Information Span in Credit Market Competition *

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Abstract

We develop a credit market competition model that distinguishes between the information span (breadth) and signal precision (quality) of data, capturing the emerging trend in fintech/non-bank lending where traditionally subjective ("soft") information becomes more objective and concrete ("hard"). In a model with multi-dimensional fundamentals, two banks equipped with similar data processing systems possess hard signals about the borrower's hard fundamentals, and the specialized bank, who further interacts with the borrower, can also assess the borrower's soft fundamentals. Increasing the span of the hard information hardens soft information, enabling the data processing systems of both lenders to evaluate some of the borrower's soft fundamentals. We show that hardening soft information levels the playing field for the non-specialized bank by reducing its winner's curse. In contrast, increasing the precision or correlation of hard signals often strengthens the informational advantage of the specialized bank.

JEL Classification: G21, L13, L52, O33, O36
Keywords: Banking competition, Winner's curse, Specialized lending, Information technology, Big data

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1 Introduction

As a crucial intermediary sector in modern economies, commercial banks serve as the main conduit between savers and credit-worthy borrowers, leveraging a broad spectrum of information. The diverse array of lending-related information includes financial data on customers, collateral evaluations, and market and economic trends, not to mention state-of-the-art data analytics. Improvements and changes in information technology affect the quality and type of information available to banks. While there have been many developments that have improved the quality of information available to banks, the trend in big data technology that transforms qualitative or subjective assessments into quantifiable and objective metrics, known as "hardening soft information" (Liberti and Petersen, 2019; Hardik, 2023) is a fairly recent phenomenon. In this paper, we are interested in how the hardening of soft information affects the equilibrium credit market and what differentiates this trend from previous technological innovations.

The farming industry provides an illustrative example of how technology can transform traditional lending practices. In the past, farm loans required extensive in-person visits from specialized loan officers, who leveraged their expertise to evaluate the borrower's abilities and farm infrastructure quality. This hands-on approach was necessary to evaluate these "soft" fundamentals, as the officers needed to directly observe factors like crop rotation techniques, pest management strategies, and barn conditions to accurately assess loan risks. Today, satellite imaging and AI-enabled data analysis allow lenders to gather some of these insights remotely via computerized "hard" data. While on-site assessments still provide important insight, new technologies have expanded access to farm data, demonstrating how technology can expand the information span of "hard signals" without entirely disrupting specialized but "soft" human expertise.

As the example above highlights, the remarkable recent and ongoing technological advancements have the potential to alter the information available to participants in the credit market and significantly impact the industrial landscape of the banking sector. For instance, as shown in Blickle, Parlatore, and Saunders (2023), many lenders specialize in certain industries and companies by providing customized financial services and pricing, often by diligently collecting and analyzing information about individual firms/industries. Nevertheless, the prevailing literature (Broecker, 1990; Marquez, 2002; He, Huang, and Zhou, 2023) on information-based credit market competition predominantly focuses on a signal structure that covers a one-dimensional fundamental with binary realizations, overlooking the nuances of the aforementioned intricate economics potentially embedded in the rich categories of information.

We incorporate a novel information structure into an otherwise conventional credit market competition model, offering an economic framework to analyze the welfare implications of the evolving landscape in information technology and competition among banks. In the model, the borrower quality depends on multiple fundamental states, which broadly belong to two categories—

"hard" states and "soft" states as distinguished by the type of information technology capable of assessing these states. Before making lending decisions, lenders can access private signals about these two states. We refer to a signal that reflects the borrower's hard states as a "hard-information-based signal" or simply a "hard signal," and likewise, a signal that reflects the borrower's soft states as a "soft-information-based signal" or a "soft signal." Crucially, hard states might overlap with soft states, so hard and soft signals might be correlated. This correlation, and its potential implications on credit market competition, are the main innovation relative to the model in the companion paper Blickle, He, Huang, and Parlatore (2024).

Our framework highlights the difference between the breadth (information span) and quality (signal precision) of data. The overlap between soft and hard fundamental states allow us to define the "information span" (of a hard signal) naturally. When hard states cover more fundamental states that are critical to the borrower's quality, the information span of the hard signal expands, and this expansion captures the core idea of "hardening soft information" in the context of credit market competition. In contrast, the precision of data is about enhancing the accuracy of the hard signals in assessing the same characteristics. While improvements in both the span and precision of information are typically associated with technological advances, we show that they have vastly different impacts on credit market outcomes.

In our model of credit market competition, as we outline in Section 2, a specialized bank competes with a non-specialized bank. Each lender has a private hard signal about the hard fundamental states that stems from data processing. Additionally, the specialized lender has access to a soft signal about the borrower's soft states. We assume that the hard signal is binary and decisive in that each lender makes an offer only if it receives a positive realization of it. The soft signal—which differentiates our paper from existing models such as (Broecker, 1990) and (Marquez, 2002)—is continuous and guides the fine-tuned interest rate offering of the specialized bank. Besides analytical convenience, this loan-making rule of the specialized bank matches well with the observed lending practices. Essentially, in our model, the specialized bank acquires two signals, one being "principal" while the other being "supplementary." The former determines whether to lend and the latter affects the detailed pricing terms.[1]

Section 3 fully characterizes the competitive credit market equilibrium with specialized lending in closed form. As in Blickle, He, Huang, and Parlatore (2024), our model has a unique equilibrium, which can fall into two distinct categories depending on whether the non-specialized bank makes zero profits. In the "zero-weak" equilibrium, the winner's curse faced by the non-specialized "weak" bank causes it to randomly withdraw from competition upon receiving a positive hard signal and earn zero profits. In the "positive-weak" equilibrium, the winner's curse is less severe so the non-specialized weak bank always participates in the loan market upon a positive hard signal and earns

[1] Alternatively, the principal signal represents the result of a credit screening test, while the supplementary signal serves the role of internal ratings (of borrowers who are qualified for credit).

positive profits.

Our main analysis, in Section 4, examines how the span of hard information affects the equilibrium in the credit market. In our model, the information technology available to the lenders affects their screening technology and their beliefs about the information of the other lender (through strategic considerations), which determines the severity of the winner's curse. In general, an expansion in the information span of the hard signal reduces Type II errors from hard-information-based screening for both specialized and non-specialized lenders. This economic force, however, is stronger for the latter, because this expansion enables the non-specialized lender to learn about its specialized opponent's soft signal. Put differently, a greater span of hard information increases the overlap between hard and soft states, thereby leveling the playing field by reducing the winner's curse faced by the non-specialized bank due to the specialized opponent's soft signal. When the information span is sufficiently large, the non-specialized lender starts to make positive profits.

As one of the main results of the paper, we compare an increase in the span of hard information with two other types of informational technology advancement: an increase in the signal precision of each hard signal, and an increase in the correlation between the two hard signals. We show that while an increase in the span of hard information levels the playing field for the non-specialized bank, an increase in the precision of hard information or the correlation of hard private signals tends to amplify the informational edge of the specialized bank, especially when the hard signal is sufficiently informative.

Why is it important to distinguish among these different aspects of information technologies? We stress that the significant advance in information technology benefits both types of lenders equally; specialized and established banks can adopt these technologies just as effectively as non-specialized banks and new fintech entrants. However, the fast-growing empirical literature on fintechs (see, e.g. Berg, Fuster, and Puri, 2022) seem to suggest that the new technology has helped relatively weaker (fintech) lenders to catch up, intensifying the credit market competition. Building a model with asymmetric lenders but symmetric technology improvement, our theory clarifies that it is enlarging the information span, not the mere improvement of "signal precision," that can deliver the desired empirical pattern in a robust way. As elucidated in our opening motivating example of "loans to the farming industry," big data technology empowers non-specialized lenders to utilize "hardened soft information." As we show, an improved signal precision allows both lenders to have a more precise evaluation of borrower quality, while enlarging the information span provides the non-specialized lender direct insights into its opponent's pricing strategy: the former tends to reinforce the position of specialized lender, while the latter serves the role of "leveling the playing field."

It is also worth highlighting that these distinctions emerge from our examination of credit market competition with specialized lending. Our model with asymmetric lenders with different types of information is a practically relevant setting; as demonstrated by Blickle, He, Huang, and Parlatore

(2024), banks with asymmetrical private information are needed to match the empirical patterns in loan pricing. From this perspective, this paper offers an analytical framework to derive potentially distinct implications of different aspects of information technology on loan pricing, which provide useful guidance for future empirical research.

The process of "hardening soft information," which expands the span of hard information, has important implications for the equilibrium credit allocation and the resulting welfare. The behavior of the loan approval and non-performing rates of the specialized bank is governed by the reduction in Type II errors, which causes them to increase and decrease, respectively. As a result, we find that total welfare, measured as the expected surplus from projects that are funded, is always increasing in the span of hard information. However, the interest rates charged by the lenders depend on whether the reduction in the winner's curse or the improvement in the screening technology dominates. Interestingly, we show that given a lower signal precision, Bank A's profits could also increase in information span η in the parameter range of positive-weak equilibrium. This highlights the feature that we directly model technology improvement, so both the specialized and nonspecialized lenders enjoy the benefit from the same technology improvement.

Throughout the paper, we make one important modeling choice of hard information technology, which takes the entire binary hard fundamental as input to generate a binary signal. As a model extension we consider an alternative way of modeling the hardening of soft information by introducing a third signal on hardened soft fundamentals. We analytically illustrate the mapping between these two different information technologies and show that this alternative modeling delivers similar economic implications to our baseline modeling.

Literature Review

Lending market competition and common-value auctions. Our paper is built on Broecker (1990) which studies lending market competition with screening tests with symmetric lenders (i.e., with the same screening abilities). Hauswald and Marquez (2003) study the competition between an inside bank that can conduct credit screenings and an outside bank without such access. He, Huang, and Zhou (2023) consider competition between asymmetric lenders with different screening abilities under open banking when borrowers control access to data. Asymmetric credit market competition can also naturally arise from the bank-customer relationship, as a bank knows its existing customers better than a new competitor does.[2] In these models, for analytical tractability it is often assumed that private screening yields a binary signal and lenders participate in bidding only following the positive signal realization.

Building on the framework established in our companion paper Blickle, He, Huang, and Parlatore (2024), our paper considers competition between asymmetric lenders with multiple information

[2]This idea was explored by a two-period model in Sharpe (1990) where asymmetric competition arises in the second period (with the corrected analysis of a mixed-strategy equilibrium offered by Von Thadden (2004)). A similar analysis is present in Rajan (1992).

sources. In both articles a non-specialized lender has access to a private "hard" signal about the borrower's credit quality, while the specialized lender receives not only an independent private "hard" signal but also a "soft" signal, both of which are informative about the borrower's credit quality. The distinction is that in Blickle, He, Huang, and Parlatore (2024), hard (soft) signals reflect *independent* borrower characteristics that drive the loan quality. This paper, however, allows these underlying states to overlap with each other, resulting in correlated hard and soft signals. The correlation and its implications of "hardening soft information" on credit market competition are the main innovation relative to Blickle, He, Huang, and Parlatore (2024).

Fundamentally speaking, credit market competition is an application of common-value auctions, and notably, the auction literature typically allows for general signal distributions (other than the binary signal in the aforementioned papers).[3] For instance, Riordan (1993) extends the N-symmetric-lender model in Broecker (1990) to a setting with continuous private signals. In terms of modeling, our framework can be viewed as a combination of Broecker (1990) (hard information) and Milgrom and Weber (1982) (soft information). It is worth highlighting that lenders are each privately informed with hard information; this hence breaks the Blackwell ordering of the information of two lenders in Milgrom and Weber (1982),[4] resulting in a problem that is considerably more challenging. What is more, the economics revealed by a setting with multi-dimensional information can be fundamentally different, as highlighted by the distinction between information precision and information span discussed in Section 2.2.

In a closely related paper, Karapetyan and Stacescu (2014) argue that sharing borrower's "hard" information (say default history) in fact increases the incumbent bank's incentive to further acquire "soft" information regarding borrower's quality.[5] Although their model also involves the stronger bank having more than one private signal, one important difference is that in Karapetyan and Stacescu (2014) there is always a strict Blackwell ordering of information between two lenders, simply because the hard information becomes public after sharing. In contrast, conditionally independent hard signals in our model allow for the possibility of having a profitable weaker lender, yielding much richer empirical predictions regarding welfare analysis.

Specialization in lending. There is a growing literature documenting specialization in bank lending; the early work includes Acharya, Hasan, and Saunders (2006). Paravisini, Rappoport, and Schnabl (2023) shows that Peruvian banks specialize their lending across export markets benefiting bor-

[3]The early papers on this topic include Milgrom and Weber (1982) and Engelbrecht-Wiggans, Milgrom, and Weber (1983), and later papers such as Hausch (1987); Kagel and Levin (1999) explore information structures where each bidder has some private information, which is the information structure adopted in Broecker (1990). For an early empirical paper on asymmetric bidders, see Hendricks and Porter (1988).

[4]More precisely, one bidder knows strictly more than the other bidder. In this setting, one can show that the under-informed bidder always makes zero profit; see also Engelbrecht-Wiggans, Milgrom, and Weber (1983).

[5]In that paper, the information that banks are sharing, which is hard information, is not that soft information that banks acquire at a cost. If sharing leads incumbent banks to lose their edge, they should have a stronger incentive to acquire soft information (which cannot be shared).

rowers who obtain credit from their specialized banks. Based on data for US stress-tested banks, Blickle, Parlatore, and Saunders (2023) documents that specialization is linked with lower interest rates and better performance in the industry of specialization, pointing to a strong link between specialization in lending and informational advantages. Our paper contributes to this literature by providing a framework that can rationalize these patterns allowing us to understand the economic mechanisms behind them and their implications more deeply.

The nature of soft/hard information in bank lending. The existing literature on soft and hard information (e.g., Stein, 2002; Liberti and Petersen, 2019) emphasizes that the latter is easily verifiable and hence transferable (within an organization); for instance, Bertomeu and Marinovic (2016) and Corrao (2023) model "soft" information via a cheap talk game a la Crawford and Sobel (1982) where the messages are soft and carry no intrinsic meaning themselves.[6] Since we do not explicitly model communications within or across banks, whether the information is verifiable is irrelevant to the core economics that our model aims to capture. However, complementing the traditional way of modeling hard/soft information which focuses on communication (e.g., Bertomeu and Marinovic, 2016; Corrao, 2023), our paper highlights the novel concept of "information span" that is necessary to understand the recent phenomenon where certain soft information becomes hardened. Furthermore, similar to Karapetyan and Stacescu (2014) where hard information can be shared, as hard information is transferable and can be analyzed by anyone, once soft information gets hardened into verifiable data, it also becomes accessible to non-specialists. This levels the playing field for non-specialized lenders, a development often conducive to welfare improvement in our analysis.

Fintech. Our paper connects to the growing literature on fintech disruption.[7] Empirical studies document the use of alternative data in fintech lending, which is consistent with our emphasis on the increasing span of hard information.[8] In particular, Huang, Zhang, Li, Qiu, Sun, and Wang (2020) shows that unconventional data from the Alibaba platform, such as business transactions, customer ratings, and consumption patterns improve credit assessment. Our paper emphasizes that the recent development of cashless payments increases the scope of firms that could be assessed by hard information (Ghosh, Vallee, and Zeng, 2022), and perhaps more importantly, the combination of payments and big data technology enlarges the span of hard information.

[6] For related empirical studies, see Liberti and Mian (2009), Paravisini and Schoar (2016). Recently, based on Harte Hanks data, He, Jiang, Xu, and Yin (2023) shows a significant rise in IT investment within the U.S. banking sector over the past decade, particularly among large banks. They also establish a causal link between communication IT investments and banks' capacity to generate and transmit soft information, which motivates our modeling of the soft signal as the outcome of interactions with borrowers.

[7] See Berg, Fuster, and Puri (2022); Vives (2019), e.g. for a review of bank-fintech competition.

[8] Examples of alternative data include phone device and spelling (Berg, Burg, Gombović, and Puri, 2020), mobile phone logs (Agarwal, Alok, Ghosh, and Gupta, 2020). Along the line of our model with different dimensions of information, Huang (2023) develops a theoretical framework wherein the importance of information concerning underlying qualities varies between collateral-backed bank lending and revenue-based fintech lending such as Square.

2 Model Setup

We consider a credit market competition model with two dates and one good. There are two ex-ante symmetric lenders (banks), indexed by $j \in \{A, B\}$ and one borrower firm; everyone is risk neutral.

2.1 The Setting

Project. At $t = 0$, the firm needs to borrow one dollar to invest in a (fixed-scale) risky project that pays a random cash flow y at $t = 1$. The cash flow realization y depends on the project's quality denoted by $\theta \in \{0, 1\}$. For simplicity, we assume that

$$y = \begin{cases} 1 + \overline{r} & \text{when } \theta = 1 \\ 0 & \text{when } \theta = 0, \end{cases} \tag{1}$$

where $\overline{r} > 0$ is exogenously given so only the good project pays off. We will later refer to $\overline{r}$ as the interest rate cap or the return of a good project. The project's quality θ is the firm's private information at $t = 0$, and the prior probability of a good project is $q \equiv \mathbb{P}(\theta = 1)$. Later we will use "project success," "good project" and/or "good borrower" interchangeably to refer to $\theta = 1$. The project quality θ is unobservable.

Hard and soft states. The project's success $\theta \in \{0, 1\}$ depends on two fundamental states, one being "*h*ard" denoted by θ_h and the other being "*s*oft" denoted by θ_s. Importantly, θ_h and θ_s are potentially correlated, and the correlation is related to the span of hard information technology.

We assume both fundamental states are binary so that $\theta_h \in \{0, 1\}$ and $\theta_s \in \{0, 1\}$, with

$$q_h \equiv \mathbb{P}(\theta_h = 1), \text{ and } q_s \equiv \mathbb{P}(\theta_s = 1).$$

When Section 2.2 introduces information technologies that allow banks to observe signals (regarding θ_h and θ_s), a crucial distinction between these states is that hard signals contain information only about θ_h while soft signals contain information about θ_s.

Multi-dimensional fundamental states and information span Following the O-ring theory of economic development (Kremer, 1993), we model the hard and soft states by a setting with multi-dimensional fundamental states. As a main contribution of our paper, this offers a novel way to study the "span" of the information available to banks.

More specifically, suppose that the success of the project θ depends on N characteristics in the following multiplicative way:

$$\theta = \prod_{n=1}^{N} \theta_n = \overbrace{\prod_{n=1}^{N_h^h} \theta_n \cdot \underbrace{\prod_{n=N_h^h+1}^{N_h^h+N_s^h} \theta_n}}^{\theta_h} \underbrace{\cdot \prod_{n=N_h^h+N_s^h+1}^{N} \theta_n}_{\theta_s}. \tag{2}$$

We assume that $\{\theta_n\}$ follow independent Bernoulli distributions, i.e., $\theta_n = 1$ with probability $q_n \in [0,1]$ for all $n = 1, ..., N$; they capture "(unobservable) characteristics" that are critical to the ultimate success of the project, such as product quality, market and funding conditions, the regulatory environment, etc. As shown in (2), the hard state θ_h covers the first $N^h \equiv N_h^h + N_s^h$ characteristics while the soft state covers the last $N - N_h^h$. Importantly, hard and soft states overlap across the middle N_s^h characteristics, which leads to correlated fundamental states. Later we will vary N_s^h—i.e., the span of hard information—and study the implication of this on credit market competition.

Since the order of characteristics plays no role in the analysis, it is without loss of generality to analyze a simplified setting with three independent fundamental states as follows:

$$\theta = \overbrace{\theta_h^h \cdot \underbrace{\theta_s^h}}^{\theta_h} \underbrace{\cdot \theta_s^s}_{\theta_s}, \tag{3}$$

with priors denoted by

$$q_h^h \equiv \mathbb{P}\left(\theta_h^h = 1\right), \quad q_s^h \equiv \mathbb{P}\left(\theta_s^h = 1\right), \quad \text{and} \quad q_s^s \equiv \mathbb{P}\left(\theta_s^s = 1\right).$$

When $\theta_h^s = 1$ with probability one (i.e., $q_h^s = 1$ or $N_s^h = 0$ in Eq. (2)), this model degenerates to independent hard and soft fundamental states as in Blickle, He, Huang, and Parlatore (2024).

Although our characterization of the credit market equilibrium is general, our main economic analysis focuses on the specification in Eq. (3). As we will explain shortly, θ_h^h in Eq. (3) captures those fundamental states that are already "hard" even before the information technology progress, θ_s^h captures those states that were used to be "soft" but now can be covered by hard information thanks to the technology progress, while θ_s^s captures those states that remain soft.

Credit market competition. At date $t = 0$, given its private information about the borrower's project quality (see Section 2.2), each bank j can choose to make a take-it-or-leave-it offer to the borrower firm or to make no offer (i.e., exit the lending market). An offer consists of a fixed loan amount of one and an interest rate r. The borrower firm accepts the offer with the lowest rate if it receives multiple offers.

