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Margins as Canaries in the Coal Mine

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Abstract

Central clearing counterparties (CCPs) manage counterparty risk by requiring clearing members to post margins. This paper explores the role of margins as "canaries in the coal mine:" By inducing defaults of fragile counterparties before contract maturity, margin calls enable CCPs to transfer these contracts to other counterparties, thereby preserving risk sharing. Our model reveals a pecking order of CCP risk management tools. When fragility is low, loss sharing among original counterparties suffices. When fragility is high, such that defaults at contract maturity would trigger cascading failures among clearing members, the CCP optimally complements loss sharing with margins. It is optimal to use margins as canaries when the balance sheets of fragile counterparties are severely impaired. Our findings highlight the complementary nature of CCP risk management tools: margins, loss sharing, and counterparty replacement.

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1 Introduction

Since the 2008 financial crisis, the architecture of the financial system has been recognized as a central determinant of systemic risk. Post-crisis regulatory reforms established central clearing counterparties (CCPs) as a cornerstone of global financial infrastructure. As of 2024, about 80% of the notional outstanding of global over-the-counter (OTC) derivatives were centrally cleared, representing more than $400 trillion.[1] As a result, the functioning of CCPs is now a fundamental cornerstone of derivative markets and crucial for financial stability. At the same time, our understanding of how central clearing affects the allocation of risk in financial markets remains limited.

In this paper, we provide a tractable model of risk sharing and central clearing and characterize the three mechanisms through which CCPs manage risk: (i) margins, (ii) counterparty replacement, and (iii) loss sharing. We show that margins (i.e., collateral that clearing members must post to cover potential losses) are critical to CCP risk management and complement counterparty replacement and loss sharing. Indeed, margins play a significant role in practice. The total amount of margin held by CCPs exceeds $1.5 trillion, with *daily* flows of more than $30 billion.[2]

The existing literature has emphasized the role of margins in ring-fencing assets, thereby increasing resources available upon default and reducing counterparties' default incentives (Biais et al., 2016). We highlight another key function of margins as 'canaries in the coal mine': margin calls at interim dates can induce fragile counterparties to default before contract maturity, allowing the CCP to replace these counterparties. The benefit of early replacement is that the contracts can then be transferred to other counterparties while risk sharing remains possible. This reduces the losses imposed on existing clearing members and can prevent a crisis scenario in which the CCP is overwhelmed by default losses at contract maturity. Overall, our results highlight the complementary nature of margins, counterparty replacement, and loss sharing as risk management tools before contract maturity.

In our model, risk-averse protection buyers hedge endowment risk by trading centrally-cleared derivatives with risk-neutral protection sellers. The terms of the derivative contract, including margin requirements and loss-sharing arrangements, are determined endogenously from an optimal contracting problem. Following an interim signal about underlying risks, some protection sellers become fragile due to privately observed shocks to their balance sheets. Fragile counterparties may be unable to make contractual payments

[1]Source: BIS OTC derivatives statistics, available at https://data.bis.org/topics/OTC_DER.

[2]Source: CCP Global (2025) "Public Quantitative Disclosure PQD Quarterly Trends Report 2024 Q4 Data".

at maturity. If sufficiently many fragile sellers default, this can overwhelm the CCP's ex-post loss-sharing capacity, leading to cascading failures among clearing members—and ultimately, CCP failure.

Our main result is that CCPs can prevent this scenario by using margin calls to screen counterparties. In this case, margins are used to induce (rather than prevent) default. Fragile sellers unable to meet margin requirements default, allowing the CCP to transfer their positions to healthier counterparties. The advantage of early replacement is that it preserves loss-absorbing capacity, preventing CCP failure.

In contrast, we show that it is not optimal to use margins when only a few clearing members are fragile. In this case, loss sharing at the CCP suffices to maintain full risk sharing: non-defaulting clearing members can absorb the losses of defaulting members without defaulting themselves. Given that posting margins entails an opportunity cost, it is optimal not to use them when they are not strictly necessary.

Only once fragility is more widespread are margins optimally used to enhance risk-sharing capacity—either to ring-fence assets or as canaries. The traditional ring-fencing role of margin preserves value by securing collateral from potential defaulters, reducing deadweight losses when defaults occur at contract maturity. In contrast, the canary margins create value by revealing private information about counterparty fragility, enabling dynamic reallocation of risk to those best able to bear it. Crucially, these two roles require different margin levels: canary margins must be set high enough to induce defaults, while ring-fencing margins are lower. When the balance sheets of fragile sellers are severely impaired, the canary role dominates and the CCP optimally sets margins such that some fragile clearing members default and are replaced.

Our model highlights the costs and benefits of the three principal risk management tools employed by CCPs: margins, counterparty replacement, and loss sharing. The central insight is that during a contract's lifetime, these tools are complements rather than substitutes: margin calls enable replacement by identifying fragile counterparties, while sharing default losses increases the value of replacement. In contrast, at contract maturity margins and loss sharing are substitutes because ring-fencing a larger share of defaulters' assets reduces the default losses to be shared. Moreover, counterparty replacement at contract maturity is not useful because the cost of replacement would at least equal the cash flow due to be paid, leaving no scope for risk sharing. Early counterparty replacement preserves risk-sharing opportunities because the final payment remains uncertain, making market participants willing to provide insurance.

Hence, the efficiency of counterparty replacement depends critically on market conditions and information structure. We identify three key determinants. First, replacement requires sufficient heterogeneity among market participants: replacement counterparties must have adequate resources to assume defaulted positions. Second, the timing of fragility shocks matters due to the Hirshleifer (1971) effect—the more information revealed about underlying risks, the less risk sharing replacement can provide. Early intervention via canary margins is therefore more valuable than late intervention. Third, replacement becomes more attractive when fragile sellers are severely impaired, as ring-fencing provides limited value when counterparties have few assets to seize.

Our framework yields several testable predictions and policy implications. First, optimal margin requirements follow a pecking order: when fragile sellers are few, ex-post loss sharing suffices and margins are unnecessary; for intermediate levels of fragility, canary margins that induce replacement can be optimal; when fragility is too widespread, however, even replacement cannot maintain full insurance. Second, our model explains why clearing member defaults often occur following margin calls rather than at contract maturity, as observed in the 2018 default of power trader Einar Aas at Nasdaq Clearing. Third, margin procyclicality—raising margins during stressed conditions—can be efficient when it facilitates replacement, provided sufficiently healthy market participants are available to absorb positions. This challenges the conventional view that procyclical margins necessarily amplify systemic stress.

These findings have important implications for CCP regulation. Current policy debates focus on calibrating margins to cover potential losses while avoiding destabilizing procyclicality, but this perspective is incomplete: margins should also be evaluated for their role in facilitating efficient counterparty replacement. Regulatory constraints that cap margin increases during stress may inadvertently prevent optimal screening, potentially increasing systemic risk. Moreover, CCP access to diverse replacement counterparties is crucial for financial stability—a consideration largely absent from current regulatory frameworks.

Our paper contributes to the literature on financial market architecture and systemic risk (Allen and Gale, 2000; Zawadowski, 2013; Acemoglu et al., 2015) and frictions in derivatives markets (Bolton and Oehmke, 2015; Biais et al., 2021; Allen and Wittwer, 2023). Specifically, our analysis shows how interim replacement of counterparties can prevent financial contagion. Margin calls enable counterparty replacement by serving as an ex-post screening device, relating our analysis to the literature on collateral as a screening device in financial markets (Bester, 1985; Besanko and Thakor, 1987). While this literature has focused on collateral as a screening device at contract origination, we show that margins as canaries serve as ex-post screening devices after contracts have been entered. The dynamic nature of canary margins generates additional considerations, such as the

Hirshleifer effect. Our results also extend the traditional focus on collateral as a means to raise borrowing capacity to finance investment (Holmstrom and Tirole, 1997; Rampini and Viswanathan, 2010; Donaldson et al., 2020). Instead of focusing on borrowing capacity, we highlight the use of margins to increase risk-sharing capacity.

Our paper is also related to the literature on CCPs as insurers of (counterparty) risk (Acharya and Bisin, 2014; Biais et al., 2016; Capponi et al., 2022b; Cucic, 2022; Kuong and Maurin, 2023). While these studies have mainly stressed that CCPs may require margins to reduce default incentives, we highlight a distinct role: margins as a screening device to separate fragile from safe counterparties.[3] Other studies on CCPs have focused on the role of netting and the demand for clearing (Duffie and Zhu, 2011; Kubitza et al., 2024; Chebotarev, 2025) and settlement (Koeppl et al., 2012).

In practice, counterparty replacement is implemented via auctions. While we abstract from auction design by assuming a competitive market for replacement, related work studies the design features of CCP auctions (Vuillemey, 2023; Huang and Zhu, 2024). Moreover, several empirical studies document the impact of central clearing on counterparty risk and financial markets (Loon and Zhong, 2014; Boissel et al., 2017; Bernstein et al., 2019; Vuillemey, 2020) as well as margining practices (Capponi et al., 2022a; Grothe et al., 2023).[4]

2 Model

We develop a simple model of centrally cleared financial contracts used to hedge endowment risk. A natural interpretation of these contracts is centrally cleared derivatives, for example, swaps.

Model overview. There are three dates, $t = 0, 1, 2$. At $t = 0$, protection buyers enter risk-sharing contracts (which we refer to as "derivatives") with protection sellers. The contracts are centrally cleared by a CCP. At $t = 1$, a signal regarding the likelihood that a protection seller will have to make a payment on the derivative contract is observed. At the same time, some protection sellers suffer an adverse shock to their balance sheets. In response, the CCP can require sellers to post margin on their contracts. Sellers with weak balance sheets may default on the interim margin call. If this happens, the CCP can transfer these contracts to other counterparties. At $t = 2$, the derivative contracts

[3] In an extension with strategic defaults, we show that the roles of margins as screening and as incentive device co-exist.

[4] See Menkveld and Vuillemey (2021) for a survey of the literature on central clearing.

pay off, and protection sellers with weak balance sheets may default on their payment obligations.

We now describe each element of the model in detail.

Protection buyers and sellers. There is a mass one of protection buyers and a corresponding mass of protection sellers. Protection buyers are identical, with a twice-differentiable concave utility function u over consumption at $t = 2$. They face endowment risk $\tilde{e}$ at $t = 2$. For simplicity, we assume that $\tilde{e}$ can only take one of two values: $\overline{e} = 1$ with probability $\pi \in (0, 1)$ and $\underline{e} = 0$ with probability $1 - \pi$. The endowment risk $\tilde{e}$ is the same for all protection buyers (i.e., it represents aggregate risk).

Protection buyers can purchase insurance against their endowment risk from risk-neutral protection sellers. Each protection seller is endowed with one unit of an asset (representing their "balance sheet"), whose payoff is realized at $t = 2$. There are two types of assets: low and high quality. Asset quality is unknown at $t = 0$ and privately revealed to protection sellers at $t = 1$, as specified in more detail below.

Interim information. At the beginning of $t = 1$, before margin calls are made and sellers might default, a public signal $\tilde{s}$ about the endowment risk $\tilde{e}$ becomes available. For example, if the risk $\tilde{e}$ stems from the oil price at $t = 2$, $\tilde{s}$ can be interpreted as the oil price at $t = 1$. The signal is correct with probability λ,

$$\lambda = \mathbb{P}(\overline{s} \mid \overline{e}) = \mathbb{P}(\underline{s} \mid \underline{e}). \tag{1}$$

Following Bayes Rule, the probability of a good endowment realization $\overline{e}$ is then updated to

$$\overline{\pi} = \mathbb{P}(\overline{e} \mid \overline{s}) = \frac{\lambda\pi}{\lambda\pi + (1-\lambda)(1-\pi)} \text{ or } \underline{\pi} = \mathbb{P}(\overline{e} \mid \underline{s}) = \frac{(1-\lambda)\pi}{(1-\lambda)\pi + \lambda(1-\pi)}. \tag{2}$$

We assume that $\frac{1}{2} < \lambda < 1$, so that observing $\overline{s}$ is positive news, implying that $\underline{\pi} < \pi < \overline{\pi} < 1$. The larger λ, the more informative the signal. The restriction that $\lambda < 1$ ensures that the signal is not perfectly revealing.

Contracts, margins, and central clearing. Protection buyers and sellers enter a derivative contract at $t = 0$, which is centrally cleared by the CCP. Clearing transforms each bilateral contract between one buyer and one seller into two cleared contracts: one between the seller and the CCP, specifying a transfer T_S, and one between the CCP and the buyer, specifying a transfer T_B. Positive transfers represent payments to the CCP. Negative transfers represent payments made by the CCP.

The contractual transfers T_B and T_S are agreed upon at $t = 0$ and realized at $t = 2$. These transfers can be made conditional on all public information, including the realizations of the buyers' endowment risk $\tilde{e}$ and the public signal $\tilde{s}$. Because the CCP is a counterparty to every contract, it pools all transfers from buyers, sellers, and (if counterparties default and are replaced at the interim date) outsiders. As a result, the transfer received by protection buyer j does not depend on the transfer made by its original counterparty, but on the transfers made by all clearing members, which determine the resources available to the CCP. Therefore, the transfers T_S and T_B can be interpreted as the total (net) transfers jointly determined by the initial bilateral derivative contracts and pooling of contracts at the CCP (in particular, loss sharing as described below).[5]

The CCP's budget constraint requires that aggregate transfers to protection buyers are matched by aggregate transfers from protection sellers and outside sellers in each state. For example, if no protection seller defaults, the CCP's budget constraint is given by

$$T_B(\tilde{s}, \tilde{e}) + T_S(\tilde{s}, \tilde{e}) = 0 \quad \forall \tilde{s}, \tilde{e}. \tag{3}$$

Margins. In addition to the transfers T_S and T_B, the CCP can require that protection sellers post some of their assets as margin at $t = 1$. The margin call can be made contingent on all public information at $t = 1$ (i.e., the signal $\tilde{s}$). We denote the amount of high-quality assets to be deposited in the margin account by $\alpha(s)$, $s \in \{\underline{s}, \bar{s}\}$. Because of their lower value, a larger amount of low-quality assets is required to satisfy the margin call.

A margin call serves to ring-fence a fraction of sellers' assets. Defaulting sellers lose all posted margin, which is seized by the CCP and distributed among surviving clearing members. However, posting margin is costly: assets in the margin account earn a lower return, resulting in a per-unit opportunity cost of $1 - k$. Therefore, the CCP makes a margin call only if doing so increases the CCP's resources after accounting for the opportunity cost of margins.

Defaults. Equilibrium payments by protection sellers are subject to default. Default occurs when protection sellers are either unable to honor their payment obligations at

[5] For example, protection buyers might trade a credit default swap (CDS) with protection sellers, which provides an insurance payment if the underlying defaults (state $\underline{e}$). In turn, if the underlying does not default, then protection buyers pay an insurance premium (state $\bar{e}$). These payments are conditional on the original's counterparty survival. After centrally clearing the contract, transfers T^B and T^S to the CCP are independent of the original counterparty's survival. However, the transfers now include the contributions to cover the CCP's overall default loss in addition to the CDS insurance premium or payment.

$t = 2$ or unable to fulfill their margin requirements at $t = 1$. Defaults imply the loss of clearing membership and the termination of any existing derivative contracts. The CCP can use the margin posted by defaulted sellers to make payments to other sellers or buyers. In contrast, its access to defaulters' balance sheets in excess of the margin account is limited. Specifically, the CCP can recover only a fraction $\rho \in (0, 1)$ of non-collateralized assets from defaulters. The lower ρ, the lower is the CCP's ability to enforce its claim against defaulters' non-collateralized assets. The remaining fraction $1 - \rho$ of uncollateralized assets are lost, creating a deadweight cost of default.

While protection sellers are ex-ante identical, the value of their assets is subject to an idiosyncratic shock at $t = 1$. We denote by $R(\tilde{s}) > 0$ the payoff of high-quality assets conditional on the signal $\tilde{s}$. For simplicity, we assume that after a positive signal $\tilde{s} = \bar{s}$, seller resources $R(\bar{s})$ are sufficient to cover contractual derivative payments for all sellers. There is therefore no default risk after a good signal.[6] In contrast, after a negative signal $\tilde{s} = \underline{s}$, there is counterparty risk. Specifically, we assume that a mass γ of sellers becomes fragile because their assets turn out to be of low quality.[7] This fragility shock reduces the payoff of fragile sellers' assets from $R(\underline{s})$ to $\phi R(\underline{s})$, where $\phi \in [0, 1)$. Assets of "safe" sellers, which are not affected by the fragility shock, remain high-quality and continue to pay $R(\underline{s})$. We assume that $R(\underline{s}) > \pi$, which ensures that safe sellers have sufficient resources to write a frictionless full insurance contract. The realization of the fragility shock is privately known to sellers but cannot be observed by the CCP. It is therefore not feasible to write a contract conditional on seller type.

Contract replacement, outsiders, and loss sharing. When a protection seller defaults on the margin call at $t = 1$, the CCP terminates the contract and proceeds to sell the position of the defaulted clearing member to outsiders, who then replace the defaulted seller as counterparties. Outsiders are not present at $t = 0$ and can therefore be interpreted as clearing members (or other market participants) who do not specialize in trading derivative contracts on $\tilde{e}$. To reflect this, we allow for the possibility that transferring defaulted contracts to outside sellers generates a (deadweight) replacement cost of $C \geq 0$ per contract.[8]

[6] Relaxing this assumption would result in an additional resource constraint after the good signal. This change would not affect our main results.

[7] Therefore, the informativeness of the signal λ reflects the correlation between seller fragility and information about the derivative (e.g., the price of the underlying). If λ is high, sellers are more fragile when the adverse endowment realization $\underline{e}$ becomes more likely. Correlation between counterparty fragility and exposure to this counterparty is commonly referred to as wrong-way risk.

