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The Co-Pricing Factor Zoo

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Abstract

We analyze 18 quadrillion models for the joint pricing of corporate bond and stock returns. Strikingly, we find that equity and nontradable factors alone suffice to explain corporate bond risk premia once their Treasury term structure risk is accounted for, rendering the extensive bond factor literature largely redundant for this purpose. While only a handful of factors, behavioral and nontradable, are likely robust sources of priced risk, the true latent stochastic discount factor is *dense* in the space of observable factors. Consequently, a Bayesian Model Averaging Stochastic Discount Factor explains risk premia better than all low-dimensional models, in- and out-of-sample, by optimally aggregating dozens of factors that serve as noisy proxies for common underlying risks, yielding an out-of-sample Sharpe ratio of 1.5 to 1.8. This SDF, as well as its conditional mean and volatility, are persistent, track the business cycle and times of heightened economic uncertainty, and predict future asset returns.

Keywords: Macro-finance, asset pricing, corporate bonds, bond-stock co-pricing, factor zoo, factor models, Bayesian methods.

JEL Classification Codes: G10; G12; G40; C12; C13; C52.

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Wherever there is risk, it must be compensated to the lender by a higher premium or interest.

— J. R. McCullough (1830, pp. 508–9)

In their seminal paper, Fama and French (1993) set themselves to "examine whether variables that are important in bond returns help to explain stock returns, and vice versa." Thirty years later, the equity literature has produced its own, independent, 'factor zoo' (Cochrane (2011)), while the corporate bond literature has effectively returned to square one with Dickerson, Mueller, and Robotti (2023) showing that there is no satisfactory (observable) factor model for that asset class.[1] Hence, to date, a model for the *joint* pricing of corporate bonds and stocks has escaped discovery—we fill this gap.

Generalizing recent methodological advances in Bayesian econometrics (Bryzgalova, Huang, and Julliard (2023)) to handle heterogeneous asset classes, we comprehensively analyze all observable factors and models proposed to date in the bond and equity literature. Our method allows us to not only study models or factors in isolation, but also consider all of their possible combinations, resulting in over 18 quadrillion models stemming from the joint zoo of corporate bond and stock factors. And we do so while relaxing the cornerstone assumptions of previous studies: the existence of a unique, low-dimensional, correctly specified and well-identified factor model.

Ultimately, this allows us to pinpoint the robust sources of priced risk in both markets, and a novel benchmark Stochastic Discount Factor (SDF) that prices both asset classes, significantly better than all existing models, both in- and out-of-sample. Remarkably, our analysis reveals that once corporate bonds' Treasury term structure risk is accounted for, stock and nontradable factors alone suffice to explain corporate bond risk premia—rendering the extensive bond factor literature largely redundant for this purpose.

First, we find that the 'true' latent SDF of bonds and stocks is *dense* in the space of observable bond and stock factors—literally dozens of factors, both tradable and nontradable, are necessary to span the risks driving asset prices. Yet, the SDF-implied maximum Sharpe ratio is not excessive, indicating that, as we confirm in our analysis, multiple bond and stock factors proxy for common sources of fundamental risk. Importantly, density of the SDF implies that the sparse models considered in the previous literature are affected by severe misspecification

[1] More precisely, they document that all low dimensional linear factor models in the previous literature add little spanning to a simple *bond* version of the Capital Asset Pricing Model, the CAPMB. At the same time, they show that the CAPMB is in itself an unsatisfactory pricing model.

and, as we show, rejected by the data and outperformed by the most likely SDF components that we identify.

Second, a Bayesian Model Averaging Stochastic Discount Factor (BMA-SDF) over the space of all possible models (including bond, stock, and nontradable factors) explains (jointly and separately) corporate bond and equity risk premia better than all existing models and most likely factors, both in- and out-of-sample. Moreover, the BMA-SDF's conditional mean and volatility—hence, the implied conditional Sharpe ratio achievable in the economy—have clear business cycle patterns. In particular, the volatility of the SDF increases sharply at the onset of recessions and at times of heightened economic uncertainty. That is, the estimated SDF behaves as one would expect from the intertemporal marginal rate of substitution of an agent exposed to the risks arising from general economic conditions and market turmoil.

Third, the predictability of the first and second moments of the SDF suggests time-varying risk premia in the economy and predictability of asset returns with lagged SDF information. We verify this by running predictive regressions of future asset returns on the conditional variance of the BMA-SDF, alone and interacted with the conditional mean of the SDF, as implied by the Hansen and Jagannathan (1991) representation of the conditional SDF. We not only find that lagged SDF information is highly significant in predicting future asset returns, but also that the amount of explained time series variation in monthly and annual returns is much larger than what is achievable with canonical predictors. This result is remarkable for two reasons. First, the BMA-SDF is *not* by construction geared toward predicting future returns: it is instead identified only under the restriction that a valid SDF should explain the cross-section of risk premia—not the time series of returns. Second, it offers an important validation of our estimation of the SDF: if risk premia are time-varying, future returns should be predictable with lagged SDF information, and that is exactly what our BMA-SDF delivers.

Fourth, we show theoretically that, to construct a tradable portfolio that captures the SDF-implied maximum Sharpe ratio achievable in the economy, one should focus on the posterior expectation of the market prices of risk of *all* factors, rather than on the factors' posterior probabilities (or some ancillary selection statistic), which have been the focus of the previous literature. Such an approach can correctly recover the pricing of risk even if the observed factors are only noisy proxies of the true, yet latent, sources of risk priced in the market. *In the data*, this yields a trading strategy with a time-series out-of-sample annualized Sharpe ratio of 1.5 to 1.8 (despite only yearly rebalancing) in an evaluation period (July 2004 to December 2022) that spans both the Global Financial Crisis and the COVID pandemic.

Fifth, we shed light on which factors, and which types of risk, are reflected in the cross-section of bond and equity risk premia. We find that only a handful of factors should be in the SDF with high probability. In particular, two factors meant to capture the bond and stock post-earnings announcement drift anomalies, PEADB and PEAD, respectively, are very likely sources of priced risk in the joint cross-section of bond and stock returns.[2] In addition to these two behavioral sources of risk, the other most likely components of the SDF are all nontradable in nature, and are a proxy for the slope of the Treasury yield curve (YSP), the AAA/BAA yield spread (CREDIT), and the idiosyncratic equity volatility (the IVOL of Campbell and Taksler (2003)). As we show, these factors alone are enough to price the cross-section of bonds and stocks better than canonical observable factor models. Nevertheless, the importance of *individual* factors should not be overstated. Even excluding the most likely factors when constructing it, the BMA-SDF strongly outperforms these individual factors and *all* low dimensional factor models—from the celebrated Fama and French (1993) model to the latest arrival in the zoo (Dick-Nielsen et al. (2025)). This superior performance occurs because the true latent SDF is dense and demands large compensations for risks that are not fully spanned by just a handful of individual observable factors. Furthermore, we find that both discount rate and cash-flow news are sources of priced risk, and yield sizeable contributions (albeit larger for the former) to the Sharpe ratio of the latent SDF.

Sixth, we demonstrate that a portion of corporate bond risk premia serves as compensation for their implicit Treasury term structure risk. Once this component is removed, the factors proposed in the tradable bond factor zoo have very little residual information content for characterizing the SDF: in this case, a BMA-SDF constructed only with stock and nontradable factors can explain the joint cross-section of bonds and stocks as well as our full BMA-SDF. This finding extends and explains the result in van Binsbergen et al. (2025), who show that once corporate bond returns are adjusted for duration risk, the equity CAPM has higher explanatory power for bond risk premia than benchmark bond models. Furthermore, we show that the empirical success of the bond factor zoo in the previous literature is largely driven by its ability to price the Treasury term structure risk—a component of bond risk premia that tradable stock

[2] The post-earnings announcement drift phenomenon is the observation, first documented in equity markets, whereby firms experiencing positive earnings surprises subsequently earn higher returns than those with negative earnings surprises. See, e.g., Hirshleifer and Teoh (2003), Della Vigna and Pollet (2009), Hirshleifer et al. (2011) and Nozawa et al. (2025) for the microfoundations of this phenomenon.

factors do not capture.

Finally, we conduct extensive robustness checks. Most notably, we show that: (i) altering the priors regarding the relative importance of bond versus stock factors, or equivalently a potential 'alpha mismeasurement' phenomenon in bond market data, has only a limited effect on the posterior probabilities of the factors and the pricing performance of the BMA-SDF; (ii) a BMA-SDF estimated with a prior that imposes sparsity—overwhelmingly the focus of the previous literature—performs worse than our baseline BMA-SDF, yet still improves upon competing models; (iii) as our theoretical results imply, *removing* the most likely factors from the estimation—a challenging test for the method—leads to only minor deterioration in the performance of the BMA-SDF in- and out-of-sample; (iv) all findings remain materially unchanged across *hundreds* of sets of corporate bond and stock in-sample test assets—we identify a similar set of most likely factors, consistent market prices of risk, and stable in-sample asset pricing performance; (v) out-of-sample, the pricing performance of the BMA-SDF is superior across *millions* of alternative cross-sections of stocks and bonds; (vi) lastly, the results are robust to extending, by dozens of factors, both the stock and bond factor zoos that we consider in our baseline estimation (to maximize the time-series sample size), to varying sample and subsample estimations, and to using a multiplicity of different corporate bond datasets.

The remainder of the paper is organized as follows. Below, we review the most closely related literature and our contribution to it. Section 1 describes the data used in our analysis, while Section 2 outlines our Bayesian SDF method and its properties for inference, selection, and aggregation. Section 3 presents our empirical findings, and Section 4 contains extensive robustness checks. Section 5 concludes. Additional details and results are reported in the Appendix and the Internet Appendix.

Closely related literature. Our research contributes to the active and growing body of work that critically reevaluates existing findings in the empirical asset pricing literature using robust inference methods. Following Harvey et al. (2016), a large literature has tried to understand which existing factors (or their combinations) drive the cross-section of returns. In particular, Gospodinov et al. (2014) develop a general method for misspecification-robust inference, while Giglio and Xiu (2021) exploit the invariance principle of PCA and recover the risk premium of a given factor from the projection on the span of latent factors driving a cross-section of returns. Similarly, Dello Preite et al. (2025) recover latent factors from the residuals of an asset pricing model, effectively completing the span of the SDF. Feng et al. (2020) combine cross-sectional asset pricing regressions with the double-selection LASSO of Belloni et al. (2014) to provide valid inference on the selected sources of risk when the true SDF is sparse. Kozak et al. (2020) use a ridge-based approach to approximate the SDF and compare sparse models based on principal components of returns. Our approach instead identifies a dominant pricing model—if such a model exists—or a BMA across the space of all models, even if the true model is not sparse in nature, hence cannot be proxied by a small number of factors. Furthermore, and importantly, our work focuses on the *co-pricing* of corporate bond and stock returns, hence shedding light on both the common, as well as the market specific, sources of risk.

As Harvey (2017) stresses in his American Finance Association presidential address, the factor zoo naturally calls for a Bayesian solution—and we adopt one. In particular, we generalize the Bayesian method of model estimation, selection, and averaging developed in Bryzgalova, Huang, and Julliard (2023) to handle heterogeneous asset classes.

Numerous strands of the literature rely on Bayesian tools for asset allocation, model selection, and performance evaluation. Our approach is most closely linked to Pástor and Stambaugh (2000) and Pástor (2000) in that we assign a prior distribution to the vector of pricing errors, and this maps into a natural and transparent prior for the maximal Sharpe ratio achievable in the economy. Barillas and Shanken (2018) also extend the prior formulation of Pástor and Stambaugh (2000) and provide a closed-form solution for the Bayes factors when all factors are tradable in nature. Chib et al. (2020) show that the improper prior formulation of Barillas and Shanken (2018) is problematic, and provide a new class of priors that leads to valid comparisons for tradable factor models. As in these papers, our model and factor selection is based on posterior probabilities, but our method is designed to work with both tradable and *nontradable* factors—as we show, the latter are a first-order source of priced risk in the joint space of corporate bonds and stock returns.

Our work is closely related to the literature that stresses the optimality of Bayesian model averaging for a very wide set of optimality criteria (see, e.g., Schervish (1995) and Raftery and Zheng (2003)).[3] We highlight that Bayesian model averaging *over the space of models* can be expressed as model averaging *over the space of factors*. This allows us to show that posterior

[3]In particular, BMA is "optimal on average," i.e., no alternative method can outperform the BMA for all values of the true unknown parameters. Furthermore, a BMA-SDF can be microfounded thanks to the equivalence between an economy populated by agents with heterogeneous beliefs and a Bayesian representative agent setting (Heyerdahl-Larsen et al. (2023)).

factor probabilities (which the previous Bayesian asset pricing literature has overwhelmingly focused on) and posterior market prices of risk (across the space of models) have very different information content. In particular, as we demonstrate, it is the latter, not the former, that tells us how to construct tradable portfolios that achieve the BMA-SDF-implied maximum Sharpe ratio. In the data, this yields a trading strategy with an (annualized) out-of-sample Sharpe ratio of 1.5 to 1.8. Most importantly, our approach can deal with a very large factor space, is not affected by the common identification failures that invalidate inference in asset pricing (see, e.g., Kan and Zhang (1999a,b), Kleibergen (2009), and Gospodinov et al. (2019)), and provides an optimal method for aggregating the pricing information stemming from the joint zoo of corporate bond and equity factors even if only noisy proxies of the true fundamental risks are available.

In the complete market benchmark, the pricing measure should be consistent across asset classes, and equilibrium models normally yield nontradable state variables. Therefore, we focus on the co-pricing of corporate bonds and stocks, and consider jointly a very broad collection of potential sources of risk that extends well beyond the set of bond and stock tradable factors that have been studied in isolation in the previous literature. Hence, our paper speaks to the large literature on co-pricing, originated with the seminal work of Fama and French (1993), and market segmentation of bonds and stocks (see, e.g., Chordia et al. (2017), Choi and Kim (2018), or Sandulescu (2022)). In particular, our paper is related to the body of work that explores whether equity market risk proxies (see, e.g., Blume and Keim (1987) and Elton et al. (2001)), equity volatilities (see, e.g., Campbell and Taksler (2003) and Chung et al. (2019)), and equity-based characteristics (see, e.g., Fisher (1959), Giesecke et al. (2011), and Gebhardt et al. (2001)) are likely drivers of corporate bond returns, and on the commonality of risks across markets (see, e.g., He et al. (2017), Lettau et al. (2014), and Chen et al. (2024)).

Overall, we find that factors in both the corporate bond and equity zoos are needed for the joint pricing of both asset classes, and stock factors do carry relevant information to explain bond returns. Yet, there is substantial overlap between the risks spanned by these two markets. That is, multiple bond and stock factors are noisy proxies for common underlying sources of risk. Nevertheless, as we show, corporate bond risk premia include an implicit compensation for Treasury term structure risk—a risk that the bond factor zoo, and nontradable factors proposed therein, price very well, while equity factors do not. And once this term structure risk component is removed, tradable bond factors become largely unnecessary for the joint pricing of bonds and stocks.

Several theoretical contributions stress that real economic activity and the business cycle should be among the drivers of bond risk premia (see, e.g., Bhamra et al. (2010), Khan and Thomas (2013), Chen et al. (2018), and Favilukis et al. (2020)). Echoing both the general equilibrium model predictions of Gomes and Schmid (2021) and the empirical findings of Elton et al. (1995) and Elkamhi et al. (2023), we show that the BMA-SDF conditional first and second moments have a clear business cycle pattern and peak during recessions and at times of heightened economic uncertainty, and that nontradable factors (especially proxies of the economic cycle such as the slope of the yield curve), are salient components of the pricing measure.[4] Furthermore, we show that the business cycle properties of the BMA-SDF and its volatility are predictable, and predict—as theory implies in this case—future asset returns, generating a substantial degree of time variation in conditional risk premia.

Our work also relates to behavioral biases and market frictions in asset pricing. In particular, complementing the evidence of Daniel et al. (2020) and Bryzgalova et al. (2023) for the equity market, we show that the post-earnings announcement drifts of both bonds (see Nozawa et al. (2025)) and stocks are extremely likely drivers of corporate bond and stock risk premia. Furthermore, we show that cash-flow and discount rate news (see, e.g., Vuolteenaho (2002), Cohen et al. (2002), Zviadadze (2021), and Delao et al. (2025)) are both important drivers of risk premia in the joint cross-section of bonds and stocks, but the latter are responsible for a larger share of the volatility of the co-pricing SDF.

1 Data

Our analysis relies on a combination of corporate bond and stock data, which we present below and in more detail in Internet Appendix IA.1. As academic research relies on various sources for corporate bond data, we are careful to estimate our model across *all* datasets available to us to ensure that our results are neither driven by the data source nor the choice of bond or stock test assets (see the discussion in Section 4.4).

[4] Elton et al. (1995) show that adding fundamental macro-risk variables (such as GNP, inflation and term spread measures) significantly improves pricing performance relative to equity and bond market index models. Elkamhi et al. (2023) show that the long-run consumption risk measure of Parker and Julliard (2003) yields a one-factor model with significant explanatory power for corporate bonds, and such an SDF, as documented in Parker and Julliard (2005), has a very strong business cycle pattern.

1.1 Corporate bond data and corporate bond returns

Our baseline results in the main text are based on the constituents of the corporate bond data set from the Bank of America Merrill Lynch (BAML) High Yield (H0A0) and Investment Grade (C0A0) indices made available via the Intercontinental Exchange (ICE) from January 1997 to December 2022. For the period from January 1986 to December 1996, we augment the data using the Lehman Brothers Fixed Income (LBFI) database.[5] These data are then merged with the Mergent Fixed Income Securities Database (FISD) to obtain additional bond characteristics. After merging the two datasets and applying the standard filters, our bond-level data spans 37 years, resulting in a total of 444 monthly observations. Our corporate bond sample is representative of the U.S. market and, once merged with CRSP equity data, covers 75% of the total stock market capitalisation of all listed firms on average (see Figure IA.3 of the Internet Appendix).[6]

In the baseline analysis, we use *excess* bond returns defined as the total bond return minus the one-month risk-free rate of return.[7] In addition, we follow van Binsbergen et al. (2025) and repeat our analysis with *duration-adjusted* returns, whereby we subtract the return on a portfolio of duration-matched U.S. Treasury bonds from the total bond return. We do not further winsorize, trim, or augment the underlying bond return data in any way, avoiding the biases that such procedures normally induce (Duarte et al. (2025) and Dickerson et al. (2024)).

1.2 The joint factor zoo

We use all factors in published papers for which a monthly time series matching our sample is publicly available. Our bond-specific factor zoo includes 16 tradable bond factors. From the equity literature, we include an additional 24 tradable factors. This set is smaller than the tradable equity factor zoo in Bryzgalova et al. (2023) as for several of their 34 tradable factors, an updated series is not publicly available. Moreover, we exclude factors for which authors did not provide sufficient information for exact replication.[8] Our nontradable zoo comprises 14

[5]We follow van Binsbergen et al. (2025) and begin the LBFI sample in 1986. Prior to 1986, bonds in the LBFI database are predominantly investment grade (91% of bonds) with 67% of all bonds priced with matrix pricing (i.e., the prices are not actual dealer quotes).

[6]See Internet Appendix IA.1 for a detailed description of the databases and associated cleaning procedures. Therein, we also discuss the additional datasets used for robustness tests.

[7]We use the one-month risk-free rate from Kenneth French's website.

[8]The excluded factors are all among the *least* likely components of the equity SDF in Bryzgalova et al. (2023). Nevertheless, we consider *all* of their factors in our robustness analysis.

factors, many of which have previously been used to study stock returns.

Overall, in our baseline analysis, we consider 54 factors—40 tradable and 14 nontradable—yielding $2^{54} \approx 18$ quadrillion models. In Section 4.4.3, we extend this to include dozens of additional factors available over varying subsamples, for a grand total of 91 candidate pricing factors. All factors are described in Table A.1 of Appendix A.[9] Internet Appendix IA.1.3 analyzes the robustness of bond factors with respect to data source and calculation method.

1.3 In-sample bond and stock test assets

For our in-sample (IS) estimation of the BMA-SDF, we construct a set of 50 bond portfolios that are sorted on various bond characteristics to ensure a sufficiently broad cross-section. The first 25 portfolios are double-sorted on credit spreads and bond size, while the remaining 25 portfolios are double-sorted on bond rating and time-to-maturity. All portfolios are value-weighted based on the market capitalization of the bond issue, defined as the bond dollar value multiplied by the number of outstanding units of the bond. For the stock test assets, we rely on a set of 33 portfolios and anomalies very similar to those used in Kozak et al. (2020) and Bryzgalova et al. (2023).[10]

In addition, we include the 40 tradable factors as Barillas and Shanken (2017) emphasize that factors included in a model should price any factor excluded from the model. This, along with the use of a nonspherical pricing error formulation (i.e., GLS) also imposes (asymptotically) the restriction of factors pricing themselves. For the estimation of the co-pricing BMA-SDF, we naturally include both bond and stock tradable factors, while we only include the respective bond and stock tradable factors to estimate the bond- and stock-specific BMA-SDFs.

In summary, our baseline cross-section comprises a wide array of 50 bond and 33 stock portfolios, as well as the underlying 40 tradable factors, for a total of 123 IS test assets.

1.4 Out-of-sample bond and stock test assets

To test the out-of-sample (OS) asset pricing efficacy of the BMA-SDF estimated on the IS test assets, we employ a broad cross-section of additional corporate bond, stock, and U.S. Treasury bond portfolios. For bonds, we use decile-sorted portfolios on: (i) bond historical

[9] All factors are publicly available from the authors' personal websites and public repositories, listed therein. We make our 16 tradable bond factors available on the companion website: openbondassetpricing.com

[10] These are publicly available from Chen and Zimmermann (2022) and Jensen et al. (2023), and replicable using CRSP and Compustat. See jkpfactors.com.

95% value-at-risk, (ii) duration, (iii) bond value (Houweling and Van Zundert (2017)), (iv) bond book-to-market (Bartram et al. (2025)), (v) long-term reversals (Bali et al. (2021a)), (vi) momentum (Gebhardt et al. (2005b)), as well as the bond version of the 17 Fama-French industry portfolios—totaling 77 bond-based portfolios.

For stocks, we include decile-sorted portfolios on: (i) earnings-to-price, (ii) momentum, (iii) long-term reversal, (iv) accruals, (v) size (measured by market capitalization), (vi) equity variance, in addition to the equity version of the 17 Fama-French industry portfolios (following Lewellen et al. (2010)), also resulting in 77 stock-based portfolios.

For U.S. Treasury bonds, we use monthly annualized continuously compounded zero-coupon yields from Liu and Wu (2021). We price the U.S. Treasury bonds each month using the yield curve data and then compute monthly discrete excess returns across the term structure as the total return in excess of the one-month Treasury Bill rate. Our set of OS U.S. Treasury portfolios consists of 29 portfolios, ranging from 2-year Treasury notes up to 30-year Treasury bonds in increments of one year.

In summary, our baseline OS test assets comprise 154 bond and stock portfolios (77 each) from the 14 distinct cross-sections discussed above.[11] We not only use the joint cross-section, but we also construct $2^{14} - 1 = 16,383$ possible unique combinations of OS cross-sections.[12] For robustness, we conduct OS pricing tests with the Jensen et al. (2023) and the Dick-Nielsen et al. (2025) bond and stock anomaly data.

2 Econometric method

This section introduces the notation and summarizes the methods employed in our empirical analysis. We consider linear factor models for the SDF and focus on the SDF representation since we aim to identify the factors that have pricing ability for the joint cross-section of corporate bond and stock returns.[13]

We first review the frequentist estimation and the inference problems that arise therein in the presence of weak identification caused by weak and useless factors. We then summarize

[11] All are available from Kenneth French's webpage and Cynthia Wu's webpage.

[12] Further details about factors and in- and out-of-sample test assets, as well as links to the data sources, can be found in Table IA.II of the Internet Appendix.

[13] Recall that a factor might have a significant risk premium even if it is not part of the SDF, just because it has non-zero correlation with the true latent SDF. Hence, in order to identify the pricing measure, focusing on the SDF representation is the natural choice.

the Bayesian method proposed by Bryzgalova, Huang, and Julliard (2023) to address the weak identification problem, present our extension of the approach to handle different asset classes, and introduce a more flexible prior structure. Finally, we establish a set of important new properties for the Bayesian model averaging of the SDF, and illustrate its mechanics in finite samples with a simulation study.

2.1 Frequentist estimation of linear factor models

We begin by introducing the notation used throughout the paper. The returns of N test assets, which are long-short portfolios, are denoted by $\boldsymbol{R}_t = (R_{1t} \dots R_{Nt})^\top$, $t = 1, \dots T$. We consider K factors, $\boldsymbol{f}_t = (f_{1t} \dots f_{Kt})^\top$, $t = 1, \dots T$, that can be either tradable or nontradable. A linear SDF takes the form $M_t = 1 - (\boldsymbol{f}_t - \mathbb{E}[\boldsymbol{f}_t])^\top \boldsymbol{\lambda_f}$, where $\boldsymbol{\lambda_f} \in \mathbb{R}^K$ is the vector containing the market prices of risk (MPRs) associated with the individual factors. Throughout the paper, $\mathbb{E}[X]$ or μ_X denote the unconditional expectation of an arbitrary random variable X.

In the absence of arbitrage opportunities, we have that $\mathbb{E}[M_t \boldsymbol{R_t}] = \boldsymbol{0_N}$; hence, expected returns are given by $\boldsymbol{\mu_R} \equiv \mathbb{E}[\boldsymbol{R}_t] = \boldsymbol{C_f \lambda_f}$, where $\boldsymbol{C_f}$ is the covariance matrix between $\boldsymbol{R}_t$ and $\boldsymbol{f}_t$, and prices of risk, $\boldsymbol{\lambda_f}$, are commonly estimated via the cross-sectional regression

$$\boldsymbol{\mu_R} = \lambda_c \boldsymbol{1_N} + \boldsymbol{C_f \lambda_f} + \boldsymbol{\alpha} = \boldsymbol{C\lambda} + \boldsymbol{\alpha}, \tag{1}$$

where $\boldsymbol{C} = (\boldsymbol{1_N}, \boldsymbol{C_f})$, $\boldsymbol{\lambda}^\top = (\lambda_c, \boldsymbol{\lambda_f}^\top)$, λ_c is a scalar average mispricing (equal to zero under the null of the model being correctly specified), $\boldsymbol{1_N}$ is an N-dimensional vector of ones, and $\boldsymbol{\alpha} \in \mathbb{R}^N$ is the vector of pricing errors in excess of λ_c (equal to zero under the null of the model).

Such models are usually estimated via GMM, MLE or two-pass regression methods (see, e.g., Hansen (1982), Cochrane (2005)). Nevertheless, as pointed out in a substantial body of literature, the underlying assumptions for the validity of these methods (see, e.g., Newey and McFadden (1994)), are often violated (see, e.g., Kleibergen and Zhan (2020) and Gospodinov and Robotti (2021)), and identification problems arise in the presence of a *weak* factor (i.e., a factor that does not exhibit sufficient comovement with any of the assets, or has very little cross-sectional dispersion in this comovement, but is nonetheless considered a part of the SDF). These issues, in turn, lead to incorrect inferences for both weak and strong factors, erroneous model selection, and inflate the canonical measures of model fit.[14]

[14] These problems are common to GMM (Kan and Zhang (1999a)), MLE (Gospodinov et al. (2019)), Fama-MacBeth regressions (Kan and Zhang (1999b), Kleibergen (2009)), and even Bayesian approaches with flat priors for risk prices (Bryzgalova et al. (2023)).

2.2 The Bayesian solution

Albeit robust frequentist inference methods have been suggested in the literature for specific settings, our task is complicated by the fact that we want to parse the entire zoo of bond and stock factors, rather than estimate and test an individual model. Furthermore, we aim to identify the best specification—*if* a dominant model exists—or aggregate the information in the factor zoo into a single SDF if no clear best model arises. Therefore, we extend the Bayesian method proposed in Bryzgalova, Huang, and Julliard (2023) (BHJ), since it is applicable to both tradable and nontradable factors, can handle the entire factor zoo, is valid under misspecification, and is robust to weak inference problems. This Bayesian approach is conceptually simple, since it leverages the naturally hierarchical structure of cross-sectional asset pricing, and restores the validity of inference using transparent and economically motivated priors.

Consider first the time-series layer of the estimation problem. Without loss of generality, we order the K_1 tradable factors first, $\boldsymbol{f}_t^{(1)}$, followed by K_2 nontradable factors, $\boldsymbol{f}_t^{(2)}$; hence $\boldsymbol{f}_t \equiv (\boldsymbol{f}_t^{(1),\top}, \boldsymbol{f}_t^{(2),\top})^\top$ and $K_1+K_2 = K$. Denote by $\boldsymbol{Y_t} \equiv \boldsymbol{f_t} \cup \boldsymbol{R_t}$ the union of factors and returns, where $\boldsymbol{Y_t}$ is a p-dimensional vector.[15] Modelling $\{\boldsymbol{Y_t}\}_{t=1}^T$ as multivariate Gaussian with mean $\boldsymbol{\mu_Y}$ and variance matrix $\boldsymbol{\Sigma_Y}$, and adopting the conventional diffuse prior $\pi(\boldsymbol{\mu_Y}, \boldsymbol{\Sigma_Y}) \propto |\boldsymbol{\Sigma_Y}|^{-\frac{p+1}{2}}$, yields the canonical Normal-inverse-Wishart posterior for the time series parameters $(\boldsymbol{\mu_Y}, \boldsymbol{\Sigma_Y})$ in equations (A.11) and (A.12) of Appendix B.

The cross-sectional layer of the inference problem allows for misspecification of the factor model via the average pricing errors $\boldsymbol{\alpha}$ in equation (1). We model these pricing errors, as in the previous literature (e.g., Pástor and Stambaugh (2000) and Pástor (2000)), as $\boldsymbol{\alpha} \sim \mathcal{N}(\boldsymbol{0_N}, \sigma^2\boldsymbol{\Sigma_R})$, yielding the cross-sectional likelihood (conditional on the time series parameters)

$$p(\text{data}|\boldsymbol{\lambda}, \sigma^2) = (2\pi\sigma^2)^{-\frac{N}{2}}|\boldsymbol{\Sigma_R}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2\sigma^2}(\boldsymbol{\mu_R} - \boldsymbol{C\lambda})^\top \boldsymbol{\Sigma_R}^{-1}(\boldsymbol{\mu_R} - \boldsymbol{C\lambda})\right\}, \tag{2}$$

where, in the cross-sectional regression, the 'data' are the expected risk premia, $\boldsymbol{\mu_R}$, and the factor loadings, $\boldsymbol{C} \equiv (\boldsymbol{1_N}, \boldsymbol{C_f})$. The above likelihood can then be combined with a prior for risk prices (presented below) to obtain a posterior distribution that informs inference and model selection.

Note that the assumption of a Gaussian conditional cross-sectional likelihood in equation (2) is not strictly necessary, and we could, in principle, use an alternative formulation (albeit, in most cases, this would cause us to lose many of the closed-form results that make our

[15] If one requires the tradable factors to price themselves, then $\boldsymbol{Y_t} \equiv (\boldsymbol{R_t}^\top, \boldsymbol{f}_t^{(2),\top})^\top$ and $p = N + K_2$.

method able to handle such high-dimensional models and parameter spaces). Nevertheless, there are two key reasons why Gaussianity is the most preferable assumption. First, the canonical quasi-maximum likelihood estimation property applies (Bollerslev and Wooldridge (1992)): that is, the likelihood in equation (2) yields consistent estimates even if the true distribution is not Gaussian. Instead, different distributional assumptions would yield consistency only if we "guess" the right distribution. Hence, Gaussianity is the *robust* choice. Second, consider estimating the model $\boldsymbol{R}_t = \boldsymbol{C}\boldsymbol{\lambda} + \boldsymbol{\varepsilon}_t$. Denoting with $\mathbb{E}_T$ the sample analogue of the unconditional expectation operator, we have $\mathbb{E}_T[\boldsymbol{R}_t] = \boldsymbol{C}\boldsymbol{\lambda} + \mathbb{E}_T[\boldsymbol{\varepsilon}_t]$. This implies that the pricing error $\boldsymbol{\alpha}$ should be equal to $\mathbb{E}_T[\boldsymbol{\varepsilon}_t]$. But the latter, under very general central limit theorem conditions (see, e.g., Hayashi (2000)), follows (under the null of the model) the limiting distribution $\boldsymbol{\alpha}|\boldsymbol{\Sigma_R} \sim \mathcal{N}(\boldsymbol{0_N}, \frac{1}{T}\boldsymbol{\Sigma_R})$. Hence, the Gaussian likelihood encoding in equation (2) not only ensures consistent estimates but is also a natural choice that guarantees compatibility of our hierarchical Bayesian modeling with frequentist asymptotic theory.

To handle model and factor selection, we introduce a vector of binary latent variables $\boldsymbol{\gamma}^\top = (\gamma_0, \gamma_1, \ldots, \gamma_K)$, where $\gamma_j \in \{0, 1\}$. When $\gamma_j = 1$, the j-th factor (with associated loadings $\boldsymbol{C_j}$) should be included in the SDF, and should be excluded otherwise.[16] In the presence of potentially weak factors and, hence, unidentified prices of risk, the posterior probabilities of models and factors are not well defined under flat priors.

To solve this issue, BHJ introduce an (economically motivated) prior that, albeit not informative, restores the validity of posterior inference. In particular, the uncertainty underlying the estimation and model selection problem is encoded via a (continuous spike-and-slab) mixture prior, $\pi(\boldsymbol{\lambda}, \sigma^2, \boldsymbol{\gamma}, \boldsymbol{\omega}) = \pi(\boldsymbol{\lambda} \mid \sigma^2, \boldsymbol{\gamma})\pi(\sigma^2)\pi(\boldsymbol{\gamma} \mid \boldsymbol{\omega})\pi(\boldsymbol{\omega})$, where

$$\lambda_j \mid \gamma_j, \sigma^2 \sim \mathcal{N}\left(0, r(\gamma_j)\psi_j\sigma^2\right). \tag{3}$$

Note the presence of three new elements, $r(\gamma_j)$, $\pi(\boldsymbol{\omega})$ and ψ_j, in the prior formulation.

First, $r(\gamma_j)$ captures the 'spike-and-slab' nature of the prior formulation. When the factor should be included, we have $r(\gamma_j = 1) = 1$, and the prior, the 'slab,' is just a diffuse distribution centred at zero. When instead the factor should not be in the model, $r(\gamma_j = 0) = r \ll 1$, the prior is extremely concentrated—a 'spike' at zero. As $r \to 0$, the prior spike is just a Dirac distribution at zero, hence it removes the factor from the SDF.[17]

[16] In the baseline analysis, we always include the common intercept in the cross-sectional layer, that is, $\gamma_0 = 1$. Nevertheless, we also consider $\gamma_0 = 0$, i.e., no common intercept, in the robustness analysis.

[17] We set $r = 0.001$ in our empirical analysis.

Second, the prior $\pi(\boldsymbol{\omega})$ not only gives us a way to sample from the space of potential models, but also encodes belief about the sparsity of the true model using the prior distribution $\pi(\gamma_j = 1|\omega_j) = \omega_j$. Following the literature on predictor selection, we set

$$\pi(\gamma_j = 1|\omega_j) = \omega_j, \quad \omega_j \sim Beta\left(a_\omega, b_\omega\right). \tag{4}$$

Different hyperparameters a_ω and b_ω determine whether one a priori favors more parsimonious models or not. The prior expected probability of selecting a factor is $\frac{a_\omega}{a_\omega+b_\omega}$ and we set $a_\omega = b_\omega = 1$ in the benchmark case, that is, we have a uniform (flat) prior for the model dimensionality and each factor has an ex ante expected probability of being selected equal to 50%.[18]

Third, the Bayesian solution to the weak factor problem in BHJ is to set

$$\psi_j = \psi \times \widetilde{\boldsymbol{\rho}}_j^{\top} \widetilde{\boldsymbol{\rho}}_j, \tag{5}$$

where $\widetilde{\boldsymbol{\rho}}_j \equiv \boldsymbol{\rho}_j - \left(\frac{1}{N}\sum_{i=1}^{N} \rho_{j,i}\right) \times \mathbf{1}_{\boldsymbol{N}}$, $\boldsymbol{\rho_j}$ is an $N \times 1$ vector of correlation coefficients between factor j and the test assets, and $\psi \in \mathbb{R}_+$ is a tuning parameter that controls the degree of shrinkage across all factors. That is, factors that have vanishing correlation with asset returns, or extremely low cross-sectional dispersion in their correlations (hence cannot help in explaining cross-sectional differences in returns), have a low value of ψ_j and are therefore endogenously shrunk toward zero. Instead, such a prior has no effect on the estimation of strong factors since these have large and dispersed correlations with the test assets, yielding a large ψ_j and consequently a diffuse prior.

Finally, for the cross-sectional variance scale parameter, σ^2, estimation and inference can be based on the canonical diffuse prior $\pi(\sigma^2) \propto \sigma^{-2}$. As per Proposition 1 of Chib et al. (2020), since the parameter σ is common across models and has the same support in each model, the marginal likelihoods obtained under this improper prior are valid and comparable.

The above hierarchical system yields a well-defined posterior distribution from which all the unknown parameters and quantities of interest can be sampled. Nevertheless, the prior formulation of BHJ might be overly restrictive when applied, as in our empirical analysis, to different asset classes jointly. To illustrate this, consider the case in which (as in our empirical application) all factors are standardized, and note that equations (3) to (5) then yield the following (squared) prior Sharpe ratio (SR) for each factor $f_{k,t}$:

$$\mathbb{E}_\pi[SR^2_{f_k} \mid \sigma^2] = \frac{a_\omega}{a_\omega + b_\omega} \psi \sigma^2 \tilde{\boldsymbol{\rho}}_k^{\top} \tilde{\boldsymbol{\rho}}_k, \text{ as } r \to 0.$$

[18] However, we could set for instance, $a_\omega = 1$ and $b_\omega >> 1$ to favor sparser models.

This implies that two factors with the same (sum of squared) demeaned correlations with asset returns will have exactly identical prior Sharpe ratios. This feature is unsatisfactory when considering factors proposed for pricing different asset classes, as the maximum Sharpe ratio achievable in different market segments might actually be quite different. We relax this constraint in the next subsection by introducing a new, more flexible prior formulation that preserves the robustness of the estimator to weak and spurious factors.

2.3 A spike-and-slab prior for heterogeneous classes of factors

We now generalize the prior specification in equation (3). As in BHJ, we formalize a continuous spike-and-slab prior that, using the correlation between factors and asset returns, endogenously solves the problems arising from weak factor identification. However, unlike them, we introduce an additional hyperparameter that researchers can use to encode their prior belief about how much of the SDF Sharpe ratio in the data can be captured with factors coming from, respectively, the bond and stock factor zoos. Specifically, we formulate a spike-and-slab prior for the vector of all factors' market prices of risk as[19]

$$\boldsymbol{\lambda}|\sigma^2, \boldsymbol{\gamma} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \boldsymbol{D}^{-1}). \tag{6}$$

For illustrative purposes, consider first the case in which we have only two types of factors under consideration: K_1 bond-market-based factors (ordered first) and $K - K_1$ stock-market-based factors (ordered last). In this case we can encode our prior beliefs about which factors are more likely drivers of observed risk premia by setting $\boldsymbol{D}$ as a diagonal matrix with elements c (the prior precision for the intercept), $[(1+\kappa)r(\gamma_1)\psi_1)]^{-1}$, ..., $[(1+\kappa)r(\gamma_{K_1})\psi_{K_1}]^{-1}$, $[(1-\kappa)r(\gamma_{K_1+1})\psi_{K_1+1}]^{-1}$, ..., $[(1-\kappa)r(\gamma_K)\psi_K]^{-1}$. The ψ_j elements are defined as in equation (5) and endogenously solve the problems arising from weak factors. Similarly, $r(\gamma_j)$, as before, captures the spike-and-slab nature of the prior formulation.

The new hyperparameter $\kappa \in (-1, 1)$ encodes the prior belief about which class of factors is more likely to explain the Sharpe ratio of asset returns. To see this, consider the case in which both factors and returns are standardized (as in our empirical implementation). In this case:

$$\frac{\mathbb{E}_\pi\left[SR^2_{\boldsymbol{f}}|\boldsymbol{\gamma}, \sigma^2\right]}{\mathbb{E}_\pi\left[SR^2_{\boldsymbol{\alpha}}|\sigma^2\right]} = \frac{\psi}{N}\left[(1+\kappa)\sum_{k=1}^{K_1} r(\gamma_k)\tilde{\boldsymbol{\rho}}_k^\top \tilde{\boldsymbol{\rho}}_k + (1-\kappa)\sum_{k=K_1+1}^{K} r(\gamma_k)\tilde{\boldsymbol{\rho}}_k^\top \tilde{\boldsymbol{\rho}}_k\right],$$

where $SR_{\boldsymbol{f}}$ and $SR^2_{\boldsymbol{\alpha}}$ denote, respectively, the Sharpe ratios achievable with all factors and the

[19] More precisely, the first element of $\boldsymbol{\lambda}$ is the coefficient associated with the common cross-sectional intercept, while the remaining elements are the market prices of risks of the factors under consideration.

Sharpe ratio of the pricing errors.

The above implies that the only free 'tuning' parameters in our setting, ψ and κ, have straightforward economic interpretations and can be transparently set. To see this, first consider $\kappa = 0$ (the homogeneous prior specification). In this case (with a uniform prior of factor inclusion), the expected prior Sharpe ratio achievable with the factors is just $\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2] = \frac{1}{2}\psi\sigma^2 \sum_{k=1}^{K} \tilde{\boldsymbol{\rho}}_k^\top \tilde{\boldsymbol{\rho}}_k$ as $r \to 0$. Hence, prior beliefs about the achievable Sharpe ratio with the factors fully pin down ψ.[20] When instead $\kappa \neq 0$, the prior is heterogeneous across types of factors, and this parameter encodes our prior expectation about which type of factors explains a larger share of the Sharpe ratio of the asset returns. As $\kappa \to 1^-$ ($\kappa \to -1^+$), the prior becomes concentrated on only bond (stock) factors being able to explain the Sharpe ratio of asset returns. For example, setting $\kappa = 0.5$ encodes the prior belief that, ceteris paribus, bond factors explain a $\frac{1+\kappa}{1-\kappa} = 3$ times as large a share of the squared Sharpe ratio than equity factors.

More generally, we can flexibly encode prior beliefs about the saliency of more than two categories of factors by setting $\boldsymbol{D} = \tilde{\boldsymbol{D}} \times \boldsymbol{\kappa}$, where $\tilde{\boldsymbol{D}}$ is a diagonal matrix with elements c, $(r(\gamma_1)\psi_1)^{-1}$, ..., $(r(\gamma_K)\psi_K)^{-1}$ and $\boldsymbol{\kappa}$ is a conformable column vector with elements 1, $1 + \kappa_1, \ldots, 1 + \kappa_K$ such that $\sum_{k=1}^{K} \kappa_j = 0$ and $0 < |\kappa_j| < 1 \; \forall j$.

Note that this general prior encoding maintains the same assumption of *exponential tails* for all factors (given the Gaussian formulation in equation (6)). And there is a very good reason for this: useless factors generate heavy-tailed cross-sectional likelihoods (in the limit, the likelihood is an improper "uniform" on $\mathbb{R}$), with peaks for the market prices of risk that deviate toward infinity. But, as first pointed out by Jeffreys (1961), as the peak of a thick-tailed likelihood moves away from the exponential-tail prior, the posterior distribution eventually *reverts back to the prior*. Hence, in our setting, the exponential tails of the prior play an important role: they shrink the price of risk of useless factors toward zero.

The transparency and interpretability of our prior formulation allows us, in the empirical analysis, to report results for various prior expectations of the Sharpe ratio achievable in the economy,[21] prior probability of factor inclusion, shares of the prior Sharpe ratio achievable with the different types of factors that we consider, and account for a potential "mismeasurement alpha" in the corporate bond data.

[20] Without a uniform prior for the SDF dimensionality, the prior Sharpe ratio value becomes $\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2] = \frac{a_\omega}{a_\omega + b_\omega}\psi\sigma^2 \sum_{k=1}^{K} \tilde{\boldsymbol{\rho}}_k^\top \tilde{\boldsymbol{\rho}}_k$ as $r \to 0$. Hence, beliefs about the prior Sharpe ratio and model dimensionality fully pin down the hyperparameters.

[21] More precisely, we report results for different prior values of $\sqrt{\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2]}$.

Furthermore, note that pure 'level' factors—i.e., factors that have no explanatory power for cross-sectional differences in asset returns but capture the average level of risk premia across assets—can be accommodated by removing the free intercept in the SDF (since it would be collinear with a pure level factor) and using simple correlations (instead of cross-sectionally demeaned ones) in equation (5), i.e. setting $\psi_j = \psi \times \boldsymbol{\rho}_j^\top \boldsymbol{\rho}_j$. We consider this particular case among our robustness exercises, and it leaves our main findings virtually unchanged.

2.4 Model and factor selection and aggregation

Our Bayesian hierarchical system defined in the previous subsections yields a well-defined posterior distribution from which all the unknown parameters and quantities of interest (e.g., R^2, SDF-implied Sharpe ratio, and model dimensionality) can be sampled to compute posterior means and credible intervals via the Gibbs sampling algorithm described in Appendix B. Most importantly, these posterior draws can be used to compute posterior model and factor probabilities, and, hence, identify robust sources of priced risk and—*if* such a model exists—a dominant model for pricing assets.

Model and factor probabilities can also be used for aggregating optimally, rather than selecting, the pricing information in the factor zoo. For each possible model $\boldsymbol{\gamma}^m$ that one could construct with the universe of factors, we have the corresponding SDF: $M_{t,\boldsymbol{\gamma}^m} = 1 - \left(\boldsymbol{f}_{t,\boldsymbol{\gamma}^m} - \mathbb{E}[\boldsymbol{f}_{t,\boldsymbol{\gamma}^m}]\right)^\top \boldsymbol{\lambda}_{\boldsymbol{\gamma}^m}$. Therefore, we construct a BMA-SDF by averaging all possible SDFs using the posterior probability of each model as weights:

$$M_t^{BMA} = \sum_{m=1}^{\bar{m}} M_{t,\boldsymbol{\gamma}^m} \Pr\left(\boldsymbol{\gamma}^m | \text{data}\right), \tag{7}$$

where $\bar{m}$ is the total number of possible models.[22]

The BMA aggregates information about the true latent SDF over the space of all possible models, rather than conditioning on a particular model. At the same time, if a dominant model exists (a model for which $\Pr\left(\boldsymbol{\gamma}^m | \text{data}\right) \approx 1$), the BMA will use that model alone. Importantly, pricing with the BMA-SDF is robust to the problems arising from collinear loadings of assets on the factors, since any convex linear combination of factors with collinear loadings has exactly the same pricing implications. Moreover, the BMA-SDF can be microfounded, as in Heyerdahl-Larsen et al. (2023), thanks to the equivalence of a log utilities and heterogeneous beliefs economy with a representative agent using the Bayes rule. Furthermore, BMA aggregation is

[22]See, e.g., Raftery et al. (1997) and Hoeting et al. (1999).

optimal under a wide range of criteria, but in particular, it is *optimal on average*: no alternative estimator can outperform it for all possible values of the true unknown parameters.[23] Finally, since its predictive distribution minimizes the Kullback-Leibler information divergence relative to the true unknown data-generating process, the BMA aggregation delivers the most likely SDF given the data, and the estimated density is as close as possible to the true unknown one, even if all of the models considered are misspecified.

Importantly, the BMA has particularly appealing properties when applied to the construction of the SDF. To see this, note that the BMA-SDF defined in equation (7)—thanks to the linearity of the models considered—can be rewritten as a weighted sum over the space of factors, rather than over the space of models. That is:

$$M_t^{BMA} = 1 - \sum_{j=1}^{K} \underbrace{\mathbb{E}[\lambda_j|\text{data}, \gamma_j = 1] \Pr(\gamma_j = 1|\text{data})}_{\equiv \mathbb{E}[\lambda_j|\text{data}]} (f_{j,t} - \mathbb{E}[f_{j,t}]) , \quad \text{as } r \to 0. \tag{8}$$

This expression makes clear that the weight attached to each factor in the BMA-SDF is driven by two elements. First, the probability of the factor being a "true" source of priced risk, $\Pr(\gamma_j = 1|\text{data})$. Hence, naturally, when a factor is more likely (given the data) to drive asset risk premia, it features more prominently in the BMA-SDF. Second, when a factor commands a large market price of risk in the models that include it, i.e. when $\mathbb{E}[\lambda_j|\text{data}, \gamma_j = 1]$ is large, it will, ceteris paribus, have a larger role in the BMA-SDF. These two forces are jointly captured in $\mathbb{E}[\lambda_j|\text{data}]$, the posterior expectation of the market price of risk given the data *only*, i.e., independently of the individual models.

This property of the BMA-SDF implies that, when parsing the factor zoo, there are two quantities of key interest. First, $\Pr(\gamma_j = 1|\text{data})$, as we want to discern which variables are more likely, given the data, to be fundamental sources of risk and, hence, should be included in our theoretical models for explaining asset returns. Second, and arguably as important, $\mathbb{E}[\lambda_j|\text{data}]$, as this quantity pins down how salient the given factor is in the BMA approximation of the SDF. Furthermore, $\mathbb{E}[\lambda_j|\text{data}]$ yields the weights that should be assigned to the factors in a portfolio that best approximates the true latent SDF. For these reasons, we track both quantities in our empirical analysis.

Furthermore, this implies that posterior probabilities of factors that are not true sources of fundamental risk will not necessarily tend to zero if they nevertheless help span the true

[23] See, e.g., Raftery and Zheng (2003) and Schervish (1995).

latent risks driving asset returns. That is, it might well be the case that, for a given factor, the posterior probability of being part of the SDF ($\Pr(\gamma_j = 1|\text{data})$) is smaller than the prior one—hence indicating that the data do not support the factor being a fundamental risk—while at the same time its estimated posterior market price of risk ($\mathbb{E}[\lambda_j|\text{data}]$) is substantial, since the factor helps the BMA-SDF span the risks in asset returns. This is not a contradiction, but rather an important element of strength of our method.

To illustrate these properties, consider the case in which the "true" SDF contains only one factor. That is, $M_t^{true} = 1 - \lambda_f f_{t,true}$, where f_{true} is the true source of fundamental risk and to simplify exposition, we employ the normalizations $\mathbb{E}[f_{t,true}] = 0$ and $\text{var}(f_{t,true}) = 1$. Note that under this innocuous normalization the risk premium and market price of risk of the factor coincide, i.e. $\lambda_{true} = \sqrt{\text{var}(M_t^{true})} = -\text{cov}(M_t^{true}, f_{t,true}) = \mu_{true}$. Consistent with the postulated one factor structure, the vector of test assets' excess returns $\boldsymbol{R}_t$ follows the process

$$\boldsymbol{R}_t = \boldsymbol{\mu_R} + \boldsymbol{C} f_{t,true} + \boldsymbol{w}_{\boldsymbol{R},t},$$

where $\boldsymbol{w}_{\boldsymbol{R},t} \perp f_{t,true}$ and $\mathbb{E}[\boldsymbol{w}_{\boldsymbol{R},t}] = \boldsymbol{0}$. Hence, it follows that the true factor prices perfectly (in population) the asset returns, as $\boldsymbol{\mu_R} = -\text{cov}(M_t^{true}, \boldsymbol{R}_t) = \boldsymbol{C}\lambda_{true}$.

Suppose further that there are a set of factors, "noisy proxies" of the true factor f_{true}, that the researcher considers as potential sources of fundamental risk,

$$f_{j,t} = \delta_j f_{t,true} + \sqrt{1-\delta_j^2}\, w_{j,t}, \quad |\delta_j| < 1,$$

for each noisy proxy j, with $w_{j,t} \perp f_{t,true}$ and $w_{j,t} \overset{\text{iid}}{\sim} (0,1)$. Note that in this handy encoding δ_j captures both the correlation between the true source of risk and the j-th noisy proxy and the latter's signal-to-noise ratio (as $\sqrt{\text{var}(f_{j,t})} = 1$ by construction).

Suppose that a researcher tests the pricing ability of the j-th noisy proxy by considering the misspecified SDF $\widetilde{M}_{j,t} = 1 - \tilde{\lambda}_j f_{j,t}$. We then have that the misspecified SDF prices the test assets perfectly in population (as long as the noise in the factor is "classical," i.e. $w_{j,t} \perp \boldsymbol{w}_{\boldsymbol{R},t}$):

$$\boldsymbol{\mu_R} = -\text{cov}(\widetilde{M}_{j,t}, \boldsymbol{R}_t) = \boldsymbol{C}\delta_j\tilde{\lambda}_j \quad \text{with} \quad \tilde{\lambda}_j = \lambda_{true}/\delta_j. \tag{9}$$

That is, the noisy proxy seems indistinguishable from the true factor in its pricing ability for the test assets, and it yields an estimated market price of risk (in population) that is larger (in absolute terms) than that of the true factor.[24]

Nevertheless, our method will detect such factor as a noisy proxy since our hierarchical

[24] Furthermore, $|\tilde{\lambda}_j| \to \infty$ as $|\delta_j| \to 0$, in yet another manifestation of the weak factor problem.

Bayesian framework requires factors to self-price. To see this, note that the true risk premium of the noisy proxy is $\mu_j \equiv -\text{cov}(M_t^{true}, f_{j,t}) \equiv \delta_j \lambda_{true}$, while instead the misspecified SDF that prices the cross-section of test assets yields an implied risk premium for the factor given by $\tilde{\mu}_j := -\text{cov}(\widetilde{M}_{j,t}, f_{j,t}) = \tilde{\lambda}_j = \lambda_{true}/\delta_j$. Thus, the noisy proxy will fail to self-price, since $|\tilde{\mu}_j| > |\mu_j| \ \forall |\delta_j| < 1$, and its self-mispricing will be proportional to $|\frac{1}{\delta_j^2} - 1|$.

This implies that, once the candidate factors are added to the set of test assets, factors that have a higher correlation (δ_j) with the true source of risk will have overall better performance in the cross-sectional likelihood in equation (2). Moreover, since $\tilde{\mu}_j \underset{|\delta_j|\to 1}{\longrightarrow} \mu_j$, noisy proxies with a higher signal-to-noise ratio will tend to have higher posterior probabilities. Importantly, the BMA-SDF is more robust in recovering the pricing of risk than other canonical estimators. The reason being that, as per equation (9), simple cross-sectional estimation with the noisy proxy included in the SDF yields an upward biased market price of risk for this factor, $\mathbb{E}[\lambda_j|\text{data}, \gamma_j = 1]$. Nevertheless, due to the self-pricing restriction that the noisy proxy will not satisfy, the posterior probability of such factors, $\Pr(\gamma_j = 1|\text{data})$, will be strictly smaller than one. This, in turn, will counteract the upward bias in the market price of risk since the factor enters the BMA in equation (8) with a weight equal to $\mathbb{E}[\lambda_j|\text{data}, \gamma_j = 1] \Pr(\gamma_j = 1|\text{data})$ (as $r \to 0$).

Note that this analytical example of the properties of our estimator is without loss of generality. For instance, a misspecified SDF with multiple noisy proxies will also yield an upward-biased measure of the market price of risk. Consequently, the misspecified SDF will not be able to satisfy the self-pricing restriction of the factors; hence, it will achieve a posterior probability strictly smaller than one. Therefore, this upward biased measure of the market price of risk implied by the misspecified SDF will be counteracted in the BMA in equation (8) by a $\Pr(\boldsymbol{\gamma}^m|\text{data}) << 1$.

But are these population (hence asymptotic) properties of our method likely to hold in a finite sample? We address this question with a realistic simulation exercise.

2.4.1 Simulation

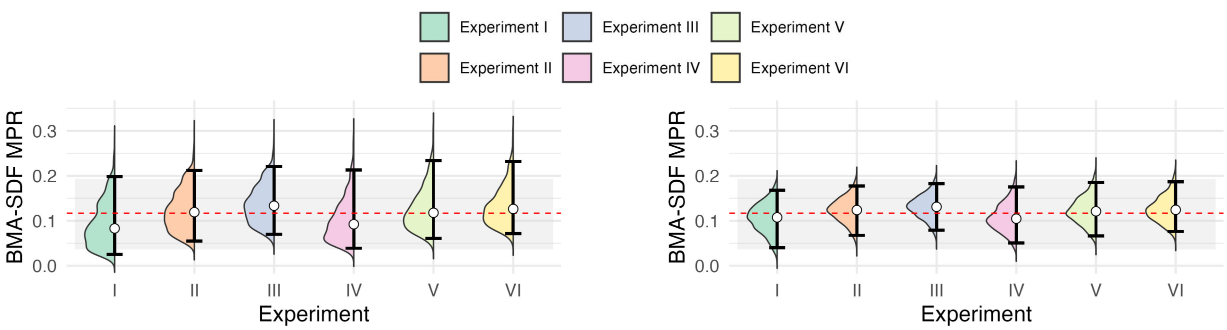
We calibrate a single (pseudo-true) useful factor (f_{true}) that mimics the pricing ability of the HML factor in the cross-section of the 25 Fama-French size and book-to-market portfolios. That is, we consider a setting with a partially misspecified pricing kernel (as HML yields sizable pricing errors in the cross-section used for calibration). To make the estimation challenging, we always include a useless factor (as this breaks the validity of canonical estimation methods), and consider noisy proxies with different correlations with the useful factor. In each experiment we include a variable number of noisy proxies f_j, $j = 1, ..., 4$ with correlations with the pseudo-true factor equal to 0.4, 0.3, 0.2, and 0.1, respectively. Further details of the simulation design are reported in Internet Appendix IA.2.

Simulation results are reported in Figure 1 for different sample sizes and a prior Sharpe ratio of 60% of the ex post maximum Sharpe ratio in the simulated samples. Results for different priors and sample sizes are reported in the Internet Appendix. We conduct six experiments. In the first three (experiments I to III), the pseudo-true factor is included among the candidate factors, while in the latter three (experiments IV to VI) only its noisy proxies are included.

Panel A of Figure 1 reports the BMA-SDF-implied market price of risk for several simulation designs in time series samples with only 400 monthly observations. The horizontal red dashed line denotes the Sharpe ratio of the pseudo-true factor, while the shaded grey area denotes the frequentist 95% confidence region for the market price of risk of the HML factor estimated via GMM in the (true) cross-section of 25 size and book-to-market portfolios with 665 monthly observations. Remarkably, the BMA-SDF estimator accurately recovers the market price of risk of the SDF not only when the pseudo-true factor is included among the candidate pricing factors (experiments I to III), but also when *only* noisy proxies of the true source of risk are included (experiments IV to VI). Moreover, the estimates are sharp—the distributions of the BMA-MPRs across simulation runs have 95% coverage areas very similar to the ones obtained (without accounting for model uncertainty) in the much longer true sample. Furthermore, as the time series sample size increases, Panel B of Figure 1 illustrates that the BMA estimates of the MPRs of the SDF become progressively more concentrated on the pseudo-true value, and converge to it in the large sample (see Panel B of Figure IA.9 of the Internet Appendix), even if only noisy proxies of the true source of risk are among the factors considered.

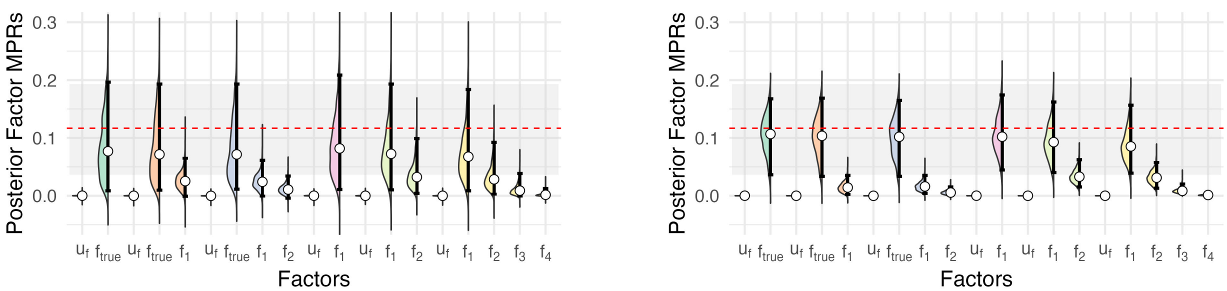
That is, our method can correctly recover the pricing of risk in the economy even when the true source of risk is not among the set of tested factors. Nevertheless, as illustrated in Panels C to F of Figure 1, this goal is achieved by the BMA in two different ways, depending on whether the pseudo-true factor is included among the tested ones or not.

First, when the pseudo-true factor is among the tested ones (experiments I to III), its estimated MPR (Panels C and D) is concentrated on the pseudo-true value, and converges to it as the time series sample size increases (as per Figure IA.9 of the Internet Appendix), and its posterior probability of being part of the SDF becomes progressively closer to one. On the



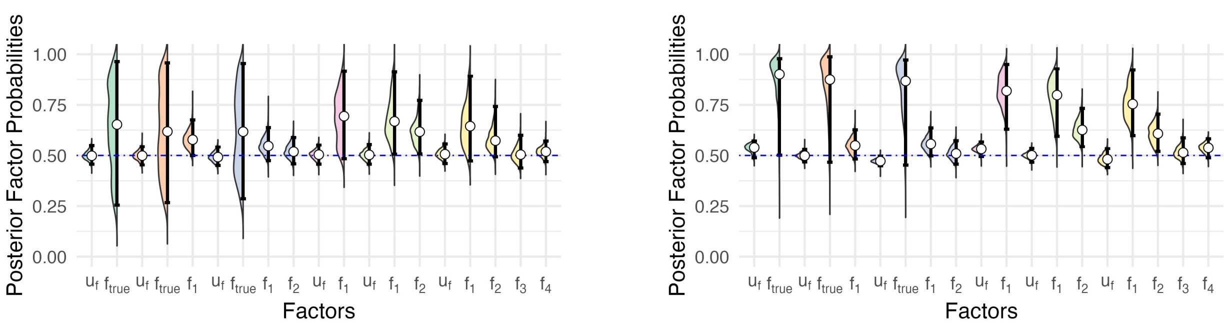
A: BMA-SDF market price of risk, $T = 400$

B: BMA-SDF market price of risk, $T = 1600$



C: Factors' market price of risk, $T = 400$

D: Factors' market price of risk, $T = 1600$



E: Factors' posterior probabilities, $T = 400$

F: Factors' posterior probabilities, $T = 1600$

Figure 1: Simulation evidence with useless factors and noisy proxies

Simulation results from applying our Bayesian methods to different sets of factors. Each experiment is repeated 1,000 times with the specified sample size (T). The data-generating process is calibrated to match the pricing ability of the HML factor (as a pseudo-true factor) for the Fama-French 25 size and book-to-market portfolios. Horizontal red dashed lines denote the market price of risk of HML, and the grey shaded area the frequentist 95% confidence region of its GMM estimate in the historical sample of 665 monthly observations. The prior is set to 60% of the ex post maximum Sharpe ratio. Simulation details are in Internet Appendix IA.2. Half-violin plots depict the distribution of the estimated quantities across the simulations, with black error bars denoting centered 95% coverage, and white circles denoting median values, across repeated samples. In all experiments we include a useless factor (u_f), while the pseudo-true factor (f_{true}) is included only in experiments I to III. In each experiment we include a variable number of noisy proxies f_j, $j = 1, ..., 4$ with correlations with the pseudo-true factor equal to, respectively, 0.4, 0.3, 0.2, and 0.1. The factors considered in the various experiments are:

Experiment I: u_f and f_{true}.
Experiment II: u_f, f_{true} and f_1.
Experiment III: u_f, f_{true}, f_1 and f_2.
Experiment IV: u_f, and f_1.
Experiment V: u_f, f_1 and f_2.
Experiment VI: u_f, f_1, f_2, f_3 and f_4.

contrary, the estimated MPRs of the noisy proxies are small and tend to zero as the sample size increases. Similarly, the market price of risk of the useless factor is effectively shrunk to zero. Note that while the posterior probability of the pseudo-true factor goes to one as the sample size increases, the probabilities of the useless factor and noisy proxies do revert to their prior value (Panels E and F). This might seem counterintuitive at first, but it is exactly what should be expected: as the posterior MPR of a given factor goes to zero, the fit of a model that includes that factor becomes indistinguishable from the one of a model that does not include said factor. Hence, the posterior probability of a factor whose MPR is sharply estimated to be close to zero should revert to its prior value—exactly what our method delivers. Note also that such factors, as shown in equation (8), will have zero weight in the BMA-SDF (as $\mathbb{E}[\lambda_j|\text{data}] \to 0$).

Second, when the pseudo-true factor is *not* among the tested factors (experiments IV to VI), the BMA-SDF still correctly recovers the overall price of risk (Panels A and B), but does so by assigning non-zero MPRs (Panels C and D), and posterior probabilities above their prior values, to the noisy proxies. Furthermore, as in the above-derived analytical results, noisy proxies more correlated with the pseudo-true factor have higher posterior probabilities and MPRs. Nevertheless, even asymptotically (Panel F of Figure IA.9 of the Internet Appendix), the posterior probability of the noisy proxies will not tend to one—as discussed above, thanks to the self-pricing restriction imposed by our estimator. This also implies that the BMA will not simply select the "best" noisy proxy. Instead, it will use multiple proxies in order to maximize the signal, and minimize the noise, that noisy proxies bring to the table.

The robustness of this last result should not be overstated. In the presence of the true factor among the tested ones, the data will always overcome the prior and converge to the truth under standard conditions (see, e.g., Schervish (1995, Thm. 7.78)). Nevertheless, when the true factor is *not* among the tested ones *and* the prior encodes a very high degree of shrinkage (via a very small prior Sharpe ratio), we should expect an attenuation bias in the BMA-SDF-implied MPR in the economy. This is due to the fact that, in the presence of only noisy proxies, no linear combination of them will be able to perfectly price (even asymptotically) both test assets and the factors themselves. Hence, the data will always provide some support for the case in which none of the factors should be included in the SDF, in turn reducing the BMA estimation of the overall MPR achievable with the factors (see, e.g., Panel B of Figure IA.10 of the Internet Appendix). This does not imply that one should prefer very little or no shrinkage at all, as this is crucial to preempt weak and useless factors from invalidating inference. Hence, exactly as we do in our empirical exercises, one should analyze the sensitivity of the results to the prior degree of shrinkage.

The above theoretical and simulation-based results stress the robustness of our method in both a large and small sample. Furthermore, they highlight that factor posterior probabilities and market prices of risk carry different, yet salient, information. Hence, both quantities should be tracked and analyzed (as we do in our empirical exploration). For instance, one might find that a given factor has both a posterior probability below its prior value—hence, it is unlikely to be a source of fundamental risk—and a large posterior MPR—since it is highly correlated with the true sources of priced risk, and it will consequently have a large weight in the BMA approximation of the true latent SDF in equation (8). In a nutshell, posterior probabilities tell us which factors should be included in a theoretical model given the data, since they identify the most likely sources of priced risk, while instead posterior market prices of risk tell us which factors should be included (and with what weight) in a portfolio that best approximates the true latent SDF and delivers the maximum achievable Sharpe ratio with the factors at hand.

3 Estimation results

In this section, we apply the hierarchical Bayesian method to a large set of factors proposed in the previous bond and equity literature. Overall, we consider 40 tradable and 14 nontradable factors, yielding $2^{54} \approx 18$ quadrillion possible models for the combined bond and stock factor zoo. In Sections 3.1 and 3.4 we only consider returns for the bond portfolios in excess of the short-term risk-free rate (calculated as outlined in Section 1.1). In Section 3.3, we also use duration-adjusted excess returns, as well as U.S. Treasury portfolios, to disentangle the credit and Treasury term structure components of corporate bond returns.

3.1 Co-pricing bonds and stocks

We now consider the pricing power of the 54 factors to gauge the extent to which the cross-section of corporate bond and stock returns is priced by the joint factor zoo. The IS test assets include the 50 bond and 33 stock portfolios described in Section 1.3 in addition to the 40 tradable factor portfolios (for a total $N = 123$). Throughout, we use the continuous spike-and-slab approach described in Section 2. To report the results, we refer to the priors as a fraction of the ex post maximum Sharpe ratio in the data, which is equal to 5.4 annualized for the joint cross-section of portfolios, from a very strong degree of shrinkage (20%, i.e., a prior annualized

Sharpe ratio of 1.0), to a very moderate one (80% or a prior annualized Sharpe ratio of 4.2). Given that the results demonstrate considerable stability across a wide range of prior Sharpe ratio values, we present selected findings for a prior set at 80% of the ex post maximum, as this choice tends to yield the best out-of-sample performance.[25]

3.1.1 The co-pricing SDF

We start by asking which factors are likely components of the latent SDF in the economy. Figure 2 reports the posterior probabilities (given the data) of each factor (i.e., $\mathbb{E}[\gamma_j|\text{data}], \forall j$) for different values of the prior Sharpe ratio achievable with the linear SDF (expressed as a percentage of the ex post maximum Sharpe ratio). See Table A.1 of Appendix A for a detailed description of the factors.

Recall that we have a uniform (hence flat) prior for the model dimensionality and each factor has an ex ante expected probability of being selected equal to 50%, depicted by the dashed horizontal line in Figure 2. Several observations are in order. First, with some notable exceptions, most factors proposed in the corporate bond and equity literatures have (individually) a posterior probability of being part of the SDF that is below its prior value of 50%. That is, given the data, they are unlikely sources of fundamental risks.

Second, given that their posterior probabilities are above the prior 50% value for the entire range of prior Sharpe ratios considered, five factors are identified as likely sources of fundamental risk in the bond and equity markets. In particular, there is strong evidence for including two tradable factors, PEADB and PEAD (i.e., respectively, the bond and stock post-earnings announcement drift factors), as a source of priced risk in the SDF. Partially, this is a surprising result, as PEADB has not specifically been proposed as a priced risk factor in the previous literature. Nozawa et al. (2025) are the first to document a post-earnings announcement drift in corporate bond prices, and they rationalize their finding with a stylized model of disagreement. They also show that a strategy that purchases bonds issued by firms with high earnings surprises and sells bonds of firms with low earnings surprises generates sizable Sharpe ratios and large risk-adjusted returns. On the other hand, Bryzgalova et al. (2023) and Avramov et al. (2023) find strong evidence that the *stock market* post-earnings announcement drift (PEAD) factor of Daniel et al. (2020) exhibits a particularly high posterior probability of being part of the SDF for stock returns. In fact, PEAD is the only other tradable factor with a posterior

[25] Additional results for different values of the prior Sharpe ratio are reported in Table A.2 of Appendix C.

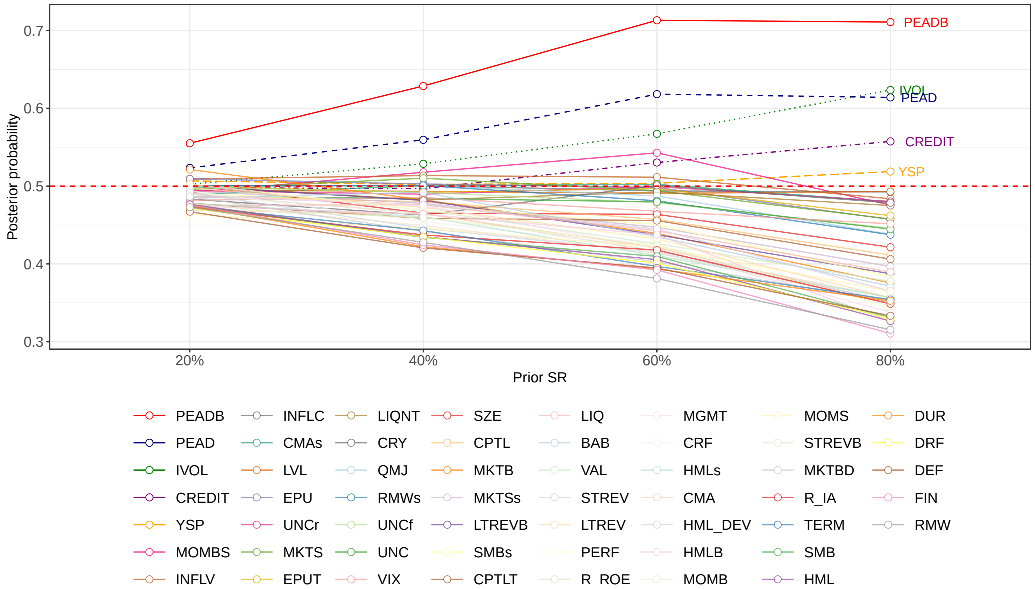


Figure 2: Posterior factor probabilities: Co-pricing factor zoo.

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, of the 54 bond and stock factors described in Appendix A. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are shown for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_{\boldsymbol{f}}^2 \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. Labels are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

probability of being part of the SDF that prices the joint cross-section of corporate bond and stock returns that is above 50%. That is, the only two tradable factors with high posterior probabilities are the bond and stock versions of the post-earnings announcement drift. Note that, in equilibrium models in which rational agents with limited risk-bearing capacity face behavioural asset demand, the drivers of the latter become part of the pricing measure—exactly as we find (see, e.g., De Long et al. (1990)). Note also that, as shown in Table IA.III of the Internet Appendix, these are the tradable factors with the highest Sharpe ratio in our full sample. Moreover, PEADB has the highest Sharpe ratio among bond factors when the sample is split in half, while PEAD has the highest Sharpe ratio among stock factors in the first half, and one of the highest in the second half of the sample (see Table IA.IV of the Internet Appendix).[26]

Furthermore, the *nontradable* idiosyncratic equity volatility factor (IVOL) of Campbell and Taksler (2003) is supported by the data as a fundamental source of priced risk. Interestingly,

[26]Despite its reduced *time series* predictability in most recent data (see, e.g., Martineau (2022)), we document remarkable stability of the post-earnings announcement drift for forming long-short corporate bond and stock portfolios across subsamples in Internet Appendix IA.4. That is, the *cross-sectional* predictability of the post-earnings announcement drift within a portfolio context remains robust and does not appear to be driven by micro-cap stocks.

the rationale behind this factor closely connects bond and stock markets: as per the seminal insight of Merton (1974), equity claims are akin to a call option on the value of the assets of the firm, while the debt claim contains a short put option on the same. Consequently, Campbell and Taksler (2003) suggest, changes in the firm's volatility should be expected to affect bond and stock prices.[27]

Additionally, two more *nontradable* factors have posterior probabilities of being part of the SDF above 50% for all values of the prior Sharpe ratio: the slope of the Treasury yield term structure (YSP, Koijen et al. (2017)), a well-known predictor of business cycle variation, and the AAA/BAA yield spread (CREDIT, Fama and French (1993)), a common metric of the risk compensation differential between safer and riskier securities. Interestingly, the term premium and default risk factors are originally suggested in Fama and French (1993) exactly for the purpose of co-pricing bonds and stocks.