2.2 Information Technology and Information Span

Information technology corresponds to mappings from some fundamental states to signals. We will introduce two types of signals, each modeled as a specific mapping from its corresponding fundamental state θ_h or θ_s to a bank-specific signal realization. To capture specialized lending, we assume that both lenders have *hard-information*-based private signal h^j for $j \in \{A, B\}$ about θ_h while only specialized bank A has the *soft-information*-based private signal s about θ_s. Figure 1 provides a summary of information technology.

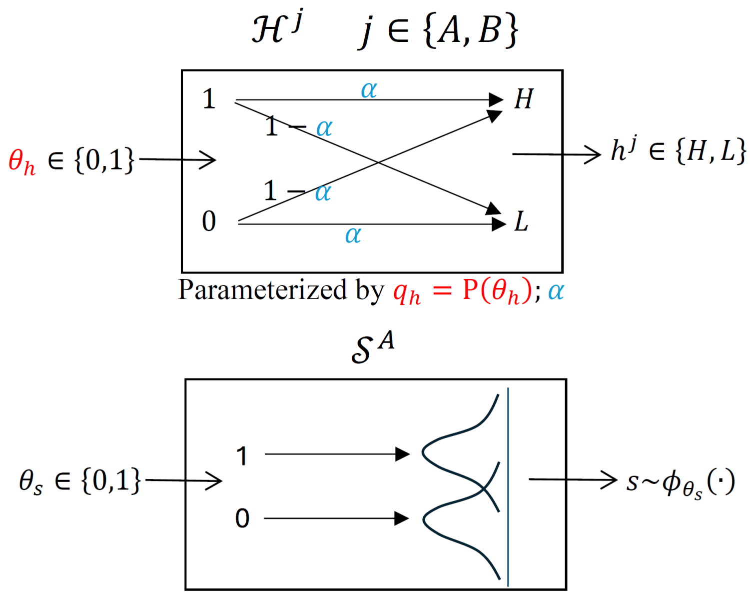

Figure 1: Information Technology, Hard (top panel) and Soft (bottom panel)

Hard signals. We assume that both lenders have access to "hard" data (which include both financial and operating data in the past as well as "alternative data" that become available following the big data technology), which they can process to produce a *hard-information*-based private signal h^j about the firm's fundamental state θ_h. We call these information "hard" signals. For tractability, we assume that these hard signals are binary, i.e., $h^j \in \{H, L\}$, with a realization H (L) being a positive (negative) signal of θ_h. (Binary hard signal is related to the assumption that hard signals are "decisive;" see Section 2.4.) Conditional on the (relevant) state, hard signals are independent across lenders.

More specifically, as illustrated in the top panel of Figure 1, the hard signal technology, denoted by $\mathcal{H}^j$, takes the binary fundamental state $\theta_h \in \{0, 1\}$—which could vary as the information span changes as will be introduced next—as input, and generate a binary signal $h^j \in \{H, L\}$ as an output. Following most of the literature with exogenous symmetric information technology (Broecker, 1990;

Marquez, 2002), we assume that

$$\mathbb{P}\left(h^j = H\,|\theta_h = 1\right) = \mathbb{P}\left(h^j = L\,|\theta_h = 0\right) = \alpha \text{ for } j \in \{A, B\}\,. \tag{4}$$

Here, $\alpha \in \left(\frac{1}{2}, 1\right)$ measures the precision of the hard signal, and captures equal probabilities of Type I and Type II errors. Given the binary fundamental state θ_h, the hard signal technology $\mathcal{H}^j$ thus can be summarized by two parameters: the prior of input $q_h = \Pr(\theta_h)$, and the signal's precision α given in (4). Finally, we assume that conditional on θ_h the hard signals $h^j = \mathcal{H}^j(\theta_h)$ are independent across two banks.[9]

Span (of hard) information Define

$$\eta \equiv 1 - \Pr(\theta_s^h = 1) = 1 - q_s^h > 0. \tag{5}$$

We call η the information span (of hard signals). Corresponding to a larger N_s^h in (2) (or θ_s^h becomes more important in (3)), an expansion of the coverage of θ_h leads to a smaller q_s^h and hence a greater η. All else equal, the larger η, the broader the span of hard information h^j's, and the greater the hard signal's information content (and capturing more of information that was soft previously, i.e., θ_s^h).

The information span η controls the input θ_h to the hard signal technology $\mathcal{H}^j$. This is the key distinction between our paper and the existing literature (Broecker, 1990; Marquez, 2002). More specifically, before soft information gets hardened the input is $\theta_h = \theta_h^h$ with a prior of $q_h = q_h^h$ while after this technology improvement the input becomes $\theta_h = \theta_h^h\theta_s^h$ with a prior of $q_h = q_h^h q_s^h = (1-\eta)q_h^h$; see (3). Importantly, from the perspective of any hard signal technology $\mathcal{H}^j$, this only changes the prior of the binary input θ_h, i.e., $q_h(\eta) = (1-\eta)\,q_h^h$, while keeping the precision α constant.[10]

The binary structure of the hard signal captures the coarseness with which much of the hard information is used in practice.[11] The main insight that the information span stemming from the big data technology trend differs from the precision of information is robust to a more general non-binary hard signal structure. We intentionally assume that both lenders have the same hard information technology because we are interested in how different aspects of information technology

[9] In the companion paper Blickle, He, Huang, and Parlatore (2024), we consider a general (binary) information technology where hard signals are potentially correlated.

[10] Many papers that adopt the binary-fundamental-binary-signal structure, including Marquez (2002), conduct the comparative statics on the prior of the project quality, with the implicit assumption that the signal precision can be kept at a constant. We do acknowledge that the interpretation of constant precision depends on the particular micro-foundation of information technology. For instance, an alternative hard signal technology could take two fundamentals θ_h^h and θ_s^h as input and two signals as output. This will necessarily complicate the analysis, and it is unclear this treatment will help which bank more.

[11] For example, credit scores are binned in five ranges even though scores are computed at a much granular level and go from 300 to 850.

improvement affect the relative market power when both lenders enjoy the technology advancement symmetrically.[12]

Soft signal. Additionally, we endow Bank A with a signal s, which captures the bank being "specialized" in the firm. Similar to Blickle, He, Huang, and Parlatore (2024) we assume that the signal s is continuous. Our preferred interpretation of this additional signal is as a *soft-information*-based private signal, collected after due diligence or face-to-face interactions with the borrower after on-site visits. Besides mathematical convenience, the continuous distribution captures soft information resulting from research tailored to the particular borrower and, therefore, allows for a more granular assessment of the borrower's quality.

Similar to the hard signal, the soft signal technology should be viewed as a mapping $\mathcal{S}^A$ from the soft fundamental state $\theta_s \in \{0, 1\}$ to a variable s that is correlated with θ_s, as shown in the bottom panel of Figure 1. It is without loss of generality to directly work with the posterior probability of the soft state being good $\theta_s = 1$ given the soft signal realization, i.e.,

$$s \equiv \Pr[\theta_s = 1|s] \in [0, 1]. \tag{6}$$

We denote the density function of s by $\phi(s)ds \equiv \mathbb{P}(s \in (s, s + ds))$, which satisfies $\int_0^1 \phi(s)\, ds = 1$ and the prior consistency $\int_0^1 s\phi(s)\, ds \equiv q_s$.

For later exposition purposes, our numerical examples consider $s = \Pr[\theta_s = 1|\theta_s + \epsilon] = \mathbb{E}[\theta_s|\theta_s + \epsilon]$ where $\epsilon \sim \mathcal{N}(0, 1/\tau)$ with τ capturing the signal-to-noise ratio of Bank A's soft information technology. This soft signal precision τ captures similar economics as α, and we stress it has different implications from the information span parameter η.

In light of Figure 1, one can derive the density of s conditional on $\theta_s = 1$, which we denote by $\phi_1(s) \equiv \phi(s|\,\theta_s = 1)$. Using the short-hand notation $s \in ds$ for $s \in (s, s + ds)$, we have

$$\phi_1(s) \equiv \frac{1}{ds}\mathbb{P}(s \in ds|\theta_s = 1) = \frac{\mathbb{P}(\theta_s = 1|s \in ds) \cdot \frac{1}{ds}\mathbb{P}(s \in ds)}{\mathbb{P}(\theta_s = 1)} = \frac{s \cdot \phi(s)}{q_s}. \tag{7}$$

Similarly, we can calculate

$$\phi_0(s) \equiv \phi(s|\,\theta_s = 0) = \frac{(1 - s)\phi(s)}{1 - q_s}.$$

As s is the posterior expectation of θ_s and a higher value of s is "good news" (Milgrom, 1981), these two densities, i.e., $\phi_1(\cdot)$ and $\phi_0(\cdot)$, satisfy the strict Monotone Likelihood Ratio Property (MLRP).

2.3 Discussions on Modelling and Related Literature

Our model departs from the literature in several ways that warrant some discussion.

[12] In the companion paper Blickle, He, Huang, and Parlatore (2024), we consider a general (binary) information technology that is potentially asymmetric between lenders.

Hardening soft information. The concept of information span η allows us to model "hardening soft information." To see this, consider Eq. (3) in Section 2.1. There, the first term θ_h^h captures those fundamental states that are already "hard" even before the information technology progresses; we call them "always hard" fundamentals. The second term θ_s^h captures those states that were used to be "soft" but now can be covered by hard signals thanks to the technology progress; the coverage of these "hardened soft" fundamentals grows with information span η. Finally, θ_s^s captures those states that remain soft; and we call them "always soft" fundamentals. Essentially, technological advancement (e.g. big data and machine learning) enables lenders to acquire pertinent hard objective data points, i.e., hard signals h^j for both lenders, about these "hardened soft" fundamentals θ_h^s, which previously could only be collected through human interactions and were accessible only to the specialized lender.

Hard information technology. In general information technology corresponds to mappings from some fundamental states to signals, and as usual, there are potentially important modeling choices in specifying the details of the (hard) information technology.[13] As the top panel of Figure 1 illustrates, the hard information technology takes the entire binary hard fundamental θ_s as input and generates a binary signal as output. But this is not the only way in a setting of multi-dimensional fundamental states; given our hard fundamental $\theta_h = \theta_s^h \theta_s^s$, another natural modeling is to keep the original hard and soft signals (h^j's and s) and introduce additional signals of the hardened soft fundamental θ_h^s. Section 5.1 considers this alternative and demonstrates that our economic implications are qualitatively similar to our baseline modeling.

Information span versus signal precision. The information span η is a key parameter in our analysis. By incorporating multi-dimensional information, our model highlights the distinction between the information span η and information precision (α for h-signal and τ for s-signal). Take α as an example; recall that α measures the quality/precision of hard information while η measures the scope/breadth of hard information. Both are significant aspects of the astonishing technological advancement in the past decades but with important differences. When the computer was introduced, it was faster and easier to process and compile bank statements. This improvement in processing made information more precise but did not change its scope much. However, the use of "big data," a distinctive trend in information technology during the last decade, has changed what can be digitized as hard information (think of Amazon predicting consumer preferences). As many scholars have argued, big data technology has expedited the process of "hardening soft information" by converting subjective or qualitative data (soft information) into more objective or quantifiable (hard) metrics; for recent evidence in the banking industry, see for example in Hardik (2023). By incorporating multi-dimensional information, our model allows us to study the distinction between

[13] Information design along the line of Kamenica and Gentzkow (2011); Bergemann and Morris (2016) addresses this issue but is beyond the scope of this paper.

these two aspects, which, as we explain shortly, have distinctive economic implications regarding credit market competition.

Endogenous information structure. Throughout we take the lenders' asymmetric information technologies as given. Blickle, He, Huang, and Parlatore (2024) endogenize this asymmetric information technology in a symmetric setting with two firms, a and b, where Bank A (B) endogenously becomes specialized by acquiring both hard and soft signals for firm a (b), while non-specialized Bank B (A) only acquires the "hard" signal of the firm a (b). There, we highlight a key difference when acquiring these two types of signals: a one-time investment—for example, installing IT equipment and software—enables lender j to receive two hard signals, one for each firm, whereas soft information must be collected separately for each firm. This is connected to the next point regarding the modeling of soft/hard information.

Connection to the literature of soft/hard information. The literature on soft and hard information (e.g., Stein, 2002; Liberti and Petersen, 2019) often emphasizes that the latter is easily verifiable and hence transferable (within an organization); for instance, Bertomeu and Marinovic (2016) and Corrao (2023) model "soft" information via a cheap talk game a la Crawford and Sobel (1982) where the messages are soft and carry no intrinsic meaning themselves. Since we do not explicitly model communications within or across banks, whether the information is verifiable is irrelevant to the core economics that our model aims to capture.

Nevertheless, our information technology discussed above, i.e., hard signals are available for both lenders while only the specialized lender has access to the soft signal, connects to this traditional view of soft information. Exactly due to the non-verifiable nature of soft information, loan officers often need to collect it individually and possess the expertise to interpret it, whereas verifiable hard information can be processed by anyone in a rather routine way. What is more, when soft information becomes hardened so that the IT equipment and software can analyze it from data, naturally some soft information becomes verifiable and hence available to non-specialists. This is exactly the logic in Karapetyan and Stacescu (2014) where hard information can be shared while soft cannot.

2.4 Decisive Hard Signals and Parametric Assumptions.

For tractability reasons, similar to Blickle, He, Huang, and Parlatore (2024), throughout our analysis we assume that the hard signal is "decisive" for participation: Bank j participates if and only if it receives $h^j = H$. For the specialized Bank A, the hard signal serves as "pre-screening," in the sense that the bank rejects the borrower upon receiving an L signal, while upon an H signal it makes a pricing decision based on its soft signal s. In other words, for the specialized lender, the "principal" signal is the one that determines whether to lend, and the "supplementary" one helps its loan pricing.[14]

To ensure that the pre-screening hard signal is "decisive," throughout the paper we impose the following parameter restrictions.

Assumption 1. ***(Decisive Hard Signals)***

- *Bank A rejects the borrower upon an L hard signal, regardless of any soft signal s:*

$$q_h (1-\alpha) \bar{r} < (1-q_h) \alpha; \tag{8}$$

- *Bank B is willing to participate if and only if its hard signal $h^B = H$:*

$$q\alpha\bar{r} > (q_h - q)\alpha + (1-q_h)(1-\alpha). \tag{9}$$

Assumption 1 says that the hard signal has to be sufficiently strong (informative) to serve as pre-screening of loan applications for both lenders. Condition (8), states that it is not profitable for Bank A to compete upon receiving a hard signal L even when the soft signal reveals that the soft fundamental θ_s is good with certainty. This condition implies that Bank B, which only has the hard signal and is uncertain about the realization of the soft fundamental, also chooses not to compete upon receiving $h^B = L$. Condition (9) states that upon $h^B = H$, Bank B is willing to lend at the highest possible interest rate if it is the monopolist lender. This condition also implies that Bank A, which also receives a soft signal, is willing to lend at the highest interest rate if it is the monopolist lender upon $h^A = H$ if it also observes high enough realizations of its soft signal.

2.5 Credit Market Equilibrium Definition

We now formally define the credit market equilibrium with specialized lending, along the line of Blickle, He, Huang, and Parlatore (2024).

Bank strategies. Conditional on the hard signal, we define the space of interest rate offers to be $\mathcal{R} \equiv [0, \bar{r}] \cup \{\infty\}$. Here, $\bar{r}$ is the exogenous maximum interest rate (or project return, see Section 2.1) and ∞ captures the strategy of not making an offer.

For Bank A, we denote its pure strategy by $r^A(s) : \mathcal{S} \to \mathcal{R}$, which induces a distribution of its interest offerings denoted by $F^A(r) \equiv \Pr\left(r^A \leq r\right)$ according to the underlying distribution of the

[14] Alternatively, the principal signal represents the result of a credit screening test, while the supplementary signal serves the role of internal ratings (of borrowers who are qualified for credit). This ranking portrays the key role played by hard information for large banks when dealing with new borrowers. Indeed, as documented on page 1677 of Crawford, Pavanini, and Schivardi (2018), Italian large banks list the factors they consider in assessing any new loan applicant's creditworthiness, with the following order of importance: i) hard information from the central bank (financial statement data); ii) hard information from Credit Register; iii) statistical-quantitative methods; iv) qualitative information (i.e., bank-specific soft information codifiable as data); v) availability of guarantees; and vi) first-hand information (i.e., branch-specific soft information).

soft signal. (At this point we take as given that Bank A uses pure strategy, though we formally prove this result in Proposition 1). Finally, the endogenous support of the equilibrium interest rates offered when making an offer is a sub-interval of $[0, \overline{r}]$. Therefore, with a slight abuse of terminology, we refer to that sub-interval as the "support" of the interest rate distribution even though loan rejection ($r = \infty$) could also occur along the equilibrium path. Bank B randomizes its interest rate offerings conditional on a positive hard signal in equilibrium, with an endogenous cumulative distribution $F^B(r) \equiv \Pr\left(r^B \leq r\right)$. Since domain of offers includes $r = \infty$ which captures rejection, it is possible that $F^B(\overline{r}) = \mathbb{P}\left(r^B < \infty | h^B = H\right) \leq 1$.

The borrower picks the lower rate from two competing lenders (if multiple loan offers are available). For instance, conditional on both banks receiving positive hard signals, if Bank B quotes r^B, then its winning probability $1 - F^A\left(r^B\right)$ equals the probability that Bank A with soft signal s offers a rate that is higher than r^B, which includes the event that Bank A rejects the borrower ($r^A(s) = \infty$), presumably because of an unfavorable soft signal.[15] When $r^A = r^B = \infty$, the borrower fails to get the loan.

Definition 1. (Credit market equilibrium) A competitive equilibrium in the credit market (with decisive general signals) consists of the following lending strategies and borrower choice:

1. A lender j rejects the borrower or $r^j = \infty$ upon $g^j = L$ for $j \in \{A, B\}$; upon $g^j = H$,
   i) Bank A offers $r^A(s) : [0, 1] \to \mathcal{R} \equiv [0, \overline{r}] \cup \{\infty\}$ to maximize its expected lending profits given $g^A = H$ and s, which induces a distribution function $F^A(r) : \mathcal{R} \to [0, 1]$;
   ii) Bank B offers $r^B \in \mathcal{R}$ to maximize its expected lending profits given $g^B = H$, which induces a distribution function $F^B(r) : \mathcal{R} \to [0, 1]$;
2. Borrower chooses the lower offer $\min\{r^A, r^B\}$.

As standard, there exists an endogenous lower bound of interest rate $\underline{r} > 0$, so that the two distributions $F^j(\cdot)$, $j \in \{A, B\}$ share a common support $[\underline{r}, \overline{r}]$ (besides ∞ as rejection). The following lemma is standard in the literature and shows that resulting equilibrium strategies in our setting are well-behaved.

Lemma 1. ***(Equilibrium Structure)*** *In any credit market equilibrium*

a. The two lenders' interest rate distributions $F^j(\cdot)$, $j \in \{A, B\}$ are smooth over $[\underline{r}, \overline{r})$, i.e. no gap and atomless, so that they admit well-defined density functions

b. At most only one lender can have a mass point at $\overline{r}$.

[15] Upon ties, i.e. $r^A = r^B < \infty$, borrowers randomly choose the lender with probability one-half, although the details of the tie-breaking rule do not matter as ties occur as zero-measure events in equilibrium.

3 Credit Market Equilibrium Characterization

In this section we solve for the credit market equilibrium with specialized lending and potentially overlapping information spans. Blickle, He, Huang, and Parlatore (2024) characterizes the credit market equilibrium under two key conditions: i) binary hard ("general" in that paper) signals are decisive, and ii) the two binary and one continuous signals are conditionally independent when success (i.e., independent conditional on the project's success). Our setting with arbitrary information span satisfies both conditions and therefore can be viewed as a special case of the general information structure in Proposition 4 in Blickle, He, Huang, and Parlatore (2024). For this reason, our exposition of this section will be less formal and instead focus on illustrating the key properties of the equilibrium, especially the differences from the special case of $\eta = 1$ in Blickle, He, Huang, and Parlatore (2024).

