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By

**Gong Cheng  
Eric Jondeau  
Benoit Mojon  
Dimitri Vayanos**

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# The Impact of Green Investors on Stock Prices\*

Gong Cheng , Eric Jondeau<sup>1</sup>, Benoit Mojon<sup>2</sup>, and Dimitri Vayanos<sup>3</sup>

<sup>1</sup>HEC Lausanne and Swiss Finance Institute

<sup>2</sup>Bank for International Settlements

<sup>3</sup>London School of Economics, CEPR and NBER

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## Abstract

We study the impact of green investors on stock prices in a dynamic equilibrium model where investors are green, passive or active. Green investors track an index that progressively excludes the stocks of the brownest firms; passive investors hold a value-weighted index of all stocks; and active investors hold a mean-variance efficient portfolio of all stocks. Contrary to the literature, we find large drops in the stock prices of the brownest firms and moderate increases for greener firms. These effects occur primarily upon the announcement of the green index's formation and continue during the exclusion phase. The announcement effects imply a first-mover advantage to early adopters of decarbonisation strategies.

**JEL:** G12, G23, Q54

**Keywords:** Green investment, Asset pricing, Financial Institutions, Climate Change

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\*chengjacques@gmail.com, eric.jondeau@unil.ch, benoit.mojon@bis.org, d.vayanos@lse.ac.uk. We thank participants in the BIS Research Meeting and the NGFS Expert Network on Research webinar series for their constructive comments, and Jingtong Zhang for excellent research assistance. The project started when Gong Cheng was at the BIS. The views presented in this paper are those of the authors and do not necessarily reflect those of the BIS.

# 1 Introduction

In the fight against climate change, the role of financial asset owners and managers is widely debated. As large institutional investors are investing in financially diversified portfolios, they hold shares of firms with high greenhouse gas (GHG) emissions and thus contribute to global warming by financing polluting activities. An increasing number of corporate initiatives have sought to promote net zero investment in recent years. Central Banks, through the Network for Greening the Financial System (NGFS), have also been reflecting on greening their investment portfolios. Two broad approaches promoting green investment prevail. Investors can divest from the brownest firms (divestment) or influence the transition of these firms to greener operations through their votes at annual general meetings (engagement).

A key question that drives investors' consideration of divestment versus engagement is whether divestment raises the cost of capital of brown firms and thereby influences brown firms' future business development. In particular, shares of brown stocks sold by green investors will be bought by less climate-conscious investors. As a result, the impact of divestment on stock prices will depend on the willingness of these other investors to absorb additional shares of brown stocks. If there are few green investors relative to their counterparts, then the price impact will be small. This point is made by [Berk and van Binsbergen \(2022\)](#) in a static equilibrium model. Their calibration assumes a small community of green investors and shows that the change in the cost of capital resulting from divestment is insignificant.

Recent trends among financial investors suggest that the community of green investors might be far from small. Several initiatives have been launched by institutional investors to promote net zero investment, some of them under the umbrella or as partner of the United Nations Framework Convention on Climate Change.<sup>1</sup> These initiatives have committed to

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<sup>1</sup>These groups include the Net Zero Asset Managers (NZAM) Initiative, the Net Zero Asset Owner (NZAO) Alliance, the Glasgow Financial Alliance for Net Zero (GFANZ), the Climate Action 100+, the Paris Aligned Asset Owners (PAAO), the Institutional Investors Group on Climate Change (IIGCC) among

transitioning investment portfolios to net zero emissions by 2050, which is consistent with a temperature rise of 1.5°C. Altogether, they represent a large share of the assets under management in the world, although not all of their assets are managed in the perspective of the net zero target. In such a situation, it might be that the counterparties of green investors require a significant price discount to buy brown stocks.

In this paper, we use a dynamic equilibrium model to determine the impact on stock prices of institutional investors reducing their exposure to the firms with the highest GHG emissions. We consider three categories of investors. Active investors hold a mean-variance efficient portfolio of all stocks. Passive investors hold a value-weighted index of all stocks and represent the large community of non-green passive institutional investors. Green investors hold a value-weighted index that progressively excludes brown firms. The brownest firms are excluded first, and every year an additional set of firms based on their carbon emissions is excluded. The green index replicates the strategy of a portfolio with a decreasing carbon footprint. Indices excluding brown stocks progressively are referred to as “net zero” or “Paris consistent” indices, and have been growing over time, as have the funds tracking them. As brown stocks are excluded progressively from the green index, active investors buy them at a discounted price that implies higher expected returns. Passive investors keep holding the market portfolio during this process and do not switch to an index that includes more brown stocks. Our model builds on [Buffa, Vayanos and Woolley \(2022\)](#) and [Jiang, Vayanos and Zheng \(2022\)](#).<sup>2</sup>

We solve for equilibrium analytically, both for the transition phase and the ensuing steady state. For a given calibration of the model and decarbonisation trajectory of the green index, we determine the dynamics of equilibrium prices and portfolio holdings. This approach allows

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many others.

<sup>2</sup>[Buffa, Vayanos and Woolley \(2022\)](#) study how tracking error constraints of asset managers affect equilibrium stock prices, and [Jiang, Vayanos and Zheng \(2022\)](#) study the effect on equilibrium stock prices of the growth of passive investing. Neither paper considers green investors and green indices. Moreover, both papers focus on steady states and do not consider transition dynamics arising from gradual exclusion of stocks from indices.

us to assess the impact on expected and realised returns of different scenarios regarding the proportion of green investors.

In our baseline Scenario 1, the population of investors is split between 30% green investors, 50% passive investors and 20% active investors. These percentages broadly correspond to the current fractions of the various investor types, as we argue in Section 4. The green index excludes progressively the 10% of the brownest firms over ten years: the brownest 1% in the first year, the next brownest 1% in the second year, and so on. We find a substantial drop in the stock prices of the brown firms that are excluded and an increase in the prices of greener firms. These effects occur primarily when the exclusion strategy is announced, and continue during the exclusion phase. By the end of the exclusion phase, after ten years, the prices of the brownest firms decline by 5.61% on average, while the prices of non-excluded firms rise by 0.71%. Overall, the cost of capital of the brownest firms rises by 20 basis points (bp) compared to the non-excluded firms.

We consider two additional scenarios in both of which the fraction of green investors grows over time from 30% to 60% over ten years. We keep the fraction of active investors to 20%, thus assuming that the fraction of passive (non-green) investors shrinks over time. In Scenario 2, where all other parameters are as in Scenario 1, the prices of the brownest firms decline by 9.91% by the end of the exclusion phase, and the cost of capital of these firms rises by 38 bp compared to non-excluded firms. In Scenario 3, we additionally introduce climate transition risk that affects the brownest firms the most. The effects of exclusion become significantly larger: because a portfolio of brown firms loads up on non-diversifiable climate transition risk, active investors require higher compensation to buy more shares of these firms from green investors. The prices of the brownest firms decline by 14.03% by the end of the exclusion phase, and the cost of capital of these firms rises by 61 bp compared to non-excluded firms.

The large effects at the announcement date imply a first-mover advantage to green in-

vestors who adopt the decarbonisation strategy at an early stage. We compare the returns of a strategy that invests only in green firms to one that invests only in brown firms, from before the announcement to the end of the ten-year exclusion phase. The green strategy outperforms the brown strategy over the ten years, despite the higher expected return that brown stocks earn during the exclusion phase. Because of that higher expected return, however, the brown strategy can outperform over longer horizons.

Our paper is related to the literature on green investing and in particular on the impact that such investment choices might have on stock prices. [Bonnefon, Landier, Sastry and Thesmar \(2022\)](#) investigate investors' ethical preferences and argue that two types of preferences drive responsible investors: in the first case, investors are reluctant to invest in companies that do not have a business model in line with their own moral values (value-alignment); in the second case, the primary drive of responsible investors is the societal outcomes of their investment choices (impact-seeking). This distinction often gives rise to the so-called divestment versus engagement (or exit versus voice) debate.

Theoretical models addressing the impact of divestment include [Pastor, Stambaugh and Taylor \(2021\)](#), [Pedersen, Fitzgibbons and Pomorski \(2021\)](#) and [Zerbib \(2022\)](#). The main mechanism is that brown assets should pay a higher expected return because green investors are reluctant to hold them and therefore active investors must be induced to buy them. [Berk and van Binsbergen \(2022, BvB\)](#) quantify this mechanism and find that the price effect of divestment is insignificant. This is because the depressing effect of green investors on the prices of brown assets is mitigated by a large number of active investors taking advantage of the low prices to purchase these assets. The effects that we uncover are larger than those in BvB by an order of magnitude. This is because we not only assume a larger fraction of green investors than in BvB, but we also drop the unrealistic assumption that all non-green investors are active. Because of the passive investors in our model, the elasticity of the demand function faced by green investors is smaller than in BvB by an order of magnitude

and is closer to empirical estimates in the literature (e.g., [Gabaix and Koijen, 2021](#)).

The empirical literature presents a mixed panorama on the impact of divestment on stock prices. [Bolton and Kacperczyk \(2021\)](#) find that in the cross-section of U.S. stocks the level of carbon emissions affects stock returns significantly and gives rise to a carbon premium. A possible interpretation of this finding is that net-zero regulations target primarily activities with the highest emissions. [Bolton and Kacperczyk \(2022\)](#) find higher stock returns associated with higher levels and growth rates of carbon emissions in all sectors and most countries. [Hsu, Li and Tsou \(2023\)](#) study the interplay between industrial pollution and asset pricing, unveiling a “pollution premium” whereby firms with higher toxic emission intensities yield superior average returns due to their exposure to higher environmental policy uncertainty. [Becht, Pajuste and Toniolo \(2023\)](#) analyse another mechanism in which divestment is a form of disapproval, highlighting its significant influence on the stock prices of high carbon emitters, including firms not directly targeted by such actions. Their research underscores the transformative nature of divestment, evolving from a moral stance to a strategic exercise in risk management with far-reaching implications in the stock market.<sup>3</sup>

A few studies investigate the formulation of benchmark portfolios tailored for net-zero strategies. [Bolton, Kacperczyk and Samama \(2021\)](#) propose a tracking error relative to standard, business as usual benchmarks, albeit at the cost of extensive portfolio rebalancing.<sup>4</sup> Conversely, [Jondeau, Mojon and Pereira Da Silva \(2021\)](#) and [Cheng, Jondeau and Mojon \(2022\)](#) advocate an approach that maintains weights closely aligned with bench-

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<sup>3</sup>Other papers explore the impact of green policies adopted by banks or capital-market investors on firms’ financing avenues and their investment behaviour. [Green and Vallee \(2023\)](#) observe that in the coal industry, exit strategies by banks correlate with a reduction in debt issuance by firms engaged with those financial institutions, with limited substitution towards other banks or equity markets. [Kacperczyk and Peydro \(2022\)](#) similarly report that high-emissions firms experience a contraction in bank credit following their lending banks’ commitment to green lending policies, yet note an absence of improvement in those firms’ environmental performance.

