SYSTEMIC RISK IN FINANCIAL NETWORKS REVISITED: THE ROLE OF MATURITY

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Summary

- N banks, two period claims on each other
  - summarized in vectors $F_{i\rightrightarrows}$ (with elements $F_{i\to j}$)
  - period 2 cash flows y_i

Summary

- N banks, two period claims on each other
  - summarized in vectors $F_{i\rightrightarrows}$ (with elements $F_{i\rightarrow j}$)
  - period 2 cash flows y_i
- There is an interim period, where i might be subject to liquidity shock:
  - raise short term credit against pledgeable period 2 assets:

$$\sigma_i \ell \leq \theta y_i + \sum_{j, j \neq i} \frac{F_{j\rightarrow i}}{F_{j\rightrightarrows}} R_{j\rightrightarrows} \tag{1}$$

  - θy_i seizable part,
  - $R_{j\rightrightarrows}$ is what j will pay-out to its creditors
    - $F_{j\rightrightarrows}$ if no default
    - $\frac{F_{j\rightarrow i}}{F_{j\rightrightarrows}}$ is pro-rata share in case of default.
  - if liquidated, $(1-\theta) y_i$ is lost
  - there is no cost of default: $R_{j\rightrightarrows} < F_{j\rightrightarrows}$.

- Solution: fixed point in $R_{i\rightrightarrows}$, e.g. with three banks

$$R_{1\rightrightarrows} = \max\{0, \min\{\theta y_i - \sigma_i \ell + \frac{F_{2\to 1}}{F_{2\rightrightarrows}} R_{2\rightrightarrows} + \frac{F_{3\to 1}}{F_{3\rightrightarrows}} R_{3\rightrightarrows}, F_{1\rightrightarrows}\}\}$$

$$R_{2\rightrightarrows} = \max\{0, \min\{\theta y_i - \sigma_i \ell + \frac{F_{1\to 2}}{F_{1\rightrightarrows 2}} R_{1\rightrightarrows} + \frac{F_{3\to 2}}{F_{3\rightrightarrows}} R_{3\rightrightarrows}, F_{1\rightrightarrows}\}\}$$

$$R_{3\rightrightarrows} = \max\{0, \min\{\theta y_i - \sigma_i \ell + \frac{F_{1\to 3}}{F_{1\rightrightarrows}} R_{1\rightrightarrows} + \frac{F_{2\to 1}}{F_{2\rightrightarrows}} R_{2\rightrightarrows}, F_{1\rightrightarrows}\}\}$$

where the $\max$ is the liquidation decision and the $\min$ is the default decision.

- Main observations:
  1. in the interim period, being owed helps as one can promise that received cash-flow to short-term creditors as short-term credit is senior. (compare (1) to $F_{j\rightrightarrows} = R_{j\rightrightarrows} = 0$) $\Longrightarrow$ an ex-ante more connected network, larger $F_{j\rightrightarrows} = F$ helps: as whoever is hit will be owed a lot
  2. as a consequence, the fact that gross positions are large compared to net positions makes sense
  3. Symmetric network tend to be worse than exponential: let us see why...

Example I: 2 shocks $\rightarrow$ 2 defaults in complete network

- $\theta y = 1$, $\ell = \frac{8}{5}$, symmetric network $\Longrightarrow \frac{F_{j\rightarrow i}}{F_{j\rightrightarrows}} = \frac{1}{2}$ for each bank. Suppose 1 and 2 are shocked: $\sigma_1 = \sigma_2 = 1, \sigma_3 = 0$

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$$R_{1\rightrightarrows} = \max\{0, \min\{-\frac{3}{5} + \frac{1}{2}R_{2\rightrightarrows} + \frac{1}{2}R_{3\rightrightarrows}, F\}\}$$
$$R_{2\rightrightarrows} = \max\{0, \min\{-\frac{3}{5} + \frac{1}{2}R_{1\rightrightarrows} + \frac{1}{2}R_{3\rightrightarrows}, F\}\}$$
$$R_{3\rightrightarrows} = \max\{0, \min\{1 + \frac{1}{2}R_{1\rightrightarrows} + \frac{1}{2}R_{2\rightrightarrows}, F\}\}$$

- Problem: shocked banks need $\frac{3}{5}$ each
- if 1 from bank 3 distributed equally $\rightarrow$ none of them survive

$\Rightarrow$ fixed point:

$$R_{1\rightrightarrows} = \max\{0, \min\{-\frac{3}{5} + \min\left(\frac{1}{2}, \frac{F}{2}\right), F\}\} = 0$$

$$R_{2\rightrightarrows} = \max\{0, \min\{-\frac{3}{5} + \min\left(\frac{1}{2}, \frac{F}{2}\right), F\}\} = 0$$

$$R_{3\rightrightarrows} = \max\{0, \min\{1, F\} = \min(1, F)$$

Example II: 2 shocks $\rightarrow$ 1 default in exponential network

- Suppose

$$F = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & \frac{1}{2} \\ 1 & \frac{1}{2} & 0 \end{bmatrix}$$

the first bank is owed more than the second, which is owed more than the last.