3.1 Bank Profits and Optimal Strategies

Joint Distributions of Signals and Posterior

To start, we define the joint and posterior probabilities of project success $\theta = 1$ of a collection of certain events. We use the ordered subscript $\{h^A h^B\} \in \{HH, HL, LH, LL\}$ to denote the events of the corresponding hard signal realizations, where HL stands for Bank A's (B's) hard signal being H (L). Denote by $\overline{p}_{h^A h^B}$ the joint probability of any collection of hard signal realization; here, the "bar" indicates "taking the average over all possible soft signal realizations." For instance,

$$\overline{p}_{HH} \equiv \mathbb{P}\left(h^A = H, h^B = H\right) = q_h \alpha^2 + (1 - q_h)(1 - \alpha)^2 . \tag{10}$$

Similarly, we denote by $\overline{\mu}_{h^A h^B}$ the posterior of project success conditional on $h^A h^B$; for instance

$$\overline{\mu}_{HH} \equiv \frac{\mathbb{P}\left(h^A = H, h^B = H, \theta = 1\right)}{\mathbb{P}\left(h^A = H, h^B = H\right)} = \frac{q_h \alpha^2}{q_h \alpha^2 + (1 - q_h)(1 - \alpha)^2} \cdot q_s^s . \tag{11}$$

Competing lenders also need to assess the probabilities of hard signals together with the soft signal. Denote by $p_{h^A h^B}(s)\, ds \equiv \mathbb{P}\left(h^A, h^B, s \in ds\right)$ the joint probability of the two hard signals being $h^A h^B$ and $s \in ds$ (i.e., the soft signal s falls in the interval $(s, s + ds)$). Similarly, $\mu_{h^A h^B}(s)$ denotes the posterior probability of project success, i.e., the fundamental state $\theta = 1$, conditional on the hard signal realizations and the soft signal:

$$\mu_{h^A h^B}(s) = \mathbb{P}\left(\theta = 1 \middle| h^A, h^B, s\right) = \frac{\mathbb{P}\left(\theta = 1, h^A, h^B, s \in ds\right)}{\mathbb{P}\left(h^A, h^B, s \in ds\right)} . \tag{12}$$

And, under the multiplicative structure in Eq. (3), project success $\theta = 1$ implies that $\theta_h = \theta_s = 1$, which allows us to derive the joint probability of $\mathbb{P}\left(\theta=1, h^A, h^B, s \in ds\right)$ as

$$p_{h^A h^B}(s)\,\mu_{h^A h^B}(s) = \underbrace{\mathbb{P}(\theta=1)}_{q} \cdot \mathbb{P}\left(h^A \,|\theta_h=1\right) \cdot \mathbb{P}\left(h^B \,|\theta_h=1\right) \cdot \underbrace{\phi(s|\,\theta_s=1)}_{\phi_1(s)}. \tag{13}$$

This result points to conditional independence when success, i.e., all signals, including hard and soft, are independent conditional on project success $\theta = 1$. We will come back to this point later when we derive the equilibrium.

Bank A's Strategy

Consider the problem of Bank A when it observes a positive hard signal $h^A = H$ and a soft signal s. If Bank A chooses to exit the lending market by quoting $r = \infty$, its expected profits are given by $\pi^A(r=\infty, s) = 0$. If Bank A chooses to participate in the lending market by offering and interest rate $r \in [\underline{r}, \overline{r}]$, its expected profits are given by

$$\pi^A(r,s) \equiv \underbrace{p_{HH}(s)}_{h^A=H, h^B=H, s} \underbrace{\left[1 - F^B(r)\right]}_{A \text{ wins}} \left[\mu_{HH}(s)(1+r) - 1\right] + \underbrace{p_{HL}(s)}_{h^A=H, h^B=L, s} \left[\mu_{HL}(s)(1+r) - 1\right], \tag{14}$$

where the first term takes into account the expected payoff conditional on winning the borrower when Bank B participates in the market and the second term accounts for the likelihood that Bank B receives a low signal. More specifically, Eq. (14) considers that Bank A cannot observe Bank B's hard signal h^B when making an offer. With probability $p_{HH}(s)$, both banks get favorable hard signals H, and Bank A wins with probability $\left[1 - F^B(r)\right]$ if it offers r, whereas with probability p_{HL} Bank B receives a low hard signal and Bank A faces no competition for the borrower. Since Bank B randomizes its strategy upon $h^B = H$, from Bank A's perspective winning the price competition is not informative about the borrower's quality. But, whether Bank B participates in the loan market or not affects Bank A's expected quality of the borrower; this economic force is captured by $\mu_{HH}(s)$ and $\mu_{HL}(s)$, which we have introduced in Section 3.1.

Given the profit function defined above, Bank A's optimal interest rate offering is $r^A(s) \equiv \arg\max_{r \in \mathcal{R}} \pi^A(r,s)$. As shown in Blickle, He, Huang, and Parlatore (2024), Bank A's equilibrium pricing strategy $r^A(s)$ is decreasing in s, hits the interest rate cap $\overline{r}$ when the soft signal worsens, and in general will jump to ∞ for sufficiently low s. Formally, $\hat{s} \equiv \sup\left\{s|\, r^A(s) = \overline{r}\right\}$; that is to say, $\hat{s}$ is the highest realization of the soft signal such that Bank A quotes $\overline{r}$.[16] And, we define $x \leq \hat{s}$ as the threshold such that $\pi^A(\overline{r}, x) = 0$; that is to say, Bank A rejects the borrower for all

[16] Throughout the paper we adopt the convention that $\sup\{\emptyset\} = \inf\{[0,1]\} = 0$.

$s < x$ so that $\hat{s} \equiv \sup \left\{ s | \, r^A(s) = \infty \right\}$. Note that $x = \hat{s}$ could occur along the equilibrium path. Given these definitions, it is straightforward to show that $r^A(s) = \overline{r}$ for $s \in [x, \hat{s})$, and $r^A(s) = \infty$ for $s \in [0, x)$. In sum, we can define the inverse function (correspondence) of $r^A(s)$ to be

$$s^A(r) \equiv \begin{cases} r^{A(-1)}(r), & \text{when } r \in [\underline{r}, \overline{r}), \\ [x, \hat{s}), & \text{when } r = \overline{r}, \\ [0, x), & \text{when } r = \infty. \end{cases} \tag{15}$$

We take the convention that $r^A(x) = \overline{r}$ when $\hat{s}$ coincides with x.

Bank B's Strategy

For the non-specialized lender B a standard winner's curse ensues because the outcome of competition against the specialized Bank A is informative about θ_s. More specifically, besides the possibility of the competitor's unfavorable hard information faced by Bank A, the non-specialized lender B who wins the price competition also infers that $r^A(s) > r^B$, which implies $s < s^A\left(r^B\right)$. Taking these unfavorable inferences into account, Bank B's lending profits when quoting r are

$$\pi^B(r) \equiv \int_0^{s^A(r)} \underbrace{p_{HH}(t)}_{h^A = h^B = H, t} \left[\mu_{HH}(t)(r+1) - 1\right] dt + \underbrace{\overline{p}_{LH}}_{h^A = L, h^B = H} \left[\overline{\mu}_{LH}(r+1) - 1\right]. \tag{16}$$

The first term in Eq. (16) accounts for the event in which Bank A competes and the second term considers the case in which Bank A receives a low hard signal and does not participate. Note that Bank B infers the project's quality based on the event of "winning the borrower," since given an offer r Bank B wins the borrower only when Bank A receives an unfavorable soft signal realization $t < s^A(r)$. Hence, Bank B updates its posterior about θ in the event of winning the borrower. Importantly, since the span of hard and soft information can overlap, the updating is not only about the soft fundamental θ_s, as in Blickle, He, Huang, and Parlatore (2024), but also about the hard fundamental θ_h.

Therefore, Bank B's strategy $F^B(\cdot)$ maximizes its expected payoff

$$\max_{F^B(\cdot)} \int_{\mathcal{R}} \pi^B(r) \, dF^B(r). \tag{17}$$

With mixed strategies, profit-maximizing Bank B is indifferent between any action on its support.

3.2 Credit Market Equilibrium

We follow the same derivation as in Blickle, He, Huang, and Parlatore (2024) to derive the credit market equilibrium with specialized lending, which is characterized in the proposition below.

Proposition 1. ***(Credit Market Equilibrium)*** *In the credit market equilibrium, Bank A follows a pure strategy as in Definition 1. In this unique equilibrium, lenders reject borrowers upon a low hard signal realization $h^j = L$ for $j \in \{A, B\}$. Otherwise (i.e., when $h^j = H$), their strategies are characterized as follows:*

1. *Bank A with soft signal s offers*

$$r^A(s) = \begin{cases} \min\left\{ \frac{\pi^B + \int_0^s p_{HH}(t)dt + \overline{p}_{LH}}{\int_0^s p_{HH}(t)\cdot\mu_{HH}(t)dt + \overline{p}_{LH}\overline{\mu}_{LH}} - 1, \overline{r} \right\}, & \text{for} \quad s \in [x, 1] \\ \infty, & \text{for} \quad s \in [0, x). \end{cases} \tag{18}$$

The equation pins down $\underline{r} = r^A(1)$, For $s \in (\hat{s}, 1]$ where $\hat{s} = \sup s^A(\overline{r})$, $r^A(\cdot)$ is strictly decreasing with its inverse function $s^A(\cdot) = r^{A(-1)}(\cdot)$.

2. *Bank B makes an offer with cumulative probability given by ($\mathbf{1}_{\{X\}} = 1$ if X holds)*

$$F^B(r) = \begin{cases} 1 - \frac{\int_0^{s^A(r)} t\phi(t)dt}{q_s}, & \text{for} \quad r \in [\underline{r}, \overline{r}), \\ 1 - \mathbf{1}_{\{\pi^B=0\}} \cdot \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s}, & \text{for} \quad r = \overline{r}. \end{cases} \tag{19}$$

When $\pi^B = 0$, $F^B(\overline{r}) = F^B(\overline{r}^-) \leq 1$ is the probability that Bank B makes the offer (and with probability $\frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt$ it withdraws by quoting $r^B = \infty$); when $\pi^B > 0$, $F^B(\overline{r}) = 1$ and there is a point mass of $\frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt$ at $\overline{r}$.

3. *The equilibrium Bank B's profit is given by*

$$\pi^B = \left[\hat{\pi}^B\left(\overline{r}; s^A(\overline{r}) = s_A^{be}\right)\right]^+, \tag{20}$$

where s_A^{be} satisfies $\hat{\pi}^A\left(\overline{r}, s_A^{be}; F^B(\overline{r}) = \int_{s_A^{be}}^1 \frac{s\phi(s)dt}{q_s} ds\right) = 0$ with auxiliary functions $\hat{\pi}^B(\cdot;\cdot)$ and $\hat{\pi}^A(\cdot,\cdot;\cdot)$ defined in Appendix.

Similar to Milgrom and Weber (1982), it is relatively easy to solve for Bank A's equilibrium strategy by invoking Bank B's indifference condition that it has to make the same profit π_B across all rates on the support $[\underline{r}, \overline{r}]$. Plugging in $r = r^A(s)$ in (16) we have,

$$\pi^B(r) = \underbrace{\left[\int_0^s p_{HH}(t)\mu_{HH}(t) + \overline{p}_{LH}(t)\overline{\mu}_{LH}\right]}_{\text{borrowers who repay}} \left(1 + r^A(s)\right) - \underbrace{\left(\int_0^s p_{HH}(t)dt + \overline{p}_{LH}\right)}_{\text{lending amount}}. \tag{21}$$

Solving for $r^A(s)$ yields (18) in Proposition 1 which further takes into account of necessary truncation on interest rate cap $\overline{r}$. Although the derivation of Bank B's equilibrium strategy is more involved, conceptually it is quite simple: B's equilibrium strategy needs to support $r^A(\cdot)$ in (18) to be Bank A's optimal strategy. Specifically, as shown below, (22) gives Bank A's first-order condition (FOC) that balances the lower probability of winning against a higher payoff from served borrowers, and this holds for $s^A(r)$ so that r is optimal at this signal:

$$F^{B\prime}(r)\underbrace{p_{HH}(s^A(r))\left[\mu_{HH}(s^A(r))(1+r)-1\right]}_{A\text{'s marginal borrowers}}=\underbrace{\left[1-F^B(r)\right]p_{HH}(s^A(r))\mu_{HH}(s^A(r))+p_{HL}(s^A(r))\mu_{HL}(s^A(r))}_{A\text{'s existing borrowers}}. \tag{22}$$

On the other hand, to maximize (16), Bank B' FOC is

$$\left[-s^{A\prime}(r)\right]\cdot\underbrace{p_{HH}\left(s^A(r)\right)\left[\mu_{HH}\left(s^A(r)\right)(1+r)-1\right]}_{B\text{'s marginal borrowers}}=\underbrace{\int_0^{s^A(r)}p_{HH}(t)\,\mu_{HH}(t)\,dt+\overline{p}_{LH}\overline{\mu}_{LH}}_{B\text{'s existing borrowers}}. \tag{23}$$

Similarly, when Bank B marginally cuts its quote, it gets $(-s^{A\prime}(r))dr$ additional borrowers of quality $\mu_{HH}(s^A(r))$ given competition (which occurs with probability $p_{HH}(s^A(r))$), and this is exactly offset by the marginal lower payoff from Bank B's existing borrowers.

Two key further steps allow us to derive $F^B(r)$ based on (22)-(23). First, note that both lenders are competing on the same marginal borrower (type), i.e., $p_{HH}(s^A(r))\left[\mu_{HH}\left(s^A(r)\right)(1+r)-1\right]$; so we can cancel this term. Second, conditional independence when success, i.e., all signals are independent conditional on project success $\theta=1$,[17]implies that

$$p_{HL}\left(s^A(r)\right)\mu_{HL}\left(s^A(r)\right)=\frac{1-\alpha}{\alpha}p_{HH}\left(s^A(r)\right)\mu_{HH}\left(s^A(r)\right), \tag{24}$$

so the second term on right hand side of (22) only depends on the event of $\left\{h^A=h^B=H, s=s^A(r)\right\}$. Applying these two steps, we obtain

$$F^{B\prime}(r)\left[\int_0^{s^A(r)}p_{HH}(t)\,\mu_{HH}(t)\,dt+\overline{p}_{LH}\overline{\mu}_{LH}\right]=-s^{A\prime}(r)\left[\frac{1}{\alpha}-F^B(r)\right]p_{HH}\left(s^A(r)\right)\mu_{HH}\left(s^A(r)\right),$$

[17] Formally, because of the multiplicative structure in (2), we have

$$\mathbb{P}\left(h^A,h^B,s\in ds\middle|\,\theta=1\right)=\mathbb{P}\left(h^A,h^B,s\in ds\middle|\,\theta_h=\theta_s=1\right)=\mathbb{P}\left(h^A\middle|\,\theta_h=1\right)\cdot\mathbb{P}\left(h^B\middle|\,\theta_h=1\right)\cdot\mathbb{P}\left(s\in ds\middle|\,\theta_s=1\right).$$

Relating to (13), it implies that $\frac{p_{HL}(s)\mu_{HL}(s)}{p_{HH}(s)\mu_{HH}(s)}=\frac{\mathbb{P}(h^B=L|\theta=1)}{\mathbb{P}(h^B=H|\theta=1)}=\frac{1-\alpha}{\alpha}$, i.e., (24). Intuitively, conditional on success ($\theta=1$), for Bank A seeing signal s does not affect the likelihood ratio of its opponent to receive H or L hard signals. This is not true conditional on project failure.

which can be simplified further to

$$\frac{d}{dr}\overbrace{\left[\frac{\frac{1}{\alpha}-F^B(r)}{\int_0^{s^A(r)} p_{HH}(t)\,\mu_{HH}(t)\,dt+\overline{p}_{LH}\overline{\mu}_{LH}}\right]}^{\text{equals a constant}}=0. \tag{25}$$

It is then clear that the term inside the bracket of (25) has to be independent of r, which allows us to derive Bank B's equilibrium strategy (19) in Proposition 1 after imposing proper boundary conditions.

Based on the intuition that two asymmetrically informed lenders are competing on the same marginal borrower and conditional independence when success (which are the two key steps mentioned above), Blickle, He, Huang, and Parlatore (2024) offer a detailed explanation on why this key ODE (25) holds in equilibrium.

Finally, the equilibrium characterization point 3) in Proposition 1 highlights a key difference between the two types of equilibrium: one with $\pi^B = 0$—we call it the zero-weak equilibrium—and the other with $\pi^B > 0$ so that only Bank B places a positive mass on the interest rate cap $\overline{r}$—we call it the positive-weak equilibrium as the weak bank earns positive profits. In the zero-(positive-)weak equilibrium, only Bank A (Bank B) places a positive mass on the interest rate cap $\overline{r}$. This captures the competition at the interest rate cap $\overline{r}$, exactly reflecting point c) in Lemma 1—otherwise, lenders will undercut each other at this point. We will soon show that, as information span η increases (due to the hardening of soft information), the non-specialized lender benefits more and a positive-weak equilibrium is more likely to arise.

4 Credit Market Competition Equilibrium

Our model is designed to understand how changes in the span of information affect the equilibrium in the credit market, highlighting the differences between hardening soft information and shifts in other characteristics of information technology, such as the precision of signals. In this section, we first explore how an increase in the span of the hard signals changes the inference problem of Bank B about its opponent's information set and, through it, the equilibrium in the credit market. We then contrast how bank profits respond to an increase in the span of hard information and to an increase in its precision. Finally, we focus on the implications an increase in the span of the hard signal has on the allocation of credit and welfare.

4.1 Information Span and Equilibrium Illustration

The lenders' beliefs about their opponent's information are key determinants of the equilibrium. In this section, we show how the span of hard information affects these beliefs, which then helps us understand the effects of hardening soft information on the credit market equilibrium.

Span of Hard Information

The information span η determines the extent of overlap between the hard and soft fundamentals, which determines the correlation between hard and soft signals. When $\eta = 0$, there is no overlap between hard and soft fundamentals and the hard and soft signals are independent. When $\eta > 0$, the hard and soft fundamentals (and the signals about them) are correlated. We illustrate the effect of changes in the span of hard information on each lender's beliefs by focusing on two particular events. This analysis is the foundation of our discussion of the effects of "hardening soft information" on the credit market equilibrium.

Two positive hard signals. We start with the event $\{H, H, s\}$, where two lenders receive positive hard signals, (potentially) competing against each other, and Bank A receives a soft signal s. The probability of this event is

$$p_{HH}(s) = q\alpha^2\phi_1(s) + \left(1 - q_h^h\right)(1-\alpha)^2\phi(s) + \underbrace{q_h^h}_{\theta_h^h=1}\left[\underbrace{\left(1 - q_s^h\right)}_{\theta_s^h=0}(1-\alpha)^2 + \underbrace{\left(q_s^h - q_s\right)}_{\theta_s^h=1,\theta_s^s=0}\alpha^2\right]\phi_0(s) \tag{26}$$

The first term captures the probability of the event $\{H, H, s\}$ when the project is good ($\theta = 1$ which occurs with probability q). This event is independent of the information span η as hard and soft signals are conditionally independent when the project is good (see footnote 17). The remaining two terms refer to the cases in which the project is bad ($\theta = 0$), which can occur when one of the states θ_h^h, θ_s^h, or θ_s^s takes a value of zero.

The second term captures the events with $\theta_h^h = 0$. This term is independent of the span η. In this case, $\theta_h = 0$ irrespective of θ_s^h. Note that while the likelihood of HH when $\theta = 1$ or $\theta_h^h = 0$ (i.e., the first or the second term in (26)) is independent of η, both terms depend on the precision of the hard signal because α determines the conditional probability of receiving hard signal H.

The third term in Eq. (26) captures the novelty of our modeling, i.e., how the hardening of soft fundamental affects the likelihood of two banks competing in the credit market. This term includes two scenarios: i) when $\theta_s^h = 0$, both the hard fundamental and the soft fundamental fail $\theta_h = \theta_s = 0$, so HH occurs with probability $(1-\alpha)^2$ and the soft signal density is $\phi_0(s)$; and ii) when $\theta_s^h = 1$ but $\theta_s^s = 0$, the hard fundamental succeeds $\theta^h = 1$ (therefore HH occurs with probability α^2) but the soft fundamental fails (the soft signal density is $\phi_0(s)$).

Using $\eta = 1 - q_s^h$ and simplifying the terms on α, we can rewrite the joint probability of $\{H, H, s\}$

as follows.

$$p_{HH}(s) = q\alpha^2\phi_1(s) + \left(1 - q_h^h\right)(1-\alpha)^2\phi(s) + \underbrace{\left[(1-q_s)\alpha^2 - \eta(2\alpha - 1)\right]}_{\downarrow \text{ in } \eta \text{ as } \alpha > \frac{1}{2}} q_h^h \phi_0(s). \tag{27}$$

The key message in Eq. (27) is captured by the last term: a broader information span reduces the probability of the competition scenario HH when soft fundamentals are unsuccessful $\theta_s = 0$. Before soft information becomes hardened, i.e., when $\eta = 0$, the state θ_s is discernible only through the soft signal s. Technological advancements that harden soft information, i.e., increases in η, allow lenders to learn about θ_s from hard signals. When competing in the credit market, an increase in η affects how lenders update their beliefs, especially for the non-specialized lender B who does not observe a direct signal of s but understands that competition occurs in the event of HH. As the overlapping state θ_s^h generates a positive correlation between soft and hard signals, two positive hard signal realizations (the event of competition under HH) lead Bank B to update its beliefs about the opponent lender's soft signal distribution upward when soft information is hardened.

