Financial Constraints, Precautionary Saving and Firm Dynamics

By

Andrea Caggese

DISCUSSION PAPER NO 338

FINANCIAL MARKETS RESEARCH CENTRE
FORMERLY FINANCIAL MARKETS GROUP

LONDON SCHOOL OF ECONOMICS

E·S·R·C
ECONOMIC & SOCIAL RESEARCH COUNCIL

ISSN 0956-8549-338

Financial Constraints, Precautionary Saving and Firm Dynamics

By

Andrea Caggese

DISCUSSION PAPER NO 338

LSE FINANCIAL MARKETS GROUP

DISCUSSION PAPER SERIES

January 2000

Andrea Caggese is a PhD student in the department of Economics at the London School of Economics and a Research Assistant at the FMG. Any opinions expressed here are those of the author and not necessarily those of the Financial Markets Group.

Financial Constraints, Precautionary Saving and Firm Dynamics*

Andrea Caggese

London School of Economics and Financial Markets Group

10 September 1999

Abstract

This paper proposes a structural model that analyses the way financing constraints affect investment, consumption and saving decisions of the entrepreneur of a small/medium firm. The entrepreneur may face financing constraints because he cannot precommit to repay debt, unless the debt is secured by collateral. In addition he cannot retain all earnings, because a fraction of returns is non tradable and can only be consumed. These assumptions generate an overinvestment and a precautionary saving effect: the proportion of wealth allocated between risky projects and safe assets depends on future expected financing problems. The model explains why small firms are on average more financially constrained, despite all firms are ex ante identical regarding their ability to access external finance. Model's simulations are shown to be consistent with the empirical evidence about financing constraints and firm dynamics: at the micro level, firm investment depends on cash flow variations not related to changes in expected profitability. At the aggregate level, small firms experience more procyclical variation in sales, investment and short term debt than larger firms do. Another interesting result is that credit availability is more effective than interest rate in propagating monetary policy for financially constrained (small) firms, while interest rate is more effective for unconstrained (large) firms.

*I am most grateful to my supervisor Nobuhiro Kiyotaki for his encouragement and valuable feedback on my research. I would like also to thank Margaret Bray, Bob Nobay and the other paticipants to the FMG PhD seminar for their valuable comments on earlier versions of the paper. All errors are my own responsibility. Research support from the Financial Markets Group is gratefully acknowledged. Please address all correspondence to: a.caggese@lse.ac.uk or Financial Markets Group, Room G207, 20 Kingsway, London, WC2B 6LH

1 Introduction

With perfect capital markets a firm can always raise external funds to finance all the projects with a positive expected net present value. This is not possible if some imperfections are present in the financial markets. Under asymmetric information or contract incompleteness (imperfect enforceability) the moral hazard problem limits the availability of debt (Stiglitz and Weiss, 1981; Besanko and Thakor, 1986; Milde and Riley, 1988; Hart and Moore, 1998). In addition adverse selection increases equity financing costs so that they overcome expected profits of feasible investment projects (Myers and Majluf, 1984).

These theories predict that financing constraints should influence real activity both at individual and aggregate level. At the individual level, if external finance is limited, retained earnings become the main source of funds, and firm investment is a function of internal finance availability rather than of expected productivity of capital. At the aggregate level financing constraints can amplify and propagate the effects of initial real and monetary shocks, through three channels: i) the financial accelerator effect: productive firms can only invest if internal finance is available. Hence at the beginning of a downturn the reduction in cash flow depresses investment (bernanke Gertler and Gilchrist, 1996); ii) the asset price effect: when the borrowing capacity of a firm depends on the collateral value of its assets, at the beginning of a downturn the drop in asset prices reduces borrowing and investment (Kiyotaki and Moore, 1997; Ortalo Magne', 1997; Bernanke, Gertler and Gilchrist, 1998); iii) the flight to quality effect: during a downturn banks increase collateral requirements, thereby reducing loans to borrowers facing financing constraints. All three effects have opposite direction during an upturn.

Recent empirical work produced a considerable evidence supporting this view. At the aggregate level, using size as a proxy for access to credit markets, it has found that small manufacturing firms experience more procyclical variation in sales, inventories, and short term debt than larger firms do (Bernanke, Gertler and Gilchrist, 1996). At the microeconomic level, empirical work is based mainly on panel data estimation technique. It tests variations of the neoclassical quadratic adjustment cost model augmented with financial frictions, and shows that investment is excessively sensitive to internal finance: cash flow influences investment more than its informational content about firm's fundamentals would predict, according to the neoclassical model (Among many others see Whited (1992); Hubbard, Kashayap and Whited (1995); Jaramillo, Schiantarelli and Weiss (1996) and Gilchrist and Himmelberg (1998)).

The limit of this literature is the absence of a theoretical framework able to explain both microeconomic and aggregate evidence. On the one hand empirical work on the aggregate effects of financing constraints is based on generic considerations on which firms are more likely to be constrained and what are the likely effects of constraints. The assumption that dimension is a proxy for financing constraints is imposed rather than founded on a theory of firm behaviour at the micro level. On the other hand microeconomic analysis is based on reduced form estimation, and does not explicitly solve the dynamic investment problem of a constrained firm to derive its aggregate implications.

This paper aims to fill this gap in the literature, with a model that analyses the way financing constraints affect the choice between consumption, investment and precautionary saving for the entrepreneur of a small/medium firm. The model is solved using a numerical method, and the simulation of the investment and saving path of the firm is shown to be consistent with microeconomic empirical evidence.

More importantly, the model explains why expected financing constraints can affect current investment decisions, through a precautionary saving effect, and why this effect is stronger for small rather than large firms.

Finally, the solution of the model is used to simulate an artificial economy, with many firms heterogeneous in terms of size and profitability. The responses of such economy to unexpected real and monetary shocks are consistent to all main macroeconomic stylised facts.

The remaining of the paper is organised as follows: chapter 2 discusses macroeconomic evidence on the effects of financing constraints, chapter 3 illustrates the model, and chapter 4 shows simulation results.

2 Evidence of the effects of financing constraints.

Empirical work investigating the presence of financing constraints at the macroeconomic level is based on two steps:

i) identification of firms that are more likely to be constrained in the access of external finance; ii) inspection of the behaviour of constrained vs unconstrained firms across different phases of the business cycle regarding sales, debt and inventories.

Kashap, Stein and Wilcox (1993), Gertler and Gilchrist (1993,1994) and Oliner and Rudebush (1996) compare the behaviour of small versus large manufacturing firms after Romer dates, that represent episodes of tight monetary policy that led to a recession. Dimension is used as a proxy of financing problems. Bernanke Gertler and Gilchrist (1996) conduct a similar study. They inspect quarterly data disaggregated at firm level, and are able to control for industry effects and to use bank dependence as an alternative criterion to identify financially constrained firms.

They observe that after a monetary policy tightening short term debt increases for large firms, which increase the supply of commercial papers, while it decreases for small firms. Moreover during the downturn that follows such monetary action sales drop earlier for small firms, which also substantially decrease inventories, while large firms maintain inventories at an higher level. As a result sales, short term debt, inventories and the inventory/sales ratio are more procyclical for small than for large firms.

This behaviour is observed also during business cycle fluctuations not directly related to monetary actions. Bernanke Gertler and Gilchrist's (1996) computations show that one third of aggregate fluctuations can be accounted for by the difference between small and large firms.

It is interesting to compare this evidence with a similar analysis on a panel data of Italian small and medium manufacturing firms (Caggese, 1998). On the one hand Italian data are annual, therefore less useful in capturing business cycle effects, on the other hand they provide additional qualitative information[1], enabling to directly identify financially constrained firms. The aggregate behaviour of Italian firms is consistent with the US evidence: i) the difference in short term bank loans over total assets between constrained and unconstrained firms increases in boom years (1984 and 1988-1989) and decreases in the long 1990-93 recession (see figure 1A); ii) constrained firms exhibit higher growth rates of total sales during booms, and lower during recessions. This regularity is more pronounced for the subset of profitable firms that declare to have problems in obtaining long term financing and to have not enough collateral. These seem also to lead both upturns and downturns (see figures 2A and 3A).

3 A model of investment with private income and collateral constraint.

The purpose of this section is to build a simple dynamic model of consumption and investment with endogenous financing constraints.

I am thinking of the entrepreneur (henceforth E) of a small-medium firm. E is the maximising agent of the dynamic problem, and can be interpreted as the owner/manager of the firm[2].

E is risk neutral, and maximises the expected discounted sum of future consumption. He can invest in the firm, in a concave technology with capital as the only factor of production. Output of the firm depends on the unobservable value of a parameter θ, that follows a stationary stochastic process.

External finance availability for E is limited. Equity financing is not available, and debt financing is limited by the fact that firm's output is non verifiable (Hart and Moore, 1998). E

[1] Firms declared, in a survey, the problems they faced in financing investments in 1989-1991.

[2] Even though, as it will be mentioned later, the model is such that E could be reinterpreted as a manager that acts weighting shareholders' and his own private interest.

cannot commit to use earnings to repay the debt, he can only commit to use next period depreciated capital. Financial markets are otherwise perfect, and the banking sector is composed by a large number of competitive banks. Therefore E can borrow and lend one period debt at the market interest rate, and the upper bound to its borrowing capacity is the collateral value of firm's capital.

Consumption is modelled in the following way: a share of firm's output is non tradable (private). It cannot be stored and has to be consumed in the period. The tradable (financial) share of output can be either saved or consumed. One possible justification for this assumption is that E receives some non monetary return from running the firm[3]. This modelling choice is also consistent with the fact that the share of a small/medium firm's profits consumed by its E is much larger than observed dividends. It is known that small firms almost never distribute dividends. Hence income for E must come partly from wage costs and partly from tax evasion. Therefore dividends are not a good way to represent his consumption choices. In this model they represent the choice to consume a share of public output in addition to the private one, and consistently with empirical evidence we show that the optimal dividend is always 0.

Firm's technology is therefore the following:

$$y_t^T = p_t \theta_t k_t^\alpha \tag{1}$$

$$y_t^{NT} = \eta y_t^T \tag{2}$$

$$0 < \alpha < 1; \eta > 0$$

k is capital. p is output price. y^T is tradable and y^{NT} nontradable output. η determines the relative weights of public and private output

In addition, at the beginning of each period E faces an exogenous probability to retire.

With probability γ he continues activity, observes the realisation of θ and decides the level of investment, that will be productive the next period.

With probability $1-\gamma$ he retires, liquidates the assets of the firm and consumes a_t, the total net worth[4].

$$a_t = (1+\eta)y_t^T + p_t^i(1-\delta)k_t - b_t \tag{3}$$

[3]Another way to justify this assumption is to interpret E as a manager that receives private benefits (Jensen, 1986) proportional to firm's output. His objective is to maximise his intertemporal utility, and he receives from the shareholders a wage proportional to tradable profits. Linearity in preferences implies that his objective is to maximise a weigthed average of tradable and non tradable output.

[4]This is equivalent to say that, after retiring, E perceives a perpetual rent of ra_t

$$w_t = a_t - y_t^{NT} \tag{4}$$

$$0 < \delta < 1$$

a_t and w_t are respectively *total* and *financial* net worth of the firm. p^i is the price of capital good and δ is the depreciation rate. Prices are assumed to be deterministic. b is face value of the loans that have to be repaid at time t.