<sup>4</sup>[Cenedese, Han and Kacperczyk \(2023\)](#) examine the construction of net zero portfolios and introduce the Distance-to-Exit (DTE) metric to evaluate the carbon-transition risk confronting companies. Their findings reveal that firms with elevated DTE values typically exhibit higher valuation ratios but lower expected returns, underscoring that DTE effectively captures carbon-transition risk.

marks, adjusting only for firms with extreme carbon emissions. The former paper adopts a forward-looking stance, assuming constant future carbon emissions. The latter paper adopts a backward-looking stance to construct a net zero benchmark over the recent period. In both methodologies, decarbonisation can be achieved at a relatively low cost in terms of financial performance and tracking error. Indeed, because of the extreme asymmetry in the distribution of carbon emissions, it is sufficient to divest from a small number of firms to achieve a large reduction in a portfolio's carbon footprint.

The rest of this paper is organised as follows. Section 2 presents the model. Section 3 solves for equilibrium prices and portfolio holdings. Section 4 describes the calibration of the model and the three scenarios. Section 5 presents the results for the three scenarios. Section 6 discusses policy implications and concludes. Proofs are in Appendix A.

## 2 Model

Time  $t$  is continuous and goes from zero to infinity. The riskless rate is exogenous and equal to  $r > 0$ . There are  $K$  groups of  $N$  firms each. All firms in the same group have the same (unmodelled) level of GHG emissions. Firms in group  $K$ , with the highest indices  $n = (K - 1)N + 1, \dots, KN$ , have the highest emissions and are excluded from the index first. Firms in group  $K - 1$ , with the second highest indices  $n = (K - 2)N + 1, \dots, (K - 1)N$ , have the second highest emissions and are excluded second, and so on.

The stock of firm  $n = 1, \dots, KN$ , referred to as stock  $n$ , pays dividend flow  $D_{nt}$  per share and is in supply of  $\eta_n > 0$  shares. The dividend flow of stock  $n$  is

$$D_{nt} = \bar{D}_n + b_n^s D_t^s + b_n^c D_t^c + D_{nt}^i, \quad (2.1)$$

where  $\{\bar{D}_n, b_n^s, b_n^c\}_{n=1, \dots, KN}$  are constants and  $\{D_t^s, D_t^c, D_{nt}^i\}_{n=1, \dots, KN}$  are stochastic processes. We refer to  $\bar{D}_n$  as the constant component of the dividend flow,  $b_n^s D_t^s$  as the systematic



component,  $b_n^c D_t^c$  as the climate component and  $D_{nt}^i$  as the idiosyncratic component. The systematic component is the product of a factor  $D_t^s$  times a factor loading  $b_n^s \geq 0$ . The factor  $D_t^s$  follows the square-root process

$$dD_t^s = \kappa^s (\bar{D}^s - D_t^s) dt + \sigma^s \sqrt{D_t^s} dB_t^s, \quad (2.2)$$

where  $\{\kappa^s, \bar{D}^s, \sigma^s\}$  are positive constants and  $B_t^s$  is a Brownian motion. The climate component is the product of a factor  $D_t^c$  times a factor loading  $b_n^c \geq 0$ . The factor  $D_t^c$  follows the square-root process

$$dD_t^c = \kappa^c (\bar{D}^c - D_t^c) dt + \sigma^c \sqrt{D_t^c} dB_t^c, \quad (2.3)$$

where  $\{\kappa^c, \bar{D}^c, \sigma^c\}$  are positive constants and  $B_t^c$  is a Brownian motion. The factors  $D_t^s$  and  $D_t^c$  are both systematic. We interpret the former as corresponding to the standard business-cycle risk and the latter as corresponding to climate transition risk. Climate transition risk refers to the uncertainty associated with the transition towards a low-carbon economy. It can arise from policies to mitigate climate change and achieve environmental sustainability goals, and the impact that these policies have on different firms. In Section 4, we relate the exposure to climate transition risk to the distribution of firms' carbon emissions. The idiosyncratic component follows the square-root process

$$dD_{nt}^i = \kappa_n^i (\bar{D}_n^i - D_{nt}^i) dt + \sigma_n^i \sqrt{D_{nt}^i} dB_{nt}^i, \quad (2.4)$$

where  $\{\kappa_n^i, \bar{D}_n^i, \sigma_n^i\}_{n=1, \dots, KN}$  are positive constants and  $\{B_{nt}^i\}_{n=1, \dots, KN}$  are Brownian motions. All Brownian motions are independent. By possibly redefining factor loadings, we set the long-run means  $\bar{D}^s$  and  $\bar{D}^c$  of the systematic factors to one. By possibly redefining the supply  $\eta_n$ , we set the long-run mean  $\bar{D}_n + b_n^s + b_n^c + \bar{D}_n^i$  of the dividend flow of stock  $n$  to

one for all  $n$ . With these normalisations, we can write the dividend flow of stock  $n$  as

$$D_{nt} = 1 + b_s^n(D_t^s - 1) + b_n^c(D_t^c - 1) + (D_{nt}^i - \bar{D}_n^i). \quad (2.5)$$

The square-root specification (2.2)–(2.4) ensures that dividends and prices are always positive and the volatility of dividends per share increases with the level of dividends per share.<sup>5</sup>

Agents are competitive and form overlapping generations living over infinitesimal time intervals. Each generation includes active investors, passive investors and green investors. Active investors can invest in the riskless asset and in the stocks without constraints. Passive investors and green investors can invest in the riskless asset and in a stock portfolio that tracks an index. The index is a broad index for passive investors and a narrower one for green investors. Passive and green investors do not observe the values of the dividend flows (2.2)–(2.4) and make their investment decisions in expectation over these values.

The broad index includes all firms. The green index includes a set  $\mathcal{G}_t$  of firms that decreases with time  $t$ . At  $t = 0$ , all firms are included. At  $t = T$ , firms  $n = (K - 1)N + 1, \dots, KN$ , i.e., in group  $K$ , are dropped. At  $t = 2T$ , firms  $n = (K - 2)N + 1, \dots, (K - 1)N$ , i.e., in group  $K - 1$ , are also dropped. The process continues until  $t = K'T$  for  $K' < K$ , when firms  $n = (K - K')N + 1, \dots, (K - K' + 1)N$ , i.e., in group  $K'$ , are the last to be dropped. Times  $T, 2T, \dots, K'T$  correspond to rebalancing times for green investors.

The broad and the green indices are capitalisation-weighted, i.e., they weigh firms according to their market capitalisation. Therefore, the number of shares  $\eta_{Int}$  that the broad index includes of any firm  $n$  is proportional to the number of shares  $\eta_n$  issued by the firm. By possibly rescaling the broad index, we set  $\eta_{Int} = \eta_n$ . Likewise, the number of shares  $\eta_{Gnt}$  that the green index includes of any firm  $n \in \mathcal{G}_t$  is proportional to  $\eta_n$ . By possibly rescaling the green index, we set  $\eta_{Gnt} = \eta_n$  for  $n \in \mathcal{G}_t$ . Since  $\eta_{Gnt} = 0$  for  $n \notin \mathcal{G}_t$ , we can write  $\eta_{Gnt}$

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<sup>5</sup>A geometric Brownian motion specification for dividends, which is commonly assumed in asset-pricing models, would also imply these properties. We adopt the square-root specification because it yields closed-form solutions.

for all  $n$  as  $1_{n \in \mathcal{G}_t} \eta_n$ .

We denote by  $W_{At}$ ,  $W_{It}$  and  $W_{Gt}$  the wealth of an active investor, a passive investor and a green investor, respectively, at time  $t$ , by  $z_{Ant}$ ,  $z_{Int}$  and  $z_{Gnt}$  the number of shares of firm  $n$  that these agents hold, and by  $\mu_{At}$ ,  $\mu_{It}$  and  $\mu_{Gt}$  the measure of these agents. A passive investor holds  $z_{Int} = \lambda_{It} \eta_n$  shares of firm  $n$ , and a green investor holds  $z_{Gnt} = \lambda_{Gt} \eta_{Gnt}$  shares of the firm, where  $\lambda_{It}$  and  $\lambda_{Gt}$  are proportionality coefficients that the agents choose optimally. The coefficients  $(\lambda_{It}, \lambda_{Gt})$  do not depend on the values of the dividend flows, which passive and green investors do not observe, but can depend on time. We assume that  $(\lambda_{It}, \lambda_{Gt})$  are constant in each of the intervals between rebalancing times  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$  and  $[K'T, \infty)$ , and denote their values by  $(\lambda_{Ik}, \lambda_{Gk})$  and  $(\lambda_{IK'}, \lambda_{GK'})$ , respectively. We likewise assume that  $(\mu_{At}, \mu_{It}, \mu_{Gt})$  are constant in each of these intervals, and denote their values by  $(\mu_{Ak}, \mu_{Ik}, \mu_{Gk})$  and  $(\mu_{AK'}, \mu_{IK'}, \mu_{GK'})$ , respectively.

The budget constraint of agent type  $i = A, I, G$  is

$$dW_{it} = \left( W_{it} - \sum_{n=1}^{KN} z_{int} S_{nt} \right) r dt + \sum_{n=1}^{KN} z_{int} (D_{nt} dt + dS_{nt}) = W_{it} r dt + \sum_{n=1}^{KN} z_{int} dR_{nt}^{sh}, \quad (2.6)$$

where  $dW_{it}$  is the infinitesimal change in wealth and  $dR_{nt}^{sh} \equiv D_{nt} dt + dS_{nt} - r S_{nt} dt$  is the share return of stock  $n$  in excess of the riskless rate. Agents have mean-variance preferences over  $dW_{it}$ . Active investors, who observe  $\{D_{nt}\}_{n=1, \dots, KN}$ , maximise the objective function

$$\mathbb{E}_t(dW_{At}) - \frac{\rho}{2} \text{Var}_t(dW_{At}) \quad (2.7)$$

over conditional mean and variance at time  $t$ . Passive and green investors, who do not observe  $\{D_{nt}\}_{n=1, \dots, KN}$ , maximise the objective function

$$\mathbb{E}_t^u(dW_{it}) - \frac{\rho}{2} \text{Var}_t^u(dW_{it}), \quad (2.8)$$

for  $i = I, G$ , over unconditional mean and variance at time  $t$ . The objective functions (2.7) and (2.8) can be derived from any Von Neumann–Morgenstern utility  $u$ , as shown in Buffa, Vayanos and Woolley (2022).