Third, there are a few factors for which the posterior probability is roughly equal to the prior (implying that at least some of these factors are likely to be weakly identified at best), and there is a large set of factors that are *individually* unlikely to be sources of fundamental risk in the SDF pricing the joint cross-section of bond and stock returns. In particular, besides PEADB and PEAD, *all* tradable bond and stock market factors are individually unlikely to capture fundamental risk in the SDF. For instance, with a prior Sharpe ratio set to 80% of the ex post maximum, the posterior probabilities for 30 of the 40 tradable bond and stock factors are below 40% (see Figure 2 as well as the top panel of Figure 4). Nevertheless, as shown theoretically and in the simulation in Section 2.4, and discussed extensively below, this does *not* imply that these factors, *jointly*, do not carry relevant information to characterize the true latent SDF.

Notably, the stock as well as the bond market factors (MKTS and MKTB, respectively) both exhibit posterior probabilities below 50% for almost the full range of prior Sharpe ratios for the joint cross-section of returns. Nevertheless, when separately pricing the cross-sections of stock and bond returns with only the factors in their respective zoos, both market indices become likely components of the SDF: for all prior levels in the MKTS case, and for all but one in the MKTB case (see Tables IA.V and IA.VI of the Internet Appendix). This confirms the finding that the equity market index contains valuable information for pricing stocks in an unconstrained SDF based on stock factors only (as in Bryzgalova et al. (2023)). However,

[27] See, e.g., Dickerson et al. (2025) for a model of the correlation of bonds and stocks of the same firm.

when the space of potential factors is expanded to include both stock and bond factors, without dimensionality restrictions on the SDF as we do in our baseline co-pricing exercise, models with MKTS (and more so in the MKTB case) overall perform worse than denser models containing factors from both zoos. That is, the information in the market indices appears to be spanned by the other factors in the zoo. This finding is unlikely to be driven by the market indices acting as "level" or "weak" factors since asset returns display large and well-dispersed loadings on these factors, and the market prices of risk they command are substantial when included in the SDF (see Table A.2 of Appendix C and the bottom panel of Figure 4). Moreover, we show in Internet Appendix IA.3.1 that removing the free intercept, and the prior penalization of pure level factors, leaves *all* of the above results virtually unchanged.

Given the focus of most (yet not all) of the previous literature on selecting models characterized by a small number of factors, the above findings raise the question of whether the handful of most likely factors that we have identified are enough to capture the span of the true, latent, SDF that jointly prices bonds and stocks. Moreover, are factors less likely to be sources of fundamental risk really devoid of useful pricing information? Since our Bayesian method does not ex ante impose the existence of a unique, low-dimensional, and correctly specified model—all assumptions that are needed with conventional frequentist asset pricing methods—we can formally answer these questions.

The top panel of Figure 3 reports the posterior dimensionality of the SDF in terms of observable factors to be included in it, and the bottom panel shows the posterior distribution of the Sharpe ratios achievable with such an SDF. It is evident that the *sparse* models suggested in the previous literature have very weak support in the data, and are misspecified with very high probability, as a substantial number of factors is needed to capture the span of the true latent SDF: the posterior median number of factors is 22 with a centered 95% coverage of 15 to 29 factors. In fact, the posterior probability of a model with less than 10 factors is virtually zero, indicating that the quest for a sparse, unique, SDF model among the observable factors in the joint bond and stock factor zoo is misguided at best.

But, as often argued, wouldn't a *dense* SDF imply an unrealistically high Sharpe ratio achievable in the market? The bottom panel of Figure 3 highlights that the SDF-implied Sharpe ratio is not unrealistically large (recall that the ex post maximum Sharpe ratio in the data is 5.4), suggesting that many factors are likely to span a lot of common risks. Furthermore, Table 1 shows that albeit the most likely (top five) factors to be included in the SDF for pricing

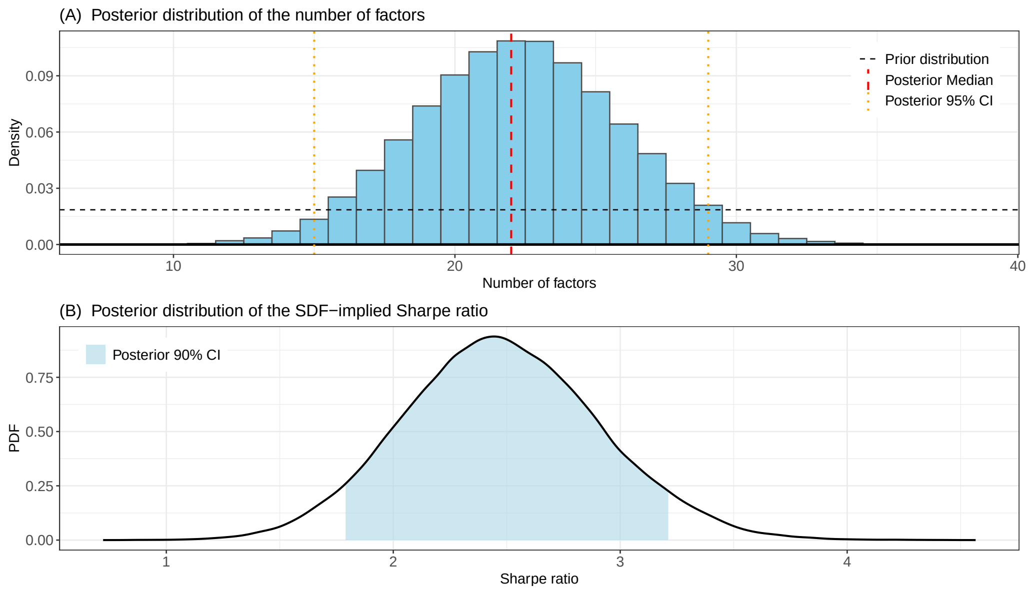


Figure 3: Posterior SDF dimensionality and Sharpe ratios: Co-pricing factor zoo.

Posterior distributions of the number of factors to be included in the co-pricing SDF (top panel) and of the SDF-implied Sharpe ratio (bottom panel), computed using the 54 bond and stock factors described in Appendix A. The prior distribution for the j^{th} factor inclusion is a Beta(1, 1), yielding a flat prior for the SDF dimensionality depicted in the top panel. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the 83 bond and stock portfolios and 40 tradable factors described Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

bonds and stocks (jointly or separately) are responsible for a substantial share of the Sharpe ratio (e.g., $\mathbb{E}[SR_f|\text{data}]$ ranges from 0.78 to 1.46 for a 60% to 80% prior), the share of the SDF squared Sharpe ratio generated by these factors alone ($\mathbb{E}\left[SR_f^2/SR_m^2|\text{data}\right]$) is quite limited. This means that there is substantial additional priced risk in the factor zoo that is *not* captured by the most likely factors. That is, the *less* likely factors are noisy proxies for latent fundamental risks and are needed, *jointly*, to provide an accurate characterization of the risks priced by the true latent SDF. This feature of the data arises not only when jointly pricing bonds and stocks (Panel A), but also when separately focusing on the pricing of the two asset classes using their respective factor zoos (Panels B and C).

As shown in Section 2.4, if a dominant, low-dimensional, model is not supported by the data—as the above evidence implies—we can optimally aggregate the pricing information in the factor zoo by constructing a Bayesian model averaging of all possible models. Moreover, the *model averaging* is equivalent to a *factor averaging*, where the weights of the individual factors are simply the factors' posterior market prices of risk ($\mathbb{E}[\lambda_j|\text{data}]$). Hence, large posterior market prices of risk reveal which factors (true sources of risk or noisy proxies) are useful in

Table 1: Most likely (top five) factor contribution to the SDF

	Panel A: Co-pricing SDF				**Panel B**: Bond SDF				**Panel C**: Stock SDF			
Total prior SR:	20%	40%	60%	80%	20%	40%	60%	80%	20%	40%	60%	80%
$\mathbb{E}[SR_f\|\text{data}]$	0.26	0.57	1.06	1.24	0.28	0.71	1.10	1.46	0.17	0.42	0.78	1.10
$\mathbb{E}\left[\frac{SR_f^2}{SR_m^2}\|\text{data}\right]$	0.13	0.20	0.32	0.28	0.34	0.57	0.65	0.70	0.12	0.22	0.35	0.42

Posterior mean of implied Sharpe ratios achievable with the most likely (top five) factors, $\mathbb{E}[SR_f|\text{data}]$, and their share of the SDF squared Sharpe ratio, $\mathbb{E}\left[SR_f^2/SR_m^2|\text{data}\right]$. Panels A, B and C report results using the corresponding factor zoos, for the co-pricing, bond-only, and stock-only BMA-SDFs, respectively. Top five co-pricing factors are PEADB, IVOL, PEAD, CREDIT and YSP. Top five bond factors are PEADB, CREDIT, MOMBS, YSP and IVOL. Top five stock factors are PEAD, IVOL, MKTS, CMAs and LVL. The total prior Sharpe ratio is expressed as a share of the ex post maximum Sharpe ratio of the test assets.

approximating the true, latent, SDF.

In Figure 4 we list all 54 factors in increasing order of posterior probabilities (i.e., $\Pr(\gamma_j = 1|\text{data})$, top panel), for a prior Sharpe ratio of 80% of the maximum ex post Sharpe ratio, along with the corresponding annualized posterior means of the price of risk of the factors ($\mathbb{E}[\lambda_j|\text{data}]$, bottom panel). Posterior probabilities and market prices of risk for different priors are tabulated in Table A.2 of Appendix C.

All five factors with posterior probabilities higher than their prior values (i.e., PEADB, IVOL, PEAD, CREDIT and YSP) command substantial market prices of risk, implying a considerable weight in a portfolio that best approximates the true latent SDF. Hence, not only does the data support their inclusion in the SDF, but they also play an important role in its BMA estimate.

Out of the next fifteen factors with the highest (individual) posterior probabilities, ten are nontradable in nature. Nevertheless, the risk prices of several of these nontradable factors are small and, in some cases, effectively shrunk toward zero. This is due to the fact that these are likely *weak factors* in the joint cross-section of corporate bond and stock returns and, consequently, carry a near-zero weight in the portfolio that approximates the SDF.[28] The occurrence of weak factors, which, in fact, is most common among the nontradable ones, causes identification failure and invalidates canonical estimation approaches (e.g., GMM, MLE, and two-pass regressions). This is *not* an issue for our Bayesian method, which restores inference by design, by regularizing the marginal likelihood. Furthermore, for these factors, both shown theoretically and in the simulation in Section 2.4, the posterior probabilities revert to their

[28]That is, their correlations with the test assets are small and have little cross-sectional dispersion. See, e.g., Gospodinov et al. (2019) and Kleibergen (2009) for a formal definition of weak and level factors.

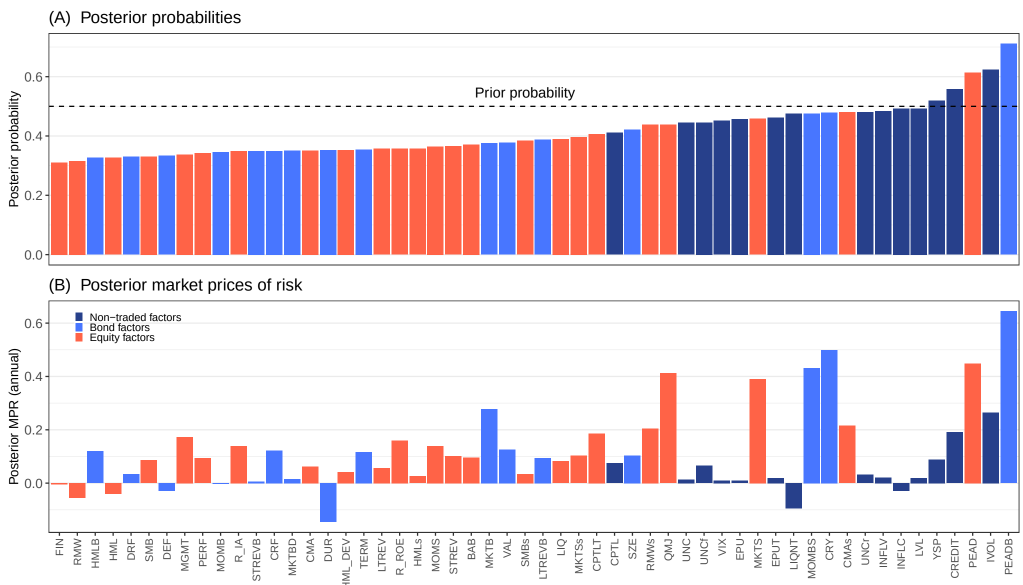


Figure 4: Posterior factor probabilities and risk prices: Joint factor zoo (excess bond returns).

The figure reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and stock factors described in Appendix A. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the 83 stock and bond portfolios and the 40 tradable factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

prior value as the market prices of risk tend to zero.

Interestingly, several factors with posterior probabilities below their prior values—hence unlikely sources of fundamental risk—do carry very sizeable posterior market prices of risk. For example, the equity market index factor carries the third largest MPR among equity factors and the sixth largest among the tradable ones. Section 2.4 informs us exactly how to interpret such findings: these are factors that the data do not support as being fundamental sources of risk (hence the posterior probability being below the prior value), but that nevertheless have a high correlation with the true latent priced risk and, hence, feature prominently in the BMA-SDF to provide an accurate approximation of the true latent SDF.

This aggregation property of the BMA-SDF is clearly displayed in Figure IA.12 of the Internet Appendix, where we plot the cumulative SDF-implied Sharpe ratio when subsequently adding factors ordered by their (individual) posterior probability. While the Sharpe ratio increases with the number of factors, some factors seem to contribute more to the implied Sharpe ratio than others. For example, the factors ranked 6 to 9 (LVL, INFLC, INFLV, UNCr) do not appear to add much individually, while the Sharpe ratio increases markedly once factors 10 (CMAs) and 11 (CRY) are included. This is because many factors are potentially noisy proxies for the same fundamental sources of risk that are important for the SDF. As shown in Section 2.4, factors that are useful noisy proxies for a particular fundamental source of risk not fully spanned by individual factors will exhibit nonzero market prices of risk (or portfolio weights). However, the Sharpe ratio only substantially increases once the first of the factors spanning (at least partially) a common risk is included in the analysis. In contrast, subsequent factors spanning the same risk generate a much smaller increase in the Sharpe ratio due to the enhanced signal extraction of the common risk. Further examining the four factors in positions 8 to 11, these are all nontradable in nature and related to inflation, interest rates, and uncertainty. Similarly, factors in positions 16 to 19 are all related to different measures of macroeconomic uncertainty. While it is important to include all of these factors in the SDF to increase the signal to noise ratio of latent fundamental risk, their individual marginal contribution to the Sharpe ratio may be minimal as they share common spanning. This is highlighted by the posterior confidence interval in the figure. As more factors are added sequentially, one might expect the posterior uncertainty to increase, as the uncertainty about the individual market prices of risk is compounded in the SDF. Nevertheless, the opposite occurs in Figure IA.12—overall, the posterior confidence region *shrinks* as factors are added. Moreover, the last few factors have virtually no effect on the posterior mean of the Sharpe ratio, but they do reduce the confidence region significantly, as the BMA aggregation increases the signal to noise ratio.

3.1.2 Cross-sectional asset pricing

We now turn to the asset pricing performance of the BMA-SDF based on the joint cross-section and factor zoos, as well as based on bond and stock portfolios separately. In Table 2 we report results for in-sample cross-sectional pricing using various performance measures, while out-of-sample results are summarized in Table 3. The in-sample assets for the joint cross-section in Panel A of Table 2 are the 83 portfolios of bonds and stocks (described in Section 1.4) plus 40 tradable factors. Panels B and C focus only on bonds (50 portfolios and 16 bond tradable factors) and stocks (33 anomaly portfolios and 24 stock tradable factors), respectively. The out-of-sample test assets in Table 3 comprise 77 bond portfolios and 77 stock portfolios (described in Section 1.4), which are considered jointly in Panel A and separately in Panels B and C.

When assessing the pricing performance, we compare our BMA-SDF for different levels of prior Sharpe ratio shrinkage with the performance of a number of benchmark models. In particular, we consider the bond CAPM (CAPMB), the stock CAPM, the Fama and French

(1993) five-factor model (FF5), the intermediary asset pricing model of He et al. (2017) (HKM), the PCA-based SDF of Kozak et al. (2020) (KNS) and the risk premia PCA approach of Lettau and Pelger (2020) (RPPCA).[29] In addition, since most of the previous literature focuses on selection (rather than aggregation) of pricing factors, we also include the respective 'top' factor models (TOP) from our Bayesian analysis that comprise only the five factors with the highest posterior probabilities (for the joint cross-section for example, this is a five-factor model with PEADB, IVOL, PEAD, CREDIT, and YSP). All the benchmark model SDFs are estimated via a GLS version of GMM.[30] Note that for the cross-sectional OS pricing, we do not refit the BMA-SDF or the other benchmark models to the new data. Instead, we use the estimated parameters from the respective IS pricing exercises.

For the in-sample pricing in Table 2, a few observations are in order. First and foremost, the BMA-SDF using moderate shrinkage (80% of the prior Sharpe ratio) outperforms virtually all benchmark models on almost all dimensions considered, with the best alternative model being KNS. Second, no low dimensional model performs well. This should not come as a surprise given the discussion in Section 3.1.4, which implies that all low-dimensional models are both misspecified with a very high probability and are strongly rejected by the data. In fact, the performance of both the bond and stock CAPM is rather disappointing compared to the BMA-SDF. Moreover, popular models such as FF5 and HKM do not perform particularly well either. Third, the low dimensional TOP factor model, albeit better performing than the low dimensional models from the literature, delivers inferior pricing compared to the BMA-SDF with moderate shrinkage, once again pointing out that aggregation of factors, rather than selection, is preferred by the data. This highlights that just the most likely factors are not sufficient to provide an accurate characterization of the risks spanned by the true latent SDF. Fourth, the results are fairly consistent across the three panels. Apart from the BMA-SDF, KNS, and RPPCA deliver consistently better IS pricing performance than the low dimensional models.

The co-pricing BMA-SDF performs exceptionally well out-of-sample (see Panel A of Table 3). While KNS is a close contender regarding in-sample performance, the BMA-SDF strongly dominates KNS out-of-sample. In Internet Appendix IA.3.2 we show that the strong OS performance of the co-pricing BMA-SDF is not driven by the specific, yet rich, selection

[29]The SDFs of both KNS and RPPCA are re-estimated using our data and the methods proposed in the original papers. Details of the estimation for all benchmark models are reported in Appendix D.

[30]See Appendix D for further details.

Table 2: In-sample cross-sectional asset pricing performance

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Co-pricing bonds and stocks											
RMSE	0.214	0.203	0.185	0.167	0.260	0.278	0.258	0.259	0.230	0.166	0.197
MAPE	0.167	0.154	0.139	0.125	0.194	0.221	0.198	0.192	0.171	0.126	0.132
R^2_{OLS}	0.155	0.240	0.367	0.487	−0.244	−0.426	−0.233	−0.238	0.023	0.489	0.282
R^2_{GLS}	0.106	0.168	0.232	0.285	0.078	0.083	0.087	0.078	0.263	0.176	0.267
Panel B: Pricing bonds											
RMSE	0.180	0.148	0.121	0.104	0.209	0.214	0.201	0.206	0.162	0.192	0.091
MAPE	0.129	0.109	0.091	0.079	0.146	0.135	0.143	0.146	0.128	0.111	0.067
R^2_{OLS}	0.196	0.455	0.638	0.733	−0.083	−0.134	−0.006	−0.049	0.347	0.088	0.794
R^2_{GLS}	0.211	0.299	0.381	0.444	0.172	0.195	0.238	0.175	0.549	0.071	0.419
Panel C: Pricing stocks											
RMSE	0.230	0.241	0.236	0.220	0.292	0.264	0.275	0.292	0.352	0.162	0.175
MAPE	0.186	0.189	0.181	0.166	0.229	0.211	0.221	0.226	0.294	0.133	0.141
R^2_{OLS}	0.023	−0.075	−0.029	0.103	−0.570	−0.282	−0.392	−0.574	−1.288	0.515	0.433
R^2_{GLS}	0.145	0.213	0.287	0.353	0.120	0.118	0.130	0.121	0.330	0.311	0.493

The table presents the cross-sectional in-sample asset pricing performance of different models pricing bonds and stocks jointly (Panel A), bonds only (Panel B) and stocks only (Panel C), respectively. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors with an average posterior probability greater than 50%. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. By panel the models are estimated with the respective factor zoos and test assets. Test assets are the 83 bond and stock portfolios and the 40 tradable bond and stock factors (Panel A), the 50 bond portfolios and 16 tradable bond factors (Panel B), and the 33 stock portfolios and 24 tradable stock factors (Panel C), respectively. All are described in Section 1. All data are standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

of test assets in our baseline analysis presented here. In particular, we compare the performance of the BMA-SDF vis-à-vis the closest competitor, KNS, across $2^{14} - 1 = 16,383$ OS cross-sections. Depending on the measure of fit (i.e., R^2_{GLS}, R^2_{OLS}, RMSE, and MAPE), the BMA-SDF outperforms KNS in 96.6% to 99.9% of all OS cross-sections we consider.

Additionally, note that, as shown in Internet Appendix IA.3.2, the pricing ability of the BMA-SDF significantly outperforms, in- and out-of-sample, not only the benchmark models in Tables 2 and 3, but also a much broader set of additional benchmark models designed specifically to price the bond and stock cross-sections individually.[31]

[31]In Table IA.XII of the Internet Appendix we consider an expanded set of benchmarks that includes the

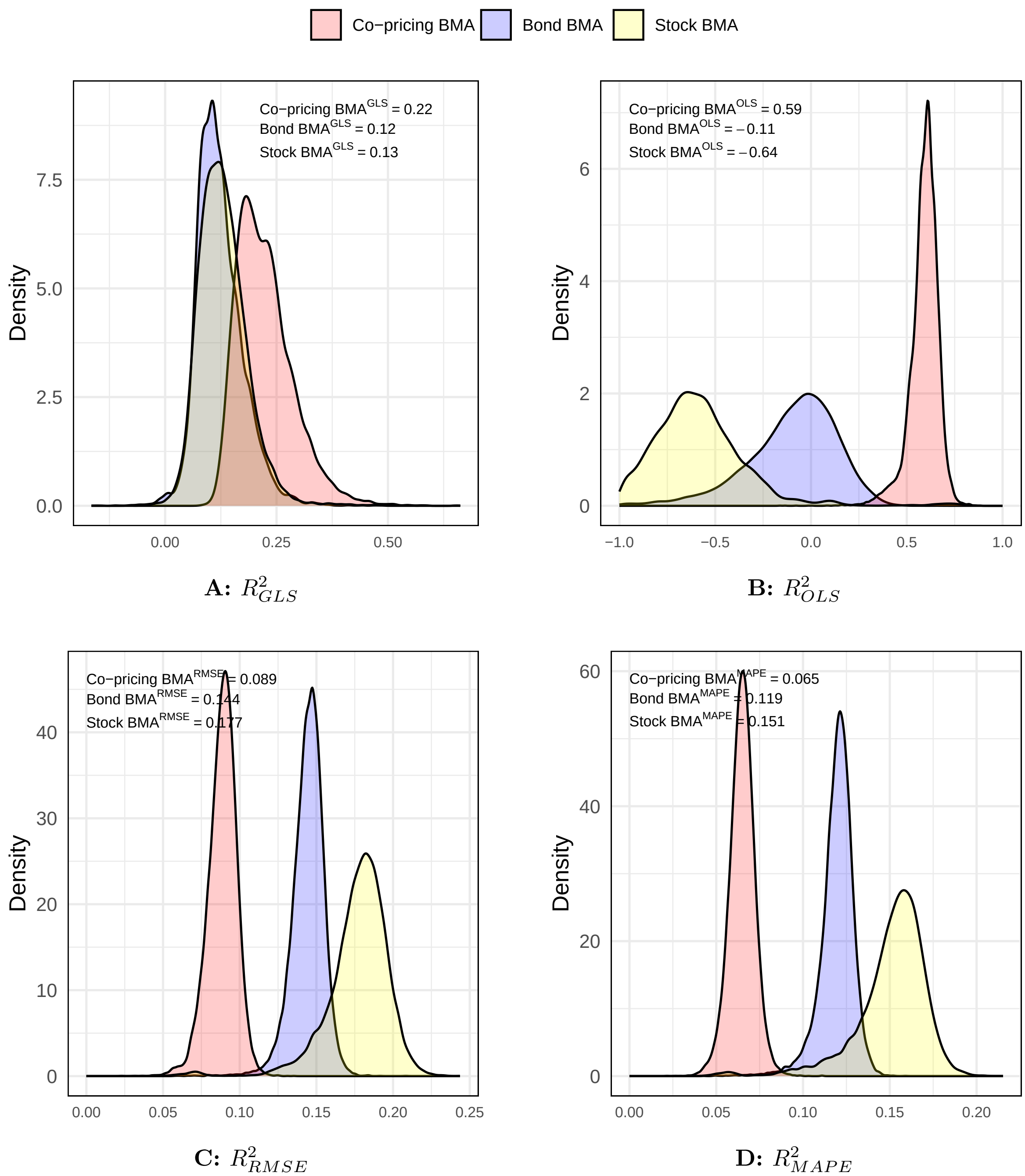


Figure 5: Pricing out-of-sample stocks and bonds with different BMA-SDFs.

This figure plots the distributions of R^2_{GLS}, R^2_{OLS}, RMSE and MAPE in Panels A, B, C and D respectively across 16,383 possible bond and stock cross-sections using the 14 sets of stock and bond test assets ($2^{14} - 1 = 16,383$) priced using the respective BMA-SDF (the empty set is excluded). The models are first estimated using the baseline set of IS test assets and then used to price (with no additional parameter estimation) each set of the 16,383 OS combinations of test assets. The red distributions correspond to the pricing performance of the co-pricing BMA-SDF. The blue (yellow) distributions correspond to the pricing performance of the bond (stock) only BMA-SDF. The BMA-SDFs are computed with a prior Sharpe ratio value set to 80% of the ex post maximum Sharpe ratio of the IS test assets. All data are standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table 3: Out-of-sample cross-sectional asset pricing performance

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Co-pricing bonds and stocks											
RMSE	0.114	0.102	0.095	0.090	0.224	0.154	0.139	0.223	0.171	0.160	0.153
MAPE	0.081	0.074	0.069	0.065	0.192	0.129	0.102	0.190	0.135	0.143	0.130
R^2_{OLS}	0.357	0.489	0.557	0.603	−1.478	−0.161	0.053	−1.444	−0.442	−0.268	−0.159
R^2_{GLS}	0.038	0.070	0.098	0.124	0.028	0.034	0.036	0.028	0.090	0.065	0.028
Panel B: Pricing bonds											
RMSE	0.123	0.116	0.110	0.106	0.129	0.128	0.140	0.133	0.102	0.114	0.100
MAPE	0.090	0.085	0.081	0.079	0.094	0.092	0.104	0.098	0.084	0.083	0.073
R^2_{OLS}	0.051	0.156	0.237	0.296	−0.051	−0.029	−0.231	−0.112	0.342	0.180	0.375
R^2_{GLS}	0.019	0.056	0.081	0.102	−0.004	0.024	−0.032	−0.007	0.101	0.066	0.045
Panel C: Pricing stocks											
RMSE	0.105	0.088	0.077	0.070	0.123	0.119	0.116	0.124	0.149	0.078	0.104
MAPE	0.078	0.067	0.062	0.057	0.089	0.085	0.082	0.091	0.115	0.060	0.082
R^2_{OLS}	0.298	0.508	0.620	0.683	0.032	0.099	0.136	0.019	−0.422	0.613	0.305
R^2_{GLS}	0.090	0.160	0.227	0.280	0.103	0.065	0.099	0.107	0.079	0.207	0.072

The table presents the cross-sectional out-of-sample asset pricing performance of different models pricing bonds and stocks jointly (Panel A), bonds only (Panel B) and stocks only (Panel C), respectively. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors with an average posterior probability greater than 50%. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. The models are first estimated using the baseline IS test assets. The resulting SDF is then used to price (with no additional parameter estimation) each set of the OS assets. The IS test assets are the same as in Table 2. OS test assets are the combined 154 bond and stock portfolios (Panel A), as well as the separate 77 bond and stock portfolios (Panels B and C). All are described in Section 1. All data are standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

Given the findings in Tables 2 and 3 that bonds and stocks can be accurately priced *separately* with BMA-SDFs constructed based only on their respective factor zoos, a natural question is whether only bond or stock factors are sufficient to price *jointly* both asset classes. We answer this question in Figure 5 where we compare the OS pricing performance of the co-pricing BMA-SDF (in red, from Panel A of Table 2) to that of BMA-SDFs constructed separately with only

models of Bai et al. (2019), van Binsbergen et al. (2025), Bali et al. (2021b), Chung et al. (2019), Carhart (1997), Hou et al. (2015), Fama and French (2015) (with and without the addition of the momentum factor), Daniel et al. (2020), and the DEFTERM specification of Fama and French (1993). In addition, in Figures IA.14 and IA.15 of the Internet Appendix, we report an extensive comparison of the BMA-SDF performance relative to the Dick-Nielsen et al. (2025) five-factor corporate bond model.

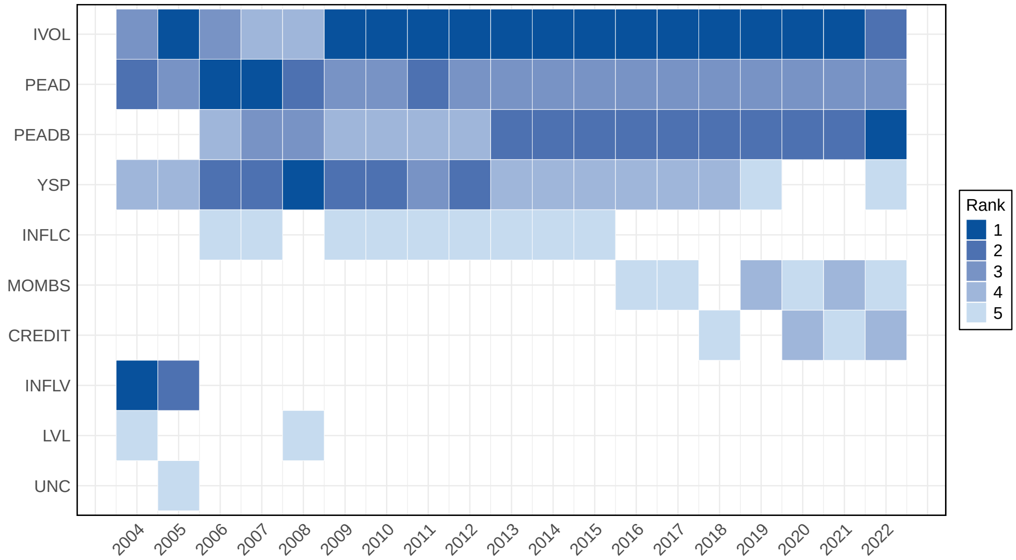
bond (in blue, from Panel B of Table 2) and stock (in yellow, from Panel C of Table 2) factors, respectively. As test assets, we again utilize the 16,383 combinations of our OS bond and stock cross-sections. Throughout, the co-pricing BMA-SDF exhibits significantly lower pricing errors and considerably higher R^2s compared to the bond-only or stock-only BMA-SDFs. That is, in order to price the joint cross-section of bond and stock excess returns, we require information from both factor zoos.

In Internet Appendix IA.3.2 we show that the co-pricing BMA-SDF can also effectively price the bond and stock cross-sections *separately*, indicating that the superior performance of the co-pricing BMA-SDF is not simply a result of its ability to price one cross-section better than the other. Furthermore, the *asset-class-specific* BMA-SDFs price their respective cross-sections very well. However, information from the bond factor zoo alone is insufficient to price the cross-section of stock returns, and conversely, information from the stock factor zoo is inadequate to price the cross-section of corporate bond excess returns.

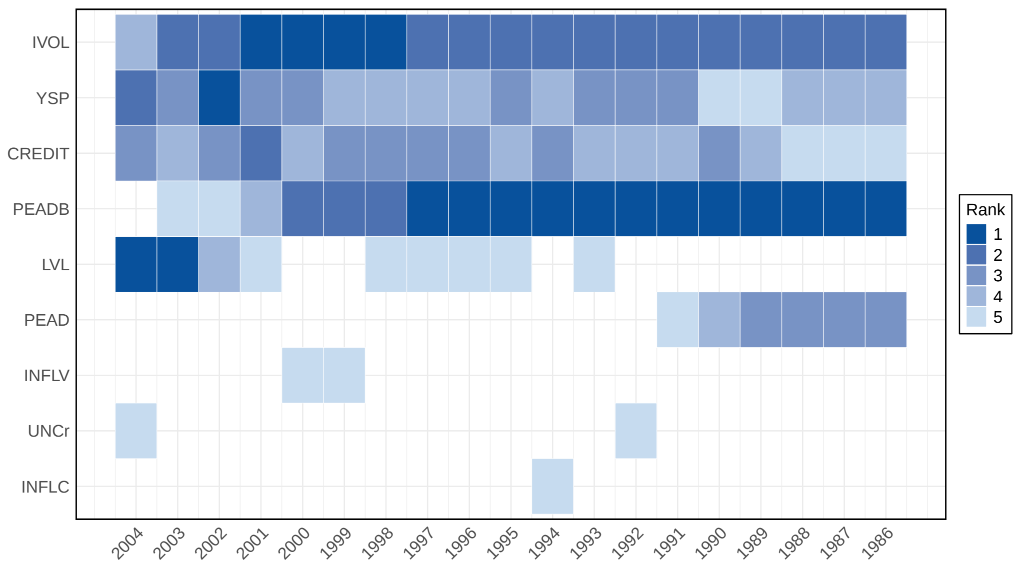
3.1.3 The saliency of factors over time

We now investigate to what extent the relevance of individual factors remains stable over time. To this end, we initially estimate our model for a shorter sample period before subsequently re-estimating the relevant quantities for progressively longer samples. Specifically, we split our sample in half, resulting in two sub-samples with 222 monthly observations each. We first estimate the model for the first subsample spanning July 1986 to June 2004, and then re-estimate it every year, adding twelve new observations at each iteration. Similarly, we estimate backward in time starting with the second subsample from December 2022 to July 2004 and add one year of data at every step. We follow our methodology described in Section 2 and, throughout, we fix the shrinkage at 80% of the corresponding ex post maximum Sharpe ratio for the respective window. We present the results for the five top factors (based on their posterior probability) in two heatmaps in Figure 6 for the forward (Panel A) and backward estimation (Panel B), respectively, with a higher rank reflected by a darker shade of blue. The top factors ranked by their market prices of risk are also presented in heatmaps in Figure IA.17 of the Internet Appendix.

Overall, the relevant factors remain remarkably stable. The top five factors from Figure 2, PEADB, IVOL, PEAD, CREDIT, and YSP, all feature prominently in both Panels A and B of Figure 6. Similarly, factors that exhibit high market prices of risk in the bottom panel of



A: Expanding forward estimation



B: Expanding backward estimation

Figure 6: Time-varying factor importance.

The figure highlights the top five factors over time, ordered by their posterior probabilities $\mathbb{E}[\gamma_{j,t}|\text{data}_t]$, and the number of times they are present in the top five, estimated using expanding samples going forward (Panel A) and backward (Panel B) in time. We use half of the sample as the initial window ($T = 222$) and then re-estimate the model every year with an expanding sample. The factors are ordered by the total number of times they are present in the 'top five.' The results are shown for prior level of Sharpe ratio shrinkage set to 80% of the ex post maximum up until year t.

Figure 4 such as PEADB, CRY, MOMBS, or QMJ, remain highly ranked over a wide range of estimation windows in Figure IA.17 of the Internet Appendix. When considering rankings based on market prices of risk, the stock market factor MKTS becomes particularly relevant for the backward estimation while it remains just outside the top five for the full sample. Overall, the results based on time-varying windows largely align with the full sample results presented earlier.

3.1.4 Which risks?

Next, we further decompose the posterior dimensionality of the SDF and its implied Sharpe ratio to better understand which types of risk are likely to be part of the true latent pricing measure and to what extent different factors capture common information.

Table 4 presents the decomposition of the posterior SDF dimensionality and Sharpe ratio split between nontradable and tradable bond and stock factors for different prior values. Panel A reports results for the pricing of the joint cross-section of stock and corporate bond returns using factors from both zoos to construct the SDF. Instead, Panels B and C focus, respectively, on the separate pricing of corporate bonds and stocks using only factors from their respective zoos. Several salient patterns are evident.

First, Panel A shows that an accurate characterisation of the pricing measure requires an SDF that is dense not only in the overall space of observable factors (as per the top Panel of Figure 3), but also over the individual subspaces of nontradable as well as bond and stock tradable factors: the posterior mean number of factors is about 7 for nontradable factors, 6 to 8 for bond, and 9 to 12 for stock tradable factors. Furthermore, this density of the SDF is not driven by the co-pricing task: even pricing only bonds (Panel B) or stocks (Panel C) requires about 7 nontradable, 6 to 8 (for bonds) or 10 to 12 (for stocks) tradable factors, respectively.

Second, each of the three categories of factors is *economically* important. Focusing on the moderate prior shrinkage case (i.e., 80% of the ex post achievable Sharpe ratio) in Panel A, the posterior mean of the annualized Sharpe ratio ascribable to the various types of factors ($\mathbb{E}[SR_f|\text{data}]$) is 1.12 for nontradable factors, and 1.51 and 1.77, respectively, for tradable bond and stock factors. Third, there is substantial common priced information across the categories of factors, as the sum of the Sharpe ratios generated by the three sets of factors (for example $1.12 + 1.51 + 1.77 = 4.40$ in Panel A) is much larger than the average posterior SDF-implied Sharpe ratio (which is around 2.5 in the bottom panel of Figure 3). This overlap in risks captured by different types of factors is particularly strong among tradable factors, where the sum of the Sharpe ratios of bond and stock factors in the SDF is $1.51 + 1.77 = 3.28$, while the

Table 4: BMA-SDF dimensionality and Sharpe ratio decomposition by factor type

	Total prior SR				Total prior SR			
	20%	40%	60%	80%	20%	40%	60%	80%
Panel A: Co-pricing BMA-SDF								
	Nontradable factors				Tradable factors			
Mean	6.97	6.94	6.97	6.80	19.52	18.78	17.84	15.51
5%	4	4	4	4	14	14	13	10
95%	10	10	10	10	25	24	23	21
$\mathbb{E}[SR_f \vert \text{data}]$	0.21	0.43	0.70	1.12	0.86	1.44	1.91	2.27
$\mathbb{E}\big[\frac{SR_f^2}{SR_m^2} \vert \text{data}\big]$	0.08	0.11	0.15	0.23	0.94	0.93	0.90	0.84
	Tradable bond factors				Tradable stock factors			
Mean	7.85	7.50	7.21	6.32	11.67	11.28	10.63	9.19
5%	4	4	4	3	8	7	7	5
95%	11	11	11	10	16	15	15	13
$\mathbb{E}[SR_f \vert \text{data}]$	0.56	0.95	1.28	1.51	0.66	1.13	1.50	1.77
$\mathbb{E}\big[\frac{SR_f^2}{SR_m^2} \vert \text{data}\big]$	0.43	0.43	0.43	0.39	0.59	0.60	0.57	0.53
Panel B: Bond BMA-SDF								
	Nontradable factors				Tradable factors			
Mean	6.98	6.95	7.02	7.03	7.81	7.77	7.38	6.41
5%	4	4	4	4	5	5	4	3
95%	10	10	10	10	11	11	11	10
$\mathbb{E}[SR_f \vert \text{data}]$	0.18	0.37	0.60	0.97	0.52	0.92	1.25	1.45
$\mathbb{E}\big[\frac{SR_f^2}{SR_m^2} \vert \text{data}\big]$	0.15	0.18	0.22	0.33	0.86	0.83	0.78	0.66
Panel C: Stock BMA-SDF								
	Nontradable factors				Tradable factors			
Mean	6.98	7.02	6.92	7.02	11.82	11.54	11.11	9.81
5%	4	4	4	4	8	7	7	6
95%	10	10	10	10	16	16	15	14
$\mathbb{E}[SR_f \vert \text{data}]$	0.14	0.29	0.47	0.79	0.60	1.03	1.39	1.70
$\mathbb{E}\big[\frac{SR_f^2}{SR_m^2} \vert \text{data}\big]$	0.08	0.10	0.14	0.23	0.94	0.93	0.92	0.87

The table reports posterior means of number of factors (along with the 90% confidence intervals), implied Sharpe ratios $\mathbb{E}[SR_f|\text{data}]$, and the ratio of SR_f^2 to the total SDF-implied squared Sharpe ratio $\mathbb{E}\big[SR_f^2/SR_m^2|\text{data}\big]$ for different subsets of factors. Subsets are tradable and nontradable factors, and within tradables we further separate bond and stock factors. Panels A, B and C report results for the co-pricing, bond-only and stock-only BMA-SDFs, respectively, using the corresponding factor zoos.

posterior mean Sharpe ratio for *all* tradable factors jointly is approximately 2.27. The degree of common spanning of priced risks can be formally assessed by focusing on the estimated share of the squared Sharpe ratio of the SDF generated by the different types of factors, $\mathbb{E}\big[\frac{SR_f^2}{SR_m^2}|\text{data}\big]$. Summing the shares in Panel A ascribable to, respectively, nontradable (0.23) and tradable bond (0.39) and stock (0.53) factors yields a total of 1.15, i.e., more than 100%, indicating substantial commonality among the fundamental risks spanned by the different types of factors.

Furthermore, the sum of the shares for bond and stock factors ($0.39 + 0.53 = 0.92$) is much larger than the share due to all tradable factors jointly (0.84). That is, tradable bond and stock factors capture, at least partially, the same underlying sources of priced risk. Similarly, summing the shares of squared Sharpe ratios ascribable to nontradable and tradable factors in Panels A to C yields 1.07, 0.99, and 1.1, indicating some common spanning between tradable and nontradable factors driven mostly by equity factors.

Since, in the cross-sectional layer of our estimation method (encoded by the likelihood function in equation (2)), the "regressors" are the loadings in the $N \times K$ matrix of covariances between test assets and factors ($\boldsymbol{C}$), the degree of commonality in pricing implications of the factors in the zoo can be gauged by performing a principal component analysis on the matrix $\boldsymbol{C}^T\boldsymbol{C}$ (in the OLS case, or $\boldsymbol{C}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{C}$ in the GLS case). In Figure IA.20 of the Internet Appendix, we perform such an analysis and document that the largest five principal components of the factor loadings explain more than 99% of their *cross-sectional* variation (in the OLS case, and more than 80% in the GLS case). That is, overall, the findings of this section highlight that the factor zoo is akin to a jungle of noisy proxies for common underlying sources of risk.

Given the salience of tradable factors for the BMA-SDF outlined above, with their share of the squared Sharpe ratio of the SDF in the two-thirds to four-fifths range, a natural question is what types of risks these factors capture. Using the method pioneered by Campbell and Shiller (1988) and extended by Vuolteenaho (2002), we classify the tradable factors into those that relate more to discount rate (DR) news and those for which, instead, cash-flow (CF) news is more important.[32] Internet Appendix IA.5 details the empirical (VAR) methodology used for categorizing our 40 tradable bond and stock factors as (mostly) driven by either discount rate or cash-flow news. Therein, we also demonstrate, with extensive robustness tests, that the decomposition remains quite stable across alternative approaches.

Table 5 decomposes, for a range of prior values, the contribution to the SDF dimensionality and Sharpe ratio of the tradable factors, primarily related to DR news on one hand and to CF news on the other. Panel A reports results for the joint pricing of bonds and stocks with all factors, while Panels B and C focus on the two asset classes and factor zoos separately. The left and right four columns pertain to DR and CF news, respectively. First, DR news factors marginally dominate the composition of the co-pricing BMA-SDF in Panel A. The average factor-implied Sharpe ratios, $\mathbb{E}[SR_f|\text{data}]$, of the DR news-driven factors are consistently higher

[32] See Koijen and Van Nieuwerburgh (2011) and more recent work by Zviadadze (2021).

Table 5: Discount rate vs. cash-flow news

	Discount rate news				Cash-flow news			
	Total prior SR				Total prior SR			
	20%	40%	60%	80%	20%	40%	60%	80%
Panel A: Co-pricing BMA-SDF, tradable bond and stock factors								
Mean	9.81	9.60	9.28	8.20	9.71	9.18	8.56	7.31
5%	6	6	6	5	6	6	5	4
95%	14	13	13	12	13	13	12	11
$\mathbb{E}[SR_f\|\text{data}]$	0.65	1.19	1.70	2.10	0.60	1.06	1.45	1.77
$\mathbb{E}\left[\frac{SR_f^2}{SR_m^2}\|\text{data}\right]$	0.58	0.67	0.75	0.75	0.51	0.55	0.57	0.56
Panel B: Bond BMA-SDF, tradable bond factors								
Mean	4.97	5.05	4.94	4.43	2.85	2.72	2.44	1.98
5%	2	3	2	2	1	1	1	0
95%	8	8	7	7	5	5	4	4
$\mathbb{E}[SR_f\|\text{data}]$	0.44	0.85	1.21	1.43	0.28	0.50	0.64	0.69
$\mathbb{E}\left[\frac{SR_f^2}{SR_m^2}\|\text{data}\right]$	0.67	0.74	0.75	0.65	0.35	0.32	0.27	0.21
Panel C: Stock BMA-SDF, tradable stock factors								
Mean	5.01	4.91	4.79	4.38	6.81	6.63	6.31	5.43
5%	2	2	2	2	4	4	3	2
95%	8	7	7	7	10	10	9	9
$\mathbb{E}[SR_f\|\text{data}]$	0.37	0.73	1.11	1.48	0.47	0.83	1.16	1.44
$\mathbb{E}\left[\frac{SR_f^2}{SR_m^2}\|\text{data}\right]$	0.44	0.54	0.65	0.72	0.65	0.66	0.69	0.68

The table reports posterior means of number of factors (along with the 90% confidence intervals), implied Sharpe ratios $\mathbb{E}[SR_f|\text{data}]$, and the ratio of SR_f^2 to the total SDF-implied squared Sharpe ratio $\mathbb{E}\left[SR_f^2/SR_m^2|\text{data}\right]$ for discount rate and cash-flow news driven tradable factors, respectively.

than those of their CF-driven counterparts. This translates into a significantly higher proportion of the total implied Sharpe ratio being driven by DR-related factors. For a prior level equal to 80% of the ex post achievable Sharpe ratio, DR-driven factors account for 75% of the total squared Sharpe ratio of the SDF, compared to 56% for the CF-driven factors. Second, when considering the corporate bond BMA-SDF (Panel B), the total Sharpe ratio is predominantly driven by bond factors related to DR news. The factor-implied Sharpe ratio $\mathbb{E}[SR_f|\text{data}]$ and $\mathbb{E}\left[\frac{SR_f^2}{SR_m^2}|\text{data}\right]$ for DR-driven factors are nearly double that of the CF-driven factors. Finally, when considering only stock factors (Panel C), both DR and CF news appear to play an equally important role, providing very similar contributions to the Sharpe ratio of the BMA-SDF.

In Internet Appendix IA.5.3, we discuss the estimated positioning of the individual factors on the spectrum of DR and CF news. Interestingly, the two most likely tradable components of the BMA-SDF, the post-earnings announcement drift factors in bonds and stocks, PEAD and

PEADB, are primarily driven by DR news.[33]

3.2 Trading the BMA–SDF

We now investigate the implementability of the BMA-SDF as a trading strategy and compare its performance to tradable benchmark strategies.

Portfolio weights for the tradable strategies are constructed by normalizing the posterior means of the MPRs of the SDF representations to sum to one in each specification. Since all benchmark models are exclusively based on tradable factors, we constrain the BMA-SDF to use only such factors. This means that our approach, de facto, focuses on a lower bound for the trading performance of the BMA-SDF since nontradable factors in the BMA-SDF command a non-trivial Sharpe ratio (see Table 4). To facilitate comparison, all tradable portfolio strategies are normalized to have the same volatility as the equity market index.

The IS results are presented in Panel A of Table 6. The IS Sharpe ratio of the tradable BMA-SDF ranges from 1.99 (20% shrinkage) to 2.85 (80% shrinkage). The closest competitor is the KNS model, which delivers an IS Sharpe ratio of 2.57. The TOP γ and λ models, using the top five *tradable* factors by posterior probability and MPR, respectively, also perform well with Sharpe ratios of 2.14 and 2.15 respectively. Note also that the tradable version of the BMA-SDF tends to exhibit much less negative skewness and thinner tails than the other benchmark strategies.

In Panel B of Table 6, we examine the time series OS performance of the same set of tradable portfolios. To conduct this exercise, the out-of-sample period is July 2004 to December 2022—a particularly challenging one as it contains both the Great Recession as well as the contraction during the COVID pandemic.

We use the first half of our baseline data (January 1986 to June 2004) as the training sample for the initial estimation of the tradable portfolio weights. These weights are used to form the portfolios that are held over the first 12 months out-of-sample. Recursively, after one year, the training sample is expanded by twelve months; the portfolio weights are recomputed using the resulting MPRs in the expanded training sample, and the performance of the portfolios is assessed over the following twelve months (yielding, in total, 222 months of OS history).

Strikingly, the OS performance of the BMA-SDF portfolio in Panel B of Table 6 is now significantly greater than any other model considered. The Sharpe ratio of the BMA-SDF

[33] See e.g., Penman and Yehuda (2019) for a discussion on how earnings reports contain both discount rate and cash-flow news.

Table 6: Trading the BMA-SDF and benchmark models

	BMA-SDF prior Sharpe ratio				TOP γ	TOP λ	KNS	RPPCA	FF5	HKM	MKTB	MKTS	EW
	20%	40%	60%	80%									
Panel A: In-sample – 1986:01 to 2022:12 ($T = 444$)													
Mean	31.38	38.94	43.43	45.03	33.77	34.04	40.54	39.59	12.20	8.37	10.18	8.29	19.42
SR	1.99	2.46	2.75	2.85	2.14	2.15	2.57	2.51	0.77	0.53	0.64	0.52	1.23
IR	1.73	2.28	2.52	2.59	1.94	1.90	2.33	2.18	0.02	0.34	−0.47	0.29	–
Skew	0.76	0.73	0.54	0.31	0.47	0.44	0.51	0.90	−0.70	−0.65	−0.71	−0.78	−0.29
Kurt	3.55	3.08	2.47	2.00	2.53	2.54	2.98	3.07	3.41	1.91	4.68	2.22	4.63
Panel B: Out-of-sample – 2004:07 to 2022:12 ($T = 222$)													
Mean	22.72	25.73	27.17	27.90	20.59	23.41	20.36	23.01	5.90	7.12	8.22	8.71	17.15
SR	1.46	1.65	1.74	1.79	1.32	1.50	1.31	1.48	0.38	0.46	0.53	0.56	1.10
IR	0.98	1.24	1.38	1.46	1.40	1.37	0.85	1.07	−0.27	−0.26	−0.04	−0.21	–
Skew	0.30	0.04	−0.10	−0.13	−0.62	0.17	−1.19	−0.60	−1.59	−0.37	−0.93	−0.54	−1.06
Kurt	2.39	3.59	4.06	3.77	5.77	2.38	11.97	7.74	10.60	1.51	5.42	1.28	7.22

In-sample (Panel A) and out-of-sample (Panel B) performance of the co-pricing BMA-SDF tradable portfolio across prior SR levels, the 'TOP' model factors portfolios, the latent co-pricing factor models (KNS and RPPCA), notable benchmark models (FF5, HKM, MKTS, MKTB) and the equally-weighted portfolio (EW) of all (40) tradable factors. The in-sample weights for the tradable portfolios are formed scaling the (posterior means of the) MPRs to sum to one in each specification considered. The Top γ (λ) model uses the MPRs from the most likely (highest absolute MPRs) factors with 80% shrinkage. These factors are: PEADB, PEAD, CMAs, CRY and MOMBS (γ) and PEADB, MOMBS, CRY, PEAD and CMAs (λ). For KNS, the weights are obtained directly from the Kozak et al. (2020) procedure. For RPPCA, FF5 and HKM, the weights are estimated via GMM. In Panel B, the results are strictly out-of-sample. An expanding window is used with an initial window of 222 months to conduct the estimation. These weights are then used to invest in the factors over the next 12 months. Thereafter, we re-estimate the models in an expanding fashion every year. The Top model input factors change dynamically at each estimation. For KNS, we re-conduct the two-fold cross-validation at every estimation to pin down the optimal parameters. For RPPCA, we re-estimate the PCs at every estimation. The Mean is annualized and presented in percent. The Sharpe ratio and Information ratio are annualized. The benchmark factor to compute the IR is the EW factor. Skew and Kurt are skewness and kurtosis, respectively. The models are estimated with the 83 bond and stock portfolios and the 40 tradable bond and stock factors as described in Section 1. For the BMA-SDFs, we report results for a range of prior Sharpe ratio values that are set as 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the relevant portfolios and factors. In Panel B, this ratio changes with the expanding window. The IS period is 1986:01 to 2022:12 ($T = 444$) and the OS period is 2004:07 to 2022:12 ($T = 222$).

portfolio is approximately 1.8 (80% shrinkage). Moreover, all of the BMA-SDF specifications convincingly outperform the equally weighted (EW) portfolio of tradable factors, which has a SR of 1.1 and is known to be exceedingly difficult to beat (DeMiguel et al., 2009).

But is this robust OS economic performance of the BMA-SDF portfolio due to just a handful of lucky episodes? Figure 7 depicts, in log scale, the cumulative returns of investing \$1 in the OS BMA-SDF strategy along with notable benchmarks. For ease of comparison, portfolio returns are scaled to have a constant volatility equal to that of the stock market factor (MKTS).

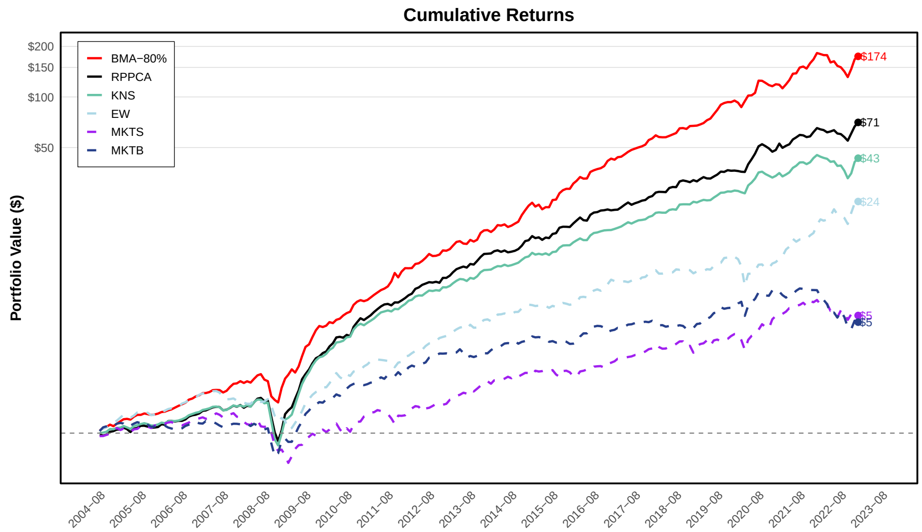


Figure 7: Out-of-sample investing in the BMA-SDF tradable portfolio and benchmark models.

Out-of-sample cumulative return of investing \$1 in the co-pricing BMA-SDF tradable portfolio with 80% SR prior, the latent factor models KNS and RPPCA, the stock and bond market factors, MKTS and MKTB, and an equally-weighted factor portfolio EW. An expanding window is used with an initial window of 222 months to conduct the estimation. These weights are then used to invest in the factors which are held constant over the next 12 months. Thereafter, we re-estimate the models in an expanding fashion every year. For KNS, we re-conduct the two-fold cross-validation every estimation to pin down the optimal parameters. For RPPCA, we re-estimate the PCs every estimation. The models are estimated to price the 83 bond and stock portfolios and the 40 tradable bond and stock factors ($N = 123$) as described in Section 1. The out-of-sample evaluation period is 2004:07 to 2022:12 ($T = 222$).

Out-of-sample, the BMA-SDF (80% shrinkage) tradable portfolio is the clear winner with a cumulative dollar value over the investment period of \$174 versus \$71 for RPPCA. Furthermore, in virtually any multi-year sub-period, the slope (and hence the log return) of the tradable BMA-SDF strategy is higher than that of any of the alternative strategies, stressing that the outperformance is extremely stable out-of-sample, and not just driven by a few lucky events.

3.3 The information content of the two factor zoos

As shown in Section 3.1.2 (see Tables 2 and 3), although one can construct well-performing BMA-SDFs to price bonds and stocks separately using the information in their respective zoos, the joint pricing of these assets requires information from both sets of factors (see Figure 5). In this section, we demonstrate that this result arises from the fact that corporate bond returns reflect not only a component related to compensation for exposure to credit risk, but also a *Treasury term structure* risk premium that is not captured by equity-based factors.

To illustrate this point, we now turn our focus to bond returns in excess of duration-matched portfolios of U.S. Treasuries. More precisely, for every bond i, we construct the following

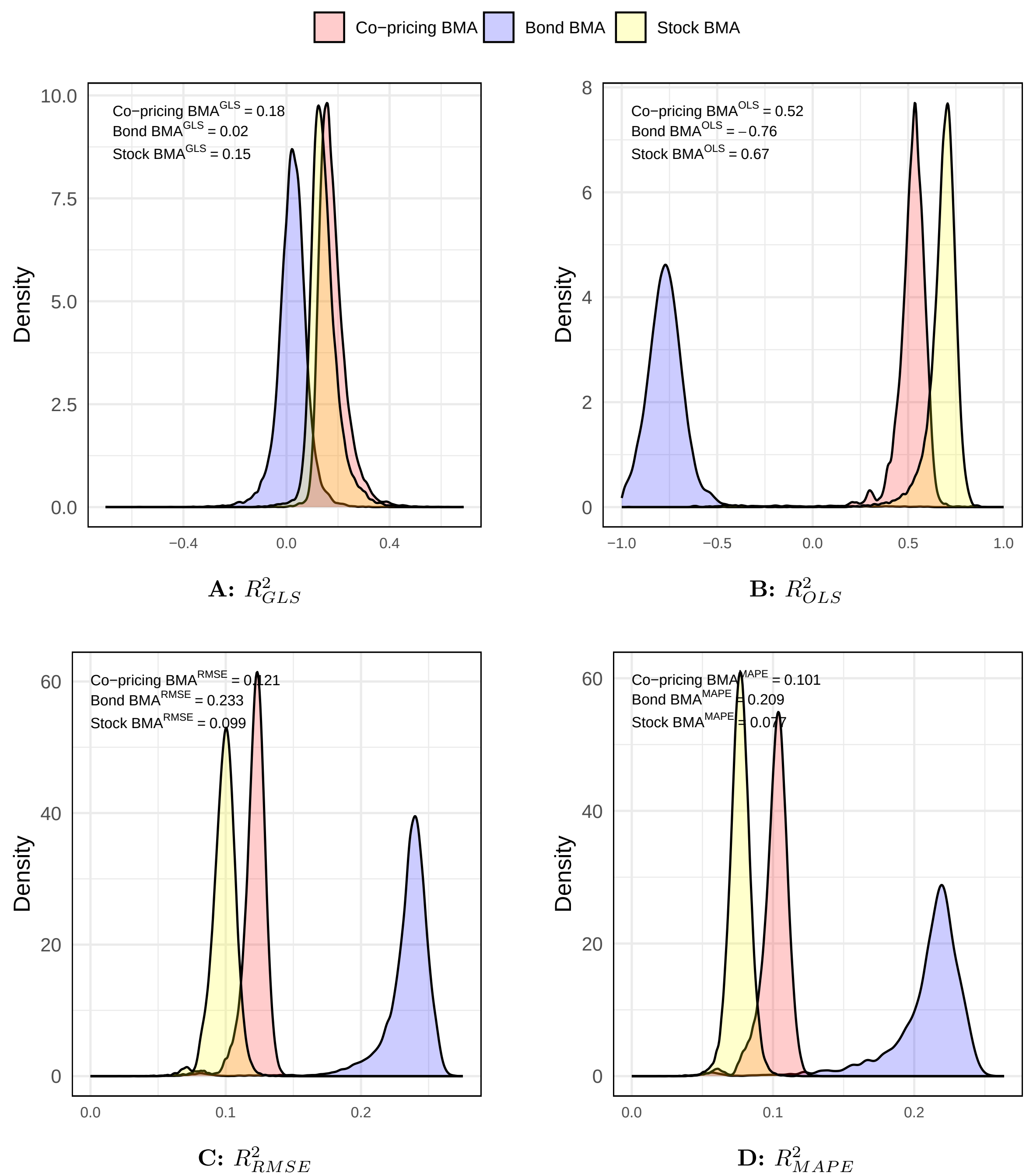


Figure 8: Pricing the joint cross-section of stock and duration-adjusted bond returns.

This figure plots the distributions of R^2_{GLS}, R^2_{OLS}, RMSE and MAPE in Panels A, B, C and D respectively across 16,383 possible bond and stock cross-sections using the 14 sets of stock and bond test assets ($2^{14} - 1 = 16,383$) priced using the respective BMA-SDF (the empty set is excluded). All bond test assets (IS and OS) and factors are formed with duration-adjusted returns defined in equation (10). The models are first estimated using the baseline set of IS test assets and then used to price (with no additional parameter estimation) each set of the 16,383 OS combinations of test assets. The red distributions correspond to the pricing performance of the co-pricing BMA-SDF. The blue (yellow) distributions correspond to the pricing performance of the bond (stock) only BMA-SDF. The BMA-SDFs are computed with a prior Sharpe ratio value set to 80% of the ex post maximum Sharpe ratio of the IS test assets. All data are standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

duration-adjusted return

$$\underbrace{R_{bond\,i,t} - R^{Treasury}_{dur\,bond\,i,t}}_{\text{Duration-adjusted return}} \equiv \underbrace{R_{bond\,i,t} - R_{f,t}}_{\text{Excess return}} - \underbrace{\left(R^{Treasury}_{dur\,bond\,i,t} - R_{f,t}\right)}_{\text{Treasury component}}, \tag{10}$$

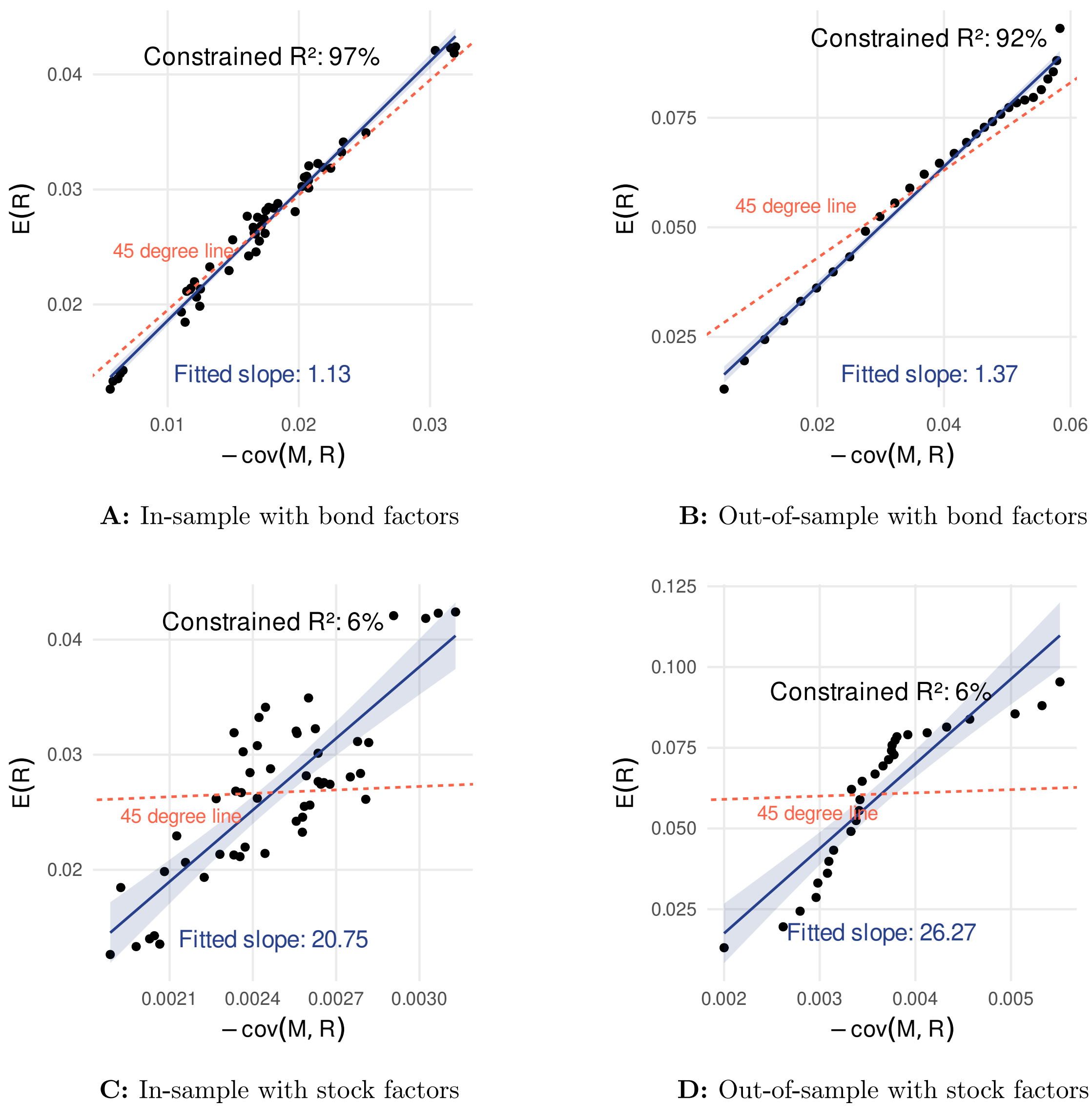
where $R_{bond\,i,t}$ is the return of bond i at time t, $R_{f,t}$ denotes the short-term risk-free rate, and $R^{Treasury}_{dur\,bond\,i,t}$ denotes the return on a portfolio of Treasury securities with the same duration as bond i (constructed as in van Binsbergen et al. (2025), see also Internet Appendix IA.6). As is evident from the right-hand side of equation (10), the duration adjustment removes the implicit Treasury component from the bond excess return, hence isolating the remaining sources of risk compensation that investing in a given bond entails.

Figure 8 reports the distribution of OS measures of fit (R^2_{GLS}, R^2_{OLS}, RMSE, and MAPE) across 16,383 possible bond and stock cross-sections using the 14 sets of stock and bond test assets for three different BMA-SDFs based on (i) bond factors only, (ii) stock factors only, and (iii) both bond and stock factors. The contrast with Figure 5 is stark: once bond returns are adjusted for duration, the BMA-SDF based solely on equity information prices jointly bonds and stocks as effectively as the co-pricing BMA-SDF that additionally includes bond factors. That is, the information content of the bond factor zoo becomes largely irrelevant for co-pricing once the Treasury component of bond returns is removed.

This last finding raises a natural question: why do we need the bond factors for co-pricing assets in the absence of the duration adjustment? As we are about to demonstrate, bond factors price the Treasury component of corporate bond returns.

Panel A of Figure 9 summarizes the in-sample pricing of the Treasury component of corporate bond returns using the Treasury component bond BMA-SDF based only on the bond factor zoo. That is, as in-sample test assets we use the Treasury component of the 50 bond portfolios and estimate the model using the 14 nontradable and the 16 tradable bond factors.[34] The pricing (evaluated at the posterior mean of the SDF) is nearly perfect, with a cross-sectional (constrained) R^2_{OLS} of about 97%. Similarly, Panel B shows that the out-of-sample pricing of a cross-section of Treasury excess returns using the same BMA-SDF is also nearly perfect, with a constrained R^2_{OLS} of 92% and average excess returns and SDF-implied risk premia aligning

[34] We do not include the factors among the test assets so that the evaluation of fit is based only on the ability to explain the Treasury component. Expected risk premia of portfolios in the figure are proxied by their time series averages (on the vertical axis), while the SDF-implied ones are computed as minus the covariance of (the posterior mean of) the BMA-SDF, hence imposing the theoretical restriction coming from the fundamental asset pricing equation. The constrained R^2 is computed imposing a unit slope and zero intercept.



A: In-sample with bond factors

B: Out-of-sample with bond factors

C: In-sample with stock factors

D: Out-of-sample with stock factors

Figure 9: Pricing the Treasury component of corporate bond returns.

Plots of sample averages of excess returns for Treasury portfolios, on the y-axis, against BMA-SDF-implied risk premia, computed as minus the covariance between portfolio returns and the (posterior mean of the) BMA-SDF, constructed using the nontradable factors plus only bond (Panels A and B) or stock (Panels C and D) factors, on the x-axis. Panels A and C: excess returns are the Treasury component from equation (10), $R^{Treasury}_{dur\,bond\,i,t} - R_{f,t}$, using the 50 IS bond portfolio test assets. Panels B and D: 29 Treasury portfolios of excess returns on Treasury securities with maturities spanning 2 to 30 years. All are described in Section 1. The Treasury component bond and stock BMA-SDFs are estimated using the 50 IS portfolios and the respective bond and stock factors in addition to the 14 nontradable factors described in Appendix A. For either estimation we do not impose self-pricing for the stock and bond factors. OLS R^2s are from a constrained regression that sets the slope coefficient to one. The sample period is 1986:01 to 2022:12 ($T = 444$).

closely around the 45-degree line.

In contrast, Panels C and D of Figure 9 report the same cross-sectional pricing exercises using the BMA-SDF based only on stock factors. Clearly, the equity-based BMA-SDF is not able to price the Treasury component of corporate bond returns, neither in- nor out-of-sample, yielding extremely low measures of fit (the constrained R^2_{OLS} is only 6%) and slope coefficients very far from the theoretical value.

The above highlights that the bond factor zoo is necessary for co-pricing bonds and stocks because the factors proposed in the corporate bond literature price well the Treasury component implicit in corporate bond returns—a component that stock factors fail to price. However, once this component is accounted for—as in the case of duration-adjusted bond returns—co-pricing can effectively be achieved using only equity information.

But does one really need our Bayesian machinery comprising quadrillions of models to uncover this phenomenon? The answer, resoundingly, is yes. As highlighted in Tables IA.XVI and IA.XVII (Panels B and D) of the Internet Appendix, unlike our BMA-SDF, canonical equity-based factor models do quite a poor job in pricing corporate bond returns *even after removing their Treasury component* (with small, and mostly negative, measures of fit, and significantly larger pricing errors than the BMA-SDF). This is due to the fact that both the co-pricing and stock-based BMA-SDFs that price duration-adjusted corporate bonds (and stocks) are *dense* in the space of both tradable and nontradable factors (as per Panel C of Table 4). That is, the link between duration-adjusted bond returns and equity market factors extends far beyond the one between these assets and just the equity market index (van Binsbergen et al. (2025)) or just a handful of factors. Consequently, and importantly, the reward for holding this risk is a multiple of that for the market index alone (with posterior annualized Sharpe ratios of about 1.4 to 1.7 just for the tradable component, as per Panel C of Table 4). Furthermore, the high degree of (posterior) factor density of the equity-based SDF that prices duration-adjusted bond returns implies that canonical inference based on low-dimensional models is unreliable (due to misspecification) and affected by a severe omitted variable problem (Giglio and Xiu (2021)). For example, in Figure IA.26 of the Internet Appendix, we test the equity CAPM as a pricing model for the duration-adjusted bond returns. We do so in both SDF and "beta" representations using (unlike previous literature) robust estimation methods. As the figure highlights, using robust inference, one would not find a statistically significant link between duration-adjusted bond returns and the equity market index in such a heavily misspecified setting.

Moreover, the Treasury component of corporate bonds is also economically important. The ex post (annualized) maximum Sharpe ratio of the duration-matched Treasury portfolios in equation (10) is approximately 1.48. As illustrated in the bottom panel of Figure IA.22 of the Internet Appendix, this is roughly the posterior mode of the Sharpe ratio achievable with the BMA-SDF that prices the Treasury component only with factors in the corporate bond factor zoo. Note also that, as depicted in the top panel of the figure, even for pricing just this Treasury component, the required SDF is quite *dense*, with a median number of factors equal to 14 and a posterior 95% C.I. ranging from 8 to 19 factors. Furthermore, as shown in Table IA.XVIII of the Internet Appendix, the required SDF is dense in the space of both nontradable and tradable factors.

Mirroring the analysis in Section 3.1, we can assess which factors are more likely to price the Treasury component individually, and how factors should be optimally combined to achieve a portfolio that captures the priced risks in these assets. Figure IA.23 of the Internet Appendix reports the posterior factor probabilities and market prices of risk implied by the pricing of the Treasury component of corporate bond returns using the corporate bond factor zoo (the prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio). Overwhelmingly, the most likely factors are nontradable: five out of the six factors with posterior probability higher than the prior value are nontradable. Furthermore, largely, these factors are the same as those that appear most likely when co-pricing bonds and stocks (the top three being YSP, CREDIT and LVL, followed by the IVOL factor). Moreover, these nontradable factors command large market prices of risk and the probability of zero nontradable factors being in the bond BMA-SDF that prices the Treasury component of corporate bond returns is virtually zero (or 0.014%).