The Bellman equation of the dynamic problem is the following:

$$V_t\left(w_t, k_t, \theta_t\right) = \underset{k_{t+1}, b_{t+1}}{MAX}\ \gamma E_t\left\{x_t + \frac{1}{R}E_t\left[V_{t+1}(w_{t+1}, k_{t+1}, \theta_{t+1})\right]\right\} + (1-\gamma)a_t \tag{5}$$

such that:

$$x_t = a_t - k_{t+1} + \frac{b_{t+1}}{R} \tag{6}$$

$$x_t \geq y_t^{NT} \tag{7}$$

$$b_{t+1} \leq \tau p_{t+1}^I k_{t+1} \tag{8}$$

$$0 < \tau \leq \delta; 0 < \gamma < 1$$

Equation 6 is the budget constraint, and equations 7 and 8 are the constraints on consumption and borrowing respectively. x_t is consumption at time t. k_{t+1} and b_{t+1}, the next period capital and debt repayment, are the control variables[5]. $R = 1 + r$, where r is the market interest rate, assumed constant. τ is the fraction of capital that can be used as collateral. It is important to note that w_t, not a_t, is the state variable of the problem. By substituting recursively in 5, the value function can be expressed as a dynamic lagrangean (Chow, 1997):

$$V_0\left(w_0, k_0, \theta_0\right) = \underset{k_{t+1}, b_{t+1}}{MAX}\ E_t\left\{\sum_{t=0}^{\infty}\left(\frac{\gamma}{R}\right)^t\left\{\gamma x_t + (1-\gamma)a_t + \mu_t\left(x_t - y_t^{NT}\right) + \lambda_t\left[\tau p_{t+1}^I k_{t+1} - b_{t+1}\right]\right\}\right\} \tag{9}$$

subject to (6) and to the standard Khun-Tucker conditions on lagrange multipliers μ_t and λ_t. Substituting (6) in (9) by x_t, the first order condition with respect to b_{t+1} is the following:

$$\mu_t = R\lambda_t + \gamma E_t\left(\mu_{t+1}\right) \tag{10}$$

[5]Investment at time t is: $i_t = k_{t+1} - (1-\delta)\,k_t$. Since there are no adjustment costs it is equivalent, in term of notation simplicity, to use i_t or k_{t+1} as control variable.

This can be solved recursively obtaining:

$$\frac{\mu_t}{R} = E_t \left(\sum_{j=0}^{\infty} \gamma^j \lambda_{t+j} \right) \tag{11}$$

Equation 11 shows that the value of the multiplier associated to the consumption constraint is equal to the expected value of the discounted sum of the multipliers associated to all future collateral constraints.

Both multipliers are non negative, and $\mu_t > 0$ if and only if $E_t \left(\sum_{j=0}^{\infty} \gamma^j \lambda_{t+j} \right) > 0$. Sufficient condition for $E_t \left(\sum_{j=0}^{\infty} \gamma^j \lambda_{t+j} \right) > 0$ is that there is a positive probability of realising in future negative "tradable" profits[6]. In this case, no matter how profitable is on average the firm and how much wealth is accumulated, there is always some chance to loose it after a long enough series of bad outcomes. This result implies that the constraint on consumption always binds. The interpretation is that, as long as there is some positive probability to be constrained in future, E always prefers to save rather than to consume, since postponing consumption reduces the expected costs of financing constraints.

This simplifies the problem. The consumption constraint (7) holds with equality, and can be substituted in (5) and (6). Moreover $x_t = y_t^{NT}$ implies that the consumption at time t is predetermined at time $t-1$, and we can consider the value function for E at the beginning of the period *after* consuming y_t^{NT} and before deciding k_{t+1} and b_{t+1}. This implies that w and θ are the only relevant state variables:

$$V_0(w_0, \theta_0) = \underset{k_{t+1}, b_{t+1}}{MAX} \; E_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{\gamma}{R} \right)^t \left\{ \frac{\gamma}{R} \left[\gamma y_{t+1}^T + (1-\gamma) a_{t+1} \right] + \lambda_t \left[\tau p_{t+1}^I k_{t+1} - b_{t+1} \right] \right\} \right\} \tag{12}$$

s.t.

$$p_t^i k_{t+1} = w_t + \frac{b_{t+1}}{R} \tag{13}$$

Equation 13 is the budget constraint, and it means that net public wealth plus additional borrowing has to finance next period capital.

3.1 The precautionary saving effect.

An important feature of this simple model is that in equilibrium E's investment and saving decisions are affected by future expected financing constraints. This is because E chooses a level of capital that maximises *total* profits rather than *tradable* profits. The concave technology implies that this level is too high with respect to the level that maximises tradable profits. The

[6] This will be true in general unless the stochastic process θ_t has very low variance.

result is an overinvestment problem. E cares about private benefits and expands the scale of production over the level efficient in tradable terms.

In this situation expected financing constraints increase the importance of tradable versus non tradable output. E prefers to invest less to generate more tradable profits and reduce the likelihood to be constrained in future.

As an example, we consider an E without present and expected financing constraints, that has to decide whether to save p_t^i or to buy one additional unit of capital:

time t: invest $\Delta k_{t+1} = 1$ time t+1:

$$w.p.\gamma \qquad \begin{array}{l} \Delta_I x_{t+1} = \alpha p_{t+1} \eta \theta_{t+1} k_{t+1}^{\alpha-1} \\ \Delta_I w_{t+1} = \alpha p_{t+1} \theta_{t+1} k_{t+1}^{\alpha-1} + p_{t+1}^i (1-\delta) \end{array}$$

$$w.p.1-\gamma \quad \Delta_I x_{t+1} = \alpha p_{t+1} (1+\eta) \theta_{t+1} k_{t+1}^{\alpha-1} + p_{t+1}^i (1-\delta)$$

time t: Save $\Delta b_{t+1}/R = -p_t^i$ time t+1:

$$w.p.\gamma \qquad \begin{array}{l} \Delta_S x_{t+1} = 0 \\ \Delta_S w_{t+1} = R p_t^i \end{array}$$

$$w.p.1-\gamma \quad \Delta_S w_{t+1} = R p_t^i$$

Since $\beta = 1/R$, present and future consumption are perfect substitute, and optimality implies:

$$\Delta_I x_{t+1} + \Delta_I w_{t+1} = \Delta_S x_{t+1} + \Delta_S w_{t+1} \tag{14}$$

Substituting the expressions in equation 14 we get the first order condition at time t:

$$(1+\eta) E_t (MPK_{t+1}) = U_t \tag{15}$$

$E_t (MPK_{t+1}) = \alpha p_{t+1} E_t (\theta_{t+1}) k_{t+1}^{\alpha-1}$ is the expected *tradable* marginal productivity of capital, and $(1+\eta) E_t (MPK_{t+1})$ is the expected *total* marginal productivity of capital.

$U_t = R p_t^i - p_{t+1}^i (1-\delta)$ is the user cost of capital. I call k_{t+1}^{PM} the level of capital that satisfies equation 15. k_{t+1}^{PM} is the optimal level of capital when markets are perfects, and E is never constrained, today or in future.

$$k_{t+1}^{PM} = \left[\frac{(1+\eta) \alpha p_{t+1} E_t (\theta_{t+1})}{U_t} \right]^{\frac{1}{1-\alpha}} \tag{16}$$

Proposition 1 *Conditional on continuation, a level of investment of k_{t+1}^{PM} generates negative expected marginal tradable profits.*

The proof is straightforward. Since $\eta > 0$ it follows that $E_t (MPK_{t+1}) < U_t$, the marginal cost is greater than the marginal productivity of capital. I call k_{t+1}^* the level of investment that maximises tradable profits, which satisfies the standard condition $E_t(MPK_{t+1}) = U_t$.

$$k^*_{t+1} = \left[\frac{\alpha p_{t+1} E_t(\theta_{t+1})}{U_t}\right]^{\frac{1}{1-\alpha}} \tag{17}$$

$k^{PM}_{t+1} > k^*_{t+1}$ because of the overinvestment problem. $k^{PM}_{t+1} - k^*_{t+1}$ increases in η, as private output has a greater weight. I define $L_{t+1}(k_{t+1}) = U_t - E_t(MPK_{t+1})$ as the marginal loss in public profits. $L_{t+1}(k^*_{t+1}) = 0$ by definition, while from proposition 1 it follows that $L_{t+1}(k^{PM}_{t+1}) > 0$.

3.2 The formal solution.

Equation (12) is the dynamic lagrangean of the maximisation problem. Adding to it the budget constraint (13), with the associated multiplier ϕ_t, and taking the derivatives with respect to b_{t+1} and k_{t+1}, yields the following first order conditions:

$$\phi_t = RE_t\left[\sum_{j=0}^{\infty}\gamma^j\lambda_{t+j}\right] + \gamma \tag{18}$$

$$\lambda_t \tau p^i_{t+1} - \phi_t p^i_t + \frac{\gamma}{R}(1+\eta-\gamma)E_t(MPK_{t+1}) + \frac{\gamma}{R}(1-\gamma)(1-\delta)p^i_{t+1} + \tag{19}$$

$$+\frac{\gamma}{R}E_t\left(\phi_{t+1}\right)\left[E_t(MPK_{t+1}) + (1-\delta)p^i_{t+1}\right] + \frac{\gamma}{R}\Psi_{t+1} = 0$$

Equation 18 shows that the shadow cost of a binding budget constraint is equal to the discounted sum of expected future shadow costs of a binding collateral constraint. That is, the expected cost of future collateral constraints increases the required expected rate of return on capital. Substituting (18) into (19), and rearranging, we get equation (20). The following equations (20-23) determine the optimal k_{t+1}:

$$(1+\eta)E_t(MPK_{t+1}) = U_t + \frac{R}{\gamma}\left[Rp^i_t - \tau p^i_{t+1}\right]\lambda_t + RE_t(L_{t+1})E_t\left(\sum_{j=0}^{\infty}\gamma^j\lambda_{t+1+j}\right) - \Psi_{t+1} \tag{20}$$

$$S_t \geq k_{t+1} \tag{21}$$

$$\lambda_t \geq 0 \tag{22}$$

$$[S_t - k_{t+1}]\lambda_t = 0 \tag{23}$$

$S_t = w_t/[p_t^i - \frac{\tau}{R}p_{t+1}^i]$ is the borrowing capacity of the firm. Ψ_{t+1} is the following covariance factor: $\Psi_{t+1} = \partial cov\left(R\sum_{j=0}^{\infty}\gamma^j\lambda_{t+1+j} + \gamma, p_{t+1}\theta_{t+1}k_{t+1}^{\alpha}\right)/\partial k_{t+1}$. It can be shown to be positive. $RE_t(L_{t+1})E_t\left(\sum_{j=0}^{\infty}\gamma^j\lambda_{t+1+j}\right)$ is the overall cost of overinvestment, and depends on the expected value of the collateral constraint in future in case of continuation. From (20-23) it follows that the solution to the problem is a capital level $\widetilde{k}_{t+1}$ equal to:

$$\widetilde{k}_{t+1}(w_t,\theta_t) = \begin{cases} k_{t+1}^o(w_t,\theta_t) & \text{if } k_{t+1}^o(w_t,\theta_t) < S_t(w_t,\theta_t) \\ S_t(w_t,\theta_t) & \text{otherwise} \end{cases} \tag{24}$$

k_{t+1}^o is the optimal unconstrained capital level[7] when (21) holds with strict disequality. From (22) and (23) we have that k_{t+1}^o satisfies (20) with $\lambda_t = 0$:

$$(1+\eta)E_t(PMPK_{t+1}) = U_t + RE_t(L_{t+1})E_t\left(\sum_{j=0}^{\infty}\gamma^j\lambda_{t+1+j}\right) - \Psi_{t+1} \tag{25}$$

It is possible to show[8] that $k_{t+1}^* < k_{t+1}^o < k_{t+1}^{PM}$. Intuitively the bigger is $E_t\left[\sum_{j=0}^{\infty}\gamma^j\lambda_{t+1+j}\right]$, the expected future financing problems, the bigger is the cost of overinvestment, and k_{t+1}^o is pushed towards k_{t+1}^*. This is like a precautionary saving effect. When E has bad news about the future, and expects to have financing problems, he will save more now to increase his borrowing capacity, even if he is not actually constrained. Reducing capital below k_{t+1}^o is like rationalising production (i.e. reducing employment, postponing nonessential investment, closing less productive branches, etc.).