[8] In practice, this cost includes the cost of documentation and accreditation for clearing membership and other replacement costs that arise when transferring defaulted derivative contracts to a new counterparty.

Similar to the initial protection sellers, outside sellers are risk-neutral and have assets that pay $R_O > 0$ at $t = 2$.[9] R_O can be interpreted as reflecting aggregate financial conditions at $t = 1$: the smaller R_O, the lower the CCP's ability to find suitable counterparties to take over defaulted contracts. Because defaults at $t = 1$ occur after a negative signal realization $\underline{s}$, outsiders generally need to be compensated for taking on the expected liability associated with the open positions of a defaulted clearing member. Therefore, even though defaulted counterparties can be replaced at $t = 1$, the CCP incurs a default loss that must be borne by the surviving clearing members.

Equilibrium We consider equilibria that maximize the expected utility of protection buyers, subject to the CCP's budget constraints and sellers' resource and participation constraints. This is a natural outcome when the CCP maximizes the welfare of its members (e.g., because the CCP is member-owned) and protection sellers are competitive.[10]

3 First Best: No Defaults

In this section, we consider the benchmark case in which protection seller resources are abundant. In this case, there are no defaults and the first-best outcome is achieved. This case serves as a benchmark for identifying inefficiencies that arise from limited resources and defaults.

Because there are no defaults, no (costly) margins are used. There is also no need to transfer contracts to outside sellers at $t = 1$. The derivative contract specifies transfers from buyers and sellers to the CCP, T_B and T_S, to maximize buyers' expected utility,

$$\mathbb{E}[u(\tilde{e} - T_B(\tilde{s}, \tilde{e}))], \tag{4}$$

subject to the CCP's budget constraint (3) and the sellers' participation constraint. Given no defaults and no margins, the sellers' expected payoff upon entering the contract at $t = 0$ is $\mathbb{E}[\tilde{\phi} R(\tilde{s})] - \mathbb{E}[T_S(\tilde{s}, \tilde{e})]$, where $\tilde{\phi}$ denotes the fragility shock (equal to $\tilde{\phi} = \phi$ for fragile sellers after observing $\underline{s}$, and $\tilde{\phi} = 1$ otherwise) and $R(\tilde{s})$ is the asset payoff (equal

[9] In an extension, we consider an alternative setting without outside sellers. In this extension, all protection sellers are present from $t = 0$. At $t = 1$, sellers learn whether they are safe, regular, or fragile (i.e., there are now three rather than two seller types at the interim date). Safe sellers then take the role of outsiders and take over the defaulted contracts of fragile sellers.

[10] In our model, the CCP's objective does not conflict with the planner's objective. It is, therefore, not necessary to model the CCP's balance sheet beyond its margin account. Several prior studies analyze conflicts of interest between CCPs and protection buyers (see Huang, 2018; Kuong and Maurin, 2023). It would be an interesting avenue for future research to explore the interaction of such conflicts with contract replacement.

to either $R(\underline{s})$ or $R(\bar{s})$). If sellers do not enter the contract, their expected payoff equals $\mathbb{E}[\tilde{\phi}R(\tilde{s})]$. The protection sellers' participation constraint is therefore

$$\mathbb{E}[T_S(\tilde{s},\tilde{e})] \leq 0. \tag{5}$$

Proposition 1 (First-best contract)**.** *When seller resources are abundant, the optimal contract provides full insurance to protection buyers, is actuarially fair, and does not depend on the signal. No margins are used, and outside sellers remain inactive. The transfers from buyers to the CCP are given by*

$$T_B(\tilde{s},\bar{e}) = 1 - \pi \text{ and } T_B(\tilde{s},\underline{e}) = -\pi, \tag{6}$$

and those from sellers to the CCP by

$$T_S(\tilde{s},\bar{e}) = -(1 - \pi) \text{ and } T_S(\tilde{s},\underline{e}) = \pi. \tag{7}$$

The first-best contract implements full insurance through transfers that effectively move resources from buyers to sellers in the good state $\bar{e}$ and from sellers to buyers in the bad state $\underline{e}$. Transfers are such that buyers are fully insured: buyer consumption is constant across states and equal to the expected endowment π. Note that the CCP is superfluous in this case. The CCP's sole role is to intermediate default-free payments between buyers and sellers. Since all parties can meet their obligations, there is no counterparty risk.

4 Central Clearing Without Replacement

We now consider the case in which protection sellers have limited resources. We first examine contracts with default risk but without replacement of counterparties. We begin by characterizing the transfers and margins after a positive signal at $t = 1$.

Proposition 2 (Full insurance and no margins after a positive signal)**.** *Conditional on a good signal, $\tilde{s} = \bar{s}$, the optimal contract requires no margins and provides full insurance to protection buyers: buyer consumption $\tilde{e} - T_B(\tilde{e},\bar{s})$ is constant across all endowment realizations $\tilde{e}$.*

Conditional on a good signal, resources are abundant and the optimal contract therefore transfers the full endowment risk from buyers to sellers. Consequently, buyers' final consumption does not depend on the endowment realization. Given the absence of defaults after a good signal, it is optimal not to use margins because they would impose unnecessary costs (since $k < 1$). To simplify notation, in what follows we therefore denote by α the margin requirement after a bad signal (i.e., $\alpha := \alpha(\underline{s})$).[11]

After a bad signal, protection sellers may default. We first consider contracts for which defaults do not occur along the equilibrium path.

No defaults. The sellers' participation constraint requires that expected transfers compensate for the opportunity cost of posting margin:

$$0 \leq -\mathbb{P}(\underline{s})\alpha(1-k)R(\underline{s}) - \mathbb{E}[T_S]. \tag{8}$$

To prevent default, the sellers' resource constraints must be satisfied. The available resources consist of the sellers' initial assets that are not posted as margin as well as the assets in their margin account. A margin call requires sellers to post α units of high-quality assets as margin. Safe sellers can post high-quality assets to satisfy the margin call. Fragile sellers, who do not have high-quality assets on their balance sheet, need to post $\frac{\alpha R}{\phi R} = \frac{\alpha}{\phi} > \alpha$ units of low-quality assets to satisfy the margin call. Sellers of type $j \in \{S, F\}$ do not default at $t = 1$ if their available resources are sufficient to cover the margin call:

$$\alpha R(\underline{s}) \leq \phi_j R(\underline{s}) =: \bar{t}_{j,1}, \tag{9}$$

where $\phi_j \in \{\phi, 1\}$ is the fragility shock suffered by sellers of type j.[12]

Sellers of type j do not default at $t = 2$ if their available resources $\bar{t}_{j,2}$ are sufficient to make the contractual transfer $T_S(\underline{s}, \tilde{e})$:

$$T_S(\underline{s}, \tilde{e}) \leq \underbrace{\left(1 - \frac{\alpha}{\phi_j}\right)\phi_j R(\underline{s})}_{\text{Uncollateralized assets}} + \underbrace{\alpha k R(\underline{s})}_{\text{Margin account}} =: \bar{t}_{j,2}. \tag{10}$$

If fragile sellers are sufficiently unconstrained, the optimal contract implements unconditional full insurance at actuarially fair prices:

Proposition 3 (Full insurance without defaults)**.** *The first-best contract described in Proposi-*

[11] While this section focuses on central clearing without replacement, Proposition 2 applies more generally to the cases with and without replacement of fragile sellers.

[12] In our model, defaults are driven by limited resources. In an extension, we show that the results naturally extend to a setting with strategic defaults, in which defaults are driven by default incentives.

tion 1 can be implemented if and only if the fragility shock is not too severe:

$$\phi R(\underline{s}) \geq \pi. \tag{11}$$

In the case of Proposition 3, the resources of fragile sellers are sufficiently large to ensure that they do not default. Therefore, there is sufficient risk-sharing capacity to implement the first-best full insurance contract. In the following, we focus on situations in which fragile sellers are constrained, such that the first-best contract cannot be implemented.

Assumption 1. *Fragile sellers' resources are sufficiently limited such that the first-best contract is not feasible: $\phi R(\underline{s}) < \pi$.*

When the first-best contract is not feasible, there are two options. The first is to write a partial insurance contract that prevents default. The second is to write a full insurance contract that involves default in some states. We now consider each in turn.

Partial insurance without defaults. When the transfers under the first-best contract exceed fragile sellers' resources, one option is to prevent default by reducing the insurance payment made by sellers. To ease notation, in what follows we denote by $u(\tilde{s}, \tilde{e}) = u(\tilde{e} - T_B(\tilde{s}, \tilde{e}))$ the buyer's utility in state $(\tilde{s}, \tilde{e})$.

Proposition 4 (Partial insurance without defaults)**.** *Under Assumption (1), the optimal contract when fragile sellers do not default provides partial insurance to buyers, with full insurance except in the worst state $(\underline{s}, \underline{e})$:*

$$u(\underline{s}, \underline{e}) < u(\underline{s}, \overline{e}) = u(\overline{s}, \underline{e}) = u(\overline{s}, \overline{e}). \tag{12}$$

Fragile sellers' resource constraint (10) is binding in state $(\underline{s}, \underline{e})$ at $t = 2$. No margin is used, and the contract is actuarially fairly priced.

The contract in Proposition 4 prevents fragile sellers from defaulting by reducing the transfers sellers are required to make. This reduction in seller transfers implies that the contract provides only partial insurance to buyers. Margins are not used because, through their opportunity cost, they reduce the resources available to sellers who do not default in equilibrium.

Full insurance with defaults. Instead of preventing fragile sellers from defaulting, it can be optimal to allow fragile sellers to default. Due to the pooling of contracts within the CCP, safe sellers step in to make the contractual payments of defaulted fragile sellers. This loss-sharing arrangement is priced into the contract ex ante via the sellers' participation constraint. In addition, the CCP can use assets seized from defaulting sellers to make contractual payments. While the CCP can seize all margin posted by defaulted sellers, due to enforcement frictions it can seize only a fraction ρ of defaulting sellers' uncollateralized assets. Therefore, when fragile sellers default in state $(\underline{s}, \underline{e})$ at $t = 2$, the CCP's budget constraint in this state is

$$T_B(\underline{s}, \underline{e}) + \underbrace{(1-\gamma)T_S(\underline{s}, \underline{e})}_{\text{Surviving (safe) sellers}} + \underbrace{\gamma \left[\alpha k R(\underline{s}) + \rho \left(1 - \frac{\alpha}{\phi}\right) \phi R(\underline{s})\right]}_{\text{Defaulting (fragile) sellers}} = 0. \tag{13}$$

In all other states, the CCP's budget constraint is given by Equation (3), as before.

When fragile sellers default, they lose all their assets. Therefore, the sellers' participation constraint becomes

$$\begin{aligned}
\mathbb{E}[\tilde{\phi} R(\tilde{s})] \leq \mathbb{P}(\bar{s}) & \underbrace{(R(\bar{s}) - \mathbb{E}[T_S \mid \bar{s}])}_{\text{No defaults after a positive signal}} \\
& + (1-\gamma)\mathbb{P}(\underline{s}) \underbrace{(R(\underline{s}) - \mathbb{E}[T_S \mid \underline{s}] - (1-k)\alpha R(\underline{s}))}_{\text{Safe sellers: no defaults}} \\
& + \gamma \mathbb{P}(\underline{s}) \underbrace{(\mathbb{P}(\bar{e} \mid \underline{s})(\phi R(\underline{s}) - T_S(\underline{s}, \bar{e}) - (1-k)\alpha R(\underline{s})) + \mathbb{P}(\underline{e} \mid \underline{s}) \cdot 0)}_{\text{Fragile sellers: defaults in state } (\underline{s}, \underline{e})},
\end{aligned} \tag{14}$$

which, after rearranging, yields

$$0 \leq -\mathbb{E}[T_S] - \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) + \gamma \mathbb{P}(\underline{s}, \underline{e}) \left(T_S(\underline{s}, \underline{e}) + ((1-k)\alpha - \phi) R(\underline{s})\right). \tag{15}$$

Equation (15) shows that sellers require compensation for the opportunity cost of posting margin, offset by the benefit of defaulting. This benefit equals the difference between the payment by safe sellers, $T_S + (1-k)\alpha R(\underline{s})$, and the payment by defaulted sellers, $\phi R(\underline{s})$. The resource constraint at $t = 1$ remains the same as before and is given by Equation (9).

Proposition 5 (Full insurance with defaults at $t = 2$)**.** *Under Assumption (1) and if the mass of fragile sellers is sufficiently small,*

$$\gamma \leq \frac{R(\underline{s}) - \pi}{R(\underline{s})(1 - (\mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \rho)\phi)} =: \overline{\gamma}^{NR}, \tag{16}$$

the optimal contract with defaults of fragile sellers in state $(\underline{s}, \underline{e})$ provides full insurance to buyers.

If

$$\mathbb{P}(\underline{s},\underline{e})\gamma(1-\rho) < \mathbb{P}(\underline{s})(1-k), \tag{17}$$

then margins are not used and the markup paid by buyers compensates for the deadweight cost of defaults: $\mathbb{E}[T_B] = m = \gamma\mathbb{P}(\underline{s},\underline{e})(1-\rho)\phi R(\underline{s})$. *No sellers default at* $t = 1$.

Condition (17) captures the trade-off in using margins. On one hand, margins reduce the deadweight cost of default, which arises because the fraction $1-\rho$ of non-collateralized assets is lost in default. A larger margin requirement reduces this efficiency loss in state $(\underline{s},\underline{e})$ for the mass γ of fragile sellers. However, posting margins imposes an opportunity cost $1-k$ on all sellers after a negative signal. When Condition (17) holds, this opportunity cost effect dominates and margins are not used.

Proposition 5 shows that, even if fragile sellers default at $t = 2$, safe sellers can still provide full insurance to buyers. This is possible if safe sellers have sufficient resources, which is the case if the mass of defaulting sellers is not too large (Condition (16)). If the mass of defaulting sellers is too large, safe sellers' balance sheets are not sufficient to absorb the default losses in state $(\underline{s},\underline{e})$. If this were the case, the contract in Proposition 5 would lead to contagious defaults: Defaults by fragile sellers would trigger a second round of defaults by safe sellers, as the loss sharing required to implement full insurance would exceed safe sellers' resources.

Partial insurance with defaults. In the following, we assume that the mass of fragile sellers is large, such that safe sellers cannot absorb all losses from defaults of fragile sellers under the full-insurance contract. In this situation, writing the full-insurance contract would lead to cascading defaults: the defaults of fragile sellers would overburden the safe sellers, causing them to default as well. The CCP would fail.

Assumption 2. *Assume that* $\gamma > \overline{\gamma}^{NR}$, *such that full insurance is not resource compatible without replacement of fragile sellers.*

Proposition 6 (Partial insurance with defaults at $t = 2$)**.** *Under Assumption (2), the optimal contract when fragile sellers default in state* $(\underline{s},\underline{e})$ *provides partial insurance to buyers. The contract is not actuarially fair but entails a markup paid by buyers to sellers of*

$$m = \underbrace{\mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s})}_{\text{Opportunity cost of margins}} + \underbrace{\gamma\mathbb{P}(\underline{s},\underline{e})\left(\phi - \alpha - \rho\left(1-\frac{\alpha}{\phi}\right)\phi\right)R(\underline{s})}_{\text{Net deadweight loss from defaults}}. \tag{18}$$

If margins are used as part of the optimal contract, the margin call is determined by the first-order condition

$$u'(\underline{s},\overline{e})\mathbb{P}(\underline{s})(1-k) = u'(\underline{s},\overline{e})\mathbb{P}(\underline{s},\underline{e})\gamma(1-\rho) \\ + (u'(\underline{s},\underline{e}) - u'(\underline{s},\overline{e}))\mathbb{P}(\underline{s},\underline{e})(\gamma(k-\rho) - (1-\gamma)(1-k)). \tag{19}$$

Fragile sellers do not default at $t = 1$ if and only if $\alpha \leq \phi$.

The optimal margin requirement determined by Equation (19) trades off the opportunity cost of posting margin (left-hand side) with the benefits of margins (right-hand side). Margins are potentially beneficial for two reasons. First, they reduce the deadweight cost of defaults by ring-fencing assets early on. Specifically, a one-unit increase in α reduces the deadweight-cost component in the markup, $\phi - \alpha - \rho(1 - \alpha/\phi)\phi$, by $1 - \rho$. Second, margins can increase the CCP's available resources after a negative signal. On the one hand, a larger margin requirement increases the recoverable resources from fragile sellers by $\gamma(k-\rho)$ per unit, which is positive if the return on margins exceeds the CCP's recovery rate on defaulters' assets, $k > \rho$. On the other hand, a larger margin reduces the available resources provided by safe sellers by $(1-\gamma)(1-k)$ (see Equation (10)). Margins increase resources in state $(\underline{s},\underline{e})$ if the first effect dominates. This is the case if there are sufficiently many fragile sellers, $\gamma > \frac{1-k}{1-\rho}$.

The following proposition compares the two partial insurance contracts analyzed above. These contracts differ in whether fragile sellers default at $t = 2$. We show that it is optimal to let fragile sellers default if they are very fragile (i.e., when ϕ is low, making their assets severely impaired).

Proposition 7 (Partial insurance with and without defaults)**.** *The optimal contract with defaults of fragile sellers at $t = 2$ provides strictly more expected utility to buyers than that without defaults when fragile sellers are sufficiently impaired, $\phi < \phi^*$, where $\phi^* > 0$.*

5 Central Clearing with Replacement

This section presents the main results of our analysis. We characterize contracts in which the CCP replaces fragile sellers at the interim date $t = 1$. Since the CCP cannot distinguish between safe and fragile sellers, it must induce sellers to reveal their type. This is accomplished through a margin call at $t = 1$ that causes fragile sellers (but not safe sellers) to default on their contracts. The defaulting sellers are then replaced by outsiders, who take over the defaulted contracts for a fee.

