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# The Structure of Leveraged Buyouts and the Free-Rider Problem

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**Abstract**

We study the structure of public firm buyouts in a model that features both the Berle-Means problem (lack of incentives) and the Grossman-Hart problem (holdout). We find that bootstrapping, debt in excess of funding needs, and upfront fees to bidders are socially optimal and increase buyout premiums. These elements make LBO financing tantamount to a "management contract" arranged by an outside manager to receive cash and incentives to manage a firm—except the cash is funded by excess debt imposed on the firm. Our model also rationalizes why PE firms collect fees from their equity partnerships *and* directly from target firms.

JEL Classification: G34, G32.
Keywords: Leveraged buyouts, bootstrap acquisitions, tender offers, free-rider problem, debt overhang, private equity

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"Leverage" refers to the fact that the company being purchased is forced to pay for... its own acquisition... If this sounds like an odd arrangement, that's because it is. (Kosman 2012, para.8).

## 1 Introduction

Leveraged buyouts (LBOs) and private equity (PE) transformed the governance of public companies. Describing this transformation, Jensen (1989) saw LBOs as undoing the "conventional model of corporate governance—dispersed public ownership, professional managers without substantial equity holdings, a board of directors dominated by management-appointed outsiders" (p.62)—by (re)unifying ownership and control. A model which encapsulates this view needs diffuse ownership, scope for ownership structure to alter incentives, and debt.[1] The crux of such a model is that it features two manifestations of the free-rider problem. On the one hand, dispersed shareholders passively reap gains from anyone else's effort to improve firm value (Berle and Means 1932), which is why a buyout could help. On the other hand, each shareholder is individually disinclined to sell shares for less than the expected post-buyout value (Grossman and Hart 1980), which can frustrate the buyout.

In this paper, we show that a model that features both of these canonical free-rider problems predicts a combination of bootstrapping, "excessive" debt, and upfront fees as an efficient buyout structure. These three elements are the defining characteristics of many LBOs. Bootstrapping refers to the practice whereby the target company is forced to pay for its own acquisition through debt financing - in effect, the company being purchased funds its own takeover.[2] "Excessive" debt refers to leverage levels that exceed what would be needed to finance the acquisition, often reaching 60% to 90% of deal value. Upfront fees are substantial payments made directly to PE firms by target companies at the time of the acquisition, often exceeding the PE firms' own capital contributions to the deals.

[1] This corporate governance view of public-to-private LBOs dominates the accounts by Jensen (1988), Shleifer and Vishny (1990), Holmstrom and Kaplan (2001), and Kaplan and Stromberg (2009). The only existing analysis that incorporates all of the above elements is Section 6 in Müller and Panunzi (2003). Internet Appendix F compares their analysis with ours in detail.

[2] Coined by Jerome Kohlberg, Henry Kravis, and George Roberts, this term comes from the metaphor "pulling oneself up by one's bootstraps" for succeeding with few means, indicating that this buyout tactic allows bidders to gain control of firms with negligible funds of their own.

Although these features may appear to be merely financial engineering designed to extract wealth from other stakeholders, we show that they can arise as components of an efficient buyout structure.

One might think that there are obvious efficiency explanations for these three elements in the existing literature, but this is not the case. The conventional argument for LBOs—debt imposes discipline on managers and raises incentives to generate value (Jensen 1986; Innes 1990)— neither requires nor predicts them. Harnessing the incentive benefits of debt does not require bootstrapping (as noted by Müller and Panunzi 2004). Because it is not a sui generis theory of takeovers, it also does not predict buyout leverage to be generically divergent from corporate leverage outside of takeovers (as stressed by Axelson et al. 2013); in particular, it does not imply the former to be systematically higher. Last, this approach does not explain deal fees either. On the contrary, it predicts that acquirers maximize their skin in the game and hence would take larger equity stakes instead. (Internet Appendix C elaborates on these points and related arguments.)

While these elements could in principle be simply irrelevant or a matter of convenience, due to their lack of footing in standard LBO theory, they are conversely often *criticized* as LBO features that enable PE firms to extract wealth from other stakeholders. The arguments underlying this wealth transfer theory (as articulated in, e.g., Shleifer and Summers (1988) and Perotti and Spier (1993)) indeed depend on bootstrapping.[3] Bootstrap acquisitions involve two steps: A bidder creates a shell company that issues debt to fund a takeover bid. After a successful bid, the target is merged with the shell company, thereby assuming the debt that financed its own takeover. Without this second-step merger, the shell company would be a holding company that would retain the debt.[4] The gist of the wealth-transfer theory is that the second step forces other target stakeholders to share the liability for the takeover debt, thereby transferring wealth to the bidder. To LBO critics, this is the explanation for bootstrapping, *over*leveraging of targets, and safe extraction of profits through fees by PE firms—that is, all three aforementioned elements (see notably

[3]For empirical studies on such transfers, see Asquith and Wizman (1990), Ippolito and James (1992), Warga and Welch (1993), Brown et al. (2009), Billett et al. (2010), and Eisenthal-Berkovitz et al. (2020).

[4]Debt can be held in targets (OpCo debt), PE funds (FundCo debt), or in-between holding companies (HoldCo debt). HoldCo debts allow to separate fund-level and deal-level financing without bootstrapping.

Applebaum (2019, p.4), but also Appelbaum and Batt (2014, 2018) and Kosman (2009, 2012)). That none of the three elements is integral to the conventional LBO argument reinforces this suspicion.

Currently, there exists only one argument in the existing literature that offers a partial *defense* of bootstrapping. Müller and Panunzi (2004) show that bootstrapping reduces the Grossman-Hart free-rider problem by shifting wealth from target shareholders to bidders, thus promoting takeover activity. Still, conditional on a takeover, leveraging up is at best a zero-sum transfer—or worse with bankruptcy costs: Thus, high leverage is inefficient. Their theory moreover predicts that high leverage leads to low buyout premia and therefore that bidding competition decreases leverage. Both of these predictions are counterfactual (e.g., Jarrell et al. 1988; Kaplan and Stein 1993; Andrade and Kaplan 1998; Holmstrom and Kaplan 2001). Müller and Panunzi themselves acknowledge that their theory neither justifies nor predicts LBO-style debt levels.

In this paper, we *combine* the theory of tender offers with the incentive theory of LBOs. Building on Grossman and Hart (1980)'s tender offer setting, we incorporate two further features. First, as in Müller and Panunzi (2004), the bidder can raise external financing for the takeover bid. This can explain bootstrapping, but on its own, counterfactually predicts small takeover premia and cannot normatively justify high leverage. Second, as in Burkart et al. (1998), the bidder exerts unobservable effort to improve the value of the target firm. This endogenous value creation alone causes the bidder to buy as few shares as possible and to minimize value creation because the value produced by her efforts is appropriated by the free-riding target shareholders. We find that the *interaction* between these ingredients can rationalize bootstrapping, excessive debt, and upfront fees as an efficient buyout structure. In particular, the more takeover gains the bidder wants to extract through debt, the more value do buyout creditors require her to create. As a result, there is a positive equilibrium relationship between the level of debt, the stake acquired by the bidder, and value creation. Any cap on bootstrapping or leverage hence reduces the bidder's willingness to concentrate ownership and improve firm value (by more).

Because of this incentive effect, bootstrapping can actually benefit target shareholders. The equilibrium supply of debt financing is determined by a debt overhang constraint that requires a wedge between the firm's value and its debt, such that the equity is sufficiently in the money for the bidder to find it worthwhile to provide the required effort. This wedge is the expected post-takeover share value that target shareholders extract via the takeover premium. The wedge increases with the amount of takeover debt when the incentive effect of debt is sufficiently strong to dominate its wealth-transfer effect. So, while bootstrapping shifts rents to bidders, target shareholders can benefit from its indirect effect on incentives. This squares the idea of bootstrapping as a wealth transfer away from target shareholders with the evidence on large target returns in LBOs (which has in fact been viewed as prima facie evidence against said idea (e.g., Eckbo and Thorburn 2013, Section 8.4.5)).

By contrast, the equity returns cannot constitute a source of profit for the bidder since free-riding target shareholders extract those through the bid price; the equity stake merely provides her with the incentives that enable her to raise debt funding. She instead extracts her profit through "fees" that are funded by the debt levied on the firm. Hence, the bidder optimally maximizes buyout leverage and deal fees. In the optimal structure, the bidder's equity returns just equal her (opportunity) cost of effort to ensure incentive compatibility and thus debt financing, which maximizes the fees she extracts upfront. Phalippou et al. (2018) document that fees collected from target firms[5] represent a sizable part of PE firms' revenues and, on average, more than 6% of the equity invested by their PE funds. This is large considering most PE firms contribute only 1-5% of the capital in their funds; in Brown and Volckmann (2024), their average (median) contribution rate is 3.5% (2%).

The optimally financed bid in our model resembles a pitch for a management contract: A competing management team (bidder) arranges an array of contracts (bid and financing) to take over as managers of the target firm in exchange for compensation that consists of cash (fees) and equity incentives (stock or carried interest)—with the cash portion financed by imposing extra debt on the firm. These LBO properties are the same as predicted by the wealth-transfer theory, but in our theory, they are efficient.

We provide four model extensions to check the robustness of our results and to derive additional insights. First, we show that competition among bidders forces them to lever up

[5]PE firms collect these so-called transaction and monitoring fees *directly* from their target companies. These fees are distinct from the carried interest and fund management fees that PE firms collect from their partnerships with outside equity investors (c.f., Phalippou et al. 2018, Fig.1).

more, exert more effort, and create more value (Section 4). Competition thus reinforces the predicted positive link between takeover debt, bid premium, and post-takeover firm value. Second, since firm value in our baseline model is a deterministic function of bidder effort, we show that our key results are also valid when it is a stochastic function of effort (Section 5.1). Third, we formalize the moral hazard problem as private benefit extraction (Burkart et al. 1998) instead of costly effort provision (Section 5.2). In this variation, our results are reminiscent of Jensen (1986)'s free cash flow theory. Fourth, we show that our main results also hold for richer contracts between bidders, i.e., PE firms/general partners, and outside equity investors, i.e., limited partners (Internet Appendix G). In this extension, our theory can explain why PE firms collect fees both from their equity partnerships and directly from targets. It also shows that carried interest contracts can increase a bidder's debt capacity.

Last, we discuss to what extent our results apply to negotiated takeovers (Section 5.3). Deals negotiated with controlling owners or incumbent managements—in private targets, divisional buyouts, and takeover activism—depend less on the elements highlighted in our paper; in particular, they might involve less leverage, larger equity injections by PE firms, and more *post*-LBO or HoldCo debts (fn.4). Patterns along these lines are documented in Boucly et al. (2011) and Cohn et al. (2022).[6] Since these types of buyouts have grown more prevalent over time, it is also noteworthy that the use of HoldCo and FundCo debts has risen (Brown et al. 2021) and that the "excessiveness" of buyout leverage relative to public comparables has declined over the last decades (Liu and Xiong 2024).

Our paper unifies the incentive theory of LBOs and the theory on the free-rider problem in takeovers by combining two governance problems of widely held firms in a single model: managerial moral hazard (Berle and Means 1932; Jensen and Meckling 1976) and holdout behavior (Grossman and Hart 1980; Bradley 1980). The interaction of these theories yields an *efficiency* argument for controversial features of LBOs such as bootstrapping, "excess" leverage, and upfront fees.

Our theory reverses three key takeaways from Müller and Panunzi (2004), namely that

[6] Cohn et al. (2022, p.3) conclude "financial engineering is not a first-order source of value creation in private firm buyouts," and Applebaum (2019, p.3) notes, "smaller PE funds typically acquire small and medium-sized enterprises. . . [and] use relatively low levels of debt." Private firm buyouts serve different objectives than public firm buyouts (Boucly et al. 2011; Chung 2011; Cohn et al. 2022; Davis et al. 2021).

bootstrapping harms target shareholders, efficient levels of bootstrapped debt are small, and bidder competition reduces bootstrapping. The analysis most closely related to ours is Section 6 in Müller and Panunzi (2003), but it is targeted at different questions, and most importantly, does not imply that the conclusions in Müller and Panunzi (2004) are turned on their head (see Internet Appendix F).

The literature on tender offers identifies a number of mechanisms that allows bidders to exclude free-riding target shareholders from part of the takeover gains: dilution (Grossman and Hart 1980), toeholds (Shleifer and Vishny 1986), squeeze-outs (Yarrow 1985; Amihud et al. 2004), and debt (Müller and Panunzi 2004). A natural caveat of these mechanisms is that, because they harm target shareholders conditional on a bid, the latter prefer to limit their use even as doing so deters some takeovers. We identify an exception to this principle. In our model, buyout debt induces aggregate gains and can divide them to mutual benefit, such that target shareholders oppose limiting this exclusion mechanism.

As mentioned, the conventional argument for LBOs relies on the optimality of debt in standard capital structure models. The various caveats of this perspective have motivated other theories of buyout debt based on factors beyond the individual control transaction, such as the financing of PE funds (Axelson et al. 2009) or the reputation of repeat acquirers like PE funds vis-à-vis lenders (Malenko and Malenko 2015). In contrast, our LBO theory remains focused on determinants at the individual transaction level. A separate strand of theories studies the role of debt in bidding contests, which we discuss in Section 4.

Metrick and Yasuda (2010, Section 2.3) discuss that PE firms collect large transaction fees and monitoring fees directly from the targets, but note that it is not exactly clear what role they play given that PE firms can also be remunerated through management fees and carried interest fees from their equity partnerships. Phalippou et al. (2018, Section 2.3.1) speculate on the role of fees collected from the targets based on various contracting models, while noting that no incentive theory in the existing literature explicitly speaks to such fees (as opposed to, e.g., carried interest fees).

The remainder of the paper is organized as follows. Section 2 describes the model and solves for the equilibrium. Section 3 presents our main results in the single-bidder setting. Bidder competition is considered in Section 4. Section 5 presents the two model extensions where firm value is a stochastic function of bidder effort and the moral hazard problem is private benefit extraction in lieu of effort provision. It also briefly discusses what relevance our analysis has for negotiated buyouts of public firms. The Appendix contains proofs and examples for specific functional assumptions. Supplementary analyses, including the model extension with richer equity-based contracts and the comparison with Müller and Panunzi (2003), are in the Internet Appendix.

## 2 Leveraged Buyouts with Free-riding and Moral Hazard

We study a tender offer with financing in which the source of takeover gains is an improvement in incentives, while the distribution of the gains is subject to free-riding behavior. It is the first model in the tradition of Grossman and Hart (1980) in which optimal financing includes both debt and outside equity.

### 2.1 Model

**Source of takeover gains.** A widely held firm ("target") faces a potential acquirer ("bidder"). If the bidder gains control, she generates a value improvement $V(e)$ over the firm's status quo value, which is normalized to 0. Generating value requires effort $e \in \mathbb{R}_0^+$, imposing a private cost $C(e)$ on the bidder. Effort is unobservable. Current shareholders, being dispersed, lack the coordination and individual incentives to exercise control and bring about such improvements themselves (the Berle-Means problem). It does not matter for our results whether the effort is provided after the buyout or during the preparation of the bid (such as assessing target suitability and potential improvements) as long as the effort is unobservable. Our assumption that $e$ represents post-takeover effort is made purely for expositional convenience.

We assume a linear value improvement function $V(e) = \theta e$ where $\theta > 0$ is the marginal return to effort. The cost function is twice differentiable, strictly increasing, and strictly convex, i.e., $C'(e) > 0$ and $C''(e) > 0$ for all $e \geqslant 0$. We assume $C(0) = 0$, $\lim_{e\to 0} C'(e) = 0$, and $\lim_{e\to\infty} C'(e) = +\infty$ to focus attention on strictly positive, finite post-takeover values.

$V$ and $C$ are commonly known.[7] The one-to-one mapping from $e$ to $V$ in our model allows for indirect contracting on $e$. We ignore this possibility and view modeling deterministic $V$ as a simplification. The issue is less salient if we model $e$ as ex ante effort prior to a bid, and absent if we model $V$ as a random variable whose distribution depends on $e$ (which we do in Section 5.1).

**Distribution of takeover gains.** To gain control, the bidder must purchase at least half of the target shares by way of a tender offer. The incumbent management is assumed to be unwilling or unable to counterbid; alternatively, it may be part of the investor group that makes the offer to buy out the current shareholders.

Each target shareholder is non-pivotal for the takeover outcome. The consequent free-riding behavior frustrates the takeover unless the bidder can exclude target shareholders from part of the takeover gains (the Grossman-Hart problem). We focus on the exclusion mechanism identified by Müller and Panunzi (2004): debt collateralized by target assets. Since debt is senior, shareholders are excluded from future cash flow pledged to the lenders, while the bidder extracts the present value of those cash flows in the form of a loan prior to the bid.

Specifically, the bidder is wealth-unconstrained but can nonetheless raise equity or debt funding for the bid from outsiders. She can choose to pledge a fraction $(1 - \gamma) \in [0, 1]$ of the cash flow from the acquired target shares to outside investors in exchange for some amount $F^E$ of equity financing, and promise outside creditors a debt repayment $D \geqslant 0$ in exchange for some amount $F^D$ of debt financing. We abstract from exclusion mechanisms other than debt, so a profitable bid requires $F^D > 0$ and bootstrapping. We can ignore "non-bootstrapped" debt without loss of generality.

We assume risk-neutrality and zero discount rates for all agents.

**Sequence of events.** There are three stages. In stage 1, the bidder makes a take-it-or-leave-it cash bid to acquire target shares at a price $p$ per share and chooses how to finance

[7] Assuming linear $V$ is without loss of generality; all results can be translated to concave $V$. Suppose $V : [0, +\infty) \to \mathbb{R}$ is a twice differentiable, strictly increasing, and concave function. The game we consider is isomorphic to a game in which the bidder chooses $y$ (instead of $e$) with $\theta y = V(e)$. In the latter game, the bidder's post-takeover objective function is $\alpha[\theta y - D]^+ - C(V^{-1}(\theta y))$, where $V^{-1}$ denotes the inverse function of $V$. Since the inverse of a strictly increasing, strictly concave function is a strictly increasing, strictly convex function, the composition $C \circ V^{-1}$ satisfies the assumptions postulated for $C$ in our model.

the bid. The financing is publicly observable. The bid is conditional, that is, it becomes void if less than half of the shares are tendered.

In stage 2, target shareholders non-cooperatively decide whether to tender their shares. The shareholders are homogeneous and atomistic such that no one is pivotal. Concretely, we assume that a unit mass of shares is dispersed among an infinite number of shareholders whose individual holdings are equal and indivisible. Shareholder $i$'s tendering strategy maps the offer terms into a probability that she tenders her shares, $\beta_i : (\gamma, D, p) \to [0, 1]$. It is without loss of generality to focus on symmetric strategies and drop index $i$. So, by the law of large numbers, $\beta$ shares are traded in a successful bid.

In stage 3, if less than half the shares are tendered, the takeover fails. Otherwise, the bidder pays $\beta p$ for the fraction $\beta$ of shares tendered and gains control. Net of the fraction $1-\gamma$ financed by outside investors, the bidder then owns an "inside" equity stake $\alpha \equiv \gamma\beta$, and chooses her effort level $e \geqslant 0$ to maximize her post-takeover payoff $U(\alpha, D, e)$. So, her post-takeover strategy is a function $e : (\alpha, D) \to \mathbb{R}^+$. Last, firm value and all payoffs are realized (see Figure 1).

**Interpretation.** An LBO is carried out by a group of investors that may comprise a PE firm and incumbent management, or a consortium of PE firms. These investors take large equity positions in the target and active roles in management or on the board (Kaplan and Stromberg 2009, p.130f). In our model, they are represented by the "bidder" whose cost of effort represents the (opportunity costs of) time and effort invested by those agents.

PE firms raise equity funding for the buyouts through PE funds. This funding typically comes from large institutional investors such as pension funds, endowments, and insurance companies (Kaplan and Stromberg 2009). These *limited partners*—unlike PE firms who are *general partners*—assume no active role in the target firms and are represented by the "outside equity investor" in our model.