[7] k_{t+1}^o is different from k_{t+1}^{PM}. In fact perfect markets imply financial constraints are never binding, while k_{t+1}^o is the optimal level of capital when the constraint is not binding today but will bind in future with a positive probability.

[8] When w_t increases, the likelyhood of being financially constrained decreases. That is, $E_t\left\{\sum_{j=0}^{\infty}\lambda_{t+j+1}\right\} \to 0$. In this case also $\Psi_{t+1} \to 0$, and from (25) follows that $\lim_{E_t\left\{\sum_{j=0}^{\infty}\lambda_{t+1+j}\right\}\to 0} k_{t+1}^o = k_{t+1}^{PM}$. Moreover when $E_t\left\{\sum_{j=0}^{\infty}\lambda_{t+1+j}\right\} \to \infty$ equation (25) is satisfied only if $E_t(L_{t+1}) = 0$. This implies that $\lim_{E_t\left\{\sum_{j=1}^{\infty}\lambda_{t+j}\right\}\to\infty} \widetilde{k}_{t+1} = k_{t+1}^*$.

4 Numerical solution and simulation.

The model cannot be solved analytically, therefore here I provide a numerical solution obtained discretising the value function and iterating the Bellman equation until convergence is achieved[9]. I model θ as a two state symmetric markow process: $\theta_t \in \{\theta_L, \theta_H\}$, with $\theta_H > \theta_L$. Transition probabilities are:

	$\theta_{t+1} = \theta_L$	$\theta_{t+1} = \theta_H$
$\theta_t = \theta_L$	ρ	$1 - \rho$
$\theta_t = \theta_H$	$1 - \rho$	ρ

with $.5 < \rho < 1$. For simplicity prices are assumed constant and normalised to 1. Chosen parameters values are:

$\gamma = 0.95$ $\alpha = 0.6$ $R = 1.01$ $p_t = p_t^i = p_{t+1}^i = 1$

$\delta = 0.7$ $\tau = 0.3$ $\rho = 0.7$ $\eta = 0.81$

$\theta_L = 1.7$ $\theta_H = 2.3$

The relatively high value of δ is motivated by the fact that capital, being costlessly adjustable, is more similar to nondurable working capital than to durable fixed capital. η indicates that non-tradable output is 81% of the tradable output. The higher is η, the larger is $k_{t+1}^{PM} - k_{t+1}^*$ and the lower are expected tradable profits at k_{t+1}^{PM}. 0.81 is the value that generates zero tradable profits at k_{t+1}^{PM} conditional on $\theta_{t+1} = \theta_H$. Since $k_{t+1}^o < k_{t+1}^{PM}$, such value means that in general the firm will obtain positive profits conditional on 'good news' ($\theta_{t+1} = \theta_H$), and negative profits conditional on bad news ($\theta_{t+1} = \theta_L$).

Figure 1 shows the two policy functions $\widetilde{k}_{t+1}(w_t \mid \theta_t = \theta_H)$ and $\widetilde{k}_{t+1}(w_t \mid \theta_t = \theta_L)$. For low values of w_t the constraint is binding at time t, and $\widetilde{k}_{t+1} = S_t$. This corresponds to the section of the policy function that is a diagonal line, with slope $1/\left(p_t^i - \frac{\tau}{R}p_{t+1}^i\right)$. The denominator of the slope is the required downpayment for a unit of purchased capital when the constraint is binding. In the remaining part of the policy function $\widetilde{k}_{t+1} = k_{t+1}^o < k_{t+1}^{PM}$. The distance between $\widetilde{k}_{t+1}$ and k_{t+1}^{PM} measures the intensity of the precautionary saving effect.

Figure 1 shows that, when θ_t increases, the binding constraint region expands and the precautionary saving region shrinks. This feature is generated by the persistency of θ. Higher θ_t means also higher values in future, and E expects to increase financial profits and financial wealth, thereby reducing future probability of being constrained.

4.1 Simulation: single firm.

Figure 2 shows the time path of $\widetilde{k}_t$ for 50 periods, with a sequence of expansion and contraction phases. Firm's idiosyncratic shock is θ_H for 15 periods, then θ_L for 20 periods, and then

[9] Details are provided in the appendix B.

again θ_H for the last 15 periods. I simulate two firms with different initial endowment w_0, but otherwise identical. The firm with high w_0 has an almost zero probability to be constrained in future. Hence it behaves like an unconstrained firm, and $\widetilde{k} \simeq k^{PM}\ \forall t$. The firm with low w_0 has a binding collateral constraint only in periods 1-4 (start-up phase) and 35-36 (expanding phase after a sequence of bad outcomes). In the other periods precautionary saving generates higher investment volatility with respect to the 'unconstrained' firm.

The effects of precautionary saving on the relation between investment, profitability and financial wealth are clear in figure 3, that shows the time paths of capital and of net tradable wealth. Capital has a positive correlation with expected profitability, but also with changes in net tradable wealth (cash flow) not related to changes in expected profitability. This is due to the precautionary saving effect. In the first 15 periods E accumulates wealth. As he becomes more confident about the future, he increases the amount of wealth invested in the risky asset (firm's capital), reducing precautionary saving. The situation is reversed in the next 20 periods of 'bad luck'. Here precautionary saving is responsible for much of the contraction of investment in periods 16-28.

Hence the model shows an investment excess sensitivity to internal finance, confirming empirical results based on panel data investment estimations, even thought here the relation is asymmetrical and nonlinear, and it depends on the level of wealth as well.

4.2 Simulation: aggregate.

In this section I will use the solution of the model to simulate an artificial economy with many heterogeneous firms. This is a very simple 'partial equilibrium' economy, because both interest rate and prices are exogenous. The purpose of this exercise is to verify that the simulated economy confirms stylised facts about the business cycle mentioned earlier. I will show that small firms react more to unexpected shocks because of financial problems, despite all firms are ex ante identical and subject to the same constraints.

The procedure is simple: I simulate an economy with 10000 firms, for 200 periods. Each firm's productivity evolves accordingly to the value of θ_{it}, that is independent across firms and path dependent across time. Firms exit when their E retire, and new firms enter, but the total number of firms is constant. Each period aggregate statistics for small and large firms are computed, to analyse the effects of an unexpected macroeconomic shock that hits after 100 periods.

The aim of this section is then to analyse firm dynamics in a simple economy with heterogeneous agents. It is very important to note that such dynamics are very sensitive to technology assumptions. From this respect one limitation of the model presented in chapter three is that the idiosyncratic shock is stationary, and firm's technology is concave. Therefore E wants to expand only up to the expected steady state size of the firm, that depends on the unconditional expectation $E(\theta_{t+1}) = (\theta_H + \theta_L)/2 = \overline{\theta}$. This corresponds to a situation where returns to scale are decreasing, or are constant/increasing but E's 'know-how' is essential for the firm[10].

A consequence of this limitation is that there exists a straightforward way to generate higher volatility of small firms. It is sufficient to impose the same $\overline{\theta}$ to all firms, and to assume that new firms start activity with a very small endowment. Given that there is an ongoing entry and exit of firms, small firms will include younger firms that are constrained because are in the start-up phase, while large firms will include older firms that are less constrained because they are on average closer to the steady state.

Therefore, in order to generalise the analysis, I consider another dimension of firms heterogeneity, regarding the value of $\overline{\theta}$. To keep things simple, I simulate 10000 firms that belong to two types:

Type1: $\overline{\theta}^1 = 2 \qquad \theta_L^1 = 1.7 \qquad \theta_H^1 = 2.3 \qquad k^{PM1}\left(\theta_L^1\right) = 14 \qquad k^{PM1}\left(\theta_H^1\right) = 18.9$

Type 2: $\overline{\theta}^2 = 2.25 \quad \theta_L^2 = 1.9125 \quad \theta_H^2 = 2.5875 \quad k^{PM2}\left(\theta_L^2\right) = 18.8 \quad k^{PM1}\left(\theta_H^2\right) = 25.4$

Type one firms have smaller $\overline{\theta}$, but are otherwise equal, also in terms of risk, as:

$$\frac{\left(\theta_H^1 - \theta_L^1\right)}{\overline{\theta}^1} = \frac{\left(\theta_H^2 - \theta_L^2\right)}{\overline{\theta}^2}$$

The fraction of type 1 and 2 among the 10000 firms is such that the two types on average produce the same aggregate level of output. The firms are then selected into small and large groups according to net tradable wealth[11].

Given the two dimensional heterogeneity of firms, we expect each group to include a mix of types: some mature or successful type 1 firms will be in the large group, while some young or expanding type 2 firms will be in the small group. A type one small firm is a firm that is happy to be small, while a type 2 small firm is either a young growing firm, or a firm that shrank after a period of bad luck and negative profits.

The other parameters are the same used in previous numerical solution. Moreover each period closing firms are substituted by new firms of the same type.

Finally, for a newly created firm, initial endowment is the following:

$$w_0 = \frac{k^{PM_z}\left(\theta_H^z\right)}{\left(p_t^i - \frac{\tau}{R}p_{t+1}^i\right)}$$

[10]The concave technology can be interpreted as a constant/increasing return to scale technology where the E supplies an essential fixed amount of labour. That is, $\theta_t = \widehat{\theta}_t \overline{L}^{\beta}$, where $\widehat{\theta}_t$ is the stochastic component, $\overline{L}$ is E's essential labour supply and $\alpha + \beta \geq 1$.

[11]Similar results are obtained using different selection criteria, like capital level or output.

where $z \in \{1, 2\}$ is firm's type. This ensures that new firms have enough resources to finance the highest possible capital level, and the constraint is not binding in the first period of life. Assuming a smaller value of w_0 does not change the qualitative results of the analysis, it only increases their quantitative importance. Therefore this assumption has the purpose of emphasising the importance of existing firms dynamics and of the precautionary saving effect versus the effect of a binding constraint in the start-up phase.

Figures 4 to 8 show the *cumulative rate of growth of capital* for small and large firms, before and after unanticipated macroeconomic shocks. Figures 9 and 10 show the *cumulative rate of growth of the capital/sales ratio*. Following the empirical literature (Gertler and Gilchrist, 1993 and 1994, and Bernanke, Gertler and Gilchrist, 1996), I select firms according to a fixed percentile in the cumulative size distribution function. Each period the firms below the 50% percentile are selected in the small firms group.

Figure 4 considers a temporary reduction in output of 10%. Since the model does not have any aggregate uncertainty, the observed variability in the aggregate growth rates before the shock depends largely on the entry/exit dynamics.

The immediate negative impact on investment after the shock is around 5% for small firms and 2% for large firm. The reason for the difference is that lower output reduces profits and tradable wealth. Some firms reduce investment because the wealth shock pushes them in the constrained region, while other firms reduce investments because of precautionary saving reasons. Both effects are stronger for small firms, that are relatively less wealthy. After the first negative reaction firms slowly return to the previous steady state. The reduction in cumulative growth rates takes between 15 and 20 periods to disappear. Such persistency depends on the precautionary saving effect. As long as firms are on average less wealthy, with respect to the before-shock situation, they have higher expected probability to be constrained in future, and will save more, until they reach the before-shock average wealth level.

Figure 5 considers a symmetric positive shock. Both small and large firms increase investment, but relatively less, in absolute value, than after the negative shock. The reason of the asymmetry is that few firms experience an "actually binding" constraint before the shock (see figure 3). Hence the positive shock's impact is caused almost exclusively by the precautionary saving effect, while the negative shock's impact is caused by a mix of precautionary saving and of binding constraint.