The bottom panel of Figure IA.23 of the Internet Appendix tells us which portfolios to form to capture the common risk priced in these cross-sections. Interestingly, in addition to the most likely factors, the bond market index (MKTB) and the traded term structure risk factor (TERM, i.e., the difference between the monthly long-term government bond return and the one-month T-Bill rate of return, Fama and French (1992)) feature prominently in the BMA-SDF with, respectively, the second and third largest portfolio weights—and the largest among tradable factors. That is, these factors are not likely fundamental sources of risk, but they appear correlated with the true sources.

This finding also explains the success of the MKTB factor in Dickerson et al. (2023). As

Internet Appendix IA.7 also confirms, the bond market index commands a large risk premium in its own market, but it is not likely to be part of the true latent SDF. Nevertheless, it commands a substantial risk premium as compensation for being highly correlated with the latent fundamental risks in the bond market, particularly the Treasury component, making it advantageous in a portfolio designed to capture these risks (as per BMA-SDF weights).

Note that, at least in nominal terms, the cash flows of Treasury bonds are perfectly known in advance. Hence, arguably, we would expect discount rate news to be the primary drivers of their priced risk (Chen and Zhao (2009)). Given the flexibility of our general prior introduced in Section 2.3, we can use our discount-rate and cash-flow news decomposition of the factors to encode this prior belief about the relative importance of DR versus CF news. In particular, we can use the $\frac{\mathbb{V}(\text{Ndr})}{\mathbb{V}(u)}$ estimates for each factor to compute the (normalized) κ weighting vector to inform the prior: DR factors are assigned a positive weight, while CF factors receive a negative weight. This encodes prior beliefs that traded bond factors classified as being driven (relatively) more by DR news, ceteris paribus, explain a larger portion of the squared Sharpe ratio compared to factors driven by CF news. We report the posterior factor probabilities and market prices of risk implied by the pricing of the Treasury component of corporate bond returns with this DR factor tilt in Figure IA.24, and the corresponding pricing statistics in Table IA.XIX of the Internet Appendix. Obviously, this tilt makes the DR factors individually more likely, pushing the likelihood of the MKTB factor above the prior value, but the pricing results are overall very similar to those with the more diffuse prior encoding, with only a very minor improvement in OS pricing performance and a small perturbation of the portfolio composition as outlined in the bottom panel of Figure IA.24.

Our analysis also sheds light on the degree of integration between equity and corporate bond markets. First, as illustrated by the generalized canonical correlation (GCC) analysis in Figure IA.8 of the Internet Appendix, there is substantial commonality—in the time series dimension—between bond and stock returns, with the first GCC being just under 75% (Panel C). Furthermore, upon removing the Treasury component from bond returns, the GCC analysis remains virtually unchanged (Panel E), once again suggesting that the Treasury component has distinct drivers compared to the risks spanned in equity markets.

Second, Table 4 shows both evidence of an overlap between the latent risks captured by equity and bond factors (in Panel A, the sum of the Sharpe ratios achievable with either of the two sets of tradable factors is *larger* than the Sharpe ratios achievable with these tradable factors jointly), but also of separation between the risks priced in the two markets (as the maximum Sharpe ratios achievable with the BMA-SDFs that use only equity or bond factors to separately price the two markets, Panels B and C, are smaller than the Sharpe ratios of the co-pricing BMA-SDF that uses the same factors jointly, in Panel A).

Third, the evidence that equity factors can price corporate bond returns once their Treasury component is accounted for (Figure 8), and that stock factors cannot price this Treasury component (Figure 9), suggests both a segmentation between equity risk and Treasury markets and a commonality between stock markets and credit risk in bonds.

Fourth, in Table IA.XX of the Internet Appendix, we report the time series correlations between (the posterior means of) BMA-SDFs constructed with equity and bond factors, jointly and separately, to price (jointly and separately) stock returns, bond excess returns, duration-adjusted bond returns, and the Treasury component of corporate bond returns. Therein, the correlation between the bond-factors BMA-SDF that prices the Treasury component of bond returns and the stock-factors BMA-SDF that prices equity returns stands out as particularly low: 0.172 (80% shrinkage). For comparison, the correlations between the co-pricing BMA-SDF and the bond- and stock-only BMA-SDFs that price these asset classes separately are all well in excess of 70%. Once again, this suggests that the (partial) evidence of segmentation between equity and bond markets is driven by the Treasury component in the latter.

Hence, overall, we find both evidence of commonality of risks priced in the two markets—net of Treasury effects—and hence of integration, and of a degree of segmentation generated by the implicit loading of corporate bonds on Treasury-related risks.

3.4 The economic properties of the co-pricing SDF

We now turn to assessing the economic properties of the co-pricing BMA-SDF. Figure 10 depicts the time series of the BMA-SDF (that is, its posterior mean), along with its conditional time series mean (estimated using an ARMA(3,1) model selected via both the Akaike and the Bayesian Information Criteria, AIC and BIC). Both the SDF and its conditional mean exhibit clear business cycle behavior as they increase during expansions and tend to peak right before recessions, being substantially reduced during economic contractions. Moreover, as highlighted in Panel A of Figure IA.27 of the Internet Appendix, the BMA-SDF is highly predictable: virtually all of its autocorrelation coefficients are statistically significant at the 1% level up to 20 months ahead, and the p-value of the Ljung and Box (1978) test of joint significance is zero

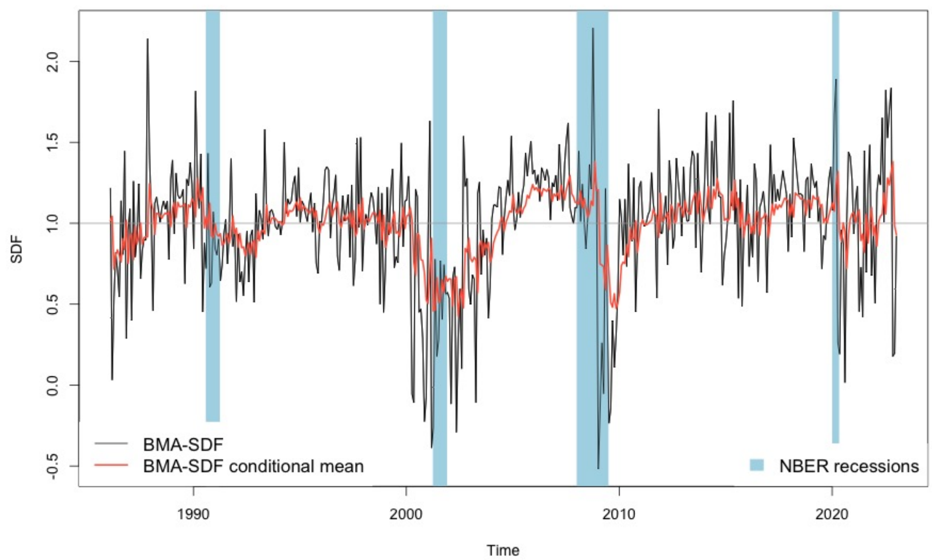


Figure 10: The co-pricing SDF and its conditional mean.

The figure plots the time series of the (posterior mean of the) co-pricing BMA-SDF and its conditional mean. The conditional mean is obtained by fitting an ARMA(3,1) to the BMA-SDF whereby the order of the ARMA is selected using the AIC and the BIC. Shaded areas denote NBER recession periods. The sample period is 1986:01 to 2022:12 ($T = 444$).

at this horizon. Additionally, about one-fifth of its time series variance is explained by its own lags (23% for the best AR specification and 19% for the best ARMA specification according to the BIC).

Note also that, as shown in Figure IA.28 of the Internet Appendix, none of the other celebrated SDF models come close to displaying such a level of business cycle variation and persistency: the KNS SDF has about 11% of its time series variation being predictable by its own history, while this number drops to 4% for RPPCA, and is only 2% to 3% for FF5 and CAPMB, and zero for HKM and CAPM. Remarkably, as shown in Panel A of Table IA.XXI of the Internet Appendix, the SDFs with a higher degree of persistency, KNS and RPPCA, are exactly the ones with the highest degree of correlation with the BMA-SDF (0.78 and 0.55, respectively), and are the closest competitors for the BMA-SDF in the pricing exercises in Section 3.1. Instead, SDFs that perform significantly worse in cross-sectional pricing have both little time series persistency and correlations with the BMA-SDF in the 0.16 to 0.29 range.

Obviously, the nontradable factors in the BMA-SDF play an important role in generating a pronounced business cycle pattern and a high degree of predictability. Nevertheless, even when removing the nontradable factors from the BMA-SDF, the resulting SDF remains predictable, with 5% to 10% of its time series variation explained by its own lags, and a highly significant

Ljung and Box (1978) test statistic even up to 20 months ahead. Furthermore, note that the five most likely factors in the SDF (PEAD, IVOL, PEADB, CREDIT, YSP) explain only about 47% of the time series variation of the BMA-SDF, further confirming the dense nature of the pricing kernel. Individually, only PEADB and IVOL explain marginally more than 20% of the time series variation of the SDF, with the other factors accounting individually for 3% to 7%.

Recall that the variance of the SDF is equal to the squared Sharpe ratio achievable in the economy. Hence, whether our filtered SDF implies time-varying compensation for risk can be elicited by analyzing the predictability of its volatility. As pointed out in Engle (1982), the presence of volatility clustering can be assessed, without taking a parametric stance on the variance process, by simply analyzing the serial correlation of the squared one-step-ahead forecast errors, since these are consistent (yet noisy) estimates of the latent conditional variance. Note that, for instance, such a variance proxy has been used extensively in the macrofinance literature (see, e.g., Bansal et al. (2005), Bansal et al. (2012), Beeler and Campbell (2012), and Chen (2017)), and squared forecast errors of returns are commonly used as a proxy for latent conditional variances.

Panel B of Figure IA.27 of the Internet Appendix reports the empirical autocorrelation function of the squared forecast errors of the co-pricing BMA-SDF. Most of the autocorrelation coefficients are statistically significant at the 1% level up to seven months ahead. Moreover, the Ljung and Box (1978) test strongly rejects the joint null of zero autocorrelations up to 20 months into the future (the p-value of the test is zero). That is, not only does the first moment of our filtered SDF exhibit substantial predictability, but so does its second moment, suggesting time-varying risk compensation in the economy.

To tackle the question of whether the SDF-implied time variation in risk compensation (i.e., the economy-wide conditional Sharpe ratio) that we uncover makes economic sense, we fit a simple GARCH(1,1) (see Bollerslev (1986)) process to our BMA-SDF.[35] Figure 11 presents the estimated conditional volatility of the SDF, revealing striking results. The implied conditional Sharpe ratio is not only highly countercyclical but also exhibits pronounced spikes during periods of market turbulence and heightened economic uncertainty. These include Black Monday, the Asian financial crisis, the burst of the dot-com bubble, the 9/11 terrorist attacks, the Iraq

[35] We estimate the process based on the posterior mean of the BMA-SDF. Ideally, one would estimate the volatility process for each draw of the SDF and for each possible model, and then compute the posterior average of these 'draws' for the volatility process. Nevertheless, since GARCH estimation requires numerical optimisation, the ideal approach is unfeasible in our model space with quadrillions of models.

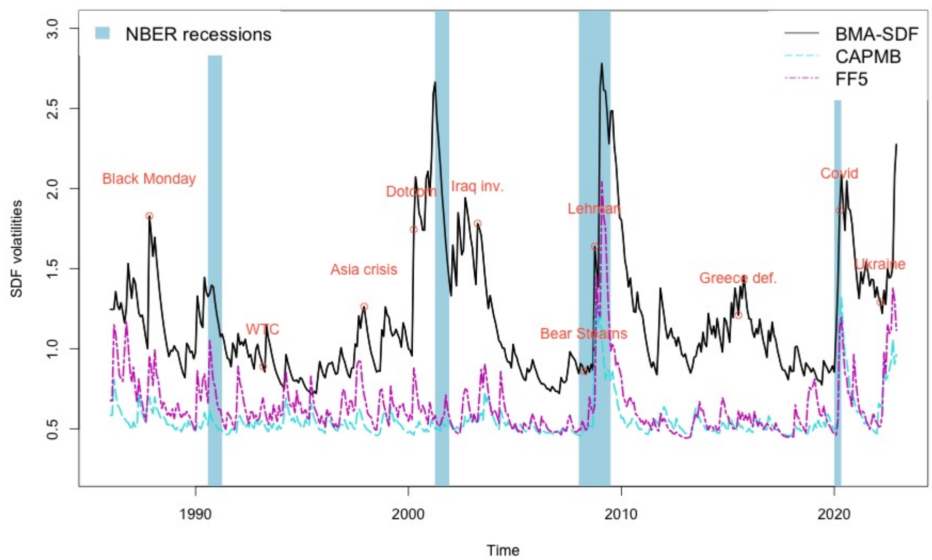


Figure 11: Volatility of the co-pricing BMA-SDF.

The figure plots the annualized volatility of the co-pricing BMA-SDF along with the volatilities of the CAMPB and FF5 SDFs. The volatility of the BMA-SDF is obtained by fitting an ARMA(3,1)-GARCH(1,1) to the posterior mean of the co-pricing BMA-SDF whereby the specification is selected via the AIC and the BIC. The GARCH quasi-maximum likelihood coefficient estimates are:

$$\sigma^2_{t+1} = \omega + \alpha\epsilon^2_t + \beta\sigma^2_t$$

	ω	α	β
Estimate	0.01	0.15	0.81
Robust SE	0.00	0.04	0.06

CAPMB is the bond single-factor model using MKTB, and FF5 is the five-factor model of Fama and French (1993). Estimation details for the benchmark models are given in Appendix D. The volatilities of the SDFs are also computed using a GARCH(1,1) model after selecting an ARMA mean process using the AIC and the BIC. Shaded areas denote NBER recession periods. The sample period is 1986:01 to 2022:12 ($T = 444$).

invasion, the great financial crisis, the Greek default and subsequent eurozone debt crisis, the COVID pandemic, and the aftermath of Russia's invasion of Ukraine. Note that the estimated GARCH coefficients imply a highly persistent conditional volatility, with deviations from the mean exhibiting a half-life of approximately 16.6 months.[36]

As per Panel A in Table 4, nontradable factors account for about a quarter of the SDF variance. Hence, a natural question is whether the SDF volatility pattern depicted in Figure 11 is simply due to tradable factors. We evaluate this conjecture by removing all tradable factors from the BMA-SDF and re-estimating the volatility process of this nontradable-only SDF. We find that the resulting volatility process remains very persistent (with a half-life of 12.3 months), with pronounced business cycle variation and reaction to periods of heightened eco-

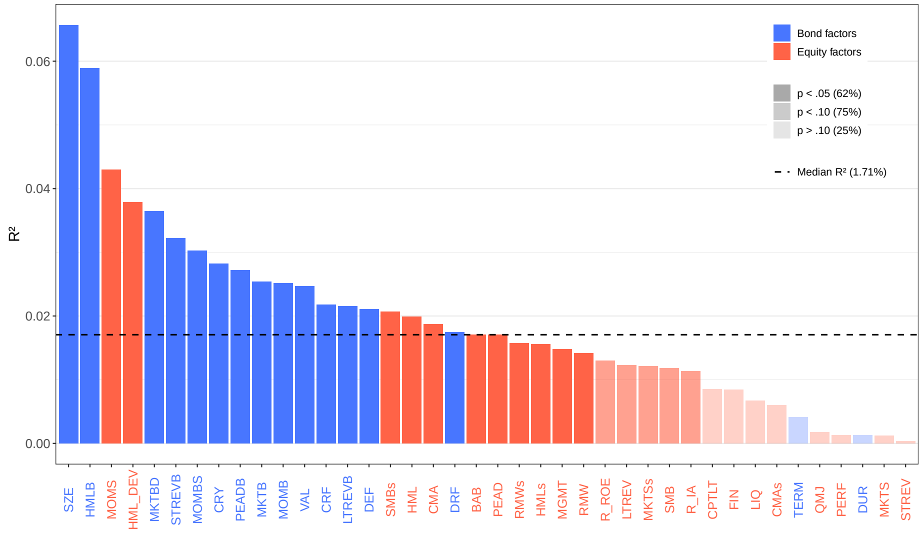
[36] Recall that the half-life of a GARCH(1,1) process is defined as $1+\frac{\ln(1/2)}{\ln(\alpha+\beta)}$ where α and β denote, respectively, the coefficients on lagged squared error and variance.

nomic uncertainty (see Figure IA.29 of the Internet Appendix). Moreover, it has a correlation with the volatility of the BMA-SDF in Figure 11 of about 62%. That is, both tradable and nontradable components of the BMA-SDF are characterized by a very persistent volatility with a clear business cycle pattern.

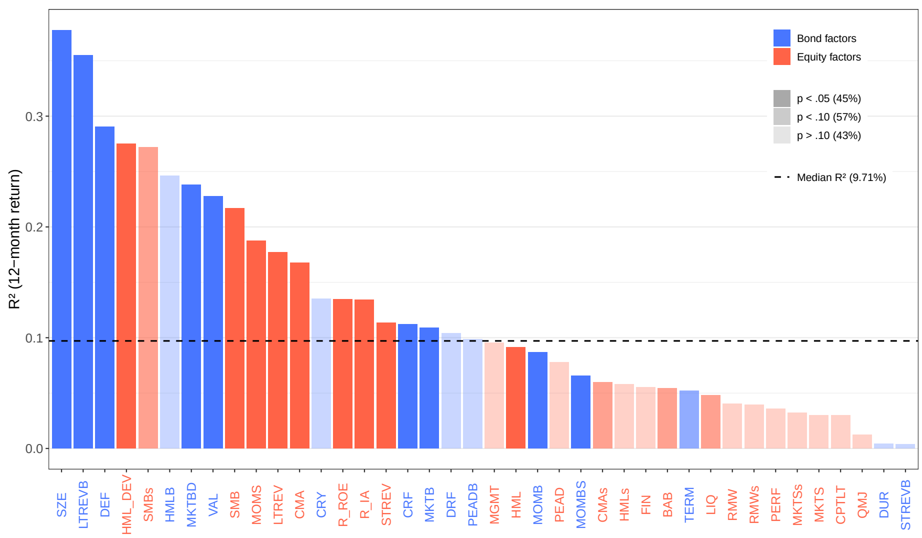
But is the strong countercyclical behavior of the BMA-SDF volatility, and its sharp increase during periods of economic uncertainty, just a mechanical byproduct of it loading on several tradable factors? Figure 11 suggests that this is not the case. Focusing on the celebrated five-factor model of Fama and French (1993) and the bond CAPM (which is not incrementally outperformed by alternative models considered in Dickerson et al. (2023)), we apply the same procedure of estimating their SDF coefficients and computing the implied conditional volatilities using a GARCH specification (after fitting a mean model based on the AIC and the BIC). The estimated volatilities for these two SDF models in Figure 11 make clear that the use of tradable factors in the SDF does not mechanically deliver our findings for the BMA-SDF: both the cyclical pattern and the reaction to periods of heightened economic uncertainty are much less pronounced for the FF5 model, and even more so for the CAPMB. This is formally measured in Figure IA.30 of the Internet Appendix that shows that the half-life of volatility shocks to the FF5 SDF model is only 4.2 months, and for the CAPMB it is just 3 months. Finally, Figure IA.31 of the Internet Appendix depicts the residual of the linear projection of the BMA-SDF estimated volatility on the estimated volatilities of the KNS, RPPCA, CAPM, CAPMB, HKM and FF5, with the residual showing a strong business cycle pattern and being particularly large and positive during periods of high economic uncertainty, suggesting that these alternative SDF models do not sufficiently capture these states despite being based on tradable factors.

The observed business cycle variations and predictability in both the first and second moments of the SDF would imply, within a structural model, time-varying and predictable risk premia for tradable assets. Therefore, we now turn to testing this *time series* prediction of our BMA-SDF identified from *cross-sectional* pricing.

The precise functional form of the predictive relation between current SDF moments and future asset returns does depend on the postulated model. Nevertheless, as shown in Bryzgalova et al. (2024), the Hansen and Jagannathan (1991) conditional SDF projections on the space of returns imply a (log) linear SDF driven by two factors: the innovations to the SDF and the product of the conditional mean of the SDF and the same innovations. Therefore, assuming a contemporaneous linear relationship between asset returns and the SDF yields a simple linear



A: Predictability of monthly log returns



B: Predictability of twelve months cumulative log returns

Figure 12: Predictability of tradable factors with lagged SDF information.

The figure shows the R^2s of predictive regressions of factor returns on the previous month estimates of the co-pricing BMA-SDF conditional variance and conditional variance interacted with the conditional mean. Panel A shows R^2s for one-month ahead predictions while Panel B shows R^2s for one-year ahead predictions. The volatility of the BMA-SDF is obtained by fitting an ARMA(3,1)-GARCH(1,1) to the posterior mean of the co-pricing BMA-SDF whereby the specification is selected via the AIC and the BIC. To account for the overlapping nature of the observations in Panel B, we construct robust standard errors by (i) using a Bartlett kernel (Newey and West (1987)) with 15 lags, (ii) constructing a sandwich estimate of the covariance matrix, and (iii) applying a degrees of freedom correction. The 40 predicted tradable factors are described in Appendix A.

dependence of conditional risk premia on two variables: (i) the conditional variance of the SDF and (ii) the product of this conditional variance with the conditional mean of the SDF.

Leveraging this insight, we run predictive regressions of asset (log) returns between time $t-1$

and t, as well as cumulated (log) returns between $t-1$ and $t+12$, on SDF information observed at time $t-1$: $\mathbb{E}_{t-1}[M_t] \times \mathbb{E}_{t-1}[\sigma_t^2]$ and $\mathbb{E}_{t-1}[\sigma_t^2]$, where the conditional mean is constructed, as in Figure 10, by fitting an ARMA(3,1) process (the preferred specification according to the BIC), and the conditional variance is obtained from the GARCH(1,1) estimates depicted in Figure 11 (and also selected via the BIC).

As test assets to be predicted, we employ the bond and stock factors used in our cross-sectional analysis since these are generally hard to predict and should, according to the previous literature, demand sizable risk premia.

Figure 12 summarizes the predictability results. In Panel A, we report the R^2 values for the one-month-ahead predictions, and in Panel B the same for the cumulative twelve-month-ahead predictive regressions. We encode, via shading, the joint statistical significance of the regressors as implied by an F-test of the regression coefficients.

The results are striking. For the majority of test assets, we find that information embedded in the lagged SDF significantly predicts future asset returns. At the monthly horizon shown in Panel A, this predictability is statistically significant in 75% of cases at the 10% confidence level and in 62% of cases at the 5% significance level. Second, the amount of predictability is economically large, albeit not unrealistically so: for the statistically significant cases it ranges from 1.1% to 6% at the monthly horizon (Panel A). At the twelve-month horizon (Panel B) the median R^2 is about 10%, with many factors having more than one-fifth of their time series variation being predictable. Moreover, even with an extremely conservative approach to constructing the covariance matrix, the F-test yields statistically significant results in about 60% of cases at the 10% level and 45% of cases at the 5% level.[37]

4 Robustness

In this section, we discuss an extensive array of robustness exercises that all confirm our main findings.

4.1 Factor tilting

Our novel spike-and-slab prior in Section 2.3 allows us to assign a heterogeneous degree of prior shrinkage to the different types of factors by setting the hyper-parameter κ to values different

[37]We construct conservative standard errors by (i) using a Bartlett kernel (Newey and West (1987)) with 15 lags, (ii) constructing a sandwich estimate of the covariance matrix, and (iii) applying a degrees of freedom correction to account for the relatively small sample of independent observations.

from zero. This parameter encodes our prior belief about the share of the SDF Sharpe ratio generated by the respective types of factors.

Consider $\kappa \in \{-0.5, 0.5\}$. Setting $\kappa = 0.5$ for bond factors implies the belief that, ceteris paribus, they explain a share of the squared Sharpe ratio of the SDF that is $\frac{1+\kappa}{1-\kappa} = 3$ times as large as the share of stock factors. This represents a substantial departure from the homogeneous prior setting (i.e., $\kappa = 0$). Nevertheless, since our prior is not dogmatic and does not impose a hard threshold, it can be falsified if the data do not conform with it.

Figure IA.32 of the Internet Appendix reports the posterior factor probabilities estimated with the tilted prior in favor of either bond or stock factors. Remarkably, the factors that we identify as more likely given the data in Section 3.1.1 still have posterior probabilities above the prior value in 9 out of 10 cases. That is, the likelihood of the data is quite informative for these more likely factors, and the prior perturbation has only a limited effect on the posterior probabilities. Similarly, the posterior market prices of risk depicted in Figure IA.33 of the Internet Appendix highlight that the set of factors that features more prominently in the co-pricing BMA-SDF is largely unchanged, albeit their individual posterior λs do vary in the expected directions.

For a *sparse* SDF, we would expect these perturbations of the posterior λs to have a substantial impact on its pricing ability. For a *dense* SDF that combines multiple noisy proxies for common underlying sources of risk, we should expect instead a much more muted effect (as also implied by our simulation results in Section 2.4.1). Table IA.XXII of the Internet Appendix summarizes the resulting in- and out-of-sample pricing performance of the tilted BMA-SDF for our baseline cross-section of test assets. Overall, the effect of the prior tilting is quite small but unambiguous in direction: as we tilt toward *either* type of factor, the out-of-sample pricing ability deteriorates. This strengthens the results in Section 3.3: for the co-pricing of bond and stock excess returns, we need information from both factor zoos. Consequently, over-reliance on either type of factor worsens the BMA-SDF performance.

Interestingly, as shown in Table IA.XXIII of the Internet Appendix, where we consider the separate pricing of bond and stock excess returns, the deterioration in pricing performance is stronger for equities when tilting the prior in favor of bond factors—again reinforcing the result in Section 3.3 of a much more limited information content in the bond factor zoo relative to the equity one.

Finally, we revisit our findings on co-pricing when bond returns are duration-adjusted. The results in Section 3.3 strongly suggest that, once the Treasury component of bond returns is accounted for, the bond factor zoo becomes largely redundant. If this were truly the case, we would expect that tilting the prior in favor of stock (bond) factors should actually improve (worsen) the pricing ability of the BMA-SDF. This is exactly what Figure IA.34 of the Internet Appendix highlights. Unambiguously, as we tilt the prior away from bond factors, the out-of-sample measures of cross-sectional fit improve. Furthermore, an extreme tilt in favor of stock factors (see Figure IA.35 of the Internet Appendix) maximizes the pricing ability of the BMA-SDF. This reinforces our previous finding: bond factors are largely redundant for co-pricing bond and stock portfolios once the Treasury component of the latter is accounted for.

4.2 Imposing sparsity

Recall that in our method, beliefs regarding SDF density are encoded through a Beta-distributed prior probability of factor inclusion: $\pi(\gamma_j = 1|\omega_j) = \omega_j \sim Beta(a_\omega, b_\omega)$. In our baseline estimations, we assign a Beta$(1,1)$ prior distribution to this quantity—equivalent to a uniform prior on [0, 1] and analogous to the flat prior implicit in canonical frequentist inference. This specification reflects our decision not to take an ex ante stance on whether the SDF should be sparse or dense.

However, the literature commonly assumes a high degree of sparsity, either ex ante or through specification selection, typically favoring factor models with approximately five factors. Our framework accommodates such beliefs in a flexible, non-dogmatic manner by choosing the prior mean and variance of ω_j, $\mathbb{E}[\omega_j] = \frac{a_\omega}{a_\omega + b_\omega}$ and $\text{Var}(\omega_j) = \frac{a_\omega b_\omega}{(a_\omega+b_\omega)^2(a_\omega+b_\omega+1)}$.

Using appropriate a_ω and b_ω, we can concentrate the prior on model dimensions typical in the literature. Specifically, we set $a_\omega \approx 3.54$ and $b_\omega \approx 34.66$ to achieve two objectives: (i) the prior expectation of included factors, $\mathbb{E}[\omega_j] \times K$, yields the canonical five-factor model, and (ii) the prior two standard deviation credible interval encompasses models with zero to ten factors (since $\text{Var}(\omega_j) = (2.5/K)^2$).

Results using this sparsity-favoring prior appear in Internet Appendix IA.9.2. Three key findings emerge. First, Table IA.XXIV of the Internet Appendix shows that the factors with posterior probabilities exceeding the prior value (that is, 9.26%) are essentially identical to those in our baseline estimates. The only exception occurs under the lowest prior shrinkage, where PEAD's posterior probability drops below this threshold—an expected outcome given this prior's reduced ability to control confounding effects from weak factors. Second, Table

IA.XXV of the Internet Appendix demonstrates that the BMA-SDF's pricing performance under the sparsity-favoring prior remains superior to alternative specifications in the literature, particularly out-of-sample. Third, despite this relative advantage, imposing sparsity degrades the performance of the BMA-SDF compared to our baseline findings in Tables 2 and 3. This deterioration is expected: as Figure 3 and Table 4 demonstrate, the data strongly support a dense SDF. Consequently, artificially imposing sparsity necessarily worsens the performance of BMA-SDF, as our results confirm. These findings highlight once more that the quest for a pricing kernel in the previous literature, focusing on low-dimensional observable factor models, relies on a stringent assumption not supported by the data.

4.3 Estimation *excluding* the most likely factors

The empirical findings in Section 3.1 strongly suggest that the joint zoo of bond and stock factors resembles a jungle of noisy proxies for common underlying sources of risk. As we show theoretically in Section 2.4 and in simulation in Section 2.4.1, if this characterization is accurate, our BMA-SDF method should provide a good approximation of the true latent SDF even when factors capturing fundamental risk sources are removed from the candidate set.

To assess whether this robustness property holds in the data, we remove the factors identified as most salient for characterizing the true latent SDF, construct a BMA-SDF using the remaining factors, and evaluate its pricing ability both in- and out-of-sample. We perform this exercise by removing three different factor sets: (i) the top five factors ranked by posterior probabilities; (ii) the top five factors ranked by posterior weights in the BMA-SDF (i.e., factors with the largest posterior market prices of risk); and (iii) the union of factors from sets (i) and (ii). This constitutes a stringent test of our method, as we remove the factors individually identified as the most informative about priced risk in the economy.

Internet Appendix IA.9.3 reports the empirical results. Remarkably, the BMA-SDF constructed with this limited information set still strongly outperforms canonical models from the literature both in- and out-of-sample (see Table IA.XXVI of the Internet Appendix). As shown in Figures IA.36 to IA.38 of the Internet Appendix, this performance is achieved by increasing the posterior weights, $\mathbb{E}[\lambda_j|\text{data}]$, of several noisy proxies in the BMA-SDF—precisely what our theoretical and simulation results in Section 2.4 predict.

However, we do observe some minor degree of deterioration in the performance of the BMA-SDF, particularly when minimal prior shrinkage is applied. This is again an expected outcome given this prior's reduced ability to control confounding effects from weak factors. However, even in the most extreme case, this reduction remains moderate, with out-of-sample R^2 measures dropping by only 8% in the worst-case scenario.

Overall, these results confirm both the soundness and robustness of our method in recovering pricing information from the factor zoo and our finding that most factors are noisy proxies of common underlying risk sources.

4.4 Estimation uncertainty

Finally, we show that our asset pricing results are robust across (i) different corporate bond data, (ii) varying bond and stock cross-sections and (iii) different factor zoos and sample periods. The detailed results are presented in Internet Appendix IA.10.

4.4.1 Varying corporate bond data

In Internet Appendix IA.1 we describe different sources for corporate bond data used for academic research and, in particular, we show the robustness of the bond factors with respect to the data source and calculation method. In this section we confirm that the asset pricing implications are also robust to the choice of corporate bond data. Detailed results are presented in Internet Appendix IA.10.1.

In particular, we compare the pricing performance of the co-pricing BMA-SDF across five different sets of corporate bond data: (i) our baseline LBFI/BAML ICE bond-level data, (ii) the LBFI/BAML ICE firm-level data, (iii) the LBFI/BAML ICE bond-level data but using only quotes (i.e., removing matrix prices), (iv) the transaction-based WRDS TRACE data, and (v) the transaction-based DFPS TRACE data. That is, with each of these datasets, we re-estimate the co-pricing BMA-SDF using the 83 test assets and 54 tradable and nontradable factors. Across estimations, only the 50 IS bond test assets and tradable bond factors change.

First, the ex post Sharpe ratios across all shrinkage levels are very closely aligned, as shown in Table IA.XXVII of the Internet Appendix. Second, the posterior probabilities across the data sets are very consistent. On average, eight out of the ten most likely factors (including the top five) match the baseline results from Section 3.1.1 (see Figure IA.39 of the Internet Appendix). Finally, the in- and out-of-sample asset pricing performance of the BMA-SDF is fairly consistent across corporate bond data sets and, most importantly, the BMA-SDF still emerges as the dominant model across all estimations, with a tight spread between min and max values (see Figures IA.40 (IS) and IA.41 (OS) of the Internet Appendix).

4.4.2 Varying cross-sections

Our baseline estimate of the BMA-SDF is specific to the test assets that we describe in Section 1. We now vary the cross-section of test assets and re-estimate the co-pricing BMA-SDF for *hundreds* of alternative sets of test assets across bonds and stocks.

Specifically, we include the 153 long-short equity anomalies provided by Jensen et al. (2023) and the corporate bond counterparts from Dick-Nielsen et al. (2025) for a joint corporate bond and stock cross-section of 306 anomalies. From this very large cross-section, we then randomly sample anomaly pairs to generate 100 in-sample co-pricing cross-sections. Each sampled cross-section consists of 25 bond and 25 stock portfolios from the same underlying anomaly characteristic. Together with the 40 tradable bond and stock factors, we use 90 IS test assets for the estimation. In Figure IA.42 of the Internet Appendix we present the average posterior probabilities (Panel A) and the market prices of risk (Panel B), along with their respective minimum and maximum values across the 100 estimations, with the Sharpe ratio shrinkage set to 80% of the ex post maximum. IVOL, PEADB, and PEAD still emerge as the most probable factors for inclusion in the SDF—consistent with the results documented in Figure 2 and Table A.2 of Appendix C.

In Figures IA.43 (IS) and IA.44 (OS) of the Internet Appendix we present the averages, minima and maxima of the R^2_{GLS} (Panel A) and R^2_{OLS} (Panel B) values across the 100 sets of test assets for the BMA-SDF across our four Sharpe ratio shrinkage levels and the additional models we consider in Tables 2 and 3. The BMA-SDF with 60% and 80% Sharpe ratio shrinkage, as well as the 'TOP' model, outperform all other models, confirming the results presented in Section 3.1.2. Finally, we also obtain similar results when we switch the IS and OS test assets (i.e., instead of evaluating the pricing performance on the OS test assets, we use them to estimate the BMA-SDF); these results are presented in Figure IA.45 and Table IA.XXVIII of the Internet Appendix. For all sets of IS test assets, our results remain materially the same. That is, we identify a similar set of factors that should be included in the co-pricing SDF, estimate consistent market prices of risk, and obtain similar in- and out-of-sample asset pricing performance for the BMA-SDF.

To further assess the OS performance of our approach, we evaluate its pricing ability across *millions* of potential OS cross-sections of bond and stock portfolios. Again, we use the Jensen et al. (2023) and the Dick-Nielsen et al. (2025) anomaly data. We form OS cross-sections with 50 and 100 portfolios (i.e., 25 and 50 anomaly pairs, for stocks and bonds), respectively. From the 306 anomalies, we sample the respective cross-section one million times and evaluate the OS pricing performance using the BMA-SDFs estimated with the baseline set of test assets in Panel A of Table 2. The results are presented in Table IA.XXIX of the Internet Appendix with the BMA-SDF and the TOP factors model ounce again outperforming their competitors.

4.4.3 Varying factor zoos and sample periods

Finally, we check the robustness of our results regarding the expansion of the factor zoo and the alteration of the sample periods. First, in order to expand the set of stock and nontradable factors included in our analysis, we consider a shorter sample (ending in December 2016) to include all 51 stock factors considered in Bryzgalova et al. (2023) as well as their stock portfolio of IS test assets to re-estimate the co-pricing BMA-SDF. Second, we extend the corporate bond factor zoo from 16 to 29 factors by adding the 13 Dick-Nielsen et al. (2025) composite bond return factors formed with equity characteristics. Third, we restrict the sample period to the TRACE era (from 2002 onward only) and include the tradable liquidity factor (LRF) from Bai et al. (2019) and the two nontradable illiquidity factors from Lin et al. (2011). Fourth, we estimate the models for the pre-TRACE period (1986 to 2002) and repeat the analysis using the split used by van Binsbergen et al. (2025) who consider pre- and post-2000 data. Finally, we estimate the models on an extended time series starting in 1977, resulting in a total of 549 observations in the time series.

The posterior probabilities and market prices of risk for these estimations are reported in Figures IA.46 to IA.48 of the Internet Appendix, with associated asset pricing results documented in Table IA.XXX (IS) and IA.XXXI (OS) of the Internet Appendix. The IS and OS asset pricing results for the pre- and post-TRACE and pre- and post-2000 sample splits are reported in Tables IA.XXXII and IA.XXXIII of the Internet Appendix, respectively.

Overall, the results are remarkably robust, and our BMA-SDF generally outperforms competing models, independently of how we cut the data.

As we show, both theoretically and in simulation in Section 2.4, the stability of the findings is to be expected given the robust inference method we use: if individual factors are combinations of signal (about the fundamental sources of risk) plus "noise" (their unpriced component, see, e.g., Daniel et al. (2020)), the BMA-SDF provides an optimal aggregation scheme that maximizes the signal-to-noise ratio of the resulting SDF. Hence, albeit perturbations of the data might alter the signal-to-noise ratio of individual factors, this effect is largely mitigated in the BMA-SDF that our method delivers, rendering such issues, as the data confirms, of second order concern for our analysis.

5 Conclusion

We generalize the Bayesian estimation method of Bryzgalova et al. (2023) to handle multiple asset classes, developing a novel understanding of factor posterior probabilities and model averaging in asset pricing, and we apply it to the study of over 18 quadrillion linear factor models for the joint pricing of corporate bond and stock returns.

Strikingly, decomposing bond excess returns into their credit and Treasury components reveals that nontradable and tradable *stock* factors are largely *sufficient* for pricing the credit component, making the bond factor literature effectively redundant for this purpose. Conversely, tradable *bond* factors (along with nontradable ones) remain necessary for pricing the Treasury component—a risk that stock factors do not seem to capture.

Overall, we find that the true latent SDF is *dense* in the space of observable nontradable and tradable bond and stock factors. Importantly, this implies that *all* low dimensional observable factor models proposed to date are affected by severe misspecification and rejected by the data.

Individually, only very few factors should be included in the SDF with high probability. Most notably, two tradable behavioral factors capturing the post-earnings announcement drift in bonds and stocks exhibit posterior probabilities above their prior value, along with nontradable factors such as the slope of the Treasury yield curve, the AAA/BAA yield spread, and the idiosyncratic equity volatility. However, these factors capture only a fraction of the risks priced in the joint cross-section of bonds and stocks, and literally dozens of other factors, both tradable and nontradable, are necessary—jointly—to span the risks driving asset prices. Nevertheless, the SDF-implied maximum Sharpe ratio is not extreme because the many factors necessary for an accurate characterization of the latent SDF are multiple noisy proxies for common underlying sources of risk.

A Bayesian Model Averaging over the space of all possible Stochastic Discount Factor models aggregates this diffuse pricing information optimally and outperforms all existing models in explaining—jointly and individually—the cross-section of corporate bond and stock returns, both in- and out-of-sample. Furthermore, leveraging the fact that the Bayesian averaging over the space of models is equivalent to an averaging over the space of factors, we show that the BMA-SDF yields a *tradable* strategy with a time-series *out-of-sample* Sharpe ratio of 1.5 to

1.8, with only yearly rebalancing, in the challenging evaluation period spanning July 2004 to December 2022.

The BMA-SDF exhibits a distinctive business cycle behavior, and persistent and cyclical first and second moments. Furthermore, its volatility increases sharply during recessions and at times of heightened economic uncertainty, suggesting time variation in conditional risk premia. And indeed, we find that lagged BMA-SDF information is a strong and significant predictor of future asset returns.

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Appendix

A The factor zoo list

We list all 54 bond, stock and nontradable factors we consider in Table A.1 along with a detailed description of their construction, associated reference, and data source.

Table A.1: List of factors for cross-sectional asset pricing. This table lists all tradable bond, stock as well as the nontradable factors used in the main paper. For each of the factors, we present their identification index (Factor ID), a description of the factor construction, and the source of the data for downloading and/or constructing the factor time series.

Factor ID	Factor name and description	Reference	Source
	Panel A: Tradable corporate bond factors		
CRF	Credit risk factor. Equally-weighted average return on two 'credit portfolios': CRF_{VaR}, and CRF_{REV}. CRF_{VaR} is the average return difference between the lowest-rating (i.e., highest credit risk) portfolio and the highest-rating (i.e., lowest credit risk) portfolio across the VaR95 portfolios. CRF_{REV} is the average return difference between the lowest-rating portfolio and the highest-rating portfolio across quintiles sorted on bond short-term reversal.	Bai et al. (2019)	Open Source Bond Asset Pricing
CRY	Bond carry factor. Independent sort (5×5) to form 25 portfolios according to ratings and bond credit spreads (CS). For each rating quintile, calculate the weighted average return difference between the highest CS quintile and the lowest CS quintile. CRY is computed as the average long-short portfolio return across all rating quintiles.	Hottinga et al. (2001), Houweling and Van Zundert (2017)	Open Source Bond Asset Pricing
DEF	Bond default risk factor. The difference between the return on the market portfolio of long-term corporate bond returns (the Composite portfolio on the corporate bond module of Ibbotson Associates) and the long-term government bond return.	Fama and French (1992) and Gebhardt et al. (2005a).	Amit Goyal's website
DRF	Downside risk factor. Independent sort (5×5) to form 25 portfolios according to ratings and 95% value-at-risk (VaR95). For each rating quintile, calculate the weighted average return difference between the highest VaR5 quintile and the lowest VaR5 quintile. DRF is computed as the average long-short portfolio return across all rating quintiles.	Bai et al. (2019)	Open Source Bond Asset Pricing
DUR	Bond duration factor. Independent sort (5×5) to form 25 portfolios according to ratings and bond duration (DUR^B). For each rating quintile, calculate the weighted average return difference between the highest DUR^B quintile and the lowest DUR^B quintile. DUR is computed as the average long-short portfolio return across all rating quintiles.	Gebhardt et al. (2005a) and Dang et al. (2023).	Open Source Bond Asset Pricing
HMLB	Bond book-to-market factor. Independent sort (2×3) to form 6 portfolios according to bond size and bond book-to-market (BBM), defined as bond principal value scaled by market value. For each size portfolio, calculate the weighted average return difference between the lowest BBM tercile and the highest BBM tercile. HMLB is computed as the average long-short portfolio return across the two size portfolios.	Bartram et al. (2025)	Open Source Bond Asset Pricing

LTREVB	Bond long-term reversal factor. Dependent sort ($3 \times 3 \times 3$) to form 27 portfolios according to ratings, maturity, and the 48-13 cumulative previous bond return (LTREV^B). For each rating quintile, the factor is computed as the average return differential between the portfolio with the lowest LTREV^B and the one with the highest LTREV^B within the rating and maturity portfolios. LTREVB is computed as the average long-short portfolio return across the nine rating-maturity terciles.	Bali et al. (2021a)	Open Source Bond Asset Pricing
MKTB	Corporate Bond Market excess return. Constructed using bond returns in excess of the one-month risk-free rate of return.	Dickerson et al. (2023)	Open Source Bond Asset Pricing
MKTBD	Corporate Bond Market duration-adjusted return. Constructed using bond returns in excess of their duration-matched U.S. Treasury bond rate of return.	van Binsbergen et al. (2025)	Open Source Bond Asset Pricing
MOMB	Bond momentum factor formed with bond momentum. Independent sort (5×5) to form 25 portfolios according to ratings and the 12-2 cumulative previous bond return (MOM). For each rating quintile, calculate the weighted average return difference between the highest MOM quintile and the lowest MOM quintile. MOMB is computed as the average long-short portfolio return across all rating quintiles.	Gebhardt et al. (2005b)	Open Source Bond Asset Pricing
MOMBS	Bond momentum factor formed with equity momentum. Independent sort (5×5) to form 25 portfolios according to ratings and the 6-1 cumulative previous equity return (MOMs). For each rating quintile, calculate the weighted average return difference between the highest MOMs quintile and the lowest MOMs quintile. MOMBS is computed as the average long-short portfolio return across all rating quintiles.	Hottinga et al. (2001), Gebhardt et al. (2005b) and Dang et al. (2023)	Open Source Bond Asset Pricing
PEADB	Bond earnings announcement drift factor. Independent sort (2×3) to form 6 portfolios according to market equity and earnings surprises (CAR), computed according to Chan et al. (1996). For each firm size portfolio, calculate the weighted average return difference between the highest CAR terciles and the lowest CAR tercile. PEADB is computed as the average long-short portfolio return across the two firm size portfolios.	Nozawa et al. (2025)	Open Source Bond Asset Pricing
STREVB	Bond short-term reversal factor. Independent sort (5×5) to form 25 portfolios according to ratings and the prior month's bond return (REV). For each rating quintile, calculate the weighted average return difference between the lowest REV quintile and the highest REV quintile. STREVB is computed as the average long-short portfolio return across all rating quintiles.	Khang and King (2004) and Bali et al. (2021a)	Open Source Bond Asset Pricing
SZE	Bond size factor. Dependent sort (3×3) to form 3 portfolios according to ratings and then with each rating tercile another 3 portfolios on bond size (SIZE). Bond size is defined as bond price multiplied by issue size (amount outstanding). For each rating tercile, calculate the weighted average return difference between the lowest SIZE tercile and the highest SIZE tercile. SZE is computed as the average long-short portfolio return across all rating terciles.	Hottinga et al. (2001) and Houweling and Van Zundert (2017)	Open Source Bond Asset Pricing
TERM	Bond term structure risk factor. The difference between the monthly long-term government bond return and the one-month T-Bill rate of return.	Fama and French (1992) and Gebhardt et al. (2005a).	Amit Goyal's website
VAL	Bond value factor. Independent sort (2×3) to form 6 portfolios according to bond size and bond value (VAL^B). VAL^B is computed via cross-sectional regressions of credit spreads on ratings, maturity, and the 3-month change in credit spread. The percentage difference between the actual credit spread and the fitted ('fair') credit spread for each bond is the VAL^B characteristic. For each size portfolio, calculate the weighted average return difference between the highest VAL^B tercile and the lowest VAL^B tercile. VAL is computed as the average long-short portfolio return across the two size portfolios.	Correia et al. (2012) and Houweling and Van Zundert (2017)	Open Source Bond Asset Pricing

Panel B: Tradable stock factors			
BAB	Betting-against-beta factor, constructed as a portfolio that holds low-beta assets, leveraged to a beta of 1, and that shorts high-beta assets, de-leveraged to a beta of 1.	Frazzini and Pedersen (2014)	AQR data library
CMA	Investment factor, constructed as a long-short portfolio of stocks sorted by their investment activity.	Fama and French (2015)	Ken French website
CMAs	CMA with a hedged unpriced component.	Daniel et al. (2020)	Kent Daniel website
CPTLT	The value-weighted equity return for the New York Fed's primary dealer sector not including new equity issuance.	He et al. (2017)	Zhiguo He website
FIN	Long-term behavioral factor, predominantly capturing the impact of share issuance and correction.	Daniel et al. (2020)	Kent Daniel website
HML	Value factor, constructed as a long-short portfolio of stocks sorted by their book-to-market ratio.	Fama and French (1992)	Ken French website
HML_DEV	A version of the HML factor that relies on the current price level to sort the stocks into long and short legs.	Asness and Frazzini (2013)	AQR data library
HMLs	HML with a hedged unpriced component.	Daniel et al. (2020)	Kent Daniel website
LIQ	Liquidity factor, constructed as a long-short portfolio of stocks sorted by their exposure to LIQ_NT.	Pástor and Stambaugh (2003)	Robert Stambaugh website
LTREV	Long-term reversal factor, constructed as a long-short portfolio of stocks sorted by their cumulative return accrued in the previous 60-13 months.	Jegadeesh and Titman (2001)	Ken French website
MGMT	Management performance mispricing factor.	Stambaugh and Yuan (2017)	Global factor data website
MKTS	Market excess return.	Sharpe (1964) and Lintner (1965)	Ken French website
MKTSs	Market factor with a hedged unpriced component.	Daniel et al. (2020)	Kent Daniel website
MOMS	Momentum factor, constructed as a long-short portfolio of stocks sorted by their 12-2 months cumulative previous return.	Carhart (1997), Jegadeesh and Titman (1993)	Ken French website
PEAD	Short-term behavioral factor, reflecting post-earnings announcement drift.	Daniel et al. (2020)	Kent Daniel website
PERF	Firm performance mispricing factor.	Stambaugh and Yuan (2017)	Global factor data website
QMJ	Quality-minus-junk factor, constructed as a long-short portfolio of stocks sorted by the combination of their safety, profitability, growth, and the quality of management practices.	Asness et al. (2019)	AQR data library
RMW	Profitability factor, constructed as a long-short portfolio of stocks sorted by their profitability.	Fama and French (2015)	Ken French website
RMWs	RMW with a hedged unpriced component.	Daniel et al. (2020)	Kent Daniel website
R_IA	Investment factor, constructed as a long-short portfolio of stocks sorted by their investment-to-capital.	Hou et al. (2015)	Lu Zhang website
R_ROE	Profitability factor, constructed as a long-short portfolio of stocks sorted by their return on equity.	Hou et al. (2015)	Lu Zhang website
SMB	Size factor, constructed as a long-short portfolio of stocks sorted by their market cap.	Fama and French (1992)	Ken French website
SMBs	SMB with a hedged unpriced component.	Daniel et al. (2020)	Kent Daniel website
STREV	Short-term reversal factor, constructed as a long-short portfolio of stocks sorted by their previous month return.	Jegadeesh and Titman (1993)	Ken French website
Panel C: Nontradable corporate bond and stock factors			
CPTL	Intermediary capital nontradable risk factor. Constructed using AR(1) innovations to the market-based capital ratio of primary dealers, scaled by the lagged capital ratio.	He et al. (2017)	Zhiguo He's website
CREDIT	Bond credit risk factor. Difference between the yields of BAA and AAA indices from Moody's. Also computed with our own data as the difference between the average yield of BAA and (AAA+AA) rated bonds. See Section IA.11 of the Internet Appendix for further computational details.	Fama and French (1993)	Amit Goyal's website or FRED for AAA and BAA indices.
EPU	Economic Policy Uncertainty. First difference in the economic policy uncertainty index.	Baker et al. (2016) and Dang et al. (2023)	FRED
EPUT	Economic Tax Policy Uncertainty. First difference in the economic tax policy uncertainty index.	Baker et al. (2016) and Dang et al. (2023)	FRED

INFLC	Shocks to core inflation. Unexpected core inflation component captured by an ARMA(1,1) model. Monthly core inflation is calculated as the percentage change in the seasonally adjusted Consumer Price Index for All Urban Consumers: All Items Less Food and Energy which is lagged by one-month to account for the inflation data release lag.	Fang et al. (2024)	FRED
INFLV	Inflation volatility. Computed as the 6-month volatility of the unexpected inflation component captured by an ARMA(1,1) model. Monthly inflation is calculated as the percentage change in the seasonally adjusted Consumer Price Index for All Urban Consumers (CPI) which is lagged by one-month to account for the inflation data release lag.	Kang and Pflueger (2015) and Ceballos (2023)	FRED
IVOL	Idiosyncratic equity volatility factor. Cross-sectional volatility of all firms in the CRSP database in each month t.	Campbell and Taksler (2003)	CRSP
LVL	Level term structure factor. Constructed as the first principal component of the one- through 30-year CRSP Fixed Term Indices U.S. Treasury Bond yields.	Koijen et al. (2017)	CRSP Indices
LIQNT	Liquidity factor, computed as the average of individual-stock measures estimated with daily data (residual predictability, controlling for the market factor)	Pástor and Stambaugh (2003)	Robert Stambaugh's website
UNC	First difference in the Macroeconomic uncertainty index.	Ludvigson et al. (2015) and Bali et al. (2021b)	Sydney Ludvigson's website
UNCf	First difference in the Financial economic uncertainty index.	Ludvigson et al. (2015)	Sydney Ludvigson's website
UNCr	First difference in the Real economic uncertainty index.	Ludvigson et al. (2015)	Sydney Ludvigson's website
VIX	First difference in the CBOE VIX.	Chung et al. (2019)	FRED
YSP	Slope term structure factor. Constructed as the difference in the five and one-year U.S. Treasury Bond yields.	Koijen et al. (2017)	CRSP Indices

B Posterior sampling

The posterior of the time series parameters follows the canonical Normal-inverse-Wishart distribution (see, e.g., Bauwens, Lubrano, and Richard (1999)) given by

$$\boldsymbol{\mu_Y} | \boldsymbol{\Sigma_Y}, \mathbf{Y} \sim \mathcal{N}\left(\hat{\boldsymbol{\mu}}_{\boldsymbol{Y}}, \ \boldsymbol{\Sigma_Y}/T\right), \tag{A.11}$$

$$\boldsymbol{\Sigma_Y} | \mathbf{Y} \sim \mathcal{W}^{-1}\left(T-1, \ \sum_{t=1}^{T}\left(\mathbf{Y_t} - \widehat{\boldsymbol{\mu}}_{\boldsymbol{Y}}\right)\left(\mathbf{Y_t} - \widehat{\boldsymbol{\mu}}_{\boldsymbol{Y}}\right)^{\top}\right), \tag{A.12}$$

where $\hat{\boldsymbol{\mu}}_{\boldsymbol{Y}} \equiv \frac{1}{T}\sum_{t=1}^{T}\mathbf{Y_t}$, $\mathcal{W}^{-1}$ is the inverse-Wishart distribution, $\boldsymbol{Y} \equiv \{\mathbf{Y}_t\}_{t=1}^{T}$, and note that the covariance matrix of factors and test assets, $\boldsymbol{C_f}$, is contained within $\boldsymbol{\Sigma_Y}$.

Define $\boldsymbol{D} = \tilde{\boldsymbol{D}} \times \boldsymbol{\kappa}$ where $\tilde{\boldsymbol{D}}$ is a diagonal matrix with elements c, $(r(\gamma_1)\psi_1)^{-1}$, ..., $(r(\gamma_K)\psi_K)^{-1}$ and $\boldsymbol{\kappa}$ is a conformable column vector with elements $1, 1+\kappa_1, \ldots, 1+\kappa_K$ such that $\sum_{k=1}^{K}\kappa_j = 0$ and $0 < |\kappa_j| < 1 \ \forall j$.It then follows that, given our prior formulations, the posterior distributions of the parameters in the cross-sectional layer $(\boldsymbol{\lambda}, \boldsymbol{\gamma}, \boldsymbol{\omega}, \sigma^2)$, conditional on the draws of $\boldsymbol{\mu_R}$, $\boldsymbol{\Sigma_R}$, and $\boldsymbol{C}$ from the time series layer, are:

$$\boldsymbol{\lambda}|\text{data}, \sigma^2, \boldsymbol{\gamma}, \boldsymbol{\omega} \sim \mathcal{N}\left(\hat{\boldsymbol{\lambda}}, \hat{\sigma}^2(\hat{\boldsymbol{\lambda}})\right), \tag{A.13}$$

$$\frac{p(\gamma_j = 1|\text{data}, \boldsymbol{\lambda}, \boldsymbol{\omega}, \sigma^2, \boldsymbol{\gamma_{-j}})}{p(\gamma_j = 0|\text{data}, \boldsymbol{\lambda}, \boldsymbol{\omega}, \sigma^2, \boldsymbol{\gamma_{-j}})} = \frac{\omega_j}{1-\omega_j}\frac{p(\lambda_j|\gamma_j = 1, \sigma^2)}{p(\lambda_j|\gamma_j = 0, \sigma^2)}, \tag{A.14}$$

$$\omega_j|\text{data}, \boldsymbol{\lambda}, \boldsymbol{\gamma}, \sigma^2 \sim Beta\left(\gamma_j + a_\omega, 1 - \gamma_j + b_\omega\right), \tag{A.15}$$

$$\sigma^2|\text{data}, \boldsymbol{\omega}, \boldsymbol{\lambda}, \boldsymbol{\gamma} \sim \mathcal{IG}\left(\frac{N+K+1}{2}, \frac{(\boldsymbol{\mu_R} - \boldsymbol{C\lambda})^\top \boldsymbol{\Sigma_R}^{-1}(\boldsymbol{\mu_R} - \boldsymbol{C\lambda}) + \boldsymbol{\lambda}^\top \boldsymbol{D\lambda}}{2}\right), \tag{A.16}$$

where $\hat{\boldsymbol{\lambda}} = (\boldsymbol{C}^\top \boldsymbol{\Sigma_R}^{-1} \boldsymbol{C} + \boldsymbol{D})^{-1} \boldsymbol{C}^\top \boldsymbol{\Sigma_R}^{-1} \boldsymbol{\mu_R}$, $\hat{\sigma}^2(\hat{\boldsymbol{\lambda}}) = \sigma^2(\boldsymbol{C}^\top \boldsymbol{\Sigma_R}^{-1} \boldsymbol{C} + \boldsymbol{D})^{-1}$ and $\mathcal{IG}$ denotes the inverse-Gamma distribution.

Hence, posterior sampling is achieved with a Gibbs sampler that draws sequentially the time series layer parameters ($\boldsymbol{\mu_R}$, $\boldsymbol{\Sigma_R}$, and $\boldsymbol{C}$) from equations (A.11) and (A.12), and then, conditional on these realizations, draws sequentially from equations (A.13) to (A.16).

C Probabilities and risk prices across prior Sharpe ratios

We report the full list of posterior probabilities and the associated annualized risk premia (in Sharpe ratio units) which complements the results from Figure 2 in Table A.2.

D Benchmark asset pricing models

We benchmark the performance of the BMA-SDF against several frequentist asset pricing models as well as other latent factor models. In the following, we provide the estimation details for the models that are compared to the BMA-SDF in Section 3.1. A larger set of comparison benchmark models is considered in Internet Appendix IA.3.2.

CAPM and CAPMB. The single-factor equity CAPM and the bond equivalent CAPMB. The CAPM is the value-weighted equity market factor from Kenneth French's webpage. The bond CAPM (CAPMB) is the value-weighted corporate bond market factor. We estimate factor risk prices using a GLS version of GMM (see, e.g., Cochrane (2005, pp. 256–258)).

FF5. The original five-factor model of Fama and French (1993) that includes the MKTS, SMB and HML factors from Fama and French (1992) and the default (DEF) and term structure

Table A.2: Posterior factor probabilities and risk prices for the co-pricing factor zoo

	Factor prob., $\mathbb{E}[\gamma_j\|\text{data}]$				Price of risk, $\mathbb{E}[\lambda_j\|\text{data}]$			
	Total prior Sharpe ratio				Total prior Sharpe ratio			
Factors	20%	40%	60%	80%	20%	40%	60%	80%
PEADB	0.555	0.629	0.713	0.711	0.054	0.213	0.446	0.645
PEAD	0.523	0.559	0.618	0.614	0.035	0.138	0.297	0.449
IVOL	0.502	0.529	0.567	0.623	0.010	0.043	0.108	0.265
CREDIT	0.498	0.497	0.530	0.557	0.008	0.033	0.084	0.191
YSP	0.507	0.502	0.504	0.519	0.003	0.014	0.034	0.088
MOMBS	0.492	0.518	0.543	0.476	0.059	0.200	0.366	0.432
INFLV	0.509	0.514	0.511	0.484	0.002	0.007	0.014	0.022
INFLC	0.500	0.501	0.494	0.492	−0.001	−0.004	−0.011	−0.028
CMAs	0.489	0.500	0.502	0.480	0.015	0.061	0.131	0.215
LVL	0.495	0.493	0.491	0.493	0.000	0.002	0.006	0.019
EPU	0.509	0.503	0.498	0.457	0.001	0.004	0.008	0.009
UNCr	0.494	0.490	0.499	0.480	0.001	0.004	0.012	0.032
MKTS	0.496	0.510	0.494	0.458	0.055	0.173	0.289	0.391
EPUT	0.500	0.492	0.497	0.462	0.003	0.009	0.016	0.019
LIQNT	0.501	0.482	0.492	0.475	−0.003	−0.013	−0.039	−0.095
CRY	0.483	0.463	0.501	0.479	0.049	0.151	0.334	0.500
QMJ	0.499	0.501	0.487	0.438	0.072	0.193	0.321	0.412
RMWs	0.500	0.501	0.481	0.438	0.025	0.077	0.141	0.205
UNCf	0.499	0.492	0.479	0.446	−0.002	−0.001	0.018	0.065
UNC	0.487	0.484	0.480	0.445	−0.001	-0.000	0.005	0.014
VIX	0.482	0.485	0.468	0.452	0.000	0.002	0.005	0.010
SZE	0.502	0.465	0.464	0.421	0.006	0.026	0.061	0.104
CPTL	0.487	0.480	0.457	0.411	0.016	0.046	0.067	0.074
MKTB	0.521	0.482	0.439	0.376	0.091	0.188	0.248	0.278
MKTSs	0.494	0.478	0.447	0.397	0.015	0.038	0.064	0.103
LTREVB	0.500	0.482	0.437	0.387	0.016	0.051	0.079	0.094
SMBs	0.491	0.476	0.450	0.384	0.004	0.016	0.029	0.034
CPTLT	0.478	0.459	0.456	0.406	0.023	0.068	0.130	0.186
LIQ	0.475	0.476	0.443	0.390	0.005	0.025	0.053	0.082
BAB	0.485	0.492	0.435	0.372	0.021	0.054	0.076	0.097
VAL	0.501	0.469	0.426	0.378	0.016	0.056	0.099	0.126
STREV	0.487	0.476	0.445	0.365	0.009	0.034	0.071	0.101
LTREV	0.498	0.473	0.432	0.357	0.009	0.031	0.052	0.057
PERF	0.503	0.469	0.433	0.343	0.048	0.104	0.120	0.093
R_ROE	0.490	0.465	0.416	0.357	0.049	0.103	0.135	0.159
MGMT	0.490	0.475	0.420	0.338	0.058	0.125	0.162	0.173
CRF	0.494	0.454	0.421	0.349	0.015	0.052	0.093	0.123
HMLs	0.478	0.461	0.411	0.357	0.004	0.011	0.021	0.026
CMA	0.469	0.464	0.421	0.351	0.028	0.063	0.077	0.063
HML_DEV	0.492	0.446	0.414	0.353	0.001	0.002	0.014	0.041
HMLB	0.475	0.464	0.438	0.326	0.038	0.104	0.148	0.120
MOMB	0.472	0.459	0.424	0.346	−0.002	−0.007	−0.005	−0.003
MOMS	0.464	0.445	0.422	0.365	0.020	0.057	0.095	0.139
STREVB	0.478	0.449	0.414	0.349	0.003	0.007	0.011	0.007
MKTBD	0.487	0.442	0.403	0.351	0.014	0.029	0.029	0.015
R_IA	0.473	0.437	0.418	0.349	0.034	0.079	0.120	0.140
TERM	0.474	0.443	0.397	0.354	0.027	0.058	0.085	0.116
SMB	0.476	0.434	0.410	0.331	0.010	0.044	0.079	0.086
HML	0.477	0.435	0.405	0.327	0.003	−0.016	−0.037	−0.040
DUR	0.475	0.422	0.393	0.352	0.010	−0.021	−0.081	−0.146
DRF	0.471	0.435	0.401	0.330	0.039	0.068	0.069	0.034
DEF	0.467	0.421	0.395	0.333	0.000	−0.007	−0.021	−0.030
FIN	0.476	0.424	0.392	0.311	0.034	0.035	0.015	−0.004
RMW	0.473	0.428	0.381	0.315	0.027	0.019	−0.018	−0.055

The table reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and stock factors described in Appendix A. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The factors are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

(TERM) factors introduced in Fama and French (1993). We estimate factor risk prices using a GLS version of GMM (see, e.g., Cochrane (2005, pp. 256–258)).

HKM. The intermediary capital two-factor asset pricing model of He, Kelly, and Manela (2017). Includes the MKTS factor from Fama and French (1992) and the value-weighted (tradable version) of the intermediary capital factor, CPTLT in excess of the one-month risk-free rate. We estimate factor risk prices using a GLS version of GMM (see, e.g., Cochrane (2005, pp. 256–258)).

KNS. The latent factor model approach of Kozak et al. (2020). For each in-sample bond, stock or co-pricing cross-section, we select the optimal shrinkage level and number of factors chosen by twofold cross-validation. Given our data has a time series length of $T = 444$, the first sample is simply January 1986 to June 2004 and the second sample is July 2004 to December 2022.

RPPCA. The risk premia PCA methodology of Lettau and Pelger (2020). We use five principal components. In our main estimation used for the baseline results, we set γ from their equation (4) equal to 20. Changing this parameter to 10, or a lower value, does not quantitatively affect pricing performance.

Internet Appendix for:

The Co-Pricing Factor Zoo

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Abstract

This Internet Appendix provides additional information, tables, figures, and empirical results supporting the main text.

Contents of the Internet Appendix

IA.1 Details on data sources, factors and test assets

In this section we first describe in detail the various sources for corporate bond data and test assets before we briefly discuss the coverage of our bond and stock data sample. Next, we assess the robustness of the corporate bond factors for different construction methods and data sources. Finally, we provide a list of the bond and stock test assets in Table IA.II.

IA.1.1 Corporate bond databases

First, we describe the sources of corporate bond data. All data filters below are applied verbatim across all of the bond databases we consider. Across all databases, we filter out bonds with maturity less than one year. Furthermore, for consistency, across all databases, we define bond ratings as those provided by Standard & Poors (S&P). We include the full spectrum of ratings (AAA to D) but exclude unrated bonds. Irrespective of the data source, we *do not* winsorize or trim bond returns in any way.

IA.1.1.1 Mergent Fixed Income Securities Database

The Mergent Fixed Income Securities Database (FISD) contains bond issue and issuer characteristic data. We apply the standard filters used in the extant literature to the FISD data:

1. Only keep bonds that are issued by firms domiciled in the United States of America, `COUNTRY_DOMICILE == 'USA'`.
2. Remove bonds that are private placements, `PRIVATE_PLACEMENT == 'N'`.
3. Only keep bonds that are traded in U.S. Dollars, `FOREIGN_CURRENCY == 'N'`.
4. Bonds that trade under the 144A Rule are discarded, `RULE_144A == 'N'`.
5. Remove all asset-backed bonds, `ASSET_BACKED == 'N'`.
6. Remove convertible bonds, `CONVERTIBLE == 'N'`.
7. Only keep bonds with a fixed or zero coupon payment structure, i.e., remove bonds with a floating (variable) coupon, `COUPON_TYPE != 'V'`.
8. Remove bonds that are equity linked, agency-backed, U.S. Government, and mortgage-backed, based on their `BOND_TYPE`.
9. Remove bonds that have a "non-standard" interest payment structure or bonds not caught by the variable coupon filter (`COUPON_TYPE`). We remove bonds that have an `INTEREST_FREQUENCY` equal to -1 (N/A), 13 (Variable Coupon), 14 (Bi-Monthly), and 15 and 16 (undocumented by FISD). Additional information on `INTEREST_FREQUENCY` is available on page 60 to 67 of the FISD Data Dictionary 2012 document.

IA.1.1.2 Bank of America Merrill Lynch Database

The Bank of America Merrill Lynch (BAML) data is made available by the Intercontinental Exchange (ICE) and provides daily bond price quotes, accrued interest, and a host of pre-computed corporate bond characteristics such as the bond option-adjusted credit spread (OAS), the asset swap spread, duration, convexity, and bond returns in excess of a portfolio of duration-matched Treasuries. The ICE sample spans the time period January 1997 to December 2022 and includes constituent bonds from the ICE Bank of America High Yield (H0A0) and Investment Grade (C0A0) Corporate Bond Indices.

BAML ICE bond filters. We follow van Binsbergen et al. (2025) and take the last quote of each month to form the bond-month panel. We then merge the ICE data to the filtered Mergent FISD data. The following ICE-specific filters are then applied:

1. Only include corporate bonds, `Ind_Lvl_1 == 'corporate'`
2. Only include bonds issued by U.S. firms, `Country == 'US'`
3. Only include corporate bonds denominated in U.S. dollars, `Currency == 'USD'`

BAML ICE bond returns. Total bond returns are computed in a standard manner in ICE, and no assumptions about the timing of the last trading day of the month are made because the data is quote based, i.e., there is always a valid quote at month-end to compute a bond return. This means that each bond return is computed using a price quote at exactly the end of the month, each and every month. This introduces homogeneity into the bond returns because prices are sampled at exactly the same time each month. ICE only provides bid-side pricing, meaning bid-ask bias is inherently not present in the monthly sampled prices, returns and credit spreads. The monthly ICE return variable is (as denoted in the original database) `trr_mtd_loc`, which is the month-to-date return on the last business day of month t.

IA.1.1.3 Lehman Brothers Fixed Income Database

The Lehman Brothers Fixed Income (LBFI) database holds monthly price data for corporate (and other) bonds from January 1973 to December 1997. The database categorizes the prices as either quote or matrix prices and identifies whether the bonds are callable or not. However, as per Chordia et al. (2017), the difference between quote and matrix prices or callable and non-callable bonds does not have a material impact on cross-sectional return predictability. Hence, we include both types of observations. In addition, the LBFI data provides key bond details such as the amount outstanding, credit rating, offering date, and maturity date. For the main results, we use the LBFI data from January 1986 to December 1996.

LBFI bond filters. As for the other databases, we merge the LBFI data to the pre-filtered Mergent FISD data and then apply the following LBFI-specific filters following Elkamhi et al. (2023):

1. Only include corporate bonds classified as 'industrial,' 'telephone utility,' 'electric utility,' 'utility (other),' and 'finance,' as per the LBFI industry classification system, `icode == {3 | 4 | 5 | 6 | 7}`.

2. Remove the following dates for which there are no observations or valid return data, `date == {1975-08 | 1975-09 | 1984-12 | 1985-01}`.

LBFI bond returns. The LBFI data includes corporate bond returns that have been precomputed. The accuracy is empirically verified by Elkamhi et al. (2023).

LBFI additional filters. We follow Bessembinder et al. (2008) and Chordia et al. (2017) and apply the following filters to the LBFI data to account for potential data errors:

1. Remove observations with large return reversals, defined as a 20% or greater return followed by a 20% or greater return of the opposite sign.

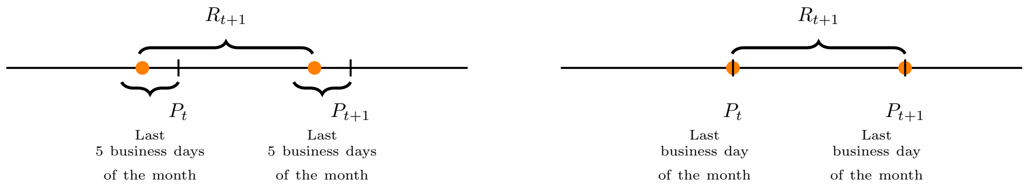
2. Remove observations if the prices appear to bounce back in an extreme fashion relative to preceding days. Denote R_t as the month t return, we exclude an observation at month t if $R_t \times R_{t-k} < -0.02$ for $k = 1, \ldots, 12$.

3. Remove observations if prices do not change for more than three months, i.e., $\frac{P_t}{P_{t-3}} - 1$ `!= 0`, where P is the quoted or matrix price.

IA.1.1.4 Trade Reporting and Compliance Engine Database

For many researchers, the Trade Reporting and Compliance Engine (TRACE) database is the main source of corporate bond data as it is available through Wharton Research Data Services (WRDS TRACE) from August 2002 to December 2022. An alternative version of the TRACE data (DFPS TRACE) is processed by Dick-Nielsen et al. (2025) and provided online via Christian Stolborg's website. The DFPS TRACE data also assumes a return is valid if there are available bond prices in the last five business days of month t and $t+1$. The data is then checked for erroneous data points, and 292 data points are discarded. See Appendix B of Dick-Nielsen et al. (2025) for additional details. The data is also available from August 2002 but ends in December 2021.

TRACE returns. A key difference between quote- (e.g., BAML ICE) and transaction-based (e.g., TRACE) databases is that for the latter transaction prices might not land on the very last business days of consecutive months t and $t+1$, implying that prices may not align with month-end CRSP equity signals. As a result, assumptions are required as to what kind of sampling criterion should be used to compute a monthly time series of bond returns. Consistent with Dickerson, Robotti, and Rossetti (2024), we use the bond return variable denoted `RET_L5M` from WRDS TRACE which recognizes a valid monthly bond return if the bond trades within the 5-day window toward the end of months t and $t+1$, respectively. Mechanically, this implies a monthly time series of bond returns that is not strictly contiguous, i.e., in month t the bond could be traded on the third last business day and in month $t+1$ the same bond may trade on the very last business day. Although quote-based databases are not a 'panacea' for corporate



A: Transaction prices

B: Quotes

Figure IA.1: Calculating bond returns using transaction- and quote-based data.

Panel A shows the timing of how prices are sampled to calculate monthly returns for the transaction-based WRDS TRACE data. The designated 'end-of-the-month' transaction price P_t and P_{t+1} must be within the last five business days of the month. The pseudo 'month-end' return is then computed with these clean prices and any accrued interest. Panel B shows the timing for a bond return calculation using quote-based prices in the BAML ICE and LBFI data. Price quotes are available on the very last business day of each month, resulting in a contiguous monthly return series.

bond data issues, they do allow for bond returns to be consistently computed because a valid month-end quote is always available.

Figure IA.1 illustrates the timing of prices used to compute 'monthly' bond returns with any version of the WRDS TRACE data vs. the BAML ICE quote-based data. In Panel A, a monthly transaction return is valid if a bond trades within the last five days of months t and $t + 1$. Missing returns `NaN` are recorded if, for example, a bond trades in the middle of month t and then only again on the last business day of month $t + 1$. In Panel B, contiguous returns can be computed because a valid indicative quote is available from the pool of dealers that are queried by BAML ICE, thus, bond return calculations are aligned with their analogue for stocks in CRSP.

We use WRDS TRACE as well as DFPS TRACE for our robustness tests that are discussed in Section 4 and in Internet Appendices IA.1.3 and IA.10.1.