To further illustrate this point, we compute $\phi(s\,|HH)$, i.e., the conditional density of s given HH, i.e., the event where lenders compete. To make the point clearer, we set $q_h^h = 1$ so that $q = q_s$ and hard signals only reflect the overlapping state θ_s^h, and the resulting conditional density of the soft signal s is

$$\begin{aligned} \phi(s\,|HH) &= \frac{q_s\alpha^2\phi_1(s) + \left[\left(1-q_s^h\right)(1-\alpha)^2 + \left(q_s^h - q_s\right)\alpha^2\right]\phi_0(s)}{(1-q_s^h)(1-\alpha)^2 + q_s^h\alpha^2} \\ &= \phi_0(s) + \overbrace{\frac{\alpha^2}{\alpha^2 - (2\alpha-1)\eta}}^{\uparrow \text{ in } \eta \text{ as } \alpha > \frac{1}{2}} \cdot q_s\left[\phi_1(s) - \phi_0(s)\right]. \end{aligned} \tag{28}$$

Without hardening soft information ($\eta \to 0$) as in Blickle, He, Huang, and Parlatore (2024), independent hard and soft signals imply that

$$\phi(s\,|HH) = (1-q_s)\phi_0(s) + q_s\phi_1(s) = \phi(s). \tag{29}$$

When hard information becomes broader, this conditional density puts more weight on the favorable distribution $\phi_1(s)$, as suggested by the greater coefficient $\frac{\alpha^2}{\alpha^2-(2\alpha-1)\eta}$. Put differently, given the monotone likelihood ratio property, we know $\phi_1(s) - \phi_0(s) > 0$ for relatively high soft signals. Hence, the conditional density $\phi(s\,|HH)$ increases with η for high values of s implying that a favorable soft signal is more likely to arise (upon HH) when the span of hard information is broader. In contrast, for relatively low soft signals with $\phi_1(s) - \phi_0(s) < 0$, the opposite occurs so the conditional density $\phi(s|HH)$ decreases with η. Finally, one can show that the coefficient

$\frac{\alpha^2}{\alpha^2-(2\alpha-1)\eta}$ in (28) increases with α, i.e., the updating towards $\phi_1(s)$ is stronger when HH is more precise about underlying states.

This effect on the conditional distribution on Bank A's soft signal implies that a larger information span also reduces the winner's curse for Bank B from Bank A's private soft signal. In turn, this implies that the non-specialized bank benefits more from technological advancements that harden soft information. To see this, note that, for Bank B, winning the bids $r^B < r^A$ upon competing (HH) indicates that the opponent's soft signal assessment is unfavorable, i.e., $s < s^A\left(r^B\right)$ as $s^A(\cdot)$ is decreasing. However, a larger η leads to an upward update of the opponent's signal s given the event of positive hard signals $\{HH\}$, which attenuates this winner's curse.

Opposite hard signals. When studying the equilibrium of credit market competition, it is also important to understand the events where two lenders receive opposite hard signals—more specifically when one bank's hard signal is positive while the opponent lender's signal is negative. As illustrated in the lenders' profit functions π^A in (14) and π^B in (16), these events represent the critical economic force behind the "winner's curse" in models with hard signals only (Broecker, 1990; He, Huang, and Zhou, 2023).[18] Going through steps similar to (26), one can calculate the probabilities for these events as

$$p_{HL}(s) = p_{LH}(s) = \alpha(1-\alpha)\phi(s). \tag{30}$$

Interestingly, this probability is only affected by the precision of the hard signal α, but not by the span of hard information η. The effect of α is natural: when α increases, hard signals become more precise, h^A and h^B become more correlated, and opposite hard signals $h^A \neq h^B$ are less likely to arise.

The observation that the information span η does not enter (30) relies on the symmetry of the hard information technology (i.e., same Type I and II errors). Intuitively, this symmetry implies that independent of the realization of θ_s^h, the probability of $h^A \neq h^B$ is always $\alpha(1-\alpha)$. Because no information about the fundamental is revealed from the disagreement events HL or LH, the distribution of the soft signal conditional on opposite hard signals remains the unconditional one:

$$\phi(s\,|HL) = \phi(s\,|LH) = \phi(s). \tag{31}$$

As discussed below, this property facilitates our later analytical proof and helps us highlight the economic mechanism behind our results more clearly.

[18]In our model, as in Blickle, He, Huang, and Parlatore (2024), the non-specialized lender B faces a winner's curse even in the event of HH because of the soft signal received by the specialized lender only.

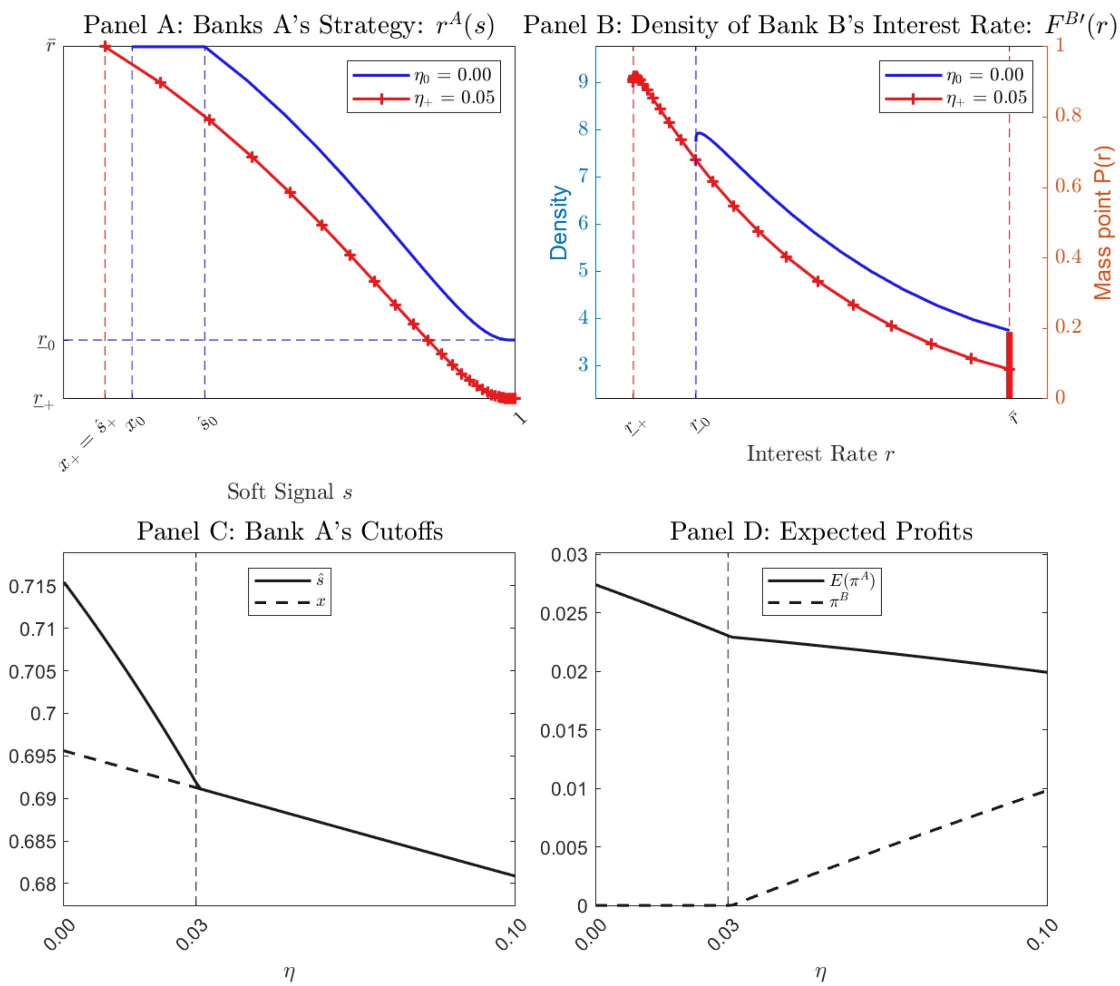

Figure 2: **Equilibrium strategies and profits for information span η.** Panel A depicts $r^A(s)$ as a function of s and Panel B plots $F^{B\prime}(r)$ as a function of r; strategies for $\eta_+ = 0.05$ are depicted in red with markers while strategies with $\eta_0 = 0$ are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders, both as a function of η. Parameters: $\bar{r} = 0.36$, $q_h = 0.8$, $q_s = 0.9$, $\alpha = 0.7$, and $\tau = 1$.

Credit Market Equilibrium and Information Span

Figure 2 illustrates how the credit market equilibrium responds to changes in the span of hard information η. For ease of exposition, we assume that Bank A's soft signal s is obtained from observing a noisy version of θ_s, i.e., $\theta_s + \epsilon$, so that

$$s = \mathbb{E}\left[\theta_s | \theta_s + \epsilon\right]. \tag{32}$$

Here, $\epsilon \sim \mathcal{N}(0, 1/\tau)$ indicates white noise, with the precision parameter τ capturing the signal-to-noise ratio of Bank A's soft information technology.

The top two panels in Figure 2 plot both lenders' pricing strategies conditional on making an offer, with Panel A plotting Bank A's $r^A(s)$ as a function of s and Panel B the density $dF^{B\prime}$ for Bank B. We have plotted the equilibrium pricing strategies for two levels of information span η: the baseline $\eta_0 = 0$, and a higher $\eta_+ = 0.05$. Overall, with a greater η, Bank B becomes more aggressive as its distribution of offered rate shifts downward (Panel B), with a lower equilibrium lower bound $\underline{r}_+ < \underline{r}_0$. This is consistent with the premise that hardening soft information levels the playing field of the non-specialized lender in our model.

As we have explained in Section 3.1, in equilibrium $r^A(s)$ decreases in s—that is to say, when the specialized Bank A receives a more favorable soft signal about credit quality, it bids more aggressively with a lower rate to win the borrower over its opponent. In Panel A we observe that the entire curve $r^A(s)$ shifts downward in response to the more aggressive bidding by Bank B.

Panel C plots the two soft signal cut-offs for the specialized Bank A, i.e., $\hat{s}$ at which it starts quoting $\overline{r}$ and x at which it starts rejecting the borrower. For sufficiently large η, $\hat{s}$ and x coincide reflecting a zero probability mass on the interest rate cap $\overline{r}$.

Finally, Panel D plots the expected profits—$\mathbb{E}(\pi^A)$ and π^B—for two lenders; when η goes up, the non-specialized lender becomes relatively stronger, leading to a strictly positive π^B as shown in Panel D. In other words, we have a positive-(zero-) weak equilibrium when η is relatively high (low); and this is why we put subscript "+" for the larger η in Panel A-B.

To piece all panels together, consider the competition at interest rate $\overline{r}$. As shown in Panel A-B, for a low information span $\eta_0 = 0$ so that $\pi^B = 0$ in equilibrium, Bank A has a point mass at $\overline{r}$ (corresponding to $s \in (x, \hat{s})$ as in Panel C) but Bank B does not, while for a high $\eta_+ = 0.05$ so that $\pi^B > 0$ then the opposite holds. The underlying economics is rather straightforward. Thanks to the big data technology that hardens soft information, a sufficiently large η leads to a positive-weak equilibrium where the non-specialized Bank B places a point mass on $\overline{r}$, enjoying some "local monopoly power" as it is the only lender when Bank A rejects the borrower upon $s < \hat{s} = x$. Importantly, this is still profitable for Bank B: for a sufficiently large η, the non-specialized Bank B faces a relatively minor winner's curse due to the opponent's soft signals (see earlier discussion in this section). In contrast, for a smaller span η, we are in a zero-weak equilibrium, where the specialized Bank A places a point mass on this interest rate (when $s \in (x, \hat{s})$, as shown in Panel C) while the non-specialized Bank B withdraws.

Last but not least, it is important to recognize that Bank A's profits can also increase with the information span η in the parameter range of positive-weak equilibrium. Figure 3 shows such an example in which case Bank A's expected profits increase with η in Panel B and contrasts it with the case in which the opposite holds shown in Panel A (which is just Panel D in Figure 2). This example highlights that the way in which we model hardening soft information implies the same technological improvement for the specialized and non specialized banks. Comparing the parameters that lead to these two cases, that $\mathbb{E}(\pi^{\mathbb{A}})$ increases with η is more likely to arise when

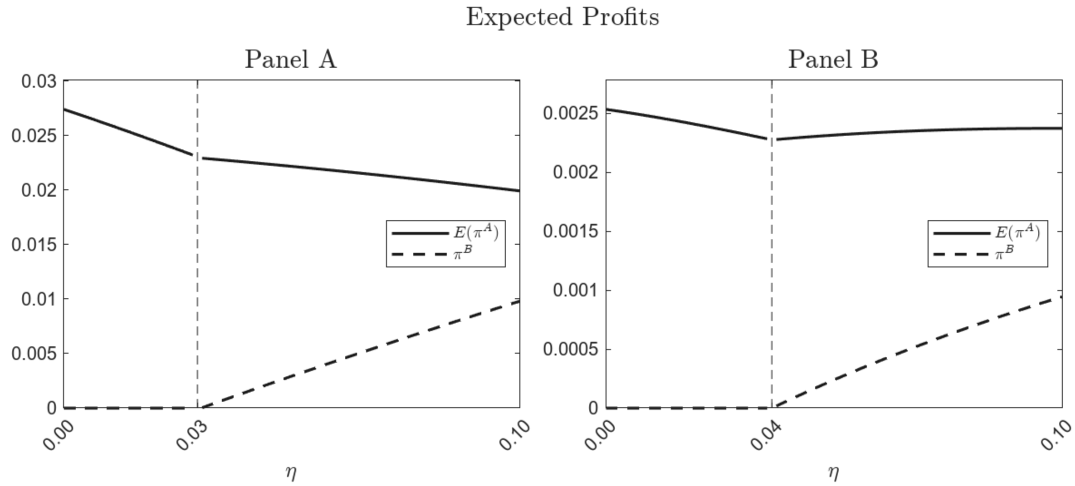

Figure 3: **Expected lender profits.** Panel A and Panel B show expected lender profits as a function of the span of hard information η under different primitive settings. The solid lines correspond to Bank A while the dashed lines correspond to Bank B. Parameters: Panel A, $\bar{r} = 0.36$, $q_h = 0.8$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.7$, $\tau = 1$; Panel B, $\bar{r} = 0.33$, $q_h = 0.8$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.6$, $\tau = 0.1$.

signal precisions are low. When the precision of the hard signal α is relatively low, the credit market is less competitive as lenders bid less aggressively due to the high uncertainty in screening. When the precision of the soft signal τ is relatively low, Bank A, who initially has an imprecise soft signal about the soft fundamental θ_s, benefits more as hardening soft information also helps in learning about θ_s. Hence, Bank A's profits increase as the technology improvement dominates the intensified competition from Bank B. In later welfare analysis in Section 4.3, we will show shortly that in this case likely every agent in the entire sector enjoys a higher surplus (hence a Pareto improvement).

Remark. ***(Hardening soft information)*** *Throughout the paper, we use the hardening of soft information as an example of technological change that can increase the span of hard information. We do this for two reasons. First, to fix ideas and provide a concrete setting in which our model applies. Second, because of the practical relevance of the example in the current "Big Data" environment. However, our results are broader and apply to any circumstance in which access to information is democratized and characteristics previously accessible only to a monopolist are now "learnable" by all market participants.*

4.2 Bank Profits: Information Span vs. Information Precision

A key advantage of our model is that it allows us to distinguish between different aspects of information technology. In this subsection, we compare the effect of changes in the information span of hard signals on bank profits to that of changes in the precision of hard signals.

Information Span and Bank Profits

We start by formally showing that an enlarged information span—i.e., a greater η—levels the playing field by benefiting the non-specialized Bank B relatively more than the specialized Bank A. The following proposition states our result.

Proposition 2. *(Hardening soft information on equilibrium profits)*

1. *The equilibrium profits of the non-specialized lender π^B are increasing in η. This implies that there exists a cutoff $\hat{\eta}$ so that when $\eta > \hat{\eta}$ the credit market features a positive-weak equilibrium with $\pi^B > 0$.*

2. *In the region of positive-weak equilibrium, the impact of η on Bank B's profits dominates that on Bank A's profits:*

$$\frac{d\pi^B}{d\eta} > \frac{d}{d\eta}\mathbb{E}\left[\pi^A\right]. \tag{33}$$

We start by explaining the underlying mechanism of point 2) in Proposition 2, which reveals interesting economics. The two lenders' equilibrium profits can be decomposed in two terms, depending on whether banks are competing for the lender, as follows.

$$\mathbb{E}[\pi^A] = \int_0^1 \pi^A\left(r^A(s), s\right) ds = \underbrace{\int_0^{\hat{s}} \left[\pi^A(\overline{r}, s)\right]^+ ds}_{\text{non-competing case}} + \underbrace{\int_{\hat{s}}^1 \pi^A\left(r^A(s), s\right) ds}_{\text{compete against Bank } B}, \tag{34}$$

$$\pi^B = \int_{\underline{r}}^{\overline{r}} \pi^B dF^B(r) = \underbrace{\pi^B \cdot \left[1 - F^B(\overline{r}^-)\right]}_{\text{non-competing case}} + \underbrace{\int_{\underline{r}}^{\overline{r}} \underbrace{\pi^B(r)}_{\text{constant } \pi^B} \frac{s^A(r)\,\phi\left(s^A(r)\right)}{q_s}\left(-s^{A\prime}(r)\right) dr}_{\text{compete against Bank } A}. \tag{35}$$

Note, (34) takes $\pi^A\left(r^A(s), s\right)$ in (14) as given which includes the density $\phi(s)$ already, and (35) uses the expression of equilibrium $F^B(r)$ in (19). The second term in both equations represents profits when lenders engage in direct competition by offering interest rates $r \in [\underline{r}, \overline{r})$. Since $ds = \left[-s^{A\prime}(r)\right] dr$, we should compare the integrand $\pi^B\left(r^A(s)\right)\frac{s\phi(s)}{q_s}$ against $\pi^A\left(r^A(s), s\right)$, where the adjustment of $\frac{s\phi(s)}{q_s}$ for π^B reflects Bank B's equilibrium probability density (i.e., $F^{B\prime}(r) = \frac{s\phi(s)}{q_s}$).

The above discussion motivates us to study the following object which is the difference of integrands in (34)-(35):

$$\Delta\pi(s;\eta) \equiv \pi^B\left(r^A(s)\right)\frac{s\phi(s)}{q_s} - \pi^A\left(r^A(s), s\right). \tag{36}$$

We aim to show that for every s, we have $\frac{d\Delta\pi(s;\eta)}{d\eta} > 0$, i.e., the impact of η on density-adjusted π^B always dominates that of π^A. Suppose that this holds; then because in a positive-weak equilibrium the first non-competing term in (34) is zero for Bank A, it follows that $0 = d\pi^B/d\eta > d\mathbb{E}\left[\tilde{\pi}^A\right]/d\eta$, which is our desired claim.[19] Note, as we can show that the dominance holds point-wisely, it is stronger than the statement on expectation in point 2) in Proposition 2.

The key observation is that, in equilibrium, both lenders make the same revenue but face different costs (i.e., the probability of lending). Specifically, we have

$$\pi^A\left(r^A(s), s\right) = \underbrace{\left\{\left[1 - F^B\left(r^A(s)\right)\right] p_{HH}(s)\,\mu_{HH}(s) + p_{HL}(s)\,\mu_{HL}(s)\right\}}_{\text{borrowers who repay}}\left(1 + r^A(s)\right) - \underbrace{\left\{p_{HH}(s)\left[1 - F^B\left(r^A(s)\right)\right] + p_{HL}(s)\right\}}_{\text{lending amount}}, \tag{37}$$

$$\pi^B\left(r^A(s)\right)\frac{s\phi(s)}{q_s} = \frac{s\phi(s)}{q_s}\underbrace{\left[\int_0^s p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]}_{\text{borrowers who repay}}\left(1 + r^A(s)\right) - \frac{s\phi(s)}{q_s}\underbrace{\left[\int_0^s p_{HH}(t)\,dt + \overline{p}_{LH}\right]}_{\text{lending amount}}. \tag{38}$$

In the proof of Proposition 2, we show that the first terms in both Eqs. (37) and (38) are equal given the equilibrium strategy $F^B(\cdot)$ in (19) and joint probability for a good borrower in (13). Therefore the profit differential between Bank A and Bank B, which can proxy for the competitiveness of the credit market, is given by

$$\Delta\pi(s;\eta) = \frac{1}{q_s}\left[s\phi(s)\int_0^s p_{HH}(t)\,dt - p_{HH}(s)\int_0^s t\phi(t)\,dt\right] + \left[\frac{s\phi(s)}{q_s}\overline{p}_{LH} - p_{HL}(s)\right]. \tag{39}$$

As can be seen from Eq. (39), the degree of competition depends on the span of hard information η only through the banks' lending probability. As explained in Eq. (30) in Section 4.1, η does not affect the probability of lenders receiving opposite hard signals, captured by the second term in parentheses in Eq. (39). Hence, the effect of η is captured by the change in the probability of HH where both lenders receive positive hard signals and (potentially) compete for the borrower.

In addition, the information span η does not affect the probability of making loans to a good borrower. To see this, recall that conditional on a good borrower, the probability of receiving a favorable hard signal H (screening) is independent of the span of characteristics assessed but determined by the precision. Competition is not affected either, as hard and soft signals are conditionally independent given the good type (see footnote 17), and the equilibrium Bank B's strategy $F^B(\cdot)$ in Eq. (19) does not rely on η. Therefore, η does not affect the type I error but affects the competitiveness of the credit market through the type II error.

