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Putting the Price in Asset Pricing

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Putting the Price in Asset Pricing

Abstract

We propose a novel way to estimate a portfolio's *abnormal price*, the percentage gap between price and the present value of dividends computed with a chosen asset pricing model. Our method, based on a novel identity, resembles the time-series estimator of abnormal returns, avoids the issues in alternative approaches, and clarifies the role of risk and mispricing in long-horizon returns. We apply our techniques to study the cross-section of price levels relative to the CAPM, finding that a single characteristic dubbed *adjusted value* provides a parsimonious model of CAPM-implied abnormal price.

Keywords: price level, mispricing metric, novel identity, stochastic discount factor, CAPM

JEL classification: G12, G14, G32

How well does an asset pricing model explain the observed *price levels* of stocks? And which stock characteristics signal model-specific *abnormal price*, the deviation of price from the present value of future dividends?[1] These are central questions in finance, since stock price levels can drive the financing and investment decisions of firms as well as the portfolio decisions of long-term buy-and-hold investors. Understanding stock price levels can also reveal whether abnormal returns on a given expected return anomaly are earned on price convergence or divergence.

The techniques with which to study asset price levels at our disposal are extremely limited. Since discount rates vary over time, abnormal price cannot simply be inferred from short-horizon abnormal returns (alphas), the literature's traditional focus. Cohen, Polk, and Vuolteenaho (2009) proposed an approximate estimator of abnormal price that nonetheless relies on strong assumptions about the data. An estimator based on directly discounting subsequent dividends and a terminal cash flow, as in van Binsbergen, Boons, Opp, and Tamoni (2023), can be significantly biased and makes statistical inference challenging due to a serious overlapping observations issue.

Our contribution is to develop a new estimator of abnormal price. The estimator resembles the time-series alpha estimator and avoids the issues in alternative approaches. Applying our techniques, we document empirical facts about the cross-section of stock price levels. (i) The CAPM explains some, but not all, of the cross-sectional variation in stock price levels. (ii) Net issuance, investment, and beta predict significant CAPM-implied abnormal price. (iii) The classic momentum strategy bets on overpriced stocks. (iv) A single characteristic that we dub adjusted value provides a parsimonious model of CAPM abnormal price.

We define a portfolio's average formation-period δ—which we also call delta, abnormal price, or simply mispricing—as follows:

$$\delta \equiv E\left[\frac{P_t - V_t}{P_t}\right] \quad \text{with} \quad V_t = E_t\left[\sum_{j=1}^{\infty} \widetilde{M}_{t,t+j} D_{t+j}\right], \tag{1}$$

where t is the portfolio-formation period, P_t is the portfolio's market price at t, and V_t is the present

[1] Abnormal price could signal either a misspecification of the model of risk or an actual distortion in asset price levels. That is, like abnormal returns, abnormal price is subject to the joint hypothesis problem emphasized in Fama (1970).

value of post-formation dividends $\left\{D_{t+j}\right\}_{j=1}^{\infty}$ with respect to the candidate cumulative stochastic discount factor (SDF) $\widetilde{M}_{t,t+j}$ implied by a chosen asset pricing model of risk. Abnormal price (δ) is a price-level analogue of short-horizon abnormal return (α) in a return analysis and is specific to the asset pricing model used to specify $\widetilde{M}$, just like α.[2]

We estimate δ via a novel exact identity expressing today's abnormal price using post-formation buy-and-hold excess returns. For intuition, consider a portfolio that is currently overpriced relative to an asset pricing model. If that overpricing undergoes a subsequent "correction," the capital gain component of future returns must be low on a risk-adjusted basis. If, on the other hand, the price remains elevated—which our identity also allows for—the dividend yield component of future returns must be low on a risk-adjusted basis. In both cases, we expect initial overpricing to be reflected in lower subsequent risk-adjusted returns, an idea our identity formalizes.

Applying our novel identity, we derive a calendar-time estimator of δ, denoted $\widehat{\delta}$, based on the portfolio's post-formation buy-and-hold excess returns:

$$\widehat{\delta}=\frac{1}{T}\sum_{t=1}^{T}\widetilde{\delta}_t \quad \text{with} \quad \widetilde{\delta}_t=-\sum_{j=1}^{J}\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}, \tag{2}$$

where t here indexes the month in which returns occur (i.e., calendar time) and the $(t-j)$ argument in the subscript indicates that the particular cumulative capital gain ($\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$) or one-month excess return ($R^e_{(t-j),t}$) is earned on buying and holding a portfolio formed at time $t-j$. Using $J=15$ years (180 months) turns equation (2) into an estimator of δ, whereas using $J=1$ month reduces it to the conventional alpha estimator, $\widehat{\alpha}=\frac{1}{T}\sum_{t=1}^{T}\widetilde{M}_tR^e_t$.[3]

Our abnormal price estimator in equation (2) has three advantages relative to estimators in the literature. First, estimating $\widehat{\delta}$ with excess (rather than gross) returns prevents measurement errors in the candidate $\widetilde{M}$ from materially biasing $\widehat{\delta}$, since it forces $\widetilde{M}$ in equation (2) to act mainly as a risk adjustment rather than a time discount. This benefit is similar to the way using excess returns in the expected return framework allows researchers to focus on risk adjustment rather than time

[2]This perspective on $\widetilde{M}$ is similar to that of Hansen and Jagannathan (1991, 1997): δ_t measures the abnormal component of P_t relative to a potentially misspecified candidate model of risk, $\widetilde{M}$.

[3]This correspondence holds after a sign flip and an interest rate adjustment $1/\overline{\widetilde{M}_t}$.

discount (p.9 of Cochrane (2009)). Second, equation (2) avoids using overlapping observations and minimizes serial correlations, since each $\widetilde{\delta}_t$ combines the contemporaneous time-t returns on portfolios formed in prior periods ($t-1$, $t-2$, $t-3$, ...). Third, our estimator implies a natural measure of long-horizon return that can be decomposed into "risk" and "abnormal price" (δ) components, facilitating the parallel between the expected return and price frameworks. In contrast, the aforementioned dividend-based approach of van Binsbergen et al. (2023) and others leads to a bias, has statistical inference that is challenging due to monthly discounted sums of future dividends resulting in overlapping 15-year windows, and does not imply a natural measure of long-horizon risk and return.

What is the statistical power of our price-level test? Since each time-series delta observation $\widetilde{\delta}_t$ in equation (2) is a discounted cross-sectional sum of 15 years of post-formation returns, our Monte Carlo analysis confirms that in order to reject the null of no mispricing, δ (abnormal price) must be roughly 10 times larger in absolute value than the annualized α (abnormal return) required for significance in a traditional return test. For example, if return volatility were such that an annualized α of 2 to 4 percent achieves significance, we need a δ of roughly 20 to 40 percent to find significance.[4]

Applying equation (2), we study cross-sectional variation in stock prices with respect to the CAPM, providing a foundation for multifactor refinements in subsequent research. Following Korteweg and Nagel (2016), we specify a loglinear candidate SDF and choose the SDF parameters to make the market portfolio's in-sample abnormal price (δ) and abnormal return (α) zero, just as the time-series *return* regression restricts a model's factor portfolios to have zero in-sample α's. Our method does not assume the portfolio's factor betas to be constant over the post-formation months and instead allows these betas—computed implicitly as part of estimation—to vary over different post-formation months. Based on Monte Carlo analysis, we recommend GMM standard errors with a Newey-West bandwidth of 24 months.

Our initial analysis focuses on two signals recently studied in the price-level context: book-

[4]Interestingly, this finding provides conceptual and statistical grounds supporting Black (1986)'s conjecture that markets are efficient if prices are within a factor of two of intrinsic value.

to-market (B/M) and *quality* which, according to Golubov and Konstantinidi (2019) and Asness, Frazzini, and Pedersen (2019) respectively, predict α precisely because they proxy for δ. In isolation, however, we find that neither is a statistically significant signal of CAPM δ (see Cohen, Polk, and Vuolteenaho (2009) for similar evidence on B/M). A quintile sort on *quality* generates an estimated δ variation of just 11.4 percentage points, whereas B/M generates a larger but statistically insignificant spread of 27.2 percentage points.

Next, we combine information about price, profitability, and CAPM risk to develop a simple novel characteristic signaling abnormal price.[5] This signal—dubbed *adjusted value*—generates variation in δ that is economically large at 51.9 percentage points and statistically significant. Hence, adjusted value generates the sort of price variation that should attract long-term buy-and-hold CAPM investors and challenge researchers developing asset pricing models of price levels.

Turning to portfolios sorted on seven other characteristics (net issuance, investment, accruals, beta, size, momentum, and profitability), we document four main findings. First, net issuance is a robust signal of abnormal price relative to the CAPM, consistent with firm managers timing the stock market based on the perceived mispricing of their stocks relative to the CAPM. Second, investment and beta also generate price-level variation not explained by the CAPM. Third, on average, momentum bets on overpriced stocks despite their short-term CAPM alpha being positive. Finally, adjusted value subsumes the ability of net issuance and investment to generate significant δ variation, indicating that adjusted value is a parsimonious signal of CAPM abnormal price.

Relation to the literature. This paper applies a novel identity linking abnormal price to subsequent returns to develop an estimator of abnormal price resembling the time-series regression for abnormal returns. Our novel identity mapping model-specific abnormal price to subsequent returns allows one to estimate a portfolio's unconditional abnormal price without a structural assumption on the evolution of abnormal returns, clearly departing from the existing identities in Cohen, Polk, and Vuolteenaho (2009) or van Binsbergen and Opp (2019). Our identity achieves this benefit within a general SDF framework by putting market price in the denominator of our

[5] Our new signal draws from the ideas developed in Frankel and Lee (1998), Piotroski (2000), Vuolteenaho (2002), Piotroski and So (2012), Novy-Marx (2013), Asness, Frazzini, and Pedersen (2019), and several others. The primary advantage of our proposed measure is its simplicity.

definition of mispricing, $\delta_t = 1 - V_t/P_t$, which allows subsequent returns—which are inversely related to initial price—to appear in the numerator of the identity in an additively separable manner. Furthermore, unlike the cumulative abnormal return (CAR) or buy-and-hold abnormal return (BHAR) constructs, our delta measure has an exact interpretation as the price deviation from the present value of cash flows, or, equivalently, abnormal price from the perspective of buy-and-hold investors.

On the empirical front, Cohen, Polk, and Vuolteenaho (2009) study the cross-section of price levels, focusing on B/M-, size-, and beta-sorted portfolios, and apply the calendar-time reformulation to price-level analysis to avoid the overlapping observations issue. Cho (2020) documents that institutional trading of return anomalies can generate abnormal price with respect to a baseline model, as trading by those intermediaries turns alphas into higher betas with intermediary factors. Belo, Xue, and Zhang (2013) use the relation between Tobin's q and investment under constant returns to scale to study the cross-section of price levels. Belo, Gala, Salomao, and Vitorino (2022) introduce labor and heterogeneous capital inputs to the Belo, Xue, and Zhang framework to structurally decompose the sources of firm value.

Chernov, Lochstoer, and Lundeby (2021) discipline popular linear factor models by requiring them to price their own factors at multiple horizons. Keloharju, Linnainmaa, and Nyberg (2021) document that discount rates on stocks tend to converge over time. van Binsbergen et al. (2023) estimate abnormal price for 57 anomalies and correlates those estimates with investment as done in Polk and Sapienza (2009). They estimate δ based on post-formation dividends and a terminal value, which, as mentioned above, suffers from a potentially large bias from misspecifying the yield curve component of the candidate SDF and is exposed to serious autocorrelation issues. Chen and Kaniel (2021) develop a new methodology to study long-horizon expected returns.

Organization of the paper. After explaining the drawbacks of existing approaches (Section 1), we develop a novel identity (Section 2) and a new estimator of abnormal price (Section 3). We then present data and our empirical results on B/M, *quality*, and *adjusted value* (Section 4), extend our analysis to other characteristic sorts (Section 5), and conclude (Section 6).

1 Why a New Estimator of Abnormal Price?

This section discusses the drawbacks of existing techniques that our novel estimator addresses. We begin by specifying the asset pricing environment.

1.1 Asset pricing environment

Consider a portfolio formed at time t with post-formation dividends $\left\{D_{t+j}\right\}_{j=1}^{\infty}$ and a *candidate* stochastic discount factor (SDF) $\left\{\widetilde{M}_{t+j}\right\}_{j=1}^{\infty}$ that may or may not be the true SDF. We want to compare the portfolio's market price P_t to the present value of post-formation dividends discounted using $\widetilde{M}$, which we denote by V_t:

$$V_t = \sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{t,t+j} D_{t+j}\right], \tag{3}$$

where $\widetilde{M}_{t,t+j} = \Pi_{s=1}^{j} \widetilde{M}_{t+s-1,t+s}$ is the cumulative candidate SDF. There is a base asset whose return, denoted R_b, satisfies the fundamental asset pricing equation with respect to $\widetilde{M}$ in all periods:

$$E_{t+j-1}\left[\widetilde{M}_{t+j}\left(1+R_{b,t+j}\right)\right] = 1 \quad \forall j. \tag{4}$$

In our particular CAPM implementation, a natural choice for the base asset b is the market portfolio itself.[6] Hence, we compute excess returns with respect to returns on the market portfolio rather than the Treasury bill.

We define (conditional) *abnormal price*, denoted δ_t, as the percentage deviation of price from present value:

$$\delta_t = \frac{P_t - V_t}{P_t}. \tag{5}$$

Hence, $\delta_t > 0$ if the portfolio is overpriced, and $\delta_t < 0$ if it is underpriced. The range of values δ_t can take is $(-\infty, 1]$, the opposite of the range of abnormal returns, $[-1, \infty)$. Define *log abnormal*

[6] For a multifactor implementation of our estimator, future research may need to think carefully about the most appropriate base asset, given that Chernov, Lochstoer, and Lundeby (2021) show that popular unconditional multifactor models cannot simultaneously price the short- and long-horizon returns on their own factors.

price as

$$\delta_t^{log} = \log(P_t) - \log(V_t). \tag{6}$$

We use $\delta \equiv E[\delta_t]$ and $\delta^{log} \equiv E[\delta_t^{log}]$ to denote the unconditional mean of abnormal price and log abnormal price, respectively.

1.2 Cohen, Polk, and Vuolteenaho (2009) (CPV)

Using the decomposition of Campbell and Shiller (1988),

$$\delta_t^{log} \approx -\sum_{j=1}^{\infty} \rho^{j-1} E_t[r_{t+j}] - E_t[r_{V,t+j}], \tag{7}$$

where $r_t \equiv \log(P_t + D_t) - \log(P_{t-1})$ and $r_{V,t} \equiv \log(V_t + D_t) - \log(V_{t-1})$ denote log returns on price and value, respectively, and $\rho < 1$ is a parameter. Equation (7) and joint lognormality implies that long-horizon returns are related to mean log abnormal price as follows:

$$\begin{aligned} E\left[\sum_{j=1}^{\infty} \rho^{j-1} R_{t+j}\right] \approx\; & E\left[\sum_{j=1}^{\infty} \rho^{j-1} R_{b,t+j}\right] + E\left[\sum_{j=1}^{\infty} \rho^{j-1} Cov_{t+j-1}(r^e_{V,t+j}, -\widetilde{M}_{t+j})\right] \\ & + \frac{1}{2} E\left[\sum_{j=1}^{\infty} \rho^{j-1} \{Var_{t+j-1}(r_{t+j}) - Var_{t+j-1}(r_{V,t+j})\}\right] - \delta^{log}. \end{aligned} \tag{8}$$

We provide a detailed derivation of this equation in Appendix C.2. Note that CPV estimate δ^{log} using a closely related equation (their Equation 9) in the cross-section of portfolios:

$$E\left[\sum_{j=1}^{J} \rho^{j-1} R_{k,t+j}\right] = \lambda_0 + \lambda_1\, \beta_k^{CF} + u_k, \tag{9}$$

where k indexes a portfolio, β_k^{CF} is measured by regressing the portfolio's long-horizon cash flows on that of the market, with the horizon capped at J.[7]

[7] CPV highlight that tests of the CAPM may be distorted when there is market-wide mispricing. Their use of a ROE CAPM, as motivated by the Vuolteenaho (2002) decomposition, nicely avoids this concern. Of course, we can similarly use a ROE-based SDF in our return-based identity approach. However, mispricing in *firm-level* returns does not hinder us when using the distorted covariance between returns and the candidate SDF to estimate δ based on our identity. See Appendix C.6 for more details and a simple example that illustrates this point.

In addition to the potentially large measurement errors when estimating β_k^{CF}, justifying equation (9) under the null where $r_v = r$ requires strong intertemporal restrictions that guarantee

$$E\left[\sum_{j=1}^{\infty}\rho^{j-1}Cov_{t+j-1}(r_{t+j}^{e}, r_{t+j}^{mkt})\right] = Cov\left(\sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}^{e}, \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}^{mkt}\right), \tag{10}$$

in which case $\lambda_1 = b_1 Var\left(\rho^{j-1}r_{t+j}^{mkt}\right)$ if the candidate SDF is given by $\widetilde{m}_t = b_0 - b_1 r_t^{mkt}$ for the log market return r^{mkt}. The simplest way to guarantee equation (10) is to assume that returns are independently and identically distributed (i.i.d.). However, assuming that returns are i.i.d. is not only inconsistent with the properties of return data but also obviates the need to study differences between abnormal price and short-horizon abnormal returns in the first place. Indeed, estimating both sides of equation (10) in our sample reveals a large discrepancy arising from (i) a portfolio's conditional market beta covarying with market volatility (which affects the left-hand side) and (ii) the long-run reversal effect generating negative cross-autocovariances between portfolio excess returns and market returns.

1.3 A direct discount of post-formation cash flows

A potential estimator of δ is to directly discount post-formation dividends and a terminal cash flow:

$$\delta = E\left[\frac{P_t - V_t}{P_t}\right] \approx 1 - E\left[\sum_{j=1}^{J}\widetilde{M}_{t,t+j}\frac{D_{t+j}}{P_t}\right] - E\left[\widetilde{M}_{t,t+j}\frac{P_{t+J}}{P_t}\right]. \tag{11}$$

Suppose one takes equation (11) to the data using the method of moments estimator $\widehat{\delta}_t^{CF}$:

$$\widehat{\delta}^{CF} = \frac{1}{T}\sum_{t=1}^{T}\widetilde{\delta}_t^{CF} \quad \text{with} \quad \widetilde{\delta}_t^{CF} = 1 - \sum_{j=1}^{J}\widetilde{M}_{t,t+j}\frac{D_{(t),t+j}}{P_{(t),t}} - \widetilde{M}_{t,t+j}\frac{P_{(t),t+J}}{P_{(t),t}}, \tag{12}$$

where the notation $X_{(t),t+j}$ indicates a time $t+j$ realization of X from buying and holding a portfolio formed at time t. The resulting dividend-based estimator of δ in equation (12) is potentially biased and also subject to a serious overlapping observations problem.

To see the bias point, restate the dividend-based estimator in equation (12) using post-formation returns (see Appendix C.6 for the equivalence):

$$\widehat{\delta}^{CF} = \frac{1}{T}\sum_{t=1}^{T}\widetilde{\delta}_t^{CF} \text{ with } \quad \widetilde{\delta}_t^{CF} = -\widetilde{M}_{t,t+j-1}\frac{P_{(t),t+j-1}}{P_{(t),t}}(\widetilde{M}_{t+j}(1+R_{(t),t+j})-1). \tag{13}$$

Here, getting $\widetilde{M}_{t+j}(1+R_{(t),t+j})-1$ close to zero on average requires getting both the risk premium and the interest rate parts of $\widetilde{M}$ right, and measurement error in the interest rate part of $\widetilde{M}$ could bias $\widetilde{M}_{t+j}(1+R_{(t),t+j})-1$ in the same direction for all periods, leading to a large bias in $\hat{\delta}^{CF}$. Importantly, simply introducing a T-bill rate into $\widetilde{M}$ is not a remedy for this issue. Indeed, van Binsbergen et al. (2023) grant that applying their dividend-based estimator to a simple strategy of rolling over T-bills for 15 years results in a $\hat{\delta}^{CF}$ of around 0.50 (i.e., 50%), despite the strategy requiring relatively little risk adjustment, highlighting the challenge of dealing with time discounts in price-level analyses.[8] (They propose correcting for such a bias through bootstrap and a structural model of how returns are distributed.) Applying our excess-return-based estimator in equation (2) to the same 15-year roll-over T-bill strategy results in a $\widehat{\delta}$ of just 0.001 (i.e., 0.1%) without having to apply a separate bias adjustment. We return to this point about the bias in Section 3.2 after we formally introduce our excess-return-based estimator.

Second, to understand the overlapping samples issue, note that the covariance between $\widetilde{\delta}_t^{CF}$ observations s months apart is

$$Cov(\widetilde{\delta}_t^{CF}, \widetilde{\delta}_{t+s}^{CF}) = \sum_{j=s+1}^{J} Cov(\frac{\widetilde{M}_{t,t+j}}{P_{(t),t}}D_{(t),t+j}, \frac{\widetilde{M}_{t+s,t+j}}{P_{(t+s),t+s}}D_{(t+s),t+j}) + other\ terms. \tag{14}$$

Since both $\widetilde{\delta}_t^{CF}$ and $\widetilde{\delta}_{t+s}^{CF}$ depend on dividend realizations in periods $t+s+1$ through $t+J$, the cross-sectional covariance in dividend yields results in severe time-series covariances. For example, for the growth portfolio (lowest B/M portfolio in a quintile sort), the time-series of δ observations based on the cash-flow approach ($\widetilde{\delta}_t^{CF}$) have 1-, 5-, and 15-year sample autocorrelations

[8] In our sample, the event-time gross-return approach estimates the $\hat{\delta}^{ET,gross}$ of the T-bill roll-over strategy to be 0.497 (49.7%), similar to the number in van Binsbergen et al. However, for other portfolios, we find that the event-time gross-return approach typically finds estimated mispricing that is similar to our calendar-time excess-return approach.

of 0.78, 0.34, and −0.86 whereas the corresponding autocorrelations based on our approach are 0.15, −0.05, and 0.01 (Table A3 in the Internet Appendix). Furthermore, unlike returns, dividends on a stock are extremely serially correlated over time such that the event-time-to-calendar-time rearrangement does not sufficiently lower the serial correlation in the $\widetilde{\delta}_t^{CF}$ observations. This high serial correlation of a dividend-based approach makes standard errors imprecise and inference unreliable.[9] We also discuss this issue further in Section 3.2.

1.4 Simple long-horizon returns

If short-horizon α cannot proxy for price-level δ, could we use simple long-horizon abnormal return measures such as the cumulative abnormal return (CAR) or buy-and-hold abnormal return (BHAR) instead? The issue with these measures is that they put equal weight on all future abnormal returns and do not differentiate among abnormal returns earned in different time periods or states of nature. As explained in the next section, our δ estimator appropriately discounts the abnormal returns earned in different periods and states, which can lead to a very different magnitude of estimated δ.

One may argue that the CAR or the BHAR could generate the direction of price distortion and associated statistical significance that tend to be correct under the *null* hypothesis. However, measuring the exact magnitude of price-level δ under the alternative hypothesis is important for firm managers and investors who wish to use our novel estimation approach and the magnitude of the resulting δ estimate to inform their investment/issuance decisions. For example, firm managers may wish to quantify the extent to which a particular characteristic is historically a signal of δ from the CAPM perspective. Similarly, long-term buy-and-hold investors would like to understand the magnitude of the δ associated with the stocks they bought. Indeed, equity analyst reports provide a price target and the magnitude of estimated δ on a stock. When those investors undertake that analysis, they would not want to rely on methods like CAR or BHAR, which do not have an interpretable magnitude under the alternative of non-zero δ and, as we emphasize and document in our analysis in Table A5 in the Internet Appendix, can have magnitudes that depart substantially from δ estimated using our approach.

[9] Panel D of Figure 2 makes this point using Monte Carlo analysis.

2 The Mispricing Identity

Under the asset pricing environment specified in the previous section, we derive a novel identity that yields our new estimator of abnormal price in Section 3.

2.1 The law of motion for abnormal price

The first step is to derive a simple law of motion for conditional abnormal price, δ_t. Equation (3) and the law of iterated expectations imply that the fundamental asset pricing equation holds for V_t with respect to $\widetilde{M}$:

$$1 = E_t\left[\widetilde{M}_{t+1}\frac{V_{t+1}+D_{t+1}}{V_t}\right]. \tag{15}$$

Next, using equation (5) to substitute the empirically unobserved quantities V_t and V_{t+1} with $V_t = (1-\delta_t)P_t$ and $V_{t+1} = (1-\delta_{t+1})P_{t+1}$,

$$\begin{aligned}\delta_t &= 1 - E_t\left[\widetilde{M}_{t+1}(1+R_{t+1})\right] + E_t\left[\widetilde{M}_{t+1}\tfrac{P_{t+1}}{P_t}\delta_{t+1}\right] \\ &= -E_t\left[\widetilde{M}_{t+1}R^e_{t+1}\right] + E_t\left[\widetilde{M}_{t+1}\tfrac{P_{t+1}}{P_t}\delta_{t+1}\right],\end{aligned} \tag{16}$$

where the last equality uses equation (4) to express mispricing δ_t at time t in terms of excess return $R^e_{t+1} = R_{t+1} - R_{b,t+1}$ at time $t+1$ and mispricing δ_{t+1} at time $t+1$.

Equation (16) is intuitive. It says that overpricing (underpricing) at time t is either "paid out" as a negative (positive) abnormal return, $E_t\left[\widetilde{M}_{t+1}R^e_{t+1}\right]$, or contributes to overpricing (underpricing) at time $t+1$. The correct discount factor on δ_{t+1} is the SDF times the capital gain because δ_{t+1} has been normalized by P_{t+1}. δ_{t+1} matters more at time t if it arises in a state in which P_{t+1} is high (hence the capital gain term) or has a higher present value (hence the SDF term).

2.2 The identity: linking abnormal price to subsequent returns

Our identity is derived under the relatively mild assumption of no explosive bubbles in prices.

Assumption 1. *Price is not explosive with respect to the candidate* $\widetilde{M}$: $\lim_{j\to\infty} E_t\left[\widetilde{M}_{t,t+j}P_{t+j}\right] = 0$.

By definition, an analogous condition on value V also holds: $\lim_{j\to\infty} E_t\left[\widetilde{M}_{t,t+j}V_{t+j}\right] = 0$. Hence, Assumption 1 implies that the present value of the deviation in price from value at the limit $j \to \infty$ is zero:

$$\lim_{j\to\infty} E_t\left[\widetilde{M}_{t,t+j}\left(P_{t+j} - V_{t+j}\right)\right] = 0. \tag{17}$$

This condition is not restrictive, as it allows for most types of price deviations from value, including a permanent gap (e.g., $\delta_{t+j} = \delta \neq 0\,\forall j$), which our identity correctly detects.[10]

Iterating equation (16) forward and imposing Assumption 1 to set $\lim_{j\to\infty} E_t\left[\widetilde{M}_{t,t+j}\frac{P_{t+j}}{P_t}\delta_{t+j}\right] = 0$ expresses abnormal price as a discounted sum of subsequent excess returns.

Lemma 1. ***(Mispricing identity).*** *Under Assumption 1, a portfolio's abnormal price δ_t is the negative of the sum of expected subsequent excess returns discounted by the price-weighted SDF:*

$$\delta_t \equiv \frac{P_t - V_t}{P_t} = -\sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right], \tag{18}$$

where $\frac{P_{t+j-1}}{P_t}$ and R^e_{t+j} are, respectively, the portfolio's cumulative capital gains from time t to $t+j-1$ and excess returns at time $t+j$. Hence, mean (unconditional) abnormal price, denoted δ, equals

$$\delta \equiv E\left[\frac{P_t - V_t}{P_t}\right] = -\sum_{j=1}^{\infty} E\left[\widetilde{M}_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right]. \tag{19}$$

Equation (18) can also be stated with conditional abnormal returns. By the law of iterated expectations, $E_t\left[\widetilde{M}_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right] = E_t\left[\widetilde{M}_{t,t+j-1}\frac{P_{t+j-1}}{P_t}E_{t+j-1}\left[\widetilde{M}_{t+j}\right]E_{t+j-1}\left[\frac{\widetilde{M}_{t+j}}{E_{t+j-1}\left[\widetilde{M}_{t+j}\right]}R^e_{t+j}\right]\right] = E_t\left[\widetilde{M}_{t,t+j}\frac{P_{t+j-1}}{P_t}\alpha_{t+j}\right]$, where α_{t+j} denotes the conditional abnormal return.

Corollary 1. ***(The identity in abnormal returns).*** *Today's abnormal price δ_t is the expectation of a simple discounted sum of subsequent conditional abnormal returns:*

$$\delta_t = -\sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{t,t+j}\frac{P_{t+j-1}}{P_t}\alpha_{t+j}\right], \tag{20}$$

[10] Appendix C.1 in the Internet Appendix gives an example illustrating this point.

where α_{t+j} is the time $t+j$ abnormal return conditional on information at time $t+j-1$.