Figures 6 and 7 consider a monetary policy tightening. Figure 6 considers a permanent reduction of τ from 0.3 to 0.2. This shock is like a credit crunch, as it reduces the borrowing capacity of all firms. Also in this case small firms are more affected, with an immediate reduction of 6.5% versus a reduction of 3% for large firms. Once again this result depends both on binding constraint and precautionary saving. In addition we can observe that after the permanent shock firms do not go back to the previous steady state. In fact the reduction in τ means that there is higher risk to be constrained in future, and small firms become permanently more "prudent" than large firms do.

Figure 7 shows the effect of a permanent increase in interest rate. This shock has three distinct effects:

a) The optimal level of capital k^o_{t+1} decreases, because the user cost of capital U_t increases. This reduces $\widetilde{k}_{t+1}$ of an unconstrained firm.

b) The borrowing capacity S_t decreases, because the downpayment $p^i_t - \frac{\tau}{R}p^i_{t+1}$ increases. This reduces $\widetilde{k}_{t+1}$ of a constrained firm.

c) The distance between k^{PM}_{t+1} and k^*_{t+1} decreases. Production is more efficient, in the sense that return on capital in tradable terms is higher. Expected rate of financial wealth accumulation is also higher, and this implies that precautionary saving decreases over time.

Large firms are on average less constrained: for the majority of them $\widetilde{k}_{t+1} = k^o_{t+1}$ and, more importantly, the difference $S_t - k^o_{t+1} > 0$ is usually large. This means that for them effect (a) should prevail on effect (b). On the contrary, small firms have on average a smaller difference $S_t - k^o_{t+1}$, and should be more affected by (b) than (a). Moreover, the magnitude of effect (b) depends crucially on τ. The larger is τ, the more levered is the economy, the more sensible S_t is to interest rate shock, the bigger is (b) relative to (a).

Figures 7 and 8 summarise these considerations. Figure 7 shows that, with the chosen $\tau = 0.3$, the economy has low leverage, and effect (a) prevails on (b). The consequence is that the immediate reaction to the interest rate shock is stronger for large firms. This result is not really surprising, as it confirms the credit view (Bernanke, 1983): if financial imperfection are present in the economy, and affect firms behaviour, the availability of credit may be a more important monetary policy propagation channel than interest rate.

After the shock the distance between small and large firms increases, because of effect (c).

Figure (8) considers and economy with higher leverage, with $\tau = 0.8$ and $\delta = 1$[12]. In this case the situation is reversed, and small firms immediate reaction to the shock is stronger.

We considered so far the effects of macroeconomic shocks on investment. The effect on output can be shown to be qualitatively the same, consistently with empirical evidence. More interestingly, Figures (9) and (10) show that also the capital/output ratio, after a negative shock, declines more for small firms. Capital in this model does not have adjustment costs, is like 'working capital', and this result is therefore consistent with the fourth stylised fact mentioned earlier, about the procyclicality of inventory investment/sales ratio.

[12]Such values are not consistent with the theoretical assumptions, that imply the restriction $\tau \leq (1 - \delta)$. Hence this particular figure is just a "simulation excercise" that would need to be justified by a different set of assumptions, like the ability of E to collateralise also a fraction of output.

5 Conclusions

I illustrated a model of investment with collateral constraint and private income, that analyses the trade off between consumption and investment for the entrepreneur of a small medium firm. E wants to expand production above the profit maximising level, to benefit of the private share of output. In this situation future expected financing constraints affect his choices today, forcing him to reduce the scale of activity to generate more financial earnings and improve his financial solidity.

The model explains why small firms are on average more financially constrained, despite all firms are ex ante identical regarding their ability to access external finance. It generates firm dynamics where constrained (small) firms are shown to be more volatile in the business cycle. After a contraction period, the constraint is binding, and a positive productive shock generates a large increase in investment because of the financial accelerator effect. On the contrary, after an expansion period, a negative shock reduces investment mainly through a precautionary saving effect. The model therefore explains the great importance of consumers and businesses confidence indexes, showing that, while binding constraints matter in upturns, expected financial constraints may have an important depressing effect on investment in downturns.

The model is solved using numerical method, and the micro and aggregate simulations are shown to be consistent with empirical evidence about firm dynamics. Other interesting results are: i) credit availability is more effective than interest rate in propagating monetary policy for financially constrained (small) firms, while interest rate is more effective for unconstrained (large) firms; ii) The model suggests an explanation to the procyclicality of inventories/sales ratio for small firms.

References

[1] Bernanke, B., 1983, nonmonetary effects of the Financial Crisis in Propagation of the Great Depression, American Economic Review 73, 257-76.

[2] Bernanke, B., Gertler, M. and S. Gilchrist, 1996, The Financial Accelerator and the Flight to Quality, The Review of Economics and Statistics 78, 1-15.

[3] Bernanke, B., Gertler, M. and S. Gilchrist, 1998, Credit Market Frictions and Cyclical Fluctuations", Forthcoming in Handbook of Macroeconomics, North-Holland, Amsterdam.

[4] Besanko, D. and A.V. Thakor, 1987, Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Credit Markets, International Economic Review, 28, 671-89.

[5] Caggese, A., 1998, "An Empirical Investigation on Financial Constraints, Firms Heterogeneity and Business Cycle", Mimeo, London School of Economics.

[6] Chow, G.C., 1997, Dynamics Economics, Oxford University Press.

[7] Gertler, M. and S. Gilchrist, 1993, The Role of Credit Market Imperfections in the Monetary Transmission Mechanism: Arguments and Evidence, Scandinavian Journal of Economics, 43-64.

[8] Gertler, M. and S. Gilchrist, 1994, Monetary Policy, Business Cycle, and the Behaviour of Small Manufacturing Firms, Quarterly Journal of Economics 109, 309-340.

[9] Gilchrist, S. and C. Himmelberg, 1998, Investment, Fundamentals and Finance, N.B.E.R. Working Paper n. 6652.

[10] Jaramillo, F., Schiantarelli, F. and A. Weiss, 1996, Capital Market Imperfections Before and After Financial Liberalization: An Euler Equation Approach to Panel Data for Ecuadorian Firms, Journal of Development Economics 51, 367-386.

[11] Jensen, M., 1986, Agency Costs of Free Cash Flow, Corporate Finance and Takeovers, American Economic Review 76, 323-29.

[12] Kashap, A., Stein, J. and D. Wilcox, 1993, The Monetary Transmission Mechanism: Evidence From the Composition of External Finance, American Economic review 83, 78-98.

[13] Kiyotaki, Nobuhiro, and Moore, John, 1997, Credit Cycles, Journal of Political Economy, 105, 211-248.

[14] Hart,O and J.H. Moore, 1998, Default and Renegotiation: a Dynamic Model of Debt, Quarterly Journal of Economics 113, 1-41.

[15] Hubbard, G.R., A.K. Kashyap and T.Withed, 1995,Internal Finance and Firm Investment, Journal of Money Credit and Banking, 27, 683-701.

[16] Milde, H. and Riley, J.G., 1988, Signalling in Credit Markets, Quarterly Journal of Economics, 103, 101-29.

[17] Myers, S.C. and N.S. Majluf, 1984,Corporate Financing Decisions When Firms Have Investment Information That Investment Do Not, Journal of Financial Economics 13, 187-221.

[18] Oliner, S.D. and Rudebusch, G.D.,1996, Is There a Broad Credit Channel for Monetary Policy?, Federal Reserve Bank of San Francisco Economic Review 1, 3-13.

[19] Ortalo-Magne', Francois, 1997, Fluctuations in a Life-cycle Economy: Do Credit Constraints Matter?, Mimeo, L.S.E.

[20] Stiglitz, J. and A.Weiss, 1981, Credit Rationing in Markets with Imperfect Information, American Economic Review 71, 912-927.

[21] Whited, T.M., 1992, Debt, Liquidity Constraints, and Corporate Investment: Evidence from Panel Data, The Journal of Finance 47, 1425-60.