More specifically, broader hard information (higher η) reduces Type II errors—i.e., making loans to a bad borrower—when both banks compete (HH). As η increases, hard signals assess

[19] The total effect of $d\pi^B/d\eta$ should also take into account the first non-competing term for Bank B; but point 1) in Proposition 2 shows that this term is positive.

more characteristics for both lenders and the event HH indicates that the overall borrower quality is more likely to be good, thereby reducing Type II errors ($Pr(\theta = 1|\theta_h = 1)$ is higher). This improvement is more pronounced for the non-specialized lender, Bank B, which is initially subject to more serious Type II errors. In the case of competition (HH), Bank B suffers from the winner's curse due to its opponent's soft signal, i.e., winning indicates that Bank A's soft signal is low $t \in (0, s)$. However, as the information span increases, the competition event HH indicates that the opponent's soft signal is more likely to be associated with good fundamentals, which attenuates Bank B's concern about the winner's curse. In sum, hardening soft information helps Bank B avoid lending to lemons more than it helps Bank A, which has information about these states through its soft signal.

Hence, the span of hard information η affects the level of lender profits as summarized in point 1) in Proposition 2. When the information span is limited ($\eta < \hat{\eta}$), Bank A maintains a substantial information advantage and enjoys local monopoly power (bidding $\overline{r}$ when $s \in (x, \hat{s})$), which enables it to compete aggressively even though broader hard information benefits the opponent more. In this range, Bank B's equilibrium profits stay at zero as heightened competition exactly offsets the gains from technology. Once η rises above the threshold $\hat{\eta}$, Bank A's information advantage shrinks to the extent that it loses the local monopoly power and becomes the break-even lender when receiving $\hat{s}(\eta)$. In this case, Bank A competes less aggressively in response to the technological advancement and so Bank B starts to make positive profits.

Information Precision and Bank Profits

The economic implications of changes in information precision in our model are drastically different from those coming from changes in the span of hard information. Two parameters capture the information precision, one being the hard signal precision α and the other the soft signal precision τ. It is quite transparent that an increase in the soft signal precision gives a greater advantage to the specialized lender, opposite to the effect of a greater η which levels the playing field for the non-specialized lender.

The effect of a hard signal precision α is a bit more involved and in general non-monotone. To understand the non-monotonicity, it is useful to consider two extreme cases. In a general auction setting with asymmetric bidders, the uninformed bidder makes zero profit as shown in Milgrom and Weber (1982). When $\alpha = 0.5$ so that the hard signal is completely uninformative,[20] the model is identical to Milgrom and Weber (1982) where the uninformed lender B ignores the realization of h^B, randomizes its bids, and makes zero profit in equilibrium. On the other extreme when $\alpha = 1$, hard information becomes a public signal and we are back to Milgrom and Weber (1982) again upon the realization $h^A = h^B = H$ and updated prior, and Bank B still makes zero profits in this

[20] Although this limiting case violates Assumption 1 which requires hard signals to be sufficiently strong, we have a well-defined equilibrium in this case where both lenders ignore the hard signals.

limiting case. In general, for intermediate values of $\alpha \in (0, 1)$, a positive-weak equilibrium (with $\pi^B > 0$) could arise.

This non-monotonicity with respect to α, compared to the monotonicity with respect to η shown in Proposition 2, implies that information precision is qualitatively different from information span. The following proposition provides a formal counterpart of Proposition 2 in the vicinity of $\alpha = 1$. We focus on this extreme case not only because it is more analytically tractable but more importantly because our analysis rests on the assumption of hard signals being decisive (Assumption 1).

Proposition 3. *(Hard information precision on bank profits) When the precision of hard information $\alpha \to 1$.*

1. *The equilibrium profit of non-specialized lender $\pi^B \to 0$, i.e., a zero-weak equilibrium arise.*
2. *Suppose that $q_h > 0.5$. In the vicinity of $\alpha \to 1$, the impact of α on Bank A's profit dominates that on Bank B's profit:*

$$\frac{d}{d\alpha}\mathbb{E}\left[\pi^A\right] > \frac{d\pi^B}{d\alpha} = 0. \tag{40}$$

In the above proposition, point 1) naturally follows from Milgrom and Weber (1982) given the discussion above. Point 2) makes a further theoretical point: in the vicinity of $\alpha = 1$, an increase in hard signal precision helps Bank A gain more profits. To see this, following the same calculation in (37)-(38), we reach the same profit wedge as in (39), where the first term captures the profit wedge for HH when lenders compete.[21] Suppose the hard fundamental prior $q_h > 0.5$ is relatively high, which is empirically relevant.[22] When hard signals become more precise so α increases, lenders are more likely to compete ($h^A = h^B = H$) than disagree and not compete ($h^A \neq h^B$). Since Bank A is endowed with an additional soft signal and hence more advantageous in the case of competition, its profits increase as α increases.

Figure 4 displays the same variables as Figure 2, plotting the comparative statics on the hard signal precision α. First, Panels A and B illustrate lenders' equilibrium pricing strategies, showing that lenders set more aggressive rates (lower rates) for $\alpha_+ > \alpha_0$. When α increases from $\alpha_+ = 0.8$ to $\alpha_0 = 0.9$, both lenders are competing more fiercely by quoting lower interest rates, so the equilibrium turns from a positive-weak one to a zero-weak one (this is why we call the larger α as α_0). However, as demonstrated in Panel D, the non-specialized lender B's profits π^B is non-monotone in α. This aligns with the discussion preceding Proposition 3, that $\pi^B = 0$ at the two limiting cases, $\alpha = \frac{1}{2}$ and $\alpha = 1$. In Panel C, the cutoff strategies of Bank A generally decrease as α increases; this reflects the standard learning effect—Bank A, receiving a more accurate positive

[21]Strictly speaking this term is for $s \geq \hat{s}$ only; but the the same logic applies to $s \in (\hat{s}, x)$ where Bank A quotes $\bar{r}$ always. Also, the second term on opposite hard signals matters; but in the proof of Proposition 3 we show that the effect on HH dominates.

[22]This parameter is empirically relevant because, in the data, the non-performing loan rate—which is about 5% as documented in Blickle, He, Huang, and Parlatore (2024)—is quite low.

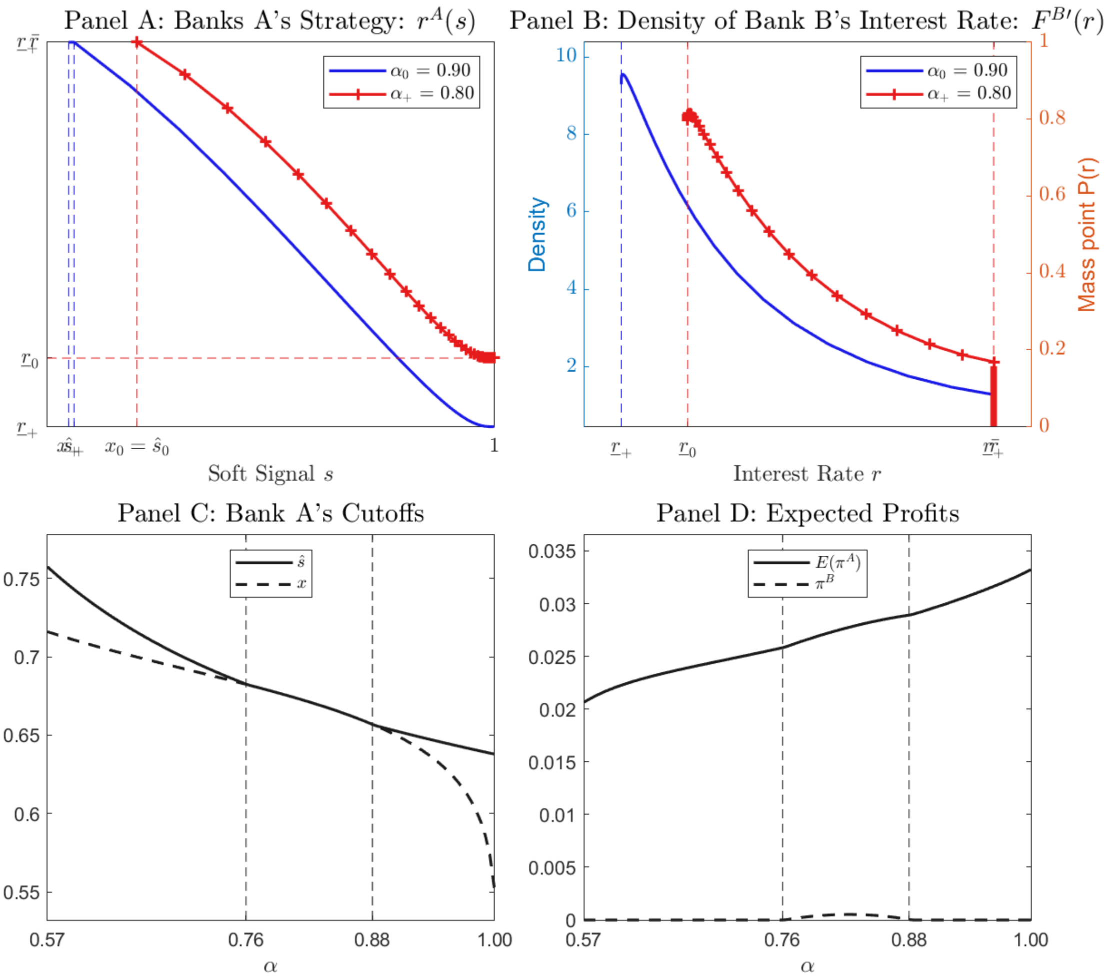

Figure 4: **Equilibrium strategies and profits for hard signal precision α.** Panel A depicts $r^A(s)$ as a function of s and Panel B plots $F^{B\prime}(r)$ as a function of r; strategies for $\alpha_+ = 0.8$ are depicted in red with markers while strategies with $\alpha_0 = 0.9$ are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders, both as a function of α. Parameters: $\bar{r} = 0.36$, $q_h = 0.8$, $q_s = 0.9$, $\eta = 0.02$, and $\tau = 1$.

signal, withdraws at a weaker soft signal. Notably, $\hat{s}$ and x coincide for mid-values of α, which is consistent with the non-monononicity of π^B.

4.3 Credit Allocation and Welfare

We now analyze the effect of the hardening of soft information on the allocation of credit and welfare through the lens of our model. We focus on three aggregate markers of credit market health: loan approval rate, non-performance rate, and probability of funding for high- and low-quality borrowers. We also investigate the expected NPV of a funded project as a measure of total welfare in the banking sector. Figure 5 shows these equilibrium outcomes as a function of the span of hard information η.

Two effects govern the comparative statics in Figure 5. On the one hand, an increase in η decreases the probability of getting a positive hard signal as there are more fundamental states covered by the hard signal and any one of them failing makes the loan quality low. On the other hand, the higher the span of hard information the higher the correlation between the hard signals and the soft signal received by the specialized bank, which tilts the distribution of the soft signal conditional on both banks competing, $\phi(s|HH)$, towards higher signals. This implies that as η increases, the non-specialized lender knows that when there is competition its specialized opponent is more likely with high soft signals, leading to an attenuated winner's curse for the non-specialized bank.

Panel A depicts the expected loan approval rates for two lenders. There, the change in the conditional distribution of the soft signal for Bank A dominates the decrease in the probability of getting a positive hard signal and hence, the expected loan approval of Bank A (solid line) increases in η. For Bank B, the effect of an increase in η on its approval rate (dashed line) depends on whether it makes zero or positive profits in equilibrium. In a zero-weak equilibrium, the reduction in the winner's curse for Bank B increases the likelihood of Bank B competing for the borrower after receiving a positive hard signal, pushing the approval rate upwards. In a positive-weak equilibrium, Bank B always participates and the effect of a lower winner's curse is dampened. In Panel A, the effect of η on the winner's curse for Bank B dominates for values of $\eta < 0.03$ (zero-weak) while the opposite holds for $\eta > 0.03$ (positive-weak). The jump in Bank B's loan approval rate when switching from a zero-weak to a positive-weak equilibrium mirrors the jump in Bank B's participation upon receiving $h^B = H$, which goes from being less than one when $\pi^B = 0$ to being one when $\pi^B > 0$.

Panel B shows the non-performing rates of loans made by Bank A (solid line) and Bank B (dashed line). As one may have expected, the non-performing rate for both banks decreases with the information span (within one equilibrium type). A higher information span improves the screening technology of the banks (reduces Type II errors) and increases the average quality of the loans in the banks' portfolios. The jump in the non-performing rate of Bank B follows from the jump in Bank B's participation upon receiving a positive hard signal when the equilibrium switches from zero-weak to positive-weak (and starts quoting $\bar{r}$ in point mass). Consequently, Bank B's incremental borrowers are of relatively low quality because it only wins competition when the opponent receives low soft signals $s < \hat{s}$.

Panel C plots the probability of good (solid line) and bad (dashed line) borrowers receiving funding in equilibrium. Without any strategic concerns, one would expect that a higher η improves lenders' screening technologies and so the probability of funding good loans rises while the probability of making a bad loan falls. This is indeed the case in Panel C when the equilibrium is in the positive-weak regime for $\eta > 0.03$. In a zero-weak equilibrium, there is an additional effect

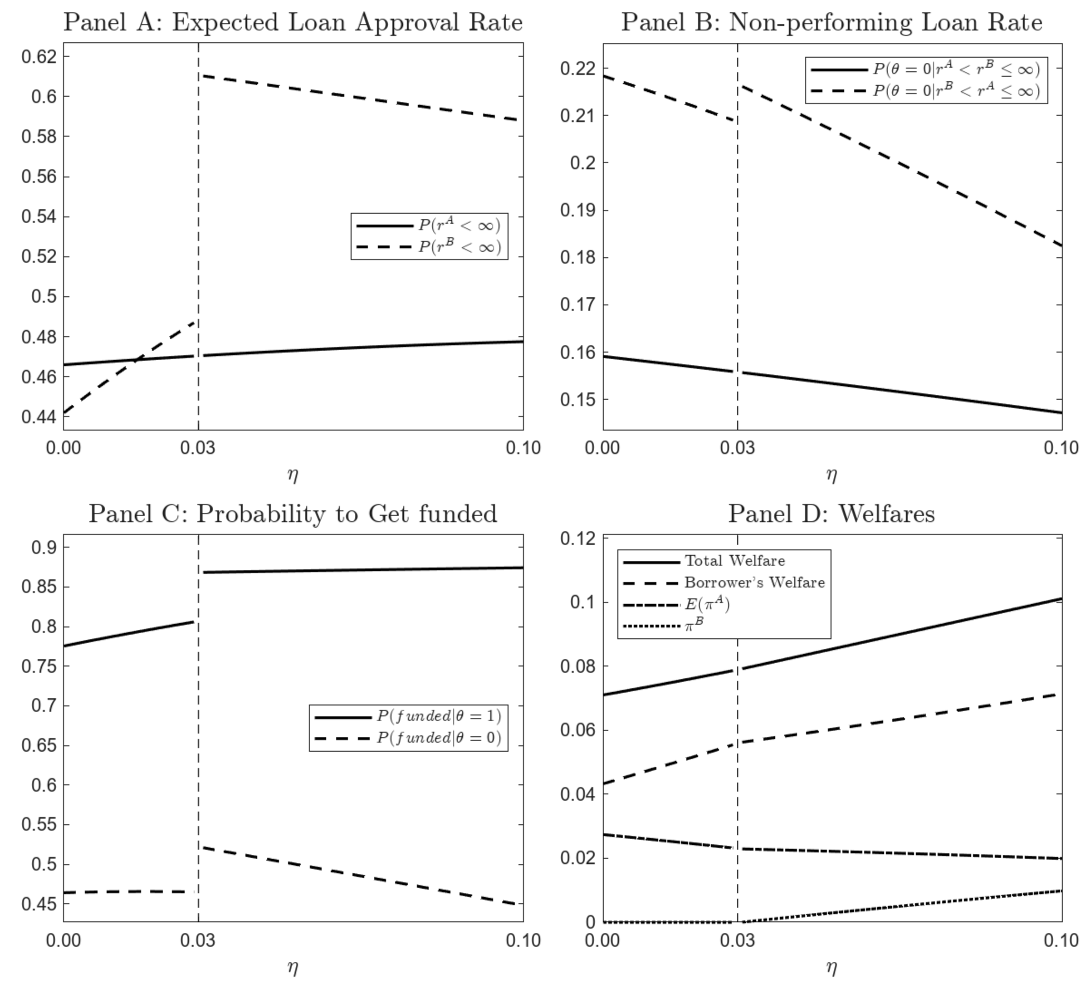

Figure 5: **Credit allocation and welfare.** Panel A and Panel B show the expected loan approval and non-performing rates, respectively. The solid lines correspond to Bank A while the dashed lines correspond to Bank B. Panel C depicts the probability of getting funded for a high-quality borrower (solid line) and a low-quality borrower (dashed line). Panel D illustrates aggregate welfare (solid line), borrower surplus (dashed line), and lender profits. All variables are depicted as a function of the span of hard information η Parameters: $\bar{r} = 0.36$, $q_h = 0.8$, $q_s = 0.9$, $\tau = 1$ (top two panels) and $\alpha_u = \alpha_d = \alpha = 0.7$ (bottom two panels).

that tends to make bad loans more likely to be funded as the span of hard information increases. As discussed above, a higher η attenuates the winner's curse, and Bank B's participation upon receiving a positive hard signal increases, leading to a greater probability of Bank B extending loans to good and bad borrowers. In the figure, this effect dominates and the probability of bad loans being funded increases with η for $\eta < 0.03$. The jumps in the figures follow from the jump in Bank B's participation as mentioned when discussing Panels A and B above.

Panel D shows aggregate welfare measured as the expected net present value (NPV) of a funded project, as well as the surplus to each agent in this sector. As the span of hard information increases and the banks' Type II errors decrease, the expected quality and the NPV of the loans made by banks go up. Note, welfare is continuous in η when we switch from a zero-weak to a positive-weak equilibrium; this is because, at the knife-edge parameter of equilibrium type switching, both Bank B and the borrower make zero profits.[23] Hence, despite a jump in quantity, these additional loans correspond to zero NPV projects on average, and therefore total welfare increases continuously as the span of information widens.

In general, borrowers benefit from the technological improvement of broader hard information. As lender screening becomes more efficient and competition intensifies, good-type borrowers are more likely to be funded (Panel C) and receive lower rates. (We normalize the surplus of bad-type borrowers to zero).[24] When $\eta < \hat{\eta} = 0.03$, the equilibrium is zero-weak and Bank A's expected profits decrease according to Proposition 2. In this case, the improvement in total welfare all accrues to borrowers, and there is additional transfer from banks to borrowers. When $\eta \geq 0.03$, the equilibrium is positive-weak and Bank B also enjoys a higher surplus from an increase in the information span η.

Finally, recall that in Panel D of Figure 5, all welfare goes up except the specialized Bank A in the range of positive-weak equilibrium. Is it possible that an increase in information span leads to a Pareto improvement for all agents in this sector? The answer is yes. As shown in Panel B of Figure 3 in Section 4.1, Bank A's profits could also increase in information span. Highlighting the feature that we directly model technology improvement, both the specialized and nonspecialized lenders enjoy the same technology improvement, especially when signal precisions before hardening soft information are low. As a result, broader hard information, which is an important form of information technology improvement in the recent decade, leads to a Pareto improvement of all sectors and everyone enjoys a higher surplus.

5 Model Extensions and Discussions

In this section, we consider several model extensions. First, the open banking initiative (He, Huang, and Zhou, 2023) implies that lenders' hard information signals are likely to become more and more correlated; our model can be easily adapted to incorporate this aspect of change in information technology. Second, so far we have adopted one particular hard information technology as illustrated in Figure 1. As robustness, we explicitly model the signal on hardened soft fundamental θ_s^h (potentially generated by Big Data technology) and show that both the equilibrium characterization

[23] The discrete jump of loans is made with a rate of $\bar{r}$, so that borrowers receive no surplus from these loans. Recall we rule out non-pledgeable income of borrowers; otherwise, there will be an upward jump in total welfare which includes the borrower's non-pledgeable income.

[24] See He, Huang, and Zhou (2023) for an analysis that includes the welfare of both good and bad types of borrowers in the context of open banking regulation.

