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Internal versus External Liquidity: Investment Efficiency under Market Frictions

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Abstract

This paper analyses how firms respond to liquidity shocks when asset prices are endogenously determined through matching frictions. Building on Holmstrom and Tirole (1998), our key innovation is that pecuniary externalities become technological when interacting with pledgeable asset constraints: fire-sale prices reduce effective pledgeable income and distort both investment scale and effort incentives, even without nominal changes to credit limits. Firms sort endogenously into internal liquidity (hoarding) and external liquidity (asset sales) based on heterogeneous shock probabilities. The competitive equilibrium features excessive reliance on external liquidity because firms fail to internalise how their strategy choice depresses fire-sale prices for others. Introducing asymmetric information amplifies these distortions: pooled debt contracts attract lower-productivity firms into external strategies, weakening effort incentives more severely. Investor behaviour creates a further feedback loop, with competitive liquidity provision paradoxically depressing fire-sale returns and distorting firm strategies. Policy interventions can restore efficiency: liquidity guarantees dominate under symmetric information, while government asset purchases at socially efficient prices improve outcomes under asymmetric information without exacerbating adverse selection.

JEL Classification: E22; E44; G01; G32; D58

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1 Introduction

Liquidity shocks pose fundamental challenges for firms operating under financial constraints. When cash flow shortfalls arise, whether from unexpected costs, delayed payments, or adverse market conditions, firms must decide how to fund continuation. This paper examines a critical but understudied dimension of liquidity management: the choice between hoarding internal reserves and relying on external asset sales, and how this choice creates systemic inefficiencies through endogenous asset price formation. The trade-off is stark. Internal liquidity, holding cash reserves, provides insurance against shocks but forces firms to sacrifice profitable investment opportunities. External liquidity, investing fully and selling assets if needed, maximises initial scale but exposes firms to fire-sale risk when many sellers meet few buyers. This tension becomes acute during crises: the 2008 financial panic, the March 2020 "dash for cash," and the 2022 UK gilt market turmoil all featured simultaneous liquidity demands that depressed asset prices and amplified real economic distortions.

Existing research has illuminated important pieces of this puzzle but not how they fit together. Holmström and Tirole (1998) established the canonical framework for analysing firm liquidity demand under moral hazard, showing how pledgeable asset constraints limit both investment scale and continuation financing. However, their analysis takes asset prices as exogenous parameters, leaving open how market conditions for distressed sales emerge endogenously from firms' collective strategies. The fire sales literature, pioneered by Shleifer and Vishny (1992) and extended by Kiyotaki and Moore (1997), demonstrated that asset sales during distress generate pecuniary externalities—each seller's actions depress prices for others. The foundational insight that pecuniary externalities can generate real inefficiencies when markets are incomplete comes from Greenwald and Stiglitz (1986). More recently, Lorenzoni (2008) and Dávila and Korinek (2018) formalised when such price effects create genuine inefficiencies rather than mere redistributions, distinguishing "collateral externalities", where constraints depend directly on prices, from "distributive externalities" arising purely from wealth transfers. Yet this literature focuses primarily on ex post crisis dynamics without modelling the firm's strategic choice between alternative liquidity provision mechanisms.

Recent work on liquidity hoarding (Gale and Yorulmazer 2013; Kahn and Wagner 2021) emphasises how one institution's cash reserves affect economy-wide liquidity availability. However, their focus is on the quantity of available liquidity rather than how investor participation affects asset prices and how firms sort between strategies based on these endogenous prices.

This paper integrates three features that prior work has studied separately: (i) endogenous firm sorting between internal liquidity (cash hoarding) and external liquidity (planned asset sales) based on heterogeneous shock probabilities; (ii) matching frictions in asset markets where strategic buyers (hoarders with unused reserves) compete with financial investors through proportional rationing; and (iii) pecuniary externalities that become technological by affecting effective pledgeable income, even though nominal pledgeability remains fixed.

The mechanism works as follows. Firms differ in their liquidity shock probability, v. At date $t = 0$, each firm borrows up to its pledgeable asset limit P and chooses whether to hold cash reserves L (investing only $P - L$ in productive capital) or invest the full amount P, planning to sell assets if a shock occurs at date $t = 1$. Asset prices are determined by matching between distressed sellers (non-hoarders hit by shocks), strategic buyers (hoarders who avoided shocks), and financial investors who allocate portfolios between productive capital and liquidity provision. This creates a pecuniary externality that becomes technological through its interaction with pledgeable asset constraints. While the nominal constraint P remains fixed, it reflects enforcement technology—anticipated fire-sale losses reduce effective pledgeable income to $P_{\text{effective}} = P - (1 - v)L/q$. When fire-sale prices q^F fall due to poor market conditions, more assets must be sold to raise required liquidity L, tightening the real borrowing constraint and distorting both investment scale and effort incentives. Critically, each firm choosing external liquidity doesn't internalise that this choice contributes to lower q^F, which tightens effective constraints for all external-strategy firms.

We establish four key findings: First, the competitive equilibrium features a systematic bias toward external liquidity: too many firms rely on asset sales (the threshold v_c^* is too low) because they don't internalise how their strategy choice affects fire-sale severity for others. This inefficiency persists even though firms rationally anticipate equilibrium prices, the externality operates through strategic complementarity in liquidity provision. Second, introducing asymmetric information about firm productivity amplifies these distortions. Under pooled debt contracts, external liquidity both (i) attracts lower-productivity firms (adverse selection: the marginal type satisfies $a^{**} < a^*$) and (ii) weakens effort incentives more severely than internal liquidity. The social surplus from marginal entrepreneurs becomes more negative under external strategies, generating excessive entry of low-quality firms and compounding the fire-sale externality with selection problems. Third, investor behaviour creates a feedback loop that amplifies crisis dynamics. Investors choosing liquidity allocation M at date $t = 0$ don't internalise how this affects firm sorting (v^*), which determines the buyer-seller ratio at date $t = 1$. The competitive equilibrium features excessive investor liquidity provision chasing fire-sale returns, which paradoxically depresses those returns and distorts firm strategies. Fourth, policy interventions can restore efficiency, but the optimal tool depends critically on the information structure. When firm productivity is observable, liquidity guarantees, allowing firms to borrow L against future returns without violating date-0 pledgeable asset constraints—eliminate fire sales while preserving first-best effort incentives. When productivity is private information, government asset purchases at socially efficient prices improve outcomes without exacerbating adverse selection.

The paper proceeds as follows. Section 2 presents the baseline model, contrasting internal and external liquidity strategies. Section 3 introduces asset market dynamics and matching frictions. Section 4 adds financial investors and analyses equilibrium asset pricing. Section 5 formalises the externality mechanism, comparing competitive equilibrium to the social planner's solution. Section 6 extends the model to asymmetric information. Section 7 evaluates policy interventions. Section 8 concludes.

2 Baseline Model of Liquidity Management[1]

Entrepreneurs initially choose investment in a project or technology with which they are endowed. They must also choose a financial policy and effort to maximise expected wealth. Firms are subject to liquidity shocks and must take this into account when choosing their strategies. Firms invest in capital at date $t = 0$ that yields a return at date $t = 2$. At an intermediate date $t = 1$, firms are faced with a liquidity shock, which is intrinsic to the project. Investment is financed with borrowing. Firm's investment policy is constrained by capital market constraints, in particular the total level of financing is constrained by credit rationing because of limited pledgeable assets, P. Let the face value of debt be D, the promised payment to financiers. Funds raised can be used to finance investment, K, or held as liquid cash, C. The investment yields a return at date $t = 2$ of aXK, with probability $\theta(E)$, return rate $X \geq 1$ and productivity parameter $a > 1$. The productivity parameter at this stage is assumed to fixed and common to all entrepreneurs, so at this stage we are assuming that $aX > 1$. The success probability $\theta(E)$, is a function of effort, with $\theta' > 0$ and $\theta'' < 0$ and cost of effort E. Liquid assets, C, can be used to insure against a negative liquidity shock,

[1]As the paper uses a lot of notation, the appendix includes a glossary of the main terms used.

L, which occurs with probability $(1 - v)$ at date $t = 1$.[2] The liquidity shock and its probability of occurring are a characteristic of the entrepreneurs project and are independent of any financing choices. The marginal cost of funds and the return on cash holdings is zero.

The alternative to the above investment and funding strategy is not to hold cash and to invest more in which case, in the event of a liquidity shock of size L, to cover the cost of the shock the firm must sell assets. The amount of assets that must be sold will depend upon the market for assets and hence the price of assets in the event of the shock. The price achieved will be worse if many firms are faced with the same shock and firms in a similar industry would be the best buyers. If the shock is idiosyncratic, the price will be denoted by q^I, which will be higher than if the shock is systemic, in which case the price is $q_F \leq q_I$. This difference will be positive and bigger, the greater the degree of industry specificity. Note that the price q^F matters to all firms. Individual firms will act to maximise profits and only be concerned with the marginal impact of q_I. However, the price of the assets for sale will reflect the average behaviour of affected firms.

2.1 Timing, Assets, and Financial Constraints

At date $t = 0$, entrepreneurs are endowed with initial pledgeable assets P. These represent the maximum that can be credibly promised to outside financiers, reflecting limited enforcement capacity: only a fraction of expected returns can be pledged, as in Holmström and Tirole (1998). The constraint $B \leq P$ thus represents the upper bound on initial borrowing capacity. Throughout the paper, we assume this constraint binds, so that $B = P$.

Pledgeable income versus project value. It is crucial to distinguish between pledgeable assets and the physical project. The pledgeable asset constraint P reflects what can be credibly committed to lenders at date $t = 0$; this determines initial borrowing capacity. In contrast, the project itself generates a (possibly higher) expected return $\theta(E)aXK$ at date $t = 2$, where K is capital invested, a is productivity, $X \geq 1$ is the return rate, and $\theta(E)$ is the success probability that depends on entrepreneurial effort E. The gap between project value and pledgeable income arises from moral

[2] The liquidity shock L is assumed fixed and independent of investment scale K for tractability, capturing working capital needs (payroll, supplier payments) largely independent of capital stock. Alternatively, we could interpret $L = \lambda K$ as a fraction of capital; results are unchanged.