Analogous to defaults at $t = 2$, the CCP holds a claim on the assets of sellers that default at $t = 1$. This claim equals the expected transfer $\mathbb{E}[T_S \mid \underline{s}]$ (i.e., the mark-to-market value of the contract at $t = 1$). We assume that, to satisfy its claim, the CCP can seize a fraction ρ of a defaulted seller's uncollateralized assets, leading to a payoff of $\min\{\rho\phi R(\underline{s}), \mathbb{E}[T_S \mid \underline{s}]\}$. In what follows, we assume that ϕ is sufficiently low such that $\rho\phi R(\underline{s}) \leq \mathbb{E}[T_S \mid \underline{s}]$, which implies that the CCP incurs a default loss. The remaining fraction $1 - \rho$ of the defaulting seller's assets is lost, resulting in a deadweight default cost of $(1 - \rho)\phi R(\underline{s})$ at $t = 1$.

Contracts of defaulted sellers are transferred to outsiders. This means that outsiders take over the contractual payment obligation T_S in return for a fee of p. In addition, counterparty replacement generates an exogenous replacement cost C.

Including net transfers made by outsiders and replacement costs, the CCP's budget constraint following a bad interim signal and the default and replacement of fragile sellers becomes

$$T_B + (1-\gamma)T_S + \underbrace{\gamma\rho\phi R(\underline{s})}_{\text{Resources from defaulters}} + \underbrace{\gamma T_S - \gamma(p + C)}_{\text{Additional resources from replacement}} = 0. \tag{20}$$

Outsiders are willing to enter the contract if the payment they receive exceeds the expected transfer,

$$p \geq \mathbb{E}[T_S \mid \underline{s}]. \tag{21}$$

In equilibrium, this participation constraint binds, so that outsiders receive a fee of $p = \mathbb{E}[T_S \mid \underline{s}]$.

Protection sellers' ex-ante participation constraint takes into account the possibility of default at $t = 1$, which means that, with probability $\mathbb{P}(\underline{s})\gamma$, no transfers are made but sellers' assets are lost:

$$0 \leq -\mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma\left(\mathbb{E}[T_S \mid \underline{s}] - \phi R(\underline{s})\right) - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s}). \tag{22}$$

This constraint shows that, in addition to expected transfer payments, sellers are compensated for losing their assets upon default at $t = 1$, $\mathbb{P}(\underline{s})\gamma\phi R(\underline{s})$, and for the expected opportunity cost of posting margin, $\mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s})$.

Using the CCP's budget constraint and the participation constraints for protection sellers and outsiders, we now derive the transfers for contracts that fully insure the buyers' endowment risk.

Proposition 8 (Full insurance transfers with replacement of fragile counterparties)**.** *The transfers of a full insurance contract with default by fragile sellers at $t = 1$ are given by*

$$T_S(\bar{s}, \bar{e}) = -(1-\pi) - m \tag{23}$$

$$T_S(\bar{s}, \underline{e}) = \pi - m \tag{24}$$

$$T_S(\underline{s}, \bar{e}) = -(1-\pi) - m + L^{CCP} \tag{25}$$

$$T_S(\underline{s}, \underline{e}) = \pi - m + L^{CCP}. \tag{26}$$

The contract is not actuarially fair but entails a markup paid by buyers to sellers of

$$m := \mathbb{E}[T_B] = \mathbb{P}(\underline{s})(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\alpha R(\underline{s})), \tag{27}$$

and the CCP's default loss after the bad signal is given by

$$L^{CCP} := \gamma(p + C - \rho\phi R(\underline{s})) = \gamma \frac{\pi - \underline{\pi} - m + C - \rho\phi R(\underline{s})}{1-\gamma}. \tag{28}$$

Proposition 8 shows that, under full insurance with interim default and replacement, the contractual transfers for sellers can be decomposed into three components. The first component is a frictionless insurance contract against the endowment risk $\tilde{e}$, where sellers pay π or receive $1-\pi$ depending on the endowment realization. The second component is a constant markup m that buyers pay to sellers. The markup compensates sellers for the deadweight replacement cost C, the deadweight default cost $(1-\rho)\phi R(\underline{s})$ per defaulted seller, and the opportunity cost of posting margin $(1-k)\alpha R(\underline{s})$ per surviving seller. The third component is a loss-sharing contribution that sellers make to cover the CCP's default loss L^{CCP} after a bad signal realization.

The CCP's default loss equals the cost of replacing defaulted sellers, which comprises the payment to outsiders p and the deadweight replacement cost C, net of the assets recovered from defaulted sellers by the CCP, $\rho\phi R(\underline{s})$. Equation (28) shows that the default loss is larger when the signal is more informative (i.e., when $\pi - \underline{\pi}$ is large). The net transfers made by outsiders that take over defaulted contracts are equal to $T_S - p$, which corresponds to a frictionless insurance contract against $\tilde{e}$ conditional on a bad signal, with a net transfer of $\underline{\pi}$ or $-(1-\underline{\pi})$. Therefore, the more informative the interim signal, the less additional risk sharing is provided by outsiders. Equation (28) also shows that the default loss is smaller when the CCP can recover more assets from defaulted sellers (i.e., ρ is larger, assuming that fragile sellers are left with some assets after the fragility shock, $\phi > 0$).

The full insurance contract described above requires fragile sellers to default at $t = 1$, since replacement is otherwise impossible. The interim margin call is crucial for achieving this: it induces fragile (but not safe) sellers to default by setting margin requirements that only fragile sellers cannot meet, enabling the CCP to replace fragile counterparties. The margin therefore serves as a canary in the coal mine that reveals hidden fragility among clearing members before contract maturity when replacement is still possible.

Fragile sellers default at $t = 1$ if the margin requirement exceeds their resources, as described by Equation (9).[13] Therefore, fragile sellers default at $t = 1$ when the fraction of assets to be posted as margin exceeds the fragility shock:

$$\alpha > \phi. \tag{29}$$

Because posting margin entails an opportunity cost, it is optimal to set the lowest margin that induces the sellers to default at $t = 1$. Therefore, it is optimal to set the margin to just exceed ϕ:

$$\alpha^* := \inf\{\alpha : \alpha > \phi\}. \tag{30}$$

Note that the role of margins differs depending on whether defaults occur at $t = 1$ or $t = 2$. As shown in the previous section, when defaults occur at $t = 2$, margins reduce the deadweight cost of default by ring-fencing assets for the CCP. However, this mechanism does not apply when defaults occur at $t = 1$, since defaults occur before margin is posted. Instead, the role of margins at $t = 1$ is to induce fragile sellers to default, thereby enabling counterparty replacement.

The optimal transfers are chosen to provide full insurance to buyers, anticipating the replacement of fragile sellers via a margin call of α^* at $t = 1$.

Proposition 9 (Full insurance with replacement of fragile counterparties)**.** *The optimal contract with default and replacement of fragile sellers at $t = 1$ provides full insurance if*

$$\gamma \leq \overline{\gamma}^R. \tag{31}$$

It is $\gamma^R \in (0, 1)$ if $\phi < \phi^R$, with $\phi^R > 0$.

[13] Fragile sellers might want to prevent posting margins by selling their positions to outsiders after a negative signal but before the margin call at $t = 1$. This possibility can be ruled out by assuming that fragile sellers' assets are not sufficient to compensate outsiders, namely that $\phi R(\underline{s}) < \mathbb{E}[T_S \mid \underline{s}]$.

The optimal margin, which incentivizes fragile sellers to default at $t = 1$, is given by

$$\alpha = \alpha^* = \phi. \tag{32}$$

The outsiders' resources are sufficient if, and only if, $R_O \geq \underline{\pi}$. This condition is satisfied if $R_O \geq \pi$.

Proposition 9 demonstrates that when the economy contains a sufficiently small proportion of sellers ($\gamma \leq \overline{\gamma}^R$), the full-insurance contract with replacement and defaults at $t = 1$ is optimal and resource-compatible for sellers.

Because the contract's present value changes following the interim signal, defaults generate losses for the CCP, which are absorbed by safe sellers. If the mass of fragile sellers were too large ($\gamma > \overline{\gamma}^R$), safe sellers would lack the capacity to absorb these default losses, rendering the full-insurance contract resource-incompatible.

Outsiders can fulfill their contractual payments if their resources exceed the insurance payment required for full insurance conditional on the negative signal, $\underline{\pi}$. Since $\underline{\pi} < \pi$, outsiders need fewer resources than safe sellers would require. Consequently, it suffices for outsiders to possess at least the same level of resources as safe sellers.

Proposition 9 also establishes a condition ensuring that the threshold on γ is economically meaningful. This requires a sufficiently severe fragility shock (small ϕ). Under this condition, safe sellers can provide insurance against seller's endowment risk and absorb the CCP's default losses even when only a negligible fraction of sellers defaults. In the absence of this condition, two degenerate cases emerge: replacement becomes either universally infeasible across all $\gamma \in (0, 1)$ due to insufficient resources among safe sellers, or universally feasible due to high asset recovery rates (high ϕ).

The optimal margin α^* equals the fragility shock ϕ. This highlights the role of margins as *canaries in the coal mine*: the smaller the difference between fragile and safe sellers, the larger the optimal margin required to distinguish between seller types.

We now investigate the conditions under which the optimal contract with replacement (characterized by Propositions 8 and 9) provides greater utility than the contract without early replacement of fragile counterparties. Although replacement improves risk sharing, it is not unambiguously efficient. First, the risk sharing provided by outsiders is constrained by the Hirshleifer effect: outsiders can provide insurance only after information about the endowment risk has been revealed. Second, replacement of fragile counterparties is costly: it requires safe sellers to post margins and triggers default and replacement costs. Consequently, a contract *without* early default and replacement may provide greater utility to buyers.

To assess the efficiency of replacement, we assume that the full-insurance contract with replacement is feasible on the interval $(0, \overline{\gamma}^R]$:

Assumption 3. *Assume that $\phi < \phi^R$ and $\underline{\pi} \leq R_O$.*

We are now in a position to compare the optimal contracts with and without replacement.

Proposition 10 (Efficiency of Counterparty Replacement)**.**

(1) If $C < \hat{C}$, the contract with counterparty replacement enables full insurance for a larger fraction of fragile sellers ($\overline{\gamma}^{NR} < \overline{\gamma}^R$), where $\hat{C} > 0$ when $\phi < \hat{\phi}$.

(2) If $\overline{\gamma}^{NR} < \gamma \leq \overline{\gamma}^R$ and $m^R - m^{NR} \leq g^{NR}$, counterparty replacement is efficient, where m^R and m^{NR} denote the markups with and without replacement, respectively, and $g^{NR} > 0$ denotes the consumption risk premium under the optimal contract without replacement.

The first part of Proposition 10 shows that replacement increases the CCP's risk-sharing capacity when deadweight replacement costs C are sufficiently small. Specifically, we compare the maximum proportion of fragile sellers γ for which full insurance can be achieved without replacement ($\overline{\gamma}^{NR}$) with the corresponding threshold with replacement ($\overline{\gamma}^R$). When replacement costs C are small, replacement allows the CCP to implement full risk sharing for a larger proportion of fragile counterparties: $\overline{\gamma}^{NR} < \overline{\gamma}^R$. This result highlights the key benefit of replacement: transferring contracts from fragile sellers to outsiders generates additional risk sharing and thereby reduces the loss-sharing burden on safe sellers.

The second part of Proposition 10 shows that when the mass of fragile sellers γ lies in the interval $(\overline{\gamma}^{NR}, \overline{\gamma}^R]$ and the markup with replacement is sufficiently small, replacement of fragile counterparties is efficient. In this case, the optimal contract with replacement implements greater risk sharing (full insurance) than the contract without replacement (partial insurance). The markup with replacement m^R is composed of deadweight replacement costs C, default costs $(1-\rho)\phi R(\underline{s})$, and opportunity costs of margins $(1-k)\alpha R(\underline{s})$ (see Proposition 8). In contrast, without replacement, the markup m^{NR} is either equal to zero because no seller defaults or it is composed of default costs and opportunity costs of margins. Therefore, the markup without replacement may be lower especially when no sellers default or margin requirements are low. Nonetheless, risk-averse buyers are willing to pay a strictly positive premium $g^{NR} > 0$ to eliminate the consumption risk that is present in the absence of replacement. Thus, replacement is efficient when the markup differential $m^R - m^{NR}$ is smaller than g^{NR}, as in this case the risk-sharing benefits outweigh the additional costs. This condition is trivially satisfied when the markup differential is negative.[14]

In contrast, when there are few fragile sellers ($\gamma \leq \overline{\gamma}^{NR}$), replacement is inefficient. In this case, the optimal contract without replacement already provides full insurance, making replacement unnecessary since it would only add costs without improving risk sharing. Therefore, it is efficient to forgo replacement and rely on ex-post loss sharing at $t = 2$.

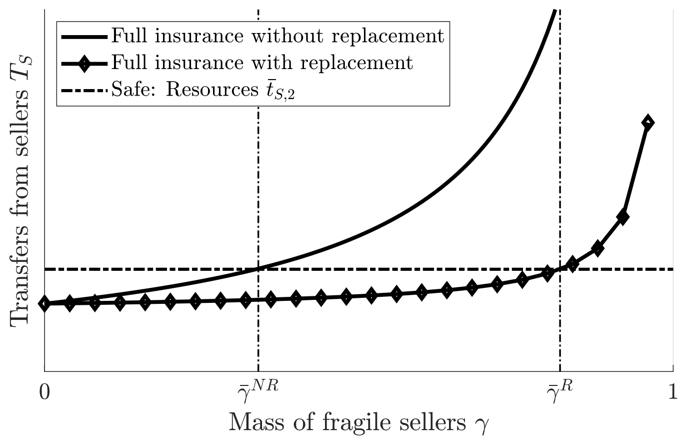

Figure 1: Full Insurance Pecking Order.

The figure plots the transfers $T_S(\underline{s}, \underline{e})$ made by sellers to the CCP after a negative signal in state $(\underline{s}, \underline{e})$ on contracts that provide full insurance (y-axis) against the ex-ante probability γ of sellers to become fragile at $t = 1$ (x-axis). The dashed line represents the maximum transfer $\bar{t}_{S,2}$ that safe sellers are able to make. The solid line represents transfers without replacement. It exceeds $\bar{t}_{S,2}$ for $\gamma > \overline{\gamma}^{NR}$, in which case the contract is not feasible. The line with diamonds represents transfers with replacement. It exceeds $\bar{t}_{S,2}$ for $\gamma > \overline{\gamma}^{R}$, in which case the contract is not feasible. In the interval $(0, \overline{\gamma}^{NR}]$ full insurance without margins is feasible without replacement and, thus, replacement is not efficient. In the interval $(\overline{\gamma}^{NR}, \overline{\gamma}^{R}]$ full insurance without margins is feasible only with replacement.

Figure 1 illustrates the resulting pecking order of full-insurance contracts. On the interval $[0, \overline{\gamma}^{NR}]$, it is optimal to rely on ex-post loss sharing among surviving clearing members to implement full insurance. On the interval $(\overline{\gamma}^{NR}, \overline{\gamma}^{R}]$, full insurance with replacement is efficient provided that the risk-sharing benefit exceeds the required contract markup. When $\gamma > \overline{\gamma}^{R}$, safe sellers lack sufficient resources to implement full insurance with replacement and absorb the resulting default losses, so risk sharing must be reduced to prevent safe sellers from defaulting at $t = 2$.

[14] If deadweight replacement costs C are small and the CCP's recovery rate ρ is high, the markup with replacement may be smaller than without replacement. This occurs because only safe sellers post margin, reducing the opportunity costs of margin.

We illustrate this pecking order of contracts in the following example, focusing on the case of very fragile sellers.

Example 1 (Very fragile sellers)**.** *We consider the case where $\phi = 0$ (i.e., fragile sellers lose all their assets due to the fragility shock) and $C = 0$ (i.e., there are no deadweight costs of replacement). In this case,*

$$\overline{\gamma}^R = \frac{R(\underline{s}) - \pi}{R(\underline{s}) - \underline{\pi}} \tag{33}$$

$$\text{and } \overline{\gamma}^{NR} = \frac{R(\underline{s}) - \pi}{R(\underline{s})}. \tag{34}$$

If $\overline{\gamma}^{NR} < \gamma \leq \overline{\gamma}^R$, the optimal contract with replacement and defaults at $t = 1$ is resource-compatible and dominates the optimal contract without replacement.

Example 1 illustrates that replacement at $t = 1$ is efficient when there are sufficiently many fragile sellers (high γ) and both fragile sellers' default costs ϕ and deadweight replacement costs C are sufficiently small. In this case, replacement enables the CCP to provide full insurance to protection buyers with a modest margin requirement and, therefore, a small markup. The optimal contract without replacement provides only partial insurance to buyers and is dominated by the contract with replacement.

The example also highlights that the maximum default risk for which full insurance is feasible, $\overline{\gamma}^{NR}$, is increasing in $\underline{\pi}$, implying that a more informative signal at $t = 1$ (which decreases $\underline{\pi}$) reduces the benefits of replacement by limiting the scope for risk sharing. To formalize this insight, the following proposition examines more generally how signal informativeness affects the value of counterparty replacement.

Proposition 11 (Signal informativeness)**.** *The additional risk-sharing capacity achieved by replacement, $\overline{\gamma}^R - \overline{\gamma}^{NR}$, is decreasing in signal informativeness λ.*

Signal informativeness determines the strength of the Hirshleifer (1971) effect: the more information revealed about $\tilde{e}$, the more sensitive the value of the derivative contract becomes to the signal. This implies that $\pi - \underline{\pi}$ is larger, leading to more limited risk sharing with outsiders. This effect is visible in the CCP's default loss upon a bad signal, which increases with signal informativeness and therefore requires a larger contribution from original sellers to loss sharing (see Proposition 8). In Proposition 11, we show that the Hirshleifer (1971) effect impairs the CCP's risk-sharing capacity: the less risk can be shared with outsiders, the smaller the maximum incentive-compatible proportion of fragile sellers $\overline{\gamma}^R$ that ensures incentive compatibility of replacement. Therefore, counterparty replacement is most useful as an early intervention tool (i.e., before a full-blown crisis) and in situations in which the fragility shock that hits counterparties is imperfectly correlated with the value of their derivative positions (see Section 6.2 for a more detailed discussion).