The rationale for the green index and the gradual exclusion of the most polluting firms is operational. Institutional investors, such as foundations and pension funds, aiming to decrease the carbon footprint of their portfolios might be hesitant to implement rapid changes given their obligation to maintain a tracking error relative to a benchmark. A gradual approach spreads the impact on tracking error over multiple years while facilitating a swift reduction in emissions for the overall portfolio. The strategy adopted by green investors in our model closely aligns with the strategies for “Paris aligned” or “net zero” indices or funds.<sup>6</sup>

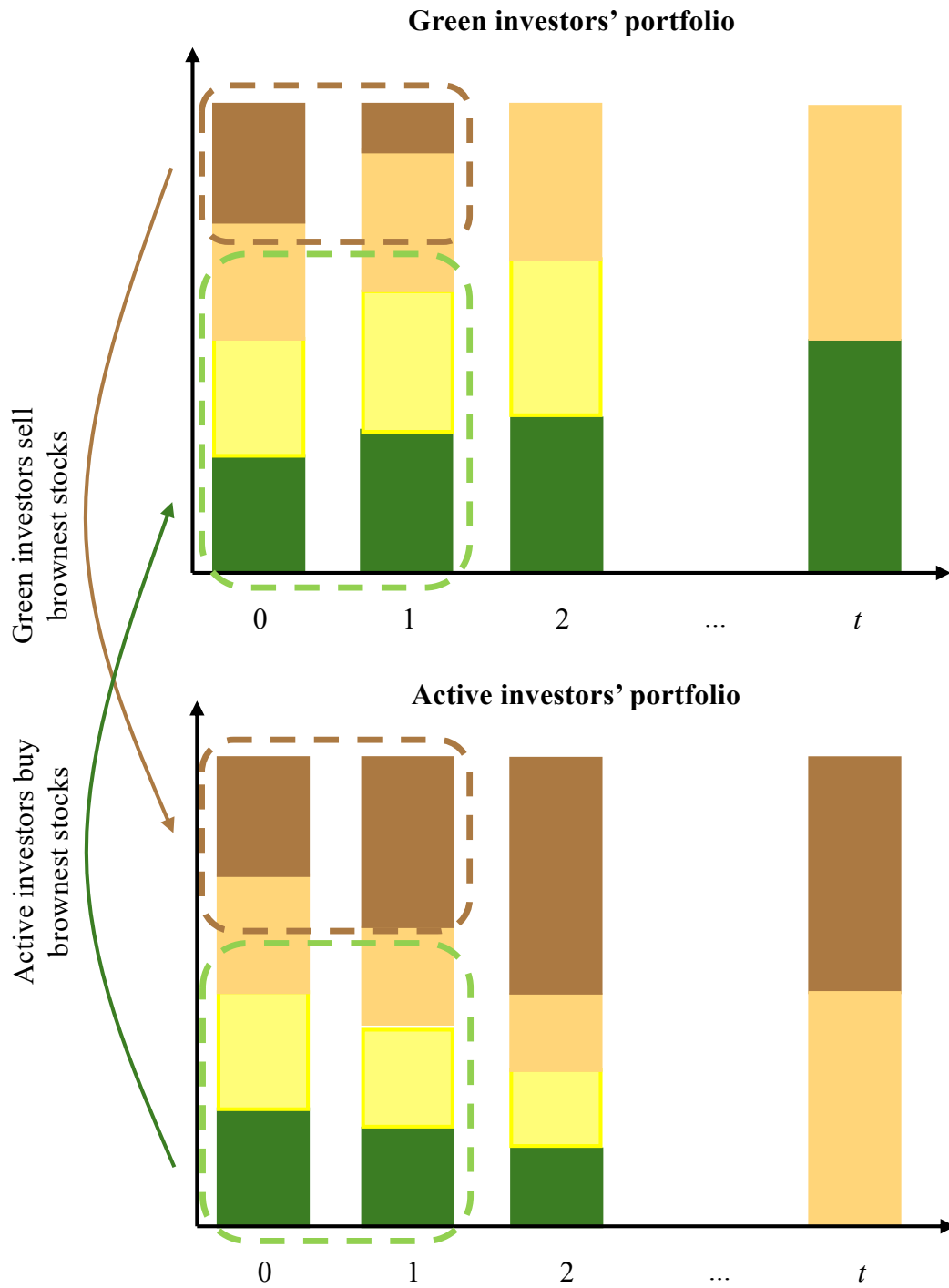
Figure 1 summarises the model by illustrating the portfolio rebalancing between green and active investors. The figure assumes four groups of stocks, with the last two groups (most polluting firms) being progressively excluded by green investors. In Year 0, both active and green investors hold one quarter of their wealth in each of the four groups. In Year 1, green investors sell a fraction of the shares of the most polluting firms (brown bars), which are bought by active investors, and reallocate the sales proceeds to the rest of their portfolio proportionately. The exclusion of the most polluting firms goes on until Year 2, when these firms are completely excluded from the portfolio of green investors. In Year 3, green investors sell a fraction of the shares of the next most polluting firms (beige bars). That exclusion goes on until Year  $t$ , when the green investors’ portfolio consists only of the first two groups (least polluting firms). The active investors’ portfolio in Year  $t$  consists instead only of the last two groups.

Alternative exclusion strategies to those assumed in our model and shown in Figure 1

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<sup>6</sup>MSCI and S&P have launched the MSCI Climate Paris Aligned Indexes family and the Paris Aligned & Climate Transition indexes family, respectively. Among others, Amundi, Lyxor, and iShares have launched ETFs or funds based on Paris aligned indices.

Figure 1: Asset exclusion and exchange between green investors and active investors



could be envisioned. For example, green investors could direct the proceeds from selling brown firms toward the least polluting firms in their portfolio instead of reallocating to the rest of their portfolio proportionally. Such strategies would strengthen the price impact that we find.

### 3 Equilibrium

We look for an equilibrium where the price  $S_{nt}$  of stock  $n$  is

$$S_{nt} = \bar{S}_{nt} + b_n^s S_t^s(D_t^s) + b_n^c S_t^c(D_t^c) + S_{nt}^i(D_{nt}^i), \quad (3.1)$$

where  $\bar{S}_{nt}$  is a deterministic function of  $t$ ,  $S_t^s(D_t^s)$  is a deterministic function of  $t$  and  $D_t^s$ ,  $S_t^c(D_t^c)$  is a deterministic function of  $t$  and  $D_t^c$ , and  $S_{nt}^i(D_{nt}^i)$  is a deterministic function of  $t$  and  $D_{nt}^i$ . The function  $\bar{S}_{nt}$  represents the present value of the constant component of dividends. The functions  $b_n^s S_t^s(D_t^s)$ ,  $b_n^c S_t^c(D_t^c)$ , and  $S_{nt}^i(D_{nt}^i)$  represent the present value of the systematic, climate and idiosyncratic components, respectively. Assuming that  $S_t^s(D_t^s)$ ,  $S_t^c(D_t^c)$ , and  $S_{nt}^i(D_{nt}^i)$  are twice continuously differentiable, we can write the share return  $dR_{nt}^{sh}$  of stock  $n$  as

$$\begin{aligned} dR_{nt}^{sh} &= (\bar{D}_n + b_n^s D_t^s + b_n^c D_t^c + D_{nt}^i) dt + (d\bar{S}_{nt} + b_n^s dS_t^s(D_t^s) + b_n^c dS_t^c(D_t^c) + dS_{nt}^i(D_{nt}^i)) \\ &\quad - r (\bar{S}_{nt} + b_n^s S_t^s(D_t^s) + b_n^c S_t^c(D_t^c) + S_{nt}^i(D_{nt}^i)) dt \\ &= \mu_{nt} dt + \sum_{j=s,c} b_n^j \sigma^j \sqrt{D_t^j} \frac{\partial S_t^j(D_t^j)}{\partial D_t^j} dB_t^j + \sigma_n^i \sqrt{D_{nt}^i} \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} dB_{nt}^i, \end{aligned} \quad (3.2)$$

where

$$\mu_{nt} \equiv \frac{\mathbb{E}_t(dR_{nt}^{sh})}{dt} = \bar{D}_n + \frac{d\bar{S}_{nt}}{dt} - r \bar{S}_{nt}$$

$$\begin{aligned}
& + \sum_{j=s,c} b_n^j \left[ D_t^j + \kappa^j (1 - D_t^j) \frac{\partial S_t^j(D_t^j)}{\partial D_t^j} + \frac{1}{2} (\sigma^j)^2 D_t^j \frac{\partial^2 S_t^j(D_t^j)}{\partial (D_t^j)^2} + \frac{\partial S_t^j(D_t^j)}{\partial t} - r S_t^j(D_t^j) \right] \\
& + D_{nt}^i + \kappa_n^i (\bar{D}_n^i - D_{nt}^i) \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} + \frac{1}{2} (\sigma_n^i)^2 D_{nt}^i \frac{\partial^2 S_{nt}^i(D_{nt}^i)}{\partial (D_{nt}^i)^2} + \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial t} - r S_{nt}^i(D_{nt}^i)
\end{aligned} \tag{3.3}$$

is the instantaneous expected share return of stock  $n$ , and the second step in (3.2) follows from (2.2)–(2.4) and Ito's lemma.

Using (2.6) and (3.2), we can write the objective function (2.7) of active investors as

$$\sum_{n=1}^{KN} z_{Ant} \mu_{nt} - \frac{\rho}{2} \left[ \sum_{j=s,c} \left( \sum_{n=1}^{KN} z_{Ant} b_n^j \right)^2 (\sigma^j)^2 D_t^j \left[ \frac{\partial S_t^j(D_t^j)}{\partial D_t^j} \right]^2 + \sum_{n=1}^{KN} z_{Ant}^2 (\sigma_n^i)^2 D_{nt}^i \left[ \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} \right]^2 \right]. \tag{3.4}$$

Active investors maximise (3.4) over positions  $\{z_{Ant}\}_{n=1,\dots,KN}$ . Their first-order condition is

$$\mu_{nt} = \rho \left[ \sum_{j=s,c} b_n^j \left( \sum_{m=1}^{KN} z_{Amt} b_m^j \right) (\sigma^j)^2 D_t^j \left[ \frac{\partial S_t^j(D_t^j)}{\partial D_t^j} \right]^2 + z_{Ant} (\sigma_n^i)^2 D_{nt}^i \left[ \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} \right]^2 \right] \tag{3.5}$$

and equates the instantaneous expected share return  $\mu_{nt}$  of stock  $n$  to the stock's contribution to instantaneous portfolio return variance times the risk-aversion coefficient  $\rho$ .