When a specific buyout deal materializes, PE firms contribute some of the capital from the PE funds as equity to finance the buyout. This equity financing is complemented with debt financing. The debt makes up the lion's share of the funds, covering 60 to 90 percent of the buyout value (Kaplan and Stromberg 2009). The parties providing the debt funding

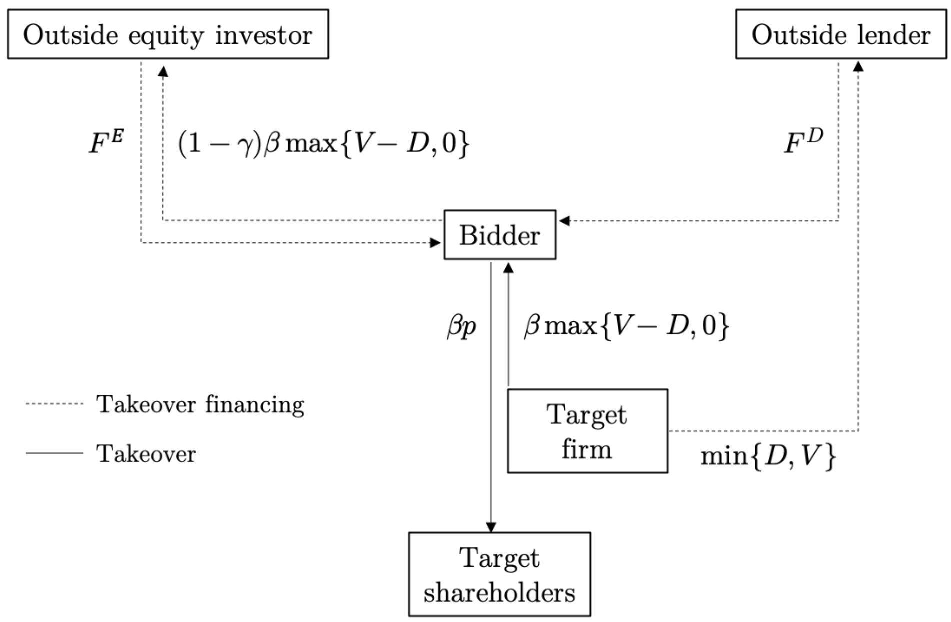


Figure 1: This summarizes the payments vis-à-vis a successful bidder in our model. Consider a management buyout as illustration: Incumbent managers and a PE firm together are the "bidder," limited partners in the buyout fund are the "outside equity investor," and bondholders or a loan syndicate are the "outside lender." Debt funds being disbursed to the bidder but repaid directly by the target firm is the key effect of bootstrapping.

are the "outside lender" in our model.

The debt is raised at *deal* (rather than fund) level. This allows it to be collateralized by the assets of the *target* through a bootstrap acquisition: In a first step, a shell company is created and funded from the aforementioned sources of buyout financing to bid for a majority of the target shares. If the bid succeeds, the second step merges the target with the shell company such that the former's assets are matched with the latter's debt. Consequently, all equity investors receive, in our model notation, parts of $[V(e) - D]^+$. Without the second step, shell company shareholders and target shareholders would instead receive $[\beta V(e) - D]^+$ and $(1-\beta)V(e)$, respectively.

How much equity the *active* investor group acquires in the merged company depends on the fraction $1-\gamma$ of outside equity financing and the fraction $\beta$ of shares tendered by the initial target shareholders: $\alpha = \gamma\beta$. Unless $\alpha = 1$, the roles of $\beta$ and $\gamma$ are partially interchangeable; though, a given $\gamma$ implies $\alpha \geqslant \gamma/2$, as a successful bid requires $\beta \geqslant 1/2$.[8]

[8]This is why constructing $\alpha$ from $\gamma$ and $\beta$ matters: Without $\gamma$, $\alpha$ has a lower bound of $1/2$. Not only is this counterfactual but it creates an artificial kink at $\alpha = 1/2$, which makes the model less tractable.

With $\gamma \in [0,1]$, the bidder can implement any $\alpha \in [0,1]$. In going-private buyouts, every initial shareholders is bought out ($\beta = 1$); in cash-outs, some of them retain their shares ($\beta < 1$). Distinguishing these cases is not important as only $\alpha$ matters for our results.

Our model allows for $\alpha$ to be fully chosen at the deal level and for $\alpha \to 0$, which will be optimal if $D \to 0$. In practice, outside equity financing is typically raised through capital commitments to PE funds before specific deals materialize. Moreover, PE firms or general partners ("bidder") and limited partners ("outside investors") are not given simple equity shares, but the former receive contingent payments once the returns to the latter meet a pre-specified threshold. Internet Appendix G shows that introducing richer contracts between the bidder and outside equity investors does not alter the central insights. Our baseline model prediction that low $D$ push the optimal $\alpha$ (and so the value improvement) to 0 is best interpreted to the effect that a buyout without debt is not lucrative.

**Optimality of debt.** We should spell out what feature of debt makes its use optimal in our model. In the second step of the bootstrap acquisition—the merger—debt reduces the target's expected share value from $V(e)$ to $[V(e) - D]^+$ by virtue of taking priority over equity. In effect, this rolls part of the debt burden otherwise carried only by the bidder and her equity co-investors onto target minority shareholders. This dilution of the claims of those shareholders who retain shares in the first step—the tender offer—overcomes the free-rider problem. The dilution-by-priority effect of debt has also been noted in the context of bargaining between firms and labor unions (Bronars and Deere 1991; Perotti and Spier 1993). Priority is necessary and sufficient for debt to play this role. Since this is the only property of debt crucial to our results, abstracting from more complex securities (e.g., convertible debt) is without loss of generality. What matters in our setting is that, at the end of stage 3, there are buyout financiers (i.e., the lenders) who are paid out before target shareholders.

Understanding the role of debt matters for applying our model in the right context. Our result on the efficiency of buyout debt and the prediction of "excessive" leverage hold to the extent that bidders rely on debt to extract takeover gains. Hence, leverage should be less excessive for private targets where price bargaining allocates gains and debt would play only its classic incentive role.

## 2.2 Equilibrium

We solve the model by backward induction in three subsections corresponding to the stages of the game. We focus on buyout debt $D$ and the bidder's stake $\alpha$, which characterize the post-buyout capital and ownership structure. Unlike in a standard financing model, there are no wealth constraints that call for outside funds. Still, the bidder will resort to outside funding due to an interaction between effort choice, tendering decisions, and financing (in fact, to an extent that wealth constraints would not bind, as we will show). For all lemmas and propositions throughout the paper that are not fully derived in the text, the proofs are in Appendix A.

### 2.2.1 Value Creation

After a successful bid, the bidder's equity stake is $\alpha$ and the target assumes the acquisition debt (of face value) $D$. The bidder then chooses effort $e$ to maximize the value of her equity stake in the levered firm net of private effort costs, $U(\alpha, D, e) \equiv \alpha[V(e) - D]^+ - C(e)$.

This objective function is not globally concave in $e$. Let $e_D$ satisfy $V(e_D) = D$. For $e \in [0, e_D)$, equity is "out of the money" because $V(e) < D$, and so $U(\alpha, D, e) = -C(e)$ which is strictly decreasing in $e$. For $e \geqslant e_D$, $U(\alpha, D, e) = \alpha[V(e) - D] - C(e)$ since equity is "in the money." Under our assumptions about $V$ and $C$, this is strictly concave and the first-order condition, $\alpha V'(e) = C'(e)$, has a unique, strictly positive solution, hereafter denoted by $e^+(\alpha)$.

Because $U(\alpha, D, e)$ is not globally concave, $e^+(\alpha)$ need not be a global optimum. Specifically, given that $\frac{\partial U}{\partial e} < 0$ for $e \in [0, e_D)$, it is possible that $U(\alpha, D, e^+(\alpha)) < 0$. If so, the bidder's optimal effort is $e = 0$. To summarize the above arguments:

**Lemma 1.** *The bidder's optimal effort is* $e^*(\alpha, D) = e^+(\alpha) > 0$ *if*

$$\alpha[V(e^+(\alpha)) - D] - C(e^+(\alpha)) \geqslant 0 \tag{1}$$

*where $e^+(\alpha)$ is the solution to*

$$\alpha V'(e^+(\alpha)) = C'(e^+(\alpha)) \tag{2}$$

*Otherwise, she makes no effort to improve target firm value, i.e., $e^*(\alpha, D) = 0$.*

Lemma 1 replicates established wisdom within our takeover setting. Too much leverage leads to a debt overhang problem that undermines a (controlling) shareholder's incentives to improve firm value (Myers 1977). Here, this occurs when condition (1) is violated. Value creation incentives also decrease with the fraction of equity that is dispersedly held (Berle and Means 1932; Jensen and Meckling 1976). Conversely, ownership concentration increases firm value: Conditional on (1), the optimal effort $e^+(\alpha)$ and resultant firm value $V(e^+(\alpha))$ increase in $\alpha$ (by the envelope theorem).

The novel element of Lemma 1 is that the two effects interact in condition (1). Whether a debt overhang problem emerges depends not only on the debt level $D$ but also on the level of ownership concentration $\alpha$. The intuition is simple: The bidder's incentives derive from a *levered* equity *stake* $\alpha[V(e^+(\alpha)) - D]$. While $D$ lowers the total value of equity, $\alpha$ determines the bidder's share of that total value. Consequently, the firm can maintain more debt without eroding the bidder's incentives when the latter owns more of its equity. This interaction between $\alpha$ and $D$ is key to our results.

### 2.2.2 Tendering Decisions

As Lemma 1 indicates, the only post-takeover ownership and capital structure that ensures first-best incentives is fully concentrated ownership without any leverage: $(\alpha, D) = (1, 0)$. In an ideal market for corporate control, the bidder can restore this structure. We discuss next how free-riding by the dispersed target shareholders distorts the bidder's preferences regarding $\alpha$ and $D$.

Suppose target shareholders face a cash bid $p$ (partly) financed with debt $D$. Being non-pivotal, an individual shareholder tenders only if $p \geqslant V(e^*(\hat{\alpha}, D))$ where $\hat{\alpha}$ denotes her beliefs about the bidder's post-takeover equity stake. Because tendering decisions depend on individual beliefs, no dominant strategy equilibrium exists. In a rational expectations equilibrium, beliefs are consistent with the outcome, so shareholders tender only if

$$p \geqslant [V(e^*(\alpha, D)) - D]^+. \tag{3}$$

That is, target shareholders tender their shares only if they extract at least the full increase in share value the bidder will generate. This is known as the free-rider condition.

Previous work has analyzed two special cases of (3). Müller and Panunzi (2004) study a model with exogenous post-takeover values where (3) becomes $p \geqslant (V - D)^+$ and show that the bidder maximizes $D$. Burkart et al. (1998) study a model with endogenous post-takeover values but without debt where (3) reduces to $p \geqslant V(e^*(\alpha, 0))$, and show that the bidder minimizes $\alpha$. These results share a common logic: the bidder aims to reduce the right-hand side of (3), i.e., the post-takeover share value that target shareholders extract through the price. As we shall see, a model in which $D$ and $\alpha$ are jointly chosen overturns some of the key predictions of the aforementioned papers.

Before we derive the stage-2 subgame equilibrium, note that (3) is merely a necessary condition for a successful bid; a failed bid, in which an insufficient number of shares is tendered, can always be supported as a self-fulfilling equilibrium outcome. To focus on the interesting case, we assume shareholders tender whenever the free-rider condition is weakly satisfied, thus selecting the Pareto-dominant success equilibrium whenever it exists.

Let the post-takeover share value the bidder will create for a given stake $\alpha$ and debt $D$ be denoted by $E(\alpha, D)$, and her *equilibrium* post-takeover equity stake by $\alpha^*(p, D)$. Since a successful bid implies that $\beta \in [1/2, 1]$ shares are tendered, the bidder's post-takeover stake $\alpha$ lies in the interval $[\gamma/2, \gamma]$ for a given outside equity financing share $1 - \gamma$. Hence, the post-takeover share value must lie between $E(\gamma/2, D)$ and $E(\gamma, D)$. In the subsequent lemma, we omit describing the subgame equilibrium for bids that we can rule out a priori: bids that fail for any set of beliefs ($p < E(\gamma/2, D)$) and bids that could be undercut without affecting any other decision ($p > E(\gamma, D)$).

**Lemma 2.** *Any bid $p \in [E(\gamma/2, D), E(\gamma, D)]$ succeeds, and $\alpha^*(p, D) = \alpha_p$ where $\alpha_p$ satisfies $p = E(\alpha_p, D)$.*

Target shareholders tender shares until the expected post-takeover share value, which increases with the bidder stake, matches the bid price. Thus, as in Burkart et al. (1998), supply is upward-sloping: the fraction of shares tendered increases with the bid price. In equilibrium, the bidder endogenously ends up with a stake for which the free-rider condition (3) holds with equality.

### 2.2.3 Bid and Financing

The bidder's ex ante profit is $\alpha E(\alpha_p, D) - \beta p - C(e) + F^E + F^D$. It comprises the value of the equity stake she expects to acquire, less takeover payment and effort cost, and outside funds she raises for the bid. She maximizes this by choosing the bid $p$, outside equity financing $\{\gamma, F^E\}$, and debt financing $\{D, F^D\}$ subject to (1), (2), (3), and the following participation constraints: Outside equity investors demand

$$F^E \leqslant \beta(1-\gamma)E(\alpha_p, D). \tag{4}$$

Outside lenders demand $F^D \leqslant \min[D, V(e)]$. Since debt overhang constraint (1) requires $V(e) > D$, this reduces to

$$F^D \leqslant D. \tag{5}$$

We assume perfect competition among outside financiers such that they only break even. Hence, (4) and (5) hold with equality. Substituting these binding participation constraints in the bidder's ex ante profit yields $\beta[E(\alpha, D) - p] - C(e) + D$.

Recall from Lemma 2 that free-rider condition (3) endogenously binds; shares will be tendered until $E(\alpha_p, D) = p$. Recall further from Lemma 1 that, subject to (1), the post-takeover effort $e^+(\alpha)$ satisfies (2). To demarcate the novel element in our analysis from existing results, we first state how these two constraints—binding free-rider condition (3) and first-order condition (2) for effort—affect the bidder. Plugging these constraints into her ex ante profit gives

$$D - C(e^+(\alpha)). \tag{6}$$

This replicates the known insights that debt $D$ enables the bidder to extract private gains and that a larger equity stake $\alpha$ is unattractive because it induces her to incur higher effort costs, while all gains in share value accrue to target shareholders. This also shows that the bidder's ex ante problem essentially reduces to choosing the post-buyout ownership and capital structure $(\alpha, D)$.[9]

The novel element is the restriction debt overhang constraint (1) imposes jointly on $D$ and $\alpha$. This constraint cannot be slack in equilibrium. The bidder could otherwise lower $\alpha$ while preserving $D$. This would increase her profit, as (6) shows. Using binding constraint (1) to replace $D$ in (6) reduces the bidder's stage-1 choices to a univariate problem:

$$\max_{\alpha \in [0,1]} \mathcal{W}(\alpha) - \frac{C(e^+(\alpha))}{\alpha} \tag{P}$$

where $\mathcal{W}(\alpha) \equiv V(e^+(\alpha)) - C(e^+(\alpha))$ is the total surplus created by the buyout. In Section 3, we use this representation of the problem to explain the role of debt. We conclude this section by establishing equilibrium existence (though not uniqueness).

**Lemma 3.** *If the bidder's profit under (P) is negative, she makes no bid. Otherwise, she succeeds with a bid such that (1)-(5) bind and $\alpha$ solves (P).*

## 3 Bootstrapping, Leverage, and Upfront Fees

Before deriving our main results, it is worth reiterating that there is no wealth constraint in our model; the bidder is capable of achieving the first-best outcome by fully self-financing the bid. Frictions in the buyout process keep her from doing so. Our results concern how financing affects this process—not only the post-buyout capital structure—making this a theory of *buyout* debt.

We will make statements about the *causal* effect of bootstrapping by using the thought experiment of an *exogenous* limit on bootstrapped debt (Propositions 1, 2, and 5). The main normative insight is that such a restriction is inefficient even though bootstrapping is a rent extraction strategy. The positive predictions are that buyout debt is bootstrapped, "excessive," and beneficial not only to the bidder via upfront fees but likely also to target

[9] This is why it is without loss of generality to abstract from cash-equity bids and restricted bids. The same objective function obtains (i) for cash-equity bids with $1 - \alpha$ being the fraction of post-takeover equity offered to target shareholders as payment combined with cash or (ii) for cash bids in which the number of shares the bidder offers to acquire is restricted to $\alpha$.

shareholders via larger takeover premiums.

### 3.1 Ownership-Debt Relationship

We first consider how bootstrapping affects total surplus $\mathcal{W}(\alpha) = V(e^+(\alpha)) - C(e^+(\alpha))$. Although this expression depends only on the bidder's equity stake $\alpha$, the latter is linked to debt $D$ through debt overhang constraint (1). This constraint binds in equilibrium, yielding

$$D = V(e^+(\alpha)) - \frac{C(e^+(\alpha))}{\alpha}. \tag{1*}$$

As shown in the proof of the next result, (1*) defines $D$ as a strictly increasing function of $\alpha$. Intuitively, to avoid a debt overhang problem, a higher debt level $D$ requires a larger bidder stake $\alpha$. The latter leads in turn to a higher surplus $\mathcal{W}(\alpha)$.[10]

Now imagine some hypothetical exogenous cap $\bar{D}$ on the amount of debt. Our formal result interprets how removing this limit affects $\mathcal{W}(\alpha)$ as the causal effect of bootstrapping on takeover surplus.

**Proposition 1.** *Bootstrapping increases takeover surplus.*

This result is not obvious as the primary aim of bootstrapping is to transfer rents from target shareholders to bidders. In Müller and Panunzi (2004), conditional on a bid, bootstrapping is a zero-sum transfer, or inefficient when there are exogenous bankruptcy costs. The interaction of the free-rider problem with the moral hazard is key to Proposition 1.

On the equity side, the fact that owning a larger stake creates stronger incentives to create value is a *dis*incentive to buy shares when faced with the free-rider problem. While the bidder is more incentivized to provide effort when acquiring a larger stake, the target shareholders appropriate the added value through the bid price. All else equal, the bidder hence prefers low $\alpha$.

On the debt side, the supply of funds depends on the value lenders expect to be created. To raise more debt, the bidder must commit to generate more value. A larger equity stake provides that commitment, as captured in the ownership-debt function. If this demand

[10] The inverse interpretation is that takeover debt makes bidders willing to buy more equity. We prefer the first interpretation in light of the bidder's profit function (6), whereby she would at the margin want to increase $D$ and decrease $\alpha$ (were it not for debt overhang constraint (1*)).

for commitment prevails over the bidder's preference for low $\alpha$, debt is used in equilibrium. A cap on takeover debt would impede this indirect benefit of bootstrapping on incentives.

Empirically, following leveraged buyouts, managers own more equity and active owners dominate boards (Kaplan 1989). Our theory posits that the lenders' willingness to provide debt depends on how much "skin in the game" such inside shareholders assume in the firm. We are undaware of empirical evidence that speaks directly to this mechanism.[11] However, there is evidence in another context consistent with it. Anderson, Mansi, and Reeb (2003) find that founding family ownership in public firms is associated with lower costs of debt, suggesting reduced debt-equity conflicts as a reason. Lagaras and Tsoutsoura (2015) find similar effects in a natural experiment. They also document that for 17% of family firms in their data, lenders explicitly require the founding family to maintain a certain percentage of ownership or control.

## 3.2 Debt (Constraints) as Sharing Rule

We now turn to how the surplus $\mathcal{W}(\alpha)$ is split between the bidder and target shareholders. The ownership-debt function (1*) pins the equity value down as a "wedge" that must be kept between firm value and debt to avoid debt overhang: $V(e^+(\alpha)) - D = \frac{C(e^+(\alpha))}{\alpha}$. This reveals that the bidder's profit in (P),

$$\mathcal{W}(\alpha) - \frac{C(e^+(\alpha))}{\alpha},$$

equals total surplus less the wedge, which target shareholders extract through the price. How the wedge varies with $\alpha$ determines how increases in $\mathcal{W}(\alpha)$ are allocated.