6 Discussion and Policy Implications

CCPs manage counterparty default risk using three tools: margins, replacement, and loss sharing. In the following, we discuss the implications of our results for each of these.

6.1 Margin Setting by CCPs

Our model highlights a novel role of margin requirements as canaries in the coal mine. It is useful to briefly reflect on the key differences and similarities between the two economic roles of margins: as ring-fenced collateral versus as an early intervention mechanism to replace counterparties.[15]

Margins as ring-fencing. Conventional wisdom views margins as a tool to increase the resources available to CCPs in the event that some clearing members default. For example, Hull (2012, p. 30) describes the role of margins as follows: "The whole purpose of the margining system is to ensure that funds are available to pay traders when they make a profit." According to this view, margins enable the CCP to seamlessly seize collateral from defaulted counterparties, thereby increasing the resources available to pay those who are owed payments.

Note, however, that seizing collateral improves outcomes only in cases where uncollateralized loss-sharing arrangements are insufficient to make counterparties whole. As long as safe clearing members can provide sufficient resources to the CCP, loss sharing among clearing members ensures that contractual payments are made, even when fragile clearing members default. This is the case as long as there are not too many fragile clearing members ($\gamma \leq \overline{\gamma}^{NR}$ in Figure 1). In this case, margins are not needed.

In contrast, when there are sufficiently many fragile clearing members ($\gamma > \overline{\gamma}^{NR}$ in Figure 1), default losses exceed the surviving clearing members' resources and it becomes beneficial to use margins. By ring-fencing some of the defaulters' assets, margins increase

[15] The literature has discussed a third role of margins: reducing incentives to default. As shown by Biais et al. (2016), this "incentive role" of margins emerges when clearing members are subject to a moral hazard problem in managing their default risk. Because of the absence of moral hazard in our framework, this incentive role does not arise in our model.

the resources the CCP can seize from defaulters and reduce the deadweight costs resulting from defaults.

Margins as canaries. Alternatively, the CCP can increase resources by replacing fragile clearing members with less fragile clearing members before contractual payments are due. Clearing membership requirements partly achieve this goal by ensuring that clearing members exhibit low *observable* default risk, such as a high credit rating. However, changes in credit quality are usually reflected in ratings only with a significant time lag and, at high frequency, remain *unobservable* to the CCP.

Employing margins as canaries in the coal mine overcomes this obstacle. A sufficiently large margin call identifies fragile clearing members by forcing them to default. Because these defaults occur before contract maturity, the CCP can replace fragile clearing members with new counterparties, thereby increasing the CCP's resources. CCPs acknowledge this value of using margins as canaries: "While sufficient time should be granted to avoid unnecessary liquidity strains for market participants to meet the ITD [intraday] margin calls, allowing excessive time for them to arrange funding may impede the timely identification of a default. This is particularly true when a CM is experiencing solvency issues and struggling to meet collateral requirements, potentially leading to a default."[16]

As with the ring-fencing role of margins, using a margin call as a canary is not required when loss sharing among safe clearing members generates sufficient resources for the CCP to honor contractual payments ($\gamma \leq \overline{\gamma}^{NR}$ in Figure 1). In contrast, when the CCP's resources are constrained, it can be efficient to use canary margins.

To induce defaults, margin requirements must be sufficiently large. This implies that canary margins generally exceed ring-fencing margins, resulting in higher opportunity costs. In addition, using margins as canaries leads to deadweight costs resulting from the defaults of fragile clearing members. Therefore, the total costs of canary margins can exceed those of margins used to ring-fence collateral. However, these additional costs can be outweighed by the benefits of replacement, namely that outside sellers provide additional risk-sharing capacity. This benefit is particularly large when, at the time of the fragility shock, relatively little information about the underlying risk has been revealed (low λ), as outsiders can only insure risks that have not yet materialized.

Distinguishing ring-fencing and canary margins. In general, ring-fencing margins and canary margins differ in two key dimensions: canary margins are larger and are explicitly

[16]CCP Global, Re "CPMI-IOSCO Report on Streamlining variation margin in centrally cleared markets-examples of effective practices", 13 May, 2024

designed to trigger defaults. While clearing member defaults are infrequent events, recent defaults have indeed been associated with margin calls rather than payments at contract maturity.[17]

The relative effectiveness of these margin types depends on the degree of counterparty fragility. When clearing members retain substantial balance sheet capacity, ring-fencing margins efficiently secure collateral without triggering defaults. However, when balance sheets become severely impaired—precisely when ring-fencing becomes ineffective due to insufficient collateralizable resources—canary margins become the optimal tool. Even relatively small margin requirements will induce fragile counterparties to default, enabling their replacement with better-capitalized outsiders. Thus, the CCP's optimal margin strategy shifts from ring-fencing to screening as counterparty fragility increases.

Initial and variation margin. Practitioners distinguish between two types of margin requirements. Initial margin is based on the risk (e.g., volatility or Value-at-Risk) of the position as well as the default risk of the counterparty. Initial margin is posted at initial contracting and can be updated throughout the life of the contract. Variation margin is based on the market value of the contract (often zero at inception) and is usually adjusted on a daily basis.

In the context of our model, variation margin at $t = 1$ equals the expected liability arising from the original (bilateral) derivative contract, which equals the expected total net transfers less the loss-sharing contributions:

$$VM_1 = \mathbb{E}[T_S \mid \underline{s}] - L^{CCP} = \pi - \underline{\pi} - m. \tag{35}$$

Under the contract with replacement, the canary margin equals ϕ.[18] Therefore, when fragile counterparties are sufficiently impaired ($\phi \leq \pi - \underline{\pi} - m$), variation margin alone is sufficient to trigger their default at $t = 1$. Otherwise, an additional initial margin is needed to induce default and replacement of fragile counterparties. Combining both cases, the total initial margin demanded at $t = 1$ to trigger replacement of fragile counterparties is given by

$$IM_1 = \max\{\phi - (\pi - \underline{\pi} - m), 0\}. \tag{36}$$

This initial margin can be interpreted as a credit risk add-on that is required only after some sellers have suffered a fragility shock. However, unlike the conventional ring-

[17] See Section 6.2 for an example.

[18] More precisely, the margin is arbitrarily close to ϕ, see Equation (30).

fencing role of margins, this margin in the contract described in Section 5 functions as a screening device to induce default and enable replacement of fragile sellers with healthier counterparties.

Procyclicality. Policymakers are concerned about margin procyclicality, namely that larger margins during turbulent times may trigger defaults when other market participants are also more fragile.[19] Our model suggests a more nuanced perspective on this concern. First, higher margins can be a means for CCPs to deal with fragility of clearing members by enabling replacement. However, an important precondition is that alternative counterparties ("outsiders") are sufficiently unconstrained ($R_O \geq \underline{\pi}$). Thus, the effects of procyclicality depend on the heterogeneity of fragility across market participants. Procyclical margins can be detrimental if all market participants are fragile, but beneficial when some market participants remain safe.

Second, the effectiveness of counterparty replacement depends on how correlated a fragility shock to clearing members is with the risks of the contracts cleared by the CCP. Replacement counterparties are less willing to insure endowment risk when more information about this risk has been revealed (i.e., λ is high). This is likely a particular concern during crisis times (e.g., credit default swaps during the 2008 global financial crisis). Accordingly, procyclical margins are more likely to be beneficial as a tool for early intervention rather than in the midst of a crisis.

Overall, our analysis therefore suggests that optimal margin-setting policies should account for both the heterogeneity of market participants and the information environment when evaluating procyclical adjustments.

6.2 Counterparty Replacement

A key result of our analysis is that default and subsequent replacement of counterparties can improve risk sharing and prevent cascading defaults of clearing members. To demonstrate the practical relevance of this insight, we now briefly discuss how counterparty replacement is implemented in practice.

CCP rules. CCP rules prescribe the conditions that trigger the default of a clearing member and the procedures to be followed in the event of defaults. In particular, if a clearing member is (or appears to be) unable to meet its obligations to the CCP (such as posting

[19] For example, see the final report on the "Review of the RTS with respect to the procyclicality of CCP margin" of the European Securities and Markets Authority (2023).

margin), the CCP may terminate the clearing membership (i.e., put the clearing member into default). In the event of such a default, the CCP can take several steps to neutralize the defaulter's positions. One of these is to enter offsetting positions or to transfer the defaulter's positions to an alternative counterparty, which replaces the defaulted counterparty and effectively takes the role of the "outsiders" in our model.

This replacement mechanism occurs regularly in practice. For example, in 2018, the Swedish clearinghouse Nasdaq Clearing AB declared the power trader Einar Aas in default after he failed to post required margins. The CCP then conducted an auction to transfer the trader's positions to a new counterparty–essentially implementing the counterparty replacement mechanism that our model shows can improve overall risk sharing.[20]

The timing of counterparty replacement. Our model shows that the benefits of replacement crucially depend on the timing of defaults and replacement. In our framework, this corresponds to interpreting the informativeness of the signal λ as capturing the proximity to contract maturity. The earlier fragile counterparties default, the larger the remaining scope for risk sharing. In the extreme case where fragile counterparties default at contract maturity (equivalent to $\lambda = 1$), replacement is not useful because outsiders would be willing to enter the contract only if they were paid exactly the cash flow that is due to be paid by them. Instead, when replacement occurs before contract maturity, the final payment remains risky and thus outsiders are willing to provide insurance against this risk.

Moreover, replacement is more beneficial when relatively unconstrained outsiders are available to take on the positions of defaulted clearing members ($R_O \geq \underline{\pi}$). Only then is it possible for outsiders to absorb the positions of defaulters without increasing their own default risk. Thus, replacement is most useful when defaults by fragile counterparties are idiosyncratic and uncorrelated with the constraints of other market participants. If outsiders are constrained exactly when fragile sellers default, replacement is less likely to improve outcomes. This suggests that the effectiveness of canary margins depends critically on market-wide stress conditions.

[20]See https://view.news.eu.nasdaq.com/view?id=b342623f1e259ec1892c5e1763ad7de15&lang=en, https://www.finanstilsynet.no/en/news-archive/news/2018/nasdaq-clearing-ab-default-of-clearing-member/, and https://www.bis.org/publ/qtrpdf/r_qt1812x.htm for the Einar Aas default. Similarly, Klein and Co. Futures, Inc. failed to meet a margin call to the New York Clearing Corporation (NYCC), with liquidation expenses leading to a default loss of approximately $6 million (as described in ICE's 10-Q filing from July 2007). See McPartland and Lewis (2017) for examples of other clearing member defaults.

6.3 Loss sharing

Finally, our analysis provides some perspectives on default losses and how they are shared within a CCP.

Default losses and the default waterfall. If the market value of the contracts to be transferred is negative, the counterparties that take on these contracts need to be compensated for the (negative) value of these positions. Therefore, when the CCP transfers a defaulter's positions to new counterparties, the CCP usually incurs a default loss. In our model, this default loss is increasing in $\pi - \underline{\pi}$, which is larger the more informative the signal at $t = 1$. This loss reflects the Hirshleifer effect: more informative signals reduce the insurance that outsiders can provide, increasing the compensation they require to assume defaulted positions.

CCPs absorb default losses according to a predetermined default waterfall. First, the defaulter's resources, such as previously posted collateral, are used to the extent that the CCP can access them. Second, the CCP typically provides a layer of its own equity to absorb additional losses. Our model abstracts away from the CCP's equity contribution, which is typically small in practice.[21] All remaining default losses are then absorbed by the surviving clearing members through loss-sharing arrangements. This loss sharing is the focus of our theoretical analysis.

Loss sharing rules. In our model, default losses should optimally be allocated to risk-neutral protection sellers, who are better positioned to bear risk than risk-averse protection buyers and thus willing to insure them against the risk of CCP default losses. This allocation principle provides a benchmark for evaluating actual CCP loss-sharing arrangements.

In practice, the counterparties that are best able to bear this risk are typically dealers with large balance sheets. However, current loss-sharing mechanisms often imply the opposite. For example, some CCPs assign losses proportionally to net exposures. Dealers acting as intermediaries often have close-to-zero net exposure, so that they contribute little to loss sharing relative to the size of their portfolios.[22]

[21] Kuong and Maurin (2023) show that when the interests of CCPs and their members are not perfectly aligned, equity provides incentives for CCPs to monitor counterparty risk. Our model abstracts from such agency conflicts by assuming the CCP maximizes member welfare, allowing us to focus on the screening role of margins.

[22] Kubitza et al. (2024) highlight a related point driven by a different mechanism. They show that allocating default losses to end-investors rather than dealers reduces incentives to use central clearing. They also provide a model that highlights a CCP's incentives to assign losses to end-investors in order to attract large clearing volumes from dealers.

7 Conclusion

This paper reveals that margin requirements at central clearing counterparties serve a critical but previously unrecognized function: they act as canaries in the coal mine, inducing early defaults that enable the replacement of fragile counterparties while risk-sharing capacity remains available. Thus, rather than merely securing collateral to cover losses, margins create value by revealing hidden fragility and facilitating the dynamic reallocation of risk to those best able to bear it. Our model demonstrates that optimal margin policy requires balancing multiple objectives and, in particular, preserving systemic risk-sharing capacity and enabling efficient counterparty replacement. This insight is particularly relevant for understanding margin procyclicality: higher margins during stressed periods may enhance rather than impair financial stability when they facilitate the substitution of weaker counterparties with stronger ones.

Our findings have broad implications for the post-crisis financial architecture that places CCPs at the center of derivatives markets. The effectiveness of central clearing depends not only on the CCP's ability to mutualize losses among surviving members, but critically on its capacity to identify and replace weak counterparties before their failure triggers systemic distress. This perspective suggests that regulatory frameworks should consider not just the level of margins and default resources, but also the diversity and depth of potential replacement counterparties available to CCPs. As derivatives markets continue to evolve and CCPs assume ever-greater systemic importance, understanding the optimal design of central clearing is essential for financial stability.

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A Proofs

A.1 First Best

Proof of Proposition 1. Without abundant resources, the optimal contract maximizes expected utility of buyers (4) subject to the participation constraint of protection sellers (5) and the CCP's budget constraints (3). The first-order conditions are

$$-\mathbb{P}(\tilde{s},\tilde{e})u'(\tilde{s},\tilde{e}) + \eta(\tilde{s},\tilde{e}) = 0 \quad \forall \tilde{s},\tilde{e} \tag{A.1}$$

$$-\mathbb{P}(\tilde{s},\tilde{e})\xi + \eta(\tilde{s},\tilde{e}) = 0 \quad \forall \tilde{s},\tilde{e}, \tag{A.2}$$

where $\eta(\tilde{s},\tilde{e})$ is the Lagrange multiplier on the budget constraint (3) in state $(\tilde{s},\tilde{e})$ and ξ that on the participation constraint (5). Using the second set of first-order conditions in the first yields

$$-\mathbb{P}(\tilde{s},\tilde{e})u'(\tilde{s},\tilde{e}) + \mathbb{P}(\tilde{s},\tilde{e})\xi = 0 \tag{A.3}$$

$$\Leftrightarrow u'(\tilde{s},\tilde{e}) = \xi. \tag{A.4}$$

Therefore, the participation constraint is binding and buyer utility is constant across states, i.e., buyers are fully insured. This implies

$$\tilde{e}(\tilde{s},\tilde{e}) - T_B(\tilde{s},\tilde{e}) = \underbrace{\mathbb{E}[\tilde{e}]}_{=\pi} - \mathbb{E}[T_B] \quad \forall \tilde{s},\tilde{e}. \tag{A.5}$$

The budget constraints (3) jointly with the binding participation constraint (5) imply that

$$\mathbb{E}[T_B] = 0, \tag{A.6}$$

i.e., the contract is actuarially fair. Therefore, full insurance implies the following transfers from buyers

$$1 - T_B(\tilde{s}, \bar{e}) = \pi \;\Leftrightarrow\; T_B(\tilde{s}, \bar{e}) = 1 - \pi \tag{A.7}$$

$$\text{and } 0 - T_B(\tilde{s}, \underline{e}) = \pi \;\Leftrightarrow\; T_B(\tilde{s}, \underline{e}) = -\pi \tag{A.8}$$

and from sellers (using the budget constraints)

$$T_S(\tilde{s}, \bar{e}) = -T_B(\tilde{s}, \bar{e}) = -(1 - \pi) \tag{A.9}$$

$$\text{and } T_S(\tilde{s}, \underline{e}) = -T_B(\tilde{s}, \underline{e}) = \pi. \tag{A.10}$$

Neither using margins nor outside sellers can improve risk sharing. Because both are costly, it is optimal to not use them. □

A.2 Central Clearing Without Replacement

A.2.1 Without Defaults

Proof of Proposition 2. The optimal contract maximizes expected buyer utility (4) subject to the participation and resource constraints of protection sellers and the CCP budget constraints. Because sellers do not default conditional on a good signal, their participation constraint is given by

$$\mathbb{E}[\tilde{\phi} R(\tilde{s})] \leq \mathbb{P}(\bar{s})\mathbb{E}[(1 - \alpha(\bar{s}))R(\bar{s}) + \alpha(\bar{s})kR(\bar{s}) - T_S(\bar{s}, \tilde{e}) \mid \bar{s}] + \mathbb{P}(\underline{s})\mathbb{E}[f_1(\underline{s}, \tilde{e}) \mid \underline{s}] \tag{A.11}$$

where $f_1(\underline{s}, \tilde{e})$ is a function of transfers and margins conditional on a bad signal, $T_S(\underline{s}, \tilde{e})$ and $\alpha(\underline{s})$. The CCP's budget constraints conditional on a good signal are

$$T_S(\bar{s}, \tilde{e}) + T_B(\bar{s}, \tilde{e}) = 0 \quad \forall \tilde{e}, \tag{A.12}$$

whereas the budget constraints conditional on a bad signal do neither depend on $T_S(\bar{s}, \tilde{e})$, $T_B(\bar{s}, \tilde{e})$, nor $\alpha(\bar{s})$. Similarly, resource constraints after a negative signal $\underline{s}$ do neither depend on $T_S(\bar{s}, \tilde{e})$, $T_B(\bar{s}, \tilde{e})$, nor $\alpha(\bar{s})$.