Market clearing requires that the aggregate demand of active investors, passive investors and green investors equals the supply coming from the issuing firm:

$$\mu_{At} z_{Ant} + \mu_{It} \lambda_{It} \eta_n + \mu_{Gt} \lambda_{Gt} 1_{n \in \mathcal{G}_t} \eta_n = \eta_n \Rightarrow z_{Ant} = \frac{1 - \mu_{It} \lambda_{It} - \mu_{Gt} \lambda_{Gt} 1_{n \in \mathcal{G}_t}}{\mu_{At}} \eta_n. \tag{3.6}$$

Substituting  $z_{Ant}$  from (3.6) into (3.5) and conjecturing that the functions  $S_t^s(D_t^s)$ ,  $S_t^c(D_t^c)$  and  $S_{nt}^i(D_{nt}^i)$  are affine increasing in  $D_t^s$ ,  $D_t^c$  and  $D_{nt}^i$ , respectively, i.e.,

$$S_t^s(D_t^s) = a_{0t}^s + a_{1t}^s D_t^s, \tag{3.7}$$

$$S_t^c(D_t^c) = a_{0t}^c + a_{1t}^c D_t^c, \quad (3.8)$$

$$S_{nt}^i(D_{nt}^i) = a_{n0t}^i + a_{n1t}^i D_{nt}^i, \quad (3.9)$$

for  $(a_{0t}^s, a_{1t}^s, \{a_{n0t}^i, a_{n1t}^i\}_{n=1, \dots, KN})$  positive functions of  $t$ , we find

$$\begin{aligned} & \bar{D}_n + \frac{d\bar{S}_{nt}}{dt} - r\bar{S}_{nt} + \sum_{j=s,c} b_n^j \left[ D_t^j + \kappa^j a_{1t}^j (1 - D_t^j) + \frac{da_{0t}^j}{dt} + \frac{da_{1t}^j}{dt} D_t^j - r(a_{0t}^j + a_{1t}^j D_t^j) \right] \\ & + D_{nt}^i + \kappa_n^i a_{n1t}^i (\bar{D}_n^i - D_{nt}^i) + \frac{da_{n0t}^i}{dt} + \frac{da_{n1t}^i}{dt} D_{nt}^i - r(a_{n0t}^i + a_{n1t}^i D_{nt}^i) \\ & = \rho \left[ \sum_{j=s,c} b_n^j \left( \sum_{m=1}^{KN} \frac{1 - \mu_{It} \lambda_{It} - \mu_{Gt} \lambda_{Gt} 1_{m \in \mathcal{G}_t} \eta_m b_m^j}{\mu_{At}} \right) (\sigma^j a_{1t}^j)^2 D_t^j \right. \\ & \left. + \frac{1 - \mu_{It} \lambda_{It} - \mu_{Gt} \lambda_{Gt} 1_{n \in \mathcal{G}_t} \eta_n (\sigma_n^i a_{n1t}^i)^2}{\mu_{At}} D_{nt}^i \right]. \end{aligned} \quad (3.10)$$

Equation (3.10) is affine in  $(D_t^s, D_t^c, D_{nt}^i)$ . Identifying linear terms in  $D_t^j$  for  $j = s, c$  and recalling that  $(\lambda_{It}, \lambda_{Gt}, \mu_{At}, \mu_{It}, \mu_{Gt})$  are constant in each of the intervals  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$  and  $[K'T, \infty)$  yields a Ricatti ordinary differential equation (ODE) in  $a_{1t}^j$  in each of these intervals. The solution in the interval  $[K'T, \infty)$  is constant. The solution in each interval  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$  is time-varying. Identifying linear terms in  $D_{nt}^i$  yields an ODE of the same type in  $a_{n1t}^i$ . Identifying constant terms yields a linear ODE in each interval.

**Proposition 3.1.** *The equilibrium price function has the form (3.1) with  $S_t^s(D_t^s)$ ,  $S_t^c(D_t^c)$  and  $S_{nt}^i(D_{nt}^i)$  given by (3.7), (3.8) and (3.9), respectively. The function  $a_{1t}^j$  for  $j = s, c$  is given by  $a_{1t}^j = \bar{a}_{1K'}^j$  for  $t \in [K'T, \infty)$  and*

$$a_{1t}^j = \frac{\bar{a}_{1k}^j \left( g_k^j a_{1,(k+1)T}^j + \frac{1}{\bar{a}_{1k}^j} \right) e^{\left( g_k^j \bar{a}_{1k}^j + \frac{1}{\bar{a}_{1k}^j} \right) [(k+1)T-t]} - \frac{1}{\bar{a}_{1k}^j} \left( \bar{a}_{1k}^j - a_{1,(k+1)T}^j \right)}{\left( g_k^j a_{1,(k+1)T}^j + \frac{1}{\bar{a}_{1k}^j} \right) e^{\left( g_k^j \bar{a}_{1k}^j + \frac{1}{\bar{a}_{1k}^j} \right) [(k+1)T-t]} + g_k^j \left( \bar{a}_{1k}^j - a_{1,(k+1)T}^j \right)} \quad (3.11)$$



for  $[kT, (k+1)T)$  and  $k = 0, \dots, K' - 1$ , where

$$\bar{a}_{1k}^j \equiv \frac{2}{r + \kappa^j + \sqrt{(r + \kappa^j)^2 + 4g_k^j}},$$

$$g_k^j \equiv \rho \left( \sum_{m=1}^{KN} \frac{1 - \mu_{Ik}\lambda_{Ik} - \mu_{Gk}\lambda_{Gk}1_{\{m \leq (K-k)N\}} \eta_m b_m^j}{\mu_{Ak}} \right) (\sigma^j)^2.$$

The function  $a_{1nt}^i$  is given by  $a_{1nt}^i = \bar{a}_{n1K'}^i$  for  $t \in [K'T, \infty)$  and

$$a_{1nt}^i = \frac{\bar{a}_{n1k}^i \left( g_{nk}^i a_{n1,(k+1)T}^i + \frac{1}{\bar{a}_{n1k}^i} \right) e^{\left( g_{nk}^i \bar{a}_{n1k}^i + \frac{1}{\bar{a}_{n1k}^i} \right) [(k+1)T-t]} - \frac{1}{\bar{a}_{n1k}^i} \left( \bar{a}_{n1k}^i - a_{n1,(k+1)T}^i \right)}{\left( g_{nk}^i a_{n1,(k+1)T}^i + \frac{1}{\bar{a}_{n1k}^i} \right) e^{\left( g_{nk}^i \bar{a}_{n1k}^i + \frac{1}{\bar{a}_{n1k}^i} \right) [(k+1)T-t]} + g_{nk}^i \left( \bar{a}_{n1k}^i - a_{n1,(k+1)T}^i \right)}, \quad (3.12)$$

where

$$\bar{a}_{n1k}^i \equiv \frac{2}{r + \kappa_n^i + \sqrt{(r + \kappa_n^i)^2 + 4g_{nk}^i}},$$

$$g_{nk}^i \equiv \rho \frac{1 - \mu_{Ik}\lambda_{Ik} - \mu_{Gk}\lambda_{Gk}1_{\{n \leq (K-k)N\}} \eta_n (\sigma_n^i)^2}{\mu_{Ak}}.$$

The function  $\bar{S}_{nt} + \sum_{j=s,c} b_n^j a_{0t}^j + a_{n0t}^i$  is given by

$$\bar{S}_{nt} + \sum_{j=s,c} b_n^j a_{0t}^j + a_{n0t}^i = \frac{\bar{D}_n}{r} + \sum_{j=s,c} b_n^j \kappa_n^j \int_t^\infty a_{1t'}^j e^{-r(t'-t)} dt' + \kappa_n^i \bar{D}_n^i \int_t^\infty a_{n1t'}^i e^{-r(t'-t)} dt'. \quad (3.13)$$

The values of  $(\lambda_{Ik}, \lambda_{Gk})$  for  $k = 0, \dots, K'$  are determined from the first-order conditions of passive and green investors in Appendix A.

## 4 Calibration and Scenarios

We consider 500 firms, categorised into 100 groups of 5 firms each ( $K = 100$  and  $N = 5$ ). Group 1 is the least polluting and Group 100 the most polluting. Firm characteristics are

identical except for the loadings on the climate transition risk factor, which depend on firms' GHG emissions. The horizon of the decarbonisation strategy is ten years ( $K' = 10$ ). Firms in Group 100 are excluded from the green index first, in Year 1 ( $T = 1$ ). Firms in Group 91 are excluded last, in Year 10. All in all, 50 firms are excluded, which amount to 10% of all firms.

The calibration of the rate of exclusion aligns with recent empirical findings on the cross-sectional characteristics of carbon emissions and net zero investment strategies. Carbon emissions exhibit a Pareto distribution with a heavy right tail. [Jondeau, Mojon and Pereira Da Silva \(2021\)](#) find that excluding the most polluting firms representing 1% of market capitalisation yields an average reduction of 15% of the portfolio's carbon emissions.<sup>7</sup> Moreover, if the 10% of the most polluting firms are excluded, the average reduction of the portfolio's carbon emissions rises to 65%. Therefore, our scheme of excluding 1% of the most polluting firms every year for ten years yields a cumulative 65% reduction of portfolio emissions, or equivalently a per-year reduction by 10% (because  $65\% \approx 1 - (1 - 10\%)^{10}$ ).

The fractions of the three types of investors are a key input in our calibration. We consider three scenarios, as outlined in Table 1. In Scenario 1 we assume that the fractions are constant over time and equal to  $(\mu_G, \mu_I, \mu_A) = (30\%, 50\%, 20\%)$ . Setting the fraction  $\mu_I$  of passive investors to 50% aligns with evidence on the massive shift from active to passive strategies. Bloomberg report that equity index mutual funds and ETFs constituted 54% of U.S. equity mutual funds' assets under management as of the end of 2020.<sup>8</sup>

The effective fraction of passive investors is even larger when considering active funds that track their benchmarks closely. [Chinco and Sammon \(2022\)](#) estimate that passive investors under this broader definition comprised 37.8% of all investors in the U.S. stock market in 2020. They arrive at their estimate using trading volumes around index additions

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<sup>7</sup>According to a widely cited report by the Climate Disclosure Project or CDP published in 2017, 70.6% of global GHG emissions since 1988 are due to 100 companies. See <https://www.cdp.net/en/articles/media/new-report-shows-just-100-companies-are-source-of-over-70-of-emissions>.

<sup>8</sup>See <https://www.bloomberg.com/professional/blog/passive-likely-overtakes-active>.

Table 1: Key parameters for the three scenarios

	Green investors	Passive investors	Active investors	Climate transition risk
Scenario 1	$\mu_G = 30\%$	$\mu_I = 50\%$	$\mu_A = 20\%$	$b_n^c = 0$
Scenario 2	$\mu_G = 30\% \rightarrow 60\%$	$\mu_I = 50\% \rightarrow 20\%$	$\mu_A = 20\%$	$b_n^c = 0$
Scenario 3	$\mu_G = 30\% \rightarrow 60\%$	$\mu_I = 50\% \rightarrow 20\%$	$\mu_A = 20\%$	$b_n^c > 0$

and deletions.

The effective fraction of passive investors rises even further when considering long-term investors who do not respond aggressively to price changes even though they do not track benchmarks closely. [Kojien, Richmond and Yogo \(2020\)](#) find that the ownership share of passive and long-term investors exceeded 50% in 2019. [Van der Beck and Jaunin \(2021\)](#) estimate that the share of inelastic investors is close to 26%, while the share of purely passive investors is around 39%.

Setting the fraction  $\mu_G$  of green investors to 30% aligns with estimates from the Global Sustainable Investment Alliance, indicating approximately 36% of sustainable assets under management in 2020 ([GSIA, 2022](#)), although [OECD \(2023\)](#) contends that this may be an overestimate.<sup>9</sup> The fraction  $\mu_A$  of active investors is set to  $\mu_A = 1 - \mu_I - \mu_G = 20\%$ .

In Scenario 2, we assume that the fraction  $\mu_G$  of green investors grows over time, starting at an initial 30% in Year 1 and reaching 60% in Year 10. We assume that the fraction  $\mu_A$  of active investors remains constant and equal to 20% and the fraction  $\mu_I$  passive investors decreases from 50% in Year 1 to 20% in Year 10.