Figure 1
Policy functions

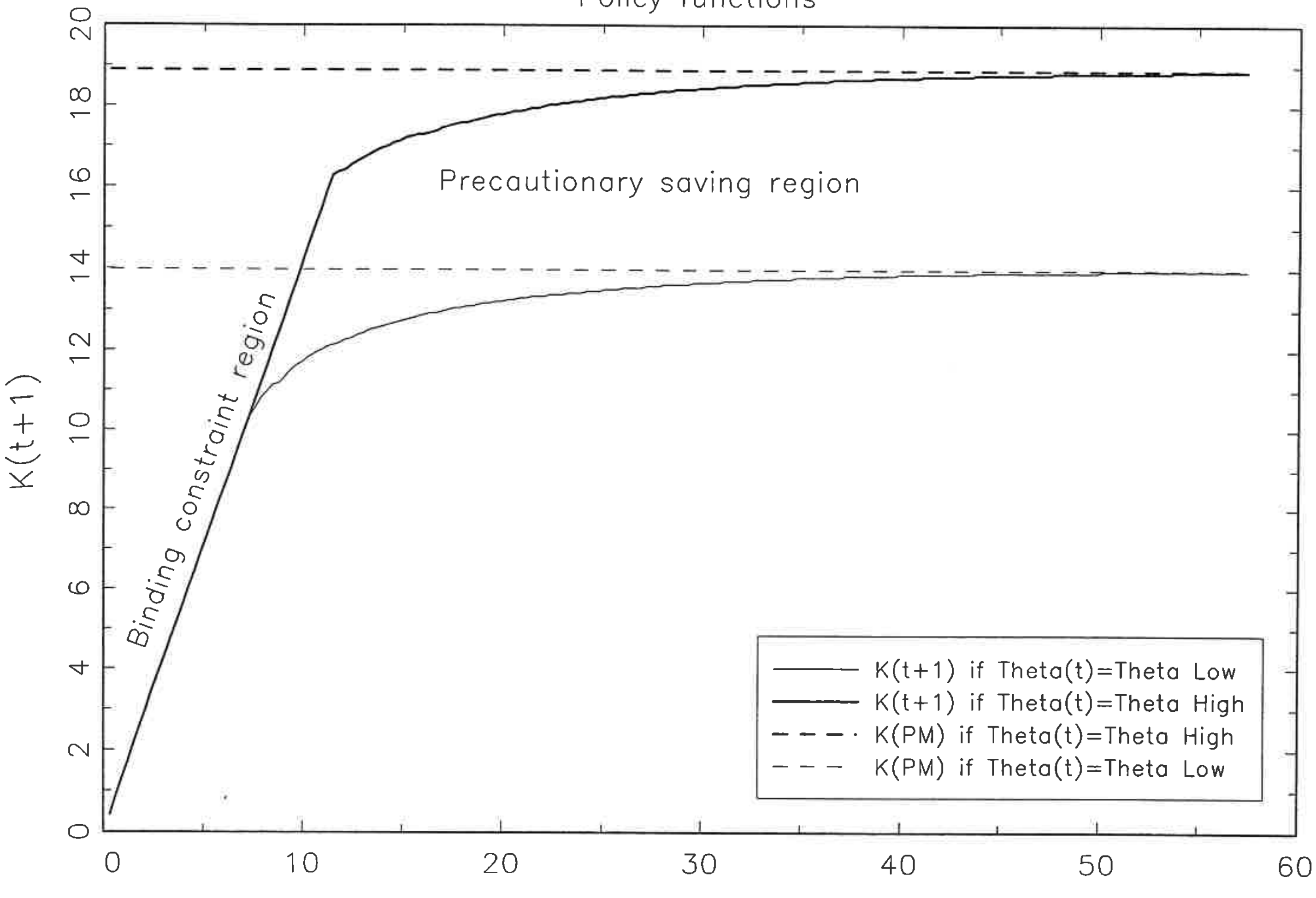


Figure 2
Time path of investment

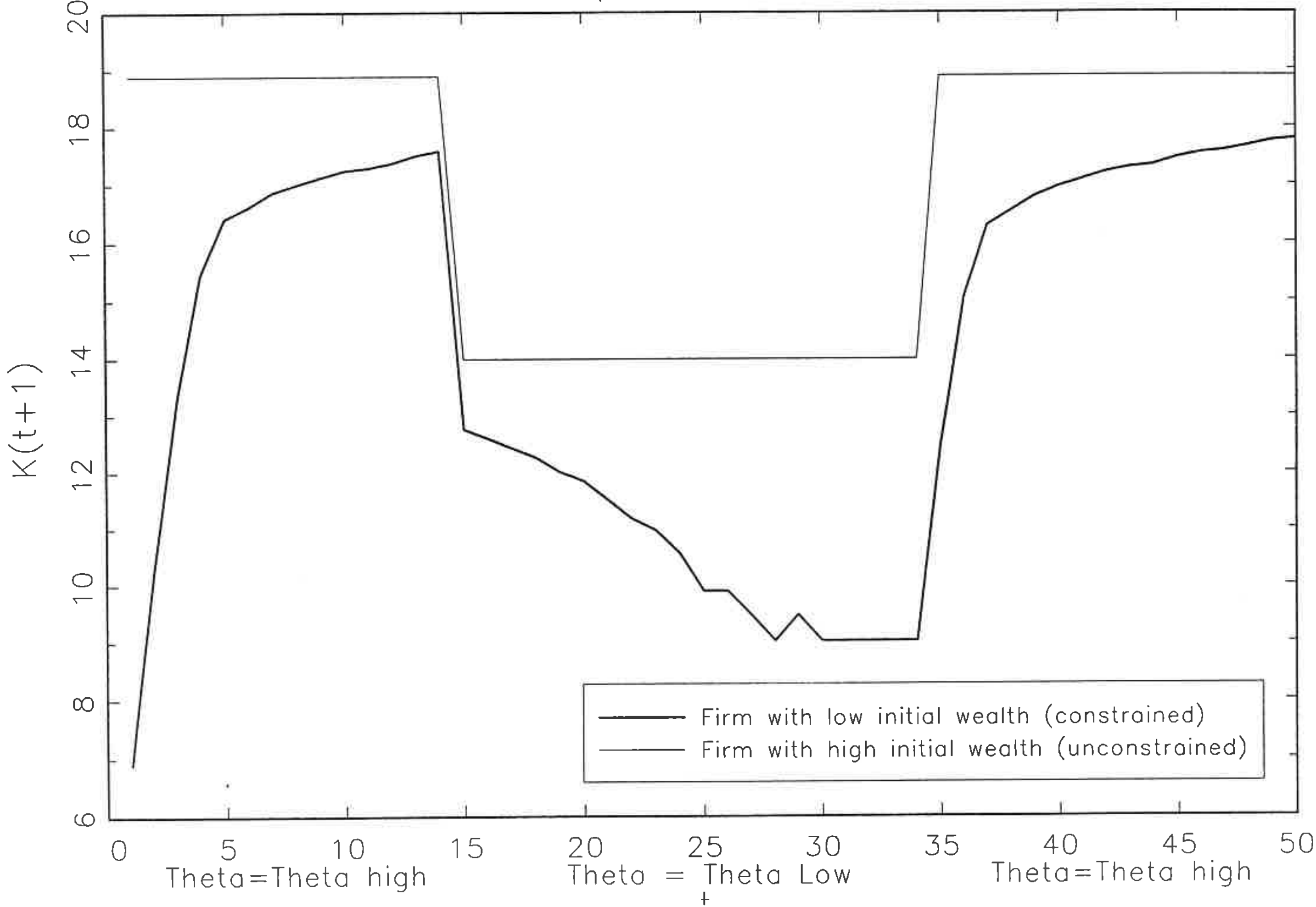


Figure 3
Time paths of investment and net financial wealth

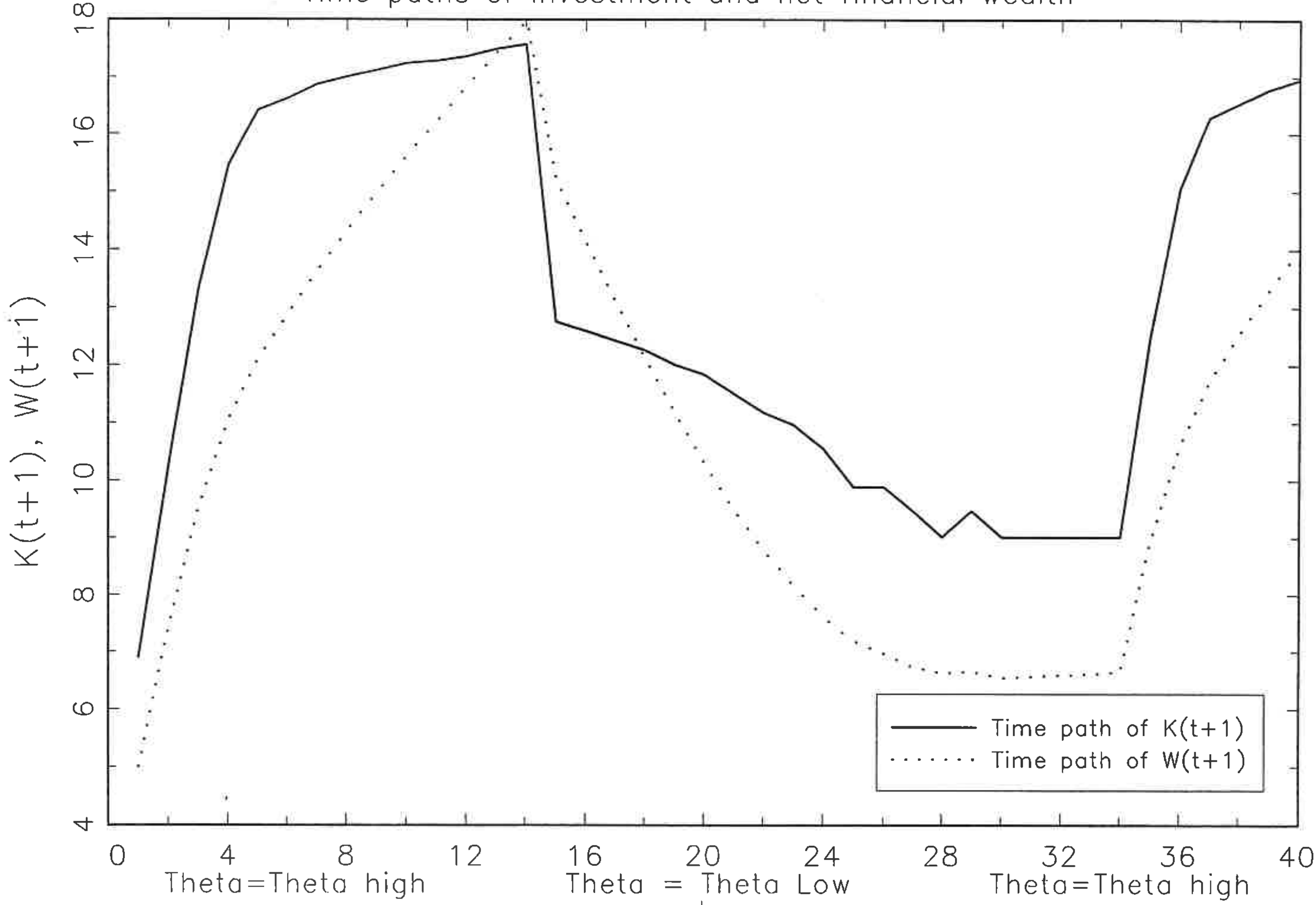


Figure 4


Unanticipated temporary reduction in output of 10%
Cumulative growth rates
0.01
0.00
−0.01
−0.02
−0.03
−0.04
−0.05
−20
−15
−10
−5
0
5
10
15
20
25
30
Time
Large Firms
Small Firms