Equation (20) is intuitive. The economic surplus, relative to a candidate SDF $\widetilde{M}$, from a buy-and-hold strategy on a portfolio is the net present value of all subsequent abnormal payoffs: $V_t - P_t = E_t\left[\sum_{j=1}^{\infty} \widetilde{M}_{t,t+j} X_{t+j}^{Abnormal}\right]$. The abnormal payoff at $t+j$ is $X_{t+j}^{Abnormal} = P_{t+j-1}\alpha_{t+j}$, the conditional abnormal return from $t+j-1$ to $t+j$, converted into monetary payoff through a multiplication by price at time $t+j-1$. Finally, divide both sides by P_t and change sign to arrive at equation (20).

We note in passing that equation (20) is fundamentally different from an identity van Binsbergen and Opp (2019) exploit in their quantitative analysis:

$$P_t = \sum_{j=1}^{\infty} E_t\left[\frac{\widetilde{M}_{t,t+j}}{\Pi_{k=1}^{j}\left(1+\alpha_{t+k}^{*}\right)} D_{t+j}\right], \tag{21}$$

where $1+\alpha_{t+k}^{*} = E_{t+k-1}\left[\widetilde{M}_{t+k}\left(1+R_{t+k}\right)\right]$.[11] Since equation (21) writes price in the numerator of the left-hand side, abnormal returns appear in the denominator of the identity. This choice is innocuous for the structural approach taken in their paper but does not render a simple expression for unconditional abnormal price as in equation (19).

2.3 Theoretical implications of the abnormal price identity

Equation (20) clarifies the exact relation between price-level δ and subsequent buy-and-hold short-horizon αs missing in the literature, summarized as Corollary 2.

Corollary 2. ***(Implications of the identity).*** *Ceteris paribus, ex-ante abnormal price δ is larger if subsequent abnormal returns*

1. *are larger*
2. *are more persistent*
3. *occur sooner*

[11] This is a discrete-time version of their continuous-time expression. See Appendix C.3 for further details.

4. occur in more valuable states

5. occur after relatively large capital gains

While the first two points are relatively obvious (i.e., they are present in the Campbell-Shiller based approach of CPV, at least to some degree), the last three points emphasize that to recover ex-ante abnormal price, one must correctly discount the subsequent abnormal returns according to the time and state in which they occur. Put differently, the correct way to aggregate subsequent abnormal returns to arrive at ex-ante abnormal price is clearly distinct from existing long-run return measures such as CAR or BHAR, which do not distinguish between abnormal returns earned in the near vs. distant future or those earned in more valuable vs. less valuable cumulative states.

Mathematically, the time discount (#3) arises because the no-explosive-bubble condition implies $\lim_{j\to\infty} E_t\left[\widetilde{M}_{t,t+j}\frac{P_{t+j-1}}{P_t}\right]=0$.[12] Intuitively, an abnormal return that arises in the distant future matters less for the stock's current price level, since such an abnormal return only affects the discounting of the dividends to be earned in the periods that follow the abnormal return, not the dividends to be earned prior to the timing of the abnormal return. In contrast, an abnormal return in the immediate future affects the discounting of all future dividends and therefore matters more for today's stock price level. The state discount (#4 and #5) arises because the conditional abnormal return, α_{t+j}, is earned on P_{t+j-1}, which has a large present value if either the cumulative capital gain has been large or the cumulative candidate SDF is high.[13] While we speculate that our abnormal price identity can be applied in other ways, the present paper applies the identity to derive a return-based estimator of abnormal price discussed next.

3 A Return-Based Estimator of Abnormal Price

This section derives a return-based estimator of a portfolio's mean formation-period abnormal price, $\delta \equiv E[\delta_t] = E\left[(P_t - V_t)/P_t\right]$, and studies its statistical properties using Monte Carlo.

[12] CPV effectively incorporate a time discount through the Campbell-Shiller discount parameter although their approach does not account for the differences in cash-flow duration across different stocks.

[13] Since the time $t+j$ component of the cumulative candidate SDF, $\widetilde{M}_{t+j}$, is orthogonal to α_{t+j} by definition, it only generates a time discount. The covariance between α_{t+j} and the past cumulative state, $\widetilde{M}_{t,t+j-1}$, could be nonzero.

3.1 The identity in calendar time

To get to our abnormal price estimator, we first rearrange the terms of the identity in Lemma 1 Equation (19) to obtain an equivalent *calendar-time* expression for unconditional abnormal price δ.[14]

Lemma 2. ***(Calendar-time expression for unconditional*** δ***).*** *Consider a series of portfolios formed every period based on a predetermined rule (e.g., characteristic cutoffs). Then, the portfolio's unconditional formation-period abnormal price,* $\delta \equiv E\left[\frac{P_t - V_t}{P_t}\right]$*, equals the unconditional expectation of the sum of appropriately discounted time-t excess returns on all portfolios formed between periods* $t-\infty$ *and* $t-1$*:*

$$\delta = -E\left[\sum_{j=0}^{\infty} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right], \tag{22}$$

where $\frac{P_{(t-j),t}}{P_{(t-j),t-j}}$ *and* $R^e_{(t-j),t}$ *denote, respectively, the time-t realizations of the cumulative capital gain and one-period excess return on the portfolio formed at* $t-j$*. The stochastic discount* $\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$ *tends to place a greater weight on portfolios formed in the recent past.*

Proof. Applying equation (19) to a multi-asset portfolio implies that a portfolio's time-t_0 ex-ante conditional abnormal price is $\delta_{t_0} = -\sum_{j=0}^{\infty} E_{t_0}[\widetilde{M}_{t_0,t_0+j}\frac{P_{(t_0),t_0+j-1}}{P_{(t_0),t_0}} R^e_{(t_0),t_0+j}]$, where (t_0) in a subscript indicates that the quantity is from a buy-and-hold strategy on a portfolio formed at t_0. Take an unconditional expectation of both sides of the expression and use calendar time $t \equiv t_0 + j$ (the time when the excess returns are realized) to obtain the expression in equation (22). □

Figure 1 visualizes the difference between an event-time and a calendar-time approach. The original identity in equation (19) shows that unconditional δ is the expectation of the event-time δ_t that appropriately discounts the post-formation buy-and-hold monthly excess returns on a growth

[14] Fama (1998) emphasizes the usefulness of calendar-time techniques in his discussion of the literature on post-event, long-horizon abnormal returns. Cohen, Polk, and Vuolteenaho (2009) modify a calendar-time approach to have portfolio weights decline as a function of time-from-event so that the return on the resulting portfolio approximates a buy-and-hold investor's experience in price-level units. We thank Rob Rogers for suggesting a similar approach for our estimation of delta using our exact identity.

portfolio formed today, as illustrated in Panel A. In contrast, the equivalent calendar-time identity in equation (22) shows that unconditional δ also equals the expectation of the calendar-time δ_t that appropriately discounts today's realizations of monthly excess buy-and-hold returns on growth portfolios formed in the past, as illustrated in Panel B.

3.2 A new estimator of abnormal price

In practice, truncating the infinite-horizon sum in equation (22) at some large J provides a good approximation, since $E\left[\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}\right]$ converges to zero as j gets large. This convergence is because both the discount part ($\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$) and the conditional abnormal return part ($E_{t-1}\left[\widetilde{M}_t R^e_{(t-j),t}\right]$) of the expression converge to zero as $j \to \infty$ due to the no-explosive-bubble condition and the long-term convergence in (abnormal) returns of stocks (Keloharju, Linnainmaa, and Nyberg (2021)), respectively. We find that $J = 15$ years works well both empirically and in simulations (see Figure A4 in the Internet Appendix).

Our estimator of unconditional δ is therefore the sample analogue of the true unconditional δ in equation (22) with the truncation of the infinite sum at a large finite J.

Corollary 3. ***(A return-based estimator of abnormal price).*** *The unconditional abnormal price δ of a portfolio formed on a predetermined rule (e.g., "value portfolio") can be estimated by*

$$\widehat{\delta} = \frac{1}{T}\sum_{t=1}^{T}\widetilde{\delta}_t \quad \text{with} \quad \widetilde{\delta}_t = -\sum_{j=1}^{J}\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}, \tag{2}$$

for $J = 15$ years, where the time-t observation of δ is the sum of appropriately discounted time-t excess returns on all portfolios formed on the characteristic over the last J periods. The expression coincides with the true δ in equation (22) as $J, T \to \infty$.

An important advantage of our estimator in equation (2) is that using excess returns rather than gross returns for estimation makes our mispricing estimates less subject to a potential bias. This can be best explained by drawing an analogy with the expected return framework. Researchers typically estimate time-series abnormal returns using the excess-return formulation, $E\left[\frac{\widetilde{M}_t}{E[\widetilde{M}_t]}R^e_t\right]$, not the gross-return formulation, $E\left[\widetilde{M}_t(1+R_t)-1\right]$, since the former only requires the SDF to explain the risk premium component of excess returns, whereas the latter requires the SDF to explain both the risk premium and the interest rate components of gross returns.[15] Since our excess-return estimator of abnormal price uses $\widetilde{\delta}_t = -\sum_{j=1}^{J}\left(M_{t-j,t-1}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}\right)\widetilde{M}_t R^e_{(t-j),t}$, getting $\widehat{\delta}$ close to zero mostly hinges on getting $\widetilde{M}_t R^e_{(t-j),t}$ close to zero on average through the risk premium component of $\widetilde{M}$, whereas the interest rate component of $\widetilde{M}$ only affects how $\widetilde{M}_t R^e_{(t-j),t}$ is time-discounted back to the portfolio formation period through $\widetilde{M}_{t-j,t-1}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$. Thus, our excess return formulation ensures that measurement error in the interest rate component of $\widetilde{M}$ does not materially affect our $\widehat{\delta}$.

Indeed, the strategy that rolls over the one-month T-bill return for 15 years has drastically different CAPM delta estimates depending on whether or not the excess return method is employed. Table 1 shows that using the proposed calendar-time excess-return method (top left corner) leads to an estimated delta of 0.1%, consistent with how restrictions used to estimate the candidate SDF parameters include an implicit assumption on the T-bill return being correct. The delta remains relatively small at 2.8% even if we use the event-time excess-return method that does not take advantage of the calendar-time formulation (top right corner of Table 1).[16]

In contrast, the calendar-time gross-return method and the dividend-discount method (equivalent to the event-time gross-return method) lead to large estimated deltas of 45.7% and 49.7%, respectively (bottom left and right corners of Table 1; the latter estimate is similar to the one found in van Binsbergen et al. (2023)). The biases remain just as large even if we use candidate SDFs estimated using the excess return method, implying that the problem arises with the delta estimator itself, not in the restrictions used to estimate the candidate SDF coefficients.

Another advantage of working with this return-based calendar-time estimator of δ is that its

[15] Cochrane (2009): "In fact, much asset pricing focuses on excess returns. Our economic understanding of interest rate variation turns out to have little to do with our understanding of risk premia, so it is convenient to separate the two phenomena by looking at interest rates and excess returns separately" (p.9).

[16] The following restrictions are used to estimate the candidate SDF parameters. (1) Calendar-time excess-return method: $\widetilde{M}_t(1+R^{mkt}_t)=1$ and $\widehat{\delta}=0$ for the market portfolio. (2) Event-time excess-return method: $\widetilde{M}_t(1+R^{mkt}_t)=1$ and $\widehat{\delta}^{ET,excess}=0$. (3) Calendar-time gross-return method: $\widetilde{M}_t(R^{mkt}_t-R_{f,t})=0$ and $\widehat{\delta}^{CT,gross}=0$. (4) Dividend discount method: $\widetilde{M}_t(R^{mkt}_t-R_{f,t})=0$ and $\widehat{\delta}^{CF}=0$.

time-series observations, $\widetilde{\delta}_t$, have very little serial correlation, simplifying the inference problem. To see this, consider estimating a value portfolio's δ. Then, the time-t observation of the value portfolio's calendar-time δ is the discounted sum of time-t excess returns on all value portfolios formed over the last J periods: $\widetilde{\delta}_t = -\sum_{j=1}^{J} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}$. Hence, even if value portfolios formed over the span of past few years have cross-sectionally correlated buy-and-hold one-period returns at time t, by putting all these time-t returns into a single observation $\widetilde{\delta}_t$, the expression ensures that this cross-sectional return correlation increases the variance of $\widetilde{\delta}_t$ instead of generating serial correlation in $\widetilde{\delta}_t$. On the other hand, since the stochastic discount part of the formula, $\widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$, must always remain positive and multiplies an excess return $R^e_{(t-j),t}$ that fluctuates around zero, the overlapping nature of the stochastic discount expression does not generate an empirically discernible serial correlation in $\widetilde{\delta}_t$.

3.3 Decomposing long-horizon return: risk vs. abnormal price

Similarly to how (short-horizon) expected return can be decomposed into an abnormal return and a risk premium, a measure of long-horizon return can be written in terms of abnormal price, long-horizon risk, and a cumulative state adjustment component. To see this, apply the covariance identity, $E[XY] = E[X]E[Y] + Cov(X,Y)$, to equation (2).

Corollary 4. ***(Long-horizon return and long-horizon risk).*** *Estimated abnormal price $\widehat{\delta}$ is a deviation of long-horizon return from long-horizon risk, adjusted by the cumulative state in which the risk-return distortion arises:*

$$\underbrace{\sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] E_T\left[\widetilde{M}_t\right] E_T\left[R^e_{(t-j),t}\right]}_{\text{“long-horizon return"}} = -\widehat{\delta} + \underbrace{\sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] Cov_T\left(R^e_{(t-j),t}, -\widetilde{M}_t\right)}_{\text{“long-horizon risk"}} \underbrace{- \sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] Cov_T\left(\frac{\phi_{(t-j),t-1}}{E_T\left[\phi_{(t-j),t-1}\right]}, \widetilde{M}_t R^e_{(t-j),t}\right)}_{\text{“cumulative state adjustment"}} \tag{23}$$

where subscript T denotes a sample moment over T periods and $\phi_{(t-j),t-1} \equiv \widetilde{M}_{t-j,t-1} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$ discounts post-formation returns in more distant future or less important states more heavily.

Long-horizon return summarizes the term structure of discount rates on the portfolio's cash flows. It puts more (less) weight on returns in the more imminent (distant) future, since in a dividend discount model, return in an imminent future (e.g., R_{t+1}) discounts all future cash flows, whereas return in a more distant future (e.g., R_{t+j}) discounts a smaller subset of cash flows that arise subsequently: $P_t = E_t\left[\frac{D_{t+1}}{1+R_{t+1}} + \frac{D_{t+2}}{(1+R_{t+1})(1+R_{t+2})} + \ldots + \frac{D_{t+j}}{(1+R_{t+1})(1+R_{t+2})\ldots(1+R_{t+j})} + \ldots\right]$. By the same logic, *long-horizon risk* applies different time discounts to the term structure of risk premia. *Cumulative state adjustment* accounts for the way the cumulative *state* in which returns deviate from risk premia—not just the *time* at which those deviations happen—matters for abnormal price.

To obtain a measure of *risk-neutral abnormal price*, simply set $\widetilde{M}_t$ in equation (60) to be one.[17]

Definition 1. ***(Risk-neutral abnormal price).*** *Estimated risk-neutral abnormal price, denoted* $\widehat{\delta}^{RN}$*, equals*

$$\widehat{\delta}^{RN} = -\sum_{j=1}^{J} E_T\left[\widetilde{M}_{t-j,t-1}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}\right]. \tag{24}$$

For instance, when excess returns are taken with respect to the market portfolio, risk-neutral abnormal price measures the extent to which the term structure of portfolio returns differs from the term structure of market portfolio returns, where the weight on each excess return depends on the time and the cumulative state in which it occurs.

3.4 Implementation

To take our estimator in equation (2) to data, we use the CAPM to specify the candidate SDF.[18] While analyzing short-horizon returns requires choosing risk factors, analyzing price levels requires choosing both risk factors and the functional form of the candidate SDF. We follow Korteweg and Nagel (2016) and use a loglinear SDF specification, a natural choice for price-level

[17]We thank an anonymous referee for this suggestion. One could alternatively set $\widetilde{M}_t$ to be the inverse of the average monthly risk-free rate, but the difference is very small at our monthly frequency.

[18]Our analysis in subsequent sections will provide a direction for future research on multifactor models of prices. In addition, it would be natural to check whether the intertemporal CAPM specification of Campbell, Giglio, Polk, and Turley (2018), which incorporates stochastic volatility into the ICAPM framework of Campbell and Vuolteenaho (2004), significantly reduces the pricing errors relative to the CAPM in standard SDF return tests. We leave a multifactor analysis of price levels to future work.

analysis given its ability to explain short-horizon returns and prices simultaneously:

$$\widetilde{M}_{t-j,t} = \exp\left(b_0 j - b_1 \sum_{s=0}^{j-1} r_{t-s}^{mkt}\right), \tag{25}$$

where $r_t^{mkt} = \log\left(1+R_t^{mkt}\right)$ is the log market return and (b_0, b_1) are constant parameters, and $\widetilde{M}_t \equiv \widetilde{M}_{t-1,t} = \exp\left(b_0 - b_1 r_t^{mkt}\right)$. We choose b_0 and b_1 to explain the market portfolio's prices and returns perfectly in sample, which makes our approach analogous to the conventional time-series approach to estimating abnormal returns. Specifically, the following two moment conditions pin down b_0 and b_1:

$$\begin{aligned} 0 &= E\left[\widetilde{M}_t(1+R_t^{mkt})\right] - 1 \\ 0 &= \sum_{j=1}^{J} E[\widetilde{M}_{t-j,t} \frac{P_{t-1}^{mkt}}{P_{t-j}^{mkt}} \left(R_{(t-j),t}^{mkt} - R_{f,t}\right)], \end{aligned} \tag{26}$$

where $R_{(t-j),t}^{mkt}$ is the time-t return on market portfolio formed at $t-j$. The Internet Appendix shows that the estimated values of b_0 and b_1 vary depending on the choice of the number of post-formation years J that we include in our estimate of δ, but not dramatically so, with confidence intervals at each horizon covering the point estimates of other horizons (Table A2). These results are consistent with the CAPM evidence from Chernov, Lochstoer, and Lundeby (2021).

We compute excess returns on a characteristic-based portfolio with respect to returns on the market portfolio, exploiting the CAPM implication that the market should be correctly priced and the fact that the market has a zero in-sample δ with respect to the model SDF. Benchmarking test assets against the market reduces the sensitivity of the time-series of δ observations to market return shocks, reducing the volatility of the resulting series and improving the precision of our estimates (as in Campbell, Giglio, and Polk (2013), Campbell et al. (2018), and Korteweg and Nagel (2022)). Simulation shows that taking excess returns with respect to the market portfolio reduces the estimator's volatility by seven percent. An added benefit of this approach could be that the resulting estimates of δ may be less affected by the near-money feature of the T-bill (Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2016)).[19]

[19] Note also that any convenience yield properties of the short-term US government debt could still affect our results

We use the generalized method of moments (GMM) standard errors and p-values that account for the time-series and cross-sectional covariances in the data as well as the uncertainty in estimating the SDF parameters. Our Monte Carlo analysis in the next subsection shows that the Bartlett kernel of Newey and West (1987) with a bandwidth of 24 months works well, given the low level of autocorrelation observed in the time series of $\tilde{\delta}_t$.

3.5 Monte Carlo analysis

To study the performance of our delta estimator, we simulate a numerical model that resembles the one in Korteweg and Nagel (2016). The model, explained in greater detail in Appendix C.5, generates realistic moments that resemble those of the high adjusted value quintile portfolio and those of the market portfolio (see Table A1 in the Internet Appendix).

We first analyze bias. Since the choice of candidate SDF parameters b_0 and b_1 affect $\widehat{\delta}$ in a highly nonlinear fashion, the uncertainty arising from having to estimate b_0 and b_1 using the base asset (market portfolio) can lead our estimated $\widehat{\delta}$ to deviate from the true δ. Panel A of Figure 2 shows that such a bias, if any, leads to a small attenuation in the estimated $\widehat{\delta}$ and a more conservative rule in rejecting an asset pricing model.

Next, turning to size and power, Table 2 shows that our estimator under the null tends to under-reject the null relative to the 5% significance level. The table also shows that our price-level test has statistical power similar to a conventional return test. As equation (2) implies, abnormal price δ is a discounted sum of 15 years of post-formation returns. This means that the magnitude of δ required for statistical significance is roughly 10-to-12 times that for annualized α, consistent with δ being a discounted sum of future αs over roughly 15 years. Panel B of Figure 2 visualizes this rule of thumb.

Panel C and D highlight the main issue with doing inference on δ using cash flows, which is that the resulting large serial correlation makes the number of independent time-series observations in

through its impact on candidate SDF parameter estimates, since the market portfolio's δ in equation (26) is estimated using its excess return with respect to the Treasury-bill rate. In practice, computing excess returns with respect to the T-bill return makes only a small difference to the point estimates.

the cash-flow approach small and the range of standard errors wide (Panel D). Our return-identity-based approach addresses this problem and tends to estimate standard errors accurately.

4 Asset Pricing Tests on Prices

4.1 Data

We combine monthly stock price data from the Center for Research in Security Prices (CRSP), annual accounting data from CRSP/Compustat Merged (CCM), and the pre-Compustat book equity data from Davis, Fama, and French (2000) to create our basic dataset. We compute the gross market portfolio return, the factor in our candidate SDF, as the sum of the market excess return and the one-month Treasury bill rate from Kenneth French's data library.

Our tests estimate the δs of diversified portfolios to minimize the impact of idiosyncratic returns. We typically form value-weight quintile portfolios by sorting stocks on single characteristic and applying NYSE cutoffs. When double-sorting, we form nine value-weight portfolios by sorting stocks independently on each characteristic and applying 30% and 70% NYSE cutoffs.

Estimating a portfolio's δ requires data on post-formation returns and capital gains over 180 months (15 years). Hence, for a portfolio formed at t, we track its monthly buy-and-hold returns and capital gains over $t+1, \ldots, t+180$.[20] That is, as illustrated in Figure 1, the post-formation returns on a portfolio formed in t are $R_{(t),t+1}$, $R_{(t),t+2}$, . . . , $R_{(t),t+3}$, following the diagonal arrow pointing southeast, where $R_{(t),t+j}$ denotes the time-$t+j$ return on buying and holding a portfolio formed at time t.

In summary, we construct three-dimensional data for each sorting characteristic: buy-and-hold monthly returns (and capital gains) over J post-formation months for T different months in which the post-formation return data are available for the full J holding periods, all together on N different portfolios. In contrast, a conventional short-horizon return test uses two dimensional data, since it

[20]Post-formation returns at $t+j$ for the portfolio formed at t are from a buy-and-hold strategy that does not reinvest dividends into the same or different stocks. Appendix C.4 in the Internet Appendix explains how to apply our identity correctly to a portfolio of stocks in an empirical analysis.

tracks just $J = 1$ month post-formation returns for T different months on N different portfolios.

Our baseline analysis uses post-formation returns over 1948m6–2022m12, where 1933m6 is the first month in which most of the characteristics other than accruals can be computed and 1948m6 is the first month in which the full 15 years of calendar-time observations of post-formation returns and capital gains are available (the horizontal arrows in Figure 1). Our modern subsample analysis uses 1972m6–2022m12. Table 3 provides descriptive statistics for the portfolios formed from a univariate sort on each of the nine characteristics we consider in the rest of the paper. Appendix B (Internet Appendix) provides further details on data construction.

4.2 Initial analysis on book-to-market and quality

Our initial analysis studies the book-to-market equity ratio (B/M) and $quality$, which the recent literature argues proxies for abnormal price. Golubov and Konstantinidi (2019) decompose B/M into the market-to-value ratio (abnormal price) and the value-to-book ratio using within-industry cross-sectional regressions of equity values on firm fundamentals, finding that the return predictability of B/M stems from the abnormal price component. Asness, Frazzini, and Pedersen (2019) measure $quality$, a z-score measure based on sixteen characteristics that rewards profitable, fast-growing, safe, and high-payout stocks and find that quality predicts price-level distortions, measured by cumulative abnormal returns over five years.

Looking first at returns, Rows 1 and 2 of Table 3 show that the CAPM does a poor job explaining the cross-section of returns on quintile portfolios sorted on B/M or $quality$. The long-short portfolio based on B/M generate an annualized CAPM alpha of 3.5% and the long-short portfolio based on $quality$ generates an annualized alpha of 5.5%. In conjunction with the fact that both B/M and $quality$ are relatively persistent characteristics, these two anomalies are natural candidates to generate significant price-level errors as well.

To provide a formal analysis of prices, we estimate quintile δs and the difference between the two extreme quintile portfolios (Hi - Lo), along with t-statistics (in parentheses) and p-values for $J \in \{1\text{mo}, 1\text{yr}, 3\text{yrs}, 5\text{yrs}, 10\text{yrs}, 15\text{yrs}\}$. The $J = 1$ month estimates recovers results close to the conventional time-series return regression results in Table 3 after the appropriate sign change, but with a loglinear model of the SDF, whereas $J = 15$ years (180 months) proxies for price-level results given by $J \to \infty$.[21] The intermediate values of J allow us to see how the performance of the asset pricing model changes as the return horizon increases gradually from 1 month to 15 years.

The results for B/M in the left columns of Table 4 Panel A show that value stocks are undervalued relative to growth stocks only from the perspective of CAPM investors with a short investment horizon of $J = 1$ month or one year. Beyond an investment horizon of 1 year, B/M is a weak signal of CAPM δ. In particular, the price-level result with $J = 15$ years in the last row shows that value stocks are 27.2 percentage points underpriced relative to growth stocks but with a t-statistic of 0.96.

Turning to *quality*, the right columns of Table 4 Panel A shows that high-quality stocks are undervalued and low-quality stocks are overvalued from the perspective of CAPM investors with an investment horizon of $J = 3$ year or less. However, for $J = 5$ or more years, the estimated δs are statistically indistinguishable from zero for all quality-sorted portfolios and imply that the market price correctly accounts for the quality difference. For example, for $J = 15$, we find that high-quality stocks are only 11.4 percentage points underpriced relative to low-quality stocks with an associated t-statistic of 0.51. Our finding based on an exact definition of price distortion δ is contrary to the conclusion drawn by Asness, Frazzini, and Pedersen (2019), whose analysis instead studies either cumulative five-year abnormal returns or a cross-sectional regression of the M/B ratio on quality.

Part of the reason that we find insignificant δ is that accounting for market risk reduces the CAPM-implied δ. Comparing Panels A and B of Table 4 for $J = 1$ month shows that accounting for market risk leads to a larger spread in risk-adjusted returns than that in simple returns. In contrast, at $J = 15$ years, CAPM δs are smaller in magnitude than their corresponding risk-neutral δ for both B/M and *quality*, albeit to a small extent for B/M. Figure 3 makes this point graphically: whereas CAPM-implied risk and average excess returns tend to have an anomalous negative relation over a one-month post-formation horizon, they have a flat (for *quality*) or positive (for B/M) relation

[21] Figure A4 in the Internet Appendix shows that $J = 15$ years captures most, if not all, of the consequences of the post-formation abnormal returns associated with characteristics we study.

over a long horizon.[22]

Figure 4 plots the post-formation behavior of excess returns and market betas to analyze why risk and return tend to have a less anomalous relation over a long horizon. These plots show that a contributing factor for the smaller spread in δs for B/M-sorted portfolios is that value stocks have slightly higher betas than growth stocks from year five, allowing the higher long-horizon risk of value stocks to exceed that of growth stocks.[23] As for *quality*-sorted portfolios, junk stocks have persistently higher betas than quality stocks, which lines up with the fact that junk stocks have higher returns than quality stocks for most of their 15-year post-formation horizon.

4.3 Primary sorting characteristic: *adjusted value*

A more powerful test on price levels requires test assets that a priori are likely to exhibit large variation in δ. One way to generate more spread in δ is by adjusting the traditional value signal (B/M) for the effect of profitability and risk.

Intuitively, the present-value logic says that a stock could be cheap (i.e., B/M high) because it is (i) expected to have low dividend growth, (ii) risky, or (iii) truly undervalued. And for (i), profitable stocks with high expected future returns on equity will tend to have faster dividend growth as more earnings per book equity gets plowed back. Therefore, cheap stocks that are nonetheless (i) profitable and (ii) safe are likely underpriced, an idea we capture in *adjusted value*:

$$adjusted\ value \equiv \underbrace{z(B/M)}_{\text{value (cheap)}} + \underbrace{z(\text{Prof})}_{\text{profitable}} - \underbrace{z(\text{Beta})}_{\text{risky}}, \tag{27}$$

where z denotes the z-score of the characteristic's cross-sectional rank.

Formally, the loglinear present-value model of Vuolteenaho (2002) implies that a stock's CAPM-implied log value ($v_t = \log V_t$) is the log book value (b_t) plus expected log clean-surplus returns on

[22] Figure 3 plots the long-horizon components of risk and return in equation (60) for direct comparison with the corresponding short-horizon measures. We thank an anonymous referee for this suggestion. Figure A8 shows that the cumulative state adjustment is not a negligible component of δ, although this component typically has an absolute magnitude below 10% and does not have a strong univariate cross-sectional relation to δ.