In Scenario 3, we introduce climate transition risk while maintaining the fractions of investors as in Scenario 2. All firms exhibit positive exposure to climate transition risk ( $b_n^c > 0$  for all  $n$ ), with the firms with higher carbon emissions having higher exposure

<sup>9</sup>Net-zero investors are a subset of green investors in the above estimates and their share is significantly smaller. According to the Phoenix Capital’s impact database, as of February 2023, 729 net zero aligned funds (from 325 organisations) have raised 289 billion euros of capital. 58% of these funds are open to investment. See <https://phenixcapitalgroup.com/download-impact-report-march-23-net-zero>.

to that risk. We set the loading  $b_n^c$  on the climate transition risk factor to  $\frac{b^c}{(100+\alpha-g(n))^\gamma}$ , where  $g(n)$  is the group to which firm  $n$  belongs and  $(b^c, \alpha, \gamma)$  are scalar parameters. This specification allows us to capture, via the parameters  $(\alpha, \gamma)$ , the heavy right tail in the distribution of carbon emissions across firms, whereby the 1% of the most polluting firms are estimated to account for 15% of total emissions and the 10% of the most polluting firms to account for 65% of total emissions (Jondeau, Mojon and Pereira Da Silva (2021)). In line with these estimates, we assume that the sum of climate loadings  $b_n^c$  across the firms in Group 100 is 15% of the sum of climate loadings across all firms, and that the sum of climate loadings across the firms in Groups 91 to 100 is 65% of the sum of climate loadings across all firms. The climate loading  $b_n^c$  increases from 0.00022 for least polluting firms (Group 1) to 0.0087 for the tenth most polluting firms (Group 91), to 0.037 for the second most polluting firms (Group 99), and to 0.1 for the most polluting firms (Group 100).

The parameter  $b^c$  determines the magnitude of the effect of climate risk on expected returns. We measure that effect by the difference between the expected return of Group 100 and Group 1 firms in the absence of green investors, and set that difference to 70 bp. By comparison, Bolton and Kacperczyk (2022) estimate cross-sectional differences exceeding 100 bp. Such differences can arise in our model when the price impact of green investors is included.

Most of the remaining parameters are set following Jiang, Vayanos and Zheng (2022) and are summarised in Table 2. The riskless rate is set to  $r = 3\%$ . The mean-reversion parameters  $\kappa^s$ ,  $\kappa^c$  and  $\kappa_n^i$  are set to 4%. The parameters  $b_n^s$  and  $\bar{D}_n^i$  are set to generate stock CAPM  $R$ -squareds that lie around 20%, as in the data. The number of shares  $\eta$  of each stock and the risk aversion parameter  $\rho$  are set to generate expected excess stock returns that lie between 3% and 5%. The diffusion parameter  $\sigma^s = 1.4$  is set to maximise stock return volatilities, which are somewhat low in our model and lie between 15% and 20%.

Returns are computed from share returns by dividing by the share price. Expected

Table 2: Model parameters

		Volatility	Beta	Mean-reversion	Dividend
Business cycle risk	Scenarios 1,2 Scenario 3	$\sigma^s = 1.4$	$b_n^s = 0.82$ $b_n^s = 0.72$	$\kappa^s = 0.04$	$\bar{D}^s = 1$
Climate transition risk	Scenarios 1,2 Scenario 3	$\sigma^c = 1.4$	$b_n^c = 0$ $b_n^c = \frac{1.36}{(105.83 - g(n))^{1.87}}$	$\kappa^c = 0.04$	$\bar{D}^c = 1$
Idiosyncratic risk		$\sigma_n^i = \sigma^s \sqrt{\bar{D}_n^i}$		$\kappa_n^i = 0.04$	$\bar{D}_n^i = 0.18$
Riskless rate		$r = 3\%$			
Number of shares		$\eta = 0.001$			
Risk aversion		$\rho = 1$			
Number of groups		$K = 100$	$K' = 10$	$N = 5$	

returns, return volatilities, CAPM betas and CAPM  $R$ -squareds are computed by taking expectations with respect to the stationary (unconditional) distribution of the stochastic processes  $\{D_t^s, D_t^c, D_{nt}^i\}_{n=1,\dots,KN}$ .

## 5 Numerical Results

### 5.1 Scenario 1: Modest Fraction of Green Investors

Scenario 1 corresponds to a modest fraction  $\mu_G = 30\%$  of green investors that remains constant over time. Figure 2 shows the evolution of prices and expected returns of three representative firms: firms that are in Group 100 and thus excluded in Year 1 (first period of exclusion), firms that are in Group 91 and thus excluded in Year 10 (last period of exclusion) and firms that are in Groups 1 to 90 and thus not excluded. Table 3 shows summary statistics of the change in the prices, in the cost of capital (i.e., expected returns), and in the volatility of the three representative firms. It also shows the realized return of holding these firms from before the announcement of the exclusion strategy to the end of the ten-year exclusion phase.

We compute the realized return by compounding the price change upon announcement with the expected returns during the exclusion phase.

Table 3: Results - Scenario 1

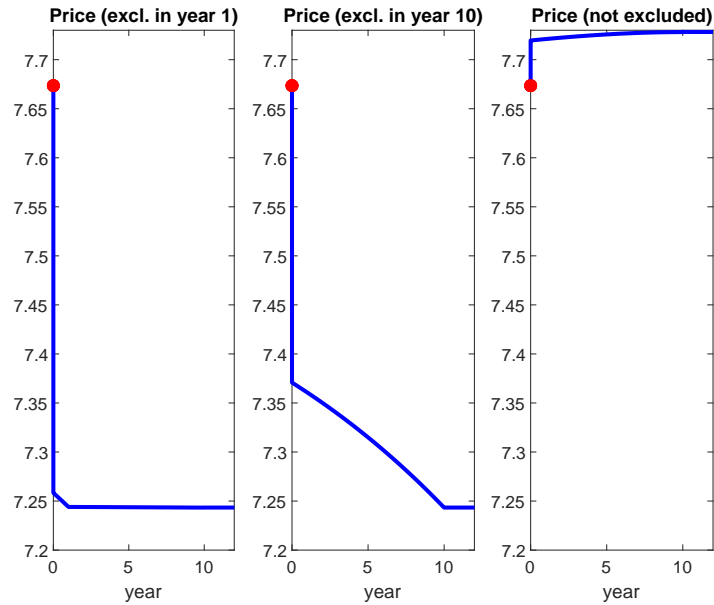
	Firms excluded in Year 1 (Group 100)	Firms excluded in Year 10 (Group 91)	Non-excluded firms (Groups 1-90)
$\Delta$ Price at $t = 0$	-5.41%	-3.95%	0.60%
$\Delta$ Price at $t = 10$	-5.61%	-5.61%	0.71%
$\Delta$ Cost of capital at $t = 0$	0.13%	0.11%	-0.01%
$\Delta$ Cost of capital at $t = 10$	0.18%	0.18%	-0.02%
Realised return over 10 years	4.38%	4.48%	4.80%
Return volatility at $t = 0$	20.16%	20.43%	20.11%
Return volatility at $t = 10$	20.11%	20.11%	20.14%

Prices and expected returns respond instantaneously to the announcement of the exclusion strategy. The prices of the excluded firms drop, with the firms in Group 100 dropping the most. The prices of the firms in Group 100 drop by 5.41% and the prices of the firms in Group 91 drop by 3.95%. In contrast, the prices of the firms in Groups 1 to 90 rise by 0.60%. The expected returns of the excluded firms increase to attract active investors. The instantaneous increase is largest for the firms in Group 100, by 13 bp. The expected returns of the firms in Group 91 rise by 11 bp and the expected returns of firms in Groups 1 to 90 drop by 1 bp. After ten years, at the end of the exclusion phase, the prices of the firms in Groups 100 to 91 drop cumulatively by 5.61% on average and the prices of the firms in Groups 1 to 90 rise by 0.71%. Over the same period, the cost of capital of the firms in Groups 100 to 91 increases cumulatively by 18 bp, while that of the firms in Groups 1 to 90 decreases by 2 bp, reflecting the higher cost of financing of brown firms relative to green firms.

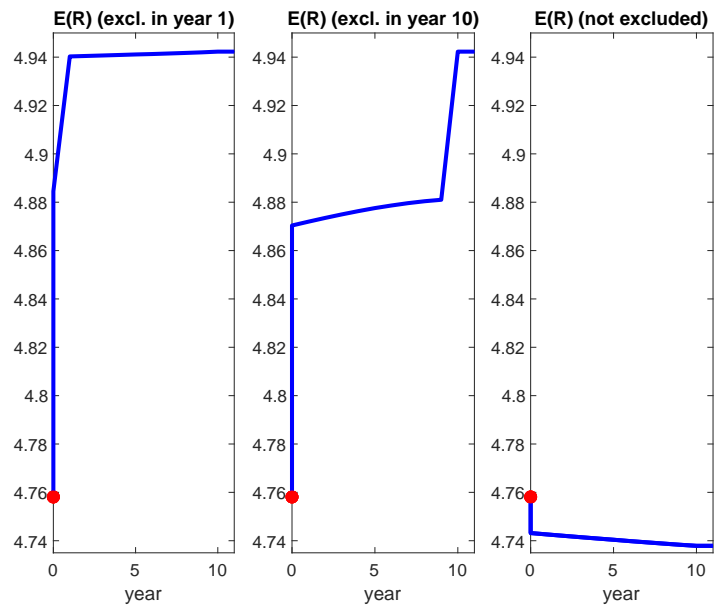
The effect of green investors' exclusion strategy on the cost of capital is large. In compar-

Figure 2: Impact of exclusion in Scenario 1

Panel A: Price impact



Panel B: Expected return impact



Note: The price impact is measured as the percentage change in the price of a given group of firms relative to the price before the announcement of the exclusion strategy. The expected return impact is measured as the change in the expected return of a given group of firms relative to the expected return before the announcement of the exclusion strategy.

ison, BvB find an insignificant effect of less than 0.5 bp in the cost of capital of brown firms relative to green firms. In our model the difference reaches 20 bp, more than 40 times as large as in BvB. Two main factors explain this difference. First, in BvB’s baseline calibration, green investors represent 2% of total wealth, whereas they represent 30% of wealth in our model. Even with a much higher fraction of green investors (33%), BvB find a small effect on the cost of capital, equal to 10.6 bp. To obtain a 20 bp effect in their framework, the fraction of green investors should equal 50% of total wealth. Second and more importantly, BvB assume that the remaining 98% of the market is composed of active investors, who can thus buy all brown assets that green investors want to sell. In contrast, we assume a much smaller share of active investors (20%), with the remaining 50% of investors being passive and not changing the composition of their portfolio away from market weights. This set-up results in a low elasticity of the demand facing green investors.

Even though brown firms earn a higher expected return than green firms after the announcement, their realized ten-year return is lower because of the initial price drop. The realized return of firms in Group 100 over the ten-year period is 4.38%, while it is 4.80% for firms in Groups 1 to 90. Our analysis thus implies a first-mover advantage of green investing. Table 3 also reports return volatility for the various groups of firms. Because all firms have the same dividend exposure to business-cycle risk, differences between groups are limited.

We conduct a sensitivity analysis of the baseline results. Increasing the fraction of green investors ( $\mu_G$ ) does not change their price impact if the fraction of active investors ( $\mu_A$ ) increases in the same proportion. Indeed, varying the fraction of passive investors from  $\mu_G = 10\%$  to  $\mu_G = 90\%$  while keeping the same relative fraction of green and active investors ( $\mu_G/\mu_A = 2/3$ ) yields the same price impact.