The partial derivative of the Lagrangian with respect to $\alpha(\bar{s})$ is $-\mathbb{P}(\bar{s})(1 - k)\xi R$, where ξ is the multiplier on the participation constraint. Due to a binding participation constraint (shown below), it is $-\mathbb{P}(\bar{s})(1 - k)\xi R < 0$, and, therefore, no margin is used after a good signal: $\alpha(\bar{s}) = 0$.

The first-order conditions with respect to $T_B(\bar{s}, \tilde{e})$ and $T_S(\bar{s}, \tilde{e})$, respectively, are

$$-\mathbb{P}(\bar{s}, \tilde{e})u'(\bar{s}, \tilde{e}) + \eta(\bar{s}, \tilde{e}) = 0 \tag{A.13}$$

$$\eta(\bar{s}, \tilde{e}) - \xi \mathbb{P}(\bar{s}, \tilde{e}) = 0, \tag{A.14}$$

where $\eta(\bar{s},\tilde{e})$ is the Lagrange multiplier on the budget constraint in state $(\bar{s},\tilde{e})$.

Re-arranging (A.13) and (A.14) gives

$$-\mathbb{P}(\bar{s},\tilde{e})u'(\bar{s},\tilde{e})+\xi\mathbb{P}(\bar{s},\tilde{e})=0 \tag{A.15}$$

$$\Leftrightarrow \xi=u'(\bar{s},\tilde{e}). \tag{A.16}$$

Therefore, $u'(\bar{s},\underline{e})=u'(\bar{s},\bar{e})$, which implies that

$$\tilde{e}-T_B(\tilde{e},\bar{s})=\mathbb{E}[\tilde{e}-T_B(\tilde{e},\bar{s})] \quad \forall\tilde{e}. \tag{A.17}$$

□

Proof of Proposition 3. The optimal contract maximizes expected buyer utility (4) subject to the CCP's budget constraints (3), and the participation constraint (8) and resource constraints (9) and (10) of protection sellers. Therefore, the partial derivatives of the Lagrangian with respect to $T_B(\tilde{s},\tilde{e})$, $T_S(\tilde{s},\tilde{e})$, and α, respectively, are

$$-\mathbb{P}(\tilde{s},\tilde{e})u'(\tilde{s},\tilde{e})+\eta(\tilde{s},\tilde{e}), \tag{A.18}$$

$$\eta(\tilde{s},\tilde{e})-\xi\mathbb{P}(\tilde{s},\tilde{e})-(\sigma_{S,2}+\sigma_{F,2})1_{(\tilde{s},\tilde{e})=(\underline{s},\underline{e})}, \tag{A.19}$$

$$-\xi\mathbb{P}(\underline{s})(1-k)R(\underline{s})-(\sigma_{S,2}+\sigma_{F,2})(1-k)R(\underline{s})-(\sigma_{S,1}+\sigma_{F,1}), \tag{A.20}$$

where $\eta(\tilde{s},\tilde{e})$ is the Lagrange multiplier on the budget constraint in state $(\tilde{s},\tilde{e})$, $\sigma_{j,t}$ that on the resource constraint for sellers of type j at time t, and ξ that on the participation constraint.

Risk sharing We focus on an equilibrium in which no resource constraint is binding, $\sigma_{j,t}=0$. In this case, (A.20) implies that there is no interior solution for α but, instead, there is no margin call: $\alpha=0$. For an interior solution, (A.18) and (A.19) are equal to zero. Combining these yields

$$-\mathbb{P}(\tilde{s},\tilde{e})u'(\tilde{s},\tilde{e})+\xi\mathbb{P}(\tilde{s},\tilde{e})=0 \tag{A.21}$$

$$\Leftrightarrow u'(\tilde{s},\tilde{e})=\xi. \tag{A.22}$$

Therefore, the participation constraint is satisfied with equality and buyer consumption is independent of the signal and endowment risk realization, $\tilde{e}-T_B=\mathbb{E}[\tilde{e}-T_B]$.

Transfers The budget constraints (3) imply that

$$\mathbb{E}[T_B]=-\mathbb{E}[T_S], \tag{A.23}$$

which we use in the participation constraint (8):

$$0 = -\mathbb{E}[T_S] = \mathbb{E}[T_B]. \tag{A.24}$$

Using this above and that $\mathbb{E}[\tilde{e}] = \pi \cdot 1 + (1-\pi) \cdot 0 = \pi$ gives the transfers of the first-best contract in Proposition 1.

No defaults at $t = 1$ Because $\phi_F < \phi_S = 1$, if the resource constraint (9) is satisfied for fragile sellers, it also holds for safe sellers. Because $\alpha = 0$ and $\phi_F \geq 0$, it is $\alpha \leq \phi_F$ and, thus, there are no defaults at $t = 1$.

No defaults at $t = 2$ Because $\phi_F < \phi_S = 1$, available resources at $t = 2$ are smaller for fragile sellers: $\bar{t}_{F,2} < \bar{t}_{S,2}$. Therefore, if the resource constraint (10) is satisfied for fragile sellers, it also holds for safe sellers. Because $T_S(\underline{s}, \bar{e}) \leq T_S(\underline{s}, \underline{e})$ it is sufficient that

$$T_S(\underline{s}, \underline{e}) \leq \bar{t}_{F,2} = \phi R(\underline{s}), \tag{A.25}$$

which, using the optimal transfer, is equivalent to

$$\pi \leq \phi R(\underline{s}). \tag{A.26}$$

□

Proof of Proposition 4. We consider an equilibrium in which the resource constraint (10) at $t = 2$ for fragile sellers is binding. Because $\phi_F < \phi_S$, safe sellers have access to more resources and, thus, the resource constraint is not binding for safe sellers. We assume that resources at $t = 1$ are sufficient such that there are no early defaults.

Risk sharing Consider the first order conditions from the proof of Proposition 3. (A.19) then implies

$$\eta(\underline{s}, \underline{e}) - \xi \mathbb{P}(\underline{s}, \underline{e}) - \sigma_{F,2} = 0 \tag{A.27}$$

$$\text{and} \quad \eta(\tilde{s}, \tilde{e}) - \xi \mathbb{P}(\tilde{s}, \tilde{e}) = 0 \quad \forall (\tilde{s}, \tilde{e}) \neq (\underline{s}, \underline{e}). \tag{A.28}$$

Therefore, combining with (A.18) yields

$$\xi = \frac{\eta(\tilde{s}, \tilde{e})}{\mathbb{P}(\tilde{s}, \tilde{e})} = u'(\tilde{s}, \tilde{e}) \quad \forall (\tilde{s}, \tilde{e}) \neq (\underline{s}, \underline{e}) \tag{A.29}$$

and, thus,

$$\eta(\underline{s},\underline{e}) - u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\underline{e}) - \sigma_{F,2} = 0 \tag{A.30}$$

$$\Leftrightarrow \sigma_{F,2} = \eta(\underline{s},\underline{e}) - u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\underline{e}) \tag{A.31}$$

$$\Leftrightarrow \sigma_{F,2} = \mathbb{P}(\underline{s},\underline{e})(u'(\underline{s},\underline{e}) - u'(\bar{s},\bar{e})). \tag{A.32}$$

Due to $\sigma_{F,2} > 0$, it is $u'(\underline{s},\underline{e}) > u'(\bar{s},\bar{e})$ and, therefore, using the concavity of the utility function, $u(\underline{s},\underline{e}) < u(\bar{s},\bar{e})$.

Optimal margin Using the above in the first order condition for α (A.20) gives

$$-\xi\mathbb{P}(\underline{s})(1-k)R(\underline{s}) - \sigma_{F,2}(1-k)R(\underline{s}) \tag{A.33}$$

$$= -\left[\xi\mathbb{P}(\underline{s}) + \sigma_{F,2}\right](1-k)R(\underline{s}) < 0, \tag{A.34}$$

which is strictly negative due to $\xi > 0$ and $\sigma_{F,2} > 0$. Thus, the optimal contract uses no margin.

Transfers The participation constraint (8) implies that $\mathbb{E}[T_S] = -\mathbb{P}(\underline{s})\alpha(1-k)R = 0$, using that $\alpha = 0$. Using the CCP's budget constraints (3), this implies that $\mathbb{E}[T_B] = 0$. Due to the binding resource constraint (10), the transfer in $(\underline{s},\underline{e})$ is given by

$$T_S(\underline{s},\underline{e}) = \phi R(\underline{s}) = \bar{t}_{F,2}. \tag{A.35}$$

(A.29) implies that

$$1 - T_B(\bar{s},\bar{e}) = 1 - T_B(\underline{s},\bar{e}) = -T_B(\bar{s},\underline{e}), \tag{A.36}$$

which, using the budget constraints (3) implies that

$$1 + T_S(\bar{s},\bar{e}) = 1 + T_S(\underline{s},\bar{e}) = T_S(\bar{s},\underline{e}). \tag{A.37}$$

Therefore,

$$\mathbb{E}[T_S \mid \bar{s}] = \mathbb{P}(\bar{e} \mid \bar{s})T_S(\bar{s},\bar{e}) + \mathbb{P}(\underline{e} \mid \bar{s})T_S(\bar{s},\underline{e}) \tag{A.38}$$

$$= \overline{\pi}(-1 + T_S(\bar{s},\underline{e})) + (1-\overline{\pi})T_S(\bar{s},\underline{e}) \tag{A.39}$$

$$= T_S(\bar{s},\underline{e}) - \overline{\pi}. \tag{A.40}$$

The participation constraint (8) then implies that

$$
\begin{aligned}
0 &= \mathbb{E}[T_S] && \text{(A.41)}\\
&= \mathbb{P}(\bar{s})\mathbb{E}[T_S \mid \bar{s}] + \mathbb{P}(\underline{s})\mathbb{E}[T_S \mid \underline{s}] && \text{(A.42)}\\
&= \mathbb{P}(\bar{s})(T_S(\bar{s},\underline{e}) - \overline{\pi}) + \mathbb{P}(\underline{s},\underline{e})T_S(\underline{s},\underline{e}) + \mathbb{P}(\underline{s},\bar{e})T_S(\underline{s},\bar{e}) && \text{(A.43)}\\
&= \mathbb{P}(\bar{s})(T_S(\bar{s},\underline{e}) - \overline{\pi}) + \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2} + \mathbb{P}(\underline{s},\bar{e})(T_S(\bar{s},\underline{e}) - 1) && \text{(A.44)}\\
&= (1 - \mathbb{P}(\underline{s},\underline{e}))T_S(\bar{s},\underline{e}) - \mathbb{P}(\bar{s},\bar{e}) + \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2} - \mathbb{P}(\underline{s},\bar{e}) && \text{(A.45)}\\
&= (1 - \mathbb{P}(\underline{s},\underline{e}))T_S(\bar{s},\underline{e}) - \mathbb{P}(\bar{e}) + \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2} && \text{(A.46)}\\
\Leftrightarrow T_S(\bar{s},\underline{e}) &= \frac{\pi - \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2}}{1 - \mathbb{P}(\underline{s},\underline{e})}. && \text{(A.47)}
\end{aligned}
$$

Together with (A.101), this yields

$$
\begin{aligned}
T_S(\underline{s},\bar{e}) = T_S(\bar{s},\bar{e}) = T_S(\bar{s},\underline{e}) - 1 && \text{(A.48)}\\
= \frac{\mathbb{P}(\bar{e}) - \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2}}{1 - \mathbb{P}(\underline{s},\underline{e})} - 1 && \text{(A.49)}\\
= \frac{-\mathbb{P}(\bar{s},\underline{e}) - \mathbb{P}(\underline{s},\underline{e})\bar{t}_{F,2}}{1 - \mathbb{P}(\underline{s},\underline{e})}. && \text{(A.50)}
\end{aligned}
$$

No defaults at $t = 1$ Because $\alpha = 0$, there are no defaults at $t = 1$. □

A.2.2 With Defaults

Proof of Proposition 5. If fragile sellers default in $(\underline{s},\underline{e})$, the optimal contract maximizes expected buyer utility (4) subject to the CCP's budget constraints (3) and (13), and the participation constraint (15) and the resource constraints at $t = 1$ (9) and that of safe sellers at $t = 2$ (10). The derivatives of the Lagrangian with respect to $T_B(\tilde{s},\tilde{e})$, $T_S(\tilde{s},\tilde{e})$, and α, respectively, are

$$
\begin{aligned}
&(\partial T_B(\tilde{s},\tilde{e})) \;\; -\mathbb{P}(\tilde{s},\tilde{e})u'(\tilde{s},\tilde{e}) + \eta(\tilde{s},\tilde{e}) \quad \forall(\tilde{s},\tilde{e}) && \text{(A.51)}\\
&(\partial T_S(\underline{s},\underline{e})) \;\; (1-\gamma)\eta(\underline{s},\underline{e}) - \xi\mathbb{P}(\underline{s},\underline{e})(1-\gamma) - \sigma_{S,2} && \text{(A.52)}\\
&(\partial T_S(\tilde{s},\tilde{e})) \;\; \eta(\tilde{s},\tilde{e}) - \xi\mathbb{P}(\tilde{s},\tilde{e}) \quad \forall(\tilde{s},\tilde{e}) \neq (\underline{s},\underline{e}) && \text{(A.53)}\\
&(\partial\alpha) \;\; \xi\left(-\mathbb{P}(\underline{s})(1-k) + \mathbb{P}(\underline{s},\underline{e})\gamma(1-k)\right)R(\underline{s}) + \eta(\underline{s},\underline{e})\gamma(k-\rho)R(\underline{s})\\
&\qquad - \sigma_{S,2}(1-k)R(\underline{s}) - (\sigma_{1,F} + \sigma_{1,S})R(\underline{s}), && \text{(A.54)}
\end{aligned}
$$

where η is the Lagrange multiplier on the budget constraint, $\sigma_{j,t}$ that on the resource constraint for type $j \in \{S, F\}$ at time t, and ξ that on the participation constraint.

Assume that no resource constraint is binding, i.e., $\sigma_{j,t} \equiv 0$.

Risk sharing For an interior solution with $\sigma_{j,t} \equiv 0$, (A.51), (A.52), and (A.53) all hold with equality and jointly imply that

$$\xi = \frac{\eta(\tilde{s}, \tilde{e})}{\mathbb{P}(\tilde{s}, \tilde{e})} = u'(\tilde{s}, \tilde{e}) \quad \forall(\tilde{s}, \tilde{e}). \tag{A.55}$$

Therefore, buyers are fully insured.

Optimal margin (A.54) is then equal to

$$\begin{aligned}
&\xi\left(-\mathbb{P}(\underline{s})(1-k) + \mathbb{P}(\underline{s}, \underline{e})\gamma(1-k)\right) R(\underline{s}) + \eta(\underline{s}, \underline{e})\gamma(k-\rho)R(\underline{s}) && \text{(A.56)}\\
=&\xi\left[\left(-\mathbb{P}(\underline{s})(1-k) + \mathbb{P}(\underline{s}, \underline{e})\gamma(1-k)\right) + \mathbb{P}(\underline{s}, \underline{e})\gamma(k-\rho)\right] R(\underline{s}) && \text{(A.57)}\\
=&\xi\left[-\mathbb{P}(\underline{s})(1-k) + \mathbb{P}(\underline{s}, \underline{e})\gamma(1-k+k-\rho)\right] R(\underline{s}) && \text{(A.58)}\\
=&\xi\left[-\mathbb{P}(\underline{s})(1-k) + \mathbb{P}(\underline{s}, \underline{e})\gamma(1-\rho)\right] R(\underline{s}), && \text{(A.59)}
\end{aligned}$$

which is strictly negative if, and only if,

$$\mathbb{P}(\underline{s}, \underline{e})\gamma(1-\rho) < \mathbb{P}(\underline{s})(1-k). \tag{A.60}$$

We assume in the following that this condition holds and, thus, $\alpha = 0$.