IA.1.2 Combined bond and stock data coverage

For our baseline results, we use corporate bond factors and test assets calculated from the dataset that combines the LBFI and the ICE data over the joint sample period January 1986 to December 2022, whereby we splice the data together. Before 1997 we use the LBFI data and, thereafter, we rely on ICE data. Stock factors and test asset returns are all calculated using CRSP data available through WRDS.

Our equity sample comprises close to ten thousand firms (9,994), while our corporate bond sample contains a total of 5,824 issuers. Overall, we can match 2,211 firms that have both public equity as well as corporate bonds outstanding throughout our sample period. That is, 78% of the firms in our sample do not issue corporate bonds, and 62% of the corporate bond issuers are not publicly listed. Figure IA.2 illustrates the overlap of equity and bond data in terms of the number of firms. The red-shaded set comprises all unique firms (as determined by the `PERMNO`) in the CRSP data; the blue-shaded set comprises the unique corporate bond issuers in our data set (as determined by the `ISSUER_CUSIP`). The brown intersection comprises the 2,211 corporate bond issuers that are publicly listed.

In Figure IA.3 we further put in perspective the coverage of our data in terms of market

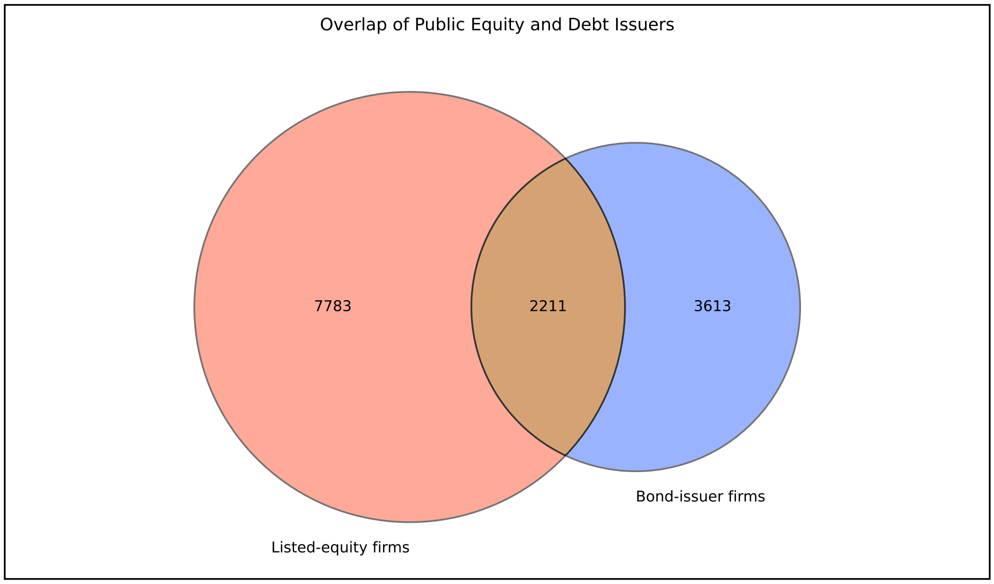


Figure IA.2: Corporate bond and equity issuers.

This figure depicts a Venn diagram where the red-shaded set comprises all 9,994 unique firms (as determined by the `PERMNO`) in the CRSP data. The blue-shaded set are all 5,824 unique issuing firms from our primary corporate bond sample as determined by the six digit `ISSUER_CUSIP`. The brown-shaded intersection comprises the 2,211 firms with outstanding corporate debt that we can match to CRSP `PERMNO` identifiers.

capitalisation. Even though 62% of the firms in the CRSP sample do not issue corporate debt, our matched sample captures around the same percentage as the S&P 500 index in terms of total U.S. market capitalisation. At the end of our sample period, the total market capitalisation of CRSP firms is USD 22.1 trillion while the market capitalisation of our corporate bond matched equity sample is only about 16% smaller with USD 18.4 trillion (see Panel A in Figure IA.3). Panel B plots the coverage in percent, defined as the equity market capitalisation of firms in the merged sample divided by the total market capitalisation of all CRSP firms. The average coverage is 74.5% but remains at or above 80% for the post-2000 period.

IA.1.3 Corporate bond factor zoo robustness

An extensive and ongoing academic debate discusses what could drive replication issues and differences in the performance of corporate bond factors. On the one hand, Dick-Nielsen et al. (2025) argue that data errors and researchers' data cleaning assumptions are the underlying *cause* of the bond replication 'crisis.' On the other hand, Dickerson et al. (2024) posit that a combination of the failure to adjust for corporate bond microstructure issues combined with *ex post* and asymmetric winsorization and/or trimming of the bond return distribution are the core drivers of the crisis.[1]

In this section we examine to what extent data choices may affect corporate bond factors. For all comparisons we re-construct 14 of our 16 corporate bond factors, excluding DEF and

[1]Recently, Jostova et al. (2024) and Li (2023) add to the debate by examining the role of outliers specifically for the corporate bond momentum factor (MOMB).

Table IA.I: The corporate bond factor zoo across data choices

Benchmark data	Alternative data	Sample period	Significant difference
LBFI/BAML ICE	LBFI/BAML ICE firm-level	1986:01–2022:12	CRY, DUR, PEADB, STREVB
LBFI Q&M	LBFI Q only	1986:01–1996:12	VAL
BAML ICE	WRDS TRACE	2002:08–2022:12	CRF
BAML ICE	DFPS TRACE	2002:08–2021:12	CRY, HMLB

The table documents which corporate bond factors exhibit significantly different average returns when comparing the benchmark combined LBFI/BAML ICE data with factors calculated at the bond level with alternatives. We compare bond factors (i) calculated using bond- vs. firm-level data; (ii) that remove matrix prices (quotes and matrix vs. quotes only); (iii) that are calculated using transaction-based WRDS TRACE; and (iv) that are calculated using transaction-based DFPS TRACE data. The factors are listed in column "Significant difference" when factor averages between the benchmark construction and the alternative are significantly different at the 5% level of significance.

TERM as they are independent of the corporate bond data.[2] We first examine differences between factors formed at the bond vs. the firm level. Then, we confirm that removing bonds with matrix prices does not materially affect our corporate bond factors and, finally, we show that the differences between factors based on quotes and factors constructed using transaction prices are negligible. The results are summarized in Table IA.I. Unless otherwise noted, the benchmark data are corporate bond factors calculated at the bond-level using the combined LBFI/BAML ICE data as discussed in Section 1 (LBFI/BAML ICE). Overall, the factor construction is very robust to the different dimensions of comparison. Changes in data (rows two through four in Table IA.I) never lead to more than two factors displaying significantly different means, although the values remain economically small. Moreover, we show in Internet Appendix IA.10.1 that even these significant differences do not affect our estimation results.

IA.1.3.1 Bond- vs. firm-level factors

To study the differences between bond- and firm-level corporate bond factors, we focus on our baseline data, the combined LBFI and BAML ICE bond data. First, we merge the corporate bond data to firm-level `PERMNO` and `GVKEY` identifiers. We then follow Choi (2013) and compute a 'representative' firm(`PERMNO`)-level return as the value-weighted average comprising all outstanding bonds for firm i over month $t+1$ using bond market capitalization weights formed at the end of month t. As in our main analysis, the sample spans 37 years from January 1986 to December 2022. Before January 1997, we merge corporate bond issuers to their `PERMNO` via the historical `NCUSIP` and manually check for errors. Thereafter, we apply the merging methodology outlined in Fang (2025).[3]

Firm-level corporate bond factors. There are benefits and costs associated with constructing factors with firm-level 'representative' bond returns. One potential benefit is that in

[2]DEF and TERM rely on the data repository of Amit Goyal.

[3]The full panel of identification variables and dates necessary to merge the data are available on https://openbondassetpricing.com/bond-compustat-crsp-link/.

a bond-level analysis, firms with a very large number of bonds are given a higher weight compared to firms with fewer or only a single bond outstanding. However, an obvious drawback is that bond-specific information may be aggregated out at the firm level. For example, firms with multiple outstanding bonds may have issued securities with different maturities or even different credit ratings. Thus, for corporate bond factors based on bond-level characteristics, bond-level returns are a natural choice for factor construction.

In Figure IA.4 we compare bond- and firm-level versions of our 14 tradable bond factors ordered by the average bond-level factor return. Panel A presents the respective average returns, while Panel B shows their differences along with associated 95% standard error bars. For most factors, the return differences are not only statistically insignificant but also economically very small—only four factors have average differences that are statistically significant at the 5% level, three of which also generate sizable economic differences. These are all factors that are by construction dependent on bond-level information such as CRY (credit spread), DUR (duration) and STREVB (bond return), i.e., these are the factors where we would not only expect a difference, but where a factor construction using bond-level data is the natural choice. For both CRY and DUR, the bond-level factor returns are around 0.10% higher per month, while for STREVB, this difference is roughly twice as high. At the other end of the spectrum, PEADB generates an additional 0.04% per month on average using firm-level as opposed to bond-level returns.

The results suggest that within a representative firm with multiple bonds outstanding, aggregating returns across the term structure appears to negatively affect factors that capture term structure phenomena (such as CRY and DUR). At the same time, using firm-level returns may be more appropriate when using a signal based on firm- or equity-level characteristics as it will be homogeneous across all of the outstanding bonds. However, as we show in Section IA.10.1, these significant differences ultimately become irrelevant as they pertain to our baseline results and the estimated BMA-SDF.

IA.1.3.2 Quotes vs. quotes & matrix prices

Over the sample period January 1986 to December 1996 the LBFI database uses matrix pricing whereas the BAML ICE database uses a combination of actual transaction prices and indicative bid-side quotes sourced from multiple dealers at 3:00pm Eastern Time (Intercontinental Exchange, 2021). Overall, 39%, 41%, and 31% of all, investment-grade and noninvestment-grade bond prices are set with matrix pricing. To assess pricing differences we follow exactly the same factor construction process as with our baseline LBFI data (including matrix-priced bonds) used in the main results and then proceed to exclude any bond that is not priced with an actual quote. In Panel A of Figure IA.5 show the factor averages over the LBFI sample period January 1986 to December 1996. The return differences are presented in Panel B.

Overall, quote- and quote-matrix-factors are very similar, with the smallest and largest average monthly differences equal to −0.022% for SZE and 0.032% for VAL, respectively. In fact, only VAL has an average return difference that is statistically significant at the 5% level. Thus, our results are consistent with Hong and Warga (2000), Choi (2013), Choi and Richardson (2016) and Chordia et al. (2017), who all find that the impact of removing bonds set with matrix prices on factor premia is quantitatively immaterial.

IA.1.3.3 Quotes vs. transaction prices

We now compare the quote-based BAML ICE factors with factors formed using the 2025 version of WRDS TRACE. The time series of the comparison is restricted to August 2002 to December 2022, starting with the commencement of the WRDS TRACE bond return data. Note that the current version of the WRDS TRACE dataset does not truncate bond returns at the +100% level although Dickerson, Robotti, and Rossetti (2024) documents that this truncation used in a prior version of the data does not result in material differences to out-of-sample factor premia.

Figure IA.6 presents the results comparing WRDS TRACE vs. BAML ICE factors. Across the 14 bond factors, all are very closely aligned. Only a single factor, the credit risk factor (CRF) of Bai et al. (2019) yields a statistically significant difference whereby the average return of the factor formed with the BAML ICE data is larger by just under 0.10% per month. In Figure IA.7 we repeat the exercise using DFPS TRACE using a sample that ends in 2021 (the last available observation in the DFPS TRACE data ends then). Not very surprisingly, the results are not very different. The differences remain economically small at under 0.10% per month, although now, CRY and HMLB exhibit statistically significantly different average returns.

While there is an ongoing debate regarding the use of quotes versus transaction prices in corporate bond research, the differences in average returns are relatively minor at the monthly rebalancing frequency and as long as the data are cleaned and processed appropriately.[4]

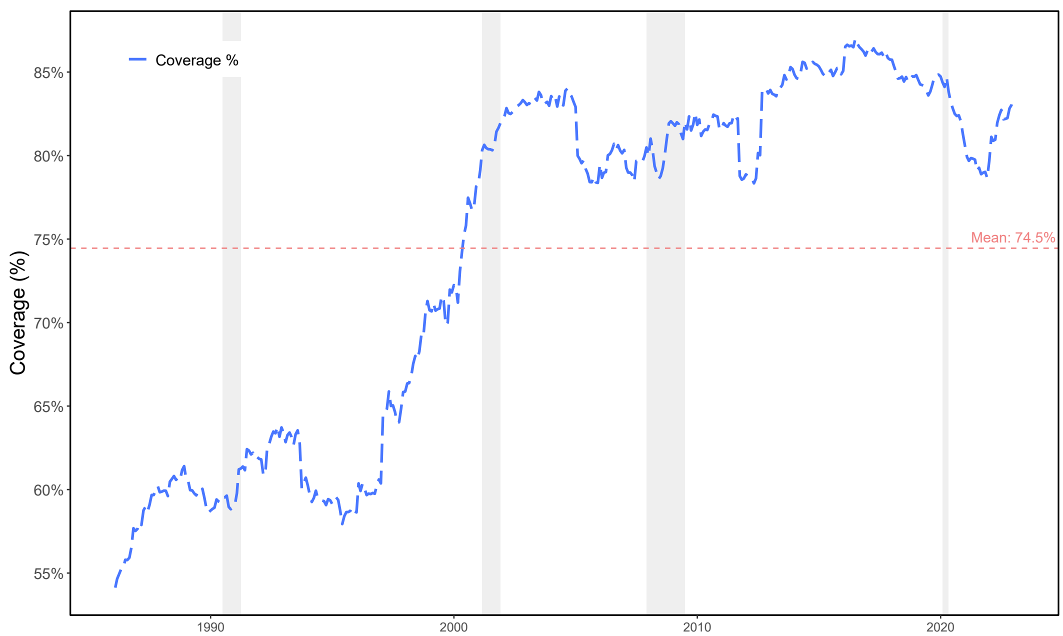
IA.1.4 In- and out-of-sample test assets

In Table IA.II we describe the in- and out-of-sample portfolio and anomaly data we use to estimate and test the BMA-SDFs and other asset pricing models we consider in the paper along with the associated reference and source. The IS corporate bond test assets are the 50 IS bond portfolios listed in Panel A in addition to the 16 tradable corporate bond factors from Panel A in Table A.1 of Appendix A. The IS stock test assets are the 33 stock portfolios listed in Panel B in addition to the 25 tradable stock factors from Panel B in Table A.1 of Appendix A.

Figure IA.8 we plot the percentage variation explained by the first five principal components of the respective test assets for stock excess returns (Panel A), corporate bond excess returns (Panel B) and duration-adjusted corporate bond excess returns (Panel D), respectively (see Internet Appendix IA.6 for details on the duration adjustment). The first principal component captures around 49%, 66% and 77% of the total variation for stock, bond and duration-adjusted bond excess returns, respectively. Panels C and E of Figure IA.8 show the generalized correlations between bond and stock portfolios. Define $\hat{v}_t^B$ and $\hat{v}_t^S$ as the top five principal components of the corporate bond and stock IS test assets. The generalized correlations between $\hat{v}_t^B$ and $\hat{v}_t^S$ are then defined as the square root of the eigenvalues of $\text{cov}(\hat{v}_t^B, \hat{v}_t^S)^\top \text{cov}(\hat{v}_t^B)^{-1} \text{cov}(\hat{v}_t^B, \hat{v}_t^S) \text{cov}(\hat{v}_t^S)^{-1}$.

[4]See Dickerson et al. (2024) for additional discussion on the differences between transaction vs. quote data

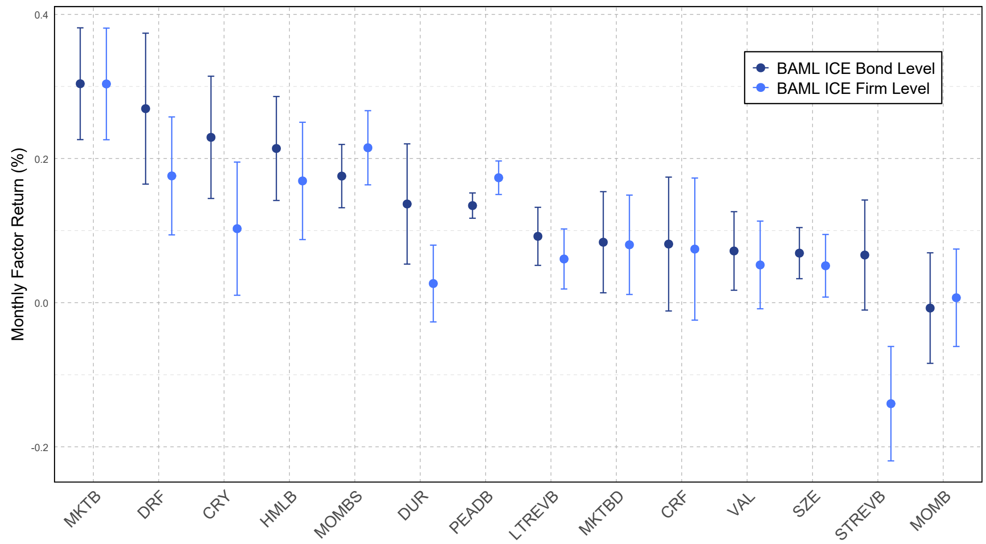
A: Total market capitalisation CRSP and matched bond-stock sample



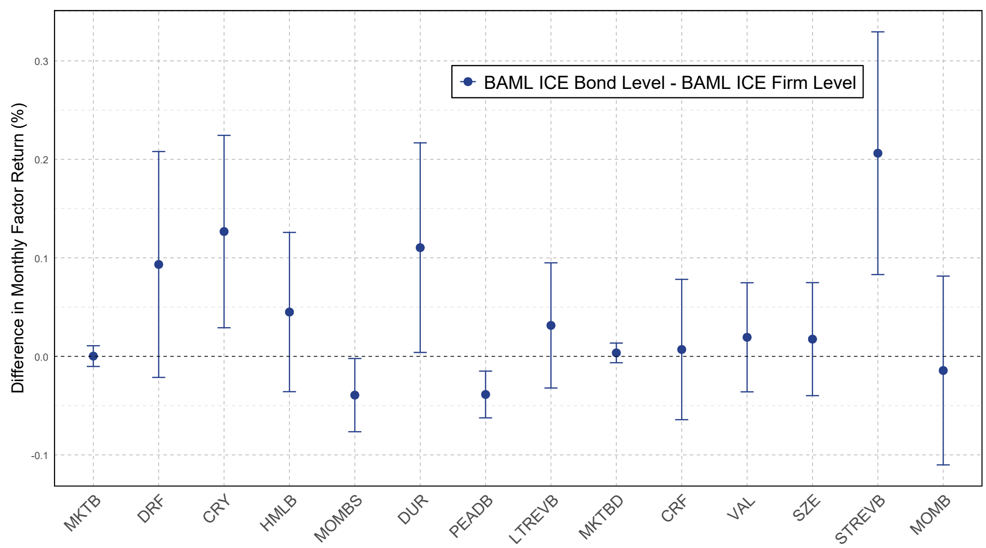
B: Percent coverage by market capitalisation

Figure IA.3: Bond and stock issuers market capitalisation.

Panel A plots the total market capitalisation (in USD trillions) of all listed firms in CRSP (red line) along with the total market capitalisation of the subset that has publicly traded debt in our merged bond-stock data sample at each month t. Panel B plots the time-varying coverage in percent, defined as the sum of the total CRSP market capitalisation divided by the market capitalisation of the firms in our corporate bond matched sample. The sample period is 1986:01 to 2022:12 ($T = 444$).



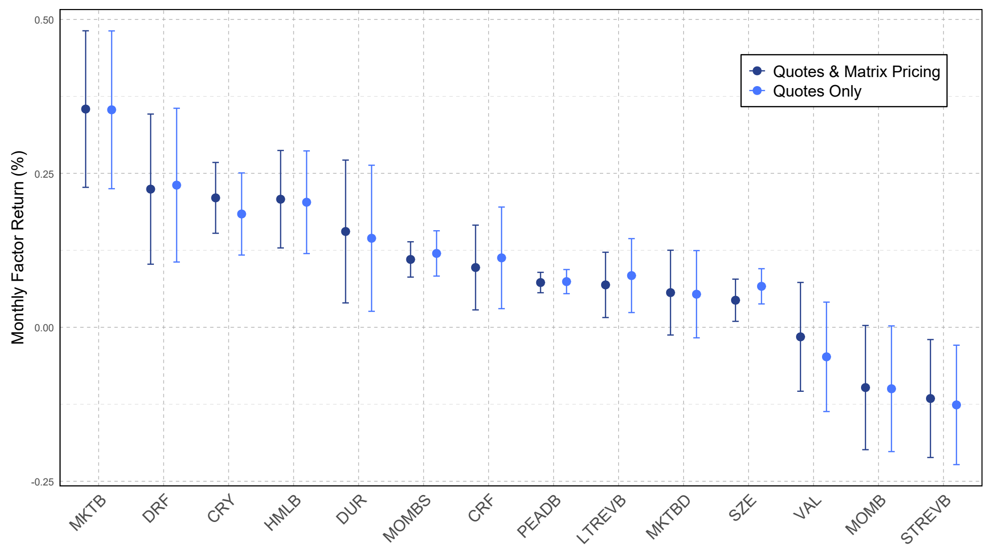
A: Average bond factor returns: bond vs. firm level



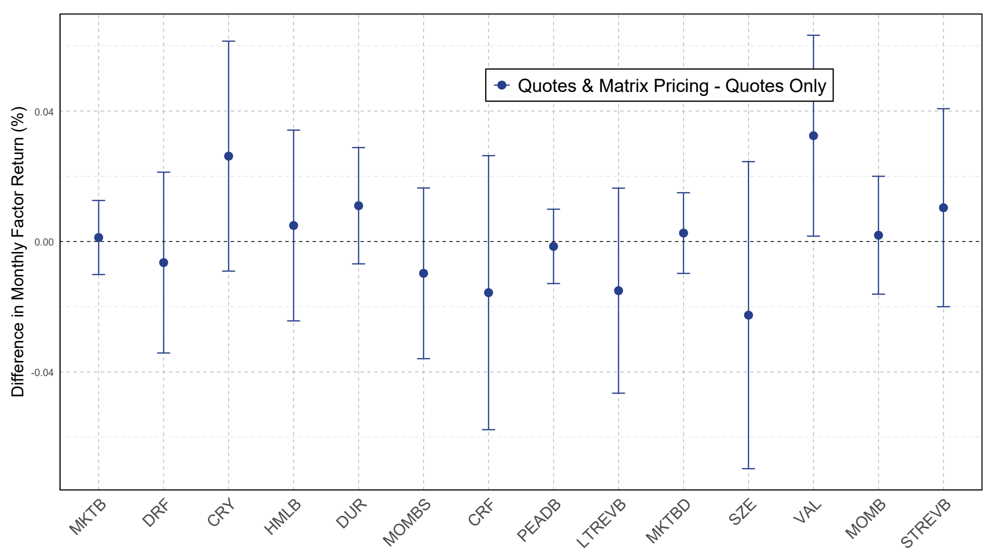
B: Return differences

Figure IA.4: Bond factor comparison: Bond- vs. firm-level.

Panel A displays the average monthly bond factor returns constructed at the bond or the firm level, respectively, using the combined LBFI/BAML ICE quote-based data. Panel B reports the average return differences in percent. The standard error bars represent the 95% confidence interval. The factors computed at the firm level use a 'representative' bond return for month $t+1$ computed as the value-weighted average return of all of a firms' bonds outstanding over month $t+1$ using bond market capitalization weights formed at month t. The sample period is 1986:01 to 2022:12 ($T = 444$).



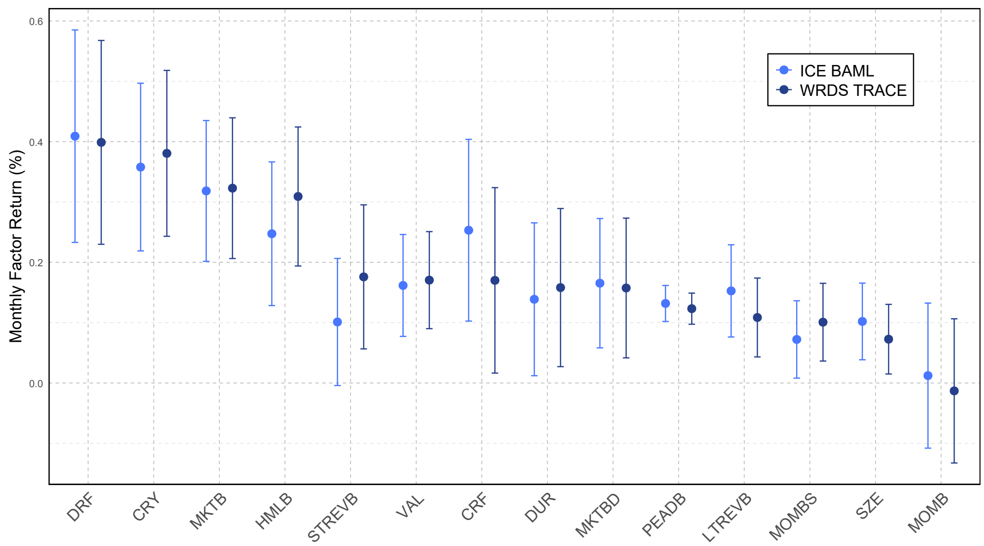
A: Average bond factor returns: quotes and matrix prices vs. quotes



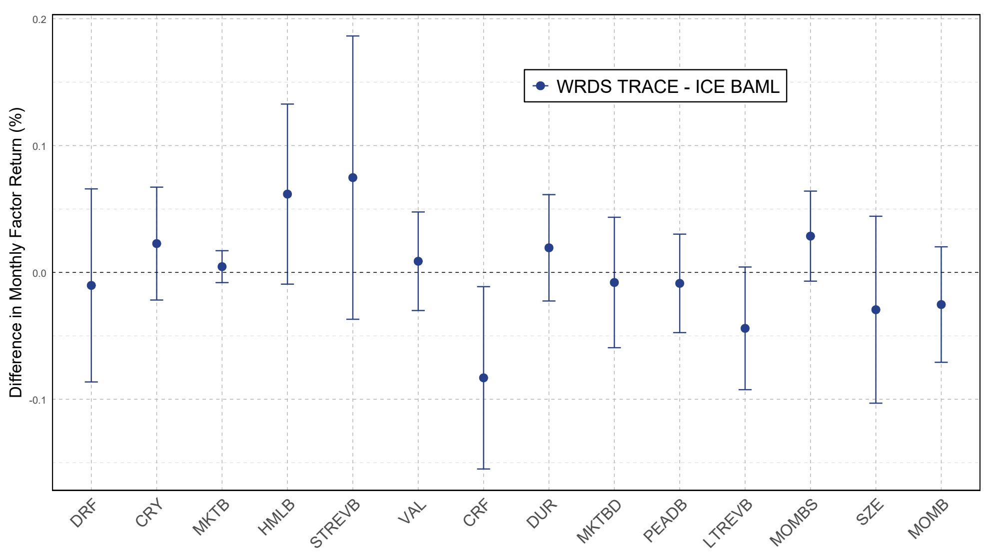
B: Return differences

Figure IA.5: Bond factor comparison: Quotes matrix prices vs. quotes only.

Panel A displays the average monthly bond factor returns constructed at the bond level with the LBFI data using returns computed with both bond price quotes as well as matrix prices and with quotes only. Panel B reports the average return differences in percent. The standard error bars represent the 95% confidence interval. The sample period is 1986:01 to 1996:12 ($T = 232$).



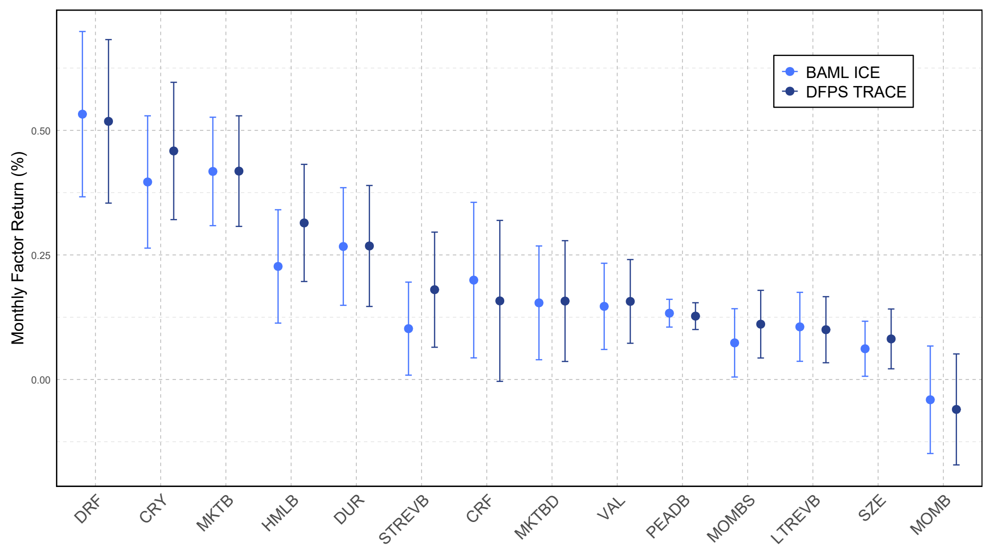
A: Average bond factor returns: BAML ICE vs. WRDS TRACE



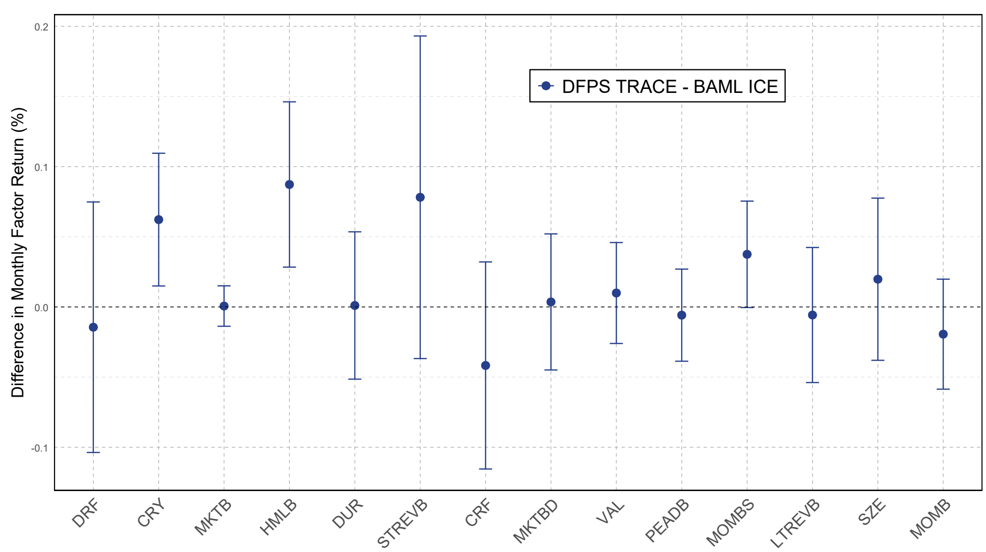
B: Return differences

Figure IA.6: Bond factor comparison: BAML ICE vs. WRDS TRACE.

Panel A displays the average monthly bond factor returns constructed at the bond level with the BAML ICE and the WRDS TRACE data, respectively. Panel B reports the average return differences in percent. The standard error bars represent the 95% confidence interval. The sample period is 2002:08 to 2022:12 ($T = 245$), starting with the first observation in WRDS TRACE.



A: Average bond factor returns: BAML ICE vs. DFPS TRACE



B: Return differences

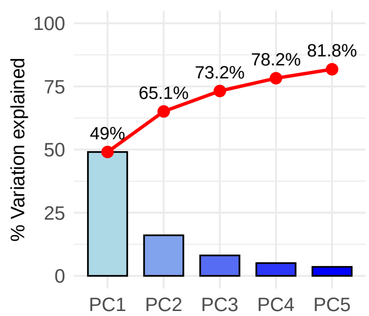
Figure IA.7: Bond factor comparison: BAML ICE vs. DFPS TRACE.

Panel A displays the average monthly bond factor returns constructed at the bond level with the BAML ICE and the DFPS TRACE data, respectively. Panel B reports the average return differences in percent. The standard error bars represent the 95% confidence interval. The sample period is 2002:08 to 2021:12 ($T = 232$), starting with the first observation in DFPS TRACE.

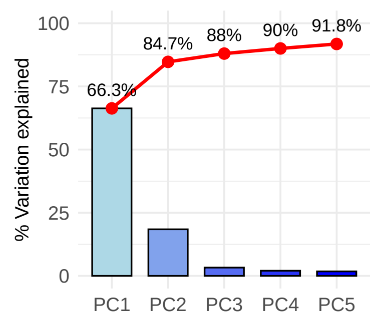
Table IA.II: List of corporate bond, stock and U.S. Treasury bond test assets. This table lists the in and out-of-sample test assets used for the baseline results in the paper. For each test asset, we present their identification (Asset ID), a description of their construction, and the source of the data for downloading and/or constructing the time series. Panel A describes the IS corporate bond portfolios/anomalies. Panel B describes the IS stock portfolios/anomalies. Panel C describes the OS corporate bond portfolios/anomalies. Panel D describes the OS stock portfolios/anomalies. Panel E describes the OS U.S. Treasury portfolios.

Asset ID	Name and description	Reference	Source
	Panel A: In-sample bond portfolios/anomalies		
25 spread/size bond portfolios	5 Bond credit spread × 5 bond market capitalization double sorted portfolios.	Nozawa (2017) and Elkamhi et al. (2023)	Open Source Bond Asset Pricing
25 rating/maturity bond portfolios	5 Bond rating × 5 bond time to maturity double sorted portfolios.	Gebhardt et al. (2005) and others	Open Source Bond Asset Pricing
	Panel B: In-sample stock portfolios/anomalies		
cash_at	CashAssets. Cash and short term investments scaled by assets.	Palazzo (2012)	Global Factor Data
ope_be	FCFBook. Operating profits-to-book equity.	Fama and French (2015)	Global Factor Data
ocf_me	CFPrice. Operating cash flow-to-market.	Desai et al. (2004)	Global Factor Data
at_turnover	Asset Turnover. Sales scaled by average of total assets.	Haugen and Baker (1996)	Global Factor Data
capx_gr2	CapIntens. CAPEX 2 year growth.	Anderson and Garcia-Feijóo (2006)	Global Factor Data
div12m_me	DP tr. Dividend yield.	Litzenberger and Ramaswamy (1979)	Global Factor Data
ppeinv_gr1a	PPE delta. Change in property, plant and equipment less inventories scaled by lagged assets.	Lyandres et al. (2008)	Global Factor Data
sale_me	SalesPrice. Sales-to-market.	William C. Barbee et al. (1996)	Global Factor Data
ret_12_7	IntermMom. Price momentum t-12 to t-7.	Novy-Marx (2012)	Global Factor Data
prc_highprc_252d	YearHigh. Current price to high price over last year.	George and Hwang (2004)	Global Factor Data
ni_me	PE tr. Earnings-to-price.	Basu (1983)	Global Factor Data
bidaskhl_21d	BidAsk. 21 day high-low bid-ask spread.	Corwin and Schultz (2012)	Global Factor Data
dolvol_126d	Volume. Dollar trading volume.	Brennan et al. (1998)	Global Factor Data
dsale_dsga	SGASales. Change sales minus change SG&A.	Abarbanell and Bushee (1998)	Global Factor Data
cop_atl1	Cash-based operating profits-to-lagged book assets.	Ball et al. (2016)	Global Factor Data
ivol_capm_252d	iVolCAPM. Idiosyncratic volatility from the CAPM (252 days).	Ali et al. (2003)	Global Factor Data
ivol_ff3_21d	iVolFF3. Idiosyncratic volatility from the Fama-French 3-factor model.	Ang et al. (2006)	Global Factor Data
rvol_21d	Return volatility.	Ang et al. (2006)	Global Factor Data
ebit_sale	ProfMargin. Operating profit margin after depreciation.	Soliman (2008)	Global Factor Data
ocf_at	PriceCostMargin. Operating cash flow to assets.	Bouchaud et al. (2019)	Global Factor Data
opex_at	OperLev. Operating leverage.	Novy-Marx (2011)	Global Factor Data
lnoa_gr1a	NetSalesNetOA. Change in long-term net operating assets.	Fairfield et al. (2003)	Global Factor Data
oaccruals_at	Operating accruals.	Sloan (1996)	Global Factor Data
at_gr1	Asset growth. Asset growth (1yr).	Cooper et al. (2008)	Global Factor Data
eqnpo_12m	Net equity payout (1yr).	Daniel and Titman (2006)	Global Factor Data
gp_at	Gross profit scaled by assets.	Novy-Marx (2013)	Global Factor Data
capex_abn	Abnormal corporate investment.	Titman et al. (2004)	Global Factor Data
noa_at	NetOA. Net operating assets to total assets.	Hirshleifer et al. (2004)	Global Factor Data
o_score	Ohlson O-score.	Dichev (1998)	Global Factor Data
niq_at	ROA. Quarterly return on assets.	Balakrishnan et al. (2010)	Global Factor Data

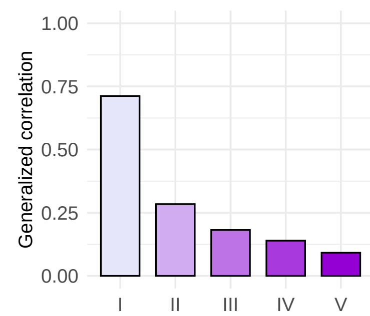
chcsho_12m	Net stock issues.	Pontiff and Woodgate (2008)	Global Factor Data
re_60_12	LRreversal. Long-run reversal.	Bondt and Thaler (1985)	Open Asset Pricing
debt_me	Lev. Market leverage.	Bhandari (1988)	Open Asset Pricing
Panel C: Out-of-sample bond portfolios/anomalies			
10× VaR portfolios	Decile sorted bond portfolios sorted on 24-month rolling 95% historical value-at-risk (VaR) defined as the second lowest return value in the rolling period.	Bai et al. (2019)	Open Source Bond Asset Pricing
10× duration portfolios	Decile sorted bond portfolios sorted on bond duration.	Gebhardt et al. (2005)	Open Source Bond Asset Pricing
10× bond value portfolios	Decile sorted bond portfolios sorted on bond market capitalization defined as bond price multiplied by bond amount outstanding.	Houweling and Van Zundert (2017)	Open Source Bond Asset Pricing
10× bond BTM portfolios	Decile sorted bond portfolios sorted on bond book-to-market (BTM) defined as the market value of the bond scaled by the par value.	Bartram et al. (2025)	Open Source Bond Asset Pricing
10× bond LTREV portfolios	Decile sorted bond portfolios sorted on bond long-term reversal defined as the sum of the bond returns from t-12 to t-48.	Bali et al. (2021a)	Open Source Bond Asset Pricing
10× bond MOM portfolios	Decile sorted bond portfolios sorted on bond momentum defined as the sum of the bond returns from t-6 to t-1.	Gebhardt et al. (2005)	Open Source Bond Asset Pricing
17× bond FF17 portfolios	17 Fama-French industry portfolios computed with bond returns.	Kelly et al. (2023)	Open Source Bond Asset Pricing
Panel D: Out-of-sample stock portfolios/anomalies			
10× E/P portfolios	Decile sorted stock portfolios sorted on the earning-to-price ratio (E/P).	Fama & French	Kenneth French's webpage
10× MOM portfolios	Decile sorted stock portfolios sorted on equity momentum.	Fama & French	Kenneth French's webpage
10× LTREV portfolios	Decile sorted stock portfolios sorted on stock long-term reversals.	Fama & French	Kenneth French's webpage
10× accruals portfolios	Decile sorted stock portfolios sorted on equity accruals.	Fama & French	Kenneth French's webpage
10× size portfolios	Decile sorted stock portfolios sorted on firm size (market capitalization).	Fama & French	Kenneth French's webpage
10× variance portfolios	Decile sorted stock portfolios sorted on earnings-to-price ratio (E\/P).	Fama & French	Kenneth French's webpage
17× stock FF17 portfolios	17 Fama-French industry portfolios computed with stock returns.	Fama & French	Kenneth French's webpage
Panel E: Out-of-sample Treasury portfolios			
29× Treasury portfolios	Monthly excess U.S. Treasury bond returns computed across the term structure using annualized continuously-compounded zero coupon yields computed as in Liu and Wu (2021). We price the U.S. Treasury Bonds each month using the yield-curve data and then compute monthly discrete excess returns across the term structure as the total return in excess of the one-month Treasury Bill rate. The portfolios span from the 2-year T Bond up until the 30-year T-Bond in increments of 1-year.	Liu and Wu (2021)	Jing Cynthia Wu's webpage



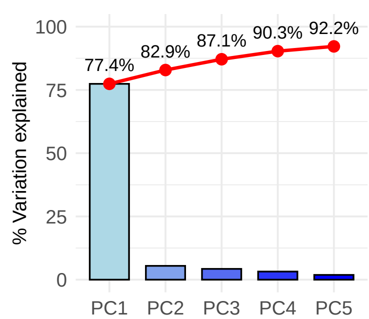
A: Stocks



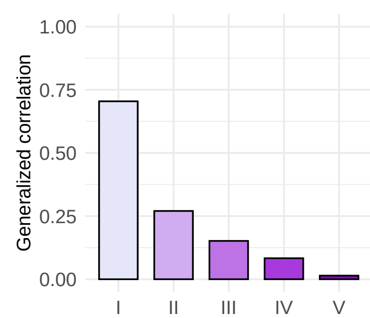
B: Corporate bonds



C: Correlation



D: Bonds (duration-adj.)



E: Correlation (duration-adj.)

Figure IA.8: Principal components and generalized correlations between bonds and stocks.

Panel A shows the percent variation explained by the first five principal components of the IS stock test assets. Panels B and D show the same information for the corporate bond portfolios constructed used bond excess and duration-adjusted bond excess returns, respectively. Panels C and E report the respective generalized (canonical) correlations between corporate bonds and stocks. See Internet Appendix IA.6 for the duration adjustment. The stock test assets comprise 33 portfolios and the 24 tradable stock factors ($N = 57$), the bond test assets comprise the 50 portfolios and 16 tradable bond factors ($N = 66$). The sample period is 1986:01 to 2022:12 ($T = 444$).

IA.2 Simulation design

We build a simple setting for a linear factor model that includes strong and weak factors and noisy proxies of the strong factors. The cross-section of asset returns is calibrated to mimic the empirical properties of 25 size and value portfolios of Fama-French. All factors and portfolio returns are generated from normal distributions. We calibrate the strong (useful) factor to mimic the HML portfolio. To generate a misspecified setting, we include the pricing errors from the GMM-OLS estimation of the model with HML as the only factor. A useless factor is simulated from an independent normal distribution with mean zero and standard deviation 1%. Noisy proxies, $f_{t,j}$, of the true factors are generated to have correlation ρ_j with the useful factor and the same variance as the latter.

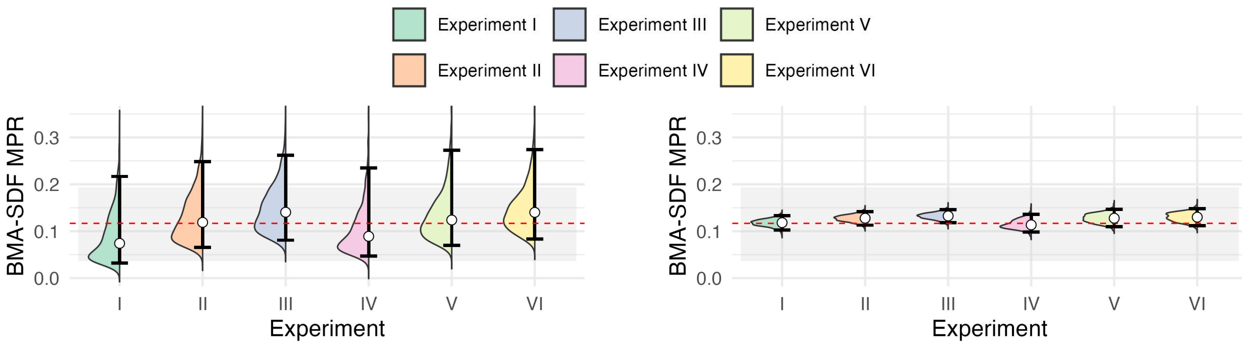
In summary,

$$f_{t,useless} \overset{\text{iid}}{\sim} \mathcal{N}(0, (1\%)^2), \qquad \begin{pmatrix} \boldsymbol{R_t} \\ f_{t,hml} \end{pmatrix} \overset{\text{iid}}{\sim} \mathcal{N}\left(\begin{bmatrix} \bar{\boldsymbol{R}} \\ \bar{f}_{hml} \end{bmatrix}, \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_{\boldsymbol{R}} & \widehat{\boldsymbol{C}}_{\boldsymbol{hml}} \\ \widehat{\boldsymbol{C}}_{\boldsymbol{hml}}^{\top} & \widehat{\sigma}^2_{hml} \end{bmatrix} \right), \text{ and}$$

$$f_{t,j} = \delta_j f_{t,hml} + \sqrt{1-\delta_j^2}\, w_{t,j}, \ |\delta_j| < 1, \ \text{ where } \ w_{t,j} \overset{\text{iid}}{\sim} \mathcal{N}(0, \widehat{\sigma}^2_{hml})$$

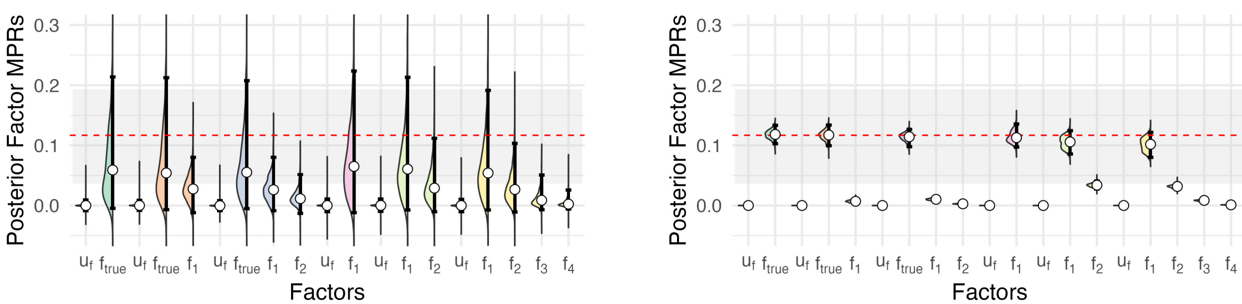
where the factor loadings, risk prices, and the variance-covariance matrix of returns and factors are equal to their sample estimates from the time series and cross-sectional regressions of the GMM-OLS procedure, applied to 25 size-and-value portfolios and HML as a factor. All the simulation parameters are estimated on monthly data from July 1963 to December 2017. For each sample size and experiment considered, we generate one thousand artificial samples, and in each artificial sample, we estimate the posterior probabilities of the factors, their posterior (mean) market prices of risk, and the BMA-SDF-implied market price of risk.

Figures IA.9 and IA.10 show some additional evidence from simulations that is discussed in Section 2.4.1.



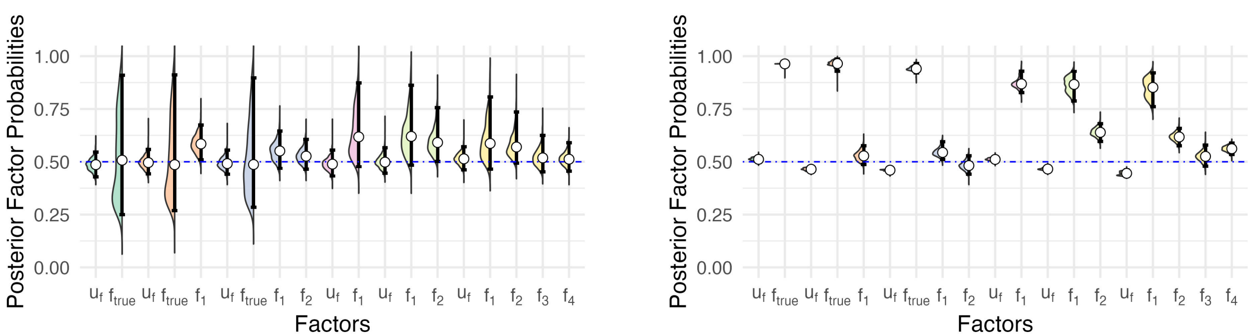
A: BMA-SDF market price of risk, $T = 200$

B: BMA-SDF market price of risk, $T = 20,000$



C: Factors' market price of risk, $T = 200$

D: Factors' market price of risk, $T = 20,000$



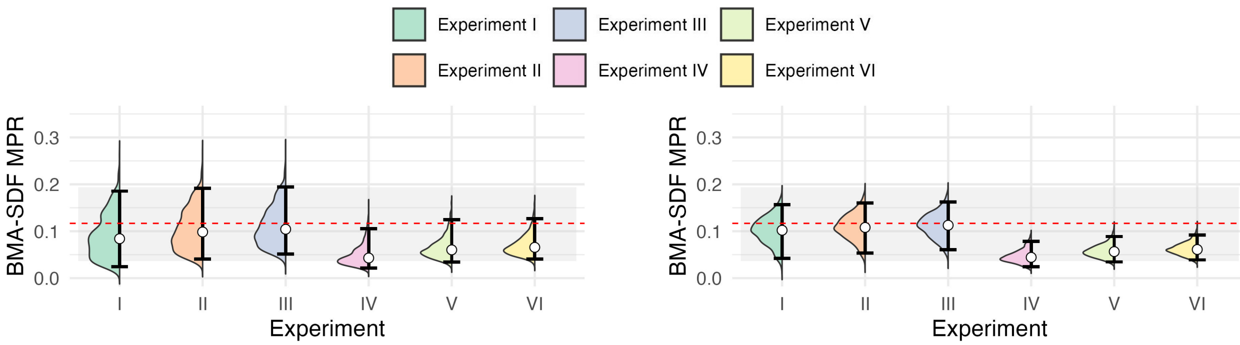
E: Factors' posterior probabilities, $T = 200$

F: Factors' posterior probabilities, $T = 20,000$

Figure IA.9: Simulation evidence in very large and very small samples.

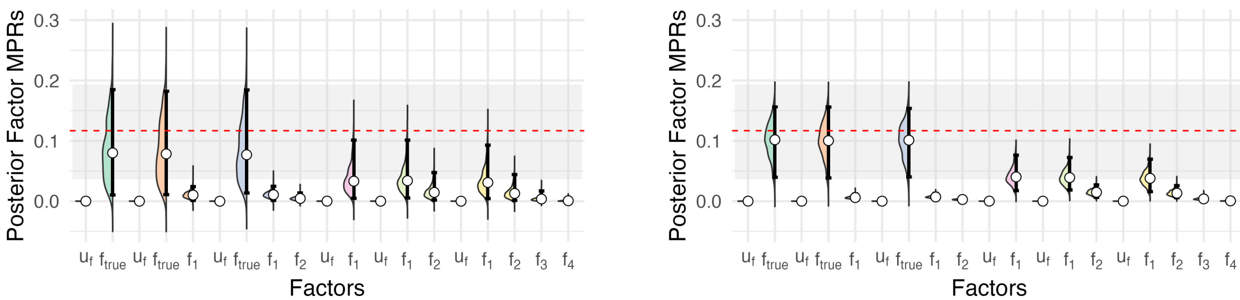
Simulation results from applying our Bayesian methods to different sets of factors. Each experiment is repeated 1,000 times with the specified sample size (T). Data generating process calibrated to match the pricing ability of the HML factor (as pseudo-true factor) for the Fama-French 25 Size and Book-to-Market portfolios. Horizontal red dashed lines denote the market price of risk of HML, and the grey shaded area the frequentist 95% confidence region of its GMM estimate in the historical sample of 665 monthly observations. The prior is set to 40% of the expost maximum Sharpe ratio. Simulation details are in Internet Appendix IA.2. Half-violin plots depict the distribution of the estimated quantities across simulation, with black error bars denoting centered 95% coverage, and white circles denoting median values, across repeated samples. In all experiments we include a useless factor (u_f), while the pseudo-true factor (f_{true}) is included only in experiments I–III. In each experiment we include a variable number of noisy proxies f_j, $j = 1, ..., 4$ with correlations with the pseudo-true factor equal to, respectively, .4, .3, .2, and .1. The factors consider in the various experiments are:

Experiment I: u_f and f_{true}.
Experiment II: u_f, f_{true} and f_1.
Experiment III: u_f, f_{true}, f_1 and f_2.
Experiment IV: u_f, and f_1.
Experiment V: u_f, f_1 and f_2.
Experiment VI: u_f, f_1, f_2, f_3 and f_4.



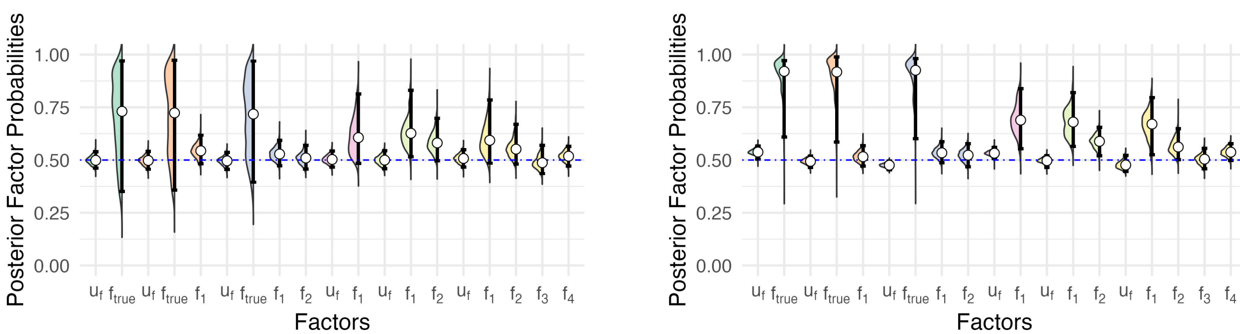
A: BMA-SDF market price of risk, $T = 400$

B: BMA-SDF market price of risk, $T = 1,600$



C: Factors' market price of risk, $T = 400$

D: Factors' market price of risk, $T = 1,600$



E: Factors' posterior probabilities, $T = 400$

F: Factors' posterior probabilities, $T = 1,600$

Figure IA.10: Simulation evidence with useless factors and noisy proxies, prior SR = 40%.

Simulation results from applying our Bayesian methods to different sets of factors. Each experiment is repeated 1,000 times with the specified sample size (T). Data generating process calibrated to match the pricing ability of the HML factor (as pseudo-true factor) for the Fama-French 25 Size and Book-to-Market portfolios. Horizontal red dashed lines denote the market price of risk of HML, and the grey shaded area the frequentist 95% confidence region of its GMM estimate in the historical sample of 665 monthly observations. The prior is set to 40% of the expost maximum Sharpe ratio. Simulation details are in Internet Appendix IA.2. Half-violin plots depict the distribution of the estimated quantities across simulation, with black error bars denoting centered 95% coverage, and white circles denoting median values, across repeated samples. In all experiments we include a useless factor (u_f), while the pseudo-true factor (f_{true}) is included only in experiments I–III. In each experiment we include a variable number of noisy proxies f_j, $j = 1, ..., 4$ with correlations with the pseudo-true factor equal to, respectively, .4, .3, .2, and .1. The factors consider in the various experiments are:

Experiment I: u_f and f_{true}.
Experiment II: u_f, f_{true} and f_1.
Experiment III: u_f, f_{true}, f_1 and f_2.
Experiment IV: u_f, and f_1.
Experiment V: u_f, f_1 and f_2.
Experiment VI: u_f, f_1, f_2, f_3 and f_4.

IA.3 Additional co-pricing results

In this section we provide additional results to complement the analysis in Section 3.1.

Table IA.III: Tradable factor performance statistics: Full sample

	SR	IR	μ	t-stat.	α	t-stat.
			Panel A: Corporate bond factors			
CRF	0.04	0.04	0.08	[0.75]	0.08	[0.69]
CRY	0.13	0.02	0.23	[2.21]	0.03	[0.41]
DEF	0.02	−0.03	0.03	[0.39]	−0.05	[−0.56]
DRF	0.12	−0.09	0.27	[2.35]	−0.09	[−1.88]
DUR	0.08	−0.15	0.14	[1.66]	−0.14	[−2.51]
HMLB	0.14	0.06	0.21	[2.44]	0.09	[1.19]
LTREVB	0.11	0.12	0.09	[2.09]	0.11	[1.97]
MKTB	0.19	-	0.30	[3.55]	-	-
MKTBD	0.06	−0.01	0.08	[1.05]	−0.02	[−0.20]
MOMB	−0.00	0.03	−0.01	[−0.10]	0.04	[0.53]
MOMBS	0.19	0.26	0.18	[3.69]	0.23	[4.36]
PEADB	0.36	0.40	0.13	[7.17]	0.14	[6.88]
STREVB	0.04	0.00	0.07	[0.95]	0.00	[−0.07]
SZE	0.09	0.11	0.07	[1.78]	0.08	[2.30]
TERM	0.12	0.01	0.36	[2.50]	0.03	[0.23]
VAL	0.06	0.06	0.07	[1.16]	0.07	[0.94]
			Panel B: Stock factors			
BAB	0.20	0.23	0.74	[3.52]	0.84	[3.55]
CMA	0.14	0.20	0.29	[2.55]	0.40	[3.45]
CMAs	0.16	0.19	0.20	[3.24]	0.24	[3.77]
CPTLT	0.11	−0.02	0.75	[2.21]	−0.08	[-0.42]
FIN	0.14	0.23	0.59	[2.78]	0.86	[4.25]
HML	0.06	0.08	0.18	[1.02]	0.25	[1.26]
HML_DEV	0.04	0.04	0.16	[0.81]	0.14	[0.68]
HMLs	0.06	0.07	0.10	[1.01]	0.12	[1.19]
LIQ	0.08	0.06	0.29	[1.52]	0.24	[1.24]
LTREV	0.06	0.05	0.17	[1.16]	0.14	[0.86]
MGMT	0.18	0.26	0.52	[3.37]	0.70	[4.33]
MKTS	0.15	-	0.69	[3.22]	-	-
MKTSs	0.17	0.12	0.56	[3.39]	0.34	[2.27]
MOMS	0.11	0.15	0.51	[2.3]	0.66	[3.36]
PEAD	0.26	0.28	0.53	[5.4]	0.56	[5.98]
PERF	0.17	0.24	0.52	[3.4]	0.66	[4.93]
QMJ	0.19	0.32	0.47	[3.45]	0.69	[6.44]
RMW	0.15	0.20	0.38	[2.95]	0.48	[3.81]
RMWs	0.21	0.20	0.31	[4.67]	0.31	[4.46]
R_IA	0.14	0.20	0.31	[2.72]	0.42	[3.55]
R_ROE	0.18	0.24	0.49	[3.58]	0.62	[5.35]
SMB	0.02	−0.01	0.06	[0.45]	−0.03	[-0.25]
SMBs	0.03	0.04	0.06	[0.58]	0.08	[0.72]
STREV	0.07	0.02	0.24	[1.69]	0.06	[0.45]

The table lists corporate bond and stock tradable factor performance statistics. SR is the Sharpe ratio, IR is the Information ratio, μ is the average return, and α is the single-factor MKTB (MKTS) alpha. All statistics are reported monthly. μ and α are reported in percent. t-statistics are reported in square brackets with Newey-West standard errors computed with four lags. The sample period is 1986:01 to 2022:12.

IA.3.1 The co-pricing SDF

Factor statistics. Tables IA.III and IA.IV provide performance statistics such as the Sharpe and Information ratio, average return μ and a one-factor α using MKTB and MKTS for the tradable bond and stock factors, respectively. The two factors with the highest Sharpe ratios in Table IA.III—PEADB with a SR of 0.36 and PEAD with a SR of 0.26—are also the two tradable factors with the highest posterior probabilities in Figure 2. For comparison, the SR of the bond and stock market factors MKTB and MKTS are 0.19 and 0.15, respectively. Table IA.IV shows the performance statistics for subsamples pre- and post-2000. PEADB displays the highest SR for a bond factor for both subsample periods, whereas PEAD is particularly strong in the first half of the sample. In the second half, the stock factors with the highest SR are BAB and RMWs with a SR of 0.21.

Posterior probabilities and market prices of risk. In Table A.2 of Appendix C we provide the full list of posterior probabilities and the associated annualized risk premia (in Sharpe ratio units) for the co-pricing factor zoo across the full range of prior Sharpe ratios we consider. Tables IA.V and IA.VI report the corresponding information for the respective bond and stock factor zoos.

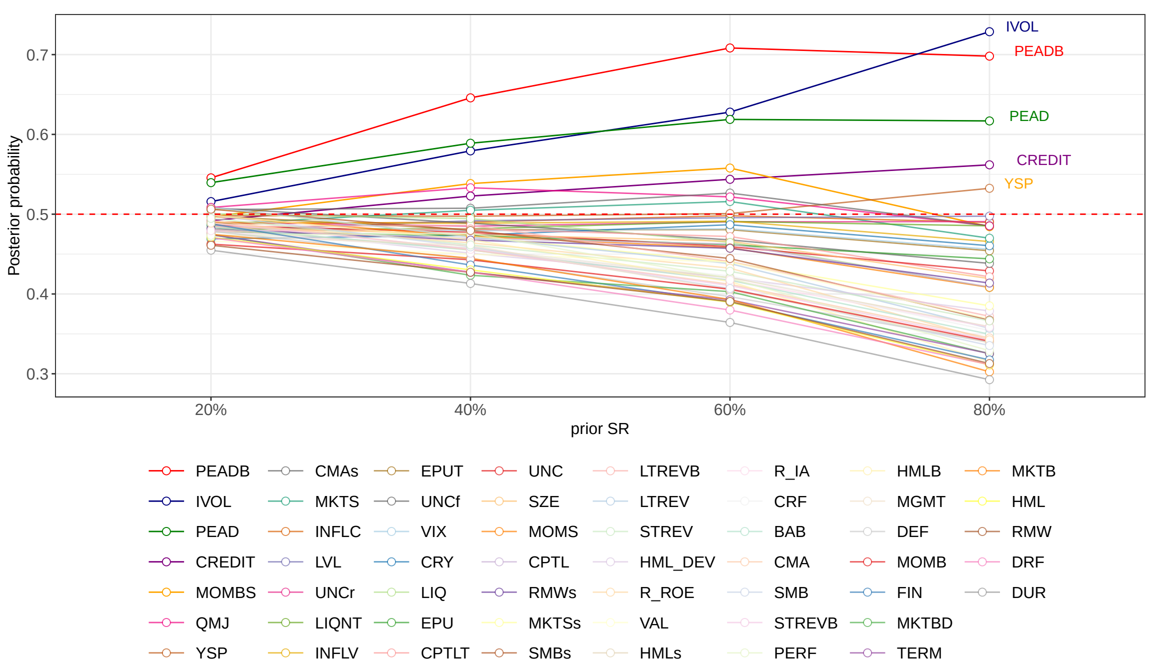


Figure IA.11: Posterior factor probabilities: Co-pricing factor zoo (no intercept).

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, of the 54 bond and stock factors described in Appendix A. All models are estimated without an intercept. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are shown for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_{\boldsymbol{f}}^2 \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. Labels are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table IA.IV: Tradable factor performance statistics: Subsamples

	1986:01–1999:12						2000:01–2022:12					
	SR	IR	μ	t-stat.	α	t-stat.	SR	IR	μ	t-stat.	α	t-stat.
Panel A: Corporate bond factors												
CRF	0.10	0.22	0.08	[1.26]	0.16	[2.31]	0.03	0.02	0.08	[0.47]	0.05	[0.29]
CRY	0.25	0.24	0.18	[2.94]	0.18	[2.63]	0.12	-0.02	0.26	[1.59]	-0.02	[-0.26]
DEF	-0.05	0.06	-0.05	[-0.73]	0.05	[0.89]	0.04	-0.04	0.08	[0.70]	-0.07	[-0.62]
DRF	0.12	-0.24	0.17	[1.48]	-0.11	[-3.05]	0.13	-0.06	0.33	[1.93]	-0.07	[-0.94]
DUR	0.09	-0.24	0.12	[1.12]	-0.13	[-2.81]	0.07	-0.13	0.15	[1.28]	-0.14	[-1.68]
HMLB	0.22	0.11	0.18	[2.48]	0.07	[1.32]	0.13	0.06	0.23	[1.74]	0.10	[0.89]
LTREVB	0.12	0.33	0.07	[1.37]	0.15	[3.37]	0.11	0.10	0.11	[1.66]	0.09	[1.27]
MKTB	0.21	-	0.29	[2.43]	-	-	0.18	-	0.31	[2.67]	-	-
MKTBD	0.06	0.12	0.05	[0.72]	0.09	[1.53]	0.06	-0.04	0.11	[0.88]	-0.06	[-0.47]
MOMB	-0.08	-0.13	-0.09	[-1.04]	-0.14	[-1.60]	0.02	0.08	0.04	[0.38]	0.14	[1.23]
MOMBS	0.33	0.36	0.11	[3.79]	0.12	[3.77]	0.19	0.27	0.21	[2.89]	0.29	[3.64]
PEADB	0.41	0.41	0.08	[4.89]	0.08	[5.09]	0.38	0.42	0.17	[6.07]	0.18	[5.85]
STREVB	-0.04	-0.03	-0.05	[-0.50]	-0.04	[-0.43]	0.07	0.02	0.13	[1.40]	0.03	[0.36]
SZE	0.08	0.13	0.03	[0.91]	0.05	[1.55]	0.10	0.11	0.09	[1.56]	0.10	[1.92]
TERM	0.14	-0.12	0.37	[1.73]	-0.14	[-1.58]	0.11	0.03	0.35	[1.84]	0.10	[0.49]
VAL	-0.01	0.24	-0.01	[-0.12]	0.14	[2.44]	0.10	0.04	0.12	[1.39]	0.05	[0.57]
Panel B: Stock factors												
BAB	0.18	0.18	0.60	[1.72]	0.60	[1.67]	0.21	0.25	0.82	[3.40]	0.93	[3.33]
CMA	0.09	0.26	0.18	[1.12]	0.42	[3.07]	0.16	0.19	0.36	[2.25]	0.41	[2.51]
CMAs	0.22	0.31	0.27	[2.78]	0.36	[3.72]	0.13	0.14	0.16	[2.06]	0.18	[2.25]
CPTLT	0.16	-0.05	1.08	[2.10]	-0.20	[-0.75]	0.08	-0.01	0.55	[1.21]	-0.04	[-0.15]
FIN	0.16	0.32	0.53	[1.93]	0.90	[3.65]	0.14	0.20	0.62	[2.15]	0.83	[3.06]
HML	0.03	0.15	0.07	[0.30]	0.33	[1.47]	0.07	0.07	0.25	[1.03]	0.26	[0.95]
HML_DEV	-0.04	0.06	-0.13	[-0.48]	0.16	[0.67]	0.08	0.06	0.34	[1.25]	0.23	[0.85]
HMLs	0.11	0.19	0.17	[1.25]	0.28	[2.04]	0.03	0.03	0.05	[0.40]	0.05	[0.37]
LIQ	0.05	0.07	0.17	[0.62]	0.22	[0.86]	0.09	0.07	0.36	[1.42]	0.29	[1.15]
LTREV	0.11	0.11	0.26	[1.26]	0.26	[1.17]	0.04	0.03	0.12	[0.60]	0.09	[0.39]
MGMT	0.18	0.36	0.41	[2.21]	0.68	[4.28]	0.18	0.23	0.58	[2.70]	0.71	[3.18]
MKTS	0.23	-	1.00	[3.04]	-	-	0.11	-	0.50	[1.74]	-	-
MKTSs	0.24	0.14	0.74	[2.72]	0.37	[1.46]	0.14	0.10	0.45	[2.30]	0.30	[1.79]
MOMS	0.32	0.28	0.99	[3.59]	0.86	[3.08]	0.04	0.09	0.22	[0.73]	0.43	[1.73]
PEAD	0.57	0.55	0.92	[8.06]	0.87	[7.68]	0.13	0.16	0.29	[2.29]	0.35	[2.79]
PERF	0.19	0.17	0.42	[2.32]	0.37	[1.86]	0.17	0.26	0.57	[2.64]	0.75	[4.54]
QMJ	0.27	0.38	0.45	[2.90]	0.60	[3.95]	0.17	0.30	0.49	[2.46]	0.68	[5.11]
RMW	0.16	0.18	0.25	[1.71]	0.28	[2.05]	0.16	0.20	0.46	[2.62]	0.57	[3.48]
RMWs	0.21	0.20	0.28	[2.80]	0.27	[2.68]	0.21	0.21	0.34	[3.88]	0.33	[3.70]
R_IA	0.17	0.35	0.31	[2.04]	0.54	[4.28]	0.13	0.16	0.31	[1.94]	0.37	[2.20]
R_ROE	0.38	0.36	0.73	[4.99]	0.68	[4.66]	0.11	0.19	0.34	[1.76]	0.51	[3.42]
SMB	-0.09	-0.13	-0.26	[-1.11]	-0.35	[-1.44]	0.08	0.06	0.26	[1.53]	0.17	[1.02]
SMBs	-0.07	-0.07	-0.14	[-0.95]	-0.13	[-0.81]	0.08	0.09	0.18	[1.31]	0.19	[1.39]
STREV	0.09	0.03	0.21	[1.07]	0.07	[0.41]	0.07	0.03	0.25	[1.34]	0.10	[0.51]

The table lists corporate bond and stock tradable factor performance statistics. SR is the Sharpe ratio, IR is the Information ratio, μ is the average return, and α is the single-factor MKTB (MKTS) alpha. All statistics are reported monthly. μ and α are reported in percent. t-statistics are reported in square brackets with Newey-West standard errors computed with four lags. The sample is split into two subperiods following van Binsbergen et al. (2025). The first sample is from 1986:01 to 1999:12, the second sample is from 2000:01 to 2022:12.

No intercept. For the baseline analysis in Section 3.1.1 we always include an intercept. In the following, we repeat the previous analysis excluding the intercept. Figure IA.11 is the

Table IA.V: Posterior factor probabilities and risk prices for the corporate bond factor zoo

	Factor prob., $\mathbb{E}[\gamma_j \mid \text{data}]$				Price of risk, $\mathbb{E}[\lambda_j \mid \text{data}]$			
	Total prior Sharpe ratio				Total prior Sharpe ratio			
Factors	20%	40%	60%	80%	20%	40%	60%	80%
PEADB	0.602	0.738	0.832	0.820	0.090	0.342	0.634	0.798
MOMBS	0.516	0.612	0.640	0.605	0.074	0.281	0.487	0.602
CREDIT	0.511	0.535	0.580	0.679	0.008	0.034	0.088	0.240
IVOL	0.499	0.510	0.535	0.558	0.005	0.018	0.045	0.112
YSP	0.497	0.506	0.525	0.567	0.003	0.013	0.035	0.100
INFLV	0.498	0.501	0.508	0.515	0.004	0.017	0.041	0.080
LIQNT	0.508	0.502	0.499	0.498	−0.002	−0.006	−0.015	−0.035
MKTB	0.513	0.523	0.512	0.443	0.068	0.179	0.288	0.359
CRY	0.479	0.487	0.513	0.509	0.036	0.123	0.276	0.451
LVL	0.492	0.493	0.489	0.510	-0.000	-0.000	-0.000	0.002
UNCf	0.510	0.504	0.497	0.468	−0.008	−0.027	−0.050	−0.076
INFLC	0.493	0.487	0.485	0.499	-0.000	−0.001	−0.003	−0.007
EPU	0.490	0.492	0.487	0.483	0.003	0.010	0.018	0.029
UNCr	0.502	0.486	0.483	0.469	-0.000	0.000	0.003	0.009
UNC	0.506	0.492	0.494	0.445	−0.004	−0.012	−0.020	−0.024
EPUT	0.489	0.466	0.480	0.475	0.004	0.012	0.027	0.053
VIX	0.483	0.483	0.482	0.449	-0.000	−0.001	−0.005	−0.012
CPTL	0.500	0.489	0.470	0.414	0.001	0.006	0.021	0.045
DRF	0.489	0.477	0.444	0.387	0.026	0.051	0.056	0.022
SZE	0.477	0.479	0.451	0.380	0.012	0.046	0.090	0.103
HMLB	0.496	0.478	0.448	0.364	0.035	0.095	0.139	0.130
STREVB	0.491	0.468	0.423	0.354	0.001	0.005	0.009	0.007
VAL	0.469	0.457	0.416	0.332	0.019	0.071	0.120	0.121
MKTBD	0.470	0.447	0.406	0.336	0.011	0.031	0.042	0.037
DUR	0.463	0.424	0.388	0.353	0.010	−0.014	−0.077	−0.160
LTREVB	0.481	0.457	0.394	0.293	0.025	0.063	0.072	0.054
DEF	0.460	0.443	0.390	0.320	−0.006	−0.021	−0.044	−0.067
MOMB	0.482	0.437	0.386	0.305	−0.005	−0.010	−0.003	0.010
TERM	0.455	0.422	0.372	0.307	0.040	0.071	0.069	0.068
CRF	0.467	0.418	0.364	0.299	0.013	0.050	0.083	0.100

The table reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 16 tradable bond and 14 nontradable factors described in Appendix A. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_{\boldsymbol{f}}^2 \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The factors are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 50 bond portfolios and the 16 tradable bond factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

no intercept analogue to Figure 2 in the paper. While the posterior probabilities are slightly different and the ordering is changed a little, the top five factors remain the same. Table IA.VII complements Table A.2 of Appendix C and provides the full list of posterior probabilities and the associated annualized risk premia (in Sharpe ratio units) for the co-pricing factor zoo across the full range of prior Sharpe ratios, estimated without an intercept. Tables IA.VIII and IA.IX report the corresponding information for the respective bond and stock factor zoos.