Fixing the fraction of passive investors to  $\mu_G = 50\%$  and varying the relative fraction of green and active investors from  $(\mu_G, \mu_A) = (10\%, 40\%)$  to  $(\mu_G, \mu_A) = (40\%, 10\%)$  changes the price impact significantly, as illustrated in Figure 3 (Panel A). For instance, a shift from



our baseline case  $(\mu_G, \mu_A) = (30\%, 20\%)$  to  $(\mu_G, \mu_A) = (40\%, 10\%)$  more than doubles the price impact. The Year 10 impact on the price of excluded firms increases from 5.61% to 12.20%, while the increase in the cost of capital rises from 18 bp to 43 bp (Panel B).

We finally investigate the effect of the exclusion strategy on the realised return of the various groups of firms (Panel C). The initial price impact dominates the expected return impact over the ten-year period for all fractions of green investors: brown firms have a lower realised return than green firms because of the initial price drop.

## 5.2 Scenario 2: Growing Fraction of Green Investors

Scenario 2 corresponds to a fraction of green investors that grows from 30% in Year 1 to 60% in Year 10. Figure 4 and Table 4 are the Scenario 2 counterparts of Figure 2 and Table 3 in Scenario 1.

Table 4: Results - Scenario 2

	Firms excluded in Year 1 (Group 100)	Firms excluded in Year 10 (Group 91)	Non-excluded firms (Groups 1-90)
$\Delta$ Price at $t = 0$	-8.94%	-7.00%	1.18%
$\Delta$ Price at $t = 10$	-9.91%	-9.91%	1.43%
$\Delta$ Cost of capital at $t = 0$	0.22%	0.20%	-0.03%
$\Delta$ Cost of capital at $t = 10$	0.34%	0.34%	-0.04%
Realized return over 10 years	4.15%	4.28%	4.84%
Return volatility at $t = 0$	20.32%	20.68%	20.09%
Return volatility at $t = 10$	20.09%	20.09%	20.14%

The effect of exclusion on prices and expected returns approximately doubles compared with Scenario 1. The prices of the firms in Group 100 drop upon announcement by 8.94% and the prices of the firms in Group 91 drop by 7.00%. In contrast, the prices of the firms

Figure 3: Impact of changing the relative fraction of green and active investors

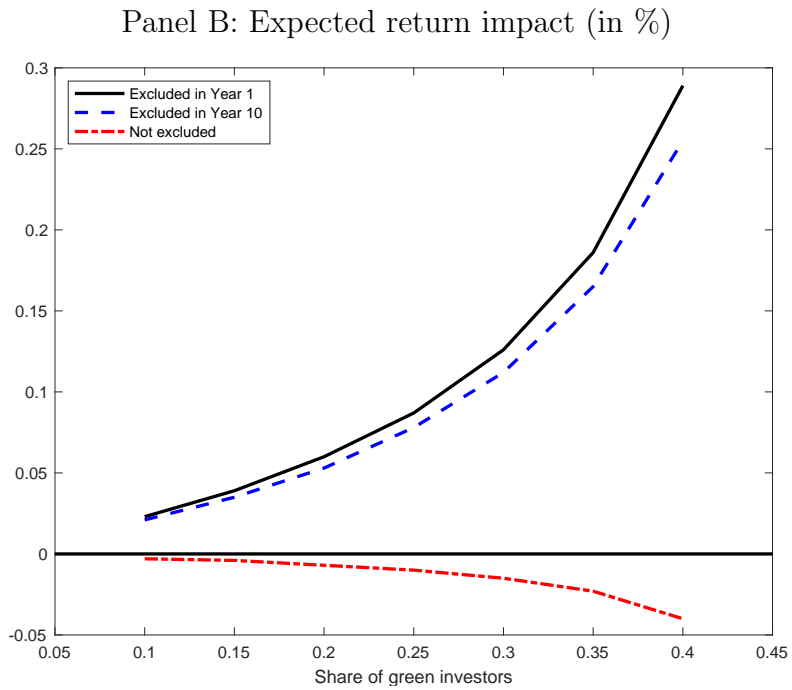
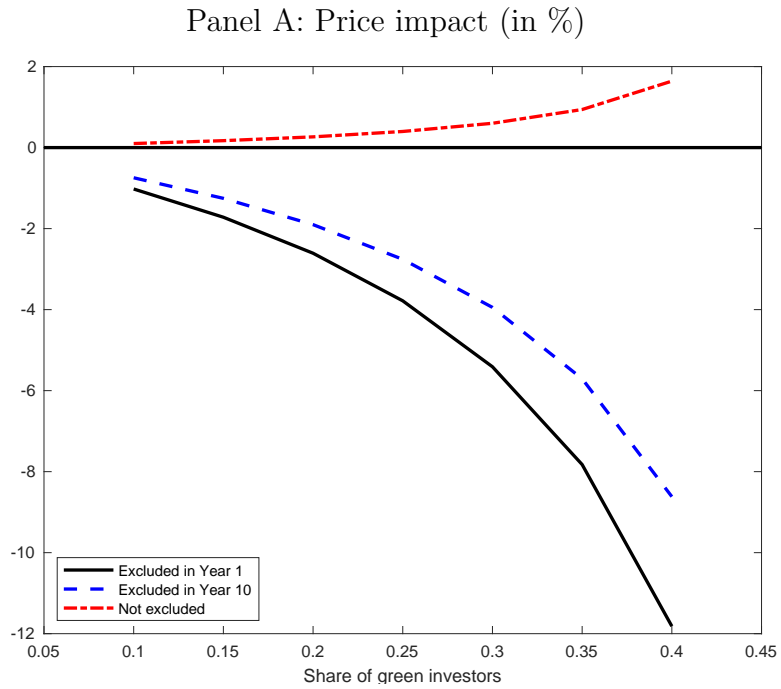
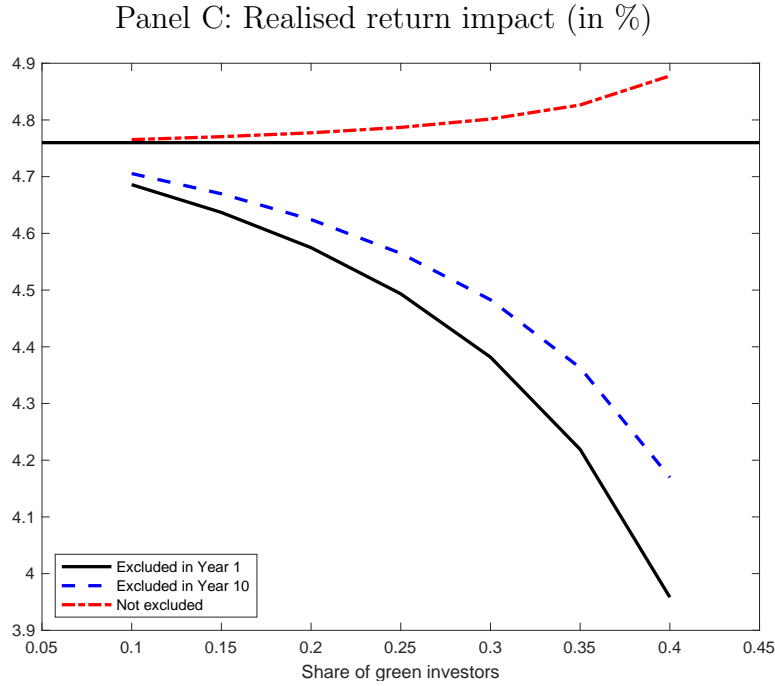


Figure 3 (Cont.): Impact of changing the relative fraction of green and active investors



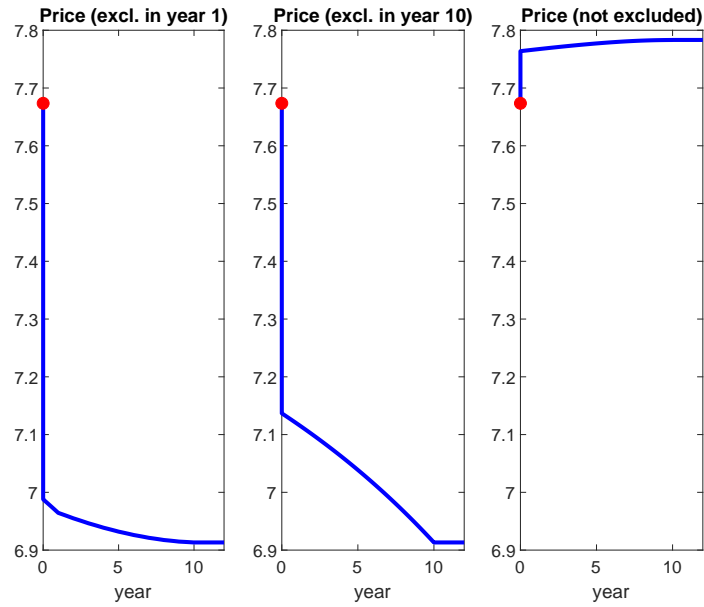
Note: The price impact is measured as the percentage change in the price of a given group of firms relative to the price before the announcement of the exclusion strategy. The expected return impact is measured as the change in the expected return of a given group of firms relative to the expected return before the announcement of the exclusion strategy. The realised return impact is measured as the percentage change in the price of a given group of firms in Year 0 plus the cumulative change in expected return over the ten years.

in Groups 1 to 90 rise by 1.18%. After ten years, the prices of the firms in Groups 100 to 91 drop cumulatively by 9.91% on average, while the prices of the firms in Groups 1 to 90 rise by 1.43%. Over the same period, the cost of capital of the firms in Groups 100 to 91 increases cumulatively by 34 bp, whereas that of the firms in Groups 1 to 90 decreases by 4 bp.

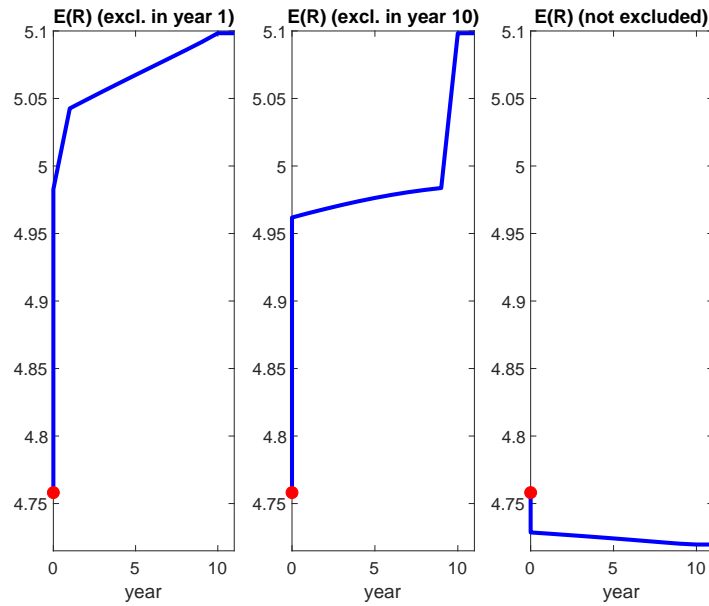
As in Scenario 1, the realized ten-year return is higher for green firms than for brown firms. The realized return of firms in Group 100 over the ten-year period is 4.15%, while it is 4.84% for firms in Groups 1 to 90. As in Scenario 1, differences in return volatility between groups are limited.