Figure 5

Unanticipated temporary increase in output of 10%

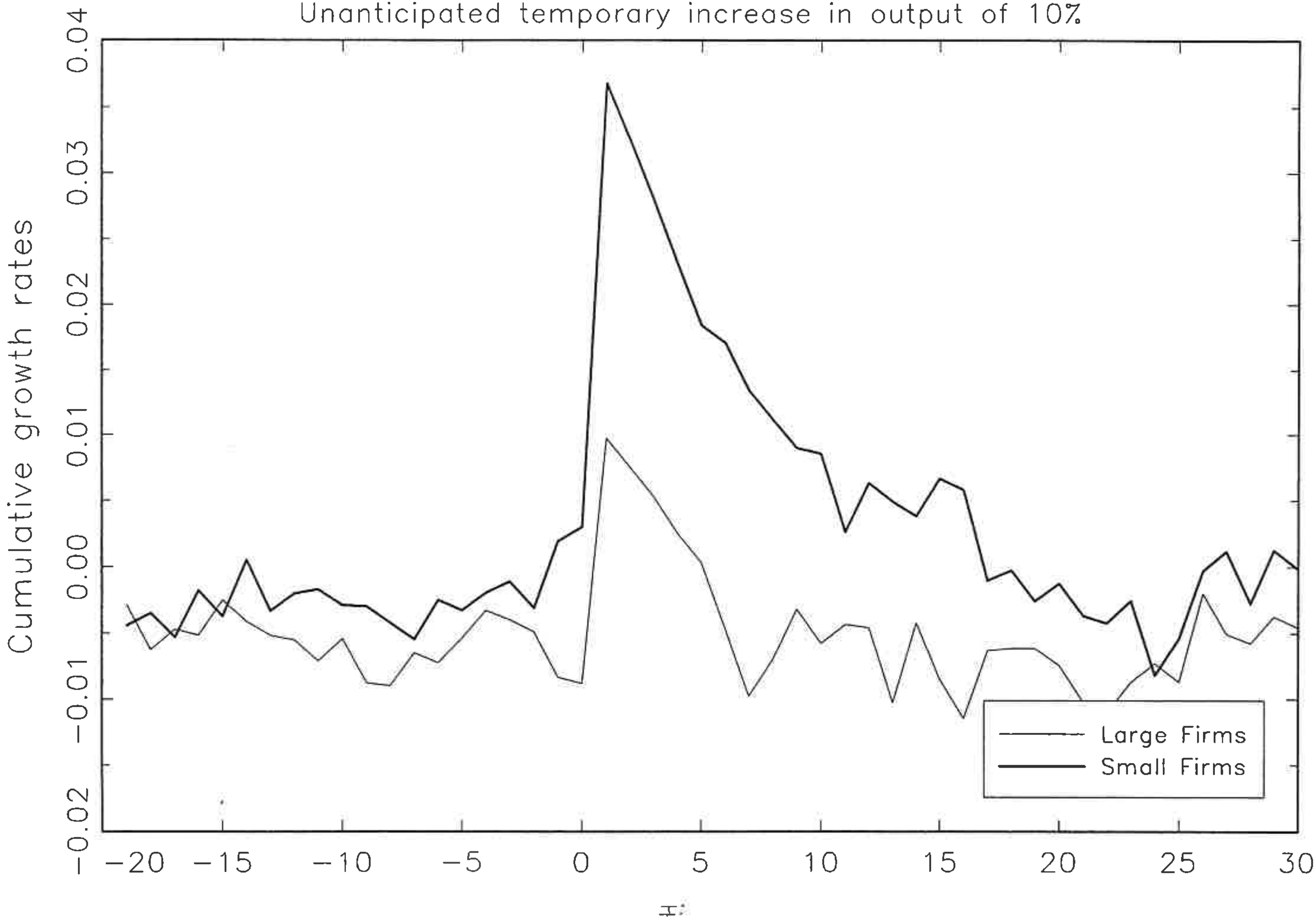


Figure 6

Unanticipated permanent reduction in tau from 0.3 to 0.2

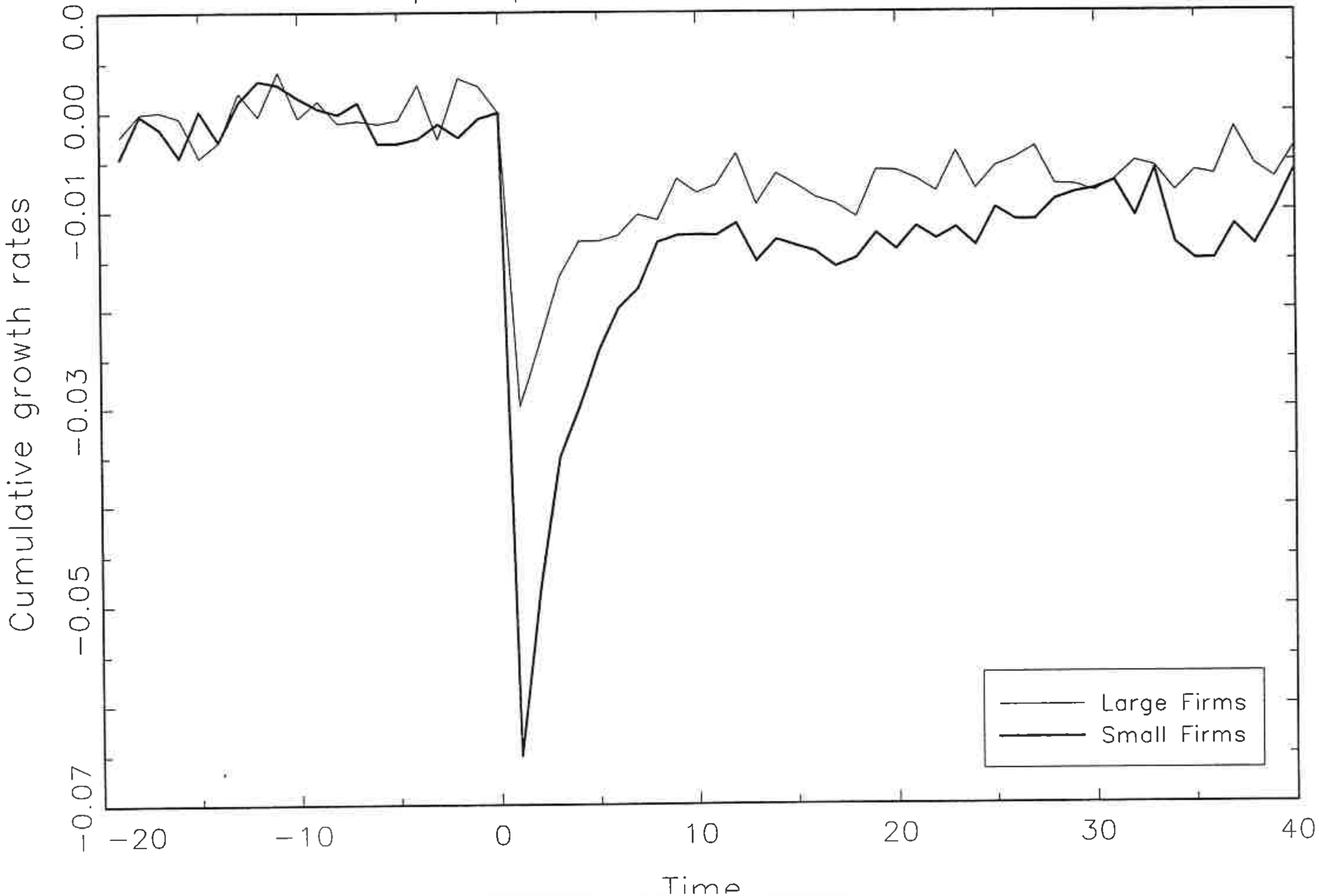


Figure 7

Unanticipated permanent increase in interest rate from 1% to 3%

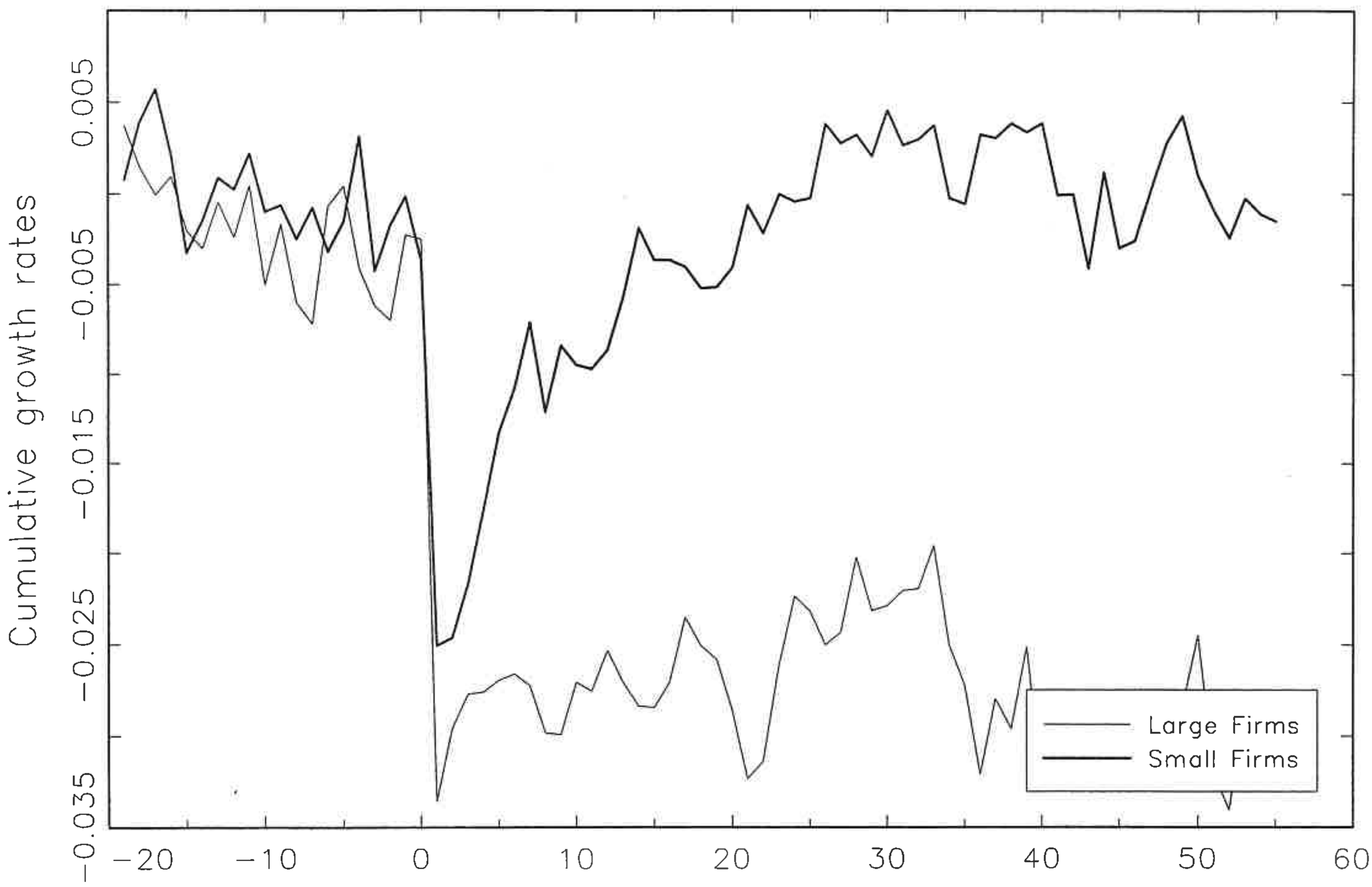


Figure 8

increase in interest rate from 1% to 3% – high leverage economy

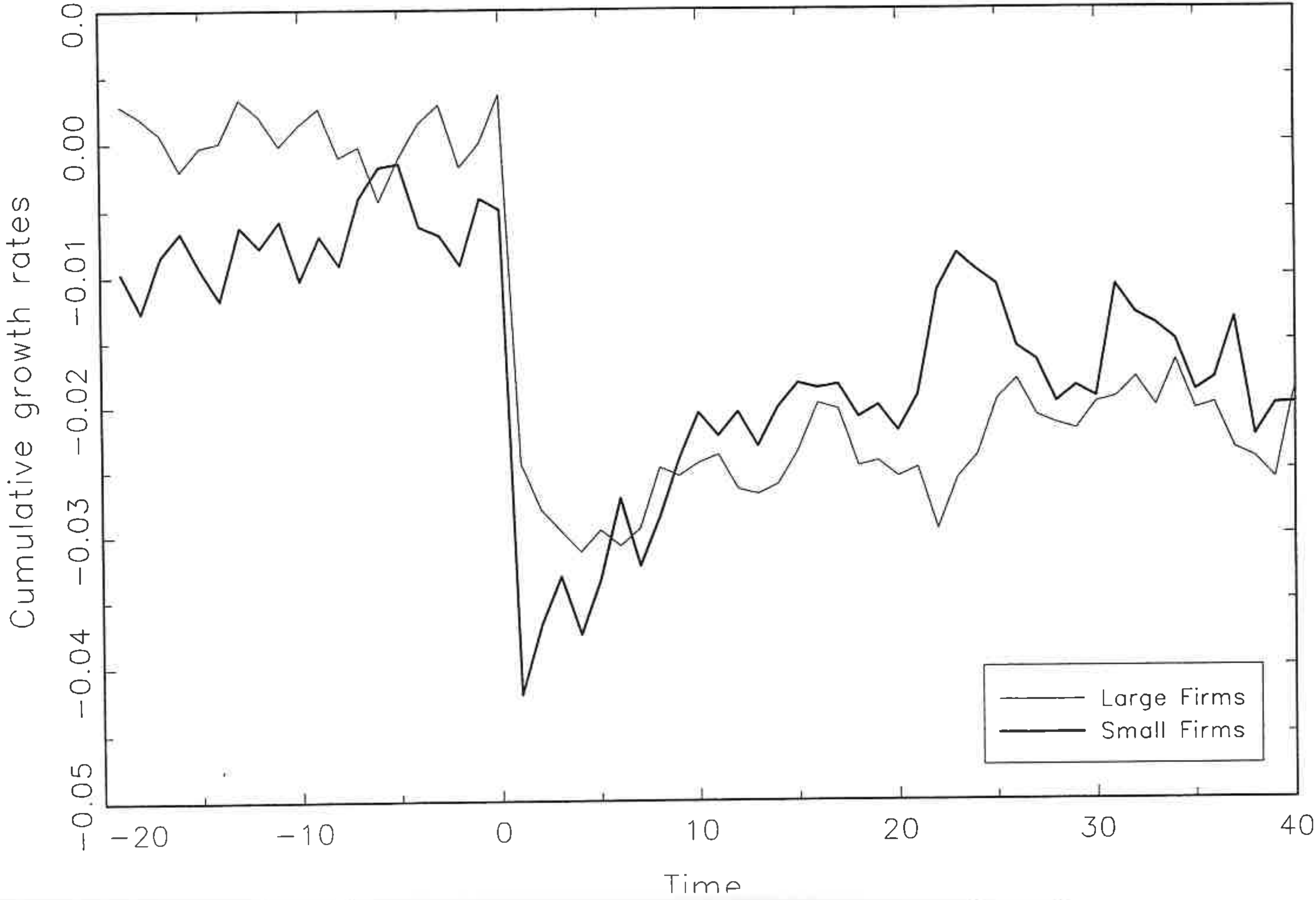


Figure 9