Example II: 2 shocks → 1 default in exponential network

- Suppose

$$F = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & \frac{1}{2} \\ 1 & \frac{1}{2} & 0 \end{bmatrix}$$

  the first bank is owed more than the second, which is owed more than the last.

- the pro-rata matrix is

$$\frac{F_{j \to i}}{F_{j \rightrightarrows}} = \begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{4}{5} & 0 & \frac{1}{5} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

- bank 1 is preferred": both bank 2 and bank 3 transfers larger share than to other bank. Similarly, Bank 2 is preferred over 3 by bank 1.

- ...implying a fixed point problem, when bank 1 and 2 are shocked

$$R_{1\rightrightarrows} = \max\{0, \min\{\theta y_i - \sigma_i \ell + \frac{F_{2\rightarrow 1}}{F_{2\rightrightarrows}} R_{2\rightrightarrows} + \frac{F_{3\rightarrow 1}}{F_{3\rightrightarrows}} R_{3\rightrightarrows}, F_{1\rightrightarrows}\}\}$$

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$$R_{3\rightrightarrows} = \max\{0, \min\{\theta y_i - \sigma_i \ell + \frac{F_{1\rightarrow 3}}{F_{1\rightrightarrows}} R_{1\rightrightarrows} + \frac{F_{2\rightarrow 1}}{F_{2\rightrightarrows}} R_{2\rightrightarrows}, F_{1\rightrightarrows}\}\}$$

- ...implying a fixed point problem, when bank 1 and 2 are shocked,

$$R_{1\rightrightarrows} = \max\{0, \min\{-\frac{3}{5} + \frac{4}{5}R_{2\rightrightarrows} + \frac{2}{3}R_{3\rightrightarrows}, 3\}\}$$
$$R_{2\rightrightarrows} = \max\{0, \min\{-\frac{3}{5} + \frac{2}{3}R_{1\rightrightarrows} + \frac{1}{3}R_{3\rightrightarrows}, \frac{5}{2}\}\}$$
$$R_{3\rightrightarrows} = \max\{0, \min\{1 + \frac{1}{3}R_{1\rightrightarrows} + \frac{1}{5}R_{2\rightrightarrows}, \frac{3}{2}\}\}$$

- now most of the excess resources of 3 goes towards bank 1, saving it, and letting bank 2 to be liquidated with even less resources (which does not cause a social loss)

- resulting in fixed point

$$R_{1\rightrightarrows} = \max\{0, \min\{-\frac{3}{5} + \frac{2}{3}\frac{36}{35}, 3\}\} = \frac{3}{35}$$
$$R_{2\rightrightarrows} = \max\{0, \min\{-\frac{3}{5} + \frac{2}{3}\frac{3}{35} + \frac{1}{3}\frac{36}{35}, \frac{5}{2}\}\} = 0$$
$$R_{3\rightrightarrows} = \max\{0, \min\{1 + \frac{1}{3}\frac{3}{35}, \frac{3}{2}\}\} = \frac{36}{35}$$

- note that all 3 defaults in period 2, (which, again, does not cause a social loss)

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- note that all 3 defaults in period 2, (which, again, does not cause a social loss)
- Similarly, if we were to shock bank 1 and 3, only bank 3 would be liquidated, and if 2 and 3 are shocked, only 3 is liquidated. (very clever!)

Comments

- A clever idea (gross vs net positions, asymmetry)
- A beautifully written (smart structure, very clear explanations)

The missing element

- one missing element: optimizing agents over debt structure $\Longrightarrow$ a Nash equilibrium
- This is missing from most of this literature since Eisenberg and Noe (2001)
- Perhaps it is doable at least in the example: Following Holmstrom and Tirole

- consider something like this:
  - suppose ex-ante identical banks endowed with A, and subject to pledgeability constraint
  - choose how much to invest (generating y)
  - and how much to borrow and lend (generating F)
  - understanding that they will get back only R after the liquidity shock
  - (with a particular bargaining mechanism over surplus)

- Is it somehow possible to generate an exponential network of F in equilibrium?
  - if in equilibrium bank 1 is liquidated less often, perhaps bank 2 and bank 3 indeed prefers to lend to bank 1
  - if bank 1 has a larger balance sheet, perhaps it wants to lend more to bank 2 and 3
  - it would be quite interesting if market mechanism would support the exponential network.
- How does this interact with heterogeneity in y?