At $t = 1$, new borrowing is infeasible because pledgeable capacity P is fully committed at $t = 0$. Asset sales work because they transfer property rights without requiring additional pledgeable income (Holmström and Tirole, 1998).

hazard: entrepreneurs must retain sufficient stake in project returns to maintain effort incentives.

When we say the borrowing constraint "binds," we mean that financiers will lend up to the pledgeable limit P, but no more, regardless of the project's fundamental value. This borrowed amount $B = P$ can be allocated in two ways:

- **Internal liquidity (hoarding):** Invest $K^h = P - L$ in the project and hold $C^h = L$ as cash reserves
- **External liquidity (non-hoarding):** Invest the full amount $K^{nh} = P$ in the project, planning to sell assets if a liquidity shock occurs

Thus $K^{nh} = K^h + L$ captures the scale advantage of external liquidity. The trade-off is between project scale and liquidity insurance. Both strategies involve the same initial borrowing ($B = P$), but allocate it differently.

At the interim date $t = 1$, firms may face a liquidity shock L that must be funded to continue the project. Firms following the internal liquidity strategy use their cash reserves $C^h = L$. Firms following the external liquidity strategy must sell a fraction α^{nh} of the ongoing project to raise cash L, where the fraction sold depends on the endogenously determined market price q (analysed in Sections 3–4). As we formalise in Section 5, when $q < 1$, the anticipated losses from fire sales reduce the effective pledgeable income available for initial investment, even though the nominal constraint P remains fixed.

2.2 Hoarding

Let K^h be capital and C^h liquid assets. The firm's liabilities are debt, B raised against pledgeable assets, P, with $B \leq P$, which we assume binds:

$$K^h + C^h = B^h = P \tag{1}$$

The NPV of projects is entrepreneurs' equity. Liquid assets raised, C^h, is liquidity hoarding that is used to cover a liquidity shock of L that occurs with probability $(1 - \nu)$ at date $t = 1$. Because the expected return on the project exceeds unity, the constraint $C^h \geq L$ will be binding, so henceforth, $C^h = L$. The firm invests K^h at date $t = 0$ in a project which generates an income at date $t = 2$ of aXK^h with probability $\theta(E)$.

To facilitate comparison between strategies, we adopt the following notation throughout:

- $K^h = P - L$: investment by hoarders (internal liquidity strategy)
- $K^{\text{nh}} = P$: investment by non-hoarders (external liquidity strategy)
- Therefore: $K^{\text{nh}} = K^h + L$

This highlights the fundamental trade-off: non-hoarders invest an additional L by forgoing cash reserves, but face asset sale risk at $t = 1$.

Assume that cash raised at date $t = 0$ to cover the liquidity shock is used at date $t = 1$ if the shock occurs. If the shock does not occur, the cash is retained in the firm and paid to equity or debt in the good state at date $t = 2$. In the bad state at date $t = 2$, there are two possibilities; either the retained cash is paid to equity or to debt, thereby reducing the face value of debt. Let us first consider that retained cash is paid to equity, then the return on equity if effort is applied is

$$\begin{aligned} U^h = \theta(E^h)v(aXK^h + L - D^h) + \theta(E^h)(1-v)(aXK^h - D^h) \\ +(1-\theta(E^h))vL - E^h = \theta(E^h)(aXK^h - D^h) + vL - E^h \end{aligned} \tag{2}$$

and the debt payment satisfies

$$\theta(E^h)D^h \geq B = K^h + L \tag{3}$$

Simplifying equation (2) by collecting terms, the firm's date $t = 0$ choice of effort E^h is chosen to maximise

$$U^h = \theta(E^h)(aXK^h - D^h) + vL - E^h \tag{4}$$

with first order condition

$$\theta'(E^h)(aXK^h - D^h) - 1 = 0 \tag{5}$$

In the alternative case where retained cash is paid to debt holders rather than equity, the entrepreneur's marginal return to effort, $\theta'(E^h)(aXK^h - D^h)$, is unchanged because the debt face value adjusts one-for-one with the reallocation of residual cash.[3] Hence the FOC (5) and the chosen effort E^h are identical under both conventions,

[3] If residual liquidity is paid to debt, the participation constraint becomes $\theta(E^h)D^h + vL \geq K^h + L$, implying $D^h = (K^h + (1-v)L)/\theta(E^h)$, compared to $D^h = (K^h + L)/\theta(E^h)$ when paid to equity. The debt face value adjusts such that $\theta(E^h)aXK^h - K^h - (1-v)L - E^h$ is identical under both allocations; since the entrepreneur's marginal return to effort depends only on $(aXK^h - D^h)$, effort choice and expected payoffs are equivalent.

yielding the *same expected return to the entrepreneur* after adjusting D^h. We use the equity-payout convention for notational economy only.

2.3 Non-Hoarding

Now suppose that there is no hoarding. Then assuming that capital raised up to the pledgeable assets threshold, P,

$$K^{nh} = B^{nh} = P \tag{6}$$

If borrowing at date $t = 0$ takes place up to this threshold, there is no scope for further borrowing at date $t = 1$ to meet the liquidity shock. The firm could choose a smaller scale of investment and so retain some borrowing capacity as an alternative to meeting the shock through asset sales but this is more expensive than asset sales.[4] Hence, debt at date $t = 0$ is issued to fund investment and the shock L is met with asset sales in the bad state at date $t = 1$, with the firm selling a fraction α^{nh} of the project. Then the equity return is:

$$\begin{aligned} U^{nh} &= \theta(E^{nh})v(aXK^{nh} - D^{nh}) \\ &+\theta(E^{nh})(1-v)(aXK^{nh} - \alpha^{nh}aXK^{nh} - D^{nh}) - E^{nh} \\ &= \theta(E^{nh})(aXK^{nh} - D^{nh}) - (1-v)\alpha^{nh}\theta(E^{nh})aXK^{nh} - E^{nh} \end{aligned} \tag{7}$$

and the investors participation constraint is:

$$\theta(E^{nh})D^{nh} \geq K^{nh} \tag{8}$$

The firm's date $t = 0$ choice of effort E^{nh} is chosen to maximise

$$U^{nh} = \theta(E^{nh})(aXK^{nh} - D^{nh}) - (1-v)\alpha^{nh}\theta(E^{nh})aXK^{nh} - E^{nh} \tag{9}$$

with first order condition

$$\theta'(E^{nh})(aXK^{nh} - D^{nh}) - (1-v)\alpha^{nh}\theta'(E^{nh})aXK^{nh} - 1 = 0 \tag{10}$$

[4] Note that the value of the asset sale at a price of $q < 1$, is $\alpha^{nh}\theta(E^{nh})aXK^{nh} = L/q$ which in turn will exceed the expected value of any additional debt finance to cover the liquidity shortfall, $\theta(E^{nh})\Delta D^{nh} = L$. However, to do this the firm needs to retain borrowing capacity of $\Delta P = L = \Delta K^{nh}$, sacrificing $\theta(E^{nh})X\Delta K^{nh}$ for sure against a potential saving of financing costs at date $t = 1$ with probability $(1-v)$

In this case, the share of the project that needs to be sold to cover the liquidity shock depends upon the price, q, of the share of the project sold, so that the necessary sale satisfies

$$q\alpha^{nh}\theta(E^{nh})aXK^{nh} = L, \text{ which can be written as } \alpha^{nh}\theta(E^{nh})aXK^{nh} = L/q \quad (11)$$

where the price q depends upon the market conditions for assets at date $t = 1$.

Here the price q depends upon the market conditions for assets at date $t = 1$. When fire-sale prices fall below fundamental value ($q < 1$), firms must sell larger fractions of their project to raise the required liquidity L, which reduces both the scale of continuation and the marginal return to effort (equation 10). This creates the pecuniary externality mechanism we analyse formally in Section 5: anticipated fire-sale losses tighten effective borrowing constraints even though the nominal pledgeable asset limit P remains fixed.

2.4 Hoarding versus Non-Hoarding

In order to compare entrepreneur returns under internal and external liquidity we first need to consider the effort choice, by comparing (5) and (10). Consider condition (10) and note that given $K^{nh} = P$ and $K^h = P - L$, then if $q = 1$, condition (10) reduces to condition (5), in which case $E^{nh} = E^h$ and $\theta(E^{nh}) = \theta(E^h)$. The return to the entrepreneur under internal liquidity in (2) would then equal that under external liquidity in (7).

However, if $q < 1$, the entrepreneur following external liquidity must sell more of the project when the liquidity event occurs (since $\alpha^{nh} = L/(q\theta(E^{nh})aXK^{nh}) > L/(aXK^{nh})$), which dilutes effort incentives through condition (10). This implies $E^{nh} < E^h$ and therefore $\theta(E^{nh}) < \theta(E^h)$.

We are now able to compare the return to the entrepreneur under hoarding and non-hoarding. Recall from (2) that the hoarder's return can be written as:

$$U^h = \theta(E^h)aXK^h - (K^h + L) + vL - E^h = \theta(E^h)aX(P - L) - P + vL - E^h \quad (12)$$

From (7), the non-hoarder's return is:

$$U^{nh} = \theta(E^{nh})aXK^{nh} - K^{nh} - (1-v)L/q - E^{nh} = \theta(E^{nh})aXP - P - (1-v)L/q - E^{nh} \quad (13)$$

Thus, hoarding will be preferred if:

$$\theta(E^h)aX(P-L) + vL - E^h > \theta(E^{nh})aXP - (1-v)L/q - E^{nh} \tag{14}$$

and non-hoarding is preferred if the inequality is reversed. Throughout, we assume the shock probability v is distributed according to $F(v)$ on the support $[\underline{v}, \bar{v}] \subset [0, 1]$, where F is continuous and strictly increasing.

Interpretation: The left side represents the net return under internal liquidity: lower investment scale $(P - L)$ but with effort E^h and certain liquidity vL when not hit by shock. The right side represents the net return under external liquidity: higher investment scale P but with lower effort E^{nh} (due to asset sale dilution) and expected fire-sale losses $(1 - v)L/q$. Note that both sides have the common term $-P$ (initial borrowing cost) which cancels.