There are two opposing effects. Holding the numerator fixed, $\frac{C(e^+(\alpha))}{\alpha}$ decreases in $\alpha$. This reflects that blockholder incentives depend on equity concentration and total equity

[11] This is a *causal* statement: in a *given* deal, creditors lend less if insider equity is *exogenously* reduced. This does not imply a positive correlation of buyout debt with post-buyout inside ownership in a cross-section of buyouts. As regards the latter, note that we do not present comparative statics. Equilibrium debt levels and the division of gains between bidder and target shareholders depend on the curvatures of $V$ and $C$ in non-trivial ways. Consequently, the comparative statics generate no clear-cut results. Hence, while we make clean statements about *causal effects* of bootstrapping, clean statements do not exist for *cross-sectional correlations* between takeover debt and other observables (such as, e.g., bid premia) driven by variation in $V$ or $C$ across deals in the data. Section E of the Internet Appendix illustrates this using an example that parametrizes $V$ as a power function.

value: active shareholders with larger stakes can dilute total equity value more without creating debt overhang problems.

By contrast, holding the denominator fixed, $\frac{C(e^+(\alpha))}{\alpha}$ increases in $\alpha$ through $C(e^+(\alpha))$. That is, the increase in equilibrium effort moderates dilution. If the bidder buys a larger equity stake as an incentive to improve firm value more, any parallel increase in debt must not undermine the required higher effort.

Target shareholders benefit from bootstrapping when the latter effect dominates. This requires equilibrium effort $e^+(\alpha)$ to be sufficiently elastic, which in turn requires that the cost function $C$ is not *too* convex. Our next result derives a sufficient condition for this to be the case, while considering how target shareholders would be affected by the removal of a hypothetical exogenous limit $\bar{D}$ on bootstrapped debt.

**Proposition 2.** *Bootstrapping increases takeover premia if $C$ is log-concave.*

In Müller and Panunzi (2004), bootstrapped debt lowers takeover premia and target shareholders may want restrictions on bootstrapping (or buyout leverage). This reflects a general point in the theory of tender offers: target shareholders prefer limits to exclusion even if that frustrates some potential bids. To our knowledge, Proposition 2 identifies the only exception to this principle. Under the stated condition, target shareholders oppose *any* restriction on bootstrapping or takeover debt.[12]

More than of pure theoretical interest, Proposition 2 squares the idea of bootstrapping as rent extraction with empirically high target returns in LBOs (e.g., Jensen 1988). Hence, the fact that takeover premiums are large and target shareholders fare well in LBOs does not disprove Müller and Panunzi (2004)'s thesis that bootstrapping is a mechanism to shift gains from target shareholders to bidders (c.f., Eckbo and Thorburn 2013, Section 8.4.5). Appendix B.1 shows examples with specific $C$ functions where high leverage ratios benefit target shareholders.

Like Proposition 1, Proposition 2 is a consequence of endogenous value creation. The

[12] Log-concavity is not a very restrictive condition and met by, inter alia, power functions $C(e) = \frac{c}{n}e^n$ and exponential functions $C(e) = \exp(e) - c$. It is tighter than needed in the sense that target shareholders can benefit even if $C$ is not *globally* log-concave. When $C$ becomes too convex, the limit $e^{+\prime}(\alpha) \to 0$ is a model with exogenous costs and values (Müller and Panunzi 2004). If we allow concave value improvement functions, an analogous condition exists for the concavity of the bidder's post-takeover objective function.

crux is that the incentive problem constrains debt—but this plays a different role than in standard financing theories where the constraint measures up against a need for outside funds. Here it determines what a bidder must leave on the table for target shareholders. Intriguingly, the incentive constraints on $D$ impose a "sharing rule" for the incentive gains from $\alpha$ such that bootstrapping can be Pareto-improving. This is the case when incentives are very sensitive to the change in ownership structure (i.e., when $C$ is not so convex as to make optimal effort too inelastic with respect to changes in $\alpha$).

### 3.3 Fees and Bidder Compensation Structure

We trace out through what channel the bidder extracts her share of the surplus $\mathcal{W}$. This is not obvious since target shareholders receive the full increase in share value, $V - D$, on any shares they retain or sell. (Outside investors merely break even.) How can the bidder make a profit if target shareholders get the full appreciation on all sold shares? The only possibility is that she does not fully pay for her stake out of her own pocket.

The bidder's financing contribution is $I_B \equiv \beta(V - D) - F^E - F^D$, where $\beta(V - D)$ is the total takeover payment, $F^E$ is outside equity funding, and $F^D$ is debt funding. By (4) and (5), which are binding in equilibrium, $F^E = \beta(1 - \gamma)(V - D)$ and $F^D = D$. Netting out outside equity, $I_B = \alpha(V - D) - D$, where $\alpha(V - D)$ is the value of the stake going to the bidder. Indeed, buyout debt $D$ lets her pay less than the value of the stake she gets.

This in itself is consistent with the bidder contributing, on balance, a positive amount of funding. However, in equilibrium, it turns out that she cashes out upfront, i.e., $I_B < 0$. By (1*), $I_B = C(e^+(\alpha)) - D$, which is the negative of bidder profit (6).

**Proposition 3.** *The bidder's net financing contribution is negative.*

The bidder cannot extract profits through her equity stake because the value of that stake is extracted by the target shareholders through the bid price. Instead, she extracts it by cashing out upfront—a cash-out financed by taking on debt that decreases the future free cash flow to equity.

The endogenous limit to this extraction is that equity cannot be diluted so much that a debt overhang arises. Thus, the equilibrium level of dilution is such that the incentives to create value are *just* preserved, as captured by the binding debt overhang constraint (1*): $\alpha(V(e^+(\alpha)) - D) = C(e^+(\alpha))$. This constraint shows that, in the optimal structure, the value of the bidder's equity is diluted until it just covers her effort cost—in fact, inducing that effort is the sole purpose of the equity stake. The bidder must in equilibrium get that stake *for free* to break even. Thus, for her to find the takeover profitable, outside funding must exceed the acquisition price to further finance upfront payouts to the bidder. These upfront fees are her profit from accepting the incentives provided by the equity stake.

As a result, the bidder's compensation structure has two components: (i) target equity she is given for free, akin to stock compensation, incentivizing her to exert the effort that outside investors bank their participation on; plus (ii) upfront fees, akin to a fixed salary, which is equal to her equilibrium rent. In Internet Appendix G, where we allow for richer contracts between the bidder and outside equity investors, this translates into a dual-fee structure under which the bidder collects performance-based fees from its partnership with outside equity investors (which depend on the equity returns), and separately, fees directly from the target (which reduces the equity returns), e.g., through transaction or monitoring fees. In some ways, this is a wash. For example, Metrick and Yasuda (2010, p.2320) write,

> While it may seem odd that funds are effectively paying themselves a fee to run companies that they own, the sharing rules with LPs can make this an indirect way for the LPs to pay the GPs for their services. From the perspective of the LPs, it should not matter whether these payments come directly through management fees or indirectly through monitoring fees, as long as the GP can create sufficient value to justify them.

Consistent with this, in practice, the fees PE firms collect directly from the target firms are sometimes partially offset by reductions in the management fees they collect from their equity partnerships. Indeed, it matters little to the outside equity investors in our model, incentive compatibility provided. However, the dual-fee structure is crucial for the bidder's ability to profit. The equity-based performance fee provides her with the incentives needed to attract outside funding. This, in turn, enables her to use debt financing to extract gains from target shareholders through upfront fees. Crucially, the ability to profit through the latter set of fees is a prerequisite for the bidder to self-impose incentives through the former set of fees. (A cap on upfront fees, like a cap on bootstrapping, is therefore inefficient.)

The LBO financing the bidder puts together thus amounts to a "management contract" whereby she gets herself "hired" by passive (debt and equity) investors to take over as the manager of the target firm for performance fees plus upfront fees. The fact that the bidder profits despite contributing no net financing is analogous to a manager earning returns to human capital, and the upfront fees (or equilibrium rent) can be interpreted as a price for her talent.

Empirically, upfront fees are common in leveraged buyouts. Müller and Panunzi (2004) cite the 1986 Revco deal where the upfront fees of $54.4 million exceeded the acquisition company's equity of $35 million (Wruck 1997), and the 1989 RJR Nabisco deal where fees amounted to $780 million (Burrough and Helyar 1990) while KKR & Co., the buyout firm behind the deal, was said to have contributed only $15 million to the deal (Knight 1988). The most comprehensive study of fees paid to PE firms directly by target companies comes from Phalippou et al. (2018). As mentioned in our introduction, their findings imply that these fees make up a significant portion of PE firms' revenues and are substantial compared both to PE firms' own capital commitments and to the fees they collect from their equity partnerships. Metrick and Yasuda (2010, Section 2.3) discuss transaction and monitoring fees and estimate that transaction fees alone typically amount to 1-2% of *total* deal value. Again this is sizable considering that equity usually covers only 20-40% of total deal value, and PE firms tend to contribute only 1-5% of that equity.

# 4 Bootstrapping and Bidding Competition

In Müller and Panunzi (2004), buyout leverage decreases when multiple bidders compete, a result that hightlights its role as a mechanism to extract gains from target shareholders. In our model, this result is reversed because the ability to extract gains increases bidders' willingness to create value. Competition induces a bidder to adopt stronger value-creation incentives along with *more* debt.

Consider two bidders who may differ in their value improvement or cost functions. To gain control of the target in this setting, a bidder must outbid her rival with an offer price that satisfies the free-rider condition.

### 4.1 Bootstrapping Increases Reservation Prices

Without loss of generality, consider bidder 2. If she succeeds, her effort will satisfy first-order condition (2) and target shareholders will tender such that free-rider condition (3) strictly binds (Sections 2.2.1 and 2.2.2). Thus, (6) still applies; bidder 2's profit can be written as $D_2 - C_2(e_2^+(\alpha_2))$.

We can characterize all offers under which bidder 2 would break even by

$$D_2 = C_2(e_2^+(\alpha_2)). \tag{7}$$

By definition, target shareholders receive the whole surplus under a break-even offer; so the break-even prices are equal to $\mathcal{W}_2(\alpha_2)$. As $\mathcal{W}_2(\alpha_2)$ is strictly increasing, bidder 2's reservation price $p_2^o$ is the break-even price under the largest $(\alpha_2, D_2)$ that is both feasible and satisfies (7), hereafter denoted by $(\alpha_2^o, D_2^o)$.

To state the causal effect of bootstrapping on the bidder's reservation price, consider an exogenous limit $\bar{D}$. If $\bar{D} < D_2^o$, the limit moves the reservation price from $\mathcal{W}_2(\alpha_2^o)$ to $\mathcal{W}_2(\bar{\alpha}_2)$ where $\bar{\alpha}_2$ solves (7) for $D_2 = \bar{D}$. Since $\alpha_2$ and $D_2$ are positively linked in (7), the new reservation price is lower, making bidder 2 a "weaker" competitor.

**Proposition 4.** *Bootstrapping strengthens competition.*

Recall that neither bidder is wealth-constrained; the role of debt financing here is not that it makes it *possible* to pay more. Its role in break-even condition (7) is to compensate bidder 2 for costs. The bidder's ability to recoup costs drives how much value she is *willing* to create, which in turn determines her reservation price.

### 4.2 Competition Increases Buyout Debt

Without loss of generality, consider bidder 1. We will first show that she does not exhaust her debt capacity in the absence of competition. Without competition, she maximizes (6) subject to (1*), that is, she solves

$$\max_{\alpha_1 \in [0,1]} D_1(\alpha_1) - C(e_1^+(\alpha_1)), \tag{8}$$

where $D_1(\alpha_1)$ is her ownership-debt function, as defined by (1*). Her maximum debt capacity, by contrast, is found by maximizing $\alpha_1$ subject to $D_1(\alpha_1) - C(e_1^+(\alpha_1)) = 0$ or is the corner value $D_1(1)$. Hence, when the solution to (8) involves $\alpha_1^* < 1$ and a strictly positive profit, bidder 1 raises less debt than she maximally could without making a loss. This is, for example, always the case when $C$ is a power function (see Appendix B.1).

**Lemma 4.** *Absent competition, bidders do not generally exhaust their debt capacity.*

Intuitively, this is a consequence of Proposition 2: If target shareholders capture part of the incentive gains due to takeover debt, bidders will generally not maximize their debt capacity. This begs the question how they adjust debt in response to competition.

Let bidder 1's optimal bid $(\alpha_1^*, D(\alpha_1^*), p_1^*)$ absent competition be profitable and feature $\alpha_1^* < 1$, so she has unused debt capacity. Without loss of generality, let bidder 1 have the higher reservation price and win. Under competition, her optimal bid must jointly satisfy debt overhang constraint (1), effort optimality condition (2), free-rider condition (3), and the competition constraint:

$$p_1 \geqslant \overline{p}_2 \tag{9}$$

We assume $\overline{p}_2 > p_1^*$, so competition is effective.

Suppose her optimal bid exactly matches bidder 2's reservation price, so (9) binds.[13] Focusing on interior solutions, where bidder 1 gets $\alpha_1 < 1$ shares, recall from Lemma 2 that free-rider condition (3) binds endogenously. (We cover corner solutions in the proof of the next result.) Substituting (2) and a binding (9) into a binding (3) yields

$$D_1 = V(e_1^+(\alpha_1)) - \overline{p}_2. \tag{10}$$

This identifies $(\alpha_1, D_1)$-pairs that take into account every optimality condition except (1).

[13] Since the objective function in (P) can be non-monotonic in $\alpha$, it is possible that bidder 1 wants to pay *strictly* more than $\overline{p}_2$. The arguments that follow in the text can also be applied to such cases with $\overline{p}_2$ replaced by $\overline{p}_2^+ = \overline{p}_2 + \Delta$ for some $\Delta > 0$.

With target shareholders' payoff fixed at $\overline{p}_2$, bidder 1's profit subject to (10) is

$$\mathcal{W}_1(\alpha_1) - \overline{p}_2.$$

As this strictly increases in $\alpha_1$, bidder 1 optimally matches $\overline{p}_2$ with the highest $\alpha_1$ subject to (10) and (1). Intuitively, if limiting target shareholders to $\overline{p}_2$ ((9)), she should maximize *surplus* subject to the other constraints. This requires increasing $\alpha$ to improve incentives to generate value ((2)) and increasing $D$ to keep the post-takeover share value at $\overline{p}_2$ (due to (3))—until further increases are infeasible due to debt constraints ((1)) or because the corner solution is reached ($\alpha_1 = 1$). This logic leads to the next result which refers to an increase in $\overline{p}_2$ as "stronger" competition.

**Proposition 5.** *Stronger competition increases bootstrapping and takeover surplus.*

Both parts of Proposition 5 are novel. In Müller and Panunzi (2004) where post-buyout values are exogenous, competition curbs bootstrapping. In incentive models with wealth constraints, competition raises the need for outside financing, which pushes the outcome further *away* from first-best incentives. In our model, bidders generally do not maximize their own incentives as much as feasible due to the free-rider problem. Competition pushes them *toward* first-best incentives, and as they generate more value, they also extract more through debt.

The effect of competition on profits is the conventional one: The added constraint (9) lowers bidder profits. Given total surplus increases, target shareholders gain. Proposition 5 thus reconciles bidding competition with high takeover leverage as well as high takeover leverage with low bidder returns—consistent with the impact competition had on premia, bidder gains, and leverage towards the end of the 1980s LBO wave (see, e.g., Holmstrom and Kaplan 2001, p.128f). If the bidders are equally competitive in our model, the winner raises her maximum feasible debt amount but *all* of the surplus goes to target shareholders, even though the debt serves to dilute the latter.

To sum up, in our theory, bootstrapping makes bidders more competitive, pushes them to use more debt, raises efficiency, and benefits target shareholders. These pro-competitive effects contrast with the role of debt in other models of bidder competition. In Chowdhry and Nanda (1993), debt financing serves to *deter* rivals. In DeMarzo et al. (2005), which extends results in Hansen (1985) and Rhodes-Kropf and Viswanathan (2000), competing bidders prefer debt to equity funding (or equivalently, paying in cash rather than in stock) because doing so *lowers* the seller's expected revenue.

# 5 Model Extensions

This section discusses model variations in which (1) the value improvement is a stochastic function of bidder effort and (2) the moral hazard problem is private benefit extraction instead of costly effort provision. We also discuss at the end what bearing our analysis has on payoffs and leverage choices in *negotiated* deals as opposed to tender offers. In Internet Appendix G, we consider a third model variation in which the sharing rule between bidders and outside equity investors can deviate from straight equity stakes. We show there that our results hold for richer equity-based compensation structures and that fine-tuning such structures can relax bidders' debt constraints, enabling them to use more buyout debt and thereby increasing their willingness to create more value. That extension also distinguishes explicitly between fees paid by the target (which dilutes equity returns) and fees paid from the equity partnerships (which depend on equity returns), and highlights why it is optimal for the PE firm to collect both in our model.

## 5.1 Stochastic Value Improvement

Building on Müller and Panunzi (2003, Section 6), we consider a setting in which the value improvement is a binary random variable. We focus here on the robustness of our results. Internet Appendix F provides an in-depth comparison with Müller and Panunzi's analysis.

The firm value in the absence of a takeover is normalized to 0. If a bid succeeds, the firm value increases to $v > 0$ with probability $q(e)$ and stays at 0 otherwise, where $e \in \mathbb{R}_0^+$ is effort provided by the bidder at a private cost $C(e) \geqslant 0$. We adopt the same success probability function $q(e) = e$ as Müller and Panunzi (2003), but instead of their quadratic cost function $C(e) = \frac{\xi e^2}{2}$, allow for a more general effort cost function $C(e)$ that satisfies

$C'(e) \geqslant 0, C''(e) \geqslant 0$ for all $e \geqslant 0$.[14] We impose Inada-style conditions $\lim_{e\to 1} C'(e) = \infty$ and $C'(0) = 0$ to abstract from corner solutions.

**Value Creation.** If the bidder gains control, she chooses her effort to solve

$$\max_{e\in[0,1]} q(e)\alpha[v - D]^+ - C(e),$$

where $\alpha$ is her equity stake and $D$ is the firm's debt. Given interior solutions, the optimal effort is pinned down by the first-order condition

$$C'(e) = \alpha(v - D). \tag{11}$$

It is instructive to define $Z \equiv \alpha(v - D)$ to stress that only the amount $Z$ the bidder gets in the success state matters for incentives. Any $Z \in [0, v]$ and associated effort level can be implemented via infinitely many payoff-equivalent $\alpha$-$D$-pairs. Debt and equity financing affect incentives in the same way. (This is a well known property of financing models with binary $v$-or-0 structures). Still, as we will show, the main results from our baseline model go through.

Denote the effort that solves (11) by $e(Z)$. There is an increasing differentiable function $f$ such that $e(Z) = f(Z)$ (given $C''(e) > 0$ and the inverse function lemma).

**Tendering Decisions.** The target shareholders' free-riding behavior equalizes, in equilibrium, the expected post-buyout share value with the bid price:

$$p = f(Z)(v - D) \tag{12}$$

where $f(Z)$ is the probability that the firm value is $v$ given the rationally expected effort.

**Bid and Financing.** The bidder's ex-ante problem is to choose $p$, $\alpha$, and $D$ to maximize $f(Z)Z - C(f(Z)) - p + f(Z)D + (1 - \alpha)f(Z)(v - D)$, where $f(Z)Z$ is the expected value

[14] Given our general cost function $C(e)$, assuming a linear $q(e)$ is without loss of generality. We use a general cost function to demonstrate that some distinctive predictions of this model variant are driven by the binary outcome structure rather than the quadratic cost function.

of the equity stake she acquires, $C(f(Z))$ is her effort cost, $p$ is the cash paid to target shareholders, $f(Z)D$ is the amount of debt funding, and $f(Z)(1-\alpha)(v-D)$ is the amount of outside equity funding. Using (12) in the objective function reduces the problem to

$$\begin{aligned} &\underset{\alpha,D}{\text{maximize}} \quad f(Z)D - C(f(Z)) \\ &\text{subject to} \\ &\qquad\qquad \alpha \in [0,1] \\ &\qquad\qquad p = f(Z)(v-D) \end{aligned} \tag{$P_q$}$$

As discussed, the effect of the bidder's $\alpha$-$D$-choice through $Z \equiv \alpha(V-D)$ on the effort level $f(Z)$ is *per se* irrelevant. But as the objective function in ($P_q$) shows, the bidder gains from raising $D$ (as a consequence of free-rider condition (12)). The fact that leverage mitigates the free-rider problem breaks her indifference between debt and outside equity (that would otherwise obtain in this model) in favor of debt.

