Contagion and Equilibria in Diversified Financial Networks

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Overview

- Networks are a prevalent characteristic of financial systems
- Companies and countries own stakes in their counterparts
- Networks can foster contagions, leading to ripple effects
- Analyzing diversified networks is important to understand *how shocks proliferate* and to assess the *stability of economic systems*.

Overview (cont'd)

- Key friction: Diversification improves resilience but increases the number of channels through which contagions can spread.
- Paper studies two types of diversified networks:
  - *Perfectly diversified network* with non-random cross-holdings
  - *Networks with random cross-holdings*
  - Equilibria are similar in both networks
- Derives dynamic equation for firm values and studies basins of attraction
- Firm valuations are classified into optimal, safe, and risky regions

Model

- Network of n *interconnected firms* with *cross-holdings* $\mathbf{C} = [C_{ij}]_{i,j=1}^{n}$
- $C_{1j} + \cdots + C_{nj} < 1$, and the remaining fraction held by outside investors
- Firms have cash endowments $(e_1, \ldots, e_n)^\top \in \mathbb{R}_+^n$
- At some stage, endowments are chosen to be equal: $e_i = e$

Model (cont'd)

- Following Elliott, Golub, and Jackson (2014), firm values satisfy

$$V = e + \mathbf{C}V - \beta 1_{\{V \leq \tau \mathbf{1}\}}$$

- This equation often replaced with *"forward" dynamics*

$$\frac{dV_t}{dt} = e - (I - \mathbf{C})V_t - \beta 1_{\{V_t \leq \tau \mathbf{1}\}}$$

- The dynamics for V_t is referred to as *natural dynamics*

- The equation is used to study contagions and solve for eq-m

Model (cont'd)

- Paper provides solution techniques in the case of *regular clique* in which

$$\mathbf{C} = \mathbf{C}^0 := \frac{c}{n}\mathbf{1}\mathbf{1}^\top$$

- In this case, asset holdings are perfectly diversified
- Also solve with *random networks* where portfolios are i.i.d. across firms
- Portfolios are drawn from distributions that have some special structure
- Equilibria turn out to be similar to those in the regular clique
- *Main result*: *random clique* equilibria $\to$ *regular clique* as n grows

Comment 1

- The forward dynamics for V_t is *exogenous*
- This dynamics might be useful as
  - solution method for finding stationary equilibria
  - refinement of stationary equilibria
- However, for studying probabilities of recovery and collapse and modeling dynamics of values, further economic motivation is desirable
- One alternative is *backward equations*

Comment 1 (cont'd)

- Pricing relations are typically *backward equations* such as

$$V_t = \frac{\mathbb{E}_t[\max(e_{t+1} + \mathbf{C}_t V_{t+1} - \beta 1_{\{V_{t+1} \leq \tau \mathbf{1}\}}; 0)]}{1 + r}$$

- Equation gives the dynamics of *market* rather than *book* values
- Dependence of $1_{\{V_{t+1} \leq \tau \mathbf{1}\}}$ on market value is consistent with evidence
- Same stationary equilibrium when $r = 0$ and e are constant
- Backward equations are also more stable

Comment 1 (cont'd)

- Solution approach: iterate backwards from large horizon T and some V_T
- One can then work with *transition probabilities* $\text{Prob}_t(V_{t+1}|V_t)$
  - (e.g., assume two-state Markov chain for e_H and e_L)
- These probabilities are closest we can get to forward dynamics
- They can be used to compute probabilities of *recovery* or *collapse*

Comment 2

- Firms treat cash positions and firm values as deterministic
- Hence, shocks to firm values V and cash positions e are *unanticipated*
- More realistically, *investors might know the distributions* of shocks
- Firms would solve portfolio choice problems to address contagions
- Consequently
  - Valuation equations with expectations seem more intuitive
  - Some reduced form portfolio optimization would be an interesting extension

Comment 2 (cont'd)

- Ideally, firms *should be allowed to optimize*
- Suppose firms are ex-ante identical and firms pay stochastic dividends
- Let firms have the same expected utility function over next period value

$$\mathbb{E}_0[u(e_i + C_i^\top V_1 - \beta 1_{\{V_{i,t} \leq \tau \mathbf{1}\}})]$$

- Firms choose portfolios $C_i = c\mathbf{1}/n \Rightarrow$ "regular clique" with *endogenous* c
- Is there a similar argument justifying *"random clique"*?

Comment 2 (cont'd)

- Firms have different cash holdings e_i in the model
  - Firms with more cash less likely to default
  - Therefore, firms are heterogeneous in term of risks
  - Hence, portfolios will load differently (and non-randomly) on firms
- How to make this consistent with "regular clique" or "random clique"?

Comment 3

- Elliott, Golub, and Jackson (2014) interpret assets as debt

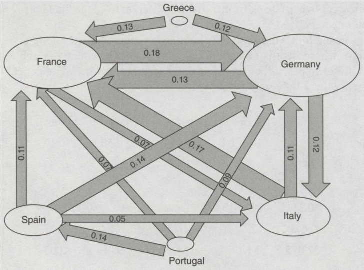

FIGURE 8. INTERDEPENDENCIES IN EUROPE

Notes: The matrix **A**, describing how much each country ultimately depends on the value of others' debt. The widths of the arrows are proportional to the sizes of the dependencies, with dependencies less than 5 percent excluded; the area of the oval for each country is proportional to its underlying asset values.

Comment 3 (cont'd)

- The paper interprets assets as equity
- Firms do hold equity of other firms
- How widespread or systemically important are cross-holdings?
- Do big companies hold shares of each other?
- Does network resemble random clique?

Comment 4

- Results seem to depend on whether states $\{V_t \leq \tau \mathbf{1}\}$ are absorbing
- Model seems to assume that they are not
- Firms may not recover from from default and close down
- This might affect the survival results and also portfolio choice of firms

Conclusion

- Main suggestions:
  - Backward equations for value dynamics
  - Incorporate portfolio choice
  - More motivating examples