Key observation: Since $K^{nh} = K^h + L$, the scale benefit of external liquidity is $\theta(E^{nh})aXL$, while its costs include lower effort reducing returns on all capital $(\theta(E^{nh}) < \theta(E^h))$, expected fire-sale losses $(1 - v)L/q$, and foregone safe returns vL from liquid reserves.

Condition (14) reveals that hoarding becomes more attractive as asset prices deteriorate (lower q), liquidity shock probability increases (lower v), project returns diminish (lower aX), or effort distortions widen (larger $\theta(E^h) - \theta(E^{nh})$). Firms facing higher shock probability thus rationally prefer insurance through cash reserves over scale expansion via full investment, particularly when secondary market conditions are poor. This trade-off generates endogenous sorting: heterogeneity in v across firms determines whether they hoard internally or rely on external asset markets, with the marginal firm at $v = v^*$ indifferent between strategies. Absent moral hazard $(\theta' \equiv 0)$ or absent price discounts $(q = 1)$, the two strategies yield identical effort and therefore no sorting; thus both features are jointly necessary for $E_{nh} < E_h$ and for a non-degenerate cutoff v^*.

With no moral hazard $(\theta' \equiv 0)$, asset sales do not dilute effort incentives, so the strategies coincide for any q. With no discount $(q = 1)$, asset sales do not dilute continuation scale, so FOCs coincide for any incentive problem. Hence, both frictions are required for $E^{nh} < E^h$ and for a non-degenerate cutoff v^*.

3 Asset Market Dynamics and Matching Frictions

We distinguish two aggregate states at $t = 1$. In a non-fire-sale state, some hoarders are liquid and act as buyers, so distressed non-hoarders can sell assets at price $q^I \leq 1$. In a fire-sale state, hoarders are also hit by shocks and cannot buy; only specialist investors purchase, driving the price down to $q^F < q^I$. We assume that entrepreneurs are distributed over an interval according to the value of v. At the value v^*, an individual entrepreneur is indifferent between internal and external liquidity. Then, from (14), at any given v^*, given monotonicity and continuity, and given that for $q < 1$, $L < L/q$, lower v entrepreneurs prefer internal liquidity.

We now introduce the possibility of a broader market for assets. Firms that prefer internal liquidity, "hoard liquidity", have the strategic advantage that if they are not hit by a liquidity shock, they will have unused liquidity and will be able to buy assets from firms that are hit by shocks and have to sell assets.

Suppose that at date $t = 1$, hoarders can buy assets from non-hoarders, if they have unused liquidity at a price of $q^I \leq 1$. The conditional (on having funds to purchase assets) probability that they can find a firm willing to sell is given by π_b so there is a probability $(1 - \pi_b)$ that they will not be able to buy assets. We also allow for possible rationing of asset sales through the rationing fraction $0 \leq f \leq 1$, to be examined later. Then the net return on investment with hoarding is

$$U^h = \theta(E^h)aXK^h - E^h + v[\pi_b(f\frac{L}{q^I} + (1-f)L) + (1-\pi_b)L] \tag{15}$$

At this stage we set $f = 1$. The net return to investment and reliance on outside liquidity being a non-hoarder is

$$U^{nh} = \theta(E^{nh})aXK^{nh} - E^{nh} - (1-v)[\pi_s\frac{L}{q^I} + (1-\pi_s)\frac{L}{q^F}] \tag{16}$$

where π_s is the conditional probability of finding a buyer, so being a seller.

At date $t = 0$, firms must decide upon a strategy: to hoard liquidity being a hoarder, or to plan to sell assets in adverse circumstances, being a non-hoarder. However, as can be seen, this will depend upon parameters and crucially upon forecast market conditions, the conditional matching probabilities. The problem with meeting liquidity needs with asset sales is matching sellers to buyers. Note that the possibility of hoarders not being able to buy assets arises if the non-hoarders do not have to sell assets because they are not hit with a liquidity shock. Similarly, the possibility that non-hoarders cannot sell at the price q^I arises if the hoarders are also hit with a liquidity shock and are not in a position to buy. In this case assets have to be sold at a fire-sale price, which we set at $q^F \geq 0$. This is made clearer through the table below.

The states in which trades between firms can take place are by definition non-fire-sale states. In fire-sale states all entrepreneurs need liquidity, so there are no buyers, only sellers. In the non-fire-sale states, in which there are sellers and buyers, if there is a match in the market between a firm with liquidity and one that is selling assets to cover a shortfall, the price of the transaction is q^I, which will be better than selling to an alternative use buyer.[5] In order to generate fire-sale and non-fire-sale states, we add some correlation of shocks, otherwise, with independent shocks sellers will always be able to find buyers.

To illustrate the correlation structure, consider the joint distribution of liquidity shocks across the population. Individual firms differ in their idiosyncratic shock probability v_i, distributed according to F(v). However, shocks are not independent: there is an aggregate component captured by correlation parameters μ and ε. These parameters determine whether liquidity needs are dispersed (allowing trades between firms) or concentrated (creating fire-sale states). Table 1 describes the four possible states based on whether each type of firm (hoarders versus non-hoarders in aggregate) faces shocks. The parameter μ captures the probability that when the aggregate state is favourable, firms avoid shocks. The parameter ε captures the probability of the systemic state where both groups face shocks simultaneously.

These are aggregate correlation states that affect the entire economy. Individual firms within each group have heterogeneous shock probabilities v_i distributed according to $F(v_i)$, but the correlation structure (μ, ε) applies uniformly across all firms. When we say 'hoarders avoid shocks' with probability μ, we mean this is the aggregate probability for the hoarder group as a whole, not the idiosyncratic probability for any individual hoarder.

Interpretation of parameters. The (μ, ε) parameterisation is a reduced form of a standard two-layer shock structure. Suppose there is an aggregate state $S \in \{0, 1\}$ with $\Pr(S = 1) = \varepsilon$, representing an economy-wide liquidity crisis. Conditional on $S = 0$ (normal state), firms face idiosyncratic shocks with probability $(1 - v_i)$; conditional on $S = 1$ (crisis state), all firms are hit. The parameter μ captures the probability that the hoarder group avoids shocks in the normal state, while ε is the

[5]We assume that prices are set competitively and not by bilateral bargaining with specialist buyers who may have bargaining power. The model could be modified to introduce a wedge between reservation prices for specialist and non-specialist buyers.

Table 1: Correlation Structure for Liquidity Shocks

Event	Joint Probability	Market Outcome
Neither firm shocked	$v\mu$	No trade needed
Only non-hoarder shocked	$v(1-\mu)$	Hoarder buys from non-hoarder (price q^I)
Only hoarder shocked	$(1-v)(1-\varepsilon)$	Hoarder uses own cash (no trade)
Both firms shocked	$(1-v)\varepsilon$	**FIRE SALE STATE** No hoarder buyers (price q^F)

Note: Fire sales occur when both types face shocks simultaneously, with probability $(1-v)\varepsilon$. In this state, hoarders need their own liquidity and cannot act as buyers, so asset prices fall to q^F (determined by investor demand only). The joint probabilities sum to unity: $v\mu + v(1-\mu) + (1-v)(1-\varepsilon) + (1-v)\varepsilon = 1$.

probability of the crisis state itself. The correlation parameters (μ, ε) are therefore independent of the cross-sectional distribution $F(v_i)$ by construction: they govern the *aggregate* state realisation, while $F(v_i)$ governs idiosyncratic exposure *conditional* on that state. This separation reflects the standard decomposition of liquidity risk into aggregate and idiosyncratic components (Brunnermeier and Pedersen, 2009), and is independent of the endogenous sorting threshold v^*.

Key insight: Fire sales occur in the state $(1-v)\varepsilon$ when both hoarders and non-hoarders experience liquidity shocks. In this state, hoarders must use their liquidity L for their own needs and cannot purchase assets from distressed non-hoarders. Asset prices therefore fall to q^F, determined solely by investor demand (introduced in Section 4).

From an individual firm's perspective choosing strategy at $t = 0$, if it chooses hoarding it expects to be liquid with probability v_i, while if it chooses non-hoarding it expects to need liquidity with probability $(1 - v_i)$. However, whether it can trade depends on the aggregate correlation states in Table 1. This generates the matching probabilities from the firm's perspective:[6]

$$\pi_b = (1-\mu) : \text{prob. a hoarder with surplus finds a distressed non-hoarder} \tag{17}$$

[6] These conditional probabilities follow from Table 1. For a hoarder that avoided a shock (which occurs with individual probability v_i), the probability of finding a non-hoarder in distress is the aggregate probability conditional on the hoarder being liquid: $\pi_b = v(1-\mu)/v = (1-\mu)$. Similarly, for a non-hoarder that was hit by a shock (individual probability $1 - v_i$), the probability of finding a liquid hoarder is: $\pi_s = (1-v)(1-\varepsilon)/(1-v) = (1-\varepsilon)$.

$$\pi_s = (1 - \varepsilon) : \text{prob. a distressed non-hoarder finds a hoarder with surplus} \tag{18}$$

3.1 Aggregation and Market Clearing without Outside Investors

At this stage, assume that the only degree of heterogeneity in the population of entrepreneurs is in the probability v. Let the total number of entrepreneurs be N. The distribution of entrepreneurs according to the variable v is given by the distribution function $F(v)$, which is continuous and strictly increasing on $[0, 1]$. We assume N is large (formally, a continuum) such that by the law of large numbers, the realised fraction of firms experiencing shocks in each group converges to its expected value.

The mass of hoarders is $F(v^*)N$. Among these hoarders, in normal aggregate states, the conditional mass of hoarders who remain liquid is $(1 - \mu)$, so the number of hoarders with surplus liquidity available is $F(v^*)N(1 - \mu)$, and the total surplus liquidity available is $L_s^h = F(v^*)N(1 - \mu)L$.