[23] That is, the insignificant delta associated with B/M is not merely an outcome of post-formation alpha decay over time but of a small reversion in post-formation alphas.

equity ($E_t roe^{cs}_{t+j+1}$) minus CAPM-implied discount rates ($E_t r_{t+j+1}$):

$$v_t \approx b_t + \sum_{j=0}^{\infty} \rho^j E_t roe^{cs}{}_{t+j+1} - \sum_{j=0}^{\infty} \rho^j E_t r_{t+j+1} \tag{28}$$

Since CAPM underpricing is the deviation of CAPM-implied value from price, it follows that a characteristic that adds the z-scores of B/M and profitability and subtracts the z-score of beta should proxy for CAPM underpricing:

$$\begin{aligned} v_t - p_t &\approx \underbrace{b_t - p_t}_{\text{book-to-market}} + \underbrace{\sum_{j=0}^{\infty} \rho^j E_t roe^{cs}{}_{t+j+1}}_{\text{profitability}} - \underbrace{\sum_{j=0}^{\infty} \rho^j E_t r_{t+j+1}}_{\text{beta}} \\ &\propto z(B/M) + z(\text{Prof}) - z(\text{Beta}), \end{aligned} \tag{29}$$

which equals *adjusted value* in equation (27).[24,25]

Our three-characteristic model of CAPM underpricing is similar in spirit to 'quality at a reasonable price' in Asness, Frazzini, and Pedersen (2019) but addresses the potential concern that a composite measure based on sixteen characteristics plus B/M can be difficult to interpret and subject to overfitting. Adjusted value is also related to the idea in Piotroski and So (2012) that one can isolate the underpricing component of B/M through its interaction with proxies for the stock's future fundamentals.[26] Although our baseline analysis uses the z-score of *current* gross profitability of Novy-Marx (2013) to measure expected *future* profitability, we explore an alternative approach that more directly proxies for future profitability as well.

[24]We use the z-score of current gross profitability to proxy for future profitability. When gross profitability data are unavailable, we use the z-score of return on equity.

[25]Cho, Kremens, Lee, and Polk (forthcoming) derive a loglinear present-value identity that links today's market-to-book equity ratio to future investment ("scale"), profitability ("yield"), and discount rates. Their more granular decomposition suggests that separately controlling for firms' scale and yield characteristics may provide a better adjustment of B/M than our current adjusted value measure.

[26]See also Frankel and Lee (1998), Piotroski (2000), Cohen, Polk, and Vuolteenaho (2003), Polk, Thompson, and Vuolteenaho (2006), Novy-Marx (2013), and Gonçalves and Leonard (2023).

4.4 *Adjusted value* and the cross-section of price levels

Table 5 shows that portfolios sorted on *adjusted value* are economically and statistically mispriced at every horizon we consider. The first row shows that high-*adjusted-value* stocks outperform low-*adjusted-value* stocks by 75 basis points a month with an associated t-statistic of 6.88. At the 15-year horizon, low-*adjusted-value* stocks are 51.9 percentage points more overpriced than their high-*adjusted-value* counterparts. The large CAPM δ difference seems to arise from the CAPM risk adjustment to long-horizon returns on high vs. low adjusted-value portfolios being too large (i.e., the ex-post security market line being too flat), since the difference in risk-neutral δ is much smaller at 8.5 percentage points. Indeed, consistent with this interpretation, Figure A7 Panel P visually confirms the strong negative relation between long-horizon risk and long-horizon return for portfolios sorted on adjusted value. Thus, Table 5 documents exactly the sort of variation a buy-and-hold CAPM investor should be interested in exploiting and represents a challenge for future asset pricing models that make understanding price-level risk a priority.[27]

We find that the exact way in which information in the three characteristics is used matters less (first column of Table A4). For instance, defining *adjusted value* in a way that puts more weight on the "price" component of adjusted value rather than putting an equal weight on all three z scores generates similar results. We also find that using VAR-implied expected future profitability z-score instead of current profitability z-score slightly improves the performance of the *adjusted value* characteristic (columns three and four in the table).[28]

At the same time, accounting for all three characteristics—B/M, profitability, and beta—is critical in isolating the large price-level variations unexplained by the CAPM. The last three columns of Table A4 shows that interacting the z-scores of only two of the three characteristics—i.e., profitability and beta, B/M and beta, and B/M and profitability—fail to generate a statistically significant CAPM δ.

[27] Appendix B.4 uses double sorts to generate variation in adjusted value.

[28] The improvement is relatively small, since profitability is one of the most persistent among the characteristics we consider, making current profitability a strong predictor of its future values (Table 3). Hence, for parsimony as well as to facilitate replication of our work, we present the *adjusted value* characteristic based on current profitability as the baseline. Appendix B.5 details how we modify the baseline approach through a VAR model of characteristic z scores to use information about expected future profitability.

5 Are Return Anomalies Price-level Anomalies?

This second and final empirical section studies the extent to which the CAPM explains price-level variation associated with nine additional characteristics known to be associated with cross-sectional variation in average returns: net issuance, investment, accruals, beta, size, momentum, and profitability. The first four are chosen for their potential conceptual link to price-level distortions, while the next three are chosen for being prominent return anomalies (in conjunction with value, investment, and the market factor, they make up the widely-used Fama-French-Carhart six-factor model).

5.1 Characteristics conceptually related to abnormal price

Certain characteristics are interesting to analyze using our abnormal price measure either due to their conceptual association with abnormal price (net issuance, investment, and accruals) vis-à-vis the endogenous choices of managers or their mechanical link to the long-horizon risk component of abnormal price in Corollary 4 (beta). We explain the conceptual link that each characteristic has to abnormal price δ and study the extent to which the characteristic is associated with price-level variation that cannot be explained by exposure to market risk that the CAPM captures.

Net share issuance

A large literature in behavioral corporate finance views securities market mispricing as a primary factor in managerial financing and investment decisions.[29] In particular, several papers document evidence that share repurchase (issuance) indicate undervaluation (overvaluation) as perceived by firm managers (e.g., Loughran and Ritter (1995); Ikenberry, Lakonishok, and Vermaelen (1995)). Nevertheless, this hypothesis has not been tested using our definition of stock mispricing that explicitly accounts for the asset pricing model of risk (in our case, the CAPM): $\delta = E\left[\left(P_t - V_t\right)/P_t\right]$.

Table 6 shows that the spread in mispricing δs associated with net issuance is indeed large.[30]

[29] Baker, Ruback, and Wurgler (2007) review this literature.

[30] Figure A7 visualizes the cross-sectional relations between long-horizon risk and long-horizon return for the return anomalies studied in this section.

Share repurchases (low net issuance) are especially strong signals of CAPM underpricing, consistent with CFOs identifying market timing as the number one reason for stock repurchases (Brav, Graham, Harvey, and Michaely (2005)). In contrast, share issuances (high net issuance) are not strongly indicative of CAPM overpricing, and this weaker result may reflect that firm CFOs use stock issuance primarily to finance investment projects (Graham and Harvey (2001)).[31] The difference in δ's across the two extreme net issuance quintiles is 23.7 percentage points with a large t-statistic of 2.72. Similarly to the *adjusted value* sort, the risk-neutral delta difference is small and statistically insignificant, indicating that the implicit CAPM long-horizon risk adjustment is an important contributor to the difference in deltas that we find. A similar finding holds for the other two significant delta characteristics discussed below, investment and beta, highlighting that the flat security line continues to play a role in our price-level analysis.[32]

Investment

Arguably the most important anomaly to study in this context is investment, given the potential link between misvaluation and the allocation of capital by firms to real investment projects. That link may occur indirectly, through the equity issuance decision (Stein (1996), Baker and Wurgler (2002), Baker, Stein, and Wurgler (2003)), or directly, through catering by the firm to investor sentiment (Polk and Sapienza (2009)). Thus, it is naturally interesting to measure whether price-levels are also anomalous for portfolios sorted on investment, as measured by asset growth. The high investment and the low investment quintiles have a statistically significant difference in δ's of 29.4 percentage points, confirming the link between investment and price-level mispricing.

Accruals

Earnings management proxied by accruals (Sloan (1996)) is an interesting phenomenon to revisit with our explicit mispricing definition, as its typical interpretation is that companies with adverse operating results manage earnings to inflate the firm's market value. Thus, if the firms are successful in managing earnings, high accruals may proxy for overpricing perceived by firm

[31] The CFOs surveyed in the study identify market timing as the number two reason for stock issuance.

[32] Since the focus of our paper is to explain our novel method and document the corresponding empirical facts, we leave a more complete analysis of the flat price-level security market line to future work.

managers. The results in Table 6 do not support this interpretation of accruals, as it is a statistically weak predictor of delta.

Beta

Equation (60) shows that long-horizon risk, defined as a discounted sum of contemporaneous covariances between excess returns and the candidate SDF $\widetilde{M}$, helps determine abnormal price δ. Hence, the persistence of market beta implies that market beta sorts have the potential to generate large variation in $\delta = E\left[(P_t - V_t)/P_t\right]$. In particular, we would find that sorts on beta generate spread in δ if the resulting spread in long-horizon risk that must generate spread in CAPM-implied V is not compensated by a corresponding spread in price (P).

Table 6 shows an estimated difference in δs of 40.9 percentage points across the high- and low-beta portfolios, consistent with the above prediction. Furthermore, this estimate is statistically significant with a p-value of 2.9%. As discussed above, the flat security market line plays a particularly important role in our analysis, and the results for beta underscore this interpretation as risk-neutral δ is of the opposite sign as risk-adjusted δ.

5.2 Prominent return anomalies

Next, we turn to three remaining prominent return anomalies. Fama and French (2015) argue that profitability and size are characteristics that are important in summarizing the cross-section of returns, and price momentum has been a prominent return anomaly since Jegadeesh and Titman (1993). To what extent are these prominent return characteristics associated with variation in price levels unrelated to CAPM price-level risk?

Size and momentum

Size and momentum are interesting to study from the price-level perspective, given that momentum strongly predicts the cross-section of average returns but is a rather transitory firm characteristic while size weakly predicts the cross-section of average returns but is a rather persistent firm characteristic. In particular, Cohen, Polk, and Vuolteenaho (2009) highlight that signal persistence is an important consideration when moving from the conventional return perspective to the price-level perspective, a point that Cochrane (2011) subsequently emphasizes.

> "For example, since momentum amounts to a very small time-series correlation and lasts less than a year, I suspect it has little effect on long-run expected returns and hence the level of stock prices. Long-lasting characteristics are likely to be more important. Conversely, small transient price errors can have a large impact on return measures" (p.1064).

Consistent with Cochrane's conjecture, Table 6 shows that momentum is not a statistically significant predictor of CAPM abnormal price: the difference in δ between the high momentum and low momentum portfolios is 22.5 percentage points with a *p*-value of 13.6%. Moreover, the point estimates suggest that high momentum stocks with large positive abnormal returns are overpriced, whereas low momentum stocks with negative abnormal returns are underpriced.[33] This finding is consistent with momentum profits often coming from continued overreaction.[34] Figure 5 visualizes how momentum's initial positive α quickly turns into negative α post formation. Low momentum stocks start having higher average returns and lower CAPM betas compared to high momentum stocks from around year 2, contributing to momentum being associated with overpricing. While size is a persistent characteristic, it generates small and statistically insignificant price-level variation unaccounted by the CAPM.

Profitability

Table 6 documents that profitability-sorted portfolios are associated with price-level errors that are statistically insignificant with an estimated δ spread of only 9.0 percentage points. The result is consistent with the observation that cross-sectional variation in the marginal product of capital, which our profitability measure could proxy for, does not necessarily imply a misallocation of

[33]The point estimates in van Binsbergen et al. (2023) are consistent with momentum stocks being overpriced as well. In contrast to our results, their analysis finds statistical significance, suggesting the importance of our method's more reliable statistical inference.

[34]Lou and Polk (2022) provide extensive analysis arguing that momentum can transition from an underreaction to an overreaction phenomenon in the presence of destabilizing activity by momentum traders. Blank, Kwon, and Tang (2023) document similar findings in the cross-section for those stocks that are held relatively more by investors who react excessively to salient public news. Our price-level measure of mispricing δ can facilitate those sorts of empirical refinements of under and overreaction phenomena.

capital (Joel, Schmid, and Zeke (2020)).

5.3 Why are (some) return anomalies not also price anomalies?

Having studied ten characteristics individually from the price-level perspective and estimated their CAPM deltas, we now study them together to understand the factors that determine whether a return anomaly is also a price anomaly.

Taking two extreme quintile portfolios from each of the ten characteristics, we run a cross-sectional regression of CAPM delta on variables motivated by our identity:

$$\delta_i = b_0 + b_1\,\alpha_i + b_2\,[\alpha_i \times \mathbf{1}(Reversal_i)] + b_3\,\beta_i + b_4\,CumStateAdj_i + \varepsilon_i, \tag{30}$$

where α is the short-horizon (1-month) alpha, $\mathbf{1}(Reversal)$ is a dummy variable that takes the value of one if the average excess return in years three-to-15 following portfolio formation is opposite in sign to the average excess return in the first post-formation month, β is the CAPM beta of the anomaly's excess monthly return in the first post-formation month, and $CumStateAdj$ is the cumulative state adjustment from the decomposition in Corollary 4.[35] We include β as an incremental predictor of delta because a large initial β is likely to predict more persistent post-formation alphas. That is, since a portfolio's CAPM β is persistent, a flat security market line could cause a portfolio starting out with a large initial β to have persistent post-formation alphas. Table A14 and Figure A9 Panel D of the Internet Appendix show that these four variables explain 94% of the cross-sectional variation in the deltas of the twenty portfolios in question.

Figure 6 plots the component of fitted delta associated with each of these four explanatory variables for all ten long-short portfolios; for example, $\widehat{b}_1 \times (\alpha_L - \alpha_S)$ is the component of long-short delta explained by short-horizon alpha and is plotted with a black bar for each anomaly. Given the high R^2 in column (4) of Table A14, the four components together explain almost all of the cross-sectional variation in delta.

Figure 6 highlights four findings. First, short-horizon alphas explain only a small fraction of

[35] Note that excess returns are measured in excess of the market return.

the variation in abnormal price. Second, the largest price anomalies—*adjusted value* and beta—are those with a large initial beta, highlighting the importance of a flat security line in our price-level analysis. Third, the other two significant price anomalies—net issuance and investment—have delta contributions coming from a large initial alpha, a large initial beta, and (for investment) a large cumulative state adjustment. Although a sort on investment leads to an eventual reversal in returns, the other components of delta are enough to offset this effect, making investment a significant price anomaly. Finally, the return anomalies that do not make the cut for a significant delta either have weak contributions from short-horizon alpha and beta (e.g., B/M, profitability, size, accruals) and/or an offsetting effect from return reversal (B/M, quality, profitability, and momentum).

5.4 Double sorts on characteristics and *adjusted value*

Our proposed *adjusted value* characteristic proxies for abnormal price by combining information in price, profitability, and beta. How well, then, does it explain variation in δ generated by other characteristics: net issuance, investment, accruals, size, and momentum? Table 7 synthesizes our analysis by examining double sorts on *adjusted value* and each of the five characteristics. Specifically, sorting stocks into three-by-three portfolios based on independent NYSE breakpoints, we report the δ and associated t-statistic for each of the nine portfolios on the left-hand side of the table. The right-hand-side of the table reports the δ's associated with the combination of the nine portfolios that results in either a characteristic-neutral portfolio that bets on *adjusted value* or a *adjusted-value*-neutral portfolio that bets on the second characteristic.

Table 7 has two important takeaways. First, across all of the rows, *adjusted value* repeatedly generates economically and statistically significant variation in CAPM δ. Hence, *adjusted value* appears to contain information about prices that is neither explained by CAPM-implied risk nor captured by another single characteristic. Second, after controlling for *adjusted value*, there is little incremental information about prices in the characteristics we study, in terms of the economic magnitude or the statistical significance. This finding is true even for net issuance and investment which both showed significant spread in δ in a univariate sort but are subsumed by adjusted value.[36]

[36]Our findings based on these double sorts suggest that adjusted value could be the natural second factor to put in a multifactor model of price levels. However, a proper treatment of all potential multifactor refinements would require a substantial expansion to the analysis. For this reason, we believe our single factor analysis based on the CAPM is one

5.5 Modern subsample

Our results and conclusions in Sections 4 and 5 continue to hold in the modern subsample, 1972m6–2022m12. Combining information in price, profitability, and CAPM risk through our composite variable *adjusted value* describes economically and statistically significant variation in CAPM abnormal price (Tables A9 and A8 in the Internet Appendix) while the other characteristics we study generally do not, with the exception of net issuance and beta (Tables A7 and A10).[37] Furthermore, in a horse race, all other characteristics are subsumed by *adjusted value* (Table A11).

5.6 Alternative approaches

How do results change if we employ alternative approaches to estimating abnormal price? First, in terms of the point estimates, we show in Figure A6 that the cash-flow approach that directly discounts the cash flows and a terminal value generates essentially the same delta estimates as the version of our return-based approach that uses gross returns (rather than excess returns) and event-time observations (rather than calendar-time observations). This equivalence confirms that the cash-flow approach does not have an inherent advantage over the return-based approach but is less desirable in finite-sample inference. (See the discussion in Sections 1.3, 3.2, and 3.5.)

Second, Table A6 shows that cumulative abnormal return (CAR) generates different results, both in terms of magnitude and statistical significance at the 5% level. Third, Figure A9 shows that short-horizon CAPM alpha, its persistence (proxied by the persistence of the characteristic associated with that alpha), and the resulting interaction explain around half of the cross-sectional variation in delta across the twenty extreme quintile portfolios we study, implying that a simple measure based on short-horizon alpha and its persistence would miss important variation in abnormal price.

that most lucidly illustrates our novel return-based price-level analysis.

[37]Investment, accruals, and profitability are defined only over the modern subsample, so their results remain the same as before.

6 Conclusion

Our novel identity precisely links ex-ante price distortion to subsequent returns and provides a new natural framework for studying the cross-section of stock price levels. Our primary tests reveal that portfolios formed on 'adjusted value,' a composite signal that extracts the mispricing component from the market-to-book ratio, generate large variation in abnormal price, our measure of price distortion. Among all other prominent return anomalies, net equity issuance, investment, and beta sorts produce significant price-level distortions relative to the CAPM, and these distortions are subsumed by adjusted value.

As a consequence, our novel abnormal price measure identifies the stocks that a buy-and-hold mean-variance investor should find attractive/unattractive. Moreover, our approach highlights where new models that aim to explain both short- and long-run patterns in markets should focus. Indeed, by providing an exact metric of the extent to which a candidate asset-pricing model explains variation in prices, we aim to advance future research in both asset pricing and corporate finance. For the former, estimates of ex-ante price distortion could provide a useful lens through which to distinguish among risk-based, behavioral-based, and institutional-friction-based explanations for well-known empirical patterns in short-horizon returns. For the latter, our measure of mispricing with respect to a risk model may refine the results of a large literature (e.g. Baker and Wurgler (2002) and Shleifer and Vishny (2003)) that aims to link a firm's investment and financing decisions to price distortions.

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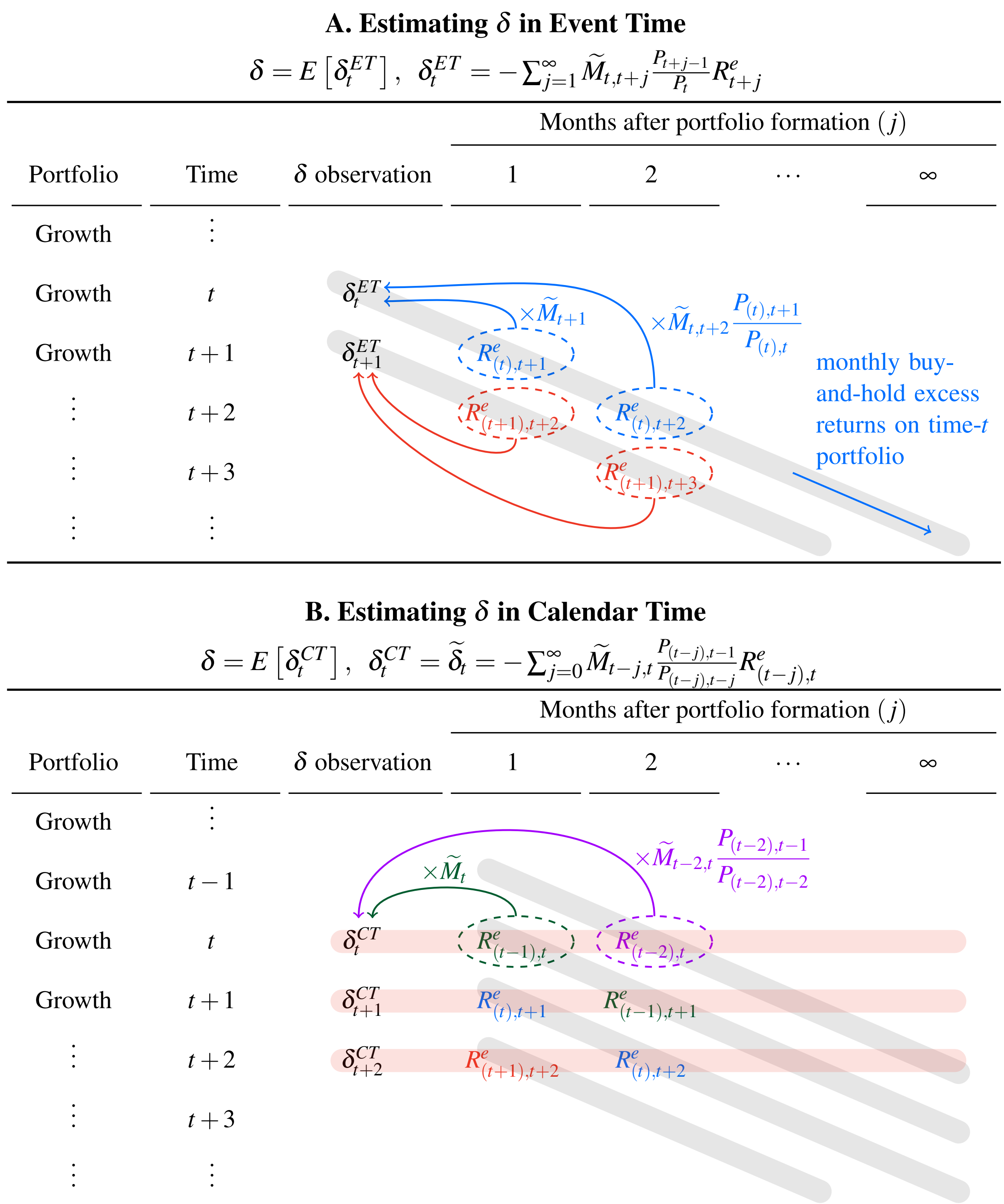


Figure 1: **Event Time vs. Calendar Time Approach to δ Estimation**

Panel A visualizes the event-time approach to estimating δ. The equation there shows that unconditional δ is the expectation of the event-time δ that appropriately discounts the post-formation buy-and-hold monthly excess returns on a growth portfolio formed today. Hence, the event-time approach takes a sum of all discounted post-formation excess returns diagonally in the southeast direction. Panel B visualizes the equivalent calendar-time approach. The equation there shows that unconditional δ also equals the expectation of the calendar-time δ that appropriately discounts today's realizations of monthly excess buy-and-hold returns on growth portfolios formed in the past. Hence, the calendar-time approach takes a sum of all discounted excess returns in the concurrent period (on portfolios formed in the past) horizontally.

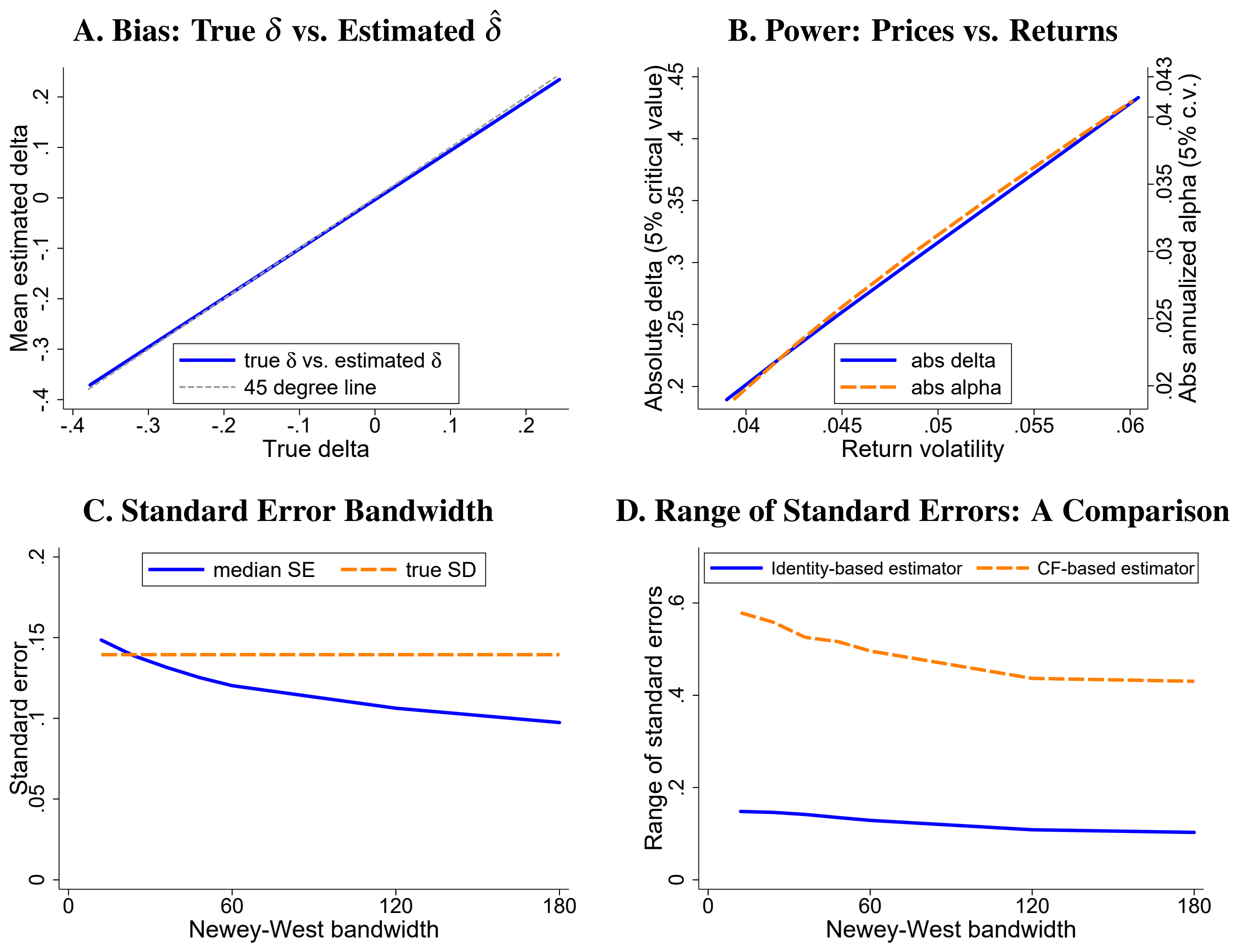


Figure 2: **Bias, Power, and Standard Errors of the Return-Identity-Based δ Estimator**

Panel A analyzes the bias of our estimator. The solid blue line relating the mean estimated $\hat{\delta}$ to the true δ from a Monte Carlo simulation coincides almost exactly with the 45-degree line in dotted gray except that a small attenuation (less than 1% point) appears in large values of δ.

Panel B shows that the magnitude of δ required for statistical significance is roughly 10-to-12 times that for annualized α, consistent with δ being a discounted sum of future αs over roughly 15 years. For each level of return volatility, the solid blue line (dotted orange line) reports the smallest absolute δ (α) needed for significance at the 5% level. The two lines roughly coincide when the scale of y axis for δ (left vertical axis) is approximately 11 times that for α (right vertical axis).

Panel C reports results from a Monte Carlo simulation analyzing whether a Newey-West standard error ("SE") with a bandwidth of around 2 years accurately estimates the true standard deviation ("SD") of $\hat{\delta}$.

Panel D shows that our return-identity-based method generates more precise standard errors than the cash-flow-based method. It reports the 10th-to-90th-percentile range of standard errors from a Monte Carlo simulation for the return-identity-based method (blue solid line) and the cash-flow-based method (dotted orange line).

The Monte Carlo simulation uses the parameter values reported in Table 2 except that the simulation for Panel B varies the volatilities of cash flow shocks and conditional δ_t shocks to generate variation in return volatility.

Figure 3: **The Risk-Return Relations in Returns and Price Levels: B/M and Quality**

The plots show that, for portfolios sorted on book-to-market or quality, the relation between long-horizon risk and long-horizon return (the right panel; Figures B and D) tends to be less anomalous than that between short-horizon risk and return (the left panel; Figures A and C). This improvement contributes to the statistically insignificant CAPM abnormal price associated with book-to-market and quality sorts. Long-horizon return and long-horizon risk summarize the term structure of post-formation average excess returns and of risk premia, respectively, and—together with the cumulative state adjustment component of abnormal price—determine the estimated $\widehat{\delta}$:

$$\underbrace{\sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] E_T\left[\widetilde{M}_t\right] E_T\left[R_t^e\right]}_{\text{long-horizon return}} = \underbrace{\sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] Cov_T\left(-\widetilde{M}_t, R_t^e\right)}_{\text{long-horizon risk}} \underbrace{-\sum_{j=1}^{J} Cov_T\left(\phi_{(t-j),t-1}, \widetilde{M}_t R_t^e\right)}_{\text{cumulative state adjustment}} - \widehat{\delta},$$

where $\phi_{(t-j),t-1} = \widetilde{M}_{t-j,t-1} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$, the expression $(t-j)$ in the subscript denotes the portfolio formation period, $J = 180$ post-formation months (15 years), and the subscript T indicates a sample moment. Short-horizon return and risk respectively denote the average and the CAPM-predicted excess returns above the market portfolio. The sample period is 1948m6–2022m12.