Figure 4: Impact of exclusion in Scenario 2

Panel A: Price impact



Panel B: Expected return impact



Note: The price impact is measured as the percentage change in the price of a given group of firms relative to the price before the announcement of the exclusion strategy. The expected return impact is measured as the change in the expected return of a given group of firms relative to the expected return before the announcement of the exclusion strategy.

### 5.3 Scenario 3: Climate Transition Risk and Growing Fraction of Green Investors

Scenario 3 allows for the climate transition risk factor, in addition to the growing fraction of green investors introduced in Scenario 2. Figure 5 and Table 5 are the Scenario 3 counterparts of Figure 2 and Table 3 in Scenario 1. In Scenario 3 we report changes for Group 1 rather than for Groups 1 to 90 because groups differ due to their different loadings on the climate transition risk factor. The differences between groups are small, however.

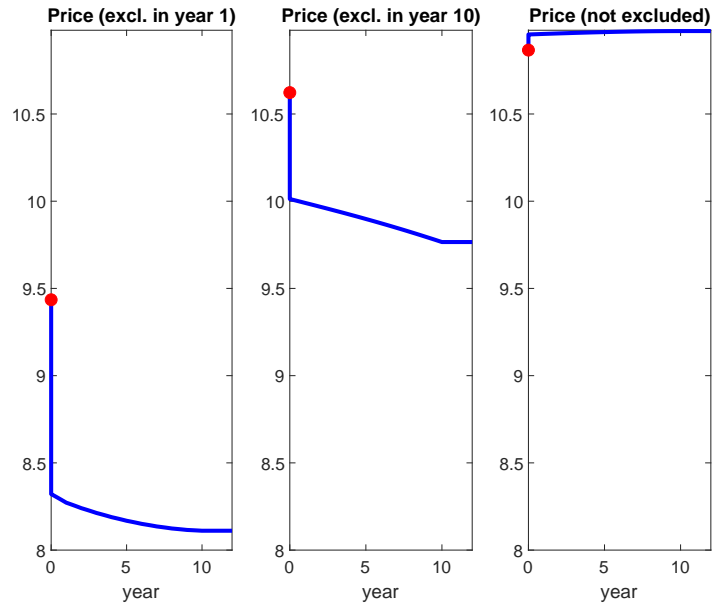
Table 5: Results - Scenario 3

	Firms excluded in Year 1 (Group 100)	Firms excluded in Year 10 (Group 91)	Non-excluded firms (Group 1)
$\Delta$ Price at $t = 0$	-11.80%	-5.74%	0.83%
$\Delta$ Price at $t = 10$	-14.03%	-8.06%	1.00%
$\Delta$ Cost of capital at $t = 0$	0.24%	0.10%	-0.01%
$\Delta$ Cost of capital at $t = 10$	0.59%	0.26%	-0.02%
Realized return over 10 years	3.29%	3.00%	3.37%
Return volatility at $t = 0$	19.53%	15.19%	14.64%
Return volatility at $t = 10$	19.07%	14.69%	14.68%

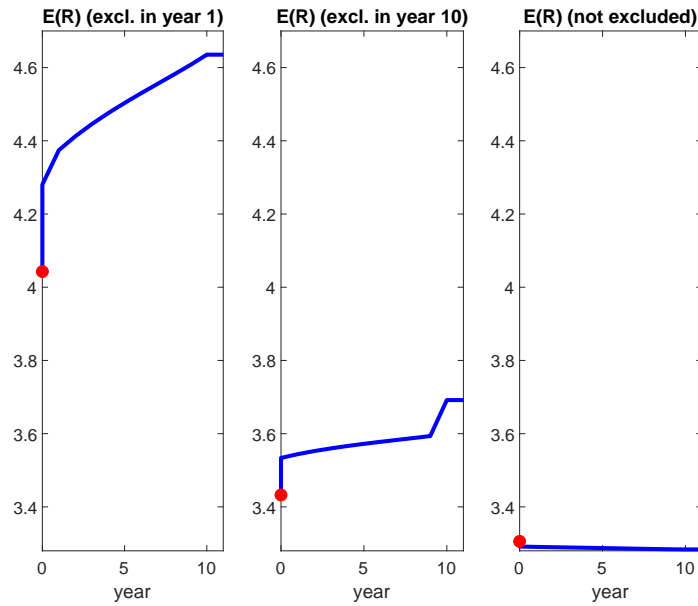
The effect of exclusion on prices and expected returns is significantly larger than in Scenario 2 for the most polluting firms and smaller for less polluting but excluded firms. The prices of the firms in Group 100 drop upon announcement by 11.8% and the prices of the firms in Group 91 drop by 5.74%. In contrast, the prices of the firms in Group 1 rise by 0.83%. After ten years, the prices of the firms in Group 100 drop cumulatively by 14.03% on average, the prices of firms in Group 91 drop by 8.06% and the prices of the firms in Groups 1 rise by 1.00%. Over the same period, the cost of capital of the firms in Group 100 increases cumulatively by 59 bp, that of the firms in Group 91 increases by 26 bp and that

Figure 5: Impact of exclusion in Scenario 3

Panel A: Price impact



Panel B: Expected return impact



Note: The price impact is measured as the percentage change in the price of a given group of firms relative to the price before the announcement of the exclusion strategy. The expected return impact is measured as the change in the expected return of a given group of firms relative to the expected return before the announcement of the exclusion strategy.

of the firms in Group 1 decreases by 2 bp.

As in Scenarios 1 and 2, the realized ten-year return is higher for green firms than for brown firms. The realized return over the ten-year period is 3.29% for firms in Group 100 and 3.00% for firms in Group 91, while it is 3.37% for firms in Group 1. The gap between Groups 100 and 1 is smaller than in Scenarios 1 and 2. An additional difference with Scenarios 1 and 2 is that volatility is significantly larger for the firms in Group 100 than for the other firms due to their high loading on the climate transition risk factor.

The intuition why exclusion hits the most polluting firms particularly hard in Scenario 3 compared to Scenarios 1 and 2 is that active investors are less willing to buy shares of brown firms because a portfolio of such firms loads up significantly on non-diversifiable climate transition risk. This effect is particularly pronounced for the most polluting firms, which are heavily exposed to the climate transition risk factor because of the heavy right tail of the distribution of factor loadings.

## 6 Conclusion

We study the impact of green investors on stock prices in a dynamic equilibrium model where three types of investors—green, passive and active—jointly determine stock prices and returns. Green investors aim to reduce their exposure to firms with the highest GHG emissions. Active investors hold a mean-variance efficient portfolio of all stocks and passive investors hold a value-weighted index of all stocks.

The decarbonisation strategy of green investors that we assume in the model reflects what the academic literature and market practitioners refer to as “Paris agreement” or “net zero” benchmark indices. The trajectory that we assume—the 1% most polluting firms are excluded every year for ten years—corresponds to an annual carbon emission reduction rate of approximately 10% for the green portfolio, given the heavy right tail of the distribution of carbon emissions. This is the necessary GHG reduction rate that green portfolios need to

generate to stay roughly on a net zero trajectory by 2050.

We find a large drop in the stock prices of the most polluting firms that are excluded by green investors and a rise in the prices of greener firms. These effects occur primarily upon the announcement of the exclusion strategy and continue during the exclusion phase. In our baseline Scenario 1 where there are 30% green investors, 50% passive investors and 20% active investors, the stock prices of the firms to be excluded in Year 1 drop by 5.41% immediately upon announcement, and the prices of the firms to be excluded in Year 10 drop by 3.95%. The changes in stock prices are reflected into variation in the cost of capital of the firms to be excluded and of those that remain in the investable space. Over a ten-year transition period, the prices of the excluded firms drop cumulatively by 5.61% and their cost of capital rises by 18 bp. These effects approximately double in Scenarios 2 and 3, where the fraction of green investor rises over time. When, in addition, the most polluting firms load heavily by climate transition risk, as is assumed in Scenario 3, exclusion has an even stronger effect on their prices and cost of capital.

We assume perfect foresight regarding the timing of exclusion and the list of firms to be excluded. This assumption contributes to the large effects upon announcement. In practice, the process may not be perfectly predictable and this may attenuate the announcement effects. The ultimate (Year 10) effects are likely to remain similar, however. The large announcement effects could trigger a rush if investors want to hedge against possible large price drops of brown stocks. As a consequence, we would expect a first-mover advantage for green investors to enter the decarbonisation strategy at an early stage.

The assumptions behind our quantitative results are far from extreme. Only a small fraction of firms would be excluded in the process, some of them only after ten years. Capital from green investors would flow from most polluting firms to less polluting firms. As the exclusion is based on the GHG emissions of individual firms and not on whether they belong to a particular sector (no sector is a priori excluded), green investors could engage in a best-



in-class approach and help the development of green technologies, including in the energy and electricity production industries.

Our analysis focuses on the impact of green investors on stock prices and does not account for linkages between stock prices and corporate investment. For example, the drop in the stock prices of the most polluting firms when they are excluded from the index could force them to cut down on investment, further accentuating the drop. Extending our analysis to incorporate real investment and its two-way feedback with stock prices is a promising direction of future research.

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# Appendix

## A Proof of Proposition 3.1

We first derive the first-order conditions of passive and green investors. Using (2.6), (3.2) and  $z_{Int} = \lambda_I \eta_n$ , we can write the objective (2.8) of passive investors as

$$\begin{aligned} & \sum_{n=1}^{KN} \lambda_{It} \eta_n \mu_n - \frac{\rho}{2} \lambda_{It}^2 \left[ \left( \sum_{n=1}^{KN} \eta_n b_n^s \right)^2 (\sigma^s)^2 \mathbb{E}_t^u \left[ D_t^s \left[ \frac{\partial S_t^s(D_t^s)}{\partial D_t^s} \right]^2 \right] \right. \\ & \left. + \left( \sum_{n=1}^{KN} \eta_n b_n^c \right)^2 (\sigma^c)^2 \mathbb{E}_t^u \left[ D_t^c \left[ \frac{\partial S_t^c(D_t^c)}{\partial D_t^c} \right]^2 \right] + \sum_{n=1}^{KN} \eta_n^2 (\sigma_n^i)^2 \mathbb{E}_t^u \left[ D_{nt}^i \left[ \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} \right]^2 \right] \right] dt, \end{aligned} \quad (\text{A.1})$$

where the expectation is taken over  $(D_t^s, D_t^c, D_{nt}^i)$ . Noting that  $\lambda_{It}^2$  is assumed to be constant in each of the intervals  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$  and  $[K'T, \infty)$ , and using (3.7)–(3.9), we find the first-order condition

$$\begin{aligned} & \sum_{j=s,c} \left( \sum_{m=1}^{KN} \eta_m b_m^j \right) \left( \sum_{m=1}^{KN} [1 - (\mu_{AK'} + \mu_{IK'}) \lambda_{IK'} - \mu_{GK'} \lambda_{GK'} 1_{\{m \leq (K-K')N\}}] \eta_m b_m^j \right) (\sigma^j \bar{a}_{1K'}^j)^2 \\ & + \sum_{m=1}^{KN} [1 - (\mu_{AK'} + \mu_{IK'}) \lambda_{IK'} - \mu_{GK'} \lambda_{GK'} 1_{\{m \leq (K-K')N\}}] \eta_m^2 (\sigma_m^i \bar{a}_{m1K'}^i)^2 \bar{D}_m^i = 0 \quad (\text{A.2}) \end{aligned}$$