Table IA.VI: Posterior factor probabilities and risk prices for the stock factor zoo

	Factor prob., $\mathbb{E}[\gamma_j \vert \text{data}]$				Price of risk, $\mathbb{E}[\lambda_j \vert \text{data}]$			
	Total prior Sharpe ratio				Total prior Sharpe ratio			
Factors	20%	40%	60%	80%	20%	40%	60%	80%
PEAD	0.520	0.579	0.665	0.701	0.034	0.141	0.332	0.552
MKTS	0.510	0.560	0.587	0.566	0.041	0.161	0.317	0.469
IVOL	0.499	0.520	0.508	0.567	0.004	0.017	0.043	0.128
LVL	0.510	0.506	0.517	0.511	0.001	0.002	0.005	0.014
CMAs	0.497	0.492	0.528	0.527	0.020	0.077	0.177	0.294
UNCr	0.499	0.507	0.503	0.509	0.001	0.003	0.009	0.028
CREDIT	0.502	0.512	0.491	0.510	-0.000	−0.001	−0.002	−0.002
EPU	0.504	0.503	0.498	0.509	−0.002	−0.006	−0.013	−0.033
INFLC	0.505	0.502	0.501	0.501	0.000	0.000	0.001	0.002
VIX	0.504	0.503	0.497	0.494	-0.000	−0.002	−0.005	−0.015
INFLV	0.501	0.499	0.494	0.500	-0.000	−0.002	−0.003	−0.005
RMWs	0.497	0.512	0.513	0.461	0.032	0.101	0.188	0.257
EPUT	0.495	0.496	0.490	0.490	0.001	0.004	0.013	0.035
UNCf	0.491	0.497	0.487	0.490	0.000	0.002	0.009	0.035
CPTL	0.500	0.500	0.486	0.476	0.017	0.058	0.103	0.159
YSP	0.494	0.494	0.490	0.482	0.001	0.003	0.008	0.023
UNC	0.487	0.489	0.484	0.491	-0.000	0.001	0.005	0.015
LIQNT	0.488	0.488	0.474	0.493	-0.000	−0.002	−0.008	−0.035
QMJ	0.491	0.485	0.484	0.473	0.049	0.129	0.244	0.390
CPTLT	0.493	0.496	0.486	0.451	0.019	0.064	0.108	0.136
MKTSs	0.526	0.500	0.468	0.410	0.018	0.049	0.073	0.081
LIQ	0.500	0.486	0.461	0.413	0.006	0.023	0.050	0.075
BAB	0.494	0.494	0.458	0.384	0.027	0.075	0.118	0.147
MGMT	0.503	0.478	0.448	0.394	0.057	0.128	0.201	0.248
STREV	0.475	0.467	0.467	0.401	0.007	0.030	0.074	0.115
MOMS	0.500	0.491	0.439	0.377	0.016	0.042	0.054	0.061
R_IA	0.497	0.469	0.452	0.388	0.030	0.068	0.106	0.127
PERF	0.503	0.481	0.450	0.370	0.038	0.094	0.131	0.137
CMA	0.492	0.467	0.433	0.370	0.025	0.056	0.071	0.067
LTREV	0.478	0.471	0.437	0.361	0.007	0.022	0.031	0.033
R_ROE	0.488	0.469	0.429	0.357	0.039	0.082	0.099	0.096
HMLs	0.485	0.461	0.421	0.366	0.006	0.016	0.019	0.011
SMBs	0.484	0.460	0.430	0.353	0.004	0.020	0.038	0.055
HML_DEV	0.481	0.458	0.410	0.375	0.005	0.017	0.053	0.122
RMW	0.502	0.453	0.414	0.325	0.034	0.042	0.019	−0.018
FIN	0.470	0.439	0.409	0.330	0.031	0.033	0.010	−0.010
SMB	0.474	0.440	0.415	0.316	0.011	0.047	0.088	0.098
HML	0.460	0.431	0.401	0.343	0.002	−0.030	−0.082	−0.131

Posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 24 tradable stock and 14 nontradable factors described in Appendix A. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The factors are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 33 stock portfolios and the 24 tradable stock factors ($N = 57$) described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table IA.VII: Posterior factor probabilities and risk prices for the co-pricing factor zoo (no intercept)

	Factor prob., $\mathbb{E}[\gamma_j \mid \text{data}]$				Price of risk, $\mathbb{E}[\lambda_j \mid \text{data}]$			
	Total prior Sharpe ratio				Total prior Sharpe ratio			
Factors	20%	40%	60%	80%	20%	40%	60%	80%
PEADB	0.539	0.632	0.693	0.690	0.059	0.231	0.458	0.644
IVOL	0.528	0.577	0.655	0.699	0.022	0.091	0.232	0.490
PEAD	0.506	0.581	0.613	0.621	0.043	0.168	0.332	0.486
MOMBS	0.533	0.576	0.580	0.516	0.077	0.258	0.440	0.506
CREDIT	0.495	0.512	0.532	0.554	0.010	0.041	0.099	0.209
YSP	0.492	0.510	0.514	0.523	0.004	0.017	0.044	0.108
UNCr	0.513	0.520	0.503	0.484	0.001	0.004	0.012	0.030
LVL	0.496	0.499	0.505	0.509	0.001	0.006	0.015	0.041
INFLC	0.503	0.499	0.501	0.499	−0.001	−0.004	−0.011	−0.028
QMJ	0.508	0.510	0.512	0.471	0.082	0.220	0.377	0.495
CRY	0.500	0.487	0.510	0.475	0.063	0.195	0.395	0.547
MKTS	0.499	0.502	0.501	0.458	0.062	0.188	0.321	0.419
LIQNT	0.486	0.486	0.493	0.486	−0.003	−0.016	−0.044	−0.105
EPUT	0.504	0.493	0.484	0.462	0.003	0.008	0.015	0.020
INFLV	0.483	0.484	0.486	0.486	0.001	0.005	0.009	0.007
VIX	0.493	0.498	0.488	0.456	−0.003	−0.008	−0.018	−0.035
CMAs	0.481	0.484	0.492	0.477	0.016	0.062	0.135	0.220
RMWs	0.513	0.502	0.475	0.443	0.031	0.087	0.152	0.221
EPU	0.503	0.490	0.473	0.457	0.000	0.001	0.002	0.003
SZE	0.491	0.488	0.482	0.427	0.007	0.029	0.067	0.100
CPTLT	0.496	0.476	0.466	0.427	0.030	0.084	0.145	0.216
UNCf	0.498	0.495	0.451	0.409	−0.009	−0.020	−0.013	0.018
UNC	0.476	0.472	0.463	0.425	−0.003	−0.008	−0.009	−0.014
CPTL	0.496	0.479	0.450	0.404	0.020	0.053	0.070	0.059
LIQ	0.495	0.480	0.464	0.390	0.007	0.031	0.065	0.095
MOMS	0.494	0.475	0.444	0.386	0.025	0.071	0.122	0.174
LTREVB	0.495	0.466	0.431	0.371	0.018	0.055	0.087	0.097
SMBs	0.489	0.463	0.435	0.370	0.006	0.022	0.036	0.041
MKTSs	0.484	0.463	0.422	0.383	0.022	0.051	0.081	0.130
VAL	0.475	0.461	0.441	0.375	0.018	0.064	0.118	0.145
MOMB	0.491	0.483	0.427	0.347	−0.001	-0.000	0.005	0.013
R_IA	0.484	0.470	0.429	0.365	0.037	0.088	0.131	0.151
BAB	0.503	0.467	0.423	0.352	0.029	0.062	0.083	0.093
PERF	0.496	0.475	0.437	0.334	0.055	0.115	0.131	0.098
R_ROE	0.493	0.476	0.414	0.354	0.055	0.118	0.153	0.178
STREV	0.477	0.463	0.425	0.363	0.012	0.044	0.085	0.120
CMA	0.489	0.466	0.422	0.347	0.033	0.070	0.078	0.064
LTREV	0.491	0.464	0.417	0.344	0.011	0.034	0.052	0.050
HMLs	0.482	0.450	0.426	0.357	0.006	0.018	0.031	0.038
STREVB	0.486	0.464	0.416	0.347	0.005	0.013	0.023	0.024
HML_DEV	0.486	0.448	0.425	0.347	0.005	0.011	0.039	0.081
DEF	0.474	0.454	0.410	0.344	0.001	−0.003	−0.015	−0.016
HMLB	0.487	0.461	0.412	0.317	0.052	0.129	0.161	0.133
MGMT	0.473	0.450	0.422	0.332	0.062	0.129	0.170	0.166
TERM	0.490	0.442	0.407	0.333	0.039	0.081	0.128	0.157
CRF	0.471	0.443	0.410	0.340	0.018	0.063	0.106	0.132
SMB	0.475	0.439	0.395	0.333	0.014	0.058	0.091	0.102
MKTBD	0.472	0.443	0.395	0.326	0.018	0.033	0.030	0.013
HML	0.462	0.440	0.402	0.331	0.004	−0.017	−0.039	−0.042
RMW	0.478	0.434	0.390	0.327	0.033	0.020	−0.028	−0.080
FIN	0.462	0.443	0.391	0.313	0.038	0.042	0.009	−0.013
MKTB	0.488	0.433	0.383	0.298	0.118	0.204	0.241	0.207
DRF	0.468	0.420	0.376	0.301	0.053	0.076	0.062	0.025
DUR	0.427	0.394	0.376	0.282	0.012	−0.025	−0.086	−0.099

The table reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and stock factors described in Appendix A. All models are estimated without an intercept. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The factors are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table IA.VIII: Posterior factor probabilities and risk prices for the corporate bond factor zoo (no intercept)

	Factor prob., $\mathbb{E}[\gamma_j\|\text{data}]$				Price of risk, $\mathbb{E}[\lambda_j\|\text{data}]$			
	Total prior Sharpe ratio				Total prior Sharpe ratio			
Factors	20%	40%	60%	80%	20%	40%	60%	80%
PEADB	0.611	0.756	0.799	0.753	0.125	0.425	0.669	0.761
MOMBS	0.584	0.727	0.729	0.605	0.174	0.571	0.812	0.746
CREDIT	0.519	0.584	0.642	0.687	0.028	0.113	0.256	0.494
IVOL	0.513	0.553	0.601	0.642	0.024	0.091	0.209	0.446
YSP	0.506	0.512	0.520	0.557	0.006	0.022	0.054	0.139
UNCf	0.516	0.530	0.511	0.459	−0.034	−0.103	−0.169	−0.225
INFLC	0.506	0.489	0.501	0.504	−0.002	−0.007	−0.018	−0.045
INFLV	0.497	0.509	0.507	0.482	0.005	0.017	0.027	0.021
LVL	0.485	0.502	0.505	0.496	0.000	0.003	0.012	0.040
UNCr	0.496	0.509	0.500	0.483	-0.000	0.000	0.004	0.012
VIX	0.499	0.503	0.499	0.468	−0.008	−0.030	−0.072	−0.133
LIQNT	0.488	0.497	0.490	0.471	−0.003	−0.013	−0.028	−0.053
CRY	0.486	0.507	0.512	0.427	0.089	0.288	0.492	0.490
EPU	0.483	0.491	0.483	0.456	0.001	0.001	−0.003	−0.008
EPUT	0.486	0.482	0.482	0.458	0.004	0.012	0.023	0.055
UNC	0.488	0.485	0.469	0.422	−0.009	−0.024	−0.036	−0.050
CPTL	0.498	0.481	0.425	0.342	−0.006	−0.018	−0.013	−0.003
VAL	0.475	0.470	0.434	0.359	0.033	0.112	0.171	0.177
SZE	0.474	0.473	0.433	0.350	0.017	0.055	0.087	0.082
CRF	0.472	0.441	0.390	0.322	0.026	0.086	0.127	0.153
MKTB	0.505	0.463	0.380	0.267	0.203	0.330	0.331	0.245
LTREVB	0.478	0.454	0.381	0.294	0.031	0.075	0.079	0.058
MOMB	0.468	0.440	0.378	0.289	0.005	0.021	0.040	0.053
STREVB	0.471	0.430	0.371	0.289	0.008	0.023	0.037	0.032
MKTBD	0.454	0.441	0.367	0.295	0.017	0.044	0.046	0.031
HMLB	0.472	0.437	0.378	0.264	0.079	0.162	0.179	0.114
TERM	0.462	0.416	0.368	0.285	0.068	0.119	0.162	0.169
DEF	0.449	0.426	0.361	0.285	−0.004	−0.017	−0.032	−0.042
DUR	0.445	0.392	0.346	0.236	−0.009	−0.097	−0.143	−0.083
DRF	0.440	0.390	0.337	0.245	0.053	0.043	0.019	−0.002

The table reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 16 tradable bond and 14 nontradable factors described in Appendix A. All models are estimated without an intercept. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The factors are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 50 bond portfolios and the 16 tradable bond factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table IA.IX: Posterior factor probabilities and risk prices for the stock factor zoo (no intercept)

	Factor prob., $\mathbb{E}[\gamma_j\|\text{data}]$				Price of risk, $\mathbb{E}[\lambda_j\|\text{data}]$			
	Total prior Sharpe ratio				Total prior Sharpe ratio			
Factors	20%	40%	60%	80%	20%	40%	60%	80%
PEAD	0.531	0.583	0.695	0.719	0.038	0.154	0.371	0.604
MKTS	0.522	0.566	0.615	0.589	0.053	0.197	0.385	0.545
IVOL	0.486	0.505	0.525	0.571	0.005	0.023	0.063	0.175
CMAs	0.505	0.509	0.529	0.528	0.022	0.086	0.191	0.316
CREDIT	0.514	0.506	0.512	0.507	-0.000	−0.001	−0.002	−0.003
UNCr	0.503	0.518	0.511	0.506	0.001	0.003	0.010	0.028
QMJ	0.505	0.504	0.511	0.511	0.063	0.170	0.318	0.502
YSP	0.503	0.503	0.510	0.511	0.001	0.003	0.010	0.030
LVL	0.508	0.512	0.494	0.501	0.001	0.004	0.010	0.027
CPTL	0.505	0.515	0.506	0.488	0.020	0.070	0.123	0.184
LIQNT	0.504	0.510	0.499	0.498	-0.000	−0.003	−0.013	−0.048
INFLC	0.497	0.500	0.501	0.513	-0.000	-0.000	0.000	0.002
VIX	0.501	0.503	0.502	0.500	−0.001	−0.002	−0.007	−0.020
EPU	0.515	0.505	0.500	0.479	−0.001	−0.005	−0.011	−0.023
INFLV	0.500	0.500	0.500	0.496	-0.000	−0.001	−0.002	−0.003
EPUT	0.483	0.504	0.493	0.501	0.001	0.007	0.018	0.050
RMWs	0.503	0.510	0.492	0.460	0.039	0.117	0.201	0.279
UNCf	0.486	0.489	0.490	0.483	−0.001	−0.002	0.003	0.019
CPTLT	0.496	0.492	0.492	0.465	0.024	0.075	0.126	0.152
UNC	0.482	0.485	0.493	0.479	0.000	0.001	0.003	0.009
MKTSs	0.504	0.512	0.466	0.412	0.019	0.054	0.077	0.092
STREV	0.485	0.479	0.481	0.446	0.010	0.041	0.093	0.155
LIQ	0.492	0.490	0.470	0.407	0.008	0.031	0.064	0.090
PERF	0.510	0.497	0.459	0.385	0.047	0.115	0.154	0.159
MGMT	0.516	0.479	0.453	0.376	0.075	0.156	0.232	0.258
BAB	0.496	0.484	0.444	0.379	0.039	0.095	0.138	0.164
MOMS	0.497	0.470	0.431	0.381	0.018	0.046	0.064	0.083
SMBs	0.491	0.464	0.428	0.363	0.007	0.027	0.053	0.072
R_ROE	0.478	0.463	0.429	0.341	0.050	0.096	0.115	0.101
LTREV	0.495	0.456	0.413	0.341	0.012	0.030	0.041	0.040
HMLs	0.489	0.441	0.423	0.347	0.008	0.020	0.031	0.021
SMB	0.474	0.453	0.416	0.347	0.018	0.070	0.116	0.131
HML_DEV	0.461	0.444	0.400	0.374	0.008	0.030	0.080	0.166
HML	0.480	0.443	0.404	0.334	0.004	−0.035	−0.095	−0.139
CMA	0.480	0.444	0.401	0.336	0.035	0.068	0.082	0.075
R_IA	0.469	0.428	0.410	0.341	0.038	0.076	0.112	0.133
FIN	0.479	0.418	0.376	0.318	0.043	0.037	0.009	−0.012
RMW	0.479	0.417	0.357	0.319	0.043	0.043	0.013	−0.034

The table reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 24 tradable stock and 14 nontradable factors described in Appendix A. All models are estimated without an intercept. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The factors are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 33 stock portfolios and the 24 tradable stock factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

Cumulative SDF-implied Sharpe ratio. In Figure IA.12 we plot the cumulative SDF-implied Sharpe ratio when subsequently adding factors ordered on their (individual) posterior probability. Overall, the Sharpe ratio is increasing in the number of factors. However, not all of them add the same amount to the implied Sharpe ratio. This is due to the fact that many factors are potentially noisy proxies for the same fundamental sources of risk that are important for the SDF. As shown in Section 2.4, factors that are useful noisy proxies for a particular fundamental source of risk not fully spanned by individual factors will display nonzero market prices of risk (or portfolio weights). The figure nicely illustrates the aggregation property of the BMA-SDF as discussed in Section 2.4.

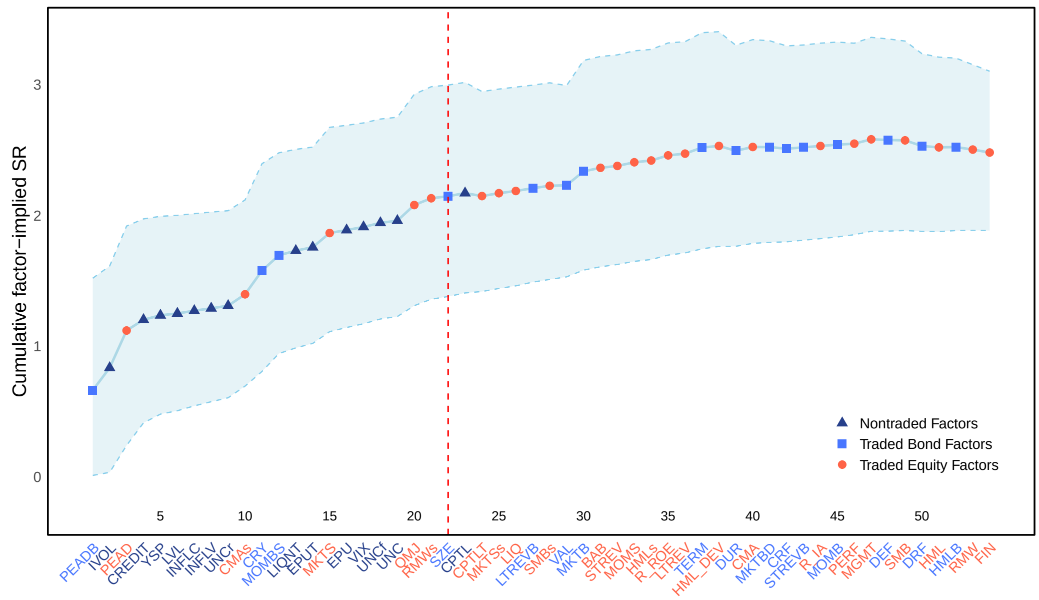


Figure IA.12: Cumulative co-pricing SDF-implied Sharpe ratio.

We incrementally compute the implied Sharpe ratio of the SDF by sequentially adding each of the 54 factors in order of their posterior probability of inclusion (see top Panel of Figure 4). We estimate the factor-implied Sharpe ratio as the annualized standard deviation of the SDF. The vertical red dashed line denotes the posterior median number of factors that should be included in the SDF. The light blue squares (red triangles) represent tradable bond (stock) factors. The dark blue circles represent nontradable factors. The light blue dashed lines and shaded area denote the centred 90% confidence interval. The 54 factors that comprise the co-pricing factor zoo are described in Appendix A. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the 83 bond and stock portfolios and the 40 tradable factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

IA.3.2 Cross-sectional asset pricing

In this section we provide additional results to complement the analysis in Section 3.1.2.

BMA-SDF vs. KNS. There is a legitimate concern that the strong OS performance of the co-pricing BMA-SDF might be driven by the particular, yet rich, selection of test assets that we use in the main text. To address this concern, we also consider the separate pricing of *all* the possible combinations of the 14 different cross-sections comprising our OS test assets. Figure IA.13 of the Internet Appendix visualizes the performance of the BMA-SDF vis-à-vis the best competitor, KNS, by depicting the distributions of different measures of fit across $2^{14} - 1 = 16,383$ OS cross-sections. For the cross-sectional R^2_{OLS}, RMSE, and MAPE, there is virtually no overlap in the distributions for the co-pricing BMA-SDF and KNS, with the former clearly besting the latter, implying that the Bayesian approach delivers strictly better OS pricing than its best competitor. There is only an overlap in the distribution when considering R^2_{GLS} as the measure of fit, yet the BMA-SDF outperforms KNS in 96.6% of the OS cross-sections and its measure of fit concentrates on much higher values.

No intercept. For the baseline analysis in Section 3.1.2 we always include an intercept. In the following, we repeat the previous analysis excluding the intercept. Tables IA.X (IS) and IA.XI (OS) complement Tables 2 (IS) and 3 (OS) by reporting the in- and out-of-sample cross-sectional pricing performance of all models we consider with an estimation that excludes the intercept. Qualitatively, results remain unchanged although most measures of fit for the BMA-SDFs improve at least marginally when the intercept is excluded in the estimation.

Additional asset specific models. Following on from the discussion above, we show in Table IA.XII how well the BMA-SDF performs vis-à-vis an additional set of bond and stock factor models. For pricing the cross-section of bond excess and duration-adjusted returns, we compare the in- and out-of-sample performance of the BMA-SDF to (i) the modified three-factor model of Bai et al. (2019) including MKTB, DRF, and CRF bond factors (BBW3), (ii) the two-factor decomposed bond market factor model from van Binsbergen et al. (2025) (DCAPM), (iii) the DEFTERM model of Fama and French (1993), (iv) the MACRO model of Bali et al. (2021b) comprising MKTB and macroeconomic uncertainty UNC, and (v) the six-factor CWW model of Chung et al. (2019) that adds innovations to the VIX index as a sixth factor to the FF5 model of Fama and French (1993). To price the cross-section of excess stock returns, we consider (i) the Carhart (1997) four-factor model that adds MOMS to the Fama and French (1992) three-factor model (FFC4), (ii) the Hou et al. (2015) four-factor model (HXZ4), (iii) the five-factor model of Fama and French (2015) which augments their three-factor model with the RMW and CMA factors ($FF5^{2015}$), (v) the $FF5^*$ model of Daniel et al. (2020) which removes unpriced risk from the original FF5 factors, and (vi) the FF6 model which augments the FF5 model with MOMS.

In addition to the models listed above and examined in Table IA.XII, we explore the latest (five-)factor corporate bond model proposed by Dick-Nielsen et al. (2025) that includes bond market, bond age, 1-year firm CAPEX growth, stock momentum and within-firm value as factors based on corporate bond returns. Again, we consider a large set of out-of-sample test

Table IA.X: In-sample cross-sectional asset pricing performance (no intercept for BMA-SDF)

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Co-pricing bonds and stocks											
RMSE	0.209	0.201	0.185	0.165	0.260	0.278	0.258	0.259	0.230	0.166	0.197
MAPE	0.158	0.149	0.135	0.121	0.194	0.221	0.198	0.192	0.171	0.126	0.132
R^2_{OLS}	0.195	0.254	0.369	0.495	−0.244	−0.426	−0.233	−0.238	0.023	0.489	0.282
R^2_{GLS}	0.051	0.129	0.204	0.266	0.078	0.083	0.087	0.078	0.263	0.176	0.267
Panel B: Pricing bonds											
RMSE	0.171	0.130	0.104	0.091	0.209	0.214	0.201	0.206	0.162	0.192	0.091
MAPE	0.116	0.093	0.078	0.069	0.146	0.135	0.143	0.146	0.128	0.111	0.067
R^2_{OLS}	0.277	0.578	0.732	0.796	−0.083	−0.134	−0.006	−0.049	0.347	0.088	0.794
R^2_{GLS}	0.096	0.241	0.337	0.392	0.172	0.195	0.238	0.175	0.549	0.071	0.419
Panel C: Pricing stocks											
RMSE	0.240	0.258	0.249	0.231	0.292	0.264	0.275	0.292	0.365	0.162	0.175
MAPE	0.192	0.201	0.189	0.172	0.229	0.211	0.221	0.226	0.304	0.133	0.141
R^2_{OLS}	−0.066	−0.229	−0.145	0.015	−0.570	−0.282	−0.392	−0.574	−1.457	0.515	0.433
R^2_{GLS}	0.060	0.146	0.237	0.317	0.120	0.118	0.130	0.121	0.299	0.311	0.493

The table presents the cross-sectional in-sample asset pricing performance of different models pricing bonds and stocks jointly (Panel A), bonds only (Panel B) and stocks only (Panel C), respectively. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors with an average posterior probability greater than 50%. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. By panel the models are estimated with the respective factor zoos and test assets. All BMA-SDFs are estimated without an intercept. Test assets are the 83 bond and stock portfolios and the 40 tradable bond and stock factors (Panel A), the 50 bond portfolios and 16 tradable bond factors (Panel B), and the 33 stock portfolios and 24 tradable stock factors (Panel C), respectively. All are described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

assets to compare the model performance to our BMA-SDF (where we consider both the co-pricing as well as the bond only BMA-SDF) and we visualize the results using the distributions of different measures of fit. In Figure IA.14 we consider the $2^7 - 1 = 127$ possible combinations of our OS corporate bond portfolios and in Figure IA.15 we repeat the analysis for one million sets of 50 OS test assets based on combinations of corporate bond portfolios formed with the Jensen et al. (2023) characteristics. Throughout, we first estimate the models using the baseline set of IS test assets and then we use the resulting SDF to price the respective OS test assets *without* re-estimation. The green distributions correspond to the pricing performance of the DFPS model, while the red and blue distributions correspond to the pricing performance of the co-pricing and corporate bond only BMA-SDF, respectively. While there is substantial overlap in the distributions of all measures of fit for the 127 combinations of our baseline OS bond

Table IA.XI: Out-of-sample cross-sectional asset pricing performance (no intercept for BMA-SDF)

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Co-pricing bonds and stocks											
RMSE	0.111	0.101	0.094	0.088	0.224	0.154	0.139	0.223	0.171	0.160	0.153
MAPE	0.080	0.075	0.069	0.065	0.192	0.129	0.102	0.190	0.135	0.143	0.130
R^2_{OLS}	0.391	0.498	0.568	0.614	−1.478	−0.161	0.053	−1.444	−0.442	−0.268	−0.159
R^2_{GLS}	0.032	0.070	0.104	0.133	0.028	0.034	0.036	0.028	0.090	0.065	0.028
Panel B: Pricing bonds											
RMSE	0.120	0.110	0.105	0.101	0.129	0.128	0.140	0.133	0.102	0.114	0.100
MAPE	0.087	0.080	0.077	0.076	0.094	0.092	0.104	0.098	0.084	0.083	0.073
R^2_{OLS}	0.087	0.233	0.299	0.360	−0.051	−0.029	−0.231	−0.112	0.342	0.180	0.375
R^2_{GLS}	0.056	0.107	0.133	0.158	−0.004	0.024	−0.032	−0.007	0.101	0.066	0.045
Panel C: Pricing stocks											
RMSE	0.102	0.087	0.078	0.072	0.123	0.119	0.116	0.124	0.163	0.078	0.104
MAPE	0.076	0.068	0.063	0.059	0.089	0.085	0.082	0.091	0.127	0.060	0.082
R^2_{OLS}	0.334	0.515	0.614	0.666	0.032	0.099	0.136	0.019	−0.696	0.613	0.305
R^2_{GLS}	0.054	0.133	0.208	0.264	0.103	0.065	0.099	0.107	0.100	0.207	0.072

The table presents the cross-sectional out-of-sample asset pricing performance of different models pricing bonds and stocks jointly (Panel A), bonds only (Panel B) and stocks only (Panel C), respectively. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors with an average posterior probability greater than 50%. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. The models are first estimated using the baseline IS test assets. All BMA-SDFs are estimated without an intercept. The resulting SDF is then used to price (with no additional parameter estimation) each set of the OS assets. The IS test assets are the same as in Table IA.X. OS test assets are the combined 154 bond and stock portfolios (Panel A), as well as the separate 77 bond and stock portfolios (Panels B and C). All are described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

test assets, the BMA-SDF outperforms the DFPS model in 60% to 93% of the cross-sections, depending on whether we consider the bond only or the co-pricing BMA-SDF and depending on the measure of fit. For the one million sets of OS test assets, the results become considerably stronger: the bond only BMA-SDF outperforms the DFPS model in nearly 100% of the cross-sections based on the cross-sectional R^2_{GLS} and in over 80% of the cross-sections for the other measures of fit.

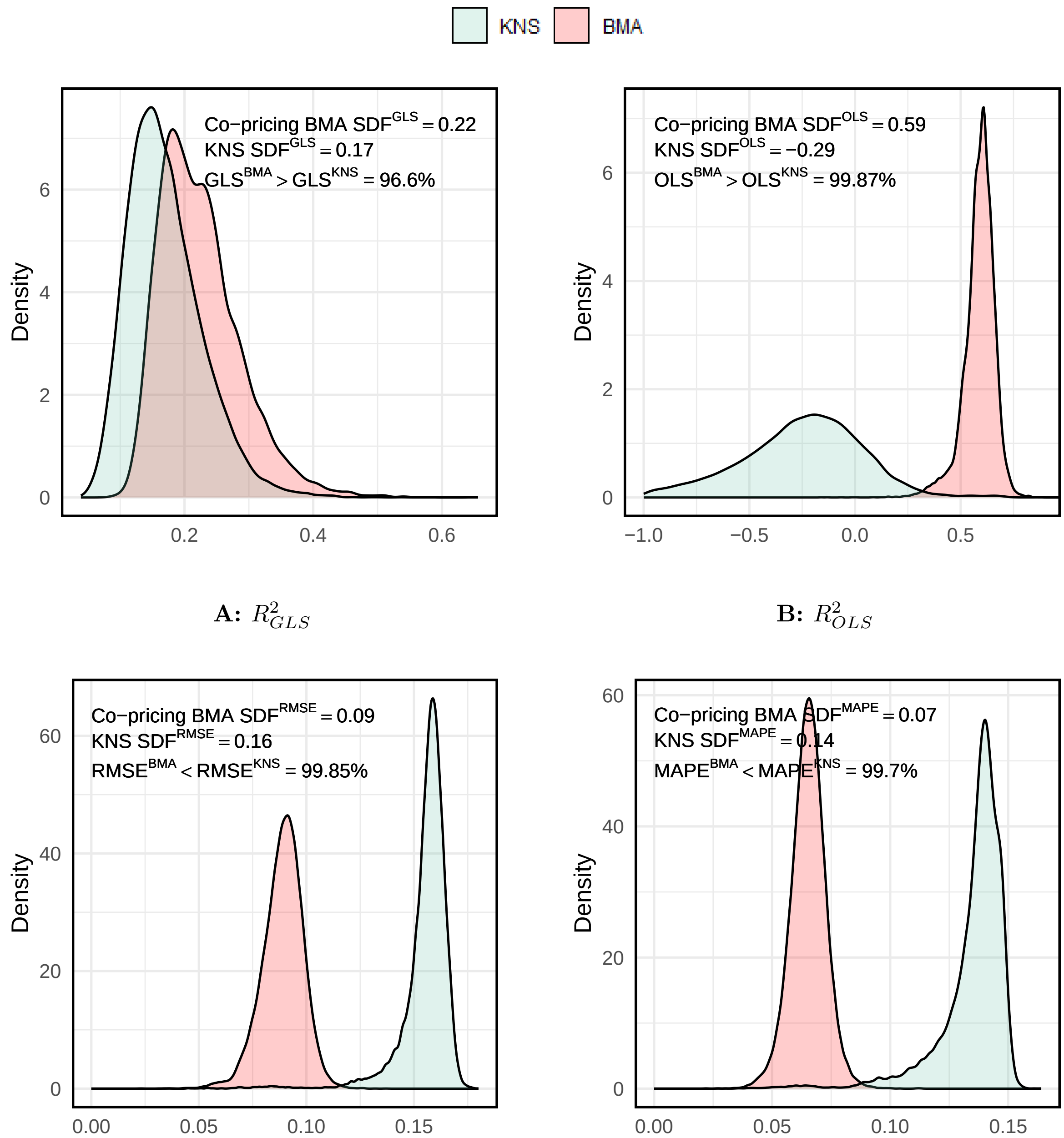
In summary—and not very surprisingly given the results in Section 3.1.2—our BMA-SDFs outperform all additional models originally designed to price the individual bond and stock cross-sections, respectively.

Table IA.XII: Cross-sectional asset pricing performance: Additional models

	BMA-SDF	Bond factor models					Stock factor models				
	BMA-80%	BBW3	DCAPM	DEFTERM	MACRO	CWW	FFC4	HXZ4	$FF5^{2015}$	$FF5^{*}$	FF6
Panel A: In-sample co-pricing stocks and bonds											
RMSE	0.167	0.270	0.250	0.220	0.279	0.258	0.236	0.283	0.247	0.272	0.242
MAPE	0.125	0.217	0.192	0.171	0.222	0.198	0.174	0.236	0.193	0.217	0.193
R^2_{OLS}	0.487	−0.342	−0.158	0.103	−0.438	−0.231	−0.029	−0.478	−0.125	−0.367	−0.083
R^2_{GLS}	0.285	0.087	0.080	0.077	0.083	0.087	0.091	0.116	0.111	0.127	0.117
Panel B: Out-of-sample co-pricing stocks and bonds											
RMSE	0.090	0.147	0.145	0.144	0.150	0.139	0.227	0.272	0.229	0.152	0.234
MAPE	0.065	0.124	0.117	0.115	0.125	0.102	0.203	0.253	0.203	0.121	0.210
R^2_{OLS}	0.603	−0.068	−0.035	−0.018	−0.111	0.049	−1.544	−2.648	−1.580	−0.135	−1.697
R^2_{GLS}	0.124	0.040	0.028	0.025	0.035	0.036	0.034	0.022	0.049	0.031	0.051
Panel C: Out-of-sample pricing stocks											
RMSE	0.076	0.114	0.117	0.115	0.119	0.102	0.079	0.084	0.089	0.097	0.075
MAPE	0.057	0.082	0.085	0.083	0.085	0.072	0.058	0.065	0.068	0.072	0.059
R^2_{OLS}	0.629	0.171	0.117	0.156	0.097	0.327	0.597	0.549	0.489	0.403	0.641
R^2_{GLS}	0.276	0.127	0.064	0.046	0.061	0.114	0.151	0.196	0.186	0.119	0.208
Panel D: Out-of-sample pricing bonds											
RMSE	0.101	0.123	0.136	0.140	0.127	0.138	0.134	0.122	0.127	0.122	0.130
MAPE	0.074	0.092	0.099	0.103	0.092	0.100	0.098	0.087	0.092	0.090	0.095
R^2_{OLS}	0.354	0.050	−0.164	−0.243	−0.015	−0.197	−0.136	0.058	−0.012	0.055	−0.073
R^2_{GLS}	0.107	0.045	0.020	0.015	0.033	0.009	−0.048	−0.061	0.019	0.031	−0.019

Panel A presents the cross-sectional in-sample asset pricing performance of different bond and stock asset pricing models. Panels B, C and D present the out-of-sample asset pricing performance for the joint, bond and stock cross-sections, respectively. For bonds we consider five models: (i) the modified three-factor model of Bai et al. (2019) including MKTB, DRF, and CRF bond factors (BBW3), (ii) the two-factor decomposed bond market factor model from van Binsbergen et al. (2025) (DCAPM), (iii) the DEFTERM model of Fama and French (1993), (iv) the MACRO model of Bali et al. (2021b) comprising MKTB and macro economic uncertainty UNC, and (v) the six-factor CWW model of Chung et al. (2019) that adds innovations to the VIX index as a sixth factor to the FF5 model of Fama and French (1993). For stocks we consider six models: (i) the Carhart (1997) four-factor model that adds MOMS to the Fama and French (1992) three-factor model (FFC4), (ii) the Hou et al. (2015) four-factor model (HXZ4), (iii) the five-factor model of Fama and French (2015) which augments their three-factor model with the RMW and CMA factors ($FF5^{2015}$), (v) the $FF5^{*}$ model of Daniel et al. (2020) which removes unpriced risk from the original FF5 factors, and (vi) the FF6 model which augments the FF5 model with MOMS. IS test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors. OS test assets are the combined 154 bond and stock portfolios (Panel B), as well as the separate 77 bond and stock portfolios (Panels C and D). All are described in Section 1. All models are first estimated using the baseline IS test assets (Panel A) and then used to price (with no additional parameter estimation) each set of OS assets (Panels B to D). We use GMM-GLS to estimate factor risk prices for bond and stock specific factor models. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

Separate pricing of bonds and stocks. In Section 3.1.2 we show that we need information from the joint factor zoo to price the joint cross-section of stock and bond excess returns. Here, we examine whether the co-pricing BMA-SDF can also price well bonds and stocks *individually*. In Figure IA.16 we report OS R^2_{GLS} and R^2_{OLS} for the separate pricing of these two asset classes using the $2^7 - 1 = 127$ possible combinations of our OS corporate bond portfolios in Panels A and B, and the same number of combinations of OS stock portfolios in Panels C and D, respectively. Clearly, the co-pricing BMA-SDF can individually price the respective bond and stock cross-sections well, implying that the superior performance of the co-pricing BMA-SDF is not due to the fact that it prices one cross-section better than the other. Nevertheless, the asset-class-specific BMA-SDFs also price the respective cross-sections very well. That is, using only information from the bond market factor zoo delivers a pricing performance for the cross-section of bond excess returns that is only marginally worse than the one achievable with the co-pricing BMA-SDF. Similarly, the stock-only BMA-SDF does price stock returns very well OS. However, the respective factor zoos fail at "cross-pricing." Clearly, the information in the bond factor zoo alone is insufficient to price the cross-section of stock returns and, vice versa, information from the stock market is not sufficient to price the cross-section of corporate bond excess returns.



A: R^2_{GLS}

B: R^2_{OLS}

C: RMSE

D: MAPE

Figure IA.13: Pricing out-of-sample stocks and bonds with BMA-SDF and KNS.

This figure plots the distributions of R^2_{GLS}, R^2_{OLS}, RMSE and MAPE in Panels A, B, C and D, respectively, across 16,383 possible OS bond and stock cross-sections using the 14 sets of bond and stock test assets ($2^{14} - 1 = 16,383$). KNS stands for the SDF estimation of Kozak, Nagel, and Santosh (2020), with tuning parameter and number of factors chosen by twofold cross-validation. The models are first estimated using the baseline IS test assets and the resulting SDF is then used to price (with no additional parameter estimation) each set of the 16,383 OS combinations of test assets. The BMA-SDF is computed with a prior Sharpe ratio value set to 80% of the ex post maximum Sharpe ratio of the IS test assets. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

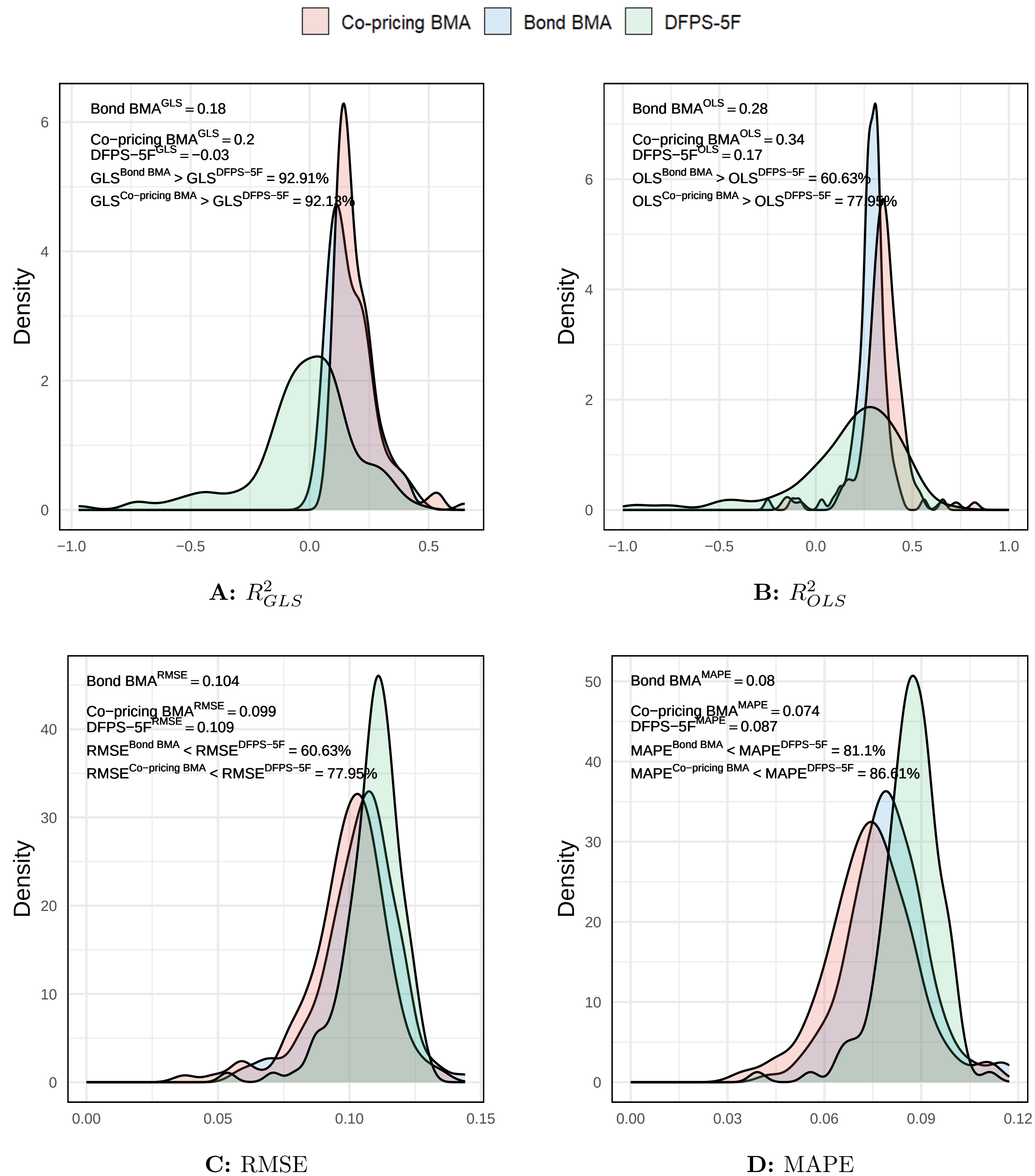


Figure IA.14: Pricing 127 sets of out-of-sample bond portfolios with BMA-SDF and DFPS.

This figure plots the distributions of R^2_{GLS}, R^2_{OLS}, RMSE and MAPE in Panels A, B, C and D, respectively, across 127 possible bond cross-sections using the 7 sets of bond test assets ($2^7 - 1 = 128$). DFPS stands for the Dick-Nielsen et al. (2025) five-factor corporate bond model that includes the following factors (all formed with bond returns): bond market, bond age, 1-year firm CAPEX growth, stock momentum and within-firm value. The models are first estimated using the baseline set of IS test assets and then used to price (with no additional parameter estimation) each set of the 127 combinations of corporate bond test assets. The green distributions correspond to the pricing performance of the DFPS model. The red (blue) distributions correspond to the pricing performance of the co-pricing (corporate bond only) BMA-SDF. The BMA-SDFs are computed with a prior Sharpe ratio value set to 80% of the ex post maximum Sharpe ratio of the IS test assets. All data are standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

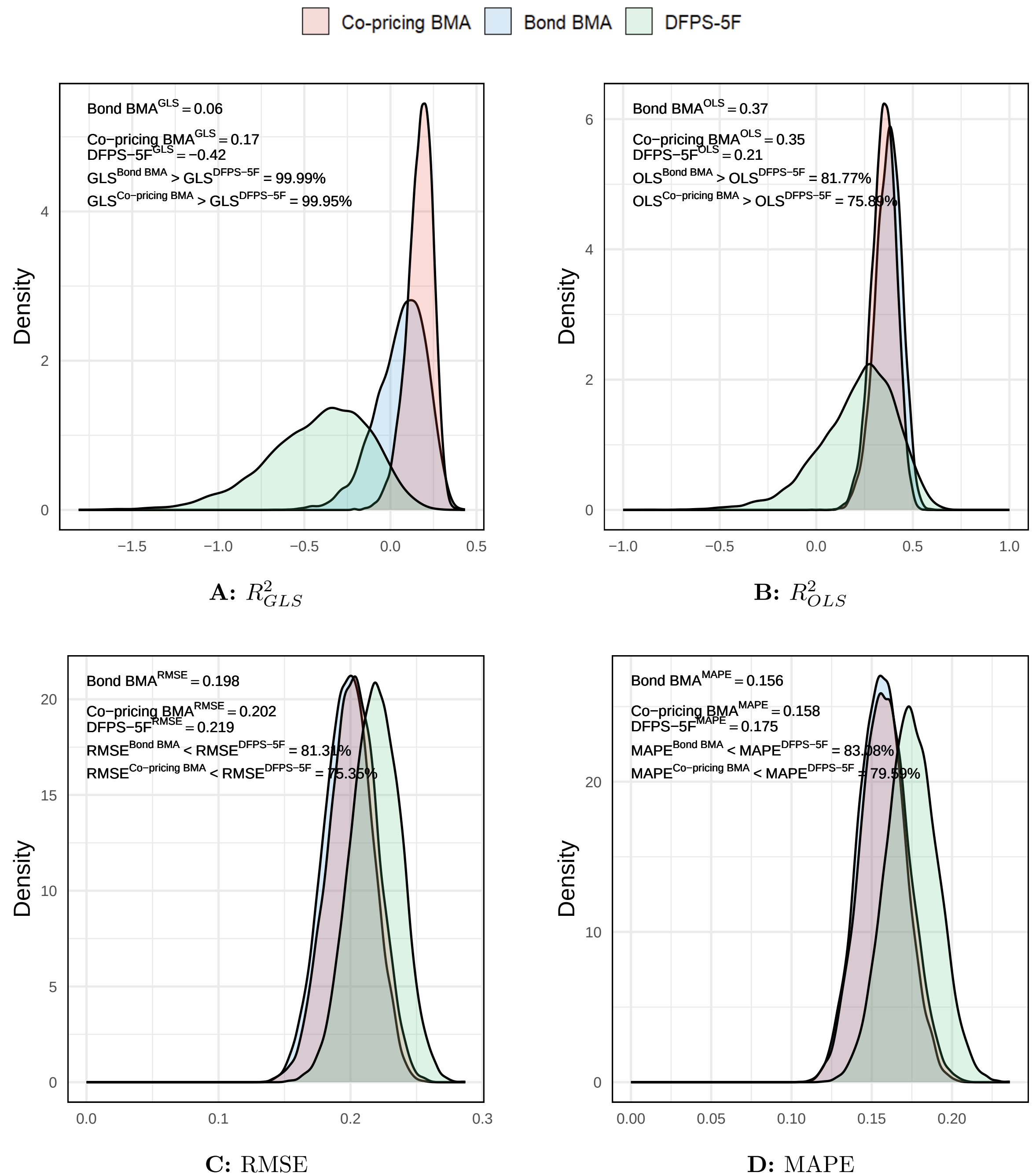
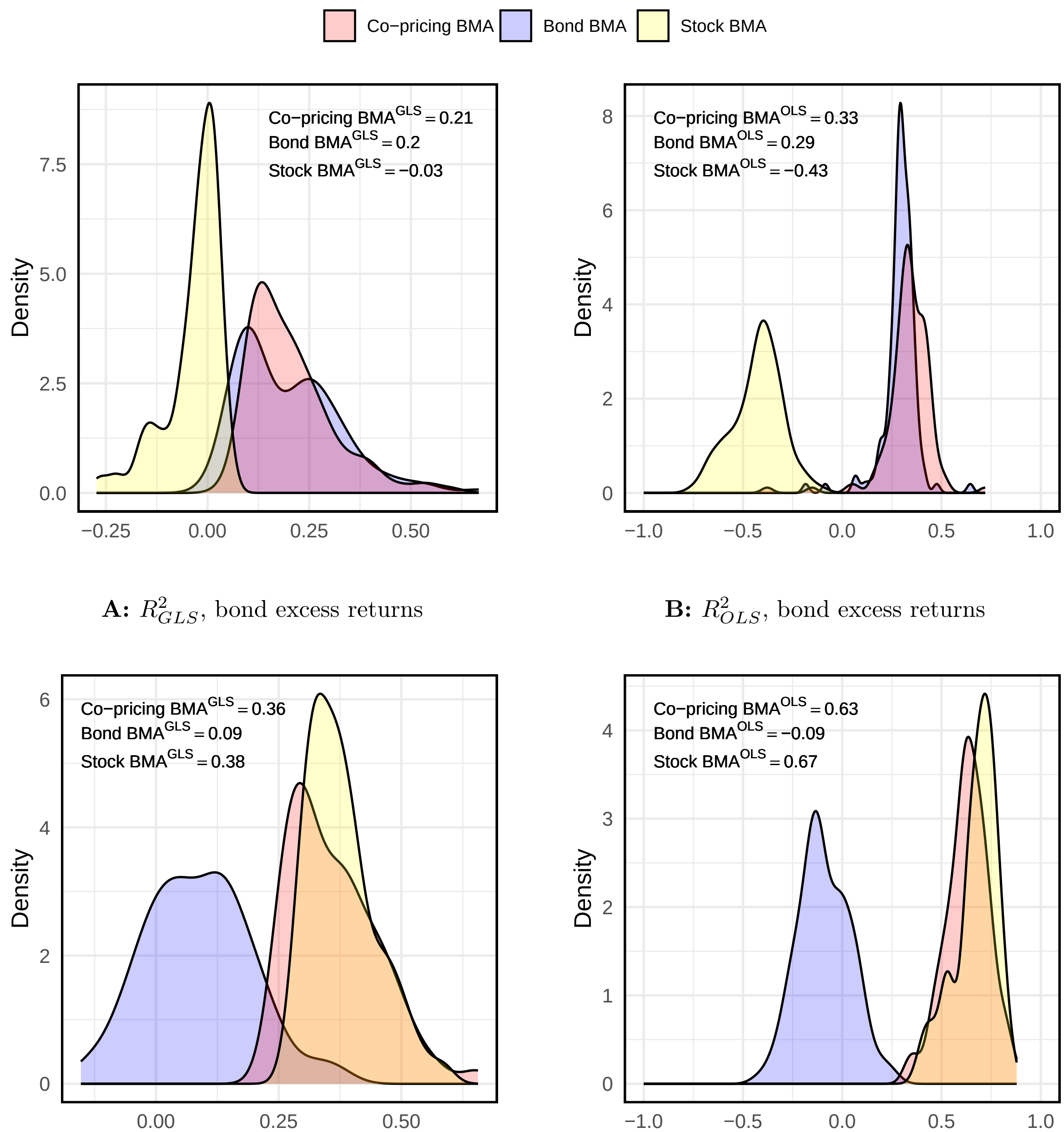


Figure IA.15: Pricing millions out-of-sample bond portfolios with BMA-SDF and DFPS.

This figure plots the distributions of R^2_{GLS}, R^2_{OLS}, RMSE and MAPE in Panels A, B, C and D, respectively, across one million possible combinations of corporate bond portfolios formed with the Jensen et al. (2023) characteristics *without* re-estimating the respective SDFs. Each combination of corporate bond OS test assets is set to have $N = 50$. DFPS stands for the Dick-Nielsen et al. (2025) five-factor corporate bond model that includes the following factors (all formed with bond returns): bond market, bond age, 1-year firm CAPEX growth, stock momentum and within-firm value. The models are first estimated using the baseline set of IS test assets and then used to price (with no additional parameter estimation) each set of the one million OS corporate bond test assets. The green distributions correspond to the pricing performance of the DFPS model. The red (blue) distributions correspond to the pricing performance of the co-pricing (corporate bond only) BMA-SDF. The BMA-SDFs are computed with a prior Sharpe ratio value set to 80% of the ex post maximum Sharpe ratio of the IS test assets. All data are standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).



A: R^2_{GLS}, bond excess returns

B: R^2_{OLS}, bond excess returns

C: R^2_{GLS}, stock excess returns

D: R^2_{OLS}, stock excess returns

Figure IA.16: Separate out-of-sample pricing of bond and stock excess returns.

This figure plots the distributions of R^2_{GLS} (Panels A and C) and R^2_{OLS} (Panels B and D) across 127 possible bond (Panels A and B) and stock (Panels C and D) cross-sections using the 7 sets of bond and stock test assets ($2^7 - 1 = 128$), respectively. All BMA-SDFs are first estimated using the baseline set of the respective IS test assets described in Section 1 for the co-pricing, bond and stock factor zoos respectively. The BMA-SDFs are then used to price (with no additional parameter estimation) each set of the 127 OS combinations of test assets. The red distributions corresponds to the pricing performance of the co-pricing BMA-SDF. The blue (yellow) distribution corresponds to the pricing performance of the bond and stock only BMA-SDFs, respectively. The BMA-SDFs are computed with a prior Sharpe ratio value set to 80% of the ex post maximum Sharpe ratio of the IS test assets. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

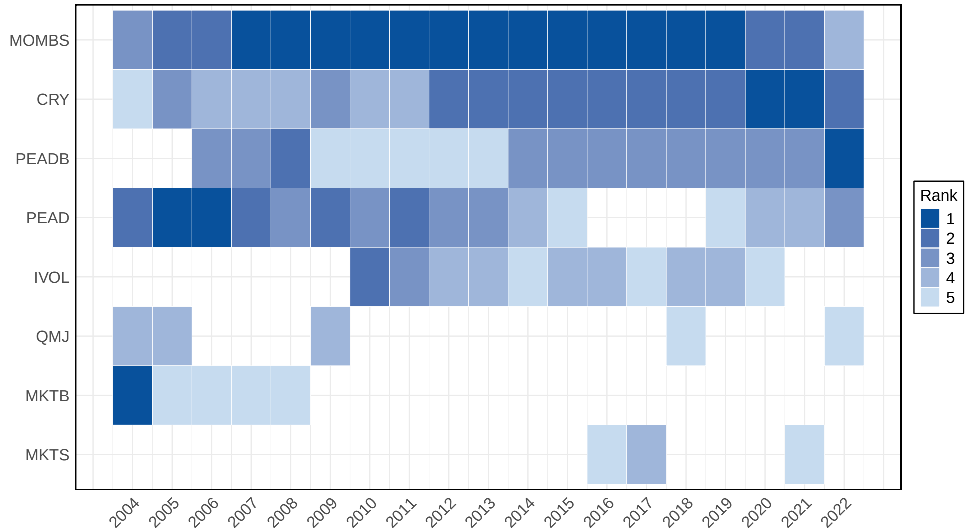
IA.3.3 The saliency of factors over time

In this section we provide additional results to complement the analysis in Section 3.1.3 of the paper. To investigate the importance of factors over time, we split our sample in half for two sub-samples with 222 monthly observations each. We first estimate the model for the first subsample spanning July 1986 to June 2004 and then re-estimate every year adding twelve new observations at each iteration. Similarly, we go backwards in time starting with the second subsample from July 2004 to December 2022 and add one year of data at every step. We follow our methodology described in Section 2 and, throughout, we fix the shrinkage at 80% of the corresponding ex post maximum Sharpe ratio for the respective window.

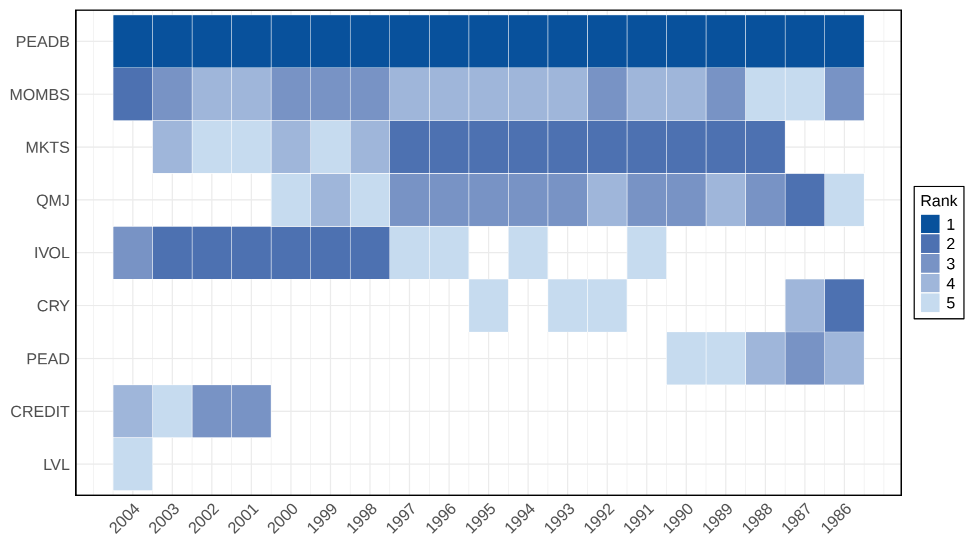
While Figure 6 presents the top factors based on their posterior probability, Figure IA.17 provides the rankings based on the market prices of risk estimates. In Figures IA.18 (forward estimation) and IA.19 (backward estimation) we plot the time series of posterior probabilities (top panel) and market prices of risk (bottom panel) for the most and least likely factors in Figure 2. Overall, the results remain very consistent over time.

IA.3.4 Commonality in pricing

We gauge the degree of commonality in pricing implications of the factors in the zoo by performing a principal component analysis on the matrix $\boldsymbol{C}^T\boldsymbol{C}$ (in the OLS case, or $\boldsymbol{C}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{C}$ in the GLS case). In the cross-sectional layer of our estimation method (encoded by the likelihood function in equation (2)), the "regressors" are the loadings in the $N \times K$ matrix of covariances between test assets and factors ($\boldsymbol{C}$). $\boldsymbol{C}^T\boldsymbol{C}$ captures how factors project onto the space of returns (and vice versa), and its PCs are closely related to the Canonical Correlation Analysis (CCA) of returns and factors. The SVD of $\boldsymbol{C}$ (properly normalized) yields the canonical variables, and the eigendecomposition of $\boldsymbol{C}^\top\boldsymbol{C}$ yields the squared canonical correlations (as eigenvalues) and canonical directions for the factors. Since in the SVD of $\boldsymbol{C}$ we get linear combinations of the returns as the left singular vectors, and linear combinations of the factors as right singular vectors, the PCA of $\boldsymbol{C}^\top\boldsymbol{C}$ is really about finding the linear combinations of returns and factors that are maximally correlated with each other. This is a natural approach for identifying the footprint of common sources of priced risks in standard asset pricing settings (as, e.g., in Ross (1976), Chamberlain and Rothschild (1983), and Giglio and Xiu (2021)), as the common risks would manifest themselves as exploding eigenvalues in both returns and factors. The results of the analysis are summarized in Figure IA.20 and they are striking. The largest five principal components of the factor loadings explain more than 99% of their *cross-sectional* variation (in the OLS case, and more than 80% in the GLS case). This highlights that the factor zoo is akin to a jungle of noisy proxies of common underlying sources of risk.



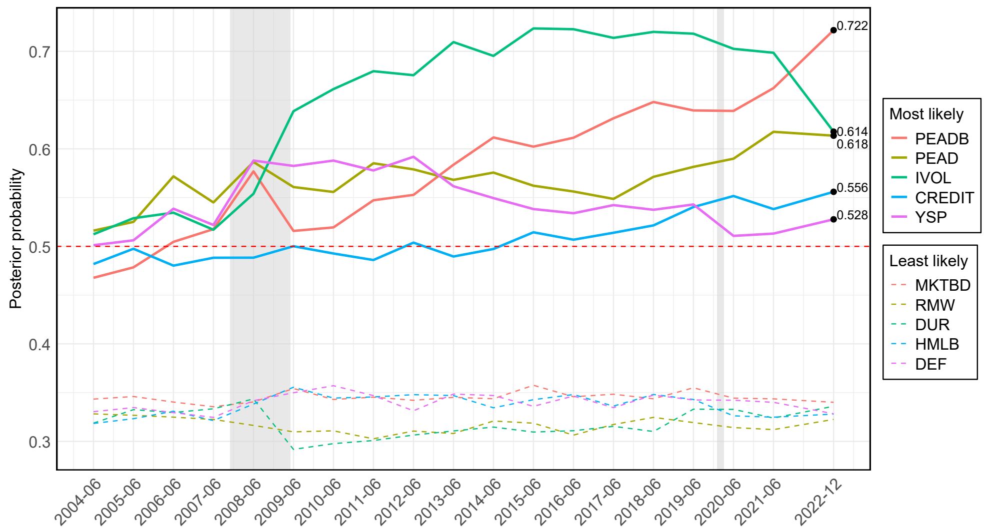
A: Expanding forward estimation



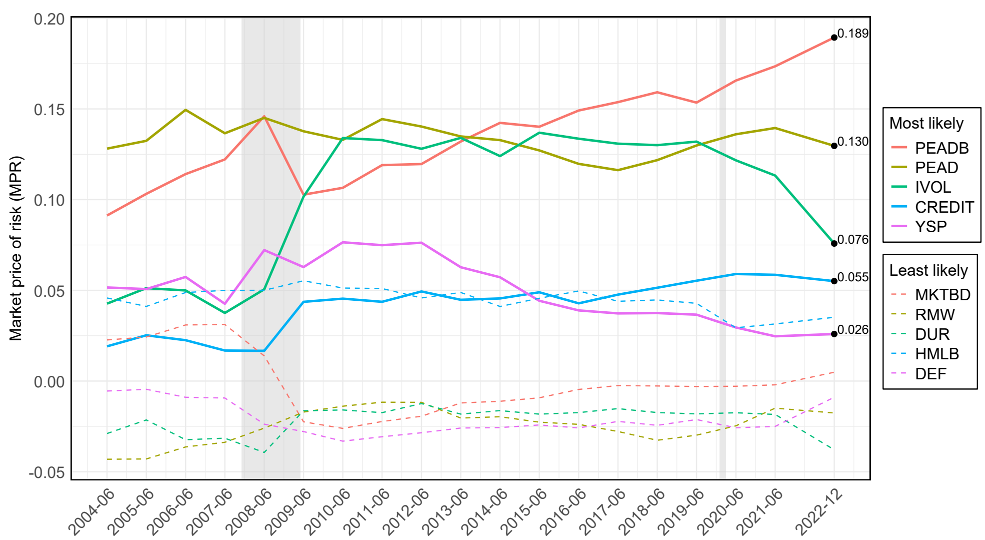
B: Expanding backward estimation

Figure IA.17: Time-varying factor importance based on the market price of risk

The figure highlights the top five factors over time, ordered by their posterior market prices of risk $\mathbb{E}[\lambda_{j,t}|\text{data}_t]$, and the number of times they are present in the top five, estimated using expanding samples going forward (Panel A) and backward (Panel B) in time. We use half of the sample as the initial window ($T = 222$) and then re-estimate the model every year with an expanding sample. The factors are ordered by the total number of times they are present in the 'top five.' The results are shown for prior level of Sharpe ratio shrinkage set to 80% of the ex post maximum up until year t.



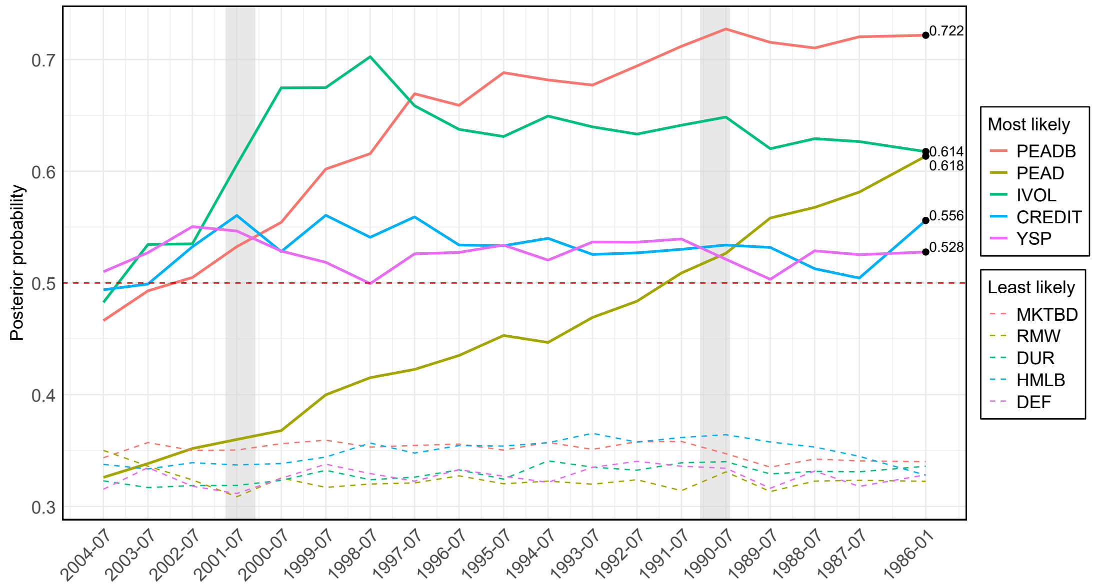
A: Expanding forward estimation – time-varying posterior probabilities.



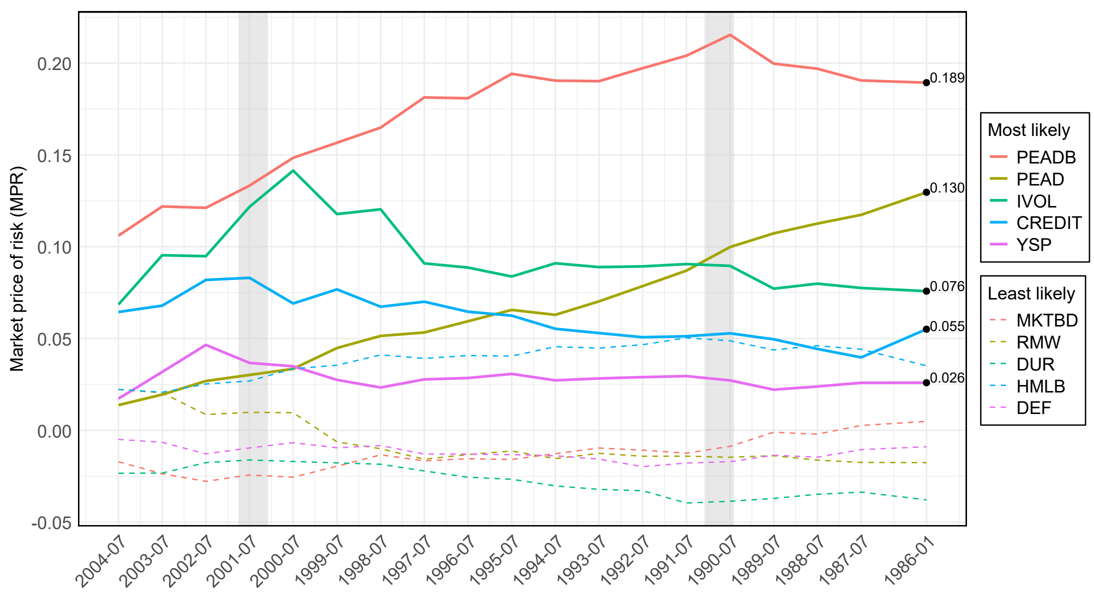
B: Expanding forward estimation – time-varying market prices of risk (MPR).

Figure IA.18: Time-varying posterior probabilities and market prices of risk (forward expansion).

Time-varying posterior probabilities, $\mathbb{E}[\gamma_{j,t}|\text{data}_t]$ (Panel A), and the posterior mean of (annualized) risk prices, $\mathbb{E}[\lambda_{j,t}|\text{data}_t]$ (Panel B), of the most (least) likely five factors estimated with an expanding window (forward in time). We use half of the sample as the initial window ($T = 222$), implying the first estimation begins in July 2004. The model is re-estimated every 12-months. The results are shown for prior level of Sharpe ratio shrinkage set to 80% of the ex-post maximum. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. The test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors described in Section 1.



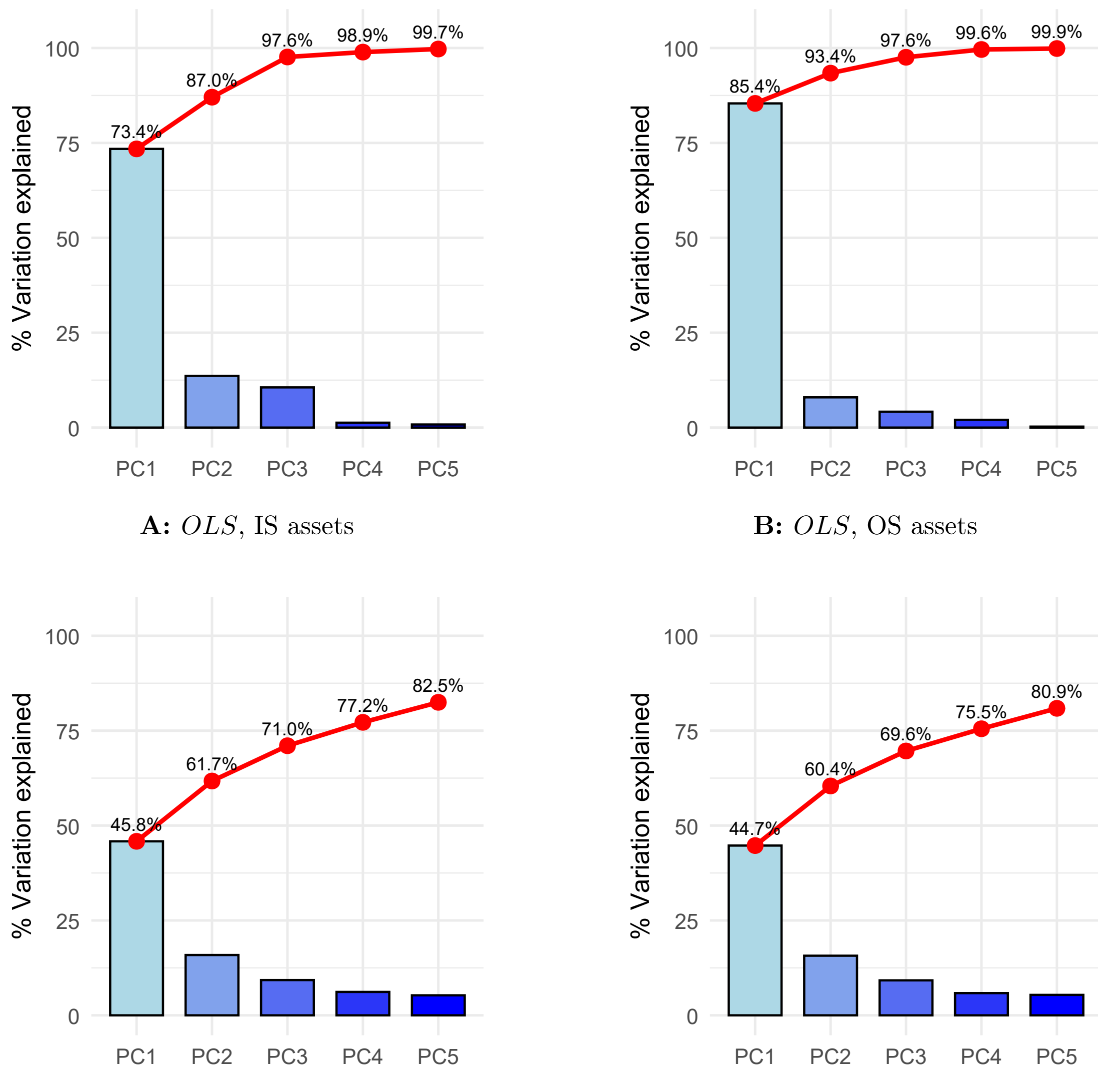
A: Expanding backward estimation – time-varying posterior probabilities.



B: Expanding backward estimation – time-varying market prices of risk.

Figure IA.19: Time-varying posterior probabilities and market prices of risk (backward expansion).

Time-varying posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$ (Panel A), and the posterior mean of (annualized) risk prices, $\mathbb{E}[\lambda_j|\text{data}]$ (Panel B), of the most (least) likely five factors estimated with an expanding window (backward in time). We use half of the sample as the initial window ($T = 222$), implying the first estimation begins in July 2004. The model is re-estimated every 12 months. The results are shown for a prior level of Sharpe ratio shrinkage set to 80% of the ex post maximum. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. The test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors described in Section 1.



A: *OLS*, IS assets

B: *OLS*, OS assets

C: *GLS*, IS assets

D: *GLS*, OS assets

Figure IA.20: The factor jungle: Commonality in cross-sectional pricing.

Principal component decomposition of the matrix $H = \hat{\boldsymbol{C}}^{\top} W \hat{\boldsymbol{C}}$ where $\hat{\boldsymbol{C}} \in \mathbb{R}^{N \times K}$ denotes the posterior mean of the covariance matrix of factors and returns, and W is either an identity matrix (OLS case) or the inverse of the (posterior mean of the) covariance matrix of the test assets (GLS case). Since in the SVD of $\hat{\boldsymbol{C}}$ we get linear combinations of the returns as the left singular vectors, and linear combinations of the factors as right singular vectors, the PCA of $\boldsymbol{C}^{\top}\boldsymbol{C}$ finds the linear combinations of returns and factors that are maximally correlated with each other. We estimate H with the in- and out-of-sample co-pricing test assets and the factor zoo with self-pricing as in the main text. The IS test assets in Panels A and C are the 83 bond and stock portfolios and 40 tradable bond and stock factors. The OS test assets in Panels B and D are the combined 154 bond and stock portfolios. Throughout, we use the co-pricing factor zoo comprising the 40 tradable and 14 nontradable factors. All are described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

IA.4 The PEAD factor

Recent work by Martineau (2022) documents that, in the time series, the (stock) PEAD effect has diminished in recent years. While the author raises interesting points regarding the decay in time series predictability of PEAD, he does not comment on the robustness of using PEAD to form long-short portfolios (i.e., the cross-sectional predictability of PEAD within a portfolio context). In this section, we document that this dimension of the PEAD factor remains robust and is not driven purely by micro-cap stocks. In addition, we confirm the same result for the corporate bond version of the PEAD factor (i.e., PEADB).

