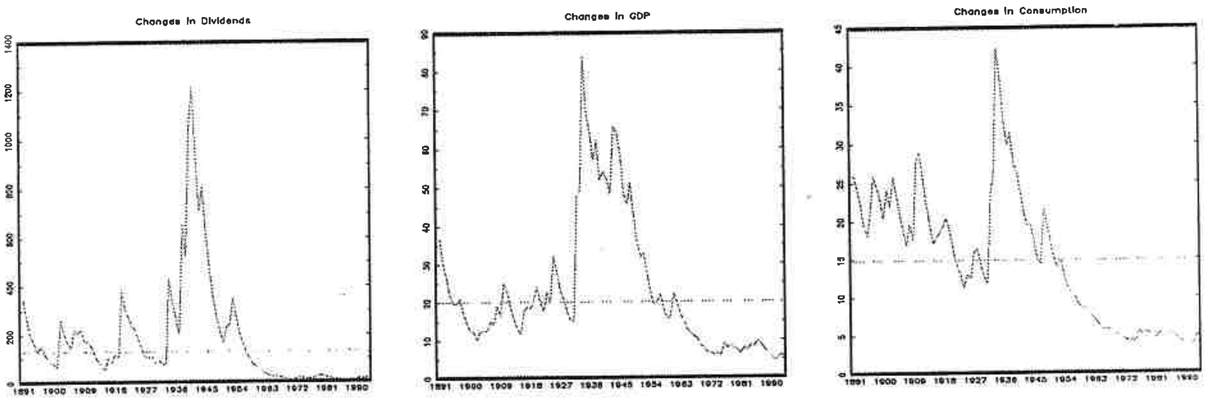
Estimated Probabilities of Being in the Low Volatility State



Note: The diagrams plot the time series of the estimated probability of being in the low-volatility state in a two-state Markov switching model with state-dependent mean and volatility parameters. The model was estimated for changes in the logarithm of real per-capita dividends, GDP, and consumption.

Figure 5

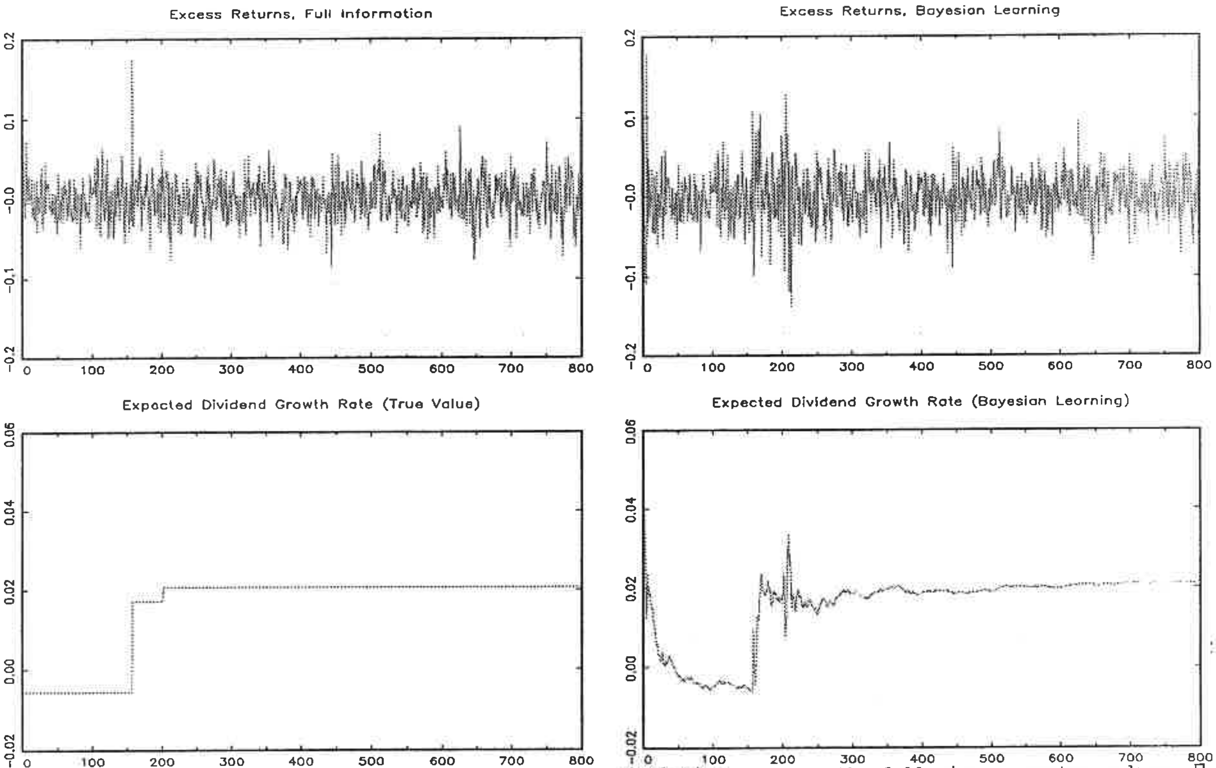
Conditional Volatility from GARCH(1,1)Model