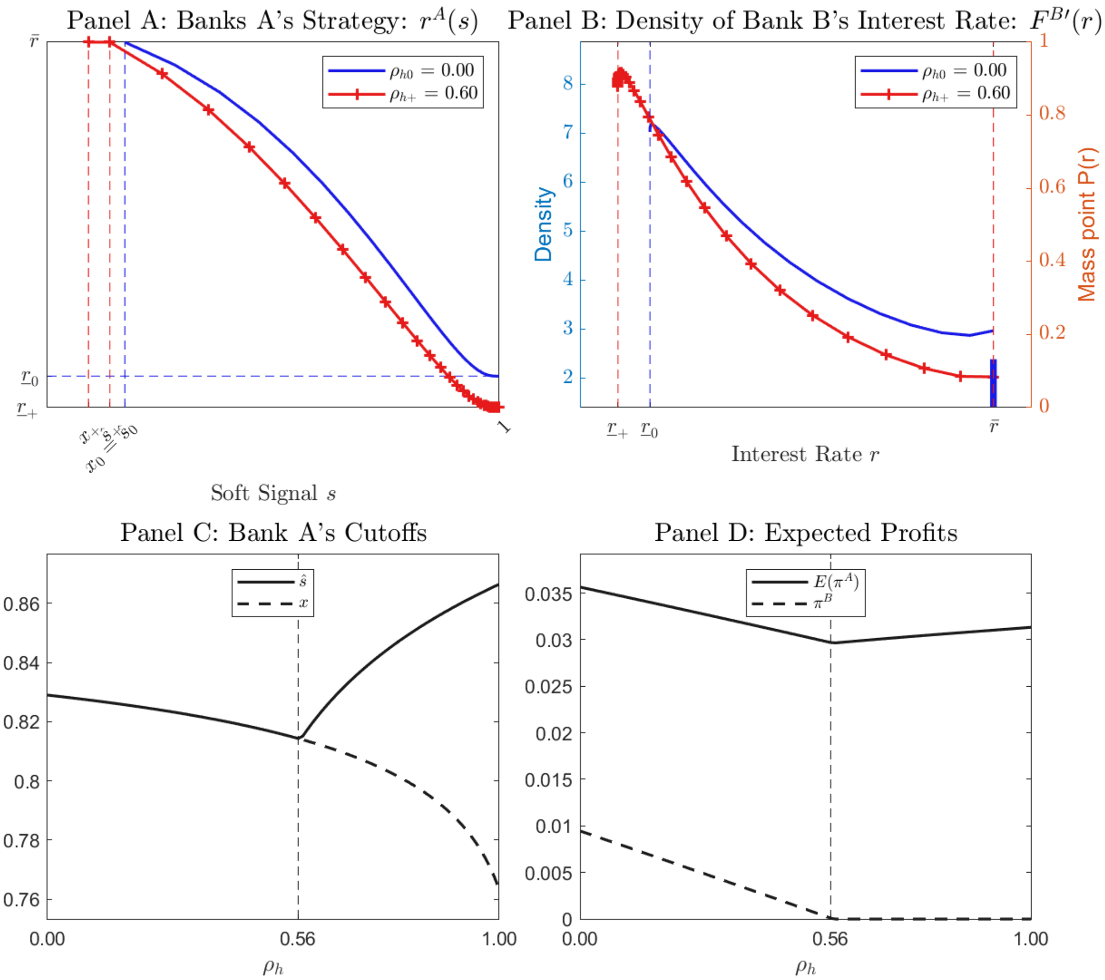

Figure 6: **Equilibrium strategies and profits for hard signal correlation ρ^h.** Panel A depicts $r^A(s)$ as a function of s and Panel B plots $F^{B\prime}(r)$ as a function of r; strategies for $\rho^{h+} = 0.6$ are depicted in red with markers while strategies with $\rho^{h0} = 0$ are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders, both as a function of ρ_h. Parameters: $\bar{r} = 0.45$, $q_h = 0.8$, $q_s = 0.9$, $\eta = 0$, $\alpha = 0.7$, and $\tau = 1$.

and the key economic takeaways are robust to this alternative modeling of the hard information technology.

5.1 Correlated Hard Signals

Another widely acknowledged aspect of information technology advancement is that the lenders' hard information signals become more correlated. For example, the open banking regulation enables sharing financial data with potential lenders under customer consent (He, Huang, and Zhou, 2023; Babina, Buchak, De Marco, Foulis, Gornall, Mazzola, and Yu, 2022), and as a result, lenders' assessments become more alike. In this section, we extend our model to capture this effect and show that making signals more "public" and hardened soft information have different effects on the equilibrium in the credit market.

To further illustrate the potentially different aspects of information technology advancement, we extend our model to allow for correlated hard signals as follows. With probability $\rho^h \in [0,1]$ lenders receive the same binary signal realization $h^c \in \{H, L\}$, while with probability $\left(1-\rho^h\right)$ each lender receives an independent binary hard signal. This captures the recent technology trend that the lenders' hard information signals become increasingly correlated; for instance, open banking regulation studied in He, Huang, and Zhou (2023) and Babina, Buchak, De Marco, Foulis, Gornall, Mazzola, and Yu (2022). We provide a detailed analysis of this extension in Appendix A.5.

Panels C and D in Figure 6 provide comparative statics with respect to the correlation $\rho_h \in [0,1]$ of hard signals across two lenders. We observe in the bottom two panels on Figure 6 that a larger ρ^h leads to a zero-weak equilibrium more likely to occur. In the extreme case in which $\rho^h = 1$, the hard signal becomes a public signal, and Bank B who becomes effectively uninformed ends up with zero profit (Milgrom and Weber, 1982, as discussed in Section 4.2). From this perspective, it is interesting to observe that the economic implications of ρ_h, which is more about data sharing, are qualitatively similar to that of changes in signal precision studied in Section 4.2 but opposite to information span highlighted in this paper.

5.2 Signal on Hardened Soft Fundamental θ_s^h

Information technology corresponds to mappings from some fundamental states to signals, and as usual, there are potentially important modeling choices in specifying the details of the (hard) information technology. As illustrated in the top panel of Figure 1, we have adopted a technology that takes the entire hard fundamental θ_s as input and produces a binary signal as output. However, this is not the only way to do this in a setting with multi-dimensional fundamental states. More specifically, given our hard fundamental $\theta_h = \theta_s^h \theta_s^s$, another natural way to model "hardening soft information" is to keep the original hard and soft signals (h^A, h^B, and s) the same, introduce additional signals of the hardened soft fundamental θ_s^h, and study the impact of these additional signals on credit market equilibrium.

Denote by h_s^j the lender j's signal of θ_h^s. We call it hardened soft signal, which takes a binary value with $h_s^j \in \{H, L\}$. For traceability, we assume that they are also decisive just as in Section 2.4, so that both lenders reject the borrower if $h_s^j = L$.

We can generally allow for any correlation ρ_s^h between two hardened soft signals, as modeled in Section 5.1. For illustration purposes, however, we assume that $\rho_s^h = 1$; essentially, the hardened soft signal becomes public. More specifically, we assume $h_s^A = h_s^B = h_s^c$ where h_s^c takes a value of H (L) with probability $\alpha_s \in (\frac{1}{2}, 1)$ conditional on $\theta_s^h = 1$ ($\theta_s^h = 0$). In practice, the signals generated by Big Data technology are indeed increasingly correlated across users, and this assumption captures this trend in its stark form. In fact, in the limiting case $\alpha_s \to 1$, h_s^j which reveals θ_s^h perfectly will be the same across two lenders for any ρ_s^h. Appendix A.6 provides a full analysis for any general ρ_s^h, following the same framework as in Section 5.1.

That $h_s^A = h_s^B = h_s^c$ is public, together with the assumption that h_s^j's are decisive, simplifies the analysis greatly. Conceptually, because lenders understand that they compete only when $h_s^c = H$ which is informative about the hardened soft fundamentals θ_s^h, this changes the effective distribution of soft signal s to $\phi\left(s|h_s^c = H\right)$. This in turn affects the credit market equilibrium outcome.

Similar to Section 3.1, we introduce $p_{HHH}\left(t\right) \equiv \mathbb{P}\left(h^A = H, h^B = H, h_s^c = H, s \in ds\right)$ as the joint probability of all three hard signals (h^A, h^B, h_s^c) being H and the soft signal s falling in the interval $(s, s+ds)$; we can define analogously $p_{HLH}\left(s\right)$, and finally $\overline{p}_{LHH}\left(t\right)$ the joint probability of $\left\{h^A = L, h^B = h_s^c = H\right\}$. Then, Bank B's lending profits when quoting r and $h^B = h_s^B = H$, is similar to (16):

$$\pi^B\left(r\right) = \int_0^{s^A(r)} \underbrace{p_{HHH}(t)}_{h^A=h^B=h_s^c=H,t} \left[\mu_{HHH}\left(t\right)\left(r+1\right)-1\right]dt + \underbrace{\overline{p}_{LHH}}_{h^A=L,h^B=h_s^c=H} \left[\overline{\mu}_{LHH}\left(r+1\right)-1\right]. \quad (41)$$

And, Bank A's profit when quoting r and $\left\{h^A = H, h_s^A = H, s\right\}$ is similar to (14):

$$\pi^A\left(r,s\right) = \underbrace{p_{HHH}(s)}_{h^A=h^B=h_s^c=H,s} \underbrace{\left[1-F^B\left(r\right)\right]}_{A \text{ wins}} \left[\mu_{HHH}(s)\left(1+r\right)-1\right] + \underbrace{p_{HLH}(s)}_{h^A=h_s^c=H,h^B=L,s} \left[\mu_{HLH}(s)\left(1+r\right)-1\right]. \quad (42)$$

In Appendix A.6 we show that the above profits are isomorphic (up to a constant) to those in Blickle, He, Huang, and Parlatore (2024) with independent fundamentals (and signals), once we replace the relevant distributions—say $\theta_s = 1$ or s—to be those conditional on $h_s^c = H$. As a result, we can derive similar analytical characterizations of the credit market equilibrium under the alternative modeling.

More importantly, the alternative modeling of hardened soft signal delivers quite similar economics as in our baseline. In Section 4.1 we have illustrated that the key mechanism of hardening soft information is to help the non-specialized lender avoid the winner's curse given a low Bank A's soft signal. Under the alternative modeling, one can calculate the distribution of soft signal conditional on positive realization of h_s^c, i.e.,

$$\phi\left(s|h_s^c = H\right) = \phi_0\left(s\right) + \overbrace{\frac{\alpha_s}{\alpha_s - \left(2\alpha_s - 1\right)\eta}}^{\uparrow \text{ in } \eta \text{ as } \alpha_s > \frac{1}{2}} \cdot q_s\left[\phi_1\left(s\right) - \phi_0\left(s\right)\right]. \quad (43)$$

Comparing it to $\phi\left(s|h^A = h^B = H\right)$ in Eq. (28), the only difference arises from the perfectly correlated hardened soft signal h_s^c here versus the conditionally independent h^A, h^B in the baseline (once their information about θ_h^h is shut down). Essentially, observing a positive (public) hardened soft signal helps both lenders update the belief about s upward, therefore reducing the winner's curse. This economic insight is robust even if the hardened soft signal is independent (conditional on θ_s^h).

6 Concluding Remarks

One of banks' main roles in the economy is producing information to allocate credit. In this paper, we show that the nature of the banks' information technology affects the credit market equilibrium and the degree of competition among banks. More specifically, we explore how the recent trend in Big Data technology that transforms qualitative or subjective assessments into quantifiable and objective metrics, known as hardening soft information, affects credit market outcomes in the presence of specialized lenders.

It is important to note that a priori, the significant advance in information technology should benefit all lenders, including specialized and established banks as well as non-specialized lenders and new fintech challengers; in fact, large banks might front-run in their IT investment in the past decade (He, Jiang, Xu, and Yin, 2023). However, the fast-growing empirical literature on fintechs (see, e.g. Berg, Fuster, and Puri, 2022) seem to suggest that the new technology has helped relatively weaker (fintech) lenders to catch up, intensifying the credit market competition.

We build a novel model with asymmetric lenders but symmetric technology improvement to study information span, and its implications on credit market competition. Our model highlights the crucial difference between information span, which captures "breadth" of data, and signal precision, which captures the "quality" of data. This distinction is crucial in understanding the changing landscape in the credit market due to technological advances related to data gathering and processing that lead to the hardening of soft information. Our theory clarifies that it is enlarging the information span, not the mere improvement of "signal precision," that can deliver the desired empirical pattern in a robust way; in fact, the former tends to reinforce the position of specialized lender while the latter serves the role of "leveling the playing field."

References

Acharya, Viral V., Iftekhar Hasan, and Anthony Saunders, 2006, Should banks be diversified? evidence from individual bank loan portfolios, *The Journal of Business* 79, 1355–1412.

Agarwal, Sumit, Shashwat Alok, Pulak Ghosh, and Sudip Gupta, 2020, Financial inclusion and alternate credit scoring for the millennials: role of big data and machine learning in fintech, *Business School, National University of Singapore Working Paper, SSRN* 3507827.

Babina, Tania, Greg Buchak, Filippo De Marco, Angus Foulis, Will Gornall, Francesco Mazzola, and Tong Yu, 2022, Customer data access and fintech entry: Early evidence from open banking, *Working Paper*.

Berg, Tobias, Valentin Burg, Ana Gombović, and Manju Puri, 2020, On the rise of fintechs: Credit scoring using digital footprints, *Review of Financial Studies* 33, 2845–2897.

Berg, Tobias, Andreas Fuster, and Manju Puri, 2022, Fintech lending, *Annual Review of Financial Economics* 14, 187–207.

Bergemann, Dirk, and Stephen Morris, 2016, Bayes correlated equilibrium and the comparison of information structures in games, *Theoretical Economics* 11, 487–522.

Bertomeu, Jeremy, and Iván Marinovic, 2016, A theory of hard and soft information, *The Accounting Review* 91, 1–20.

Blickle, Kristian, Zhiguo He, Jing Huang, and Cecilia Parlatore, 2024, Information-based pricing in specialized lending, .

Blickle, Kristian, Cecilia Parlatore, and Anthony Saunders, 2023, Specialization in banking, *FRB of New York Staff Report No. 967*.

Broecker, Thorsten, 1990, Credit-worthiness tests and interbank competition, *Econometrica* pp. 429–452.

Corrao, Roberto, 2023, Mediation markets: The case of soft information, Discussion paper Working Paper.

Crawford, Gregory S, Nicola Pavanini, and Fabiano Schivardi, 2018, Asymmetric information and imperfect competition in lending markets, *American Economic Review* 108, 1659–1701.

Crawford, Vincent P, and Joel Sobel, 1982, Strategic information transmission, *Econometrica: Journal of the Econometric Society* pp. 1431–1451.

Engelbrecht-Wiggans, Richard, Paul R. Milgrom, and Robert J. Weber, 1983, Competitive bidding and proprietary information, *Journal of Mathematical Economics* 11, 161–169.

Ghosh, Pulak, Boris Vallee, and Yao Zeng, 2022, Fintech lending and cashless payments, in *Proceedings of Paris December 2021 Finance Meeting EUROFIDAI-ESSEC*.

Hardik, Nimbark, 2023, Digitalisation promotes adoption of soft information in sme credit evaluation: the case of indian banks, *Digital Finance* pp. 1–32.

Hausch, Donald B, 1987, An asymmetric common-value auction model, *RAND Journal of Economics* pp. 611–621.

Hauswald, Robert, and Robert Marquez, 2003, Information technology and financial services competition, *Review of Financial Studies* 16, 921–948.

He, Zhiguo, Jing Huang, and Jidong Zhou, 2023, Open banking: Credit market competition when borrowers own the data, *Journal of Financial Economics* 147, 449–474.

He, Zhiguo, Sheila Jiang, Douglas Xu, and Xiao Yin, 2023, Investing in lending technology: It spending in banking, *University of Chicago, Becker Friedman Institute for Economics Working Paper*.

Hendricks, Kenneth, and Robert H Porter, 1988, An empirical study of an auction with asymmetric information, *The American Economic Review* pp. 865–883.

Huang, Jing, 2023, Fintech expansion, *Available at SSRN 3957688*.

Huang, Yiping, Longmei Zhang, Zhenhua Li, Han Qiu, Tao Sun, and Xue Wang, 2020, Fintech credit risk assessment for smes: Evidence from china, .

Kagel, John H, and Dan Levin, 1999, Common value auctions with insider information, *Econometrica* 67, 1219–1238.

Kamenica, Emir, and Matthew Gentzkow, 2011, Bayesian persuasion, *American Economic Review* 101, 2590–2615.

Karapetyan, Artashes, and Bogdan Stacescu, 2014, Information sharing and information acquisition in credit markets, *Review of Finance* 18, 1583–1615.

Kremer, Michael, 1993, The o-ring theory of economic development, *The quarterly journal of economics* 108, 551–575.

Liberti, Jose M., and Atif R. Mian, 2009, Estimating the effect of hierarchies on information use, *The Review of Financial Studies* 22, 4057–4090.

Liberti, José María, and Mitchell A Petersen, 2019, Information: Hard and soft, *Review of Corporate Finance Studies* 8, 1–41.

Marquez, Robert, 2002, Competition, adverse selection, and information dispersion in the banking industry, *Review of Financial Studies* 15, 901–926.

Milgrom, Paul, and Robert J Weber, 1982, The value of information in a sealed-bid auction, *Journal of Mathematical Economics* 10, 105–114.

Milgrom, Paul R, 1981, Good news and bad news: Representation theorems and applications, *The Bell Journal of Economics* pp. 380–391.

Paravisini, Daniel, Veronica Rappoport, and Philipp Schnabl, 2023, Specialization in bank lending: Evidence from exporting firms, *Journal of Finance* 78, 2049–2085.

Paravisini, Daniel, and Antoinette Schoar, 2016, The incentive effect of scores: Randomized evidence from credit committees,, *NBER Working Paper, 19303.*

Rajan, Raghuram G, 1992, Insiders and outsiders: The choice between informed and arm's-length debt, *Journal of finance* 47, 1367–1400.

Riordan, Michael H., 1993, Competition and bank performance: A theoretical perspective, in C. Mayer, and X. Vives, ed.: *Capital Markets and Financial Intermediation* . pp. 328–343 (Cambridge University Press: Cambridge).

Sharpe, Steven A, 1990, Asymmetric information, bank lending, and implicit contracts: A stylized model of customer relationships, *Journal of Finance* 45, 1069–1087.

Stein, Jeremy C., 2002, Information production and capital allocation: Decentralized versus hierarchical firms, *The Journal of Finance* 57, 1891–1921.

Vives, Xavier, 2019, Digital disruption in banking, *Annual Review of Financial Economics* 11, 243–272.

Von Thadden, Ernst-Ludwig, 2004, Asymmetric information, bank lending and implicit contracts: the winner's curse, *Finance Research Letters* 1, 11–23.

A Technical Appendices

A.1 Credit Competition Equilibrium

Proof of Lemma 1

Proof. Note that the property of no gap implies common support $[\underline{r}, \overline{r}]$, because if a bank's interest rate offering has a larger lower bound or a smaller upper bound interest rate than its competitor's, this is one example of gaps in the first bank's support.

To show that the distributions have no gap, suppose that, say, the support of F^B has a gap $(r_1, r_2) \subset [\underline{r}, \overline{r}]$.[25] Then F^A should have no weight in this interval either, as any $r^A(s) \in (r_1, r_2)$ will lead to the same demand for Bank A and so a higher r will be more profitable. At least one lender does not have a mass point at r_1 (it is impossible that both distributions have a mass point at $\overline{r}_1$), under which its competitor has a profitable deviation by revising r_1 to $r \in (r_1, r_2)$ instead. Contradiction.

Regarding point mass, suppose that one distribution, say F^B has a mass point at $\tilde{r} \in [\underline{r}, \overline{r})$. Then Bank A would not quote any $r^A(s) \in [\tilde{r}, \tilde{r} + \epsilon]$ and it would strictly prefer quoting $r^A = \tilde{r} - \epsilon$ instead. In other words, the support of F^A must have a gap in the interval $[\tilde{r}, \tilde{r} + \epsilon]$. This contradicts the property of no gaps which we have shown. Finally, it is impossible that both distributions have a mass point at $\overline{r}$.

□

A.2 Proof of Proposition 1

Proof. This part proves that Bank A's equilibrium interest rate quoting strategy as a function of soft signal $r^A(s)$ is always decreasing; this implies that the FOC that helps us derive Bank A's strategy also ensures the global optimality.

Write Bank A's value $\Pi^A(r, s)$ as a function of its interest rate quote and soft signal, in the event of $h^A = H$ and s. (We use π to denote the equilibrium profit but Π for any strategy.) Recall that Bank A solves the following problem:

$$\max_r \Pi^A(r, s) = \underbrace{p_{HH}(s)}_{h^A=H, h^B=H, s} \underbrace{\left[1 - F^B(r)\right]}_{A \text{ wins}} \left[\mu_{HH}(s)(1+r) - 1\right] + \underbrace{p_{HL}(s)}_{h^A=H, h^B=L, s} \left[\mu_{HL}(s)(1+r) - 1\right] \tag{44}$$

[25]The same argument follows if the support of F^A has a gap in the conjectured equilibrium, and then for Bank B, any quotes within the gap lead to the same demand of the same posterior quality of customers, where Bank B updates its belief from Bank A's strategy.

with the following FOC:

$$0 = \Pi_r^A (r(s), s)$$

$$= \underbrace{p_{HH}(s)\left[-\frac{dF^B(r)}{dr}\right]}_{\text{lost customer}} \left[\underbrace{[\mu_{HH}(s)(1+r)-1]}_{\text{customer return}}\right] + \underbrace{p_{HH}(s)\left[1-F^B(r)\right]}_{\text{customer}} \underbrace{\mu_{HH}(s)}_{\text{MB of customer}} + p_{HL}(s)\mu_{HL}(s). \tag{45}$$

One useful observation is that on the support, it must hold that $\mu_{HH}(s)(1+r)-1>0$; otherwise, $\mu_{HL}(s)(1+r)-1<\mu_{HH}(s)(1+r)-1\leq 0$, implying that Bank A's profit is negative (so it will exit).