The mass of non-hoarders is $(1 - F(v^*))N$. In normal states, the fraction of non-hoarders hit by shocks is $(1-\varepsilon)$, so the number needing liquidity is $(1-F(v^*))N(1-\varepsilon)$, and the total liquidity demanded in normal states is $L_d^{nh} = (1 - F(v^*))N(1 - \varepsilon)L$. In fire-sale states, the fraction of non-hoarders hit by shocks is ε, so the number needing liquidity is $(1 - F(v^*))N\varepsilon$, and the total liquidity needed in fire-sale states is $(1 - F(v^*))N\varepsilon L$.

These aggregate quantities will determine equilibrium prices in Section 3.2 (without investors) and Section 4 (with investors). Note that the joint probabilities sum to unity: $v\mu + v(1 - \mu) + (1 - v)(1 - \varepsilon) + (1 - v)\varepsilon = 1$. While firms within each group have heterogeneous v_i, distributed according to $F(v_i)$ on $[0, 1]$, there are four aggregate states for the liquidity event: one where neither firm suffers a liquidity event with probability $v\mu$; two where one does and the other does not with combined probability $v(1-\mu)+(1-v)(1-\varepsilon)$; and one where both do with probability $(1-v)\varepsilon$.

We now turn to the market clearing conditions. In both normal and fire-sale states, distressed non-hoarders, sellers, need to raise L in cash per firm. To obtain this cash, they sell assets at the prevailing market price q. In normal states hoarders deploy $L_s^h = F(v^*)N(1 - \mu)L$. The total cash needed by distressed non-hoarders is $L_d^{nh} = (1 - F(v^*))N(1 - \varepsilon)L$. Each distressed firm raises L by selling assets worth L/q^I, where $q^I \leq 1$ reflects the discount. Absent other buyers, in equilibrium in non-fire-sale (normal) states supply must equal demand, so the price in these states must be consistent with market clearing

$$F(v^*)N(1-\mu)L = (1-F(v^*))N(1-\varepsilon)L \tag{19}$$

or $F(v^*)/(1-F(v^*)) = (1-\varepsilon)/(1-\mu)$. Here, we are using the average shock probabilities within each group for market clearing in normal states. Then to ensure that hoarders are induced in sufficient numbers to hoard liquidity, asset sellers (non-hoarders), who must sell to raise L, have to take a discount so $q^I < 1$ to clear the market. The term L/q^I represents how much in asset value must be sold per firm to raise L in cash.

In the fire-sale states, that occur when hoarders are also hit by a shock, there is no trade. Non-hoarders want to sell $(1-F(v^*))N\varepsilon L$ but there are no buyers and so $q^F = 0$. Clearly, in this case non-hoarders, with no buyers in distressed sales, are severely disadvantaged. Given $q^F = 0$, the value of q^I determines the strategy, to hoard or not to hoard, by condition (14) of an individual entrepreneur with a given value of v (our only source of heterogeneity and hence price sensitivity). The equilibrium value of q^I is one which, when forecasted rationally, induces a marginal $v = v^*$ that satisfies the equilibrium condition (19).

Note that if $(1-v)\varepsilon$ is high and q^F is low, zero here, then low v firms will avoid being exposed to fire sale outcomes by hoarding liquidity, so v^* will be higher, which will benefit those high v firms that do not hoard, who will achieve better prices for assets in non-fire-sale states. This degenerate outcome, a consequence of the absence of outside buyers rather than a substantive prediction, motivates the introduction of specialist investors in Section 4, whose participation transforms q^F into an endogenous equilibrium object and is necessary for a well-defined interior solution.

4 Investor Behaviour and Asset Price Formation

The role of outside investors (specialists) in providing liquidity and influencing asset prices has been explored in Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2002). Our model extends this by incorporating investor competition with hoarders and analysing the implications for equilibrium asset pricing and firm strategy.

In equilibrium, the willingness of investors to allocate liquidity toward purchasing assets in fire-sale states depends critically on the expected return from such purchases. This return must be competitive with alternative uses of their capital, such as investment in productive assets. Consequently, asset prices, particularly in fire-sale conditions, must adjust to ensure that investors are sufficiently incentivised to participate.

An endogenous value of fire-sale assets requires a market for fire-sale assets, which depends upon there being buyers. We assume that there exist specialist investors who are in the market for assets. The expected (risk-adjusted) return they get from assets held for fire-sale purchases must equal the alternative return on these funds. Then in fire-sale states, they must buy all assets using available funds, M, from the total value from of selling firms, L_d^{nh}, so $M = L_d^{nh}$. In non-fire-sale states, they compete with hoarders to buy assets, $M = L_d^{nh} - L_s^h$, where the total mass of hoarders is determined in equilibrium.

The proportional allocation rule $f = \frac{L_s^h}{L_s^h + M}$ determines how notional liquidity supplies translate into *effective* demand for assets at date $t = 1$. Hoarders arrive in the market with a notional liquidity supply $L_s^h = F(v^*)\,N(1-\mu)\,L$, which depends only on which hoarders avoided idiosyncratic shocks. Investors bring a fixed liquidity position M, chosen at date $t = 0$ on the basis of expected returns. Once the liquidity shock is realised, neither group can adjust its cash position; total notional liquidity available for purchases is therefore predetermined at $L_s^h + M$. Because liquidity is inelastic at date $t = 1$, price adjustment alone cannot clear markets. Actual asset purchases must therefore be rationed in proportion to each buyer type's share of notional liquidity: hoarders obtain effective demand $f\,L_s^h$, while investors obtain $(1 - f)M$.

This allocation rule admits two microfoundations. First, under a random-matching interpretation, each distressed seller draws a buyer with probability proportional to that buyer type's share of total liquidity. Second, in an OTC-market interpretation, liquidity determines order priority, so that buyers with greater committed funds execute trades with proportionally higher probability. Importantly, proportional rationing differs from pure price adjustment. If prices alone governed allocation, hoarders would crowd out investors in normal states—when hoarders are liquid and dominate demand, while in fire-sale states investors would be forced to absorb the entire supply when hoarders are also distressed. Anticipating this asymmetry, investors would choose a much lower liquidity position M at date $t = 0$. Proportional rationing prevents this commitment problem by guaranteeing investors a stable execution share $(1 - f)$ even in normal states, thus sustaining interior investment in liquidity and supporting an equilibrium with positive fire-sale prices q^F.[7]

[7] Prices q^I do adjust to some extent, but cannot fully clear markets given fixed buyer liquidity. This captures the realistic feature that liquidity provision must be committed before knowing

In fire-sale states, hoarders need their own liquidity and cannot participate as buyers, so $f = 0$ and only investors provide liquidity. The fire-sale price q^F is then determined solely by investor demand.

Investors have wealth W that they allocate between liquidity, M, with return R, and investment in productive capital, which generates a return $\phi(W - M)$, with $\phi(0) = 0$, $\phi' > 0$ and $\phi'' < 0$. The expected return R from holding liquidity for asset purchases depends on the probability of trading in different states, prices in those states (q_I in normal states, q_F in fire-sale states) and rationing when competing with hoarders, with their allocation being $(1 - f)$. Their optimisation problem is

$$\max\{RM + \phi(W - M)\} \tag{20}$$
$$\text{where } R = F(v^*) + (1 - F(\nu^*))[(1 - \varepsilon)(1 - f)\frac{L}{q_I} + \varepsilon\frac{L}{q_F} + (1 - \varepsilon)fL]$$

Equivalently, R can be decomposed as $R = (1-\varepsilon)(1-f)\cdot r_I + \varepsilon\cdot r_F$, where $r_I = L/q_I - L$ and $r_F = L/q_F - L$ are the state-contingent net returns from asset purchases in normal and fire-sale states respectively. The first-order-condition for the choice of M is

$$R = \phi' \tag{21}$$

This equation solves for M,

$$M = W - \phi'^{-1}(R) \tag{22}$$

This implies that the return on asset purchases must equal the marginal return on productive investment. The return R itself depends on the equilibrium asset prices in both normal and fire-sale states, denoted q_I and q_F respectively. In fire-sale states, investors are the sole buyers, and must absorb the entire supply of assets from distressed firms. To clear the market, the price q_F must be sufficiently low to make the expected return attractive. In normal states, investors compete with hoarders, and the price reflects both demand sources. The extent of rationing between hoarders and investors affects the equilibrium price and thus the return R.

4.1 Equilibrium with Outside Investors

We have already seen that equilibrium asset prices must adjust to induce optimal investor liquidity provision, creating a feedback loop: investor expectations about prices influence their liquidity allocation M, which in turn affects actual prices (q^I, q^F)

aggregate demand.

and firm strategy (v^*), demonstrating how price formation is endogenous to investor participation with implications for efficiency and policy.

An equilibrium with investors consists of: Firm strategy threshold v^*; Asset prices q_I (normal states) and q_F (fire-sale states); Investor liquidity M; and the rationing parameter f. These are jointly determined by four conditions: firm optimization, investor optimization, market clearing in normal states, and market clearing in fire-sale states and the definition of f. Investor liquidity M affects asset prices q_I and q_F, which in turn influence firm strategy and hence v^*, altering the composition of buyers and sellers in asset markets:

Condition 1: Firm Optimisation. At date $t = 0$, the marginal entrepreneur v^* is indifferent between strategies (equation (14) as an equality). This condition ties down v^*

Condition 2: Investor Optimization. At date $t = 0$, investors solve the problem in (20) and choose M to satisfy equation (22),

$$M = W - \phi'^{-1}(R),$$

with $R = F(v^*) + (1 - F(\nu^*))[(1 - \varepsilon)(1 - f)\frac{L}{q_I} + \varepsilon\frac{L}{q_F} + (1 - \varepsilon)fL]$.