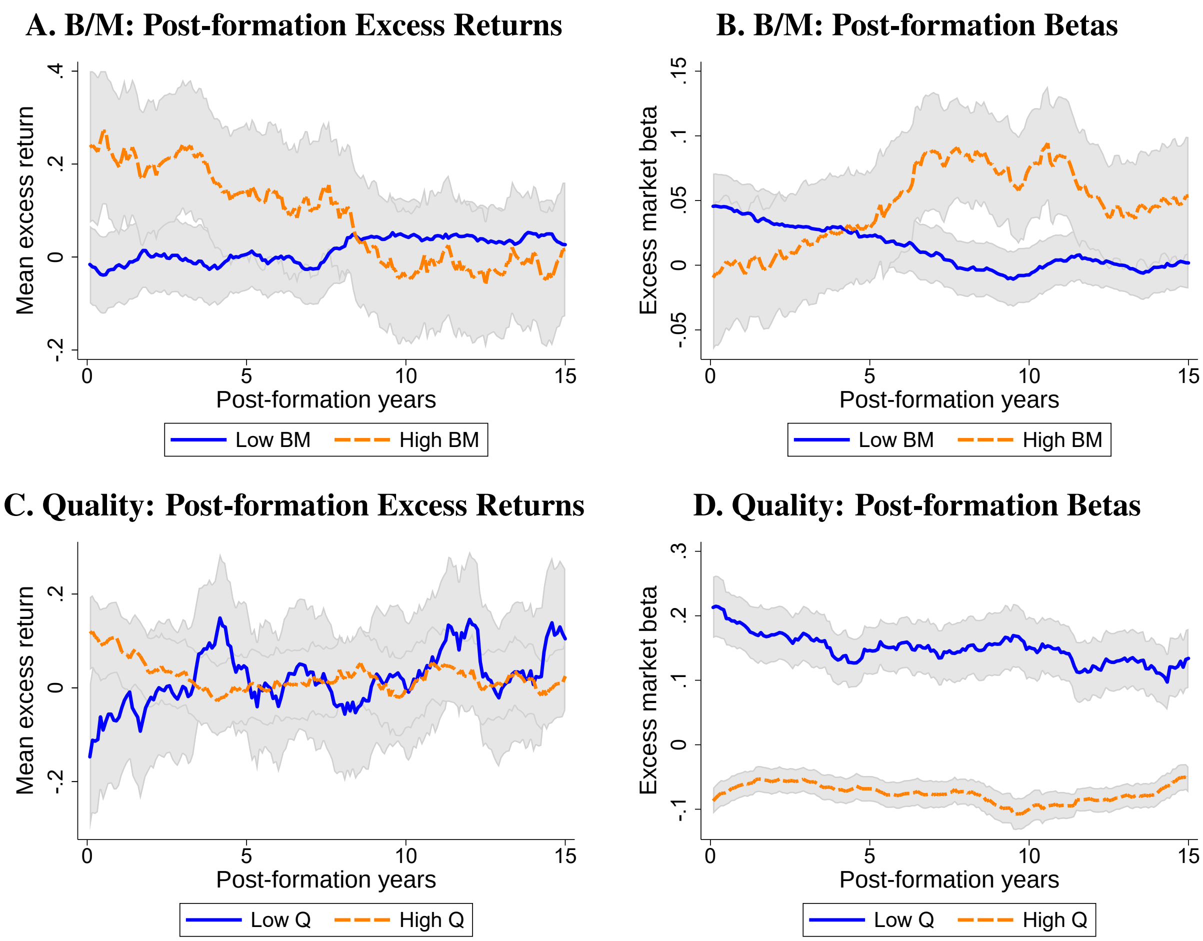


Figure 4: **Post-formation Behavior of Return and Risk: B/M and Quality**

The plots study the post-formation behavior of returns and CAPM betas of extreme quintile B/M or *quality* portfolios. The top plots (Panels A and B) show that the post-formation beta of value (high *BM*) stocks exceeds that of growth (low *BM*) stocks from around year 5, consistent with value stocks having higher post-formation mean returns than growth stocks until around year 8. The bottom plots (Panels C and D) show that junk (low *quality*) stocks have higher mean returns than quality stocks from around year 3, consistent with junk stocks having higher betas than quality stocks post formation. Excess returns used in the left panel are taken relative to post-formation returns on the market portfolio. The sample period is 1948m6–2022m12.

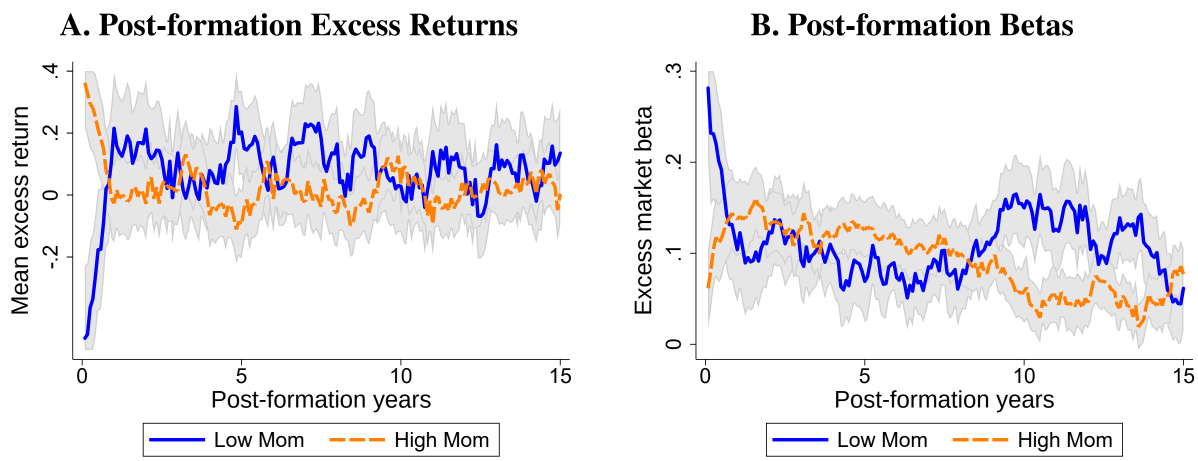


Figure 5: **Post-formation Behavior of Return and Risk: Momentum**

The plots study the post-formation behavior of returns and CAPM betas of extreme quintile momentum portfolios. They show that the anomalous pattern of high momentum stocks having higher excess returns but lower market betas than low momentum stocks quickly reverses such that low momentum stocks have higher excess returns and lower betas on those excess returns than high momentum stocks from around year 2 post portfolio formation. Excess return is measured relative to the post-formation returns on the market portfolio. The sample period is 1948m6–2022m12.

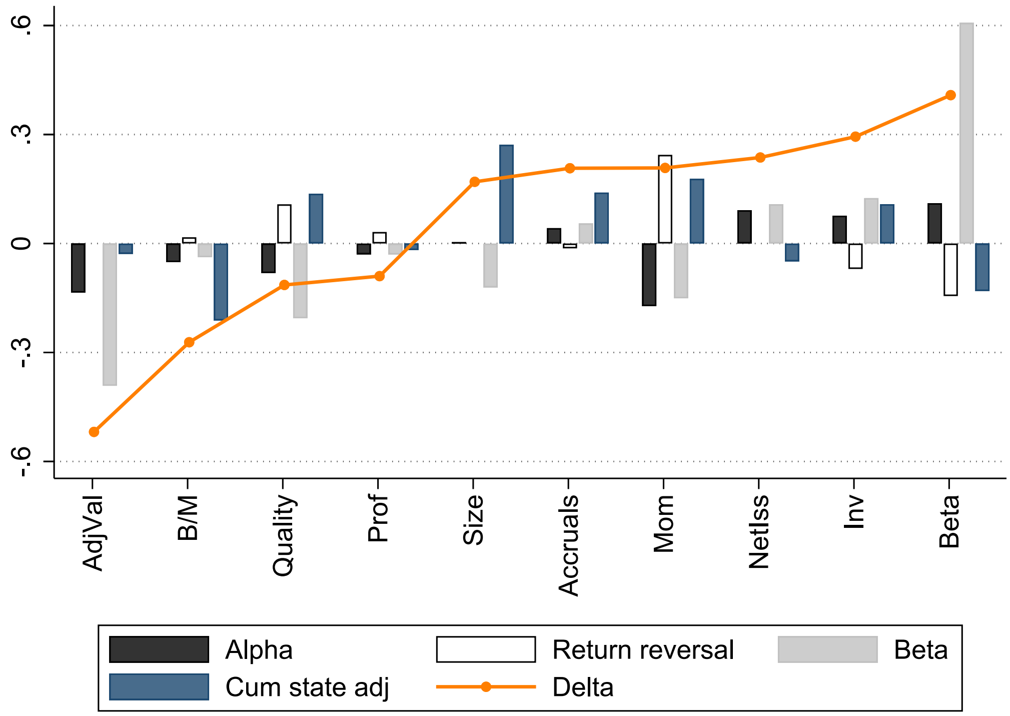


Figure 6: **Why Are (Some) Return Anomalies Not Also Price Anomalies?**

The figure plots the extent to which short-horizon alpha (+), post-formation return reversal (−), short-horizon beta (+), and cumulative state adjustment (+) explain portfolio long-short delta. The fitted values are based on a cross-sectional regression of delta on these four explanatory variables (Table A14; see also Figure A9), and each vertical bar measures the component of fitted value due to a particular explanatory variable. The sum across the four vertical bars for each characteristic portfolio is the fitted delta, with any remaining difference representing the regression residual. Overall, the figure shows that short-horizon beta contributes the most to delta for the two largest price anomalies (adjusted value and beta), whereas the other two statistically significant price anomalies (net issuance and investment) are driven by multiple factors. The other characteristics are not associated with a significant abnormal price typically because different components offset each other's effect (e.g., short-horizon alpha is offset by a subsequent reversal in returns). Return reversal is a dummy variable that takes the value of one if the average excess return in years three-to-15 following portfolio formation is opposite in sign to the average excess return in the first post-formation month. Short-horizon beta is the portfolio's one-month market beta immediately following portfolio formation and matters because beta tends to be persistent and large betas tend not to be accompanied by large excess returns (i.e., the security market line is flat). Cumulative state adjustment is defined in Section 3.3.

Table 1: **Estimated CAPM Abnormal Price (δ) of a T-bill Rollover Strategy**

This table presents the estimated CAPM δ, computed based on four different methods, of a strategy that rolls over the 1-month Treasury bill return for 15 years. The calendar-time excess return method proposed in the present study (#1) ensures that the T-bill rollover strategy has a near-zero estimated CAPM δ. The event-time excess return method (#2) also leads to a small CAPM δ of the T-bill strategy (but the corresponding $\widetilde{\delta}_t^{ET,excess}$ has large serial correlation). The gross-return method based on excess returns (#3) or the direct discount of cash flows (#4, equivalent to the event-time gross return method) leads to large estimated CAPM δs of the T-bill rollover strategy of around 50%. For each method, the candidate SDF's two parameters are estimated by imposing the restriction that the estimated delta of the market portfolio is zero as well as the complementary restriction that either the market's gross one-month return (methods #1 and #2) or its excess one-month return (methods #3 and #4) is perfectly explained.

- #1 Calendar-time excess return (the present paper):

$$\widehat{\delta} = \tfrac{1}{T}\textstyle\sum_{t=1}^{T} \widetilde{\delta}_t \quad \text{with} \quad \widetilde{\delta}_t = -\textstyle\sum_{j=1}^{J} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}$$

- #2 Event-time excess return:

$$\widehat{\delta}^{ET,excess} = \frac{1}{T}\sum_{t=1}^{T} \widetilde{\delta}_t^{ET,excess} \quad \text{with} \quad \widetilde{\delta}_t^{ET,excess} = -\sum_{j=1}^{J} \widetilde{M}_{t,t+j} \frac{P_{(t),t+j-1}}{P_{(t),t}} R^e_{(t),t+j}$$

- #3 Calendar-time gross return:

$$\widehat{\delta}^{CT,gross} = \tfrac{1}{T}\textstyle\sum_{t=1}^{T} \widetilde{\delta}_t^{CT,gross} \quad \text{with} \quad \widetilde{\delta}_t^{CT,gross} = -\textstyle\sum_{j=1}^{J} \widetilde{M}_{t-j,t-1} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} \left(\widetilde{M}_t(1+R_{(t-j),t}) - 1\right)$$

- #4 Direct discount of cash flows (equivalent to the event-time gross return method):

$$\widehat{\delta}^{CF} = \tfrac{1}{T}\textstyle\sum_{t=1}^{T} \widetilde{\delta}_t^{CF} \quad \text{with} \quad \widetilde{\delta}_t^{CF} = 1 - \textstyle\sum_{j=1}^{J} \widetilde{M}_{t,t+j} \frac{D_{(t),t+j}}{P_{(t),t}} - \widetilde{M}_{t,t+j} \frac{P_{(t),t+J}}{P_{(t),t}}$$

		Calendar Time vs. Event Time	
		Calendar Time	Event Time
Excess Return vs. Gross Return	Excess Return	#1 Calendar-time excess return (Cho and Polk) $\widehat{\delta} = 0.001$	#2 Event-time excess return $\widehat{\delta}^{ET,excess} = 0.028$
	Gross Return	#3 Calendar-time gross return $\widehat{\delta}^{CT,gross} = 0.457$	#4 Direct discount of cash flows $\widehat{\delta}^{CF} = 0.497$

Table 2: **Size and Power of the Return-identity-based Estimator of Abnormal Price**

Panel A provides a Monte Carlo analysis of the size and power of our return-based estimator of abnormal price δ. We choose the parameters in the simulation model to match the key moments of the high-*adjusted-value* portfolio. Using $B = 2$ years leads to a conservative rejection rate of 2.4% as opposed to the 5% significance level. Under the alternative in which the portfolio is underpriced on average by 36.6% with respect to the candidate SDF, the null is rejected 75% of the time.

Panel B examines the size and power of a conventional abnormal return test, showing that an annualized alpha of around 3.2% (12×0.26bp) paired with GMM standard errors with no lag has a similar statistical power as a price-level test on a delta of -36.6% and GMM standard errors with a NW lag of 2 years. For all tests, we use 1,000 simulations of the same number periods as in the actual data ($1,074$ months spanning 89.5 years).

Panel A. Mispricing ($J = 15$ years)

	Size ($\delta = 0$) and Power ($\delta = -0.366$)							
		Newey-West $se(\hat{\delta})$ with a block length of B yrs						
	True $\sigma(\hat{\delta})$	1 year	2 yrs	3 yrs	4 yrs	5 yrs	10 yrs	15 yrs
Size	0.047	0.016	0.024	0.027	0.035	0.042	0.072	0.098
Power	0.750	0.686	0.750	0.796	0.818	0.835	0.873	0.891

Panel B. Abnormal Return ($J = 1$ month)

	Size ($\alpha = 0$) and Power ($\alpha = 0.0026$)							
		Newey-West $se(\hat{\alpha})$with a block length of B months						
	True $\sigma(\hat{\alpha})$	0 mo	3 mo	6 mo	12 mo	24 mo	36 mo	60 mo
Size	0.051	0.056	0.053	0.050	0.054	0.058	0.064	0.069
Power	0.606	0.720	0.717	0.715	0.714	0.697	0.690	0.690

Table 3: **Descriptive Statistics on Stock Characteristics**

The table describes the ten characteristics we use to study CAPM-implied abnormal price. Column 2 reports the sample period over which post-formation returns for $j = 1$ through 180 months are available. Columns 3–5 report the CAPM alphas of the lowest and highest portfolio quintiles as well as the difference in the alphas between the two portfolios. We report t-statistics based on heteroskedasticity-robust standard errors in parentheses. Column 6 reports Persistence, the value-weighted probability that the characteristic decile of a stock in the portfolio does not change after a year. The remaining columns report the time-series average of the pairwise cross-sectional correlations among the characteristics. We use quintile numbers to compute these correlations to ensure that the correlations are not driven by outliers. Excess return is taken with respect to market returns. See Appendix B for a detailed description of the ten characteristics.

		CAPM alpha				Correlation									
	Sample	Lo	Hi	Hi-Lo	Persistence	B/M	Qlty	AdjVal	Size	Mom	NI	Beta	Inv	Prof	Acc
Book-to-market	Jun48-Dec22	-0.04	0.25	0.29	0.67	1.00									
		(-0.99)	(2.87)	(2.46)											
Quality	Jun48-Dec22	-0.28	0.17	0.46	0.56	-0.31	1.00								
		(-3.84)	(4.56)	(4.57)											
Adjusted value	Jun48-Dec22	-0.29	0.47	0.76	0.59	0.44	0.21	1.00							
		(-5.58)	(6.39)	(7.15)											
Size	Jun48-Dec22	0.03	0.01	-0.02	0.89	-0.35	0.26	-0.30	1.00						
		(0.23)	(0.29)	(-0.15)											
Momentum	Jun48-Dec22	-0.64	0.33	0.97	0.23	-0.00	0.11	0.05	0.10	1.00					
		(-5.74)	(4.39)	(5.81)											
Net issuance	Jun48-Dec22	0.24	-0.28	-0.51	0.49	-0.15	-0.33	-0.18	0.04	-0.03	1.00				
		(5.39)	(-5.24)	(-6.23)											
Beta	Jun48-Dec22	0.24	-0.38	-0.62	0.63	-0.15	-0.18	-0.66	0.15	-0.03	0.14	1.00			
		(3.58)	(-4.67)	(-4.94)											
Investment	Jun72-Dec22	0.25	-0.18	-0.43	0.35	-0.26	0.08	-0.18	0.15	-0.00	0.16	0.10	1.00		
		(3.30)	(-2.88)	(-3.75)											
Profitability	Jun72-Dec22	-0.08	0.09	0.17	0.77	-0.28	0.42	0.30	-0.05	0.02	-0.07	0.06	0.03	1.00	
		(-0.90)	(1.31)	(1.29)											
Accruals	Jun72-Dec22	0.04	-0.20	-0.24	0.37	-0.04	-0.09	-0.03	-0.00	-0.03	0.04	-0.01	0.28	-0.04	1.00
		(0.57)	(-3.08)	(-2.14)											

Table 4: **Pricing B/M- or $Quality$-sorted Portfolios: Returns vs. Prices**

The table shows that the book-to-market equity ratio (B/M) and *quality* are not statistically significant univariate signals of abnormal price relative to the CAPM (the last row), although they are significant signals of abnormal one-month returns (the first row). *Quality* follows the definition in Asness, Frazzini, and Pedersen (2019). We form value-weight quintile portfolios based on NYSE breakpoints and track post-formation returns for 15 years. In the first "return" row, δ measures -1 times the average one-month abnormal return:

$$\delta(1) \quad = \quad -E\left[\widetilde{M}_t R_t^e\right].$$

Positive (negative) δ here means negative (positive) one-month abnormal return. The reported δs in the last row are estimated values of abnormal price defined as

$$\delta = E\left[\frac{P_t - V_t}{P_t}\right] \quad \approx \quad \delta(180) = -E\left[\sum_{j=1}^{180} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right],$$

where $(t-j)$ denotes the portfolio formation month and t denotes the month in which returns are realized. In the remaining rows, the reported δ estimates illustrate how the estimated $\delta(J) = -E\left[\sum_{j=1}^{J} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$ changes as J takes values less than 180. We use the candidate SDF implied by the CAPM, $\widetilde{M}_{t-j,t} = \exp\left(b_0 j - b_1 \sum_{s=0}^{j-1} r^{mkt}_{t-s}\right)$, where r_t^{mkt} is log market returns and b_0 and b_1 are chosen to make the market portfolio's prices ($\delta = 0$) and returns ($\delta(1) = 0$) correct in sample. We report t-statistics (in parentheses) and p-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF. The sample period is 1948m6–2022m12.

Panel A. CAPM δ

	B/M							$Quality$						
	$\delta \times 100$							$\delta \times 100$						
J	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
1mo	0.04	0.02	-0.14	-0.16	-0.24	-0.28	0.017	0.29	0.04	0.06	-0.03	-0.17	-0.46	0.000
("return")	(0.97)	(0.44)	(-2.65)	(-2.30)	(-2.77)	(-2.38)		(3.85)	(0.65)	(1.51)	(-0.86)	(-4.55)	(-4.58)	
1yr	0.52	-0.23	-1.22	-2.38	-2.81	-3.33	0.055	2.49	0.61	0.71	-0.65	-1.88	-4.37	0.001
	(0.78)	(-0.43)	(-1.40)	(-2.19)	(-2.34)	(-1.92)		(2.78)	(0.71)	(1.19)	(-1.52)	(-3.31)	(-3.24)	
3yrs	0.70	-0.79	-2.07	-6.43	-7.89	-8.58	0.124	4.69	1.31	1.44	-1.17	-3.79	-8.48	0.034
	(0.30)	(-0.56)	(-0.77)	(-2.00)	(-2.22)	(-1.54)		(1.90)	(0.53)	(0.99)	(-0.84)	(-2.05)	(-2.12)	
5yrs	1.90	-0.59	-3.53	-9.68	-12.64	-14.53	0.163	4.19	1.37	1.39	-1.28	-3.74	-7.93	0.264
	(0.44)	(-0.25)	(-0.82)	(-1.77)	(-1.97)	(-1.39)		(1.02)	(0.31)	(0.49)	(-0.63)	(-1.09)	(-1.12)	
10yrs	4.58	1.30	-1.42	-14.42	-17.61	-22.19	0.277	7.83	4.70	3.03	-1.78	-5.27	-13.10	0.407
	(0.55)	(0.28)	(-0.15)	(-1.41)	(-1.41)	(-1.09)		(0.86)	(0.57)	(0.43)	(-0.69)	(-0.74)	(-0.83)	
15yrs	5.98	1.59	-2.14	-15.31	-21.19	-27.18	0.337	6.09	8.25	4.49	-4.13	-5.32	-11.41	0.609
("price")	(0.49)	(0.23)	(-0.16)	(-1.08)	(-1.26)	(-0.96)		(0.48)	(0.71)	(0.41)	(-1.14)	(-0.52)	(-0.51)	

Panel B. Risk-neutral δ

	B/M							$Quality$						
	$\delta \times 100$							$\delta \times 100$						
J	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
1mo	0.02	0.04	-0.09	-0.13	-0.24	-0.25	0.027	0.15	0.03	0.08	-0.03	-0.12	-0.26	0.008
("return")	(0.38)	(0.95)	(-1.84)	(-1.94)	(-2.84)	(-2.21)		(2.00)	(0.63)	(1.92)	(-0.73)	(-3.07)	(-2.66)	
15yrs	5.50	-0.90	-5.93	-13.25	-24.44	-29.94	0.123	-13.32	0.17	2.73	-4.21	6.27	19.58	0.186
("price")	(0.64)	(-0.18)	(-0.70)	(-1.24)	(-2.11)	(-1.54)		(-1.46)	(0.02)	(0.37)	(-1.12)	(0.98)	(1.32)	

Table 5: **Pricing *Adjusted-value*-sorted Portfolios: Returns vs. Prices**

The table shows that *adjusted value*, our proxy for the value-to-price ratio (V/P), generates cross-sectional variation in price levels (the last rows) and returns (the first row) not explained by the CAPM. *Adjusted value* combines B/M, profitability, and beta by taking the sum of their z scores: *Adjusted value* $= z(B/M) + z(\text{Prof}) - z(\text{Beta})$. We form value-weight quintile portfolios based on NYSE breakpoints for *adjusted value* and track post-formation returns for 15 years. In the first "return" row, δ measures -1 times the average one-month abnormal return:

$$\delta(1) \quad = \quad -E\left[\widetilde{M}_t R_t^e\right].$$

Positive (negative) δ here means negative (positive) one-month abnormal return. The reported δs in the last row are estimated values of abnormal price defined as

$$\delta = E\left[\frac{P_t - V_t}{P_t}\right] \quad \approx \quad \delta(180) = -E\left[\sum_{j=1}^{180} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right],$$

where $(t-j)$ denotes the portfolio formation month and t denotes the month in which returns are realized. In the remaining rows, the reported δ estimates illustrate how the estimated $\delta(J) = -E\left[\sum_{j=1}^{J} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$ changes as J takes values less than 180. We use the candidate SDF implied by the CAPM, $\widetilde{M}_{t-j,t} = \exp\left(b_0 j - b_1 \sum_{s=0}^{j-1} r_{t-s}^{mkt}\right)$, where r_t^{mkt} is log market returns and b_0 and b_1 are chosen to make the market portfolio's prices ($\delta = 0$) and returns ($\delta(1) = 0$) correct in sample. We report t-statistics (in parentheses) and p-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF. The sample period is 1948m6–2022m12.

	$\delta \times 100$							
J	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)	$[\text{Hi - Lo}]^{RN}$
1mo	0.29	-0.00	-0.17	-0.23	-0.46	-0.75	0.000	-0.39
("return")	(5.46)	(-0.01)	(-3.43)	(-4.01)	(-6.11)	(-6.88)		(-3.82)
1yr	3.24	0.10	-1.96	-3.04	-5.02	-8.26	0.000	-4.12
	(4.39)	(0.16)	(-3.01)	(-3.51)	(-5.24)	(-5.75)		(-3.03)
3yrs	7.08	0.76	-4.91	-8.06	-12.28	-19.37	0.000	-6.98
	(3.54)	(0.44)	(-2.24)	(-3.35)	(-4.34)	(-4.69)		(-1.74)
5yrs	9.34	2.45	-6.90	-12.17	-18.92	-28.26	0.000	-8.95
	(3.59)	(1.05)	(-2.03)	(-3.66)	(-3.57)	(-4.15)		(-0.05)
10yrs	14.72	0.44	-9.04	-20.74	-26.88	-41.60	0.000	-11.17
	(3.79)	(0.13)	(-2.07)	(-3.49)	(-3.05)	(-3.63)		(-0.62)
15yrs	18.45	2.32	-13.15	-29.92	-33.41	-51.86	0.002	-8.54
("price")	(3.33)	(0.47)	(-2.80)	(-2.46)	(-2.69)	(-3.14)		(-0.47)

Table 6: **Pricing Anomaly-sorted Portfolios**

The table reports estimated abnormal price with respect to the CAPM for portfolios sorted on characteristics conceptually linked to abnormal price or prominent return anomaly characteristics. For each characteristic, we form value-weight quintile portfolios based on NYSE breakpoints and track post-formation returns for 15 years. Hi (Lo) denotes stocks with the highest (lowest) value of the characteristic. The reported δs are estimated values of abnormal price defined as $\delta = E\left[\frac{P_t - V_t}{P_t}\right]$. We report *t*-statistics (in parentheses) and *p*-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF. The sample period is 1948m6–2022m12 except for investment, accruals, and profitability, which have a sample period of 1972m6–2022m12.

	$\delta \times 100$							
Sort	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)	[Hi - Lo]^{RN}
Net issuance	-16.45	-4.24	2.02	-0.18	7.22	23.67	0.006	8.62
	(-2.67)	(-0.40)	(0.52)	(-0.03)	(1.08)	(2.72)		(0.93)
Investment	-17.61	-17.82	-2.96	8.99	11.80	29.41	0.035	16.68
	(-2.33)	(-2.50)	(-0.68)	(1.67)	(1.43)	(2.11)		(1.08)
Accruals	0.18	-11.66	0.18	4.88	20.89	20.71	0.222	9.40
	(0.02)	(-1.97)	(0.03)	(0.93)	(1.98)	(1.22)		(0.59)
Beta	-22.63	-15.89	-4.95	5.28	18.21	40.85	0.029	-26.86
	(-1.88)	(-2.19)	(-1.11)	(1.00)	(2.05)	(2.18)		(-1.21)
Size	-13.44	-16.91	-20.64	-13.50	3.54	16.98	0.549	54.62
	(-0.53)	(-0.92)	(-1.19)	(-1.14)	(1.04)	(0.60)		(1.91)
Momentum	-16.49	-7.67	-3.68	2.56	4.31	20.80	0.136	22.48
	(-1.15)	(-1.89)	(-1.10)	(0.53)	(0.74)	(1.49)		(1.85)
Profitability	13.35	-9.30	-14.43	-4.96	4.37	-8.98	0.805	2.25
	(0.63)	(-0.66)	(-1.27)	(-0.41)	(0.24)	(-0.25)		(0.08)

Table 7: **Incremental Information About Prices:** ***Adjusted Value*** **vs. Others**

The table shows that controlling for *adjusted value*, our proxy for the value-to-price ratio (V/P), subsumes the ability of other characteristics to predict CAPM abnormal price. In contrast, *adjusted value* retains its ability to predict CAPM abnormal price when controlling the other characteristic in question. To draw this conclusion, we form nine value-weight portfolios based on 30% and 70% NYSE breakpoints for *adjusted value* and, independently, the 30% and 70% NYSE breakpoints for the second sorting characteristic specified in column one. We study the five characteristics that do not comprise *adjusted value*. *Adjusted value* combines B/M, profitability, and beta by taking the sum of their z scores: *Adjusted value* $= z(B/M) + z(\text{Prof}) - z(\text{Beta})$. The left-hand side of the table reports the estimated δ and associated t-statistic for each portfolio. The right-hand-side of the table reports the δs associated with the combination of the portfolios that results in either a characteristic-neutral portfolio that bets on *adjusted value* or a *adjusted-value*-neutral portfolio that bets on the second characteristic. We report t-statistics (in parentheses) and p-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF. The sample period is 1948m6–2022m12 except for investment and accruals, which has a sample period of 1972m6–2022m12.