Lemma 2. *Consider s_1, s_2 in the interior domain with corresponding interest rate quote r_1 and r_2. The marginal value of quoting r_2 for type $s=s_1$, i.e. $\Pi_r^A(r_2,s_1)$, has the same sign as s_2-s_1.*

Proof. Evaluating the FOC of type s_1 but quoting r_2:

$$\begin{aligned}\Pi_r^A(r_2,s_1) =& p_{HH}(s_1)\left[-\frac{dF^B(r_2)}{dr}\right][\mu_{HH}(s_1)(1+r_2)-1] \\ &+ p_{HH}(s_1)\left[1-F^B(r_2)\right]\mu_{HH}(s_1)+p_{HL}(s_1)\mu_{HL}(s_1).\end{aligned} \tag{46}$$

FOC at type s_2 yields

$$\begin{aligned}0 = \Pi_r^A(r_2,s_2) =& p_{HH}(s_2)\left[-\frac{dF^B(r_2)}{dr}\right][\mu_{HH}(s_2)(1+r_2)-1] \\ &+ p_{HH}(s_2)\left[1-F^B(r_2)\right]\mu_{HH}(s_2)+p_{HL}(s_2)\mu_{HL}(s_2),\end{aligned}$$

or

$$\frac{dF^B(r_2)}{dr} = \frac{p_{HH}(s_2)\left[1-F^B(r_2)\right]\mu_{HH}(s_2)+p_{HL}(s_2)\mu_{HL}(s_2)}{p_{HH}(s_2)[\mu_{HH}(s_2)(1+r_2)-1]}. \tag{47}$$

Plugging in this term to (46), $\Pi_r^A(r_2,s_1)$ becomes

$$\begin{aligned}\Pi_r^A(r_2,s_1) &= \left[\phi_1(s_1) - \frac{p_{HH}(s_1)}{p_{HH}(s_2)}\cdot\frac{\mu_{HH}(s_1)(1+r_2)-1}{\mu_{HH}(s_2)(1+r_2)-1}\cdot\phi_1(s_2)\right]\left\{\overline{p}_{HH}\overline{\mu}_{HH}\left[1-F^B(r_2)\right]+\overline{p}_{HL}\overline{\mu}_{HL}\right\} \\ &= \frac{p_{HH}(s_1)\phi_1(s_2)-\phi_1(s_1)p_{HH}(s_2)}{p_{HH}(s_2)[\mu_{HH}(s_2)(1+r_2)-1]}\left\{\overline{p}_{HH}\overline{\mu}_{HH}\left[1-F^B(r_2)\right]+\overline{p}_{HL}\overline{\mu}_{HL}\right\},\end{aligned} \tag{48}$$

where $\overline{p}_{h^Ah^B} \equiv \mathbb{P}\left(h^A,h^B\right)$, $\overline{\mu}_{h^Ah^B} \equiv \mathbb{P}\left(\theta=1\middle|h^A,h^B\right)$ are defined in Section 3.1 and $\phi_1(s) \equiv \phi(s\,|\theta_s=1) = \frac{s\phi(s)}{q_s}$ is the conditional density of soft signal (also $\phi_0(s)\equiv\phi(s\,|\theta_s=0)=\frac{(1-s)\phi(s)}{q_s}$). Hence, the sign of $\Pi_r^A(r_2,s_1)$ depends on $p_{HH}(s_1)\phi_1(s_2)-p_{HH}(s_2)\phi_1(s_1)$ because the denominator is positive as we noted right after Eq. (45).

Last, we argue that the sign of $p_{HH}(s_1)\phi_1(s_2) - p_{HH}(s_2)\phi_1(s_1)$ depends on $s_2 - s_1$. Note that the event HH correlates with the soft signal s only via θ_s^h which affects θ_s, we have

$$p_{HH}(s) = \mathbb{P}(\theta_s = 1, HH)\,\phi_1(s) + \mathbb{P}(\theta_s = 0, HH)\,\phi_0(s),$$

i.e., two positive hard signals HH are no longer informative about the soft signal s once we condition on the soft state θ_s. Using this observation,

$$p_{HH}(s_1)\phi_1(s_2) - p_{HH}(s_2)\phi_1(s_1) = \mathbb{P}(\theta_s = 0, HH)\,\phi_0(s_1)\,\phi_0(s_2)\left[\frac{\phi_1(s_2)}{\phi_0(s_2)} - \frac{\phi_1(s_1)}{\phi_0(s_1)}\right], \qquad (49)$$

which shares the same sign as $s_2 - s_1$. □

Lemma 2 has three implications. First, as long as $r^A(\cdot)$ is (strictly) increasing in some segment, then Bank A would like to deviate in this segment. To see this, suppose that $r_1 > r_2$ when $s_1 > s_2$ for s_1, s_2 arbitrarily close. Because Lemma 1 has shown that Bank A's strategy is smooth, r_2 is arbitrarily close to r_1. Then $\Pi_r^A(r_2, s_1) < 0$, implying that the value is convex and the Bank A at s_1 (who in equilibrium is supposed to quote r_1) would like to deviate further.

Second, the monotonicity implied by Lemma 2 helps us show that Bank A uses a pure strategy. To see this, for any $s_1 > s_2$ that induce interior quotes $r_1, r_2 \in [\underline{r}, \overline{r})$, however close, in equilibrium we must have $\sup r^A(s_1) < \inf r^A(s_2)$ by monotonicity. Combining this with Part 3 of Lemma 1, i.e., the induced distribution $F^A(\cdot)$ is atomless except for at $\overline{r}$ and has no gaps, we know that Bank A must adopt a pure strategy in the interior of $[\underline{r}, \overline{r})$, or for $s \leq \hat{s}$. Finally, the following argument shows pure strategy for $s < \hat{s}$: i) randomize over $s = 0$ is a zero-measure set; and ii) on $s > \hat{s}$ Bank A can either quote $\overline{r}$ or ∞, which, generically, gives different values (and hence rules out randomization).

Third, if $r^A(\cdot)$ is decreasing globally over $\mathcal{S}$, then the FOC is sufficient to ensure global optimality. Consider a type s_1 who would like to deviate to $\check{r} > r_1$; then

$$\Pi^A(\check{r}, s_1) - \Pi^A(r_1, s_1) = \int_{r_1}^{\check{r}} V_r^A(r, s_1)\, dr.$$

Given the monotonicity of $r(s)$, we can find the corresponding type $s(r)$ for $r \in [r_1, \check{r}]$. From Lemma 2 we know that

$$\Pi_r^A(r, s_1) \propto \frac{\phi_1(s(r))}{\phi_0(s(r))} - \frac{\phi_1(s_1)}{\phi_0(s_1)}$$

which is negative given $s(r) < s_1$. Therefore the deviation gain is negative. Similarly, we can show a negative deviation gain for any $\check{r} < r_1$.

Next we show that $r^A(\cdot)$ is single-peaked over the space of $\mathcal{S} = [0, 1]$.

Lemma 3. *Given any exogenous* $\pi^B \geq 0$, $r^A(\cdot)$ *single-peaked over* $\mathcal{S} = [0,1]$ *with a maximum point.*

Proof. When $r \in [\underline{r}, \overline{r})$, the derivative of $r^A(s)$ with respect to s is

$$\frac{dr^A(s)}{ds} = \frac{p_{HH}\phi(s)\left(\overbrace{\int_0^s p_{HH}(t)\left[\mu_{HH}(t) - \mu_{HH}(s)\right]dt}^{M_1(s)<0,\text{and} M_1'(s)<0} + \overbrace{\overline{p}_{LH}\overline{\mu}_{LH} - \left(\pi^B + \overline{p}_{LH}\right)\mu_{HH}(s)}^{M_2(s)?0,\text{ but} M_2'(s)<0}\right)}{\left(\int_0^s p_{HH}(t)\cdot\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right)^2}.$$

As $\mu_{HH}(t) < \mu_{HH}(s)$ for $t \in [0,s)$, the first term in the bracket $M_1(s) < 0$, and

$$M_1'(s) = -\frac{\partial \mu_{HH}(s)}{\partial s}\int_0^s p_{HH}(t)\,dt < 0.$$

For $M_2(s) = \overline{p}_{LH}\overline{\mu}_{LH} - \left(\pi^B + \overline{p}_{LH}\right)\mu_{HH}(s)$, it has an ambiguous sign, but is decreasing in s. This implies that $M_1(s) + M_2(s)$ decreases with s. It is easy to verify that $M_1(0) + M_2(0) > 0$ and $M_1(1) + M_2(1) < 0$. Therefore $r^A(s)$ first increases and then decreases, therefore single-peaked. ☐

Suppose that the peak point is $\tilde{s}$; then Bank A should simply charge $r(s) = \tilde{r}$ for $s < \tilde{s}$ for better profit. This is the standard "ironing" technique and we therefore define the following ironed strategy formally (here, we also take care of the capping $r \leq \overline{r}$):

$$r^A_{ironed}(s) \equiv \sup_{t\in[s,1]} \min\left(r^A(t), \overline{r}\right).$$

By definition $r^A_{ironed}(s)$ is monotone decreasing.

We now argue that in equilibrium, π^B and $\underline{r}$ adjust so that $r^A(\cdot)$ is always monotonely decreasing over $[x,1]$. (Since we define $r^A(s) = \infty$ for $s < x$, monotonicity over the entire signal space $[0,1]$ immediately follows.) There are two subcases to consider.

1. Suppose that $\tilde{r} = \overline{r}$. In this case, $r^A(s) = \frac{\pi^B + \int_0^s p_{HH}(t)dt + \overline{p}_{LH}}{\int_0^s p_{HH}(t)\cdot\mu_{HH}(t)dt + \overline{p}_{LH}\overline{\mu}_{LH}} - 1$ used in Lemma 2 and 3 does not apply to $s < \tilde{s}$ because the equation is defined only over the left-closed-right-open interval $[\underline{r}, \overline{r})$. Instead, $r^A(s)$ in this region is determined by Bank A's optimality condition: as r^A does not affect the competition from Bank B (which equals $F^B(\overline{r}^-)$), Bank A simply sets the maximum possible rate $r^A(r) = \overline{r}$ unless it becomes unprofitable (for $s < x$). (This is our zero-weak equilibrium with $\pi^B = 0$, and there is no "ironing" in this case.)

2. Suppose that $\tilde{r} < \overline{r}$; then bank A quotes $\tilde{r}$ for all $s < \hat{s}$. But this is not an equilibrium—Bank A now leaves a gap in the interval $[\tilde{r}, \overline{r}]$, contradicting with point 3) in Lemma 1 (there, we rule out gaps in equilibrium). Intuitively, Bank B always would like to raise its quotes inside

$[\tilde{r}, \overline{r}]$ to $\overline{r}$; there is no "ironing" in this case. (This is our positive-weak equilibrium with $\pi^B > 0$.)

Therefore, we have shown that Bank A uses a pure strategy $r^A(s)$ that decreases in s. Bank A's equilibrium strategy Eq. (18) is then derived from Bank B's indifference condition.

Equilibrium strategy F^B In this part, we derive equilibrium strategy taking π^B and $\hat{s}$ as given.

Bank A's equilibrium strategy Eq. (18) in $s \in [\hat{s}, 1]$ maximizes its profit and so satisfies the following first order condition (FOC):

$$p_{HH}(s)\left(-\frac{dF^B(r)}{dr}\right)[\mu_{HH}(s)(r+1)-1] + \left\{p_{HH}(s)\left[1-F^B(r)\right]\mu_{HH}(s) + p_{HL}(s)\mu_{HL}(s)\right\} = 0. \tag{50}$$

Bank B's equilibrium quotes $r \in [\underline{r}, \overline{r})$ maximizes its expected profits and satisfy the following FOC:

$$\underbrace{\left(-s^{A\prime}(r)\right)p_{HH}\left(s^A(r)\right)}_{\text{B's additional borrowers}}[\mu_{HH}(s)(r+1)-1] = \underbrace{\int_0^{s^A(r)} p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}_{\text{B's existing borrowers}}.$$

Plug this condition into Bank A's optimality condition (50), and we have

$$\left(-\frac{dF^B(r)}{dr}\right)\frac{\int_0^{s^A(r)} p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}{-s^{A\prime}(r)} + p_{HH}(s)\left[1-F^B(r)\right]\mu_{HH}(s) + p_{HL}(s)\mu_{HL}(s) = 0,$$

which could be rearranged as

$$-\frac{d}{ds}\left\{\frac{1-F^B(r)}{\int_0^s p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}\right\} = -\frac{\frac{d}{ds}\left\{\int_0^s p_{HL}(t)\mu_{HL}(t)dt\right\}}{\left[\int_0^s p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]^2}.$$

Using the conditional independence of signals, the right-hand-side of the above equation is

$$\begin{aligned}-\frac{\frac{d}{ds}\left\{\int_0^s p_{HL}(t)\mu_{HL}(t)dt\right\}}{\left[\int_0^s p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]^2} &= -\frac{\frac{d}{ds}\left\{\int_0^s \frac{p_{HL}(t)\mu_{HL}(t)}{p_{HH}(t)\mu_{HH}(t)}p_{HH}(t)\,\mu_{HH}(t)\,dt\right\}}{\left[\int_0^s p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]^2} \\ &= \frac{1-\alpha}{\alpha}\frac{d}{ds}\left\{\frac{1}{\int_0^s p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}\right\}.\end{aligned}$$

Hence, Bank B's equilibrium strategy satisfies the following key ordinary differential equation,

$$\frac{d}{ds}\left[\frac{\frac{1-\alpha}{\alpha} + 1 - F^B(r)}{\int_0^s p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}\right] = 0, \tag{51}$$

so

$$\frac{\overline{p}_{HH}\overline{\mu}_{HH}\left[1-F^{B}\left(r\right)\right]+\overline{p}_{HL}\overline{\mu}_{HL}}{\int_{0}^{s}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt+\overline{p}_{LH}\overline{\mu}_{LH}}=K.$$

Using the boundary condition $F^{B}\left(\underline{r}\right)=0$, we solve for the constant

$$K=\frac{\overline{p}_{HH}\overline{\mu}_{HH}+\overline{p}_{HL}\overline{\mu}_{HL}}{\overline{p}_{HH}\overline{\mu}_{HH}+\overline{p}_{LH}\overline{\mu}_{LH}}=1.$$

Therefore, for $r\in[\underline{r},\overline{r})$, we have

$$F^{B}\left(r\right)=1-\frac{\int_{0}^{s(r)}t\phi\left(t\right)dt}{q_{s}},$$

and if $\pi^{B}>0$ ($\pi^{B}=0$), we have $F^{B}\left(\overline{r}\right)=1$ ($F^{B}\left(\overline{r}\right)=1-\frac{\int_{0}^{\hat{s}}t\phi(t)dt}{q_{s}}$).

π^{B} and boundary condition $\hat{s}$ We define the following auxilary functions

$$\hat{\pi}^{B}\left(\overline{r};s^{A}\left(\overline{r}\right)=\check{s}\right)=\int_{0}^{\check{s}}p_{HH}\left(t\right)\left[\mu_{HH}\left(t\right)\left(\overline{r}+1\right)-1\right]dt+\overline{p}_{LH}\left[\overline{\mu}_{LH}\left(\overline{r}+1\right)-1\right], \tag{52}$$

which is Bank B's profits when assuming that quoting $r^{B}=\overline{r}$ wins Bank A of type $s\in[0,\check{s}]$ in competition (HH). We define s_{B}^{be} as the threshold where Bank B's auxilary profits break even,

$$\hat{\pi}^{B}\left(\overline{r};s^{A}\left(\overline{r}\right)=s_{B}^{be}\right)=0. \tag{53}$$

In addition, we define the following auxilary profit function for Bank A,

$$\begin{aligned}&\hat{\pi}^{A}\left(\overline{r},\check{s};F^{B}\left(\overline{r}\right)=\int_{\check{s}}^{1}\frac{s\phi\left(s\right)}{q_{s}}ds\right)\\ =&p_{HH}\left(\check{s}\right)\underbrace{\int_{0}^{\check{s}}\frac{s\phi\left(s\right)}{q_{s}}ds}_{=1-F^{B}(\overline{r})}\left[\mu_{HH}\left(\check{s}\right)\left(1+\overline{r}\right)-1\right]+p_{HL}\left(\check{s}\right)\left[\mu_{HL}\left(\check{s}\right)\left(1+r\right)-1\right],\end{aligned} \tag{54}$$

which assumes that Bank A receiving soft signal $\check{s}$ wins with probability $\int_{0}^{\check{s}}\frac{s\phi(s)}{q_{s}}ds$ when quoting $\overline{r}$. We define s_{A}^{be} as the threshold where Bank A's auxilary profits break even,

$$\hat{\pi}^{A}\left(\overline{r},s_{A}^{be};F^{B}\left(\overline{r}\right)=\int_{s_{A}^{be}}^{1}\frac{s\phi\left(s\right)}{q_{s}}ds\right)=0. \tag{55}$$

First, we argue that equilibrium $\hat{s}\equiv\arg\sup_{s}\left\{s:r^{A}\left(s\right)\geq\overline{r}\right\}$ either equals s_{A}^{be} or s_{B}^{be}. To see this, if $\pi^{B}=0$, we have $\hat{s}=s_{B}^{be}$ by construction. If $\pi^{B}>0$, then Bank B always makes an offer upon H, i.e., $F^B(\overline{r}) = 1$. We also know that $F^B(\overline{r}^-) = 1 - \frac{\int_0^{s^A(r)=\hat{s}^+} t\phi(t)dt}{q_s} < 1$, because Bank A must reject the borrower when s realizes as close to 0 and $\hat{s} > 0$. Hence, $F^B(r)$ has a point mass at $\overline{r}$. It follows that $F^A(r)$ is open at $\overline{r}$: $\hat{s} = x$ and $\pi^A\left(r^A(\hat{s})|\hat{s}\right) = 0$, which is exactly the definition of s_A^{be} and so $\hat{s} = s_A^{be}$.

Now we prove the claim in this lemma. In the first case of $s_B^{be} < s_A^{be}$, we have $\hat{s} \le s_A^{be}$ and thus Bank A's probability of winning when quoting $r^A = \overline{r}$ is at most $\frac{\int_0^{s_A^{be}} t\phi(t)dt}{q_s} \ge \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s} = 1 - F^B(\overline{r}^-)$. The definition of s_A^{be} says that Bank A upon s_A^{be} breaks even when quoting $r^A\left(s_A^{be}\right) = \overline{r}$ and facing this most favorable winning probability $\frac{\int_0^{s_A^{be}} t\phi(t)dt}{q_s}$. Then upon a worse soft signal $s_B^{be} < s_A^{be}$, Bank A must reject the borrower because offering $\overline{r}$ leads to losses, which rules out $\hat{s} = s_B^{be}$. According to our earlier observation of $\hat{s} = s_B^{be}$ or s_A^{be}, we have $\hat{s} = s_A^{be}$ and $\pi^B > 0$ in this case, where π^B is the same as Eq. (52).

In the second case of $s_B^{be} \ge s_A^{be}$, we have $\hat{s} \le s_B^{be}$. When Bank B quotes $r^B = \overline{r}$, the marginal type that Bank B wins over is at most s_B^{be}. The definition of s_B^{be} says that Bank B breaks even when quoting $r^B = \overline{r}$ and facing this most favorable winning probability at marginal type s_B^{be}. Then if the competition from A were more aggressive, say the marginal type quoting $r^A = \overline{r}$ is $s_A^{be} \le s_B^{be}$, Bank B would make a loss when quoting $\overline{r}$, so $\hat{s} = s_A^{be}$ cannot support an equilibrium. Hence, in this case, $\hat{s} = s_B^{be}$ and $\pi^B = 0$. From the definition of s_A^{be}, Bank A's equilibrium break-even condition $0 = \pi^A(\overline{r}|x)$, and the fact that $s_B^{be} \ge s_A^{be}$ in this case, we have

$$
\begin{aligned}
0 =& \frac{\int_0^{s_A^{be}} p_{HH}(s)\,ds}{q_s}\left[\mu_{HH}\left(s_A^{be}\right)(1+\overline{r}) - 1\right] + p_{HL}\left[\mu_{HL}\left(s_A^{be}\right)(1+\overline{r}) - 1\right] \\
=& \frac{\int_0^{s_B^{be}} p_{HH}(s)\,ds}{q_s}\left[\mu_{HH}(x)(1+\overline{r}) - 1\right] + p_{HL}\left[\mu_{HL}(x)(1+\overline{r}) - 1\right] \\
\ge& \frac{p_{HH}\int_0^{s_A^{be}} t\phi(t)\,dt}{q_s}\left[\mu_{HH}(x)(1+\overline{r}) - 1\right] + p_{HL}\left[\mu_{HL}(x)(1+\overline{r}) - 1\right].
\end{aligned}
$$

Hence, $x \le s_A^{be} \le s_B^{be} = \hat{s}$. □

A.3 Proof of Proposition 2

Lemma 4. *The break-even soft signals s_A^{be} and s_B^{be} defined in Eq. (55) and (53) satisfy*

$$\frac{\partial s_A^{be}}{\partial \eta} < 0, \quad \frac{\partial s_B^{be}}{\partial \eta} < 0.$$

Proof. The definition of s_A^{be} or $\hat{\pi}^A\left(\overline{r}, s_{be}^A; \frac{\int_0^{s_{be}^A} t\phi(t)dt}{q_s}\right) = 0$ corresponds to an implicit function of η

and s^A_{be},

$$\hat{\pi}^A(\eta, s) \equiv p_{HH}(s)\frac{\int_0^s t\phi(t)\,dt}{q_s}\left[\mu_{HH}(s)(1+\overline{r})-1\right] + p_{HL}(s)\left[\mu_{HL}(s)(1+\overline{r})-1\right]$$

$$= \underbrace{\frac{\int_0^s t\phi(t)\,dt}{q_s}}_{\text{soft info, indept of }\eta}\left[\underbrace{p_{HH}(s)\mu_{HH}(s)}_{\text{Type 1, indept of }\eta}(1+\overline{r}) - \underbrace{p_{HH}(s)}_{\text{decrease in }\eta}\right] + \underbrace{p_{HL}(s)\mu_{HL}(s)}_{\text{Type 1, indept of }\eta}(1+\overline{r}) - p_{HL}(s).$$

We first analyze the key terms' monotonicity in η. Note that the joint events of signal realizations and good project (Type 1 error) is independent of η,

$$p_{HH}(s)\mu_{HH}(s) = q\alpha^2 \cdot \frac{s\phi(s)}{q_s},$$
$$p_{HL}(s)\mu_{HL}(s) = q\alpha(1-\alpha) \cdot \frac{s\phi(s)}{q_s}.$$

In addition, $p_{HH}(s)$ decreases with η as shown in Eq. (27). The remaining term $p_{HL}(s)$ is independent of η as shown in Eq. (30).