Condition 3: Normal Market Equilibrium. At date $t = 1$, if the market is normal the supply from distressed no-hoarders is $L_d^{nh} = (1-F(v^*))N(1-\varepsilon)L$ and the demand from hoarders and investors is $L_s^h = F(v^*)N(1-\mu)L$ plus M. In this case there will be excess demand and rationing, with $f = L_s^h/(M+L_s^h)$. Then the equilibrium condition is:

$$(1 - f)M + fL_s^h = L_d^{nh} \tag{23}$$

Using our matching probabilities, when a non-hoarder is selling, $L_d^{nh} = (1-F(v^*))N(1-\varepsilon)L$; he either sells to an investor who has funds available of M, or to a hoarder, who also has funds available of $L_s^h = F(v^*)N(1 - \mu)L$. But due to rationing, the probability of a match with a hoarder must be reduced. Substituting for L_d^{nh} and L_s^h into (22) yields:

$$(1 - f)M + fF(v^*)N(1 - \mu)L = (1 - F(v^*))N(1 - \varepsilon)L \tag{24}$$

which implies,

$$M = \frac{[(1 - F(v^*))(1 - \varepsilon) - fF(v^*)(1 - \mu)]NL}{1 - f} \tag{25}$$

Condition 4. Fire Sale Equilibrium. In the fire-sale case, $f = 0$, and

$$M = (1 - F(v^*)\varepsilon NL \tag{26}$$

These conditions immediately yield explicit expressions for the equilibrium asset prices. In fire-sale states, investors must absorb the entire supply of distressed assets; from equation (26),

$$q_F = \frac{(1 - F(v^*))NL\varepsilon}{M} \tag{27}$$

A lower M or a larger mass of distressed sellers $(1-F(v^*))N\varepsilon$ depresses q_F, consistent with the fire-sale mechanism. In normal states, buyers consist of both investors and liquid hoarders; from equation (24),

$$q_I = \frac{(1 - F(v^*))NL(1 - \varepsilon)}{M + F(v^*)N(1 - \mu)L} \tag{28}$$

The denominator captures the total liquidity available from both buyer types, so $q_I > q_F$ whenever hoarders are present as buyers (i.e. whenever $F(v^*)N(1-\mu)L > 0$). Both prices are decreasing in the mass of distressed sellers and increasing in available buyer liquidity, making the comparative statics of Proposition 2 transparent. In this case the expected return, R, will need to be the right value, where the demand from investors on the left-hand-side depends on expectations of returns. This equation determines q_F. Note that to ensure that investors commit sufficient funds, M, to buy assets, the prices q^F and q^I will have to be sufficiently low. In particular, the extent of rationing in the normal market will impact the extent of firesale discounts to induce the equilibrium value of M satisfying (23). Thus, when investors choose liquidity M, they affect not only fire-sale prices but also which firms become hoarders versus non-hoarders (through v^*, satisfying equation (14)).

The market clearing conditions require careful interpretation. In both normal and fire-sale states, distressed non-hoarders need to raise L in cash per firm. To obtain this cash, they sell assets at the prevailing market price q. In normal states (equation 25): the left side represents total cash available from buyers , investors deploy $(1 - f)M$ while hoarders deploy $fF(v^*)N(1 - \mu)L$. The right side is the total cash needed by distressed non-hoarders:$(1-F(v^*))N(1-\varepsilon)L$. Each distressed firm raises L by selling assets worth L/q_I, where $q_I < 1$ reflects the discount.

In fire-sale states (equation 27): Investors' cash M must equal the total liquidity needs of all distressed firms. Since investors pay L per firm to acquire assets worth L/q_F, we have $M = (1 - F(v^*))N\varepsilon L$. Equivalently, $M/q_F = (1 - F(v^*))N\varepsilon L/q_F$

represents the total fundamental value of assets acquired by investors. In both states, each distressed firm sells value L/q to raise cash L. Hence the investor return in state $s \in \{I, F\}$ is $1/\hat{q}_s - 1$. Equations (24) and (26) pin down the equilibrium prices q_I and q_F given (v^*, M). The FOC $R = \phi'(W - M)$ then determines the equilibrium liquidity M, since R depends on q_I and q_F through the state-contingent return terms L/q_I and L/q_F. Lower prices raise these returns, inducing greater investor participation until the FOC is satisfied.

To sum up, investor liquidity M affects fire-sale prices q_F. This changes the marginal firm's strategy (v^*), which affects the mass of potential buyers (hoarders) versus sellers. This in turn determines normal-state prices q_I through matching adjusted for competition between hoarders and investors. This externality, through the matching channel, is not internalised by the market mechanism.

Proposition 1 (Equilibrium Characterization with Endogenous Sorting) *An equilibrium with investors consists of: (i) a threshold v^* such that firms with $v < v^*$ choose internal liquidity (hoarding) while firms with $v > v^*$ choose external liquidity; (ii) asset prices q^I in normal states and q^F in fire-sale states; (iii) investor liquidity allocation M; and (iv) rationing parameter $f = L_s^h/(M + L_s^h)$. These are jointly determined by:*

(a) Firm indifference at v^*: The marginal firm satisfies condition (14) as an equality

(b) Investor optimization: $M = W - \phi'^{-1}(R)$ from equation (20), where R equals the expected return from asset purchases across all states

(c) Normal state market clearing: $(1-f)M + fF(v^*)N(1-\mu)L = (1-F(v^*))N(1-\varepsilon)L$ from equation (25)

(d) Fire-sale state market clearing: $M = (1 - F(v^*))N\varepsilon L$ from equation (26)

5 Pecuniary and Technological Externalities in Asset Sales

In decentralised asset markets, firms and investors act competitively, taking prices as given and ignoring the broader impact of their decisions on market outcomes. However, when asset sales depress prices, they impose pecuniary externalities on other firms by tightening financing constraints and reducing continuation values. These externalities become technological when they interact with constraints, such as limited pledgeable assets, that affect real decisions like effort, investment scale, and liquidity strategy. This section formalises the nature of these externalities and shows how competitive equilibrium leads to excessive reliance on external liquidity, inefficient asset pricing, and suboptimal investment.

5.1 Effective versus Nominal Pledgeability

A key concept in our analysis is the distinction between nominal and effective pledgeable income. The nominal constraint P represents the maximum borrowing capacity determined by what can be credibly committed to lenders at $t = 0$. This constraint remains fixed throughout—it is a technological parameter reflecting enforcement capacity.

However, when firms choose external liquidity strategies, they rationally anticipate potential fire sales at $t = 1$. Consider a firm planning to invest the full amount $K^{nh} = P$ at date $t = 0$. With probability $(1 - v)$ this firm will face a liquidity shock requiring asset sales at the fire-sale price $q^F < 1$. To raise the required liquidity L, the firm must sell assets with fundamental value L/q^F. The fire-sale discount—the excess of L/q^F over L—represents a deadweight loss of $L(1/q^F - 1) = L(1 - q^F)/q^F$ per unit of liquidity raised.

From the ex-ante perspective of date $t = 0$, this creates an expected loss of $(1 - v) \cdot L/q^F$. While the firm can still borrow up to P initially, the expected value of the external liquidity strategy is diminished by anticipated fire-sale losses. We capture this through **effective pledgeable income**:

$$P_{\text{effective}} = P - (1 - v)\frac{L}{q^F} \tag{29}$$

The interpretation is straightforward: while the firm can still borrow up to P initially, the expected fire-sale loss $(1 - v)L/q^F$ effectively reduces the net resources available for productive investment. For instance, if fire-sale prices fall to $q^F = 0.5$, firms must sell assets worth $2L$ to raise liquidity L, reducing effective pledgeable income by $(1 - v) \cdot 2L$ compared to the nominal constraint P. When q^F falls (worse fire-sale conditions), more assets must be sold to raise L, increasing the expected loss and tightening the effective constraint.

Importantly, P itself never changes, it is the market price q^F that varies with aggregate conditions, thereby altering effective pledgeability even though nominal pledgeability is fixed. This is the mechanism through which price effects (pecuniary externalities) become real effects (technological externalities): changes in fire-sale prices q^F affect actual investment and effort decisions through their impact on $P_{\text{effective}}$, even though the fundamental constraint P remains unchanged. This aligns with the "collateral externality" identified by Dávila and Korinek (2018).

The effective constraint $P_{\text{effective}}$ operates directly through the effort incentive compatibility condition. Recall from equation (10) that the non-hoarder's effort choice satisfies:

$$\theta'(E^{nh})(aXK^{nh} - D^{nh}) - (1 - v)\alpha^{nh}\theta'(E^{nh})aXK^{nh} - 1 = 0 \tag{30}$$

where α^{nh} is the fraction of the project that must be sold to cover the liquidity shock. From equation (11), this fraction satisfies:

$$\alpha^{nh} = \frac{L}{q^F\theta(E^{nh})aXK^{nh}} \tag{31}$$

The asset sale term in equation (10) directly reduces the marginal return to effort. When fire-sale prices fall (lower q^F), the required sale fraction α^{nh} rises through equation (11), which reduces the effective return to effort and weakens incentives. This creates a feedback loop[8]:

$$q^F \downarrow \Longrightarrow \alpha^{nh} \uparrow \Longrightarrow \text{ marginal return to effort} \downarrow \Longrightarrow E^{nh} \downarrow \Longrightarrow \theta(E^{nh}) \downarrow \Longrightarrow \alpha^{nh} \uparrow$$

This feedback is amplified in equilibrium because each firm choosing external liquidity contributes to the mass of sellers in fire-sale states, depressing q^F further through equation (26) an effect no individual firm internalises.

The system converges to an equilibrium with depressed effort, reduced success probability, and greater asset dilution than would occur under better market conditions. In the absence of fire-sale risk, optimal effort would satisfy $\theta'(E^{FB})(aXK^{nh} - D^{nh}) = 1$. But with fire-sale exposure, equation (10) implies that the marginal return to effort is diluted by the term $(1 - v)\alpha^{nh}\theta'(E^{nh})aXK^{nh}$, leading to $E^{nh} < E^{FB}$ and therefore $\theta(E^{nh}) < \theta(E^{FB})$.