	Adjusted value sort									*Adj val* sort (Second sort neutral)	Second sort (*Adj val* neutral)
	Low			2			High			$\frac{1}{3}*((H1+H2+H3)$	$\frac{1}{3}*((L3+23+H3)$
Second sort →	1	2	3	1	2	3	1	2	3	$-(L1+L2+L3))$	$-(L1+21+H1))$
Net issuance	10.68	16.63	11.63	-18.61	-8.66	-6.78	-37.70	-33.00	-28.97	-46.20	7.17
	(1.26)	(3.40)	(1.45)	(-2.53)	(-1.39)	(-1.12)	(-3.16)	(-2.63)	(-1.80)	(-2.80), [0.005]	(0.83), [0.408]
Investment	0.55	16.97	19.88	-25.06	-10.01	-3.90	-44.06	-41.42	-35.40	-52.76	16.39
	(0.05)	(2.17)	(2.29)	(-2.95)	(-1.42)	(-0.44)	(-2.80)	(-2.67)	(-2.16)	(-2.95), [0.003]	(1.62), [0.106]
Accruals	8.33	12.13	26.95	-14.54	-11.05	-1.05	-43.42	-33.93	-46.72	-57.16	9.60
	(0.79)	(1.90)	(2.46)	(-1.86)	(-1.23)	(-0.10)	(-1.94)	(-2.58)	(-2.16)	(-2.80), [0.005]	(0.81), [0.416]
Size	4.71	-3.82	15.69	-8.55	-24.56	-9.37	-41.36	-36.79	-33.66	-42.80	5.95
	(0.18)	(-0.25)	(3.25)	(-0.36)	(-1.49)	(-1.32)	(-1.49)	(-1.99)	(-2.34)	(-2.66), [0.008]	(0.25), [0.804]
Momentum	2.56	15.06	16.37	-25.54	-12.59	-4.44	-48.68	-33.60	-29.43	-48.56	18.06
	(0.34)	(2.26)	(2.54)	(-2.35)	(-2.09)	(-0.63)	(-2.06)	(-2.76)	(-2.28)	(-2.78), [0.005]	(1.61), [0.107]

Internet Appendix

A Supplementary Literature Review

A.1 Additional related work

An important motivation for studying price levels is the link between stock price levels and corporate financing or investment decisions as explored by Stein (1996), Baker and Wurgler (2002), Baker, Stein, and Wurgler (2003), Shleifer and Vishny (2003), Cohen, Polk, and Vuolteenaho (2009), Polk and Sapienza (2009), van Binsbergen and Opp (2019), and Whited and Zhao (forthcoming) among others. For example, Polk and Sapienza (2009) study how price distortion relates to corporate investment, using discretionary accruals to proxy for price distortion, and van Binsbergen and Opp (2019) study the link in a quantitative model of a production economy to study how abnormal returns on anomaly characteristics affect output. Dessaint, Olivier, Otto, and Thesmar (2021) find evidence that the beta anomaly's CAPM abnormal price as perceived by firm managers affects the M&A decision. Gormsen and Huber (2023) and Gormsen and Huber (2022) explore how firms' perceived costs of capital relate to factor models and affect corporate investment.[38]

The asset pricing literature also explored the difference in the type of information that expected returns and price levels have about capital market efficiency. Shiller (1984) writes, "because real returns are nearly unforecastable, the real price of stocks is close to the intrinsic value . . . is one of the most remarkable errors in the history of economic thought" (pp. 458–459). Summers (1986) provides a numerical example that illustrates this argument and Campbell (2018) shows how an expected return that follows a persistent AR(1) process may leave little room for return predictability despite a large variance in the dividend-price ratio. Pastor and Veronesi (2003) show that high price levels may not be a signal of capital market inefficiency but of increased uncertainty about future profitability. More recently, Liu, Moskowitz, and Stambaugh (2021) use a restriction on the price distortion process to revisit factor models of expected returns and Baba Yara, Boons, and Ta-

[38] Taking the link between price distortion and equity financing as given, Cho and Salarkia (2020) show that firms' equity issuance and repurchases in the face of apparent model-specific price distortion reveals the CAPM as the model most likely used by firms.

moni (2020) study the extent to which the permanent and transitory components of characteristics differently describe the cross-section of long-horizon average returns.[39]

Other strands of literature study price levels and long-horizon returns for different reasons. First, these quantities are important for the portfolio decision of long-term investors. For instance, Cochrane (2014) develops a mean-variance characterization of a stream of long-run payoffs that is useful even when risks and expected returns vary through time.[40] Second, Vuolteenaho (2002), Cohen, Polk, and Vuolteenaho (2003), Cochrane (2011), De La O and Myers (2021), and Cho et al. (forthcoming) among many others study valuations, expected returns, and cash flows through the lens of an identity in the spirit of Campbell and Shiller (1988). Third, Lee, Myers, and Swaminathan (1999), Bartram and Grinblatt (2018), Gerakos and Linnainmaa (2018), Asness, Frazzini, and Pedersen (2019), Golubov and Konstantinidi (2019), and Favero, Melone, and Tamoni (2020) take different approaches to come up with proxies for price distortion. Finally, Koijen, Richmond, and Yogo (2022) use a structural demand-based approach to study how different types of investors affect equity valuations.

Cumulative abnormal returns (CARs) or buy-and-hold abnormal returns (BHARs) are used extensively in the corporate finance literature. Barber and Lyon (1997), Kothari and Warner (1997), Fama (1998), Lyon, Barber, and Tsai (1999), Brav (2000), and Bessembinder, Cooper, and Zhang (2018) have critically evaluated these approaches.

A.2 Detailed Response to van Binsbergen et al. (2023)

Internet Appendix C of van Binsbergen, Boons, Opp, and Tamoni (2023) (vBBOT) compares their dividend-based event-time approach to our calendar-time, (excess) return-based approach. This type of comparison is useful, as it allows us to reflect further on the advantages and (potential) disavantages of our proposed method. However, the first two (out of three) issues they point out apply to methodological aspects of an older version of our paper (Dec 2020) that are not present

[39]Other recent papers on the topic of market efficiency and price levels include Bai, Philippon, and Savov (2016), Dávila and Parlatore (2020), Joel, Schmid, and Zeke (2020), and Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2020).

[40]See also Kandel and Stambaugh (1996), Campbell and Viceira (1999), Barberis (2000), and Viceira (2001) among several others.

in our analysis. The third issue they point out is a weakness particular to their dividend-based approach and only helps highlight the strength of our excess-return-based approach. The last issue they point out is a matter of taste as to whether one prefers a simple or log measure of abnormal price and has little empirical consequence. We explain these points in further detail below.

1. Risk-free rate in the candidate SDF.

First, vBBOT argue that our candidate SDF does not discount returns more in times of high risk-free rates. They base this critique on an older version of our paper (Dec 2020) that used the excess return on the market as a factor in the candidate SDF:

$$\widetilde{M}_t = b_0 - b_1 R_t^{mkt,e}, \tag{31}$$

where $R_t^{mkt,e}$ denotes market return in excess of the risk-free rate.

However, since that draft, we have switched to using the the gross return on the market in our loglinear SDF setup employed by Korteweg and Nagel (2016).

$$\widetilde{M}_t = \exp(b_0 - b_1 \log(1 + R_t^{mkt})), \tag{32}$$

Since the gross market return includes the risk-free rate as a component, this candidate SDF does apply a large discount on returns in times of high risk-free rates, not just in times of high risk premium.[41]

The candidate SDF in vBBOT follows

$$\widetilde{M}_t = \exp(-\log(1 + R_{f,t}) + b_0 - b_1 \left(\log(1 + R_t^{mkt}) - \log(1 + R_{f,t]})\right)), \tag{33}$$

where $R_{f,t}$ is the risk-free rate known at time $t-1$ and realized at time t. Whereas the risk-free rate component is embedded in the gross market return in equation (32), equation (33) disentangles the risk-free rate and the (log) market risk premium components of the SDF. However, we find that

[41] In earlier drafts, e.g., Apr 2021, we used a linearized SDF based on the gross market return: $\widetilde{M}_t = b_0 - b_1 R_t^{mkt}$, where R_t^{mkt} is the gross market return.

empirically using the candidate SDF in equation (33) has little effect on either the estimates of δ or their associated p-values.

2. Computing excess returns with respect to the risk-free rate vs. market return

The second critique of vBBOT also does not apply to our paper's methodology. They argue that computing excess returns with respect to the Treasury bill rate as part of our identity,

$$\delta \equiv E\left[\frac{P_t - V_t}{P_t}\right] = -\sum_{j=1}^{\infty} E\left[\widetilde{M}_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right], \tag{34}$$

requires the candidate SDF to explain the T-bill rate conditionally. However, since the Dec 2020 draft, we have switched to using the market rather than the Treasury bill as the base asset with which to compute excess returns R^e_{t+j}. Since we estimate CAPM-based δ's, computing excess returns against the market is a natural methodological choice, given that the CAPM implies that the market is correctly priced, and follows other research (e.g., Campbell et al. (2018) and Korteweg and Nagel (2022)) that also prices returns in excess of the market.

3. Inability of vBBOT to price Treasury bill strategies

vBBOT argue that both their dividend-based event-time framework and our framework are subject to a large bias that results from having to estimate SDF parameters in a finite sample. They rely on a back-of-the-envelope calculation to show that using their dividend-based method means that the strategy of rolling over T-bill investments for 15 years has an estimated δ of more than 50%, despite the fact that their candidate SDF is designed to price T-bill rates conditionally (Internet Appendix D.1 of vBBOT). We confirm that in our sample, applying an event-time *gross-return* approach to a strategy that rolls over T-bills results in an estimated δ of 0.497 (49.7%), similar to the number in vBBOT (which they then hope to correct through a bootstrapping adjustment).

We find that this source of bias has little effect on δ estimates based on our approach. When we apply our novel calendar-time, *excess-return* approach, the same roll-over strategy that is dramatically mispriced by vBBOT has an estimated δ of only 0.1% in our full sample (1948m6–2022m12) and 2.3% in the modern subsample (1972m6–2022m12). This finding is consistent with Figure 2A in our main paper, which confirms, based on Monte Carlo analysis, that our δ estimates are close to being unbiased.[42]

What is it about our approach ensures that strategies such as rolling over the T-bill do not have an artificially inflated δ estimate? Our method is not vulnerable to this defect in vBBOT's method primarily because our estimated δ aggregates future *excess* returns R^e_{t+j} (see equation (33) above) rather than gross returns.[43] To see why our approach is immune to this concern, suppose, for the sake of argument, that we bring equation (34) to data by computing excess returns against the T-bill rate. Then, by definition, the T-bill roll-over strategy earning the T-bill rate in each period has zero excess return in all periods and must have an estimated δ of zero.

In practice, of course, we compute excess returns against the market rather than the T-bill. Even so, the use of excess returns rather than gross returns in this way helps reduce the impact of measurement errors in a candidate $\widetilde{M}$ on our δ estimates. We explain this in detail in Section 3.2 of the main paper.

4. Log vs. simple mispricing

vBBOT choose to estimate the log abnormal price of Cohen, Polk, and Vuolteenaho (2009) (equation (6)) rather than the simple abnormal price measure estimated in our paper. They implicitly acknowledge, however, that one definition is not inherently superior to another. We prefer working with simple abnormal price measure, as it allows us to develop a nonparametric estimator of abnormal price with several desirable properties, whereas log abnormal price does not.

[42]Note that in our Monte Carlo, the interest rate is held fixed for the sake of tractability. We leave a generalization of our Monte Carlo model to allow for time-varying interest rates for future research.

[43]Note that the evidence in Figure A6 only confirms the equivalence between the event-time, dividend-based approach and an event-time, *gross-return* approach. Our main approach uses the calendar-time excess-return expression, which helps minimize the bias as well as serial correlations.

B Empirical Appendix

B.1 Basic data adjustments

We use domestic common stocks (CRSP share code 10 or 11) listed on the three major exchanges (CRSP exchange code 1, 2, or 3). We replace missing prices with the average bid-ask price when available and drop observations with missing share or price information in the previous month. We code missing returns as zero returns and add delisting returns to returns. If delisting returns are missing, but the CRSP delisting code is 500 or between 520 and 584, we use -35% (-55%) as the delisting returns for NYSE and AMEX stocks (for NASDAQ stocks) (Shumway (1997) and Shumway and Warther (1999)). To compute capital gains, we use the CRSP split-adjustment factor (CFACPR) to ensure that capital gains are not affected by split events. We use NYSE breakpoints when sorting stocks throughout the analysis and always study value-weight portfolios.

B.2 Characteristics and portfolios

An important stock characteristic is the book-to-market-equity (B/M) ratio computed each year in June. B/M ratio is the stock's book value of equity in the previous fiscal year divided by its market value of equity in December of the previous calendar year. Book value of equity is defined as stockholders' equity SEQ (Compustat item 144) plus balance sheet deferred taxes and investment tax credit $TXDITC$ (item 35) minus book value of preferred stock ($BE = SEQ + TXDITC - BPSTK$). Book value of preferred stock $BPSTK$ equals the preferred stock redemption value $PSTKRV$ (item 56), preferred stock liquidating value $PSTKL$ (item 10), preferred stock $PSTK$ (item 130), or zero depending on data availability. If SEQ is unavailable, we set it equal to total assets AT (item 6) minus total liabilities LT (item 181). If $TXDITC$ is unavailable, it is assumed to be zero. In the pre-Compustat period, we use the book equity data from Davis, Fama, and French (2000). We treat zero or negative book values as missing.

Another stock characteristic used in our preliminary analysis is the quality measure of Asness, Frazzini, and Pedersen (2019) defined as a z-score based on four characteristics—profitability, growth, safety, and payout ratio—that determine the market-to-book ratio in a Gordon growth model and in the absence of mispricing: quality $= z\left(z_{\text{profitability}} + z_{\text{growth}} + z_{\text{safety}} + z_{\text{payout ratio}}\right)$. The four characteristic z scores are in turn obtained as an equal weighted average of z scores based different measures of each characteristic. When some of the underlying measures are missing, the z score is taken over all available measures. In the pre-Compustat period, we use the book equity numbers that Davis, Fama, and French (2000) collected from the Moody's Industrial, Public Utility, Transportation, and Bank and Finance Manuals to calculate measures that require book equity data. Quality is computed once a year at the end of June and requires the past six years of data in order to compute z_{growth}. See Asness, Frazzini, and Pedersen (2019) for further details.

As discussed in the main body of the paper, our core analysis uses a three-characteristic model of the value-to-price ratio named *adjusted value*. We simply add the z scores of B/M and profitability and subtract the z score of beta. For profitability, we use the z score of gross profitability when available, and the z score of return on equity otherwise.

We also examine portfolios sorted by seven additional characteristics: size, momentum, net issuance, beta, profitability, investment, and accruals. The first four characteristics can be computed in the pre-Compustat period, whereas the last three characteristics are available only in the post-Compustat period. Size is market equity calculated at the end of each month. Momentum is calculated is the cumulative gross return over the previous 12 months excluding the month before the portfolio formation and is also computed at the end of each month. Net issuance is calculated annually at the end of each June and is the split-adjusted growth in shares outstanding over the previous 12 months. Beta is the trailing 3-year market beta (minimum of 2 years) calculated each month based on overlapping 3-day returns.

Profitability is computed each year in June. Gross profitability ("profitability") in calendar year y equals sales *SALE* (Compustat item 12) minus cost of goods sold *COGS* (item 41) in fiscal year $y-1$ over total assets in fiscal year $y-1$ Asset growth ("investment") is also computed each year in June, and investment in calendar year y is total assets in fiscal year $y-1$ divided by total assets in fiscal year $y-2$. Accruals measures the degree to which earnings come from non-cash sources and is defined according to Sloan (1996).

B.3 The GMM

To estimate the deltas of characteristic-sorted portfolios, write the sample moments and the GMM restriction as

$$\boldsymbol{g}_T(\boldsymbol{b}) = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{u}_t(\boldsymbol{b})$$

$$\boldsymbol{A}\boldsymbol{g}_T(\boldsymbol{b}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

where the first two moments set the market portfolio's alpha and delta with respect to the candidate SDF to be zero:

$$\boldsymbol{u}_t(\boldsymbol{b}) = \begin{pmatrix} \widetilde{M}_t R_t^{mkt,e} & -\sum_{j=1}^{J}\widetilde{M}_{t-j,t-1}\frac{P^{mkt}_{(t-j),t-1}}{P^{mkt}_{(t-j),t-j}}\left(\widetilde{M}_t\left(1+R^{mkt}_{(t-j),t}\right)-1\right) & \tilde{\delta}_{1,t} & \ldots & \tilde{\delta}_{N,t} \end{pmatrix}'$$

$$\boldsymbol{A} = \begin{pmatrix} J & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \end{pmatrix}.$$

Recall that we model one-period candidate SDF as $\widetilde{M}_t = \exp\left(b_0 - b_1 r_t^{mkt}\right)$ with r_t^{mkt} denoting log market return and cumulative candidate SDF as $\widetilde{M}_{(t-j),t} = \exp\left(b_0 j - b_1 r^{mkt}_{(t-j),t}\right)$ with $r^{mkt}_{(t-j),t}$ denoting log market return from $t-j$ to t. Hence, the asymptotic variance-covariance matrices of the parameters and the sample moments are

$$Var\left(\sqrt{T}\hat{\boldsymbol{b}}\right) = (AD)^{-1}ASA'(AD)^{-1,'}$$

$$Var\left(\sqrt{T}\boldsymbol{g}_T\left(\hat{\boldsymbol{b}}\right)\right) = \left[I_{N+2} - D(AD)^{-1}A\right]S\left[I_{N+2} - D(AD)^{-1}A\right]'$$

and the finite-sample variance estimates are

$$\hat{V}\left(\hat{\boldsymbol{b}}\right) = \frac{1}{T}\left(A\hat{D}\right)^{-1}A\hat{S}A'\left(A\hat{D}\right)^{-1,'}$$

$$\hat{V}\left(\boldsymbol{g}_T\left(\hat{\boldsymbol{b}}\right)\right) = \frac{1}{T}\left[I_{N+2} - \hat{D}\left(A\hat{D}\right)^{-1}A\right]\hat{S}\left[I_{N+2} - \hat{D}\left(A\hat{D}\right)^{-1}A\right]',$$

where S is the spectral density matrix and

$$D = E\left[\frac{\partial \boldsymbol{u}_t(\boldsymbol{b})}{\partial \boldsymbol{b}'}\right]$$

is estimated by $\hat{D}$, which equals

$$T^{-1}\sum_{t=1}^{T}\begin{bmatrix} \widetilde{M}_t(1+R_t^{mkt}) & -r_t^{mkt}\widetilde{M}_t(1+R_t^{mkt}) \\ -\sum_{j=1}^{J} j\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}^{mkt}}{P_{(t-j),t-j}^{mkt}}R_{(t-j),t}^{mkt,e} & \sum_{j=1}^{J} r_{t-j,t}^{mkt}\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}^{mkt}}{P_{(t-j),t-j}^{mkt}}R_{(t-j),t}^{mkt,e} \\ -\sum_{j=1}^{J} j\widetilde{M}_{t-j,t}\frac{P_{1,(t-j),t-1}}{P_{1,(t-j),t-j}}R_{1,(t-j),t}^{e} & \sum_{j=1}^{J} r_{t-j,t}^{mkt}\widetilde{M}_{t-j,t}\frac{P_{1,(t-j),t-1}}{P_{1,(t-j),t-j}}R_{1,(t-j),t}^{e} \\ \vdots & \vdots \\ -\sum_{j=1}^{J} j\widetilde{M}_{t-j,t}\frac{P_{N,(t-j),t-1}}{P_{N,(t-j),t-j}}R_{N,(t-j),t}^{e} & \sum_{j=1}^{J} r_{t-j,t}^{mkt}\widetilde{M}_{t-j,t}\frac{P_{N,(t-j),t-1}}{P_{N,(t-j),t-j}}R_{N,(t-j),t}^{e} \end{bmatrix}.$$

with $r_{0,0}^{mkt}$ defined to be 0. The spectral density matrix is estimated as follows:

$$\hat{S}_T\left(\hat{\boldsymbol{b}}\right) = \hat{\Gamma}_0 + \sum_{b=1}^{B}\frac{B-b}{B}\left(\hat{\Gamma}_b + \hat{\Gamma}_b'\right)$$

where B is the Newey-West bandwidth and

$$\hat{\Gamma}_0 = \frac{1}{T}\sum_{t=1}^{T}\left(\boldsymbol{u}_t\left(\hat{\boldsymbol{b}}\right) - \overline{\boldsymbol{u}}\left(\hat{\boldsymbol{b}}\right)\right)' \times \left(\boldsymbol{u}_t\left(\hat{\boldsymbol{b}}\right) - \overline{\boldsymbol{u}}\left(\hat{\boldsymbol{b}}\right)\right)$$

$$\hat{\Gamma}_b = \frac{1}{T}\sum_{t=b+1}^{T}\left(\boldsymbol{u}_t\left(\hat{\boldsymbol{b}}\right) - \overline{\boldsymbol{u}}_{t\geq b+1}\left(\hat{\boldsymbol{b}}\right)\right)' \times \left(\boldsymbol{u}_{t-b}\left(\hat{\boldsymbol{b}}\right) - \overline{\boldsymbol{u}}_{t\leq T-b}\left(\hat{\boldsymbol{b}}\right)\right).$$

B.4 Adjusted value in a double sort

Recall that δ measures the percentage deviation of value from price, which can be written as a product of book equity over market price (B/M) and the present value of cash flows over book equity (V/B):

$$\delta_t = 1 - \frac{V_t}{P_t} = 1 - \frac{B_t}{M_t} \times \frac{V_t}{B_t}, \tag{35}$$

where for convenience we equate market value M with per-share price P. Hence, sorting stocks on both B/M and V/B should generate a large variation in δ.

Although V/B is unobserved, the loglinear present-value model of Vuolteenaho (2002) shows that it can be written as a spread between future expected profitability and CAPM-implied discount rates. Hence, we simply model V/B as the spread between the z-score of profitability and the z-score of market beta today:

$$\frac{V}{B} \propto \text{ Profitability Spread} \equiv z(\text{Prof}) - z(\text{Beta}), \tag{36}$$

Consistent with our prior, double sorting stocks based on B/M and profitability spread, our simple proxy for V/B, resurrects the ability of these characteristics to explain larger variation in δs (Table A5). Furthermore, the variation in δ across the two dimensions of the table is consistent with our conjecture in Figure A5.

Abnormal price δ declines as we move from left to right, which amounts to holding B/M fixed while increasing profitability spread, and profitability spread appears to be an especially informative predictor of CAPM abnormal price among low-B/M (growth) stocks. Similarly, δ declines as we move from top to bottom, which amounts to holding profitability spread fixed while increasing B/M, and this variation leads to statistically significant differences in abnormal price among the middle tercile profitability spread stocks. Moving diagonally from the top left to the bottom right generates the largest variation in δs. We estimate low profitability spread, low-B/M stocks to be 57.6 percentage points more overpriced than high profitability spread, high-B/M stocks with a p-value of 0.0%.

B.5 *Adjusted value* based on expected future profitability

Equation (29) shows that, strictly speaking, the value-to-price ratio depends on expected future profitability rather than current profitability. Hence, we estimate a VAR model in which

$$x_{t+1} = Ax_t$$

where x_t is a vector of z-scores of B/M, profitability, beta, and investment (which is the order we use to form the column vector). The resulting VAR coefficients is as follows:

$$A = \begin{pmatrix} .89 & -.07 & .02 & .02 \\ -.15 & .80 & .01 & -.10 \\ .00 & -.01 & .92 & .03 \\ -.22 & -.02 & .07 & .31 \end{pmatrix}$$

It is easy to show that the discounted sum of future profitability can be written as a linear combination of the four current characteristics:

$$FutureProf \equiv 1_{\text{prof}} A \left(I - \rho A\right)^{-1} x_t,$$

where 1_{prof} denotes a vector of zeros and the one in the row that corresponds to profitability. We use the z-score of the last expression as our profitability z-score that feeds into an alternative *adjusted value* measure in Table A4.

B.6 Other proxies of misvaluation

Although we use a relatively simple three-characteristic signal of abnormal price dubbed *adjusted value*, one may wonder how well existing measures fare against *adjusted value* to predict abnormal price in the data. We examine two characteristics having been suggested as proxies for abnormal price: the analyst-forecast-based measure of Frankel and Lee (1998) and the market-multiples-based measure of Golubov and Konstantinidi (2019). Both signals have a relatively short sample period, and the signal based on analyst forecasts is limited by the availability of analyst forecast data. The market-multiples-based approach requires within-industry cross-sectional regressions, which can have a very small cross-section of data in the 1970s and earlier. Hence, our analysis is restricted to (roughly) the same sample periods used in the original papers, starting in the mid 1970s, giving us post-formation return data from 1991m6 to 2022m12.[44]

[44] A related measure by Bartram and Grinblatt (2018) was defined from year 1987, which would lead to an even shorter sample period, so we do not consider the measure.

Table A12 shows that the V/P signal based on analyst forecasts does not predict CAPM mispricing in the direction we expect, consistent with the observation of Chen and Zimmermann (2021) that the (short-horizon) return performance reported in Frankel and Lee (1998) is difficult to replicate: "The most notable likely predictors in terms of reproduction difficulty come from Frankel and Lee (1998)" (p.23). The same table shows that although the signal based on market multiples generates a larger variation in CAPM δs, it is also not significant in such a short sample period. It is possible that this signal proxies for mispricing in a longer sample, although even in Golubov and Konstantinidi (2019), their market-multiples-based V/P signal is not a stronger signal of abnormal return than B/M itself (e.g., see the second panel of Figure 1 of their paper, which shows the value-weight returns on their V/P signal—called firm-specific error—are lower than those of B/M). Of course, part of their relatively weak performance could be a feature of the sample.

B.7 Incremental information about prices when controlling for adjusted value: book-to-market, profitability, and beta

Table 7 studies the incremental information about prices contained in characteristics other than ones used to construct adjusted value. Here, we also examine characteristics we *do* use to construct adjusted value. Table A13 shows that controlling for adjusted value reduces the ability of the other characteristic to predict CAPM mispricing. In particular, the δ variation associated with beta drops dramatically. Controlling for adjusted value, however, increases the magnitude of mispricing associated with profitability. On the other hand, controlling for one of the underlying characteristics does not affect the economic magnitude or the statistical significance of δ associated with adjusted value. However, the substantial correlation between the two signals means that controlling for beta no longer makes adjusted value a statistically significant predictor of CAPM mispricing.

C Theory Appendix

C.1 Mispricing identity: an illustrative example

An example shows how our identity correctly recovers the initial price deviation with respect to a candidate SDF, even when there is *no subsequent correction* in mispricing.

An asset pays a perpetual dividend of $D_{t+j} = \lambda$ and has a constant price of $P_{t+j} = 1$ in all periods. We posit a candidate SDF that explains the constant risk-free rate $R_f > \lambda$, even if it may not explain the returns and prices of the asset in consideration:

$$\widetilde{M}_{t+j} = \frac{1}{1+R_f}$$

Then, the asset's present value of dividends with respect to $\widetilde{M}$ is $V_t = \lambda/R_f$, so the asset has a positive abnormal price with respect to the candidate SDF:

$$\delta_t = \frac{P_t - V_t}{P_t} = \frac{R_f - \lambda}{R_f} > 0.$$

That is, the asset is overpriced with respect to $\widetilde{M}$.

Does our excess-return-based identity correctly recover the same level of overpricing? Applying our identity and using the risk-free asset as the base asset,

$$\delta_t = -\sum_{j=1}^{\infty} \widetilde{M}_{t,t+j} \frac{P_{t+j-1}}{P_t} R^e_{t+j} = -\sum_{j=1}^{\infty} \frac{1}{\left(1+R_f\right)^j} \left(\lambda - R_f\right) = \frac{R_f - \lambda}{R_f},$$

so the answer is yes!

Intuitively, how does our formula correctly recover initial overpricing even if overpricing does not get corrected in the long run? The reason is that if price stays high, the dividend yield component of future return is (abnormally) low, and our formula detects that as signal of initial overpricing.

To be more specific, overpricing with respect to $\widetilde{M}$ lowers the dividend yield from R_f to λ, which leads to lower return and lower excess return. The artificially low dividend yield is detected by perpetually negative excess returns, which our identity discounts back to the present to arrive at overpricing of one.