in  $[K'T, \infty)$ , and

$$\begin{aligned} & \sum_{j=s,c} \left( \sum_{m=1}^{KN} \eta_m b_m^j \right) \left( \sum_{m=1}^{KN} [1 - (\mu_{Ak} + \mu_{Ik}) \lambda_{Ik} - \mu_{Gk} \lambda_{Gk} 1_{\{m \leq (K-k)N\}}] \eta_m b_m^j \right) (\sigma^j)^2 \int_{kT}^{(k+1)T} (a_{1t}^j)^2 dt \\ & + \sum_{m=1}^{KN} [1 - (\mu_{Ak} + \mu_{Ik}) \lambda_{Ik} - \mu_{Gk} \lambda_{Gk} 1_{\{m \leq (K-k)N\}}] \eta_m^2 (\sigma_m^i)^2 \bar{D}_m^i \int_{kT}^{(k+1)T} (a_{m1t}^i)^2 dt = 0 \end{aligned} \quad (\text{A.3})$$

in  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$ . We can likewise write the objective (2.8) of green investors as

$$\begin{aligned}
& \sum_{n=1}^{KN} \lambda_{It} 1_{\{n \in \mathcal{G}_t\}} \eta_n \mu_n - \frac{\rho}{2} \lambda_{It}^2 \left[ \left( \sum_{n=1}^{KN} 1_{\{n \in \mathcal{G}_t\}} \eta_n b_n^s \right)^2 (\sigma^s)^2 \mathbb{E}_t^u \left[ D_t^s \left[ \frac{\partial S_t^s(D_t^s)}{\partial D_t^s} \right]^2 \right] \right. \\
& + \left. \left( \sum_{n=1}^{KN} 1_{\{n \in \mathcal{G}_t\}} \eta_n b_n^c \right)^2 (\sigma^c)^2 \mathbb{E}_t^u \left[ D_t^c \left[ \frac{\partial S_t^c(D_t^c)}{\partial D_t^c} \right]^2 \right] \right. \\
& + \left. \sum_{n=1}^{KN} 1_{\{n \in \mathcal{G}_t\}} \eta_n^2 (\sigma_n^i)^2 \mathbb{E}_t^u \left[ D_{nt}^i \left[ \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} \right]^2 \right] \right] dt, \tag{A.4}
\end{aligned}$$

where the expectation is taken over  $(D_t^s, D_t^c, D_{nt}^i)$ . The first-order condition is

$$\begin{aligned}
& \sum_{j=s,c} \left[ \left( \sum_{m=1}^{KN} 1_{\{m \leq (K-K')N\}} \eta_m b_m^j \right) \right. \\
& \times \left. \left( \sum_{m=1}^{KN} [1 - \mu_{IK'} \lambda_{IK'} - (\mu_{AK'} + \mu_{GK'}) \lambda_{GK'} 1_{\{m \leq (K-K')N\}}] \eta_m b_m^j \right) (\sigma^j \bar{a}_{1K'}^j)^2 \right] \\
& + \sum_{m=1}^{KN} [1 - \mu_{IK'} \lambda_{IK'} - (\mu_{AK'} + \mu_{GK'}) \lambda_{GK'} 1_{\{m \leq (K-K')N\}}] \eta_m^2 1_{\{m \leq (K-K')N\}} (\sigma_m^i \bar{a}_{m1K'}^i)^2 \bar{D}_m^i = 0 \tag{A.5}
\end{aligned}$$

in  $[K'T, \infty)$ , and

$$\begin{aligned}
& \sum_{j=s,c} \left[ \left( \sum_{m=1}^{KN} 1_{\{m \leq (K-k)N\}} \eta_m b_m^j \right) \right. \\
& \times \left. \left( \sum_{m=1}^{KN} [1 - \mu_{Ik} \lambda_{Ik} - (\mu_{Ak} + \mu_{Gk}) \lambda_{Gk} 1_{\{m \leq (K-k)N\}}] \eta_m b_m^j \right) (\sigma^j)^2 \int_{kT}^{(k+1)T} (a_{1t}^j)^2 dt \right] \\
& + \sum_{m=1}^{KN} [1 - \mu_{Ik} \lambda_{Ik} - (\mu_{Ak} + \mu_{Gk}) \lambda_{Gk} 1_{\{m \leq (K-k)N\}}] \eta_m^2 1_{\{m \leq (K-k)N\}} (\sigma_m^i)^2 \bar{D}_m^i \int_{kT}^{(k+1)T} (a_{m1t}^i)^2 dt = 0 \tag{A.6}
\end{aligned}$$

in  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$ .

We next determine  $a_{1t}^j$  for  $j = s, c$ . Identifying terms in  $D_t^j$  in (3.10) yields the ODE

$$1 - (r + \kappa^j)a_{1t}^j - g_k^j(a_{1t}^j)^2 + \frac{da_{1t}^j}{dt} = 0. \quad (\text{A.9})$$

When  $k = 0, \dots, K' - 1$ , (A.9) is defined over  $t \in [kT, (k+1)T)$ , and when  $k = K'$ , (A.9) is defined over  $t \in [K'T, \infty)$ . When  $k = K'$ , we look for a constant solution of (A.9), corresponding to the steady state. Such a solution  $\bar{a}_{1K'}^j$  must satisfy the quadratic equation

$$1 - (r + \kappa^j)\bar{a}_{1K'}^j - g_{K'}^j(\bar{a}_{1K'}^j)^2 = 0. \quad (\text{A.10})$$

Equation (A.10) has two solutions if

$$(r + \kappa^j)^2 + 4g_{K'}^j > 0,$$

which we assume. We focus on the smaller solution, which is the continuous extension of the unique solution when  $g_{K'}^j = 0$ , and is as in the proposition. When  $k = 0, \dots, K' - 1$ , we solve (A.9) recursively with terminal condition  $\lim_{t \rightarrow (k+1)T} a_{1t}^j = a_{1,(k+1)T}^j$ . We find

$$\begin{aligned} \frac{da_{1t}^j}{dt} &= g_k^j(a_{1t}^j)^2 + (r + \kappa^j)a_{1t}^j - 1 \\ \Rightarrow \frac{da_{1t}^j}{dt} &= (a_{1t}^j - \bar{a}_{1k}^j)(g_k^j a_{1t}^j + \frac{1}{\bar{a}_{1k}^j}) \\ \Rightarrow \frac{da_{1t}^j}{(a_{1t}^j - \bar{a}_{1k}^j)(g_k^j a_{1t}^j + \frac{1}{\bar{a}_{1k}^j})} &= dt \\ \Rightarrow \frac{da_{1t}^j}{g_k^j \bar{a}_{1k}^j + \frac{1}{\bar{a}_{1k}^j}} \left( \frac{1}{a_{1t}^j - \bar{a}_{1k}^j} - \frac{g_k^j}{g_k^j a_{1t}^j + \frac{1}{\bar{a}_{1k}^j}} \right) &= dt \\ \Rightarrow \log \left( \frac{a_{1,(k+1)T}^j - \bar{a}_{1k}^j}{g_k^j a_{1,(k+1)T}^j + \frac{1}{\bar{a}_{1k}^j}} \right) - \log \left( \frac{a_{1t}^j - \bar{a}_{1k}^j}{g_k^j a_{1t}^j + \frac{1}{\bar{a}_{1k}^j}} \right) &= \left( g_k^j \bar{a}_{1k}^j + \frac{1}{\bar{a}_{1k}^j} \right) [(k+1)T - t] \\ \Rightarrow \frac{g_k^j a_{1t}^j + \frac{1}{\bar{a}_{1k}^j}}{a_{1t}^j - \bar{a}_{1k}^j} &= \frac{g_k^j a_{1,(k+1)T}^j + \frac{1}{\bar{a}_{1k}^j}}{a_{1,(k+1)T}^j - \bar{a}_{1k}^j} e^{\left( g_k^j \bar{a}_{1k}^j + \frac{1}{\bar{a}_{1k}^j} \right) [(k+1)T - t]}, \end{aligned}$$

which yields (3.11).

We next determine  $a_{n1t}^i$ . Identifying terms in  $D_{nt}^i$  in (3.10) yields the ODE

$$1 - (r + \kappa_n^i) a_{n1t}^i - g_{nk}^i (a_{n1t}^i)^2 + \frac{da_{n1t}^i}{dt} = 0. \quad (\text{A.11})$$

When  $k = 0, \dots, K' - 1$ , (A.11) is defined over  $t \in [kT, (k+1)T)$ , and when  $k = K'$ , (A.11) is defined over  $t \in [K'T, \infty)$ . When  $k = K'$ , we look for a constant solution of (A.11). Proceeding as for  $a_{1t}^j$ , we find  $\bar{a}_{n1K'}^i$  in the proposition. When  $k = 0, \dots, K' - 1$ , we solve (A.11) recursively with terminal condition  $\lim_{t \rightarrow (k+1)T} a_{n1t}^i = a_{n1, (k+1)T}^i$ . Proceeding as for  $a_{1t}^j$ , we find (3.12).

Identifying the remaining terms yields the ODE

$$\bar{D}_n + \frac{d\bar{S}_{nt}}{dt} - r\bar{S}_{nt} + \sum_{j=s,c} b_n^j \left( \kappa^j a_{1t}^j + \frac{da_{0t}^j}{dt} - r a_{0t}^j \right) + \kappa_n^i a_{n1t}^i \bar{D}_n + \frac{da_{n0t}^i}{dt} - r a_{n0t}^i = 0 \quad (\text{A.12})$$

in the function  $\bar{S}_{nt} + \sum_{j=s,c} b_n^j a_{0t}^j + a_{n0t}^i$ . Its solution is

$$\begin{aligned} \bar{S}_{nt} + \sum_{j=s,c} b_n^j a_{0t}^j + a_{n0t}^i &= \int_t^\infty \left( \bar{D}_n + \sum_{j=s,c} b_n^j \kappa^j a_{1t'}^j + \kappa_n^i a_{n1t'}^i \bar{D}_n \right) e^{-r(t'-t)} dt' \\ &= \frac{\bar{D}_n}{r} + \sum_{j=s,c} b_n^j \kappa^j \int_t^\infty a_{1t'}^j e^{-r(t'-t)} dt' + \kappa_n^i \bar{D}_n \int_t^\infty a_{n1t'}^i e^{-r(t'-t)} dt'. \end{aligned} \quad (\text{A.13})$$

For  $t \in [K'T, \infty)$ , the solution is constant and equal to

$$\bar{S}_n + \sum_{j=s,c} b_n^j a_0^j + a_{n0}^i = \frac{\bar{D}_n + \sum_{j=s,c} b_n^j \kappa^j \bar{a}_{1K'}^j + \kappa_n^i \bar{D}_n \bar{a}_{n1K'}^i}{r}.$$