**Lemma 5.** *In equilibrium,* $\alpha^* = 1$, $D^* = \frac{v}{2}$, *and* $p^* > 0$.

We now show that analogues of Propositions 1 and 2 from our baseline model hold in this model variant with uncertainty. Recall that $\bar{D}$ denotes an exogenously imposed limit on (bootstrapped) takeover debt.

**Proposition 6.** *Any limit* $\bar{D} < \frac{v}{2}$ *reduces takeover surplus. For* $C(e) = \frac{c}{2}e^2$ *and* $c > v$, *it also reduces takeover premia.*

The first part replicates the result that bootstrapping is socially optimal. As for the second part, recall that the effect of bootstrapping on target shareholders depends on the sensitivity of the bidder's effort to the financing structure, which in turn is determined by the curvature of effort cost function $C$ (Section 3.2). Here, we show that for the quadratic cost function—used by Müller and Panunzi (2003, Sec.6)—bootstrapping *benefits* target shareholders.

Propositions 3 and 5 of our baseline model do not have analogues in this model variant; this is not a general consequence of introducing uncertainty but an artefact of the binary outcome structure, as we explain in Internet Appendix F.

## 5.2 Private Benefit Extraction

Private benefits are another source of bidder gains which, already without buyout leverage, reduces the free-rider problem (Grossman and Hart 1980; Burkart et al. 1998).

Let $V$ be the potential value improvement the bidder can create upon completion of a takeover. As before, we normalize the value under incumbent management to 0. As in Burkart et al. (1998), the bidder chooses an allocation $\phi \in [0, 1]$ that generates security benefits $(1-\phi)V$, which are distributed among all shareholders, and private benefits $d(\phi)V$ for herself. Let $d(\phi)$ be twice continuously differentiable, strictly increasing, and strictly concave with $d(0) = 0$ and $d'(0) = 1$. Thus, private benefit extraction is inefficient. Both $V$ and $d(\phi)$ are common knowledge.

**Extraction Choice.** If the bid succeeds, the target firm assumes the buyout debt $D$ and the bidder's equity stake is $\alpha$. The bidder then chooses $\phi$ to maximize the combined value of her equity stake and her private benefits. Define $U(\alpha, D, \phi) \equiv \alpha[(1-\phi)V - D]^+ + d(\phi)V$. The bidder's post-takeover decision problem is

$$\max_{\phi \in [0,1]} U(\alpha, D, \phi)$$

Let $\phi^*(\alpha, D)$ denote the optimal extraction rule for the bidder. A unique $\phi^*(\alpha, D)$ exists due to our assumptions.

The bidder's objective function $U$ is not globally concave in $\phi$. Let $\bar{\phi} \in [0, 1]$ be such that $(1 - \bar{\phi})V = D$, i.e., $\bar{\phi} = \frac{V-D}{V}$. For $\phi > \bar{\phi}$, equity is "out of the money," and $U(\alpha, D, \phi) = d(\phi)V$, which is strictly increasing in $\phi$. Thus, the bidder optimally extracts the maximum ($\phi = 1$). For $\phi < \bar{\phi}$, equity is "in the money," and $U(\alpha, D, \phi) = \alpha[(1 - \phi)V - D] + d(\phi)V$, which is strictly concave in $\phi$. Here, the first-order condition, $d'(\phi) = \alpha$, has a unique, strictly positive solution, which we denote $\phi(\alpha)$. The next result follows directly from these observations.

**Lemma 6.** *The bidder's optimal diversion choice is* $\phi^*(\alpha, D) = \phi(\alpha) > 0$ *if*

$$\alpha[(1 - \phi(\alpha))V - D] + d(\phi(\alpha))V \geqslant d(1)V \tag{13}$$

*where $\phi(\alpha)$ is the solution to*

$$d'(\phi(\alpha)) = \alpha. \tag{14}$$

*Otherwise, she chooses $\phi^*(\alpha, D) = 1$.*

**Tendering Decisions.** As in the other model variants, target shareholders' free-riding behavior equalizes the expected post-takeover share value with the bid price:

$$p = [(1 - \phi^*(\alpha, D))V - D]^+ \equiv E(\alpha, D) \tag{15}$$

where $\phi^*(\alpha, D)$ is the bidder's optimal extraction choice as rationally anticipated by the target shareholders.

**Bid and Financing.** The bidder's ex ante profit is $\alpha E(\alpha, D) + d(\phi)V + F^E + F^D - \beta p$. It comprises the value of her expected equity stake, her expected private benefits, and the outside funding she receives for the bid less the takeover payment. She maximizes this by choosing bid $p$, debt financing $\{D, F^D\}$, and outside equity financing $\{\gamma, F^E\}$ subject to (13), (14), (15), and investors' participation constraints. We assume that outside investors are competitive and merely break even:

$$F^E = \beta(1 - \gamma)E(\alpha, D) \quad \text{and} \quad F^D = D. \tag{16}$$

Substituting these constraints as well as (14) and (15) into the bidder's profit yields

$$D + d(\phi(\alpha))V. \tag{17}$$

This shows that there are *two* exclusion mechanisms in this setting: leverage and private benefit extraction. But these mechanisms endogenously conflict with each other, as per debt overhang constraint (13). Lenders only agree to a proposed $\{\alpha, D\}$ that satisfies (13). If (13) is violated, lenders rationally anticipate the bidder to choose $\phi = 1$ and hence do not finance the bid. In parallel to the effort model, (13) defines an $\alpha$-$D$-relationship for feasible financing. To raise more debt financing, the bidder must take on more equity to improve her incentives—here, to extract fewer private benefits—such that debt repayment is feasible. In short, extracting rents through debt requires forgoing private benefits.

The debt overhang constraint cannot be slack at the optimum. Otherwise, the bidder can lower $\alpha$ while preserving $D$ and so raise her profit (17). Using the binding constraint (13) to replace $D$ in (17) reduces the bidder's stage-1 choices to the univariate problem

$$\max_{\alpha\in[0,1]} \mathcal{W}(\alpha) - \frac{d(1)V - d(\phi(\alpha))V}{\alpha} \tag{P$_\phi$}$$

where $\mathcal{W}(\alpha) \equiv (1-\phi(\alpha))V + d(\phi(\alpha))V$ is the total surplus generated by the takeover conditional on the bidder's optimal extraction choice. We can establish equilibrium existence (though not uniqueness without further specifying private benefit function $d$).

**Lemma 7.** *The bidder succeeds with a bid such that (13)-(16) hold and $\alpha$ solves ($P_\phi$). If the solution is $\alpha^* = 0$, then $\phi^* = 1$, $D^* = 0$ and the bidder's profit is $d(1)V$.*

Parameters for which solutions identified by Lemma 7 involve $D^* > 0$ and $\alpha^* > 0$ are easy to find, so takeover leverage generally plays a role. To understand why the solution to ($P_\phi$) is often interior, i.e., why neither exclusion mechanism dominates the other, note that both are costly to the bidder. On one hand, private benefit extraction entails deadweight losses. On the other hand, the use of buyout leverage requires the bidder to leave a positive post-buyout equity value (to avoid debt overhang), which target shareholders extract via the takeover premium. In choosing her optimal exclusion strategy, the bidder trades off larger deadweight losses against a larger premium. Target shareholders hence benefit from bootstrapping as an alternative to private benefit extraction. Without bootstrapping, the bidder implements $\phi = 1$ and their payoff (i.e., the takeover premium) is 0.

While takeover premia are costly to the bidder, they are merely redistributive from a social perspective. Imposing a limit on bootstrapping—which shifts the bidder's exclusion strategy toward private benefit extraction—is thus weakly inefficient and strictly so if the unconstrained solution to ($P_\phi$) is $\alpha^* > 0$ and the exogenous limit $\overline{D}$ would create a binding constraint. We summarize the above arguments in the below result for which we formally consider the effect of removing such an exogenous limit.

**Proposition 7.** *When $\alpha^* > 0$, bootstrapping increases takeover surplus as well as takeover premia, and the bidder's net financing contribution is negative.*

Proposition 7 replicates the insights from the effort model in Section 3 for parameters under which the bidder at least partly prefers buyout leverage to private benefit extraction. To offer more intuition for when this is likely to be the case, consider a specification for $d$. Let $d$ belong to the following class of power functions:

$$d(\phi) = \left(\phi - \frac{\kappa\phi^n}{n}\right) V \tag{18}$$

where $n \geqslant 2$ and $\kappa$ parametrizes the inefficiency, or deadweight loss, of diversion. For this class of functions, the solution in Lemma 3 is unique and allows us to conduct comparative statics with respect to the inefficiency of diversion.

**Proposition 8.** *For (18), the optimal debt level is increasing in $\kappa$.*

When diversion entails larger deadweight losses, the bidder's exclusion strategy relies more on debt. That is, a buyout is more highly levered and improves incentives more when private benefit extraction is more inefficient. High leverage corrects severe inefficiencies.

The model variant with private benefit extraction maps well into narratives of LBOs reducing managerial agency problems such as empire-building and diversion of free cash flow (Jensen 1986). Under this interpretation, $\phi$ denotes how much of the firm's resources managers misuse and $\phi - d(\phi)$ how much value is thereby wasted. By Propositions 7 and 8, if the deadweight loss of such misuse is large, the optimal disciplinary takeover is a highly leveraged, bootstrapped buyout that disburses part of the efficiency gains to the bidder in upfront fees.

It is noteworthy that debt is not necessary to make buyouts feasible in this alternative model with private benefit extraction. In contrast to both Müller and Panunzi (2003, 2004) and our effort model, private benefit extraction already makes a bid profitable for bidders. Here, the social gains of buyout debt originate purely from improved incentives. The crux is that bootstrapping provides bidders with rents *from improving (as opposed to reducing)* post-buyout value.

## 5.3 Negotiated Buyouts

In our model there are no wealth constraints and it is free-riding behavior that prevents the first-best outcome. Absent the free-rider problem, such as when a merger is negotiated with the target management, bargaining would lead to some merger price that implements the first-best outcome and splits the surplus to mutual benefit.

Yet, the ability to extract gains through debt in a tender offer may affect the division of merger gains since resorting to a tender offer is the bidder's threat in the negotiations. Having the LBO structure in her arsenal would shift bargaining power to the bidder in the negotiations, even if the resulting merger need not be as highly leveraged as a threatened tender offer would have been. That said, the threat of the tender offer may lack credibility unless the bidder has procured requisite financing agreements from lenders (who have to conduct their own due diligence). If so, it could be that such agreements find their way into the merger deal. Suppose the optimal tender offer (threat) involves lending agreements that would generate a debt-to-value ratio of $\frac{D*}{V*}$ in our model (e.g., as the solution to (P)). The negotiated merger may then end up with a debt-to-value ratio of $\frac{D*}{V^{fb}}$.

Whether or not the tender offer threat requires lending agreements that influence the eventual merger deal, our theory predicts that leveraged buyouts consummated through tender offers are more highly leveraged and rely more on bootstrapping, i.e., debt raised at the target level ("OpCo debt") rather than at the level of private equity funds ("FundCo debt") or intermediary holding companies ("HoldCo debt"). Or put conversely, we would expect that leverage ratios are lower for *negotiated* buyouts of public companies and for buyouts of *private* companies. As these types of buyouts have become more common over the last decades, buyout leverage ratios should have decreased and the use of HoldCo and FundCo debt should have increased.

# 6 Conclusion

This paper combines the incentive theory of buyout debt with the theory on the free-rider problem in takeovers of dispersedly held firms. This combined theory predicts "excessive" levels of debt (beyond financing needs) raised via bootstrapping, paired with upfront fees collected by bidders directly from the target firms, as a financing structure that is socially optimal and increases buyout premiums.

Our analysis expands on a prominent line of reasoning in corporate governance theory. Set against Berle and Means (1932)'s thesis that dispersed ownership empowers managers, Manne (1965) proposed a direct remedy: the threat of a takeover to reunify ownership and control. Such takeovers must reconsolidate ownership to improve *incentives* and overcome the *holdout behavior* among the dispersed shareholders (Jensen and Meckling 1976; Grossman and Hart 1980). Bootstrapping targets (to such a degree that PE firms cash out early) is, as we show, a buyout design that achieves both objectives simultaneously, immune even to the usual caveat that target shareholders want to limit the means bidders use to extract gains. This makes bootstrapped debt a silver bullet against free-riding and possibly crucial to implementing Manne's vision of disciplinary takeovers.

Surely, this does not dispel concerns that extreme buyout leverage can entail costs, such as a higher risk of financial distress or negative externalities borne by other stakeholders. But we offer efficiency arguments for controversial LBO features to counterbalance some of the concerns. In fact, we can explain why some takeovers are so extremely leveraged based on arguments that combine two canonical strands of takeover theory, and importantly, do *not* apply to capital structure choice outside of takeovers.

# Appendix

## A Proofs Omitted in the Text

### Proof of Lemma 2

For every $p \in [E(\gamma/2, D), E(\gamma, D)]$, there exists a unique $\alpha_p \in [\gamma/2, \gamma]$ such that $E(\alpha_p, D) = p$. Every shareholder tenders for $\hat{\alpha}_i < \alpha_p$, retains her shares for $\hat{\alpha} > \alpha_p$, and is indifferent between tendering and retaining for $\hat{\alpha} = \alpha_p$. ■

### Proof of Lemma 3

The objective function is continuous in $\alpha$ and its domain is compact. Hence there exists an $\alpha \in [1/2, 1]$ that solves (P). If the profit under this solution is positive, the bidder makes a successful bid. Otherwise, she abstains from a takeover. ■

### Proof of Proposition 1

The proof is composed of two lemmas. One establishes that a binding debt overhang constraint entails a positive relationship between $\alpha$ and $D$. The other shows that the debt overhang constraint binds in equilibrium also when an exogenous cap $\bar{D}$ limits the bidder's choice of $D$.

**Lemma A.1.** *A binding debt overhang constraint defines $D$ as a strictly increasing function of $\alpha$.*

*Proof.* As per (1*), define $D(\alpha) \equiv V(e^+(\alpha)) - \frac{C(e^+(\alpha))}{\alpha}$. We have:

$$\begin{aligned} D'(\alpha) &= V'(e^+(\alpha))e^{+\prime}(\alpha) + \frac{1}{\alpha^2}C(e^+(\alpha)) - \frac{1}{\alpha}C'(e^+(\alpha))e^{+\prime}(\alpha) \\ &= \big(V'(e^+(\alpha)) - \frac{1}{\alpha}C'(e^+(\alpha))\big)e^{+\prime}(\alpha) + \frac{1}{\alpha^2}C(e^+(\alpha)) \\ &= \frac{1}{\alpha^2}C(e^+(\alpha)) > 0. \end{aligned}$$

The third equality holds because $\alpha V'(e^+(\alpha)) - C'(e^+(\alpha)) = 0$ by (2). The fact that $D(\alpha)$ is strictly increasing implies the same for its inverse function. □

Let $\overline{D}$ be an *exogenous* upper bound on debt, that is, the bidder is only allowed to issue $D \in [0, \overline{D}]$. Let $(\alpha^*, D^*)$ denote the optimal post-takeover bidder stake $\alpha^*$ and debt level $D^*$ in the absence of the exogenous upper bound on $D$.

Since the exogenous debt limit is non-binding for $\overline{D} > D^*$, we restrict attention to $\overline{D} \leqslant D^*$. The next lemma shows that the debt overhang constraint is *always* binding in equilibrium even when there is an exogenous cap on debt.

**Lemma A.2.** *In equilibrium, the bidder chooses* $(\alpha, D)$ *such that* $D \leqslant \overline{D}$ *and* $\alpha D = \alpha V(e(\alpha)) - C(e(\alpha))$.

*Proof.* First suppose $D < \overline{D}$. By the endogenous debt overhang constraint, $\alpha D \leqslant \alpha V(e(\alpha)) - C(e(\alpha))$. If $\alpha D < \alpha V(e(\alpha)) - C(e(\alpha))$, the bidder can increase $D$ by some $\varepsilon > 0$ so that $D + \varepsilon < \overline{D}$ and $\alpha(D + \varepsilon) < \alpha V(e(\alpha)) - C(e(\alpha))$, which strictly increases the bidder's profit. Thus, this yields a contradiction.

Now suppose $D = \overline{D}$ but $D < \bar{D}(\alpha)$ where, for said $\alpha$, $\bar{D}(\alpha) = V(e(\alpha)) - \frac{C(e(\alpha))}{\alpha}$ is the *endogenous* debt capacity where the debt overhang constraint would be binding. Since $\bar{D}'(\alpha) > 0$ by Lemma A.1 and $D < \bar{D}(\alpha)$, there is an $\varepsilon > 0$ such that $\alpha' = \alpha - \varepsilon$ satisfies $D < \bar{D}(\alpha') = V(e(\alpha')) - \frac{C(e(\alpha'))}{\alpha'}$. Because $C(e(\alpha))$ is increasing in $\alpha$, it then follows that $D - C(e(\alpha')) > D - C(e(\alpha))$, so the bidder obtains a strictly higher profit. Thus, this too leads to a contradiction. □

Lemma A.2 implies that the imposition of a binding exogenous cap $\bar{D}$ causes the debt overhang constraint (1*) to be binding at some lower level of debt $D \leqslant \bar{D} < D^*$. Lemma A.1 consequently implies that the imposition of $\overline{D}$ leads to a smaller bidder stake $\alpha$. We will use these lemmas also in the proof of Proposition 2.

To conclude this proof, note that $\mathcal{W}(\alpha)$ is strictly increasing in $\alpha$. By lowering $\alpha$, the imposition of a binding exogenous cap hence reduces takeover surplus. ■

### Proof of Proposition 2

For reference, we state a result from one variable calculus (e.g., Rudin 1964, p. 114):

**Lemma A.3.** *Let $f : (0, +\infty) \to \mathbb{R}$ be a differentiable function such that $f'(x) > 0$ for all $x \in (0, +\infty)$. Then $f$ is strictly increasing on $(0, +\infty)$ and has a differentiable inverse function $g$ with*

$$g'(f(x)) = \frac{1}{f'(x)}$$

*for all $x \in (0, +\infty)$. If $f : (0, +\infty) \to \mathbb{R}$ is twice differentiable and such that $f''(x) > 0$ for all $x \in (0, +\infty)$ then its inverse $g$ is also twice differentiable and we have*

$$g''(f(x)) = -\frac{f''(x)}{(f'(x))^3}$$

*for all $x \in (0, +\infty)$.*

We turn to the main proof. As in the proof of Proposition 1, suppose the bidder may issue $D \in [0, \overline{D}]$ where $\overline{D}$ is an exogenous limit. We restrict attention to $\overline{D} \leqslant D^*$ where the limit matters.

We know that debt overhang constraint (1) binds in equilibrium with or without a limit on $D$ and that such a limit causes a decrease in the bidder's post-buyout stake $\alpha$ (Lemmas A.1 and A.2 in the proof of Proposition 1). For the current proposition, it hence suffices to establish whether or when target shareholders benefit from larger $\alpha$, conditional on (1) binding.

As shown in the main text, when (1) binds, target shareholders' payoff is $\frac{C(e^+(\alpha))}{\alpha}$. Target shareholders benefit from larger $\alpha$ if

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\alpha}\frac{C(e^+(\alpha))}{\alpha} &= \frac{C'(e^+(\alpha))e^{+\prime}(\alpha)}{\alpha} - \frac{C(e^+(\alpha))}{\alpha^2} \\ &= \frac{\theta}{\alpha}\left[\frac{C'(e^+(\alpha))}{C''(e^+(\alpha))} - \frac{C(e^+(\alpha))}{C'(e^+(\alpha))}\right] \geqslant 0. \end{aligned}$$

The second equality above holds by Lemma A.3, whereby if $e^+(\alpha) > 0$, then $e^{+\prime}(\alpha) = \frac{\theta}{C''(e^+(r))}$. A sufficient condition for the last inequality to hold globally is log-concavity of $C$, i.e., $C(e)C''(e) \leqslant [C'(e)]^2$ for all $e > 0$.[15]

[15]Note that $\frac{C(e^+(\alpha))}{\alpha}$ is an average cost per share, but $\alpha$ is not the direct argument in $C$. If $C$ were a *direct* function of $\alpha$, a sufficient condition for the average cost to be increasing is that marginal cost exceeds average cost. Log-concavity matters for first-order condition (2) to ensure that $e^+(\alpha)$ is sufficiently elastic with respect to $\alpha$.