- Dividend yield with respect to the "correct" value of $V_{t+j} = \lambda / R_f$:

$$\frac{D_{t+j}}{V_{t+j-1}} = R_f$$

- Dividend yield with respect to market price $P_{t+j} = 1$:

$$\frac{D_{t+j}}{P_{t+j-1}} = \lambda$$

- Return: $R_{t+j} = \underbrace{\frac{P_{t+j} - P_{t+j-1}}{P_{t+j-1}}}_{\text{capital gain}} + \underbrace{\frac{D_{t+j}}{P_{t+j-1}}}_{\text{dividend yield}} = \frac{1-1}{1} + \lambda = \lambda$

- Excess return: $R^e_{t+j} = \lambda - R_f$

- Return-identity-based $\delta_t = -\sum_{j=1}^{\infty} \widetilde{M}_{t,t+j} \frac{P_{t+j-1}}{P_t} R^e_{t+j} = \dfrac{R_f - \lambda}{R_f}$

In the absence of overpricing, the dividend yield would be R_f such that excess returns and return-based delta would be zero.

In contrast, if the asset's price does come down to be consistent with $\widetilde{M}$, the capital gain component of return is abnormally low, and our formula detects the corresponding low excess return as a sign of initial overpricing.

As a sidenote, our formula also does not rely on the candidate SDF being the true SDF. However, We need the base-asset return to satisfy the fundamental asset pricing equation with respect to the candidate SDF.

C.1.1 *Mispricing identity: the special case of zero dividend*

Our excess-return-based identity continues to be valid in the special case when an asset pays zero dividend and there is permanent mispricing with respect to a candidate SDF. This amounts to setting $\delta = 0$ in the previous example.

Suppose we are still interested in computing abnormal price with respect to the candidate SDF, $\widetilde{M}_{t+j} = \frac{1}{1+R_f}$. An "asset" with permanently zero dividend has a positive price $P_{t+j} > 0$ in all $j \geq 0$, leading to a permanent overpricing of

$$\delta_{t+j} = 1 - \frac{V_{t+j}}{P_{t+j}} = 1$$

in all periods, including the initial period at t.

Does our excess-return-based identity correctly recover the same level of overpricing? Applying our identity,

$$\delta_t = -\sum_{j=1}^{\infty} \frac{1}{\left(1+R_f\right)^j} \left(-R_f\right) = \frac{R_f}{R_f} = 1,$$

so the answer is again a resounding yes. Intuitively, initial overpricing is reflected in subsequent negative excess returns of $-R_f$, which our identity correctly discounts to the present to find $\delta_t = 1$.

C.2 The estimator in Cohen, Polk, and Vuolteenaho (2009)

CPV proposes estimating average log abnormal price. Based on the Campbell and Shiller (1988) decomposition,

$$\delta_t^{log} \approx -\sum_{j=1}^{\infty} \rho^{j-1} E_t[r_{t+j}] - E_t[r_{V,t+j}], \tag{7}$$

where $r_t \equiv \log(P_t + D_t) - \log(P_{t-1})$ and $r_{V,t} \equiv \log(V_t + D_t) - \log(V_{t-1})$ denote log returns on price and value, respectively, and $\rho < 1$ is a parameter. Since $E_{t+j-1}[e^{r_{V,t+j}+\widetilde{m}_{t+j}}] = 1$ and $E_{t+j-1}[e^{r_{b,t+j}+\widetilde{m}_{t+j}}] = 1$, the conditional joint normality of the log quantities implies

$$E_{t+j-1}[r_{V,t+j}] = E_{t+j-1}[r_{b,t+j}] + \frac{1}{2}Var_{t+j-1}(r_{b,t+j}) - \frac{1}{2}Var_{t+j-1}(r_{V,t+j}) + Cov_{t+j-1}(r^e_{V,t+j}, -\widetilde{m}_{t+j}), \tag{37}$$

where r_V^e denotes log return on value in excess of the base asset return. Plugging this into equation (7), using the approximation $E_{t+j-1}[R_{t+j}] \approx E_{t+j-1}[r_{t+j}] + \frac{1}{2}Var_{t+j-1}(r_{t+j})$, applying unconditional expectation, and rearranging,

$$E\left[\sum_{j=1}^{\infty}\rho^{j-1}R_{t+j}\right] \approx E\left[\sum_{j=1}^{\infty}\rho^{j-1}R_{b,t+j}\right] + E\left[\sum_{j=1}^{\infty}\rho^{j-1}Cov_{t+j-1}(r_{V,t+j}^{e}, -\widetilde{m}_{t+j})\right] + \frac{1}{2}E\left[Var_{t+j-1}(r_{t+j}) - Var_{t+j-1}(r_{V,t+j})\right] - \delta^{log}. \quad (8)$$

This decomposition motivates CPV to estimate δ^{log} using a closely related equation (their Equation 9) in the cross-section of portfolios, where k indexes a portfolio and the horizon is capped at J:

$$E\left[\sum_{j=1}^{J}\rho^{j-1}R_{k,t+j}\right] = \lambda_0 + \lambda_1\, \beta_k^{CF} + u_k, \quad (9)$$

where β_k^{CF} is measured by regressing the portfolio's long-horizon cash flows on that of the market.

Besides the potentially large measurement errors in estimated β_k^{CF}, two additional difficulties arise. First, under the null where $r_v = r$, justifying equation (9) requires strong intertemporal restrictions that guarantee

$$E\left[\sum_{j=1}^{\infty}\rho^{j-1}Cov_{t+j-1}(r_{t+j}^{e}, r_{t+j}^{mkt})\right] = Cov\left(\sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}^{e}, \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}^{mkt}\right), \quad (10)$$

in which case $\lambda_1 = b_1 Var\left(\rho^{j-1}r_{t+j}^{mkt}\right)$ if the candidate SDF is given by $\widetilde{m}_t = b_0 - b_1 r_t^{mkt}$ for log market return r^{mkt}. The simplest way to guarantee equation (10) is to assume that returns are independently and identically distributed (i.i.d.). However, in a world with i.i.d. returns, it makes little sense to explore the distinctions between abnormal price and short-horizon abnormal returns.

To quantify the extent of the problem, rewrite the conditional variance on the left-hand side of equation (10) as $r_{t+j}^{e} = \beta_{t+j-1}r_{t+j}^{mkt} + u_{t+j}$, which implies that the value of the left-hand side can be estimated as

$$E\left[\sum_{j=1}^{\infty}\rho^{j-1}\beta_{t+j-1}\sigma_{mkt,t+j-1}^{2}\right] \quad (38)$$

with β_{t+j-1} and $\sigma^2_{mkt,t+j-1}$ denoting the portfolio's time $t+j$ conditional return beta and the market portfolio's time $t+j$ conditional variance, respectively. We estimate the conditional return beta using past 36 months' return data and realized market variance using daily market returns over the month. The right-hand side of the equation can be estimated from the observed values of portfolio and market returns.

We find that there is indeed a large empirical difference between the two sides of equation (10), 0.192 (left-hand side) vs. 0.132 (right-hand side) for the high abnormal profitability quintile portfolio. Since we estimate the log candidate SDF's loading on the log market return to be larger than 3, in the context of CPV's equation (7), this translates into an estimation error in log abnormal price of more than 18 percentage points. Two forces contribute to the left-hand side of equation (10) being larger than the right-hand side in the high abnormal profitability portfolio. First, the portfolio's unconditional beta is less than one but tends to rise in times of high market volatility. This fact makes the left-hand side larger (more positive) than it would be under i.i.d. Second, the right-hand side of equation (10) involves cross-autocovariances between portfolio excess log returns and market log returns that are likely to be negative due to the long-term reversal effect. This fact pushes the right-hand side to be smaller.

Second, under the alternative where $r_v \neq r$, interpreting the error term u in equation (9) as δ^{log} in equation (8) requires that the long-horizon sum of volatility of log returns is the same for price and value:

$$E\left[\sum_{j=1}^{\infty}\rho^{j-1}Var_{t+j-1}(r_{t+j})\right] = E\left[\sum_{j=1}^{\infty}\rho^{j-1}Var_{t+j-1}(r_{V,t+j})\right]. \tag{39}$$

This fact can add to the bias in estimated δ^{log} under the alternative in which mispricing shocks with respect to $\widetilde{M}$ makes r substantially more volatile than r_V.[45]

C.3 Comparison to the abnormal return identity in van Binsbergen and Opp (2019)

van Binsbergen and Opp (2019) use a different identity to link price to subsequent abnormal re-

[45] Having the correct estimate of abnormal price under the alternative is important, just as the error term in an asset pricing regression for returns can be interpreted as abnormal returns under the alternative.

turns:

$$P_t = E_t\left[\int_t^{\infty} \widetilde{M}_{t,t+\tau} e^{-\int_t^{\tau}\alpha_u^* du} d\Pi_t\right],$$

or in discrete time,

$$P_t = \sum_{j=1}^{\infty} E_t\left[\frac{\widetilde{M}_{t,t+j}}{\Pi_{k=1}^{j}\left(1+\alpha_{t+k}^*\right)} D_{t+j}\right], \tag{40}$$

where $1+\alpha_{t+k}^* \equiv E_t\left[\widetilde{M}_{t+1}\left(1+R_{t+1}\right)\right]$. van Binsbergen and Opp use the term "mispricing wedge" to refer to the stochastic cumulation of abnormal returns, $1/\left[\Pi_{k=1}^{j}\left(1+\alpha_{t+k}^*\right)\right]$, which is different from the definition of ex-ante abnormal price $\delta_t = E_t\left[1-V_t/P_t\right]$ we introduce in our paper.

To derive their discrete-time identity in equation (40), begin with

$$1+\alpha_{t+1}^* \equiv E_t\left[\widetilde{M}_{t+1}\left(1+R_{t+1}\right)\right] = E_t\left[\widetilde{M}_{t+1}\frac{P_{t+1}+D_{t+1}}{P_t}\right].$$

Rearranging terms and iterating forward,

$$\begin{aligned} P_t &= E_t\left[\tfrac{\widetilde{M}_{t+1}}{1+\alpha_{t+1}^*} D_{t+1} + \tfrac{\widetilde{M}_{t+1}}{1+\alpha_{t+1}^*} P_{t+1}\right] \\ &= E_t\left[\tfrac{\widetilde{M}_{t+1}}{1+\alpha_{t+1}^*} D_{t+1} + \tfrac{\widetilde{M}_{t+1}\widetilde{M}_{t+2}}{\left(1+\alpha_{t+1}^*\right)\left(1+\alpha_{t+2}^*\right)} D_{t+2} + \ldots\right] \\ &= \textstyle\sum_{j=1}^{\infty} E_t\left[\tfrac{\widetilde{M}_{t,t+j}}{\Pi_{k=1}^{j}\left(1+\alpha_{t+k}\right)} D_{t+j}\right]. \end{aligned}$$

To see what equation (40) implies about unconditional ex-ante abnormal price we define, write

$$\delta = E\left[1-V_t/P_t\right] = \sum_{j=1}^{\infty} E\left[\widetilde{M}_{t,t+j}\left(\frac{1}{\Pi_{k=1}^{j}\left(1+\alpha_{t+k}^*\right)}-1\right)\frac{D_{t+j}}{P_t}\right]. \tag{41}$$

Compared to the analysis based on the Campbell-Shiller approximation in equation (7), equation (41) helpfully clarifies that the "mispricing wedge," $1/\left[\Pi_{k=1}^{j}\left(1+\alpha_{t+k}^*\right)\right]$, has to be stochastically discounted using with the cumulative SDF to arrive at ex-ante mispricing.

However, it is not obvious how to take equation (41) to data to estimate δ using returns. For instance, one could rewrite equation (41) in terms of returns,

$$\delta = \sum_{j=1}^{\infty} E\left[\widetilde{M}_{t,t+j}\left(\frac{1}{\Pi_{k=1}^{j} E_{t+k-1}\left[\widetilde{M}_{t+k}\left(1+R_{t+k}\right)\right]}-1\right)\frac{D_{t+j}}{P_t}\right], \tag{42}$$

but the conditional expectation in the denominator, $E_{t+k-1}[\,]$, prevents one from taking equation (42) to data without making additional assumptions about which state variables help forecast the time series of conditional abnormal returns. Our identity circumvents this issue by making the intentional decision to use a definition of mispricing that has price P_t in the denominator, which results in subsequent abnormal returns appearing in the numerator and leads to our expression for unconditional mispricing in equation (19) as well as our return-based calendar-time estimator in equation (2).

C.4 Portfolio δ

In practice, one would typically estimate the δ of a portfolio of stocks, which requires expressing the portfolio δ as a function of post-formation capital gains and returns on the portfolio. These capital gains and returns should be those based on a buy-and-hold strategy that does not rebalance the portfolio (or equivalently, use the original weight times the stock's cumulative capital gain to rebalance the portfolio every month). If $w_{i,t}$ is the portfolio weight on security i at the time of portfolio formation t,

$$\begin{aligned}
\delta_t &= \sum_{i=1}^{N} w_{i,t}\,\delta_{i,t} \\
&= \sum_{i=1}^{N} w_{i,t}\left(-\sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{t,t+j}\frac{P_{i,t+j-1}}{P_{i,t}}R^e_{i,t+j}\right]\right) \\
&= -\sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{t,t+j}\sum_{i=1}^{N}\left(w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}R^e_{i,t+j}\right)\right] \\
&= -\sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{t,t+j}\sum_{i\in N_{t+j}}\left(w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}R^e_{i,t+j}\right)\right],
\end{aligned} \tag{43}$$

where N_{t+j} denotes the set of firms surviving (not delisted) at the end of $t+j-1$ and therefore have return data for $t+j$. Hence,

$$\begin{aligned}\delta_t &= -\sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{t,t+j}\left(\sum_{i\in N_{t+j}} w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}\right)\left(\sum_{i\in N_{t+j}} \frac{w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}R^e_{i,t+j}}{\sum_{i\in N_{t+j}} w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}}\right)\right] \\ &= -\sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{t,t+j}\frac{P_{t+j-1}}{P_t}R^e_{t+j}\right],\end{aligned} \tag{44}$$

where

1. we normalize the time t portfolio price P_t to be 1.
2. the buy-and-hold time $t+j-1$ portfolio price is $P_{t+j-1} = \sum_{i\in N_{t+j}} w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}$.
3. the buy-and-hold portfolio weight on asset i between $t+j-1$ and $t+j$ is

$$w_{i,t+j} = \frac{w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}}{\sum_{i\in N_{t+j}} w_{i,t}\frac{P_{i,t+j-1}}{P_{i,t}}}$$

4. the buy-and-hold portfolio excess return is then given by $R^e_{t+j} = \sum_{i\in N_{t+j}} w_{i,t+j}R^e_{i,t+j}$.

C.5 Monte Carlo analysis

We analyze our estimator's statistical properties and compare them to those of alternative approaches by simulating the asset market. We do this by adopting the model used in the Monte Carlo analysis of Korteweg and Nagel (2016) (KN) to our purposes.[46]

As in KN, the log one-period (candidate) SDF and log market returns follow, respectively,

$$\widetilde{m}_t = b_0 - b_1 r_t^{mkt} \tag{45}$$

$$r_t^{mkt} = r_f + b_1\sigma^2 - \frac{1}{2}\sigma^2 + \sigma\varepsilon_t, \tag{46}$$

where b_0 and b_1 are parameters, r_f is the constant log risk-free rate, σ is the volatility of log

[46] We present the model in a similar manner to KN for an easy comparison and specify the cash flow and mispricing processes.

market return, and $\varepsilon_t \sim N(0,1)$ are i.i.d. so that r_t^{mkt} also is. Given this setup, the candidate SDF explains market returns and the risk-free rate if $b_0 = -r_f + b_1(r_f + b_1\sigma^2 - \frac{1}{2}\sigma^2) - \frac{1}{2}b_1^2\sigma^2$, which we assume.[47] To keep the model lean, we assume that there is single market portfolio and do not model how market portfolios formed in different periods could be different due to IPOs, delistings, and net issuance.[48]

Since market returns are i.i.d., the market has a constant log price-dividend ratio y^{mkt}, which we define as the log of one plus the ratio of price to dividend. Then, the log dividend growth, $\Delta d_t^{mkt} = \log(D_t^{mkt}/D_{t-1}^{mkt})$, follows[49]

$$\Delta d_t^{mkt} = r_t^{mkt} - y^{mkt} + \log\left(\exp\left(y^{mkt}\right) - 1\right), \tag{47}$$

which allows us to back out y^{mkt} from $E[\Delta d_t^{mkt}]$. The constant price-dividend ratio also implies that the log capital gain follows the same process as the log dividend growth.

Next, we specify the returns on a characteristic-based portfolio's present value. The portfolio formed at $(t-j)$ for $j \geq 1$ has a time-t log return on value of

$$r_{v,(t-j),t} = r_f + \beta_v(r_t^{mkt} - r_f) + \frac{1}{2}\beta_v\sigma^2 - \frac{1}{2}(\beta_v^2\sigma^2 + \sigma_\eta^2) + \eta_{(t-j),t}, \tag{48}$$

where $\boldsymbol{\eta}_t = \begin{pmatrix} \eta_{(t-1),t} & \cdots & \eta_{(t-J),t} \end{pmatrix}' \sim MVN\left(0, \sigma_\eta^2\Gamma_\eta\right)$ and Γ_η is a J-by-J cross-sectional correlation matrix with $\rho_\eta^{|i_1-i_2|+|j_1-j_2|}$ as the correlation between entries (i_1, j_1) and (i_2, j_2) in the matrix. $\boldsymbol{\eta}_t$ has zero time-series *auto*correlations. This correlation structure ensures that cross-sectional correlations among the portfolio's post-formation returns fall as the difference in the

[47]Furthermore, equation (46) implies that b_1 is pinned down by choosing r_f, σ^2, and the average log market return $E[r_t^{mkt}]$: $b_1 = (E[r_t^{mkt}] - r_f)/\sigma^2 + 1/2$

[48]One can think of that as the market portfolio being a single Lucas tree. Practically, this assumption means that investors in our model receive the same return and cash flows at each t regardless of when the investor began buying and holding the market portfolio. Of course, this implication is not true in reality, but we find that it is a reasonable approximation, since the correlation between market returns and post-formation market returns tends to be extremely high. The sample correlation between returns on the market portfolio formed a month ago versus 15 years ago (the largest gap) is 97.8% over 1948m6–2022m12.

[49]To see this, $r_t^{mkt} = \log\left(P_t^{mkt} + D_t^{mkt}\right) - \log P_{t-1}^{mkt} = \log\left(1 + P_t^{mkt}/D_t^{mkt}\right) + \log\left(D_t^{mkt}/D_{t-1}^{mkt}\right) - \log\left(P_{t-1}^{mkt}/D_{t-1}^{mkt}\right) = y^{mkt} + \Delta d_t^{mkt} - \log\left(\exp\left(y^{mkt}\right) - 1\right)$.

portfolio formation periods increases. It is easy to check that the expected return on $r_{v,(t-j),t}$ is consistent with the candidate SDF and market return processes. We also assume that the portfolio's dividend growth has a β_v exposure to the market dividend growth and an expected value of $\beta_v E[\Delta d_t^{mkt}]$.

Since portfolio returns on value are i.i.d. over time (though not in the cross-section), the portfolio should have a constant log value-dividend ratio of y. This means that the portfolio's log dividend growth follows

$$\Delta d_{(t-j),t} = r_{v,(t-j),t} - y + \log\left(\exp\left(y\right) - 1\right). \tag{49}$$

This equation and the fact that we assume $E[\Delta d_{(t-j),t}] = \beta_v E[\Delta d_t^{mkt}]$ pins down the value of y and of $\Delta d_{(t-j),t}$. Under the null of a correct SDF, there is no mispricing such that the log return is the return on value and price is the intrinsic value: $r = r_v$ and $P = V$. In this case, a constant price-dividend ratio also means that capital gain again equals the dividend growth. This is the process used to examine size.

To examine power, we need to allow for mispricing in the characteristic-based portfolio. We do this by specifying log abnormal price, $\delta_t^{log} = -\log(1-\delta_t) = \log(P_t/V_t)$. We allow the formation-period abnormal price to be autocorrelated across time:

$$\delta_{(t),t}^{log} = (1-\phi_{init})\overline{\delta}^{log} + \phi_{init}\delta_{(t-1),t-1}^{log} + e_t, \tag{50}$$

where $e_t \sim N(0, \sigma_e^2)$ such that portfolio-formation period log abnormal price tends to mean revert to $\overline{\delta}^{log}$. In each simulation, we draw the first portfolio-formation period abnormal price $\delta_{(0),0}^{log}$ from a normal distribution with mean $\overline{\delta}^{log}$ and variance $(1-\phi_{init}^2)^{-1}\sigma_e^2$. On the other hand, post-formation log abnormal price tends to converge to zero:

$$\delta_{(t-j),t}^{log} = \phi_{post}\delta_{(t-j),t-1}^{log} + u_{(t-j),t} \;\text{ for }\; j \geq 1, \tag{51}$$

where $\boldsymbol{u}_t = \begin{pmatrix} u_{(t-1),t} & \cdots & u_{(t-J),t} \end{pmatrix}' \sim MVN\left(0, \sigma_u^2\Gamma_u\right)$ is cross-sectionally correlated across portfolios formed in different time periods (different j's) and Γ_u is a J-by-J matrix with elements $\rho_u^{|i_1-i_2|+|j_1-j_2|}$ as the correlation between entries (i_1, j_1) and (i_2, j_2) in the matrix. We allow ϕ_{init} and ϕ_{post} to be different. Both e_t and $\boldsymbol{u}_t$ have zero time-series autocorrelations. Simple algebra implies that the return on price in the presence of mispricing is

$$r_{(t-j),t} = \Delta d_{(t-j),t} + \log\left(\exp\left(\delta^{log}_{(t-j),t}\right)(\exp(y)-1)+1\right) - \delta^{log}_{(t-j),t-1} - \log(\exp(y)-1) \quad (52)$$

and that the log capital gain is

$$\log\left(\frac{P_t}{P_{t-1}}\right) = \Delta d_t + \delta^{log}_t - \delta^{log}_{t-1}. \quad (53)$$

We choose the parameters of the model to match the key moments of the market portfolio and the high abnormal profitability portfolio, which serves as our benchmark for the characteristic-based portfolio under the alternative. Table A1 compares a number of key moments from simulations and data.

C.6 Estimation in the presence of mispricing in firm-level returns

Cohen, Polk, and Vuolteenaho (2009) explain that tests of the CAPM may be distorted when there is market-wide mispricing. Their use of a ROE CAPM, as motivated by the Vuolteenaho (2002) decomposition, nicely avoids this. Of course, we can also use a ROE-based SDF in our return-based identity approach. Mispricing in *firm-level* returns, on the other hand, does not hinder us when using the distorted covariance between returns and the candidate SDF to estimate δ based on our identity.

The easiest way to see that mispricing in firm-level returns does not hinder us from using the covariance between distorted returns and the candidate SDF when estimating δ is to recognize that the direct discount of cash flows is equivalent to an event-time, gross-return version of our return-based-identity (i.e., a version of our identity that does not exploit the calendar-time reformulation and the excess return restriction used in the paper). Therefore, the direct discount of cash flows is not superior to our method in the presence of firm-level mispricing.

The event-time return-identity-based formula for $\delta(J)$ can be written as

$$\delta(J) = -\sum_{j=1}^{J} E\left[\widetilde{M}_{t,t+j-1}\frac{P_{(t),t+j-1}}{P_{(t),t}}(\widetilde{M}_{t+j}\left[(1+R_{(t),t+j})-(1+R_{b,(t),t+j})\right])\right], \quad (54)$$

where R_b denotes the return on the base asset (for which we use the market portfolio). This equation is the basis for an event-time return-identity-based estimator of abnormal price that we could use were it not for the serial correlation or the time discount issue:

$$-\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{J}\left[\widetilde{M}_{t,t+j}\frac{P_{(t),t+j-1}}{P_{(t),t}}R^{e}_{(t),t+j}\right]. \quad (55)$$

Since $1 = E_{t+j-1}\left[\widetilde{M}_{t+j}(1+R_{b,(t),t+j})\right]$ by definition, we can also write

$$\delta(J) \;=\; -\sum_{j=1}^{J} E\left[\widetilde{M}_{t,t+j-1}\frac{P_{(t),t+j-1}}{P_{(t),t}}(\widetilde{M}_{t+j}(1+R_{(t),t+j})-1)\right] \quad (56)$$

The sample analogue of equation (56) is

$$\begin{aligned}
&-\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{J}\left[\widetilde{M}_{t,t+j-1}\frac{P_{(t),t+j-1}}{P_{(t),t}}(\widetilde{M}_{t+j}(1+R_{(t),t+j})-1)\right]\\
=\;&-\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{J}\left[\widetilde{M}_{t,t+j}\frac{P_{(t),t+j}+D_{(t),t+j}}{P_{(t),t}}-\widetilde{M}_{t,t+j-1}\frac{P_{(t),t+j-1}}{P_{(t),t}}\right]\\
=\;&\frac{1}{T}\sum_{t=1}^{T}\left[1-\sum_{j=1}^{J}\widetilde{M}_{t,t+j}\frac{D_{(t),t+j}}{P_{(t),t}}-\widetilde{M}_{t,t+j}\frac{P_{(t),t+J}}{P_{(t),t}}\right], \quad (57)
\end{aligned}$$

which is the sample delta expression for the cash-flow method. Figure A6 verify empirically that the two methods generate identical point estimates.

Since equation (55) can be stated using returns or using cash flows, taking (57) to the data cannot provide any additional advantage in terms of improving the point estimate. In contrast, using the calendar-time expression for equation (55) has the advantage of having low serial correlation in its time-series observations.

To show exactly how this equivalence works, we next analyze a simple example. Consider a three-period setting $(t = 0, 1, 2)$ with two states at time 1 with equal probability $(s_1 = L, H)$ and two cumulative states at time 2 due to the two time-1 states. The candidate SDF follows the following dynamics:

$$\widetilde{M}_t = \begin{array}{ccc} \underline{t=1} & & \underline{t=2} \\ 1+\mu \text{ if } s_1 = H & & 1 \text{ if } s_1 = H \\ & \longrightarrow & \\ 1-\mu \text{ if } s_1 = L & & 1 \text{ if } s_1 = L \end{array}$$

where $\mu > 0$. The market return is the inverse of the candidate SDF: $R_t^{mkt} = \widetilde{M}_t^{-1} - 1$.

There is a stock portfolio paying no cash flow other than a deterministic liquidating dividend of V at time 2. Since $\widetilde{M}_t$ has a conditional mean of one in all periods, the stock's correct price with respect to the candidate SDF is $V_t = V$ in all periods.

Besides analyzing the case with no mispricing with respect to $\widetilde{M}$, we also consider two cases of mispricing. Case 1 is when there is an overvaluation by a factor of $(1-\mu)^{-1}(1+2\varepsilon)$ in the low-M state at time 1, but the price is correct in all other periods and states. Hence, there is no ex-ante mispricing at time 0. Case 2 is when there is the same overvaluation in the low-$\widetilde{M}$ state at time 1, AND the time-0 price also takes the resulting distorted market beta into account: $P_0 = E_0\left[\widetilde{M}_1 P_1\right] = 0.5V + 0.5(1-\mu)\left(V(1-\mu)^{-1}(1+2\varepsilon)\right) = V(1+\varepsilon)$. Hence, in Case 2, there is an initial overpricing of $(P_0 - V_0)/P_0 = \varepsilon/(1+\varepsilon)$ at time 0.

		$t=0$	$t=1$	$t=2$
D_t			0	V
P_t	No mispricing	V	V	0
	Mispricing: Case 1	V	V if $s_1 = H$ $V(1-\mu)^{-1}(1+2\varepsilon)$ if $s_1 = L$	0
	Mispricing: Case 2	$V(1+\varepsilon)$	V if $s_1 = H$ $V(1-\mu)^{-1}(1+2\varepsilon)$ if $s_1 = L$	0

Now consider computing the initial abnormal price measured using either the conventional cash-flow expression or our return-based identity:

$$\delta_{CF,t} = 1 - E_t\left[\widetilde{M}_{t+1}\frac{D_{t+1}}{P_t}\right] - E_t\left[\widetilde{M}_{t,t+2}\frac{D_{t+2}+P_{t+2}}{P_t}\right]. \tag{58}$$

$$\delta_t = -E_t\left[\widetilde{M}_{t+1}R^e_{t+1}\right] - E_t\left[\widetilde{M}_{t,t+2}\frac{P_{t+1}}{P_t}R^e_{t+2}\right], \tag{59}$$

where the excess return is with respect to the market return. We want to check that one can rely on either formula to correctly find the initial abnormal price, whether or not there is a distorted covariance with the candidate SDF.

No mispricing

In this case,

$$\delta_{CF,0} = 1 - E_t\left[\widetilde{M}_{t,t+2}\frac{D_{t+2}}{P_t}\right] = 1 - \frac{V}{V} = 0.$$

Also,

$$\delta_0 = -E_0\left[\widetilde{M}_1\left(0 - R^{mkt}_1\right)\right] = -\frac{1}{2}\left[(1+\mu)\left((1+\mu)^{-1}-1\right) + (1-\mu)\left((1-\mu)^{-1}-1\right)\right] = 0$$

so that we recover the initial abnormal price of zero in both cases.