The effective pledgeable income $P_{\text{effective}}$ thus summarises how anticipated fire-sale losses tighten the *real* constraint on investment and effort through their effect

[8]Convergence of this feedback loop follows from the concavity of $\theta(\cdot)$ and the boundedness of α^{nh}. Formally, define the fixed-point mapping $\Phi(\alpha^{nh}) \equiv L/(q^F\theta(E^{nh}(\alpha^{nh}))aXK^{nh})$, where $E^{nh}(\alpha^{nh})$ solves the effort FOC (10) for given α^{nh}. Since $\theta'' < 0$, effort E^{nh} is decreasing and concave in α^{nh}, so Φ is increasing and bounded above by 1 (the entire project cannot be sold). By Brouwer's fixed point theorem, a fixed point exists. Uniqueness follows if $|\Phi'(\alpha^{nh})| < 1$, which holds when the effort response to dilution is sufficiently small relative to the direct scale effect, a condition satisfied in a neighbourhood of the competitive equilibrium.

on equations (10) and (11), even though the *nominal* enforcement constraint P remains fixed. This is precisely how pecuniary externalities (falling prices q^F) become technological externalities (reduced real activity).

5.2 Competitive Equilibrium versus Social Optimum

In the competitive equilibrium, investors allocate liquidity M to asset purchases based solely on private returns, without accounting for how their actions affect asset prices and, through them, the financing constraints of other firms. Entrepreneurs make their strategy choice taking prices as given; with fair competitive financial terms, they choose the strategy that yields maximum expected wealth: for given v, $U^h(v) \gtreqless U^{nh}(v)$. The marginal entrepreneur, with $v = v^*$, is indifferent: $\max\{U^h(v^*), U^{nh}(v^*)\}$.

The social planner's problem maximises total social surplus. Assuming financiers break even, this is the sum of all active entrepreneurs' expected wealth and the expected returns of investors:

$$
\begin{aligned}
\Omega = &\int_{\underline{v}}^{v^*} \left[\theta(E^h)aXK^h - (K^h + L) - E^h + v\left[\pi_b\left(f\frac{L}{q^I} + (1-f)L\right) + (1-\pi_b)L\right]\right] dF(v) \\
&+ \int_{v^*}^{\bar{v}} \left[\theta(E^{nh})aXK^{nh} - K^{nh} - E^{nh} - (1-v)\left[\pi_s\frac{L}{q^I} + (1-\pi_s)\frac{L}{q^F}\right]\right] dF(v) \\
&+ RM + \phi(W - M)
\end{aligned} \tag{32}
$$

where $\pi_b = (1-\mu)$ and $\pi_s = (1-\varepsilon)$ are the matching probabilities from equations (17)–(18).

The planner is constrained by the hoarders' and non-hoarders' pledgeable income constraints (equations 1 and 6), investor optimization (equation 22), market clearing (equations 24 and 26), and effort incentive compatibility. The planner chooses the threshold v^* (determining firm sorting), investor liquidity M, and implicitly the asset prices q^I, q^F to maximise Ω subject to these constraints.

Taking the derivative of Ω with respect to M and using the envelope theorem:

$$
\begin{aligned}
\frac{d\Omega}{dM} = &\int_{\underline{v}}^{v^*} v\pi_b\left[Lf\left(\frac{-dq^I/dM}{(q^I)^2}\right) + \frac{L}{q^I}\frac{df}{dM} - L\frac{df}{dM}\right] dF(v) \\
&+ \int_{v^*}^{\bar{v}} (1-v)\left[\pi_s L\left(\frac{dq^I/dM}{(q^I)^2}\right) + (1-\pi_s)L\left(\frac{dq^F/dM}{(q^F)^2}\right)\right] dF(v) + R - \phi' = 0
\end{aligned} \tag{33}
$$

The first integral represents the effect on hoarders: how investor liquidity M affects the prices they receive and the rationing they face when selling unused liquidity in normal states. The second integral captures the effect on non-hoarders: how M affects prices in both normal states (through dq^I/dM) and crucially in fire-sale states (through dq^F/dM). The final terms $(R-\phi')$ represent the private investor's first-order condition from equation (21).

The key insight is that competitive investors satisfy $R = \phi'$ (equation 21) without internalising the price effects dq^I/dM and dq^F/dM that appear in the planner's condition (33). The derivative $dq^F/dM > 0$ is particularly important: more investor liquidity improves fire-sale prices, since investors are the sole buyers in fire-sale states (equation 27). However, private investors ignore how their liquidity provision affects q^F, which in turn affects the effective pledgeable income $P_{\text{effective}}$ and effort incentives for all firms choosing external liquidity.

In equilibrium, the fire-sale effect in the second integral dominates, making the sum of the integrals negative. This implies that at the competitive equilibrium M_c where $R = \phi'$, we have $d\Omega/dM|_{M=M_c} < 0$. Therefore the socially optimal investor liquidity satisfies $M_s < M_c$: the competitive equilibrium features excessive investor liquidity provision. Competitive investors chase fire-sale returns without recognising that their collective liquidity provision depresses asset prices and induces excessive reliance on external liquidity strategies.

Proposition 2 (Inefficiency Through Effective Pledgeability Constraints) *The competitive equilibrium features systematic inefficiency because firms choosing external liquidity fail to internalise how their strategy choice affects fire-sale prices q^F, which operates through the following channels:*

(a) Direct effect on effective pledgeability: *Lower q^F reduces effective pledgeable income through $P_{\textit{effective}} = P - (1-v)L/q^F$ (equation 30), tightening the real borrowing constraint even though nominal P remains fixed.*

(b) Distortion of effort incentives: *The anticipated asset sale fraction $\alpha^{nh} = L/(q^F\theta(E^{nh})aXK^{nh})$ in equation (10) reduces the marginal return to effort. When q^F falls due to congestion in fire-sale markets, this dilution effect strengthens through the feedback loop described in Section 5.1, leading to equilibrium effort $E^{nh} < E^{FB}$ and success probability $\theta(E^{nh}) < \theta(E^{FB})$.*

(c) Externality mechanism: *Each firm choosing external liquidity contributes to the mass of potential sellers $(1-F(v^*))N\varepsilon$ in fire-sale states, which depresses q^F through the market-clearing condition (equation 27): $M = (1-F(v^*))\varepsilon NL$. For given investor liquidity M, more external-liquidity firms (lower v^*) imply lower fire-sale prices q^F. Since individual firms take q^F as given when choosing strategy at $t = 0$,* *they don't internalise this effect on others.*

(d) Excessive external liquidity and investor provision: *The competitive equilibrium features:*

- *Threshold v_c^* too low relative to social optimum v_s^*: $v_s^* > v_c^*$, or equivalently $F(v_s^*) > F(v_c^*)$, meaning too few firms hoard*
- *Investor liquidity M_c exceeds social optimum M_s: $M_c > M_s$, as private investors ignore price sensitivities $\partial q^I/\partial M$ and $\partial q^F/\partial M$ in the planner's FOC (equation 33)*

(e) Comparative statics of the inefficiency wedge $(v_s^* - v_c^*)$*:*

- *Increases in systemic risk (ε) or fire-sale severity (lower q_c^F): both amplify the externality through the same channel, more firms face simultaneous shocks or deeper discounts, forcing larger asset sales α_{nh}, which dilutes effort incentives more severely and tightens $P_{effective}$ for all external-liquidity firms through equations (10), (11), and (29).*
- *Ambiguous in investor wealth W: higher W improves liquidity provision through equation (22), which raises q^F and reduces fire-sale severity; however, better prices make external liquidity more attractive, potentially inducing more firms to rely on asset sales (lower v^*), which partially offsets the benefit.*
- *Increases when density $f(v^*)$ is high: more firms near the margin amplifies distortion because a small change in fire-sale conditions shifts a larger mass of firms between strategies, magnifying the aggregate externality.*
- *Magnitude proportional to $(1 - F(v^*))\varepsilon N \times \partial q^F/\partial M$ (equation 34): the wedge depends on both the mass of external-liquidity firms exposed to fire sales $(1 - F(v^*))N\varepsilon$ and the sensitivity of fire-sale prices to investor liquidity $\partial q^F/\partial M$, capturing the interaction between firm sorting and market depth.*

The welfare loss equals the integral over $[v_c^*, v_s^*]$ of firms inefficiently exposed to fire sales, plus the deadweight loss from depressed asset prices affecting all external firms. The intuition is that pecuniary externalities become technological precisely because they affect the effective constraint $P_{effective}$ that governs real decisions through effort and investment choices.

6 Asymmetric Information and Pooled Financing Distortions

The preceding analysis assumed homogeneous firm productivity, isolating the core fire-sale externality mechanism. We now introduce private information about entrepreneurial ability, captured by productivity parameter $a > 1$. When lenders cannot observe productivity, they must offer pooled debt contracts with uniform terms, creating adverse selection: lower-quality firms benefit from subsidised terms while higher-quality firms cross-subsidise them. Crucially, this subsidy is larger under external liquidity because fire-sale exposure magnifies the wedge between average and marginal borrower quality. We show that the marginal productivity type choosing external liquidity satisfies $a^{**} < a^*$, compounding the excessive fire-sale exposure from Section 5 with adverse selection of low-quality firms into the most distortionary strategy.

6.1 Internal Liquidity with Asymmetric Information

Consider first how private information affects the internal liquidity (hoarding) strategy from Section 2.2. Let productivity a be distributed according to $G(a)$, continuous and strictly increasing on $[\underline{a}, \overline{a}]$. Since entrepreneurs have no incentive to reveal their type, financiers offer pooled debt contracts with uniform terms. The entrepreneur's return remains $U^h = \theta(E^h)(aXK^h - \overline{D}^h) + vL - E^h$ as in equation (4), but the face value of debt now reflects pooled financing. Rather than the individual constraint $\theta(E^h)D^h \geq K^h + L$, financiers require that the average borrower breaks even:

$$\rho = \int_{a^*}^{\overline{a}} \theta(E^h(a))\overline{D}^h \frac{dG(a)}{1 - G(a^*)} = B = K + L \tag{34}$$

where a^* is the marginal type indifferent between entering and staying out, and $\overline{D}^h$ is the common face value charged to all hoarders.