Finally, we want to verify that there exist log-concave $C$ for which $D^* > 0$, that is, for which the bidder is inclined to use debt (make a bid) such that the exogenous debt limit could be binding. In Appendix B, we show that this is the case, for example, for power functions.[16] ■

**Proof of Proposition 5**

**Interior solution.** Equation (10) defines bidder 1's debt as a strictly increasing function of her equity stake. We denote this function by

$$D_1^c(\alpha_1) \equiv V(e_1^+(\alpha_1)) - \overline{p}_2.$$

It represents $(\alpha_1, D_1)$ that take into account all optimality conditions except (1), or more specifically, for which (2) holds and (3) and (9) strictly bind.

Recall that, as per (1*),

$$D_1(\alpha_1) \equiv V_1(e_1^+(\alpha_1)) - \frac{C_1(e_1^+(\alpha_1))}{\alpha_1}$$

represents all $(\alpha_1, D_1)$ for which (1) strictly binds.

As established in the main text, bidder 1 optimally matches bidder 2's reservation price by maximizing $\alpha$ subject to (1) and (10). The solution is the highest $\alpha_1$ where

$$D_1^c(\alpha_1) \leqslant D_1(\alpha_1),$$

which we hereafter denote by $\alpha_1^{**}$.

The previous inequality is slack at the single-bidder optimum $\alpha_1^*$:

$$D_1^c(\alpha_1^*) = V_1(e^+(\alpha_1^*)) - \overline{p}_2 < V_1(e^+(\alpha_1^*)) - p_1^* = D_1(\alpha_1^*),$$

[16]One can also find conditions under which the bidder's equilibrium profit is globally increasing in $\alpha$. A sufficient condition for this is that C is log-convex (see Internet Appendix D). That said, *global* conditions on $C$ are much more restrictive than needed for bootstrapping to create Pareto gains. For example, it is simple to construct such a setting with cost functions that have alternating log-convex and log-concave segments.

where the inequality follows from $p_1^* = \frac{C_1(e_1^+(\alpha_1^*))}{\alpha_1^*}$ and effective competition ($\overline{p}_2 > p_1^*$). Thus, $\alpha_1^{**} > \alpha_1^*$. That is, competition increases bidder 1's takeover debt compared to the single-bidder case.

For a given $\overline{p}_2$, suppose $\alpha_1^{**} < 1$. Does bidder 1 use even more takeover debt when bidder 2's reservation price increases to $\overline{p}_2^\epsilon > \overline{p}_2$? One can show that this is the case by relabeling $\alpha_1^{**}$ as $\alpha_1^*$, $\overline{p}_2$ as $p_1^*$, and $\overline{p}_2^\epsilon$ as $\overline{p}_2$ and retracing the previous arguments. In doing so, an important observation is that debt overhang constraint (1) binds for any optimal non-corner winning bid; for $\alpha_1^{**} < 1$, $D_1^c(\alpha_1^{**}) = D_1(\alpha_1^{**})$.

**Corner solution.** Suppose bidder 1 matches bidder 2's reservation price with a bid that leads to $\alpha_1 = 1$. At $\alpha_1 = 1$, the free-rider condition can be slack. Still, as bidder 1 buys all shares at a price equal to $\overline{p}_2$, her profit is $\mathcal{W}(1) - \overline{p}_2$, which is the maximum value of the profit function $\mathcal{W}(\alpha) - \overline{p}_2$ used in the arguments in the text. Thus, the result that bidder 2's presence increases bidder 1's takeover debt, if $\alpha_1^* < 1$, is valid also when the winning bid is a corner solution. Once in the corner solution, bidder 1 can meet further increases in $\overline{p}_2$ by reducing debt but, equivalently, also by raising $p_1$ without a change in debt. ■

### Proof of Lemma 5

Using (12) again, this time to replace $f(Z)$, the problem can be rewritten

$$\underset{p,D}{\text{maximize}} \quad \frac{pD}{v-D} - C\left(\frac{p}{v-D}\right)$$

subject to

$$\alpha \in [0,1]$$

$$p = f(Z)(v-D)$$

Partially differentiating w.r.t. $p$ and $D$ yields

$$\frac{d\Pi}{dp} = \frac{D}{v-D} - C'\left(\frac{p}{v-D}\right)\frac{1}{v-D},$$

and

$$\frac{d\Pi}{dD} = \frac{p}{v-D} + \frac{Dp}{(v-D)^2} + C'\left(\frac{p}{v-D}\right)\frac{p}{(v-D)^2}.$$

The first order condition $\frac{d\Pi}{dp} = 0$ implies

$$C'\left(\frac{p}{v-D}\right) = D \tag{A.1}$$

Inserting this in the partial w.r.t. $D$ gives

$$\frac{d\Pi}{dD} = \frac{p}{v-D} + \frac{2pD}{(v-D)^2} > 0. \tag{A.2}$$

The four conditions (11)-(A.2) pin down the optimal financing choice. First, if we rewrite (11) as $C'(f(Z)) = Z$, we see that (11) and (A.1) imply $f(D) = \frac{p}{v-D}$, or

$$p = f(D)[v-D].$$

Combining the latter equation with (12) implies $f(D)[v-D] = f(Z)[v-D]$. Since $f$ is invertible, this implies $D = Z$. As $Z \equiv \alpha[D-v]$, this defines $\alpha$ as an increasing function of $D$, i.e., $\alpha = \frac{D}{D-v}$. By (A.2), the upper bound $\alpha = 1$ is optimal. If $\alpha = 1$, then $D = \frac{v}{2}$ and $p = f\left(\frac{v}{2}\right)\frac{v}{2} > 0$. ■

**Proof of Proposition 6**

*First part*: As shown in the proof of Lemma 5, the bidder's optimal strategy is to maximize $D$ and set $\alpha = \frac{D}{D-v}$. Absent an exogenous debt limit, the optimum is hence given by the upper bound on $\alpha$, i.e., $\alpha = 1$ and the associated debt level $D = \frac{v}{2}$. With the exogenous debt limit $\bar{D} < \frac{v}{2}$, the optimum is instead given by the upper bound on $D$, i.e., $D = \bar{D}$ and the associated equity share $\bar{\alpha} \equiv \frac{\bar{D}}{\bar{D}-v}$. The decreases in $D$ and $\alpha$ have opposite effects on the bidder's incentives. To see the net effect, insert $\bar{D}$ and $\bar{\alpha}$ into the first-order condition for the optimal effort (11). This yields

$$C'(e) = \bar{\alpha}[\bar{D} - v] = \bar{D}, \tag{A.3}$$

and so $e(\bar{D}) = f(\bar{D})$ where $f$ is increasing in $\bar{D}$.

*Second part*: Continuing from above, for any given $\bar{D} \in (0, \frac{v}{2})$, we have $D = \bar{D}$ and $\bar{\alpha} \equiv \frac{\bar{D}}{\bar{D}-v}$. Retracing steps from the proof of Lemma 5, we then have $p = f(\bar{D})[v - \bar{D}]$ in equilibrium. For the present proof, we must determine the sign of

$$\frac{dp}{d\bar{D}} = f'(\bar{D})(v - \bar{D}) - f(\bar{D}).$$

Now assume $C(e) = \frac{ce^2}{2}$, and let $c > v$ to focus on (ensure) an interior solution to the effort problem. Then $C'(e) = ce$. The inverse $f$ of $C'$ is $f(x) = \frac{x}{c}$. With this,

$$\frac{dp}{d\bar{D}} = f'(\bar{D})(v - \bar{D}) - f(\bar{D}) = \frac{1}{c}(v - \bar{D}) - \frac{1}{c}\bar{D}.$$

Hence $\frac{dp}{d\bar{D}} \geqslant 0$ if and only if $v - \bar{D} \geqslant \bar{D}$, which holds if and only if $\bar{D} \leqslant \frac{v}{2}$. Recall that the optimal debt level in the absence of a limit is $D^* = \frac{v}{2}$. It follows from $D^* = \frac{v}{2}$ and $\frac{dp}{d\bar{D}} > 0$ for all $\bar{D} \leqslant \frac{v}{2}$ that any debt limit $\bar{D} < \frac{v}{2}$ (i.e., any limit that would be binding) reduces target shareholder wealth. ■

### Proof of Lemma 7

The objective function is continuous in $\alpha$ and its domain is compact. Hence there exist $\alpha \in [0, 1]$ that solve ($P_\phi$). If the solution is $\alpha = 0$, it follows from (13) that $D = 0$ and from (14) that $\phi = 1$. The bidder's profit in this case is $d(1)V$. ■

### Proof of Proposition 7

The proof of the first part builds on two lemmas. One establishes that a binding debt overhang constraint entails a positive relationship between $\alpha$ and $D$. The other shows that the debt overhang constraint binds in equilibrium also when an exogenous cap $\bar{D}$ limits the bidder's choice of $D$.

**Lemma A.4.** *A binding debt overhang condition (13) defines debt as an increasing function of the ownership stake $\alpha$.*

*Proof.* Let (13) hold with equality and rearrange to $D = (1-\phi(\alpha))V + \frac{d(\phi(\alpha))V - d(1)V}{\alpha}$. Differentiating with respect to $\alpha$ yields

$$\begin{aligned} D'(\alpha) &= -\phi'(\alpha)V + \frac{d'(\phi(\alpha))\phi'(\alpha)V}{\alpha} - \frac{d(\phi(\alpha))V - d(1)V}{\alpha^2} \\ &= \frac{d'(\phi(\alpha))\phi'(\alpha)V - \alpha\phi'(\alpha)V}{\alpha} + \frac{d(1)V \ - d(\phi(\alpha))V}{\alpha^2} \\ &= \frac{d(1)V \ - d(\phi(\alpha))V}{\alpha^2} > 0 \end{aligned}$$

□

Let $(\alpha^*, D^*)$ denote the optimal bidder stake $\alpha^*$ and debt level $D^*$ in the absence of the exogenous upper bound on $D$. Assume the exogenous limit binds: $\overline{D} < D^*$.

**Lemma A.5.** *In equilibrium, the bidder chooses* $(\alpha, D)$ *such that* $D \leqslant \overline{D}$ *and* $\alpha D = \alpha(1-\phi(\alpha))V + d(\phi(\alpha))V - d(1)V$.

*Proof.* First suppose $D < \overline{D}$. By the endogenous debt overhang constraint, $\alpha D \leqslant \alpha(1 - \phi(\alpha))V + d(\phi(\alpha))V - d(1)V$. If $\alpha D < \alpha(1-\phi(\alpha))V + d(\phi(\alpha))V - d(1)V$, the bidder can increase $D$ by some $\varepsilon > 0$ so that $D + \varepsilon < \overline{D}$ and $\alpha(D+\varepsilon) < \alpha(1-\phi(\alpha))V + d(\phi(\alpha))V - d(1)V$, which strictly increases the bidder's profit. Thus, this yields a contradiction.

Now suppose $D = \overline{D}$ but $D < \bar{D}(\alpha)$ where, for said $\alpha$, $\bar{D}(\alpha) = (1-\phi(\alpha))V + \frac{d(\phi(\alpha))V - d(1)V}{\alpha}$ is the *endogenous* debt capacity where the debt overhang constraint would be binding. Since $\bar{D}'(\alpha) > 0$ by Lemma A.4 and $D < \bar{D}(\alpha)$, there is an $\varepsilon > 0$ such that $\alpha' = \alpha - \varepsilon$ satisfies $D < \bar{D}(\alpha') = (1-\phi(\alpha'))V + \frac{d(\phi(\alpha'))V - d(1)V}{\alpha'}$. Because $d(\phi(\alpha))$ is a decreasing function of $\alpha$, it then follows that $D + d(\phi(\alpha'))V > D + d(\phi(\alpha))V$, so the bidder obtains a strictly higher profit. Thus, this too leads to a contradiction. □

Finally, to prove the first part of Proposition 7, note that

$$\mathcal{W}'(\alpha) = d'(\phi(\alpha))\phi'(\alpha)V - \phi'(\alpha)V = V\phi'(\alpha)[d'(\phi(\alpha)) - 1] > 0.$$

The last inequality follows because $(i)$ $\phi'(\alpha) < 0$ and $(ii)$ $d'(\phi(\alpha)) < 1$ for all $\phi(\alpha) > 0$ due to the strict concavity of $d$. Thus, as imposing the debt limit $\overline{D}$ reduces the bidder's optimal $\alpha$ (as per Lemmas A.4 and A.2), total surplus decreases because the bidder resorts to more private benefit extraction, which leads to more deadweight losses.

We now prove the second part of the proposition. Absent takeover debt, $\phi^* = 1$ and the takeover premium is $p^* = 0$. By contrast, if $D > 0$ and $\alpha > 0$ in equilibrium, $\phi(\alpha) < 1$ and the binding debt overhang constraint (13) implies a post-takeover share value of $(1 - \phi(\alpha))V - D = \frac{d(1) - d(\phi(\alpha))}{\alpha} V > 0$. By the free-rider condition (15), this equals the takeover premium.

Last, we prove the third part of the proposition. The stage-1 external financing flows to the bidder net of the (cash) acquisition price are $F^E + F^D - \beta E(\alpha, D)$. Using the outside investors' break-even constraints from (16), this reduces to $D - \alpha E(\alpha, D)$ as outside equity financing and the value of the outside equity stake cancel out (recall $\alpha = \gamma\beta$). The debt overhang constraint (13) holds with equality in equilibrium. Subtracting $D$ on both sides and rearranging the binding constraint yields

$$\underbrace{D^* - \alpha^*(((1 - \phi^*)V - D^*)}_{D^* - \alpha^* E(\alpha^*, D^*)} = d(\phi^*)V + D^* - d(1)V. \tag{A.4}$$

(with * indicating equilibrium values). The left-hand side is equal to the net external financing flows from above. On the right-hand side, $d(\phi^*)V + D^*$ equals the bidder's profit when $\alpha^* > 0$ and $D^* > 0$ (cf. (17)), whereas $d(1)V$ is her profit for $\alpha = D = 0$. If $\alpha^* > 0$, the right-hand side is positive by revealed preference, in which case (A.4) implies that the external financing flows net of the acquisition price are positive (i.e., the bidder's net financing contribution is negative). ■

### Proof of Proposition 8

We begin by showing that the bidder's ex-ante problem has a unique solution under the diversion function specified in (18).

**Lemma A.6.** *Let $f : (a, b) \to \mathbb{R}$, $g : [a, b) \to \mathbb{R}$ and $h : (a, b) \to \mathbb{R}$ be functions such that $f(x) = g(x)h(x)$ for all $x \in (a, b)$, where $h(x) > 0$ for all $x \in (a, b)$ and $g(a) \geqslant 0$ and $\lim_{x \to b} f(x) < 0$. Assume there is an $x^* \in (a, b)$ such that $g'(x^*) = 0$ and $g'(x) > 0$ for all*

$x \in (a, x^*)$ *and* $g'(x) < 0$ *for all* $x > x^*$*. Then there is a unique* $y \in (a, b)$ *with* $f(y) = 0$ *and* $f(x) > 0$ *for all* $x < y$ *and* $f(x) < 0$ *otherwise.*

*Proof.* We first show that $f(x) > 0$ for all $x \in (a, x^*]$. To see this, note that $g(x) > 0$ for all $x \in (a, x^*]$ since $g(a) \geqslant 0$ and $g$ is strictly increasing on $(a, x^*)$. Thus, since $h(x) > 0$ for all $x \in (a, b)$ it follows that $f(x) = g(x)h(x) > 0$ for all $x \in (a, x^*]$. Further, since $f(x^*) > 0$ and $\lim_{x \to b} f(x) < 0$ there is an $y \in (x^*, b)$ with $f(y) = 0$, and since $g$ is strictly decreasing on $(x^*, 1)$ it follows that $f(x) > 0$ for all $x < y$ and $f(x) < 0$ for all $x > y$. □

We now establish uniqueness.

**Lemma A.7.** *For the diversion function defined in (18), the bidder's ex-ante problem has a unique solution* $\alpha^* \in (0, 1)$*.*

*Proof.* From the calculations above, it follows that for all $\alpha \in (0, 1)$

$$\begin{aligned} \Pi'(\alpha) &= D'(\alpha) + d'(\phi(\alpha))\phi'(\alpha)V \\ &= \frac{V}{\alpha^2}\left[d(1) - d(\phi(\alpha)) + \alpha^3\phi'(\alpha)\right]. \end{aligned}$$

Now define

$$\begin{aligned} g(\alpha) &\equiv d(1) - d(\phi(\alpha)) + \alpha^3\phi'(\alpha) \\ h(\alpha) &\equiv \frac{V}{\alpha^2} \end{aligned}$$

so that $\Pi'(\alpha) = g(\alpha)h(\alpha)$ for all $\alpha \in (0, 1)$. We will show that $g(\cdot)$ and $h(\cdot)$ satisfies the premises of lemma A.6. It is clear that $h(\alpha) > 0$ for all $\alpha \in (0, 1)$ and that $g(0) = d(1) - d(\phi(0)) \geqslant 0$ (since $d(\cdot)$ is increasing and $\phi(0) \leqslant 1$). Note also that

$$\lim_{\alpha \to 1} \Pi'(\alpha) = \lim_{\alpha \to 1} \frac{V}{\alpha^2}\left[d(1) - d(\phi(\alpha)) + \alpha^3\phi'(\alpha)\right] < 0,$$

since $\lim_{\alpha \to 1} \phi'(\alpha) = -\infty$. We next compute the first derivative of $g$.

$$g'(\alpha) = -d'(\phi(\alpha))\phi'(\alpha) + 3\alpha^2\phi'(\alpha) + \alpha^3\phi''(\alpha)$$

$$\begin{aligned} &= [3\alpha^2 - \alpha]\phi'(\alpha) + \alpha^3\phi''(\alpha) \\ &= \phi'(\alpha)\frac{\alpha}{1-\alpha}\left[(3\alpha-1)(1-\alpha) + \alpha^2\frac{n-2}{n-1}\right]. \end{aligned}$$

It is only in the last equality that we use the explicit functional form of $d$ from (18) and the resultant $\phi(\alpha) = \left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{n-1}}$ from the bidder's ex-post diversion choice. More specifically, we use that

$$\phi'(\alpha) = \frac{1}{n-1}\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{n-1}-1}\left(-\frac{1}{\kappa}\right)$$

and hence that

$$\begin{aligned} \phi''(\alpha) &= \phi'(\alpha)\left(\frac{\kappa}{1-\alpha}\right)\left(\frac{1}{n-1}-1\right)\left(-\frac{1}{\kappa}\right) \\ &= \phi'(\alpha)\left(\frac{1}{1-\alpha}\right)\left(\frac{n-2}{n-1}\right). \end{aligned}$$

Next, define $m(\alpha) \equiv [3\alpha-1][1-\alpha](n-1) + \alpha^2(n-2) = [(n-2)-3(n-1)]\alpha^2 + 4(n-1)\alpha - (n-1)$ and $l(\alpha) \equiv \phi'(\alpha)\frac{\alpha}{(1-\alpha)(n-1)}$. We show that there is a unique $\beta \in (0,1)$ such that $m(\beta) = 0$ and $m(\alpha) < 0$ for all $\alpha < \beta$ and $m(\alpha) > 0$ for all $\alpha > \beta$.

First, note that $m(0) = -(n-1) < 0$ and $m(1) = (n-2) > 0$, so the (polynomial) function $m$ has at least one zero $\beta$ in $(0,1)$. Second, since $(n-2) < 3(n-1)$ it follows that $m$ is strictly concave. Since $m$ is a quadratic polynomial (and is strictly concave) it cannot have both of its zeros in the interval (0,1), as $m(0) < 0$ would then imply that $m(1) < 0$, which is a contradiction. Hence, it must be the case that there is a $\beta \in (0,1)$ such that $m(\beta) = 0$ and $m(\alpha) < 0$ for all $\alpha < \beta$ and $m(\alpha) > 0$ for all $\alpha > \beta$.

Thus, since $m$ satisfies the property established in the previous paragraph, and since $l(\alpha) < 0$ for all $\alpha \in (0,1)$, it follows that $g'(\beta) = l(\beta)m(\beta) = 0$ and $g'(\alpha) = l(\alpha)m(\alpha) > 0$ for all $\alpha < \beta$ and $g'(\alpha) = l(\alpha)m(\alpha) < 0$ for all $\alpha > \beta$.