Mispricing Case 1 (no initial mispricing)

In this case,

$$\delta_{CF,0} = 1 - E_t\left[\widetilde{M}_{t,t+2}\frac{D_{t+2}}{P_t}\right] = 1 - \frac{V}{V} = 0.$$

Also,

$$\begin{aligned}
\delta_0 &= -\tfrac{1}{2}\left[\widetilde{M}_{1,s_1=H}\left(R_{1,s_1=H}-R^{mkt}_{1,s_1=H}\right)+\widetilde{M}_{1,s_1=L}\left(R_{1,s_1=L}-R^{mkt}_{1,s_1=L}\right)\right] \\
&\quad -\tfrac{1}{2}\widetilde{M}_{1,s_1=L}\tfrac{P_{1,s_1=L}}{P_0}\left(R_{1,s_1=L}-R^{mkt}_{2,s_1=L}\right) \\
&= -\tfrac{1}{2}\left[(1+\mu)\left(0-\left((1+\mu)^{-1}-1\right)\right)-(1-\mu)\left(\left((1-\mu)^{-1}(1+2\varepsilon)-1\right)-\left((1-\mu)^{-1}-1\right)\right)\right] \\
&\quad -\tfrac{1}{2}(1-\mu)(1-\mu)^{-1}(1+2\varepsilon)\left(\left(\tfrac{1}{(1-\mu)^{-1}(1+2\varepsilon)}-1\right)-0\right) \\
&= 0
\end{aligned}$$

Hence, we recover an initial mispricing of zero in both cases in spite of the distorted covariance with $\widetilde{M}$ due to mispricing. Our identity neutralizes this distortion through the cumulative capital gain term multiplying the cumulative candidate SDF and excess returns. Of course, this result generalizes to the case with additional periods.

Mispricing Case 2 (initial overpricing)

In this case,

$$\delta_{CF,0} = 1 - E_t\left[M_{t,t+2}\frac{D_{t+2}}{P_t}\right] = 1 - \frac{V}{V(1+\varepsilon)} = 1 - \frac{1}{1+\varepsilon} = \frac{\varepsilon}{1+\varepsilon}.$$

When applying the Cho-Polk identity, the difference from Case 1 arises in R_1 and $\frac{P_1}{P_0}$.

$$\begin{aligned}
\delta_0 &= -\tfrac{1}{2}\left[(1+\mu)\left(\left((1+\varepsilon)^{-1}-1\right)-\left((1+\mu)^{-1}-1\right)\right)-(1-\mu)\left(\left((1+\varepsilon)^{-1}(1-\mu)^{-1}(1+2\varepsilon)-1\right)-\left((1-\mu)^{-1}-1\right)\right)\right] \\
&\quad -\tfrac{1}{2}(1-\mu)(1+\varepsilon)^{-1}(1-\mu)^{-1}(1+2\varepsilon)\left(\left(\tfrac{1}{(1-\mu)^{-1}(1+2\varepsilon)}-1\right)-0\right) \\
&= -\tfrac{1}{2}(1+\mu)\left((1+\varepsilon)^{-1}-1\right)-\tfrac{1}{2}(1-\mu)\left((1+\varepsilon)^{-1}-1\right)(1-\mu)^{-1}(1+2\varepsilon) \\
&\quad -\tfrac{1}{2}(1-\mu)\left((1+\varepsilon)^{-1}-1\right)(1-\mu)^{-1}(1+2\varepsilon)\left(\tfrac{1}{(1-\mu)^{-1}(1+2\varepsilon)}-1\right) \\
&= \varepsilon/(1+\varepsilon)
\end{aligned}$$

Hence, we recover an initial abnormal price of $\varepsilon/(1+\varepsilon)$ in both cases in spite of the distorted covariance with $\widetilde{M}$ due to mispricing.

D Additional Figure and Tables

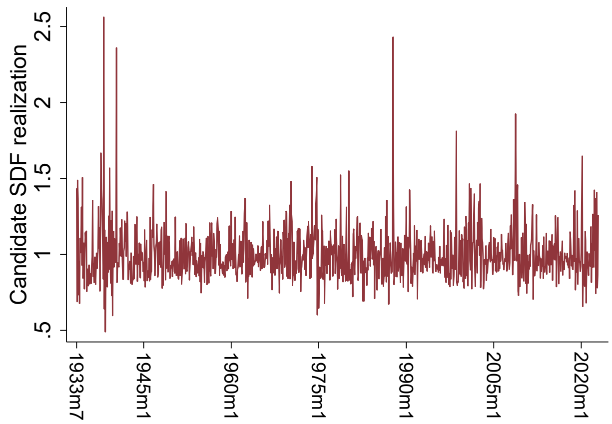


Figure A1: **Estimated CAPM-implied Candidate SDF**

The figure plots the time-series realizations of the CAPM-implied candidate SDF: $\widetilde{M}_t = \exp(b_0 - b_1 r_t^{mkt})$ with r_t^{mkt} denoting log market returns

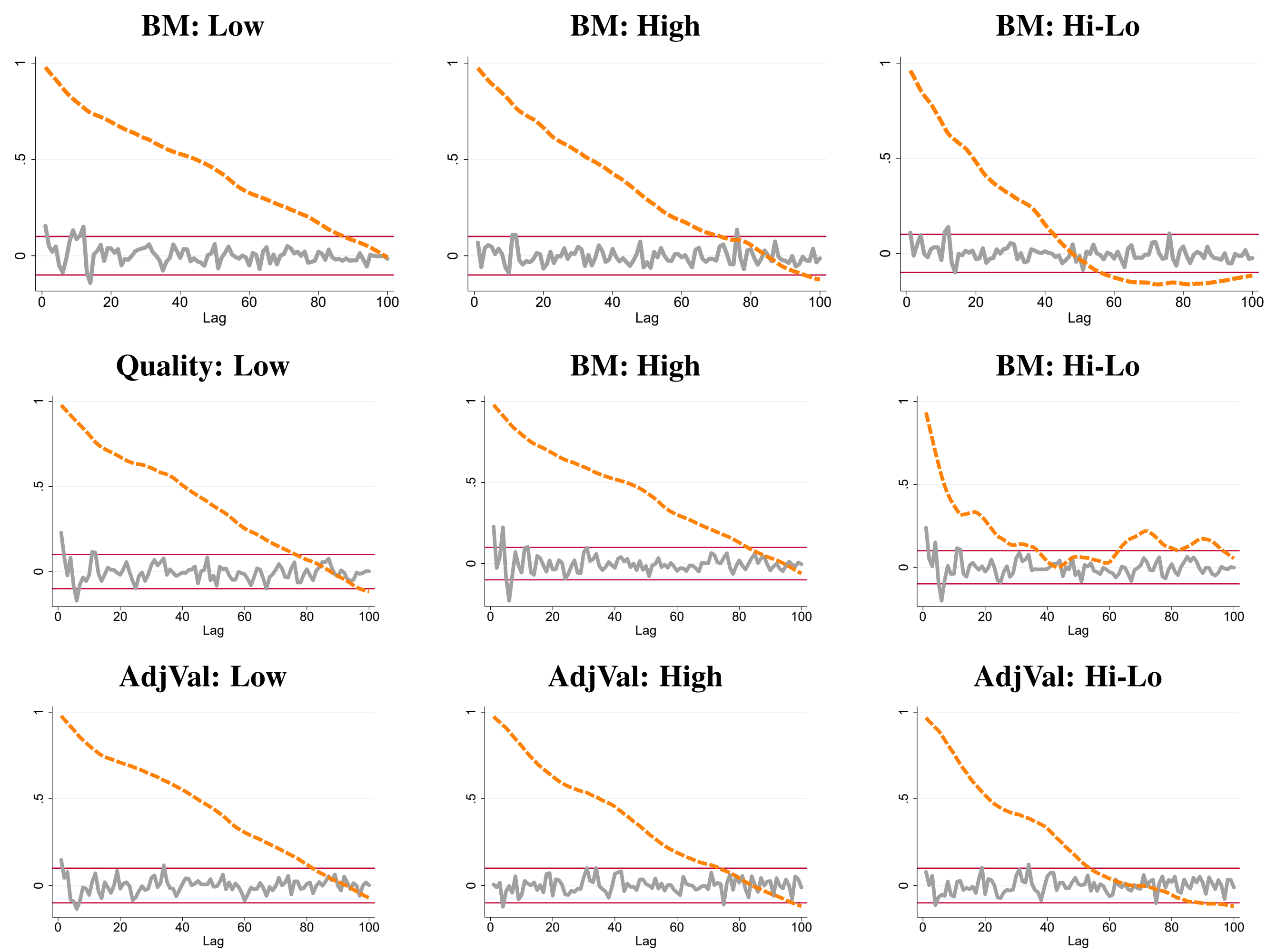


Figure A2: **Autocorrelation by Estimation Approach: B/M, Quality, and Adjusted value**

The figure reports the 1- to 100-month autocorrelations in time-series δs estimated based on the return-identity-based approach (solid grey) and the dividend-based approach using event-time (dash orange). We provide the comparison for the low-, high-, and high-minus-low quintile portfolios (in different columns) sorted on the book-to-market, quality, and adjusted value (in different rows).

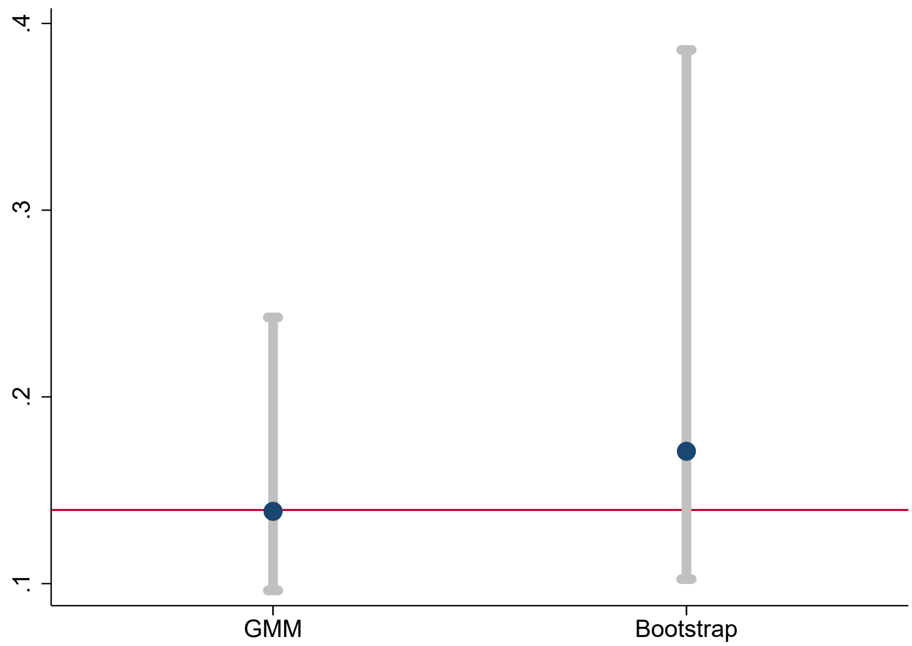


Figure A3: **GMM vs. Bootstrap Standard Errors: Monte Carlo**

The figure shows the true standard deviation of the delta estimator (red solid line), median GMM standard error (blue solid circle), median bootstrap standard error, and the 10% and 90% values for the two standard errors based on Monte Carlo. The comparison shows that GMM standard errors have a median that almost exactly matches the true standard deviation of $\widehat{\delta}$ and a much narrower confidence interval than the bootstrap standard error. Both GMM and bootstrap standard errors use a bandwidth / blocklength of 2 years.

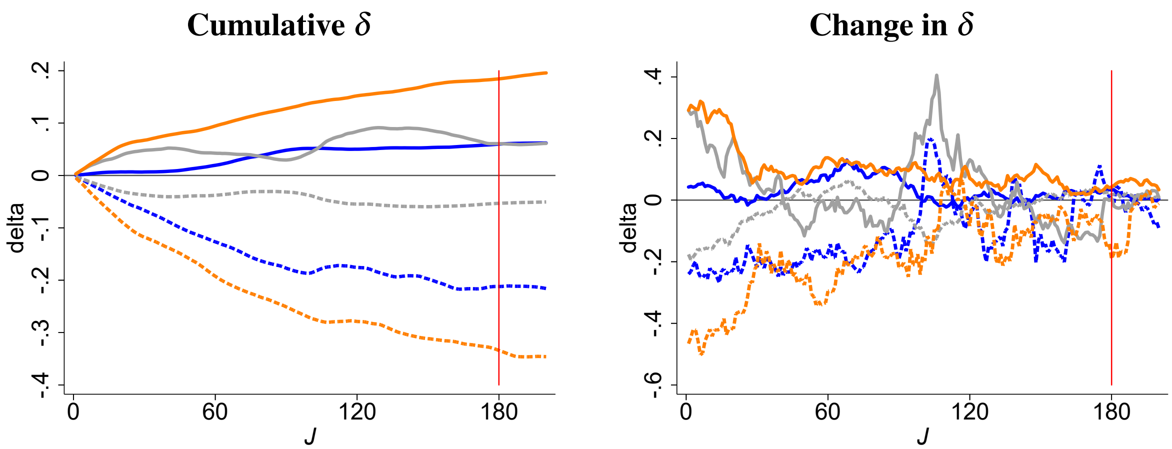


Figure A4: **Estimated δ by the Choice of J**

The left plot shows the way estimated δ changes as we vary the total number of post-formation months J used in the estimate. The right plot shows the corresponding change in δ by J. The plotted lines are for the high (dotted lines) and low (solid lines) quintile portfolios sorted on the market-to-book (blue), quality (grey), and quality-to-price (orange). The two plots suggest that estimated δs tend to plateau after $J = 15$ years (180 months).

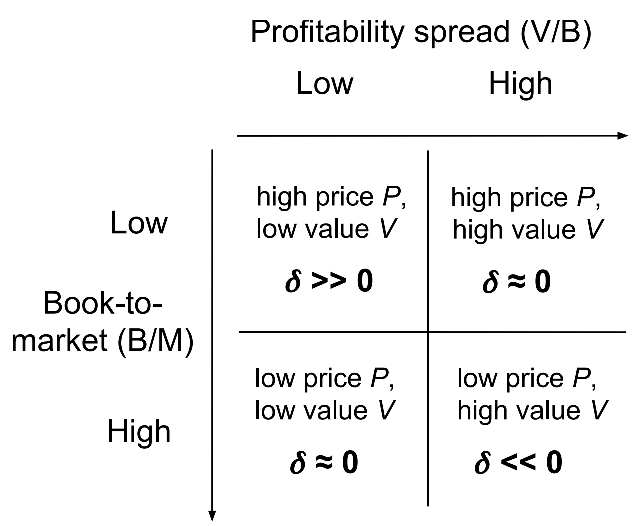


Figure A5: **A Double Sort on B/M and Profitability Spread (V/B): Illustration**

This diagram illustrates how a double sort on the book-to-market equity ratio and a proxy for the value-to-book ratio should generate large cross-sectional variation in mispricing δ. We proxy the value-to-book ratio with a two-characteristic signal dubbed *profitability spread*.

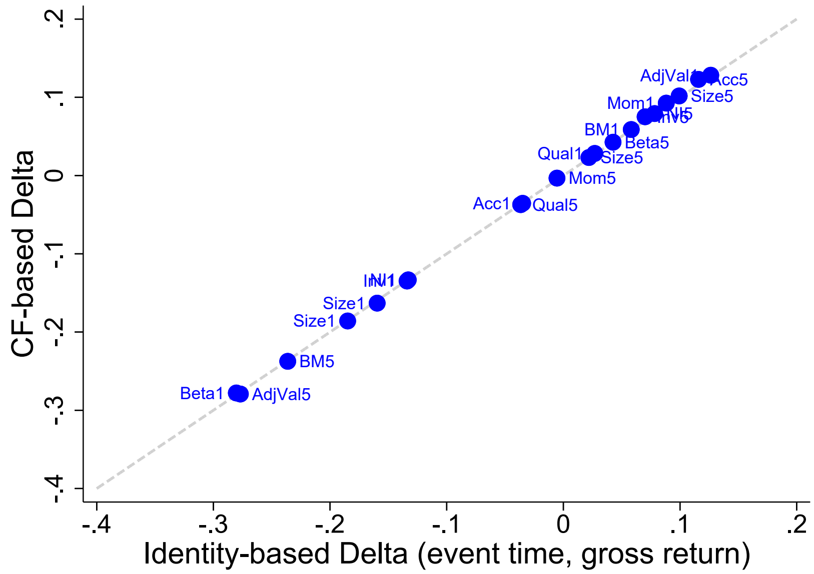
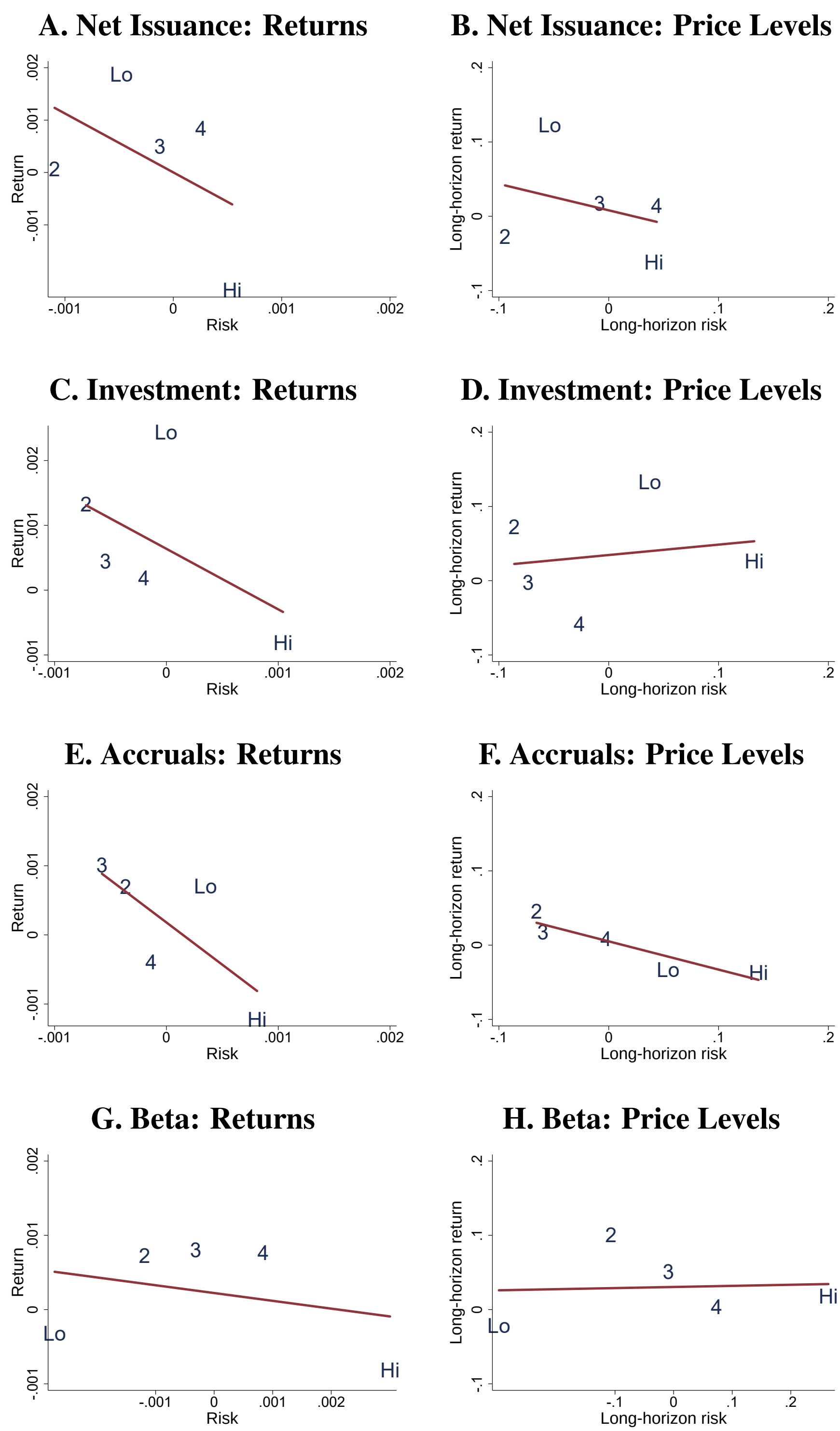


Figure A6: **Delta Based on the Return Identity vs. Cash Flows**

The figure plots the delta estimate using two theoretically equivalent approaches: the direct discount of future cash flows and a terminal value (price in 15 years; vertical axis) and the event-time, gross-return version of the return-identity-based approach (horizontal axis). The two approaches yield very similar results, with only differences of a few basis points due to measurement errors and estimation noise. The identity-based point estimates reported in our main analysis differ from those reported here due to the use of the calendar time rearrangement as well as the excess return restriction.


A. Net Issuance: Returns
Return
Risk
Lo
2
3
4
Hi
B. Net Issuance: Price Levels
Long-horizon return
Long-horizon risk
Lo
2
3
4
Hi
C. Investment: Returns
Return
Risk
Lo
2
3
4
Hi
D. Investment: Price Levels
Long-horizon return
Long-horizon risk
Lo
2
3
4
Hi
E. Accruals: Returns
Return
Risk
3
2
Lo
4
Hi
F. Accruals: Price Levels
Long-horizon return
Long-horizon risk
2
3
4
Lo
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G. Beta: Returns
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H. Beta: Price Levels
Long-horizon return
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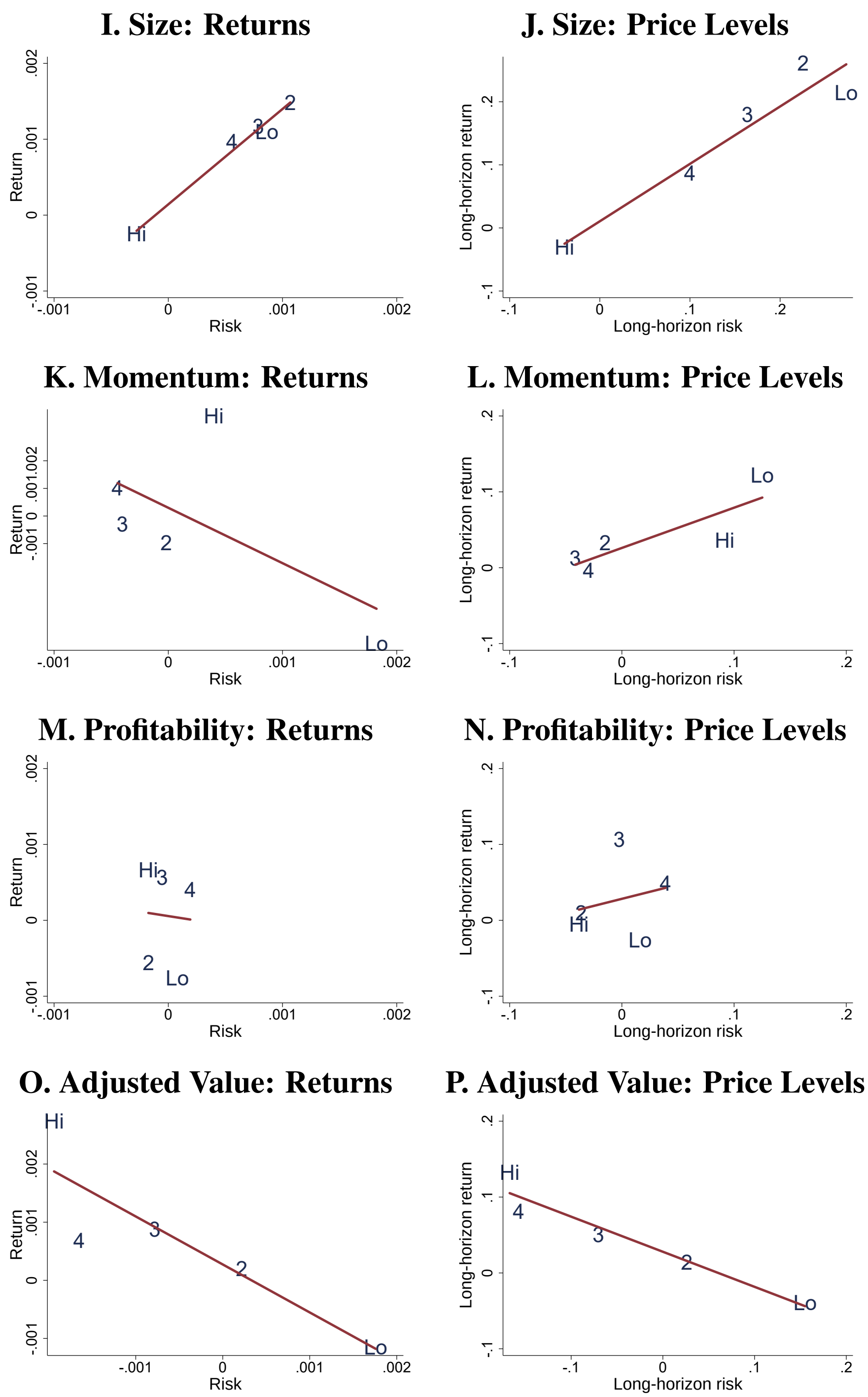

Figure A7: **The Risk-Return Relations in Returns and Price Levels (Other Return Anomalies)**

The plots, for portfolios sorted on various return anomaly characteristics, the relation between long-horizon risk and long-horizon return (the right panel) versus that between short-horizon risk and return (the left panel). Risks are measured with respect to the market portfolio. See the description in Figure 3 for more details. The sample period is 1948m6–2022m12.

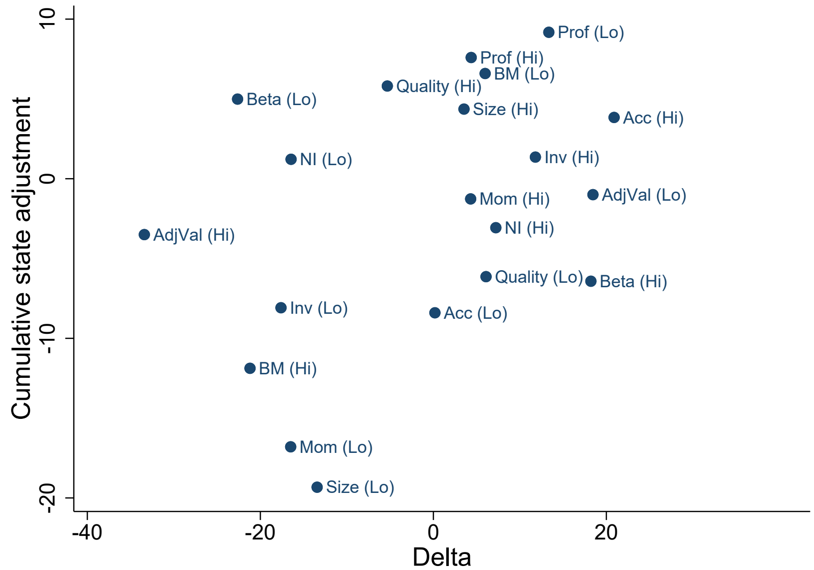

Figure A8: **Delta vs. Cumulative State Adjustment**

The figure plots the abnormal price estimate (horizontal axis) against the cumulative state adjustment (vertical axis) for the twenty extreme quintile portfolios. The cumulative state adjustment component arises from the three-way decomposition of long-horizon return presented in Corollary 4:

$$\underbrace{\sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] E_T\left[\widetilde{M}_t\right] E_T\left[R^e_{(t-j),t}\right]}_{\text{“long-horizon return"}} = -\hat{\delta} + \underbrace{\sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] Cov_T\left(R^e_{(t-j),t}, -\widetilde{M}_t\right)}_{\text{“long-horizon risk"}} \underbrace{- \sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] Cov_T\left(\frac{\phi_{(t-j),t-1}}{E_T\left[\phi_{(t-j),t-1}\right]}, \widetilde{M}_t R^e_{(t-j),t}\right)}_{\text{“cumulative state adjustment"}} \quad (60)$$

The plot shows that the cumulative state adjustment typically has an absolute magnitude below 10% and does not have a clear univariate cross-sectional relation with abnormal price δ.