Note: The diagrams plot the time series of the estimated conditional volatility from GARCH(1,1) models fitted to changes in the logarithm of real per-capita dividends, GDP, and consumption. An intercept and a single lag of the dependent variable were included in the mean equation.

Figure 6

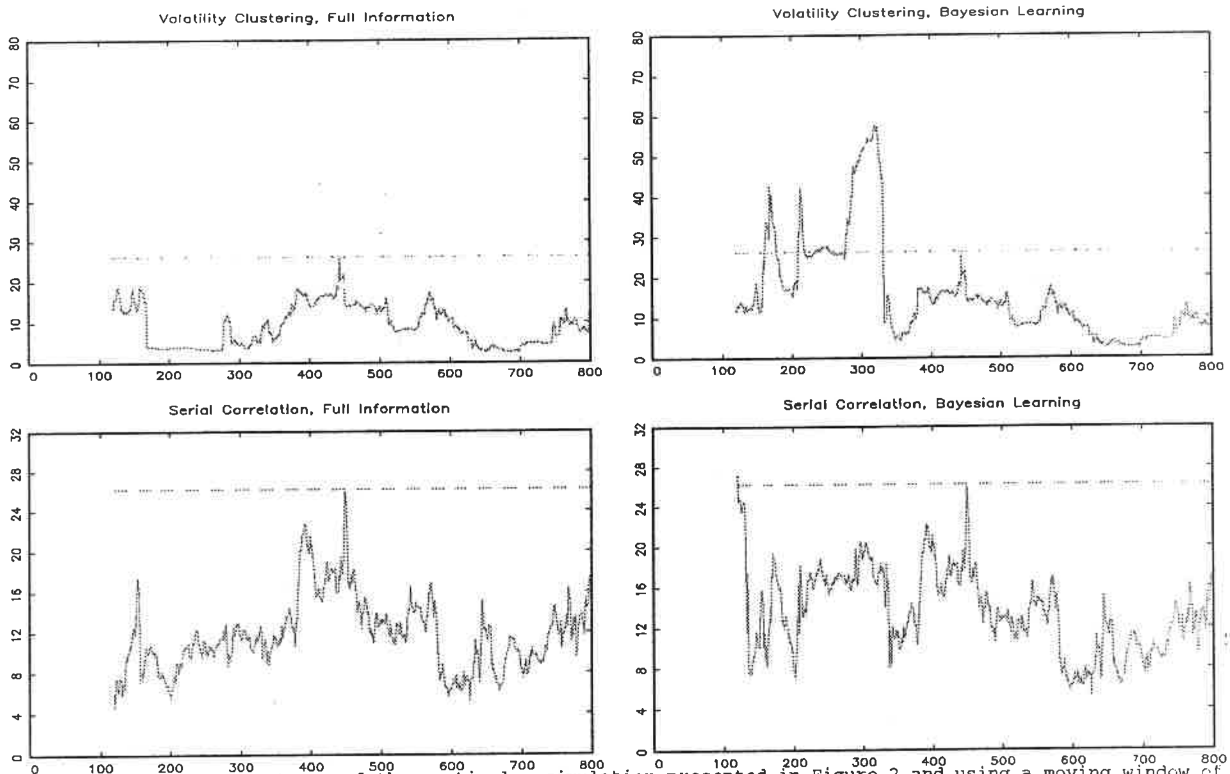
Simulated Excess Returns and Dividend Growth Rates