Finally, since $\Pi'(\alpha) = g(\alpha)h(\alpha)$ for all $\alpha \in (0,1)$ and since the premises of lemma A.6 are satisfied (as estbalished above), it follows that there is a unique $\alpha^* \in (0,1)$ such that $\Pi'(\alpha^*) = 0$ and $\Pi'(\alpha) > 0$ for all $\alpha < \alpha^*$ and $\Pi'(\alpha) < 0$ for all $\alpha > \alpha^*$. □

With uniqueness established, we analyze how the solution depends on $\kappa$. Using $d(\phi) =$

$\phi - \frac{\kappa\phi^n}{n}$ we get

$$\Pi'(\alpha) = D'(\alpha) + d'(\phi(\alpha))\phi'(\alpha)V =$$
$$\frac{V}{\alpha^2}\left[d(1) - \left(\frac{n-1+\alpha}{n}\right)\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{n-1}} - \left(\frac{\alpha^3}{1-\alpha}\right)\left(\frac{1}{n-1}\right)\left(\frac{1-\alpha}{\kappa}\right)^{\frac{1}{n-1}}\right].$$

To stress the dependence of $\Pi'(\alpha)$ on $\kappa$ we denote it as $\Pi'_\kappa(\alpha)$. We rewrite the expression above as follows

$$\Pi'_\kappa(\alpha) = A(\alpha) - B(\alpha)\kappa^{-\frac{1}{n-1}},$$

where $A(.)$ and $B(.)$ are defined as $A(\alpha) \equiv \frac{Vd(1)}{\alpha^2}$ and

$$B(\alpha) \equiv \frac{V}{\alpha^2}\left[\left(\frac{n-1+\alpha}{n}\right)(1-\alpha)^{\frac{1}{n-1}} + \left(\frac{\alpha^3}{1-\alpha}\right)\left(\frac{1}{n-1}\right)(1-\alpha)^{\frac{1}{n-1}}\right].$$

Note that $B(\alpha) > 0$. It follows that

$$\frac{d\Pi'_\kappa(\alpha)}{d\kappa} = \frac{B(\alpha)}{n-1}\kappa^{-\frac{n}{n-1}} > 0. \tag{A.5}$$

Let $\alpha^*(\kappa)$ be the unique solution to the bidder's ex-ante problem, i.e., satisfy $\Pi'_\kappa(\alpha^*(\kappa)) = 0$. By (A.5), $\kappa < \kappa'$ implies $\alpha^*(\kappa) < \alpha^*(\kappa')$ and, because $D(\alpha)$ increases in $\alpha$, also $D(\alpha^*(\kappa)) < D(\alpha^*(\kappa'))$. That is, the more inefficient diversion is, the more the bidder resorts to debt financing. ■

# B Examples with Specific Functional Forms for $C$

We consider two specific functional forms for cost function $C$: power functions and exponential functions. Power functions serve as an example of log-concave functions for which the bidder uses a strictly positive amount of debt, but not the maximum feasible amount of debt in the absence of competition (i.e., $\alpha^* \in (0,1)$ while her profit is strictly positive).[17] Exponential functions serve as an example of log-convex functions, under which a corner solution obtains: the bidder takes on the maximum stake $\alpha^* = 1$, accordingly exhausting her debt capacity even absent competition. Under either class of functions, bootstrapping

[17] These equilibrium properties obtain under all power functions except the linear one.

is Pareto-improving (though in the case of exponential functions, target shareholders gain only weakly).

**Example B.1** (Power functions)**.** Let $V(e) \equiv \theta e$ and $C(e) \equiv \frac{c}{n} e^n$ where $\theta > 0$, $c > 0$ and $n \in \mathbb{N}$ are exogenous parameters. These functions satisfy all our assumptions. It can also be shown that they generate unique solutions to (P) (proof available upon request). So, if the bidder's profit is positive under the solution to (P), there exists a unique $\langle D, \alpha, p, e \rangle$ such that $\alpha V'(e) = C'(e)$, $p = V(e) - D$, $\alpha D = \alpha V(e) - C(e)$, and $\alpha \in [1/2, 1]$ satisfying $\alpha \in \{1/2, 1\}$ or the ex ante first-order condition for (P),

$$\frac{1}{\alpha^2} C(e^+(\alpha)) = C'(e^+(\alpha)) e^{+\prime}(\alpha). \tag{B.1}$$

The specific functional form allows us to express $\langle D, \alpha, p, e \rangle$ in closed form. The first-order condition for effort $\alpha V'(e) = C'(e)$ yields $e = \left(\frac{\alpha\theta}{c}\right)^{\frac{1}{n-1}}$. The equilibrium stake $\alpha$ solves (B.1). One can show that this condition holds if and only if

$$\theta e^{+\prime}(\alpha) \left( \frac{n-1}{n} - \alpha \right) = 0,$$

which in turn holds if and only if $\alpha = 0$ (since $e^{+\prime}(0) = 0$) or $\alpha = \frac{n-1}{n}$. Of these, only $\alpha = \frac{n-1}{n}$ is admissible as a solution to (P). It is straightforward to verify that

$$D = \frac{(n-1)\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}}$$

and

$$p = \frac{\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}}.$$

Furthermore, the bidder's profit under the solution to (P) is positive since

$$\begin{aligned} D - C(e^+(\alpha)) &= \frac{(n-1)\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}} - \frac{(n-1)\theta}{n^2} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}} \\ &= \theta \left( \frac{n-1}{n} \right)^2 \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}} \geqslant 0. \end{aligned}$$

To sum up, there is a unique equilibrium in which

$$\langle D, \alpha, p, e \rangle = \left\langle \frac{(n-1)\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}}, \frac{n-1}{n}, \frac{\theta}{n} \left( \frac{(n-1)\theta}{nc} \right)^{\frac{1}{n-1}}, \left( \frac{\frac{n-1}{n}\theta}{c} \right)^{\frac{1}{n-1}} \right\rangle.$$

As power functions are log-concave for all $n \in \mathbb{N}$, (more) debt always increases post-takeover share value and target shareholder wealth (Proposition 2). The equilibrium debt-equity ratio is $D/p = n - 1$. For $n = 5$, the ratio equals 4. $\lhd$

**Example B.2** (Exponential functions.)**.** Let $V(e) \equiv \theta e$ and $C(e) \equiv \exp(e)$ with $\theta > \exp(2)$. These functions satisfy all our assumptions, and can be shown to entail unique solutions to (P) (proof available upon request). If the bidder's profit is positive under (P), there is a unique $\langle D, \alpha, p, e \rangle$ such that $\alpha V'(e) = C'(e)$, $p = V(e) - D$, $\alpha D = \alpha V(e) - C(e)$, and $\alpha \in [1/2, 1]$ either satisfying the ex ante first-order condition (B.1) or $\alpha \in \{1/2, 1\}$. The post-takeover first-order condition $\alpha V'(e) = C'(e)$ yields $e^+(\alpha) = \ln(\alpha\theta)$, which is stictly positive given $\alpha\theta > \frac{\exp(2)}{2} > 1$. Substituting $e^+(\alpha)$ into the profit function of (P) yields

$$\theta \ln(\alpha\theta) - (1 + 1/\alpha)\alpha\theta.$$

Differentiating with respect to $\alpha$ yields $\theta(1/\alpha - 1)$, which is strictly positive for all $\alpha \in [1/2, 1)$. Thus, $\alpha = 1$ is the unique solution to (P). It is straightforward to verify that

$$D = \theta \ln(\theta) - \theta$$

and

$$p = \theta.$$

Furthermore, the bidder's profit is

$$D - C(e^+(1)) = \theta(\ln(\theta) - 2),$$

which is positive since $\theta > \exp(2)$ implies $\ln(\theta) > 2$. To summarize, there is a unique equilibrium in which

$$\langle D, \alpha, p, e \rangle = \langle \theta \ln(\theta) - \theta, 1, \theta, \ln(\theta) \rangle .$$

As exponential functions are weakly log-concave, leverage is weakly Pareto-improving. With $\alpha = 1$ in equilibrium, first-best incentives are restored. The equilibrium debt-equity ratio is $D/p = \ln(\theta) - 1$. For example, if $\theta = \exp(5)$, the ratio is 4. $\lhd$

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## Internet Appendix of "The Free-Rider Problem and the Structure of Leveraged Buyouts"

## C Bootstrapping, Fees, and Conventional LBO Arguments

We discuss how bootstrapping fits into the conventional incentive argument for LBOs, notably whether that argument implies that bootstrapping improves post-buyout incentives, increases the bidder's debt capacity, or more effectively curtails post-buyout incentives to divert cash. We also briefly discuss an alternative argument for upfront fees.

**Bootstrapping.** The conventional argument for LBO debt is that it realigns managerial incentives. It is based on financing models in which a wealth-constrained owner-manager chooses an unobservable effort level to increase a firm's value. In such models, the second-best outcome requires that the owner-manager raises outside financing through debt and retains all of the equity (e.g., Jensen and Meckling 1976; Innes 1990). Applied to takeovers, this implies that, when a firm with dispersed ownership suffers from managerial agency problems, one remedy is to fully concentrate ownership in the hands of an owner-manager through a debt-financed takeover.

If one applies this prediction to a bootstrap acquisition, the post-buyout ownership and capital structure is such that the target absorbs the takeover debt $D$ and the bidder owns $\alpha = 1$ of the target shares.

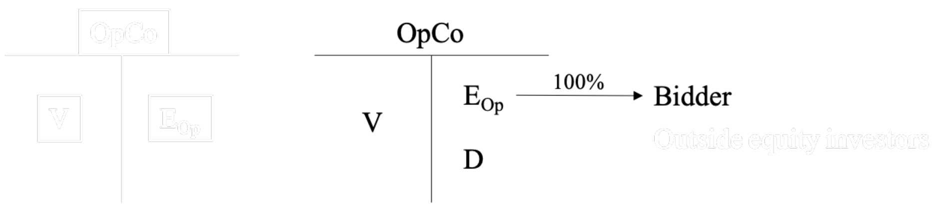


Figure O.1: Bootstrapping in a Standard Model

The bidder's post-buyout effort choice problem under this structure is

$$\max_{e}[\mathrm{E}[\tilde{V}(e)] - D]^{+} - C(e) \tag{C.1}$$

where $\tilde{V}(e)$ and $C(e)$ denote, respectively, the (possibly stochastic) firm value as a function of bidder effort and the effort cost function.

Now suppose that the takeover is executed via an acquisition company that issues the debt *without* a subsequent merger between the acquisition company and the target.

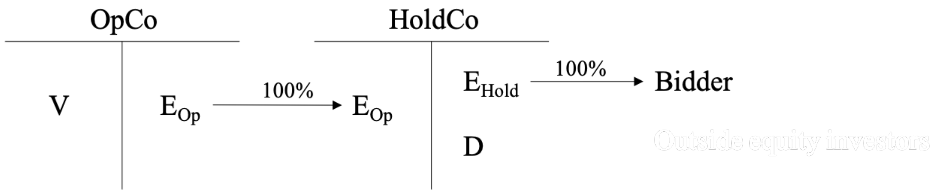


Figure O.2: Standard Model without Bootstrapping

The bidder's post-buyout effort choice problem under this structure is exactly the same as with bootstrapping, i.e., equal to (C.1). The two structures are payoff-equivalent.

The equivalence holds even if the bidder draws equity capital from a buyout fund that is, for reasons outside of the standard model, partly funded by outside investors (limited partners). In this case, the structure with bootstrapping is

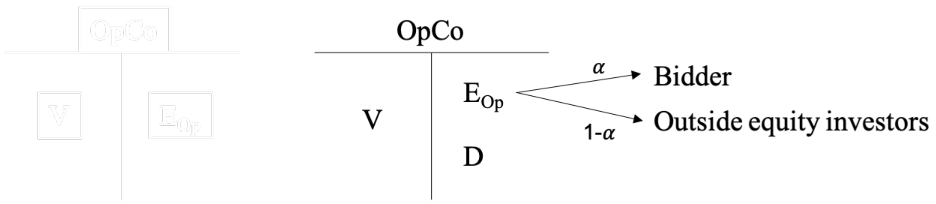


Figure O.3: Buyout with Bootstrapping

and the structure without bootstrapping is

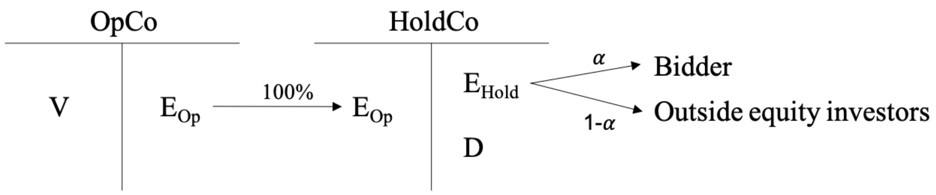


Figure O.4: Buyout without Bootstrapping

In either case, the bidder's post-buyout effort choice problem would be

$$\max_{e} \alpha[\mathrm{E}[\tilde{V}(e)] - D]^{+} - C(e)$$

It follows from these comparisons that bootstrapping is irrelevant in settings in which the *only* role of buyout financing is to optimize post-buyout incentives. This irrelevance also implies that the bidder's *debt capacity* is the same across both structures.

While we have couched the above equivalence in terms of managerial effort, it can also be cast in terms of cash flow diversion, i.e., the *free cash problem*. The idea is that directly imposing debt on the target as in Figure O.1 forces future target cash flow to be paid out to creditors, lest the firm defaults and the bidder loses control. However, the bidder's incentives to use target cash flow to repay creditors is the same under the holding structure in Figure O.2; a default under this structure would equally imply that she loses control.

The above arguments abstract, for simplicity, from other pre-existing (e.g., employee, supplier, or debt) claims on the target firm. However, the same arguments remain valid in the presence of such claims—as long as imposing the buyout debt on the target firm does *not* dilute the claims of those other target stakeholders (e.g., workers, suppliers, existing creditors). In contrast, if those claims can be diluted, the associated wealth-transfer effect induces bidders to strictly favor bootstrapping and to lever up more than the conventional incentive argument predicts. (This is the gist of the wealth-transfer theory.)

**Upfront fees.** The presumption for buyouts is that the bidder (or PE firm) acquires the target firm for price $p$ that is below the firm's post-takeover equity value $V$. It is therefore, in principle, possible for the bidder to raise such an amount of total outside financing $F$ that $p < F \leqslant V$. The bidder could then cash out $F - p > 0$ at the time of the deal, e.g., through fees, such that her net capital contribution is negative. (If the net contribution is positive, the bidder can simply reduce her contribution without a payout.) In the case of multiple buyouts (with $V > p$, else a buyout is not profitable), she could cash out upfront in all of them. Of course, this is a wash because, for *exogenously* given post-buyout values, her payoff is the same if she keeps claims in each target firm instead of selling those claims to outside investors for a (fairly priced) cashout upfront. Indeed, if we superimpose a moral hazard problem such that the post-buyout values are endogenous, the bidder prefers not to take externally-funded upfront cashouts (by the second-best argument that her incentives are optimized by minimizing outside claims). That being said, there may be situations in which cashing out early in some deals facilitates the financing of additional deals (on the extensive margin), should the fund get close to exhausting its capital commitments, even though doing so may compromise incentives (on the intensive margin, i.e., within a deal).

If so, one should expect upfront cashouts to be prevalent when a given PE fund is, perhaps suddenly, flush with potential deals. Nevertheless, even this argument cannot rationalize a *net* cash out *in aggregate*, i.e., across all the deals.

# D Log-convex Cost Functions

For reference, we first state the following auxiliary result:

**Lemma D.1.** *There is a unique differentiable function* $e : [1/2, 1] \to \mathbb{R}_{\geqslant 0}$ *such that* $\alpha V'(e(\alpha)) = C'(e(\alpha))$ *for all* $\alpha \in [1/2, 1]$ *and such that* $e'(\alpha) > 0$ *for all* $\alpha \in (1/2, 1)$.

*Proof.* Define a function $H : (0, +\infty) \to \mathbb{R}$ by $H(e) = \frac{C'(e)}{\theta}$. Clearly,

$$H'(e) = \frac{C''(e)}{\theta} > 0$$

for all $e > 0$ by our assumption that $C''(e) > 0$ for all $e \geqslant 0$. Thus $H$ satisfies the premises of Lemma A.3, and hence there is a differentiable function $G$ such that $G(H(e)) = e$ for all $e > 0$ and $H(G(y)) = y$ for all $y$ in the range of $H$. From our assumptions $\lim_{e \to 0} C'(e) = 0$ and $\lim_{e \to +\infty} C'(e) = +\infty$ and the fact that $H$ is continuous, it follows that $[1/2, 1]$ is a subset of the range of $H$, i.e., $[1/2, 1] \subseteq H((0, +\infty))$. Hence we may define $e : [1/2, 1] \to (0, +\infty)$ by $e(\alpha) := G(\alpha)$ for all $\alpha \in [1/2, 1]$. Then $\frac{C'(e(\alpha))}{\theta} = H(e(\alpha)) = H(G(\alpha)) = \alpha$ for all $\alpha \in [1/2, 1]$ and the first part of the claim follows. Let $\alpha \in (1/2, 1)$ and $e > 0$ be such that $H(e) = \alpha$, applying Lemma *A*.3 once again then yields

$$e'(\alpha) = e'(H(e)) = \frac{1}{H'(e)} = \frac{\theta}{C''(e)} > 0.$$

□

We now show that, when cost function $C$ is log-convex, bidder profit is strictly increasing in $\alpha$, thus leading to the corner solution $\alpha^* = 1$. By Lemma A.1, the debt overhang constraint (1) always binds (even with an exogenous limit on debt). When (1) binds, the profit is

$$\pi^B(\alpha) \equiv V(e^+(\alpha)) - \left[1 + \frac{1}{r}\right] C(e^+(\alpha))$$

(cf. the objective function in (P)). This is strictly increasing in $\alpha$ if

$$\begin{aligned}
\frac{\mathrm{d}\pi^B(\alpha)}{\mathrm{d}\alpha} &= V'(e^+(\alpha))e^{+\prime}(\alpha) + \frac{1}{\alpha^2}C(e^+(\alpha)) - \frac{1}{\alpha}C'(e^+(\alpha))e^{+\prime}(\alpha) - C'(e^+(\alpha))e^{+\prime}(\alpha) \\
&= \left[V'(e^+(\alpha)) - \frac{1}{\alpha}C'(e^+(\alpha))\right]e^{+\prime}(\alpha) + \frac{1}{\alpha^2}C(e^+(\alpha)) - C'(e^+(\alpha))e^{+\prime}(\alpha) \\
&= \frac{1}{\alpha^2}C(e^+(\alpha)) - C'(e^+(\alpha))e^{+\prime}(\alpha) \\
&= \frac{1}{\alpha^2}C(e^+(\alpha)) - \frac{C'(e^+(\alpha))\theta}{C''(e^+(\alpha))} \\
&= \frac{1}{\alpha^2}C(e^+(\alpha)) - \frac{C'(e^+(\alpha))C'(e^+(\alpha))}{C''(e^+(\alpha))\alpha} \\
&= \frac{1}{\alpha}\left(\frac{C(e^+(\alpha))}{\alpha} - \frac{[C'(e^+(\alpha))]^2}{C''(e^+(\alpha))}\right) > 0
\end{aligned}$$

The second equality is obtained by rearranging terms. The third equality holds since $\alpha V'(e^+(\alpha)) - C'(e^+(\alpha)) = 0$ by (2). The fourth equality follows from Lemma D.1. The fifth equality holds because $\alpha\theta = C'(e^+(\alpha))$ by (2). A sufficient condition for the last inequality to be satisfied globally is that

$$\frac{1}{\alpha}\left(\frac{C(e)}{\alpha} - \frac{[C'(e)]^2}{C''(e)}\right) > \frac{1}{\alpha}\left(C(e) - \frac{[C'(e)]^2}{C''(e)}\right) \geqslant 0$$

for all $e > 0$. The strict inequality holds for all $\alpha < 1$. The last weak inequality holds if $C$ is log-convex, i.e., if $C(e)C''(e) \geqslant [C'(e)]^2$ for all $e > 0$. For example, exponential functions satisfy this property.