Transfers The budget constraints imply that

$$\mathbb{E}[T_B] + \mathbb{E}[T_S] + \mathbb{P}(\underline{s}, \underline{e})\gamma(\rho\phi R(\underline{s}) - T_S(\underline{s}, \underline{e})) = 0. \tag{A.61}$$

Using this in the binding participation constraint yields

$$\begin{aligned}
&0 = -\mathbb{E}[T_S] - \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) + \gamma\mathbb{P}(\underline{s}, \underline{e})\left(T_S(\underline{s}, \underline{e}) + ((1-k)\alpha - \phi)R(\underline{s})\right) && \text{(A.62)}\\
\Leftrightarrow &0 = \mathbb{E}[T_B] - (\mathbb{P}(\underline{s}) - \mathbb{P}(\underline{s}, \underline{e})\gamma)(1-k)\alpha R(\underline{s}) - \gamma\mathbb{P}(\underline{s}, \underline{e})(1-\rho)\phi R(\underline{s}) && \text{(A.63)}\\
\Leftrightarrow &\mathbb{E}[T_B] = (\mathbb{P}(\underline{s}) - \mathbb{P}(\underline{s}, \underline{e})\gamma)(1-k)\alpha R(\underline{s}) + \gamma\mathbb{P}(\underline{s}, \underline{e})(1-\rho)\phi R(\underline{s}) =: m. && \text{(A.64)}
\end{aligned}$$

Full insurance implies that

$$\tilde{e} - T_B(\tilde{s}, \tilde{e}) = \mathbb{E}[\tilde{e} - T_B] = \pi - m \tag{A.65}$$

and, thus, (using the budget constraints)

$$\begin{aligned}
&1 - T_B(\tilde{s}, \bar{e}) = \pi - m && \text{(A.66)}\\
\Leftrightarrow &-T_B(\tilde{s}, \bar{e}) = \pi - 1 - m && \text{(A.67)}\\
\Leftrightarrow &T_S(\tilde{s}, \bar{e}) = -(1-\pi) - m && \text{(A.68)}
\end{aligned}$$

and

$$0 - T_B(\bar{s}, \underline{e}) = \pi - m \tag{A.69}$$
$$\Leftrightarrow T_S(\bar{s}, \underline{e}) = \pi - m \tag{A.70}$$

and

$$0 - T_B(\underline{s}, \underline{e}) = \pi - m \tag{A.71}$$
$$\Leftrightarrow (1-\gamma) T_S(\underline{s}, \underline{e}) + \gamma\rho\phi R(\underline{s}) = \pi - m \tag{A.72}$$
$$\Leftrightarrow T_S(\underline{s}, \underline{e}) = \frac{\pi - m - \gamma\rho\phi R(\underline{s})}{1-\gamma} = \pi - m + \frac{\gamma}{1-\gamma}(\pi - m - \rho\phi R(\underline{s})). \tag{A.73}$$

(No) Defaults at $t = 2$ Using (10), fragile sellers default at $t = 2$ if, and only if,

$$T_S(\underline{s}, \underline{e}) > \phi R(\underline{s}) \tag{A.74}$$
$$\Leftrightarrow \frac{\pi - m - \gamma\rho\phi R(\underline{s})}{1-\gamma} > \phi R(\underline{s}) \tag{A.75}$$
$$\Leftrightarrow \frac{\pi - \gamma \mathbb{P}(\underline{s}, \underline{e})(1-\rho)\phi R(\underline{s}) - \gamma\rho\phi R(\underline{s})}{1-\gamma} > \phi R(\underline{s}) \tag{A.76}$$
$$\Leftrightarrow \frac{\pi - \gamma(\mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \rho)\phi R(\underline{s})}{1-\gamma} > \phi R(\underline{s}) \tag{A.77}$$
$$\Leftrightarrow \pi - \gamma(\mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \rho)\phi R(\underline{s}) > (1-\gamma)\phi R(\underline{s}) \tag{A.78}$$
$$\Leftrightarrow \pi > [(1-\gamma) + \gamma(\mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \rho)]\phi R(\underline{s}) \tag{A.79}$$
$$\Leftrightarrow \pi > [1 - \gamma(1 - \rho - \mathbb{P}(\underline{s}, \underline{e})(1-\rho))]\phi R(\underline{s}) \tag{A.80}$$
$$\Leftrightarrow \pi > \underbrace{[1 - \gamma(1-\rho)(1 - \mathbb{P}(\underline{s}, \underline{e}))]}_{<1} \phi R(\underline{s}), \tag{A.81}$$

which holds because Assumption (1) implies that $\pi > \phi R(\underline{s})$.

Safe sellers do not default at $t = 2$ if, and only if,

$$T_S(\underline{s}, \underline{e}) \leq R(\underline{s}) \tag{A.82}$$
$$\Leftrightarrow \frac{\pi - \gamma(\mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \rho)\phi R(\underline{s})}{1-\gamma} \leq R(\underline{s}) \tag{A.83}$$
$$\Leftrightarrow \pi - \gamma(\mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \rho)\phi R(\underline{s}) \leq (1-\gamma) R(\underline{s}) \tag{A.84}$$
$$\Leftrightarrow \gamma(1 - \mathbb{P}(\underline{s}, \underline{e})(1-\rho)\phi - \rho\phi) R(\underline{s}) \leq R(\underline{s}) - \pi \tag{A.85}$$
$$\Leftrightarrow \gamma \leq \frac{R(\underline{s}) - \pi}{R(\underline{s})(1 - (\mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \rho)\phi)} =: \overline{\gamma}^{NR}, \tag{A.86}$$

where, in the last step, we use that

$$1-(\mathbb{P}(\underline{s},\underline{e})(1-\rho)+\rho)\phi>0 \tag{A.87}$$

$$\Leftrightarrow 1>\underbrace{(\mathbb{P}(\underline{s},\underline{e})(1-\rho)+\rho)}_{<1}\underbrace{\phi}_{<1}. \tag{A.88}$$

Because $T_S(\underline{s},\bar{e})<0$, there are no defaults in state $(\underline{s},\bar{e})$.

No defaults at $t=1$ Because $\alpha=0$, there are no defaults at $t=1$. □

Proof of Proposition 6. Consider an interior solution for T_S, T_B, and α, such that the first order conditions (A.51) to (A.54) are all equal to zero. Due to Assumption (2), safe sellers' resource constraint (10) is binding in equilibrium and, thus, $T_S(\underline{s},\underline{e})=R(\underline{s})-(1-k)\alpha R(\underline{s})$ and $\sigma_{S,2}>0$.

Risk sharing (A.53) together with (A.51) imply that

$$\xi=\frac{\eta(\tilde{s},\tilde{e})}{\mathbb{P}(\tilde{s},\tilde{e})}=u'(\tilde{s},\tilde{e}) \quad \forall(\tilde{s},\tilde{e})\neq(\underline{s},\underline{e}), \tag{A.89}$$

which implies that $u(\tilde{s},\tilde{e})=u(\bar{s},\bar{e})$ for all states $(\tilde{s},\tilde{e})\neq(\underline{s},\underline{e})$. (A.52) gives

$$(1-\gamma)\eta(\underline{s},\underline{e})-\xi\mathbb{P}(\underline{s},\underline{e})(1-\gamma)=\sigma_{S,2} \tag{A.90}$$

$$\Leftrightarrow(1-\gamma)(\mathbb{P}(\underline{s},\underline{e})u'(\underline{s},\underline{e})-u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\underline{e}))=\sigma_{S,2} \tag{A.91}$$

$$\Leftrightarrow(1-\gamma)\mathbb{P}(\underline{s},\underline{e})(u'(\underline{s},\underline{e})-u'(\underline{s},\bar{e}))=\sigma_{S,2}, \tag{A.92}$$

and, thus, using the concavity of $u(\cdot)$, it is $u(\underline{s},\underline{e})<u(\underline{s},\bar{e})=u(\bar{s},\underline{e})=u(\bar{s},\bar{e})$.

Transfers The participation constraint (15) is binding in equilibrium, which implies that

$$0=-\mathbb{E}[T_S]-\mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s})+\gamma\mathbb{P}(\underline{s},\underline{e})\left(T_S(\underline{s},\underline{e})+((1-k)\alpha-\phi)R(\underline{s})\right) \tag{A.93}$$

$$\Leftrightarrow\mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s})-\gamma\mathbb{P}(\underline{s},\underline{e})((1-k)\alpha-\phi)R(\underline{s})=-\mathbb{E}[T_S]+\gamma\mathbb{P}(\underline{s},\underline{e})T_S(\underline{s},\underline{e}), \tag{A.94}$$

which together with the budget constraints (3) and (13) implies that

$$\mathbb{E}[T_B]+\mathbb{E}[T_S]-\mathbb{P}(\underline{s},\underline{e})\gamma\left(T_S(\underline{s},\underline{e})-\alpha kR(\underline{s})-\rho\left(1-\frac{\alpha}{\phi}\right)\phi R(\underline{s})\right)=0 \tag{A.95}$$

$$\Leftrightarrow\mathbb{E}[T_B]=-\mathbb{E}[T_S]+\gamma\mathbb{P}(\underline{s},\underline{e})\left(T_S(\underline{s},\underline{e})-\alpha kR(\underline{s})-\rho\left(1-\frac{\alpha}{\phi}\right)\phi R(\underline{s})\right) \tag{A.96}$$

$$\Leftrightarrow\mathbb{E}[T_B]=\mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s})-\gamma\mathbb{P}(\underline{s},\underline{e})((1-k)\alpha-\phi)R(\underline{s})-\gamma\mathbb{P}(\underline{s},\underline{e})\left(\alpha kR(\underline{s})+\rho\left(1-\frac{\alpha}{\phi}\right)\phi R(\underline{s})\right)$$

$$\Leftrightarrow\mathbb{E}[T_B]=\left[(\mathbb{P}(\underline{s})-\gamma\mathbb{P}(\underline{s},\underline{e}))(1-k)\alpha+\gamma\mathbb{P}(\underline{s},\underline{e})\left(\phi-\alpha k-\rho\left(1-\frac{\alpha}{\phi}\right)\phi\right)\right]R(\underline{s})=:m, \tag{A.97}$$

i.e., the price of the contract is not actuarially fair but there is a markup that compensates for the cost of posting margin and for the deadweight cost of defaults. This markup is equal to

$$m = \left(\mathbb{P}(\underline{s})(1-k)\alpha + \gamma \mathbb{P}(\underline{s}, \underline{e}) \left(\phi - \alpha - \rho \left(1 - \frac{\alpha}{\phi} \right) \phi \right) \right) R(\underline{s}). \tag{A.98}$$

The transfer in $(\underline{s}, \underline{e})$ is determined by the binding resource constraint (10):

$$T_S(\underline{s}, \underline{e}) = (1-\alpha)R(\underline{s}) + \alpha k R(\underline{s}) = \bar{t}_{S,2}. \tag{A.99}$$

The result on risk sharing above implies that

$$1 - T_B(\bar{s}, \bar{e}) = 1 - T_B(\underline{s}, \bar{e}) = -T_B(\bar{s}, \underline{e}), \tag{A.100}$$

which, using the budget constraints (3) implies that

$$1 + T_S(\bar{s}, \bar{e}) = 1 + T_S(\underline{s}, \bar{e}) = T_S(\bar{s}, \underline{e}). \tag{A.101}$$

Therefore,

$$\begin{aligned} \mathbb{E}[T_S \mid \bar{s}] &= \mathbb{P}(\bar{e} \mid \bar{s}) T_S(\bar{s}, \bar{e}) + \mathbb{P}(\underline{e} \mid \bar{s}) T_S(\bar{s}, \underline{e}) & \text{(A.102)} \\ &= \overline{\pi}(-1 + T_S(\bar{s}, \underline{e})) + (1 - \overline{\pi}) T_S(\bar{s}, \underline{e}) & \text{(A.103)} \\ &= T_S(\bar{s}, \underline{e}) - \overline{\pi}. & \text{(A.104)} \end{aligned}$$

The participation constraint (15) then implies that

$$\gamma\mathbb{P}(\underline{s},\underline{e})T_S(\underline{s},\underline{e}) - \mathbb{P}(\underline{s})(1-k)\alpha R(\underline{s}) + \gamma\mathbb{P}(\underline{s},\underline{e})((1-k)\alpha - \phi)R(\underline{s}) - m$$
$$+\left[(\mathbb{P}(\underline{s}) - \gamma\mathbb{P}(\underline{s},\underline{e}))(1-k)\alpha + \gamma\mathbb{P}(\underline{s},\underline{e})\left(\phi - \alpha k - \rho\left(1-\frac{\alpha}{\phi}\right)\phi\right)\right]R(\underline{s}) = \mathbb{E}[T_S] \tag{A.105}$$

$$\Leftrightarrow \gamma\mathbb{P}(\underline{s},\underline{e})\left(T_S(\underline{s},\underline{e}) - \alpha k R(\underline{s}) - \rho\left(1-\frac{\alpha}{\phi}\right)\phi R(\underline{s})\right) - m = \mathbb{E}[T_S] \tag{A.106}$$

$$= \mathbb{P}(\bar{s})\mathbb{E}[T_S \mid \bar{s}] + \mathbb{P}(\underline{s})\mathbb{E}[T_S \mid \underline{s}] \tag{A.107}$$

$$= \mathbb{P}(\bar{s})(T_S(\bar{s},\underline{e}) - \overline{\pi}) + \mathbb{P}(\underline{s},\underline{e})T_S(\underline{s},\underline{e}) + \mathbb{P}(\underline{s},\bar{e})T_S(\underline{s},\bar{e}) \tag{A.108}$$

$$= \mathbb{P}(\bar{s})(T_S(\bar{s},\underline{e}) - \overline{\pi}) + \mathbb{P}(\underline{s},\underline{e})\bar{t}_{S,2} + \mathbb{P}(\underline{s},\bar{e})(T_S(\bar{s},\underline{e}) - 1) \tag{A.109}$$

$$= (1 - \mathbb{P}(\underline{s},\underline{e}))T_S(\bar{s},\underline{e}) - \mathbb{P}(\bar{s},\bar{e}) + \mathbb{P}(\underline{s},\underline{e})\bar{t}_{S,2} - \mathbb{P}(\underline{s},\bar{e}) \tag{A.110}$$

$$= (1 - \mathbb{P}(\underline{s},\underline{e}))T_S(\bar{s},\underline{e}) - \mathbb{P}(\bar{e}) + \mathbb{P}(\underline{s},\underline{e})\bar{t}_{S,2} \tag{A.111}$$

$$\Leftrightarrow T_S(\bar{s},\underline{e}) = \frac{\mathbb{P}(\bar{e}) - \mathbb{P}(\underline{s},\underline{e})\bar{t}_{S,2} - m + \mathbb{P}(\underline{s},\underline{e})\gamma\left(\bar{t}_{S,2} - \alpha k R(\underline{s}) - \rho\left(1-\frac{\alpha}{\phi}\right)\phi R(\underline{s})\right)}{1 - \mathbb{P}(\underline{s},\underline{e})} \tag{A.112}$$

$$\Leftrightarrow T_S(\bar{s},\underline{e}) = \frac{\mathbb{P}(\bar{e}) - \mathbb{P}(\underline{s},\underline{e})((1-\gamma)\bar{t}_{S,2} + \gamma(\alpha k + \rho\left(1-\frac{\alpha}{\phi}\right)\phi)R(\underline{s})) - m}{1 - \mathbb{P}(\underline{s},\underline{e})}. \tag{A.113}$$

Together with the result on risk sharing, this yields

$$T_S(\underline{s},\bar{e}) = T_S(\bar{s},\bar{e}) = T_S(\bar{s},\underline{e}) - 1 \tag{A.114}$$

$$= \frac{\mathbb{P}(\bar{e}) - \mathbb{P}(\underline{s},\underline{e})((1-\gamma)\bar{t}_{S,2} + \gamma(\alpha k + \rho\left(1-\frac{\alpha}{\phi}\right)\phi)R(\underline{s})) - m}{1 - \mathbb{P}(\underline{s},\underline{e})} - 1 \tag{A.115}$$

$$= \frac{-\mathbb{P}(\bar{s},\underline{e}) - \mathbb{P}(\underline{s},\underline{e})((1-\gamma)\bar{t}_{S,2} + \gamma(\alpha k + \rho\left(1-\frac{\alpha}{\phi}\right)\phi)R(\underline{s})) - m}{1 - \mathbb{P}(\underline{s},\underline{e})}. \tag{A.116}$$

Buyer transfers. In all non-default states, the CCP budget constraint implies

$$T_B(\tilde{s},\tilde{e}) = -T_S(\tilde{s},\tilde{e}), \qquad (\tilde{s},\tilde{e}) \neq (\underline{s},\underline{e}). \tag{A.117}$$

In the default state $(\underline{s},\underline{e})$, the CCP budget is

$$T_B(\underline{s},\underline{e}) + (1-\gamma)T_S(\underline{s},\underline{e}) + \gamma\left[\alpha k R(\underline{s}) + \rho\left(1-\frac{\alpha}{\phi}\right)\phi R(\underline{s})\right] = 0. \tag{A.118}$$

Using $T_S(\underline{s},\underline{e}) = \bar{t}_{S,2}$, this yields

$$T_B(\underline{s},\underline{e}) = -(1-\gamma)\bar{t}_{S,2} - \gamma\left[\alpha k + \rho\left(1-\frac{\alpha}{\phi}\right)\phi\right]R(\underline{s}). \tag{A.119}$$

For the state $(\bar{s}, \underline{e})$, we have

$$\begin{aligned}
T_B(\bar{s},\underline{e}) &= -T_S(\bar{s},\underline{e}) && \text{(A.120)}\\
&= -\frac{\mathbb{P}(\bar{e}) - \mathbb{P}(\underline{s},\underline{e})\left((1-\gamma)\bar{t}_{S,2} + \gamma\left(\alpha k + \rho\left(1-\frac{\alpha}{\phi}\right)\phi\right)R(\underline{s})\right) - m}{1-\mathbb{P}(\underline{s},\underline{e})} && \text{(A.121)}\\
&= \frac{-\mathbb{P}(\bar{e}) + \mathbb{P}(\underline{s},\underline{e})\left((1-\gamma)\bar{t}_{S,2} + \gamma\left(\alpha k + \rho\left(1-\frac{\alpha}{\phi}\right)\phi\right)R(\underline{s})\right) + m}{1-\mathbb{P}(\underline{s},\underline{e})}. && \text{(A.122)}
\end{aligned}$$

Since risk sharing implies

$$1 - T_B(\bar{s},\bar{e}) = 1 - T_B(\underline{s},\bar{e}) = -T_B(\bar{s},\underline{e}), \tag{A.123}$$

we obtain

$$\begin{aligned}
T_B(\bar{s},\bar{e}) &= T_B(\underline{s},\bar{e}) = 1 + T_B(\bar{s},\underline{e}) && \text{(A.124)}\\
&= 1 + \frac{-\mathbb{P}(\bar{e}) + \mathbb{P}(\underline{s},\underline{e})\left((1-\gamma)\bar{t}_{S,2} + \gamma\left(\alpha k + \rho\left(1-\frac{\alpha}{\phi}\right)\phi\right)R(\underline{s})\right) + m}{1-\mathbb{P}(\underline{s},\underline{e})} && \text{(A.125)}\\
&= \frac{\mathbb{P}(\bar{s},\underline{e}) + \mathbb{P}(\underline{s},\underline{e})\left((1-\gamma)\bar{t}_{S,2} + \gamma\left(\alpha k + \rho\left(1-\frac{\alpha}{\phi}\right)\phi\right)R(\underline{s})\right) + m}{1-\mathbb{P}(\underline{s},\underline{e})}. && \text{(A.126)}
\end{aligned}$$