For a given pooled payment $\overline{D}^h$, the marginal entrepreneur has ability a^* such that

$$U^h = \theta(E^h(a^*))(a^* XK^h - \bar{D}^h) + vL - E^h = 0.$$

From the effort FOC (5), the implicit function theorem gives

$$\frac{\partial E^h}{\partial a} = \frac{\theta'(E^h)X}{\Delta} > 0 \quad \text{and} \quad \frac{\partial E^h}{\partial \bar{D}^h} < 0,$$

where $\Delta = \theta''(E^h)(aXK^h - \overline{D}^h) < 0$, so $\partial\theta/\partial a > 0$ and $\partial\theta/\partial\overline{D}^h < 0$. Only entrepreneurs with $a > a^*$ apply for finance, and in the pooling equilibrium $\bar{\theta}^h = \int_{a^*}^{\bar{a}} \theta(E^h(a))dG/(1 - G(a^*)) > \theta(E^h(a^*))$. Since financiers break even, the social surplus from the marginal entrepreneur is $(\theta(E^h(a^*)) - \bar{\theta}^h)\overline{D}^h < 0$.

Moreover, in equilibrium

$$\frac{d\rho}{d\overline{D}^h} = (\overline{\theta}^h - \theta(E^h(a^*))\frac{dG(a^*)}{1-G(a^*)}) + \overline{D}^h \int_{a^*}^{\overline{a}} \frac{\partial\theta(E^h(a))}{\partial\overline{D}^h} \frac{dG(a)}{1-G(a^*)} \tag{35}$$

where in the first term (the adverseeffect), $(\overline{\theta}^h - \theta(E^h(a^*))$ is the difference between average and marginal success probability, which is positive. That is if marginal entrepreneurs are lower quality than the average, more debt attracts worse types. Moreover, $\frac{\partial\theta(E^h(a))}{\partial\overline{D}^h} < 0$ increasing debt weakens effort incentives. This term being negative reflects how higher debt reduces pledgeable income via lower effort. A separating equilibrium would require high-productivity firms to signal their type through costly underinvestment, sacrificing $\theta(E^h)aXL$ in expected returns. Since this exceeds the cross-subsidy cost under pooling for sufficiently productive firms, separation is dominated.

6.2 External Liquidity with Asymmetric Information

Now consider the alternative of external liquidity provision. The same modifications as above are required for the entrepreneur's return function and incentive constraint. For a given pooled payment $\overline{D}^{nh}$, the marginal entrepreneur has ability a^{**} such that:

$$U^{nh} = \theta(E^{nh}(a^{**}))(aXK^{nh} - \overline{D}^{nh}) - (1-v)\bar{\theta}^{nh}\alpha^{nh}aXK^{nh} - E^{nh} = 0$$

The effort FOC (10) gives, by the implicit function theorem:

$$\frac{\partial E^{nh}}{\partial a} = \frac{\theta'(E^{nh})X\left[1-(1-v)\alpha^{nh}K^{nh}\right]}{\Delta} > 0, \qquad \frac{\partial E^{nh}}{\partial\overline{D}^{nh}} < 0$$

where $\Delta = \theta''(E^{nh})\left[(aXK^{nh} - \bar{D}^{nh}) - (1-v)\alpha^{nh}aXK^{nh}\right] < 0$. Hence $\partial\theta/\partial a > 0$ (higher ability raises effort and success probability) and $\partial\theta/\partial\bar{D}^{nh} < 0$ (higher debt weakens incentives), mirroring the hoarding case but with the additional dilution term $(1-v)\alpha^{nh}$ throughout.

Since the effort dilution term $(1-v)\alpha^{nh}\theta'(E^{nh})aXK^{nh}$ in FOC (10) scales with a, the net benefit of external liquidity is decreasing in productivity: higher-a firms bear a larger marginal cost from fire-sale exposure and therefore prefer internal liquidity. It follows that the marginal entrant under external liquidity has strictly lower productivity than under internal liquidity, i.e. $a^{**} < a^*$. The principal modification is that the face value of debt now satisfies a pooled condition involving the conditional average probability of success, so $\theta(E^{nh})D^{nh} \geq B = K^{nh}$ is replaced by:

$$\rho = \int_{a^{**}}^{\bar{a}} \theta(E^{nh}(a))\, \overline{D}^{nh} \frac{dG}{1 - G(a^{**})} = B = K^{nh} \tag{36}$$

with $\overline{D}^{nh} > \overline{D}^{h}$. To confirm that $\overline{D}^{nh} > \overline{D}^{h}$, compare the two pooled break-even conditions. For hoarders, equation (34) requires $\bar{\theta}^h\, \overline{D}^{h} = K^h + L = P$, so $\overline{D}^{h} = P/\bar{\theta}^h$. For non-hoarders, equation (36) above requires $\bar{\theta}^{nh}\, \overline{D}^{nh} = K^{nh} = P$, so $\overline{D}^{nh} = P/\bar{\theta}^{nh}$. Since $a^{**} < a^*$, the non-hoarding pool contains lower-ability firms, implying $\bar{\theta}^{nh} < \bar{\theta}^h$, and therefore $\overline{D}^{nh} = P/\bar{\theta}^{nh} > P/\bar{\theta}^h = \overline{D}^{h}$.

Again, given that the financiers break even, the social surplus from the marginal entrepreneur is $(\theta(E^{nh}(a^{**})) - \overline{\theta}^{nh})\overline{D}^{nh} < 0$. Moreover, in equilibrium

$$\frac{d\rho}{d\overline{D}^{nh}} = (\overline{\theta}^{nh} - \theta(E^{nh}(a^{**}))\frac{dG}{1 - G(a^{**})}) + \overline{D}^{nh} \int_{a^{**}}^{\bar{a}} \frac{\partial \theta(E^{nh}(a^{**}))}{\partial \overline{D}^{nh}} \frac{dG}{1 - G(a^{*})} \tag{37}$$

The first term (the adverse selection effect) is positive and the second term (the incentive effect) is negative but both will be bigger than in the case of internal finance.

The first term captures the (adverse) selection effect: how the marginal entrepreneur compares to the average. If the marginal type is worse than the average, this term is positive, increasing $\overline{D}^{nh}$ worsens the pool. The second term captures the incentive effect: how increasing debt affects effort and thus success probability. This term is negative, higher debt weakens incentives. But both will be bigger than in the case of internal finance. That is, the crucial point is that $a^{**} < a^*$ means that the subsidy in pooled financing is greater for non-hoarding (external liquidity), which will move firms towards hoarding (internal liquidity). This means less investment but also lower exposure to asset sales and in particular fire-sales.

Proposition 3 (Pooling Equilibrium with Asymmetric Information) *When firm productivity $a \in [\underline{a}, \bar{a}]$ is private information, pooled debt contracts generate greater distortions under external liquidity:*

(a) The marginal firm threshold satisfies $a^{**} < a^*$ (external attracts lower types than internal) due to the interaction of asset sales with incentive constraints (equation 10)

(b) *The adverse selection effect* $(\theta^{nh} - \theta(E^{nh}(a^{**})))$ *from equation (35) exceeds* $(\theta^h - \theta(E^h(a^*)))$ *from equation (33)*

(c) *The incentive distortion* $\frac{\partial\theta(E^{nh}(a))}{\partial D^{nh}}$ *is more negative than* $\frac{\partial\theta(E^h(a))}{\partial D^h}$ *due to effort dilution from anticipated asset sales*

(d) *Total distortion in pledgeable income:* $d\theta/dD^{nh}$ *from (37) is more negative than* $d\theta/dD^h$ *from (35), implying greater cross-subsidization under external liquidity*

(e) *The social surplus from marginal entrepreneurs* $(\theta(E(a^*)) - \theta)D$ *is more negative under external liquidity, generating excessive entry of low-productivity firms*

We summarise the above result in in the following:

Result: Comparative Distortions in Pooled Debt Contracts

Let firms differ in productivity $a \in [\underline{a}, \overline{a}]$ and let debt contracts be pooled across types. Then, under external liquidity provision:

1.The adverse selection effect $(\overline{\theta}^{nh} - \theta(E^{nh}(a))$ is larger than under internal liquidity, i.e., $(\overline{\theta}^{nh} - \theta(E^{nh}(a)) > (\overline{\theta}^{h} - \theta(E^{h}(a))$ because external liquidity attracts lower-productivity firms (i.e., $a_{nh} < a_h$).

2.The incentive effect $\frac{\partial\theta(E^{nh}(a))}{\partial\overline{D}^{nh}}$ is more negative than under internal liquidity: $\frac{\partial\theta(E^{nh}(a))}{\partial\overline{D}^{nh}} < \frac{\partial\theta(E^{h}(a))}{\partial\overline{D}^{h}}$ due to greater dilution of effort incentives from asset sales and fire-sale pricing.

3. Therefore, the total distortion in pledgeable income from increasing debt is greater under external liquidity.

This result implies that pooled debt contracts are more distortionary under external liquidity. The planner should prefer internal liquidity for high-productivity firms to preserve effort incentives and reduce adverse selection.

7 Policy Interventions: Guarantees, Asset Purchases, and Market Design

As noted, at a fundamental level the model has incomplete markets. If entrepreneurs could engage in state contingent futures contracts and maintain incentives, the first-best would obtain. This would be achieved if at the initial date the firm could undertake the first-best level of investment and cover the interim liquidity shock through borrowing against the final returns on the project but without violating the pledgeable asset constraint. If, as we have argued, such contracts are not feasible without violating the pledgeable asset constraint, there may be a role for the government to step in and provide liquidity funding using its ability to guarantee and enforce loans. By the same token, if this is feasible, in eliminating the need for interim asset sales, there will be no externalities arising from the impact of investor behaviour on these markets. If this intervention is not possible, then if the government enters the market to buy assets in the event of liquidity shocks at prices consistent with social efficiency, it can restore efficiency.

In the case of asymmetric information and the equilibrium distortions in incentives and the composition of investment, the above inefficiencies are compounded by market distortions impacting the relative value of firms investment and effort choices and the size of the market. To correct these inefficiencies the government will need to impact the relative returns to the entrepreneurs strategic choices and the marginal cost of funds to all entrepreneurs. In the presence of incomplete markets and asymmetric information, decentralised asset sales generate externalities that distort firm behaviourr and aggregate investment. Two policy tools can mitigate these inefficiencies: liquidity guarantees and government asset purchases. These responses have been discussed in Hanson et al. (2011) and Caballero and Krishnamurthy (2008). However, our framework formalises the conditions under which such interventions restore efficiency by mitigating fire-sale externalities and improving selection.