# E Comparative Statics with Respect to Parameters of $V$

For reference, we first state the following auxiliary result:

**Lemma E.1.** *Let $f : (a,b) \to \mathbb{R}$ be a function such that $f(x) = h(x)g(x)$ for all $x \in (a,b)$, where $h(x) > 0$ and $g'(x) < 0$ for all $x \in (a,b)$. Then there is at most one $x \in (a,b)$ such that $f(x) = 0$. Moreover, if a point $x \in (a,b)$ such that $f(x) = 0$ exists then $f(y) > 0$ for all $y < x$ and $f(y) < 0$ for all $y > x$.*

*Proof.* Consider two arbitrary distinct points $x, y \in (a,b)$ with $f(x) = f(y) = 0$. Then $h(x) > 0$ and $h(y) > 0$ implies $g(x) = g(y) = 0$. Since $g$ is differentiable, hence also continuous on $[x, y]$, the mean value theorem (Rudin 1964, Theorem 5.10, p. 108) gives a point $z$ with $x < z < y$ and $g'(z) = 0$. This contradicts $g'(z) < 0$. The second part clearly holds since $g(y) > 0$ for all $y < x$ and $g(y) < 0$ for all $y > x$ and since $h(x)$ is strictly positive. $\square$

We will now provide a comparative statics analysis with respect to the parameters of $V$ in the setting of example B.1. This is for two reasons. First, it showcases a class of cost functions that satisfies the log concavity assumption and hence Proposition 2. Second, it allows us to contrast the *causal* effect of debt, described by Proposition 2, with the "cross-sectional" relationship between takeover debt and target shareholder wealth generated by variation in (fundamentals such as) $V$ in the debt-unconstrained equilibrium.

As in example B.1, $V(e) = \theta e$ and $C(e) = \frac{c}{n}e^n$ with $\theta > 0$, $c > 0$ and $n \geqslant 2$. Consequently, the optimal debt level, target shareholder wealth, and bidder stake as functions of $n$ are:

$$D(n) \equiv \frac{(n-1)\theta}{n}\left(\frac{(n-1)\theta}{nc}\right)^{\frac{1}{n-1}},$$

$$\pi^S(n) \equiv V(e(\alpha)) - D = \frac{\theta}{n}\left(\frac{(n-1)\theta}{nc}\right)^{\frac{1}{n-1}}.$$

$$\alpha(n) \equiv \frac{n-1}{n}$$

One can verify that $C(e)$ is convex and log-concave for all $n \geqslant 2$. Thus, Proposition 2 applies: Takeover debt has a positive causal impact on target shareholder wealth.

As a comparison, we now describe comparative statics of the equilibrium without an exogenous debt limit with respect to $n$.

**Result 1.** *Let $V(e) = \theta e$ and $C(e) = \frac{c}{n}e^n$ with $\theta > 0$, $c > 0$ and $n \geqslant 2$.*

*a. If $\theta > 2c$, then $\pi^S(n)$ is a decreasing function of $n$. If $\theta \leqslant 2c$ then there is an $n^*$ such that shareholder wealth $\pi^S(n)$ is increasing in $n$ for $n \leqslant n^*$ and decreasing in $n$ for $n > n^*$.*

*b. The optimal bidder stake $\alpha(n)$ is an increasing function of $n$.*

c. *The optimal debt level $D(n)$ is an increasing function of $n$ whenever $\frac{\theta(n-1)}{cn} \leqslant \mathrm{e}$ where $\mathrm{e} \equiv \sum_{k=0}^{\infty} \frac{1}{k!}$ and a decreasing function of $n$ for all $n$ with $\frac{\theta(n-1)}{cn} > \mathrm{e}$.*

*Proof.* For part $a$, let $f(x) \equiv \frac{\theta}{x}\left(\frac{(x-1)\theta}{xc}\right)^{\frac{1}{x-1}}$ for all $x \geqslant 2$ and $x \in \mathbb{R}$. Now,

$$f'(x) = \frac{\theta}{x}\left(\frac{(x-1)\theta}{xc}\right)^{\frac{1}{x-1}}\left[-\frac{1}{x} - \frac{1}{(x-1)^2}\log\left(\frac{(x-1)\theta}{xc}\right) + \frac{1}{(x-1)^2 x}\right] =$$

$$\frac{\theta}{x}\left(\frac{(x-1)\theta}{xc}\right)^{\frac{1}{x-1}}\left[\frac{2-x}{(x-1)^2} - \frac{1}{(x-1)^2}\log\left(\frac{(x-1)\theta}{xc}\right)\right] =$$

$$\frac{\theta}{x(x-1)^2}\left(\frac{(x-1)\theta}{xc}\right)^{\frac{1}{x-1}}\left[2 - x - \log\left(\frac{(x-1)\theta}{xc}\right)\right]$$

Denote the last term in brackets as $g(x)$. Then $g'(x) = -1 - \frac{x}{x-1}\frac{1}{x^2} = -1 - \frac{1}{x(x-1)} < 0$. If $\theta \leqslant 2c$ then $g(2) = -\log\left(\frac{\theta}{2c}\right) > 0$ and since $g(x) < 0$ for large $x$, it follows that there is a unique $x^*$ such that $g(x^*) = 0$. Since $f(x)$ is the product of a strictly positive function and $g(x)$ it follows that $f(x) > 0$ for all $x < x^*$ and $f(x) \leqslant 0$ for all $x \geqslant x^*$. If $\theta > 2c$ then $g(2) \leqslant 0$, and since $g(x)$ is strictly decreasing, it follows that $f(x) < 0$ for all $x \geqslant 2$. Since the factor multiplying $g(x)$ is positive for all $x \geqslant 2$ it follows by Lemma E.1 that there is no more than one point $x^*$ such that $g(x^*) = 0$. Moreover, by the same lemma, if such a point exists, then $f'(x) > 0$ for all $x < x^*$ and $f'(x) \leqslant 0$ for all $x \geqslant x^*$. Now, if $\theta \leqslant 2c$ then $g(2) = -\log\left(\frac{\theta}{2c}\right) \geqslant 0$ and since $g(x) < 0$ for large $x$, it follows by the intermediate value theorem (Rudin 1964, Theorem 4.23, p. 93) that there is a $x^*$ such that $g(x^*) = 0$. If $\theta > 2c$ then $g(2) < 0$, and since $g(x)$ is strictly decreasing, it follows that $f'(x) < 0$ for all $x \geqslant 2$.

For part $b$, note that $\alpha'(n) = \frac{1}{n^2} > 0$ for all $n \geqslant 2$ and $n \in \mathbb{R}$.

For part $c$, define $h(x) \equiv \frac{\theta(x-1)}{x}\left(\frac{(x-1)\theta}{xc}\right)^{\frac{1}{x-1}}$ for all $x \geqslant 2$ and $x \in \mathbb{R}$. Then

$$h'(x) = \theta\left(\frac{(x-1)\theta}{xc}\right)^{\frac{1}{x-1}}\left[\frac{1}{x(x-1)} - \frac{1}{x(x-1)}\log\left(\frac{(x-1)\theta}{xc}\right)\right].$$

By the last term in brackets, $h'(x) > 0$ if and only if $1 \geqslant \log\left(\frac{(x-1)\theta}{xc}\right)$, which in turn holds if and only if $\mathrm{e} \geqslant \frac{(x-1)\theta}{xc}$. (Note that e denotes Euler's constant $\mathrm{e} \equiv \sum_{k=0}^{\infty}\frac{1}{k!}$, not the bidder's effort.) ∎

Result 1 shows by example that there may well be no clear-cut equilibrium relationship between $\alpha$, $\pi^S(n)$, and $D$. Changes in $n$ represent variations in economic fundamentals that are plausibly unobserved in the data. The three parts of the result state that there are parameters such that $\alpha$ is decreasing, $\pi^S(n)$ is increasing, and $D$ is non-monotonic in $n$—correlations that contrast sharply with the *causal* impact of $D$ on the other variables (Propositions 1 and 2). This is to say that unobserved confounding factors, as represented by $n$ in our example, can generate correlations in the data that obscure the causal effect of takeover leverage.

# F Comparison to Section 6 in Müller and Panunzi (2003)

Section 5.1 of our paper derives two of our main results in a model variant with uncertainty and a binary outcome structure, which is based on Müller and Panunzi (2003, Sec.6). Here, we compare our analysis to theirs in detail to delineate that the main results in our paper represent novel insights.

Before we begin the comparison, we should highlight a unique property of external financing models with binary $v$-or-0 outcome structures, like the one analyzed here.

**Remark 1.** *A well-known property of financing models with binary $v$-or-0 outcomes is that debt and equity, or any other financial contract for that matter, are equivalent. The only material contract feature is how the firm value $v$ in the singular success state is split. Whether the sharing rule is defined as an equity share $\alpha$ or a debt claim $D$ is irrelevant; for any equity contract $\alpha$, there is a payoff-equivalent debt contract $D$ and vice versa. In models with only moral hazard, this equivalence renders the choice or distinction between debt and equity immaterial.*

We now proceed to discuss first the analysis and the focus of Section 6 in Müller and Panunzi (2003) and afterwards the four central results of our paper.

## F.1 Analysis and Focus of Section 6 in Müller and Panunzi (2003)

Müller and Panunzi (2003)'s analysis focuses on the comparison of the model with moral hazard to a baseline model without moral hazard. Their central result is that, with moral hazard, the equilibrium bid features $\alpha^* = 1$, $D^* = \frac{v}{2}$, and $p^* > 0$ (which we replicate in Lemma 5 of our paper). They then compare $D^* = \frac{v}{2}$ and $p^* > 0$ with the equilibrium in the absence of moral hazard, $D = v$ and $p = 0$. This comparison underlies what Müller and Panunzi (2003, 2004) sum up as their central insight from introducing moral hazard: (a) The potential debt overhang problem *reduces takeover leverage* and (b) *the reduced use of debt* in turn benefits target shareholders through *higher takeover premia*.[18] These two observations, (a) and (b), seem to reinforce the takeaway from the baseline model without moral hazard that bootstrapping harms target shareholders (at the intensive margin) and cannot explain "LBO-style debt levels" (Müller and Panunzi 2004, p.1220).

The point of our paper is that introducing moral hazard *overturns* the above takeaway of Müller and Panunzi (2004). That takeaway rests on three results in the model without moral hazard: (1) a small debt amount (to cover given takeover costs) is socially optimal, (2) takeover leverage harms target shareholders conditional on a takeover, and (3) bidding competition reduces such leverage. The extended analysis in Müller and Panunzi (2003) does not uncover that including moral hazard upends results (1)-(3) and so their takeaway. The contribution of our paper is to fill this gap (Propositions 1-5 in Section 3) and thereby to contend that a framework that combines moral hazard and rent extraction *can* explain LBO-style leverage, in our view, better than the precedent literature.

### F.2 Replicating our Results in Müller and Panunzi (2003)'s Extension

We now discuss the results of our paper within the model of Müller and Panunzi (2003, Section 6). Before doing so, it is worth emphasizing two peculiarities of the binary outcome structure. First, $\alpha^* = 1$ regardless of effort cost function $C$.[19] This is because, under the $v$-or-0 structure, debt and equity are equivalent absent the free-rider problem (Remark 1). Without this equivalence, the optimal $\alpha$ is not always the upper bound and $C$ is not irrelevant. Second, $D^* = \frac{v}{2}$, again regardless of $C$. This too is specific to the $v$-or-0 structure (see Remark 2 below). In general, optimal leverage varies with $C$ since the latter affects

[18] This emphasis is clear in the conclusion of Müller and Panunzi (2004), the opening paragraph of Section 6 in Müller and Panunzi (2003), and the main proposition in Section 6 (Müller and Panunzi 2003, Proposition 11): "[T]he raider uses less debt ex ante. This, in turn, raises the takeover premium."

[19] This means that post-takeover "inside" owners, such as (new) management and private equity firms, own 100 percent of the equity. In other words, no *outside* equity financing obtains.

the degree of moral hazard and thus the severity of the incentive constraints on financing. That is, $D^* = \frac{v}{2}$ and $\alpha^* = 1$ (independent of $C$) are "special cases" peculiar to the binary payoff structure, and by no means a general implication of introducing uncertainty.

**Remark 2.** *To explain why $D^* = v/2$ regardless of $C$ in this model, it helps to spell out how the free-rider problem affects the moral hazard problem. Taking $\alpha = 1$ to simplify matters, compare the objective in the bidder's ex post effort choice*

$$e(v - D) - C(e)$$

*to that in her ex ante financing choice, which due to the free-rider condition is*

$$eD - C(e)$$

*(see ($P_q$)). For $D < v - D$, the bidder will exert more effort ex post than is optimal for her ex ante. Indeed, note that she maximizes the value of debt ex ante but the value of equity ex post. Increasing $D$ reduces the discrepancy which, in this specific setting, is minimized (even eliminated) at $D = v - D$. So, $D^* = \frac{v}{2}$ regardless of $C$.*[20]

*To see that this is a knife-edge result, recall that, in this binary $v$-or-$0$ outcome model, debt and equity are equivalent since what matters is only how $v$ is split (Remark 1). Furthermore, effort only affects the probability that the $v$-state is realized, and so the marginal effect of effort on debt and on equity depends only on the division of $v$ in that singular state. Hence any difference in the marginal effect of effort on debt and equity—and so between the ex ante optimal and the ex post optimal effort—is eliminated by splitting $v$ equally between debt and equity in that singular state. So, $D^* = \frac{v}{2}$ for any and all $C$.*

*In a model in which firm value can assume more than one strictly positive value, the equivalence of debt and equity breaks down and this knife-edge logic does not hold. In such a model, $C$ affects which set of (states with differing) firm values is likely to be realized and so, more importantly, how effort affects debt and equity at the margin. Depending on $C$, the logic of reducing the discrepancy between the ex ante and ex post optimal effort levels*

[20] $D > \frac{v}{2}$ is suboptimal here for the standard reason that the negative incentive effect of further outside financing (more debt in this case) reduces the expected debt value, i.e., the bidder's ex ante debt capacity.

*plays out in different ranges of the firm value or states of nature, thus translating into different optimal debt levels.*[21]

We now consider the four main results of our paper within the binary $v$-or-0 setting.

**Result 1: Social efficiency** As Müller and Panunzi (2003) point out, leverage has a direct negative effect and an indirect positive effect on the bidder's effort: On one hand, holding bidder equity constant, leverage creates a debt overhang problem. On the other hand, an increase in leverage leads to an increase in bidder equity, and thus the bidder's incentives. Their analysis is focused on how the negative incentive effect reduces the use of debt: "To counteract the adverse incentive effects of high leverage ... the [bidder] reduces his debt to $D^* = \frac{v}{2}$" (p.25)—and by "reduces" they mean relative to the debt level of $D = v$ in the model without moral hazard.

A striking fact they do not discuss is that the positive effect *dominates* for $D < \frac{v}{2}$. This is not obvious as it could in principle be optimal for the bidder to substitute debt for equity in such a way that bidder profit increases while incentives stay the same (i.e., negative and positive effects cancel out) or even when incentives are on balance compromised (i.e., the negative effect slightly dominates). To fill this gap, we use a different thought experiment: the effect of an exogenous limit $\bar{D}$ on takeover debt, i.e., bootstrapping, *within* the setting with moral hazard. This leads to our first main result: In the presence of moral hazard, *any* restriction on bootstrapping is socially inefficient (i.e., the first part of Proposition 6 in our paper).

Why does the indirect positive effect dominate the direct negative effect (for $D < \frac{v}{2}$)? Since the bidder profits through debt, she wants to maximize $eD$ ex ante, but once in control of the target, she maximizes $e\alpha(v - D)$ ex post. This discrepancy results from the interaction of the free-rider problem with the moral hazard problem (cf. Remark 2). When her choice of $D$ is capped by $\bar{D}$, her ex ante incentives fall. Her optimal response is to adjust $\alpha$ downward to $\bar{\alpha}$ such that her *ex post* marginal return to effort matches her now

[21] To give a simple example, imagine a model with two possible firm values $v_l$ and $v_h > v_l$, with the higher one requiring (or being more likely for) higher effort. If effort is very costly (cheap), the split of $v_l$ ($v_h$) is likely more relevant to the marginal return of effort across debt and equity. Depending on how costly effort is, splitting $v_l$ or $v_h$ will thus be more relevant for the optimal debt level.

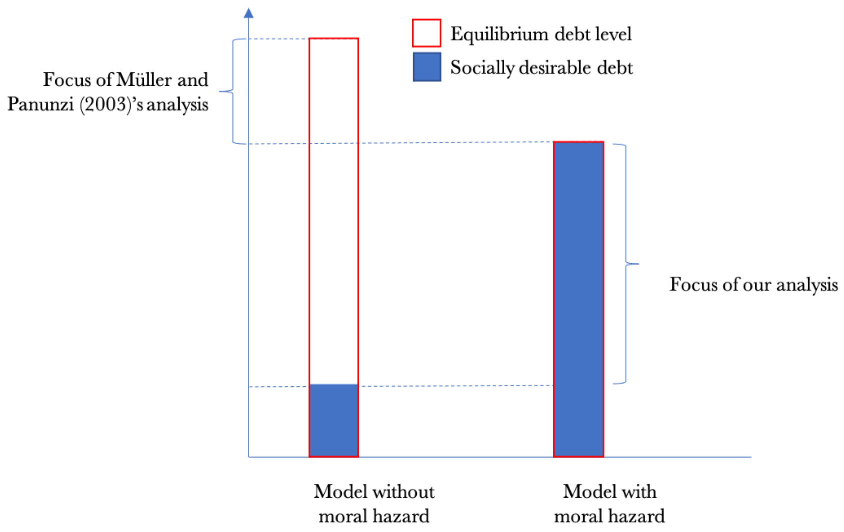


Figure O.5: Müller and Panunzi (2003) show that the equilibrium debt level is lower in the setting with moral hazard. We emphasize that the *socially desirable* debt level is *higher* in that setting: In the absence of moral hazard, part (likely, most) of the equilibrium debt reflects pure rent seeking. In the presence of moral hazard, *all* of the equilibrium debt is socially efficient because it increases total value creation.

lower *ex ante* marginal return to effort, namely $\Pi'(e) = \bar{D}$ (cf. first-order condition (A.3) in the proof of Proposition 6). Hence, her effort falls as $\bar{D}$ decreases.

Intuitively, the bidder's ex ante willingness to give herself value-creation incentives stems solely from the value she can extract through debt financing available under those incentives. As she is only willing to commit via higher $\alpha$ to creating more value ex post to the extent that she gets more debt financing ex ante, imposing limits on debt effectively reduces her willingness (to adopt incentives) to improve value.

This insight directly contradicts the conclusion in Müller and Panunzi (2004) that a "minimal amount of debt" is socially optimal. With moral hazard, *any* binding debt limit is inefficient; *all* debt used by the bidder in equilibrium is socially desirable. In Müller and Panunzi (2003)'s own model with moral hazard, this privately *and socially* optimal debt amount would be substantial at a debt-to-equity ratio of 100 percent, or $\frac{eD^*}{e[v-D^*]} = 1$.[22]

Figure O.5 summarizes the difference between Section 6 of Müller and Panunzi (2003) and our paper. The former is focused on showing that moral hazard decreases the *privately* optimal level of debt, whereas we show that the presence of moral hazard completely alters

[22]The examples in Appendix B of our paper feature even higher socially efficient leverage ratios.

the *social* optimality of debt.

Our insight that bootstrapping is socially optimal further hints at another potential qualification of the main takeaway in Müller and Panunzi (2004): If the use of debt to extract rents generates an increase in total surplus rather than being purely a redistribution, it is possible that the target shareholders may also benefit from it.

**Result 2: Sharing rule** As for target shareholders, Müller and Panunzi (2003) focus again on a comparison with the model without moral hazard, and concretely, to the fact that the decrease in leverage relative to the model without moral hazard translates into a rise in the takeover premium to $p^* > 0$ (from $p = 0$ in the model without moral hazard). This comparison *across* different settings—with and without moral hazard—reinforces the impression from Müller and Panunzi (2004) that bootstrapping harms target shareholders conditional on the buyout being realized.

But this comparison is misleading; it obscures the practically more relevant question whether bootstrapping harms target shareholders *within* a given setting, conditional on the buyout. In the setting without moral hazard, the answer is "yes." In the setting with moral hazard, the opposite can be true. Indeed, with a quadratic cost function as in Müller and Panunzi (2003), we show (in the second part of Proposition 6) that target shareholders strictly benefit from the bidder's use of bootstrapping even on the intensive margin.