Therefore,

$$T_B(\underline{s},\bar{e}) = T_B(\bar{s},\bar{e}) = \frac{\mathbb{P}(\bar{s},\underline{e}) + \mathbb{P}(\underline{s},\underline{e})\left((1-\gamma)\bar{t}_{S,2} + \gamma\left(\alpha k + \rho\left(1-\frac{\alpha}{\phi}\right)\phi\right)R(\underline{s})\right) + m}{1-\mathbb{P}(\underline{s},\underline{e})}. \tag{A.127}$$

Optimal margin The derivative of the Lagrangian with respect to α is given by (A.54), which is equal to

$$\begin{aligned}
&\xi\left(-\mathbb{P}(\underline{s})(1-k) + \mathbb{P}(\underline{s},\underline{e})\gamma(1-k)\right)R(\underline{s}) + \eta(\underline{s},\underline{e})\gamma(k-\rho)R(\underline{s}) - \sigma_{S,2}(1-k)R(\underline{s}) && \text{(A.128)}\\
=&u'(\underline{s},\bar{e})\left(-\mathbb{P}(\underline{s})(1-k) + \mathbb{P}(\underline{s},\underline{e})\gamma(1-k)\right)R(\underline{s}) + \mathbb{P}(\underline{s},\underline{e})u'(\underline{s},\underline{e})\gamma(k-\rho)R(\underline{s}) - \sigma_{S,2}(1-k)R(\underline{s})\\
=&u'(\underline{s},\bar{e})\left(-\mathbb{P}(\underline{s})(1-k) + \mathbb{P}(\underline{s},\underline{e})\gamma(1-k)\right)R(\underline{s}) + \mathbb{P}(\underline{s},\underline{e})u'(\underline{s},\underline{e})\gamma(k-\rho)R(\underline{s}) - \sigma_{S,2}(1-k)R(\underline{s})\\
=&u'(\underline{s},\bar{e})\left(\mathbb{P}(\underline{s},\underline{e})\gamma(1-\rho) - \mathbb{P}(\underline{s})(1-k)\right)R(\underline{s}) + \mathbb{P}(\underline{s},\underline{e})(u'(\underline{s},\underline{e}) - u'(\underline{s},\bar{e}))(\gamma(k-\rho) - (1-\gamma)(1-k))R(\underline{s})
\end{aligned}$$

and, thus, an interior optimum for the margin requirement is reached if, and only if,

$$\begin{aligned}
u'(\underline{s},\bar{e})\mathbb{P}(\underline{s})(1-k) =&u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\underline{e})\gamma(1-\rho)\\
&+ (u'(\underline{s},\underline{e}) - u'(\underline{s},\bar{e}))\mathbb{P}(\underline{s},\underline{e})(\gamma(k-\rho) - (1-\gamma)(1-k)). && \text{(A.129)}
\end{aligned}$$

(No) Defaults at $t = 2$ Safe sellers do not default at $t = 2$ as their resource constraint (10) is satisfied with equality. Fragile sellers default in state $(\underline{s}, \underline{e})$ at $t = 2$ if, and only if, (using the binding resource constraint of safe sellers at $t = 2$)

$$T_S(\underline{s}, \underline{e}) > \phi R(\underline{s}) - (1-k)\alpha R(\underline{s}) \tag{A.130}$$

$$\Leftrightarrow R(\underline{s}) - (1-k)\alpha R(\underline{s}) > \phi R(\underline{s}) - (1-k)\alpha R(\underline{s}) \tag{A.131}$$

$$\Leftrightarrow R(\underline{s}) > \phi R(\underline{s}) \tag{A.132}$$

$$\Leftrightarrow 1 > \phi, \tag{A.133}$$

which holds by assumption.

No defaults at $t = 1$ Using the resource constraint (9), fragile sellers do not default at $t = 1$ if

$$\alpha \leq \phi, \tag{A.134}$$

which, due to $\phi < 1$, is sufficient to ensure no defaults of safe sellers. □

Proof of Proposition 7. Consider the set of resource-compatible contracts with defaults of fragile sellers at $t = 2$ in state $(\underline{s}, \underline{e})$ and suppose that Assumption 2 holds. If $\alpha = 0$, then the markup is $m = \gamma \mathbb{P}(\underline{s}, \underline{e})(1-\rho)\phi R(\underline{s})$ and the transfer in $(\underline{s}, \underline{e})$ is equal to $T_S(\underline{s}, \underline{e}) = R(\underline{s}) - m$, which provides buyers with the following consumption:

$$0 - T_B(\underline{s}, \underline{e}) = (1-\gamma)T_S(\underline{s}, \underline{e}) + \gamma\rho\phi R(\underline{s}) \tag{A.135}$$

$$= (1-\gamma)(R(\underline{s}) - m) + \gamma\rho\phi R(\underline{s}) \tag{A.136}$$

$$= (1-\gamma)R(\underline{s}) - \gamma((1-\gamma)\mathbb{P}(\underline{s}, \underline{e})(1-\rho) - \rho)\phi R(\underline{s}). \tag{A.137}$$

In contrast, the optimal contract without defaults has $\alpha = 0$, $m = 0$, and $T_S(\underline{s}, \underline{e}) = \phi R(\underline{s})$, which provides buyers with the following consumption:

$$0 - T_B(\underline{s}, \underline{e}) = T_S(\underline{s}, \underline{e}) \tag{A.138}$$

$$= \phi R(\underline{s}). \tag{A.139}$$

Both contracts provide partial insurance with $u(\underline{s}, \underline{e}) < u(\tilde{s}, \tilde{e})$ for all $(\tilde{s}, \tilde{e}) \neq (\underline{s}, \underline{e})$. Consider the case that $\phi = 0$. Then, the markup for both contracts is $m = 0$. Therefore, the contract with defaults provides more risk sharing if, and only if,

$$(1-\gamma)R(\underline{s}) > 0, \tag{A.140}$$

which holds due to $\gamma < 1$. The result follows from continuity in ϕ. □

A.3 Central Clearing with Replacement

Proof of Proposition 8. The budget constraints (3) and (20) imply that

$$\mathbb{E}[T_B] + \mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma\rho\phi R(\underline{s}) - \mathbb{P}(\underline{s})\gamma(\mathbb{E}[T_S \mid \underline{s}] + C) = 0 \tag{A.141}$$

$$\Leftrightarrow \mathbb{E}[T_B] - \mathbb{P}(\underline{s})\gamma C + \mathbb{P}(\underline{s})\gamma\rho\phi R(\underline{s}) = -\mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma\mathbb{E}[T_S \mid \underline{s}]. \tag{A.142}$$

Using this in the binding participation constraint (22) yields

$$0 = -\mathbb{E}[T_S] + \mathbb{P}(\underline{s})\gamma\mathbb{E}[T_S \mid \underline{s}] - \mathbb{P}(\underline{s})\gamma\phi R(\underline{s}) - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s}) \tag{A.143}$$

$$\Leftrightarrow 0 = \mathbb{E}[T_B] - \mathbb{P}(\underline{s})\gamma C + \mathbb{P}(\underline{s})\gamma\rho\phi R(\underline{s}) - \mathbb{P}(\underline{s})\gamma\phi R(\underline{s}) - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s})$$

$$\Leftrightarrow 0 = \mathbb{E}[T_B] - \mathbb{P}(\underline{s})\gamma C - \mathbb{P}(\underline{s})\gamma(1-\rho)\phi R(\underline{s}) - \mathbb{P}(\underline{s})(1-\gamma)(1-k)\alpha R(\underline{s})$$

$$\Leftrightarrow \mathbb{E}[T_B] = \mathbb{P}(\underline{s})(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\alpha R(\underline{s})) =: m, \tag{A.144}$$

where m is the markup of the contract.

Full insurance implies that $\tilde{e} - T_B(\tilde{s}, \tilde{e}) \equiv \mathbb{E}[\tilde{e} - T_B(\tilde{s}, \tilde{e})]$, where the right hand side is equal to $\pi \cdot 1 + (1-\pi) \cdot 0 - m$. Thus,

$$1 - T_B(\tilde{s}, \bar{e}) = \pi - m \tag{A.145}$$

$$\text{and } 0 - T_B(\tilde{s}, \underline{e}) = \pi - m. \tag{A.146}$$

Using the CCP's budget constraints, this implies that

$$1 + T_S(\bar{s}, \bar{e}) = \pi - m \tag{A.147}$$

$$0 + T_S(\bar{s}, \underline{e}) = \pi - m \tag{A.148}$$

$$1 + T_S(\underline{s}, \bar{e}) - \gamma(\mathbb{E}[T_S \mid \underline{s}] + C - \rho\phi R(\underline{s})) = \pi - m \tag{A.149}$$

$$T_S(\underline{s}, \underline{e}) - \gamma(\mathbb{E}[T_S \mid \underline{s}] + C - \rho\phi R(\underline{s})) = \pi - m. \tag{A.150}$$

Taking the $\mathbb{P}(\cdot \mid \underline{s})$-weighted sum of the last two equations yields

$$\mathbb{P}(\bar{e} \mid \underline{s}) + \mathbb{E}[T_S \mid \underline{s}] - \gamma(\mathbb{E}[T_S \mid \underline{s}] + C - \rho\phi R(\underline{s})) = \pi - m \tag{A.151}$$

$$\Leftrightarrow \mathbb{P}(\bar{e} \mid \underline{s}) + (1-\gamma)\mathbb{E}[T_S \mid \underline{s}] = \pi - m + \gamma(C - \rho\phi R(\underline{s})) \tag{A.152}$$

$$\Leftrightarrow \mathbb{E}[T_S \mid \underline{s}] = \frac{\pi - \underline{\pi} - m + \gamma(C - \rho\phi R(\underline{s}))}{1-\gamma}. \tag{A.153}$$

Therefore, transfers are given by

$$T_S(\bar{s},\bar{e}) = -(1-\pi) - m \tag{A.154}$$

$$T_S(\bar{s},\underline{e}) = \pi - m \tag{A.155}$$

$$T_S(\underline{s},\bar{e}) = -(1-\pi) + \gamma\left(\frac{\pi - \underline{\pi} - m + \gamma(C - \rho\phi R(\underline{s}))}{1-\gamma} + C - \rho\phi R(\underline{s})\right) - m \tag{A.156}$$

$$= -(1-\pi) + \gamma\frac{\pi - \underline{\pi} - m + C - \rho\phi R(\underline{s})}{1-\gamma} - m \tag{A.157}$$

$$T_S(\underline{s},\underline{e}) = \pi + \gamma\frac{\pi - \underline{\pi} - m + C - \rho\phi R(\underline{s})}{1-\gamma} - m. \tag{A.158}$$

□

Proof of Proposition 9. The optimal contract maximizes expected buyer utility (4) subject to the CCP's budget constraints (3) and (20), and the participation constraint (22) and resource constraints of safe sellers (9) and (10) and that of fragile sellers (29). Therefore, the partial derivative of the Lagrangian with respect to $T_B(\tilde{s},\tilde{e})$, $T_S(\tilde{s},\tilde{e})$, and α, respectively, are

$$[\partial T_B(\tilde{s},\tilde{e})] \; -\mathbb{P}(\tilde{s},\tilde{e})u'(\tilde{s},\tilde{e}) + \eta(\tilde{s},\tilde{e}) \tag{A.159}$$

$$[\partial T_S(\bar{s},\tilde{e})] \; \eta(\bar{s},\tilde{e}) - \xi\mathbb{P}(\bar{s},\tilde{e}) \tag{A.160}$$

$$[\partial T_S(\underline{s},\underline{e})] \; \eta(\underline{s},\underline{e})(1-\gamma\mathbb{P}(\underline{e}\mid\underline{s})) - \eta(\underline{s},\bar{e})\gamma\mathbb{P}(\underline{e}\mid\underline{s}) - \xi\mathbb{P}(\underline{s},\underline{e})(1-\gamma) - \sigma_{S,2}(\underline{s},\underline{e}) \tag{A.161}$$

$$[\partial T_S(\underline{s},\bar{e})] \; \eta(\underline{s},\bar{e})(1-\gamma\mathbb{P}(\bar{e}\mid\underline{s})) - \eta(\underline{s},\underline{e})\gamma\mathbb{P}(\bar{e}\mid\underline{s}) - \xi\mathbb{P}(\underline{s},\bar{e})(1-\gamma) - \sigma_{S,2}(\underline{s},\bar{e}) \tag{A.162}$$

$$[\partial\alpha] \; -\xi\mathbb{P}(\underline{s})(1-\gamma)(1-k)R(\underline{s}) + \sigma_{F,1} - (\sigma_{S,2}(\underline{s},\underline{e}) + \sigma_{S,2}(\underline{s},\bar{e}))(1-k)R(\underline{s}), \tag{A.163}$$

An interior solution satisfies the conditions with equality. We assume that the resource constraints of safe sellers after a negative signal are not binding and, thus, $\sigma_{S,2}(\underline{s},\tilde{e}) \equiv 0$.

Margin An interior solution for the margin has (A.163) equal to zero:

$$\xi\mathbb{P}(\underline{s})(1-\gamma)(1-k)R(\underline{s}) = \sigma_{F,1}, \tag{A.164}$$

which, given the binding participation constraint $\xi > 0$ implies that $\sigma_{F,1} > 0$. Therefore, the optimal margin is given by $\alpha^* = \phi$.

Risk sharing Combining (A.161) and (A.162) yields

$$\begin{aligned}
&\frac{1}{\mathbb{P}(\underline{s},\bar{e})}\Big(\eta(\underline{s},\bar{e})(1-\gamma\mathbb{P}(\bar{e}\mid\underline{s}))-\eta(\underline{s},\underline{e})\gamma\mathbb{P}(\bar{e}\mid\underline{s})-\xi\mathbb{P}(\underline{s},\bar{e})(1-\gamma)\Big)\\
&-\frac{1}{\mathbb{P}(\underline{s},\underline{e})}\Big(\eta(\underline{s},\underline{e})(1-\gamma\mathbb{P}(\underline{e}\mid\underline{s}))-\eta(\underline{s},\bar{e})\gamma\mathbb{P}(\underline{e}\mid\underline{s})-\xi\mathbb{P}(\underline{s},\underline{e})(1-\gamma)\Big) &\text{(A.165)}\\
=&\frac{\eta(\underline{s},\bar{e})}{\mathbb{P}(\underline{s},\bar{e})}-\gamma\frac{\eta(\underline{s},\underline{e})+\eta(\underline{s},\bar{e})}{\mathbb{P}(\underline{s})}-\xi(1-\gamma)-\left(\frac{\eta(\underline{s},\underline{e})}{\mathbb{P}(\underline{s},\underline{e})}-\gamma\frac{\eta(\underline{s},\underline{e})+\eta(\underline{s},\bar{e})}{\mathbb{P}(\underline{s})}-\xi(1-\gamma)\right) &\text{(A.166)}\\
=&\frac{\eta(\underline{s},\bar{e})}{\mathbb{P}(\underline{s},\bar{e})}-\frac{\eta(\underline{s},\underline{e})}{\mathbb{P}(\underline{s},\underline{e})} &\text{(A.167)}\\
=&u'(\underline{s},\bar{e})-u'(\underline{s},\underline{e})=0, &\text{(A.168)}
\end{aligned}$$

where in the last step we use (A.159). Therefore, $u'(\underline{s},\bar{e})=u'(\underline{s},\underline{e})$. (A.160) together with (A.159) imply that $\xi=u'(\bar{s},\underline{e})=u'(\bar{s},\bar{e})$. Using this in (A.162) and setting it to zero yields

$$\begin{aligned}
&u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma\mathbb{P}(\bar{e}\mid\underline{s}))-u'(\underline{s},\underline{e})\mathbb{P}(\underline{s},\underline{e})\gamma\mathbb{P}(\bar{e}\mid\underline{s})-u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) &\text{(A.169)}\\
&=u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma\mathbb{P}(\bar{e}\mid\underline{s}))-u'(\underline{s},\bar{e})\gamma\mathbb{P}(\underline{s},\underline{e})\mathbb{P}(\bar{e}\mid\underline{s})-u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) &\text{(A.170)}\\
&=u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma\mathbb{P}(\bar{e}\mid\underline{s}))-u'(\underline{s},\bar{e})\gamma\mathbb{P}(\underline{e}\mid\underline{s})\mathbb{P}(\underline{s},\bar{e})-u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) &\text{(A.171)}\\
&=u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})[1-\gamma\mathbb{P}(\bar{e}\mid\underline{s})-\gamma\mathbb{P}(\underline{e}\mid\underline{s})]-u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) &\text{(A.172)}\\
&=u'(\underline{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma)-u'(\bar{s},\bar{e})\mathbb{P}(\underline{s},\bar{e})(1-\gamma) &\text{(A.173)}\\
&=0, &\text{(A.174)}
\end{aligned}$$

which is equivalent to $u'(\underline{s},\bar{e})=u'(\bar{s},\bar{e})$ and, thus, $u'(\tilde{s},\tilde{e})\equiv\xi$, i.e., buyers are fully insured. Thus, the optimal contract has the transfers from Proposition 8.

Defaults of fragile sellers For any margin $\alpha>\phi$, resources of fragile sellers are not sufficient after a negative signal at $t=1$, which induces them to default.

No defaults of safe seller Because $\alpha^*=\phi<1$, the resource constraint (9) for safe sellers at $t=1$ is satisfied and, thus, safe sellers do not default at $t=1$.