Proposition 4 (Optimal Policy Intervention) *Government intervention can restore efficiency through:*

(a) Liquidity guarantees: Allowing firms to borrow L against future returns without violating pledgeable asset constraints P eliminates fire sales and preserves first-best effort $\theta'(E^{FB}) = 1$.

(b) Asset purchases: Government buying assets at price $q^G = q^s > q^c$ in fire-sale states reduces pecuniary externalities, where q^s satisfies the social planner's FOC (34).

(c) Under symmetric information: liquidity guarantees dominate when enforcement costs are sufficiently low, since guarantees eliminate fire sales entirely and preserve first-best effort incentives; government asset purchases dominate when enforcement costs exceed the deadweight loss from fire sales, as direct market intervention avoids the need for credible commitment to repayment.

(d) Under asymmetric information: Guarantees should target high-productivity firms $(a > \hat{a})$ while asset purchases stabilise markets for all participants.

(e) *The optimal intervention satisfies: Government liquidity M^G solves $dS/dM = 0$ from equation (33), internalising price effects on firm constraints.*

The optimal intervention levels characterised in Proposition 4 align with the general framework of Dávila and Korinek (2018), who demonstrate that the optimal policy intervention requires a Pigouvian tax or subsidy equal to the marginal social cost of tightening others' constraints. In our context, this translates to a corrective subsidy for internal liquidity (hoarding) equal to the shadow value of the pledgeable asset constraint multiplied by the price impact $\partial q^F/\partial M$. Equivalently, external liquidity strategies should face a tax reflecting their contribution to fire-sale conditions. The government's optimal liquidity provision M^G that satisfies $dS/dM = 0$ from equation (33) effectively implements this Pigouvian correction by supporting asset prices at the socially efficient level $q_s > q_c$, thereby internalising the externality that private investors ignore.[9]

Liquidity guarantees allow firms to borrow against future returns without violating pledgeable asset constraints. By reducing the need for interim asset sales, guarantees eliminate fire-sale externalities, as firms no longer depress asset prices. They also preserve effort incentives, since financing is less sensitive to market conditions and improve selection, by enabling high-productivity firms to invest at scale. However, guarantees require credible enforcement and may introduce moral hazard if firms anticipate bailouts. Their effectiveness hinges on the government's ability to screen firms and enforce repayment.

Government Asset Purchases in which the government acts as a buyer in asset markets during liquidity shocks can be beneficial. By purchasing assets at socially efficient prices, the government supports asset prices, reducing pecuniary externalities. This crowds in investment, especially for firms relying on external liquidity, stabilises investor expectations and improves liquidity provision. This intervention is particularly valuable when asymmetric information leads to underpricing and adverse selection. However, it requires accurate valuation and may distort private incentives if mispriced. In equilibrium, both tools can restore efficiency, but their design must account for the nature of shocks, firm heterogeneity, and investor behaviour. The planner's optimal policy may involve a hybrid approach, guarantees for high-productivity firms and asset purchases to stabilise markets during systemic shocks.

[9] While Dávila and Korinek (2018) focus on explicit taxes and subsidies as policy instruments, our mechanism of direct government asset purchases achieves the same corrective effect through price support rather than explicit transfers. This may be more practical in crisis situations where rapid intervention is needed and firm-specific subsidies are difficult to implement.

8 Conclusion

This paper examines how firms manage liquidity shocks when financial markets are incomplete and asset prices are endogenously determined through investor participation. By integrating firm sorting between internal and external liquidity strategies with matching frictions in asset markets, we demonstrate that competitive equilibrium generates systematic inefficiencies even when all agents rationally anticipate prices. The core mechanism operates through the interaction of pledgeable asset constraints with fire-sale pricing: while firms understand that asset prices affect their effective borrowing capacity through $P_{\text{effective}} = P-(1-v)L/q$, they fail to internalise how their strategy choices collectively determine those prices. We establish four key findings. First, the competitive equilibrium features excessive reliance on external liquidity: too many firms choose asset-sale strategies (the threshold v_c^* is too low relative to the social optimum v_s^*) because they don't internalise how their choices contribute to fire-sale severity. Second, asymmetric information about firm productivity amplifies these distortions—pooled debt contracts become more distortionary under external strategies ($a^{**} < a^*$), as lower-productivity firms disproportionately choose asset sales while fire-sale prospects dilute effort incentives for all participants. Third, investor behaviour creates a feedback loop that amplifies crisis dynamics: competitive investors provide excessive liquidity ($M_c > M_s$) chasing fire-sale returns, which paradoxically depresses those returns and distorts firm strategies. Fourth, optimal policy intervention depends critically on information structure—liquidity guarantees work best under symmetric information, while government asset purchases at socially efficient prices improve outcomes under asymmetric information without exacerbating adverse selection.

A further source of inefficiency arises from the typology of liquidity shocks. If shocks were purely transitory, firms could borrow L at $t = 1$ against future receivables without violating pledgeability constraints, eliminating the need for asset sales entirely. Market incompleteness forecloses this option: the inability to write state-contingent debt contracts at $t = 0$ forces firms into the suboptimal choice between hoarding and asset sales even when shocks have positive probability of being temporary. This suggests that policy interventions enabling conditional liquidity provision, such as central bank lending facilities or government guarantees for temporary shocks, can be particularly valuable when shock persistence is uncertain, complementing the asset purchase and guarantee mechanisms analysed in Section 7.

The model generates testable predictions: (i) firms with higher liquidity shock probabilities should hold more internal reserves, creating observable heterogeneity in cash holdings across industries; (ii) fire-sale discounts should correlate with the concentration of liquidity shocks, with specialised industries facing deeper discounts during sector-specific downturns; (iii) dedicated distressed asset investors should improve prices and induce more aggressive investment by operating firms. Recent evidence from the March 2020 liquidity crisis and the 2022 UK gilt market stress supports these mechanisms.

The framework suggests that macroprudential liquidity regulation should internalise the collateral externality captured by $\partial q^F/\partial M$ in equation (33). Liquidity guarantees work best when firm quality is observable, preserving effort incentives while eliminating fire sales. Under asymmetric information, government asset purchases at socially efficient prices improve outcomes without worsening adverse selection. This implements the Pigouvian correction of Dávila and Korinek (2018).

Several extensions warrant investigation. First, dynamic reputation effects and relationship banking might partially mitigate these externalities if repeated interactions enable credible quality signaling or state-contingent liquidity provision. Second, heterogeneous investor types—from patient capital to algorithmic traders—would reveal how different horizons affect fire-sale dynamics. Third, analysing interactions with existing macro-prudential tools (capital requirements, liquidity ratios) could show whether current regulation inadvertently exacerbates fire-sale exposure.

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Appendix A: Symbol Reference Guide

Symbol	Definition
Basic Parameters	
t	Time index: $t = 0$ (initial), $t = 1$ (liquidity shock), $t = 2$ (returns)
N	Total number of entrepreneurs (continuum)
L	Size of the liquidity shock at $t = 1$
P	Pledgeable asset value (credit constraint)
a	Productivity parameter of the project
X	Return rate on capital investment ($X \geq 1$)
Investment and Financing	
K	Capital invested at date $t = 0$
K^h	Capital invested by hoarders ($= P - L$)
K^{nh}	Capital invested by non-hoarders ($= P$)
C	Liquid cash reserves held at $t = 0$
C^h	Cash reserves held by hoarders ($= L$)
B	Total borrowing raised at $t = 0$ (equals $K + C$)
D	Face value of debt (promised payment to financiers)
D^h	Face value of debt for hoarders
D^{nh}	Face value of debt for non-hoarders
Effort and Returns	
E	Entrepreneurial effort
E^h	Effort level chosen by hoarders
E^{nh}	Effort level chosen by non-hoarders
$\theta(E)$	Success probability of project, increasing in effort E
U^h	Expected return to entrepreneur under hoarding strategy
U^{nh}	Expected return to entrepreneur under non-hoarding strategy
Asset Prices and Trading	
q_I	Asset price in normal (non-fire-sale) states
q_F	Asset price in fire-sale states ($q_F < q_I \leq 1$)
α^{nh}	Fraction of project sold by non-hoarders to meet liquidity needs
f	Rationing fraction in asset markets between hoarders and investors

Symbol	Definition
Shock Probabilities and Distributions	
v	Probability that an individual firm does not suffer a liquidity shock (individual characteristic, distributed via $F(v)$ on $[\underline{v}, \bar{v}]$)
v^*	Threshold shock probability (marginal firm indifferent between strategies)
$F(v)$	Distribution function of shock probabilities (continuous, strictly increasing)
μ	Aggregate correlation parameter: when μ is high, shocks are positively correlated (both groups tend to have same shock status)
ε	Aggregate correlation parameter: probability both groups face shocks simultaneously (fire-sale state occurs with probability $(1-v)\varepsilon$)
π_b	$= (1-\mu)$: Conditional probability of successful match for hoarder-buyers
π_s	$= (1-\varepsilon)$: Conditional probability of successful match for non-hoarder-sellers
Investors	
M	Liquidity allocated by investors to asset purchases
W	Total wealth of investors
R	Expected return on liquidity allocated to asset purchases
$\phi(W-M)$	Return on productive investment by investors
Asymmetric Information (Section 6)	
a	Productivity parameter (private information), distributed via $G(a)$ on $[\underline{a}, \bar{a}]$
$G(a)$	Distribution function of productivity parameters
a^*	Marginal productivity type under internal liquidity (hoarding)
a^{**}	Marginal productivity type under external liquidity (non-hoarding)
θ^h	Average success probability for hoarders under pooled financing
θ^{nh}	Average success probability for non-hoarders under pooled financing
Effective Constraints (Section 5)	
$P_{\text{effective}}$	Effective pledgeable income: $P-(1-v)L/q_F$