To form the bond and stock PEAD factors, we first form tercile portfolios based on firm market capitalization. Thereafter, within each size tercile, we create quintile portfolios sorted on earnings announcement returns, `AnnouncementReturn`, obtained from `Open Asset Pricing`. Each PEAD factor is long in Q5 (high PEAD) and short Q1 (low PEAD), within each size tercile.[5] We denote the small, mid and large cap PEAD factor as Small, Mid and Large respectively.

Table IA.XIII reports the monthly average returns, alphas, Sharpe ratios and Information ratios. In Panel A and C, we exclude "micro-cap" stocks, by filtering out any stocks in portfolio formation month t, which have a market capitalization below the 20^{th} percentile in that month. Across all subsamples, the small- and mid-cap PEAD factors yield large monthly premia, alphas and Sharpe ratios. The cross-sectional 'anomaly decay' effect in PEAD for small caps is not present, regardless of whether micro-cap stocks are filtered out or not. For mid-cap stocks, the premia are reduced, but still economically large and statistically significant for the latter part of the sample. The large-cap PEAD factor yields a statistically significant five-factor alpha with equal weights once micro-cap stocks are excluded. Performance of the large-cap PEAD factor is diminished over later parts of the sample consistent with large-cap anomaly decay. In Panels B and D, where we include micro-cap stocks, the small-cap PEAD premium is marginally increased. The mid- and large-cap PEAD is materially unaffected. Overall, our results strongly confirm the efficacy of the PEAD factor across time and in the cross-section. Notably, the PEAD effect is still strongly present in both small- and mid-cap stocks even after excluding micro-caps.

Finally, in Table IA.XIV, we repeat the same analyses for the PEADB factor. Across all subsamples and size terciles, the PEADB factor exhibits large average returns, MKTB factor alphas, as well as Sharpe and information ratios.

[5] Daniel et al. (2020) are conservative with their choice of portfolio breakpoints and form PEAD with a two-by-three sort on size and earnings-announcement returns.

Table IA.XIII: Stock post-earnings announcement drift (PEAD) factors

	1986:01–2022:12			1986:01–1999:12			2000:01–2022:12		
	Small	Mid	Large	Small	Mid	Large	Small	Mid	Large
Panel A: Excluding micro-cap stocks (value weighted)									
Ave. Ret	1.34	0.95	0.45	1.32	1.50	0.84	1.35	0.61	0.20
t-stat	(11.45)	(8.46)	(3.14)	(10.27)	(10.66)	(4.57)	(7.88)	(3.92)	(1.03)
Alpha	1.30	0.98	0.54	1.25	1.54	0.83	1.32	0.60	0.27
t-stat	(9.29)	(7.01)	(3.90)	(9.64)	(11.26)	(4.67)	(6.54)	(3.75)	(1.54)
SR	0.54	0.40	0.15	0.79	0.82	0.35	0.47	0.24	0.06
IR	0.55	0.43	0.19	0.77	0.87	0.35	0.49	0.25	0.09
Panel B: Including micro-cap stocks (value weighted)									
Ave. Ret	1.26	0.78	0.44	1.45	1.40	0.85	1.15	0.41	0.20
t-stat	(11.23)	(7.28)	(3.25)	(11.87)	(10.41)	(4.72)	(6.98)	(2.74)	(1.04)
Alpha	1.24	0.87	0.52	1.46	1.45	0.84	1.09	0.48	0.25
t-stat	(9.68)	(6.67)	(3.98)	(13.67)	(12.61)	(4.87)	(6.16)	(3.12)	(1.52)
SR	0.53	0.35	0.15	0.92	0.80	0.36	0.42	0.16	0.06
IR	0.55	0.40	0.19	0.94	0.87	0.37	0.43	0.21	0.08
Panel C: Excluding micro-cap stocks (equally weighted)									
Ave. Ret	1.25	1.08	0.52	1.20	1.61	1.03	1.28	0.75	0.21
t-stat	(10.19)	(9.77)	(4.77)	(8.20)	(11.84)	(7.45)	(7.26)	(4.88)	(1.37)
Alpha	1.18	1.10	0.62	1.10	1.67	1.11	1.22	0.73	0.29
t-stat	(7.68)	(7.95)	(5.43)	(6.98)	(12.07)	(7.75)	(5.58)	(4.71)	(2.26)
SR	0.48	0.46	0.23	0.63	0.91	0.57	0.44	0.29	0.08
IR	0.48	0.49	0.29	0.60	0.98	0.65	0.44	0.31	0.13
Panel D: Including micro-cap stocks (equally weighted)									
Ave. Ret	1.26	0.82	0.42	1.41	1.48	0.90	1.17	0.42	0.13
t-stat	(11.28)	(7.56)	(3.97)	(11.81)	(10.78)	(6.50)	(7.11)	(2.82)	(0.88)
Alpha	1.26	0.89	0.50	1.41	1.54	0.98	1.15	0.46	0.18
t-stat	(9.91)	(6.65)	(4.76)	(13.64)	(12.55)	(6.55)	(6.48)	(3.13)	(1.48)
SR	0.54	0.36	0.19	0.91	0.83	0.50	0.43	0.17	0.05
IR	0.56	0.40	0.24	0.93	0.91	0.57	0.46	0.20	0.08

This table presents the performance of Post-Earnings Announcement Drift (PEAD) factors across different market capitalization groups (Small, Mid, Large) and sample periods. For each size group, stocks are conditionally sorted into quintiles based on PEAD. The respective PEAD factor is long Q5 and short Q1. Panel A and C exclude micro-cap stocks (bottom 20% by market cap) at the portfolio formation month t, while Panel B and D include all stocks. Panels A and B use value-weights by market capitalization, while Panels C and D use equal-weights. Ave. Ret is the average monthly return in percent. Alpha is the monthly Fama-French five-factor alpha in percent. t-statistics are reported in parentheses and are adjusted using the Newey-West procedure with 4 lags, chosen as the integer component of $T^{1/4}$ following Greene (2012). SR is the monthly Sharpe ratio. IR is the monthly information ratio (alpha divided by residual volatility).

Table IA.XIV: Corporate bond post-earnings announcement drift (PEADB) factors

	1986:01–2022:12			1986:01–1999:12			2000:01–2022:12		
	Small	Mid	Large	Small	Mid	Large	Small	Mid	Large
Panel A: Excluding micro-cap bonds (equally weighted)									
Ave. Ret	0.25	0.18	0.16	0.16	0.11	0.11	0.30	0.22	0.19
t-stat	(5.60)	(6.29)	(5.79)	(4.17)	(3.74)	(4.00)	(4.50)	(5.24)	(4.64)
Alpha	0.30	0.19	0.18	0.16	0.11	0.10	0.37	0.24	0.22
t-stat	(5.35)	(5.54)	(5.70)	(3.99)	(3.70)	(3.66)	(4.54)	(4.87)	(4.99)
SR	0.27	0.30	0.27	0.32	0.29	0.31	0.27	0.32	0.28
IR	0.33	0.33	0.32	0.33	0.27	0.29	0.35	0.36	0.35
Panel B: Including micro-cap bonds (value weighted)									
Ave. Ret	0.20	0.18	0.15	0.16	0.11	0.11	0.23	0.23	0.17
t-stat	(5.65)	(6.84)	(5.35)	(3.95)	(3.78)	(3.78)	(4.37)	(5.81)	(4.23)
Alpha	0.23	0.20	0.17	0.16	0.11	0.10	0.26	0.25	0.21
t-stat	(5.29)	(6.20)	(5.54)	(3.77)	(4.15)	(3.27)	(4.32)	(5.40)	(4.81)
SR	0.27	0.32	0.25	0.30	0.29	0.29	0.26	0.35	0.25
IR	0.31	0.36	0.29	0.30	0.29	0.27	0.32	0.40	0.31
Panel C: Excluding micro-cap bonds (equally weighted)									
Ave. Ret	0.25	0.18	0.16	0.14	0.11	0.09	0.31	0.22	0.20
t-stat	(4.64)	(6.13)	(6.13)	(3.43)	(3.57)	(4.25)	(3.82)	(5.12)	(5.04)
Alpha	0.30	0.19	0.18	0.14	0.11	0.09	0.39	0.24	0.23
t-stat	(4.83)	(5.46)	(5.43)	(3.10)	(3.52)	(3.92)	(4.26)	(4.81)	(4.89)
SR	0.22	0.29	0.29	0.26	0.28	0.33	0.23	0.31	0.30
IR	0.28	0.32	0.35	0.27	0.26	0.32	0.31	0.36	0.38
Panel D: Including micro-cap bonds (equally weighted)									
Ave. Ret	0.20	0.19	0.15	0.16	0.12	0.09	0.23	0.23	0.18
t-stat	(5.71)	(6.83)	(5.60)	(4.07)	(3.88)	(3.90)	(4.42)	(5.75)	(4.54)
Alpha	0.23	0.20	0.17	0.15	0.12	0.09	0.27	0.25	0.22
t-stat	(5.31)	(6.27)	(5.23)	(3.78)	(4.36)	(3.35)	(4.36)	(5.38)	(4.69)
SR	0.27	0.32	0.27	0.31	0.30	0.30	0.27	0.35	0.27
IR	0.31	0.36	0.32	0.31	0.30	0.29	0.32	0.39	0.35

This table presents the performance of the Corporate Bond Post-Earnings Announcement Drift (PEADB) factors across different bond market capitalization groups (Small, Mid, Large) and sample periods. For each size group, bonds are conditionally sorted into quintiles based on PEAD. The respective PEAD factor is long Q5 and short Q1. Panel A and C exclude micro-cap bonds (bottom 20% by market cap) at the portfolio formation month t, while Panel B and D include all bonds. Panels A and B use value-weights by bond market capitalization, while Panels C and D use equal-weights. Ave. Ret is the average monthly return in percent. Alpha is the monthly bond market one-factor alpha in percent. t-statistics are reported in parentheses and are adjusted using the Newey-West procedure with 4 lags, chosen as the integer component of $T^{1/4}$ following Greene (2012). SR is the monthly Sharpe ratio. IR is the monthly information ratio (alpha divided by residual volatility).

IA.5 Discount rate and cash-flow news decomposition

In this section we provide additional results to complement the analysis in Section 3.1.4. We also provide details on the decomposition of tradable factor returns into discount rate vs. cash-flow news.

IA.5.1 Tradable factor return decomposition

Vuolteenaho (2002), Cohen et al. (2002), and others decompose unexpected asset returns into an expected return (discount rate) component on the one hand and a cash-flow component on the other hand:

$$r_{t+1} - E_t r_{t+1} = \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j e_{t+1+j} - \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1+j},$$

where ΔE_{t+1} denotes the change in expectations from t to $t+1$ (i.e., $E_{t+1}(\cdot) - E_t(\cdot)$), e_{t+1} the aggregate return on equity (ROE), and r_{t+1} the log asset return. ρ is determined by the data, and in our setting is equal to 0.979, although any value between 0.95 and 1.00 does not materially affect the results. As argued by Vuolteenaho (2002), using ROE as the measure of firm cash flows is more appropriate in our case since we are dealing with both debt and equity-based tradable factors, and many firms do not pay cash-based dividends.

We define the two return components as discount rate (N_r, DR) and cash-flow news (N_{cf}, CF), respectively:

$$N_{r,t+1} = \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1+j}, \qquad N_{cf,t+1} = \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j e_{t+1+j}.$$

IA.5.2 Implementation using the VAR methodology

To empirically estimate equation IA.5.1, we implement a parsimonious vector autoregression (VAR). The behavior of the tradable factors is captured by a vector, $z_{i,t}$ of state variables. The first variable is always the tradable bond or stock factor, whilst the remaining variables could be any set of predictors that are associated with future stock or bond returns. In this respect, we use predictors that are standard in the literature. We define the vector, $z_t = [r_t, roe_t, bm_t, gz_t]$, where r_t is the tradable factor return, roe_t is the log of aggregate return on equity (ROE), bm_t is the log of the aggregate book-to-market ratio, and gz_t is the first difference of the log of the Gilchrist and Zakrajšek (2012) aggregate credit spread (GZ). Aggregate ROE is the equally-weighted average of firm-level net income (NI) scaled by one-quarter lagged book equity. Aggregate book-to-market is from Amit Goyal's data repository available here. The GZ credit spread is computed as in Gilchrist and Zakrajšek (2012).[6]

The vector of state variables, z_t is assumed to follow a first-order VAR,

$$z_{t+1} = Az_t + u_{t+1}$$

[6] We thank Yoshio Nozawa for making this data available to us.

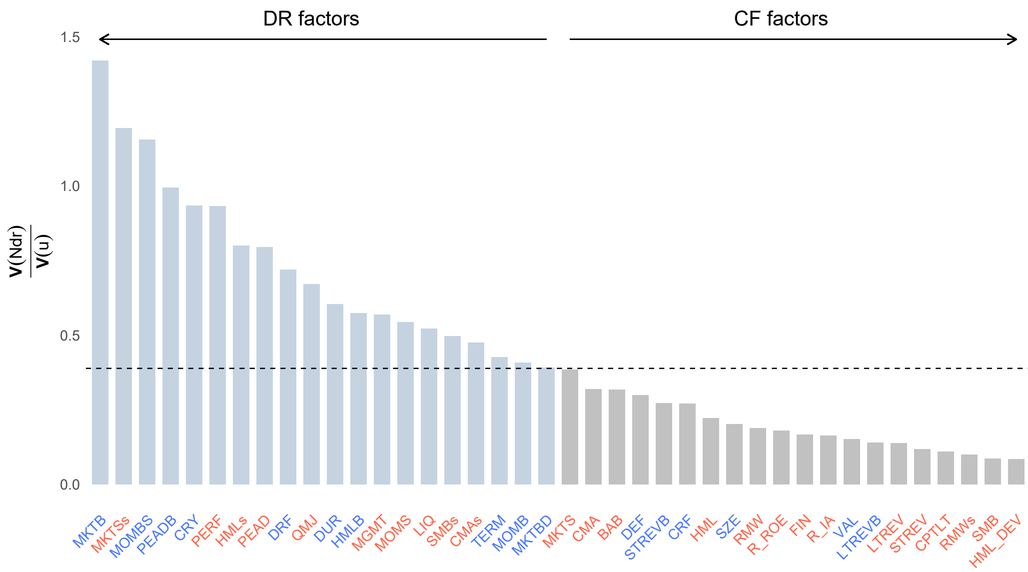


Figure IA.21: Tradable factors decomposition: Discount rate and cash-flow news.

Ordered ratios of the variance of the discount rate news component to total variance of residuals, $\mathbb{V}(Ndr)/\mathbb{V}(u)$, for each bond and stock tradable factor (estimated using equation (IA.17) in Internet Appendix IA.5). The dashed horizontal line denotes the median value of the ratio (0.39). Bond factors are displayed in blue while stock factors are displayed in red on the x-axis.

From the VAR, we estimate discount rate news as,

$$\begin{aligned} Ndr_{t+1} &= (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} = e_1' \sum_{j=1}^{\infty} \rho^j A^j u_{t+1} \\ &= e_1' \rho A (I - \rho A)^{-1} u_{t+1} = \lambda' u_{t+1}, \end{aligned} \tag{IA.17}$$

where $\lambda' = e_1' \rho A(I - \rho A)^{-1}$ and $e1$ is a vector whose first element is equal to one and zero otherwise. The cash-flow news component is computed as the residual of the total unexpected factor return and discount rate news,

$$Ncf_{t+1} = r_{t+1} - E_t r_{t+1} + Ndr = (e_1' + \lambda')u_{t+1}.$$

VAR with principal components. Given the criticisms of the VAR methodology outlined in Chen and Zhao (2009) and Chen et al. (2013), we also implement a VAR using the first five principal components extracted from 37 predictors in Amit Goyal's data library.

Thousands VARs. Finally, we perform a further extensive robustness exercise to alleviate concerns about potential data uncertainty by first fixing the number of predictors in the VAR to three. Then, we estimate 7,700 possible combinations of VARs with the set of 37 predictors.

IA.5.3 Factor decomposition

We now implement the VARs and decompose each tradable factor into the component related to either discount rate or cash-flow news across the three methods discussed above. Following Vuolteenaho (2002) and Cohen, Gompers, and Vuolteenaho (2002) we compute the variance of the discount rate news component, $\mathbb{V}(Ndr)$ and the ratio of the discount rate news variance to total unexpected factor return variance $\frac{\mathbb{V}(Ndr)}{\mathbb{V}(u)}$. To pin down a relative classification of the factors into a discount rate or cash-flow news category, we use the median level of $\frac{\mathbb{V}(Ndr)}{\mathbb{V}(u)}$ as a break-point. Factors above the break-point are classified (relatively) as more likely to capture discount rate news as opposed to cash-flow rate news. In Table IA.XV we present the $\frac{\mathbb{V}(Ndr)}{\mathbb{V}(u)}$ and the classification (DR/CF) and the 'Match' column which displays a number out of three, illustrating how often the methods predict the same classification. Importantly, the classification remains consistent across all three approaches we consider. We focus on the 'Vuolteenaho' column, since these results pertain to the baseline results presented in Section 3.1.4.

We present the results of the Vuolteenaho (2002) decomposition in Figure IA.21. The y-axis of the figure shows the proportion of residual variance of each factor estimated from the VAR model that represents discount rate news. Overall, 10 of the 16 bond factors (62%) are driven relatively more by discount rate news as opposed to cash-flow news shocks. In contrast, slightly more stock factors (14/26=53%) are driven by cash-flow news shocks. However, it is important to note that this classification is a function of our estimated VARs. Thus, just because a factor is classified as (relatively) more either DR- or CF-based, does not mean that this factor cannot capture other asset pricing phenomena.

The two most likely factors that ought to be included in the co-pricing BMA-SDF (i.e., PEAD and PEADB) are driven relatively more by discount rate news as opposed to cash-flow news. For a discussion on how PEAD and PEADB could be linked to both news sources via accounting (earnings) reports see Penman and Yehuda (2019). Most other behavioral-linked factors such as MOMBS (bond factor formed with equity momentum), PERF and MGMT (equity and management performance factor of Stambaugh and Yuan (2017)), are also classified as relatively more discount rate news-based.

Table IA.XV: Tradable factors decomposition: Discount rate and cash-flow news robustness

	$\frac{\mathbb{V}(Ndr)}{\mathbb{V}(u)}$			DR/CF classification			
Factor	Vuolteenaho	PCA	7,770 VARs	Vuolteenaho	PCA	7,770 VARs	Match
BAB	0.32	0.36	0.32	CF	CF	CF	3/3
CMA	0.32	0.66	0.33	CF	CF	CF	3/3
CMAs	0.47	1.15	0.41	DR	DR	DR	3/3
CPTLT	0.11	0.20	0.21	CF	CF	CF	3/3
CRF	0.27	0.43	0.25	CF	CF	CF	3/3
CRY	0.93	1.81	0.86	DR	DR	DR	3/3
DEF	0.30	0.93	0.71	CF	DR	DR	2/3
DRF	0.72	1.61	0.70	DR	DR	DR	3/3
DUR	0.60	0.93	0.29	DR	DR	CF	2/3
FIN	0.17	0.26	0.15	CF	CF	CF	3/3
HML	0.22	0.48	0.25	CF	CF	CF	3/3
HMLB	0.57	2.12	1.07	DR	DR	DR	3/3
HML_DEV	0.08	0.33	0.37	CF	CF	CF	3/3
HMLs	0.80	0.94	0.38	DR	DR	CF	2/3
LIQ	0.52	1.27	0.49	DR	DR	DR	3/3
LTREV	0.14	0.43	0.26	CF	CF	CF	3/3
LTREVB	0.14	0.81	0.47	CF	DR	DR	2/3
MGMT	0.57	0.99	0.43	DR	DR	DR	3/3
MKTB	1.42	2.34	0.89	DR	DR	DR	3/3
MKTBD	0.39	1.02	0.68	DR	DR	DR	3/3
MKTS	0.38	0.63	0.39	CF	CF	CF	3/3
MKTSs	1.19	1.98	0.89	DR	DR	DR	3/3
MOMB	0.41	0.74	0.41	DR	CF	CF	2/3
MOMBS	1.16	1.68	0.78	DR	DR	DR	3/3
MOMS	0.54	1.35	0.84	DR	DR	DR	3/3
PEAD	0.80	1.20	0.66	DR	DR	DR	3/3
PEADB	1.00	1.78	0.84	DR	DR	DR	3/3
PERF	0.93	1.36	0.58	DR	DR	DR	3/3
QMJ	0.67	0.99	0.38	DR	DR	CF	2/3
RMW	0.19	0.16	0.09	CF	CF	CF	3/3
RMWs	0.10	0.15	0.14	CF	CF	CF	3/3
R_IA	0.16	0.51	0.29	CF	CF	CF	3/3
R_ROE	0.18	0.68	0.38	CF	CF	CF	3/3
SMB	0.09	0.56	0.37	CF	CF	CF	3/3
SMBs	0.50	0.87	0.49	DR	DR	DR	3/3
STREV	0.12	0.10	0.11	CF	CF	CF	3/3
STREVB	0.27	0.76	0.38	CF	CF	CF	3/3
SZE	0.20	0.79	0.51	CF	CF	DR	2/3
TERM	0.43	0.73	0.43	DR	CF	DR	2/3
VAL	0.15	0.68	0.65	CF	CF	DR	2/3

This table presents variance decomposition results showing the variance of the discount rate news component to total variance of the residuals, $\mathbb{V}(Ndr)/\mathbb{V}(u)$ and classification (DR/CF) for each factor across three different approaches. The factors are ordered alphabetically. 'Vuolteenaho' uses the method proposed by Vuolteenaho (2002) using three predictors. The 'PCA' method follows the advice of Chen and Zhao (2009) and uses the first five principal components estimated using 37 predictors from Amit Goyal's website.. The '7,770 VARs' method estimates the average DR and CF components across 7,770 VARs with combinations of three predictors from the total set of 37. The 'Match' column displays how often the three methods predict the same classification.

IA.6 The Treasury component

Duration-adjusted corporate bond returns are computed for each bond i at each time t such that the resultant bond return is in 'excess' of a portfolio of duration-matched U.S. Treasury bond returns (van Binsbergen et al. (2025), Andreani et al. (2023)).

Start with the total return for corporate bond i in month t:

$$R_{i\,t} = \frac{B_{i\,t} + AI_{i\,t} + Coupon_{i\,j\,t}}{B_{i\,t-1} + AI_{i\,t-1}} - 1,$$

where $B_{i\,t}$ is the clean price of bond i in month t, $AI_{i\,t}$ is the accrued interest, and $Coupon_{i\,t}$ is the coupon payment, if any.

The bond duration-adjusted (or credit excess) return is the total bond return minus the return on a hedging portfolio of U.S. Treasury securities that has the same duration as the bond in month t. Thus, the duration-adjusted return isolates the portion of a bond's performance that is attributed to the credit risk of each bond (including other non-interest rate-related risks).

In equation (10) we define the duration-adjusted return as

$$\underbrace{R_{bond\,i,t} - R^{Treasury}_{dur\,bond\,i,t}}_{\text{Duration-adjusted return}} = \underbrace{R_{bond\,i,t} - R_{f,t}}_{\text{Excess return}} - \underbrace{\left(R^{Treasury}_{dur\,bond\,i,t} - R_{f,t}\right)}_{\text{Treasury component}}$$

where $R_{bond\,i,t}$ is the return of bond i at time t, $R_{f,t}$ denotes the short-term risk-free rate, and $R^{Treasury}_{dur\,bond\,i,t}$ denotes the return on a portfolio of Treasury securities with the same duration as bond i (constructed as in van Binsbergen et al. (2025)). The duration adjustment removes the implicit Treasury component from the bond excess return, hence isolating the remaining sources of risk compensation that investing in a given bond entails.

IA.6.1 Pricing duration-adjusted corporate bond returns

We use duration-adjusted returns to re-compute the tradable bond factor returns and returns on bond test assets. In Section 3.3 we show that once corporate bond returns are adjusted for duration, the BMA-SDF based only on equity information jointly prices (duration-adjusted) corporate bond and stock returns as well as the co-pricing BMA-SDF that additionally includes bond factors. That is, the information content of the bond factor zoo becomes largely irrelevant for co-pricing once the Treasury component of bond returns is removed. In Table IA.XVI we repeat the in- and out-of-sample cross-sectional asset pricing exercises from Tables 2 and 3, respectively. That is, we estimate the co-pricing as well as the bond BMA-SDFs using duration-adjusted corporate bond test portfolios and tradable corporate bond factors. The resulting BMA-SDFs are then again used to price (with no additional parameter estimation) the OS test assets. In Panel C the OS test assets are the combined 154 bond and stock portfolios and in Panel D they are the 77 bond portfolios as described in Section 1. The results complement the information in Figure 8 and show how our co-pricing and bond BMA-SDFs still outperform all competitors out-of-sample.

In Table IA.XVII we repeat the analysis from Table IA.XII using duration-adjusted returns to assess how the BMA-SDF performs vis-à-vis the additional set of bond and stock factor

Table IA.XVI: Cross-sectional asset pricing performance: Duration-adjusted bond returns

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: In-sample co-pricing stocks and bonds											
RMSE	0.203	0.197	0.186	0.174	0.326	0.297	0.278	0.324	0.294	0.157	0.219
MAPE	0.147	0.141	0.135	0.128	0.274	0.245	0.216	0.272	0.245	0.117	0.137
R^2_{OLS}	0.106	0.157	0.246	0.339	−1.310	−0.913	−0.675	−1.282	−0.885	0.465	−0.047
R^2_{GLS}	0.052	0.120	0.191	0.252	0.024	0.028	0.033	0.024	0.209	0.177	0.184
Panel B: In-sample pricing bonds											
RMSE	0.169	0.138	0.112	0.101	0.201	0.217	0.179	0.183	0.198	0.117	0.162
MAPE	0.103	0.088	0.080	0.076	0.120	0.119	0.088	0.111	0.143	0.069	0.110
R^2_{OLS}	0.093	0.396	0.601	0.676	−0.270	−0.484	−0.018	−0.059	−0.237	0.569	0.171
R^2_{GLS}	0.057	0.187	0.324	0.430	0.003	0.036	0.068	0.019	0.412	0.262	0.243
Panel C: Out-of-sample co-pricing stocks and bonds											
RMSE	0.178	0.158	0.138	0.125	0.121	0.168	0.106	0.120	0.342	0.159	0.112
MAPE	0.158	0.139	0.119	0.106	0.093	0.146	0.078	0.091	0.315	0.144	0.086
R^2_{OLS}	0.045	0.246	0.423	0.528	0.558	0.143	0.658	0.568	−2.525	0.235	0.624
R^2_{GLS}	0.030	0.058	0.078	0.097	0.024	0.002	0.023	0.025	−0.003	0.049	0.028
Panel D: Out-of-sample pricing bonds											
RMSE	0.086	0.080	0.080	0.081	0.095	0.091	0.086	0.086	0.103	0.082	0.128
MAPE	0.066	0.059	0.057	0.057	0.074	0.070	0.067	0.067	0.075	0.057	0.096
R^2_{OLS}	0.125	0.243	0.247	0.228	−0.070	0.014	0.120	0.119	−0.247	0.211	−0.936
R^2_{GLS}	0.018	0.042	0.055	0.065	0.009	0.009	−0.028	0.015	−0.029	0.040	−0.080

The table presents the cross-sectional in and out-of-sample asset pricing performance of different models pricing (duration-adjusted) bonds and stocks jointly (Panels A and C), and (duration-adjusted) bonds only (Panels B and D), respectively. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors with an average posterior probability greater than 50%. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of a duration matched portfolio of U.S. Treasury bonds. In Panels A and B the models are estimated with the respective factor zoos and test assets. The resulting SDF is then used to price (with no additional parameter estimation) the two sets of the OS assets in Panels C and D. IS test assets are the 83 bond and stock portfolios and the 40 tradable bond and stock factors (Panel A), and the 50 bond portfolios and 16 tradable bond factors (Panel B), respectively. OS test assets are the combined 154 bond and stock portfolios (Panel C), as well as the 77 bond portfolios only (Panel D). All are described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table IA.XVII: Cross-sectional asset pricing performance: Additional models (duration-adjusted bond returns)

	BMA-SDF	Bond factor models					Stock factor models				
	BMA-80%	BBW3	DCAPM	DEFTERM	MACRO	CWW	FFC4	HXZ4	$FF5^{2015}$	$FF5^{*}$	FF6
Panel A: In-sample co-pricing stocks and bonds											
RMSE	0.174	0.305	0.275	0.219	0.276	0.272	0.264	0.252	0.249	0.251	0.223
MAPE	0.128	0.257	0.223	0.164	0.224	0.209	0.203	0.179	0.177	0.174	0.156
R^2_{OLS}	0.339	−1.023	−0.643	−0.044	−0.660	−0.604	−0.518	−0.378	−0.350	−0.369	−0.084
R^2_{GLS}	0.252	0.030	0.026	0.023	0.030	0.034	0.038	0.064	0.058	0.074	0.065
Panel B: Out-of-sample co-pricing stocks and bonds											
RMSE	0.125	0.162	0.164	0.147	0.155	0.111	0.103	0.116	0.097	0.147	0.103
MAPE	0.106	0.138	0.142	0.124	0.132	0.082	0.077	0.091	0.075	0.124	0.079
R^2_{OLS}	0.528	0.202	0.187	0.349	0.278	0.631	0.682	0.593	0.717	0.351	0.681
R^2_{GLS}	0.097	0.002	0.010	0.004	0.002	0.020	0.034	0.039	0.057	0.044	0.059
Panel C: Out-of-sample pricing stocks											
RMSE	0.077	0.121	0.117	0.114	0.119	0.105	0.080	0.086	0.092	0.098	0.078
MAPE	0.058	0.088	0.084	0.083	0.085	0.074	0.059	0.066	0.070	0.073	0.061
R^2_{OLS}	0.618	0.056	0.117	0.160	0.091	0.288	0.590	0.522	0.463	0.383	0.615
R^2_{GLS}	0.271	0.052	0.041	0.020	0.038	0.080	0.132	0.182	0.169	0.097	0.188
Panel D: Out-of-sample pricing bonds											
RMSE	0.082	0.092	0.088	0.085	0.090	0.104	0.121	0.123	0.102	0.102	0.114
MAPE	0.061	0.071	0.067	0.065	0.070	0.080	0.094	0.097	0.079	0.080	0.089
R^2_{OLS}	0.196	−0.009	0.084	0.140	0.038	−0.277	−0.735	−0.785	−0.226	−0.222	−0.546
R^2_{GLS}	0.098	0.013	0.013	0.005	0.016	0.006	−0.055	−0.035	0.076	0.038	0.022

Panel A presents the cross-sectional in-sample asset pricing performance of different bond and stock asset pricing models. Bond factor and test asset returns are duration adjusted as per equation (10). Panels B, C and D present the out-of-sample asset pricing performance for the joint, bond and stock cross-sections, respectively. For bonds we consider five models: (i) the modified three-factor model of Bai et al. (2019) including MKTB, DRF, and CRF bond factors (BBW3), (ii) the two-factor decomposed bond market factor model from van Binsbergen et al. (2025) (DCAPM), (iii) the DEFTERM model of Fama and French (1993), (iv) the MACRO model of Bali et al. (2021b) comprising MKTB and macro economic uncertainty UNC, and (v) the six-factor CWW model of Chung et al. (2019) that adds innovations to the VIX index as a sixth factor to the FF5 model of Fama and French (1993). For stocks we consider six models: (i) the Carhart (1997) four-factor model that adds MOMS to the Fama and French (1992) three-factor model (FFC4), (ii) the Hou et al. (2015) four-factor model (HXZ4), (iii) the five-factor model of Fama and French (2015) which augments their three-factor model with the RMW and CMA factors ($FF5^{2015}$), (v) the $FF5^{*}$ model of Daniel et al. (2020) which removes unpriced risk from the original FF5 factors, and (vi) the FF6 model which augments the FF5 model with MOMS. IS test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors. OS test assets are the combined 154 bond and stock portfolios (Panel B), as well as the separate 77 bond and stock portfolios (Panels C and D). All are described in Section 1. All models are first estimated using the baseline IS test assets (Panel A) and then used to price (with no additional parameter estimation) each set of OS assets (Panels B to D). We use GMM-GLS to estimate factor risk prices for bond and stock specific factor models. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

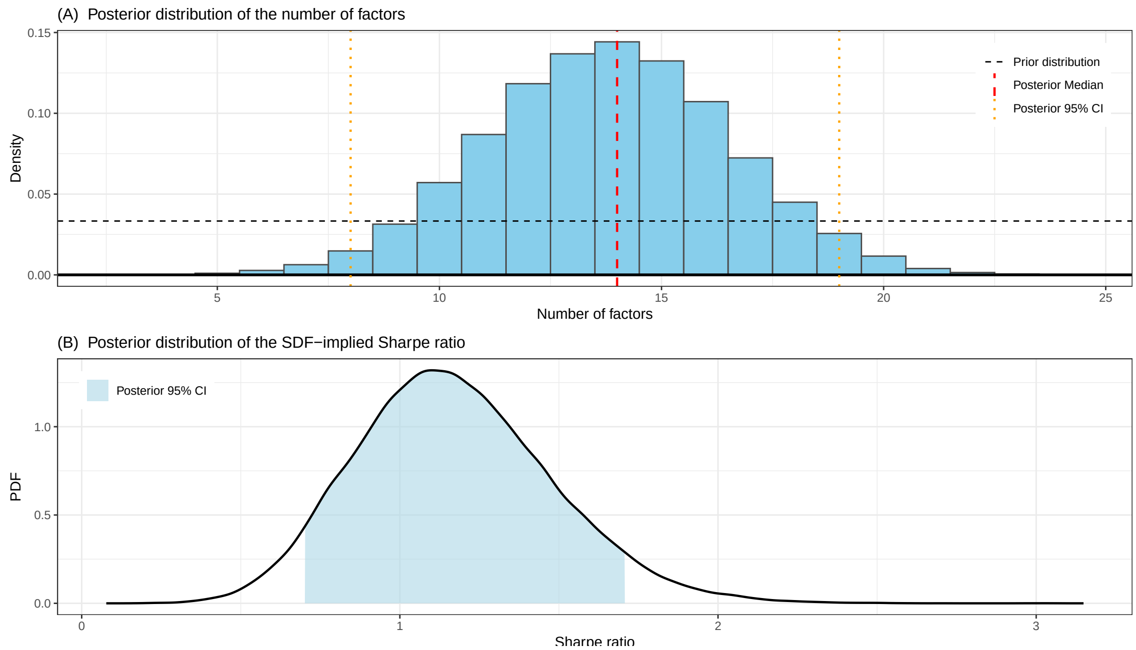


Figure IA.22: Posterior SDF dimensionality and Sharpe ratios: Treasury component.

Posterior distributions of the number of factors to be included in the bond SDF (top panel) and of the SDF-implied Sharpe ratio (bottom panel), computed using the 14 nontradable and 16 tradable bond factors described in Appendix A. The prior distribution for the j^{th} factor inclusion is a Beta(1, 1), yielding a flat prior for the SDF dimensionality depicted in the top panel. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the Treasury component of the 50 corporate bond portfolios and 16 bond tradable factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

models. Again, the BMA-SDFs outperform all additional models originally designed to price the individual bond and stock cross-sections, respectively.

Table IA.XVIII: BMA-SDF dimensionality and Sharpe ratio decomposition for Treasury component

	Total prior SR				Total prior SR			
	20%	40%	60%	80%	20%	40%	60%	80%
	Nontradable factors				Tradable factors			
Mean	7.01	6.98	6.97	6.80	7.89	7.72	7.47	7.00
5%	4	4	4	4	5	4	4	4
95%	10	10	10	10	11	11	11	10
$\mathbb{E}[SR_f \mid \text{data}]$	0.15	0.32	0.52	0.84	0.28	0.48	0.64	0.80
$\mathbb{E}\left[\frac{SR_f^2}{SR_m^2} \mid \text{data}\right]$	0.30	0.36	0.44	0.54	0.71	0.67	0.60	0.51

The table reports posterior means of number of factors (along with the 90% confidence intervals), implied Sharpe ratios $\mathbb{E}[SR_f|\text{data}]$, and the ratio of SR_f^2 to the total SDF-implied squared Sharpe ratio $\mathbb{E}\left[SR_f^2/SR_m^2|\text{data}\right]$, of the 14 nontradable and 16 tradable bond factors described in Appendix A. Test assets are the Treasury components of the 50 corporate bond portfolios described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

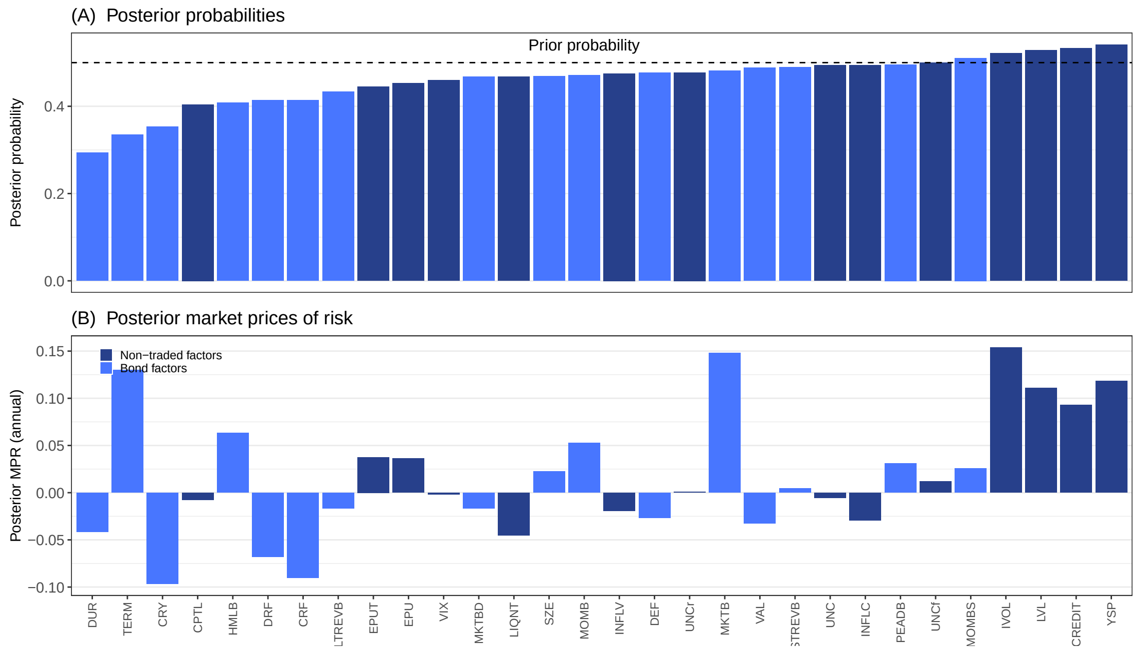


Figure IA.23: Posterior factor probabilities and risk prices: Treasury component.

Posterior probabilities (top panel), $\mathbb{E}[\gamma_j|\text{data}]$, and the corresponding posterior market prices of risk (bottom panel), $\mathbb{E}[\lambda_j|\text{data}]$, of the 14 nontradable and 16 tradable bond factors described in Appendix A. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the Treasury components of the 50 corporate bond portfolios described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

IA.6.2 Pricing the Treasury component

As per equation (10), the duration adjustment of corporate bond returns also yields Treasury components of corporate bond test assets that can be used for asset pricing exercises. In particular, we can estimate "Treasury component BMA-SDFs" using either the bond or stock factor zoos described in Appendix A (whereby the bond factors are *not* duration adjusted). For both exercises we use the Treasury component of the 50 bond portfolios as IS test assets and we do not impose self-pricing on the bond or stock factors, respectively. Figure 9 shows how the Treasury component *bond* BMA-SDF can price the Treasury component IS while the Treasury component *stock* BMA-SDF fails to do so. Mirroring the results presented in Section 3.1, Figure IA.22 shows the posterior SDF dimensionality and the distribution of Sharpe ratios when pricing the Treasury component using only the 14 nontradable and the 16 tradable bond factors (again, without self-pricing). While the median number of factors is now much lower than for the co-pricing BMA-SDF, the required SDF is still dense and low-dimensional factor models remain misspecified with very high probability even for pricing the Treasury component only. Moreover, the SDF is dense in both nontradable as well as tradable factors (see Table IA.XVIII).

In Figure IA.23 we mirror the analysis in Section 3.1 and assess which factors are more likely to price the Treasury component individually, and how factors should be optimally combined to achieve a portfolio that captures the priced risks in these assets. The top and bottom panels report the posterior factor probabilities and market prices of risk implied by the pricing of the

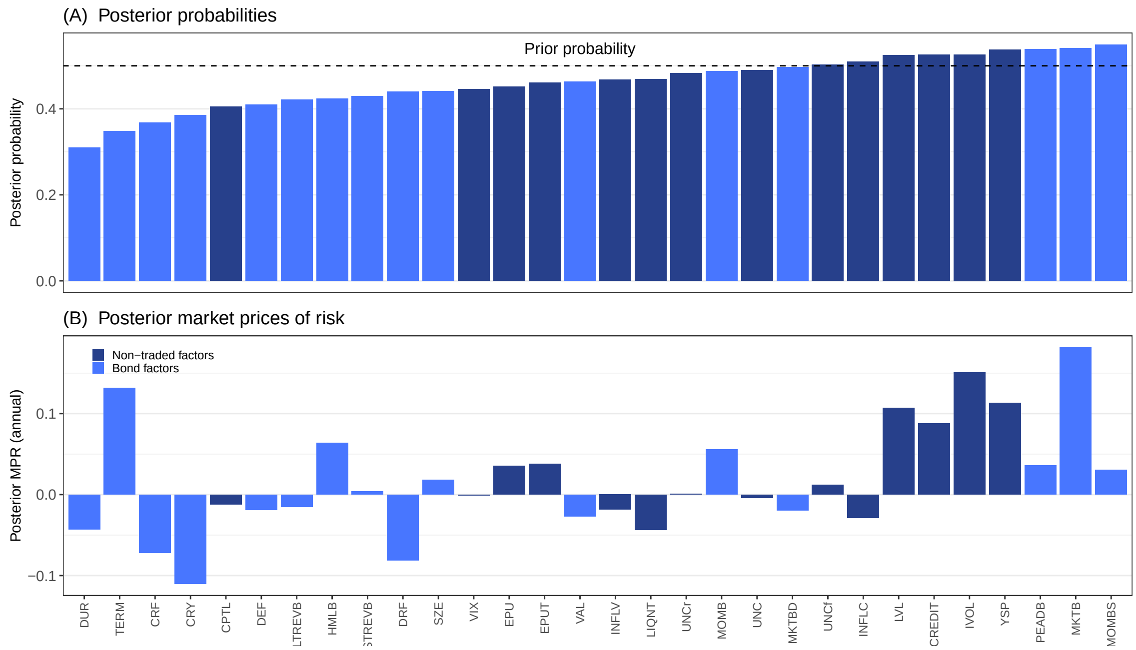


Figure IA.24: Posterior factor probabilities and risk prices: Treasury component with DR tilt.

Posterior probabilities (top panel), $\mathbb{E}[\gamma_j|\text{data}]$, and the corresponding posterior market prices of risk (bottom panel), $\mathbb{E}[\lambda_j|\text{data}]$, of the 14 nontradable and 16 tradable bond factors described in Appendix A. We tilt the prior for each factor inclusion via the κ vector discussed in Section 2.3 using weights informed by the CF and DR news decomposition discussed in Internet Appendix IA.5 whereby DR (CF) classified factors are given a positive (negative) weight. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the Treasury components of the 50 corporate bond portfolios described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

Treasury component of corporate bond returns using the Treasury component of the corporate bond factor zoo (the prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio). The first four factors with the highest posterior probability are nontradable. Furthermore, largely, these factors are the same as those that appear most likely when co-pricing bonds and stocks (the top three being YSP, CREDIT and LVL, followed by the IVOL factor). Moreover, they command large market prices of risk and the probability of zero nontradable factors being in the BMA-SDF that prices the Treasury component of corporate bond returns is virtually zero (or 0.014%).

Given the nature of the Treasury component where, at least in nominal terms, cash flows are known in advance, one would expect discount rate news to be the main driver of their priced risk (Chen and Zhao (2009)). Thus, we implement a factor tilt (see Section 2.3) whereby we assign a positive weight to DR factors and a negative weight to CF factors as given by the decomposition discussed in Internet Appendix IA.5. The top and bottom panels of Figure IA.24 report the posterior factor probabilities and market prices of risk implied by the pricing of the Treasury component of corporate bond returns using the corporate bond factor zoo without self-pricing (the prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio) and the encoded prior belief about the relative importance of DR versus CF news. The tilt towards DR factors makes them individually more likely, and for example pushes the likelihood of the

MKTB factor above the prior value. However, the pricing results remain overall very similar to the baseline estimation with the more diffuse prior encoding.

Table IA.XIX: IS and OS cross-sectional asset-pricing performance: Treasury component

	In-sample				Out-of-sample			
	20%	40%	60%	80%	20%	40%	60%	80%
Panel A: Baseline without factor tilt, GLS, $\hat{\lambda} = (C^T \Sigma_R^{-1} C + D)^{-1} C^T \Sigma_R^{-1} \mu_R$								
RMSE	0.084	0.084	0.079	0.071	0.096	0.095	0.089	0.078
MAPE	0.064	0.064	0.060	0.053	0.076	0.074	0.068	0.059
R^2_{OLS}	−0.153	−0.169	−0.039	0.177	−0.084	−0.058	0.075	0.289
R^2_{GLS}	0.045	0.087	0.131	0.194	0.081	0.134	0.186	0.259
Panel B: With DR-factor tilt, GLS, $\hat{\lambda} = (C^T \Sigma_R^{-1} C + D)^{-1} C^T \Sigma_R^{-1} \mu_R$								
RMSE	0.084	0.084	0.079	0.070	0.096	0.095	0.089	0.078
MAPE	0.064	0.064	0.060	0.053	0.076	0.074	0.068	0.059
R^2_{OLS}	−0.155	−0.163	−0.019	0.193	−0.086	−0.056	0.092	0.309
R^2_{GLS}	0.045	0.087	0.131	0.195	0.082	0.136	0.188	0.261
Panel C: BMA baseline, OLS, $\hat{\lambda} = (C^T C + D)^{-1} C^T \mu_R$								
RMSE	0.056	0.037	0.030	0.027	0.075	0.060	0.054	0.051
MAPE	0.042	0.027	0.023	0.021	0.063	0.052	0.048	0.046
R^2_{OLS}	0.479	0.774	0.850	0.881	0.342	0.586	0.660	0.694
R^2_{GLS}	−3.653	−6.475	−8.242	−9.518	0.402	0.442	0.456	0.463
Panel D: BMA with DR-factor tilt, OLS, $\hat{\lambda} = (C^T C + D)^{-1} C^T \mu_R$								
RMSE	0.055	0.037	0.030	0.027	0.074	0.059	0.054	0.051
MAPE	0.041	0.027	0.023	0.021	0.062	0.052	0.048	0.046
R^2_{OLS}	0.493	0.778	0.851	0.881	0.354	0.592	0.662	0.694
R^2_{GLS}	−3.707	−6.466	−8.222	−9.516	0.402	0.441	0.454	0.462

The table presents the cross-sectional in- and out-of-sample asset pricing performance of the Treasury component bond BMA-SDF estimated with and without a DR-factor tilt. We provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The models are first estimated using the baseline IS test assets. The resulting SDF is then used to price (with no additional parameter estimation) each set of the OS assets. Panel A provides the baseline estimation without any factor tilt ($\kappa = 0$). In Panel B we tilt the prior for each factor inclusion via the κ vector discussed in Section 2.3 using weights informed by the CF and DR news decomposition discussed in Internet Appendix IA.5 whereby DR (CF) classified factors are given a positive (negative) weight. This implies DR (CF) factors explain a relatively greater (smaller) share of the squared Sharpe ratio of the Treasury component under the prior. The IS test assets are the Treasury components of the 50 corporate bond portfolios. The OS test assets are the 29 Treasury portfolios of excess returns on Treasury securities with maturities 2 to 30 years. All are described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

This is highlighted in Table IA.XIX where we report in- and out-of-sample performance measures for the Treasury component bond BMA-SDF without (Panel A) and with (Panel B) the DR-factor tilt. The IS test assets are the Treasury components of the 50 corporate bond portfolios and the OS test assets are the 29 Treasury portfolios with maturities ranging from 2 to 30 years. All are described in Section 1. The numbers do not change materially when comparing the two panels in the table.

Finally, Table IA.XX provides the time series correlations between (the posterior means of)

Table IA.XX: BMA-SDF time series correlations: 60% and 80% SR shrinkage

	$\text{Co-pricing}_{\text{Exc.}}$	$\text{Bond}_{\text{Exc.}}$	$\text{Stock}_{\text{Exc.}}$	$\text{T-Bond}_{\text{Bond}}$	$\text{T-Bond}_{\text{Stock}}$	$\text{Co-pricing}_{\text{Dur.}}$	$\text{Bond}_{\text{Dur.}}$
$\text{Co-pricing}_{\text{Exc.}}$		0.716	0.738	0.284	0.198	0.967	0.671
$\text{Bond}_{\text{Exc.}}$	0.744		0.093	0.337	0.247	0.688	0.929
$\text{Stock}_{\text{Exc.}}$	0.725	0.113		0.122	0.035	0.730	0.098
$\text{T-Bond}_{\text{Bond}}$	0.402	0.439	0.172		0.379	0.213	0.229
$\text{T-Bond}_{\text{Stock}}$	0.272	0.325	0.064	0.533		0.182	0.252
$\text{Co-pricing}_{\text{Dur.}}$	0.964	0.712	0.708	0.351	0.243		0.729
$\text{Bond}_{\text{Dur.}}$	0.686	0.908	0.121	0.335	0.286	0.754	

This table presents the time series correlation coefficients for the co-pricing (Co-pricing), bond (Bond), and stock (Stock) BMA-SDFs estimated with excess (Exc.) and duration-adjusted (Dur.) returns, and the U.S. Treasury Bond BMA-SDFs estimated with the bond ($\text{T-Bond}_{\text{Bond}}$) or stock ($\text{T-Bond}_{\text{Stock}}$) factor zoos, respectively. The lower (upper) triangular of the table provides results for prior Sharpe ratio values set to 80% (60%) of the ex post maximum Sharpe ratio of the test assets. The sample period is 1986:01 to 2022:12 ($T = 444$).

BMA-SDFs constructed with bond and stock factors, jointly and separately, to price (again jointly and separately) bond and stock excess returns, duration-adjusted bond returns, and the Treasury component of corporate bond returns.

IA.7 Risk premia vs. market prices of risk

In this section, we show that testing a risk premium is not the same as testing a market price of risk. In fact, a factor that is not part of the SDF might command a large risk premium just because it correlates with the latter.

To show this, we report two-pass regression estimates of the risk premium attached to MKTB as the sole factor, as well as linear SDF estimates of the market price of risk in the CAPMB model used to price our baseline cross-section of corporate bonds and bond tradable factors. Furthermore, we evaluate and report the risk premium and the market price of risk from the CAPM model when pricing *duration-adjusted* corporate bond returns and factors. To understand why the two types of estimations can lead to very different outcomes, let's consider a simple example with two (demeaned) tradable risk factors only, i.e. $\boldsymbol{f}_t = [f_{1,t}, f_{2,t}]^\top$, and suppose for simplicity that their covariance matrix is

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

Suppose further that only the first factor is part of the SDF, and has a market price of risk equal to κ. That is

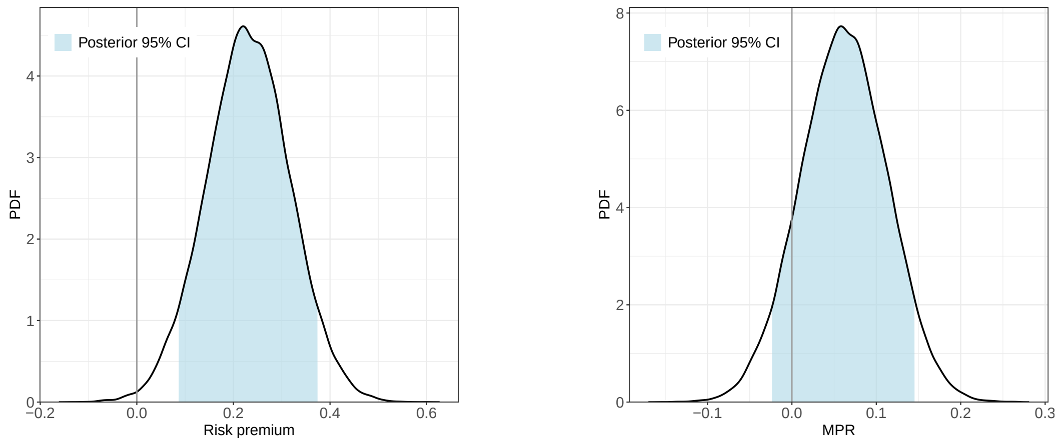
$$M_t = 1 - \boldsymbol{f}_t^\top \boldsymbol{\lambda_f} = 1 - [f_{1,t}, f_{2,t}]^\top \begin{bmatrix} \kappa \\ 0 \end{bmatrix} = 1 - f_{1,t}\kappa$$

Denoting with $\boldsymbol{\mu}_{RP} = [\mu_{RP,1}, \mu_{RP,2}]^\top$ the vector of risk premia of the factors, applying the fundamental asset pricing equation to the returns generated by the factors, we have

$$\boldsymbol{\mu}_{RP} = \Sigma \boldsymbol{\lambda_f} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} \kappa \\ 0 \end{bmatrix} = \begin{bmatrix} \kappa \\ \rho\kappa \end{bmatrix}.$$

That is, the second factor, that is *not* part of the SDF, commands nevertheless a non-zero risk premium (equal to $\rho\kappa$) as long as the factor has non-zero correlation (i.e., as long as $\rho \neq 0$) with the true risk factor—the one that is part of the SDF. This also implies that a two-pass regression method that uses the second factor as the sole driver of a cross-section of asset returns will estimate its ex post risk premium as being non-zero—in fact, the estimated risk premium for the second factor will be inflated relative to its true value. This is due to the fact that the estimated betas of f_2 will be, in population, smaller than the ones of f_1 by a factor equal to ρ. Hence, in population, the two-pass regression will yield an estimated risk premium for f_2 equal to $\rho^{-1}\kappa$ (where $|\rho| \leq 1$).

Example 1 (CAPMB pricing corporate bond excess returns.)**.** *To estimate the SDF of the CAPMB model we rely on the Bayesian-SDF estimator in Definition 1 of Bryzgalova et al. (2023). This is equivalent to the method presented in Section 2 under the null that MKTB is the only factor in the SDF with probability one and that the model is true. To put the comparison of market prices of risk and ex post risk premia estimates on the same footing, we estimate the two-pass regression using the Bayesian implementation of the Fama and MacBeth (1973) method in Bryzgalova et al. (2022). Posterior distributions of the two-pass regression ex*



A: Two-pass regression risk premium **B:** SDF-based market price of risk

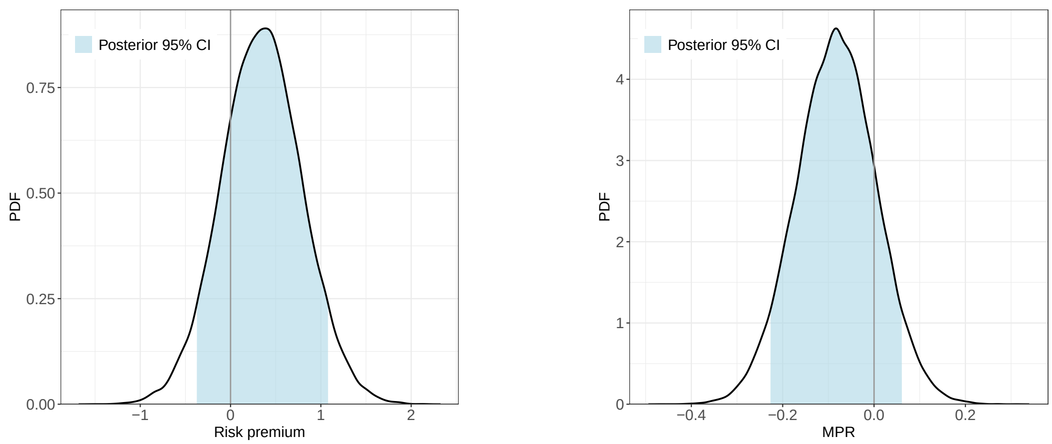
Figure IA.25: CAPMB: Two-pass regression risk premium and market price of risk.

The figure plots the posterior distributions of the two-pass regression ex post risk premium (Panel A), and SDF-based market price of risk (Panel B), of a model with MKTB as the only risk factor, i.e. CAPMB. Test assets are the 50 bond portfolios and the 16 tradable bond factors described in Section 1. The prior Sharpe ratio does not impose any shrinkage, being set to the ex post Sharpe ratio of the MKTB factor. The sample period is 1986:01 to 2022:12 ($T = 444$).

post risk premium and SDF-based market price of risk are plotted, respectively, in Panels A and B of Figure IA.25. The estimates suggest that, albeit MKTB carries a sizable and significant risk premium, it is very unlikely that the data are generated by a "true" latent SDF with MKTB as the only factor—the (Bayesian) p-value of its market price of risk being equal to zero is about 52.34%.[7]

Example 2 (CAPM pricing corporate bond duration-adjusted returns.)**.** *We follow a similar procedure, using the same set of corporate bond portfolios and factors, computed with duration-adjusted returns. Now, the null is defined such that MKTS (the stock market factor) is the only factor in the SDF with probability one and that the model is true. Posterior distributions of the two-pass regression ex post risk premium and SDF-based market price of risk are plotted, respectively, in Panels A and B of Figure IA.26. The estimates suggest that MKTS, neither carries a significant ex post risk premium (as in van Binsbergen et al. (2025, Table A8)) in this heavily misspecified setting (given our results in the main text) nor it is likely that the duration-adjusted bond return data are generated by a "true" latent SDF with the stock market factor as the only factor—the (Bayesian) p-value of its market price of risk being equal to zero is about 76.30%.*

[7] This broadly confirms the results presented in Dickerson et al. (2023). These authors show that *incrementally*, in a frequentist setting, other low dimensional models that they consider do not outperform the CAPMB. However, in itself, they also show that the CAPMB is a poor model for describing the cross-section of expected corporate bond excess returns (see their Fig. 2, on Page 11 of the published version of the paper and the R_{GLS} values reported in Table 3).



A: Two-pass regression risk premium

B: SDF-based market price of risk

Figure IA.26: CAPM: two pass-regression risk premium and market price of risk with duration-adjusted bond returns.

The figure plots the posterior distributions of the two-pass regression ex post risk premium (Panel A), and SDF-based market price of risk (Panel B), of a model with MKTS as the only risk factor, i.e. the CAPM. Test assets are the 50 duration-adjusted bond portfolios and the 16 tradable bond factors (also duration adjusted) described in Section 1. The prior Sharpe ratio does not impose any shrinkage, being set to the ex post Sharpe ratio of the MKTS factor. The sample period is 1986:01 to 2022:12 ($T = 444$).

IA.8 Economic properties

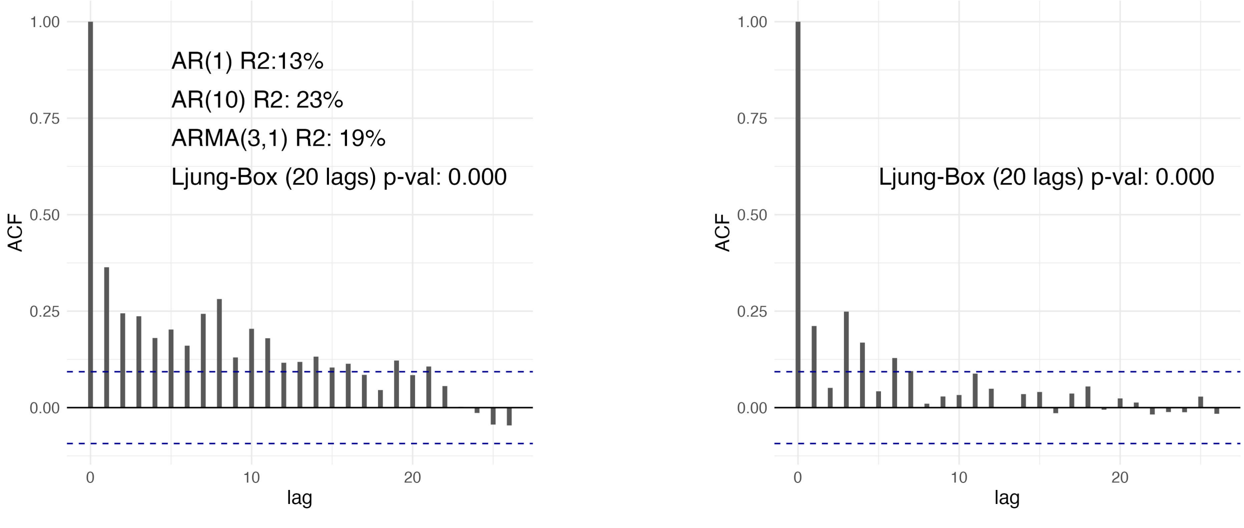
In this section we provide additional results to complement the analysis in Section 3.4.

Panel A in Figure IA.27 shows that the BMA-SDF is highly predictable: virtually all of its autocorrelation coefficients are statistically significant at the 1% level up to 20 months ahead, and the p-value of the Ljung and Box (1978) test of joint significance is zero at this horizon. Additionally, about one fifth of its time series variance is explained by its own lags (23% for the best AR specification and 19% for the best ARMA specification according to the AIC and the BIC).

Figure IA.28 shows the autocorrelations for a range of models discussed in Appendix D. As is evident, none of the other models come close to displaying the same level of business cycle variation and persistency as our BMA-SDF: the KNS SDF has about 11% of its time series variation being predictable by its own history, while this number drops to 4% for RPPCA, and its only 2% to 3%, for FF5 and CAPMB, and zero for HKM and CAPM.

Moreover, as shown in Panel A of Table IA.XXI, the SDFs with a higher degree of persistency, KNS and RPPCA, are exactly the ones with the highest degree of correlation with the BMA-SDF (0.78 and 0.55, respectively), and are the closest competitors for the BMA-SDF in the pricing exercises in Section 3.1. Instead, SDFs that perform significantly worse in cross-sectional pricing have both little time series persistency and correlations with the BMA-SDF in the 0.16 to 0.29 range.

The GARCH(1,1) coefficient estimates in Figure 11 imply a highly persistent conditional volatility, with deviations from the mean exhibiting a half-life of approximately 16.6 months. In Figure IA.29 we show that the volatility patterns of the BMA-SDF are not simply driven by the



A: Co-pricing BMA-SDF predictability **B:** Squared forecast errors of BMA-SDF

Figure IA.27: Autocorrelation functions of co-pricing BMA-SDF and forecast errors.

In Panel A we show the autocorrelation coefficients of the co-pricing BMA-SDF and in Panel B we plot its squared forecast errors. The BMA-SDF is obtained with 80% prior Sharpe ratio. The ARMA(3,1) conditional mean process is selected via the AIC and the BIC. The Ljung and Box (1978) p-value tests the null of squared autocorrelations being equal to zero. The sample period is 1986:01 to 2022:12 ($T = 444$).

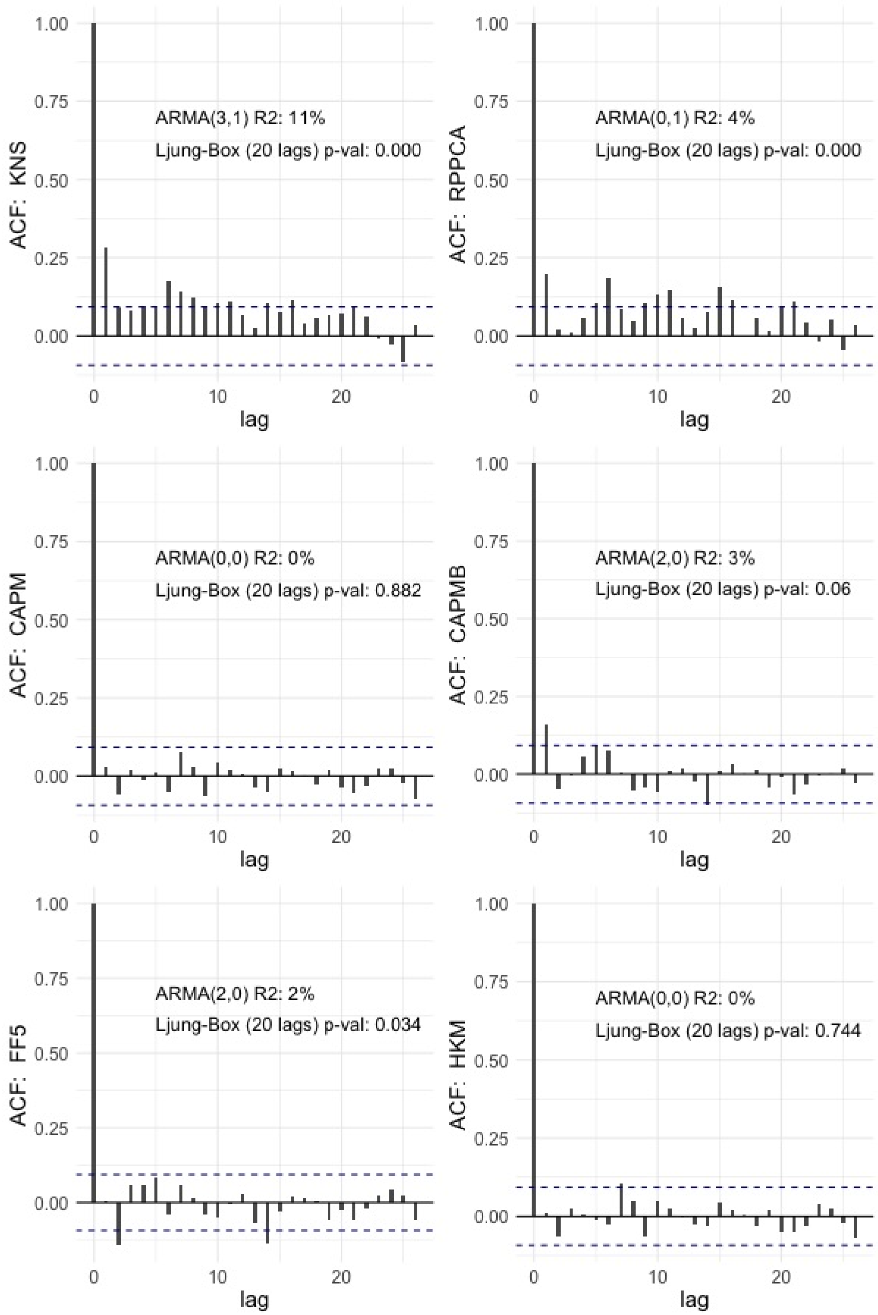


Figure IA.28: Autocorrelations functions of SDFs from alternative models.

The figure shows the autocorrelation coefficients of the SDFs estimated using KNS, RPPCA, CAPM, CAPMB, FF5 and HKM (from left to right and top to bottom). CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. The ARMA mean process for each model is selected via the AIC and the BIC. The Ljung and Box (1978) p-value tests the null of squared autocorrelations being equal to zero. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table IA.XXI: Correlation of SDF levels and volatilities

	KNS	RPPCA	CAPM	CAPMB	FF5	HKM
Panel A: SDF levels						
BMA	0.78	0.55	0.16	0.28	0.29	0.16
KNS		0.85	0.11	0.46	0.32	0.13
RPPCA			0.09	0.35	0.18	0.11
CAPM				0.42	0.70	0.98
CAPMB					0.70	0.41
FF5						0.66
Panel B: SDF estimated volatilities						
BMA	0.76	0.70	0.74	0.52	0.56	0.74
KNS		0.71	0.64	0.55	0.55	0.65
RPPCA			0.54	0.18	0.24	0.56
CAPM				0.57	0.61	0.98
CAPMB					0.75	0.57
FF5						0.58

Panel A shows the correlation matrix of the SDFs from the co-pricing BMA-SDF, KNS, RPPCA, CAPM, CAPMB, FF5 and HKM. Panel B shows the correlations for the same model of their filtered volatilities. The BMA-SDF is obtained with 80% prior Sharpe ratio. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. The BMA-SDF is obtained with 80% prior Sharpe ratio. The ARMA mean process for each model is selected using the AIC and the BIC. Volatilities are estimated using a GARCH(1,1) model. The Ljung and Box (1978) p-value tests the null of squared autocorrelations being equal to zero. The sample period is 1986:01 to 2022:12 ($T = 444$).

tradable factors by removing them from the BMA-SDF and re-estimating the volatility process of the new nontradable-only SDF. The resulting volatility process remains very persistent (with a half-life of 12.3 months), with pronounced business cycle variation and reaction to periods of heightened economic uncertainty. Moreover, the correlation of the two BMA-SDF volatility time series in Figures 11 and IA.29 is around 62%. That is, both tradable and nontradable components of the BMA-SDF are characterized by a very persistent volatility with a clear business cycle pattern.

Panel B of Figure IA.27 reports the empirical autocorrelation function of the squared forecast errors of the co-pricing BMA-SDF while the squared forecast errors for the SDFs of the KNS, RPPCA, CAPM, CAPMB, FF5 and HKM models are reported in Figure IA.30. As mentioned above, the conditional volatility of the co-pricing BMA-SDF is highly persistent, with deviations from the mean exhibiting a half-life of approximately 16.6 months. Instead, Figure IA.30 for example shows that the half-life of volatility shocks to the FF5 SDF model is only 4.21 months, and for the CAPMB it is just 3 months. That is, the use of tradable factors in the SDF does not mechanically deliver our findings for the BMA-SDF.

Finally, it seems that the alternative SDF models do not sufficiently capture business cy-

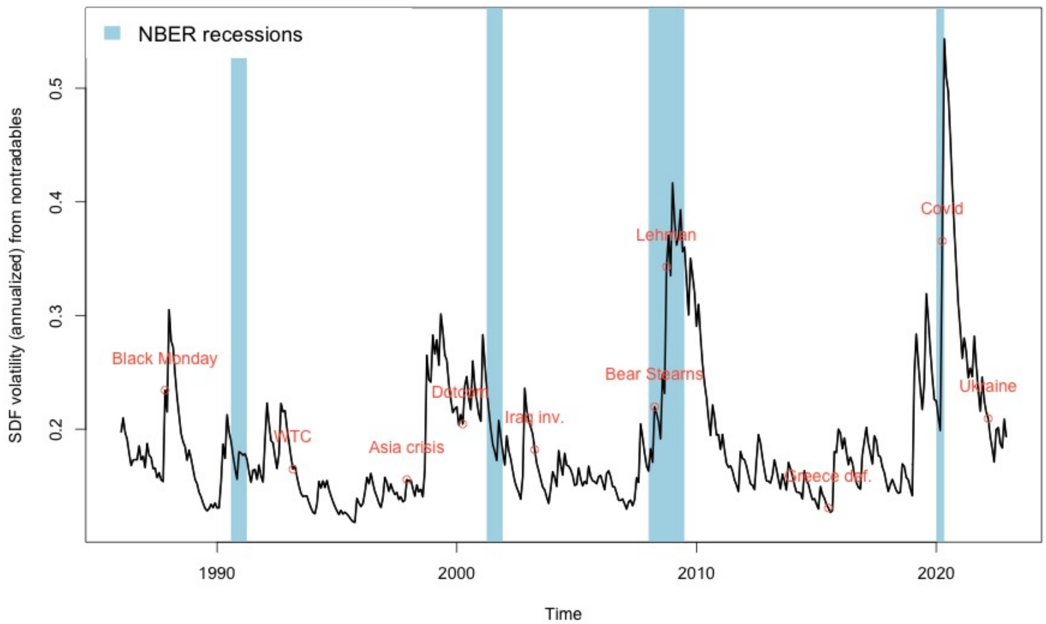


Figure IA.29: Volatility of the co-pricing BMA-SDF with only nontradable factors.

The figure plots the annualized volatility of the co-pricing BMA-SDF estimated using only nontradable factors. The volatility of the BMA-SDF is obtained by fitting an ARMA(3,1)-GARCH(1,1) to the posterior mean of the co-pricing BMA-SDF whereby the specification is selected via the AIC and the BIC. The GARCH quasi-maximum likelihood coefficient estimates are:

$$\sigma_{t+1}^2 = \omega + \alpha\epsilon_t^2 + \beta\sigma_t^2$$

	ω	α	β
Estimate	0.000202	0.142293	0.798533
Robust SE	0.000090	0.052041	0.047567

Shaded areas denote NBER recession periods. The sample period is 1986:01 to 2022:12 ($T = 444$).

cle variation and periods of high economic uncertainty. We show this by linearly projecting the estimated volatility of our co-pricing BMA-SDF on the estimated volatilities of the KNS, RPPCA, CAPM, CAPMB, FF5 and HKM models. Figure IA.31 plots the time series of the residuals, revealing that they still show a very strong business cycle variation and they exhibit similar spikes as the volatility series in Figure 11. Overall, the observed business cycle variations and predictability in both the first and second moments of the BMA-SDF would imply, within a structural model, time-varying and predictable risk premia for tradable assets.