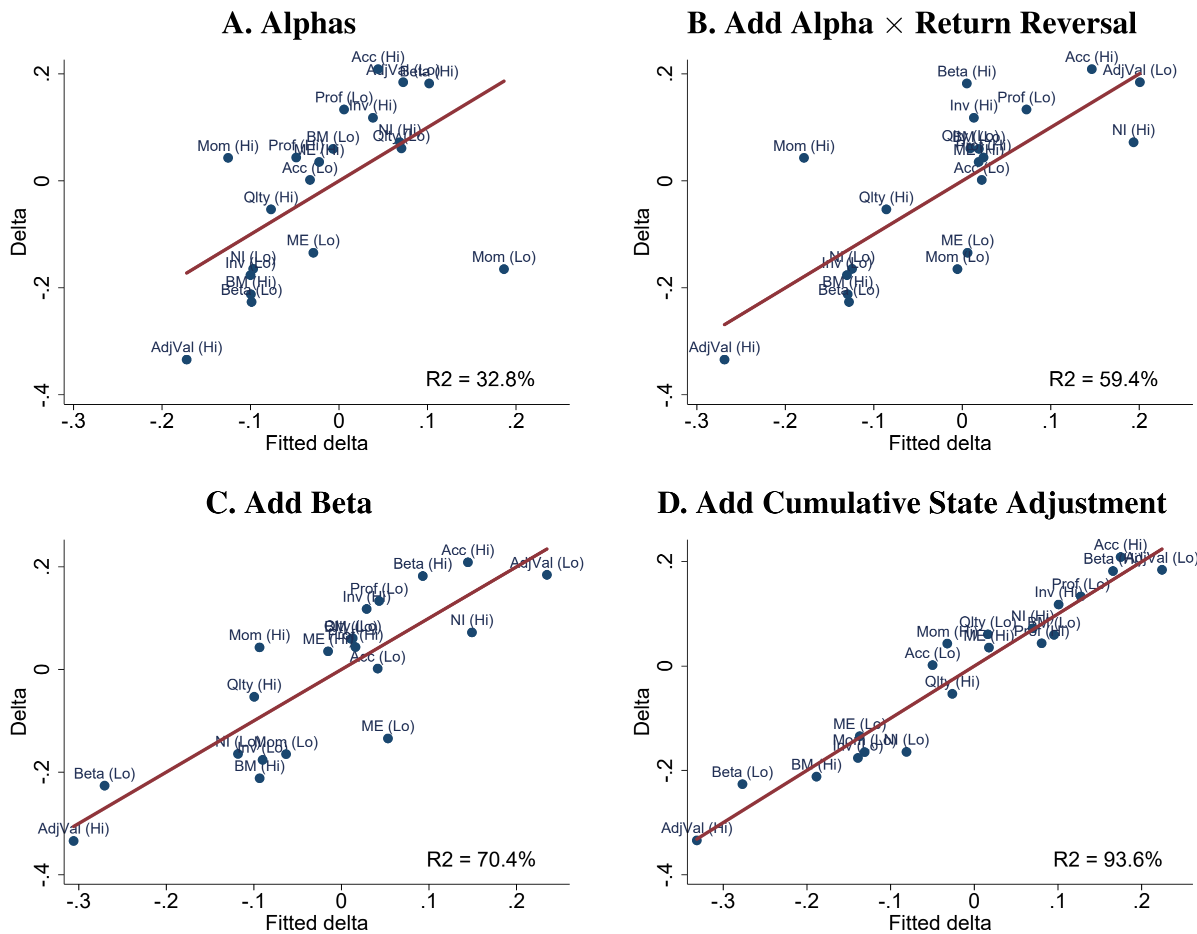

Figure A9: **Predicting Abnormal Price (δ) with Short-horizon Alpha and Other Factors**

The figures plot the cross-sectional relation between estimated delta and a fitted value based on the portfolio's short-horizon abnormal return (alpha), its interaction with a dummy variable for return reversal, short-horizon beta, and cumulative state adjustment. Panel A uses only short-horizon alpha to predict abnormal price, whereas Panel B adds the interaction of alpha and the return reversal dummy. Panel C adds short-horizon beta and Panel D also adds the cumulative state adjustment. Short-horizon alpha is the one-month abnormal return on the portfolio immediately following portfolio formation. Return reversal is a dummy variable that takes the value of one if the average excess return in years three-to-15 following portfolio formation is opposite in sign to the average excess return in the first post-formation month. Short-horizon beta is the portfolio's one-month market beta immediately following portfolio formation. Cumulative state adjustment is as defined in Section 3.3. The sample period is 1948m6–2022m12.

Table A1: **Monte Carlo Moments**

The table reports the key moments from simulations and data. The portfolio parameters are from the high adjusted value decile portfolio.

Portfolio parameter	Simulation $\delta = 0$	Simulation $\delta \neq 0$	Data	Market-level parameter	Simulation	Data
δ	0	-0.366	n/a			
$E\left[\hat{\delta}\right]$, $\hat{\delta}$	0.003	-0.360	-0.335			
σ_r	0.039	0.046	0.041	σ^{mkt}	0.043	0.043
$\bar{r}$	0.008	0.010	0.010	$\overline{r^{mkt}}$	0.0087	0.0087
β	0.830	0.830	0.827	r_f	0.0032	0.0032
$\rho(r_{(t-1),t}, r_{(t-2),t})$	0.999	0.997	0.996			
$\rho(r_{(t-1),t}, r_{(t-180),t})$	0.875	0.778	0.817			

Table A2: **Estimated Candidate SDF Parameters**

The table reports the estimated parameters of the model SDF, $\widetilde{M}_t = \exp(b_0 - b_1 r_t^{mkt})$ with r_t^{mkt} denoting log market returns, along with the 95% confidence intervals in brackets.

J	b_0	b_1
1mo	0.015 [0.000,0.030]	3.294 [1.518,5.070]
1yr	0.014 [-0.000,0.027]	3.007 [1.251,4.764]
3yrs	0.014 [-0.002,0.030]	3.127 [1.362,4.892]
5yrs	0.015 [-0.002,0.032]	3.306 [1.548,5.064]
10yrs	0.014 [-0.002,0.031]	3.184 [1.435,4.933]
15yrs	0.015 [-0.006,0.037]	3.399 [1.634,5.165]

Table A3: **Autocorrelation by Estimation Approach: All Characteristics**

The table reports the 1-month, 12-month, 60-month, and 180-month autocorrelations in time-series δs estimated based on the return-identity-based approach ("Return identity") and the dividend-based approach using event time ("Cash flow"). We provide the comparison for the low-, high-, and high-minus-low quintile portfolios sorted on all ten characteristics studied in the paper.

		Low				High				Hi - Lo			
		Autocorrelation lag											
Sort	Approach	1	12	60	180	1	12	60	180	1	12	60	180
Book-to-market	Cash flow	0.98	0.78	0.34	-0.86	0.98	0.77	0.18	-0.95	0.96	0.63	-0.14	-0.34
	Return identity	0.15	0.15	-0.05	0.01	0.07	0.11	-0.01	0.00	0.11	0.14	-0.03	0.00
Quality	Cash flow	0.98	0.77	0.26	-0.94	0.98	0.77	0.31	-0.80	0.93	0.32	0.03	-0.18
	Return identity	0.23	0.11	-0.02	0.02	0.23	0.10	-0.05	-0.00	0.24	0.11	-0.03	0.01
Adjusted value	Cash flow	0.98	0.80	0.32	-0.82	0.98	0.77	0.19	-0.95	0.97	0.70	0.04	-0.13
	Return identity	0.15	0.07	-0.09	-0.05	0.01	0.01	-0.04	-0.05	0.08	0.03	-0.06	-0.06
Net issuance	Cash flow	0.98	0.76	0.30	-0.76	0.98	0.79	0.35	-0.80	0.95	0.48	0.03	-0.32
	Return identity	0.20	0.11	-0.04	-0.00	0.23	0.09	-0.10	0.04	0.20	0.11	-0.08	0.01
Investment	Cash flow	0.98	0.82	0.24	-1.18	0.98	0.74	0.37	-0.71	0.97	0.66	-0.06	-0.24
	Return identity	0.01	-0.00	0.02	0.04	0.02	0.12	-0.07	-0.05	-0.05	0.11	-0.02	-0.00
Beta	Cash flow	0.97	0.75	0.24	-1.02	0.98	0.82	0.33	-0.80	0.96	0.65	0.01	0.07
	Return identity	0.16	0.06	-0.05	-0.05	0.11	0.12	-0.09	-0.03	0.16	0.09	-0.08	-0.05
Accruals	Cash flow	0.98	0.83	0.40	-0.70	0.98	0.77	0.30	-0.78	0.94	0.32	0.09	-0.29
	Return identity	0.08	-0.02	-0.01	-0.02	0.21	0.06	-0.09	-0.03	0.12	-0.04	-0.02	0.01
Size	Cash flow	0.98	0.80	0.23	-0.85	0.98	0.79	0.32	-0.84	0.96	0.60	-0.06	-0.51
	Return identity	0.03	0.15	-0.04	0.03	0.02	0.12	-0.06	0.01	0.03	0.15	-0.05	0.03
Momentum	Cash flow	0.97	0.76	0.17	-1.06	0.97	0.71	0.37	-0.77	0.94	0.45	-0.14	-0.44
	Return identity	0.21	0.11	-0.05	0.01	0.20	0.05	-0.09	-0.01	0.09	0.16	-0.00	0.01
Profitability	Cash flow	0.98	0.80	0.32	-0.55	0.98	0.77	0.35	-0.84	0.98	0.76	0.35	-0.08
	Return identity	0.24	-0.00	-0.02	0.03	0.12	0.15	-0.02	0.01	0.22	0.08	-0.02	0.02
Average	Cash flow	0.98	0.79	0.28	-0.87	0.98	0.77	0.31	-0.83	0.96	0.56	0.01	-0.25
	Return identity	0.15	0.08	-0.03	0.00	0.12	0.09	-0.06	-0.01	0.12	0.09	-0.04	-0.00

Table A4: **Alternative Constructions of Adjusted Value**

The table presents the deltas and p-values associated with alternative constructions of our adjusted value characteristic. p-values are based on GMM standard errors with the Newey-West kernel and a 24-month bandwidth.

	AdjVal	$z_{BM} + z_{z(Prof)-z(Beta)}$	AdjVal (Future Prof)	$z_{BM} + z_{z(FutureProf)-z(Beta)}$	$z_{Prof} - z_{Beta}$	$z_{BM} - z_{Beta}$	$z_{BM} + z_{Prof}$
delta difference	-0.519	-0.508	-0.547	-0.552	-0.394	-0.352	-0.467
(p-value)	0.002	0.005	0.001	0.003	0.164	0.134	0.130

Table A5: **Pricing B/M-and-Profitability-Spread-sorted Portfolios: A Double Sort**

The table shows that abnormal price relative to the CAPM is large for portfolios double-sorted on the book-to-market equity ratio (B/M) and profitability spread, our proxy for the value-to-book ratio: Profitability Spread $= z_{\text{prof}} - z_{\text{beta}}$, where z is a z-score. We form nine value-weight portfolios by independently sorting stocks into three B/M bins and three profitability spread bins based on the associated 30% and 70% NYSE breakpoints. We form portfolios and track post-formation returns for 15 years. The reported δs are estimated values of abnormal price defined as $\delta = E\left[\frac{P_t - V_t}{P_t}\right] \approx \delta(180) = -E\left[\sum_{j=1}^{180} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$, where $(t-j)$ denotes the portfolio formation month and t denotes the month in which returns are realized. We use the candidate SDF implied by the CAPM, $\widetilde{M}_{t-j,t} = \exp\left(b_0 j - b_1 \sum_{s=0}^{j-1} r^{mkt}_{t-s}\right)$, where r^{mkt}_t is log market returns and b_0 and b_1 are chosen to make the market portfolio's prices (δ) and returns ($\delta(1)$) correct in sample. We report t-statistics (in parentheses) and p-values (in brackets) based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF. The sample period is 1948m6–2022m12.

	$\delta \times 100$ (t-statistic) [p-value]			
	Profitability spread			
Book-to-market	Lo	2	Hi	Hi - Lo
Lo	26.7 (3.04)	4.1 (2.91)	-11.7 (1.61)	-26.4 (-1.95), [0.052]
2	18.6 (1.50)	-12.7 (2.21)	-33.1 (1.96)	-38.4 (-1.54), [0.124]
Hi	0.3 (1.49)	-34.4 (1.43)	-31.0 (1.92)	-19.3 (-0.74), [0.462]
Hi - Lo	-38.4 (-1.63), [0.104]	-51.8 (-2.11), [0.035]	-31.2 (-1.93), [0.053]	
	δ difference	t-statistic	p-value	δ^{RN} diff (t-stat)
$100 \times (\delta_{HH} - \delta_{LL})$	-57.6	-3.61	0.000	-18.5 (-0.95)

Table A6: **Abnormal Price vs. Cumulative Abnormal Return (CAR)**

The table compares the magnitudes and the p-values of abnormal price vs. CAR estimates. CAR is the conventional calendar-time cumulative abnormal return with respect to the CAPM :

$$\widehat{CAR} = \sum_{j=1}^{J} \hat{\alpha}_j.$$

When reporting CAR, we flip the sign for a better comparison with delta. To be consistent with the delta estimates, CAR uses portfolio excess returns with respect to post-formation market returns. p-values are based on GMM standard errors with the Newey-West kernel and a 24-month bandwidth.

	Delta						Cumulative abnormal return					
	δ			p-value			CAR $\times -1$			p-value		
Sort	Lo	Hi	Hi-Lo	Lo	Hi	Hi-Lo	Lo	Hi	Hi-Lo	Lo	Hi	Hi-Lo
Book-to-market	0.060	-0.212	-0.272	0.625	0.206	0.337	-0.012	-0.100	-0.088	0.868	0.390	0.567
Quality	0.061	-0.053	-0.114	0.629	0.603	0.609	0.138	-0.129	-0.266	0.088	0.019	0.054
Adjusted value	0.184	-0.334	-0.519	0.001	0.007	0.002	0.243	-0.316	-0.559	0.000	0.000	0.000
Net issuance	-0.164	0.072	0.237	0.008	0.278	0.006	-0.199	0.122	0.321	0.000	0.036	0.000
Investment	-0.176	0.118	0.294	0.020	0.154	0.035	-0.114	0.102	0.216	0.089	0.176	0.089
Accruals	0.002	0.209	0.207	0.983	0.047	0.222	0.097	0.183	0.086	0.316	0.017	0.580
Beta	-0.226	0.182	0.408	0.060	0.041	0.029	-0.311	0.302	0.613	0.001	0.000	0.000
Size	-0.134	0.035	0.170	0.595	0.300	0.549	0.026	-0.009	-0.035	0.884	0.741	0.836
Momentum	-0.165	0.043	0.208	0.248	0.460	0.136	-0.010	0.066	0.076	0.889	0.094	0.145
Profitability	0.133	0.044	-0.090	0.527	0.812	0.805	0.054	-0.025	-0.080	0.695	0.807	0.661

D.1 Modern Subsample

Table A7: **Pricing B/M- or quality-sorted Portfolios: Returns vs. Prices (Modern Subsample)**

The table shows that B/M and V/B are weak signals of abnormal price relative to the CAPM (the last row), although they tend to predict abnormal one-month returns better (the first row). It repeats Table 4 in the paper using the modern subsample: 1972m6–2022m12. We report t-statistics (in parentheses) and p-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF.

Panel A. CAPM δ

	B/M							$Quality$						
	$\delta \times 100$							$\delta \times 100$						
J	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
1mo	0.06	-0.05	-0.12	-0.18	-0.27	-0.33	0.026	0.26	-0.06	0.03	-0.04	-0.16	-0.42	0.001
("return")	(1.12)	(-0.98)	(-1.78)	(-1.98)	(-2.52)	(-2.23)		(2.76)	(-0.77)	(0.62)	(-0.96)	(-3.17)	(-3.23)	
1yr	0.82	-0.89	-1.12	-2.75	-3.69	-4.51	0.036	2.27	-0.34	0.17	-0.95	-1.53	-3.81	0.027
	(1.02)	(-1.30)	(-1.02)	(-2.04)	(-2.52)	(-2.10)		(1.99)	(-0.32)	(0.23)	(-1.84)	(-2.15)	(-2.22)	
3yrs	1.73	-2.12	-2.15	-7.54	-10.28	-12.01	0.066	4.59	0.02	0.08	-1.81	-2.84	-7.43	0.136
	(0.63)	(-1.24)	(-0.67)	(-1.97)	(-2.54)	(-1.84)		(1.48)	(0.01)	(0.05)	(-1.17)	(-1.27)	(-1.49)	
5yrs	4.16	-3.03	-4.64	-11.18	-16.26	-20.42	0.092	3.64	0.12	-0.69	-1.97	-2.38	-6.02	0.482
	(0.82)	(-1.15)	(-0.89)	(-1.72)	(-2.22)	(-1.69)		(0.73)	(0.02)	(-0.21)	(-0.87)	(-0.58)	(-0.70)	
10yrs	8.69	-2.20	-4.08	-18.14	-23.81	-32.50	0.170	5.25	2.91	-0.22	-2.22	-2.74	-7.99	0.663
	(0.90)	(-0.46)	(-0.38)	(-1.55)	(-1.65)	(-1.37)		(0.50)	(0.30)	(-0.03)	(-0.76)	(-0.33)	(-0.44)	
15yrs	10.96	-2.09	-5.97	-19.76	-28.78	-39.74	0.223	2.65	6.04	0.72	-4.61	-2.12	-4.77	0.856
("price")	(0.79)	(-0.29)	(-0.41)	(-1.23)	(-1.48)	(-1.22)		(0.18)	(0.45)	(0.06)	(-1.19)	(-0.18)	(-0.18)	

Panel B. Risk-neutral δ

	B/M							$Quality$						
	$\delta \times 100$							$\delta \times 100$						
J	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
1mo	0.04	-0.04	-0.08	-0.14	-0.25	-0.29	0.050	0.14	-0.06	0.04	-0.04	-0.10	-0.24	0.065
("return")	(0.75)	(-0.72)	(-1.30)	(-1.59)	(-2.35)	(-1.96)		(1.49)	(-0.81)	(0.85)	(-0.92)	(-1.95)	(-1.85)	
15yrs	10.41	-5.18	-9.60	-15.92	-29.27	-39.68	0.129	-14.92	-1.14	-0.86	-5.04	9.50	24.42	0.172
("price")	(0.92)	(-0.95)	(-0.92)	(-1.16)	(-1.90)	(-1.52)		(-1.37)	(-0.11)	(-0.10)	(-1.11)	(1.22)	(1.37)	

Table A8: **Pricing Portfolios Sorted on Adjusted Value: Returns vs. Prices (Modern Subsample)**

The table shows that the adjusted value signal that combines B/M, profitability, and beta into single characteristic is a strong signal of CAPM abnormal price (the last row) and abnormal returns (the first row). It repeats Table 5 in the paper using the modern subsample: 1972m6–2022m12. We report t-statistics (in parentheses) and p-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF.

	$\delta \times 100$							
J	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)	$[\text{Hi - Lo}]^{RN}$
1mo	0.22	0.01	-0.14	-0.23	-0.49	-0.71	0.000	-0.38
("return")	(3.19)	(0.17)	(-2.12)	(-2.98)	(-4.87)	(-4.87)		(-2.72)
1yr	2.63	0.21	-1.65	-3.15	-5.35	-7.99	0.000	-4.06
	(2.90)	(0.26)	(-2.00)	(-2.91)	(-4.33)	(-4.35)		(-2.32)
3yrs	6.28	1.07	-4.10	-8.42	-13.12	-19.40	0.000	-7.75
	(2.61)	(0.52)	(-1.55)	(-2.90)	(-3.83)	(-3.84)		(-1.56)
5yrs	9.41	3.07	-6.66	-13.01	-20.49	-29.90	0.000	-11.45
	(2.99)	(1.13)	(-1.65)	(-3.31)	(-3.29)	(-3.70)		(-0.87)
10yrs	15.01	1.82	-9.59	-23.86	-30.64	-45.65	0.001	-15.21
	(3.17)	(0.51)	(-1.89)	(-3.61)	(-2.94)	(-3.40)		(-0.45)
15yrs	18.92	4.26	-13.41	-33.96	-38.21	-57.12	0.002	-13.96
("price")	(2.97)	(0.78)	(-2.53)	(-2.70)	(-2.69)	(-3.10)		(-0.60)

Table A9: **Pricing B/M-and-Profitability-Spread-sorted Portfolios (Modern Subsample)**

The table shows that abnormal price relative to the CAPM is large for portfolios double-sorted on the book-to-market equity ratio and our proxy for the value-to-book ratio. It repeats Table A5 using the modern subsample: 1972m6–2022m12. We report t-statistics (in parentheses) and p-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF.

	$\delta \times 100$ (t-statistic) [p-value]			
	Profitability spread			
Book-to-market	Lo	2	Hi	Hi - Lo
Lo	24.9 (2.42)	5.4 (2.37)	-6.0 (1.31)	-30.0 (-1.91), [0.056]
2	15.3 (1.24)	-18.0 (1.80)	-39.4 (1.70)	-46.1 (-1.59), [0.111]
Hi	-5.2 (1.25)	-40.6 (1.15)	-37.2 (1.52)	-31.1 (-1.02), [0.308]
Hi - Lo	-30.9 (-1.14), [0.253]	-54.7 (-2.07), [0.038]	-32.0 (-1.77), [0.078]	
	δ difference	t-statistic	p-value	δ^{RN} diff (t-stat)
$100 \times (\delta_{HH} - \delta_{LL})$	-62.0	-3.32	0.001	-22.0 (-0.87)

Table A10: **Pricing Anomaly-sorted Portfolios (Modern Subsample)**

The table reports estimated abnormal price with respect to the CAPM for portfolios sorted on characteristics conceptually linked to abnormal price or on prominent return anomaly characteristics. This table repeats Table 6 with the modern subsample, 1972m6–2022m12, and therefore the results for investment, accruals, and profitability are the same as in Table 6. We report *t*-statistics (in parentheses) and *p*-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF.

	$\delta \times 100$							
Sort	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)	[Hi - Lo]^{RN}
Net issuance	-16.45	-4.24	2.02	-0.18	7.22	23.67	0.006	8.62
	(-2.67)	(-0.40)	(0.52)	(-0.03)	(1.08)	(2.72)		(0.93)
Investment	-17.61	-17.82	-2.96	8.99	11.80	29.41	0.035	16.68
	(-2.33)	(-2.50)	(-0.68)	(1.67)	(1.43)	(2.11)		(1.08)
Accruals	0.18	-11.66	0.18	4.88	20.89	20.71	0.222	9.40
	(0.02)	(-1.97)	(0.03)	(0.93)	(1.98)	(1.22)		(0.59)
Beta	-22.63	-15.89	-4.95	5.28	18.21	40.85	0.029	-26.86
	(-1.88)	(-2.19)	(-1.11)	(1.00)	(2.05)	(2.18)		(-1.21)
Size	-13.44	-16.91	-20.64	-13.50	3.54	16.98	0.549	54.62
	(-0.53)	(-0.92)	(-1.19)	(-1.14)	(1.04)	(0.60)		(1.91)
Momentum	-16.49	-7.67	-3.68	2.56	4.31	20.80	0.136	22.48
	(-1.15)	(-1.89)	(-1.10)	(0.53)	(0.74)	(1.49)		(1.85)
Profitability	13.35	-9.30	-14.43	-4.96	4.37	-8.98	0.805	2.25
	(0.63)	(-0.66)	(-1.27)	(-0.41)	(0.24)	(-0.25)		(0.08)

Table A11: **Incremental Information About Prices: Adjusted Value vs. Others (Modern Subsample)**

The table studies the CAPM abnormal price of portfolios that bet on a particular characteristic while controlling for the adjusted value characteristic and vice versa. The table studies the characteristics that do not comprise adjusted value based on the nine portfolios from a 3×3 independent sort. This table repeats Table 7 with the modern subsample, 1972m6–2022m12, and therefore the results for investment and accruals are the same as in Table 7. We report t-statistics (in parentheses) and p-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF.

	Adjusted value sort									*Adj val* sort (Second sort neutral)	Second sort (*Adj val* neutral)
	Low			2			High			$\frac{1}{3} * ((H1+H2+H3)$	$\frac{1}{3} * ((L3+23+H3)$
Second sort →	1	2	3	1	2	3	1	2	3	$-(L1+L2+L3))$	$-(L1+21+H1))$
Net issuance	13.37	18.60	10.28	-18.90	-7.71	-8.86	-40.57	-39.55	-31.23	-51.21	5.43
	(1.39)	(3.23)	(0.96)	(-2.34)	(-1.01)	(-1.26)	(-3.12)	(-2.89)	(-1.78)	(-2.84), [0.005]	(0.53), [0.595]
Investment	0.55	16.97	19.88	-25.06	-10.01	-3.90	-44.06	-41.42	-35.40	-52.76	16.39
	(0.05)	(2.17)	(2.29)	(-2.95)	(-1.42)	(-0.44)	(-2.80)	(-2.67)	(-2.16)	(-2.95), [0.003]	(1.62), [0.106]
Accruals	8.33	12.13	26.95	-14.54	-11.05	-1.05	-43.42	-33.93	-46.72	-57.16	9.60
	(0.79)	(1.90)	(2.46)	(-1.86)	(-1.23)	(-0.10)	(-1.94)	(-2.58)	(-2.16)	(-2.80), [0.005]	(0.81), [0.416]
Size	-3.37	-7.59	17.98	-16.99	-30.10	-8.55	-52.88	-41.92	-37.50	-46.44	15.06
	(-0.11)	(-0.40)	(3.32)	(-0.64)	(-1.53)	(-1.06)	(-1.56)	(-1.89)	(-2.48)	(-2.67), [0.007]	(0.52), [0.602]
Momentum	0.84	15.76	17.21	-29.39	-13.32	-3.57	-55.13	-39.13	-33.22	-53.76	21.37
	(0.09)	(2.11)	(2.29)	(-2.43)	(-1.96)	(-0.46)	(-2.12)	(-2.96)	(-2.32)	(-2.85), [0.004]	(1.75), [0.080]

Table A12: **Pricing Portfolios Sorted on Other Proxies for V/P (1991m6-)**

The table reports estimated abnormal price with respect to the CAPM for portfolios sorted on characteristics proposed in the literature to proxy for the value-to-price ratio. This table studies the analyst-forecast-based measure of Frankel and Lee (1998) and the market-multiples-based measure of Golubov and Konstantinidi (2019) using the sample period from 1991m6 (these signals are first available in 1976m7 and therefore the entire 15 years of post-formation return data are first available in 1991m6. We report t-statistics (in parentheses) and p-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF.

	$\delta \times 100$						
Sort	Lo	2	3	4	Hi	Hi - Lo	p(Hi - Lo)
Analyst V/P	-6.85	-0.12	13.66	-2.44	1.67	8.52	0.762
	(-0.33)	(-0.04)	(0.84)	(-0.27)	(0.19)	(0.30)	
Multiples V/P	0.04	-16.76	-32.23	-12.78	-22.41	-22.45	0.368
	(0.00)	(-1.52)	(-1.47)	(-0.93)	(-0.94)	(-0.90)	

Table A13: **Incremental Information About Prices: Adjusted Value vs. Others**

The table studies the CAPM abnormal price of portfolios that bet on a characteristic while controlling for the adjusted value characteristic and vice versa. This table studies the three characteristics comprising adjusted value using nine value-weight portfolios based on independent 30% and 70% NYSE breakpoints for both adjusted value and the second sorting characteristic specified in column one. Adjusted value combines the information in book-to-market, profitability, and beta using their z scores: $z_{B/M} + z_{Prof} - z_{Beta}$. The left-hand side of the table reports the estimated δ and associated t-statistic for each portfolio. The right-hand-side of the table reports the δs associated with the combination of the portfolios that results in either a characteristic-neutral portfolio that bets on adjusted value or a adjusted-value-neutral portfolio that bets on the second characteristic. We report t-statistics (in parentheses) and p-values (in brackets) based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF. The sample period is 1948m6–2022m12 except for profitability, which has a sample period of 1967m6–2022m12.

	Adjusted value sort									*AdjVal* sort (Second sort neutral)	Second sort (*AdjVal* neutral)
	Low			2			High			$\frac{1}{3}*((H1+H2+H3)$	$\frac{1}{3}*((L3+23+H3)$
Second sort →	1	2	3	1	2	3	1	2	3	$-(L1+L2+L3))$	$-(L1+21+H1))$
Book-to-market	12.49	19.43	8.50	-9.00	-14.92	-11.87	-36.50	-35.37	-32.59	-48.29	-0.98
	(1.93)	(1.06)	(0.30)	(-0.60)	(-1.26)	(-0.72)	(-2.04)	(-2.58)	(-2.15)	(-2.31), [0.021]	(-0.04), [0.970]
Beta	14.22	7.48	18.42	-17.04	-10.14	-12.82	-32.17	-41.34	-69.01	-60.88	-9.48
	(0.75)	(1.12)	(2.60)	(-1.74)	(-1.65)	(-0.73)	(-2.64)	(-1.83)	(-0.78)	(-1.39), [0.164]	(-0.23), [0.817]
Profitability	23.55	-3.71	20.96	-11.38	-19.73	-3.31	-26.59	-54.01	-44.94	-55.45	-4.29
	(1.08)	(-0.36)	(0.97)	(-0.65)	(-1.51)	(-0.18)	(-1.62)	(-1.89)	(-2.14)	(-2.74), [0.006]	(-0.14), [0.887]

Table A14: **Explaining Delta Using Short-horizon Alpha**

The table explains the cross-section of CAPM abnormal price (δ) based on short-horizon abnormal return (α), its interaction with a dummy variable for return reversal, short-horizon beta (β), and cumulative state adjustment (Cumul. state adj.). Return reversal is a dummy variable that takes the value of one if the average excess return in years three-to-15 following portfolio formation is opposite in sign to the average excess return in the first post-formation month. All regressors are cross-sectionally standardized for the ease of interpreting the point estimates. We use the extreme quintile portfolios for each characteristic, resulting in a cross-section of twenty observations. We report *t*-statistics (in parentheses) based on heteroskedasticity-robust standard errors.

	(1)	(2)	(3)	(4)
α	-0.09	-0.17	-0.11	-0.05
	(-1.79)	(-5.00)	(-2.34)	(-1.95)
$\alpha \times$ Reversal		0.12	0.12	0.07
		(2.74)	(4.03)	(4.78)
β			0.09	0.14
			(2.27)	(6.22)
Cumul. state adj				0.09
				(9.93)
r2	0.33	0.59	0.70	0.94