What creates the possibility that target shareholders benefit is that leverage increases not only bidder profits but also total surplus. Why would the bidder not extract more of the (added) surplus? The crux is that the negative impact of debt on her incentives (i.e., debt overhang) constrains how much debt she can raise and so how much she can extract. The financing constraint is de facto a *sharing rule* for how (any increase in) total surplus is split between bidder and target shareholders.

Müller and Panunzi (2003)'s analysis overlooks the point that in their own model with moral hazard target shareholders *gain* on the (intensive) margin as the bidder raises more debt to extract more for herself. This matters because it reconciles their rent-extraction theory of buyout debt with the empirical fact that—"in spite of" high leverage—takeover premiums are large and appear to allocate a substantial part of the takeover gains to the

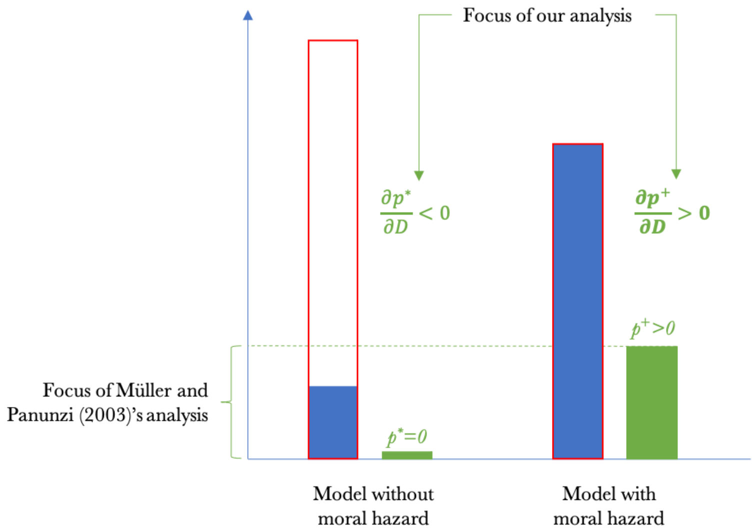


Figure O.6: Müller and Panunzi (2003) show that the takeover premium is higher in the setting with moral hazard, where the equilibrium debt level is lower. This comparison suggests that debt reduces the takeover premium. We consider comparative statics of the takeover premium with respect to debt *within* each of the two models: Debt *increases* the premium in the presence of moral hazard, while decreasing it in the absence of moral hazard.

target shareholders.

Condition $c > v$ in Proposition 6 rules out that optimal effort hits the upper bound $e = 1$. This is not a technical matter. At $e = 1$, effort and so total surplus become *inelastic* to further debt increases, which then revert to a pure rent extraction device. This suggests that the extent to which buyout leverage benefits or harms target shareholders depends on the elasticity of bidder effort to the firm's post-buyout ownership and capital structure ($\alpha$ and $D$), that is, on how much the buyout improves incentives. The results in our paper confirm this intuition (as e.g., reflected by the log-concavity condition in Proposition 2).

Figure O.6 illustrates this difference: Müller and Panunzi (2003) focus on how takeover premia differ across the settings with and without moral hazard. By contrast, we study how bootstrapping affects the takeover premium *within* each setting.

**Result 3: Bidding competition** The result that increases in buyout leverage on the intensive margin can benefit rather than harm target shareholders suggests that bidding competition may not induce bidders to reduce bootstrapping. Proposition 5 of our paper indeed reveals the opposite: In a framework with moral hazard, bidding competition generally induces bidders to use *more* debt. This contradicts a key point in Müller and Panunzi (2004)'s reasoning for why a theoretical framework based on the free-rider problem cannot explain LBO-style leverage (p.1220).

That said, Section 6 of Müller and Panunzi (2003) does not explore competition, nor can Proposition 5 of our paper be replicated within their specific model variant. The reason is that the binary $v$-or-0 structure causes post-buyout inside ownership to hit its upper bound $\alpha^* = 1$ regardless of other model parameters (see Remark 1 and the discussion prior to Remark 2). That is, "insiders" *always* end up owning 100 percent of the post-buyout equity. Not only is this empirically debatable (considering the limited partners in buyout funds) but it also rules out that more intense bidding competition pushes bidders to raise $\alpha$ (along with $D$) as part of making higher bids, which is key to Proposition 5.

In the framework of our paper, the optimal ownership structure is not invariably the upper bound ($\alpha^* = 1$) and depends on model parameters. We conjecture that, generally in models where $\alpha^* < 1$ in the absence of competition, bidding competition leads to more buyout leverage: If a bidder can structure a higher bid by either "creating less value but also extracting less" or by "creating more value but also extracting more," she prefers the latter. The reason is that, for a given intended target shareholder payoff (due to competition), bidders fare better with offers that generate a larger total surplus (which are offers with higher $\alpha$'s and higher $D$'s).

**Result 4: Upfront payout** Müller and Panunzi (2004) show that, in their model without moral hazard, upfront payouts to the bidder can occur. Though suggestive, it is not obvious that the result is robust to the inclusion of moral hazard; in financing models with moral hazard, *ex ante* cash-outs by agents who subsequently manage the firm are suboptimal from an incentive perspective.

Müller and Panunzi (2003) do not derive ex ante financing contributions or payouts in their model extension with moral hazard. If they had, the following result would have been obtained:

**Proposition F.1.** *The bidder's upfront payout equals 0.*

Thus, in Müller and Panunzi (2003)'s model extension, bidders do not receive upfront cash payouts (but they also do not contribute any net positive financing). However, this result is not due to a negative incentive effect of cash-outs but an artifact of the binary $v$-or-0 outcome structure.

Recall from Remark 2 that, under the binary outcome $v$-or-0 structure, the optimal debt level equals exactly *half* the firm value in the success state, $D^* = \frac{v}{2}$, regardless of other model parameters. With $D^* = \frac{v}{2}$, the (expected) equity value is $q(e^*)\frac{v}{2}$ where $q(e^*)$ is the rationally expected probability of the outcome $v$. Due to the free-rider condition, this equals the bid price. With the bidder buying all shares ($\alpha^* = 1$), the cash payment to target shareholders is hence $p = q(e^*)\frac{v}{2}$. On the financing side, the (expected) debt value is equally $q(e^*)\frac{v}{2}$, which in turn is the funding the bidder receives from creditors for the takeover. Thus, the debt financing exactly equals the cash transfer to target shareholders with neither an ex ante payout nor a net funding contribution from the bidder.

The framework in our paper does not reproduce the knife-edge solution of the $v$-or-0 structure. We find that, in the presence of moral hazard, upfront payouts are positive for reasons that (a) highlight the interplay of moral hazard with the free-rider problem and (b) make LBO financing isomorphic to a managerial incentive compensation contract (see Section 3.3 and Proposition 3 in our paper).

Overall, our analysis shows that, in a buyout model with moral hazard *and* free-riding, upfront payouts play a *positive* incentive role; limiting them erodes bidders' willingness to adopt more incentive-efficient financing structures, which harms (not only bidders but also) target shareholders.[23] Showing that ex ante payouts play a positive incentive role is a novel insight of our analysis.

### F.3 Summary of comparison

Müller and Panunzi (2004) conclude that they cannot explain LBO-style debt levels based on a chain of three arguments: (1) The socially optimal level of takeover debt is small, (2)

[23]By contrast, in Müller and Panunzi (2004), limiting upfront payouts to the bidder would merely reduce rent seeking, that is, lower bidder profits and benefit target shareholders.

takeover debt harms target shareholders, and (3) bidding competition thus pushes the debt level down to the social optimum (minimum needed to not frustrate the buyout). Results (1)-(3) are derived in a model with free-riding but absent moral hazard.

We show that these conclusions are invalid in the presence of moral hazard and free-riding. There, another chain of arguments applies: (a) The socially optimal level of takeover debt is *high*, (b) takeover debt *benefits* target shareholders when incentive effects are important in the buyout, and (c) bidding competition hence pushes debt levels *up*. Crucially, results (a)-(c) are not derived in Müller and Panunzi (2003), and being the opposite of (1)-(3), *do* explain LBO-style debt levels as well as the importance of (d) bootstrapping and upfront fees for the incentive efficiency of LBOs.

## G Carried Interest Provisions

Suppose outside equity investors commit callable capital before targets are found. Once a target is identified, the bidder chooses how much equity capital to draw down to fund the buyout. Let the sharing rule between bidder and outside equity investors be as follows: Out of the post-buyout equity value $E$, the bidder receives a positive payment only once the outside investors recoup the amount of outside capital $F^E$ that was drawn down.[24] Let $R(E)$ denote the bidder's performance-sensitive reward whose size may vary with $E$. Assume that, conditional on $E \geqslant F^E$, $R(E(e)) - C(E(e))$ is concave in $e$. This guarantees that the post-buyout effort choice has a unique solution provided the bidder chooses $e > 0$. This shape of $R(E)$ accommodates a range of compensation rules, such as hurdle rates, catch-ups, and other carried-interest clauses observed in practice. In particular, the fees that PE firms are in practice paid from the equity partnerships with their limited partners would be part of $R(E)$. We now show that our key insights holds for *any* $R(E)$ that meets the above assumptions.[25]

[24]In our model, this represents outside investors earning their required (or hurdle) rate of return.

[25]For a theory that can rationalize why, for buyout funds, outside equity financing is committed upfront, debt financing is sourced deal by deal, and the sharing rule between general partners and limited partners in the buyout funds deviates from simple equity shares, see Axelson et al. (2009). Their theoretical model focuses on asymmetric information problems between general and limited partners that are outside of our framework.

**Value Creation.** If the bidder acquires the firm with outside equity financing $F^E$, the compensation rule $R(E)$, and debt $D$, she subsequently chooses $e$ to maximize

$$\begin{cases} R(E(e)) - C(e) & \text{if } E(e) - R(E(e)) \geqslant F^E \\ -C(e), & \text{otherwise.} \end{cases}$$

where $E(e) = [V(e) - D]^+$. If the lower case where optimal effort is 0 had been anticipated, outside financing would not have been provided. So, any feasible buyout implements the upper case which has a unique solution given our assumptions on $R(.)$ and $C(.)$. Denote this solution by

$$e^* = \arg\max\{R(E(e)) - C(e)\} \quad s.t. \quad E(e) - R(E(e)) \geqslant F^E$$

Not only must $e^*$ meet the "equity performance" hurdle $E(e^*) - R(E(e^*)) \geqslant F^E$ or

$$R(E(e^*)) \leqslant E(e^*) - F^E, \tag{G.1}$$

but also the "debt overhang" constraint $R(E(e^*)) - C(e^*) \geqslant 0$ or

$$R(E(e^*)) \geqslant C(e^*) \tag{G.2}$$

which, via $E(e^*) = [V(e^*) - D]^+$, defines an upper bound on $D$ since $R(.)$ is increasing. Indeed, both (G.1) and (G.2) imply financing bounds: For any rationally expected $e^*$, (G.1) can be made binding by increasing $F^E$ and (G.2) can be made binding by increasing $D$.

**Tendering Decisions.** As in the other model variants, target shareholders tender their shares such that the free-rider condition endogenously binds: $p = E(e^*)$.

**Bid and Financing.** The bidder chooses the amount of equity capital $F^E$, the amount of debt funding $F^D$, and the bid price $p$ to solve

$$\max_{F^E,F^D,p} F^D + F^E - p + R(E(e^*)) - C(e^*)$$

subject to the free-rider condition and (G.1)-(G.2) from the bidder's stage-3 problem (of which the last two subsume outside investors' and creditors' break-even conditions). It is optimal to increase $F^E$ until (G.1) binds. Subject to (G.2), $F^D = D$. Hence,

$$\max_{D,p} D - p + E(e^*) - C(e^*).$$

The binding free-rider condition reduces this to

$$\max_{D,p} D - C(e^*), \tag{G.3}$$

which is exactly the same as in the baseline model. Thus, the key mechanism underlying our results is at work *for any* $R(E)$: For any rationally anticipated effort $e^*$ given $R(E)$, the bidder raises $D$ until debt overhang constraint (G.2) binds. Thus, for any compensation rule $R(E)$, optimal leverage equalizes the expected value of the equity-based incentive to the expected effort cost:

$$R(E(e^*)) = C(e^*).$$

As in the baseline model, the equity-based payments only serve to induce bidder effort but are not the source of bidder *profit*. Indeed, by increasing $D$ (until (G.2) binds), the bidder actually reduces her equity-based reward $R(E)$. Any part of $R(E)$ comes out of post-buyout equity value, which target shareholders fully extract (i.e., free-ride on), so it cannot be a source of gains. It is $D$ that provides the bidder with gains that elude free-riding, while $R(E)$ merely serves as a commitment to exert effort, which enables the bidder to raise debt financing.

The fact that (G.3) does not feature $R(E)$ means that the latter affects the outcome only through its effect on $D$ via debt overhang constraint (G.2). In other words, the shape of $R(E)$ matters for the bidder's debt capacity. Within the logic of our model, this means that an exogenous (binding) limit $\overline{D}$ on debt causes bidders to prefer incentive contracts $R(E)$ that induce a lower level of effort. Put differently, our model predicts that PE firms are willing to use higher-powered incentive contracts because they are allowed to lever up the buyouts.

Conversely, a *bidder-optimal* incentive contract $R(E)$—in the absence of a limit $\overline{D}$—is one that maximizes her debt capacity in the buyout. More precisely, since the bidder's profit is $D - C(e)$, for a given $e$ and hence $C(e)$, a bidder-optimal compensation contract $R(E)$ maximizes the bidder's debt capacity $D$ conditional on incentivizing $e$.

Suppose the bidder wants to implement $\hat{e}$, i.e., raise firm value to $V(\hat{e}) \equiv \hat{V} \in (V_0, V^{fb})$ where $V_0$ and $V^{fb}$ denote the firm's status quo value and its first-best value, respectively.[26] Consider the contract below (depicted in Figure O.7):

1. **Hurdle rate**. Let the bidder use $F^E = V_0$ in outside equity financing for the buyout. So she is paid equity-based compensation only once $E = V - D > V_0$, that is, once the outside equity investors have received their required return.

2. **Catch-up**. For $E \in [V_0, \hat{V} - D]$, let the bidder receive any marginal increase in equity value, and hence the full incremental equity appreciation $E - V_0$; since $D$ is fixed, this makes the bidder the full recipient of any *marginal* increase in $V$ within that range. It is then locally optimal for the bidder to generate $\hat{V}$ within that range. For $E \in [\hat{V} - D, V^{fb}]$, set $R'(E) < C'(\hat{e})$ such that the bidder has no incentive to prefer $e > \hat{e}$.

3. **Maximal leverage**. Set $D$ such that $\hat{V} - V_0 - D = \hat{C}$ where $\hat{C} \equiv C(\hat{e})$. This ensures that the bidder's equity-based reward $\hat{V} - V_0 - D$ at $e = \hat{e}$ just covers the associated effort cost $\hat{C}$. That is, the debt overhang constraint would bind, ensuring that $e = \hat{e}$ is also globally optimal, while maximizing $D$. This is then the maximally extractable amount of debt conditional on implementing $e = \hat{e}$, as $D = \hat{V} - V_0 - \hat{C}$ equals the

[26]In our model, $V_0$ is normalized to 0, but for the purposes of this illustration, considering $V_0 > 0$ aids mapping the example to the real world. Following Müller and Panunzi (2004), we assume, for simplicity, no initial debt and that the bidder cannot offer less than $V_0$ even with bootstrapping.

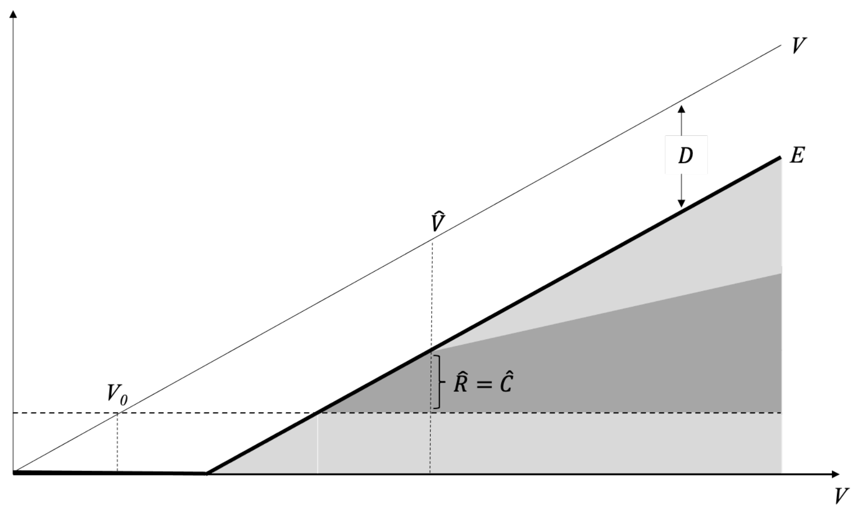


Figure O.7: A bidder-optimal contract with outside equity investors for implementing effort $\hat{e}$. For any post-buyout firm value $V$, the bidder and outside equity investors split the resulting equity value $E = V - D$. The bidder's part is shaded dark gray.

*full* surplus at $e = \hat{e}$.[27]

Under the above contract, the post-buyout equity value will equal $\hat{E} = V_0 + \hat{C}$. The free-rider condition hence imposes $p = \hat{E}$ for the acquisition of this equity. Outside equity investors contribute $F^E = V_0$. The bidder's net funding contribution is $\hat{C} - D$.

This contract maximizes buyout debt conditional on inducing a given effort $\hat{e}$. To put it differently, the bidder's equity-based reward under the above contract is, in equilibrium, the *full* value improvement net of debt rather than merely a fraction of that: the debt overhang constraint is $\hat{V} - V_0 - D \geqslant \hat{C}$ rather than $\alpha(\hat{V} - V_0 - D) \geqslant \hat{C}$ for some $\alpha \in [0, 1]$, thus maximizing the value of $D$ consistent with the inequality. Any $\hat{e}$ implementable in the baseline model with straight equity stakes can be implemented by the above contract with more debt financing, that is, more profitably for the bidder.

Compared to straight equity stakes, contracts like the above induce bidders not only to raise more debt but also to create more value. Inserting the maximally extractable debt amount $D = V - V_0 - C(e)$ into the bidder's ex ante profit $D - C(e)$ reduces the ex ante optimization problem to

$$\max_e \underbrace{V(e) - C(e)}_{\mathcal{W}} - C(e) \tag{P$_R$}$$

[27]The debt-maximizing contract is not unique; there are many ways to make $e^+$ locally optimal.

where $\mathcal{W}$ denotes total takeover surplus. The term $C(e)$ appears twice—(i) once because the bidder incurs a direct effort cost $C(e)$ to improve value and (ii) once more because the debt overhang constraint requires a positive post-takeover equity reward equaling $C(e)$ to incentivize the bidder ex post but this equity value is extracted by the target shareholders. In our baseline model, this free-riding "cost" is $\frac{C(e)}{\alpha}$ with $\alpha \leqslant 1$ instead. It follows from simple comparison that the bidder's optimal effort under the above optimization program ($P_R$) must be (weakly) higher than under the analogous optimization program (P) in our baseline model (see Section 2.2.3).

**Proposition G.1.** *Richer outside equity contracts can increase bootstrapping and thereby value creation.*

There are three main takeaways from this extension. First, the key effect identified in the baseline model is robust. Second, this richer setting highlights distinct roles played by fees PE firms collect from their partnerships with outside equity investors (which depend on equity returns) and fees they collect directly from targets (which dilute equity returns). The former provide the PE firms with the necessary incentives, whereas the latter extract buyout gains in the face of the free-rider problem. The two types of fees are interdependent and both necessary in our model. Third, fine-tuning the performance-based fee structure (i.e., the equity-based compensation) can enhance bidder incentives and thereby raise debt capacity. This increases the bidders' ability to extract gains through debt and upfront fees and, in turn, their willingness to create more value. Though we hesitate to give Proposition G.1 too much weight because, in practice, arrangements between the general and limited partners in buyout funds likely account for considerations that lie outside of our framework (see, e.g., Axelson et al. 2009). Still, even if our model may not provide a *comprehensive* analysis of factors that shape PE firms' compensation contracts, we believe that our main point is valid: A limit on bootstrapping in buyouts would reduce PE firms' willingness to use, or operate under, higher-powered incentive contracts.