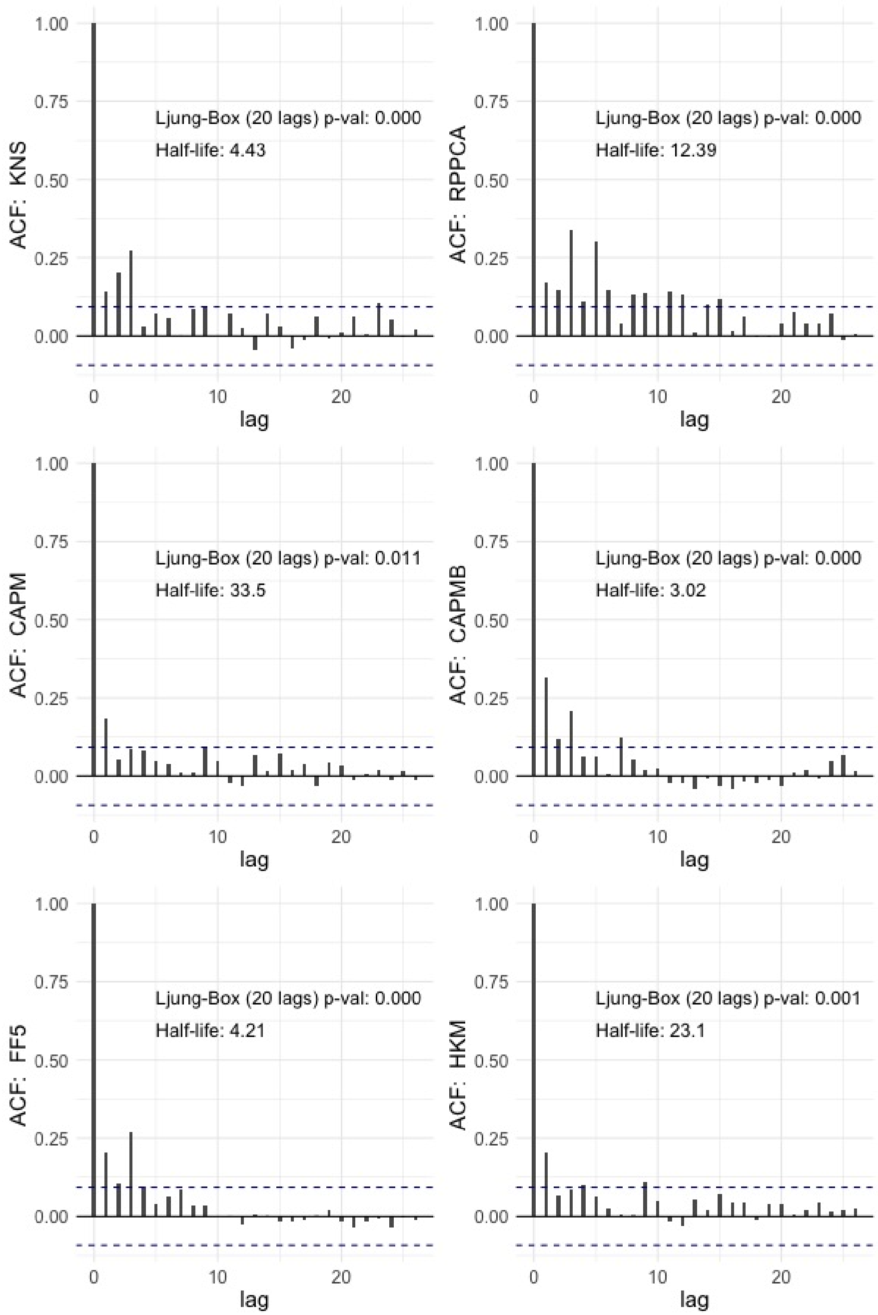


Figure IA.30: Autocorrelations of SDF squared residuals.

The figure shows the autocorrelation coefficients of the squared residuals of SDFs estimated using KNS, RPPCA, CAPM, CAPMB, FF5 and HKM (from left to right and top to bottom). CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. The ARMA mean process for each model is selected using the BIC and reported in Figure IA.28. The Ljung and Box (1978) p-value tests the null of squared autocorrelations being equal to zero. The sample period is 1986:01 to 2022:12 ($T = 444$).

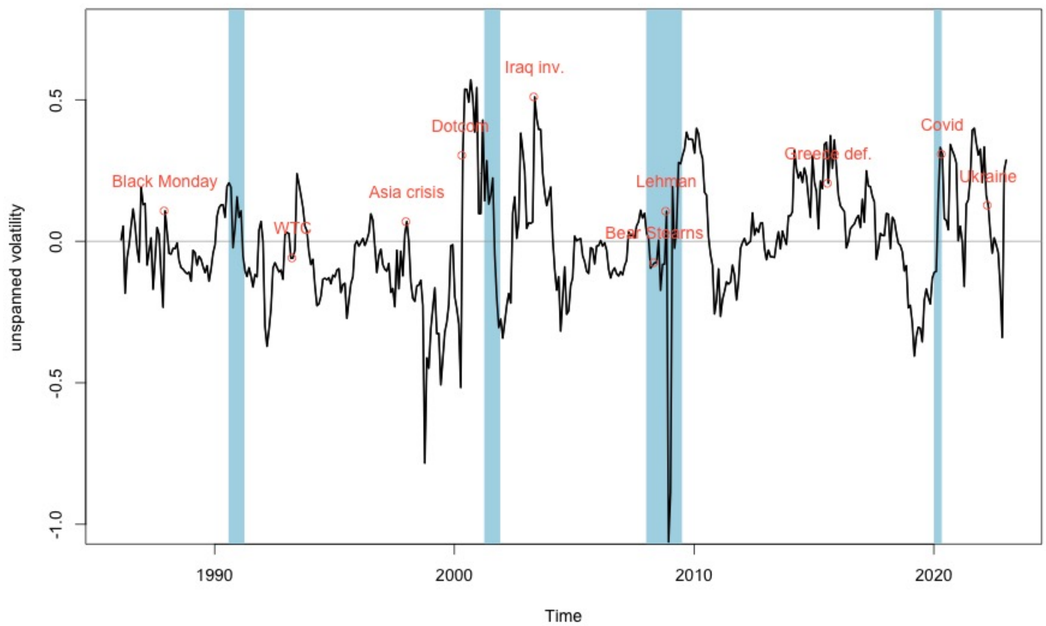


Figure IA.31: Residual volatility of the co-pricing BMA-SDF.

The figure Residuals of the linear projection of the BMA-SDF estimated volatility on the volatilities of CAPM, CAPMB, KNS, RPPCA, FF5 and HKM SDFs CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. The sample period is 1986:01 to 2022:12 ($T = 444$).

IA.9 Prior perturbation

In this section we provide additional results to complement the robustness analysis in Sections 4.1, 4.2 and 4.3 with regards to perturbations of the prior and removing the most important factors in terms of posterior probabilities and market prices of risk, respectively.

IA.9.1 Factor tilting

First, we tilt the estimation of the co-pricing BMA-SDF in favor of bond factors by setting $\kappa = 0.5$. This implies the belief that they explain a share of the squared Sharpe ratio of the SDF that is $\frac{1+\kappa}{1-\kappa} = 3$ times as large as the share of stock factors. Thereafter, we tilt toward stock factors. In Figure IA.32 we report the posterior factor probabilities estimated with the tilted priors either in favor of bond (bars with diagonal lines) or stock (bars with dots) factors, respectively. Overall, the likelihood of the data is quite informative for the posterior probabilities, especially for the nontradable factors. Posterior probabilities for bond and stock factors reflect the direction of the tilt.

Similarly, the posterior market prices of risk depicted in Figure IA.33 highlight that the set

Table IA.XXII: IS and OS cross-sectional asset pricing performance across κ tilts

	In-sample				Out-of-sample			
	20%	40%	60%	80%	20%	40%	60%	80%
Panel A: Baseline ($\kappa = 0$)								
RMSE	0.214	0.203	0.185	0.167	0.114	0.102	0.095	0.090
MAPE	0.167	0.154	0.139	0.125	0.081	0.074	0.069	0.065
R^2_{OLS}	0.155	0.240	0.367	0.487	0.357	0.489	0.557	0.603
R^2_{GLS}	0.106	0.168	0.232	0.285	0.038	0.070	0.098	0.124
Panel B: Bond factor tilt ($\kappa = 0.5$)								
RMSE	0.200	0.185	0.175	0.161	0.117	0.113	0.111	0.104
MAPE	0.152	0.139	0.130	0.119	0.085	0.090	0.091	0.085
R^2_{OLS}	0.258	0.368	0.438	0.523	0.330	0.367	0.390	0.466
R^2_{GLS}	0.106	0.168	0.224	0.272	0.040	0.072	0.096	0.119
Panel C: Stock factor tilt ($\kappa = -0.5$)								
RMSE	0.240	0.229	0.209	0.183	0.122	0.116	0.112	0.105
MAPE	0.195	0.182	0.165	0.143	0.089	0.085	0.083	0.078
R^2_{OLS}	−0.063	0.035	0.195	0.382	0.271	0.337	0.384	0.453
R^2_{GLS}	0.107	0.163	0.222	0.281	0.035	0.064	0.092	0.122

The table presents the cross-sectional in- and out-of-sample asset pricing performance of the co-pricing BMA-SDF estimated with and without factor tilts. We provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The models are first estimated using the baseline IS test assets. The resulting SDF is then used to price (with no additional parameter estimation) each set of the OS assets. Panel A provides the baseline estimation without any factor tilt ($\kappa = 0$) as discussed in Section 2.3. In Panel B we tilt the prior towards bond factors ($\kappa = 0.5$) and in Panel C towards stock factors ($\kappa = -0.5$), respectively. The factor tilts imply bond (Panel B) and stock (Panel C) factors explain a $\frac{1+\kappa}{1-\kappa} = 3$ times as large share of the squared Sharpe ratio than stock and bond factors, respectively. The IS test assets are the 83 bond and stock portfolios and the 40 tradable bond and stock factors. The OS test assets are the combined 154 bond and stock portfolios. All are described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table IA.XXIII: OS cross-sectional pricing performance for bonds and stocks across κ tilts

	Stock test assets				Bond test assets			
	20%	40%	60%	80%	20%	40%	60%	80%
Panel A: Baseline ($\kappa = 0$)								
RMSE	0.102	0.087	0.080	0.076	0.122	0.115	0.108	0.101
MAPE	0.075	0.066	0.060	0.057	0.089	0.083	0.079	0.074
R^2_{OLS}	0.330	0.513	0.591	0.629	0.064	0.171	0.267	0.354
R^2_{GLS}	0.106	0.189	0.246	0.276	0.022	0.051	0.078	0.107
Panel B: Bond factor tilt ($\kappa = 0.5$)								
RMSE	0.112	0.100	0.094	0.089	0.121	0.112	0.105	0.100
MAPE	0.082	0.073	0.069	0.065	0.088	0.081	0.076	0.073
R^2_{OLS}	0.195	0.356	0.435	0.494	0.078	0.216	0.307	0.365
R^2_{GLS}	0.088	0.148	0.187	0.216	0.036	0.073	0.097	0.118
Panel C: Stock factor tilt ($\kappa = -0.5$)								
RMSE	0.095	0.080	0.073	0.070	0.123	0.118	0.112	0.103
MAPE	0.071	0.062	0.058	0.054	0.090	0.086	0.081	0.076
R^2_{OLS}	0.419	0.591	0.655	0.687	0.050	0.116	0.215	0.334
R^2_{GLS}	0.123	0.218	0.278	0.315	0.006	0.026	0.054	0.093

The table presents the cross-sectional out-of-sample asset pricing performance of the co-pricing BMA-SDF estimated with and without factor tilts for bonds and stocks, respectively. We provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The models are first estimated using the baseline IS test assets. The resulting SDF is then used to price (with no additional parameter estimation) each set of the OS assets. Panel A provides the baseline estimation without any factor tilt ($\kappa = 0$) as discussed in Section 2.3. In Panel B we tilt the prior towards bond factors ($\kappa = 0.5$) and in Panel C towards stock factors ($\kappa = -0.5$), respectively. The factor tilts imply bond (Panel B) and stock (Panel C) factors explain a $\frac{1+\kappa}{1-\kappa} = 3$ times as large share of the squared Sharpe ratio than stock and bond factors, respectively. The IS test assets are the 83 bond and stock portfolios and the 40 tradable bond and stock factors. The OS test assets are the combined respective 77 bond and stock portfolios. All are described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

of factors that features more prominently in the co-pricing BMA-SDF is largely unchanged, albeit their individual posterior λs do vary in the expected directions. That is, market prices of risk that are very small in absolute terms are not strongly affected by the factor tilt.

In Table IA.XXII we report in- and out-of-sample performance measures for the co-pricing BMA-SDF without (Panel A) and with bond (Panel B) and stock (Panel C) factor tilts. As in Tables 2 and 3 we first estimate the co-pricing BMA-SDF on the standard 123 test assets and then use the resulting BMA-SDF to price the 154 OS test assets that are all described in Section 1. The numbers do not change materially when comparing the two panels in the table. Overall, the effect of the prior tilting is small and unambiguous in direction: as we tilt toward *either* type of factor, the out-of-sample pricing ability deteriorates. This is very much in line with the findings in Section 3.3: for the co-pricing of stock and bond excess returns, we need information from both factor zoos. Consequently, over-reliance on either type of factor worsens the BMA-SDF performance. This result is further reinforced in Table IA.XXIII where we consider the separate pricing of bond and stock excess returns using the co-pricing BMA-SDF estimated with and without factor tilts. The deterioration in out-of-sample pricing performance is stronger for stocks when tilting the prior in favor of bond factors and vice versa, although it's asymmetric, again suggesting a much more limited information content in the bond factor zoo relative to the equity one.

Next, we apply the factor tilts to price duration-adjusted bond returns. As the results in Section 3.3 suggest, once we account for the Treasury component of bond returns, the bond factor zoo becomes largely redundant. This would imply that tilting the prior in favor of stock (bond) factors should actually improve (worsen) the pricing ability of the BMA-SDF. Figure IA.34 highlights this: as the prior is tilted away from bond factors (moving from $\kappa = 0.5$ towards $\kappa = -0.5$), the OS measures of cross-sectional fit improve for the models estimated with duration-adjusted corporate bond returns.

Finally, an extreme tilt in favor of stock factors as implemented in Figure IA.35 maximizes the pricing ability of the BMA-SDF for duration-adjusted returns but performs worse for the standard corporate bond excess returns we use in our baseline analysis. Overall, this further reinforces our previous findings: the bond factor zoo is largely redundant for co-pricing bond and stock portfolios once the Treasury component of the latter is accounted for.

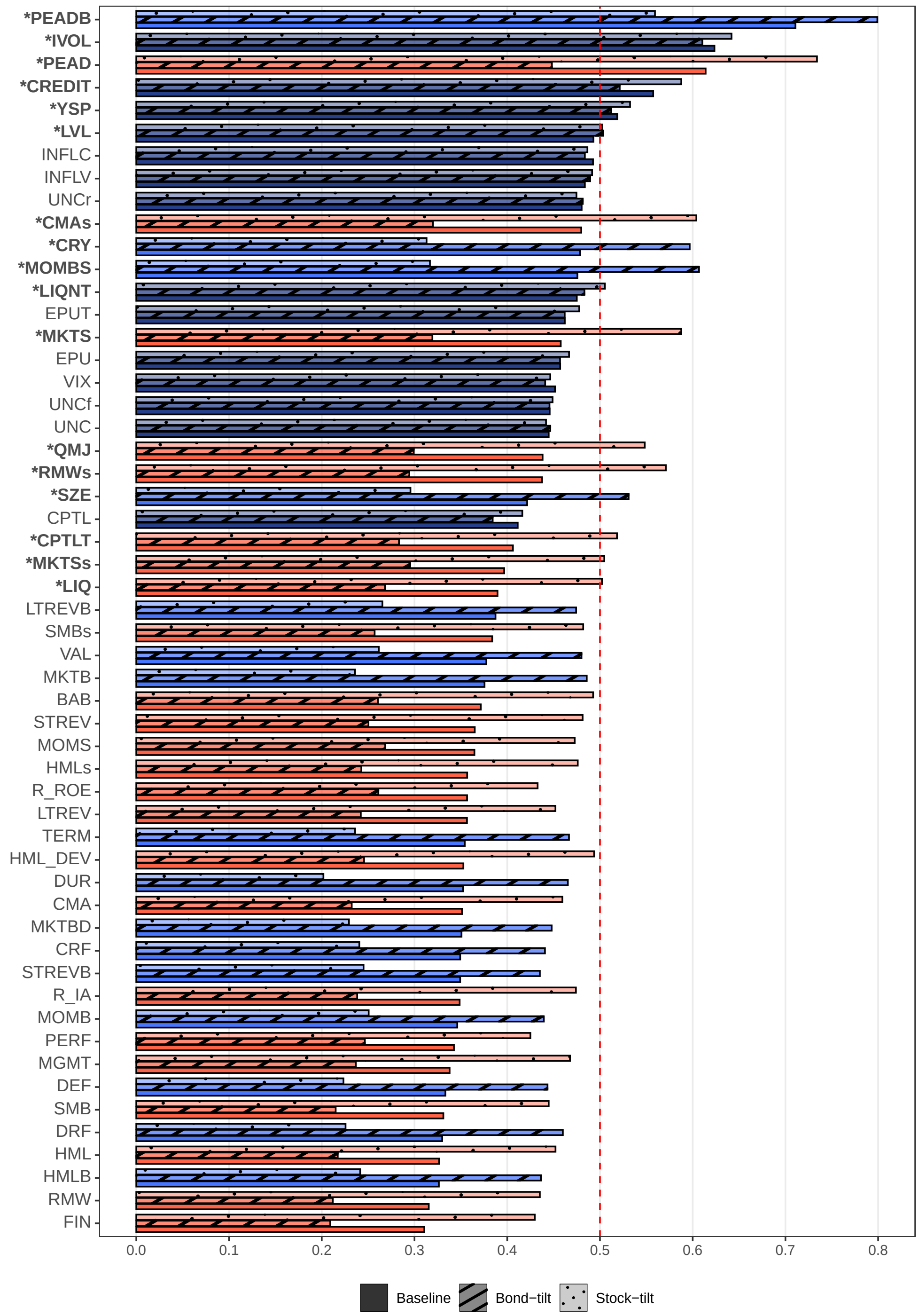


Figure IA.32: Posterior factor probabilities across κ tilts.

The figure reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$ with Sharpe ratio values set as 80% of the ex post maximum including factor tilting induced by augmenting the κ parameter described in Section 2.3, of the 54 bond and stock factors described in Appendix A. The labels are ordered by each factor's average posterior probability with κ set to 0 (no factor tilting). The bond (stock) factor tilt involves setting κ for the bond to values of $0.5(-0.5)$ respectively, which implies bond (stock) factors explain a $\frac{1+\kappa}{1-\kappa} = 3$ times as large share of the squared Sharpe ratio than stock (bond) factors. Factors with a posterior probability > 0.5, for any value of κ are in bold face and include an asterisk. The prior distribution for the j^{th} factor inclusion is a Beta(1,1), yielding a 50% prior expectation for γ_j. The sample period is 1986:01 to 2022:12 ($T = 444$).

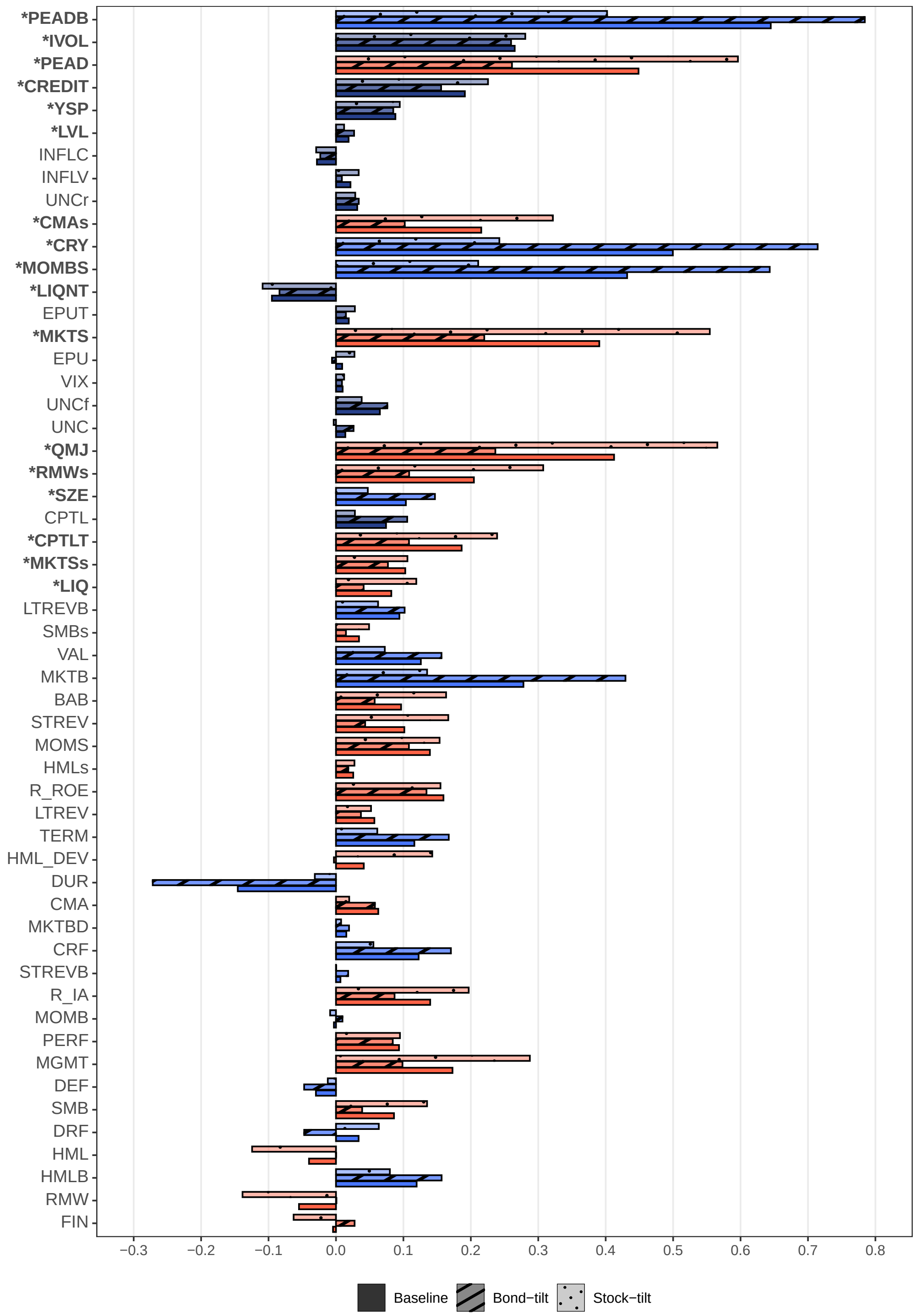
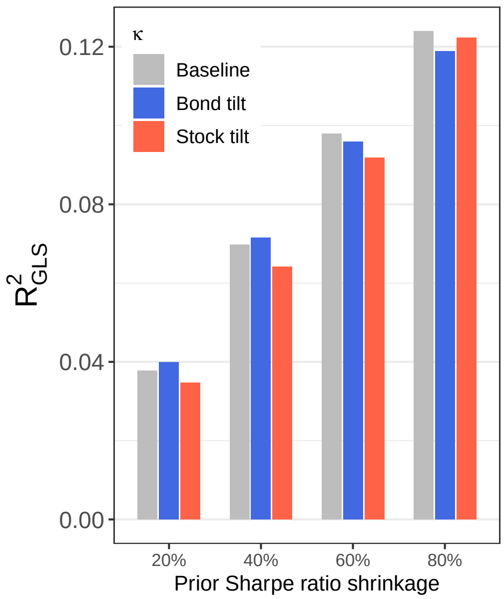
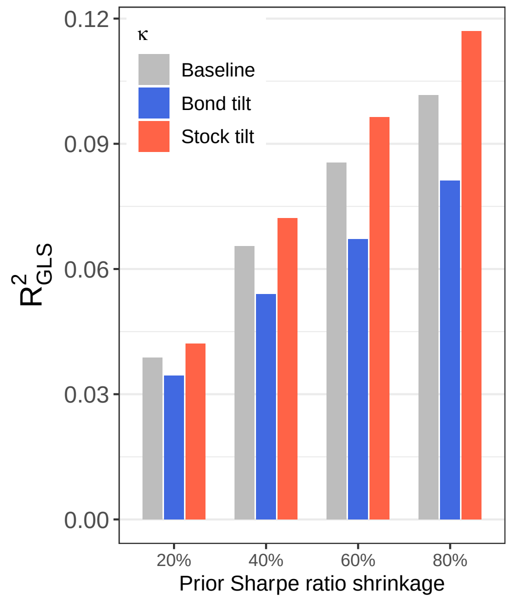


Figure IA.33: Posterior market prices of risk across κ tilts.

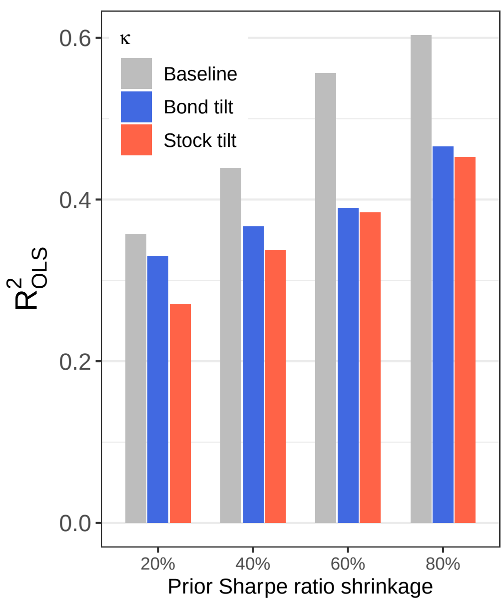
The figure reports posterior market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$ with Sharpe ratio values set as 80% of the ex-post maximum including factor tilting induced by augmenting the κ parameter described in Section 2.3, of the 54 bond and stock factors described in Appendix A. The labels are ordered by each factor's average posterior probability, $\mathbb{E}[\gamma_j|\text{data}]$ with κ set to 0 (no factor tilting). The bond (stock) factor tilt involves setting κ to values of 0.5(−0.5) respectively, which implies bond (stock) factors explain a $\frac{1+\kappa}{1-\kappa} = 3$ times as large share of the squared Sharpe ratio than equity (bond) factors. Factors with a posterior probability > 0.5, for any value of κ are in bold face and include an asterisk. The prior distribution for the j^{th} factor inclusion is a Beta(1,1), yielding a 50% prior expectation for γ_j. The sample period is 1986:01 to 2022:12 ($T = 444$).



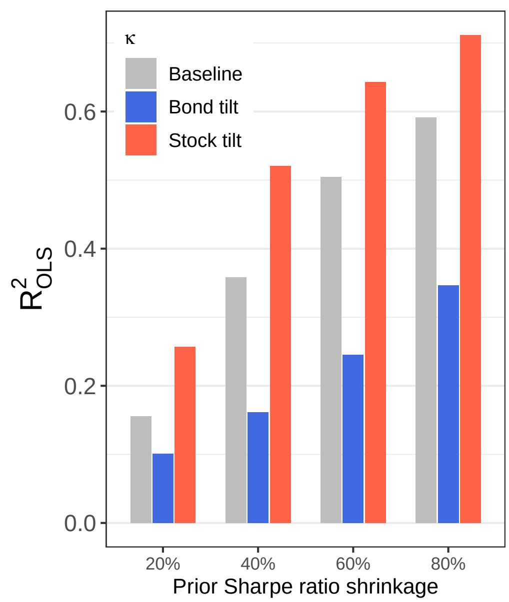
A: Excess bond returns



B: Duration-adjusted bond returns



C: Excess bond returns



D: Duration-adjusted bond returns

Figure IA.34: OS cross-sectional asset pricing performance across κ tilts.

This figure plots out-of-sample R^2_{GLS} (Panels A and B) and R^2_{OLS} (Panels C and D) of the co-pricing BMA-SDF pricing the joint cross-section of excess bond and stock returns (Panels A and C) as well as the joint cross-section of duration-adjusted bond and stock excess returns (Panels B and D), respectively. The IS test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors. In Panels B and D all bond returns are duration adjusted as per equation (10). The OS test assets are the combined 154 bond and stock portfolios and 40 tradable bond and stock factors (again calculated using duration-adjusted bond returns for Panels B and D). All are described in Section 1. Each panel provides the measures of fit for the baseline estimation without any factor tilt ($\kappa = 0$, grey bars), a bond-factor tilt ($\kappa = 0.5$, blue bars), and a stock-factor tilt ($\kappa = -0.5$, red bars), as discussed in Section 2.3, respectively. The factor tilts imply bond and stock factors explain a $\frac{1+\kappa}{1-\kappa} = 3$ times as large share of the squared Sharpe ratio than stock and bond factors, respectively.

IA.9.2 Imposing sparsity

Our method not only allows tilting factors towards a certain group (bond vs. stock as discussed in Section IA.9.1 or DR vs. CF news as discussed in Section IA.6.2) but also provides the flexibility to encode beliefs about the density of the SDF through the Beta-distributed prior probability of factor inclusion $\pi(\gamma_j = 1|\omega_j) = \omega_j \sim Beta(a_\omega, b_\omega)$. For our baseline estimations we do not take an ex ante stance on whether the SDF should be sparse or dense. However, since the extant literature overwhelmingly assumes a high degree of sparsity, typically favoring factor models with approximately five factors, we now tweak the prior mean and variance to mirror such a belief. In particular, by choosing the prior mean and variance of ω_j, $\mathbb{E}[\omega_j] = \frac{a_\omega}{a_\omega + b_\omega}$ and $\text{Var}(\omega_j) = \frac{a_\omega b_\omega}{(a_\omega+b_\omega)^2(a_\omega+b_\omega+1)}$ we can form a prior on the model dimensions that is similar to what is typically used in the literature. Setting $a_\omega \approx 3.54$ and $b_\omega \approx 34.66$ we get: (i) the prior expectation of included factors, $\mathbb{E}[\omega_j] \times K$, yields the canonical five-factor model, and (ii) the prior two standard deviation credible interval encompasses models with zero to ten factors (since $\text{Var}(\omega_j) = (2.5/K)^2$).

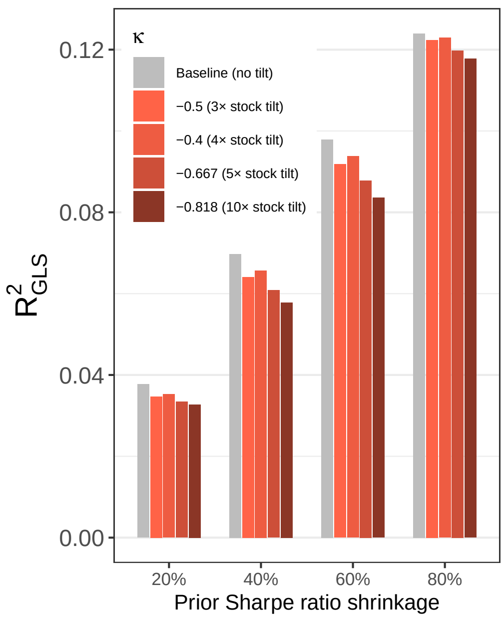
Table IA.XXIV shows that the factors with posterior probabilities exceeding the prior value (that is, 9.26%) are essentially identical to those in our baseline estimates in Table A.2. The only exception occurs under the lowest prior shrinkage, where PEAD's posterior probability drops below this threshold—an expected outcome given this prior's reduced ability to control confounding effects from weak factors. Moreover, as shown in Table IA.XXV, the pricing performance of the co-pricing BMA-SDF a sparsity-favoring prior remains superior compared to the list of models we consider in Appendix D, particularly out-of-sample. Finally, imposing sparsity degrades the performance of the BMA-SDF compared to our baseline findings in Tables 2 and 3. This is not surprising as Figure 3 and Table 4 demonstrate that the data strongly support a dense SDF.

IA.9.3 Estimation *excluding* the most likely factors

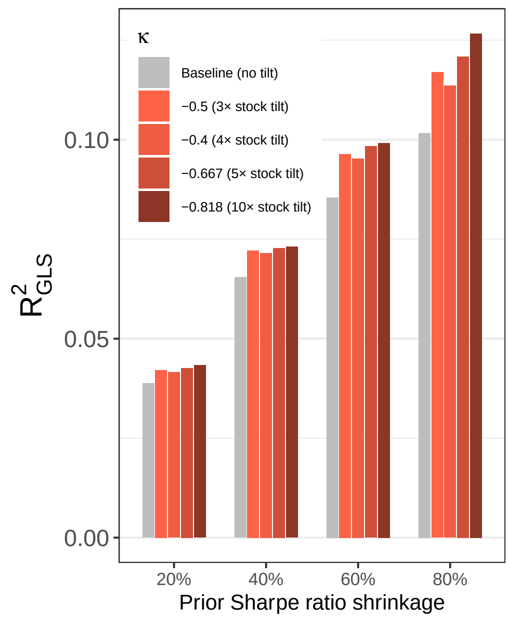
In this section we assess whether our BMA-SDF method provides a robust characterization of the true latent SDF even when factors capturing fundamental risk sources are removed from the candidate set. Thus, we remove the factors identified as most salient for characterizing the true latent SDF and construct a BMA-SDF using the remaining factors. In Table IA.XXVI we report the pricing ability of the resulting co-pricing BMA-SDF both in- and out-of-sample. In Panel A we report the results from Tables 2 (IS) and 3 (OS). In Panel B we exclude PEADB, PEAD, IVOL, CREDIT, and YSP, the top five factors in terms of probability from Table A.2. In Panel C we exclude PEADB, PEAD, CRY, QMJ, and MOMBS, the top five factors in terms of market price of risk from Table A.2. In Panel D we exclude the eight factors PEADB, PEAD, IVOL, CREDIT, YSP, PEAD, CRY, QMJ, MOMBS, the union of the factors excluded in Panels B and C.

The BMA-SDF constructed with the limited set of factors still strongly outperforms canonical models from the literature both in- and out-of-sample.

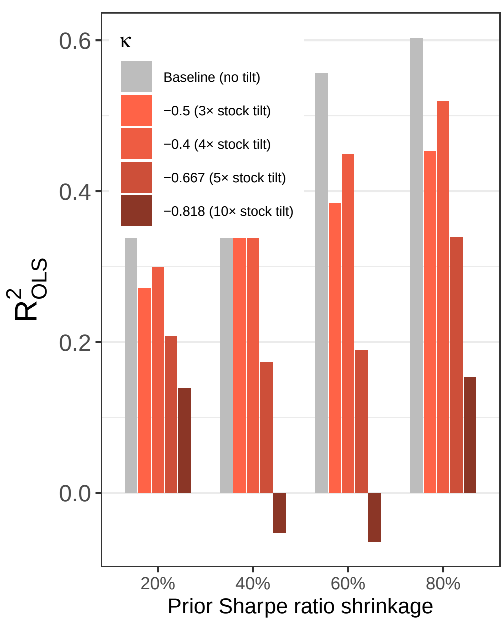
Figures IA.36 to IA.38 present the posterior factor probabilities and market prices of risk corresponding to Panels B through D in Table IA.XXVI. Removing the top factors from Table A.2 results in increased posterior weights for $\mathbb{E}[\lambda_j|\text{data}]$ of several noisy proxies in the BMA-SDF—precisely what our theoretical and simulation results in Section 2.4 predict.



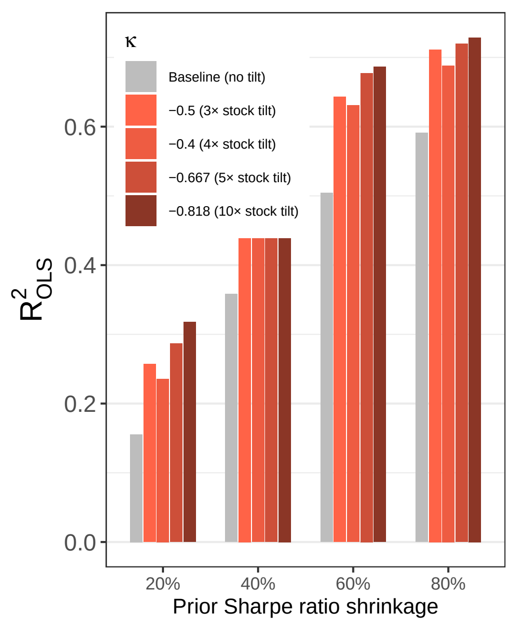
A: Excess bond returns



B: Duration-adjusted bond returns



C: Excess bond returns



D: Duration-adjusted bond returns

Figure IA.35: OS cross-sectional asset pricing performance: Favoring stock factors.

This figure plots out-of-sample R^2_{GLS} (Panels A and B) and R^2_{OLS} (Panels C and D) of the co-pricing BMA-SDF pricing the joint cross-section of excess bond and stock returns (Panels A and C) as well as the joint cross-section of duration-adjusted bond and stock excess returns (Panels B and D), respectively. The IS test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors. In Panels B and D all bond returns are duration adjusted as per equation (10). The OS test assets are the combined 154 bond and stock portfolios and 40 tradable bond and stock factors (again calculated using duration-adjusted bond returns for Panels B and D). All are described in Section 1. Each panel provides the measures of fit for the baseline estimation without any factor tilt ($\kappa = 0$, grey bars), as well as stock-factor tilts with increasing more negative κ-values ($\kappa = -0.5, -0.4, -2/3, -9/11$, increasingly dark red bars). The κ-values imply stock factors explain $\left(\frac{1+\kappa}{1-\kappa}\right)^{-1} =$ 3, 4, 5 and 10 times (respectively) as large a share of the squared Sharpe ratio than bond factors.

Table IA.XXIV: Posterior factor probabilities and risk prices imposing sparsity

	Factor prob., $\mathbb{E}[\gamma_j \| \text{data}]$				Price of risk, $\mathbb{E}[\lambda_j \| \text{data}]$			
	Total prior Sharpe ratio				Total prior Sharpe ratio			
Factors	20%	40%	60%	80%	20%	40%	60%	80%
IVOL	0.109	0.173	0.268	0.326	0.011	0.066	0.204	0.385
PEADB	0.157	0.212	0.191	0.152	0.060	0.164	0.186	0.176
YSP	0.094	0.106	0.110	0.127	0.003	0.014	0.034	0.075
CREDIT	0.099	0.121	0.123	0.104	0.008	0.034	0.063	0.076
LVL	0.091	0.088	0.088	0.079	0.001	0.002	0.005	0.010
INFLV	0.094	0.092	0.086	0.074	0.002	0.006	0.011	0.014
UNCr	0.089	0.086	0.077	0.067	0.001	0.003	0.006	0.011
INFLC	0.096	0.091	0.081	0.064	−0.001	−0.004	−0.008	−0.011
PEAD	0.103	0.114	0.095	0.064	0.027	0.064	0.070	0.064
EPUT	0.084	0.085	0.075	0.056	0.002	0.007	0.011	0.013
EPU	0.103	0.091	0.076	0.054	0.001	0.003	0.004	0.004
LIQNT	0.097	0.087	0.075	0.053	−0.003	−0.009	−0.015	−0.015
UNC	0.090	0.084	0.074	0.048	−0.001	−0.003	−0.004	−0.003
VIX	0.087	0.087	0.064	0.048	0.000	0.001	0.001	0.002
UNCf	0.093	0.079	0.062	0.041	−0.003	−0.007	−0.007	−0.005
CMAs	0.097	0.078	0.060	0.039	0.012	0.024	0.024	0.021
MKTSs	0.083	0.076	0.057	0.037	0.012	0.025	0.028	0.029
RMWs	0.089	0.077	0.054	0.034	0.018	0.032	0.030	0.029
MOMBS	0.089	0.067	0.046	0.030	0.032	0.042	0.039	0.048
SZE	0.080	0.064	0.050	0.030	0.005	0.010	0.010	0.009
BAB	0.090	0.073	0.048	0.030	0.017	0.027	0.025	0.024
LIQ	0.076	0.061	0.043	0.029	0.004	0.008	0.008	0.009
QMJ	0.093	0.066	0.042	0.027	0.037	0.042	0.039	0.058
MKTS	0.086	0.063	0.041	0.027	0.028	0.038	0.035	0.048
STREVB	0.084	0.062	0.044	0.025	0.003	0.005	0.004	0.003
LTREVB	0.078	0.058	0.039	0.025	0.009	0.014	0.012	0.012
MOMS	0.081	0.056	0.038	0.025	0.012	0.016	0.015	0.021
R_ROE	0.080	0.056	0.037	0.024	0.025	0.028	0.028	0.043
SMBs	0.083	0.058	0.041	0.024	0.002	0.003	0.003	0.002
CPTL	0.079	0.064	0.039	0.024	0.011	0.017	0.015	0.017
PERF	0.084	0.054	0.038	0.024	0.025	0.027	0.026	0.037
HMLs	0.082	0.055	0.040	0.023	0.003	0.005	0.004	0.004
STREV	0.075	0.058	0.038	0.023	0.006	0.010	0.009	0.009
MOMB	0.078	0.057	0.038	0.022	−0.001	−0.003	−0.002	−0.000
CPTLT	0.077	0.059	0.036	0.022	0.014	0.022	0.020	0.023
HMLB	0.075	0.058	0.037	0.021	0.020	0.027	0.025	0.028
CRY	0.076	0.052	0.035	0.021	0.023	0.029	0.030	0.039
LTREV	0.075	0.049	0.034	0.020	0.005	0.007	0.006	0.005
VAL	0.075	0.051	0.031	0.020	0.008	0.012	0.010	0.011
MGMT	0.080	0.050	0.032	0.020	0.027	0.026	0.027	0.041
DEF	0.070	0.047	0.032	0.020	0.000	0.001	0.001	−0.001
HML_DEV	0.073	0.047	0.030	0.019	0.002	0.005	0.003	0.001
CRF	0.069	0.045	0.031	0.019	0.006	0.009	0.009	0.014
TERM	0.074	0.048	0.030	0.019	0.014	0.015	0.014	0.022
MKTB	0.084	0.055	0.032	0.019	0.036	0.037	0.037	0.059
R_IA	0.069	0.049	0.027	0.018	0.015	0.018	0.016	0.025
CMA	0.072	0.048	0.031	0.018	0.014	0.015	0.014	0.019
SMB	0.060	0.041	0.028	0.018	0.002	0.004	0.004	0.005
RMW	0.071	0.045	0.027	0.016	0.016	0.015	0.014	0.021
FIN	0.064	0.040	0.024	0.015	0.015	0.014	0.015	0.025
HML	0.061	0.040	0.025	0.015	0.003	0.003	0.004	0.004
DRF	0.069	0.044	0.028	0.015	0.017	0.018	0.018	0.029
MKTBD	0.065	0.043	0.026	0.015	0.007	0.008	0.008	0.009
DUR	0.062	0.042	0.024	0.015	0.005	0.005	0.006	0.012

The table reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and stock factors described in Appendix A. We encode sparsity by choosing the prior mean and variance of ω_j, $\mathbb{E}[\omega_j] = \frac{a_\omega}{a_\omega+b_\omega}$ and $\text{Var}(\omega_j) = \frac{a_\omega b_\omega}{(a_\omega+b_\omega)^2(a_\omega+b_\omega+1)}$. We set $a_\omega \approx 3.54$ and $b_\omega \approx 34.66$ so that the prior expectation of how many of the K factors should be included in the SDF, $\mathbb{E}[\omega_j] \times K$, yields the canonical five-factor model; and the prior two standard deviations credible interval includes models with zero to ten factors (since $\text{Var}(\omega_j) = (2.5/K)^2$). The prior for each factor inclusion is a Beta(3.54, 34.66), yielding a prior expectation for γ_j of ~9.25%. Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The factors are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 83 bond and stock portfolios and 40 tradable bond and stock factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table IA.XXV: IS and OS cross-sectional asset pricing performance: Imposing sparsity

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: In-sample (co-pricing stocks and bonds)											
RMSE	0.221	0.213	0.207	0.199	0.260	0.278	0.258	0.259	0.232	0.166	0.214
MAPE	0.175	0.171	0.167	0.159	0.194	0.221	0.198	0.192	0.180	0.126	0.144
R^2_{OLS}	0.101	0.160	0.206	0.267	−0.244	−0.426	−0.233	−0.238	0.008	0.489	0.152
R^2_{GLS}	0.093	0.116	0.131	0.153	0.078	0.083	0.087	0.078	0.249	0.176	0.220
Panel B: Out-of-sample (co-pricing stocks and bonds)											
RMSE	0.125	0.120	0.117	0.111	0.224	0.154	0.139	0.223	0.172	0.160	0.109
MAPE	0.090	0.086	0.084	0.081	0.192	0.129	0.102	0.190	0.132	0.143	0.086
R^2_{OLS}	0.229	0.286	0.323	0.390	−1.478	−0.161	0.053	−1.444	−0.461	−0.268	0.410
R^2_{GLS}	0.029	0.042	0.057	0.078	0.028	0.034	0.036	0.028	0.099	0.065	0.030

The table presents the cross-sectional in- (Panel A) and out-of-sample (Panel B) asset pricing performance of different models pricing bonds and stocks jointly whereby the BMA-SDF is estimated with a prior tilted towards sparsity. We encode sparsity by choosing the prior mean and variance of ω_j, $\mathbb{E}[\omega_j] = \frac{a_\omega}{a_\omega + b_\omega}$ and $\text{Var}(\omega_j) = \frac{a_\omega b_\omega}{(a_\omega+b_\omega)^2(a_\omega+b_\omega+1)}$. We set $a_\omega \approx 3.54$ and $b_\omega \approx 34.66$ so that the prior expectation of how many of the K factors should be included in the SDF, $\mathbb{E}[\omega_j] \times K$, yields the canonical five-factor model; and the prior two standard deviations credible interval includes models with zero to ten factors (since $\text{Var}(\omega_j) = (2.5/K)^2$). The prior for each factor inclusion is a Beta(3.54, 34.66), yielding a prior expectation for γ_j of $\sim$9.25%. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The models are first estimated using the baseline IS test assets. The resulting SDF is then used to price (with no additional parameter estimation) each set of the OS assets. TOP includes the top five factors based on the average posterior probability. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. The IS test assets in Panel A are the 83 bond and stock portfolios and 40 tradable bond and stock factors. The OS test assets in Panel B are the combined 154 bond and stock portfolios. Throughout, we use the co-pricing factor zoo comprising the 40 tradable and 14 nontradable factors. All are described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table IA.XXVI: IS and OS cross-sectional asset pricing performance: Exclusion of top factors

	In-sample				Out-of-sample			
	20%	40%	60%	80%	20%	40%	60%	80%
Panel A: Baseline								
RMSE	0.214	0.203	0.185	0.167	0.114	0.102	0.095	0.090
MAPE	0.167	0.154	0.139	0.125	0.081	0.074	0.069	0.065
R^2_{OLS}	0.155	0.240	0.367	0.487	0.357	0.489	0.557	0.603
R^2_{GLS}	0.106	0.168	0.232	0.285	0.038	0.070	0.098	0.124
Panel B: Drop top 5 factors by their posterior probability								
RMSE	0.200	0.196	0.192	0.186	0.115	0.105	0.100	0.098
MAPE	0.161	0.152	0.145	0.139	0.082	0.076	0.073	0.070
R^2_{OLS}	0.177	0.210	0.242	0.288	0.344	0.458	0.504	0.525
R^2_{GLS}	0.107	0.149	0.189	0.223	0.035	0.060	0.083	0.102
Panel C: Drop top 5 factors by their posterior market price of risk								
RMSE	0.196	0.187	0.180	0.171	0.116	0.103	0.097	0.094
MAPE	0.157	0.144	0.134	0.125	0.082	0.074	0.070	0.068
R^2_{OLS}	0.219	0.288	0.339	0.405	0.340	0.475	0.534	0.567
R^2_{GLS}	0.098	0.140	0.174	0.205	0.033	0.056	0.077	0.101
Panel D: Drop union of excluded factors from Panels B and C								
RMSE	0.197	0.190	0.186	0.181	0.116	0.104	0.099	0.099
MAPE	0.158	0.146	0.139	0.135	0.082	0.075	0.072	0.071
R^2_{OLS}	0.210	0.265	0.300	0.337	0.336	0.467	0.513	0.520
R^2_{GLS}	0.098	0.136	0.165	0.183	0.032	0.051	0.064	0.073

The table presents the cross-sectional in- and out-of-sample asset pricing performance of the co-pricing BMA-SDF estimated using different sets of underlying factors, whereby we exclude the top five factors by their posterior probability (Panel B), their absolute market price of risk (Panel C), and the union of the top five factors by probability and market price of risk (dropping eight factors total, Panel D), respectively. We provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The models are first estimated using the baseline IS test assets. The resulting SDF is then used to price (with no additional parameter estimation) each set of the OS assets. In Panel A we report the baseline results from Tables 2 (IS) and 3 (OS). In Panel B we exclude PEADB, PEAD, IVOL, CREDIT, YSP; in Panel C we exclude PEADB, PEAD, CRY, QMJ, MOMBS; and in Panel D we exclude PEADB, PEAD, IVOL, CREDIT, YSP, CRY, QMJ, MOMBS. IS test assets are the 83 bond and stock portfolios and the respective sets of tradable stock and bond factors accounting for the exclusions described above. OS test assets are the combined 154 bond and stock portfolios. All are described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

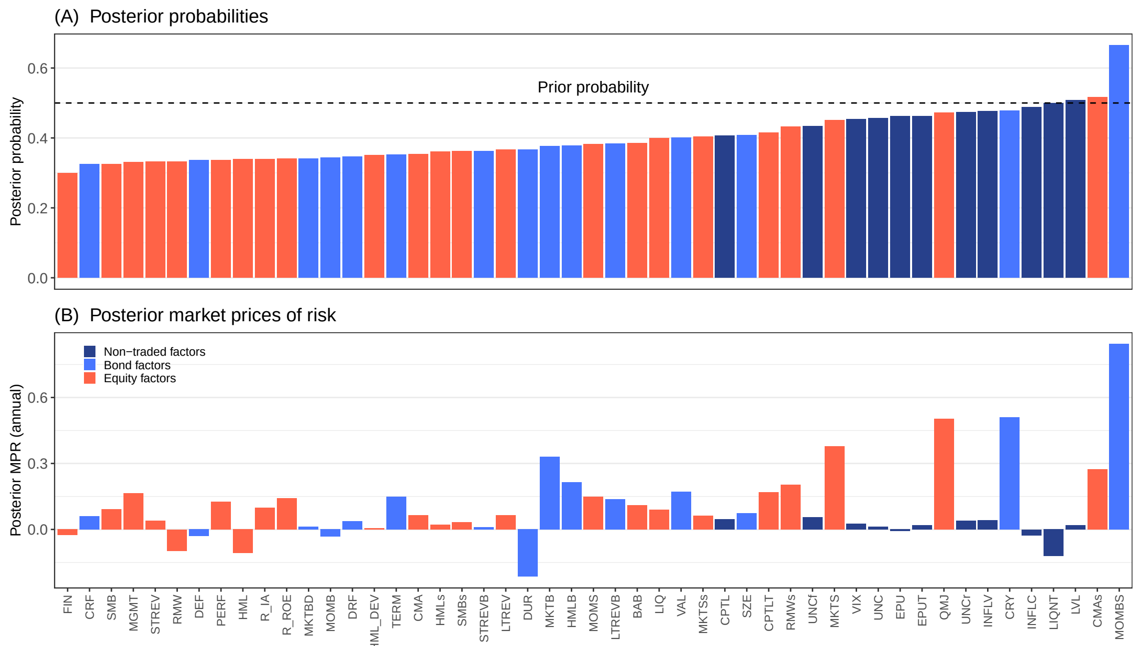


Figure IA.36: Posterior factor probabilities and risk prices excluding top factors based on posterior probability.

The figure reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of 49 bond and stock factors described in Appendix A. The following five factors are excluded based on the posterior probability of inclusion in the BMA-SDF as per Table A.2: PEADB, PEAD, IVOL, CREDIT, and YSP. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The factors are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 83 bond and stock portfolios and remaining 38 tradable bond and stock factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

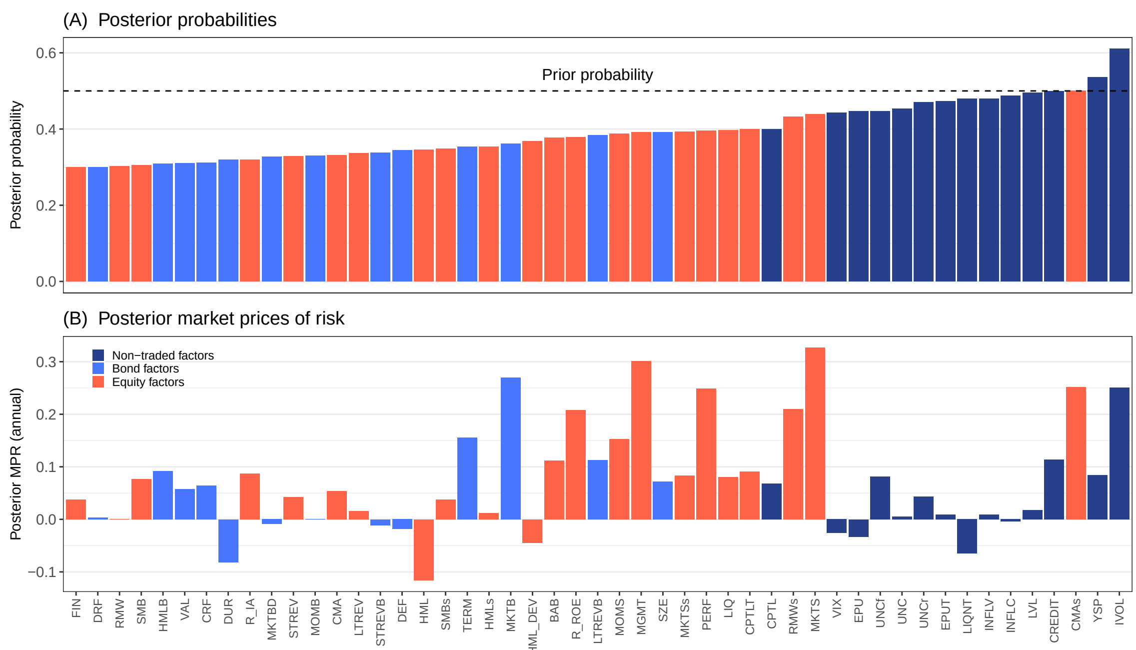


Figure IA.37: Posterior factor probabilities and risk prices excluding top factors based on market price of risk.

The figure reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of 49 bond and stock factors described in Appendix A. The following five factors are excluded based on the absolute value of the market price of risk as per Table A.2: PEADB, PEAD, CRY, QMJ, and MOMBS. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR^2_{\boldsymbol{f}} \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The factors are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 83 bond and stock portfolios and remaining 38 tradable bond and stock factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

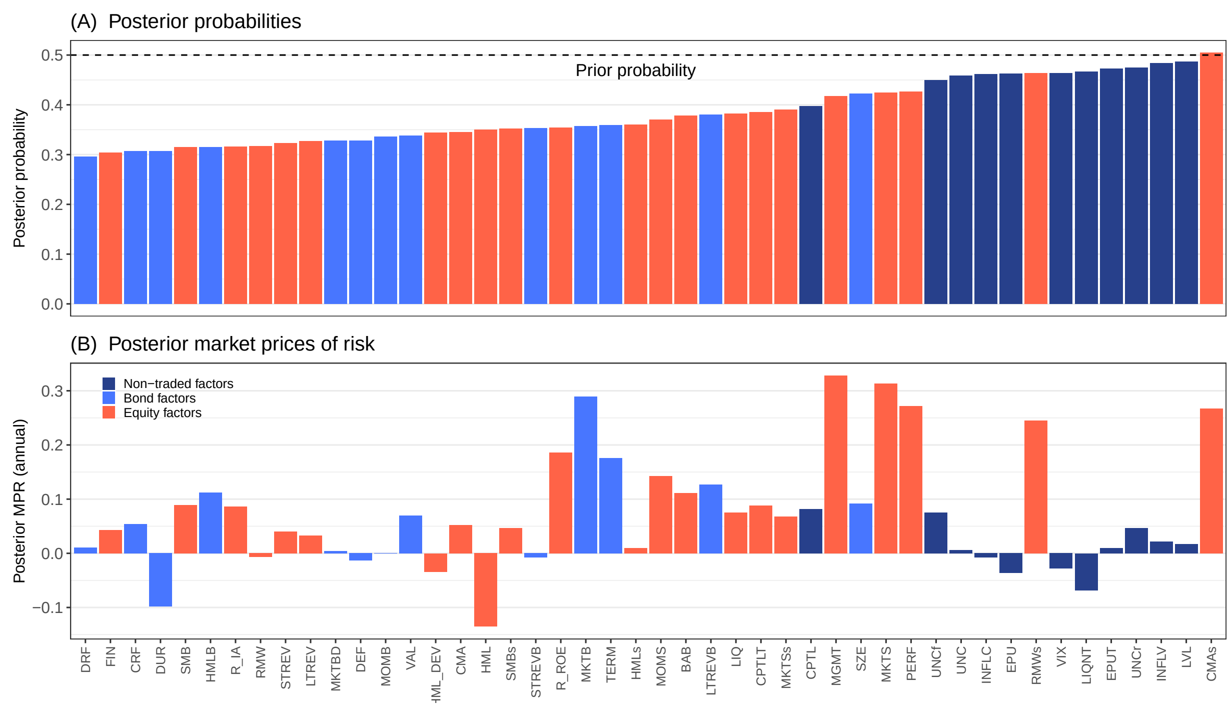


Figure IA.38: Posterior factor probabilities and risk prices excluding top factors.

The figure reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of 46 bond and stock factors described in Appendix A. The following eight factors are excluded based on the union of the top five factors ranked on the posterior probability of inclusion in the BMA-SDF and the absolute value of the market price of risk as per Table A.2: PEADB, PEAD, IVOL, CREDIT, YSP, MOMBS, QMJ, and CRY. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. Results are tabulated for different values of the prior Sharpe ratio, $\sqrt{\mathbb{E}_\pi[SR_{\boldsymbol{f}}^2 \mid \sigma^2]}$, with values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. The factors are ordered by the average posterior probability across the four levels of shrinkage. Test assets are the 83 bond and stock portfolios and remaining 35 tradable bond and stock factors described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$).

IA.10 Estimation uncertainty

In this section we provide additional results to complement the robustness analysis in Section 4.4.

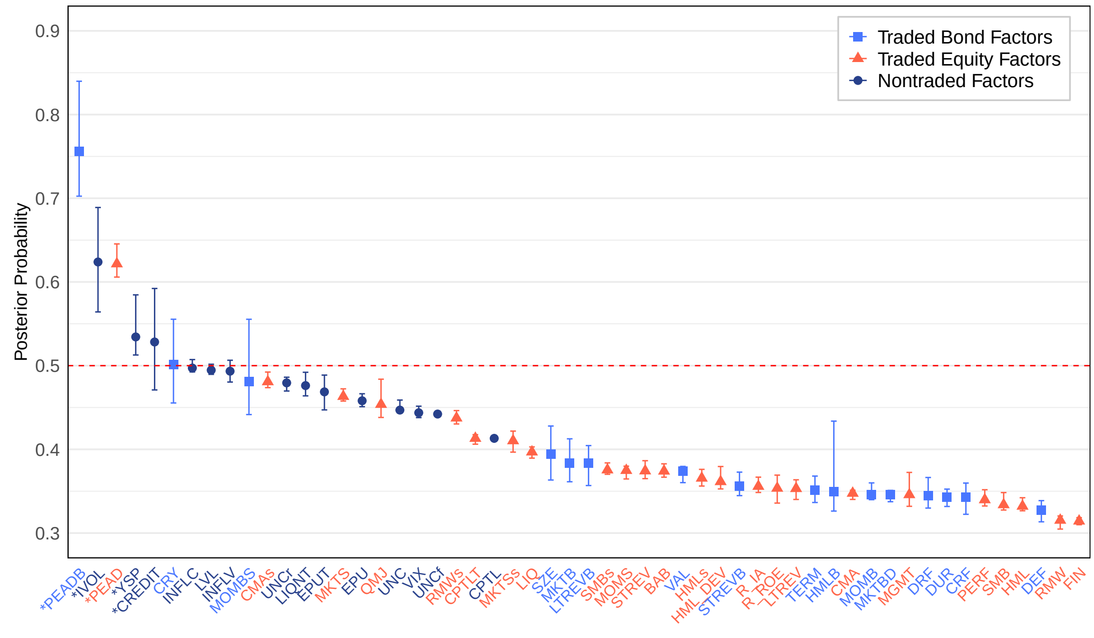
IA.10.1 Varying corporate bond data

We start by revisiting the different corporate bond datasets described in Internet Appendix IA.1. In particular, we study the pricing performance of the co-pricing BMA-SDF estimated using the baseline stock test assets, stock tradable factors, nontradable factors as well as bond test assets and bond tradable factors constructed using five different sets of corporate bond data: (i) our baseline LBFI/BAML ICE bond-level data, (ii) the LBFI/BAML ICE firm-level data, (iii) the LBFI/BAML ICE bond-level data but using only quotes (i.e., removing matrix prices), (iv) the transaction-based WRDS TRACE data, and (v) the transaction-based DFPS TRACE data. That is, we re-estimate the co-pricing BMA-SDF using the 83 test assets and 54 tradable and nontradable factors. Across estimations, only the 50 IS bond test assets and the tradable bond factors change.

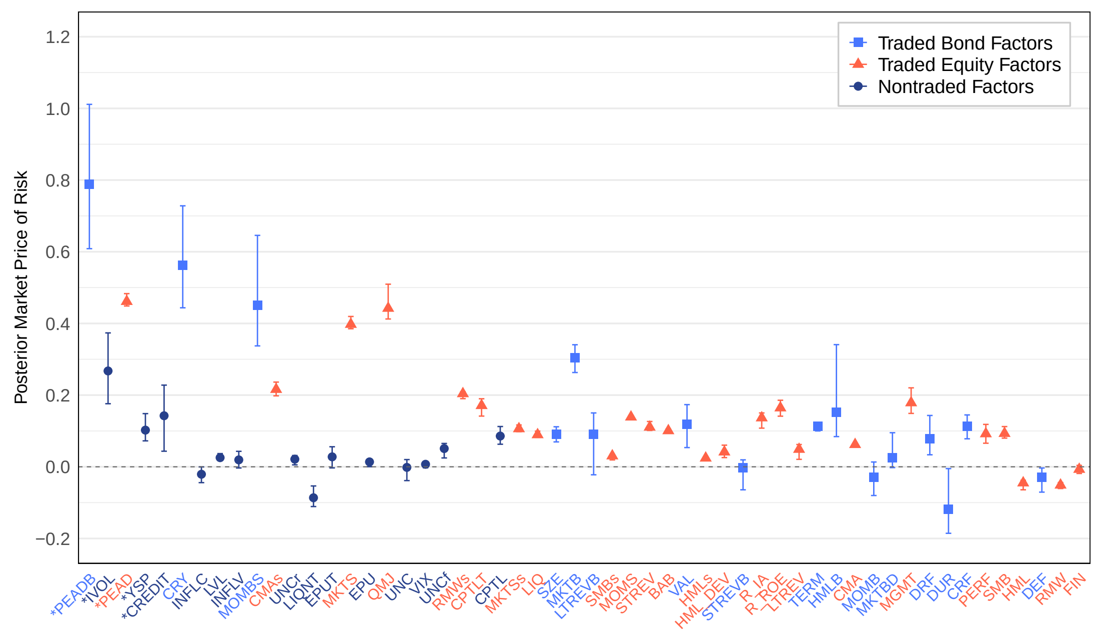
Table IA.XXVII: Ex post Sharpe ratios by corporate bond data

Data type	**Sharpe ratio**				
	20%	**40%**	**60%**	**80%**	**Max**
LBFI/BAML ICE bond-level	1.05	2.10	3.14	4.19	5.24
LBFI/BAML ICE firm-level	0.98	1.96	2.94	3.92	4.90
LBFI/BAML ICE bond-level quotes	1.03	2.06	3.08	4.11	5.14
WRDS TRACE	1.02	2.05	3.07	4.10	5.12
DFPS TRACE	1.09	2.18	3.27	4.36	5.45

This table presents the prior Sharpe Ratios at different levels of shrinkage (20%, 40%, 60%, and 80%) and the ex post maximum Sharpe ratio for the five corporate bond datasets: (i) our baseline LBFI/BAML ICE bond-level data, (ii) the LBFI/BAML ICE firm-level data, (iii) the LBFI/BAML ICE bond-level data but using only quotes (i.e., removing matrix prices), (iv) the transaction-based WRDS TRACE data, and (v) the transaction-based DFPS TRACE data. The TRACE data before July 2002 is augmented using the baseline LBFI/BAML ICE bond-level data. The 50 bond portfolios and 16 tradable bond factors are constructed using the respective bond datasets. All joint datasets include the 33 stock portfolios and 24 tradable stock factors described in Section 1. All values are annualized. The sample period is 1986:01 to 2022:12 ($T = 444$) except for DFPS TRACE where the data ends in December 2021 ($T = 432$).



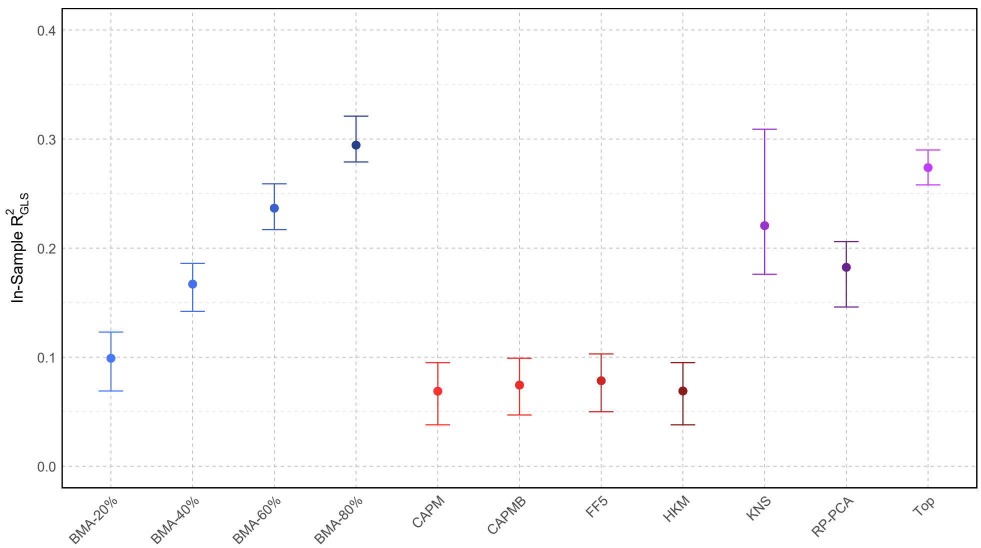
A: Posterior factor probabilities (80% shrinkage) over corporate bond datasets



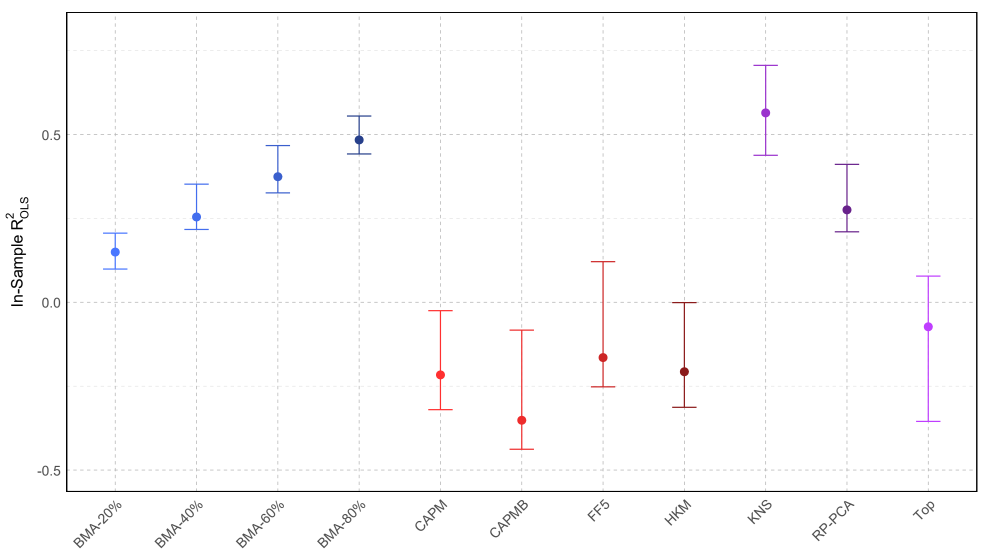
B: Posterior risk prices (80% shrinkage) over corporate bond datasets

Figure IA.39: Varying corporate bond data.

This figure plots average, minimum and maximum posterior factor probabilities, $\overline{\mathbb{E}[\gamma_j \mid \text{data}]}$ (Panel A) and market prices of risk, $\overline{\mathbb{E}[\lambda_j \mid \text{data}]}$ (Panel) for the 40 tradable and 14 nontradable factors described in Appendix A. We use five corporate bond datasets as described in Table IA.XXVII. Bond test assets are the 50 bond portfolios and 16 tradable bond factors constructed using the respective bond datasets. Stock test assets are the 33 stock portfolios and 24 tradable stock factors. All are described in Section 1. All results are for a level of shrinkage equal to 80% of the maximum ex post Sharpe ratio. The sample period is 1986:01 to 2022:12 ($T = 444$) except for DFPS TRACE where the data ends in December 2021 ($T = 432$).



A: In-sample R^2_{GLS} over corporate bond datasets



B: In-sample R^2_{OLS} over corporate bond datasets

Figure IA.40: IS asset pricing performance with varying corporate bond data.

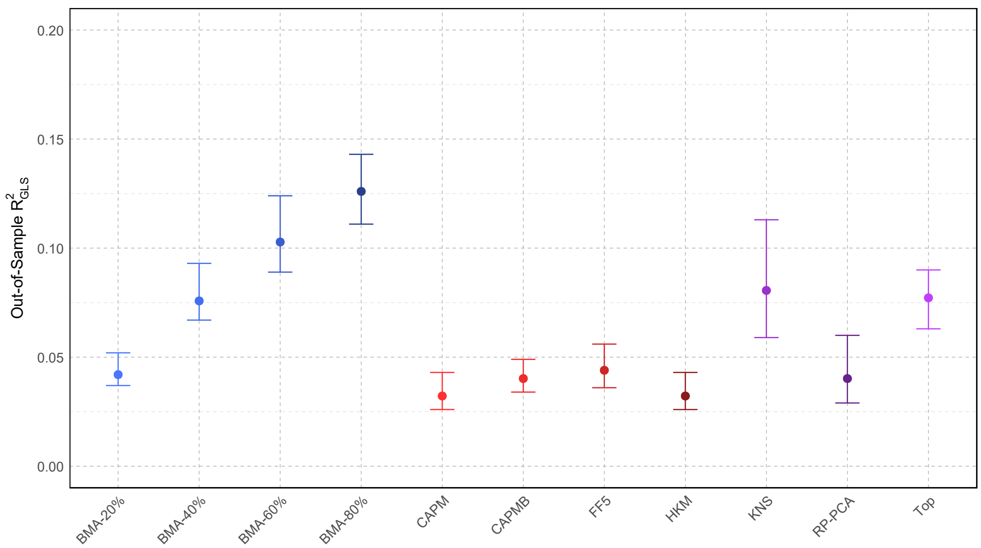
This figure plots average, minimum and maximum in-sample asset pricing performance metrics, R^2_{GLS} (Panel A) and R^2_{OLS} (Panel B), of different models pricing bonds and stocks jointly, respectively. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors based on the average posterior probability. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. For all estimations, we use five corporate bond datasets as described in Table IA.XXVII. Bond test assets are the 50 bond portfolios and 16 tradable bond factors constructed using the respective bond datasets. Stock test assets are the 33 stock portfolios and 24 tradable stock factors. All are described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$) except for DFPS TRACE where the data ends in December 2021 ($T = 432$).

We replicate the results in Section 3.1.1 across the five data samples. For consistency, we fix the sample period from January 1986 to December 2022, except for the DFPS TRACE data that ends in December 2021. That means for the two TRACE data sets we augment the data with our baseline LBFI/BAML ICE bond-level data January 1997 to July 2002 because TRACE is only available thereafter. Before January 1997, we always use LBFI (with and without matrix prices). For each dataset, the estimation yields posterior probabilities (given the data) of each factor, (i.e., $\mathbb{E}[\gamma_j|\text{data}], \forall j$) for different values of the prior Sharpe ratio achievable with the BMA-SDF (expressed as a percentage of the ex post maximum Sharpe ratio). We set the prior as a fraction (20%, 40%, 60% and 80%) of the ex post maximum Sharpe ratio given each dataset, as reported in Table IA.XXVII.

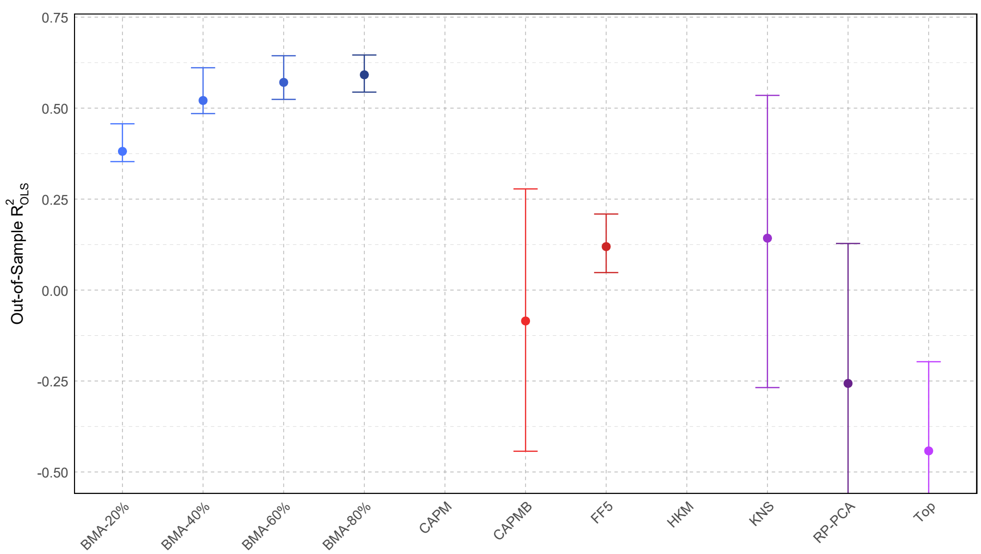
Across all five datasets, the maximum achievable Sharpe ratio is similar, ranging from 4.90 (LBFI/BAML ICE firm-level) to 5.45 (DFPS TRACE augmented with LBFI/BAML ICE bond-level). To concisely report which factors are the most likely components of the co-pricing BMA-SDF in the economy across datasets, we focus on the posterior probabilities estimated with 80% shrinkage, resulting in five 54×1 vectors of averaged posterior probabilities (given each respective dataset). In Figure IA.39 we report the means along with minimum and maximum values of posterior probabilities (Panel A) and market prices of risk (Panel B), ordered by probabilities.