The optimal transfers satisfy $T_S(\underline{s},\underline{e})>T_S(\underline{s},\bar{e})$. Therefore, to satisfy the resource constraint

(10) in both states $(\underline{s}, \underline{e})$ and $(\underline{s}, \bar{e})$, it is necessary and sufficient that

$$
\begin{aligned}
& T_S(\underline{s}, \underline{e}) \leq R(\underline{s}) - (1-k)\alpha R(\underline{s}) && \text{(A.175)} \\
\Leftrightarrow & \pi + \gamma \frac{\pi - \underline{\pi} - m + C - \rho\phi R(\underline{s})}{1-\gamma} - m \leq R(\underline{s}) - (1-k)\alpha R(\underline{s}) && \text{(A.176)} \\
\Leftrightarrow & (1-\gamma)\pi + \gamma(\pi - \underline{\pi} - m + C - \rho\phi R(\underline{s})) \\
& - (1-\gamma)m \leq (1-\gamma)R(\underline{s}) - (1-\gamma)(1-k)\alpha R(\underline{s}) && \text{(A.177)} \\
\Leftrightarrow & \pi + \gamma(R(\underline{s})(1-(1-k)\alpha) - \underline{\pi} + C - \rho\phi R(\underline{s})) \\
& - m \leq R(\underline{s}) - (1-k)\alpha R(\underline{s}) && \text{(A.178)} \\
\Leftrightarrow & \pi + \gamma(R(\underline{s})(1-(1-k)\alpha) - \underline{\pi} + C - \rho\phi R(\underline{s})) \\
& - \mathbb{P}(\underline{s})(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\alpha R(\underline{s})) \leq R(\underline{s}) - (1-k)\alpha R(\underline{s}) && \text{(A.179)} \\
\Leftrightarrow & \pi + \gamma[R(\underline{s}) - \mathbb{P}(\bar{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} + \mathbb{P}(\bar{s})C - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s})] \\
& \leq R(\underline{s}) - \mathbb{P}(\bar{s})(1-k)\alpha R(\underline{s}) && \text{(A.180)} \\
\Leftrightarrow & \gamma \leq \frac{R(\underline{s}) - \mathbb{P}(\bar{s})(1-k)\alpha R(\underline{s}) - \pi}{R(\underline{s}) - \mathbb{P}(\bar{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\bar{s})C} =: \overline{\gamma}^R, && \text{(A.181)}
\end{aligned}
$$

using that $m = \mathbb{P}(\underline{s})(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\alpha R(\underline{s}))$. The numerator is strictly positive if, and only if,

$$
\begin{aligned}
& R(\underline{s}) > \mathbb{P}(\bar{s})(1-k)\phi R(\underline{s}) + \pi && \text{(A.182)} \\
\Leftrightarrow & \phi < \frac{R(\underline{s}) - \pi}{\mathbb{P}(\bar{s})(1-k)R(\underline{s})} =: \phi^*, && \text{(A.183)}
\end{aligned}
$$

which is the condition for the resource constraint being satisfied for $\gamma = 0$. Therefore, due to $R(\underline{s}) > \pi$ by assumption, the numerator is positive for all $\phi < \phi^*$ with $\phi^* > 0$.

The denominator of $\overline{\gamma}^R$ is strictly positive if, and only if,

$$
\begin{aligned}
& R(\underline{s}) - \mathbb{P}(\bar{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\bar{s})C > 0 && \text{(A.184)} \\
\Leftrightarrow\ & R(\underline{s}) - \mathbb{P}(\bar{s})(1-k)\phi R(\underline{s}) - \underline{\pi} + \mathbb{P}(\bar{s})C > (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) && \text{(A.185)} \\
\Leftrightarrow\ & R(\underline{s}) - \underline{\pi} + \mathbb{P}(\bar{s})C > (\rho + \mathbb{P}(\underline{s})(1-\rho) + \mathbb{P}(\bar{s})(1-k))\phi R(\underline{s}), && \text{(A.186)}
\end{aligned}
$$

using $\alpha = \phi$. Assuming that the numerator is strictly positive, a sufficient condition is that

$$
\begin{aligned}
& \mathbb{P}(\bar{s})(1-k)\phi R(\underline{s}) + \pi - \underline{\pi} + \mathbb{P}(\bar{s})C > (\rho + \mathbb{P}(\underline{s})(1-\rho) + \mathbb{P}(\bar{s})(1-k))\phi R(\underline{s}) && \text{(A.187)} \\
\Leftrightarrow\ & \pi - \underline{\pi} + \mathbb{P}(\bar{s})C > (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) && \text{(A.188)} \\
\Leftrightarrow\ & \frac{\pi - \underline{\pi} + \mathbb{P}(\bar{s})C}{(\rho + \mathbb{P}(\underline{s})(1-\rho))R(\underline{s})} > \phi, && \text{(A.189)}
\end{aligned}
$$

which holds for all $C \geq 0$ if

$$\phi < \frac{\pi - \underline{\pi}}{(\rho + \mathbb{P}(\underline{s})(1-\rho))R(\underline{s})} =: \phi^{**}. \tag{A.190}$$

Therefore, a sufficient condition for both the numerator and denominator to be positive is that $\phi < \min\{\phi^*, \phi^{**}\} =: \phi^R$.

Moreover, it is

$$\overline{\gamma}^R < 1 \tag{A.191}$$
$$\Leftrightarrow R(\underline{s}) - \mathbb{P}(\bar{s})(1-k)\alpha R(\underline{s}) - \pi < R(\underline{s}) - \mathbb{P}(\bar{s})(1-k)\alpha R(\underline{s}) - \underline{\pi}$$
$$- (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\bar{s})C \tag{A.192}$$
$$\Leftrightarrow \pi - \underline{\pi} + \mathbb{P}(\bar{s})C > (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}), \tag{A.193}$$

which is satisfied if $\phi < \phi^R$.

Finally, safe sellers do not default at $t = 1$ if, and only if, $\alpha \leq 1$, which holds because $\alpha^* = \phi < 1$.

Outsiders Outsiders have assets that pay R_O at $t = 2$ and make payments $T_S - p$. Thus, their resources are sufficient to make payments on the derivative contract if, and only if,

$$T_S(\underline{s}, \underline{e}) - p \leq R_O \tag{A.194}$$
$$\Leftrightarrow T_S(\underline{s}, \underline{e}) - \mathbb{E}[T_S \mid \underline{s}] \leq R_O \tag{A.195}$$
$$\Leftrightarrow \pi - m + \gamma(\mathbb{E}[T_S \mid \underline{s}] + C - \rho\phi R(\underline{s})) - \mathbb{E}[T_S \mid \underline{s}] \leq R_O \tag{A.196}$$
$$\Leftrightarrow \pi - m + \gamma(C - \rho\phi R(\underline{s})) - (1-\gamma)\mathbb{E}[T_S \mid \underline{s}] \leq R_O \tag{A.197}$$
$$\Leftrightarrow \pi - m + \gamma(C - \rho\phi R(\underline{s})) - (1-\gamma)\frac{\pi - \underline{\pi} - m + \gamma(C - \rho\phi R(\underline{s}))}{1-\gamma} \leq R_O \tag{A.198}$$
$$\Leftrightarrow \pi - m + \gamma(C - \rho\phi R(\underline{s})) - (\pi - \underline{\pi} - m + \gamma(C - \rho\phi R(\underline{s}))) \leq R_O \tag{A.199}$$
$$\Leftrightarrow \underline{\pi} \leq R_O. \tag{A.200}$$

Thus, using $\pi > \underline{\pi}$, it is sufficient that $R_O \geq \pi$. □

Proof of Proposition 10.

(1) Using Propositions 5 and 9, it is (using $\alpha = \phi$ in case of replacement)

$$\overline{\gamma}^{NR} < \overline{\gamma}^{R} \tag{A.201}$$

$$\Leftrightarrow \frac{R(\underline{s}) - \pi}{R(\underline{s})(1 - (\mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \rho)\phi)} < \frac{R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \pi}{R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\overline{s})C} \tag{A.202}$$

$$\Leftrightarrow (R(\underline{s}) - \pi)\,\big(R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) - \pi + \pi - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) \\ + \mathbb{P}(\overline{s})C\big) < R(\underline{s})(1 - (\mathbb{P}(\underline{s}, \underline{e})(1-\rho) + \rho)\phi)(R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) - \pi) \tag{A.203}$$

$$\Leftrightarrow \Big(R(\underline{s}) - \pi\Big)\Big(\underbrace{\pi - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\overline{s})C}_{>0}\Big) \\ < \Big(\underbrace{\pi - (\rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho))\phi R(\underline{s})}_{>0}\Big)\Big(\underbrace{R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) - \pi}_{>0}\Big). \tag{A.204}$$

using that by Assumption 1, it is

$$\pi > \phi R(\underline{s}) = (\rho + (1-\rho))\phi R(\underline{s}) > (\rho + \mathbb{P}(\underline{s}, \underline{e})(1-\rho))\phi R(\underline{s}). \tag{A.205}$$

Because $\mathbb{P}(\underline{s}, \underline{e}) < \mathbb{P}(\underline{s})$, it is sufficient that

$$\Big(R(\underline{s}) - \pi\Big)\Big(\pi - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\overline{s})C\Big) \\ < \Big(\pi - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s})\Big)\Big(R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) - \pi\Big) \tag{A.206}$$

$$\Leftrightarrow \Big(R(\underline{s}) - \pi\Big)\Big(-\underline{\pi} + \mathbb{P}(\overline{s})C\Big) \\ < \Big(\pi - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s})\Big)\Big(R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) - \pi - R(\underline{s}) + \pi\Big) \tag{A.207}$$

$$\Leftrightarrow \Big(R(\underline{s}) - \pi\Big)\Big(-\underline{\pi} + \mathbb{P}(\overline{s})C\Big) \\ < \Big(\pi - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s})\Big)\Big(-\mathbb{P}(\overline{s})(1-k)\phi R(\underline{s})\Big) \tag{A.208}$$

$$\Leftrightarrow \mathbb{P}(\overline{s})C < \underline{\pi} - \frac{\pi - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s})}{R(\underline{s}) - \pi}\mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) = \hat{C}. \tag{A.209}$$

It remains to be shown that the RHS is positive:

$$\underline{\pi} - \frac{\pi - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s})}{R(\underline{s}) - \pi}\mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) > 0 \tag{A.210}$$

$$\Leftrightarrow \underline{\pi}(R(\underline{s}) - \pi) > \Big(\pi - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s})\Big)\mathbb{P}(\overline{s})(1-k)\phi R(\underline{s}) \tag{A.211}$$

It is sufficient that

$$\underline{\pi}(R(\underline{s}) - \pi) > \pi \mathbb{P}(\bar{s})(1-k)\phi R(\underline{s}) \tag{A.212}$$

$$\frac{\underline{\pi}}{\pi} \cdot \frac{(R(\underline{s}) - \pi)}{\mathbb{P}(\bar{s})(1-k)R(\underline{s})} > \phi \tag{A.213}$$

$$\tag{A.214}$$

(2) If $\gamma > \overline{\gamma}^{NR}$, then Proposition 6 implies that the optimal contract without replacement provides less than full insurance to buyers, with expected utility

$$\mathbb{E}[u(\tilde{e} - T_B)] = u(\mathbb{E}[\tilde{e}] - m^{NR} - g^{NR}), \tag{A.215}$$

where $m^{NR} = \mathbb{E}[T_B]$ is the markup of that contract and $g^{NR} > 0$ is the risk premium for the remaining consumption risk $\tilde{e} - T_B$.

If $\gamma \leq \overline{\gamma}^{R}$, then Proposition 9 implies that the optimal contract with replacement provides full insurance, with expected utility

$$\mathbb{E}[u(\tilde{e} - T_B)] = u(\mathbb{E}[\tilde{e}] - m^{R}), \tag{A.216}$$

where $m^{R} = \mathbb{E}[T_B]$ is the markup of that contract.

Therefore, replacement is efficient if, and only if,

$$u(\mathbb{E}[\tilde{e}] - m^{NR} - g^{NR}) \leq u(\mathbb{E}[\tilde{e}] - m^{R}) \tag{A.217}$$

$$\Leftrightarrow m^{R} - m^{NR} \leq g^{NR}. \tag{A.218}$$

□

Proof of Example 1. Consider the full-insurance contract with replacement when $\phi = 0$ and $C = 0$. Then,

$$\overline{\gamma}^{R} = \frac{R(\underline{s}) - \pi}{R(\underline{s}) - \underline{\pi}}, \tag{A.219}$$

$$\text{and } \overline{\gamma}^{NR} = \frac{R(\underline{s}) - \pi}{R(\underline{s})}. \tag{A.220}$$

Note that $\overline{\gamma}^{NR} < \overline{\gamma}^{R} < 1$.

Because $\gamma \leq \overline{\gamma}^{R}$, the optimal contract with replacement provides full insurance. Denote by $m^{R} = \mathbb{P}(\underline{s})(\gamma C + \gamma(1-\rho)\phi R(\underline{s}) + (1-\gamma)(1-k)\phi R(\underline{s}))$ the markup of this contract. It is equal to $m^{R} = 0$ for $\phi = 0$ and $C = 0$.

Because $\gamma > \overline{\gamma}^{NR}$, the optimal contract without replacement provides partial insurance. The markup of this contract is either equal to $m^{NR}_{\text{wo}} = 0$ in the case without defaults at $t = 2$ or equal to $m^{NR}_{\text{w}} = \mathbb{P}(\underline{s})(1-k)\alpha R \geq 0$ in the case of defaults at $t = 2$.

Thus, the contract with replacement implements strictly more risk sharing and has a (weakly) lower markup $m^R = 0 \leq \min\{m^{NR}_{\text{wo}}, m^{NR}_{\text{w}}\}$. Thus, it dominates the optimal contract without replacement. □

The following lemma will be useful for the remaining proof:

Lemma 1. *The conditional probability of the good endowment state after a low signal is decreasing with the informativeness of the signal:* $\frac{\partial \underline{\pi}}{\partial \lambda} < 0$.

Proof. It is

$$\frac{\partial \underline{\pi}}{\partial \lambda} = \frac{\partial}{\partial \lambda} \frac{(1-\lambda)\pi}{(1-\lambda)\pi + \lambda(1-\pi)} \tag{A.221}$$

$$= \frac{-\pi((1-\lambda)\pi + \lambda(1-\pi)) - (1-\lambda)\pi(1-2\pi)}{((1-\lambda)\pi + \lambda(1-\pi))^2} \tag{A.222}$$

$$= \pi \frac{-(1-\lambda)\pi - \lambda(1-\pi) + 2(1-\lambda)\pi - (1-\lambda)}{((1-\lambda)\pi + \lambda(1-\pi))^2} \tag{A.223}$$

$$= \pi \frac{-\lambda + \pi\lambda + \pi - \lambda\pi - 1 + \lambda}{((1-\lambda)\pi + \lambda(1-\pi))^2} \tag{A.224}$$

$$= -\pi \frac{1-\pi}{((1-\lambda)\pi + \lambda(1-\pi))^2} < 0. \tag{A.225}$$

□

Proof of Proposition 11. $\overline{\gamma}^R$ is decreasing with λ as

$$\frac{\partial \overline{\gamma}^R}{\partial \lambda} \tag{A.226}$$

$$= \frac{\partial}{\partial \lambda} \frac{R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \pi}{R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\overline{s})C} \tag{A.227}$$

$$= \frac{\partial \underline{\pi}}{\partial \lambda} \frac{\partial}{\partial \underline{\pi}} \frac{R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \pi}{R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\overline{s})C} \tag{A.228}$$

$$= \underbrace{\frac{\partial \underline{\pi}}{\partial \lambda}}_{<0} \underbrace{\frac{\partial}{\partial \underline{\pi}} \frac{-(-1)(R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \pi)}{(R(\underline{s}) - \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) - \underline{\pi} - (\rho + \mathbb{P}(\underline{s})(1-\rho))\phi R(\underline{s}) + \mathbb{P}(\overline{s})C)^2}}_{>0}, \tag{A.229}$$

using Proposition 1 and that Assumption 3 implies that $R(\underline{s}) > \mathbb{P}(\overline{s})(1-k)\alpha R(\underline{s}) + \pi$.

In the case without replacement:

$$\frac{\partial \overline{\gamma}^{NR}}{\partial \lambda} \tag{A.230}$$

$$= \frac{\partial}{\partial \lambda} \frac{R(\underline{s}) - \pi}{R(\underline{s})(1 - (\mathbb{P}(\underline{s} \mid \underline{e})\mathbb{P}(\underline{e})(1-\rho) + \rho)\phi)} \tag{A.231}$$

$$= \frac{\partial}{\partial \lambda} \frac{R(\underline{s}) - \pi}{R(\underline{s})(1 - (\lambda\mathbb{P}(\underline{e})(1-\rho) + \rho)\phi)} \tag{A.232}$$

$$= (-1) \frac{R(\underline{s}) - \pi}{R(\underline{s})(1 - (\lambda\mathbb{P}(\underline{e})(1-\rho) + \rho)\phi))^2} R(\underline{s})(-1)\mathbb{P}(\underline{e})(1-\rho)\phi \tag{A.233}$$

$$= \frac{R(\underline{s}) - \pi}{R(\underline{s})(1 - (\lambda\mathbb{P}(\underline{e})(1-\rho) + \rho)\phi))^2} R(\underline{s})\mathbb{P}(\underline{e})(1-\rho)\phi \geq 0. \tag{A.234}$$

Therefore,

$$\frac{\partial(\overline{\gamma}^{R} - \overline{\gamma}^{NR})}{\partial \lambda} = \underbrace{\frac{\partial \overline{\gamma}^{R}}{\partial \lambda}}_{<0} - \underbrace{\frac{\partial \overline{\gamma}^{NR}}{\partial \lambda}}_{\geq 0} < 0. \tag{A.235}$$

□