Note: The Figure presents the outcome of a particular simulation based on the following parameter values: Π = .995, g = -.02, g = .06, r = .075. The graph entitled " Full Information" assumes that investors know the true value of the dividend growth rate while the graph entitled " Bayesian Learning" is based on a model where investors estimate the current value of the growth rate recursively.

Figure 7

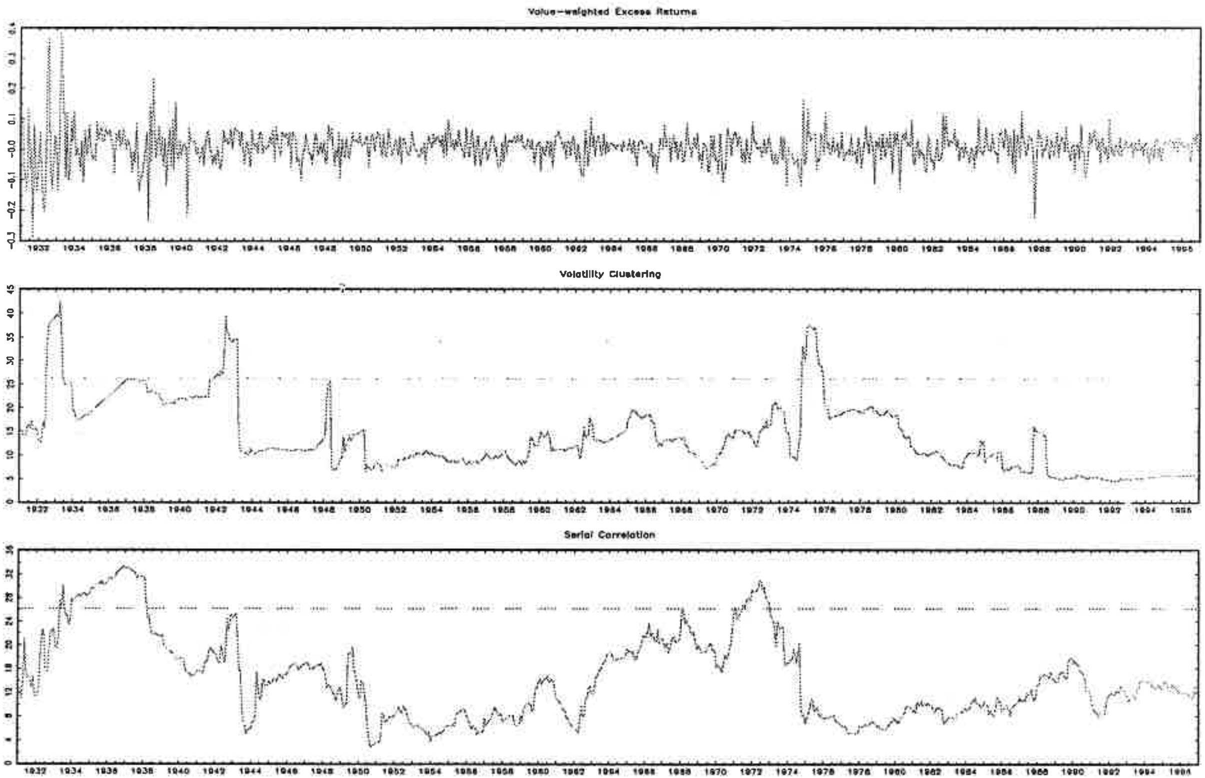
Volatility Clustering and Serial Correlation in the Excess Returns from a Particular Simulation



Note: For the excess returns of the particular simulation presented in Figure 2 and using a moving window of 120 observations, this figure shows the twelvth order LM and Ljung-Box statistics for ARCH and serial correlation. Dashed lines indicate the five percent critical value for the tests.