The average of the posterior probabilities across the five datasets yields a set of factors that are most likely to be included in the SDF that are very similar to the baseline results reported in Table C of the Appendix: eight out of ten and all top five most likely factors to be included in the SDF remain the same.

Examining the tradable factors first, both PEADB and PEAD remain the most likely to be included, with very tight min and max values. In fact, the minimum posterior probability for PEADB across the five datasets is still above the next highest value (the maximum of PEAD). Additionally, the ordering of the three most likely factors is identical to our baseline results (i.e., PEADB, PEAD and then IVOL). Turning to the nontradable factors, CREDIT, YSP and LVL are all in the top ten, again closely aligned with the results reported in the paper. Thus, overall, even though some of the tradable bond factors marginally differ across the respective datasets, this does not, on average, affect the results when considering factors individually.



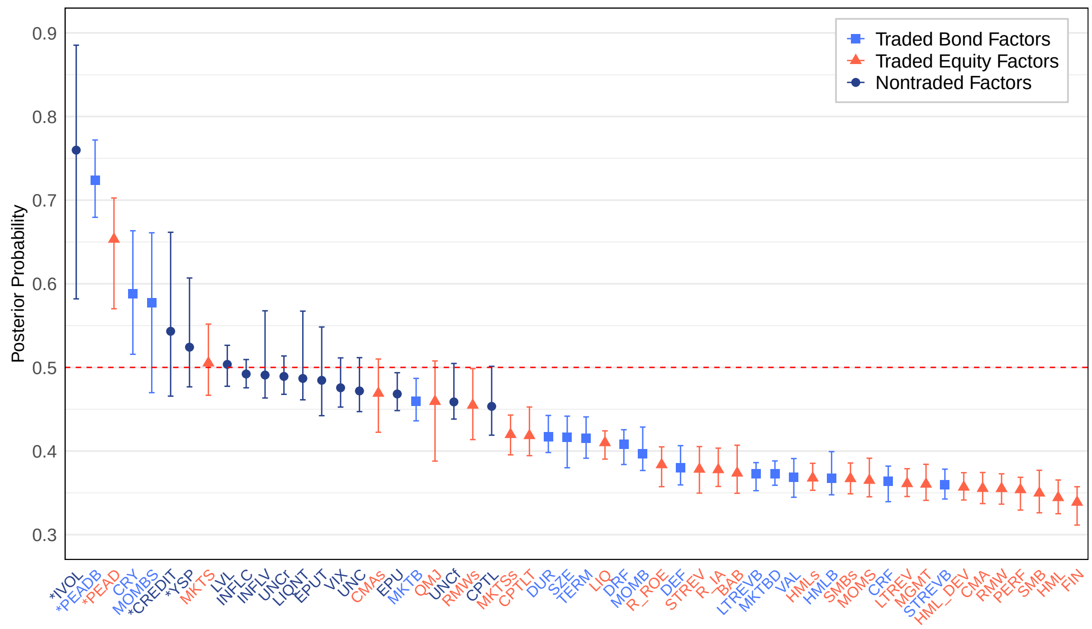
A: Out-of-sample R^2_{GLS} over corporate bond datasets



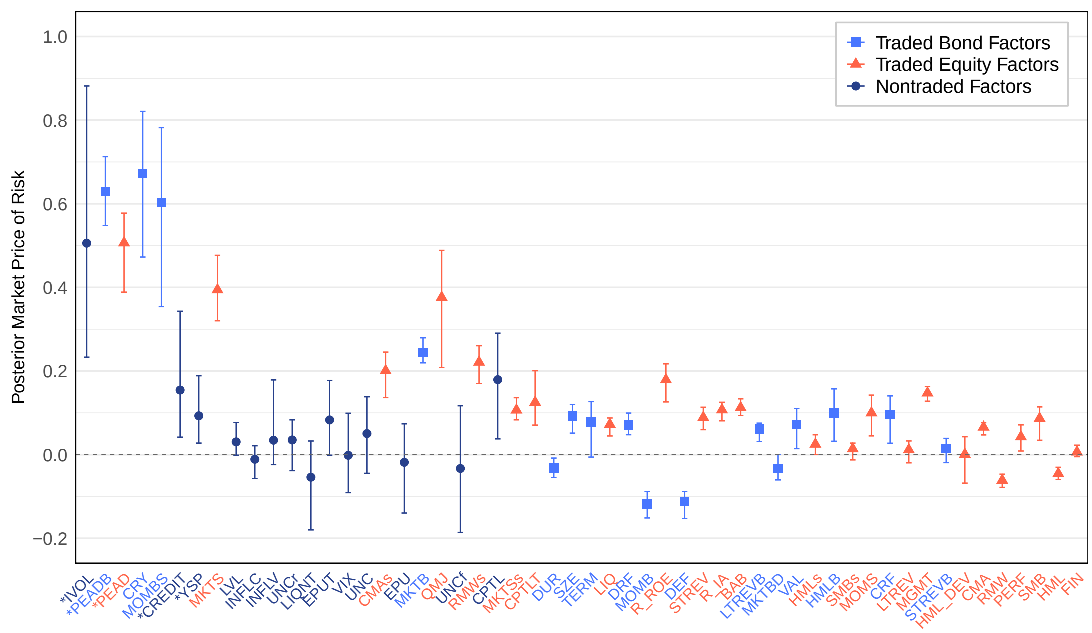
B: Out-of-sample R^2_{OLS} over corporate bond datasets

Figure IA.41: OS asset pricing performance with varying corporate bond data.

This figure plots average, minimum and maximum out-of-sample asset pricing performance metrics, R^2_{GLS} (Panel A) and R^2_{OLS} (Panel B), of different models pricing bonds and stocks jointly, respectively. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors based on the average posterior probability. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. For all estimations, we use five corporate bond datasets as described in Table IA.XXVII. The IS test assets are the same as in Figure IA.40. OS bond test assets are the 77 OS bond portfolios constructed using the respective bond datasets. Stock test assets are the 77 OS stock portfolios. All are described in Section 1. The sample period is 1986:01 to 2022:12 ($T = 444$) except for DFPS TRACE where the data ends in December 2021 ($T = 432$).



A: Posterior factor probabilities (80% shrinkage) over DFPS and JKP IS test assets



B: Posterior risk prices (80% shrinkage) over DFPS and JKP IS test assets

Figure IA.42: Varying in-sample test assets using DFPS and JKP data.

This figure plots average, minimum and maximum posterior factor probabilities, $\overline{\mathbb{E}[\gamma_j \mid \text{data}]}$ (Panel A) and market prices of risk, $\overline{\mathbb{E}[\lambda_j \mid \text{data}]}$ (Panel) for the 40 tradable and 14 nontradable factors described in Appendix A. We use 100 different sets of 50 IS test asset portfolios, randomly sampling 25 equity anomalies from Jensen et al. (2023) and 25 bond anomalies from Dick-Nielsen et al. (2025). Test assets per estimation then are the resulting 50 bond and stock portfolios plus the 40 tradable bond and stock factors. All results are for a level of shrinkage equal to 80% of the maximum ex post Sharpe ratio. Asterisks indicate factors also in the top-five using the baseline data as reported in Table A.2 of Appendix C. The sample period is 1986:01 to 2021:12 ($T = 431$, with one missing observation in August 2002).

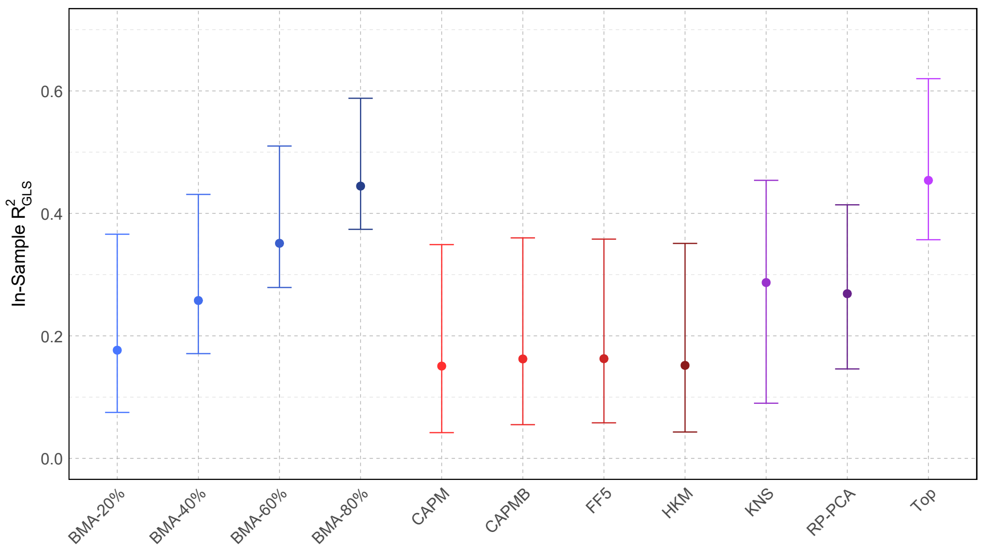
Furthermore, the in- and out-of-sample asset pricing results remain very similar to what we report in Tables 2 and 3. The aggregated results across the five datasets are presented in Figures IA.40 (IS) and IA.41 (OS). For each model we consider in Tables 2 and 3, we report the average, minimum and maximum values for the R^2_{OLS} (Panel A) and R^2_{GLS} (Panel B) asset pricing metrics. For the BMA-SDF, the spread in the metrics between minimum and maximum values is very tight and the average BMA-SDF across all five datasets outperforms the frequentist and latent (KNS and RPPCA) factor models both in- and out-of-sample for higher percentages of shrinkage of the prior Sharpe ratio.

This result, given our estimation methodology, is expected. The BMA-SDF aggregates factors to optimize the signal-to-noise ratio of the SDF. Although different datasets may alter individual factors' signal-to-noise ratios, the BMA-SDF recombines these factors to extract common pricing information while minimizing noise effects, thereby mitigating concerns about data uncertainty in our analysis.

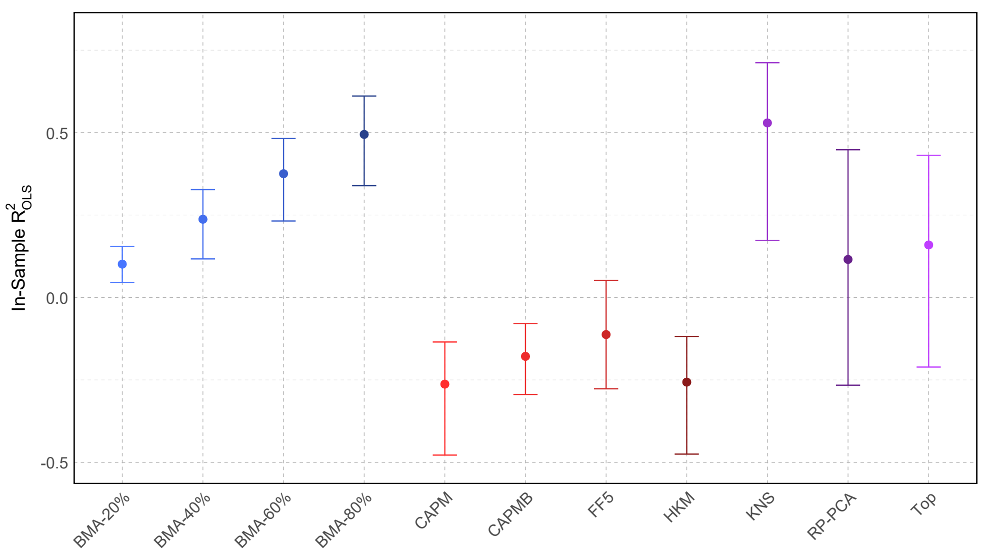
IA.10.2 Varying in-sample cross-sections

In this section we fix the corporate bond data to construct the tradable bond factors to our baseline LBFI/BAML ICE bond-level data. However, we vary the cross-sections of IS test assets using publicly available corporate bond and stock anomaly portfolio data from `Christian Stolborg's webpage` (corporate bond data associated with Dick-Nielsen et al. (2025)) and the Jensen et al. (2023) equity data repository from `jkpfactors.com`.

The DFPS bond data repository contains 153 corporate bond anomaly portfolios formed with the underlying equity characteristics from JKP. The portfolios are long-short formed using (3×3), rating $\times$ characteristic tercile sorts and span the sample period January 1984 to December 2021, with a missing row of data in August 2002. We start the sample in January 1986 to align the start date of our baseline data, resulting in $T = 431$ observations in the time series. We then extract the same 153 anomaly portfolios from the JKP data repository, resulting in a total cross-section of 306 stock and bond anomaly portfolios.



A: In-sample R^2_{GLS} over DFPS and JKP IS test assets



B: In-sample R^2_{OLS} over DFPS and JKP IS test assets

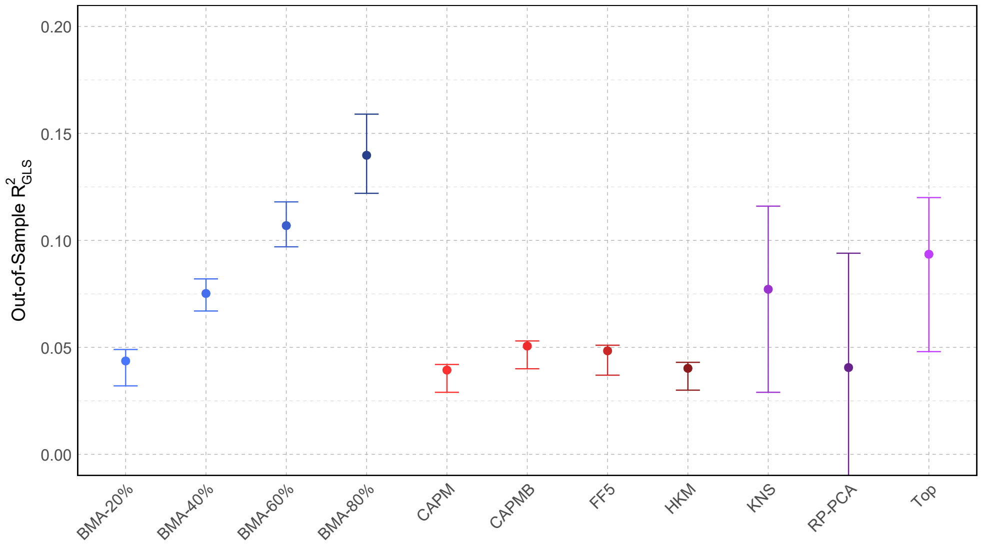
Figure IA.43: IS asset pricing performance with varying IS test assets.

This figure plots average, minimum and maximum in-sample asset pricing performance metrics, R^2_{GLS} (Panel A) and R^2_{OLS} (Panel B), of different models pricing bonds and stocks jointly, respectively. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors based on the average posterior probability. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. We perform 100 estimations for different sets of 50 IS test asset portfolios, each time randomly sampling 25 equity anomalies from Jensen et al. (2023) and 25 bond anomalies from Dick-Nielsen et al. (2025). Test assets per estimation then are the resulting 50 bond and stock portfolios plus the 40 tradable bond and stock factors. All results are for a level of shrinkage equal to 80% of the maximum ex post Sharpe ratio. C. The sample period is 1986:01 to 2021:12 ($T = 431$, with one missing observation in August 2002).

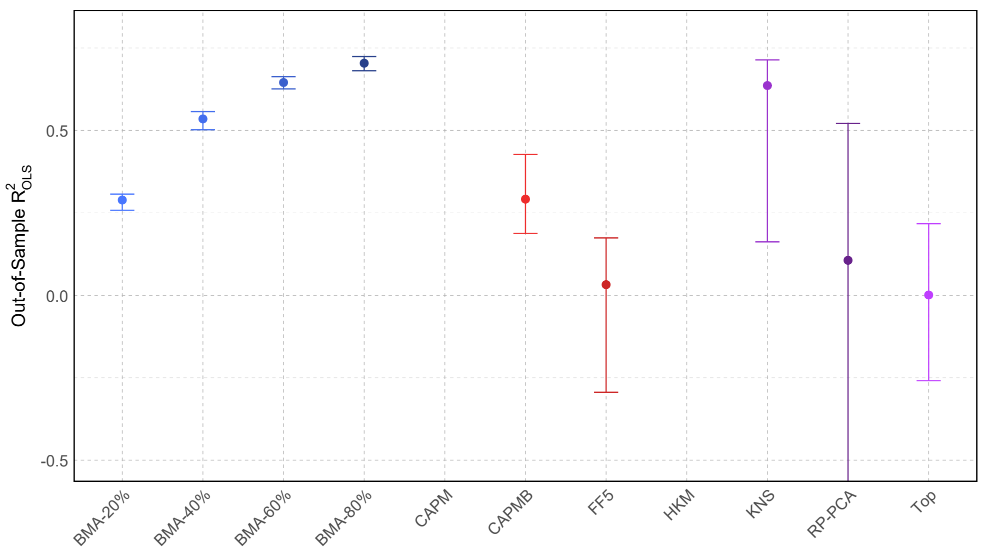
To account for estimation uncertainty, we fix the size of our total co-pricing cross-section to 50. That is, we randomly sample 25 anomalies (one bond and one stock) resulting in a co-pricing cross-section of 50 test assets. We then repeat this process 100 times and apply our hierarchical Bayesian method including the constant with Beta(1,1) priors as in Section 3. For each estimation, we store the posterior factor probabilities, market prices of risk, and in-sample asset pricing performance metrics. We also price the baseline 154 OS test assets described in Section 1 using the estimated co-pricing BMA-SDF. For ease of exposition, we again focus on an ex post Sharpe ratio shrinkage set to 80%.

Posterior probabilities and market prices of risk for hundreds of estimations. We present the average posterior probabilities and market prices of risk with associated minimum and maximum values across the 100 estimations in Panels A and B of Figure IA.42 with the Sharpe ratio shrinkage set of 80% of the ex post maximum. On the x-axis, we denote factors which are in the top five based on posterior probabilities in Table A.2 of Appendix C with a leading asterisk. Affirming the results from Section 3, the factors which are most likely to be included are very closely aligned with IVOL, PEADB and PEAD coming out on top. Other factors which are in the top 10 most likely across both sets of estimations are MOMBS, YSP, CREDIT, LVL and MKTS (i.e., 8 out of 10 are the same). These results strengthen the case of these factors being likely candidates for inclusion in the SDF from estimations that use a very different set of cross-sectional assets, with data prepared by external sources, different bond data for the test assets (DFPS TRACE), and over a slightly shorter sample period.

Asset pricing results for hundreds of estimations. In Figure IA.43 we present the IS mean, minimum and maximum R^2_{GLS} (Panel A) and R^2_{OLS} (Panel B) values across 100 estimations for the BMA-SDF across our four Sharpe ratio shrinkage levels and other benchmark models discussed in Appendix D. Based on the R^2_{GLS}, the BMA-SDF with 60% and 80% shrinkage as well as the TOP model including the top 5 most likely factors outperform KNS, RPPCA and the frequentist asset pricing models by a wide margin. Using the R^2_{OLS} as the performance metric we observe similar results with the BMA-SDF using a shrinkage level of 80% and KNS performing about the same while frequentist asset pricing models deliver negative R^2_{OLS} values.



A: Out-of-sample R^2_{GLS} over DFPS and JKP IS test assets



B: Out-of-sample R^2_{OLS} over DFPS and JKP IS test assets

Figure IA.44: OS asset pricing performance with varying IS test assets.

This figure plots average, minimum and maximum out-of-sample asset pricing performance metrics, R^2_{GLS} (Panel A) and R^2_{OLS} (Panel B), of different models pricing bonds and stocks jointly, respectively. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors based on the average posterior probability. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. The IS test assets and estimated BMA-SDFs are the same as in Figure IA.43. OS test assets are the 154 bond and stock portfolios described in Section 1. The sample period is 1986:01 to 2021:12 ($T = 431$, with one missing observation in August 2002).

Table IA.XXVIII: IS and OS cross-sectional asset pricing performance: Switching IS and OS test assets

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Switch IS to OS test assets, in-sample pricing											
RMSE	0.152	0.130	0.116	0.110	0.247	0.206	0.203	0.245	0.232	0.182	0.157
MAPE	0.106	0.090	0.083	0.081	0.203	0.152	0.144	0.199	0.172	0.116	0.099
R^2_{OLS}	0.304	0.491	0.594	0.636	−0.843	−0.281	−0.240	−0.807	−0.628	−0.004	0.258
R^2_{GLS}	0.191	0.227	0.257	0.278	0.183	0.186	0.187	0.183	0.302	0.048	0.240
Panel B: Switch IS to OS test assets, out-of-sample pricing											
RMSE	0.195	0.190	0.184	0.175	0.199	0.220	0.189	0.202	0.207	0.199	0.194
MAPE	0.149	0.146	0.141	0.132	0.136	0.170	0.140	0.136	0.161	0.148	0.123
R^2_{OLS}	0.173	0.211	0.262	0.337	0.137	−0.055	0.222	0.110	0.063	0.141	0.183
R^2_{GLS}	0.057	0.104	0.138	0.159	−0.064	−0.033	−0.071	−0.062	−0.019	0.083	0.027

The table presents the cross-sectional in- (Panel A) and out-of-sample (Panel B) asset pricing performance of different models pricing bonds and stocks jointly. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors with an average posterior probability greater than 50%. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. The SDFs are estimated using the 154 OS bond and stock test assets described in Section 1 IS test assets along with the 40 tradable bond and stock factors. The OS test assets in Panel B are the 83 bond and stock portfolios used as IS test assets in Table 2. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2022:12 ($T = 444$).

The results carry over to the OS analysis presented in Figure IA.44 we use the SDFs estimated on the 100 different cross-sections to price the baseline 154 OS test assets discussed in Section 1. Based on R^2_{GLS} (Panel A), the BMA-SDF with 60% and 80% shrinkage as well as the TOP model outperform all other models, again confirming the results presented in Section 3 for a very different sets of IS test assets used to estimate the co-pricing BMA-SDF.

Switch in- to out-of-sample test assets. We further vary the IS test assets by swapping IS and OS test assets from our baseline analysis in Section 3. Thus, the IS test assets now comprise the combined 154 OS bond and stock portfolios discussed in Section 1 plus the 40 tradable bond and stock factors. The OS test assets are then the original 83 bond and stock portfolios. The posterior factor probabilities and market prices of risk with 80% Sharpe ratio shrinkage are reported in Figure IA.45. The most likely factors still remain very consistent with IVOL, PEADB, YSP, and CREDIT and LVL, followed by PEAD. The corresponding IS

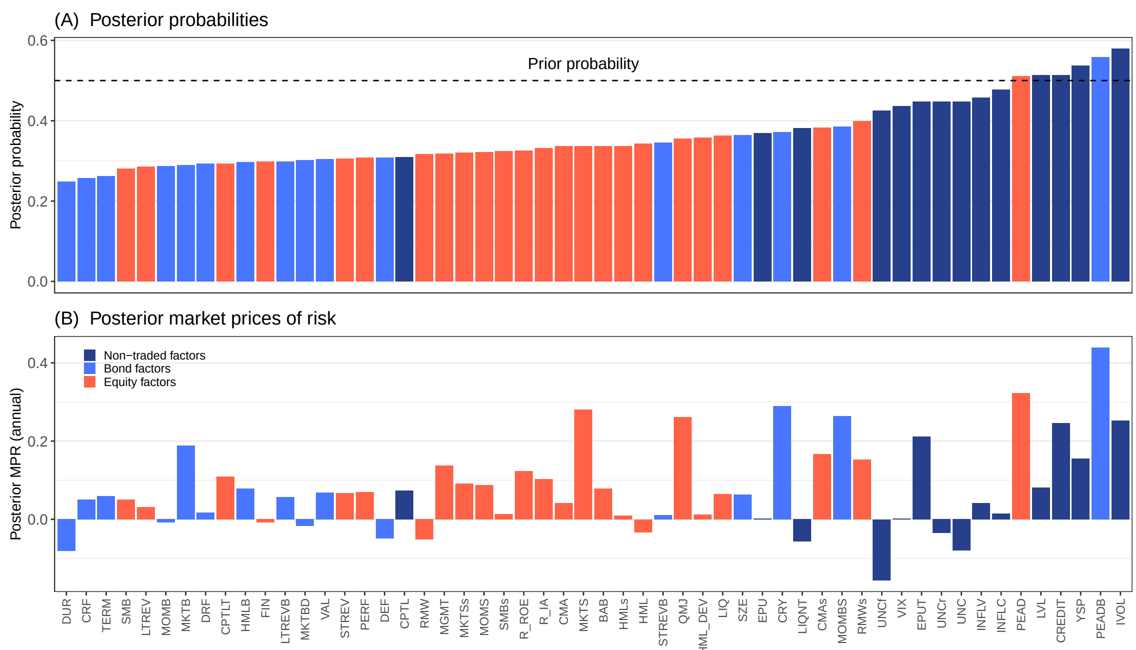


Figure IA.45: Posterior factor probabilities and risk prices when switching OS to IS test assets.

The figure reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and stock factors described in Appendix A. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. The 154 OS bond and stock test assets described in Section 1 are used as IS test assets along with the 40 tradable bond and stock factors for the estimation of the BMA-SDF. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the test assets. The sample period is 1986:01 to 2022:12 ($T = 444$).

and OS asset pricing results are reported in Table IA.XXVIII, the BMA-SDF outperforms the competition both in- and out-of-sample.

IA.10.3 Varying out-of-sample cross-sections

Next we go back to the IS co-pricing BMA-SDFs from Section 3 that are estimated using our baseline set of test assets. In addition, we again consider the additional benchmark models described in Appendix D. Equipped with the IS SDFs, we price millions of possible combinations of the Dick-Nielsen et al. (2025) and Jensen et al. (2023) bond and stock anomalies *without* re-estimating the respective SDFs. We conduct the asset pricing tests using a bootstrap approach and summarize the results in Table IA.XXIX. As discussed earlier, the DFPS and JKP dataset comprises 153 anomalies for bonds and stocks, resulting in 306 combined bond and stock anomaly portfolios. We set the size of the OS cross-section to 50 portfolios in Panel A

Table IA.XXIX: Millions of out-of-sample cross-sectional asset pricing tests

	BMA Prior Sharpe Ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: 50 OS portfolios using DFPS and JKP data											
RMSE	0.309	0.303	0.290	0.272	0.359	0.362	0.344	0.368	0.230	0.278	0.317
	[0.038]	[0.035]	[0.032]	[0.029]	[0.043]	[0.046]	[0.044]	[0.046]	[0.018]	[0.035]	[0.040]
MAPE	0.238	0.235	0.227	0.213	0.273	0.279	0.262	0.285	0.188	0.209	0.238
	[0.025]	[0.024]	[0.023]	[0.022]	[0.032]	[0.033]	[0.030]	[0.033]	[0.017]	[0.023]	[0.029]
R^2_{OLS}	0.047	0.080	0.155	0.255	−0.287	−0.310	−0.175	−0.351	0.453	0.228	−0.017
	[0.049]	[0.090]	[0.111]	[0.120]	[0.119]	[0.139]	[0.093]	[0.143]	[0.144]	[0.090]	[0.190]
R^2_{GLS}	0.086	0.149	0.215	0.281	−0.001	0.022	0.006	0.019	0.385	0.214	0.139
	[0.047]	[0.047]	[0.050]	[0.056]	[0.062]	[0.059]	[0.061]	[0.059]	[0.099]	[0.040]	[0.072]
Panel B: 100 OS portfolios using DFPS and JKP data											
RMSE	0.313	0.307	0.293	0.274	0.363	0.367	0.348	0.372	0.231	0.281	0.320
	[0.025]	[0.023]	[0.020]	[0.019]	[0.028]	[0.030]	[0.029]	[0.030]	[0.012]	[0.022]	[0.026]
MAPE	0.240	0.237	0.228	0.215	0.274	0.281	0.264	0.287	0.189	0.210	0.239
	[0.016]	[0.015]	[0.015]	[0.014]	[0.020]	[0.021]	[0.019]	[0.021]	[0.011]	[0.014]	[0.018]
R^2_{OLS}	0.048	0.084	0.162	0.264	−0.285	−0.309	−0.176	−0.348	0.472	0.231	−0.004
	[0.030]	[0.055]	[0.069]	[0.075]	[0.073]	[0.083]	[0.056]	[0.086]	[0.091]	[0.055]	[0.117]
R^2_{GLS}	0.043	0.093	0.143	0.192	−0.019	−0.011	−0.019	−0.015	0.250	0.152	0.098
	[0.050]	[0.047]	[0.046]	[0.047]	[0.062]	[0.062]	[0.061]	[0.062]	[0.068]	[0.022]	[0.052]

Using the in-sample SDFs estimated for different models pricing bonds and stocks jointly in Panel A of Table 2, we price one million of possible combinations of the Dick-Nielsen et al. (2025) and Jensen et al. (2023) bond and stock anomalies *without* re-estimating the respective SDFs. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors based on the average posterior probability. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020) and RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. We conduct the pricing tests with a bootstrap approach. We set the total number of OS test assets to 50 in Panel A and to 100 in Panel B. That is, for each bootstrap iteration we draw 25 or 50 unique anomalies from Jensen et al. (2023) (stock portfolios) and Dick-Nielsen et al. (2025) (bond portfolios), respectively. We report the average asset pricing metrics (and their standard deviation in square brackets) for the one million draws in Panels A and B. All data is standardized, that is, pricing errors are in Sharpe ratio units. The sample period is 1986:01 to 2021:12 ($T = 431$, with one missing observation in August 2002).

and to 100 portfolios in Panel B, implying that for each bootstrap iteration, we draw 25 and 50 unique anomalies, respectively. We then generate one million combinations for each cross-section size and report the average asset pricing metrics along with their standard deviations in square brackets. As in Panel A of Table 3, the BMA-SDF outperforms all other frequentist models and the latent factor models RPPCA and KNS.

IA.10.4 Varying factor zoos and sample periods

Finally, we provide results to accompany the discussion in Section 4.4.3 where we vary the factor zoos as well as the sample periods. First, we expand the set of stock and nontradable factors by including all 51 stock factors considered in Bryzgalova et al. (2023) as well as their IS test assets. To do so we have to consider a shorter sample period ending in December 2016. Second, we extend the corporate bond factor zoo by adding the 13 Dick-Nielsen et al. (2025) composite bond return factors formed with equity characteristics. Third, extend the corporate bond factor zoo again, this time by including the tradable liquidity factor LRF from Bai et al. (2019) as well as the two nontradable illiquidity factors from Lin et al. (2011). Here, we restrict the sample period to the Trace era from 2002 onwards. Fourth, we estimate the models on the maximally possible sample period starting in 1977 and resulting in a total of 549 observations in the time series. Finally, we consider two sample splits and estimate the models (i) for the pre- and post-Trace period (i.e., pre-/post-2002) and (ii) for the pre- and post-2000 period as in van Binsbergen et al. (2025).

Extended stock and nontradable factor zoo following Bryzgalova et al. (2023). We extend the cross-sectional dimension of our stock and nontradable factor zoo to match BHJ, resulting in a time series spanning January 1986 to December 2016 for a total of 372 monthly observations. The number of stock factors increases from 24 to 35, and the number of non-tradable factors from 14 to 24. We also use the 51 equity test asset portfolios from Bryzgalova et al. (2023). After combining their stock and nontradable factors with our co-pricing factor zoo, the number of factors totals 75, resulting in 37.8 sextillion possible models. We apply our hierarchical Bayesian method including the constant with Beta(1,1) priors as in Section 3 to the *joint* cross-section of stock and corporate bond excess returns. For brevity, we report

Table IA.XXX: In-sample cross-sectional asset pricing performance: Robustness

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Extended stock and nontradable factor zoo following Bryzgalova et al. (2023), 1986–2016											
RMSE	0.281	0.237	0.198	0.165	0.330	0.292	0.277	0.331	0.295	0.168	0.203
MAPE	0.232	0.187	0.154	0.130	0.279	0.221	0.212	0.280	0.244	0.121	0.128
R^2_{OLS}	0.229	0.451	0.619	0.735	−0.064	0.168	0.253	−0.071	0.149	0.724	0.597
R^2_{GLS}	0.141	0.200	0.271	0.348	0.120	0.131	0.131	0.120	0.336	0.184	0.226
Panel B: Extended bond factor zoo following Dick-Nielsen et al. (2025), 1986–2021											
RMSE	0.259	0.235	0.212	0.189	0.292	0.299	0.269	0.289	0.203	0.191	0.251
MAPE	0.212	0.185	0.165	0.145	0.221	0.233	0.202	0.218	0.161	0.142	0.167
R^2_{OLS}	0.177	0.325	0.448	0.565	−0.041	−0.094	0.112	−0.025	0.496	0.553	0.230
R^2_{GLS}	0.120	0.181	0.242	0.299	0.097	0.104	0.105	0.098	0.284	0.171	0.222
Panel C: Extended bond factor zoo using TRACE bond illiquidity factors, 2002–2022											
RMSE	0.206	0.178	0.155	0.135	0.240	0.233	0.235	0.219	0.247	0.182	0.175
MAPE	0.158	0.132	0.113	0.097	0.173	0.180	0.181	0.159	0.194	0.132	0.118
R^2_{OLS}	0.279	0.460	0.589	0.688	0.021	0.080	0.057	0.181	−0.035	0.438	0.479
R^2_{GLS}	0.056	0.085	0.120	0.158	0.054	0.053	0.057	0.056	0.242	0.022	0.110
Panel D: Extended time-series back to 1977, 1977-2022											
RMSE	0.206	0.209	0.197	0.179	0.264	0.304	0.325	0.265	0.332	0.145	0.227
MAPE	0.151	0.148	0.137	0.124	0.203	0.259	0.271	0.205	0.300	0.115	0.140
R^2_{OLS}	−0.015	−0.047	0.069	0.233	−0.675	−1.213	−1.525	−0.678	−1.642	0.495	−0.230
R^2_{GLS}	0.062	0.147	0.237	0.322	0.018	0.016	0.031	0.019	0.239	0.338	0.237

The table presents the cross-sectional in-sample asset pricing performance of different models pricing bonds and stocks jointly. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors with an average posterior probability greater than 50%. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020). If the time series dimension is smaller than the number of assets, $T < N$, we allow a small degree of overlap in the two cross-validation samples. RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. By panel the models are estimated with the respective factor zoos and test assets. In Panel A we use the 75 bond and stock factors described in Appendix A and the Appendix of Bryzgalova et al. (2023). Test assets are the 50 bond portfolios and 16 tradable bond factors described in Section 1 as well as the 26 stock anomalies and 35 tradable stock factors from Bryzgalova et al. (2023). The sample period is 1986:01 to 2016:12 ($T = 372$). In Panel B we use the 54 bond and stock factors described in Appendix A as well as the 13 composite bond factors of Dick-Nielsen et al. (2025). The 13 composite bond excess return factors are formed with the underlying *equity* characteristic data. Test assets are the 83 stock and bond portfolios and the 40 tradable factors described in Section 1 as well as the 13 composite DFPS factors. The sample period is 1986:01 to 2021:12 ($T = 432$). In Panel C we use the 54 bond and stock factors described in Appendix A plus the tradable liquidity factor LRF of Bai et al. (2019) and the two nontradable illiquidity factors AMD and PSB of Lin et al. (2011). Test assets are the 83 stock and bond portfolios and the 40 tradable factors described in Section 1 plus the tradable LRF bond factor. The sample period is 2002:10 to 2022:12 ($T = 243$). In Panel D we use the 54 bond and stock factors described in Appendix A. Test assets are the 83 stock and bond portfolios and the 40 tradable factors described in Section 1. The sample period is 1977:01 to 2022:12 ($T = 549$). All data is standardized, that is, pricing errors are in Sharpe ratio units.

Table IA.XXXI: Out-of-sample cross-sectional asset pricing performance: Robustness

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Extended stock and nontradable factor zoo following Bryzgalova et al. (2023), 1986–2016											
RMSE	0.147	0.116	0.103	0.097	0.293	0.141	0.183	0.293	0.180	0.144	0.113
MAPE	0.111	0.083	0.073	0.069	0.262	0.113	0.148	0.263	0.148	0.124	0.086
R^2_{OLS}	0.422	0.642	0.717	0.751	−1.296	0.468	0.104	−1.301	0.129	0.447	0.656
R^2_{GLS}	0.048	0.083	0.121	0.157	0.037	0.052	0.041	0.038	0.117	0.090	0.069
Panel B: Extended bond factor zoo following Dick-Nielsen et al. (2025), 1986–2021											
RMSE	0.125	0.105	0.099	0.095	0.277	0.152	0.158	0.275	0.186	0.146	0.115
MAPE	0.091	0.075	0.072	0.069	0.249	0.128	0.123	0.246	0.157	0.128	0.090
R^2_{OLS}	0.428	0.596	0.643	0.665	−1.833	0.153	0.081	−1.785	−0.271	0.221	0.514
R^2_{GLS}	0.044	0.079	0.107	0.131	0.036	0.047	0.046	0.037	0.099	0.085	0.033
Panel C: Extended bond factor zoo using TRACE bond illiquidity factors, 2002–2022											
RMSE	0.121	0.120	0.117	0.114	0.175	0.187	0.130	0.159	0.290	0.149	0.102
MAPE	0.096	0.097	0.095	0.093	0.149	0.161	0.098	0.134	0.259	0.126	0.075
R^2_{OLS}	0.030	0.048	0.100	0.143	−1.030	−1.308	−0.112	−0.671	−4.553	−0.475	0.312
R^2_{GLS}	0.008	0.022	0.036	0.048	0.005	0.004	0.012	0.010	−0.041	0.016	0.015
Panel D: Extended time-series back to 1977, 1977–2022											
RMSE	0.114	0.115	0.106	0.102	0.132	0.178	0.094	0.134	0.293	0.122	0.100
MAPE	0.097	0.101	0.091	0.088	0.106	0.161	0.064	0.108	0.268	0.111	0.076
R^2_{OLS}	−0.191	−0.210	−0.016	0.047	−0.587	−1.876	0.192	−0.639	−6.775	−0.359	0.089
R^2_{GLS}	0.040	0.086	0.118	0.138	0.021	0.015	0.030	0.019	−0.007	0.113	0.012

The table presents the cross-sectional out-of-sample asset pricing performance of different models pricing bonds and stocks jointly. For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors with an average posterior probability greater than 50%. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020). If the time series dimension is smaller than the number of assets, $T < N$, we allow a small degree of overlap in the two cross-validation samples. RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. By panel, the models are first estimated using the respective IS test assets and sample periods as per Table IA.XXX. The resulting SDF is then used to price (with no additional parameter estimation) each set of the OS assets. OS test assets are the combined 154 bond and stock portfolios described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units.

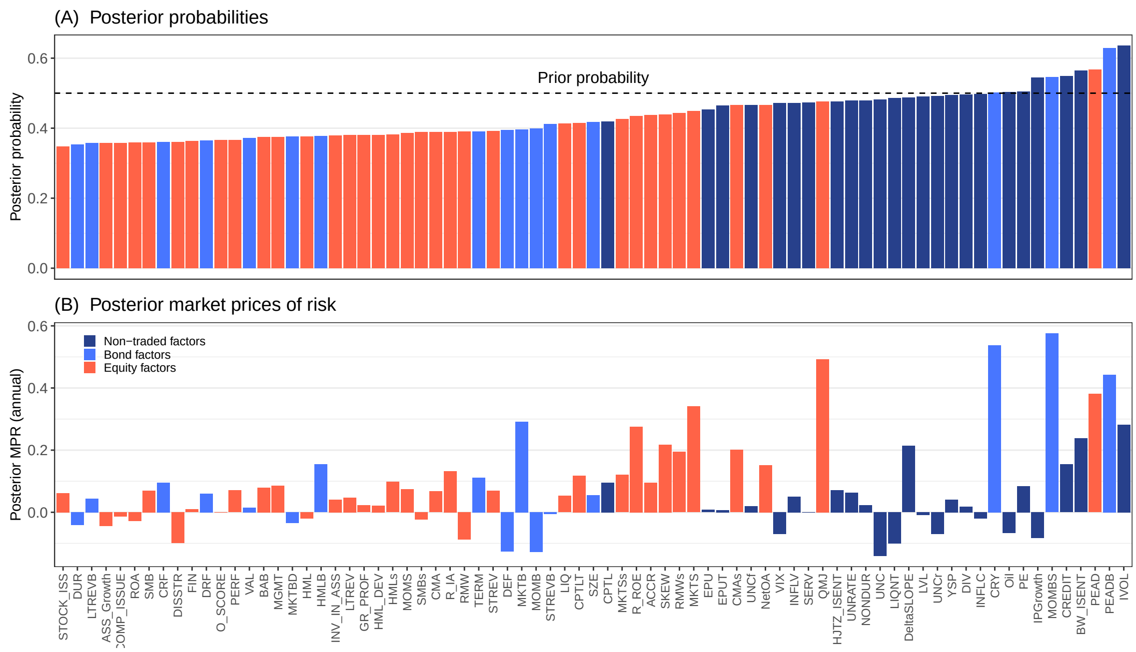


Figure IA.46: Posterior factor probabilities and risk prices: Extending the stock factor zoo.

The figure reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 75 bond and stock factors described in Appendix A and the Appendix of Bryzgalova et al. (2023). The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the 50 bond portfolios and 16 tradable bond factors described in Section 1 as well as the 26 stock anomalies and 35 tradable stock factors from Bryzgalova et al. (2023). The sample period is 1986:01 to 2016:12 ($T = 372$).

the posterior factor probabilities and market prices of risk with 80% Sharpe ratio shrinkage in Figure IA.46. Confirming the main results, the top five factors are displayed in Panel A are IVOL, PEADB, PEAD, BWI_SENT, and CREDIT (four out of five match those from Table A.2 in Appendix C). These factors also yield large posterior market prices of risk in Panel B. In addition, the BWI_SENT sentiment nontradable factor of Baker and Wurgler (2006) is a likely candidate for inclusion in the co-pricing BMA-SDF using the extended factor zoo.

The corresponding in- and out-of-sample asset pricing results are reported in Panel A of Tables IA.XXX and IA.XXXI. The BMA-SDF with a 80% Sharpe ratio shrinkage and the TOP model (comprising the factors IVOL, PEADB, PEAD, BWI_SENT and CREDIT) outperform all other models by a wide margin, both in- as well as out-of-sample.

Extended bond factor zoo following Dick-Nielsen et al. (2025). We now extend the corporate bond factor zoo to include the 13 bond factor clusters (aggregated factors) formed with underlying equity characteristic data from DFPS.[8] The sample spans the period January 1986 to December 2021 for a total of 432 monthly observations. The posterior factor probabilities and market prices of risk with 80% Sharpe ratio shrinkage are reported in Figure IA.47. Results again closely align with those reported in Section 3. Only 2 of the 13 DFPS aggregate factors are likely candidates for inclusion to the BMA-SDF. These include the composite bond factors formed with equity short-reversal, DFPS_STREV and momentum, DFPS_MOM equity characteristics. This overlaps with the factors already included in our baseline bond factor zoo (MOMBS and PEADB), both of which are formed with prior equity return data.

The corresponding in- and out-of-sample asset pricing results are reported in Panel B of Tables IA.XXX and IA.XXXI. Again, the BMA-SDF with a 80% Sharpe ratio shrinkage and the TOP model (comprising the factors IVOL, PEAD, DFPS_MOM, PEADB, and CRY) outperform all other models by a wide margin, both in- as well as out-of-sample.

Extended bond factor zoo including TRACE bond illiquidity factors. We again tweak the bond factor zoo by including three additional illiquidity factors computed using TRACE transaction data. In particular, we include the tradable liquidity risk factor LRF from Bai et al. (2019) and the Amihud (2002) (AMD) and Pástor and Stambaugh (2003) (PSB) nontradable risk factors from Lin et al. (2011). The sample is restricted to the TRACE era from October 2002 to December 2022 for a total of 243 monthly observations (with two months lost to compute the illiquidity factors). The set of IS test assets remains the largely same, we only add the tradable LRF factor. The posterior factor probabilities and market prices of risk with 80% Sharpe ratio shrinkage are reported in Figure IA.48. None of the illiquidity factors are likely candidates for inclusion in the BMA-SDF. Notably, the LRF factor is the *least* likely bond factor to be included with a market price of risk close to zero. Likewise, nontradable AMD factor is the least likely nontradable factor for inclusion. Our results echo those of, e.g., Richardson and Palhares (2019) who document a very limited illiquidity premium in corporate bond returns using characteristic portfolio sorts. The corresponding in- and out-of-sample asset pricing results are again reported in Panel C of Tables IA.XXX and IA.XXXI.

[8] This data is available for download here.

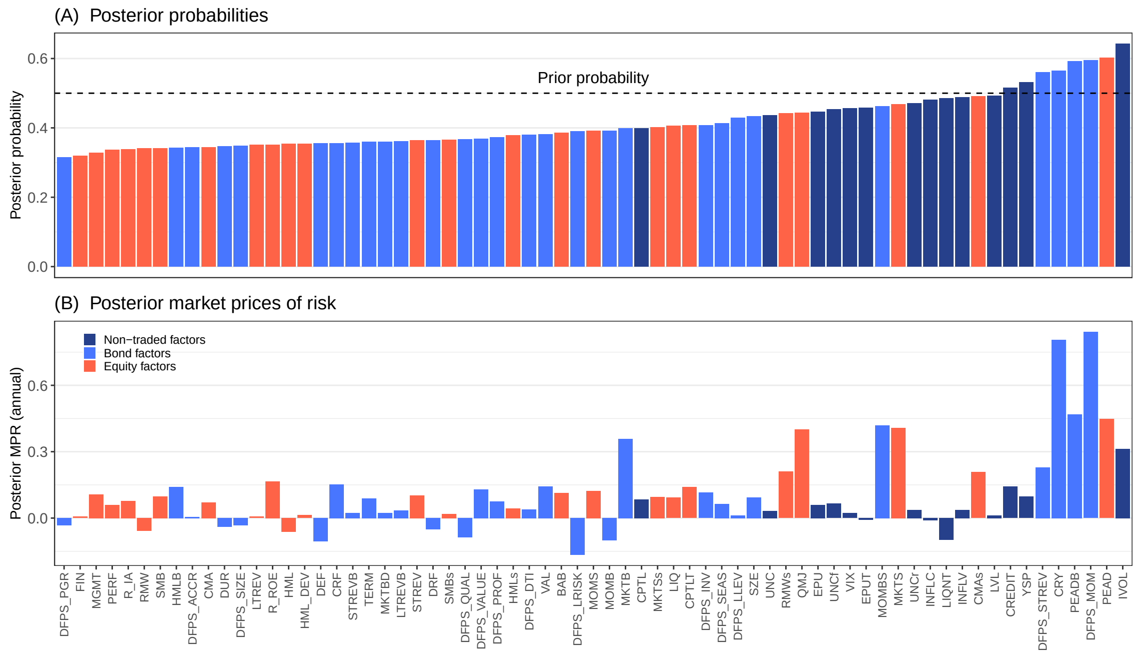


Figure IA.47: Posterior factor probabilities and risk prices: Extending the bond factor zoo I.

The figure reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and stock factors described in Appendix A as well as the 13 composite bond factors of Dick-Nielsen et al. (2025). The 13 composite bond excess return factors are formed with the underlying *equity* characteristic data. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the 83 stock and bond portfolios and the 40 tradable factors described in Section 1 as well as the 13 composite DFPS factors. The sample period is 1986:01 to 2021:12 ($T = 432$).

Extending the time series back to 1977. Using the full span of the LBFI database we extend the sample period back to January 1977 resulting in a maximum sample span of 549 monthly observations. It is important to note that the vast majority of bonds present in the data over the early period we exclude for our analysis in Section 3 are exclusively investment grade with matrix prices as opposed to quotes. Prior to 1977, the U.S. high-yield corporate bond market was primarily composed of "fallen angels," bonds originally issued as investment grade but subsequently downgraded. As such, the percentage of bonds classified as "high-yield" was lower than 5% of the total U.S. corporate bond market in terms of market capitalization before 1980. The U.S. high-yield bond market only began to truly take root in the early 1980s when large investment banks (Lehman and Drexel Burnham Lambert) began both underwriting and trading these bond issues.[9] The posterior factor probabilities and market prices of risk with

[9]See Taggart (1987) for further discussion.

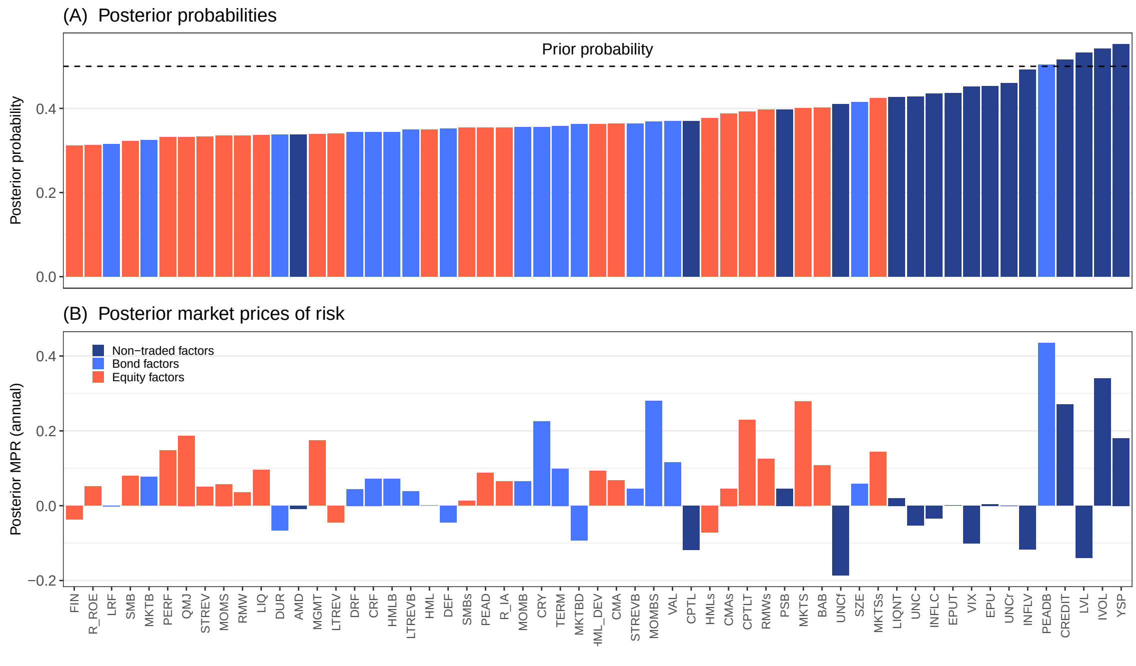


Figure IA.48: Posterior factor probabilities and risk prices: Extending the bond factor zoo II.

The figure reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and stock factors described in Appendix A plus the tradable liquidity factor LRF of Bai et al. (2019) and the two nontradable illiquidity factors AMD and PSB of Lin et al. (2011). The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the 83 stock and bond portfolios and the 40 tradable factors described in Section 1 plus the tradable LRF bond factor. The sample period is 2002:10 to 2022:12 ($T = 243$).

80% Sharpe ratio shrinkage are reported in Figure IA.49. Despite the flaws in market structure, result remain unaffected compared to what we report in Section 3. The top five factors in terms of posterior probabilities are PEADB, PEAD, CRY, MOMBS, and CREDIT. Other factors outside the top five but with a posterior probability $> 50\%$ include LVL, IVOL and YSP, again aligning with the baseline results. These factors also yield relatively large posterior market prices of risk. The corresponding in- and out-of-sample asset pricing results are again reported in Panel D of Tables IA.XXX and IA.XXXI.

Varying subsamples. Finally, we present the in- and out-of-sample cross-sectional asset pricing performance for two sample splits in Tables IA.XXXII and IA.XXXIII. In particular, we first estimate the models for the pre- and post-TRACE era, i.e., before and after July 2002 (respective Panels A and C). Second, we also split the sample into a pre- and post-2000 period

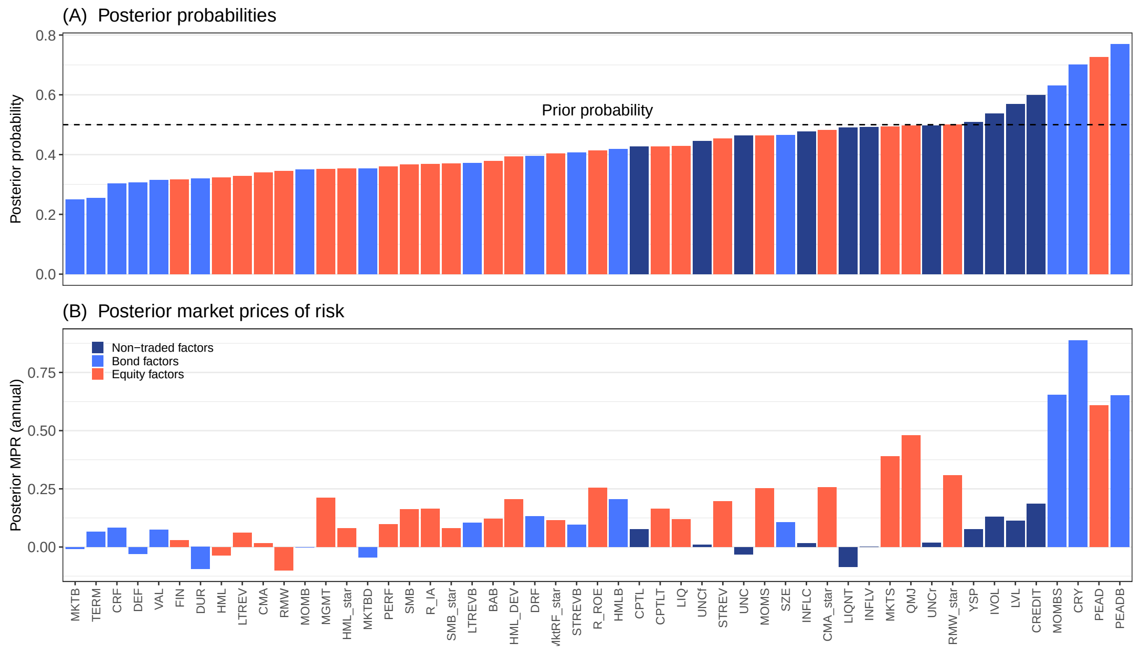


Figure IA.49: Posterior factor probabilities and risk prices: Extending the sample to 1977.

The figure reports posterior probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior means of annualized market prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, of the 54 bond and stock factors described in Appendix A. The prior for each factor inclusion is a Beta(1, 1), yielding a prior expectation for γ_j of 50%. The prior Sharpe ratio is set to 80% of the ex post maximum Sharpe ratio of the 83 stock and bond portfolios and the 40 tradable factors described in Section 1. The sample period is 1977:01 to 2022:12 ($T = 549$).

as in van Binsbergen et al. (2025) (respective Panels B and D).

The results from the full sample estimation in Tables 2 and 3 carry over to the subsamples, the BMA-SDF and TOP models outperform the other competitor models. Note, however, that the OS pricing exercise in Table IA.XXXIII is more stringent than the one in Table 3. For the full sample, only the test assets are out-of-sample. Once we have two sample splits, we perform the OS pricing not only in the cross-section but also the time series. That is, we estimate the BMA-SDF using the IS test assets for the respective sample period and then use the resulting SDF to price (with no additional parameter estimation) each set of the OS test assets over the remaining sample.

Table IA.XXXII: In-sample cross-sectional asset pricing performance across sample splits

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Pre-TRACE, 1986:01–2002:07											
RMSE	0.318	0.308	0.295	0.279	0.362	0.391	0.365	0.359	0.364	0.263	0.357
MAPE	0.237	0.229	0.220	0.210	0.259	0.315	0.276	0.257	0.276	0.201	0.243
R^2_{OLS}	0.118	0.171	0.240	0.317	−0.149	−0.336	−0.163	−0.126	−0.158	0.396	−0.115
R^2_{GLS}	0.078	0.098	0.120	0.144	0.086	0.088	0.090	0.086	0.189	0.097	0.156
Panel B: Pre-2000, 1986:01–1999:12											
RMSE	0.423	0.409	0.395	0.380	0.405	0.444	0.398	0.407	0.521	0.258	0.318
MAPE	0.303	0.292	0.281	0.271	0.295	0.350	0.300	0.296	0.423	0.203	0.223
R^2_{OLS}	0.063	0.122	0.183	0.244	0.138	−0.035	0.170	0.131	−0.423	0.651	0.471
R^2_{GLS}	0.125	0.136	0.149	0.165	0.185	0.185	0.187	0.186	0.251	0.100	0.250
Panel C: Post-TRACE, 2002:08–2022:12											
RMSE	0.206	0.175	0.153	0.133	0.239	0.232	0.237	0.216	0.240	0.180	0.171
MAPE	0.163	0.133	0.114	0.098	0.175	0.180	0.182	0.157	0.186	0.133	0.116
R^2_{OLS}	0.283	0.480	0.604	0.700	0.029	0.086	0.047	0.210	0.020	0.452	0.505
R^2_{GLS}	0.046	0.075	0.108	0.146	0.040	0.041	0.044	0.042	0.231	0.018	0.100
Panel D: Post-2000, 2000:01–2022:12											
RMSE	0.198	0.172	0.146	0.122	0.261	0.290	0.279	0.261	0.239	0.147	0.195
MAPE	0.151	0.131	0.110	0.092	0.196	0.231	0.216	0.196	0.174	0.104	0.133
R^2_{OLS}	0.234	0.423	0.585	0.708	−0.337	−0.644	−0.527	−0.332	−0.113	0.580	0.256
R^2_{GLS}	0.033	0.081	0.132	0.185	0.008	0.015	0.020	0.008	0.234	0.115	0.097

The table presents the cross-sectional in-sample asset pricing performance of different models pricing bonds and stocks jointly for different sample splits. For Panels A and C we split the sample into a pre- and post-TRACE period (1986:01–2002:07 and 2002:08–2022:12), in Panels B and D we show results for the pre- and post-2000 period (1986:01–1999:12 and 2000:01–2022:12). For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors with an average posterior probability greater than 50%. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020). If the time series dimension is smaller than the number of assets, $T < N$, we allow a small degree of overlap in the two cross-validation samples. RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. All models are estimated on the baseline IS test assets. Test assets are the 83 bond and stock portfolios and the 40 tradable bond and stock factors described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units.

Table IA.XXXIII: Out-of-sample cross-sectional asset pricing performance across sample splits

	BMA-SDF prior Sharpe ratio				CAPM	CAPMB	FF5	HKM	TOP	KNS	RPPCA
	20%	40%	60%	80%							
Panel A: Pre-TRACE											
RMSE	0.117	0.114	0.114	0.130	0.158	0.186	0.190	0.167	0.163	0.157	0.821
MAPE	0.093	0.092	0.094	0.110	0.131	0.160	0.155	0.138	0.128	0.137	0.738
R^2_{OLS}	0.073	0.129	0.121	−0.139	−0.682	−1.325	−1.440	−0.887	−0.786	−0.657	−44.455
R^2_{GLS}	0.001	0.008	0.014	0.017	−0.004	−0.003	−0.015	−0.008	−0.120	0.014	−0.284
Panel B: Pre-2000											
RMSE	0.134	0.116	0.099	0.091	0.290	0.141	0.357	0.284	0.267	0.157	0.386
MAPE	0.105	0.089	0.076	0.070	0.261	0.115	0.330	0.251	0.223	0.126	0.328
R^2_{OLS}	0.225	0.420	0.578	0.641	−2.656	0.139	−4.540	−2.486	−2.099	−0.073	−5.469
R^2_{GLS}	0.037	0.050	0.062	0.074	0.044	0.055	0.031	0.045	−0.126	−0.018	−0.305
Panel C: Post-TRACE											
RMSE	0.235	0.217	0.213	0.225	0.369	0.196	0.271	0.381	0.284	0.191	0.500
MAPE	0.177	0.164	0.162	0.175	0.312	0.141	0.209	0.321	0.216	0.135	0.454
R^2_{OLS}	0.157	0.279	0.305	0.226	−1.078	0.414	−0.121	−1.218	−0.230	0.440	−2.830
R^2_{GLS}	0.009	0.018	0.023	0.023	0.006	0.009	0.007	0.005	−0.155	0.011	0.021
Panel D: Post-2000											
RMSE	0.190	0.184	0.194	0.216	0.241	0.233	0.204	0.242	0.297	0.220	0.278
MAPE	0.138	0.132	0.141	0.163	0.181	0.198	0.164	0.182	0.233	0.190	0.246
R^2_{OLS}	0.126	0.181	0.094	−0.126	−0.398	−0.308	−0.008	−0.408	−1.132	−0.173	−0.864
R^2_{GLS}	0.006	0.010	0.013	0.015	0.007	0.007	0.006	0.007	−0.007	0.015	0.012

The table presents the cross-sectional out-of-sample asset pricing performance of different models pricing bonds and stocks jointly for different sample splits. For Panels A and C we split the sample into a pre- and post-TRACE period (1986:01–2002:07 and 2002:08–2022:12), in Panels B and D we show results for the pre- and post-2000 period (1986:01–1999:12 and 2000:01–2022:12). For the BMA-SDF, we provide results for prior Sharpe ratio values set to 20%, 40%, 60% and 80% of the ex post maximum Sharpe ratio of the test assets. TOP includes the top five factors with an average posterior probability greater than 50%. CAPM is the standard single-factor model using MKTS, and CAPMB is the bond version using MKTB. FF5 is the five-factor model of Fama and French (1993), HKM is the two-factor model of He et al. (2017). KNS stands for the SDF estimation of Kozak et al. (2020). If the time series dimension is smaller than the number of assets, $T < N$, we allow a small degree of overlap in the two cross-validation samples. RPPCA is the risk premia PCA of Lettau and Pelger (2020). Estimation details for the benchmark models are given in Appendix D. Bond returns are computed in excess of the one-month risk-free rate of return. The models are first estimated using the baseline IS test assets from Table IA.XXXII for the IS training period. The resulting SDF is then used to price (with no additional parameter estimation) each set of the OS assets for the out-of-sample period in the time series. OS test assets are the combined 154 bond and stock portfolios described in Section 1. All data is standardized, that is, pricing errors are in Sharpe ratio units.

IA.11 The nontradable CREDIT factor

The nontradable CREDIT factor is defined as the difference between aggregate corporate bond yield indices made available by FRED (the BAA index minus the AAA index), using data constructed by Moody's. The CREDIT factor is consistently included as a top factor (large posterior probability) with a sizable market price of risk across all of our estimations.

A wider BAA−AAA spread indicates that investors are *less* willing to bear credit risk. That is, before (i.e., in the build-up to) a recession, investor portfolios are re-allocated to "safer" securities, implying they are more concerned about bearing credit risk, rendering the CREDIT factor not only a useful indicator of the health of the economy, but a likely candidate for inclusion in the SDF.

Potential issues with the CREDIT factor. Unfortunately, the data made available from Moody's is opaque, and perhaps more concerning, only two firms (Microsoft and Johnson & Johnson) are included in the AAA yield index (Boyarchenko and Shachar, 2020) toward the end of the sample. Given that the data filtering process used by Moody's is not publicly available, we reached out to the economics department at `Moody's Analytics`. The full (and unedited) response from the Moody's economics department is provided below:

> *We don't currently publish a detailed methodology but it is summarized as: "Yield index for US investment grade nonfinancial corporate bonds with long-term maturities. Based on seasoned bonds with remaining maturities of at least 20 Years. Derived from pricing data on a regularly-replenished population of over 100 seasoned corporate bonds in the US market, each with current outstandings over $100 million. The bonds have maturities as close as possible to 30 years, with an average maturity of 28 years; they are dropped from the list if their remaining life falls below 20 years or if their ratings change. Bonds with deep discounts or steep premiums to par are generally excluded. All yields are yield-to-maturity calculated on a semi-annual compounding basis. Each observation is an unweighted average, with Average Corporate Yields representing the unweighted average of the corresponding Average Industrial and Average Public Utility observations." For Aaa you are correct that we currently*

only have bonds from MSFT and JNJ in the actively traded list. We periodically update a master list of eligible bonds in each ratings bucket and then exclude bonds from the active list whose ratings no longer match the bucket or other criteria.[10]

A custom made CREDIT factor. To address the core issues above, (i) opaque data filtering rules and (ii) only two firms being present in the AAA index toward the end of the sample, we re-construct our own "custom-made" high grade and BAA indices with our core dataset comprising the Lehman Brothers and ICE/BAML corporate bond datasets.

We apply the following filters to our data, which ensures a reasonable sample whilst trying to adhere to the filters supposedly applied by Moody's:

(i) Remove bonds with a market capitalization less than $100 million.

(ii) Remove bonds with a credit spread less that 0 or greater than 5,000 bps.

(iii) Remove bonds which are classified as "financials."

When constructing the AAA yield index, we include all bonds rated Aaa to Aa3, e.g., those rated `Prime` and `High Grade` with maturities from 20 to 30 years. For the BAA yield index, we keep all bonds rated Baa1 to Baa3, e.g., those rated `Lower Medium Grade` with maturities from 20 to 30 years. This construction method implies we have 24 unique firms rated Aaa to Aa3 toward the end of the sample (as opposed to only two firms). On average, from 1986 to 2022, the high grade (Aaa to Aa3) index contains 24 firms, with an average number of bonds equaling 54. For the BAA index, the sample contains an average of 123 firms with an average number of bonds equaling 255. Toward the end of the sample, the BAA index contains 198 firms.

We plot the time-series of the CREDIT factor constructed by Moody's and ourselves in Figure IA.50. The unconditional time-series correlation is equal to 0.89. The average spread for Moody's and our custom factor is 97 and 87 bps, respectively. Even though the two CREDIT factors are computed with different data filtering rules and an expanded subset of investment grade bonds for the custom version of the AAA index, the time-series dynamics are similar.

[10]We thank David Mena from Moody's Analytics, Inc for helping us with the data.

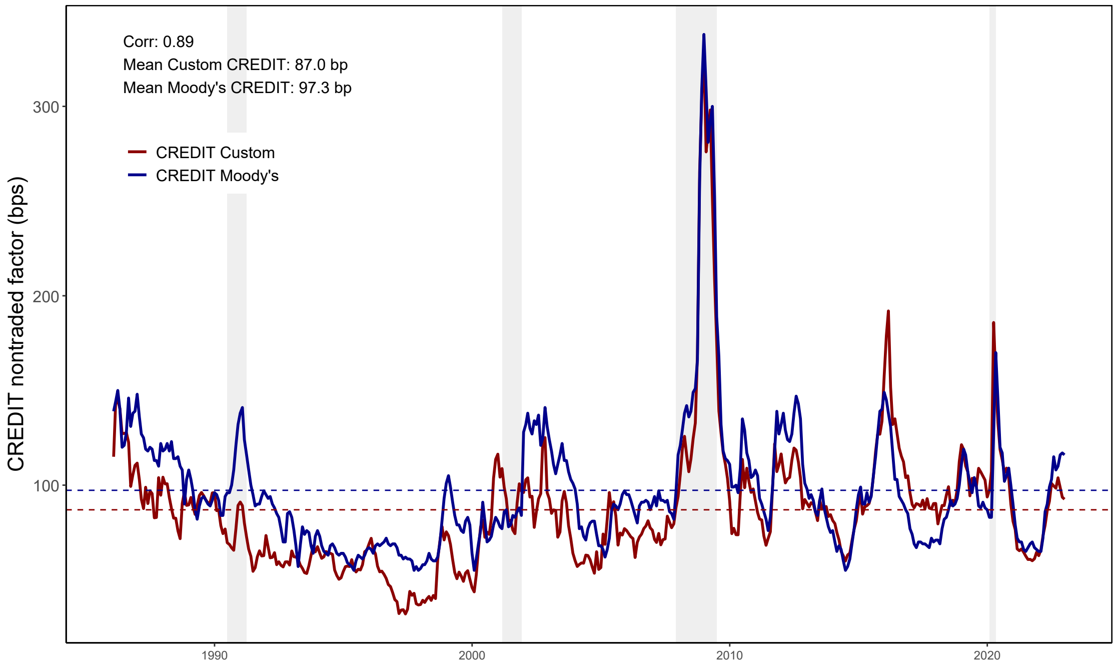


Figure IA.50: The nontradable CREDIT factor.

This figure depicts the time series of the CREDIT nontradable factor constructed with data from FRED which uses Moody's corporate bond yield data and our own data (discussed in Section IA.11 of the Internet Appendix). The Moody's CREDIT factor, `CREDIT Moody's`, is defined as the difference between the BAA and AAA corporate bond yield indices from Moody's. The exact computation of this series, and it's underlying constituent bond sample is not made available by Moody's. The Moody's sample (BAA and AAA) roughly comprises (as per Moody's Economics Department), "*seasoned bonds with remaining maturities of at least 20 Years ... Derived from pricing data on a regularly-replenished population of over 100 seasoned corporate bonds in the US market, each with current outstandings over $100 million. The bonds have maturities as close as possible to 30 years, with an average maturity of 28 years; they are dropped from the list if their remaining life falls below 20 years or if their ratings change. Bonds with deep discounts or steep premiums to par are generally excluded. All yields are yield-to-maturity calculated on a semi-annual compounding basis. Each observation is an unweighted average, with Average Corporate Yields representing the unweighted average of the corresponding Average Industrial and Average Public Utility observations.*" Our custom made CREDIT factor, `CREDIT Custom` attempts to broadly follow the guidance of Moody's. We, (i) exclude bonds from the financials sector, (ii) keep bonds with amount outstanding values greater than $100 million, (iii) Remove bonds with a credit spread less than 0 or greater than 5000 bps. For the AAA yield index, we keep all bonds rated Aaa to Aa3, e.g., those rated `Prime` and `High Grade` with maturities from 20-30 years. For the BAA yield index, we keep all bonds rated Baa1 to Baa3, e.g., those rated `Lower Medium Grade` with maturities from 20-30 years. The sample period is 1986:01 to 2022:12 ($T = 444$).

Table IA.XXXIV: IS and OS cross-sectional asset pricing performance: CREDIT factor robustness

	In-sample				Out-of-sample			
	20%	40%	60%	80%	20%	40%	60%	80%
Panel A: Baseline with Moody's BAA-AAA Yield Index								
RMSE	0.214	0.203	0.185	0.167	0.114	0.102	0.095	0.090
MAPE	0.167	0.154	0.139	0.125	0.081	0.074	0.069	0.065
R^2_{OLS}	0.155	0.240	0.367	0.487	0.357	0.489	0.557	0.603
R^2_{GLS}	0.106	0.168	0.232	0.285	0.038	0.070	0.098	0.124
Panel B: With our corporate bond yield data, BAA−(AAA+AA)								
RMSE	0.214	0.203	0.186	0.169	0.114	0.102	0.095	0.091
MAPE	0.167	0.154	0.140	0.127	0.081	0.075	0.069	0.066
R^2_{OLS}	0.151	0.240	0.361	0.476	0.357	0.486	0.551	0.593
R^2_{GLS}	0.106	0.167	0.229	0.281	0.037	0.069	0.096	0.120

The table presents the cross-sectional in- and out-of-sample asset pricing performance of the co-pricing BMA-SDF with the baseline CREDIT factor from Moody's (Panel A) and the custom-made CREDIT factor using our own yield data (Panel B). Our custom made CREDIT factor `CREDIT Custom` attempts to broadly follow the guidance of Moody's. That is, we (i) exclude bonds from the financials sector, (ii) keep bonds with amount outstanding values greater than \$100 million, (iii) remove bonds with a credit spread less than 0 or greater than 5000 bps. For the AAA yield index, we keep all bonds rated Aaa to Aa3, e.g., those rated `Prime` and `High Grade` with maturities from 20-30 years. For the BAA yield index, we keep all bonds rated Baa1 to Baa3, e.g., those rated `Lower Medium Grade` with maturities from 20-30 years. The sample period is 1986:01 to 2022:12 ($T = 444$). The posterior probabilities and the market prices of risk are included below.

	Total prior Sharpe ratio			
	20%	40%	60%	80%
Panel A: With Moody's BAA−AAA Yield Index				
$\mathbb{E}[\gamma_j\|\text{data}]$	0.498	0.497	0.530	0.557
$\mathbb{E}[\lambda_j\|\text{data}]$	0.002	0.009	0.024	0.055
Panel B: With corp. yields, BAA−(AAA+AA)				
$\mathbb{E}[\gamma_j\|\text{data}]$	0.487	0.494	0.517	0.518
$\mathbb{E}[\lambda_j\|\text{data}]$	0.001	0.006	0.015	0.034

In Panel B, CREDIT remains a factor in the top five most likely to be included in the SDF.

The BMA-SDF with the custom CREDIT factor. We now re-estimate our baseline results with the custom made CREDIT factor. We report the in-and-out-of-sample asset pricing results over the four levels of SR shrinkage in Table IA.XXXIV. Included in the table caption are the posterior probabilities and the MPR for the estimation with Moody's (Panel A) and the custom CREDIT factor (Panel B). First, the in and out-of-sample asset pricing results are close to identical with numbers changing only at the third decimal place. Second, the table in the caption documents that both the posterior probabilities and the MPRs are closely aligned, confirming results in the main text using Moody's CREDIT factor. In unreported results, we also re-estimate the BMA-SDF with the GZ spread (as opposed to the CREDIT spread) from Gilchrist and Zakrajšek (2012) and document very similar results.[11]

Why are the results so consistent? Our theoretical and simulation results (see Section 2.4) show that stability is expected from our robust inference method. Since individual factors contain both signals about fundamental risk sources and noise, the BMA-SDF optimally aggregates them to maximize the signal-to-noise ratio. While data perturbations may affect individual factors (such as the CREDIT factor), the BMA-SDF largely mitigates this impact.

[11] We thank Yoshio Nozawa for making the GZ spread data available to us.

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