Unanticipated temporary decrease in output of 10%

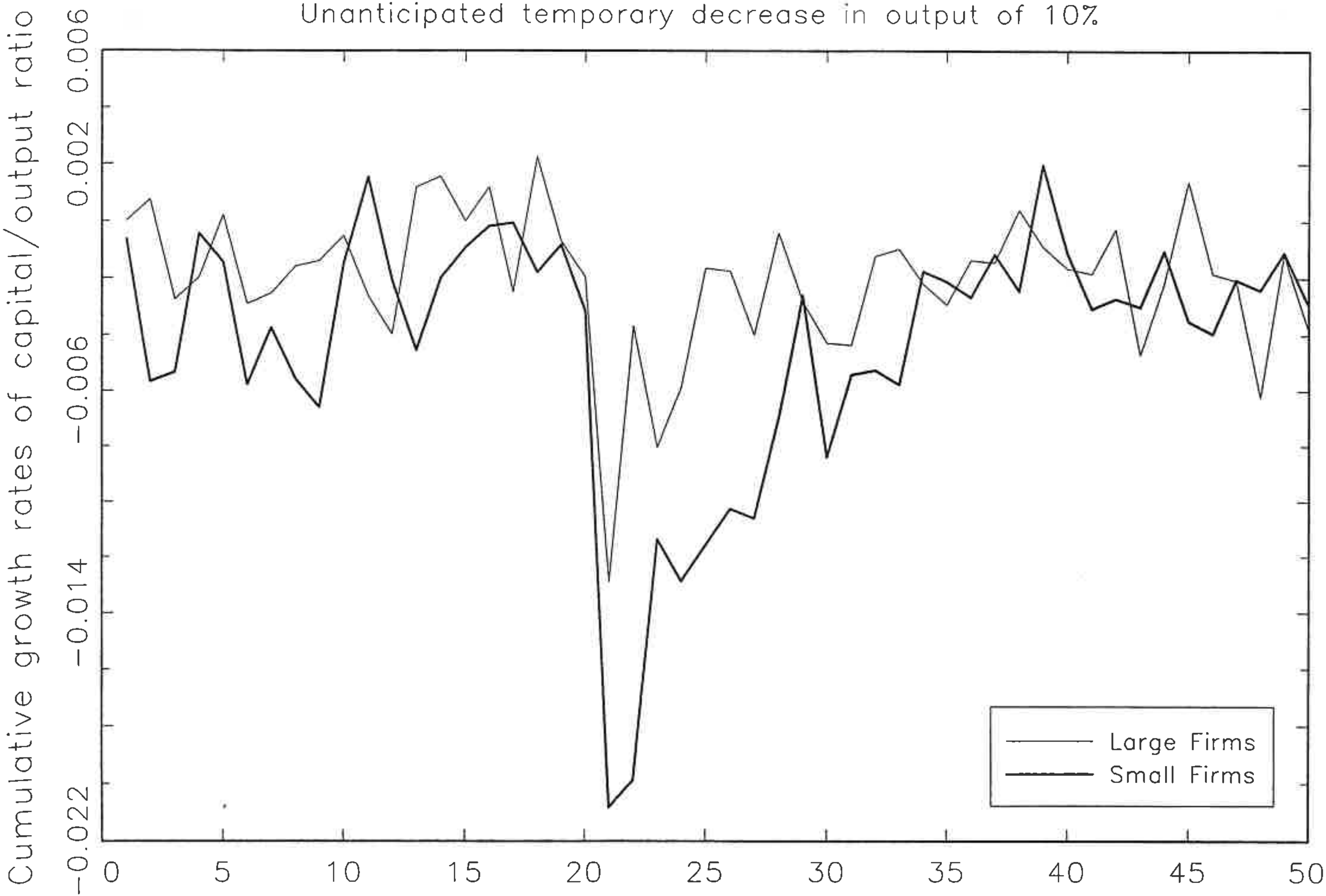
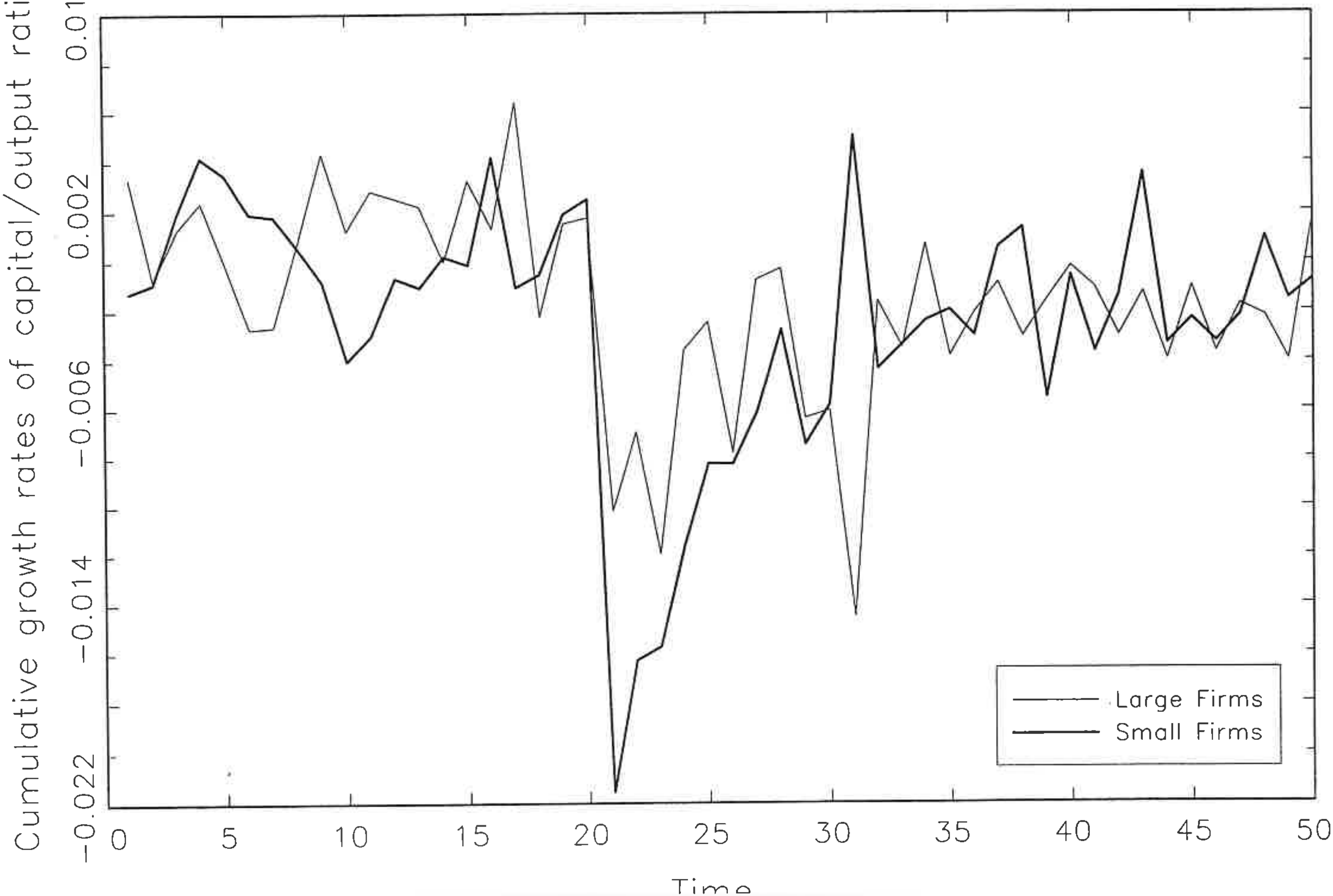


Figure 10

Unanticipated permanent decrease in tau from 0.3 to 0.2



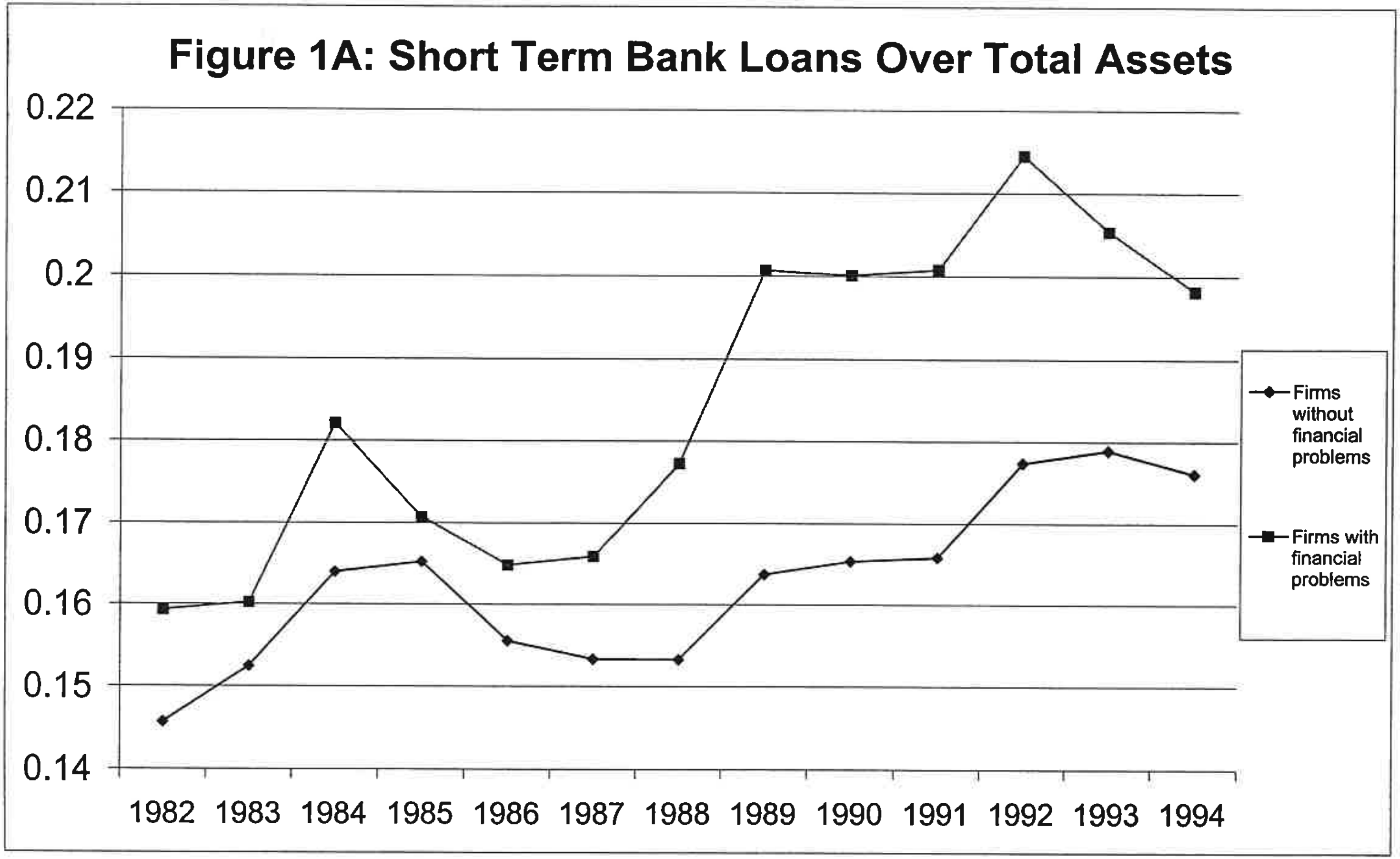


Figure 1A: Short Term Bank Loans Over Total Assets
0.22
0.21
0.2
0.19
0.18
0.17
0.16
0.15
0.14
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
Firms without financial problems
Firms with financial problems