Taken together,

$$\frac{\partial\hat{\pi}^A(\eta, s)}{\partial\eta} > 0;$$

Combining with the fact that $\frac{\partial\hat{\pi}^A(\eta,s)}{\partial s} > 0$, the implicit function theorem shows

$$\frac{\partial s^{be}_A(\eta)}{\partial\eta} < 0.$$

Part 2. The definition of s^{be}_B corresponds to an implicit function

$$0 = \hat{\pi}^B(\eta, s) = \int_0^s p_{HH}(t)\left[\mu_{HH}(t)(\overline{r}+1)-1\right]dt + \overline{p}_{LH}\left[\overline{\mu}_{LH}(\overline{r}+1)-1\right].$$

Similar as in the previous argument, Type I mistakes are constant in the span η due to the multiplicative-characteristic setting:

$$\int_0^s p_{HH}(t)\mu_{HH}(t)\,dt = q\alpha^2\Phi(s|\,\theta_s = 1) = q\alpha^2 \cdot \frac{\int_0^s t\phi(t)\,dt}{q_s},$$
$$\overline{p}_{LH}\overline{\mu}_{LH} = q\alpha(1-\alpha).$$

In addition, the probability of disagreement in hard signals is also independent of η as $\alpha^A = \alpha^B$,

$$\overline{p}_{LH} = \alpha(1-\alpha).$$

The probability that Bank B wins in competition $\int_0^s p_{HH}(t)\, dt$ decreases with η.

Taken together, we have $\frac{\partial \hat{\pi}^B(\eta,s)}{\partial \eta} > 0$. Combining with $\frac{\partial \hat{\pi}^B(\eta,s)}{\partial s} > 0$, the implicit function theorem implies that

$$\frac{\partial s_B^{be}(\eta)}{\partial \eta} < 0.$$

□

Proof. First, we show that for $s \in [\hat{s}, 1]$, we have

$$\frac{d\pi^A(r(s), s)}{d\eta} < \frac{d\left[\pi^B(r(s)) \frac{s\phi(s)}{q_s}\right]}{d\eta} \tag{56}$$

or equivalently, $\Delta\pi(s;\eta)$ defined in Eq. (36) increases in η,

$$\frac{\partial \Delta\pi(s;\eta)}{\partial \eta} = \frac{q_h^h\left[\alpha^2 - (1-\alpha)^2\right]\phi(s)}{(1-q_s)\, q_s} \int_0^s (s-t)\,\phi(t)\, dt > 0.$$

Second, we argue that η weakly increases Bank B's profits

$$\frac{d\pi^B}{d\eta} \geq 0$$

and the inequality is strict if and only if $\pi^B > 0$. When $\pi^B = 0$, from

$$\frac{d\pi^A(r(s), s)}{d\eta} < 0 = \frac{d\left[\pi^B(r(s)) \frac{s\phi(s)}{q_s}\right]}{d\eta}, \quad \text{where } s \in [\hat{s}, 1]$$

When $\pi^B > 0$, $\hat{s} = s_{be}^A$. From Lemma 4, for any $\eta_1 < \eta_2$,

$$\hat{s}(\eta_1) = s_{be}^A(\eta_1) > s_{be}^A(\eta_2) = \hat{s}(\eta_2).$$

Then when $\eta = \eta_2$, Bank A breaks even upon soft signal $\hat{s}(\eta_2)$ and makes profits upon a better soft signal $\hat{s}(\eta_1)$, i.e.,

$$\pi^A\left(r^A(\hat{s}(\eta_1)), \hat{s}(\eta_1); \eta_2\right) > 0 = \pi^A\left(r^A(\hat{s}(\eta_1)), \hat{s}(\eta_1); \eta_1\right).$$

Hence, broader information span from η_1 to η_2 increases Bank A's profit conditional on soft signal type $\hat{s}(\eta_1)$. This implies that Bank B should benefit more from broader hard information,

$$\frac{\hat{s}(\eta_1)\,\phi(\hat{s}(\eta_1))}{q_s}\left[\pi^B(\hat{s}(\eta_1);\eta_2) - \pi^B(\hat{s}(\eta_1);\eta_1)\right] > \pi^A(r(\hat{s}(\eta_1)), \hat{s}(\eta_1);\eta_2) - \pi^A\left(r^A(\hat{s}(\eta_1)), \hat{s}(\eta_1);\eta_1\right) > 0.$$

As Bank B makes a constant profit,

$$\pi^B(\eta_2) > \pi^B(\eta_1).$$

This holds for any $\eta_1 < \eta_2$ when the resulting equilibrium is positive weak, so

$$\frac{d\pi^B(\eta)}{d\eta} > 0\,(=0)$$

if $\pi^B(\eta) > 0\,(=0)$. □

A.4 Proof of Proposition 3

Proof. First, we argue that Bank A benefits more when $s \in [\hat{s}, 1]$, i.e. $\left.\frac{\partial\Delta\pi(s;\alpha)}{\partial\alpha}\right|_{\alpha\to 1} > 0$ in the vicinity of $\alpha = 1$. To see this,

$$\frac{\partial\Delta\pi(s;\alpha)}{\partial\alpha} = \int_0^s \left[\frac{s\phi(s)}{q_s}\frac{\partial p_{HH}(t)}{\partial\alpha} - \frac{\partial p_{HH}(s)}{\partial\alpha}\frac{t\phi(t)}{q_s}\right]dt + \left[\frac{\partial\overline{p}_{LH}}{\partial\alpha}\frac{s\phi(s)}{q_s} - \frac{\partial p_{HL}(s)}{\partial\alpha}\right].$$

Recall that

$$p_{HH}(s) = \left[\underbrace{\left(1-q_h^h\right)}_{\theta_h^h=0}\phi(s) + \underbrace{q_h^h\left(1-q_s^h\right)}_{\theta_h^h=1,\theta_s^h=0}\phi_0(s)\right](1-\alpha)^2 + \left[\underbrace{q_h^h q_s^h\left(1-q_s^s\right)}_{\theta_h^h=\theta_s^h=1,\theta_s^s=0}\phi_0(s) + \underbrace{q}_{\theta=1}\phi_1(s)\right]\alpha^2,$$

and then

$$\frac{\partial p_{HH}(s)}{\partial\alpha} = -2(1-\alpha)\left[\left(1-q_h^h\right)\phi(s) + q_h^h\left(1-q_s^h\right)\phi_0(s)\right] + 2\alpha\left[q_h^h q_s^h\left(1-q_s^s\right)\phi_0(s) + q\phi_1(s)\right].$$

Hence, when $\alpha \to 1$, we have

$$\frac{s\phi(s)}{q_s}\frac{\partial p_{HH}(t)}{\partial\alpha} - \frac{\partial p_{HH}(s)}{\partial\alpha}\frac{t\phi(t)}{q_s} \to 2\alpha q_h^h q_s^h\left(1-q_s^s\right)\left(\phi_1(s)\phi_0(t) - \phi_0(s)\phi_1(t)\right).$$

In addition,

$$\frac{\partial\overline{p}_{LH}}{\partial\alpha}\frac{s\phi(s)}{q_s} - \frac{\partial p_{HL}(s)}{\partial\alpha} = (1-2\alpha)\left[\frac{s\phi(s)}{q_s} - \phi(s)\right].$$

Using these terms, we have

$$\left.\frac{\partial\Delta\pi(s;\alpha)}{\partial\alpha}\right|_{\alpha\to 1} = \frac{2q_h^h q_s^h\left(1-q_s^s\right)}{q_s\left(1-q_s\right)}\phi(s)\int_0^s (s-t)\phi(t)\,dt - \left[\frac{s\phi(s)}{q_s} - \phi(s)\right].$$

Under the primitive condition of $\frac{2q_h^h q_s^h(1-q_s^s)}{1-q_s} > 1$,

$$\left.\frac{\partial \Delta\pi(s;\alpha)}{\partial\alpha}\right|_{\alpha\to 1} > \int_{\hat{s}}^{1} \phi(s)\left[\frac{1}{q_s}\int_0^s (s-t)\,\phi(t)\,dt - \left(\frac{s}{q_s}-1\right)\right] ds.$$

For the integrand, note that when $s=1$,

$$\left.\frac{1}{q_s}\int_0^s (s-t)\,\phi(t)\,dt - \left(\frac{s}{q_s}-1\right)\right|_{s=1} = \frac{1}{q_s}(1-q_s) - \left(\frac{1}{q_s}-1\right) = 0;$$

addition, the integrand decreases in s,

$$\frac{\partial\left[\frac{1}{q_s}\int_0^s (s-t)\,\phi(t)\,dt - \left(\frac{s}{q_s}-1\right)\right]}{\partial s} = \frac{1}{q_s}\left[\Phi(s)-1\right] < 0,$$

so it is positive when $s \in [\hat{s}, 1)$.

Therefore, we have shown the first part that when $s \in [\hat{s}, 1]$, $\left.\frac{\partial\Delta\pi(s;\alpha)ds}{\partial\alpha}\right|_{\alpha\to 1} > 0$. Because the equilibrium is zero-weak and α has no effect on Bank B's equilibrium profits, we have

$$\left.\frac{\partial\pi(s;\alpha)}{\partial\alpha}\right|_{\alpha\to 1} > 0.$$

In addition, similar as in Lemma 4, we have $\frac{d\hat{s}}{d\alpha} < 0$. Hence, if $\frac{dx}{d\alpha} < 0$, Bank A's expected equilibrium profits $\int_x^1 \pi^A ds$ increases in α.

In a zero-weak equilibrium, we have $\hat{s} = s_B^{be}$. For Bank A, the break-even threshold x satisfies

$$0 = \pi^A(\overline{r}, x) = p_{HH}(x)\frac{\int_0^{s_{be}^B} t\phi(t)\,dt}{q_s}\left[\mu_{HH}(x)(1+\overline{r})-1\right] + p_{HL}(x)\left[\mu_{HL}(x)(1+\overline{r})-1\right].$$

Define the Implicit function $F(\alpha, x) \equiv \pi^A(\overline{r}, x) = 0$. We calculate

$$\frac{\partial F}{\partial\alpha} = \frac{d\pi^A(\overline{r},x)}{d\alpha} = \frac{\partial\pi^A(\overline{r},x)}{\partial\alpha} + \frac{\partial s_{be}^B}{\partial\alpha}\frac{\partial\pi^A(\overline{r},x)}{\partial s_{be}^B}.$$

When $\alpha \to 1$, we can solve for x and s_B^{be}. The threshold s_B^{be} is defined from

$$0 = \hat{\pi}^B\left(\overline{r}; s_{be}^B\right) = \int_0^{s_{be}^B} p_{HH}(t)\left[\mu_{HH}(t)(\overline{r}+1)-1\right]dt \underbrace{+\overline{p}_{LH}\left[\overline{\mu}_{LH}(\overline{r}+1)-1\right]}_{\to 0 \text{ when } \alpha\to 1},$$

Hence, $\frac{\int_0^{s_{be}^B} t\phi(t)dt}{\int_0^{s_{be}^B} \phi(t)dt} = \frac{1}{1+\bar{r}}$, and similarly, $x(\alpha \to 1) = \frac{1}{1+\bar{r}}$. It follows that

$$\begin{aligned}\frac{\partial \pi^A(\bar{r}, x)}{\partial s_{be}^B} &= \frac{s_{be}^B \phi\left(s_{be}^B\right)}{q_s} p_{HH}(x)\left[\mu_{HH}(x)(1+\bar{r}) - 1\right] \\ &\to \frac{s_{be}^B \phi\left(s_{be}^B\right)}{q_s} p_{HH}(x)\left[x(1+\bar{r}) - 1\right] \to 0.\end{aligned}$$

Hence, at $x(\alpha)$ where Bank A breaks even, the only effect of α is via the direct change in technology,

$$\frac{\partial F}{\partial \alpha} = \frac{\partial \pi^A(\bar{r}, x)}{\partial \alpha} + \frac{\partial s_{be}^B}{\partial \alpha} \underbrace{\frac{\partial \pi^A(\bar{r}, x)}{\partial s_{be}^B}}_{\to 0} \to \frac{\partial \pi^A(\bar{r}, x)}{\partial \alpha}.$$

Note that

$$\frac{\partial \pi^A(\bar{r}, x)}{\partial \alpha} = \underbrace{\frac{\int_0^{s_{be}^B} t\phi(t)\,dt}{q_s} \phi(x)\left\{2q_h \alpha x(1+\bar{r}) + 2(1 - q_h - q_h)\right\}}_{\to 0} - (2\alpha - 1)\phi(x)\left[\underbrace{q_h^0 x(1+\bar{r}) - 1}_{-}\right] > 0.$$

For Bank A, α has no effect in the case of competition upon HH, because the profit conditional on winning is close to zero. Instead, α reduces the winner's curse to Bank A with signal x—the case where competitor Bank B receives $h^B = L$.

Taken together, we have $\frac{\partial F}{\partial \alpha} > 0$. Combined with $\frac{\partial F}{\partial x} = \frac{\partial \pi^A(\bar{r}, x)}{\partial x} > 0$, implicit function theorem implies $\frac{dx}{d\alpha} < 0$. □

A.5 Derivation of Correlated Hard Signals

Another aspect of information technology advancement is that the lenders' hard information signals become more correlated. Formally, with probability ρ_h, lenders receive the same signal realization $h^c \in \{H, L\}$ and

$$\mathbb{P}(h^c = H \,|\theta_h = 1) = \mathbb{P}(h^c = L \,|\theta_h = 0) = \alpha;$$

with probability $1 - \rho_h$, each receives an independent hard signal according to Eq. (4).

With more correlated hard signals or a higher ρ^h, lenders are more likely to agree on the customer quality and so more likely to compete (the event of HH). In terms of inference, the posterior upon disagreement (that comes from the uncorrelated part of the assessment) is still the prior q_h.[26] Taken together, competition becomes fiercer, because lenders are more likely to compete but not more concerned about the winner's curse.

[26] Upon competition (HH), lenders are less sure about a good quality borrower, i.e., $\mu_{HH}(\rho_h)$ decreases in ρ_h.

A.6 Signal on Hardened Soft Fundamental

Perfectly correlated hardened soft signal. First, given the hardened soft signal $h_s^c = H$, the conditional density of soft signal s is

$$\begin{aligned}
\phi\left(s \middle| h_s^c = H\right) &= \frac{\frac{1}{ds}\mathbb{P}\left(h_s^c = H, s \in ds\right)}{\mathbb{P}\left(h_s^c = H\right)} \\
&= \frac{q_h^s q_s^s \alpha_s \phi_1(s) + q_h^s\left(1-q_s^s\right)\alpha_s\phi_0(s) + \left(1-q_h^s\right)q_s^s\left(1-\alpha_s\right)\phi_0(s) + \left(1-q_h^s\right)\left(1-q_s^s\right)\left(1-\alpha_s\right)\phi_0(s)}{q_h^s\alpha_s + \left(1-q_h^s\right)\left(1-\alpha_s\right)} \\
&= \frac{q_h^s q_s^s \alpha_s}{q_h^s\alpha_s + \left(1-q_h^s\right)\left(1-\alpha_s\right)}\left[\phi_1(s) - \phi_0(s)\right] + \phi_0(s) \\
&= \phi_0(s) + \overbrace{\frac{\alpha_s}{\alpha_s - \left(2\alpha_s - 1\right)\eta}}^{\uparrow \text{ in } \eta \text{ as } \alpha_s > \frac{1}{2}} \cdot q_s\left[\phi_1(s) - \phi_0(s)\right].
\end{aligned}$$

Note that when additionally conditioning on the good soft fundamental state $\theta_s = 1$, the hardened soft signal h_s^c and soft signal s are independent so that

$$\phi\left(s \middle| h_s^c = H, \theta_s = 1\right) = \phi\left(s \middle| \theta_s = 1\right) = \phi_1(s).$$

Hence, $h_s^c = H$ reveals information about s only through reducing type II errors.

Due to the independence between the original hard signals h^A, h^B and hardened soft h_s^c , soft signal s, we introduce notations to separate the events of signal realizations. Let $\hat{p}_{h^A h^B} \equiv \mathbb{P}\left(h^A, h^B\right)$ denote probability of the hard signal realizations, and $\hat{\mu}_{h^A h^B} \equiv \mathbb{P}\left(\theta_h^h = 1 \middle| h^A, h^B\right)$ denote the posterior probability of successful θ_h^h conditional on original hard signals. Let $p_{h_s^c}(t) \equiv \mathbb{P}\left(h_s^c, s \in ds\right)$ denote the joint density of hardened soft signal h_s^c and soft signal s, and $\mu_{h_s^c} \equiv \left(\theta_s = 1 \middle| h_s^c, s \in ds\right)$ denote the posterior probability of successful θ_s given h_s^c and s. Let $\overline{\mu}_{h_s^c} \equiv \left(\theta_s = 1 \middle| h_s^c\right)$ denote the posterior probability of successful θ_s given h_s^c. Then lender's payoff function could be rewritten as

$$\begin{aligned}
&\pi^A(r, s) \\
=&p_{HHH}(s)\left[1 - F^B(r)\right]\left[\mu_{HHH}(s)(1+r) - 1\right] + p_{HLH}(s)\left[\mu_{HLH}(s)(1+r) - 1\right] \\
=&\hat{p}_{HH}p_H(s)\left[1 - F^B(r)\right]\left[\hat{\mu}_{HH}\mu_H(s)(1+r) - 1\right] + \hat{p}_{HL}p_H(s)\left[\hat{\mu}_{HL}\mu_H(s)(r+1) - 1\right] \\
\propto&\hat{p}_{HH}\left[1 - F^B(r)\right]\left[\hat{\mu}_{HH}\overline{\mu}_H\phi_1(s)(1+r) - \phi\left(s \middle| h_s^c = H\right)\right] + \hat{p}_{HL}\left[\hat{\mu}_{HL}\overline{\mu}_H\phi_1(s)(r+1) - \phi\left(s \middle| h_s^c = H\right)\right],
\end{aligned}$$

where the "proportional to" in the last equation omits a constant $\mathbb{P}\left(h_s^c = H\right)$. Compared with the benchmark setting where hard and soft signals are independent, adding the hardening soft signal plays two roles. First, $h_s^c = H$ improves screening, and lenders are more likely to have good borrowers, as seen by the term $\overline{\mu}_H\phi_1(s)$ in the above equation versus $q_s\phi_1(s)$ in the benchmark.

Second, $h_s^c = H$ updates the distribution of the soft signal, as seen by $\phi\left(s\right| h_s^c = H)$ versus $\phi\left(s\right)$ in the benchmark; note that conditional on a good project, $h_s^c = H$ is independent and uninformative about the soft signal, so under both settings its density is $\phi_1\left(s\right)$ conditional on repayment.

Similarly, for Bank B,

$$\begin{aligned}
\pi^B\left(r\right) &\equiv \int_0^{s^A(r)} p_{HHH}(t)\left[\mu_{HHH}\left(t\right)\left(r+1\right)-1\right]dt + \overline{p}_{LHH}\left[\overline{\mu}_{LHH}\left(r+1\right)-1\right]. \\
&= \hat{p}_{HH}\int_0^{s^A(r)} p_H\left(t\right)\left[\hat{\mu}_{HH}\mu_H\left(t\right)\left(r+1\right)-1\right]dt + \hat{p}_{LH}\overline{p}_H\left[\hat{\mu}_{LH}\overline{\mu}_H\left(r+1\right)-1\right] \\
&\propto \hat{p}_{HH}\int_0^{s^A(r)} \left[\hat{\mu}_{HH}\mu_H\phi_1\left(s\right)\left(r+1\right)-\phi\left(s\right| h_s^c = H)\right]dt + \hat{p}_{LH}\left[\hat{\mu}_{LH}\mu_H\left(r+1\right)-1\right].
\end{aligned}$$