Figure 8

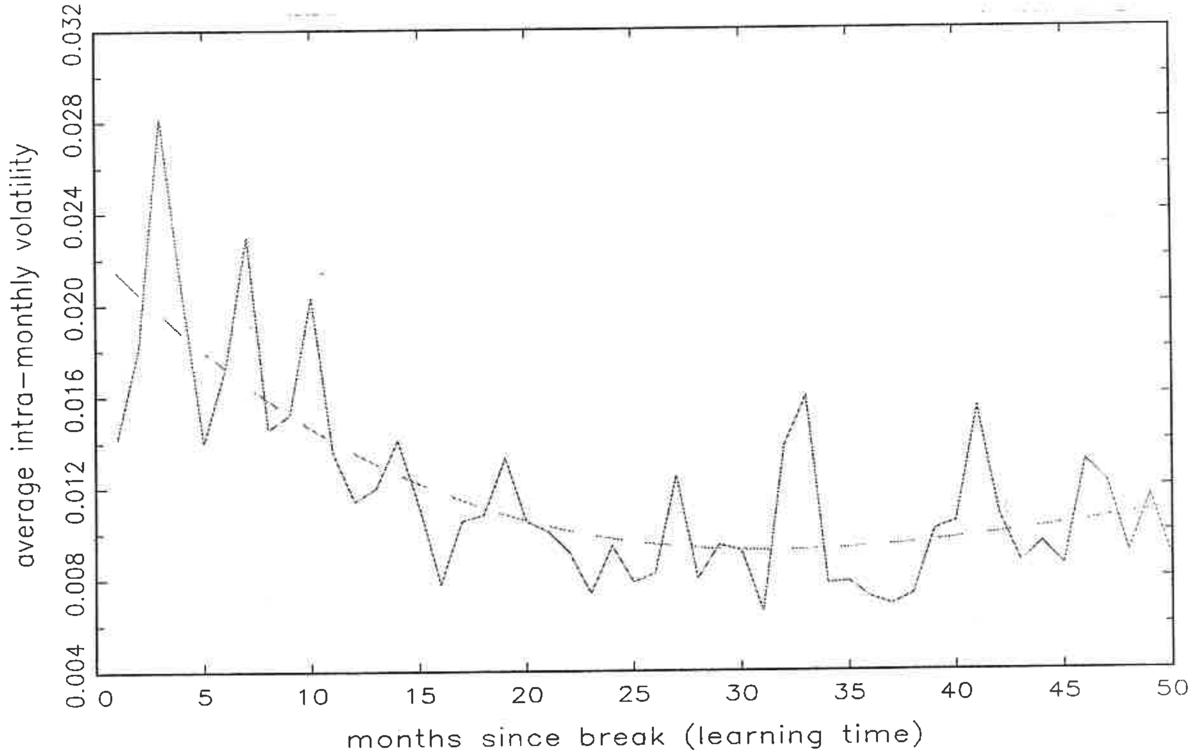
Volatility Clustering and Serial Correlation in US Stock Returns (monthly data, 1926-1997)



Note: Monthly excess returns on the value-weighted CRSP index are shown in the first window while the other windows show the estimated twelfth-order LM and Ljung Box test statistics for ARCH effects and serial correlation in returns based on a moving window of 120 observations (fewer observations between 1931 and 1935). Dashed lines indicate the five percent critical value for the tests.

<u>Figure 9</u>

Return Volatility as a Function of Time from a Structural Break



Note: This Figure plots the average of the monthly volatility (based on daily data) in US stock prices as a function of the distance from the most recent break point in the fundamentals process. Three break points (1932:1, 1943:1, 1974:9) were assumed in the calculations and an average of the monthly stock price volatility was computed across these three events. The dotted line shows a cubic approximation to the mean volatility.

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