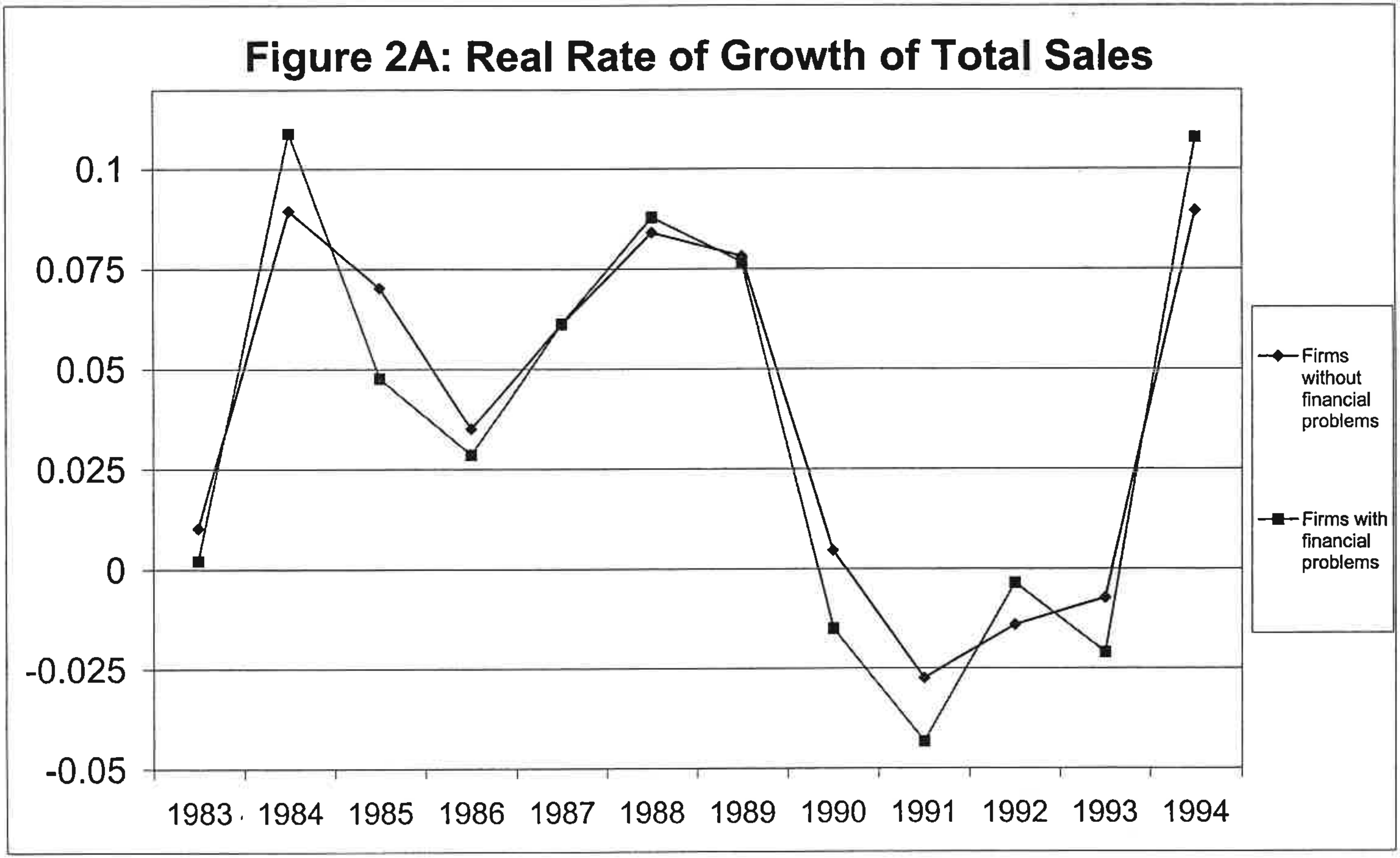
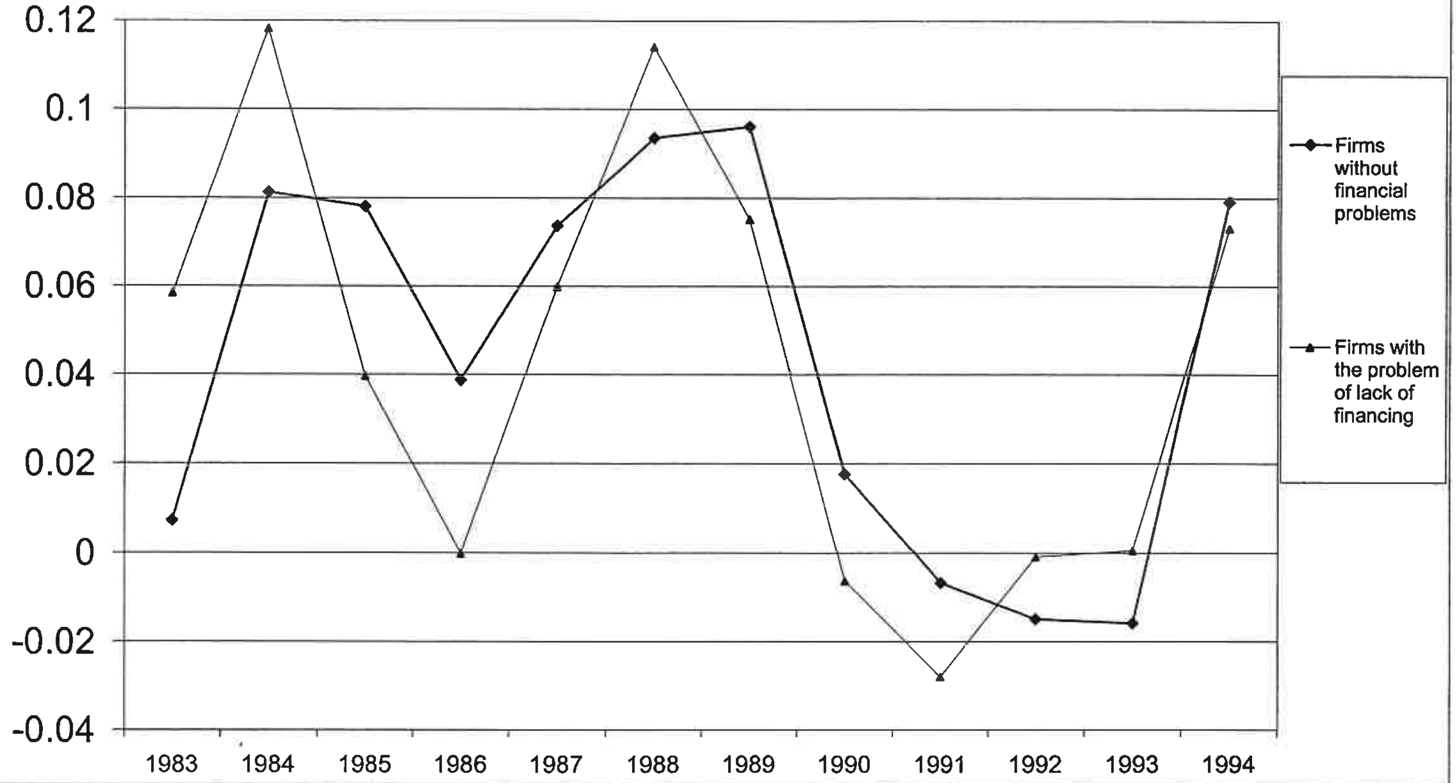


Figure 2A: Real Rate of Growth of Total Sales
0.1
0.075
0.05
0.025
0
-0.025
-0.05
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
Firms without financial problems
Firms with financial problems

Figure 3A: Real Rate of Growth of Total Sales
(only firms with positive income in years '89,'90 and '91 included)



FINANCIAL MARKETS GROUP
DISCUSSION PAPER SERIES

310 Ortalo-Magné, François, and Sven Rady, "Boom In, Bust Out: Young Households and the Housing Price Cycle", November 1998

311 Timmermann, Allan, "Structural Breaks, Incomplete Information and Stock Prices", November 1998

312 Sentana, Enrique, "Least Squares Predictions and Mean-Variance Analysis", January 1999

313 Goodhart, Charles A. E. and Haizhou Huang, "A Model of the Lender of the Last Resort", January 1999

314 Schellekens, Philip, "Conservatism at No Real Cost: Monetary Policy Delegation Reconsidered", January 1999

315 Hoffman-Burchardi, Ulrike, "Corporate Governance Rules and the Value of Control - A Study of German Dual-class Share", February 1999

316 Hoffmann-Burchardi, Ulrike, "Clustering of Initial Public Offerings, Information Revelation and Underpricing", February 1999

317 Vitale, Paolo, "Foreign Exchange Intervention and Macroeconomic Stability", March 1999

318 Alger, Ingela and Ching-to Albert Ma, "Moral Hazard, Insurance, and Some Collusion", March 1999

319 Zigrand, Jean-Pierre, "Arbitrage and Endogenous Market Integration", March 1999

320 Danielsson, Jon and Richard Payne, "Real Trading Patterns and Prices in Spot Foreign Exchange Markets", March 1999

321 Ellul, Andrew, "Inter-market Price and Volatility Impacts Generated by Large Trades: The Case of European Cross-quoted Securities", May 1999

322 Pesaran, Hashem and Allan Timmermann, "A Recursive Modelling Approach to Predicting UK Stock Returns", May 1999

323 Timmermann, Allan, "Moments of Markov Switching Models", May 1999

324 Koopman, Siem Jan and Hung Neng Lai, "Modelling Bid Ask Spreads in Competitive Dealership Markets, May 1999

325 Dahlström, Tom and Pierre Mella-Barral, "Corporate Walkout Decisions and the Value of Default", May 1999

326 Carletti, Elena, "Bank Moral Hazard and Market Discipline", May 1999

327 Freixas, Xavier, "Optimal Bail Out Policy, Conditionality and Creative Ambiguity",May 1999

328 Dessí, Roberta, "Financing Entrepreneurs: Optimal Contracts and the Role of Intermediaries", June 1999

329 Brunnermeier, Markus K. and Clemens Grafe, "Contrasting Different Forms of Price Stickiness: An Analysis of Exchange Rate Overshooting and the Beggar Thy Neighbour Policy", June 1999

330 Hoffmann-Burchadi, Ulrike, "Barbarians in Chains - Takeover Regulation and Minority Shareholder Wealth", July 1999

331 Black, Jane and Ian Tonks, "Times Series Volatility of Commodity Futures Prices", August 1999

332 Friederich, Sylvain, Alan Gregory, John Matatko and Ian Tonks, "Stock Price Patterns Around the Trades of Corporate Insiders on the London Stock Exchange", September 1999

333 Nier, Erlend, "Equity Finance, Adverse Selection and Product Market Competition", September 1999

334 Bhattacharya, Sudipto and Giovanna Nicodano, "Insider Trading, Investment, and Liquidity: A Welfare Analysis", September 1999

335 Perez-Quiros, Gabriel and Allan Timmermann, "Firm Size and Cyclical Variations in Stock Returns", September 1999

336 Goodhart, Charles A. E. and Haizhou Huang, "A Simple Model of an International Lender of Last Resort" October 1999

337 Willer, Dirk, "If You Can Not Beat Them, Join Them: Management Behaviour in the Absence of State Guaranteed Shareholder Rights", October 1999

Subject to availability, copies of these Discussion Papers can be obtained from the Financial Markets Group (Room G307, 0171 955 7002). Alternatively visit the FMG website ***http://cep.lse.ac.uk/fmg/*** where some papers are available for downloading.